ISTANBUL TECHNICAL UNIVERSITY ★ INSTITUTE OF SCIENCE AND TECHNOLOGY

REAL AND REACTIVE POWER OPTIMIZATION BY APPROXIMATE SUB-OPTIMUM TECHNIQUES

Ph.D.THESIS

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Date of Submission : 10 September 1990
Date of Approval : 21 November 1990
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NOVEMBER , 1990

ACKNOWLEDGEMENTS

I want to acknowledge my thanks to Prof.Dr.Yılmaz ÖZKAN for his generous provisions of facilities and to Prof.Dr.Kemal SARIOĞLU and to Associate Prof.Dr.Osman SEVAİOĞLU for their valuable suggestions towards the improvement of the problem.

I also want to express my sincere gratitude to Prof.İzzet GÖNENÇ and to Prof.Dr.Nesrin TARKAN for their special interest in my studies. I would like to express my appriciation to my collegues Özcan KALENDERLİ and Haspi İSMAİLOĞLU for their encouregement and various usefull suggestions while my studies.

Final thanks to my wife and childeren for their patient support.

CONTENTS

ABSTRACT

Guided by the experience that real power is more sensitive to nodal phase angles and reactive power to voltage magnitudes, a new formulation of power flow optimization is developed which exploits this decoupling and consequently reduces problem size, computer memory and solution time. The decomposition of the variables and the load flow equations results in the formation of two subproblems- one corresponding to the reactive power equations (Q-Optimization) and the other to the real power equations (P-Optimization). The two subproblems are alternatively solved until the desired accuracy is attained.

The method has considerable potantiel for real time applications and the solution of only one problem, if desired. Approximate models are preferred for both modules. The errors induced by ignored set of power flow equations, which are the major limitations common to all approximate models, are overcome by the proposed formulations.

Q-optimization subproblem is devoted to the minimization of the real power transmission losses (RPTL) and the improvement of voltage profiles by adjusting the control variables of the module, i.e. generator voltage magnitudes, VAR injection values of the shunt capacitor/reactors and tap values of the tap changing transformers. P-optimization module is used to minimize the hourly fuel costs (HFC) of the thermal generating units. It is also possible to eliminate an unfavourable increase in the RPTL throughout the latter module.

The proposed formulations are programed and the efficiency is tested by solving several test systems. The results are discussed and compared with those reported in the literature.

ÖZET

YAKLAŞIK AYRIŞTIRMA YÖNTEMİ İLE ETKİN VE TEPKİN GÜÇ OPTİMİZASYONU

Bir enerji sisteminde optimal yük akışı; sistem değişkenlerinin, güç akışını ve fiziksel kısıtlamaları sağlayacak ve belirli bir amaç ölçütünü en iyi kılacak şekilde belirlenmesidir.

1930'lu yıllarda üreteçlerin verimlilikleri oranında sırayla yüklenmeleri ile kabaca çözülmeye çalışılan bu problem, zamanla gelişen ve büyüyen enerji sistemlerinde, matematiksel olarak ele alınmış ve çeşitli çözüm yöntemleri geliştirilmiştir. Enerji iletim kayıpları gözardı edilerek ortaya atılan Eşit Birim Maliyet Yöntemi (Equal Incremental Cost Method) bu alandaki çalışmaların ilk ürünüdür. Enerji iletim kayıplarının farklı gösterilimlerine dayanan çalışmalar bugünkü B-Katsayıları yönteminin (B-Coefficients Method) ilk temellerini oluşturmaktadır.

1950'li yılların ortalarında yük akışı probleminin bilgisayarlarda çözülmesinin gündeme gelmesi, sistem eşitlik/eşitsizliklerinin ve optimizasyon işleminin birlikte ele alındığı, daha kapsamlı gösterilimlerin kurulmasına olanak sağlamıştır. 1968 yılında Dommel ve Tinney tarafından verilen matematiksel gösterilim bu yöndeki çalışmaların en önemlisi olarak bilinmekte ve günümüz yayınlarında temel referans olarak vurgulanmaktadır. Sözkonusu gösterilim;

$$f = f(x,u)$$

biçiminde verilen belirli bir amaç ölçütünü,

$$g(x,u) = 0$$

$$h(x,u) \geq 0$$

eşitlik ve eşitsizliklerini de sağlayacak şekilde, en iyi kılan kontrol değişkenlerinin belirlenmesi olarak tanımlanır. Burada **u** kontrol değişkenleri vektörünü, **x** durum değişkenleri vektörünü, **f** yakıt giderleri vb. bir amaç ölçütünü, **g** sistem eşitliklerini (yük akışı eşitlikleri) ve **h** da fiziksel kısıtlamaları göstermektedir.

Optimal kontrol teorisinden bilineceği üzere, optimal çalışma noktası

$$\mathscr{L}(x,u,\lambda) = f(x,u) + \lambda^T g(x,u)$$

genişletilmiş amaç ölçütünün tüm kontrol, durum ve λ Lagrange Çarpanlarına göre kısmi türevlerinin sıfıra eşitlenmesi ile oluşan

$$\frac{\partial \mathscr{L}}{\partial \lambda} = g(x,u) = 0$$

$$\frac{\partial \mathscr{L}}{\partial x} = \frac{\partial f}{\partial x} + \left[\frac{\partial g}{\partial x} \right]^T . \lambda = 0$$

$$\frac{\partial \mathscr{L}}{\partial u} = \frac{\partial f}{\partial u} + \left[\frac{\partial g}{\partial u} \right]^T . \lambda = 0$$

denklem takımının çözülmesi ile bulunur. h sistem eşitsizliklerinin sağlanması ise çözüm yöntemine göre değişir.

Problemin bu şekilde ortaya konmasından sonra, araştırmalar yukarıda verilen, lineer olmayan denklem takımının çözümü doğrultusunda yoğunlaşmış ve çeşitli sayısal yöntemler geliştirilmiştir. Bu arada, elektrik şebekesindeki büyüme sonucu, enerji iletim kayıplarının azaltılması ve bara gerilim genliklerinin kontrolu sorunu ortaya çıkmış ve bu amaçla tepkin (reaktif) güç akışından yararlanma olayı gündeme gelmiştir.

Geniş bir birleşik (enterkonnekte) şebekede optimizasyon işlemi oldukça karmaşık ve zaman alıcı bir olaydır. Bu nedenle 1970'li yılların sonlarında optimizasyonun, etkin (P) ve tepkin (Q) güç optimizasyonu şeklinde iki alt optimizasyon sürecine ayrıştırılması fikri ortaya atılmıştır. Böyle bir ayrıştırma, çözümü kolaylaştırması ve hızlandırmasına ek olarak, alt birimlerin farklı amaçlara uygun olarak modellenmesi kolaylığını da sağlamıştır.

Son yıllarda konuyla ilgili yapılan çalışmaların amacı daha hızlı sonuç veren ve daha az bilgisayar belleği gerektiren yöntemlerin araştırılması doğrultusunda yoğunlaşmaktadır. Bu çalışmaları;

a) Amaç ölçütünün ve/veya sistem eşitlik ve eşitsizliklerinin gösterilimi konusunda yapılan çalışmalar,
b) Sayısal yöntemlerin geliştirilmesi veya probleme uyarlanması konusunda yapılan çalışmalar

şeklinde iki ana gurupta toparlayabiliriz.

Bu çalışmada alt optimizasyon birimlerinin herbiri için birer farklı gösterilim önerilmiş, önerilen gösterilimler programlanmış ve çeşitli örnek sistemlere uygulanarak üstünlükleri araştırılmıştır. Çıkan sayısal sonuçlar irdelenmiş ve konuyla ilgili yayınlanan sonuçlarla karşılaştırılmıştır.

Daha hızlı sonuç alabilmek amacıyla gerek yük akışı ve gerekse diğer optimizasyon eşitliklerinde, fiziksel gerçeklerden yararlanarak, bazı yaklaşıklıklar yapılmıştır. Bu yaklaşıklıklar sonucunda, eşitliklerde sıkça karşılaşılan matrislerin simetrik olmaları sağlanmıştır.

Elektrik enerji sistemlerinde bir takım değişkenler arasındaki yakın ilişkiden (kuplaj) yararlanarak, sistem kontrol değişkenleri alt optimizasyon birimleri arasında paylaştırılmıştır. Yine bilinen etkin güç-bara gerilim açısı ($P-\delta$) ve tepkin güç-bara gerilim genliği ($Q-V$) ilişkisinden hareketle, alt birimlerde geçerli durum değişkenleri ve dolayısıyla da sistem eşitlikleri belirlenmiştir.

Tepkin güç optimizasyonu alt biriminde enerji iletim kayıplarının olası en küçük değere indirilmesi ve bara gerilim genliklerinin belirli sınırlar içerisinde tutulması amaçlanmıştır. Bu birim,

$$f_q = f_q(x_q, u_q)$$

biçiminde tanımlanan enerji iletim kayıplarını minimum yapan ve

$$g_q = g_q(x_q, u_q) = 0$$

eşitliklerini ve

$$h_q = h_q(x_q, u_q) \geq 0$$

eşitsizliklerini sağlayan u_q kontrol değişkenlerinin belirlenmesi olarak tanımlanır. Kontrol değişkenleri olarak üreteç bağlı baraların bara gerilim genlikleri, tepkin güç üreteçlerinin tepkin güçleri ve kademe değiştirici transformatörlerin kademe ayarları; durum değişkenleri olarak yük baralarının bara gerilim genlikleri ve üreteç tepkin güç çıkışları alınmıştır. g_q sistem eşitliklerinden h_q ise durum değişkenleri üzerindeki kısıtlamalardan oluşan vektörel eşitlik ve eşitsizliklerdir. Diğer optimizasyon birimine ilişkin kontrol değişkenleri ve durum değişkenleri sabit alınmış ve etkin güç akışı eşitlikleri dikkate alınmamıştır. Bu durum, salınım üreteci etkin gücünün amaç ölçütü olarak alınabilme olanağını sağlar. Diğer taraftan etkin güç eşitliklerinin dikkate alınmaması sonucu,

$$\Delta P = \frac{\partial P}{\partial u_q} \cdot \Delta u_q + \frac{\partial P}{\partial x_q} \cdot \Delta x_q$$

şeklinde bir artık terim oluşur. Bu artık terimlerin toplamı doğrudan etkin güç kaybını etkiler. Bu nedenle bu artık terimlerin yokedilmesi veya en küçük değere indirilmesi gerekir. Bu amaçla, etkin güç eşitliklerine ilişkin Lagrange Çarpanlarından sabit büyüklükler olarak yararlanılmıştır.

Etkin güç optimizasyonu alt biriminde,

$$f_p = f_p(x_p, u_p) = \sum_{i=1}^{NG} f_{p_i}(P_{G_i})$$

şeklinde NG adet üretecin etkin güçlerinin bir fonksiyonu olarak verilen ısıl ünitelerin saatlik yakıt giderlerini minimum yapan ve

$$g_p = g_p(x_p, u_p) = 0$$

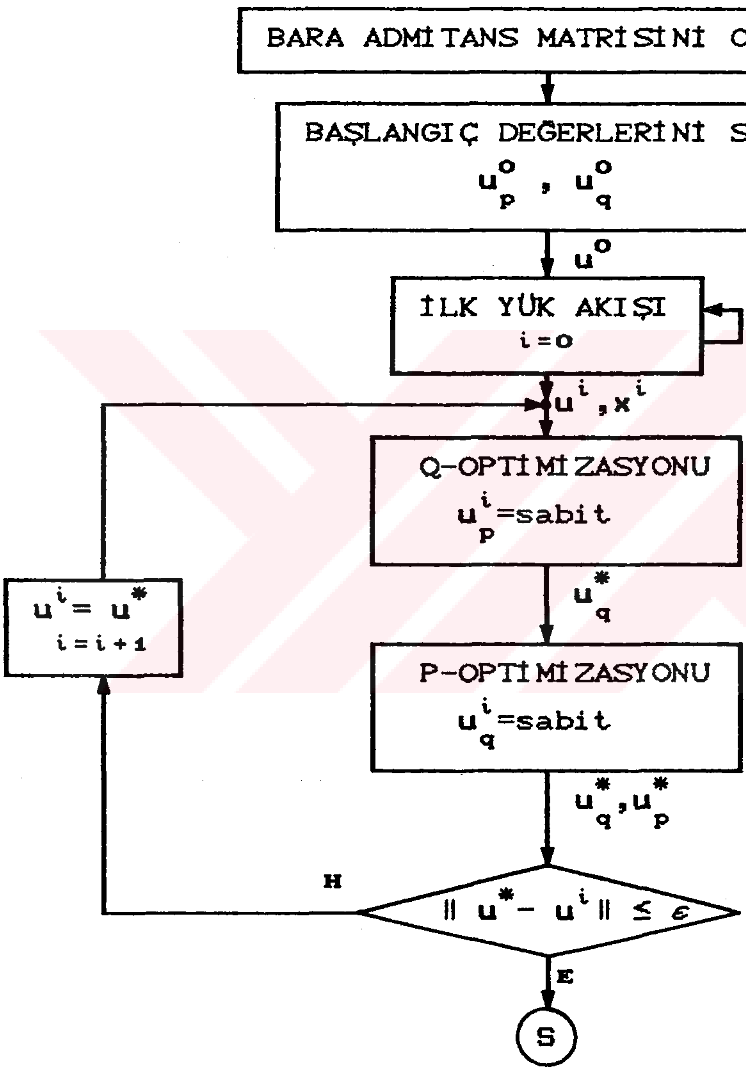
eşitliklerini ve

$$h_p = h_p(x_p, u_p) \geq 0$$

eşitsizliklerini sağlayan u_p kontrol değişkenleri hesaplanır. Kontrol değişkenleri olarak salınım üretecinin dışındaki üreteçlerin etkin güçleri; durum değişkenleri olarak da bara gerilim açıları alınmıştır. g_p sistem eşitlikleri olarak etkin güç akışları ve h_q eşitsizlikleri olarak da enerji iletim hatlarının yüklenme sınırları alınmıştır. Bu süreçte, diğer sürece ilişkin kontrol ve durum değişkenleri sabit alınmış ve tepkin güç eşitlikleri dikkate alınmamıştır. Buradaki en büyük sıkıntı enerji iletim kaybındaki belirsizliktir. Diğer birimde, yakıt giderleri de etkin güç kaybıyla birlikte bir azalma göstermesine karşın, bu birimde enerji iletim kabındaki değişim artma veya azalma yönünde olabilmektedir. Enerji kaybındaki bu artışa izin verilemeyecek bazı durumların olabileceği de dikkate alınarak, bu birim birden fazla seçeneğe olanak sağlayacak şekilde tasarlanmıştır. Bu amaçla toplam yakıt giderleri, salınım barası yakıt giderine bölünerek, normalize edilmiş birimler arasında

Lagrange Çarpanları açısından bir karşılılık sağlanmıştır. Hatta bu birimin, ek bir matematiksel işleme gerek kalmaksızın, kayıp optimizasyonu için de kullanılması olasıdır.

Optimizasyona ilişkin akış diyagramı Şekil 1'de verilmiştir.



CHAPTER I

INTRODUCTION

Optimal dispatch or economic dispatch is a computational process whereby the total generation required is allocated among the generating units available so that the constraints imposed are satisfied and the operating costs, system losses or some other appropriate objective functions are minimized. The idea of optimal dispatch had started from the time that two or more generating units were committed to take on load on a power system whose total capacities exceeded the generation required.

Prior to 1930 it was preferred to load the most efficient unit to its maximum capacity, than the second most efficient one and so on [1-3]. Afterwards Equal Incremental Cost Method was developed [2,3]. In this simplest method the sum of the hourly fuel costs (HFC) of the generating units was minimized under the equality constraint of meeting the total demand. That was a static, separable, nonlinear programming problem and the optimal loading corresponded to the equal incremental cost point off all the generators. The method is now only used for some special cases but it is still assigned to be the starting point of the problem [4-6].

The only variables considered in Equal Incremental Cost Method were the generator active powers, but the presence of the transmission network and so thus the real power transmission losses (RPTL) were ignored. It was, however, unrealistic to neglect the RPTL, particularly when long distance transmission of power was involved. This difficulty was overcome by augmenting the system constraints in such a way to include the system RPTL. However, in case the formulation of the RPTL as a function of generator real power outputs were required. It was done via B-coefficients, which is still widely used. The treatment of the RPTL was first introduced by Ward [7] and then extended by Kron [8-9] and Kirchmayer and Stag [10,11]. The following works aimed to improve the RPTL formulation [12,13].

The principle advantages of the foregoing methods are related to the simplicity of them. Memory requirements were low and programming was not difficult. However, many types of costraints were inconvenient to handle since the optimization process was carried out independently from the system equations (load flow calculations). In effect, the load flow equations must appear as equality costraints.

In the late 1950's the load flow problem made its appereance in digital computers [14]. The object of a load flow is to determine the voltages and angles at all appropriate buses of the network from which all other quantities can be calculated. An optimal load flow (OLF) is a load flow in which some objective functions are minimized, with the ordinary load flow costraints.

The general problem of OLF, subject to equality constraints, based upon the Kuhn-Tucker conditions of nonlinear programming (See Appendix A) was first formulated by Carpentier [15] and later extended by other pioneers [16-19]. His work was depended on the formation of the incremental loss terms or penalty factors from a converged load flow. The solution procedure employed for the load calculations was Gauss-Seidel Method.

The method introduced by Dommel and Tinney, which was based on load flow solutions by Newton-Raphson method [19] can be ranked as one of the most important one that has so far been developed. A number of approaches to the solution of the nonlinear programming problem introduced by them were investigated by mathematicians [20-23] and these techniques together with the improved load flow solution algorithms [24-30] were applied towards the OLF problem [31-35].

In addition, it was recognized that the redistribution of reactive power is necessary to improve the system voltage profiles and to minimize the RPTL [36-43]. Due to the size and the complexity of the problem, recent developments propose the decomposition of the optimization process into active (P) and reactive (Q) power sub-optimization problems [44-49]. This decomposition simplifies the formulation, improves the computation time and permits a certain flexibility in the types of calculations desired.

Following the decomposition idea, much effort has been devoted to the improvement of the convergence and the reduction of the computation time and the computer storage requirements. Further studies were greatly made use of the developments in numerical techniques in optimization and their applications in power system studies [50-59]. A number of articles devoted to the Q-optimization problem were based on lineer programming techniques [60-68]. Various formulations based on linear programming or gradient type nonlinear programming methods were introduced for P-optimization problem [69-75]. Improved second order methods, based on Hessian Matrix approximation, were developed to handle very large scale problems [76-83].

The security of power system operations is an additional and an important element that has arisen in the last decade and has had a profound influence especially on P-optimization problems. The problem was included in the optimization process by recent pioneers [84-88].

The foregoing formulations of the OLF assume that the data provided are absolutely precise. However, the information available is somehow stochastic in nature. In addition, in case of hydro-termal systems the water availability constraint for hydro-generation plants must also taken into account. Optimal stochastic load flows and optimal operation of hydro-termal systems were discussed in recent papers [89-92].

Load flow optimization methods can be classified in one of two categories, one providing exact solutions and the other approximate solutions. Exact methods take into account both the real and the reactive power flows in obtaining the solution. Approximate methods achieve simplified representations by ignoring either the real or the reactive power flows. Generally, the latter ones are preferred to handle the large scale problems.

In this thesis, two new formulations are introduced for sub-optimization modules. Approximate optimization methods, which are more adequate for sparse matrix implementitations, are preferred. As in other recent methods, the problem is decomposed into a Q-optimization and a P-optimization modules. The modules are formulated in such a way to make use of symmetric, sparse and diagonal dominant Jacobian matrices, obtained during the load flow calculations.

Q-Optimization module is used both for the minimization of the RPTL and the improvement of voltage profiles by adjusting associated control variables. In addition to conventional fixed control and state variable assumption, the Lagrangian variables of the adjacent module are also held fixed. This assumption mades it possible to compensate the residual term for active power mismatch equations which are ignored throughout the module. Moreover, improvement of voltage profiles is accepted as a prerequisite condition and the module is so arranged to keep the load bus voltage magnitudes within the specified limits, before minimizing the RPTL.

P-Optimization module is used to minimize the HFC of thermal generating units by adjusting associated control variables. A flexibility is provided depending upon the change in the RPTL. Even, it is also possible to reconstruct the module for the loss minimization, if desired. The cost function is normalized to achieve a correspondence between the Lagrangian variables of the modules.

The proposed method is programmed and its efficiency is tested by solving several IEEE test systems. The method is capable of handling both the minimum cost and the minimum loss problems. Each sub-optimization module is small and simple. Modelling concepts and the procedures are the same. Each sub-problem can be solved independently with the control variables of the other sub-problem held fixed. This method requires less computing time and the core storage than the exact methods. Convergence was obtained in a few iterations for all test systems.

The second chapter begins with the statement of the general optimal power dispatch problem, similar to that was introduced by Dommel and Tinney. The disadvantages of the method and so thus the necessity of decomposition is introduced and the fundamentals of decomposition are given. The third chapter is devoted to load flow calculations. Load flow equations and the solution of non-linear equations via sparse matrix methods are introduced. Some approximations on the load flow equations and their effects on the convergence characteristics are discussed. The proposed formulations for the sub-modules are introduced in the fourth chapter. Finally, Chapter V is devoted to the test results for several IEEE test systems.

CHAPTER II

DECOMPOSED OPTIMAL POWER DISPATCH

2.1. General Formulation

The first exact solution method developed by Dommel and Tinney [19] can be summerized as; minimize a scalar objective function f,

$$f = f(x,u) \tag{2.1}$$

subject to the equality constraints of the load flow equations,

$$g(x,u) = 0 \tag{2.2}$$

and the inequality constraints of the physical restrictions,

$$h(x,u) \geq 0 \tag{2.3}$$

where u, x and 0 denote the control, state and the zero vectors, respectively.

The set of necessary conditions for the minimum can be obtained by differentiating the augmented Lagrangian function,

$$\mathcal{L}\,(x,u,\lambda) = f(x,u) + \lambda^T.g(x,u) \tag{2.4}$$

with respect to all control, state and λ slack variables.

$$\frac{\partial \mathscr{L}}{\partial \lambda} = g(x,u) = 0 \qquad (2,5,a)$$

$$\frac{\partial \mathscr{L}}{\partial x} = \frac{\partial f}{\partial x} + \left[\frac{\partial g}{\partial x} \right]^T . \lambda = 0 \qquad (2.5,b)$$

$$\frac{\partial \mathscr{L}}{\partial u} = \frac{\partial f}{\partial u} + \left[\frac{\partial g}{\partial u} \right]^T . \lambda = 0 \qquad (2.5,c)$$

The handling of the inequality constraints differs due to the optimization methods used. Many existing methods of nonlinear programming fall roughly into two main categories from this point of view. Methods of Feasible Directions requires a starting point satisfying the constraints and finding a direction such that a small move in that direction violates no constraints but improves the objective function [23,50,51]. The Penalty Function Approach changes a constrained optimization problem into an unconstrained problem by adding a penalty term to the objective function, which penalizes the solution when the constraints are violated [23,50,51].

Optimal values are determined by the solution of Eqs. (2.5,a), (2.5,b) and (2.5,c). The solution methods applied can be classified as linear and nonlinear programming (solution) methods. The linear programming methods employ both the linearized objective function and the system equations. In addition, some decomposition techniques developed for large scale systems, can also be applied [50-52]. Two main types of nonlinear programming methods are widely used. In gradient type methods, updated control variables are determined via Eq. (2.5,c); where the Newton type methods employ the second order derivatives [50-52,76,77]. In this thesis, gradient type methods, together with the penalty function techniques, have been applied.

The computer time is a nonlinear function of the unknown variables and any increase in these variables results in an increase in computational effort. The complete optimization problem also requires a large computer memory. Decomposed optimal power dispatch methods are therefore more desirable for large scale power system optimizations. On the other hand decomposition permits a certain flexibility in the types of objective functions desired. That is, sub-optimization modules can be formulated for different types of objective functions.

2.2. Decomposition

Power system optimization requires both the optimization of the HFC and the RPTL. The choice of the objective function depends on the application of the test results, generation capacity and the desired impact on the system model. However, it has been a general custom of formulating the reactive power dispatch (Q-dispatch) and the active power dispatch (P-dispatch) sub-optimization modules to minimize the RPTL and the HFC, respectively. The objective functions and the associated control variables change for the sub-optimization modules. Moreover, the system equations and the state variables also change in case of approximate solution methods.

2.2.1. Reactive Power Dispatch

Reactive power dispatch is used to improve the voltage profiles and to reduce the RPTL. Thus, it has a twofold objective function. The control variables of active power dispatch, i.e. real power generations except the swing bus and phase shifter angles are assumed to be constant. Mathematical statement of the module can be given as;

$$\text{Minimize} \quad f_q = f_q(x,u_q) \tag{2.6}$$

subject to equality constraints of Eq. (2.2) and the inequality constraints of,

$$h_q(x,u_q) \geq 0 \tag{2.7}$$

Where f_q denotes the objective function, i.e. the RPTL and u_q denotes the control variables, i.e. generator voltage magnitudes, reactive power generations of VAR sources and the tap ratios of the tap changing transformers. The state vector x includes the bus voltage magnitudes of load buses and the bus voltage angles of all buses. Inequality constraint set h_q is formed due to specified voltage magnitudes, tap changing transformer taps and VAR source limits.

The RPTL is an explicit function of every voltage magnitude, angle and transformer tap ratio in the system. In order to reduce the computation time, the objective function can be taken as the real power generation of the swing bus, since the real power generations except the swing bus are assumed to be constant.

On the other hand, the improvement of voltage profiles may sometimes retards the reduction of the RPTL, especially in large power systems. In case, there must be made some modifications on f_q and h_q. In addition elimination of the real load flow equations and so thus the terms associated to bus voltage angles is also possible. These two important points will be discussed in Chapter IV.

2.2.2. Active Power Dispatch

This module is used to minimize the HFC where the control variables of the reactive power dispatch, u_q, are assumed to be constant. Mathematical statement of the problem can be given as; minimize,

$$f_p = f_p(x,u_p) = \sum_{i=1}^{NG} f_i(P_{Gi}) \tag{2.8}$$

NG: Number of Generators

subject to equality constraints of Eq. (2.2) and the inequality constraints of,

$$h_p(x,u_p) \geq 0 \tag{2.9}$$

Where f_p denotes the HFC which is the function of real power generations of individual buses (P_{Gi}) and u_p denotes the control variables, i.e. real power generations except the swing bus and the phase shifter angles. The state vector x is the same as in the reactive power dispatch. Inequality constraints include the real power generation and transmission limits.

2.2.3. Optimization Procedure

The two sub-problems are solved alternately until the desired accuracy is attained. The overall procedure for solving the decomposed optimization problem is illustrated in Figure 2.1.

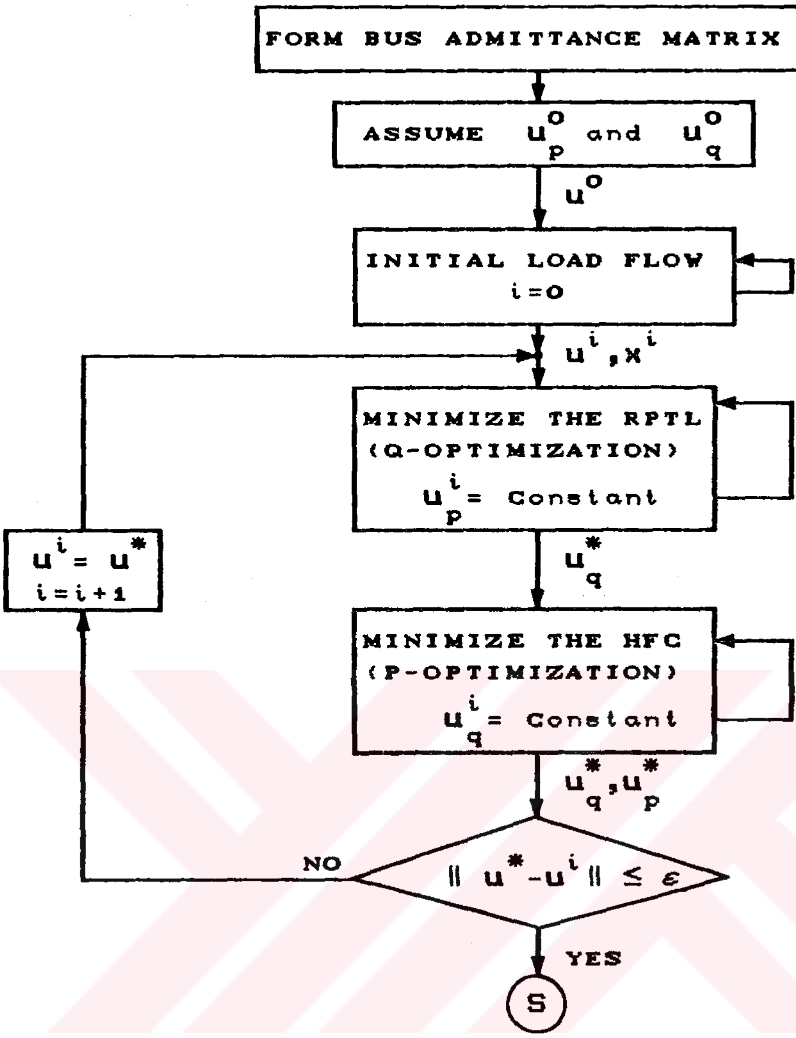


Figure 2.1 The Flow Chart of Decomposed Optimal Power Flow.

2.3. Disadvantages of Exact Decomposed Optimal Power Dispatch

Since the two sub-optimization modules employ the same state vector, both the active and reactive load flow equations must appear in $g(.,.)$. Then, a doubt about the advantages of such a decomposition arises. Although, it does provide a flexibility in the types of objective functions, it does not yield a significant decrease in computer time and memory.

The classification of the state variables due to the sub-optimization processes provides the desired decrease in computer time and memory. In case, real and reactive load flow equations can be omitted from reactive and real power dispatch sub-optimization modules, respectively. This formulation is called as approximate decomposed optimal power dispatch, which brings some additional problems to be taken into account. The details will be discussed in Chapter VI.

As it was mentioned, large scale power system optimization requires some additional efforts. The approximate formulations must satisfy the following requirements.

a) It must be appliciable for several kind of systems.
b) It must not destroy the sparse nature of the resulting matrix equations.
c) The resulting matrices must be symmetric or, if not, incidence symmetric as far as possible.

CHAPTER III

LOAD FLOW

The load flow calculations are the most time consuming process in an overall optimization procedure. Therefore, the module must be very well organized to reduce the computation time.

3.1. Load Flow Equations

The load flow in a system of NB buses (nodes) comprises NB complex equations:

$$\bar{S}^{*}_{NET_i} = \bar{V}^{*}_{i} \cdot \sum_{k=1}^{NB} \bar{Y}_{ik} \cdot \bar{V}_{k} \qquad i=1,2,..NB \tag{3.1}$$

Where $\bar{V}_i$ is the voltage phasor at bus i, $\bar{Y}_{ik} = G_{ik} + jB_{ik}$ is the element of the complex bus admittance matrix and $\bar{S}_{NET_i}$ is the net injected complex power at bus i. The superscript '*' denotes the conjugate of a complex quantity and '–' denotes a complex quantity. Equation (3.1) can be given in a matrix form,

$$\bar{S}^{*}_{NET} = \left[\text{Diag } (\bar{V}^{*}) \right] . \bar{Y} . \bar{V} \tag{3.2}$$

where,

$$\bar{V}^{T} = [\bar{V}_1 \ \bar{V}_2 \ .... \ \bar{V}_{NB}] \tag{3.3}$$

and the superscript 'T' denotes the transpose of a matrix.

Using the notation $P_{NET_i}-jQ_{NET_i}$ for the left hand side of Equation (3.1) and separating the real and imaginary parts;

$$P_{NET_i} - \mathrm{Re}\left\{ \bar{V}_i^* \cdot \sum_{k=1}^{NB} \bar{Y}_{ik} \cdot \bar{V}_k \right\} = 0 \qquad (3.4,a)$$

$$Q_{NET_i} + \mathrm{Im}\left\{ \bar{V}_i^* \cdot \sum_{k=1}^{NB} \bar{Y}_{ik} \cdot \bar{V}_k \right\} = 0 \qquad (3.4,b)$$

$$i=1,2,\ldots,NB$$

By introducing the voltage magnitudes (V_i) and voltage angles (δ_i) instead of voltage phasors;

$$P_{NET_i} - \mathrm{Re}\left\{ V_i \cdot e^{-j\delta_i} \cdot \sum_{k=1}^{NB} (G_{ik}+jB_{ik}) \cdot V_k \cdot e^{j\delta k} \right\} = 0 \qquad (3.5,a)$$

$$Q_{NET_i} + \mathrm{Im}\left\{ V_i \cdot e^{-j\delta_i} \cdot \sum_{k=1}^{NB} (G_{ik}+jB_{ik}) \cdot V_k \cdot e^{j\delta k} \right\} = 0 \qquad (3.5,b)$$

$$i=1,2,\ldots,NB$$

or in vector notation,

$$P_{NET}-P(V,\delta) = 0 \qquad (3.6,a)$$

$$Q_{NET}-Q(V,\delta) = 0 \qquad (3.6,b)$$

Each bus is characterized by four variables; P_{NET_i}, Q_{NET_i}, V_i and δ_i of which two are specified and the other two must be calculated. Depending upon the specified variables, buses can be divided into three groups:

1. Slack (Swing) bus with V_i and δ_i are specified and P_{NET_i} and Q_{NET_i} are unknown. Generally δ_i is assumed to be zero as reference. This bus will be numbered as Bus 1 throughout the thesis.
2. PV-buses with P_{NET_i} and V_i are specified and Q_{NET_i} and δ_i are unknown. Those are mostly generation buses at which the injected real power is held fixed by turbine settings. The number of PV-buses are assigned as NV (including the swing bus) of which NG of them are generator buses.
3. PQ-buses wih P_{NET_i} and Q_{NET_i} are specified and V_i and δ_i are unknown.

3.2. Load Flow Calculations

The AC load flow calculations can be done by several methods. However, Newton-Raphson (N-R) Method (also known simply as Newton's Method) is the most efficient one for the optimization procedure.

Due to the classifications of the preceeding section, among the 2NB equations given by Eqs. (3.5,a) and (3.5,b), (2NB-NV-1) of them, are primarly need to be solved. However, the VAR generation of the shunt capacitors are assumed to be fixed during the load flow calculations and are determined throughout the optimization process. Therefore, all buses except the generator ones can be treated as PQ-buses. Thus, equations those yielding fixed power injections, i.e.,

$$P_{NET_i} - P_i(V,\delta) = 0 \qquad i=2,3,\ldots,NB \qquad (3.7,a)$$

$$Q_{NET_i} - Q_i(V,\delta) = 0 \qquad i=(NG+1),\ldots,NB \qquad (3.7,b)$$

comprise the required (2NB-NG-1) equations for the load flow calculations.

If we introduce control and state variables/vectors instead of specified and unspecified variables/vectors;

$$g_p(u,x) = P_{NET} - P(u,x) = 0 \qquad (3.8,a)$$

$$g_q(u,x) = Q_{NET} - Q(u,x) = 0 \qquad (3.8,b)$$

$$[g(u,x)]^T = [g_p^T(u,x) \quad g_q^T(u,x)] \qquad (3.9)$$

where,

$$u^T = [P_{G_2} \; P_{G_3} \ldots P_{G_{NG}} \; V_1 \; V_2 \ldots V_{NG} \quad Q_{NG+1} \ldots Q_{NV}]$$

$$= [P_G^T \quad V_G^T \quad Q_C^T] \qquad (3.10)$$

$$x^T = [\delta_2 \; \delta_3 \ldots \delta_{NB} \; V_{NG+1} \ldots V_{NB}] = [\delta^T \quad V_L^T] \qquad (3.11)$$

For a given set of control vector u, the state variables are calculated iteratively by using the Newton-Raphson algorithm as,

$$x^{k+1} = x^{k} - \left[J(u,x^{k}) \right]^{-1} \cdot g(u,x^{k}) \qquad (3.12)$$

or in a decomposed form,

$$\begin{bmatrix} \delta \\ V_L \end{bmatrix}^{k+1} = \begin{bmatrix} \delta \\ V_L \end{bmatrix}^{k} - \left[J(u,x^{k}) \right]^{-1} \cdot \begin{bmatrix} g_p(u,x^{k}) \\ g_q(u,x^{k}) \end{bmatrix} \qquad (3.13)$$

where $J(u,x^{k})$ is the sparse and diagonal dominant Jacobian matrix of g_p and g_q, evaluated at iteration k and represents the slopes of the tangent hyperplanes.

$$J = \begin{bmatrix} \dfrac{\partial g_p}{\partial \delta} & \dfrac{\partial g_p}{\partial V_L} \\ \dfrac{\partial g_q}{\partial \delta} & \dfrac{\partial g_q}{\partial V_L} \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \qquad (3.14)$$

The required equations and partial derivatives are [6] ;

$$g_{p_i} = P_{NET_i} - \sum_{j=1}^{NB} V_i V_j (G_{ij} \cos \delta_{ji} - B_{ij} \sin \delta_{ji}) \qquad (3.15,a)$$

$$i = 2, 3, \ldots, NB$$

$$g_{q_i} = Q_{NET_i} + \sum_{j=1}^{NB} V_i V_j (G_{ij} \sin \delta_{ji} + B_{ij} \cos \delta_{ji}) \qquad (3.15,b)$$

$$i = (NG+1), \ldots, NB$$

$$J_{1_{ij}} = \frac{\partial g_{p_i}}{\partial \delta_j} = V_i V_j (G_{ij} \mathrm{Sin}\ \delta_{ji} + B_{ij} \mathrm{Cos}\ \delta_{ji}) \qquad (3.16,a)$$

$$J_{1_{ii}} = \frac{\partial g_{p_i}}{\partial \delta_i} = Q_i + V_i^2\ B_{ii} \qquad (3.16,b)$$

$i,j=2,3,\ldots,NB$

$$J_{2_{ij}} = \frac{\partial g_{p_i}}{\partial V_j} = -V_i (G_{ij} \mathrm{Cos}\ \delta_{ji} - B_{ij} \mathrm{Sin}\ \delta_{ji}) \qquad (3.17,a)$$

$$J_{2_{ii}} = \frac{\partial g_{p_i}}{\partial V_i} = -\ P_i/V_i - V_i G_{ii} \qquad (3.17,b)$$

$i=2,3,\ldots,NB$
$j=(NG+1),\ldots,NB$

$$J_{3_{ij}} = \frac{\partial g_{q_i}}{\partial \delta_j} = V_i V_j (G_{ij} \mathrm{Cos}\ \delta_{ji} - B_{ij} \mathrm{Sin}\ \delta_{ji}) \qquad (3.18,a)$$

$$J_{3_{ii}} = \frac{\partial g_{q_i}}{\partial \delta_i} = -\ P_i + V_i^2\ G_{ii} \qquad (3.18,b)$$

$i=(NG+1),\ldots,NB$
$j=2,3,\ldots,NB$

$$J_{4_{ij}} = \frac{\partial g_{q_i}}{\partial V_j} = V_i (G_{ij} \mathrm{Sin}\ \delta_{ji} + B_{ij} \mathrm{Cos}\ \delta_{ji}) \qquad (3.19,a)$$

$$J_{4_{ii}} = \frac{\partial g_{q_i}}{\partial V_i} = -\ Q_i/V_i + V_i\ B_{ii} \qquad (3.19,b)$$

$i,j= (NG+1),\ldots,NB$

where,

$$\text{Cos } \delta_{ji} = \text{Cos } (\delta_j - \delta_i)$$

$$\text{Sin } \delta_{ji} = \text{Sin } (\delta_j - \delta_i)$$

The iteration process given by Eq. (3.12) or Eq.(3.13) can be converted to a set of simultaneous linear equations as;

$$J(u,x^k) \cdot \left[x^{k+1} - x^k \right] = -g(u,x^k) \tag{3.20}$$

or simply;

$$Ax = b \tag{3.21}$$

3.3. Solution of Sparse Simultaneous Linear Equations

There are many methods which can be used for solving a set of simultaneous equations. Among them, optimally ordered triangular bi-factorization method is chosen. The method is particularly suitable for sparse coefficient matrices that have dominant and non-zero diagonal elements and that are either symmetric, or, if not symmetrical, have a symmetrical sparsity nature.

The method is based on finding 2n factor matrices for n^{th} order problem, such that the product of these factor matrices satisfies the requirement [25],

$$L_n \cdot L_{n-1} \cdot \ldots \cdot L_2 \cdot L_1 \cdot A \cdot R_1 \cdot R_2 \cdot \ldots \cdot R_n = I \tag{3.22}$$

where A is the original coefficient matrix, L_i is the i^{th} left-hand factor matrix, R_i is the i^{th} right-hand factor matrix and I is the unit matrix of order n.

Applying a sequence of pre-multiplying and post-multiplying processes to Eq. (3.22) gives,

$$A.R_1.R_2....R_n.L_n.L_{n-1}...L_2.L_1 = I \tag{3.23}$$

and hence,

$$R_1.R_2....R_n.L_n.L_{n-1}...L_2.L_1 = A^{-1} \tag{3.24}$$

$$x = R_1.R_2....R_n.L_n.L_{n-1}...L_2.L_1.b \tag{3.25}$$

The factor matrices have the form of;

$$L_k = \begin{bmatrix} 1 & 0 & 0 \ldots & 0 & 0 \ldots & 0 \\ 0 & 1 & 0 \ldots & 0 & 0 \ldots & 0 \\ \vdots & & & & & \\ 0 & 0 & 0 \ldots & L_{kk} & 0 \ldots & 0 \\ 0 & 0 & 0 \ldots & L_{lk} & 1 \ldots & 0 \\ \vdots & & & \vdots & & \\ 0 & 0 & 0 \ldots & L_{nk} & 0 \ldots & 1 \end{bmatrix}_{l=k+1} \tag{3.26,a}$$

$$R_k = \begin{bmatrix} 1 & 0 & 0 \ldots & 0 & 0 & 0 \ldots & 0 \\ 0 & 1 & 0 \ldots & 0 & 0 & 0 \ldots & 0 \\ \vdots & & & & & & \\ 0 & 0 & 0 \ldots & 1 & R_{kl} & R_{km} \ldots & R_{kn} \\ 0 & 0 & 0 \ldots & 0 & 1 & 0 \ldots & 0 \\ \vdots & & & & & & \\ 0 & 0 & 0 \ldots & 0 & 0 & 0 \ldots & 1 \end{bmatrix} \tag{3.26,b}$$

$l=k+1$
$m=k+2$

Elements of the factor matrices are computed throughout the Gauss Elimination process [24-28]. In case of symmetrical coefficient matrices,

$$R_{ij} = L_{ji} \tag{3.27}$$

Therefore, the required number of operations and the amount of storage space is reduced to almost a half.

The factor matrices can posses a high degree of sparsity, if the original coefficient matrix A is sparse. But, bi-factorization brings new non-zero elements, known as fill-in. Bi-factorization can be used to exploit sparsity in two ways. Firstly, by using them in junction with a suitable ordering technique, the number of fill-in produced during bi-factorization can be minimized. The amount of fill-in greatly depends on the order in which the pivotal rows (or coulomns) are selected. A dynamic ordering technique called Least Number of Connected Branches is preferred. Secondly, the method needs only the non-zero elements to be stored and processed.

Because only the non-zero elements of the original and reduced coefficient matrices are stored and processed, they are not clearly arranged in an array that is visually identical to the corresponding matrix. Instead, they are stored in, what is generally called, a compact form. It is therefore necessary to locate each of these non-zero elements during the bi-factorization process and identify it, as a particular element of the corresponding coefficient matrix. This requires indexing information, so that the relative position of each non-zero elements is established in addition to the numerical value of it.

Linked List Storage Technique [30-34] is an extremely useful scheme for solving and manipulating large sparse matrices. The storage scheme consists of two tables, assuming that the operations are performed on a column-by-column basis. The first table comprises three arrays each having a dimension of NE, where NE is the total number of non-zero elements of the coefficient matrix including the fill-in ones. The first two arrays show the value and the row of the non-zero off-diagonal element of the coefficient matrix; where the third one shows the location of the next non-zero off-diagonal element in the column. The second table also comprises three arrays, but each having a dimension of n. The arrays show the value of the diagonal elements, the location of the first non-zero elements of the column and the number of non-zero off-diagonal elements of the column, respectively.

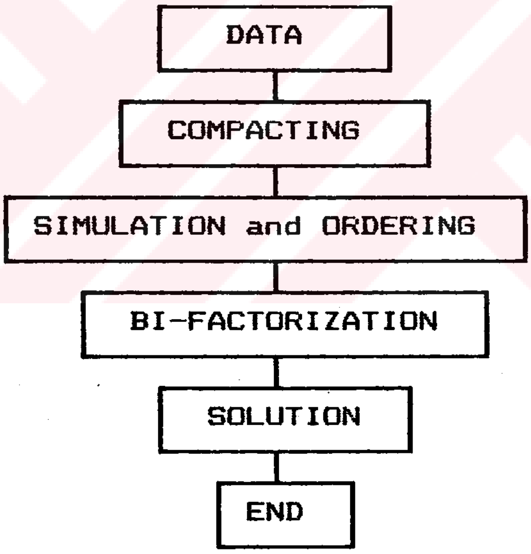


Figure 3.1 Program Flow-chart of the solution of sparse, simultaneous linear equations.

The flow chart of the program used for solving sparse, simultaneous linear equations is illustrated in Figure 3.1.

Compacting involves the generation of the non-zero elements of the coefficient matrix from the basic information of the network and storing them in a compact form. Simulation and ordering is a logic module and establishes the order of elimination. This simulation process can make the extra time spent in ordering, but it will remain unchanged for any problem having the same sparsity structure. Bi-factorization factorizes the original coefficient matrix using the information of the simulation and ordering module. Numerical solution is the final process of the calculations where the set of linear equations $Ax = b$, is solved via Eq. (3.25).

3.4. Convergence Characteristics and Initialization of Load Flow Calculations

Let us consider the Newton-Raphson updating equation (3.12) and write it as

$$\Delta x^k = -J^{-1} g^k \tag{3.28}$$

Let the eigenvalues of the J^{-1} be $\lambda_1, \lambda_2, \ldots, \lambda_n$ and let the corresponding eigenvectors be $e_1, e_2, \ldots, e_n$. Further, let the modal matrix, M, be defined as,

$$M = [e_1 \; e_2 \; \ldots. \; e_n] \tag{3.29}$$

Let us consider the case of linearly independent eigenvectors. Since the e_i's span Euclidian n-space (R^n), both Δx^k and g^k can be expressed as linear combinations of e_i, i.e.,

$$\Delta x^k = \sum_{i=1}^{n} b_i e_i \qquad (3.30,a)$$

$$g^k = \sum_{i=1}^{n} c_i e_i \qquad (3.30,b)$$

Let B denote the n-vector of scalar elements b_i and let C denote the n-vector of scalar elements c_i,

$$\Delta x^k = MB \qquad (3.31,a)$$

$$g^k = MC \qquad (3,31,b)$$

Then Eq.(3.28) can be written as

$$MB = -J^{-1}M\ C \qquad (3.32)$$

Hence,

$$B = -(M^{-1}J^{-1}M\ )\ C \qquad (3.33)$$

Since M is the modal matrix of J^{-1}, the term $M^{-1}J^{-1}M$ is a similarity transformation on J^{-1} which diagonalizes J^{-1}, i.e.,

$$B = -\ \Lambda C \qquad (3.34)$$

where,

$$\Lambda = \text{diag}\ (\lambda_i) \qquad i=1,2,\ldots,n \qquad (3.35)$$

The magnitude of B may be thought of as the length of the correction term and the magnitude of C as the length of the residuals at iteration k. Note thet if M is orthonormal,

$$\| \Delta x^k \|^2 = [\Delta x^k]^T [\Delta x^k] = B^T M^T M\ B = B^T B = \| B \|^2 \qquad (3.36)$$

Even if M is not orthonormal, it can be shown that $\| B \|$ is a measure of the norm of Δx^k [6]. Hence Eq.(3.34) states that the correction term is related to the residual term by the eigenvalues of J^{-1}.

The largest eigenvalue of J^{-1} degrade and control the convergence at each iteration. There are two main tests concerning the convergence of the N-R method. One of them states that the sufficient condition for

$$|\lambda_i|_{max} \leq 1 \qquad (3.37)$$

and the other (Newton-Kantorowitch Criterion) is based on the starting values of x [6]. Unfortunately, these tests have been found to be very conservative and difficult to apply in practice. For these reasons, the practical views of convergence of a load flow study are generally relegated to the realm of numerical calculation. That is, if convergence is obtained from the numerical implemention, there is little motivation to pursue the question further.

The simplest and perhaps the most common initialization for a N-R load flow study is the flat start ,i.e., $\delta=0$,$V_L=1.0$ p.u. An alternative initialization is based on the simple approximations on Eqs. (3.15) and (3.16). If the presence of the real parts of the bus admittance matrix is ignored, i.e., $G = 0$, and if we assume that the bus voltage angles are small enough to satisfy,

$$\mathrm{Sin}\ \delta_{ji} \cong \delta_j - \delta_i$$

$$\mathrm{Cos}\ \delta_{ji} \cong 1$$

the resulting equations and the elements of the Jacobian matrix become,

$$g_p = P_{NET} + B_1 \delta = 0 \qquad (3.38,a)$$

$$g_q = Q_{NET} + B_2 \delta = 0 \qquad (3.38,b)$$

$$J_1 = B_1 \qquad (3.39,a)$$

$$J_2 = J_3^T = 0 \qquad (3.39,b)$$

$$J_4 = B_2 \qquad (3.39,c)$$

where B_1 and B_2 are the appropriate submatrices of the imaginary part of the bus admittance matrix $\bar{Y}$.

Newton-Raphson algorithm has quadratic convergence near the solution; but, far from the solution the convergence may be eratic. This initialization is generally applied to force the variables near the solution region.

3.5. Approximate Load Flow Formulations

There are numerous modifications of the N-R formulation used for the load flow studies. Because of the dimensionality of typical load flow studies, even small improvements have potentially significant impact on central processing time and memory requirements.

The decoupled N-R load flow exploits the close dependence between active power-bus voltage phase angles (P-δ) and reactive power-bus voltage magnitudes (Q-V). This dominance makes it available the approximation, in which J_2 and J_3 submatrices are assumed small enough to be ignored. Thus the resulting iteration equations take the form of,

$$\begin{bmatrix} J_1 & 0 \\ 0 & J_4 \end{bmatrix}^k \begin{bmatrix} \Delta\delta \\ \Delta V_L \end{bmatrix}^k = -\begin{bmatrix} g_p(u,x^k) \\ g_q(u,x^k) \end{bmatrix} \tag{3.40}$$

or

$$J_1^k \, \Delta\delta^k = - g_p(u,x^k) \tag{3.41,a}$$

$$J_4^k \, \Delta V_L = - g_q(u,x^k) \tag{3.41,b}$$

J_1^k and J_4^k are diagonal dominant, sparse and incidence symmetric matrices evaluated at iteration k. This decoupling provides a considerable savings in time requirement per iteration.

On the other hand, a few additional approximations are also needed to achieve a symmetrical structure for the prescribed matrices. If the partial derivatives with respect to bus voltage magnitudes are normalized; that is, if $\partial V_{L_i}/V_{L_i}$ is used instead of ∂V_{L_i}, then,

$$J_{4_{ij}} = J_{4_{ji}} = \frac{\partial g_{q_i}}{\partial V_j} V_j = \frac{\partial g_{q_j}}{\partial V_i} V_i = V_i V_j (G_{ij} \mathrm{Sin}\ \delta_{ji} + B_{ij} \mathrm{Cos}\ \delta_{ji}) \quad (3.42)$$

Moreover, in practical power systems the following assumptions are also valid.

$$G_{ij} \mathrm{Sin}\ \delta_{ji} \ll B_{ij} \mathrm{Cos}\ \delta_{ji} \quad (3.43,a)$$

$$Q_i \ll B_{ii} V_i^2 \quad (3.43,b)$$

Final form of the elements of J_1 and J_4 are,

$$J_{1_{ij}} = J_{1_{ji}} \cong V_i V_j B_{ij} \mathrm{Cos}\ \delta_{ji} \quad (3.44,a)$$

$$J_{1_{ii}} \cong V_i^2\ B_{ii} \quad (3.44,b)$$

$i,j=2,3,\ldots,NB$

$$J_{4_{ij}} = J_{4_{ji}} = J_{1_{ij}} \quad (3.44,c)$$

$$J_{4_{ii}} = J_{1_{ii}} \quad (3.44,d)$$

$i,j=(NG+1),\ldots,NB$

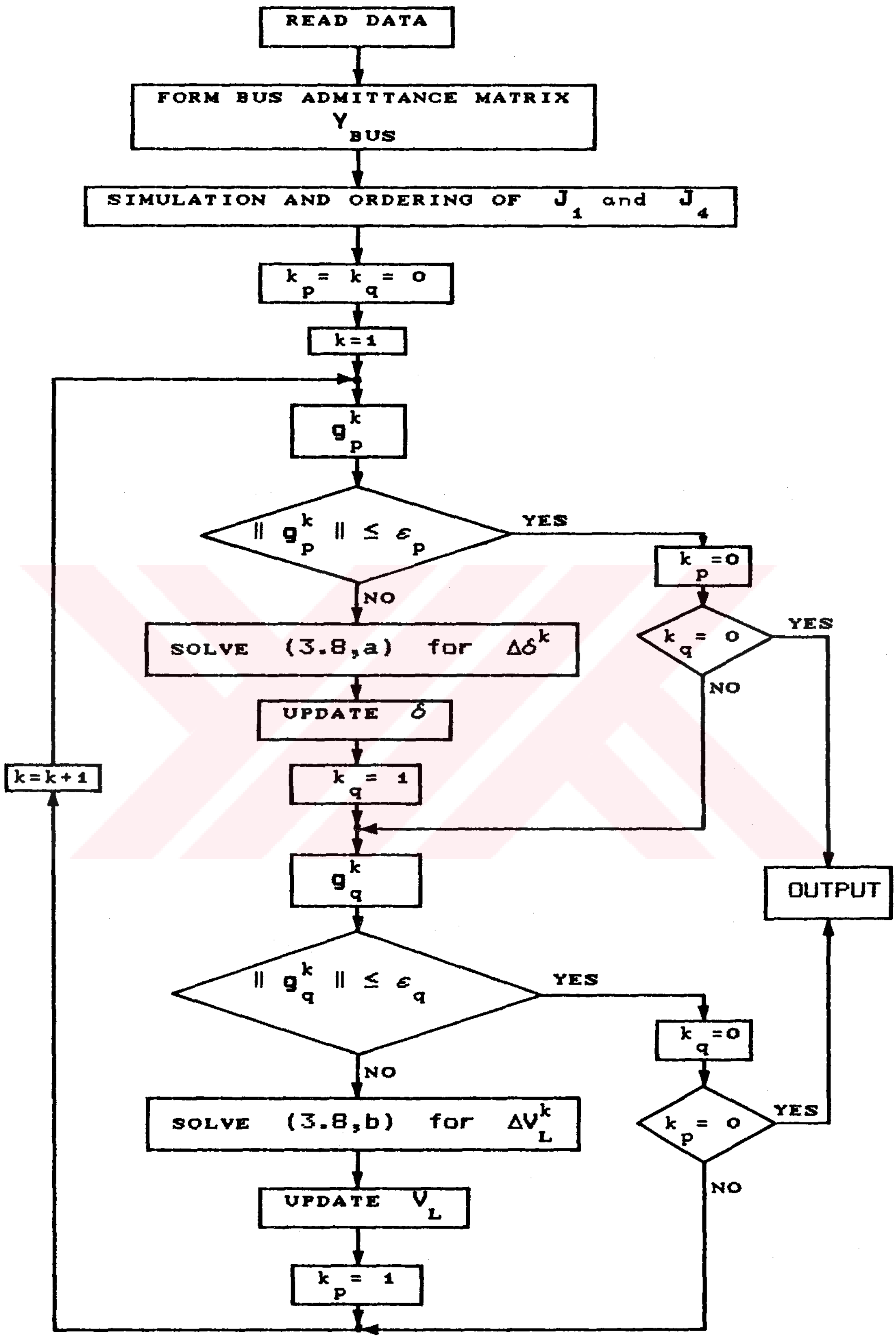


Figure 3.2 The Flow Chart of the approximate decoupled Load Flow

As a result, these approximations results in a symmetric, sparse and diagonal-dominant matrices of J_1 and J_4 . In addition, they provide an equivalance between J_4 and the associated part of J_1.

The flow chart of the approximate decoupled load flow calculations is given in Figure 3.2. The same ordering sequence can be used for both decoupled modules.

CHAPTER IV
DEVELOPMENTS IN OPTIMAL POWER DISPATCH

4.1. Reactive Power Dispatch

The problem of voltage and reactive power control in power systems is directly concerned not only with the service quality of electric energy supply to consumers, but also with the rational operation of power system on the part of electric utilities. The latter objective is for the stable operation of various kinds auxilary equipment in generating plants and substations, for the requirements of insulation level and for the prevention of coronas, through the maintenance of system voltages within proper tolerances.

The voltage and reactive power control is also required for the reduction of transmission losses, for the improvement of utilization factor of transmission lines and transformers. Effective use of various regulating devices such as voltage or reactive power controlled generators, on load tap changing transformers, shunt capacitor/reactors, makes it possible to achieve the desired objectives. The simple manual or semi-automatic voltage control in the past has evolved to a more advanced and sophisticated control taking into account the proper allocation of reactive power flows on the trunk lines of the system.

Briefly, the objective of reactive power dispatch is twofold, namelly;

a) To minimize the RPTL due to reactive power flows,
b) To keep the voltage at each bus within a specified tolerances.

The former one can automatically be done by assigning the RPTL as a cost function of the module, where the latter is achieved via inequality constraints upon the voltage magnitudes.

4.1.1. Exact Reactive Power Dispatch

Let us rewrite the formulation given in Section 2.2.1

$$\underset{\text{wrt } u_q}{\text{Minimize}} \qquad f_q = f_q(x, u_q) \tag{4.1}$$

$$\text{Subject to} \qquad g(x, u_q) = 0 \tag{4.2}$$

$$h(x, u_q) \geq 0 \tag{4.3}$$

The control vector u_q comprises the generator voltage magnitudes (V_g), VAR-injection values of shunt capacitor/reactors (Q_c) and the tap values of tap changing transformers (t). Equality constraints are both the real and reactive power mismatch equations. Most common case of inequality constraints are,

$$x_{min} \leq x \leq x_{max} \tag{4.4}$$

$$u_{q_{min}} \leq u_q \leq u_{q_{max}} \tag{4.5}$$

The first ones , Eq. (4.4) , held by changing them into equality constraints via penalty terms, i.e.,

$$w(x) = \sum_{k \in \{\mathbb{V}\}} w_k(x_k) \tag{4.6}$$

$$w_k(x_k) = \begin{cases} R_k(x_k - x_{kmin})^2 & \text{for} \quad x_k < x_{kmin} \\ 0 & \text{for} \quad x_{kmin} \leq x_k \leq x_{kmax} \\ R_k(x_k - x_{kmax})^2 & \text{for} \quad x_k > x_{kmax} \end{cases} \tag{4.7}$$

where $\{\mathbb{V}\}$ denotes the set of violating state variables and R_k denotes the weighing factor associated to the violating state variable x_k. Inequalities upon the control variables given by Eq. (4.5) are satisfied while updating them.

Since the control variables of the active power dispatching module, i.e., active power generations except the swing bus and phase shifter angles, are held fixed , f_q can also be chosen as the slack bus generation, P_1. This alternative not only provides the desired decrease described in Section 2.2.1 but also makes it possible to use the sparse vector techniques [58], together with the conventional sparse matrix techniques. However, it does require more computational effort for approximate formulations.

The resulting Lagrangian and the necessary conditions are;

$$\mathscr{L}_q = f_q + \begin{bmatrix} \lambda_p^T & \lambda_q^T \end{bmatrix} \begin{bmatrix} g_p \\ g_q \end{bmatrix} + w(x) \tag{4.8}$$

$$\frac{\partial \mathscr{L}_q}{\partial \lambda} = [\, g_p^T \qquad g_q^T \,] = 0 \qquad (4.9,a)$$

$$\frac{\partial \mathscr{L}_q}{\partial x} = \frac{\partial f_q}{\partial x} + \begin{bmatrix} \frac{\partial g_p}{\partial x} \\ \frac{\partial g_q}{\partial x} \end{bmatrix}^T \begin{bmatrix} \lambda_p \\ \lambda_q \end{bmatrix} + \frac{\partial w}{\partial x} = 0 \qquad (4.9,b)$$

$$\frac{\partial \mathscr{L}_q}{\partial u_q} = \frac{\partial f_q}{\partial u_q} + \begin{bmatrix} \frac{\partial g_p}{\partial u_q} \\ \frac{\partial g_q}{\partial u_q} \end{bmatrix}^T \begin{bmatrix} \lambda_p \\ \lambda_q \end{bmatrix} = 0 \qquad (4.9,c)$$

All the partial derivatives except those with respect to the tap ratios of TCT's were derived in the third chapter. Figure 4.1 shows an equivalent circuit of a TCT connected between the nodes i and j. The tap position is denoted as t and the transformer admittance is denoted as $\bar{y}_{ij}$.

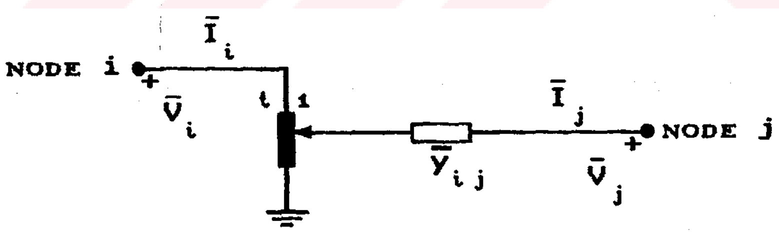


Figure 4.1 Equivalent Circuit of a TCT

Kirchoff Voltage Law results in,

$$\bar{I}_i = (\bar{V}_i - \bar{V}_j)\,\frac{\bar{y}_{ij}}{t} + \frac{1}{t}\,(\frac{1}{t} - 1)\cdot\bar{y}_{ij}\cdot\bar{V}_i$$

$$= \bar{Y}_1(\bar{V}_i - \bar{V}_j) + \bar{Y}_2\bar{V}_i \qquad (4.10,a)$$

$$\bar{I}_j = (\bar{V}_j - \bar{V}_i)\frac{\bar{y}_{ij}}{t} + (1 - \frac{1}{t})\ \bar{y}_{ij}\bar{V}_j$$

$$= \bar{Y}_1(\bar{V}_j - \bar{V}_i) + \bar{Y}_3\bar{V}_j \qquad (4.10,b)$$

An equivalent circuit yielding the Eqs. (4.10,a) and (4.10,b) is given in Figure 4.2.

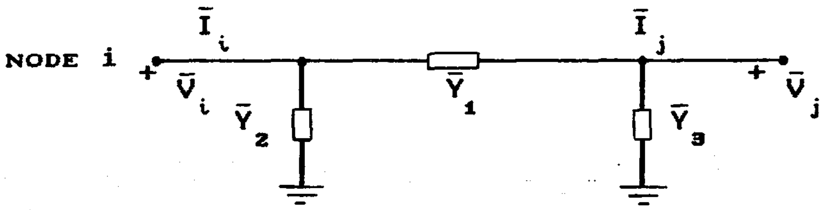


Figure 4.2 Electrical equivalent of a TCT

In order to derive the expressions for bus power mismatch sensitivities with respect to transformer taps, let $\bar{Y}'_{ii}$ and $\bar{Y}'_{jj}$ be defined as the sums of line admittances connected to bus i and j, respectively, excluding the transformer line $\bar{y}_{ij}$. Then,

$$\bar{Y}_{ii} = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}'_{ii} \qquad (4.11,a)$$

$$\bar{Y}_{jj} = \bar{Y}_1 + \bar{Y}_3 + \bar{Y}'_{jj} \qquad (4.11,b)$$

Using Eqs. (4.10,a) and (4.10,b),

$$\bar{Y}_{ii} = \bar{y}_{ij}/t^2 + \bar{Y}'_{ii} \qquad (4.12,a)$$

$$\bar{Y}_{ij} = -\ \bar{y}_{ij}/t \qquad (4.12,b)$$

$$\bar{Y}_{jj} = \bar{y}_{ij} + \bar{Y}'_{jj} \tag{4.12,c}$$

If we introduce Eqs. (4.12,a), (4.12,b) and (4.12,c) in Eqs. (3.15,a) and (3.15,b), the resulting partial derivatives can be derived as,

$$\frac{\partial g_{p_i}}{\partial t} = \frac{2V_i^2}{t^3} g_{ij} - \frac{V_i V_j}{t^2} (g_{ij} \mathrm{Cos}\ \delta_{ji} - b_{ij} \mathrm{Sin}\ \delta_{ji}) \tag{4.13,a}$$

$$\frac{\partial g_{q_i}}{\partial t} = -\frac{2V_i^2}{t^3} b_{ij} + \frac{V_i V_j}{t^2} (g_{ij} \mathrm{Sin}\ \delta_{ji} + b_{ij} \mathrm{Cos}\ \delta_{ji}) \tag{4.13,b}$$

$$\frac{\partial g_{p_j}}{\partial t} = -\frac{V_i V_j}{t^3} (g_{ij} \mathrm{Cos}\ \delta_{ji} - b_{ij} \mathrm{Sin}\ \delta_{ji}) \tag{4.13,c}$$

$$\frac{\partial g_{q_j}}{\partial t} = \frac{V_i V_j}{t^2} (g_{ij} \mathrm{Sin}\ \delta_{ji} + b_{ij} \mathrm{Cos}\ \delta_{ji}) \tag{4.13,d}$$

The solution algorithm of an exact reactive power dispatch is given in Figure 4.3

The first necessary condition of optimality corresponds to the calculation of load flows. The coefficient matrix of the Lagrangian variables is the same of the Jacobian matrix of the load flow, and therefore the factor matrices evaluated before can be used. The residual term $\| \nabla f \|_q$ is used both for the stopping criteria and the updating measure, i.e.,

$$\Delta u_q^k = c_q r_q^k \tag{4.14}$$

where c_q is a constant and r_q is an appropriate function of $\| \Delta f \|$. Details are given in Appendix A.

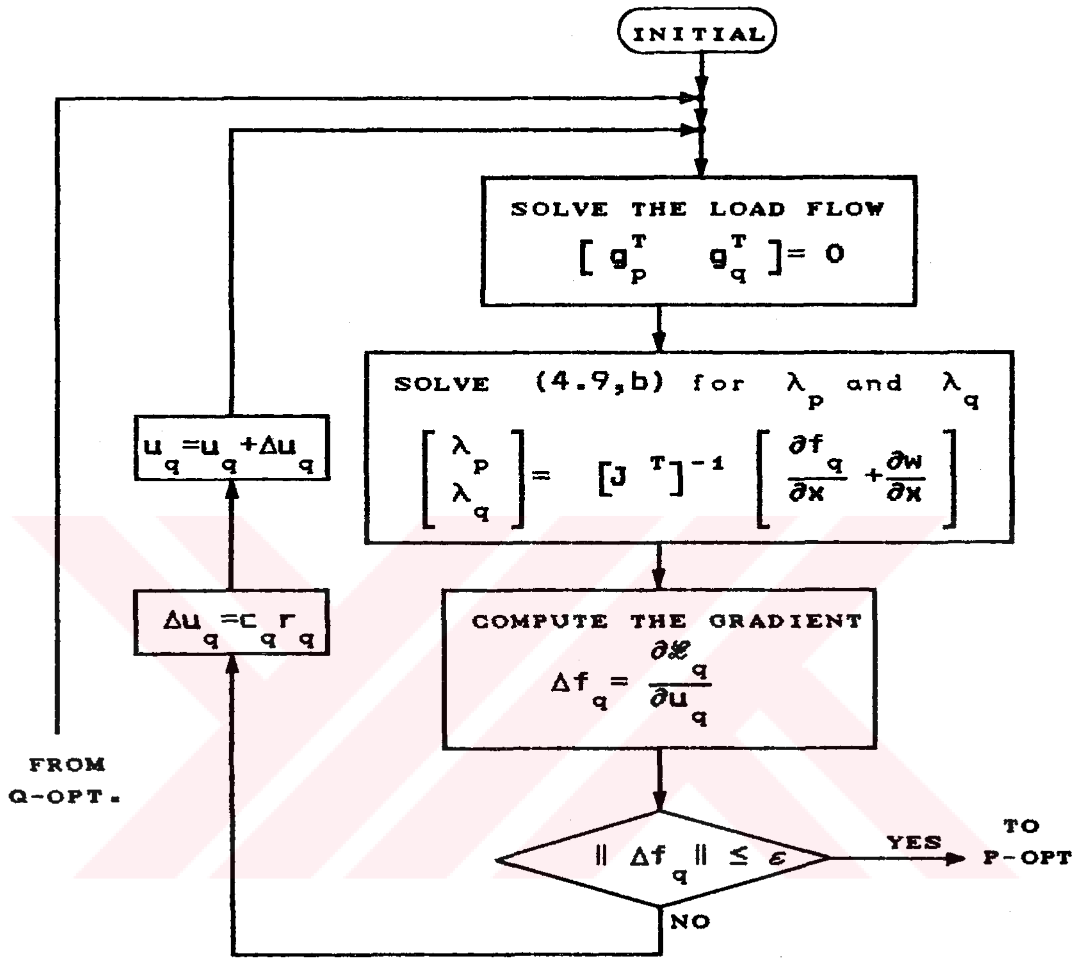


Figure 4.3 The Flow Chart of the Exact Reactive Power Dispatch

4.1.2. The Proposed Formulation

Improved load flow calculations bring the necessity of the rearrangement of Eq. (4.9,b). The elimination of the off-diagonal blocks of the Jacobien matrix, i.e. J_2 and J_3, must be adopted for the calculation of λ_p and λ_q. Let us write the Eq. (4.9,b) in a decomposed form;

$$\begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix}^T \begin{bmatrix} \lambda_p \\ \lambda_q \end{bmatrix} = - \begin{bmatrix} \dfrac{\partial f_q}{\partial \delta} \\ \dfrac{\partial f_q}{\partial V_L} V_L + \dfrac{\partial w}{\partial V_L} V_L \end{bmatrix} \qquad (4.15)$$

The Lagrangian Multipliers , λ_p and λ_q, have the physical meaning in that they represent the first order sensitivity of the objective function with respect to the active and reactive power injections at each bus [71]. That is,

$$\lambda_{p_i} = \frac{\partial f_q}{\partial P_i} \qquad i=2,3,\ldots,NB \qquad (4.16,a)$$

$$\lambda_{q_i} = \frac{\partial f_q}{\partial Q_i} \qquad i=(NG+1),\ldots,NB \qquad (4.16,b)$$

In addition, practical power system studies have shown that λ_{p_i}'s are considerably larger than λ_{q_i}'s. Therefore, Eq.(4.15) can be simplified as,

$$\lambda_p \cong - J_1^{-1} \left[\frac{\partial f_q}{\partial \delta} \right] \qquad (4.17,a)$$

$$\lambda_q \cong - J_4^{-1} \left[\frac{\partial f_q}{\partial V_L} V_L + \frac{\partial w}{\partial V_L} V_L - J_2^T \lambda_p \right] \qquad (4.17,b)$$

Note that the transpose superscript for J_1 and J_4 are omitted since they had been approximated to be symmetrical matrices.

Further improvements will be stated due to the objective function, f_q.

4.1.2.1. Derivation of f_q

Let L_{kj} denote the real power transmission losses in the line connected between the nodes k and j.

$$L_{kj} = \text{Re}\left\{(\bar{V}_k - \bar{V}_j)\ (-\bar{Y}_{kj})\ (\bar{V}_k - \bar{V}_j)^*\right\}$$

$$= -\,G_{kj}\ |V_k - V_j|^2$$

$$= -\,G_{kj}(V_k^2 + V_j^2 - 2V_kV_j\text{Cos}\ \delta_{kj}) \tag{4.18}$$

Since

$$\text{Cos}\ \delta_{kj} = (1 - \text{Sin}^2\ \delta_{kj})^{1/2} \cong 1 - \frac{1}{2}\ \text{Sin}^2\ \delta_{kj} \tag{4.19}$$

Then

$$L_{kj} \cong -\,G_{kj}(\ V_k - V_j)^2 - G_{kj}V_kV_j\ \text{Sin}^2\ \delta_{kj} \tag{4.20}$$

The voltage angles are small enough to satisfy,

$$\text{Sin}^2\ \delta_{kj} \cong (\ \delta_k - \delta_j)^2 \tag{4.21}$$

Thus,

$$L_{kj} \cong -\,G_{kj}(\ V_k - V_j)^2 - G_{kj}V_kV_j(\ \delta_k - \delta_j)^2 \tag{4.22}$$

The total RPTL in a power system of NB buses,

$$RPTL = \sum_{k=1}^{NB-1} \sum_{j=k+1}^{NB} L_{kj}$$

$$\cong \sum_{k=1}^{NB-1} \sum_{j=k+1}^{NB} \left[-G_{kj}(V_k - V_j)^2 - G_{kj} V_k V_j (\delta_k - \delta_j)^2 \right] \quad (4.23)$$

If we distinguish the terms which are sensitive to voltage magnitudes than those which are sensitive to voltage angles.

$$RPTL_v \cong \sum_{k=1}^{NB-1} \sum_{j=k+1}^{NB} -G_{kj}(V_k - V_j)^2 \quad (4.24,a)$$

$$RPTL_\delta \cong \sum_{k=1}^{NB-1} \sum_{j=k+1}^{NB} - G_{kj} V_k V_j (\delta_k - \delta_j)^2 \quad (4.24,b)$$

$$RPTL = RPTL_v + RPTL_\delta \quad (4.24,c)$$

A similar derivation and classification for slack bus active generation results in,

$$P_1 \cong V_1 \sum_{j=2}^{NB} \left[-G_{1j}(V_1 - V_j) - B_{1j} V_j \text{Sin } \delta_j \right] \quad (4.25)$$

$$P_{1v} \cong V_1 \sum_{j=2}^{NB} - G_{1j}(V_1 - V_j) \quad (4.26,a)$$

$$P_{1\delta} \cong V_1 \sum_{j=2}^{NB} - B_{1j} V_j \mathrm{Sin}\ \delta_j \tag{4.26,b}$$

$$P_1 = P_{1V} + P_{1\delta} \tag{4.27}$$

The phase differences between the voltage angles accross a transmission line is nearly proportional to the active line loading. On the other hand, bus voltage magnitudes are controlled by reactive power flow. Therefore for a particular load, the term $(\delta_k - \delta_j)$ in Eq. (4.23) does not vary very much and can be considered constant (see [45] for details).

It has been a general custom of accepting $RPTL_V$ as f_q and optimizing it by taking into account only the reactive load flow equations [43,45,47,68,73]. However, in case, active power mismatch can considerably be high depending upon the active loading and the system parameters. So, sub-optimal values after the Q-optimization module may sometimes be unrealistic. Moreover, the systems which do not require voltage magnitude improvements posses generally much higher $RPTL_\delta$ than $RPTL_V$. On the other hand if the improvement of voltage profiles is necessary, minimizing $RPTL_V$ plays a secondary role.

Instead of the conventional fixed $(\delta_k - \delta_j)$ assumption let us consider Eq. (4.16,a). Since λ_{p_i} is a measure of the first order sensitivity of the objective function with respect to the active power injection at bus i then it will be more realistic to assume a constant λ_{p_i} throughout the Q-optimization module. This holds almost all kind of power systems, since the active power generations are held fixed during this module. The second problem, i.e., ignoring the active load flow equations needs the classification of the state variables.

4.1.2.2. The Classification of State Variables

Via $P-\delta$ and $Q-V$ coupling, the state variable vector x can be divided into two sub-vectors.

$$x_q^T = [\, V_L^T \quad Q_G^T \,] \qquad (4.28,a)$$

$$x_p = \delta \qquad (4.28,b)$$

$$x^T = [\, x_q^T \quad x_p^T \,] \qquad (4.28,c)$$

The first sub-vector coordinates the reactive load flow equations and the second one coordinates the active load flow equations. However, reactive power outputs of the generators (Q_G), are calculated after the solution of the rective load flow equations. Therefore, reactive load flow calculations can be assumed to be primarly governed by the voltage magnitudes.

If we ignore the active load flow equations; that is, if we state the problem as,

$$\underset{\text{wrt } u_q}{\text{Minimize}} \quad f_q(x_q,u_q) \qquad (4.29)$$

$$\text{Subject to} \quad g_q(x_q,u_q) = 0 \qquad (4.30)$$

$$h_q(x_q,u_q) \geq 0 \qquad (4.31)$$

where x_p is held fixed, there will be a residual term for active power mismatch equations,

$$\Delta g_p = \frac{\partial g_p}{\partial x_q} \Delta x_q + \frac{\partial g_p}{\partial u_q} \Delta u_q \tag{4.32}$$

The voltage angles are required to be changed slightly to compansate this residual term,

$$\Delta g_p = \frac{\partial g_p}{\partial \delta} \Delta\delta = J_1 \Delta\delta \tag{4.33}$$

From Eq. (4.32) and Eq.(4.33),

$$\Delta\delta = \left[\frac{\partial g_p}{\partial \delta}\right]^{-1} \frac{\partial g_p}{\partial x_q} \Delta x_q + \left[\frac{\partial g_p}{\partial \delta}\right]^{-1} \frac{\partial g_p}{\partial u_q} \Delta u_q \tag{4.34}$$

Hence,

$$\frac{\partial \delta}{\partial x_q} = \left[\frac{\partial g_p}{\partial \delta}\right]^{-1} \frac{\partial g_p}{\partial x_q} \tag{4.35}$$

$$\frac{\partial \delta}{\partial u_q} = \left[\frac{\partial g_p}{\partial \delta}\right]^{-1} \frac{\partial g_p}{\partial u_q} \tag{4.36}$$

Note that x_q does not include Q_G throughout Equations (4.34)-(4.36), since g_q does not include the reactive power mismatch equations of generator buses. The term $[\partial g_p/\partial Q_C]$ is absolutely zero and the term $[\partial g_p/\partial t]$ is small enough to be negligible. Then, Eq. (4.35) and Eq. (4.36) can be combined as,

$$\left[\frac{\partial \delta}{\partial V} V \right] = \left[\frac{\partial \delta}{\partial V_G} V_G \qquad \frac{\partial \delta}{\partial V_L} V_L \right] = J_1^{-1} \bar{J}_2 \tag{4.37}$$

where

$$\bar{J}_2 = \left[\frac{\partial g_P}{\partial V_G} V_G \qquad \frac{\partial g_P}{\partial V_L} V_L \right] = \left[\frac{\partial g_P}{\partial V} V \right] \tag{4.38}$$

is the augmented form of J_2.

Partial derivatives of the objective function with respect to x_q and u_q are,

$$\frac{\partial f_q}{\partial V} V = \frac{\partial f_q}{\partial V} V + \left[\frac{\partial \delta}{\partial V} V \right]^T \frac{\partial f_q}{\partial \delta} \tag{4.39,a}$$

$$\frac{\partial f_q}{\partial Q_G} = \left[\frac{\partial \delta}{\partial Q_G} \right]^T \frac{\partial f_q}{\partial \delta} \cong 0 \tag{4.39,b}$$

$$\frac{\partial f_q}{\partial Q_C} = \left[\frac{\partial \delta}{\partial Q_C} \right]^T \frac{\partial f_q}{\partial \delta} \cong 0 \tag{4.39,c}$$

$$\frac{\partial f_q}{\partial t} = \frac{\partial f_q}{\partial t} + \left[\frac{\partial \delta}{\partial t} \right]^T \frac{\partial f_q}{\partial \delta} \cong \frac{\partial f_q}{\partial t} \tag{4.39,d}$$

The Lagrangian and the necessary conditions are,

$$\mathscr{L}_q(x_q,u_q,\lambda_q) = f_q(x_q,u_q) + \lambda_q^T g_q(x_q,u_q) + w(x_q) \tag{4.40}$$

$$\frac{\partial \mathscr{L}_q}{\partial \lambda_q} = g_q = 0 \tag{4.41,a}$$

$$\frac{\partial \mathscr{L}_q}{\partial x_q} = \left[\frac{\partial f_q}{\partial x_q} + \frac{\partial w}{\partial x_q} + \left[\frac{\partial \delta}{\partial x_q} \right]^T \frac{\partial f_q}{\partial \delta} \right] + \left[\frac{\partial g_q}{\partial x_q} \right] \lambda_q = 0 \tag{4.41,b}$$

$$\frac{\partial \mathscr{L}_q}{\partial u_q} = \left[\frac{\partial f_q}{\partial u_q} + \left[\frac{\partial \delta}{\partial u_q} \right]^T \frac{\partial f_q}{\partial \delta} \right] + \left[\frac{\partial g_q}{\partial u_q} \right]^T \lambda_q = 0 \tag{4.41,c}$$

Eq. (4.41,a) comprises reactive power load flow equations. Eq. (4.41,b) is used to calculate λ_q. From Eq. (4.35) and (4.41,b) one can easily derive that,

$$\left[\frac{\partial \delta}{\partial x_q} \right]^T \frac{\partial f_q}{\partial \delta} = \left[J_1^{-1} J_2 \right] \frac{\partial f_q}{\partial \delta}$$

$$= J_2^T \left[J_1^{-1} \right] \frac{\partial f_q}{\partial \delta}$$

$$= - J_2^T \lambda_p \tag{4.42}$$

$$\lambda_q = - J_4^{-1} \left[\frac{\partial f_q}{\partial x_q} + \frac{\partial w}{\partial V_L} V_L - J_2^T \lambda_p \right] \tag{4.43}$$

The result is the same of Eq. (4.17,b) which was derived from the approximations. A similar analysis for the control vector u_q results in,

$$\frac{\partial \mathscr{L}_q}{\partial u_q} = \frac{\partial f_q}{\partial u_q} + \frac{\partial g_p}{\partial u_q} \lambda_p + \frac{\partial g_q}{\partial u_q} \lambda_q \tag{4.44}$$

Optimization procedure is the same of that was given by Figure (4.3) and will not be reproduced here.

4.1.2.3. Improvement of Voltage Profiles

In bulk power systems, keeping the bus voltage magnitudes within specified limits possesses an important problem. The foregoing formulations were primarly based on the minimization of the RPTL, but the voltage control problem was aimed to be achieved by means of penalty terms. However, the first calculated state variables (especially load voltage magnitudes, V_L) for a given set of initial control vector may sometimes be extremely out of range. This situation results in larger penalty terms and so the minimization of the RPTL can be retarded. Moreover, the process may result an increase for the RPTL, depending on the values of the weighing factors.

There are two alternatives to overcome this inconvenience. First of them is to satisfy the limits while performing the load flow calculations. But, this process requires bus-type switching, which brings an extre time spent. Also the algorithm contradicts with the optimization process.

The second alternative, which was preferred, accomplishes the voltage control problem together with the optimization procedure. A systematic approach is proposed for the reduction of voltage magnitude deviations between the load buses. At the beginning of the module, the object is devoted to minimizing the voltage deviations. This means a change in f_q , in such a way to include only the terms related to the dominant objective.

The simplist formulation can be achieved by omitting the term f_q in Eqs. (4.8) or (4.40). Although such a modification provides the expelling of excess terms related to f_q and its partial derivatives, it is not preferable, since the remaining penalty terms are not strong enough to provide a quick convergence. Instead, $RPTL_v$ can be used either with or without the penalty terms, since it can be accepted as a measure of voltage deviations, as well. Moreover, this choice not only provides the minimization of voltage profiles but also may result a decrease in the RPTL.

By the way, it must be recognized that the most frequent case of load voltage magnitude violations are the abnormal decreases. Therefore, omitting the penalty terms results in a balance of the term $(V_k-V_j)^2$ in the lower limit. That is, load bus voltage magnitudes trend to decrease. Sample system studies have shown that either the augmented objective function

$$\bar{f}_q = RPTL_v + w(V_L) \tag{4.45}$$

or highly weighed

$$\bar{f}_q = w(V_L) \tag{4.46}$$

provide acceptable results.

Note that, at this stage there is no need to use λ_p or g_p in anyway.

4.1.2.4 Final Scheme

The twofold objective of reactive power dispatch is achieved by systematic formulation of the module.

First, voltage magnitudes are checked to be within the specified limits. In case of highly violated voltage magnitudes, the module is arranged primarly in such a way to force them within the desired limits. This is done by switching the objective function to an appropriate form. Later, the formulation given by Eqs. (4.29), (4.30) and (4.31) is used to minimize the RPTL. In the second stage f_q is choosen to be P_1. The flow chart of the proposed procedure is given in Figure 4.4.

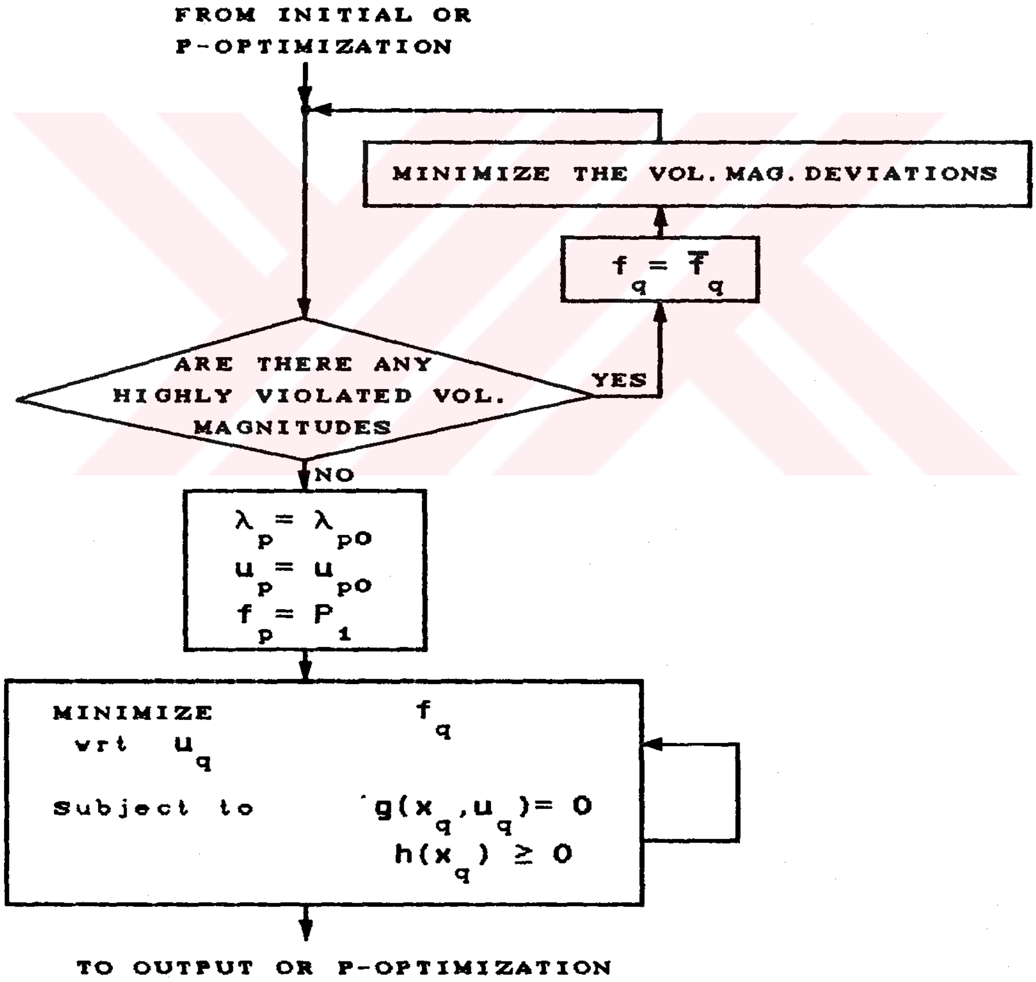


Figure 4.4 The Flow Chart of the Proposed Two Stage Reactive Power Optimization Module.

4.2. Active Power Dispatch

The main economic factor in power system operation is the cost of generating real power. It is obvious that the fuel cost is meaningful only in case of thermal generating stations, but for hydro stations the operating cost as such not meaningful. Active power dispatch problem can be partitioned into two sub-problems; optimum allocation of generators at each generating station at various station load levels and optimum allocation of generation to each station for various system load levels.

The first problem in power system parlance is called the unit commitment (UC) problem, and the second is called the load scheduling problem. One must first solve the UC problem before proceeding with the load scheduling.

It is not economical to run all the units available all the time. To determine the units of a plant that should operate for a particular load is the problem of UC [5]. This problem is of importance for thermal plants as for other types of generation such as hydro; their operating cost and start up times are negligible so that their on-off status is not important. A straightforward but highly time-consuming way of finding the most economical combination of units to meet a particular load demand, is to try all possible combinations of units that supply this load. That is, optimal unit commitment problem requires hundreds to ten thousands of solution of the optimal static dispatch problem. However, there are some simple solutions [5].

Here, it was assumed that all the generating units are ready to take on load and the unit commitment problem is not taken into consideration.

4.2.1. Fuel Cost Function

The selection of the fuel cost function depends on the accuracy with which this function has to represent the actual fuel costs and the speed with which the associated dispatch problem can be solved. The fuel cost function is derived from the fuel consumption function, which can be measured. Based on the measurements of the fuel consumption at constant powers, a polynomial can be selected to fit these points. The degree of the polynomial determines the accurracy and the speed of the method. A polynomial of degree 2 is accepted as the most suitable one [75].

$$f_{p_i}(P_{G_i}) = a_i P_{G_i}^2 + b_i P_{G_i} + c_i \tag{4.47}$$

The total HFC of NG generators,

$$f_p = \sum_{i=1}^{NG} a_i P_{G_i}^2 + b_i P_{G_i} + c_i \tag{4.48}$$

or in matrix form

$$f_p = P_G^T A P_G + B^T P_G + C \tag{4.49}$$

4.2.2. Exact Active Power Dispatch

The statement of the problem is,

$$\underset{\text{wrt } u_p}{\text{Minimize}} \qquad f_p = \sum_{i=1}^{NG} f_{p_i} = f_p(x,u_p) \tag{4.50}$$

$$\text{Subject to} \qquad g(x,u_p) = 0 \tag{4.51}$$

$$h(x,u_p) \geq 0 \tag{4.52}$$

The control vector u_p comprises the generator real power outputs except the swing machine (P_G) and the phase shifter angles (α). The presence of the phase shifters are ignored since it is not a most frequent case. Equality constraints are the same of Eq. (4.2). Physical restrictions upon the active power transmission line flows, which are so called the security limits are the additional inequality constraints of this module. Although the voltage angles can be limited to handle the security problem, improved security constrained economic dispatch methods are preferred [84-88]. For the sake of simplicity, security problem is encountered via limiting the bus voltage angles.

The flow chart of the exact active power dispatch is given in Figure 4.5. The relation between the cost function and the state variables are governed by the cost function of the swing generator, f_{p_1}, i.e.,

$$\frac{\partial f_p}{\partial x} = \frac{\partial f_p}{\partial P_1} \cdot \frac{\partial P_1}{\partial x} = \frac{\partial f_{p1}}{\partial P_1} \cdot \frac{\partial P_1}{\partial x} \tag{4.53}$$

Other calculations are straightforward and pose no computational problem. The main inconvenience of the module is the indefinetness of the behaviour of the RPTL. That is, P-optimization may result either a decrease or an increase in RPTL, while minimizing the HFC [59].

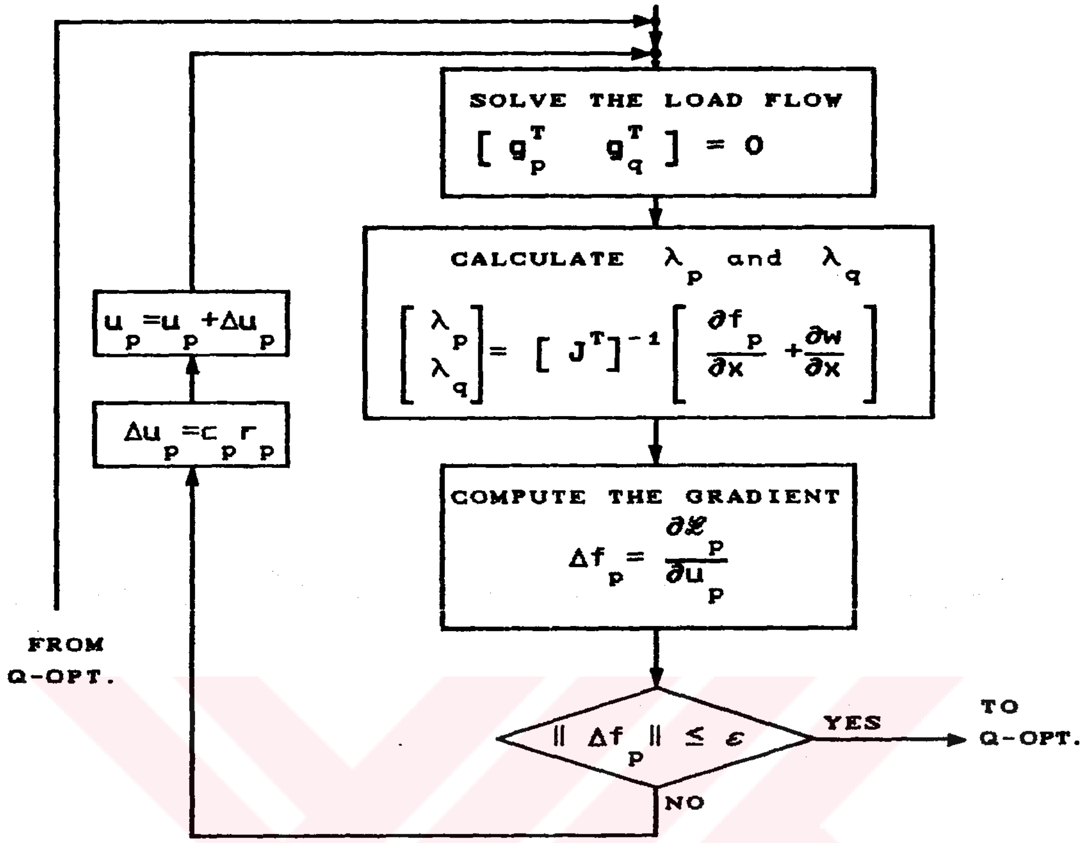


Figure 4.5 The Flow Chart of the Exact Active Power Dispatch

4.2.3. The Proposed Formulation

The second necessary condition of the optimality, i.e. partial derivatives of the Lagrangian with respect to state variables, gives;

$$\begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix}^T \begin{bmatrix} \bar{\lambda}_p \\ \bar{\lambda}_q \end{bmatrix} = - \begin{bmatrix} \frac{\partial P_1}{\partial \delta} + \frac{\partial w}{\partial \delta} \\ \frac{\partial P_1}{\partial V_L} V_L \end{bmatrix} \frac{\partial f_p}{\partial P_1} \qquad (4.54)$$

where the Lagrangian multipliers are denoted by $\bar{\lambda}_p$ and $\bar{\lambda}_q$ in order to distinguish from those of the other module.

The relation between the Lagrangian variables of the sub-optimization modules can easily be derived as,

$$\bar{\lambda}_p = \lambda_p \frac{\partial f_p}{\partial P_1} \tag{4.55,a}$$

$$\bar{\lambda}_q = \lambda_q \frac{\partial f_p}{\partial P_1} \tag{4.55,b}$$

The third condition of optimality gives,

$$\frac{\partial \mathcal{L}_p}{\partial u_p} = \frac{\partial f_p}{\partial P_G} + \begin{bmatrix} \frac{\partial g_p^T}{\partial P_G} & \frac{\partial g_q^T}{\partial P_G} \end{bmatrix} \begin{bmatrix} \bar{\lambda}_p \\ \bar{\lambda}_q \end{bmatrix} \tag{4.56}$$

But,

$$\frac{\partial g_p}{\partial P_G} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \tag{4.57,a}$$

$$\frac{\partial g_q}{\partial P_G} = [\ 0\ ] \tag{4.57,b}$$

where I is a unit matrix of order (NG-1). Hence,

$$\frac{\partial \mathcal{L}_p}{\partial u_p} = \frac{\partial f_p}{\partial P_G} + \bar{\lambda}_{p1} \tag{4.58}$$

where

$$\bar{\lambda}_{p1} = [\ \bar{\lambda}_2 \quad \bar{\lambda}_3 \quad \cdots \quad \bar{\lambda}_{NG}] \tag{4.59}$$

Because of the absence of $\bar{\lambda}_q$ in Eq. (4.58), it can easily be concluded that the omitting the reactive load flow calculations do not effect the results considerably. Moreover, normalizing the cost function of the generating units with $\partial f_p/\partial P_1$ provides an equivalance between the associated components of the λ_p and $\bar{\lambda}_p$. This is an important point, since the last calculated Lagrangian variables needed to be used in the other module without any change. Final statement of the module is,

$$\underset{\text{wrt } u_p}{\text{Minimize}} \qquad \bar{f}_p = (\partial f_p/\partial P_1)^{-1} f_p = \bar{f}_p(x_p, u_p) \tag{4.60}$$

$$\text{Subject to} \qquad g_p(x_p, u_p) = 0 \tag{4.61}$$

$$h_p(x_p) \geq 0 \tag{4.62}$$

The remaining problem is again the behaviour of the module upon the RPTL. Losses in the transmission of electrical energy results in a loss of revenue and therefore, even a small percentage of savings will be very much appriciated. The load demand, generation capacity and the dimensions of the network determine whether an increase in the RPTL is allowed or not.

Let us derive the relation between the RPTL and the variables of the module.

$$\Delta RPTL = \left[\frac{\partial RPTL}{\partial P} \right] \Delta P + \left[\frac{\partial RPTL}{\partial Q} \right] \Delta Q \tag{4.63}$$

ΔQ is small enough to be negligible, since the voltage magnitudes are held costant. Moreover, the components of the vector ΔP, which are related to the load buses, are zero. Hence,

$$\Delta RPTL = \sum_{i=2}^{NG} \frac{\partial RPTL}{\partial P_i} \Delta P_{G_i} \qquad (4.64)$$

But

$$\frac{\partial RPTL}{\partial P_i} = \frac{\partial P_1}{\partial P_i} + 1 = \lambda_{p_i} + 1 \qquad (4.65)$$

Then

$$\Delta RPTL = \sum_{i=2}^{NG} (\lambda_{p_i} + 1) \Delta P_{G_i} \qquad (4.66)$$

$$\Delta RPTL = \left[[1] + \lambda_{p1} \right] \Delta P_G = \left[[1] + \lambda_{p1} \right] \Delta u_p \qquad (4.67)$$

where

$$[1]^T = [\ 1\ 1\ 1\ \ldots\ 1\ ] \qquad (4.68)$$

But, as it was stated for the adjacent module, Δu_p is is also determined via the residual term $\partial \mathcal{L}_p / \partial u_p$, i.e., the change in control vector at k^{th} iteration,

$$\Delta u_p^k = c_p r_p^k \qquad (4.69)$$

and r_p^k is a function of $\| \Delta f_p \|$ (See Appendix A). Hence, Eq. (4.67) provides the sufficient information about the variation of the RPTL during the module. The following procedure depends on the objective desired. If an increase in the RPTL is accepted for the sake of low cost, updating of the control variables is done via Eq. 4.69. But if an increase in the RPTL is accepted in any-way, only the control variables satisfying,

$$(1+\lambda_{p_i})\ \Delta u_{p_i} \leq 0 \tag{4.70}$$

can be updated. That is,

$$\Delta u_{p_i} = \begin{cases} \Delta u_{p_i} & \text{if} \quad (1+\lambda_{p_i})\ \Delta u_{p_i} \leq 0 \\ 0 & \text{if} \quad (1+\lambda_{p_i})\ \Delta u_{p_i} \geq 0 \end{cases} \tag{4.71}$$

If there are not any control variable satisfying Eq. (4.70), either the continuation of the module can be stopped or the objective can be switched to achieve less values for the RPTL. It is obvious that the latter case results in an increase for the HFC. The flow chart of the algorithm is given in Figure 4.6.

The proposed formulations for both modules are programmed and the efficency of the method is tested by solving several IEEE test systems. The details are given in the next chapter.

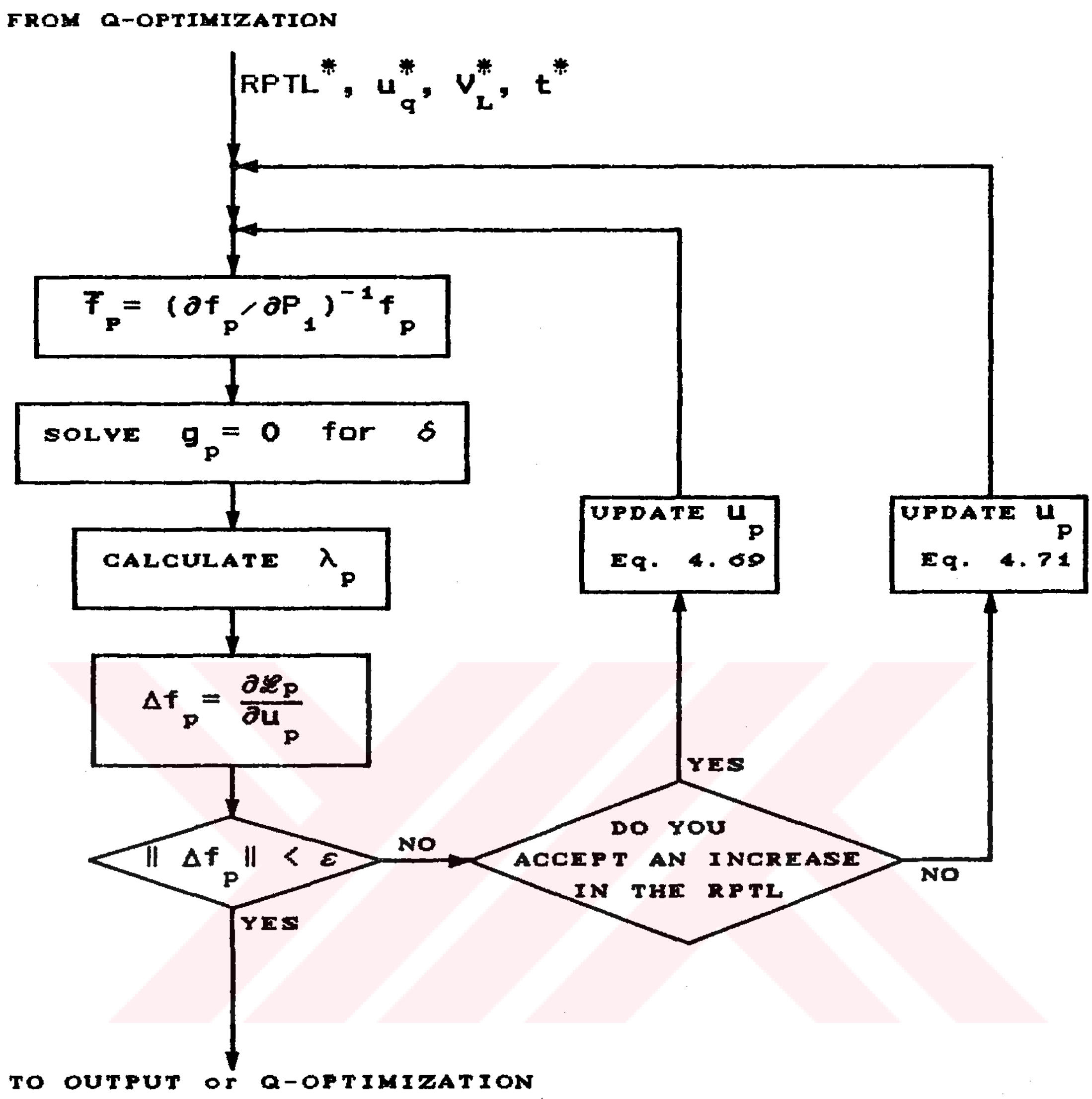


Figure 4.6 The Flow Chart of the Proposed Active Power Dispatch Algorithm

CHAPTER V
SYSTEM STUDIES

The propsed decoupled optimal load flow method is programmed in FORTRAN 77. Flexibility has been built in the program to allow for solving either the minimum cost problem or the minimum loss problem. It was run using an AT-compatible personal computer, GOUPIL G40.

5.1. Test Systems

The efficiency of the method was tested by solving 5 different test systems. They are the most common ones applied in power system optimization. All the impedance data given in this section are in per unit on a 100 MVA base.

5.1.1. 5-Bus System

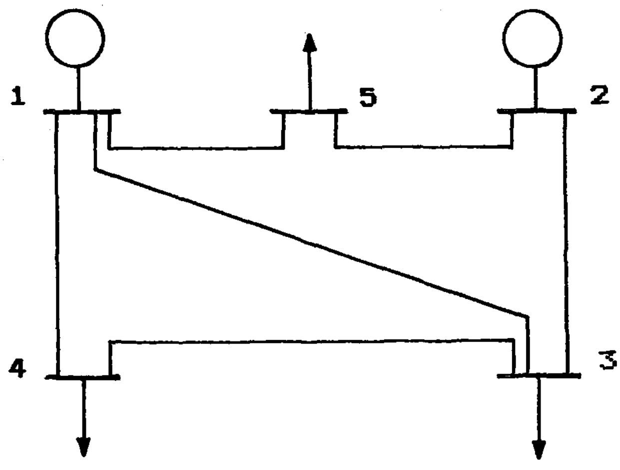


Figure 5.1 5-Bus System Line Diagram

Line diagram and the impedance data of the 5-bus test system is given in Figure 5.1. Other necessary information is illustrated in Table 5.1-5.3. This is a typical system of comparable cost function coefficients and therefore minimum loss problem needs to be solved effectivelly.

Table 5.1 5-Bus System Impedance Data

LINE NO	FROM BUS	TO BUS	LINE IMPEDANCE R [p.u.]	LINE IMPEDANCE X [p.u.]
1	1	3	0.10	0.40
2	1	4	0.15	0.60
3	1	5	0.05	0.20
4	2	3	0.05	0.20
5	2	5	0.05	0.20
6	3	4	0.10	0.40

Table 5.2 5-Bus System Load Data and the Generator Data

BUS NO	LOAD DATA		COST COEFF. DATA		
	P_L [p.u.]	Q_L [p.u.]	a	b	c
1	0.0	0.0	50.0	351.0	44.4
2	0.0	0.0	50.0	389.0	40.6
3	0.6	0.3	-	-	-
4	0.4	0.1	-	-	-
5	0.6	0.2	-	-	-

Table 5.3 Physical restrictions upon 5-bus system variables

BUS NO	V_{min}	V_{max}	P_{Gmin}	P_{Gmax}	Q_{Gmin}	Q_{Gmax}
	[p.u.]		[p.u.]		[p.u.]	
1	1.0	1.10	0.3	1.2	0.0	0.6
2	1.0	1.10	0.3	1.2	0.0	0.6
3	0.9	1.05	-	-	-	-
4	0.9	1.05	-	-	-	-
5	0.9	1.05	-	-	-	-

5.1.2. IEEE 6-Bus System

Line diagram and the impedance data of the IEEE 6-bus test system (also known as Ward and Hale's 6-bus system) is given in Figure 5.2 and the other necessary information is illustrated in Table 5.4-5.6.

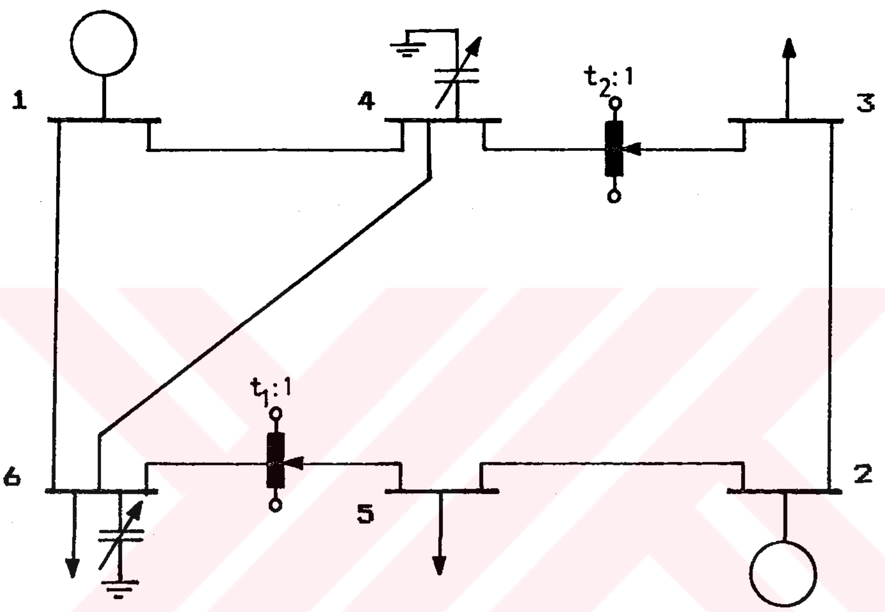


Figure 5.2 IEEE 6-Bus System Line Diagram

Table 5.4 IEEE 6-Bus System Impedance Data

LINE NO	FROM BUS	TO BUS	LINE IMPEDANCE R [p.u.]	X [p.u.]	TAP SETTING
1	1	6	0.123	0.518	–
2	1	4	0.080	0.370	–
3	4	6	0.097	0.407	–
4	6	5	0.000	0.300	1.025
5	5	2	0.282	0.640	–
6	2	3	0.723	1.050	–
7	4	3	0.000	0.133	1.100

Table 5.5 IEEE 6-Bus System Load Data and the Generator Data

BUS NO	LOAD DATA		COST COEFF. DATA		
	P_L [p.u.]	Q_L [p.u.]	a	b	c
1	0.000	0.000	500.0	100.0	0.0
2	0.000	0.000	100.0	100.0	0.0
3	0.550	0.130	–	–	–
4	0.000	0.000	–	–	–
5	0.300	0.180	–	–	–
6	0.500	0.050			

Table 5.6 Physical restrictions upon IEEE 6-bus System Variables

BUS NO	V_{min}	V_{max}	P_{Gmin}	P_{Gmax}	Q_{Gmin}	Q_{Gmax}
	[p.u.]		[p.u.]		[p.u.]	
1	1.0	1.10	0.1	1.0	-0.2	1.0
2	1.1	1.15	0.1	1.0	-0.2	1.0
3	0.9	1.00	–	–	–	–
4	0.9	1.00	–	–	0.0	0.05
5	0.9	1.00	–	–	–	–
6	0.9	1.00	–	–	0.0	0.055
$t_{1min} = t_{2min} = 0.90$ $t_{1max} = t_{2max} = 1.10$						

5.1.3. 11-Bus System

Line diagram and the impedance data of the 11-bus test system containing 5 generation buses, is given in in Figure 5.3 and the other necessary information is illustrated in Table 5.7-5.9.

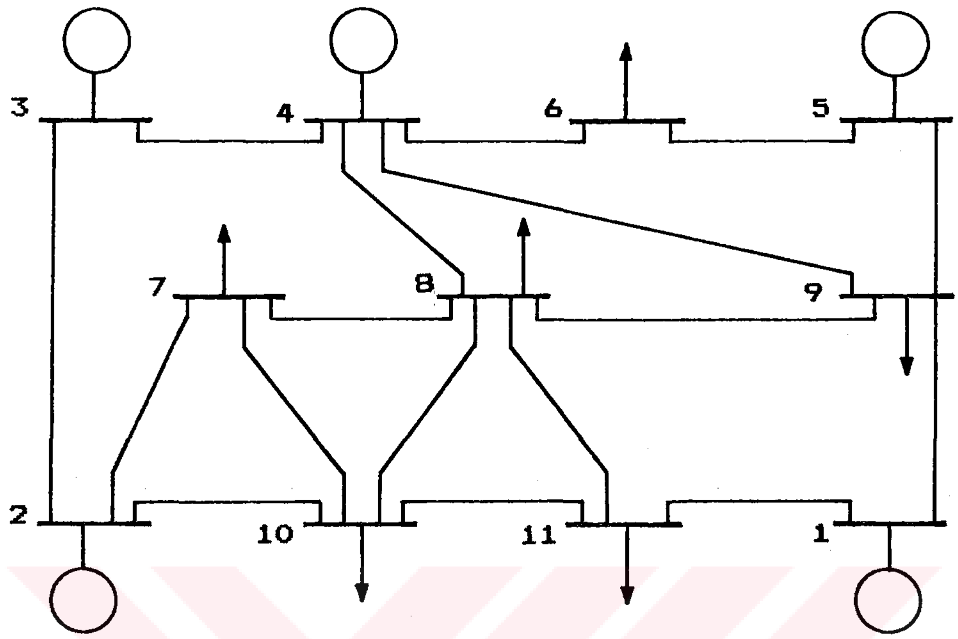


Figure 5.3 11-Bus System Line Diagram

Table 5.7 11-Bus System Impedance and Line Charging Data

LINE NO	FROM BUS	TO BUS	LINE IMPEDANCE R [p.u.]	LINE IMPEDANCE X [p.u.]	LINE CHARGING B [p.u.]
1	1	9	0.15	0.50	0.030
2	1	11	0.05	0.16	0.010
3	2	3	0.15	0.50	0.030
4	2	7	0.10	0.28	0.020
5	2	10	0.05	0.16	0.010
6	3	4	0.08	0.24	0.015
7	4	6	0.10	0.28	0.020
8	4	8	0.10	0.28	0.020
9	4	9	0.15	0.50	0.030
10	5	6	0.12	0.36	0.025
11	5	9	0.05	0.16	0.010
12	7	8	0.05	0.16	0.010
13	7	10	0.08	0.24	0.015
14	8	9	0.12	0.36	0.025
15	8	10	0.08	0.24	0.015
16	8	11	0.10	0.28	0.020
17	10	11	0.12	0.36	0.025

Table 5.8 11-Bus System Load Data and the Generator Data

BUS NO	LOAD DATA		COST COEFF. DATA		
	P_L [p.u.]	Q_L [p.u.]	a	b	c
1	0.000	0.000	50.0	360.0	40.0
2	0.000	0.000	40.0	340.0	60.0
3	0.000	0.000	40.0	340.0	60.0
4	0.000	0.000	45.0	350.0	50.0
5	0.000	0.000	45.0	350.0	40.0
6	0.100	0.020	–	–	–
7	0.400	0.100	–	–	–
8	0.900	0.450	–	–	–
9	0.700	0.350	–	–	–
10	0.250	0.050	–	–	–
11	0.250	0.050	–	–	–

Table 5.9 Physical restrictions upon 11-bus System Variables

BUS NO	V_{min}	V_{max}	P_{Gmin}	P_{Gmax}	Q_{Gmin}	Q_{Gmax}
	[p.u.]		[p.u.]		[p.u.]	
1	1.0	1.10	0.3	1.2	0.0	0.6
2	1.0	1.10	0.3	1.2	0.0	0.6
3	1.0	1.10	0.3	1.2	0.0	0.6
4	1.0	1.10	0.3	1.2	0.0	0.6
5	1.0	1.10	0.3	1.2	0.0	0.6
6	0.9	1.05	–	–	–	–
7	0.9	1.05	–	–	–	–
8	0.9	1.05	–	–	–	–
9	0.9	1.05	–	–	–	–
10	0.9	1.05	–	–	–	–
11	0.9	1.05	–	–	–	–

5.1.4. IEEE 14-Bus System

IEEE 14-bus test system containing 3 generators, 2 tap changing transformers and 3 parallel capacitive reactors is a typical moderate size test system suitable for both the minimum loss and the minimum cost problem. Line diagram is given in Figure 5.4 and the other characteristics are given in Table 5.10-5.12.

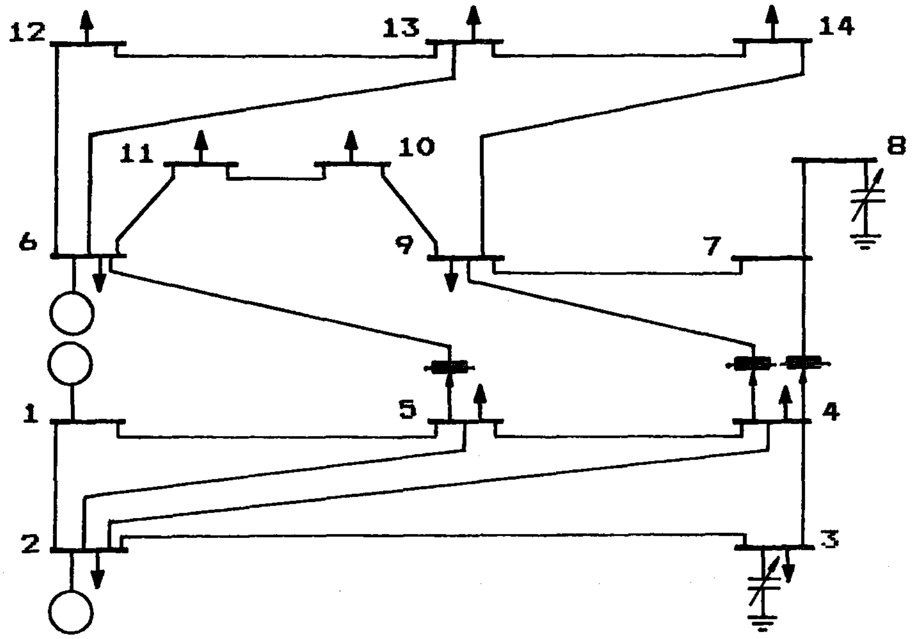


Figure 5.4 IEEE 14-Bus System Line Diagram

Table 5.10 IEEE 14-Bus System Impedance and Line Charging Data

LINE NO	FROM BUS	TO BUS	LINE IMPEDANCE R [p.u.]	LINE IMPEDANCE X [p.u.]	LINE CHARGING B [p.u.]	TAP SETTING
1	1	2	0.01938	0.05917	0.0528	–
2	2	3	0.04699	0.19797	0.0438	–
3	2	4	0.05811	0.17632	0.0374	–
4	1	5	0.05403	0.22304	0.0492	–
5	2	5	0.05695	0.17388	0.0340	–
6	3	4	0.06701	0.17103	0.0346	–
7	4	5	0.01335	0.04211	0.0128	–
8	5	6	0.00000	0.25202	0.0000	1.050
9	4	7	0.00000	0.20912	0.0000	1.050
10	7	8	0.00000	0.17615	0.0000	–
11	4	9	0.00000	0.55618	0.0000	1.050
12	7	9	0.00000	0.11001	0.0000	–
13	9	10	0.03181	0.08450	0.0000	–
14	6	11	0.09498	0.19890	0.0000	–
15	6	12	0.12291	0.25581	0.0000	–
16	6	13	0.06615	0.13027	0.0000	–
17	9	14	0.12711	0.27038	0.0000	–
18	10	11	0.08205	0.19207	0.0000	–
19	12	13	0.22092	0.19988	0.0000	–
20	13	14	0.17093	0.34802	0.0000	–

Table 5.11 IEEE 14-Bus System Load Data and the Generator Data

BUS NO	LOAD DATA		COST COEFF. DATA		
	P_L [p.u.]	Q_L [p.u.]	a	b	c
1	0.000	0.000	50.0	245.0	105.0
2	0.217	0.127	50.0	351.0	44.4
3	0.942	0.190	-	-	-
4	0.478	0.040	-	-	-
5	0.076	0.016	-	-	-
6	0.112	0.075	50.0	389.0	40.6
7	0.000	0.000	-	-	-
8	0.000	0.000	-	-	-
9	0.295	0.166	-	-	-
10	0.090	0.058	-	-	-
11	0.035	0.018	-	-	-
12	0.061	0.016	-	-	-
13	0.135	0.058	-	-	-
14	0.149	0.050	-	-	-

Table 5.12 Physical restrictions upon IEEE 14-bus System Variables

BUS NO	V_{min}	V_{max}	P_{Gmin}	P_{Gmax}	Q_{Gmin}	Q_{Gmax}
	[p.u.]		[p.u.]		[p.u.]	
1	1.0	1.10	0.5	2.0	-0.2	1.0
2	1.0	1.10	0.2	1.0	-0.4	0.5
3	0.95	1.05	-	-	-0.4	0.6
4	0.95	1.05	-	-	-	-
5	0.95	1.05	-	-	-	-
6	1.0	1.10	0.2	1.0	-0.6	0.45
7	0.95	1.05	-	-	-	-
8	0.95	1.05	-	-	-0.6	0.45
9	0.95	1.05	-	-	-	-
10	0.95	1.05	-	-	-	-
11	0.95	1.05	-	-	-	-
12	0.95	1.05	-	-	-	-
13	0.95	1.05	-	-	-	-
14	0.95	1.05	-	-	-	-

$t_{1min} = t_{2min} = t_{3min} = 0.90$

$t_{1max} = t_{2max} = t_{3max} = 1.10$

5.1.5. IEEE 30-Bus System

Table 5.13 IEEE 30-Bus System Impedance Data

LINE NO	FROM BUS	TO BUS	LINE IMPEDANCE R [p.u.]	X [p.u.]	TAP SETTING
1	1	2	0.0192	0.0575	-
2	1	3	0.0452	0.1852	-
3	2	4	0.0570	0.1737	-
4	3	4	0.0132	0.0379	-
5	2	5	0.0472	0.1983	-
6	2	6	0.0581	0.1763	-
7	4	6	0.0119	0.0414	-
8	5	7	0.0460	0.1160	-
9	6	7	0.0267	0.0820	-
10	6	8	0.0120	0.0420	-
11	6	9	0.0000	0.2080	1.050
12	6	10	0.0000	0.5560	1.050
13	9	11	0.0000	0.2080	-
14	9	10	0.0000	0.1100	-
15	4	12	0.0000	0.2560	1.050
16	12	13	0.0000	0.1400	-
17	12	14	0.1231	0.2559	-
18	12	15	0.0662	0.1304	-
19	12	16	0.0945	0.1987	-
20	14	15	0.2210	0.1997	-
21	16	17	0.0824	0.1932	-
22	15	18	0.1070	0.2185	-
23	18	19	0.0639	0.1292	-
24	19	20	0.0340	0.0680	-
25	10	20	0.0936	0.2090	-
26	10	17	0.0324	0.0845	-
27	10	21	0.0348	0.0749	-
28	10	22	0.0727	0.1499	-
29	21	22	0.0116	0.0236	-
30	15	23	0.1000	0.2020	-
31	22	24	0.1150	0.1790	-
32	23	24	0.1320	0.2700	-
33	24	25	0.1885	0.3292	-
34	25	26	0.2544	0.3800	-
35	25	27	0.1093	0.2087	-
36	28	27	0.0000	0.3960	1.050
37	27	29	0.2198	0.4153	-
38	27	30	0.3202	0.6027	-
39	29	30	0.2400	0.4533	-
40	8	28	0.6360	0.2000	-
41	6	28	0.0169	0.0599	-

IEEE 30-bus test system containing 6 generators, 4 tap changing transformers and 9 parallel capacitive reactors is the most common test system used widelly in power system optimization. Line diagram is omitted for the sake of simplicity. Necessarry information is illustrated in Table 5.13-5.15.

Table 5.14 IEEE 30-Bus System Load Data and the Generator Data

BUS NO	LOAD DATA		COST COEFF. DATA		
	P_L [p.u.]	Q_L [p.u.]	a	b	c
1	0.000	0.000	37.5	200.0	0.0
2	0.217	0.127	175.0	175.0	0.0
3	0.024	0.012	–	–	–
4	0.076	0.016	–	–	–
5	0.942	0.190	625.0	100.0	0.0
6	0.000	0.000	–	–	–
7	0.228	0.109	–	–	–
8	0.300	0.300	83.4	325.0	0.0
9	0.000	0.000	–	–	–
10	0.058	0.020	–	–	–
11	0.000	0.000	250.0	300.0	0.0
12	0.112	0.075	–	–	–
13	0.000	0.000	250.0	300.0	0.0
14	0.062	0.016	–	–	–
15	0.082	0.025	–	–	–
16	0.035	0.018	–	–	–
17	0.090	0.058	–	–	–
18	0.032	0.009	–	–	–
19	0.095	0.034	–	–	–
20	0.022	0.007	–	–	–
21	0.175	0.112	–	–	–
22	0.000	0.000	–	–	–
23	0.032	0.016	–	–	–
24	0.087	0.067	–	–	–
25	0.000	0.000	–	–	–
26	0.035	0.023	–	–	–
27	0.000	0.000	–	–	–
28	0.000	0.000	–	–	–
29	0.024	0.009	–	–	–
30	0.106	0.019	–	–	–

Table 5.15 Physical restrictions upon IEEE 30-bus System Variables

BUS NO	V_{min} [p.u.]	V_{max} [p.u.]	P_{Gmin} [p.u.]	P_{Gmax} [p.u.]	Q_{Gmin} [p.u.]	Q_{Gmax} [p.u.]
1	1.0	1.10	0.50	2.00	-0.20	2.00
2	1.0	1.10	0.20	0.80	-0.20	1.00
3	0.95	1.05	-	-	-	-
4	0.95	1.05	-	-	-	-
5	1.0	1.10	0.15	0.50	-0.15	0.80
6	0.95	1.05	-	-	-	-
7	0.95	1.05	-	-	-	-
8	1.0	1.10	0.10	0.35	-0.15	0.60
9	0.95	1.05	-	-	-	-
10	0.95	1.05	-	-	0.00	0.05
11	1.0	1.10	0.10	0.30	-0.10	0.50
12	0.95	1.05	-	-	0.00	0.05
13	1.0	1.10	0.12	0.40	-0.15	0.60
14	0.95	1.05	-	-	-	-
15	0.95	1.05	-	-	0.00	0.05
16	0.95	1.05	-	-	-	-
17	0.95	1.05	-	-	0.00	0.05
18	0.95	1.05	-	-	-	-
19	0.95	1.05	-	-	-	-
20	0.95	1.05	-	-	0.00	0.05
21	0.95	1.05	-	-	0.00	0.05
22	0.95	1.05	-	-	-	-
23	0.95	1.05	-	-	0.00	0.05
24	0.95	1.05	-	-	0.00	0.05
25	0.95	1.05	-	-	-	-
26	0.95	1.05	-	-	-	-
27	0.95	1.05	-	-	-	-
28	0.95	1.05	-	-	-	-
29	0.95	1.05	-	-	0.00	0.05
30	0.95	1.05	-	-	-	-

$$t_{1min} = t_{2min} = t_{3min} = t_{4min} = 0.90$$

$$t_{1max} = t_{2max} = t_{3max} = t_{4max} = 1.10$$

5.2. System Studies

System studies have been carried out on 5 different test systems which were illustrated in the preceeding section. Several case studies have been conducted on each test systems to confirm the different aspects of the proposed method.

Both the minimum cost problem and the minimum loss problem are tried to be solved with respect to the associated control variables. In case of minimum loss problem, an increase in the RPTL is not allowed. On the other hand, such an increase is not cared in case of minimum cost problem. Since the control variables are shared by sub-optimization modules, the resulting minimum cost/loss values may sometimes be larger than of those ones which assign all the control variables for a specific objective function.

5.2.1. 5-Bus System

Table 5.16 shows the solution to the minimum cost and the minimum loss problems for the 5-bus system. Note that one single iteration includes both the P-optimization and the Q-optimization modules.

The convergence characteristics are given in Table 5.17 and a comparison with of some other methods is illustrated in Table 5.18. It can easily be observed from Table 5.18 that, the minimum cost and the minimum loss values of the proposed method are in the same range of reported ones in the literature. By the way, it must be recognized that the values of Ref. [70] and Ref. [79] are obtained by handling all the control variables for a specified problem.

Table 5.16 Summary of the Results for the 5-Bus System

Variable		Limits		Initial State	Final States	
		Lower	Upper		Min. Cost Problem	Min. Loss Problem
P_{G1}	p.u.	0.30	1.20	0.8467	0.9514	0.8438
P_{G2}	p.u.	0.30	1.20	0.8000	0.6932	0.8000
Q_{G1}	p.u.	0.00	0.60	0.4826	0.3709	0.3748
Q_{G2}	p.u.	0.00	0.60	0.3043	0.4075	0.4005
V_1	p.u.	1.00	1.10	1.0700	1.0862	1.0853
V_2	p.u.	1.00	1.10	1.0500	1.0839	1.0857
V_3	p.u.	0.90	1.05	0.9845	1.0135	1.0142
V_4	p.u.	0.90	1.05	0.9637	0.9889	0.9893
V_5	p.u.	0.90	1.05	1.0245	1.0504*	1.0510*
δ_2	Deg.	-	-	1.799	-1.391	-0.1761
δ_3	Deg.	-	-	-4.737	-5.417	-4.693
δ_4	Deg.	-	-	-8.011	-8.174	-7.749
δ_5	Deg.	-	-	-2.814	-3.461	-2.851
Cost [$/h]				761.248	757.878	759.979
Loss [MW]				4.672	4.459	4.381
Number of Iterations					4	1
Maximum Power Mismatch					$<10^{-5}$	$<10^{-5}$
Maximum Constraint Violation					0.0004	0.001

Table 5.17 Convergence Characteristics for the 5-Bus System

ITERATION		Minimum Cost		Minimum Loss	
		Cost [$/h]	Loss [MW]	Cost [$/h]	Loss [MW]
Initial		761.2477	4.6725	761.2477	4.6725
1	Q	759.9787	4.3813	759.9787	4.3813
	P	757.8892	4.5419	759.9787	4.3813
2	Q	757.8810	4.5416	-	-
	P	757.8301	4.4757	-	-
3	Q	757.8275	4.4753	-	-
	P	757.8824	4.4605	-	-
4	Q	757.8776	4.4594	-	-
	P	757.8776	4.4594	-	-

P : After P-Optimization
Q : After Q-Optimization

Table 5.18 Comparison of 5-Bus System Studies

PROBLEM		A	B	C	D
Minimum Cost	n	4	-	6	-
	Cost [$/h]	757.88	760.95	757.72	757.71
	Loss [MW]	4.459	5.200	4.514	4.497
Minimum Loss	n	1	-	7	-
	Cost [$/h]	759.98	-	760.43	760.48
	Loss [MW]	4.381	-	4.390	4.390

n : Number of Iterations
A : Proposed Method
B : Method of Ref. [70]
C : Method of Ref. [77]
D : Method of Ref. [79]

5.2.2. IEEE 6-Bus System

Table 5.19 shows the solution to the minimum cost and the minimum loss problems for the IEEE 6-Bus System. Studies were carried out for two load levels. The convergence characteristics are given in Table 5.20 and a comparison with of some other methods is illustrated in Table 5.21. This sample system was mostly handled for minimum loss problem and therefore, comparison is done for only that problem.

This system yields a significant violation in real power transmission limits. Therefore, bus voltage angles are restricted between [-15.0-5.0] degrees. The minimum RPTL and the associated generation cost for the full load minimum loss problem are found to be favourably better than those reported in Ref. [43,60,62,63,67].

The number of iterations required for the full load minimumm cost problem is rather high because of the violation of the secure operating limits, reported above. On the other hand, the minimum generation cost was found to be 547.575 $, which compares favourably with the value of 588.609 $ reported in Ref. [73], for the same problem.

Table 5.19 Summary of the Results for the IEEE 6-Bus System.

Variable		Limits		Full Load			Half Load		
		Lower	Upper	Initial State	Final States Min.Cost	Final States Min.Loss	Initial State	Final States Min.Cost	Final States Min.Loss
P_{G1}	p.u.	0.10	1.00	0.9663	0.8578	0.9323	0.4557	0.3323	0.4440
P_{G1}	p.u.	0.10	1.00	0.5000	0.5902	0.5073	0.2500	0.3841	0.2554
Q_{G1}	p.u.	-0.20	1.00	0.3817	0.3966	0.3654	0.0485	-0.0048	0.0703
Q_{G2}	p.u.	-0.10	1.00	0.3482	0.1611	0.1900	0.2162	0.2673	0.1417
Q_{C4}	p.u.	0.00	0.05	0.0000	0.0500	0.0500	0.0000	0.0092	0.0170
Q_{C6}	p.u.	0.00	0.055	0.0000	0.0550	0.0550	0.0000	0.0035	0.0265
V_1	p.u.	1.00	1.10	1.0500	1.0990	1.0925	1.0500	1.0110	1.0329
V_2	p.u.	1.10	1.15	1.1000	1.1500	1.1500	1.1000	1.1500	1.1000
V_3	p.u.	0.90	1.00	0.8552*	1.0015*	1.0005*	0.9351	0.9774	1.0027*
V_4	p.u.	0.90	1.00	0.9525	1.0022*	1.0005*	1.0270*	0.9885	1.0027*
V_5	p.u.	0.90	1.00	0.9009	1.0005*	1.0010*	0.9962	0.9960	1.0001*
V_6	p.u.	0.90	1.00	0.9331	0.9842	0.9827	1.0173*	0.9988	0.9993
δ_2	Deg.	-15.00	5.0	-6.146	0.400	-3.178	-3.4047	0.8450	-2.0863
δ_3	Deg.	-15.00	5.0	-13.831	-10.543	-11.669	-6.1323	-4.7188	-6.0384
δ_4	Deg.	-15.00	5.0	-9.924	-7.922	-8.828	-4.6021	-3.6680	-4.6536
δ_5	Deg.	-15.00	5.0	-13.425	-9.348	-11.198	-6.2260	-3.7033	-5.8041
δ_6	Deg.	-15.00	5.0	-12.651	-9.871	-11.026	-5.8680	-4.6758	-5.7924
t_1		0.90	1.10	1.025	0.950	0.9554	1.0250	1.0192	0.9972
t_2		0.90	1.10	1.100	0.981	0.9835	1.1000	1.0202	0.9947
Cost [$/h]				638.510	547.575	604.318	180.659	141.587	175.0335
Loss [MW]				11.631	9.804	8.965	3.072	4.135	2.4507
Number of Iterations					7	2		3	3
Maximum Power Mismatch					$<10^{-5}$	$<10^{-5}$		$<10^{-5}$	$<10^{-5}$
Maximum Constraint Violation					0.0022	0.001		0	0.0027

Table 5.20 Convergence Characteristics for the IEEE 6-Bus System

ITERATION		FULL LOAD Minimum Cost Cost [$/h]	Loss [MW]	Minimum Loss Cost [$/h]	Loss [MW]	HALF LOAD Minimum Cost Cost [$/h]	Loss [MW]	Minimum Loss Cost [$/h]	Loss [MW]
Initial		638.51	11.631	638.51	11.631	180.66	3.072	180.66	3.072
1	Q	608.72	8.800	608.72	8.800	183.56	3.590	177.44	2.489
	P	549.04	9.778	603.24	8.866	142.63	4.100	175.17	2.511
2	Q	548.99	9.772	604.259	8.965	142.63	4.100	174.85	2.452
	P	547.98	9.794	604.318	8.965	141.65	4.133	175.01	2.451
3	Q	548.01	9.799	-	-	141.59	4.135	175.01	2.451
	P	547.34	9.814	-	-	141.59	4.135	175.03	2.451
4	Q	547.25	9.804	-	-	-	-	-	-
	P	547.70	9.794	-	-	-	-	-	-
5	Q	547.75	9.798	-	-	-	-	-	-
	P	547.65	9.800	-	-	-	-	-	-
6	Q	547.67	9.802	-	-	-	-	-	-
	P	547.57	9.804	-	-	-	-	-	-
7	Q	547.57	9.804	-	-	-	-	-	-
	P	547.57	9.804	-	-	-	-	-	-

Table 5.21 Comparison of IEEE 6-Bus System Studies (Minimum Loss Problem)

LOAD LEVEL		A	B	C	D	E	F
Full	Cost [$/h]	604.32	605.74	619.40	610.00	612.00	610.00
	Loss [MW]	8.965	8.930	9.757	8.943	9.222	9.900
1/2	Cost [$/h]	175.03	176.05	-	-	-	-
	Loss [MW]	2.451	2.240	-	-	-	-

A : Proposed Method
B : Method of Ref. [43]
C : Method of Ref. [60]
D : Method of Ref. [62]
E : Method of Ref. [63]
F : Method of Ref. [67]

5.2.3. 11-Bus System

Table 5.22 shows the solution to the minimum cost and the minimum loss problems for the 11-bus system. The convergence characteristics are given in Table 5.23 and a comparison with of some other methods is illustrated in Table 5.24.

The minimum generation cost and the associated loss reported in Ref. [74] are seem to be the best ones, for minimum cost problem. However, bus voltage magnitude for the sixth bus was reported to be 1.054 p.u. , which violates the upper limit more than the value of 1.0514 given in Table 5.22.

Table 5.22 Summary of the Results for the 11-Bus System

Variable		Limits Lower	Limits Upper	Initial State	Final States Min. Cost Problem	Final States Min. Loss Problem
P_{G1}	p.u.	0.30	1.20	0.7059	0.3910	0.5814
P_{G2}		0.30	1.20	0.5000	0.7217	0.7275
P_{G3}		0.30	1.20	0.5000	0.5820	0.3345
P_{G4}		0.30	1.20	0.5000	0.4930	0.5065
P_{G5}		0.30	1.20	0.5000	0.5203	0.5415
Q_{G1}	p.u.	-0.20	1.10	0.1521	0.2674	0.2229
Q_{G2}		-0.10	1.00	0.3040	0.2924	0.2764
Q_{G3}		-0.10	1.00	-0.2181*	-0.1329*	-0.0845
Q_{G4}		-0.10	1.00	0.2749	0.1714	0.1422
Q_{G5}		-0.10	1.00	0.2297	0.1241	0.1152
V_1	p.u.	1.00	1.10	1.0500	1.0761	1.0772
V_2		1.00	1.10	1.0400	1.0715	1.0714
V_3		1.00	1.10	1.0400	1.0755	1.0729
V_4		1.00	1.10	1.0500	1.0619	1.0609
V_5		1.00	1.10	1.0500	1.0479	1.0479
V_6		0.90	1.05	1.0456	1.0514*	1.0510*
V_7		0.90	1.05	0.9807	1.0073	1.0079
V_8		0.90	1.05	0.9671	0.9903	0.9908
V_9		0.90	1.05	0.9921	1.0022	1.0025
V_{10}		0.90	1.05	1.0008	1.0285	1.0290
V_{11}		0.90	1.05	1.0124	1.0394	1.0399
δ_2	Deg.	–	–	-2.518	2.916	0.145
δ_3		–	–	4.052	8.929	3.769
δ_4		–	–	0.147	4.483	1.329
δ_5		–	–	-0.792	3.188	1.193
δ_6		–	–	-1.108	3.087	0.438
δ_7		–	–	-6.907	-2.328	-4.702
δ_8		–	–	-6.790	-2.807	-5.008
δ_9		–	–	-4.359	-0.922	-2.822
δ_{10}		–	–	-5.685	-1.281	-3.557
δ_{11}		–	–	-4.265	-2.191	-3.273
Cost [$/h]				1261.557	1253.828	1254.526
Loss [MW]				10.577	10.802	9.147
Number of Iterations					4	5
Maximum Power Mismatch					$<10^{-4}$	$<10^{-4}$
Maximum Constraint Violation					0.0329	0.001

Table 5.23 Convergence Characteristics for the 11-Bus System

ITERATION		Minimum Cost		Minimum Loss	
		Cost [$/h]	Loss [MW]	Cost [$/h]	Loss [MW]
Initial		1261.557	10.5770	1261.557	10.5770
1	Q	1260.341	10.3094	1260.341	10.3095
	P	1255.240	11.0841	1254.739	10.3804
2	Q	1253.315	10.6062	1252.666	9.8698
	P	1254.345	10.9585	1253.947	9.2377
3	Q	1254.334	10.9598	1253.979	9.2375
	P	1253.817	10.8013	1254.047	9.2239
4	Q	1253.828	10.8025	1254.045	9.2243
	P	1253.828	10.8025	1254.152	9.2125
5	Q	-	-	1254.436	9.1572
	P	-	-	1254.526	9.1475

Table 5.24 Comparison of 11-Bus System Studies

Problem	Cost [$/h]			Loss [MW]		
	A	B	C	A	B	C
Min.Cost	1253.83	1251.96	1253.84	10.802	9.93	10.30
Min.Loss	1254.53	-	-	9.147	-	-

A : Proposed Method
B : Method of Ref.[74]
C : Method of Ref.[79]

5.2.4. IEEE 14-Bus System

Table 5.25 shows the solution to the minimum cost and the minimum loss problems for the IEEE 14-Bus test system. The convergence characteristics are given in Table 5.26 and the comparison with of some other methods is illustrated in Table 5.27. The values given are nearly the same with those reported in Ref. [69] and Ref. [70].

Table 5.25 Summary of the Results for the IEEE 14-Bus System

Variable		Limits		Initial State	Final States	
		Upper	Lower		Min.Cost	Min.Loss
P_{G1}	p.u.	0.50	2.00	2.1136	1.6303	1.4832
P_{G2}	p.u.	0.20	1.00	0.4000	0.6647	0.6499
P_{G6}	p.u.	0.20	1.00	0.2000	0.3839	0.5399
Q_{G1}	p.u.	-0.20	1.00	0.0301	0.0722	-0.0285
Q_{G2}	p.u.	-0.40	0.50	0.6665*	0.2409	0.4113
Q_{G6}	p.u.	-0.60	0.45	0.3080	0.4537*	0.3624
Q_{C3}	p.u.	-0.40	0.60	0.0000	0.0532	0.0427
Q_{C8}	p.u.	-0.60	0.45	0.0000	0.0445	0.0371
V_1	p.u.	1.00	1.10	1.0700	1.1000	1.0936
V_2	p.u.	1.00	1.10	1.0500	1.0791	1.0792
V_3	p.u.	0.95	1.10	0.9811	1.0252	1.0224
V_4	p.u.	0.95	1.10	1.0022	1.0462	1.0431
V_5	p.u.	0.95	1.10	1.0151	1.0642	1.0603
V_6	p.u.	1.00	1.10	1.0500	1.0629	1.0592
V_7	p.u.	0.95	1.10	1.0055	1.0591	1.0527
V_8	p.u.	0.95	1.10	1.0055	1.0664	1.0589
V_9	p.u.	0.95	1.10	0.9964	1.0404	1.0333
V_{10}	p.u.	0.95	1.10	0.9978	1.0364	1.0295
V_{11}	p.u.	0.95	1.10	1.0196	1.0454	1.0398
V_{12}	p.u.	0.95	1.10	1.0321	1.0475	1.0436
V_{13}	p.u.	0.95	1.10	1.0240	1.0415	1.0369
V_{14}	p.u.	0.95	1.10	0.9895	1.0225	1.0162
δ_2	Deg.	-	-	-4.398	-3.000	-2.859
δ_3	Deg.	-	-	-11.513	-9.623	-9.268
δ_4	Deg.	-	-	-8.924	-7.128	-6.563
δ_5	Deg.	-	-	-7.559	-5.932	-5.335
δ_6	Deg.	-	-	-11.786	-8.156	-6.174
δ_7	Deg.	-	-	-11.597	-9.361	-8.470
δ_8	Deg.	-	-	-11.597	-9.361	-8.470
δ_9	Deg.	-	-	-13.043	-10.588	-9.522
δ_{10}	Deg.	-	-	-13.109	-10.448	-9.221
δ_{11}	Deg.	-	-	-12.563	-9.440	-7.839
δ_{12}	Deg.	-	-	-12.739	-9.146	-7.235
δ_{13}	Deg.	-	-	-12.823	-9.342	-7.493
δ_{14}	Deg.	-	-	-14.032	-11.070	-9.673
t_1		0.90	1.10	0.9620	1.0618	1.0421
t_2		0.90	1.10	0.9780	0.9623	0.9628
t_3		0.90	1.10	0.9690	0.9600	0.9602
Cost [$/h]				1159.391	1134.444	1137.238
Loss [MW]				12.340	8.890	8.313
Number of Iterations					3	2
Maximum Power Mismatch					$<10^{-4}$	$<10^{-4}$
Maximum constraint Violation					0.0037	0

Table 5.26 Convergence Characteristics for the IEEE 14-Bus System

ITERATION		Minimum Cost		Minimum Loss	
		Cost [$/h]	Loss [MW]	Cost [$/h]	Loss [MW]
Initial		1159.391	12.3396	1159.391	12.3396
1	Q	1155.094	11.4129	1155.094	11.4129
	P	1136.254	9.0000	1138.486	8.6243
2	Q	1134.604	8.5961	1137.238	8.3135
	P	1134.423	8.8890	1137.238	8.3135
3	Q	1134.444	8.8900	-	-
	P	1134.444	8.8900	-	-

Table 5.27 Comparison of IEEE 14-Bus System Studies

Problem	Cost [$/h]			Loss [MW]		
	A	B	C	A	B	C
Min.Cost	1134.44	1134.86	1135.92	8.890	8.920	9.200
Min.Loss	1137.24	-	-	8.313	-	-

A : Proposed Method
B : Method of Ref. [69]
C : Method of Ref. [70]

5.2.5. IEEE 30-Bus System

IEEE 30-Bus Test System is the most common one for power system optimization applications. The solution to the minimum cost and the minimum loss problems are given in Table 5.28,a and Table 5.28,b. The convergence characteristics are illustrated in Table 5.29. The comparison with of some other methods are given in Table 5.30.

The minimum generation cost for the minimum cost problem and the minimum RPTL for the minimum loss problem are found to be better than those reported in Ref. [48, 67,73,85,88]. In addition, the required iteration num-numbers are also rather small.

Table 5.28,a Resulted Bus Voltages for the IEEE 30-Bus Test System.

Bus No	Limits		Initial State		Final States			
					Min.Cost		Min.Loss	
	V_{min}	V_{max}	V	δ	V	δ	V	δ
	p.u.		p.u.	degree	p.u.	degree	p.u.	Degree
1	1.00	1.10	1.050	0.000	1.085	0.000	1.056	0.000
2	1.00	1.10	1.040	-1.766	1.056	-3.294	1.058	-2.343
3	0.95	1.05	1.025	-3.885	1.049	-5.443	1.050	-3.910
4	0.95	1.05	1.020	-4.655	1.042	-6.578	1.049	-4.692
5	1.00	1.10	1.010	-6.506	1.034	-10.107	1.040	-6.652
6	0.95	1.05	1.013	-5.397	1.034	-7.686	1.048	-5.316
7	0.95	1.05	1.003	-6.366	1.026	-9.167	1.036	-6.348
8	1.00	1.10	1.010	-5.618	1.030	-7.935	1.053	-5.324
9	0.95	1.05	0.984	-6.994	1.018	-10.047	1.030	-6.303
10	0.95	1.05	0.961	-9.174	1.000	-11.906	0.914	-8.634
11	1.00	1.10	1.050	-4.688	1.078	-8.961	1.083	-3.096
12	0.95	1.05	0.995	-8.656	1.018	-11.241	1.020	-8.531
13	1.00	1.10	1.050	-7.120	1.061	-10.353	1.055	-7.638
14	0.95	1.05	0.976	-9.665	1.004	-12.227	1.008	-9.454
15	0.95	1.05	0.968	-9.692	1.000	-12.370	1.007	-9.586
16	0.95	1.05	0.973	-9.174	1.004	-11.826	1.011	-8.892
17	0.95	1.05	0.959	-9.413	0.997	-12.124	1.009	-8.961
18	0.95	1.05	0.952	-10.318	0.989	-12.998	0.999	-10.044
19	0.95	1.05	0.947*	-10.469	0.986	-13.160	0.997	-10.107
20	0.95	1.05	0.949*	-10.210	0.990	-12.937	1.002	-9.838
21	0.95	1.05	0.947*	-9.717	0.990	-12.439	1.004	-9.225
22	0.95	1.05	0.948*	-9.706	0.991	-12.431	1.005	-9.230
23	0.95	1.05	0.949*	-10.084	0.993	-12.869	1.003	-10.032
24	0.95	1.05	0.933*	-10.216	0.984	-12.977	0.998	-10.010
25	0.95	1.05	0.928*	-10.254	0.991	-12.763	1.005	-9.958
26	0.95	1.05	0.909*	-10.760	0.973	-13.207	0.987	-10.388
27	0.95	1.05	0.935*	-9.964	1.004	-12.347	1.017	-9.649
28	0.95	1.05	1.006	-5.856	1.028	-8.222	1.042	-5.773
29	0.95	1.05	0.913*	-11.442	0.995	-13.969	1.010	-11.247
30	0.95	1.05	0.900*	-12.513	0.979	-14.725	0.993	-11.975

Table 5.28,b Summarry of the results for the IEEE 30-Bus Test System.

Variable		Limits Upper	Limits Lower	Initial State	Final States Min.Cost	Final States Min.Loss
P_{G1}	p.u.	0.50	2.00	0.9911	1.8144	1.1088
P_{G2}		0.20	0.80	0.8000	0.5145	0.5029
P_{G5}		0.15	0.50	0.5000	0.2028	0.5000
P_{G8}		0.10	0.35	0.2000	0.1803	0.3500
P_{G11}		0.10	0.30	0.2000	0.1000	0.3000
P_{G13}		0.12	0.40	0.2000	0.1200	0.1200
Q_{G1}	p.u.	-0.20	2.00	0.0500	0.2739	-0.2925*
Q_{G2}		-0.20	1.00	0.2800	-0.0116	0.3521
Q_{G5}		-0.15	0.80	0.2215	0.3798	0.2614
Q_{G8}		-0.15	0.60	0.2512	0.2429	0.4123
Q_{G11}		-0.10	0.50	0.3350	0.3100	0.2868
Q_{G13}		-0.15	0.60	0.4182	0.3236	0.2604
Q_{C10}		0.00	0.05	0.0000	0.0108	0.0140
Q_{C12}		0.00	0.05	0.0000	0.0000	0.0000
Q_{C15}		0.00	0.05	0.0000	0.0139	0.0200
Q_{C17}		0.00	0.05	0.0000	0.0131	0.0188
Q_{C20}		0.00	0.05	0.0000	0.0201	0.0288
Q_{C21}		0.00	0.05	0.0000	0.0203	0.0293
Q_{C23}		0.00	0.05	0.0000	0.0242	0.0330
Q_{C24}		0.00	0.05	0.0000	0.0335	0.0466
Q_{C29}		0.00	0.05	0.0000	0.0387	0.0413
t_1		0.90	1.10	1.0500	1.0401	1.0377
t_2		0.90	1.10	1.0500	1.0404	1.0382
t_3		0.90	1.10	1.0500	1.0522	1.0540
t_4		0.90	1.10	1.0500	1.0100	1.0106
Cost [$/h]				901.6391	802.0598	882.4489
Loss [MW]				5.8626	9.7827	4.7789
Number of Iterations					2	3
Max.Power Mismatch					$<10^{-4}$	$<10^{-4}$
Max.Constraint Violation					0	0.0925

Table 5.29 Convergence Charecteristics for the IEEE 30-Bus System

ITERATION		Minimum Cost		Minimum Loss	
		Cost [$/h]	Loss [MW]	Cost [$/h]	Loss [MW]
Initial		901.6391	5.6882	901.6391	5.6882
1	Q	900.5922	5.3238	900.5922	5.3238
	P	803.1304	10.1100	808.8611	11.6271
2	Q	802.0598	9.7827	807.6384	11.2717
	P	802.0598	9.7827	884.1073	5.3567
3	Q	–	–	882.4499	4.7789
	P	–	–	882.4489	4.7780

Table 5.30 Comparison of IEEE 30-Bus System Studies

PROBLEM		A	B	C	D	E	F
Min. Cost	Cost [$/h]	802.06	803.89	–	804.85	802.40	802.39
	Loss [MW]	9.783	9.071	–	10.486	9.480	9.427
Min. Loss	Cost [$/h]	882.45	900.00	906.39	–	–	–
	Loss [MW]	4.779	4.821	7.442	–	–	–

A : Proposed Method
B : Method of Ref. [48]
C : Method of Ref. [67]
D : Method of Ref. [73]
E : Method of Ref. [85]
F : Method of Ref. [88]

CONCLUSIONS

This thesis has presented two new formulations for sub-optimization modules which provide fast convergence and less computer storage.The advantages of the method are as follows:

1. The method is capable of minimizing both the HFC and the RPTL together as one problem or seperately. This is achieved by decomposing the process into two sub-optimization modules.
2. Since the modules can be solved independently, the convergence and the optimal values are not considerably effected from the starting point.
3. P-optimization module can be either used for cost minimization or loss minimization without any additional computational efort.
4. The number of iterations needed to obtain the solution is small and does not vary with the size of the system.
5. During the optimization phase all intermediate results after the first iteration are feasible.
6. The test results show that the method presented yields a more accurate and optimal solution.

On the other hand; in order to achieve less computation time, inefficiencies related to the penalty factors must be overcome and fast decoupled load flow must be adopted for the problem. Moroeover, IEEE 118-Bus bigger test systems must also be tested.

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APPENDIX A

KUHN-TUCKER CONDITIONS

The most important theoretical result in the field of nonlinear programming are the Kuhn-Tucker (K-T) conditions. They form the basis for the development of many computational algorithms. In addition, the stopping criteria for many algorithms are derived directly from them.

The geometrical interpretation of the K-T conditions can be stated by means of a cone [51]. A cone is a set of points ,R, such that,

$$x \in R \qquad (cx) \in R \quad , \qquad c \geq 0 \tag{A.1}$$

A convex cone is a cone which is a convex set. It can easily be derived that the set of all nonnegative linear combinations of a finite set of vectors as a convex cone, i.e., that the

$$R = \{x \mid x = c_1 x_1 + \ldots\ldots + c_n x_n \ , \ c_i \geq 0 \ , \ i=1,2,\ldots,n\} \tag{A.2}$$

is a convex cone. The vectors $x_1, x_2, \ldots, x_n$ are called the generators of the cone.

In order to minimize a function $f(x)$, subject to

$$g_i(x) \leq 0 \quad , \ i=1,2,\ldots,n \tag{A.3}$$

at any point x^k , ∇f must lie within the cone generated by the negative gradients of the binding constraints g [50-52]. Algebrically,

$$\nabla f(x^k) = \sum_{i \in B} a_i^k \ (-\nabla g_i(x^k)) \quad , \quad a_i^k \geq 0 \tag{A.4}$$

and B is the set of indices of the binding constraints. These results may be restated to include all type of constraints by defining the coefficient,

$$a_i^k \begin{cases} 0 & \text{if} \quad g_i(x^k) < 0 \\ \geq 0 & \text{if} \quad g_i(x^k) = 0 \end{cases} \tag{A.5}$$

The K-T conditions are closely related to the classical Lagrange Multiplier results. This relation and a different statement of the conditions are given in Ref. [5], Appendix E.

APPENDIX B

FUNCTION MINIMIZATION BY CONJUGATE GRADIENTS

Let us consider the minimization by successive linear searches of a quadratic function,

$$f(x)= a + B^T x + \frac{1}{2} x^T A x \tag{B.1}$$

where A is a positive definite and symmetric. Let x^* minimize $f(x)$. Then,

$$\nabla f(x^*) = B + A x^* \tag{B.2}$$

Given a starting point x_o and a set of linearly independent directions $r_o, r_1, \ldots, r_n$, constants β_i can be found such that,

$$x^* = x^o + \sum_{i=1}^{n-1} \beta_i r_i \tag{B.3}$$

If the directions r_i are A-conjugate, i.e., satisfy

$$r_i^T A r_j = 0 \quad , i \neq j \quad , i, j = o, 1, \ldots, n-1 \tag{B.4}$$

and none are zero, then r_i are easily shown to be linearly independent and the β_i can be determined from Eq. (B.3) as follows [21,51]:

$$r_j^T A x^* = r_j^T A x_o + \sum_{i=1}^{n-1} \beta_i r_j^T A r_i \tag{B.5}$$

$$r_j^T A x^* = r_j^T A x_o + \beta_j r_j^T A r_j \tag{B.6}$$

using Eq.(B.2)

$$\beta_j = - (B + A x_o)^T \frac{r_j}{r_j^T A r_j} \tag{B.7}$$

This sequential process is called Conjugate Directions and leads, in n steps, to the minimum x^*. The method of conjugate gradients is one of the simple conjugate direction minimization techniques and requires computation of the gradient of $f(x)$ and the storage of only one additional vector. The algorithm is,

$$x^{i+1} = x^i + c_i r^i \tag{B.8}$$

where

$$r^i = - \nabla f(x^i) + \beta_i r^{i-1} \tag{B.9}$$

Due to Fletcher and Reeves [21]

$$\beta_i = \frac{\| \nabla f(x^i) \|^2}{\| \nabla f(x^{i-1}) \|^2} \tag{B.10}$$

with an arbitrary x_o and $\beta_o = 0$.

This process is guaranteed to locate the minimum of any quadratic function of n arguments in at most n iterations. For function which are not quadratic, the process is iterative rather than n-steps and a test for convergence is required.

BIOGRAPHY

Aydoğan Özdemir was born in Artvin in January, 1957. He had completed his primary education in Artvin and secondary education at Diyarbakır Anadolu Lycee in Diyarbakır. He received the BS and MS degrees from the Faculty of Electrical Engineering of Technical University of İstanbul in 1980 and 1982, respectively. After completing his military service, he entered to the Ph.D program on Electric Power Engineering in the Institute of Science and Technology of Technical University of İstanbul. He has been working at the High Voltage Laboratory of the same university since 1980. He is married and has got two children.