İSTANBUL TECHNICAL UNIVERSITY ★ INSTITUTE OF SCIENCE AND TECHNOLOGY

RELIABILITY BASED
SHIP STRUCTURAL ANALYSIS

M.Sc. Thesis by
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Programme : Naval Architecture and Marine Engineering

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FOREWORD

Realiability methods for engineering are getting more attention each day as more researchers are giving their efforts to improve the technique to get more accurate results. These methods are gaining support as an alternative to determinisitic methods which considers the systems are perfect that there is no uncertainity in the inputs. I personally felt an interest in this subject after I had taken reliability lectures from my advisor Prof. Dr. Ergin whom I am very thankful for his efforts on my study. I also thank to my family for their full support to my study.

May 2011

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LIST OF ABBREVIATIONS

BSRA **:** British Ship Research Association
CDF **:** Cumulative distribution function
DWT **:** Deadweight
IACS **:** International Association of Classification Societies
IMO **:** International Maritime Organization
ISSC **:** International Ship and Offshore Structures Congress
OMAE **:** International Conference on Ocean, Offshore and Arctic Engineering
PDF **:** Probability denstity function
RINA **:** Royal Institution of Naval Architects
SNAME **:** Society of Marine Architects and Marine Engineers
SSC **:** Ship Structure Committee
VLCC **:** Very Large Crude Carrier

LIST OF SYMBOLS

$\mathbf{A_B}$: Cross sectional area of bottom

$\mathbf{A_{BLK}}$: Cross sectional area of bulkhead

$\mathbf{A_D}$: Cross sectional area of deck

$\mathbf{A_{ei}}$: Effective cross sectional area of the ith element in compression

$\mathbf{A_j}$: Cross sectional area of the jth element in tension

$\mathbf{A_S}$: Cross sectional area of side

$\mathbf{B}$: Ship breadth

$\mathbf{b}$: Element breadth

$\mathbf{\bar{B}}$: Ratio between actual and estimated stress range

$\mathbf{b_{eff}}$: Effective element breadth

$\mathbf{C}$: Distance from local neutral axis to global neutral axis

$\mathbf{C_b}$: Block coefficient

$\mathbf{E}$: Modulus of elasticity

$\mathbf{F_Q}$: CDF of the load random variable

$\mathbf{f_Q}$: PDF of the load random variable

$\mathbf{F_R}$: CDF of the resistance random variable

$\mathbf{f_R}$: PDF of the resistance random variable

$\mathbf{F_{x_i}(x_i^*)}$: CDF of the original non normal random variable

$\mathbf{f_{x_i}(x_i^*)}$: PDF of the original non normal random variable

$\mathbf{g}$: Limit state function

$\mathbf{g_u}$: Neutral axis

$\mathbf{h}$: Height of the element

$\mathbf{I}$: Moment of inertia

$\mathbf{I_{red}}$: Reduced hull girder moment of inertia

$\mathbf{k_{sw}}$: Still water load combination factor

$\mathbf{k_w}$: Wave induced load combination factor

$\mathbf{L}$: Ship length

$\mathbf{f_Q}$: PDF of the load random variable

$\mathbf{m}$: Inverse slope of S-N curve

$\mathbf{M_{sw}}$: Random variable extreme vertical still water bending moment

$\mathbf{M_u}$: Ultimate hull girder bending moment

$\mathbf{M_w}$: Random variable extreme vertical wave bending moment

$\mathbf{N}$: Number of cycles to fail

$\mathbf{P_f}$: Probability of failure

$\mathbf{Q}$: Load term of the limit state function
$\mathbf{q_i}$: Random load
$\mathbf{R}$: Resistance term of the limit state function
$\mathbf{SM_{eff}}$: Effective section modulus
$\mathbf{SM_P}$: Plastic section modulus
$\mathbf{r_i}$: Random resistance
$\mathbf{x_i}$: Design point for ith variable
$\mathbf{x_{nl}}$: Modelling uncertainity of the nonlinear wave load
$\mathbf{x_{sw}}$: Modelling uncertainity of the still water load
$\mathbf{x_u}$: Modelling uncertainity of the ultimate strength
$\mathbf{x_w}$: Modelling uncertainity of the linear wave load
$\mathbf{Z_{dk-mean}}$: Vertical distance to the mean deck height
$\mathbf{Z_{NA-red}}$: Vertical distance to the neutral axis
$\mathbf{z_i}$: Coordinate of the ith element front the baseline
$\mathbf{Z_i}$: Reduced variable for the ith content
$\mathbf{Z_i^*}$: ith reduced variable at the design point
$\mathbf{Z_Q}$: Reduced variable of the load
$\mathbf{Z_R}$: Reduced variable of the resistance
α_i : Sensitivity factor for ith variable
β : Reliability index
ϕ : CDF of the standart normal variate
$\bar{\Delta}_F$: Value of Palmgren-Miner damage index at failure
$\Delta \mathbf{S}$: Stress range
σ_{cr} : Critical stress
σ_i : Longitudinal stress in the ith structural member
σ_Q : Standart deviation of the load
σ_R : Standart deviation of the resistance
σ_{ul} : Ultimate stress
$\sigma_{x_i}^N$: Equivalent normal standart deviation
σ_y : Yielding stress
σ_{yd} : Specified minimum yield stress of the material
μ_g : Mean value standart deviation
$\mu_{M_{serv}}$: Hull girder mean serviceable mean moment
μ_Q : Mean value of the load
μ_R : Mean value of the resistance
$\mu_{x_i}^N$: Equivalent normal mean value of the ith non normal random variable
Ω : Stress parameter
ψ : Load combination factor

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RELIABILITY BASED SHIP STRUCTURAL ANALYSIS

SUMMARY

In all engineering disciplines, reliability considerations are playing increasing role. Since failures in a system increase costs or be fatal, engineers are trying to minimize the probability of failures. Relibility calculations have also been used in naval architecture for a number of decades and seem to be much widely used in the near future. There are many aprroaches to ship constructural analysis from the reliability perspective and as time goes, target reliability levels change to due change of methods and regulations. In this thesis, the methods to calculate ultimate hull girder strength of ships will be explained. The first order reliability method with Hasofer-Lind reliability index will be used and Rackwitz-Fiessler method will be applied to find equivalent normal variables of nonnormal variables in order to use Hasofer-Lind reliability index. As an important part of the reliability methods, the different limit state functions will be shown and a comparison of results of two different limit state functions calculated by Hasofer-Lind reliability index will be done in the numerical examples section. In this section, there are also two very simple examples to express the basics of these methods. Also, basic concepts of the reliability subject and its application to ships will be mentioned briefly.

GÜVENİLİRLİK TEMELLİ GEMİ YAPI ANALİZİ

ÖZET

Tüm mühendislik dallarında, güvenilirlik analizinin geçerliliği artmaktadır. Bir yapı içerisindeki başarısızlık durumları maliyetli ve ölümcül olabildiğinden mühendisler başarısızlık olasılığını azaltmaya çalışımaktadır. Güvenilirlik hesapları gemi inşaa mühendisiliği için de onlarca yıldır kullanılıyor ve yakın zamanda daha sık kullanılacak gibi gözüküyor. Güvenilirlik açısından yapılan gemi yapı analizlerinin pek çok yaklaşımı mevcut ver hedef güvenilirlik seviyeleri yeni yöntemler ve kurallar ile değişiyor. Bu tezde gemilerin orta kesitinin en yüksek mukavemetinin hesaplanış metotları açıklanacaktır. Hasofer-Lind güvenilirlik dizini ile birinci dereceden güvenilirlik analizi yapılacak ve normal dağılım göstermeyen değerlerin Hasofer-Lind dizininde çözümü için Rackwitz-Fiessler dönüşüm prosedürü uygulanacaktır. Güvenilirliğin önemli bir konusu olarak farklı sınır durumlarında analiz yapılıp karşılaştırması nümerik örnekler bölümünde işlenecektir. Bu bölümde ayrıca konunun anlaşılabilmesi için 2 basit örnek daha gösterilmiştir. Ayrıca, güvenilirlik analizinin temel konuları ve bunların gemi yapılarına uygulanışına değinilmiştir.

1. INTRODUCTION

Conventional deterministic methods are being used by engineers for their own disciplines for centuries. Those deterministic methods assume all the elements to be ideal. But in reality, ideality cannot be achieved, hence there must be some errors. For example, an exact predesignated sized, perfectly uniform steel cannot be produced. This also generates other uncertainities such as young modulus, yielding strength and moment of inertia uncertanities. To deal with engineering uncertainites reliability techniques are applied. "Reliability is the probability that an item will perform a required function without failure under stated conditions for a stated period of time." [1]. Both probability and statistics are the backbones of reliability methods [2].

Reliability techniques were used primarily by Germans during World War 2 for their rocket development programme for the first time [2]. In 1951, Weibull published a statistical distribution for material strength and life length [3]. Later, Freudenthal contributed to structural reliability [4] [5] [6].

Probabilistic approach concept to ship structures took attention later than civil engineering applications. One of the first researches was done by Mansour in 1972 [7]. His paper presents a comprehensive probabilistic model for formulating structural desgn criteria in ships. It covers the overall structural strength of ships under the random sea loads from a probabilitic point of view. It was applied to a mariner-class ship which was analysed for both long and short terms. The results were compared to conventional methods and finally an ideal acceptable level of risk was tried to be modeled. Mansour also published a paper for solely extreme values of bending moment [8]. The probability of failure was obtained on the basis of an assumed normal probability density function of the resistive strength and deterministic still water bending moment. As a conlusion, it was suggested that when dealing in ocean structres, both sea waves and structure response was random, therefore the design should be based on the calculated risk of failure rather than deterministic quantities. Also it was advised to take into account the variability of the strength in any proposed design procedure. The results were depended on ship characteristics, loading conditions and the general sea condition. Mansour's another study on the subject of ship reliability was concerned about calculation of ship's longitudinal strength [9]. Analysis of eighteen ships of different types were made in order to serve as a preliminary investigation of the appropriate level of safety as measured by a proposed safety index. The aim of the study was to find a reliable section modulus for a ship.

A unified approach of conventional and probabilistic methods was researched by Faulkner and Sadden [10]. Two semi-probabilistic methods, which are partial safety factor approach and the safety index were included. Safety concepts were categorized in three levels by their sophistication. Effective wave height and section modulus requirements were measured by conventional deterministic methods. It was concluded that the safety level of ships were widely spread and merchant ships were safer than navy ships.

Another general investigation of general ship structrual reliability was done by Mansour and Hovem [11]. In this paper, reliability techniques were developed to determine safety levels of existing vessels. Ultimate, serviceability and fatigue limit states were also developed and applied to an existing tanker. The benefits and drawbacks of the reliability methods were expressed wich will be mentioned at the end of this chapter.

Guedes Soares et al [12] further investigated reliability based ship structural design to be transferred from the area of research to systematic application in practical design. New probabilistic methods of still water load effects were presented for tankers and container ships. The research is a good summary of the SHIPREL project which aimed to develop reliability based methods to be used for the design of ship structures, in particular by providing calibrated safety factors for new design rules. The project dealt with the primary strength and thus with the hull behaviour under longitudinal bending, in particular with the design of the midship section. Having identified the new formulations, they were applied to typical cases of container ships and tankers, demonstrating how a coherent set of rules had been developed and applied to ships with various loadings and structural behaviours. In the article, it was mentioned that the still water loads vary during voyage so they can only be described by a probability distribution. For still water loads, there was already a research which was done by Guedes Soares and Moan [13]. The data in that article was collected from 2000 voyages but was short in container ships. SHIPREL project’s main contribution to that matter was that a database form 40 container ships and 3200 voyages were collected [14]. It was also shown that there were significant uncertainities associated with the wave data which were the input to the calculations of long term wave induced load effects. Furthermore new approaches to hull girder collapse, fatigue reliability and degradation were developed.

Hull girder reliability was researched by Panov, Senjanovic and Guedes Soares specifically for oil tankers [15]. The paper discusses changes of classic and new oil tanker designs. First order reliability methods were carried out to finf the difference due to design evoluiton of oil tankers. The wave induced bending moment was derived from direct hydrodynamic analysis performed according to IACS Recommendation No 34 Standart Wave Data, Rev 1, 2000. The probability distribution of the still water bending moment was assumed based on the data from loading manuals. The analysis was performed for full load, ballast and partial loads. In conclusion, it was shown that newer oil tankers were more reliable than older ones that were designed by IACS rules. For corroded ships, the difference was smaller but new tanker were still more reliable.

Later, Parunov, Corak and Guedes Soares investigated hull girder reliability for chemical tankers [16]. The hull girder realiability was calculated by the reliability model proposed by International Maritime Organization. First order realiability method was used for both oil tanker and a chemical tanker to reflect the differences. IACS rules were carried out for evaluation of the wave induced load effects. The still water loads were defined on the basis of a statistical analysis of loading conditions form the loading manuals. As a result, it was expressed that the hull girder reliability of newly built chemical tanker was well above the upper target reliability proposed by International Maritime Organization. However, the reliability index of the corroded ship was significanlt reduced, such that it was slighly below the lower bound of allowable International Maritime Organization reliability. It was also explained that the reliability of chemical tankers were very sensitive to statistical modelling of the still water bending moment comparing to the oil tankers.

Paik and Thayamballi's study discusses the reliability assessments of ships [17]. In this paper, up to quarternary failure modes were modelled. The progress of the ship structural reliability methods briefly explained and the change of reliability levels in time was discussed. Not only that but also time dependant structural damage was dealt with. This includes corrosion and fatigue-crack model. Also, application of time dependant reliability assessment of ships was shown. To maintain the ship's safety and reliability at a certain target level few repair strategies were introduced.

Further analysis considering structural reliability of ships was done by Iijima, Fujii and Yao [18]. In this work, previously used methods of this subject were briefly explained. First order reliability method was applied for the iteration of the process whereas Monte Carlo simulations were used to calculate the probability of failure. Ultimate strength was gathered by employing a numerical code HULLST which was developed by Yao and Nikolov [19]. Still water bending moment was calculated deterministically from the loading manual. This value also changed parametrically for hogging condition. The results were got by both mean value-first order second moment method and advanced first order-second moment method with responce surface method. For sagging condition, the probability of failure of advanced first order second moment method with responce surface method was twenty times higher. Failure probability in hogging can be neglected form the practical viewpoint although there was a significant difference of two methods's results. To diminish the importance of the coefficient of variation of the still water bending moment, mean value of this moment multiplied by 1, 2, 3, 3.5 and 4 to add some randomness. In the end, it was shown that when the reliability index is lower than 3, the difference between the methods were narrowed. This result was achieved when the mean value of still water bending moment was multiplied by 4.

Britner-Gregersen and Hovem also published a paper about reliability of ship structures [20]. The study's goal was to suggest a procedure for calculating the reliability level inherent in existing ships. The suggested procedure was illustrated by analysis of three ships: a loaded 290000 tonnes VLCC, a 125000 DWT Suezmax and a 165000 DWT bulk carrier. Buckling of a ship deck in the extreme sagging conditions is considered. The reliability calculations were carried out using the software called PROBAN. The results showed that the VLCC ship was more reliable, almost fifty times reliable than the bulk carrier. The authors commented that using L-profiles instead of flatbars caused that significant difference.

Lua and Hess developed a computer program to compute probabilistic strength of the ships [21]. They used ULTSTR program which was developed to calculate deterministic ultimate strength of a ship as a base to their program which was called PULSTR. Monte Carlo simulation and first order reliability method was implemented as the probabilistic integrator. The program was written for hogging conditions only.

An ISSC report were written to establish ship rules and base for a reliability based ultimate limit state and fatigue limit state criteria for ships. The relevant characteristic features of design code formulations, the reliability methodology and the rule calibration approach are also mentioned. Formulations of ultimate and fatigue limit states are shown. Load and load effects are explained and their uncertainities for relevant limit states are evaluated. Furthermore, for a design format, safety factor calibration discussed. Because of the difficulty in implementing reliability based rules, primary objective of the report is given to the uncertainity measures [22].

Subjective (cognitive) uncertainity and objective (non-cognitive) uncertainites are two types of uncertainites that are included in structural strength analysis. Stochastic analysis are used to deal with objective uncertainity and probability theory or fuzzy set theory are used to deal with subjective uncertainities [23]. Yang and Huang studied the reliability assessment of ship structures based on a fuzzy definition of failure. They investigated the influence of high tensile steels on fuzzy reliability of ship hull girder. For the paper, a tanker of 260000 tonnes built with high tensile steel was analysed. The results were compared to Mansour and Hovem's research [11]. They found out that the tripping failure of principal member of the tanker is governing mode in failures. Also, it was said that the use of high tensile steel in ship design generally reduced the thickness of ship hulls, thus buckling strength and fatigue strength.

Nikolaidis and Kapania published a paper concerning models for calculating system reliability and redundancy [24]. There are two types of redundancy [25]. The first one is local which is the margin capacity of the structural member or joints and the demand imposed by loads. The second one is global which includes two different types: system reserve and residual strength. Reserve strength is the margin between the design load and limit state or the ultimate capacity of the overall structure to sustain the applied loads. The residual strength, on the other hand, is the remaining strength in a structure after one or more items have failed because of damage. In their study, the authors investigated the overall residual strength amongst other meanings. Probabilistic analysis of reserve strength was explained and the current problems were shown.

Guedes Soares and Garbatov studied time-variant reliability of ship structures [26]. The goal of the study was to establish a primary failure with a time variant probability model due to the effects of repair and corroded effects. In that approach, repair operations included so that the calculation of the reliability was a continous function of time. Its advantage was to calculate the change of reliability of a ship during life time so that one could decide when to do a repair operation. A numerical example was also included in the paper.

In a later research Kee Paik et al. also discussed ship ultimate strength reliability considering corrosion [27]. In the research, degradation of primary members due to general corrosion took into account for the reliability analysis. The probability of steel renewal due to corrosion was also predicted. The reliability index was calculated by using the second order reliability method. The procedure was applied to a double hull tanker and a bulk carrier. Change of section modulus and ultimate strength with time investigated. In the conlcusion, it was expressed that "the section modulus and ultimate strength of corroded hulls can potentially significantly change (decrease) with time, but hte degree of change is controllable through proper technology application, inspections and steel renewal criteria" [27]. It was also mentioned that by renewal of local members, the levl olf ultimate strength reliability increases, so that with a proper renewal criteria, target reliability levels can be maintained with time passes.

In maritime industry, many applications need use of reliability methods such as [28]:

- Development of reliability based design code requirements
- Estimation of reliability in existing structures
- Performing failure analysis that investigates the cause of structrual failure
- Comparison of alternative designs that compete with existing or conventional design concepts
- Supporting economic value analysis that identifies the trade off between cost and risk so as to minimize total expected life cycle cost
- Development of optimal maintenance strategies of aging structures

Probabilistic methods in design procure some benefits such as [11]:

- Explicit consideration and evaluation of uncertainities associated with the design variables
- Inclusion of all available relevant information in the design process
- Provision of a framework of sensitivity measures
- Provision of means for decomposition of global safety of a structure into partial safety factors associated with the individual design variables
- Provision of means for achieving uniformity of safety within a given class of structures
- Minimum ambiguity when updating design criteria
- Provision of means to weigh variables in terms of their significance
- Provision of rationale for data gathering
- Provision of guidance in novel design
- Provision of the potential to reduce weight without loss of reliability, or to improve reliability without increasing weight

There are also some drawbacks of implementing these methods [11]:

- Reliability methods require more information on environment, loads and the properties and characteristics of the structure than deterministic analysis. Some information might be unaccessible or need enormous time and effort to achieve
- Reliability methods require knowledge of probability, reliability and statistics.
- Reliability analysis did not deliver what it initially promised which is true measure of reliability. Instead it delivered notional probabilities which are good for comparative measures.

2. BASICS CONCEPTS

2.1 Failure Models

Failure is the term for a system to to define the termination of the ability to perform its required function. There are basically two terms in a system that determines whether the system is functional or not in a predefined criterion. These are the resistance or capacity and challange. When challange surpasses capacity, failure occurs [29].

2.1.1 Stress-Strength model

In this model, failure occurs when challenge exceeds capacity. Stress is a factor that tends to produce a failure as the challenge and strength is the ability of an item to resist failure due to external environment and loading.

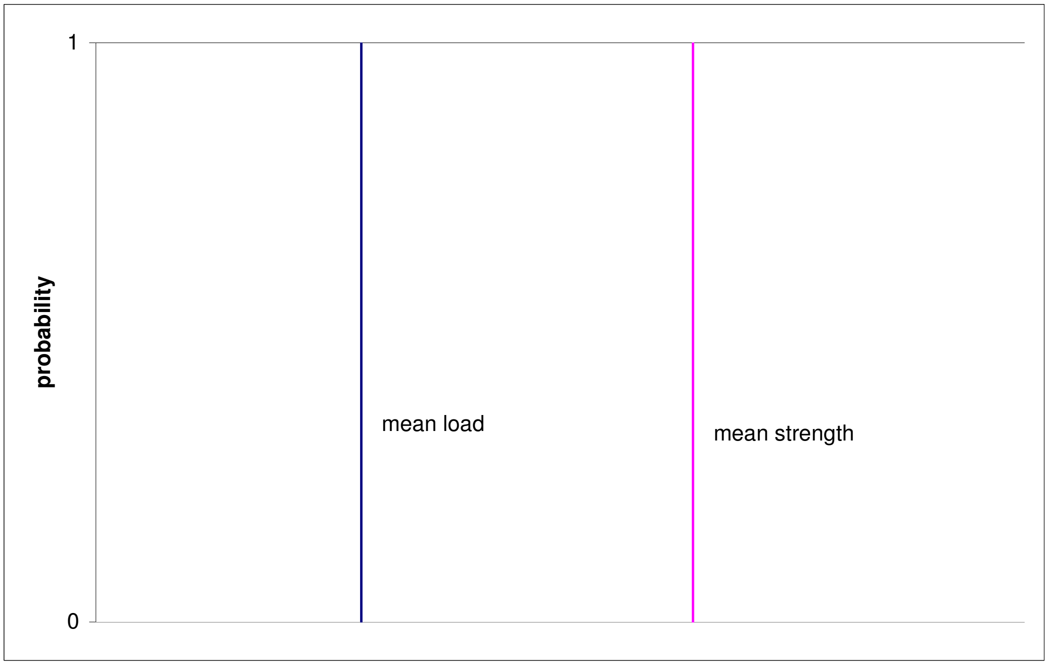

Figure 2.1 Deterministic load and strength

In figure 2.1, a case shown in which load and stregth are constant. That represents the deterministic approach where load is lesser than strength. Hence a failure will never occur.

In figure 2.2, both load and strength are distributed values. In this case, failure will occur where load overlaps strength. Probability of failure is calculated by the area under the interfering curves. There are two terms to analyse the interference: the strength (resistance) and load.

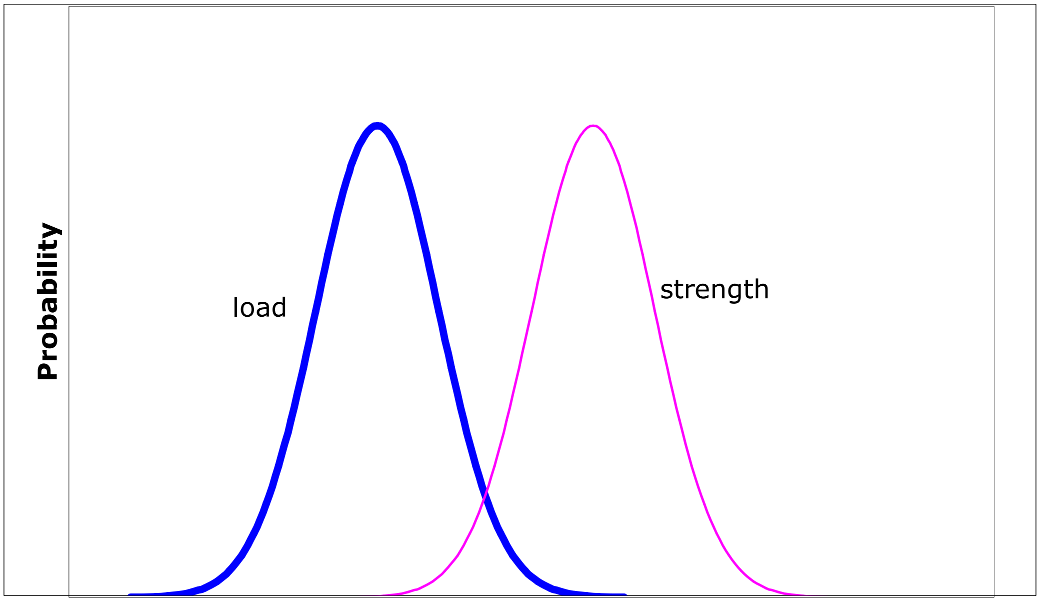

Figure 2.2 Distributed values of load and strength

$$safety\_margin = \frac{\mu_R - \mu_Q}{\sqrt{(\sigma_R^2 + \sigma_Q^2)}} \qquad \textbf{(2.1a)}$$

$$loading\_roughness = \frac{\sigma_Q}{\sqrt{(\sigma_R^2 + \sigma_Q^2)}} \qquad \textbf{(2.2b)}$$

Safety margin (or safety index) is the term which shows us how far the resistance and load curves are. Loading roughness is the standar deviation of the load.

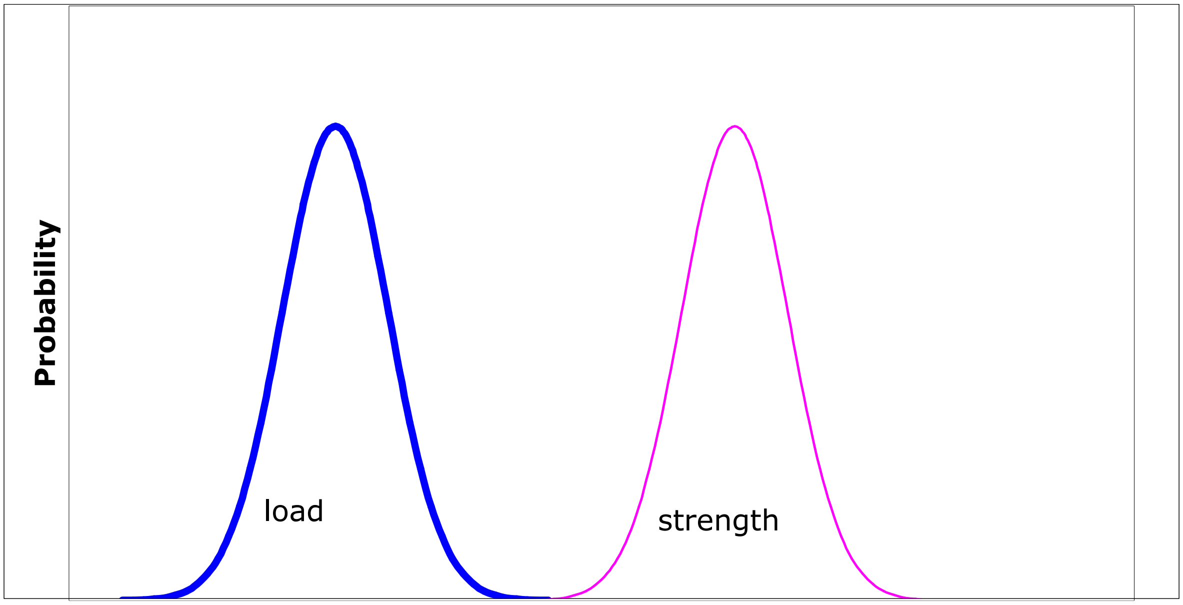

Figure 2.3 High safety margin & low loading roughness

Figure 2.3 shows a case of strength and load probability density function with high safety margin and low loading roughness. It is obvious that the failure probability is low since the overlaping tails of both curves has a very little area below them.

One can also express a case with a low safety margin and low roughness as like in figure 2.4. As expected, the probability of failure is higher than the example shown in figure 2.3.

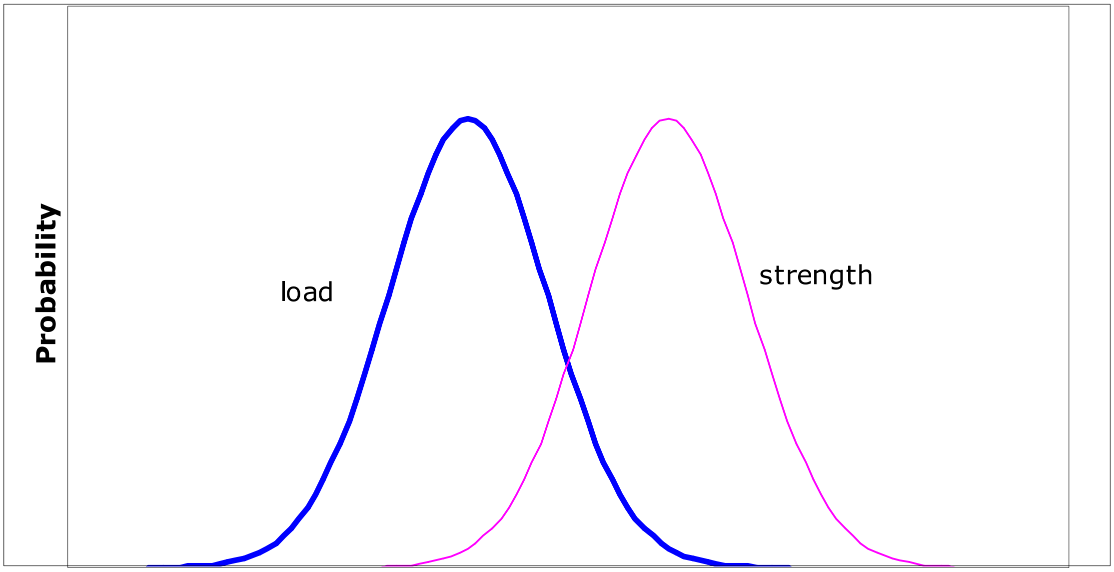

Figure 2.4 Low safety margin & low loading roughness

Another case is where safety margin is low and loading roughness is high (figure 2.5). Since the standart deviation is makes the curve more spread and curves are intersecting closer to their curve's peak, this is the worst case from an engineering perspective amongst the other cases explained above.

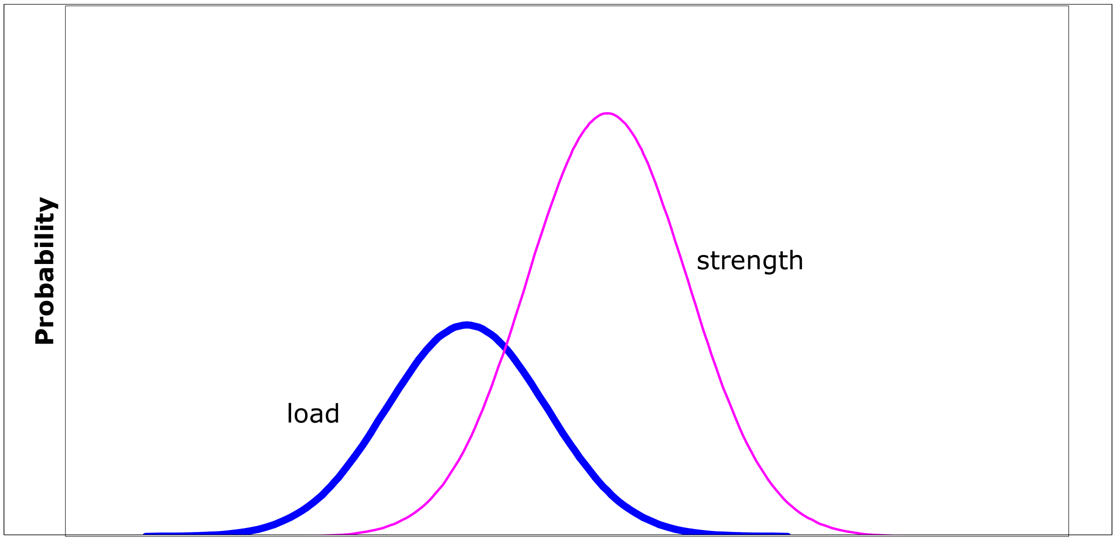

Figure 2.5 Low safety margin & high loading roughness

A good example of stress-strength model for ship structure is the relation of the ultimate collapse bending moment of the midship section of a ship and the loads affecting the section. Hull girder strength is the capacity and still water and wave induced bending moments are the challenges.

2.1.2 Damage-Endurance model

In this model, challenges causes permanant damage over time. Fail occurs as soon as the damage surpasses endurance. In reliability analysis of a corroded ship, the corrosion on the ship's elements is the challenge and the critical thickness of the ship is the endurance factor. Generally, researchers taken into account of general corrosion, which can be defined as the result of the uncoated internal surfaces' friable rust that breaks off over time to expose fresh metal to corrosive attack [27]. But there are other corrosion types such as pitting, which occurs on bottom plating or some horizontal surfaces that traps water, grooving which occurs at structural intersections where water collected or flows and weld corrosion. Wing ballast tanks are most prone to corrosion due to exposure of sea water, humidity, salty atmosphere when empty and temperature increase when exposed to sunlight [27].

2.1.3 Challenge-Response model

This model resembles a system that have many components. An element of the system may have failed, but failure occurs when that failed element causes whole system to fail [29]. That means the vital element is operating in series with the system. Items with very high reliability should be used for the critical part of a system or redundancy must be introduced which means using one or more reserve items [30]. If the reserve items are operating in parallel, it is called active redundancy. They share the load from the very beginning. If the reserve item is kept in standby to be activated when the failure occurs, then it is called passive redundancy where the reserve item has no failure probability before the activation. Partly loaded redundancy can be defined if there is a very weak failure probability concerning the reserve element [30].

There is also another concept on this matter called majority-vote system [31]. A majority element system is one consisting n independent elements which are so arranged that any m or more of the elements are required to be in the successful state for the system to be successful. Conversely, the system may be defined in that any r or more of the elements are required to be in the failed state in order to reduce the system to overall failed state.

2.2 Limit States

Limit state is a boundary between desired and undesired performance of a structure [32]. In mechanics there are generally two terms contributing to a limit state, one is the resistance and the other one is load.

$$g(R,Q) = R - Q \tag{2.2}$$

In the equation above, if R is greater than Q, then the system is safe and the system works as desired.

If there exists a big uncertainity in a system, the safety margin should be kept as large as possible.

The limit states can be briefly categorized in three groups for ships [11]. These are the ultimate strength limit state, the serviceability limit state and the fatigue limit state. The ultimate limit state can also be categorized in two groups: failure due to spread of plastic deformation and failure due to instability or buckling of longitudinal stiffeners or overall buckling of transverse and longitudinal stiffeners of main grillages.

2.2.1 Ultimate strength limit state

Ultimate strength limit state can be roughly written as:

$$\mu_{M_u} - \mu_{M_{sw}} - \mu_{M_w} \tag{2.3}$$

In equation 2.3, the ultimate hull girder moment capacity represents the resistance and the still water bending moment and wave induced bending moment are the loads.

Three failure modes due to still water and wave induced bending moments should be calculated. First of the three failure modes is deck initial yield. “The effective section modulus after the buckling should be applied because buckling of the plates in the deck occurs before inital yield” [11]. Effective section modulus can be calculated as follows:

$$SM_{eff}=\frac{1}{C}\left(I-\left(\frac{bh^3}{12}+C^2bh\right)_{deck}+\left(\frac{b_{eff}h^3}{12}+C^2b_{eff}h\right)_{deck}\right),b_{eff}=92\% \tag{2.4}$$

The critical stress is material yield strength.

The second failure mode for the ultimate limit state is the fully plastic collapse failure mode. The critical stress should be again the material yield strength. The formulation of the mean plastic section modulus is given in equation 2.5 [11]:

$$SM_P=A_Dg+2(A_S+A_{BLK})\left(\frac{D}{2}-g+\frac{g^2}{D}\right)+A_B(D-g) \tag{2.5}$$

and

$$\frac{g}{D}=\left(\frac{A_B+2(A_S+A_{BLK})-A_D}{4A_S}\right) \tag{2.6}$$

The third and final failure mode for the ultimate limit state is the buckling instability failure mode. The elastic section modulus for this one is calculated by the equation given in [34].

In this failure mode, critical stress is the buckling stress and it depends on the buckling mode [11]:

For plates between stiffeners, the plates between the longitudinal stiffeners are considered as simply supported isotropic plates under uniaxial compressive load. The plate collapse stress is:

$$\frac{\sigma_{ul}}{\sigma_y}=\begin{cases}\left(\frac{\sigma_{cr}}{\sigma_y}\right)\rightarrow\chi\geq 3.5\\ \frac{2.25}{b}-\frac{1.25}{b^2}\rightarrow 1<\chi<3.5\\ 1\rightarrow\chi\leq 1\end{cases} \tag{2.7a}$$

where

$$\chi = \frac{b}{t}\sqrt{\frac{\sigma_y}{E}} \tag{2.7b}$$

For stiffeners and effective plating, this case is determined from buckling considerations because the plate is under edge compression. There is only ultimate limit state mode of this criticial stress because when a column buckles it reaches its ultimate stress instantly. The effective plating under edge compression is determined by equation 2.8:

$$b_c = b\left(\frac{\sigma_{ul}}{\sigma_y}\right) \tag{2.8a}$$

Critical stress calculation for column buckling is as follows:

$$\sigma_{cr} = \begin{cases} \dfrac{\pi^2 E}{\left(l/r\right)^2} \rightarrow \sigma_{cr} \le \sigma_p \\ \sigma_y - \dfrac{1}{C} \rightarrow \sigma_{cr} > \sigma_p \end{cases} \tag{2.8b}$$

where

$$r = \sqrt{\frac{I}{A}} \tag{2.8c}$$

2.2.2 Serviceability limit state

The serviceability is very similar to ultimate limit state:

$$\mu_{M_{serv.}} - \mu_{M_{sw}} - \mu_{M_w} \tag{2.9a}$$

Since $\mu_{M_{serv.}}$ is determined by critical buckling stress in a serviceable limit state, relevant section modulus is calculated as shown in [34] and the critical stress is calculated as follows

$$\sigma_{cr} = \begin{cases} K_c \dfrac{\pi^2 E}{12\left(1-\nu^2\right)}\left(\dfrac{t}{b}\right)^2 \rightarrow \sigma_{cr} \le \sigma_p \\ \dfrac{C_l \sigma_y}{C_l + 1} \rightarrow \sigma_{cr} > \sigma_p \end{cases} \tag{2.9b}$$

whereas

$$C_l = \frac{\sigma_1^2}{\sigma_p\left(\sigma_y - \sigma_p\right)} \tag{2.9c}$$

2.2.3 Fatigue limit state

The fatigue limit state is relevant to repeated loading. There are two approaches to this concept. The fracture mechanics and the Palmgren-Miner approach which is based on S-N curves that are obtained by stress cycle experiments. Such curves are in the form:

$$N.\Delta S^m = C \tag{2.10a}$$

and C is determined by:

$$\log C = \log a - 2\sigma_{\log N} \tag{2.10b}$$

In equation 2.10b, $\sigma_{\log N}$ is the standart deviation of $\log N$.

The fatigue life calculation is determined based on the assumption of linear cumulative damage by the Palmgren-Miner rule. Application of this assumption implies that long-term distribution of stress range is replaced by a stress histogram consisting of a n equivalent set of constant amplitude stress blocks. The time to failure of a detail can be expressed as [35]:

$$\bar{T} = \frac{\bar{\Delta}_F \bar{C}}{\bar{B}^m \bar{\Omega}} \tag{2.10c}$$

If the long term distribution of the wave process is assumed to be a series of short term sea states that are stationary, zero-mean, Gaussian and narrow banded and if the structure is linear, the stress range will follow a Rayleigh distribution [11] and $\bar{\Omega}$ is determined as [35] [36]:

$$\Omega = \frac{\left(2\sqrt{2}\right)^{m}}{2\pi}\Gamma\left(1+\frac{m}{2}\right)\sum_{j} p_j \lambda_{0j}^{(m-1)/2}\lambda_{2j}^{1/2} \qquad \textbf{(2.10d)}$$

and the fatigue limit state turns out to be:

$$g = \frac{\bar{\Delta}_F \bar{C}}{\bar{B}^m \bar{\Omega}} - \tau \qquad \textbf{(2.10e)}$$

2.3 Probability of Failure

Probability of failure is defined as the probability that the system will fail by a certain amount of time. As stated in equation 2.1, there is a very simple function to determine if the system in safe or unsafe. Considering the R or Q is a random variable, then the probability of failure is equal to the probability that the load is greater than the resistance [33]. For every random variable available either belonging to resistance or load, the proabability of failure is summation of the variables. When the random variables are continuous, the probability of failure can be written as:

$$P_f = 1 - \int_{-\infty}^{+\infty} F_Q(r_i) f_R(r_i) dr_i \qquad \textbf{(2.11a)}$$

$$P_f = \int_{-\infty}^{+\infty} F_R(q_i) f_Q(q_i) dq_i \qquad \textbf{(2.11b)}$$

As such integrals are difficult to evaluate, numerical techniques used in order to get a result. Some of the numerical techniques will be discussed later in this current thesis. Also, some numerical examples will be shown.

2.4 Useful Probability Distributions

2.4.1 Gaussian (normal) distribution

This type of distribution is the best known and most widely used [37]. Its PDF for a continous variable X is:

$$f_x = \frac{1}{\sigma_X \sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{X - \mu_X}{\sigma_X} \right)^2 \right] \qquad -\infty < X < \infty \qquad \textbf{(2.12a)}$$

and the CDF of the normal distribution is:

$$\Phi = \int_{-\infty}^{x} \frac{1}{\sigma_X \sqrt{2\pi}} \exp\left[-\frac{(x - \mu_X)^2}{2\sigma_X{}^2} \right] dx \qquad \textbf{(2.12b)}$$

Gaussian probability distribution with the parameters of a zero mean and one standart deviation is called standart normal distribution. To standardize any normally distributed random variable the following approach is used:

$$Z = \frac{x - \mu}{\sigma} \qquad \textbf{(2.13)}$$

Then the PDF of the distribution becomes:

$$f(Z) = \frac{1}{2\pi} \exp\left(-\frac{Z^2}{2} \right) \qquad -\infty < X < \infty \qquad \textbf{(2.13a)}$$

and the CDF is:

$$\Phi(Z) = \int_{-\infty}^{Z} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{Z^2}{2} \right) dZ \qquad \textbf{(2.13b)}$$

2.4.2 Lognormal distribution

Lognormal distribution is also popular. If a random variable X has a lognormal distribution and its PDF is:

$$f_x = \frac{1}{\zeta X \sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{\ln X - \lambda}{\zeta} \right)^2 \right] \qquad x \geq 0 \qquad \textbf{(2.14)}$$

The parameter ζ determines the positivity or the negativity of the skew of the lognormal PDF. A lognormal variable of X is normal as lnX with normal mean and standart deviation. Hence, lognormal distribution can be determined from standart normal distribution tables.

Because lognormal variate is always positive, this kind of distribution is useful when any application the variate is known to be positive from the physical consideration such as the strength and fatigue life of materials, the intensity of rainfall, the volume of air traffic etc. [37].

2.4.3 Extreme type I (gumbel) distribution

Extreme value distributions are useful to characterize the probabilistic nature of the extreme values of some phenomenon over time [33]. During any specified period of time, there will be a maximum value of some phenomenon. A ship will face a maximum number of waves during a period of time. That's why the calculation of wave induced bending moment based on extreme type I distribution. The PDF and CDF of this kind of random variable is shown in equation 2.16 and 2.17 respectively:

$$f_x = \alpha \exp\left[-\exp\left(-\alpha (X - u) \right) \right] \exp\left[-\alpha (X - u) \right] \qquad \textbf{(2.15)}$$

$$F_x = \alpha \exp\left[-\exp\left(-\alpha (X - u) \right) \right] \qquad -\infty < X < \infty \qquad \textbf{(2.16)}$$

and [38]:

$$\alpha = \frac{1.282}{\sigma_X} \qquad \textbf{(2.17)}$$

$$u = \mu_X - 0.45\sigma_X \qquad \textbf{(2.18)}$$

3. METHODS

Structural reliability methods are used to check safety in a structure within a safety domain for a defined limit state function. There are many methods present that are grouped in three levels depending upon the degree of sophistication [39][40].

3.1 Level III Procedures

The results got from the level III procedure shown in equation 3.1 is exact but very difficult in practice due to requirement of high number of statistical data and high computational time [41].

$$P_f = \int_0^\infty F_R(q) f_Q(q) dq \qquad \textbf{(3.1)}$$

This procedure will not be discussed more since it is beyond the scope of this thesis.

3.2 Level II Procedures

In this method, safety checks are made at finite number of points of the safety domain boundary [41]. Only mean and standart deviation is necessary in order to calculate the probability of failure. Furthermore, unlike level III procedure, either resistance's or load's mean and standart deviation is enough to find a result.

3.2.1 First order reliability methods (FORM)

FORM is widely used in literature due to its low demand of computational resources and its accurate results to the same degree of level III procedure [41]. FORM is consisted of second moment methods such as first order second moment (FOSM) and advanced first order second moment (AFOSM) methods[42]..

3.2.1.1 First order second moment method (FOSM)

This method is also called mean value first order second moment (MVFOSM) method. It is called first order because first order terms in taylor series expansion is used, second moment because only means and variances are needed and mean value because taylor series expansion is about mean values [33].

Probability function for FOSM method is defined for normal variables as:

$$P_f = 1 - \Phi(\frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}}) \qquad \textbf{(3.2a)}$$

where

$$\mu_Z = \mu_R - \mu_Q \qquad \textbf{(3.2b)}$$

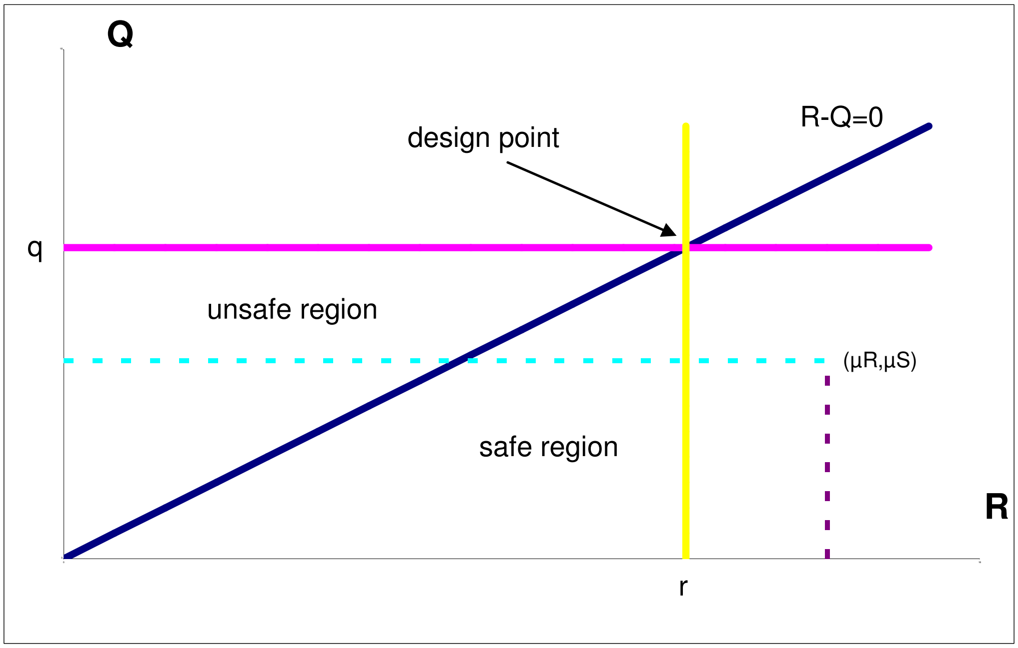

Figure 3.1 : Original coordinates

The term in paranthesis in the equation 3.2 is the reliability index:

$$\beta = (\frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}}) \qquad \textbf{(3.3a)}$$

we can also write reliability index as:

$$\beta = \frac{\mu_Z}{\sigma_Z} \tag{3.3b}$$

Thus the reliability index is the distance from the origin (z=0) to μ_z measured in standart deviation units. It means the reliability index is a measure of probability that z will be less than zero.

So that, the equation 3.2 becomes:

$$P_f = 1 - \Phi(\beta) \tag{3.4}$$

Although it is easy to use and it doesn't need much information about the distribution of random variables, there are some setbacks. The main criticism of this method is that when the limit state is non-linear which generally is, errors occur because taylor series is truncated at the linear terms and the error widens with the increase in distance from the linearising point.

More importantly, this approach gives different safety indexes to differently formulated but physically same situations. For example, if one is calculating the reliability of any kind of a beam, the answer differentiates according to using strength formulation or stress formulation as a limit state function.

3.2.1.2 Advanced first order second moment method (AFOSM)

In the early 1970s, lack of invariance problem of the MVFOSM method was overcame by the AFOSM method which was proposed by Hasofer and Lind [43].

This method is also called Hasofer-Lind method. It only serves for normal and uncorrelated variables and very commonly used. It gives more accurate results than Level III method at the same time without recoursing to convolution integral [41].

In this method, firstly, reduced variables are calculated. For instace, the reduced variable for resistance is:

$$Z_R = \frac{R - \mu_R}{\sigma_R} \tag{3.5a}$$

and reduced variable for load:

$$Z_Q = \frac{Q - \mu_Q}{\sigma_Q} \tag{3.5b}$$

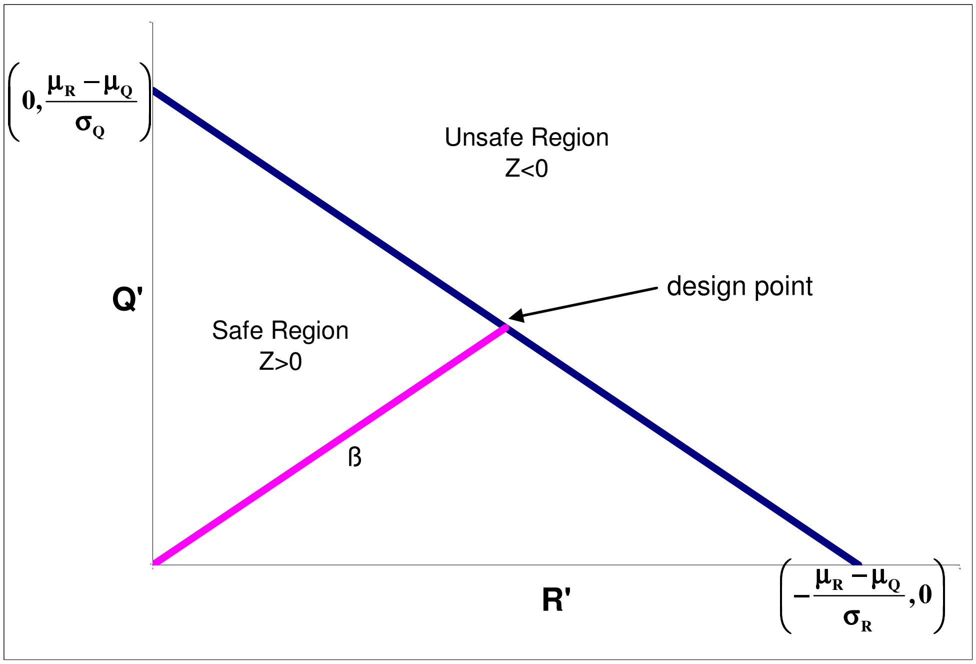

Figure 3.2 : Reduced coordinates

The minimum distance point on the limit state surface is called the design point. It is clear from figure 3.2 that if the limit state line is closer to the origin, the failure region is larger.Thus, the position of the limit state surface relative the origin in the reduced coordinate system is a measure of reliability of the system. If the variables are not non linear and are both normal we get the equation 3.3a as in the FOSM method. We can get this by simply using geometric formulations in figure 3.2.

For non linear systems, the computation of the minimum distance becomes an optimization problem.

Equations 3.5a and 3.5b can be arranged as:

$$R = \mu_R + Z_R \sigma_R \tag{3.6a}$$

$$Q = \mu_Q + Z_Q \sigma_Q \tag{3.6b}$$

Then, these variables are put into the limit state function given in equation 2.2:

$$g = \mu_R + Z_R\sigma_R - \mu_Q - Z_Q\sigma_Q \qquad \textbf{(3.7)}$$

where:

$$Z_i^* = \beta\alpha_i \qquad \textbf{(3.8)}$$

Sensitivity factor can be found by the equation:

$$\alpha_i = \frac{-(\frac{\partial g}{\partial x_i})}{\sqrt{\sum_{i=1}^{n}(\frac{\partial g}{\partial x_i})^2}} \qquad \textbf{(3.9)}$$

This method is an iterative procedure to determine the reliability index. As mentioned above, first of all, the limit state function is derived as the function of means, standart deviations and design points chosen. After that, equation 3.8 is put into the equation to express the limit state as a function of sensitivities and reliability index. Having that done, an iteration must be done in order to determine the reliability index and the sensitivities until reliability index converges. For minimal iteration steps, reliability index should be chosen around 3 and sensitivities for the resistance terms should be negative. After that, probability of failure can be found from equation 3.4.

There are two examples about beams with uniform loads in numerical examples section and also an example for ultimate strength hull girder reliability that are evaluated by hasofer-lind reliability index.

For nonnormal variables, transformation methods must be used to get an accurate result which will be discussed later in the thesis.

3.3 Level I Procedure

This method is used when safety margins are given as structural element basis by specifying a number of partial safety factors related to some predefined characteristic values of the basic variables[41].

3.4 Rackwitz-Fiessler Procedure

As mentioned earlier, Hasofer-Lind method can only be used for uncorrelated normal variables. Otherwise, transformation is needed. One of many transformation methods, the one called Rackwitz-Fiessler procedure is extensively used in the literature [42]. This method simply finds equivalent normal mean value and standart deviation at the design point. The important point is that after every iteration step, new design point should be also put in the transformation to get new normal mean and standart deviation to be used in the next iteration step of the Hasofer-Lind method.

In this method normal equivalent standart deviation is expressed as:

$$\sigma_{x_i}^N = \frac{\phi\{\phi^{-1}[F_{x_i}(x_i^*)]\}}{f_{x_i}(x_i^*)} \qquad \textbf{(3.10)}$$

and the equivalent mean value can be found by the equation:

$$\mu_{x_i}^N = x_i^* \phi^{-1}[F_{x_i}(x_i^*)]\sigma_{x_i}^N \qquad \textbf{(3.11)}$$

4. BASIC CONCEPTS ON SHIP STRUCTURAL RELIABILITY

As this thesis is concerned about the reliability of the ultimate collapse bending moment of the hull girder, all the information given in this section will be limited to it.

4.1 Limit State Function

In the previous studies many limit state functions have been defined for ultimate collapse bending moment for ship hull girders by many researchers.

For instance[16]:

$$g = x_u M_u - M_{sw} + \Psi x_w x_{nl} M_w < 0 \quad \textbf{(4.1)}$$

Another example[44]:

$$g = x_u M_u - (x_{sw} k_{sw} M_{sw} + x_w k_w M_w) \leq 0 \quad \textbf{(4.2)}$$

In the equations 4.1 and 4.2, the resistance factor is the multiplication of the uncertainity in the ultimate strength and the ultimate strength itself. The main difference between the two equations above is that there is a coefficient called uncertainity of nonlinear effects. It comes from the difference of the sagging and hogging conditions of the vertical bending moments. It is very important for containerships but not significant for oil/chemical tankers[16].

4.2 Ultimate Vertical Bending Moment Capacity

To represent the resistance term in the limit state function, other failure modes could be chosen, but the ultimate bending moment capacity of hull girder is considered the most important measure of the ship strength as it ensures that ship will not be broken in two parts even if some structural damage in longitudinal structures already occured[16].

Table 4.1 gives information about the mean value and assumed standart deviation of design variables which can be used to determine the ultimate vertical bending moment capacity of the hull girder[45].

Table 4.1: Mean value and assumed standart deviation of design variables

Variable	probability distribution	mean	COV (%)
plate thickness	normal	nominal value	0.6
thickness of stiffner plating	normal	nominal value	0.6
yield stress of HT32 steel	lognormal	314 Mpa	7
yield stress of HT36 steel	lognormal	402 Mpa	4.3
compressive residual stress	normal	calculated value	30
initial deflection in plates	normal	calculated value	50
initial deflection in stiffners	normal	0.05% of span	20
initial deflection in stiffners (tripping)	normal	0.05% of span	20

Also, there are few rule based calculations of the ultimate hull girder bending moment capacity. For instance, a method proposed by IACS[46]:

$$M_u = z_{red}\sigma_{red}10^3 \quad \textbf{(4.3)}$$

where,

$$z_{red} = \frac{I_{red}}{z_{dk-mean} - z_{NA-mean}} \quad \textbf{(4.4)}$$

Another example of formulation is given by [44]:

$$M_u = \sum_C \sigma_i A_{ei}(a_i - g_u) + \sum_T \sigma_j A_j (a_j - g_u) \quad \textbf{(4.5)}$$

There are some other formulations in chapter 2.2.1

4.2.1 Modeling uncertainity of the ultimate vertical bending moment

The main contributors to the ultimate bending moment capacitiy's uncertainity are those related to yield strength and model uncertainity of the calculation method [16]. This uncertainity is distributed by lognormal distribution and has a mean value of 1.14 and a coefficient of variation of 0.13[47].

4.3 Still Water Bending Moment Model

Still water loads are the loads that is caused by light ship, deadweight and the buoyancy[13]. In a research it is advised using normal distribution with coefficient of variation ranging between 0.46 to 0.98[48]. Further analysis indicates that departure or arrival conditions and full load, part load and ballast load conditions greatly effet this variable, thus these different conditions must be taken into account[13]. In a different research [16], normal distribution is used for one voyage and gumbel distribution for annual calculation and the ship is considered always in hogging condition.

Oil tankers have more predictable load than a bulk carrier. Bulk carriers can carry almost any goods with large variations in density[20].

There are also approximate formulas such as[44]:

$$M_{sw} = 0.015CL^2B(8.167 - C_b) \text{ for hogging} \tag{4.6a}$$

$$M_{sw} = -0.065CL^2B(0.7 + C_b) \text{ for sagging} \tag{4.6b}$$

where:

$$C = 0.0792L \text{ for } L \leq 90 \tag{4.7a}$$

$$C = 10.75 - (\frac{300 - L}{100})^{1.5} \text{ for } 90 < L \leq 300 \tag{4.7b}$$

$$C = 10.75 \text{ for } 300 < L \leq 350 \tag{4.7c}$$

$$C = 10.75 - (\frac{L - 350}{150})^{1.5} \text{ for } 350 < L \leq 500 \tag{4.7d}$$

4.4 Wave Induced Bending Moment Model

Wave-induced bending moment is treated as a random variable dependent on ship's principal characteristics, environmental influences, and operational conditions. Spectral and extreme analyses can be used to determine the extreme values and the load spectra of this load type during the design life of the ship. The outcome of this analysis can be in the form of vertical or horizontal longitudinal bending moments or stresses on the hull girder. Wave induced bending moment is taken as the mean value of the extreme wave induced bending moment the ship encounters during a predefined period of time. [50]

A formula for unrestricted worldwide service is as follows[11]:

$$M_w = 0.19CL^2BC_b \quad \text{for hogging} \qquad \textbf{(4.8a)}$$

$$M_w = -0.11CL^2B(0.7 + C_b) \quad \text{for sagging} \qquad \textbf{(4.8b)}$$

4.4.1 Modeling uncertainity of the wave bending moment

There are some factors for the prediction of the extreme wave loads on ship hulls, such as simplifications, assumptions and inaccuracies of the linear engineering models are represented by modeling uncertainty of the wave bending moment[16].

4.5 Target Reliability Levels

As mentioned before, the results that was got by reliability analysis are not absolute reliability value which depends on the analysis model and uncertainites [20]. So, the results should be compared to other analysis which takes into account similar models and uncertainites. That limits the approach not to be applied on a general basis, but only case by case for individual examples. In another research, some results were compiled and as a result, the target reliability index was advised 3.7 for new built ships and 3.0 for the lower limit of corroded structure [12]. Thayamballi and Paik's study indicates that in 1991, the average reliability index was 3.5, and in 2000, the average was 2.5 [17]. It was suggested that 2.5 was a good target reliability. Mansour suggests hogging reliability index of 2.54 and sagging index of 4.04 for tankers [51].

5. NUMERICAL EXAMPLES

In this section, two simple examples are given to clarify the Hasofer-Lind reliability index method. First one is a simple supported beam under uniform load and the second one is carrying the same uniform load but has fixed supports. The difference of the probability of failure is obvious. The last example is recalculating an alrady done research [16] by using a different limit state function defined in section 4.1.

5.1 Example 1

Calculate the Hasofer-Lind reliability index for beam shown in figure 5.1. The random variables are length, elasticity and moment of inertia which are given in table 5.1. The limit state to be considered is deflection, and the allowance deflection is specified as $L/360$. The maximum deflection is $0.013wL^4/EI$.

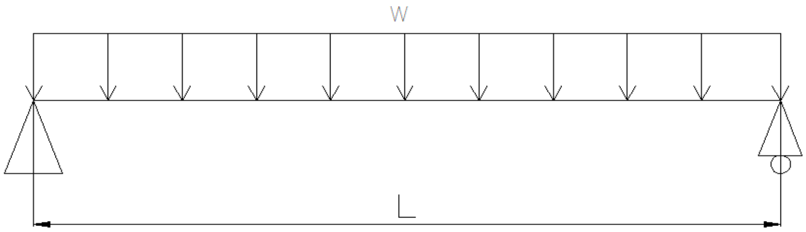

Figure 5.1 : Beam under uniform load.

Table 5.1: Variables for example one.

Variable	Mean	Standart Deviation
w	5 kN/m	0.3 kN/m
L	5 m	-
E	$2x10^7$ kN/m ^{2}	$0.5x10^7$ kN/m ^{2}
I	$9x10^{-4}$ m ^{4}	$2x10^{-4}$ m ^{4}

5.1.1 Solution

limit state function:

$$g = \frac{L}{360} - 0.013\frac{wL^4}{EI}$$

Reduced variables:

$$Z_1 = \frac{I-\mu_I}{\sigma_I} \quad Z_2 = \frac{E-\mu_E}{\sigma_E} \quad Z_3 = \frac{w-\mu_w}{\sigma_w}$$

$$I = \mu_I + Z_1\sigma_I \quad E = \mu_E + Z_2\sigma_E \quad w = \mu_w + Z_3\sigma_w$$

Substitution into limit-state function:

$$\frac{5}{360} - 0.013\frac{(5+Z_3 0.3)5^4}{(2x10^7 + Z_2 0.5x10^7)(9x10^{-4} + Z_1 2x10^{-4})} = 0$$

That derives into:

$$(2x10^7 + Z_2 0.5x10^7)(9x10^{-4} + Z_1 2x10^{-4}) - 585.9(5+Z_3 0.3) = 0$$

$$15070 + 4000Z_1 + 4500Z_2 + 1000Z_1Z_2 - 175.8Z_3$$

since $Z_i^* = \beta\alpha_i$

$$\beta = \frac{-15070}{4000\alpha_1 + 4500\alpha_2 + 1000\beta\alpha_1\alpha_2 - 175.8\alpha_3}$$

Sensitivity factors are derived from equation 3.9

The iterations start with inital values for β, α_1, α_2 and α_3.

The results of each step is given in table 5.2

Table 5.2: Iteration steps for example one

	initial	first	second	third	fourth	fifth	sixth	seventh
β	3	3.7954	3.62203	3.45257	3.33323	3.30467	3.30092	3.30034
α_1	-0.57	-0.633	-0.4535	-0.2638	-0.1849	-0.1841	-0.1903	-0.1930
α_2	-0.57	-0.772	-0.8881	-0.9627	-0.9815	-0.9819	-0.9807	-0.9801
α_3	0.57	0.0486	0.07453	0.05923	0.04808	0.04444	0.04430	0.04450

β is 3.30 so that:

$\Phi(-3.30) = 4.83x10^{-4}$

5.2 Example 2

Calculate the Hasofer-Lind reliability index for the beam shown in figure 5.2. The random variables and the values as same as the example 1. The specified deflection allowance is $L/360$ and the maximum deflection is $0.0026wL^4/EI$

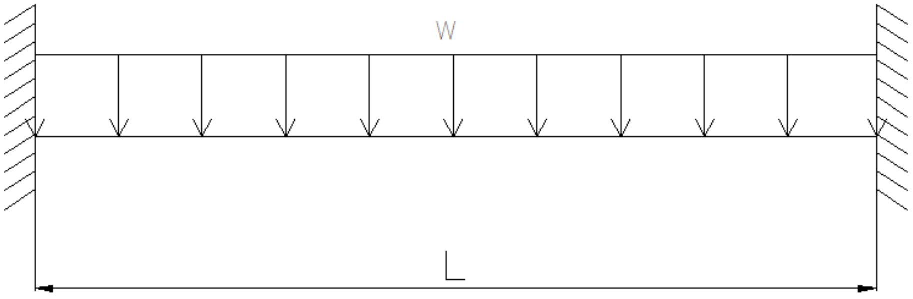

Figure 5.2 : Beam under uniform load

5.2.1 Solution

limit state function:

$$g = \frac{L}{360} - 0.0026\frac{wL^4}{EI}$$

Reduced variables:

$$Z_1 = \frac{I - \mu_I}{\sigma_I} \qquad Z_2 = \frac{E - \mu_E}{\sigma_E} \qquad Z_3 = \frac{w - \mu_w}{\sigma_w}$$

$$I = \mu_I + Z_1\sigma_I \quad E = \mu_E + Z_2\sigma_E \quad w = \mu_w + Z_3\sigma_w$$

Substitution into limit-state function:

$$\frac{5}{360} - 0.0026\frac{(5 + Z_3 0.3)5^4}{(2x10^7 + Z_2 0.5x10^7)(9x10^{-4} + Z_1 2x10^{-4})} = 0$$

That derives into:

$$(2x10^7 + Z_2 0.5x10^7)(9x10^{-4} + Z_1 2x10^{-4}) - 117.2(5 + Z_3 0.3) = 0$$

$17410+4000Z_1+4500Z_2+1000Z_1Z_2-35.16Z_3$

since $Z_i^* = \beta\alpha_i$

$$\beta = \frac{-17410}{4000\alpha_1+4500\alpha_2+1000\beta\alpha_1\alpha_2-35.16\alpha_3}$$

Sensitivity factors are derived from equation 3.9

The iterations start with inital values for β, α_1, α_2 and α_3.

The results of each step is given in table 5.3

Table 5.3: Iteration steps for example two

	initial	first	second	third	fourth	fifth	sixth	seventh
β	3	4.475186	4.5555	4.16885	3.877524	3.87356	3.867212	3.86704
α_1	-0.57	-0.63441	-0.3096	0.10641	0.029301	-0.0269	-0.02909	-0.0306
α_2	-0.57	-0.77293	-0.9506	-0.9942	-0.99955	-0.9996	-0.99954	-0.9995
α_3	0.57	0.009741	0.02012	0.01131	0.007109	0.00761	0.007995	0.00801

β is 3.87 so that:

$\Phi(-3.87) = 5.44x10^{-5}$

5.3 Example 3

The final example is application of first order reliability method to ultimate ship hull girder strength. The ship taken into accounted has been worked by Guedes Soares C., however a different approach to the limit state function will be done.

The limit-state function in [16] is used as:

$g = x_u M_u - M_{sw} - \psi x_w x_{nl} M_w \leq 0$

The limit-state function in this example is [17]

$$g = x_u M_u - (x_{sw} k_{sw} M_{sw} + x_w k_w M_w) \leq 0$$

The ship used is a chemical tanker whose main characteristics are shown in table 5.4. It is an existing ship that fully satisfies the current rules including IACS UR S11.

Table 5.4: Main characteristics of the ship

L_{BP} (m)	120
B (m)	17
D (m)	9
T (m)	7
DWT (t)	7900

The random variables are shown in table 5.5

Table 5.5: Random variables

Variable	Distribution	Mean	COV
M_{sw} (MNm)	Gumbel	102.6	0.22
M_w (MNm)	Gumbel	299.0	0.09
M_u (MNm)	Deterministic	813	
x_w	Gaussian	0.9	0.15
x_{nl}	Gaussian	0.95	0.15
x_u	Lognormal	1.14	0.13
ψ	Deterministic	0.92	

Since some of the variables are nonnormal, in order to use Hasofer-Lind reliability index, Rackwitz-Fiessler transformation method must be applied to get normal equivalents of the random variables.

Reduced random variables are calculated from equation 3.5a follows:

$$z_1 = \frac{x_u - 1.12}{0.15}$$

$$z_2 = \frac{x_{sw} - 1}{0.05}$$

$$z_3 = \frac{M_{sw} - 70.4}{24.9}$$

$$z_4 = \frac{x_w - 0.9}{0.135}$$

$$z_5 = \frac{M_w - 290.8}{23.9}$$

by rearranging the reduced variables we get:

$$x_u = 0.15 z_1 + 1.13$$

$$x_{sw} = 0.05 z_2 + 1$$

$$M_{sw} = 18.63 z_3 + 70.4$$

$$x_w = 0.135 z_4 + 0.9$$

$$M_w = 21.74 z_5 + 291.5$$

then, the limit state function becomes:

$$g = (0.15 z_1 + 1.12) M_u - [(0.05 z_2 + 1) k_{sw} (24.9 z_3 + 70.4) + (0.135 z_4 + 0.9) k_w (23.9 z_5 + 290.8)]$$

where,

$$z_i^* = \beta \alpha_i$$

$$g = (0.15 \beta \alpha_{x_u} + 1.12) M_u - [(0.05 \beta \alpha_{x_{sw}} + 1) k_{sw} (24.9 \beta \alpha_{M_{sw}} + 70.4) + (0.135 \beta \alpha_{x_w} + 0.9) k_w (23.9 \beta \alpha_{M_w} + 290.8)]$$

and the reliability index:

$$\beta = \frac{-656.956}{121.95 \alpha_{x_u} - 3.52 \alpha_{x_{sw}} - 24.9 \alpha_{M_{sw}} - 27.4806 \alpha_{x_w} - 15.057 \alpha_{M_w} - 1.245 \beta \alpha_{x_{sw}} \alpha_{M_{sw}} - 2.25855 \beta \alpha_{x_w} \alpha_{M_w}}$$

The results calculated by the limit state function shown as equation 4.2 are shown in table 5.6

It must be noted that the loads combination factors k_{sw} and k_w are 1 and 0.7 respectively.

Table 5.6: Iteration steps of example 3

	initial	first	second	third	fourth	fifth
β	3	-12.5532	5.158747	5.120681	5.10229	5.102236
α_{x_u}	0.67	-0.94127	-0.96122	-0.94619	-0.94429	-0.9442
$\alpha_{x_{sw}}$	0.5	0.041872	0.002294	0.036834	0.036799	0.036826
$\alpha_{M_{sw}}$	0.51	0.206605	0.191106	0.19331	0.194626	0.194599
α_{x_w}	0.32	0.227799	0.186894	0.219345	0.224765	0.224924
α_{M_w}	0.3	0.132953	0.067774	0.13372	0.136234	0.136633

β is 5.10 so that:

$\Phi(-5.10) = 1.7x10^{-7}$

5.3.1 Results of the original paper

Reliability index and probability of failure had in [16] are shown below. Also, the sensitivities calculated are shown in table 5.7.

Table 5.7: Sensitivities calculated in the original paper

	x_u	x_w	x_{nl}	M_w	M_{sw}
α	0.67	0.32	0.32	0.30	0.52

$\beta = 4.75$

$\phi(-4.75) = 1.02x10^{-6}$

This example analyses the ultimate collapse of the ship hull of a typical chemical tanker. The chemical tanker also exists and currently in use. The ship has corrugated centerline and transverse bulkheads, satisfying IACS rules as mentioned before. Cargo hold area is entirely mild steel and covered by epoxy coating. It can carry chemicals with a density ranging between 0.7 and 1.5 t/m^3 according to International Code for the Construction and Equipment of Ships Carrying Dangerous Chemicals in Bulk.

It should be noted that these results are annual values. The period of the analysis determines the number of waves the ship faces, hence it affects the wave induced bending moment term. The period of the analysis has no effect on other terms, hence the wave induced bending moment determines the change of the reliability level due to different intervals. But it must also be mentioned that the load combination factor of [16] also affected by the period that it increases with the interval. The load combination factors of [17] are constant numbers.

6. CONCLUSION AND RECOMMENDATIONS

The major purpose of this research was to achieve the failure probabilities so that the two different approaches to the problem could be compared. Although both study's reliability indices are relatively similar, the failure of probability of this thesis is ten times lower than [16]. It was noted that the lack of uncertainity of non linear wave load was not a significant factor in case of chemical tankers [16]. So, the difference of of these the studies could not be a result of that term. The main problem seems to be the period of the analysis, since the small interval makes the reliability index go higher, making the area under the curve smaller. This causes the absolute difference between the two results very small, but this paper's result is ten times lower. The period should be set to higher values to abate the order of the difference. The lack of information prevents to analyse in a larger interval to be put in the thesis.

In [16], the senstivity factor of the ultimate moment capacity uncertainity was the most significant amongst all. This research shows the same with an increase of the significancy of the modelling uncertainity of the ultimate moment capacity of the ship.

It was previously mentioned that the reliability analysis do not show real reliabilities, instead they give notional values to be compared amongst similar cases. That means the results do not disclose the actual number of ships which will not be able to stay in one peace in a year. So that there are some stuides to determine a target reliability level. International Maritime Organization's target reliability index for newly built tankers is 3.71, hence both Guedes Soares' and this thesis' results satisfy the IMO targets. It is also explained that a reliability index of 2.5 is also acceptable value [17]. Those target reliability indices are not calculated values, instead they were gathered by compiling the results of the analysis of the existing ships. That makes the target reliability obscure. For instance, the target reliability level was determined as 3.5 in 1991 [17]. The same source set it to 2.5 in 2000. Changes in constuction codes or techniques change the average value of reliabilities of ships. That prevents determination of a constant value. Large amounts of ships should be analysed to gather enough data to set an acceptable target level and it should be updated continuosly with time to keep the targets valid and sensible.

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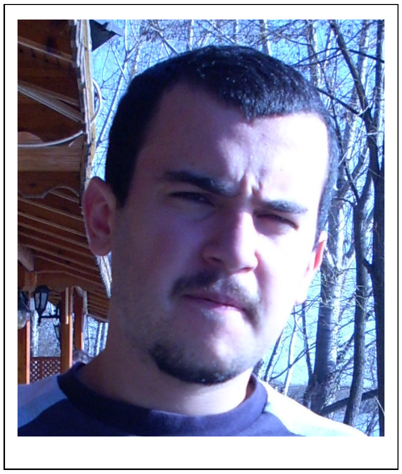

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