

ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL

**A STUDY ON STATIC AND DYNAMIC BUCKLING
ANALYSIS OF THIN WALLED COMPOSITE CYLINDRICAL SHELLS**



M.Sc. THESIS

Cansu ÖZGEN

Department of Aeronautical and Astronautical Engineering

Aeronautical and Astronautical Engineering Programme

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**Cansu ÖZGEN
(511171148)**

Department of Aeronautical and Astronautical Engineering

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Thesis Advisor: Prof. Dr. Vedat Ziya DOĞAN

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STATİK VE DİNAMİK BURKULMA ANALİZİNİ ÜZERİNE BİR ÇALIŞMA**

YÜKSEK LİSANS TEZİ

**Cansu ÖZGEN
(511171148)**

Uçak ve Uzay Mühendisliği Anabilim Dalı

Uçak ve Uzay Mühendisliği Programı

Tez Danışmanı: Prof. Dr. Vedat Ziya DOĞAN

JANUARY 2022

Cansu ÖZGEN, a M.Sc. student of ITU Graduate School student ID 511171148, successfully defended the thesis entitled “A STUDY ON STATIC AND DYNAMIC BUCKLING ANALYSIS OF THIN WALLED COMPOSITE CYLINDRICAL SHELLS”, which he/she prepared after fulfilling the requirements specified in the associated legislations, before the jury whose signatures are below.

Thesis Advisor : **Prof. Dr. Vedat Ziya DOĞAN**
Istanbul Technical University

Jury Members : **Prof. Dr. Halit Süleyman TÜRKMEN**
Istanbul Technical University

Asst. Prof. Dr. Özgür DEMİR
Yıldız Technical University

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To my family,



FOREWORD

I would like to express my sincere appreciation to my thesis supervisor Prof. Dr. Vedat Ziya Doğan for his support and guidance. I am also thankful to my family for their support, encouragement and understanding in my whole life. Finally, thanks to my friends for their valuable support.

January 2022

Cansu ÖZGEN
(Astronautical Engineer)



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ABBREVIATIONS

EOM	: Equation of Motion
FEM	: Finite Element Method
FEA	: Finite Element Analysis
LVDT	: Linear Variable Differential Transducer
SPLA	: Single Perturbation Load Approach
CFRP	: Composite Fiber Reinforced Polymer



SYMBOLS

$\epsilon_x, \epsilon_y, \gamma_{xy}$	: Strain component
$\kappa_x, \kappa_y, \kappa_{xy}$	: Curvature component
λ	: Buckling load factor
λ_{cr}	: Critical buckling load factor
N_x, N_y, N_{xy}	: Internal Loads
N_{x0}, N_{y0}	: Reference Loads in x and y direction
N_{xy0}	: Reference Loads in xy direction
Q_x, Q_y	: Shear Forces
α, β, c_1, c_2	: Parameters describing buckling shape of a shell
D	: Bending stiffness
t	: Time
u, v, w	: Displacement Vector Components
ξ, η, τ	: Normalized parameters of x, θ and t
R, a	: Radius
h	: Thickness
L/D	: Length to diameter ratio
w_i	: Initial imperfection
F	: Airy's stress function
E	: Elastic modulus
$P_c,$	: Static buckling load of perfect structure
$P_s,$	: Static buckling load of imperfect structure
$P_D,$	: Dynamic buckling load of imperfect structure
$[K_\sigma]$	: Geometric stiffness matrix
$[K]$	: Global stiffness matrix
$[R]$	: External load
F^{int}	: Internal force
F^{ext}	: External force
$\{R\}$	: Displacement vector
θ	: Lamina orientation angle

$[S]$	: Stiffness matrix
A_{ij}, B_{ij}, D_{ij}	: Stiffness matrix component
ρ	: Density
x	: Displacement
$\bar{x}$	: Initial position
ϵ	: x/L
z	: $\epsilon/\bar{\epsilon}$
m	: Number of half waves in circumferential direction
n	: Number of waves in axial direction
N'_x, N'_y, N'_{xy}	: Change force due to buckling



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A STUDY ON STATIC AND DYNAMIC BUCKLING ANALYSIS OF THIN WALLED COMPOSITE CYLINDRICAL SHELLS

SUMMARY

Thin-walled structures have many uses in many industries. Examples of these fields include: aircraft, spacecraft and rockets can be given. The reason for the use of thin-walled structures is that they have a high strength weight ratio. In order to define a cylinder as thin-walled, the ratio of radius to thickness must be more than 20, and one of the problems encountered in the use of such structures is the problem of buckling. It is possible to define the buckling as a state of instability in the structure under compressive loads. This state of instability can be seen in the load displacement graph as the curve follows two different paths. The possible behaviors; snap through or bifurcation behavior. Compressive loading that cause buckling; there may be an axial load, torsional load, bending load, external pressure. In addition to these loads, buckling may occur due to temperature change. Within the scope of this thesis, the buckling behavior of thin-walled cylinders under axial compression was examined.

The cylinder under the axial load indicates some displacement. When the amount of load applied reaches critical level, the structure moves from one state of equilibrium to another. After some point, the structure shows high displacement behavior and loses stiffness. The amount of load that the structure will carry decreases considerably, but the structure continues to carry loads. The behavior of the structure after this point is called post-buckling behavior.

The critical load level for the structure can be determined by using finite elements method. Linear eigenvalue analysis can be performed to determine the static buckling load. However, it should be noted here that eigenvalue-eigenvector analysis can only be used to make an approximate estimate of the buckling load and input the resulting buckling shape into nonlinear analyses as a form of imperfection. In addition, it can be preferred to change parameters and compare them, since they are cheaper than other types of analysis. Since the buckling load is highly affected by the imperfection, nonlinear methods with geometric imperfection should be used to estimate a more precise buckling load. It is not possible to identify geometric imperfection in linear eigenvalue analysis. Therefore, a different type of analysis should be selected in order to add imperfection. For example, an analysis model which includes imperfection can be established with the Riks method as a nonlinear static analysis type. Unlike the Newton-Rapson method, the Riks method is capable of backtracking in curves. Thus, it is suitable for use in buckling analysis. In Riks analysis, it is recommended to add imperfection in contrast to linear eigenvalue analysis. Because if the imperfection is added, the problem will be bifurcation problem instead of limit load problem and sharp turns in the graph can cause divergence in analysis. Another nonlinear method of static

phenomena is called quasi-static analysis which is used dynamic solver. The important thing to note here is that the inertial effects should be too small to be neglected in the analysis. For this purpose, kinetic energy and internal energy should be compared at the end of the analysis and kinetic energy should be ensured to be negligible levels besides internal energy. Also, if the event is solved in the actual time length, this analysis will be quite expensive. Therefore, the time must be scaled. In order to scale the time correctly, frequency analysis can be performed first and the analysis time can be determined longer than the period corresponding to the first natural frequency. For three analysis methods mentioned within this study, validation studies were carried out with the examples in the literature. As a result of each type of analysis giving consistent results, the effect of parameters on static buckling load was examined, while linear eigenvalue analysis method was used because it was also sufficient for cheaper analysis method and comparison studies. While displacement-controlled analyses were carried out in the static buckling analyses mentioned, load-controlled analyses were performed in the analyses for the determination of dynamic buckling force. As a result of these analyses, they were evaluated according to different dynamic buckling criteria. There are some of the dynamic buckling criteria; Volmir criterion, Budiansky-Roth criterion, Hoff-Bruce criterion, etc. When Budiansky-Roth criterion is used, the first estimated buckling load is applied to the structure and displacement - time graph is drawn. If a major change in displacement is observed, it can be assumed that the structure is dynamically buckled. For Hoff-Bruce criterion, the speed - displacement graph should be drawn. If this graph is not focused in a single area and is drawn in a scattered way, it is considered that the structure has moved to the unstable area. As in static buckling analyses, dynamic buckling analyses were primarily validated with a sample study in the literature.

After the analysis methods, the numerical studies were carried out on the effect of some parameters on the buckling load. First, the effect of the stacking sequence of composite layers on the buckling load was examined. In this context, a comprehensive study was carried out, both from which layer has the greatest effect of changing the angle and which angle has the highest buckling load. In addition, the some angle combinations are obtained in accordance with the angle stacking rules found in the literature. For those stacking sequences, buckling forces are calculated with both finite element analyses and analytically. In addition, comparisons were made with different materials. Here, the buckling load is calculated both for cylinders with different masses of the same thickness and for cylinders with different thicknesses with the same mass. Here, the highest force value for cylinders with the same mass is obtained for a uniform composite. In addition, although the highest buckling force was obtained for steel material in the analysis of cylinders of the same thickness, when we look at the ratio of buckling load to mass, the highest value was obtained for composite material. In addition, the ratio of length to diameter and the effect of thickness were also examined. Here, as the length to diameter ratio increases, the buckling load decreases. As the thickness increases, the buckling load increases with the square of the thickness. In addition to the effect of the length to diameter ratio and the effect of thickness, the loading time and the shape of the loading profile are also known in dynamic buckling analysis.

In addition, the critical buckling force is affected by imperfections in the structure, which usually occur during the production of the structure. How sensitive the structures are to the imperfection may vary depending on the different parameters.

The imperfection can be divided into three different groups as geometric, material and loading. Cylinders under axial load are particularly affected by geometric imperfection. The geometric imperfection can be defined as how far the structure is from a perfect cylindrical structure. It is possible to determine the specified amount of deviation by different measurement methods. Although it is not possible to measure the amount of imperfection for all structures, an idea can be gained about how much imperfection is expected from the studies found in the literature. Both the change in the buckling load on the measured cylinders and the imperfection effect of the buckling load can be measured by adding the measured amount of imperfection to the buckling load calculations. In cases where the amount of imperfection cannot be measured, the finite element can be included in the analysis model as an eigenvector imperfection obtained from linear buckling analysis and the critical buckling load can be calculated for the imperfect structure using nonlinear analysis methods. In this study, studies were carried out on how imperfection sensitivity changes under both static and dynamic loading with different parameters. These parameters are the length-to-diameter ratio, the effect of the stacking sequence of the composite layers and the added imperfection shape. The most important result obtained in the study on imperfection sensitivity is that the effect of the imperfection on the buckling load is quite high. Even geometric imperfection equal to thickness can cause the buckling load to drop by up to half.



İNCE CİDARLI KOMPOZİT SİLİNDİRİK KABUKLARIN STATİK VE DİNAMİK BURKULMA ANALİZİNİ ÜZERİNE BİR ÇALIŞMA

ÖZET

İnce cidarlı yapılar, endistrüride birçok kullanım alanına sahiptir. Bu alanlara örnek olarak; uçaklar, uzay araçları ve roketler verilebilir. İnce cidarlı yapıların kullanılma nedeni yüksek mukavemet ağırlık oranına sahip olmasıdır. Bir silindiri ince cidarlı olarak tanımlayabilmemiz için yarı çapının kalınlığına oranının 20'den fazla olması gerekir ve bu tip yapıların kullanımında karşılaşılan sorunlardan bir tanesi burkulma problemidir. Burkulmayı baskı yükleri altındaki yapıda gerçekleşen kararsızlık hali olarak tanımlamamız mümkündür. Bu kararsızlık hali yük deplasman grafiğinde eğrinin iki farklı yol izlemesi şeklinde görülebilir. Bahsi geçen davranışlar; doğrudan geçme (ing. snap through) ya da çatallaşma (ing. bifurcation) davranışlarıdır. Burkulmaya neden olan baskı yükleri; eksensel basma yükü, burulma yükü, bükülme yükü, dış veya hidrostatik basınç olabilir. Bu yüklere ek olarak sıcaklık değişiminden kaynaklı olarak da burkulma gerçekleşebilmektedir. Bu tez kapsamında, eksensel basma altındaki ince cidarlı silindirlerin burkulma davranışı incelenmiştir.

Eksenel yük altındaki silindir, bir miktar yer değiştirme gösterir. Uygulanan yük miktarı kritik seviyeye geldiğinde yapı bir denge durumundan başka bir denge durumuna geçer. Bu esnada, yapı yüksek yer değiştirme davranışı gösterir ve katılık kaybına uğrar. Bu noktadan sonra yapının taşıyacağı yük miktarı oldukça düşse de yapı yük taşımaya devam eder. Yapının bu noktadan sonraki davranışı burkulma sonrası davranış olarak isimlendirilir.

Burkulma yükünün tespiti için sonlu elemanlar yöntemi kullanılarak yapı için kritik yük seviyesi belirlenebilir. Statik burkulma yükünün belirlenmesi için doğrusal özdeğer analizi yapılabilir. Fakat, burada dikkat edilmesi gereken husus, özdeğer-özvektör analizi yalnızca burkulma yükü hakkında yaklaşık bir tahmin yapabilmek ve elde edilen burkulma şeklinin doğrusal olmayan analizlere kusur şekli olarak girdi olması amacıyla kullanılabilir. Ek olarak, diğer analiz türlerine göre daha ucuz olması bakımından parametre değiştirip karşılaştırma yapmak için de tercih edilebilir. Burkulma yükü kusurdan oldukça etkilendiği için, daha hassas bir burkulma yükünün tahmini için geometrik kusur eklenmiş doğrusal olmayan yöntemler kullanılmalıdır. Doğrusal özdeğer analizlerinde geometrik kusur tanımlamak mümkün değildir. Bu sebeple kusur ekleyebilmek için farklı bir analiz türü seçilmelidir. Örneğin, doğrusal olmayan statik analiz türü olarak Riks yöntemi ile kusur için analiz modeli kurulabilir. Riks yöntemi Newton-Rapson yönteminden farklı olarak eğrilerdeki geri dönmeleri takip etme yeteneğine sahiptir. Böylece

burkulma analizlerinde kullanılmaya uygundur. Riks analizlerinde doğrusal özdeğer analizlerinin tersine kusur eklenmesi tavsiye edilir. Çünkü kusur eklendiği takdirde problem limit yük problemi yerine çatallaşma problemi olacaktır ve grafikteki keskin dönüşler analizde ıraksamaya neden olabilmektedir. Bir başka doğrusal olmayan statik analiz yöntemi de sanki statik analiz olarak adlandırılan ve dinamik analiz çözücüsü kullanarak statik olgunun analiz edilmesidir. Burada dikkat edilmesi gereken durum ataletsel etkilerin analizde ihmal edilebilecek kadar küçük olmasıdır. Bu amaçla analiz sonunda kinetik enerji ve iç enerji karşılaştırılmalı ve kinetik enerjinin iç enerji yanında ihmal edilebilir seviyelerde olduğundan emin olunmalıdır. Ayrıca, eğer olayı gerçek süresi uzunluğunda çözdürülürse, bu analiz oldukça pahalı olacaktır. Bu sebeple, süre ölçeklendirilmelidir. Süreyi doğru bir şekilde ölçeklendirmek için ilk olarak frekans analizi yapıp, ilk doğal frekansa karşılık gelen periyottan uzun olacak şekilde analiz süresi belirlenebilir. Bu çalışma kapsamında bahsedilen üç analiz yöntemi için literatürde bulunan örneklerle doğrulama çalışması yapılmıştır. Her bir analiz türünün tutarlı sonuçlar vermesi sonucunda parametrelerin statik burkulma yükü üzerindeki etkisi incelenirken, daha ucuz bir analiz yöntemi ve karşılaştırma çalışmaları için de yeterli olması sebebiyle, doğrusal özdeğer analiz yöntemi kullanılmıştır. Bahsedilen sanki statik analizlerde yer değiştirme kontrollü analizler yapılırken, dinamik burkulma kuvvetinin tespiti için yapılan analizlerde yük kontrollü analizler yapılmıştır. Bu analizler sonucunda farklı dinamik burkulma kriterlerine göre değerlendirilmiştir. Dinamik burkulma kriterlerinden bazıları; Volmir kriteri, Budiansky-Hutchinson kriteri, Hoff-Bruce kriteri vb. Bu kriterler değerlendirirken ilk tahmini burkulma yükü yapıya uygulanır ve yer değiştirme - zaman grafiği çizdirilir. Eğer yer değiştirmede büyük bir değişim görülürse, yapı dinamik olarak burkulmaya uğradığı farz edilebilir. Bir diğer kritere göre ise hız - yer değiştirme grafiği çizdirilir. Eğer bu grafik tek bir alanda toplanmayıp dağınık bir şekilde çizdiriliyorsa yapının kararsız bölgeye geçtiği kabul edilir. Statik burkulma analizlerinde olduğu gibi dinamik burkulma analizlerinde de öncelikle literatürde örnek bir çalışma ile doğrulama çalışması yapılmıştır.

Analiz metodlarının ardından bazı parametrelerin burkulma yüküne etkisi hakkında nümerik çalışmalar yapılmıştır. İlk olarak kompozit katmanların serim açılarının burkulma yüküne etkisi incelenmiştir. Bu kapsamda, gerek hangi katmanın açısını değiştirmenin etkisinin en fazla olduğundan gerekse hangi açının daha yüksek burkulma yüküne sahip olduğuna kadar kapsamlı bir çalışma yapılmıştır. Bunlara ek olarak, literatürde bulunan açılı serim kurallarına uygun olacak şekilde elde edilen serim açılı kombinasyonları hem sonlu eleman analizleri ile hem de analitik olarak hesaplanan burkulma kuvvetleri yardımıyla hesaplanmıştır. Ayrıca, farklı malzemeler ile karşılaştırma yapılmıştır. Burada burkulma yükü hem aynı kalınlık farklı kütlelere sahip silindirler için hem de aynı kütleli farklı kalınlığa sahip silindirler için yapılmıştır. Burada aynı kütleli olan silindirler için en yüksek kuvvet değeri tekyönlü kompozit için elde edilmiştir. Ayrıca, aynı kalınlığa sahip silindirler için yapılan analizlerde en yüksek burkulma kuvveti çelik malzeme için elde edilmesine rağmen burkulma yükünün kütleyle oranına baktığımızda yine en yüksek değer tekyönlü kompozit malzeme için elde edilmiştir. Ayrıca, boy - çap oranı ve kalınlığın etkisi de incelenmiştir. Burada boy-çap oranı arttıkça burkulma yükü azalmıştır. Kalınlık arttıkça ise kalınlığın karesi ile burkulma yükü artmaktadır. Dinamik burkulma analizlerinde ise boy-çap oranının etkisi ve kalınlığın etkisine ek olarak, yükleme süresi ve yükleme profilinin şekli de incelenmiştir.

Ek olarak, kritik burkulma kuvveti yapıda bulunan ve genellikle yapının üretimi sırasında ortaya çıkan kusurlardan etkilenmektedir. Yapıların kusura ne kadar hassas olacağı farklı parametrelere bağlı olarak değişiklik gösterebilmektedir. Kusur; geometrik, malzeme ve yükleme olmak üzere üç farklı grupta ele alınabilir. Eksenel yük altındaki silindirler özellikle geometrik kusurdan oldukça etkilenmektedir. Geometrik kusur yapının kusursuz bir silindirik şekilden ne kadar uzaklaştığı olarak tanımlanabilir. Belirtilen sapma miktarını farklı ölçüm yöntemleri ile saptamak mümkündür. Her ne kadar tüm yapılar için bu kusur miktarını ölçmek mümkün olmasa da literatürde bulunan çalışmalardan ne kadar bir kusur beklendiği hakkında fikir sahibi olunabilir. Gerek kusur miktarı ölçülmüş silindirlerdeki burkulma yükündeki değişim gerekse ölçülmüş kusur miktarını burkulma yükü hesaplarına katarak burkulma yükündeki kusur etkisi ölçülebilir. Kusur miktarının ölçülemediği durumlarda ise doğrusal burkulma analizlerinden elde edilen öz vektör kusur olarak sonlu eleman analiz modeline dahil edilebilir ve bu model ile doğrusal olmayan analiz yöntemleri kullanılarak kritik burkulma yükü kusur içeren yapı için hesaplanabilir. Bu çalışmada, kusur hassasiyetinin farklı parametrelerle hem statik hem de dinamik yükleme altında nasıl değiştiği hakkında çalışma yapılmıştır. Bu parametreler; boy-çap oranı, kompozit katmanların serim açıları ve eklenen kusur şeklinin etkisidir. Kusur hassasiyeti hakkında yapılan çalışmada elde edilen en önemli sonuç kusurun burkulma yüküne etkisinin oldukça fazla olmasıdır. Kalınlık ile eşit miktardaki geometrik kusur bile burkulma yükünün yarısına kadar düşmesine sebep olabilir.



1. INTRODUCTION

Thin walled structures are used in aircraft, ships, rockets and missiles thanks to their high strength to weight ratio. Thin walled structures can be defined as a structure having one of its dimensions very small compared to the other two dimensions. The ratio of the smallest dimension to the other dimensions should be less than $1/20$. Since the buckling is defined as an instability problem that occurs under compressive loading which is applied to the thin walled structures, usage of the thin walled structures can cause buckling of the structure. The compressive load mentioned here can be axial compression, external pressure, or bending moment for cylindrical shells. Another feature of that phenomenon is that it occurs suddenly. Therefore, it is hard to estimate it. Moreover, the imperfect structure can buckle much more easier than the perfect structure. Since there is no perfect structure in the real life, it is important to consider that the effect of imperfection on the buckling load is significant. When all those concerns are considered, the need of finding a solution to buckling of slender structures arises. To deal with this problem, some small changes in the structure can help the structure not to buckle. Those changes can be adding stiffeners, changing the stacking sequence of the composite layers, or using hybrid structure which uses more than one material.

The buckling can occur under both static and dynamic compressive loading. The difference of the static and dynamic buckling is the duration of the loading. For the dynamic loading, the inertial effects become significant. The critical dynamic buckling load can be higher or lower than the critical static buckling load. In real life, the static buckling can be seen in storage tanks. For the dynamic buckling, suddenly applied thrust of rocket engine ignition can be example.

Until 1980s, aluminum materials are commonly used in many fields. However, with the improvement in the material science and the need for more strength with the less mass, advanced material such as composites started to widely used. Reducing the weight is important for the aviation industry to reduce fuel consumption. Also, layered

composites have provided the design flexibility with adjusting the stacking sequences as the strong properties in desired direction. Moreover, the composites are the chemical and corrosion resistant which are important features for underwater applications. In addition, it can provide thermal and noise isolation. Although the composite materials seem costly, it sometimes gives the chance to combine some parts and it can reduce the cost. In addition to those properties, their fatigue resistance is higher than the metals and they have longer fatigue life. However, additional to all those advantages, there are some disadvantages of composites. Refrigerated transportation, storage, and limited shelf life can be given as the examples of the drawbacks of composites. In addition, the delamination can occur between composite lamina and it can cause the reduction of the strength.

1.1 Purpose of Thesis

The motivation of this study is the investigation of the buckling problem which comes with the usage of the slender structure under static or dynamic axial compressive loading. This study involves the effect of different parameters such as; stacking sequence, thickness, material and loading duration and the shape of dynamic loading. In addition to the effects on critical buckling load for perfect structure, the main concern of this study is how the imperfection sensitivity changes with parameters. Those parameters are length to diameter ratio, stacking sequence, and initial imperfection shape.

1.2 Literature Review

With the increase in the usage of thin structure, the buckling become an important issue because of high sensitivity to the imperfection. As the result of that, this phenomenon is investigated in many cases in the history. Forerunner of the studies about the stability of the thin structures may be taken as Euler et al. [10], Timoshenko et al. [11] and Volmir [12]. After those studies, the numerous works have been made for different loadings. One of the important studies is performed by Batdof [13]. In his study, the cylinders are classified by their length. The cylinder is buckled like Euler column if it is long, diamond shape if it has medium length, and ring for short cylinder.

In 70s and 80s, the studies were more widely on thin walled structures with isotropic materials. However, developments in the material science provide composite material to become widespread in many industries. Therefore, the necessity of studies on buckling of composite structures arose. One of the oldest works about buckling of structures which are made of orthotropic material is written by Seydel [14].

Moreover, in the study of Bisagni and Cordisco, the experimental studies were made for composite cylindrical shells [15]. Torsional and axial loading is applied separately and together for different stacking sequence and fiber angles. Initial imperfection and progress of the displacement are measured by the laser scanning technique. Another experimental study example was published by Priyadarsini et al. [6]. Besides experimental study, Finite element analysis (FEA) is made for different fiber angles. Also, the initial imperfection is measured with linear variable differential transducer (LVDT), and it is added to the analysis to be able to get more realistic results. FEA is made with different method: linear eigenvalue analysis, nonlinear static analysis and explicit analysis are made.

It is possible to find some papers which investigate the stacking sequence and the optimization of the angle. One of them is a master thesis of Kalantari Osgouei [16]. In this thesis, the buckling load optimization of the stacking sequence is made by taking the composite failure theories into account. The effect of fiber angle of a basalt-carbon hybrid composite plate is investigated in the study of Dhuban et al. [17]. As a result, the analyses are performed for increasing value of the fiber angle for different stacking sequence and when the angle increases the buckling load is decreases. However, it remains constant over 50 degrees.

During the studies of buckling, the results of the experimental studies and the analytical studies are significantly different. It is realized that this difference is sourced by an initial imperfection in the system and the buckling is highly sensitive to the imperfection and calculating the buckling load with analytical methods causes to overestimate it. Imperfection can be sourced by geometry, material or/and loading. These imperfections can be sourced by production state of the specimen or test setup. For example, not having parallel end of the specimen causes to apply extra moment while loading is axial compression. Thus, the results will be affected. Addition to that, surface roughness is inevitable and causes the decrease of the buckling load. Although

the material is chosen as isotropic, it can show different behavior in different properties because of the production state. All of them affect the buckling load, especially geometric imperfection. Since theoretical investigation causes the overestimation for the buckling load, the need of analysis of the effect of imperfection is appeared. Thus, NASA space vehicle structural guideline (NASA SP-8007) presents some equations for calculating the knockdown factor [18]. Those knockdown factors are based on the experimental studies for different loading types. However, this guideline overestimates the results and the results are over-dated. Also, the knockdown factor for orthotropic materials is based on the test data for isotropic specimens. For that reason, the need for new approaches arises. Degenhardt et al used Single Perturbation Load Approach (SPLA) which is grounded on geometric and loading imperfection and stochastic approach which is grounded on variation of thickness and stiffness [19]. Recent studies use those two methods together. One of those studies considers an unstiffened cylinder under axial compression load and the found design buckling load is 45% lower than NASA's result. Therefore, that reduction provides 20% decreasing of the weight of the structure [20].

Although there are numerous paper about static buckling, finding the source about dynamic buckling is harder than static one. In the literature, some criteria are constructed to obtain dynamic buckling load. One of the first criteria is found by Volmir [21]. In his study, a simply supported plate is considered with both finite and infinite loading duration. According to that criterion, the plate is dynamically buckled when the deflection of the plate is equal or half of the thickness. Another dynamic buckling criterion is written by using the equation of motion approach. This approach is presented first time in the reference [22]. In the study of Budiansky and Hutchinson, axially compressed shell structures are modeled with rods and spring model [23]. By using similar methods, Budiansky and Roth transversely loaded cylinders are investigated [24]. At the conclusion of this study, the dynamic buckling criterion is that when the displacement-time graph is drawn for different loading values, the structure is buckled in which displacements are much greater than the other loading values. Moreover, the investigation of imperfection on dynamic buckling is made by Tamura and Babcock with the Budiansky and Roth's criterion [25]. In addition, one of the most used buckling criteria is the total energy-phase plane criterion. This

method which is mentioned in the study of Hoff and Bruce is used for determining dynamic buckling [26]. According to this criterion when the velocity-displacement graph is drawn, if the motion is stable, then the curve is not spread around equilibrium. However, when the dynamic buckling occurs, the graph will be spread which means that the displacements are huge. Also, Simitses investigated the dynamic buckling for arches, cylindrical shells and spherical caps. In that study, effect of static preloading is also explored [27].

With the development of the finite element codes, the numerical analysis with the commercial programs is used to investigate the dynamic buckling. The study of Bisagni can be given as an example [28]. In this study, both static and dynamic buckling analysis for a composite cylinder are performed and the results are compared to the analytical solutions. Dynamic buckling analysis results are evaluated with Budiansky and Roth criteria that are made for different loading time to see how the buckling load changes. Also, the geometric imperfection is added to the analysis to show how the buckling load changes. Another study for the numerical investigation of the dynamic buckling used composite conical shell under axial dynamic loading with the imperfection for two different stacking sequences [8]. The conclusion is drawn such that the characteristic of the loading (static or dynamic) is determined by the loading duration with longer or shorter than the period of the natural frequency of the structure. Also, Katarzyna Kowal–Michalska stated how the initial geometric imperfection, shape of the pulse and loading duration affects to the dynamic behavior of the plate structure [29]. Although most of the paper in the literature assume the material is in elastic range, that paper includes the strain rate dependence effect. Another investigation of dynamic buckling is made by Shaw et al. which is about composite shell with imperfection [30]. In this paper the load is applied as axial compression and torsion. Budiansky-Roth criterion and Simitses criterion are used and the results of them diverge with the increasing the amount of the imperfection. Also, the conclusion is drawn such that the imperfection sensitivity of the axial loading is higher than the torsional loading.

1.2.1 Content of the thesis

In the first chapter of the thesis, brief information about buckling and the previous studies about the static and dynamic buckling of the shell structure are mentioned. Also, scope of the thesis is explained.

In the second chapter, the analytical derivation of the static buckling load formulation is given. The derivation is made for the anisotropic materials.

Third chapter is about dynamic buckling and that chapter involves some of the most used dynamic buckling criteria.

Forth chapter includes three methods of finite element analysis. Firstly, those three methods described. Then, verification studies for all methods are made with the sources in the literature. In addition, shell geometry properties are decided and with that geometry, the effect of the some parameters are investigated.

The fifth chapter is about the sensitivity of the imperfections. In that chapter, three parameters are used to obtain how the sensitivity to the amount of imperfection changes. Those three parameters are length to diameter ratio, stacking sequence and the shape of the imperfection.

In the last chapter, conclusions drawn from that study are listed. Also, some ideas about how that study can be expanded will be introduced.

2. STATIC BUCKLING OF CYLINDRICAL SHELLS

In this section, analytical calculation of critical static buckling load of a cylindrical shell element is derived for anisotropic materials. The following derivation of the buckling load is made under the assumption of that the material is linearly elastic and small deformation is occurred. Also, thin shell theory is applied which ignores transverse shear deformations. In addition to those, the assumption of constant internal forces before buckling is made.

2.1 Buckling of Shells

Consider a small element of composite shell which is subjected to compressive load and shear loads. That portion of the composite should be away from the ends. In Figure 2.1, the internal forces on that small shell element is shown. Also, it is assumed that $N_{xy} = N_{yx}$, $M_{xy} = M_{yx}$.

The in-plane strains and the curvatures are given as [2]:

$$\begin{aligned}\epsilon_x &= \frac{\partial u}{\partial x} \\ \epsilon_y &= \frac{\partial v}{R\partial\theta} - \frac{w}{R} \\ \gamma &= \frac{\partial u}{R\partial\theta} + \frac{\partial v}{\partial x} \\ \kappa_x &= -\frac{\partial^2 w}{\partial x^2} \\ \kappa_y &= \frac{1}{R^2} \left(\frac{\partial v}{\partial\theta} - \frac{\partial^2 w}{\partial\theta^2} \right) \\ \kappa_{xy} &= \frac{1}{R} \left(\frac{\partial v}{\partial x} - \frac{\partial^2 w}{\partial x \partial\theta} \right)\end{aligned}\tag{2.1}$$

where R is the radius of cylinder. u, v, w are displacements in cylindrical coordinate system. As it is seen in Figure 2.1, the loads acting on the shell elements is;

$$\begin{aligned}N_x &= -\lambda N_{x0} \\ N_y &= -\lambda N_{y0} \\ N_{xy} &= -\lambda N_{xy0}\end{aligned}\tag{2.2}$$

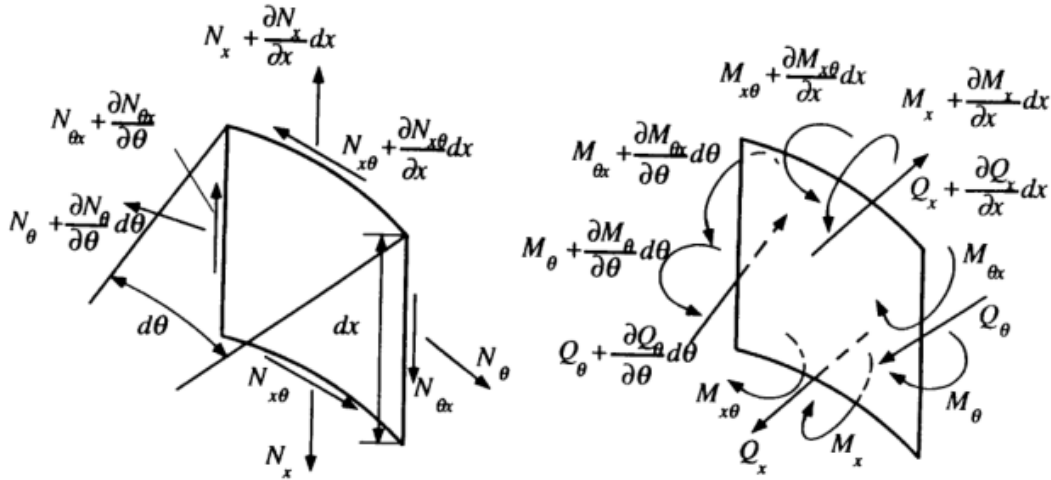


Figure 2.1 : Internal forces acting on cylinder shell element [2].

where λ is the load parameter. The value that the shell starts to buckle is called critical load parameter, λ_{cr} .

The relations between force and strain for anisotropic laminate can be given as:

$$N = S\varepsilon \quad (2.3)$$

where N is in-plane internal force vector, ε is the strain vector and S is the stiffness matrix.

$$N = \begin{pmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{pmatrix} S = \begin{pmatrix} A_{11} & A_{12} & A_{13} & B_{11} & B_{12} & B_{13} \\ A_{21} & A_{22} & A_{23} & B_{21} & B_{22} & B_{23} \\ A_{31} & A_{32} & A_{33} & B_{31} & B_{32} & B_{33} \\ B_{11} & B_{21} & B_{31} & D_{11} & D_{12} & D_{13} \\ B_{12} & B_{22} & B_{32} & D_{21} & D_{22} & D_{23} \\ B_{13} & B_{23} & B_{33} & D_{31} & D_{32} & D_{23} \end{pmatrix} \varepsilon = \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{pmatrix} \quad (2.4)$$

Force equilibrium equations on the shell element in Figure 2.1 in axial, circumferential, and radial direction are:

$$\begin{aligned} & -\frac{1}{R} \left\{ R \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial \theta} - N_y \left(\frac{\partial w}{\partial x} + \frac{\partial^2 v}{\partial x \partial \theta} \right) - R N_{xy} \frac{\partial^2 v}{\partial x^2} \right. \\ & \quad \left. + R Q_x \frac{\partial^2 w}{\partial x^2} - Q_y \left(\frac{\partial v}{\partial x} - \frac{\partial^2 w}{\partial x \partial \theta} \right) \right\} = 0 \\ & -\frac{1}{R} \left\{ \frac{\partial N_y}{\partial \theta} + R \frac{\partial N_{xy}}{\partial x} + R N_x \frac{\partial^2 v}{\partial x^2} + N_{yx} \left(\frac{\partial w}{\partial x} + \frac{\partial^2 v}{\partial x \partial \theta} \right) - Q_x \left(\frac{\partial v}{\partial x} - \frac{\partial^2 w}{\partial x \partial \theta} \right) \right. \\ & \quad \left. - Q_y \left(1 + \frac{\partial v}{R \partial \theta} - \frac{\partial^2 w}{R \partial \theta^2} \right) \right\} = 0 \\ & \frac{1}{R} \left\{ N_y + qR - R N_x \frac{\partial^2 w}{\partial x^2} + N_y \left(\frac{\partial v}{R \partial \theta} - \frac{\partial^2 w}{R \partial \theta^2} \right) + N_{xy} \left(\frac{\partial v}{\partial x} - \frac{\partial^2 w}{\partial x \partial \theta} \right) \right. \\ & \quad \left. + N_{yx} \left(\frac{\partial v}{\partial x} - \frac{\partial^2 w}{\partial x \partial \theta} \right) + R \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial \theta} \right\} = 0 \end{aligned} \quad (2.5)$$

Shear forces are determined from the moment equilibrium:

$$\begin{aligned} Q_y &= -\frac{1}{R} \left[\frac{\partial M_y}{\partial \theta} + R \frac{\partial M_{xy}}{\partial x} + R M_x \frac{\partial^2 v}{\partial x^2} + M_{yx} \left(\frac{\partial w}{\partial x} + \frac{\partial^2 v}{\partial x \partial \theta} \right) \right] \\ Q_x &= \frac{1}{R} \left[-R \frac{\partial M_x}{\partial x} - \frac{\partial M_{yx}}{\partial \theta} + M_y \left(\frac{\partial w}{\partial x} + \frac{\partial^2 v}{\partial x \partial \theta} \right) + R M_{xy} \frac{\partial^2 v}{\partial x^2} \right] \end{aligned} \quad (2.6)$$

The stretching effect of the reference surface is not included in the equations above. To add the strain of the reference surface, the variables are replaced with the following terms [2].

$$\begin{aligned} N_x &\rightarrow N_x(1 + \varepsilon_y), N_y \rightarrow N_y(1 + \varepsilon_x), N_{xy} \rightarrow N_{xy}(1 + \varepsilon_y), N_{yx} \rightarrow N_{yx}(1 + \varepsilon_x) \\ M_x &\rightarrow M_x(1 + \varepsilon_y), M_y \rightarrow M_y(1 + \varepsilon_x), M_{xy} \rightarrow M_{xy}(1 + \varepsilon_y), M_{yx} \rightarrow M_{yx}(1 + \varepsilon_x) \\ q &\rightarrow q(1 + \varepsilon_x)(1 + \varepsilon_y) \end{aligned} \quad (2.7)$$

Forces can be expressed as the summation of the internal forces and the change in the forces due to buckling [2].

$$\begin{aligned} N_x &= \lambda_{cr} N_{x0} + N'_x, M_x = \lambda_{cr} M_{x0} + M'_x \\ N_y &= \lambda_{cr} N_{y0} + N'_y, M_y = \lambda_{cr} M_{y0} + M'_y \\ N_{xy} &= \lambda_{cr} N_{xy0} + N'_{xy}, M_{xy} = \lambda_{cr} M_{xy0} + M'_{xy} \\ q &= \lambda_{cr} q_0 \end{aligned} \quad (2.8)$$

Using three force equilibrium Equation 2.5 and shear forces from moment equilibrium Equation 2.6 with the effect of the reference surface stretching (change of the variables with Equation 2.7) transform Equation 2.8 into the Equation 2.9 form.

$$\begin{aligned} \Theta_n N' + [\lambda_{cr} (N_{x0} \Theta_{N_x} + N_{y0} \Theta_{N_y} + N_{xy0} \Theta_{N_{xy0}} + M_{x0} \Theta_{M_x} \\ + M_{y0} \Theta_{M_y} + M_{xy0} \Theta_{M_{xy0}} + q_0 \Theta_q)] u = 0 \end{aligned} \quad (2.9)$$

Forces due to buckling can be written as $N' = S\varepsilon = S\Theta_e u$ and Equation 2.9 become Equation 2.10 which is a set of three equation of displacements u, v, w . The solution of that eigenvalue problem gives critical load factor λ_{cr} and the corresponding buckling shape.

$$\begin{aligned} [\Theta_n S\Theta_e + \lambda_{cr} (N_{x0} \Theta_{N_x} + N_{y0} \Theta_{N_y} + N_{xy0} \Theta_{N_{xy0}} \\ + M_{x0} \Theta_{M_x} + M_{y0} \Theta_{M_y} + M_{xy0} \Theta_{M_{xy0}} + q_0 \Theta_q)] u = 0 \end{aligned} \quad (2.10)$$

where Θ 's in the Equation 2.9 and Equation 2.10 are the differential operators. They are given in Appendix A.

The assumed displacement functions are given as;

$$\begin{aligned} u &= u_1 \cos \alpha (x + c_1 y) \sin \beta (y + c_2 x) - u_2 \sin \alpha (x + c_2 y) \cos \beta (y + c_2 x) \\ v &= v_1 \sin \alpha (x + c_1 y) \cos \beta (y + c_2 x) - v_2 \cos \alpha (x + c_2 y) \sin \beta (y + c_2 x) \\ w &= w_1 \sin \alpha (x + c_1 y) \sin \beta (y + c_2 x) + w_2 \cos \alpha (x + c_2 y) \cos \beta (y + c_2 x) \end{aligned} \quad (2.11)$$

Substituting the displacements given in Equation 2.11 into Equation 2.10 gives an eigenvalue problem (Equation 2.12), which provides the critical load parameter λ_{cr} and the buckling shape [1].

$$\left(\begin{bmatrix} O & L \\ L & O \end{bmatrix} \begin{bmatrix} S_o & S_n \\ S_n & S_o \end{bmatrix} \begin{bmatrix} O & L \\ L & O \end{bmatrix}^T - \lambda \begin{bmatrix} \phi_1[J] & \phi_2[J] \\ \phi_2[J] & \phi_1[J] \end{bmatrix} \right) \begin{Bmatrix} u_1 \\ v_1 \\ w_1 \\ u_2 \\ v_2 \\ w_2 \end{Bmatrix} = 0 \quad (2.12)$$

where u_1, v_1, u_2, v_2 are the in plane displacement amplitudes in x and y directions while w_1 and w_2 are out of plane amplitudes. The matrices $[O], [L], [J]$ and parameters ϕ_1, ϕ_2 depend on the constants α, β, c_1, c_2 and those constants specify the buckling pattern of the shell.

$$[O] = \begin{bmatrix} -\alpha & 0 & \beta & 0 & 0 & 0 \\ 0 & -\beta & \alpha & 0 & 0 & 0 \\ 1/R_x & 1/R_y & 0 & \alpha^2 + \beta^2 c_2^2 & \beta^2 + \alpha^2 c_1^2 & -2\alpha\beta(1 + c_1 c_2) \end{bmatrix} \quad (2.13)$$

$$[L] = \begin{bmatrix} \beta c_2 & 0 & -\alpha c_1 & 0 & 0 & 0 \\ 0 & \alpha c_1 & -\beta c_2 & 0 & 0 & 0 \\ 0 & 0 & 1/R_{xy} & -2\alpha\beta c_2 & -2\alpha\beta c_1 & 2(c_1 \alpha^2 + c_2 \beta^2) \end{bmatrix} \quad (2.14)$$

$$[S_o] = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & D_{66} \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} \end{bmatrix} \quad (2.15)$$

$$[S_n] = \begin{bmatrix} A & B \\ B & D \end{bmatrix} - [S_o] \quad (2.16)$$

$$[J] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2.17)$$

$$\begin{aligned} \phi_1 &= N_{x0}(\alpha^2 + \beta^2 c_2^2) + N_{xy0}2(\alpha^2 c_1 + \beta^2 c_2^2) + N_{y0}(\alpha^2 c_1 + \beta^2) \\ \phi_2 &= -2\alpha\beta(N_{x0}c_2 + N_{xy0}(1 + c_1 c_2) + N_{y0}c_1) \end{aligned} \quad (2.18)$$

For a cylinder shell, $1/R_x = 1/R_{xy} = 0$ and $R_y = R$ [1]. In Equations 2.19 and 2.20, the slope of the lines which are passing through the left and right propagation waves' maxima. It is illustrated in Figure 2.2.

$$\tan\theta_1 = \frac{\alpha + \beta c_2}{\beta + \alpha c_1} \quad (2.19)$$

$$\tan\theta_2 = \frac{\alpha - \beta c_2}{\beta - \alpha c_1} \quad (2.20)$$

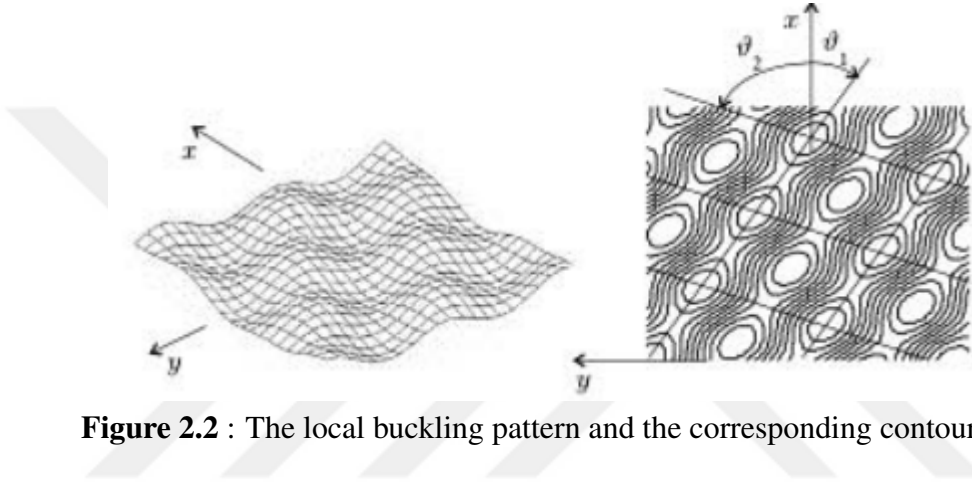


Figure 2.2 : The local buckling pattern and the corresponding contour plot [1].

To find the load parameter λ in Equation 2.12, the eigenvalue analysis must be made for different α, β, c_1, c_2 . The critical load parameter λ_{cr} is the lowest value which is calculated for different combination of α, β, c_1, c_2 . Those parameters are not completely random, they are selected according to the displacements which satisfy the boundary conditions. After choosing α, β, c_1, c_2 , the out of plane deflection can be calculated with Equation 2.21

$$w^\circ = w_1 \sin(\alpha(x + c_1 y)) \sin(\beta(y + c_2 x)) + w_2 \sin(\alpha(x + c_1 y)) \cos(\beta(y + c_2 x)) \quad (2.21)$$



3. DYNAMIC BUCKLING OF CYLINDRICAL SHELL

Dynamic buckling can be categorized as vibration buckling and pulse buckling. Vibration buckling is caused by the large oscillating load with the critical combination of loading amplitude, frequency, and the structural damping [3]. For the pulse buckling, it can be said that large deformation occurs under the transient load such as a single pulse. The dynamic buckling shape is also similar to the static buckling shape which is called as diamond shape buckling. The waves of the buckled pattern is longer in the circumferential direction and shorter in the longitudinal direction in dynamic buckling as it is seen in Figure 3.1 [3].

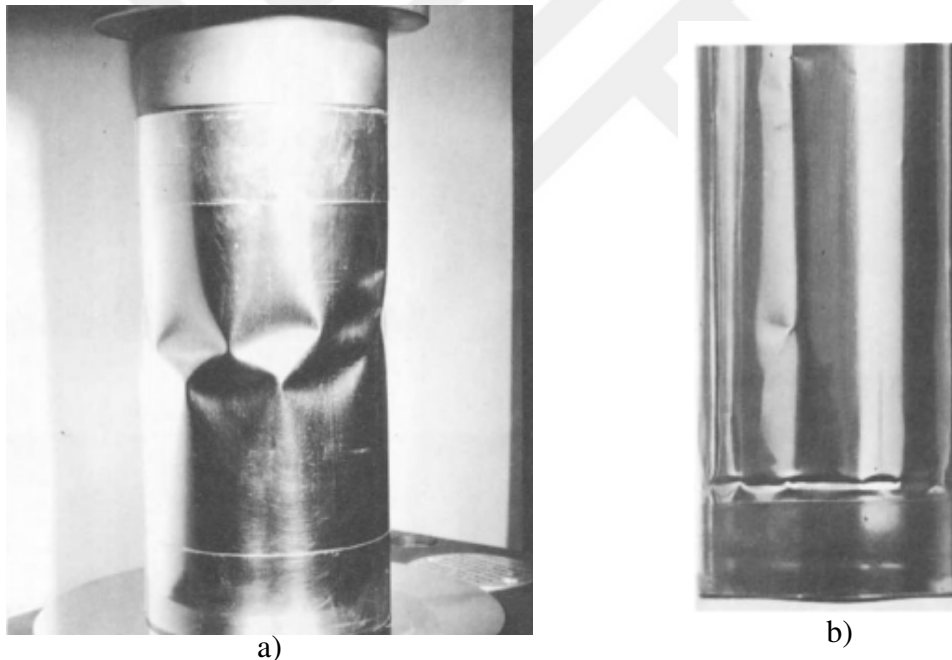


Figure 3.1 : Examples of Static and Dynamic Buckling Pattern (a) Static Buckling
(b) Dynamic Buckling [3].

For the perfect structures, the critical dynamic buckling amplitude goes to infinity [4]. Therefore, the structure which is analyzed requires to include initial imperfections.

In addition to the analytical evaluation model which was suggested by Budiansky and Hutchinson [23], some new criteria are constituted for the evaluation of the critical

value of dynamic load. In this section, some of those criteria are presented and the equation of motion approach is used to interpret the results.

3.1 Analytical Formulation of Dynamic Buckling of Cylindrical Shells

Donnell's equation for cylindrical shells can be written as in Equation 3.1 [3].

$$D\nabla^4 w + \bar{N}_x \frac{\partial^2(w + w_i)}{\partial x^2} + \rho h \frac{\partial^2 w}{\partial t^2} - \frac{1}{a} \frac{\partial^2 F}{\partial x^2} = 0 \quad (3.1)$$

$$\nabla^4 F = -\frac{Eh}{a} \frac{\partial^2 w}{\partial x^2}, \quad (3.2)$$

where $\nabla^4 = (\frac{\partial}{\partial x^2} + \frac{\partial^2}{a^2 \partial \theta^2})^2$ and $D = Eh^3/12(1 - \nu^2)$

where x is the axial coordinate, θ is the circumferential coordinate, w is the positive inward displacement, w_i is the initial imperfection, ρ is the material density, E is Young's modulus, h is the wall thickness, a is the radius, D is the flexural rigidity and F is Airy's stress function. The dimensionless parameters are given in Equation 3.3.

$$\xi = x \left(\frac{\bar{N}_x}{D} \right)^{1/2}, \eta = a\theta \left(\frac{\bar{N}_x}{D} \right)^{1/2}, \tau = \frac{\bar{N}_x}{(\rho h D)^{1/2}} t \quad (3.3)$$

Equation 3.1 and 3.2 becomes;

$$\nabla^4 w + \frac{\partial^2}{\partial \xi^2}(w + w_i) + \ddot{w} - \frac{1}{a\bar{N}_x} \frac{\partial^2 F}{\partial \xi^2} = 0 \quad (3.4)$$

$$\nabla^4 F = \frac{EhD}{a\bar{N}_x} \frac{\partial^2 w}{\partial \xi^2} \quad (3.5)$$

where $\nabla^4 = (\frac{\partial}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2})^2$ and $\ddot{w} = \frac{\partial^2 w}{\partial \tau^2}$

Simply supported boundary conditions are assumed at both ends and that boundary conditions are provided as;

$$w = \frac{\partial^2 w}{\partial x^2} = 0 \text{ at } x = 0, L \quad (3.6)$$

Equation 3.7 and 3.8 satisfies the boundary conditions in 3.6.

$$w(\xi, \eta, \tau) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn}(\tau) \sin \alpha_m \xi \sin \beta_n \eta \quad (3.7)$$

$$F(\xi, \eta, \tau) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} F_{mn}(\tau) \sin \alpha_m \xi \sin \beta_n \eta \quad (3.8)$$

where $\alpha_m = \frac{m\pi}{L}(\frac{D}{N_x})^{1/2}$, $\beta_n = \frac{n}{a}(\frac{D}{N_x})^{1/2}$

The initial imperfection is assumed;

$$w_i(\xi, \eta) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \sin \alpha_m \xi \sin \beta_n \eta \quad (3.9)$$

When Equations 3.7 - 3.9 are substituted into Equation 3.4 and 3.5, W_{mn} of normal modes [3] is;

$$\ddot{W}_{mn} + \left[(\alpha_m^2 + \beta_n^2)^2 - \alpha_m^2 + \frac{EDh}{a^2 \bar{N}_x^2} \frac{\alpha_m^4}{(\alpha_m^2 + \beta_n^2)^2} \right] W_{mn} = \alpha_m^2 a_{mn} \quad (3.10)$$

3.2 Dynamic Buckling of Cylindrical Shell under Step Loading

In this section, the study of Budiansky and Hutchinson dynamic buckling of the cylindrical shells under step load is presented [23]. To represent the cylindrical shell, a model is created by Budiansky and Hutchinson. This model consists of three-hinge and rigid-rod and it is shown in Figure 3.2. Cubic softening spring is used in this model and the characteristic is given with Equation 3.11.

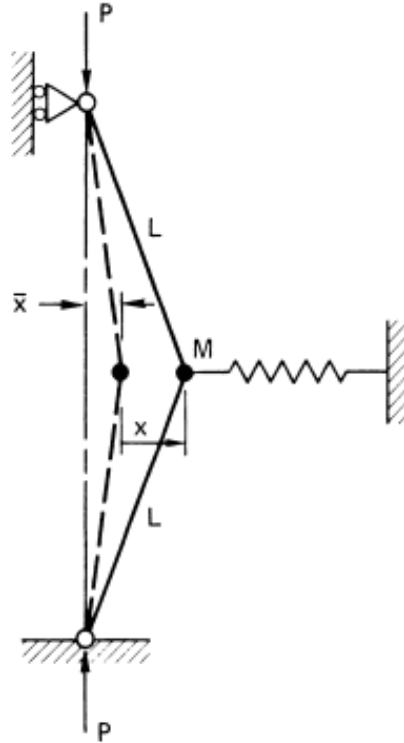


Figure 3.2 : Single degree of freedom rigid rod column with imperfection and nonlinear spring [3].

$$F = KL(\epsilon - b\epsilon^3), b > 0 \quad (3.11)$$

where $\epsilon = x/L$. Also, $\bar{\epsilon}$ is initial position and x is the displacement from the initial position.

If the assumption of small rotation is made, then ϵ and $\bar{\epsilon}$ is very small values. The critical load value of the perfect structure is $P_c = KL$. Also, static equilibrium can be expressed in Equation 3.12. Since $\bar{\epsilon}$ is the normalized initial imperfection, it is equal to zero for perfect column. In Figure 3.3, the load-displacement graph from Equation 3.12 is given for both perfect and imperfect structures. As seen from that graph, the critical load value for the perfect structure is P_c and the limit load value for imperfect structure (P_s) is below of that value.

$$\left(1 - \frac{P}{P_c}\right)\epsilon - b\epsilon^3 = \frac{P}{P_c}\bar{\epsilon} \quad (3.12)$$

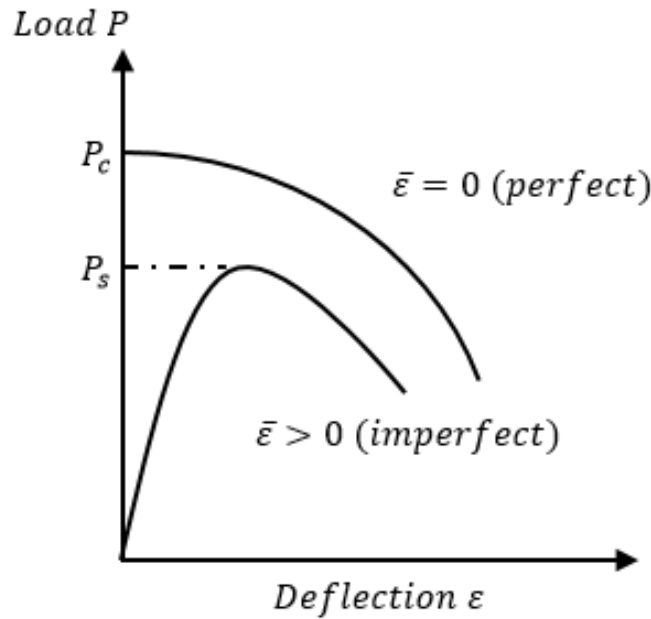


Figure 3.3 : Load-deflection curves for rigid-rod with cubic softening spring [3].

The actual static axial buckling load and the classical buckling load is represented with P_s and P_c , respectively. As mentioned before, the cylinder is represented with rod and spring. This model shows much simpler behavior than the actual behavior of the cylindrical shell. However, the term P_s/P_c can be made small enough with large b and $\bar{\epsilon}$ to account the effect of the other types of imperfection [3].

When Equation 3.12 is divided with $\bar{\epsilon}$ and the expression $z = \epsilon/\bar{\epsilon}$ is introduced and Equation 3.13 is obtained.

$$\left(1 - \frac{P}{P_c}\right)z - (b\bar{\epsilon}^2)z^3 = \frac{P}{P_c} \quad (3.13)$$

For $dP/dz = 0$, the corresponding load value will be P_s . It is seen from Equation 3.14 that P_s/P_c is only changed with $b\bar{\epsilon}^2$.

$$\left(1 - \frac{P_s}{P_c}\right)^{3/2} = \frac{3\sqrt{3}}{2}b^{1/2}\bar{\epsilon}\frac{P_s}{P_c} \quad (3.14)$$

To be able to calculate P_D , it is required that introducing the inertia into system and the load should be time dependent. Then, Equation 3.13 becomes,

$$\ddot{z} + \left(1 - \frac{P}{P_c}\right)z - (b\bar{\epsilon}^2)z^3 = \frac{P}{P_c} \quad (3.15)$$

If the loading is step load, $z = \dot{z} = 0$ at $t = 0$. When Equation 3.15 is integrated once with the information of $\dot{z} = \dot{z}dz/dz$, Equation 3.16 is obtained.

$$\dot{z}^2 + \left(1 - \frac{P}{P_c}\right)z^2 - \frac{1}{2}(b\bar{\epsilon}^2)z^4 = 2\left(\frac{P}{P_c}\right)z \quad (3.16)$$

For the loads below the dynamic buckling load, the motion is periodic and Equation 3.16 represents limit cycle in plane $z, \dot{z}$. Therefore, the maximum value of z occurs when $\dot{z} = 0$. Substitute $\dot{z} = 0$ and z_{max} in terms of z to obtain Equation 3.17.

$$\left(1 - \frac{P}{P_c}\right)z_{max}^2 - \frac{1}{2}(b\bar{\epsilon}^2)z_{max}^4 = 2\left(\frac{P}{P_c}\right)z_{max} \quad (3.17)$$

The motion diverges from the limit cycle, when the applied load reaches the critical dynamic buckling load P_D . Also, the amplitude and the period of P_D are infinite [3]. To take place of this situation, $dP/dz_{max} = 0$ should be implemented to Equation 3.17.

$$\left(1 - \frac{P_D}{P_c}\right)^{3/2} = \frac{3\sqrt{6}}{2}b^{1/2}\bar{\epsilon}\frac{P_D}{P_c} \quad (3.18)$$

To obtain an expression which is not depended on imperfection explicitly, imperfection parameter $b\bar{\epsilon}^2$ is eliminated in Equation 3.14 and Equation 3.18. The relation between static and dynamic loads is given below.

$$\frac{P_D}{P_s} = \frac{\sqrt{2}}{2} \left(\frac{P_c - P_D}{P_c - P_s} \right)^{3/2} \quad (3.19)$$

In Equation 3.19, the relation between critical dynamic and static buckling load is given.

3.3 Dynamic Buckling Approaches

3.3.1 Volmir criterion

Volmir was one of the first contributors of dynamic buckling criteria. He studied the simply supported plate under rectangular and exponentially decreasing pulses with both finite and infinite duration [21]. Bubnov - Galarkin method was used to solve the static buckling problem. In addition, deflection and load are taken as a function of time and equation of motion (EOM) is solved by the Runge - Kutta method to obtain a dynamic buckling response.

His method to determine the critical dynamic buckling load is very simple but time consuming. This method claims that when the deflection of the plate under pulse load is equal to the thickness of the plate or half of it, it can be considered that it has lost its dynamic stability [4].

3.3.2 Equation of motion approach (Budiansky - Roth)

Budiansky and Hutchinson used the displacement that is the numerical solution of the equation of motion as the criterion in their work [22], [23]. In those studies, cylindrical shells and rods that are axially compressed are investigated. Transversely loaded cylinders are presented as the similar method by Budiansky and Roth [24]. According to their method, the critical load is defined as where the turning point of the load-displacement curve. The behavior of the load displacement curves for large and small initial imperfections are given in Figure 3.4.

Also, Budiansky and Roth's criterion states that the stability loss occurs when the huge amount of deflection appears corresponding a small increase of the pulse loading [4].

In the case of the smaller peaks with more number of waves, the maximum value of the displacement gets smaller. To be able to catch that modes, Budiansky and Roth's criterion is improved via the work of Ari Gur and Simenetta [31]. Second criterion in their study can be suitable for smaller peaks. This criterion states that the decrease in maximum value of the lateral deflection occurs if pulse intensity increases slightly (Figure 3.5 b) [31].

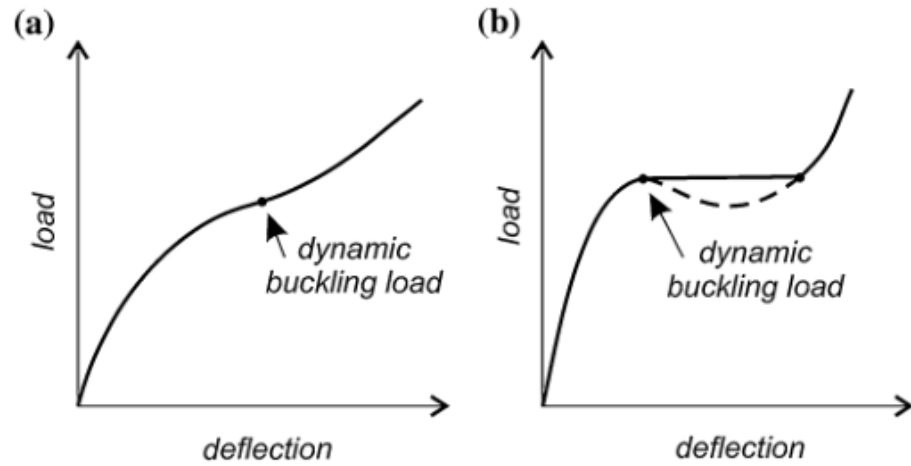


Figure 3.4 : Equilibrium paths for pulse loaded structures with (a) large and (b) small initial geometric imperfection [4].

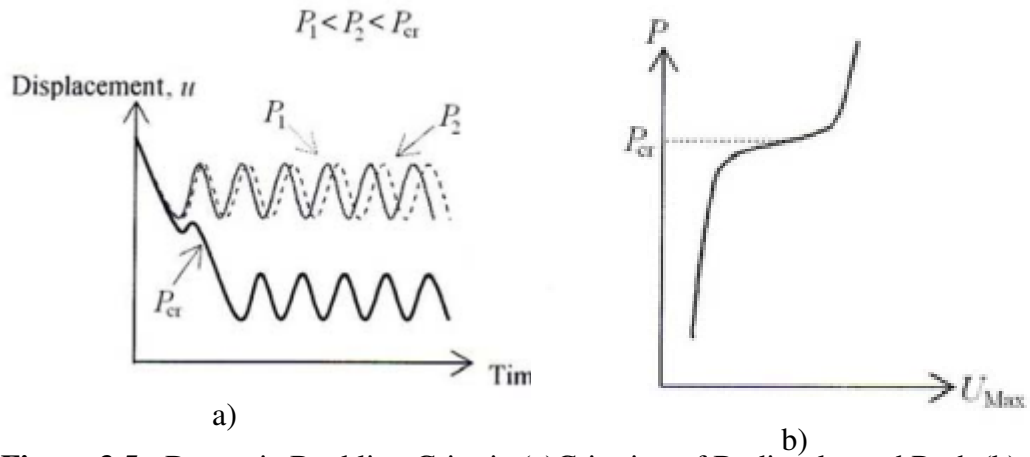


Figure 3.5 : Dynamic Buckling Criteria (a)Criterion of Budiansky and Roth (b) Criterion of Ari Gur and Simonetta [5].

3.3.3 Total energy - phase plane approach (Hoff & Bruce)

Another method to evaluate the system in terms of dynamic buckling is introduced in the study of Hoff and Bruce [26]. In this method, the displacement-velocity graph should be plotted as shown in Figure 3.6. When the displacement response of the system is small, the graphic focuses around the static equilibrium. However, when the load is high enough to make the system unstable, then the graph is spread as shown in Figure 3.6-b [5].

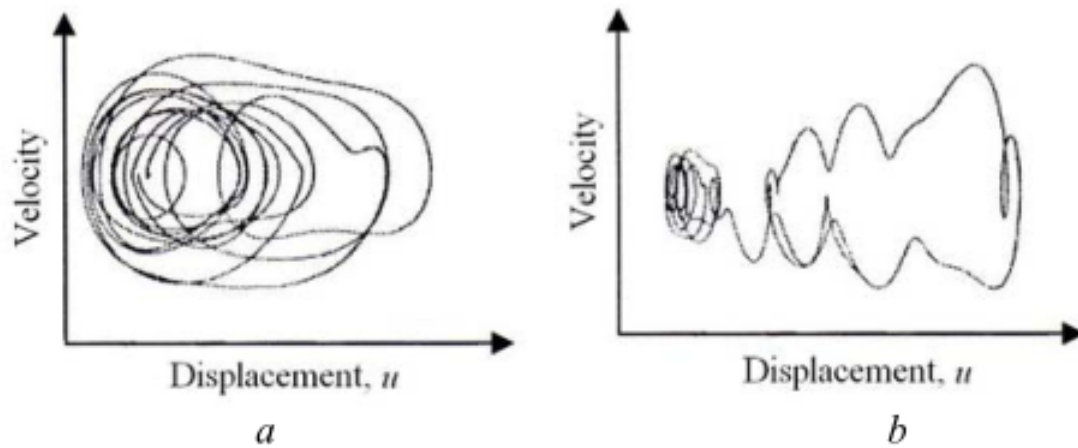


Figure 3.6 : Phase plane a) stable b)unstable motion [5].

3.3.4 Total potential energy approach (Simitses)

In the Figure 3.7 a, variation of the potential energy as a function of displacement is shown and point B shows the static equilibrium position. When the dynamic buckling load factor is higher than the applied load factor, the harmonic movement of the structure is limited around point O in Figure 3.7 a. Therefore, the distance between point A and point C can be considered as oscillation measure. It is dynamically stable if the movement is between A and C.

Figure 3.7 b illustrates change in potential energy for the case that the applied load factor is nearly equal to the dynamic load factor. The dynamic movement is again around point O, however the point C is corresponding to the critical dynamic load factor. Therefore, the movement is out of control between point O and point C.

As it is mentioned, the movement oscillates between point A and point C because the applied load factor is smaller than the dynamic load factor (Fig 3.7 a) and the maximum amplitude OC. If the load-displacement curve is drawn for static equilibrium and it is shifted to right with (OC), the new curve gives the dynamic equilibrium. The intersection point of these two curves gives the critical dynamic buckling load (Fig 3.8).

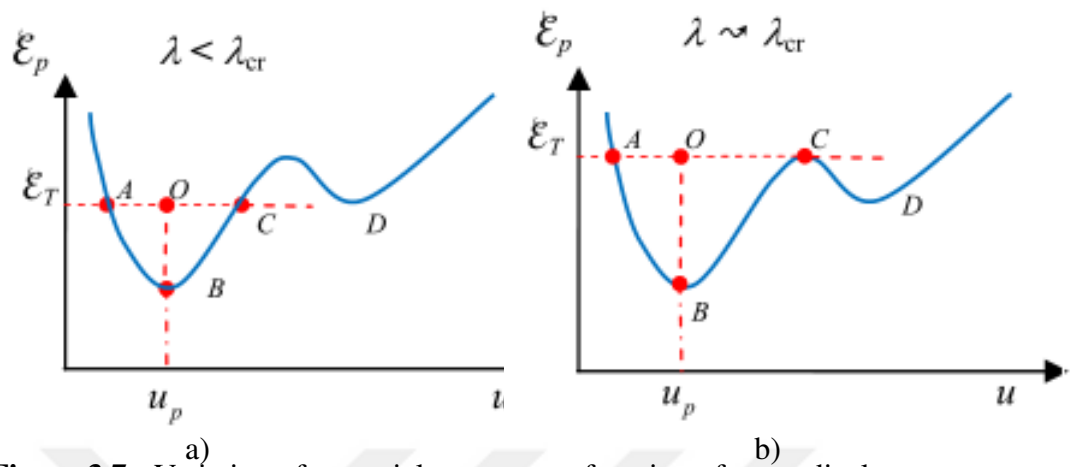


Figure 3.7 : Variation of potential energy as a function of some displacement variable: (a) stable dynamic movement (b) unstable dynamic movement [5].

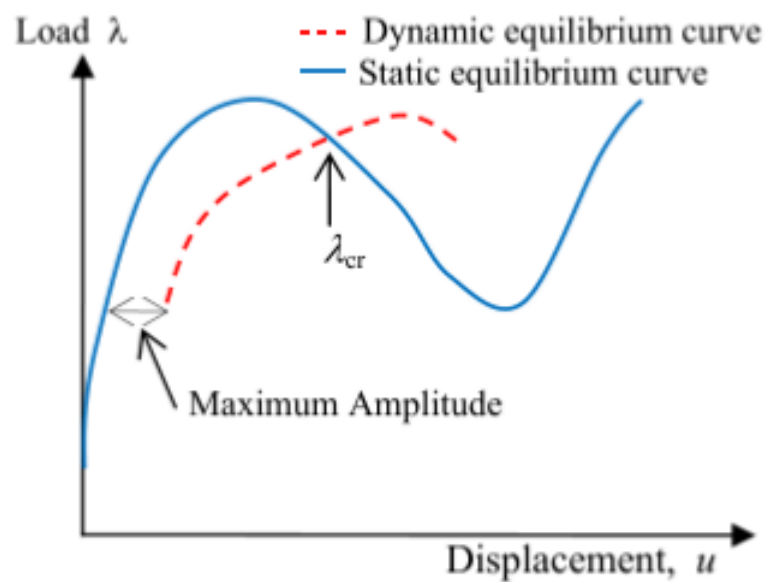


Figure 3.8 : Critical load factor at the intersection of the two curves of static and dynamic equilibrium [5].



4. FINITE ELEMENT ANALYSIS

The finite element method (FEM) is used to solve the real life problems which can have complex geometry, loading and/or material properties. Complex differential equations in many engineering fields can be solved using the FEM. Those engineering field could be solid mechanics, dynamics, thermal, and fluid mechanics analysis. Basic idea in the finite element method is to divide the problem into smaller parts and then the equation systems of these smaller parts are combined. Then, solutions of the smaller parts are assembled to solve the larger problems. There are some advantages listed as [32];

- It can provides more accurate results for the complex structure.
- It can capture local effects.
- It can include different material properties.
- It can provides good visualization of the total problem.

The FEM solution is not the exact solution. It is the approximation of the solution of the problem. The accuracy of the solution depends on the number and size of those small parts which represents the mesh density of the structure. In addition to the mesh density, element types and the simplifications of the structures when meshing the model. The boundary conditions and simulation of the loading also have an important influence on the analysis results. Therefore, the model should be carefully modeled to reduce the possible errors.

The static analysis, the structural dynamic analysis, or thermal analysis can be easily modelled with FEM tools. Additionally, stability of the structure can also be analyzed by FEM. Static buckling problems can be solved either linear eigenvalue method, Riks method, or Quasi-static method with dynamic solver. In the following sections, these methods are used to investigate the static buckling and the dynamic buckling is investigated with the explicit method.

4.1 Linear Eigenvalue Buckling Method

This type of analysis is used to determine static buckling load factor and the corresponding mode shape. In this method, the structure is assumed to be perfect. Therefore, it is not possible to add the imperfection in the analysis. Since the buckling is highly affected by the imperfection, this method may not simulate the real life buckling problems with high accuracy. However, the mode shapes obtained from the linear eigenvalue analysis can be readily used as the imperfection input. The closely spaced eigenvalue which are obtained from linear buckling analysis give a clue that the structure is highly imperfection sensitive.

Linear buckling analysis consist of two steps. In the first step, the external load to structure to create a small linear elastic deflection. In the second step, the eigenvalue analysis to obtain the buckling load of the structure as [33],

$$[K] \{D\} = \{R\} \quad (4.1)$$

where $[K]$ is the global stiffness matrix, $\{D\}$ is the displacement and $\{R\}$ is the external load.

For the buckling analysis of shells, a reference level of the load $\{R\}_{ref}$ is applied. The membrane stresses in the shell is found from the standard linear analysis. The corresponding stiffness of the reference level of loading is $[K_\sigma]_{ref}$. To obtain actual level of the loading, the reference level value is multiplied with a scalar λ .

$$[K_\sigma] = \lambda [K_\sigma]_{ref} \text{ when } \{R\} = \lambda \{R\}_{ref} \quad (4.2)$$

where $[K_\sigma]$ is the stress (geometric) stiffness matrix.

Here $[K_\sigma]$ is the function of the geometry, state of membrane stress, and the displacement field, but independent from the material properties. While global stiffness matrix $[K]_{ref}$ does not depend on the load, the geometric stiffness matrix $[K_\sigma]_{ref}$ includes the effect of the load. Buckling occurs without change of the external load at bifurcation point [33].

$$([K] + \lambda_{cr} [K_\sigma]_{ref}) \{D\}_{ref} = \lambda_{cr} \{R\}_{ref} \quad (4.3)$$

If the buckling displacement is written in terms of relative displacement, Equation 4.3 becomes,

$$([K] + \lambda_{cr} [K_\sigma]_{ref})(\{D\}_{ref} + \delta D) = \lambda_{cr} \{R\}_{ref} \quad (4.4)$$

Substituting Equation 4.3 into Equation 4.4 gives Equation 4.5.

$$([K] + \lambda_{cr} [K\sigma]_{ref}) \{\delta D\} = 0 \quad (4.5)$$

This is a typical eigenvalue problem. λ_{cr} is the smallest roots which defines the critical buckling loading value. $\{\delta D\}$ is an eigenvector and it gives the buckling mode shape for corresponding λ_{cr} .

4.1.1 Verification study for linear buckling analysis

To validate the linear buckling analysis' procedure, a study from the literature is taken as reference study [6] and the same model is constructed. The composite cylindrical shell is modeled with eight layer and each layer has the thickness of 0.125 mm. The stacking sequence is $[0/45/-45/0]_s$.

Material and geometrical properties are given in Table 4.1. In our analysis, as in the reference study, four node shell element with reduced formulation (S4R) is used with elements with element size of 5.25 mm.

Table 4.1 : Properties of the composite shell.

Property	Value
E_{11}	148500 MPa
E_{12}	9850 MPa
G_{12}	4800 MPa
ν	0.286
ρ	1700 kg/m ³
L	390 mm
R	150 mm



Figure 4.1 : Linear buckling reference source analysis result [6].

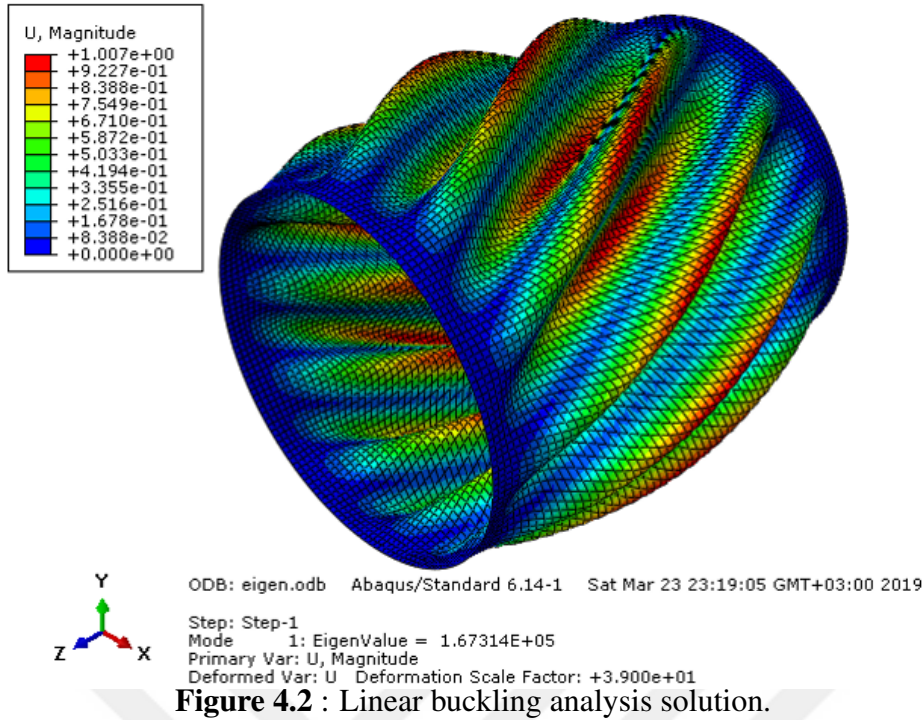


Figure 4.2 : Linear buckling analysis solution.

Figure 4.1 shows the solution of the reference study, Figure 4.2 shows the solution of present study. As it can be easily seen from figures, the same buckling shapes are obtained. In the reference study, the critical load value is found as 165 kN, while it is 167 kN in this study. The critical load and mode shapes for reference and present study are close enough, therefore the present model is validated for linear buckling analysis.

4.2 Non-linear Static Buckling Analysis

Nonlinear behavior of static buckling can be solved with Riks method which is also known as Arc-Length method. Generally, static analysis solver uses the Newton-Rapson method which is not able to follow backtracking of the nonlinear curves. On the other hand, the Riks method is a better alternative to follow nonlinear curve of post buckling behavior. However, the sharp edge can cause divergence of the analysis. Therefore, to have a smooth load-displacement curve, the addition of imperfection is recommended for that type analysis. When the imperfection is added, the load-displacement curve becomes a limit load problem. In this method, the critical load is defined as the point on the load-displacement curve where the system cannot tolerate any increase of the externally applied load, and the curve drops suddenly. Equation 4.6 is the nonlinear system of equations to be solved [7].

$$F^{int}(u) - F^{ext} = 0 \quad \text{and} \quad F^{int}(u) - \lambda q = 0 \quad (4.6)$$

Consider a point (u_o, λ_o) which satisfies the equilibrium path . While using the Arc-Length method, changes in both displacement (Δu) and force factor $(\Delta \lambda)$ are unknowns, in contrast to the Newton-Raphson method [7]. In this case, the equation can be written as [7],

$$R(u', \lambda') = F^{int}(u_0 + \Delta u) - (\lambda_0 + \Delta \lambda)q = 0 \quad (4.7)$$

The point $(u_0 + \Delta u, \lambda_0 + \Delta \lambda)$ is on the equilibrium path, that point can be possibly a solution. But, in the most cases, it is hard to estimate the solution with the first try. Therefore, the additional corrections $(\delta u, \delta \lambda)$ are made and the new point will be $(u_0 + \Delta u + \delta u, \lambda_0 + \Delta \lambda + \delta \lambda)$. Substituting that point into the main equation (Equation 4.6), then applying Taylor's expansion, and retaining only linear terms results in,

$$F^{int}(u_0 + \Delta u) + [\partial F^{int}(u)/\partial u]_{u_0 + \Delta u} \delta u - (\lambda_0 + \Delta \lambda + \delta \lambda)q = 0 \quad (4.8)$$

where $[\partial F^{int}(u)/\partial u]$ is Jacobian matrix and represented with $[K_T]$.

$$[K_T]_{u_0 + \Delta u} \delta u - \delta \lambda q = -[F^{int}(u_0 + \Delta u) - (\lambda_0 + \Delta \lambda)q] = -R(u', \lambda') \quad (4.9)$$

where δu and $\delta \lambda$ are the unknown parameters. However, vector u has a size of $N \times 1$. In this case, there are N equations and $N+1$ unknowns. Therefore, we need one more equation which is the Arc-Length Equation 4.10 [7].

$$(\Delta u + \delta u)^T . (\Delta u + \delta u) + \psi^2 (\Delta \lambda + \delta \lambda)^2 (q^T . q) = \Delta l^2 \quad (4.10)$$

Parameters ψ and Δl are selected by the user and they can be used to adjust the sensitivity. Writing Equation 4.10 and Equation 4.9 in the matrix form gives,

$$\begin{bmatrix} [K_T] & -q \\ 2\Delta u^T & 2\psi^2 \Delta \lambda (q^T . q) \end{bmatrix} \begin{bmatrix} \delta u \\ \delta \lambda \end{bmatrix} = - \begin{bmatrix} R \\ A \end{bmatrix} \quad (4.11)$$

where $R = F^{int}(u_0 + \Delta u) - (\lambda_0 + \Delta \lambda)q$ and $A = -(\Delta u^T \Delta u + \psi^2 \Delta \lambda^2 (q^T . q) - \Delta l^2)$. Until the convergence is reached, the addition of corrections $\delta \lambda$ and δu is continued. When ψ is equal to one, it is called as the Spherical Arc-Length method. Radius of arc (Δl) should be defined by user and the next point is selected as the intersection point of equilibrium path and the arc. It is shown in Figure 4.3 [7].

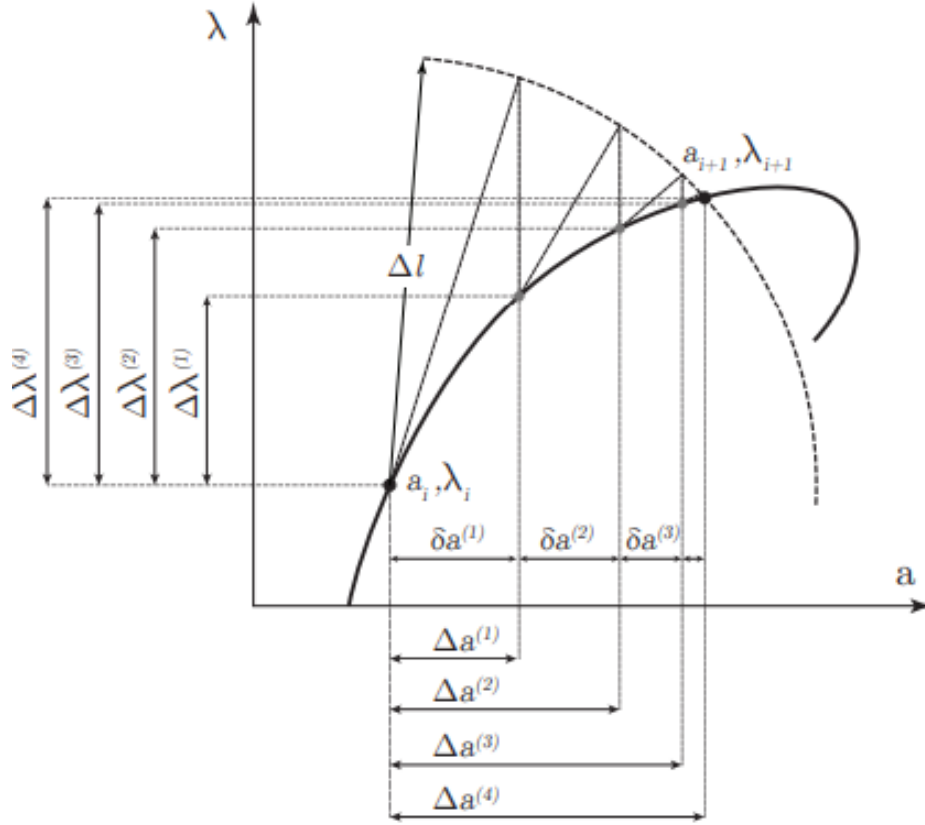


Figure 4.3 : Arc length method iterations [7].

4.2.1 Verification study for the nonlinear static buckling analysis

On the nonlinear static buckling analysis, the same geometry taken from the reference study [6] in the previous section is used. Material and geometric properties of the model are given in Table 4.1. Although the recommendation of the Riks analysis is adding the imperfection to the analysis, our analysis does not include imperfection, therefore we are only able to compare our results to the results of the linear eigenvalue analysis. The load-displacement curve which is obtained from Riks analysis is shown in Figure 4.4 and the critical point is identified as the point where the slope of the curve is suddenly changed. The load value of that point is 162 kN and the corresponding buckling shape obtained from Riks analysis is given in Figure 4.6. The buckling shape of reference study is also given in Figure 4.5. As it can be seen from two figures (Figure 4.6 and Figure 4.5), the similar buckling shapes are obtained. In addition to the buckling shape, the critical load with nonlinear analysis is found as 164 kN in the reference study while our result is 162 kN. The difference between the critical loads is about 1%.

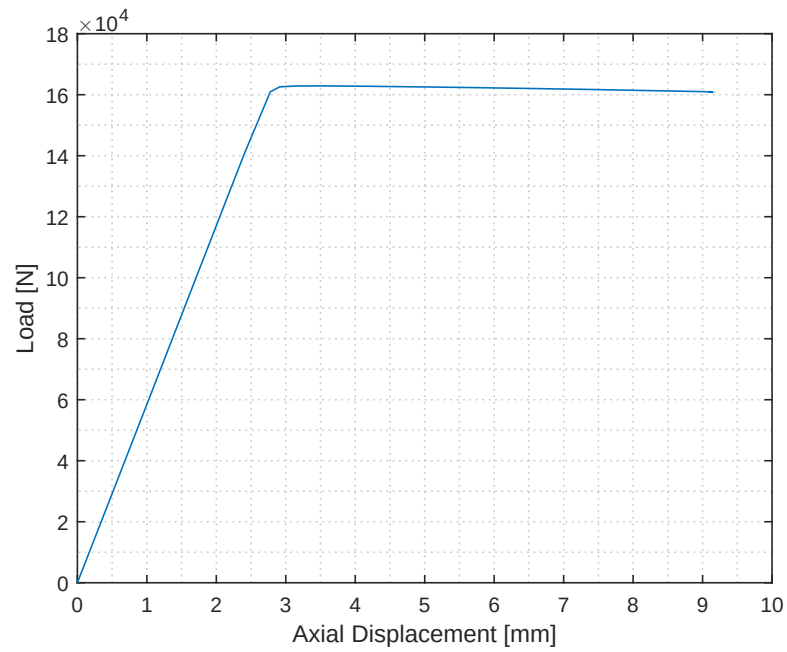


Figure 4.4 : Load displacement graph for Riks analysis.



Figure 4.5 : Reference study solution of Riks method.

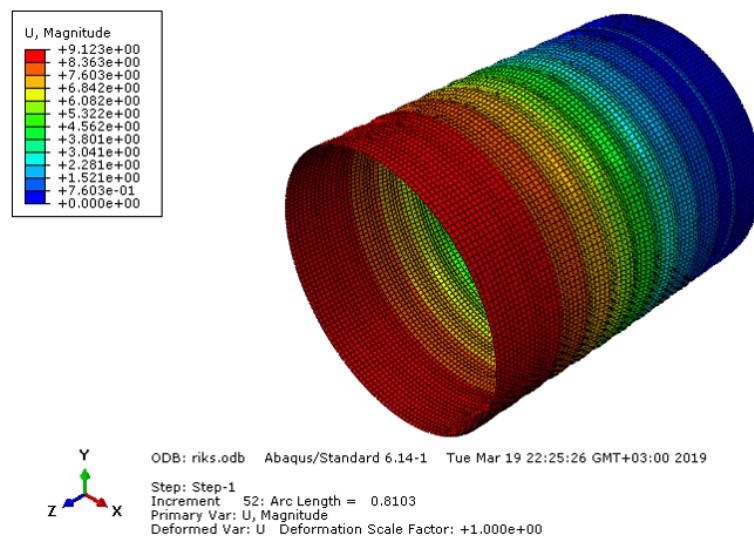


Figure 4.6 : Analysis solution of Riks method.

4.3 Quasi-Static Buckling Analysis

The basic idea of the quasi static buckling analysis is that the inertial effects can be minimized when the load is applied slow enough. In that way, dynamic explicit solver which is actually used for dynamic problems such as impact loading is used for the solution of the static problem. However, long simulation time of the motion is costly. For that reason, time scaling or mass scaling are used to minimize the inertial effect. For time scaling, the modal analysis is performed to obtain the fundamental natural frequency of the structure before explicit analysis. Then, the loading time is chosen as longer than the period of the fundamental natural frequency. Another method to reduce the elapsed time is the mass scaling. Mass scaling of entire or some elements of the model provides a larger stable time increment [34]. Although smaller time increment provides higher stability, it causes longer running time to complete whole simulation. Therefore, an optimum stable time increment should be used. The stable time increment varies with size, stiffness and density [34].

The dynamic solver is based on the integration of the equation of motion with the central difference method [34]. Using explicit solver has an advantage as it does not require tangential stiffness matrix and iterations. Moreover, the dynamic solver is able to solve large deformations, and the convergence can be obtained easier than the static solver. Since the buckling is stability problem, large deformations are seen and the static analysis may diverge. Although it provides buckling load, the post buckling behavior can be fallacious. In order to be sure of neglecting inertial effects, it should be compared to internal and kinematic energies.

4.3.1 Verification study for quasi static buckling analysis

The same reference [6] is used for verification. The loading time is assigned as 1 s. When the kinetic and internal energies are compared, it is found that the kinetic energies are very smaller than the internal energies. Thus, it can be assumed as quasi-static analysis. When the axial reaction force versus displacement is plotted, the sudden decrease occurs when the load value is 168 kN as shown in Figure 4.7 for the perfect model.

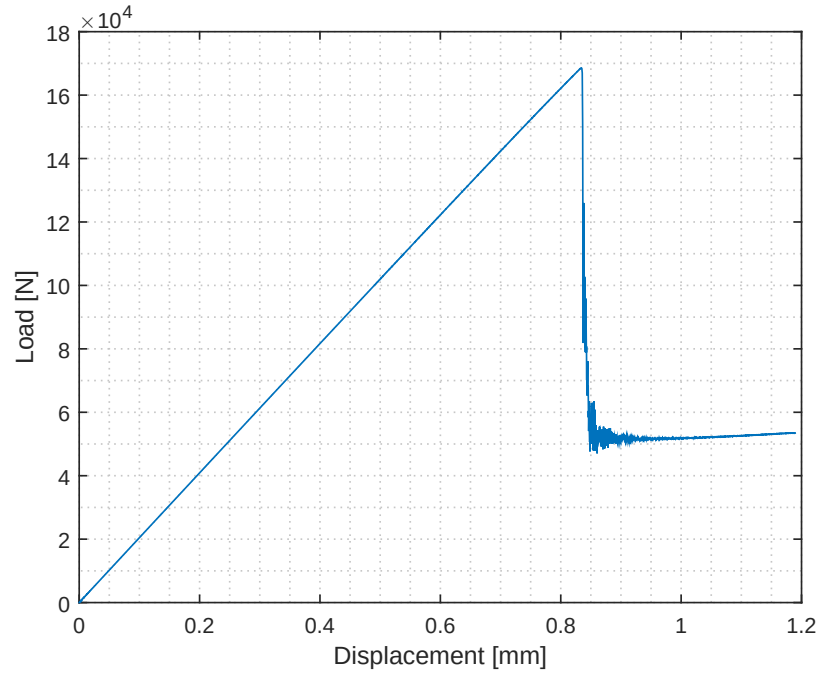


Figure 4.7 : Load displacement curve for quasi static analysis.

4.4 Dynamic Buckling Analysis

The dynamic buckling load is found with the explicit dynamic solver. The dynamic buckling solver is more suitable to solve large deformations than to the static solvers. Although the dynamic solver can be used for quasi static analysis, dynamic buckling analysis is basically different from the quasi-static analysis. As it is mentioned before, to be able to use dynamic solver to simulate static behavior, some methods could be used for minimizing the effect of inertial terms. However, for the dynamic problem, the inertial effects are not small and cannot be ignored for the dynamic problem. In the dynamic buckling analysis in this section, load control method with step load is used.

4.4.1 Verification study for dynamic buckling analysis

A verification study is made in this section to see whether the good approximation of dynamic buckling load is obtained. For that reason, "Effect of Imperfection on the Dynamic Buckling Composite Conical Shell Under Axial Impact" is used as the reference study [8]. The buckling of a laminated conical shell is investigated and its FEA model is given in Figure 4.8. The conical shell under step load consist of 8 CFRP plies with total thickness of 1 mm and it has stacking sequence of $[0^\circ/0^\circ/60^\circ/-60^\circ]_s$, where 0° is the axial direction of the shell. Geometric and material properties of the structure are given in the Table 4.2.

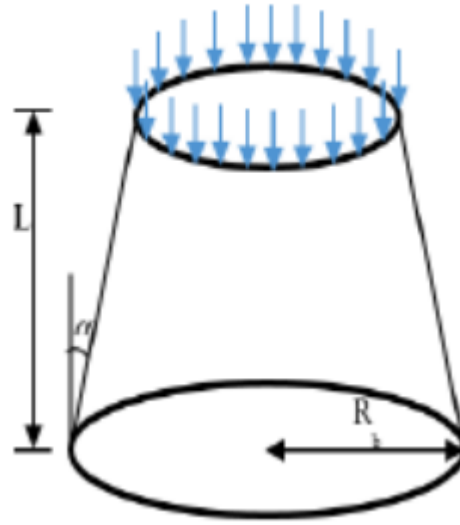


Figure 4.8 : Axially loaded conical shell.

Table 4.2 : Properties of the conical shell.

Property	Value
E_{11}	134780 MPa
E_{12}	9250 MPa
G_{12}	4800 MPa
ν	0.286
ρ	1700 kg/m ³
L	300 mm
α	15 °
R	150 mm

Before the explicit dynamic buckling analysis, modal analysis is performed to decide suitable loading time. According to the reference study [8], the first natural frequency is found as 718 Hz while it is found as 739 Hz in that study. First mode shape obtained from modal analysis result is illustrated in Figure 4.9. The difference is around 3% and it is close enough. The loading duration for dynamic analysis is determined from the first natural period. The analysis can be assumed as dynamic when the loading occurs in time duration equal or shorter time interval. For 718 Hz natural frequency, the corresponding period is around 1.4 ms. In the light of that knowledge, in the reference study, the loading duration t_l is chosen as 0.5 ms, 1 ms, 1.5 ms, 2 ms, and 5 ms. Therefore, it can be observed that the behavior for shorter and longer than the natural period of the structure. The validation study is also made in terms of imperfect model with the imperfection scale factor of 0.05.

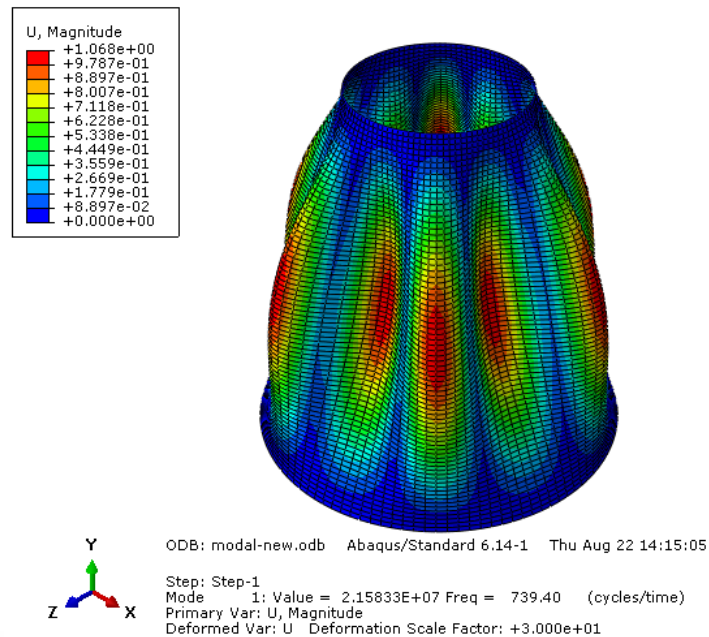


Figure 4.9 : 1.Mode of modal analysis result.

After modal analysis, the linear eigenvalue analysis are made. In the reference, critical buckling load is found as 113 kN while we found as 117 kN. Again, the error is around 3%. The result of linear eigenvalue analysis is given in Figure 4.11 and the same result for reference study is given in Figure 4.10. As it can be seen from Figure 4.11 and Figure 4.10, mode shapes of the linear buckling analysis are quite similar.

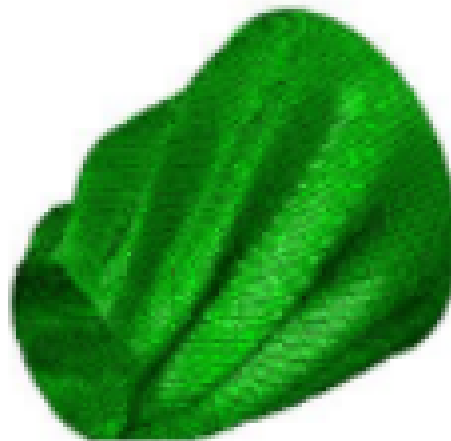


Figure 4.10 : Linear eigenvalue buckling result in the study of Vipin and Priyadarsini [8].

To determine the dynamic critical dynamic buckling load, two the dynamic buckling criteria are used. These two approaches are Budiansky - Roth and total energy-phase plane approaches. In Budiansky-Roth approach, the critical dynamic load is obtained with repetition of the explicit dynamic analysis. The displacement-time curves for

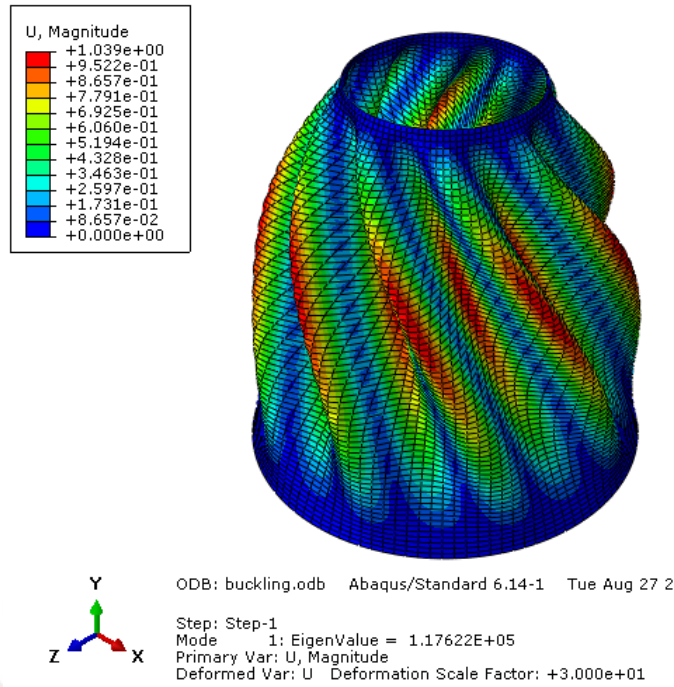


Figure 4.11 : Linear eigenvalue buckling analysis result.

different applied loads are drawn. The critical dynamic load is determined from the displacement of the structure which increases suddenly. The displacement-time history of the reference study is shown in Figure 4.12. The critical load value is seen between the 87 kN and 89 kN. It can be taken as average value of 88 kN for the perfect model.

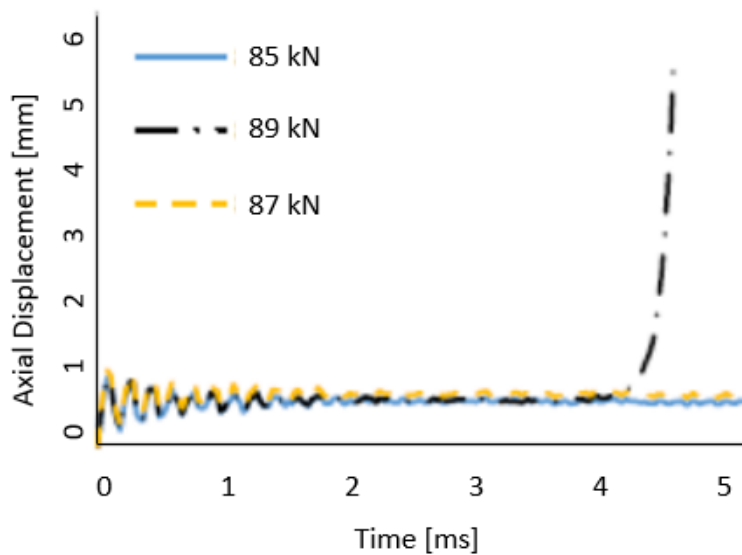


Figure 4.12 : Axial displacement for different load magnitudes on perfect composite conical shell for $t_l = 5ms$ [8].

In our representation of the same model, step loads are applied with different amplitude to the structure for the 5 ms loading duration to the perfect model. The first guess of the load amplitude is chosen and the displacement-time curve is drawn for that load

value. When the displacement value of the curve is suddenly increase, the load value is decreased.

In the second iteration, assume that the displacement value is huge. Then, the critical load is in between that two value. Try to narrow that interval as much as possible. In the analysis for the model, the displacement-time history for 89 kN and 90 kN is given in Figure 4.13. It is obviously seen that the critical dynamic buckling load is between 89 kN and 90 kN. In other words, the critical load value can be taken as 89.5 kN according to Budiansky-Roth approach.

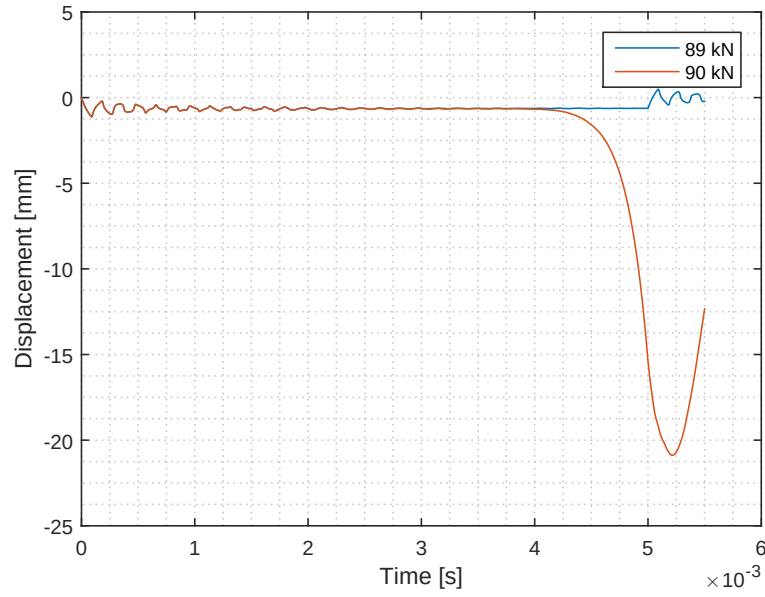


Figure 4.13 : Displacement graphic of perfect conical shell for different loading value for $t_l = 5ms$.

Another method to determine critical dynamic load is total energy - phase plane approach. In this method, when the velocity-displacement graph of the loaded edge is spread, it can be assumed that the structure become unstable. In our study, velocity-displacement graphs are given in Figure 4.14 and Figure 4.15 for 89kN and 90kN. The change in stability with the increasing loading is clearly seen.

Similar analysis steps are made for $t_l = 2ms$, and the critical load value is obtained as 115 kN with Budiansky-Roth approach (Figure 4.16) while it is found as 110 kN in the reference study. Figure 4.18 and Figure 4.17 show scattering of velocity-displacement curve when load reaches critical value. However, the critical load is not obvious for 5 ms as shown in Figure 4.14 and Figure 4.15.

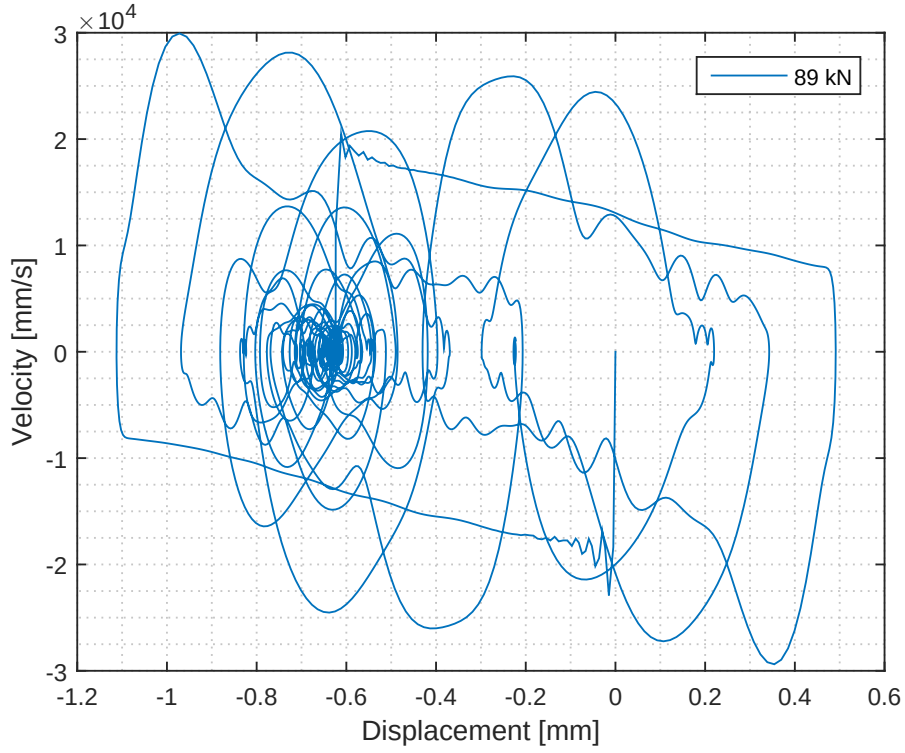


Figure 4.14 : Displacement velocity graphic for 89 kN $t_l = 5ms$.

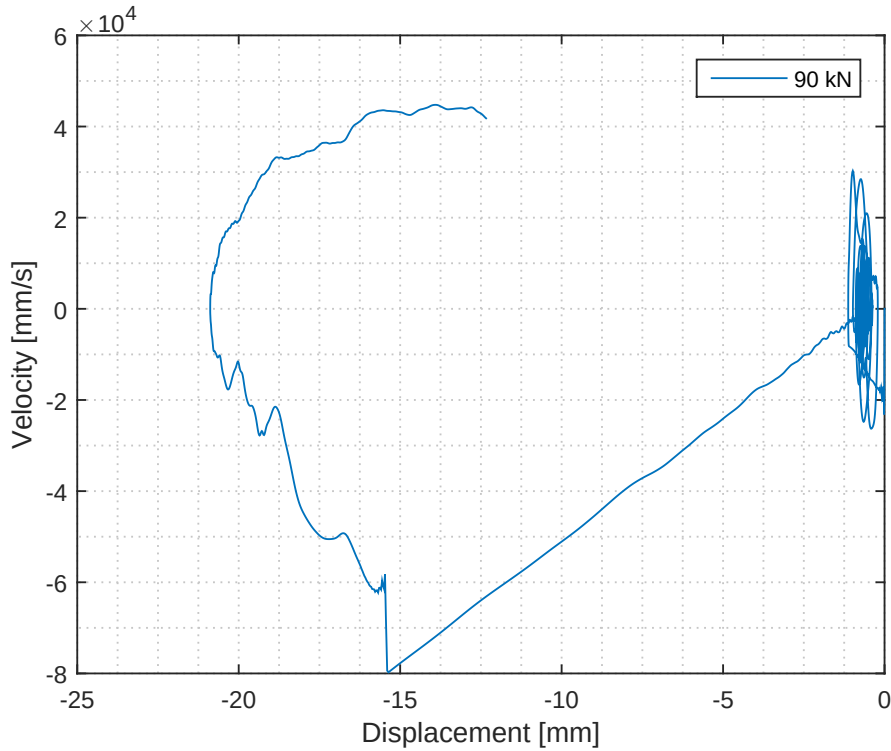


Figure 4.15 : Displacement velocity graph for 90 kN $t_l = 5ms$.

Finally, the analyses are repeated for the loading time of 1 ms. The displacement-time history is given for 175 kN and 179 kN in Figure 4.19. Although increase in the displacement is not as obvious as in 5 ms loading time, the divergence behavior of

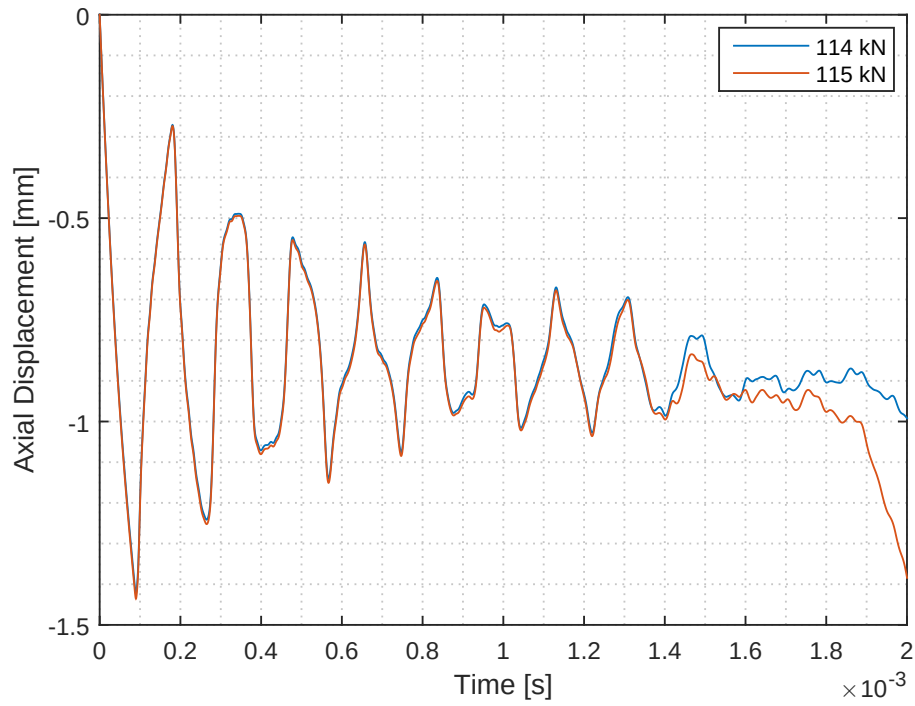


Figure 4.16 : Displacement time graph of 114 kN and 115 kN for $t_l = 2ms$.

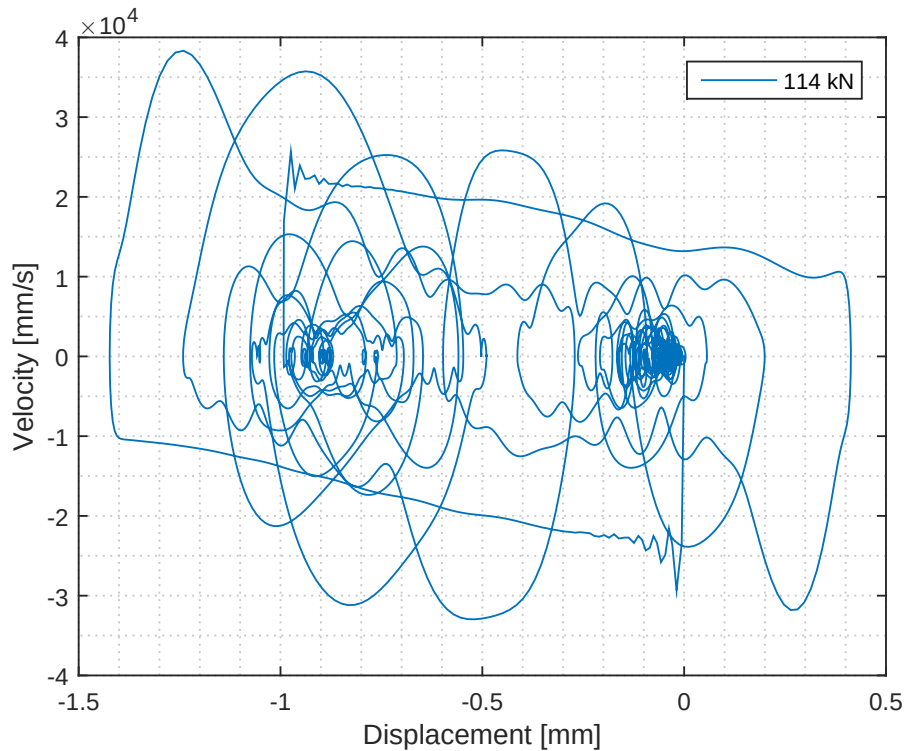


Figure 4.17 : Displacement velocity graph of 114 kN for $t_l = 2ms$.

the displacement is still seen. The velocity-displacement results by using total energy phase-plane approach in Figure 4.20 illustrates the stable and unstable behavior for 175 kN and 179 kN. Thus, these two methods reaches the same critical value of the

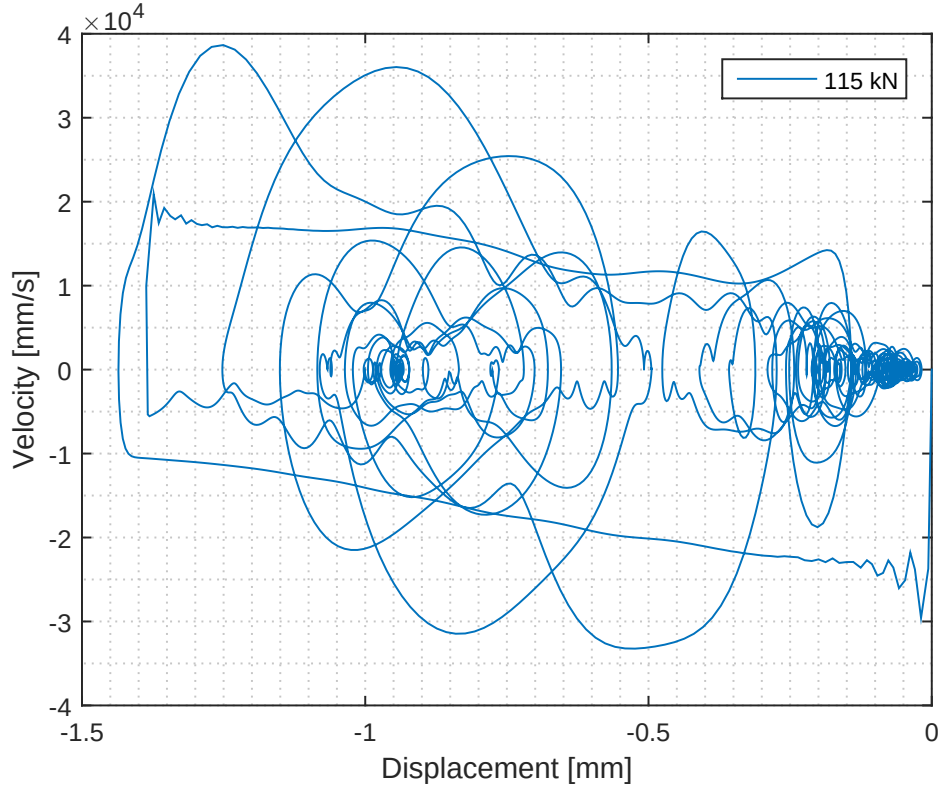


Figure 4.18 : Displacement velocity graphic of 115 kN for $t_l = 2ms$.

dynamic buckling load, and it can be taken as the average value of the interval (177 kN).

To see the effect of imperfection on dynamic buckling load, the geometric imperfection to the model is added and 0.05 scale factor is used. The displacement-time history for $t_l = 5ms$ is given in Figure 4.21 for 79kN and 80kN. When the displacement-time history is investigated, it is seen that at the 5 ms the displacement value suddenly increased to 80 kN. However, that sudden increase is not seen for 79kN. Therefore, the critical value is detected as 80 kN. The velocity-displacement graph with total energy - phase plane approach is given in Figure 4.22 for checking stability. When the velocity-displacement graph is examined, although the differences are not obvious, the scatter is larger for 80 kN.

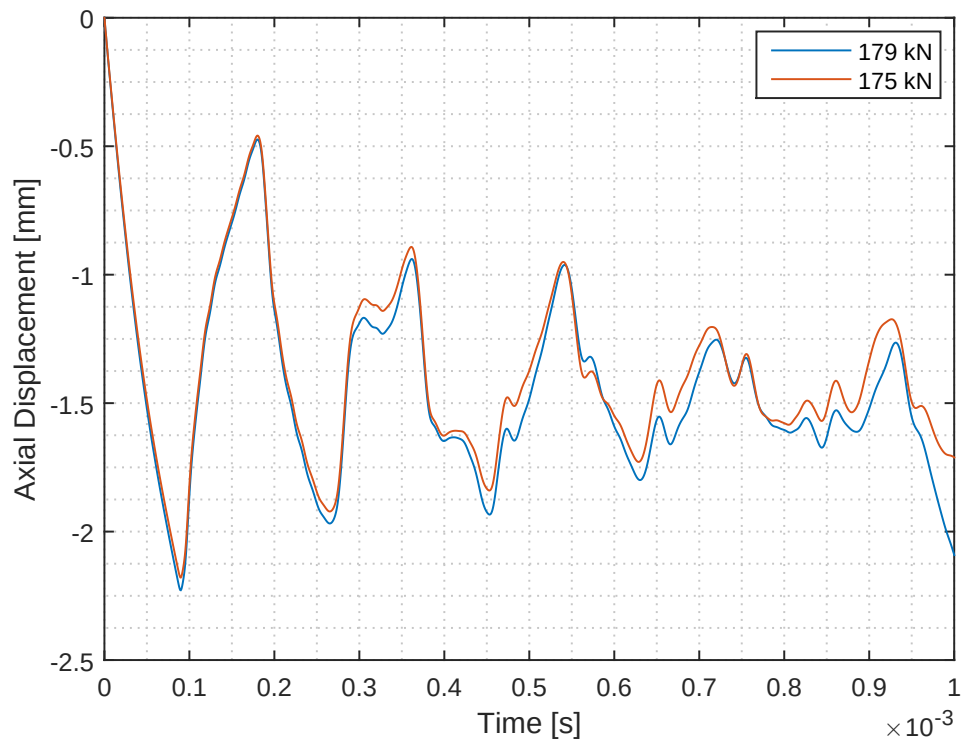


Figure 4.19 : Displacement graphic of 175 kN and 179 kN for $t_l = 1ms$.

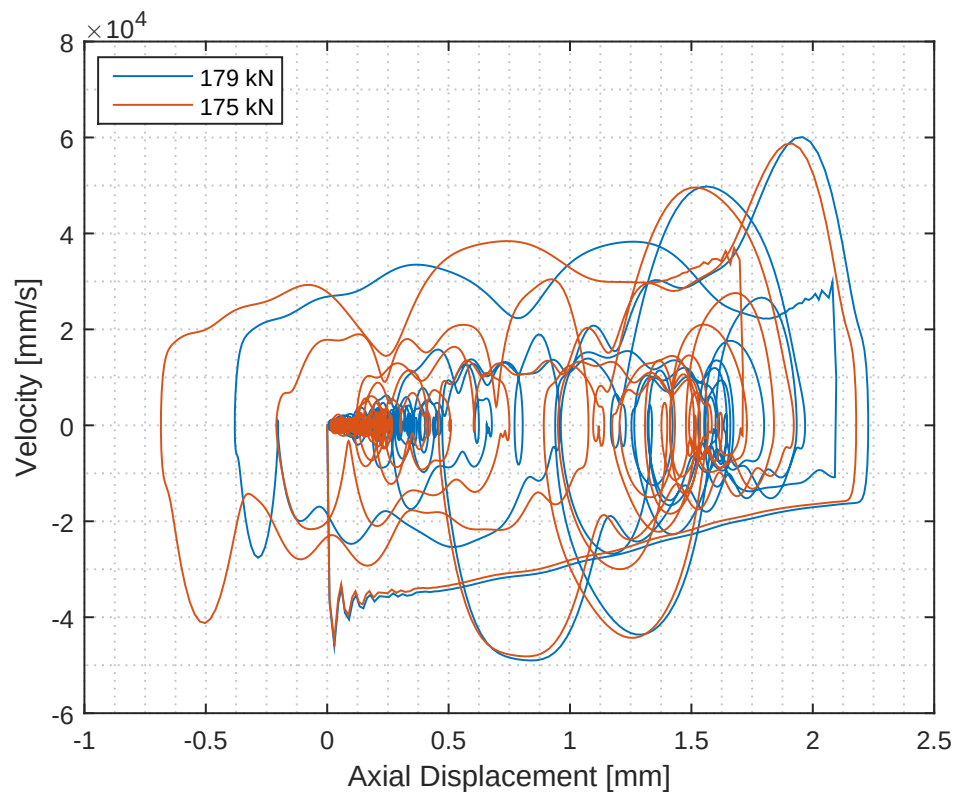


Figure 4.20 : Velocity displacement graphic of 175 kN and 179 kN for $t_l = 1ms$.

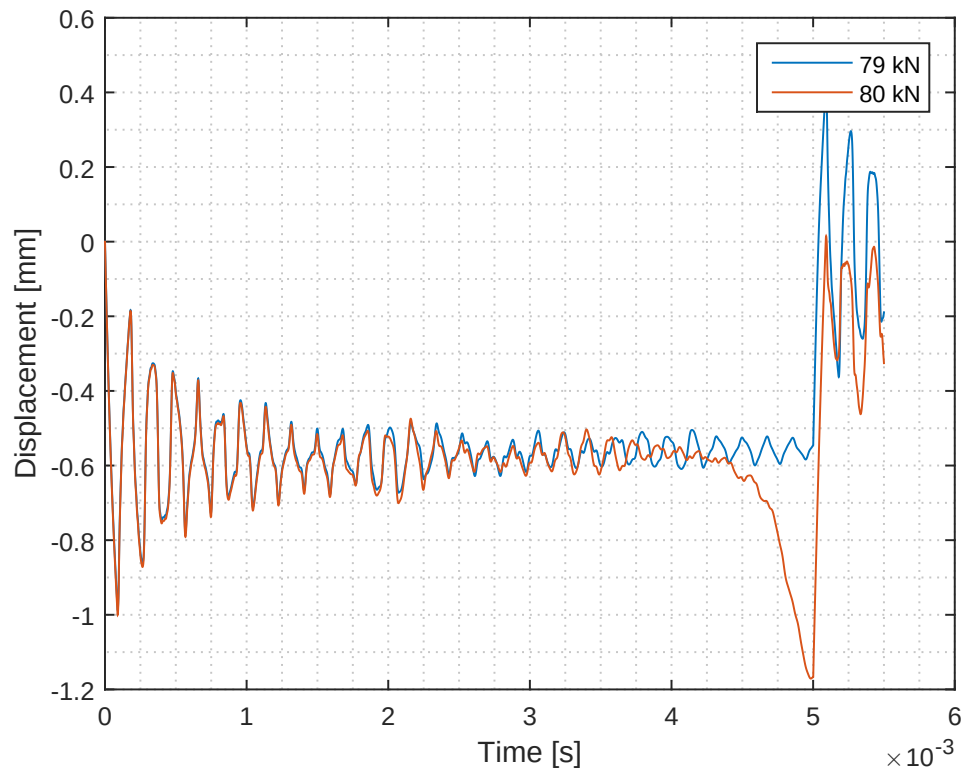


Figure 4.21 : Displacement graphic of 79kN and 80kN for $t_l = 5ms$ with 0.05 scale factor.

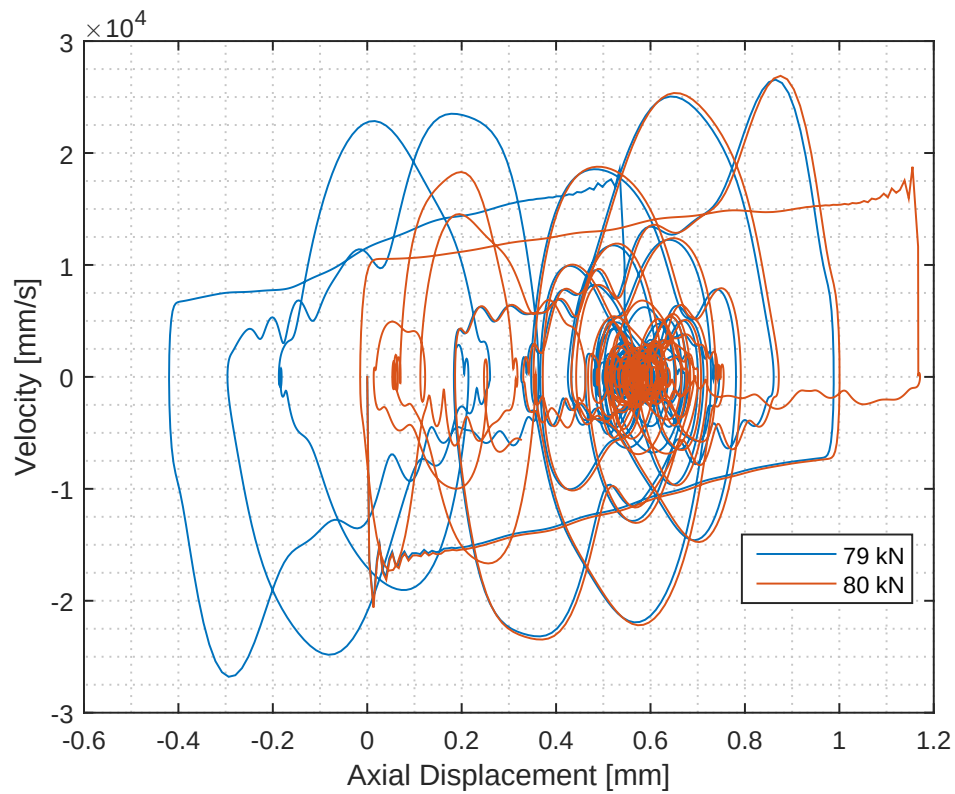


Figure 4.22 : Velocity displacement graphic of 79 kN and 80 kN for $t_l = 5ms$ with 0.05 scale factor.

4.5 FEM Model

The FEM analyses are conducted by using a commercial program, ABAQUS. In this study, a cylindrical shell is considered and effect of some parameters are investigated on this geometry. The cylinder has 250 mm radius, 500 mm length. The number of the layers is 20 with a thickness of 0.127 mm for each of layer. The axial load is applied to the edge of the shell. The material properties are taken from literature and they are given in Table 4.3. The constituted model of is shown in Figure 4.23.

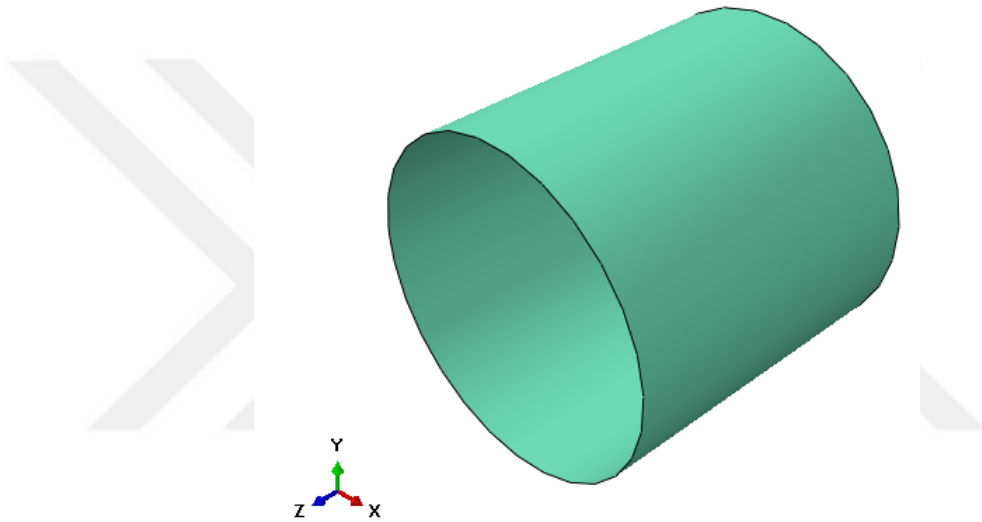


Figure 4.23 : Geometry of example structure.

Table 4.3 : Material properties [1].

E_1	E_2	G_{12}	ν_{12}
127.5 GPa	9.0 GPa	5.7 GPa	0.25

First, mesh density study is performed to choose suitable size for mesh elements. Stacking sequence of $[45/90/-45/0/45/90/-45/0/45/0]_s$ is used for mesh density studies. S4R shell elements which has four node and six degrees of freedom for each node with reduced integration are used. The analyses are performed for different mesh sizes, and buckling load values and elapsed times are compared. Results are shown in the Figure 4.24. From this study, the buckling load can be determined as reached threshold value for the mesh size 10 mm. Smaller mesh size causes the increase of the time and larger mesh size causes the increase in the error. Thus, 10 mm is determined as mesh size for the following analysis.

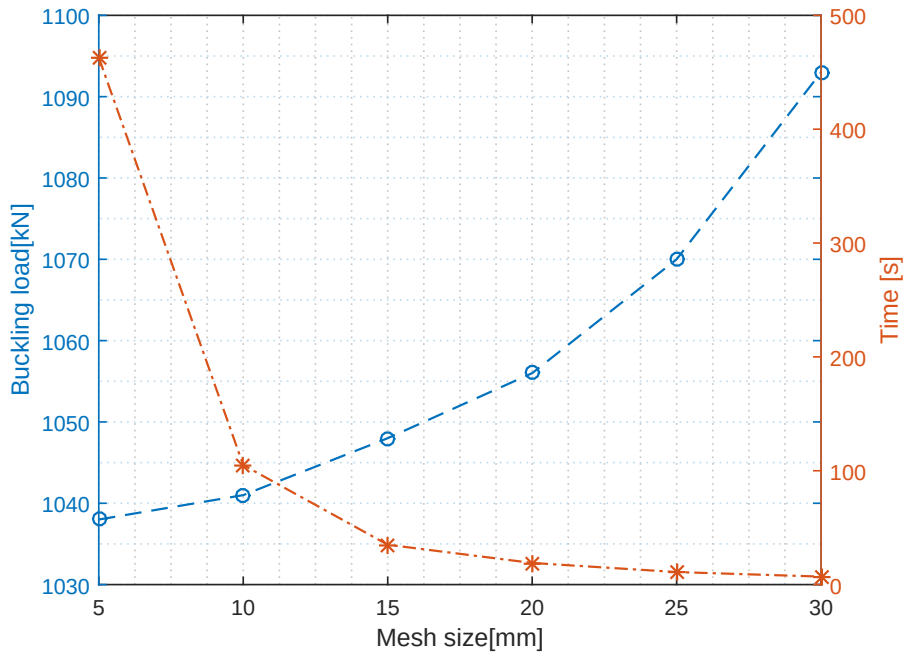


Figure 4.24 : Mesh convergence study.

4.5.1 Effect of some parameters on static Buckling

4.5.1.1 Effect of stacking sequence

To realize the effect of the stacking sequence, the buckling load of stacking sequence $[(\pm\theta^\circ)_n]$ is calculated by FEM analysis with ABAQUS for different values of θ and n which is the number of repeated $\pm\theta^\circ$. The results are given in the Table 4.4, Figure 4.25 and Figure 4.26. Also, normalized buckling load in Figure 4.26 is calculated as dividing the buckling load for current θ° to the buckling load of $\theta^\circ = 0^\circ$ to be able to compare the buckling resistance without the effect of the number of the lamina.

Table 4.4 : Critical buckling load for stacking sequence $[(\pm\theta^\circ)_n]$.

θ	n=2	n=3	n=4	n=5
0	22151	47737	83089	128961
10	25279	57504	102261	159367
20	27613	65425	118506	187248
30	26201	61675	111551	175955
40	25162	58997	106356	167457
50	25391	59206	106167	166799
60	26985	62108	111231	174251
70	29289	66511	118467	185165
80	25135	57018	101188	157082
90	21985	47042	81497	125301

According to result of these analysis, the maximum resistance to the buckling is seen around $\theta^\circ = 15^\circ$ and $\theta^\circ = 75^\circ$. In addition, the highest buckling load is found $n = 5$. In the study of Taheri-Behrooz and Omidi [9], the effect of fiber orientation with similar stacking sequence is investigated both theoretically and numerically. Similar behavior is obtained, and it is shown in Figure 4.27. It can be easily seen that the highest buckling load is obtained about $\theta^\circ = 15^\circ$ and $\theta^\circ = 70^\circ$.

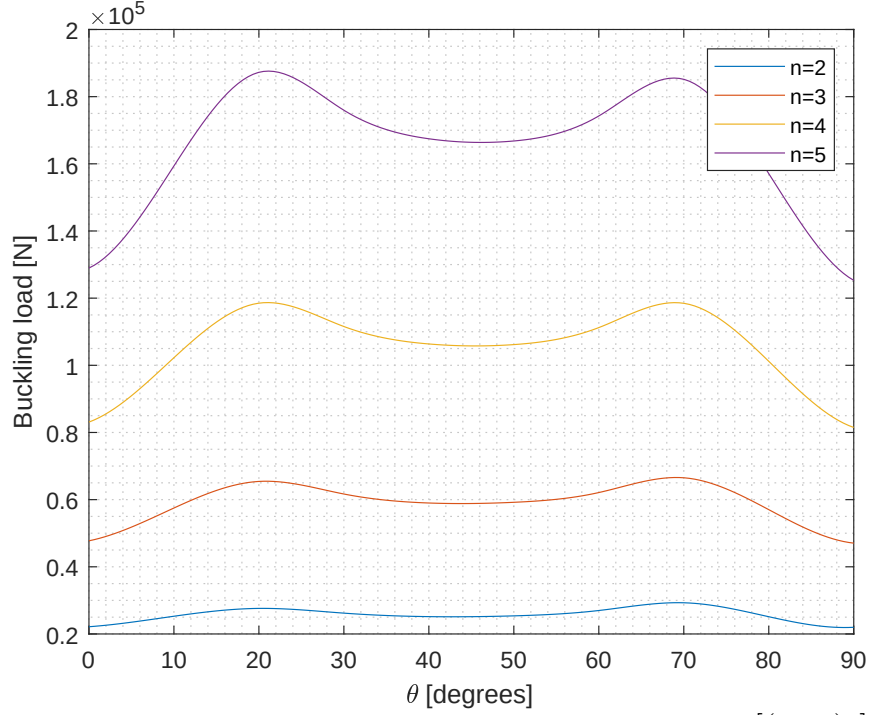


Figure 4.25 : Buckling load of the stacking sequence of $[(\pm\theta^\circ)_n]$.

Then, buckling analysis of the composite cylindrical shells with the stacking sequence $[0/\theta^\circ]_{10}$ are made and the results are given in Table 4.5. According to data in Table 4.5, when θ° increases, buckling load increases until some point. However, it slightly decrease above $\theta = 45^\circ$. In Table 4.5, analytically calculated buckling load values are also given. The analytical calculations are based on the formulas which are presented in Chapter 2 and a MATLAB code is written for this aim which is given in Appendix B. The agreement of the analytical results and FEM results is generally good. However, there are substantial differences of (25.8%) for the $[0^\circ/30^\circ]_{10}$ and (17.9%) for $[0^\circ/15^\circ]_{10}$.

In addition, some analyses are made to find out the best location of lamina (inner, middle, outer) in the laminate which will have a higher buckling load. These analyses are made for composite cylindrical shell that its geometric and material properties were

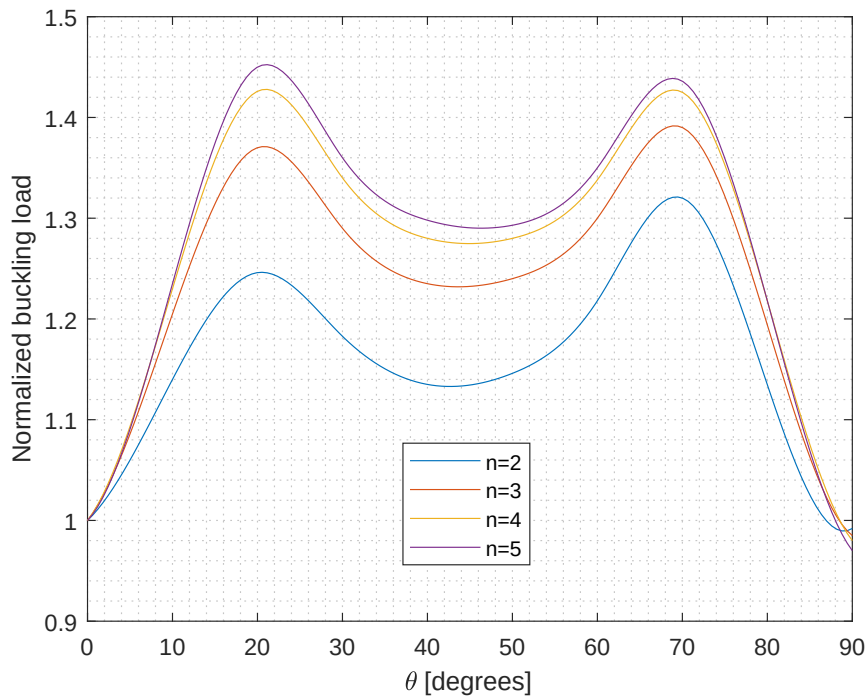


Figure 4.26 : Normalized buckling load of the stacking sequence of $[(\pm\theta)_n]$.

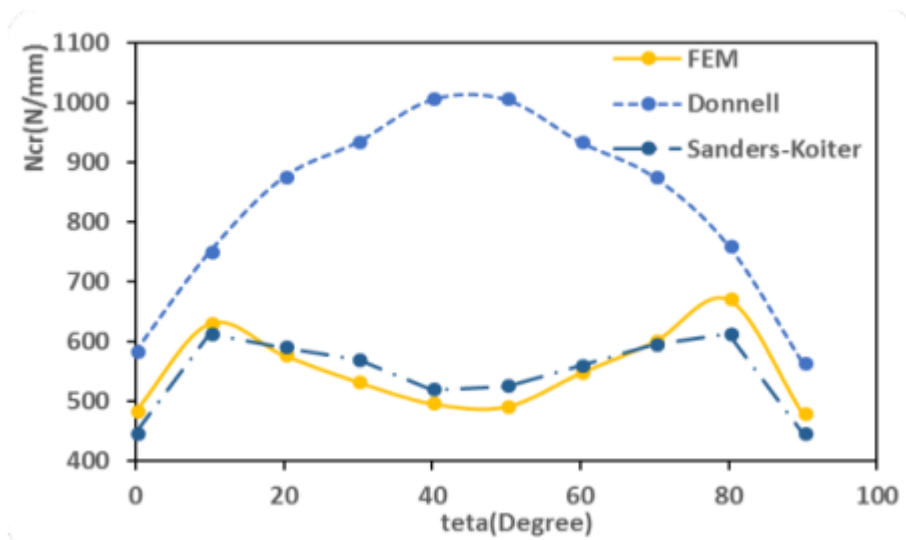


Figure 4.27 : Buckling load predicted by theories and FEM for composite $[\pm\theta/\pm\theta/\theta]_s$ [9].

Table 4.5 : Critical buckling load for stacking sequence $[0/\theta^\circ]_{10}$.

θ	FEM [N]	Analytical [N]	difference
$[0/15]_{10}$	452795	551360	17.9
$[0/30]_{10}$	553876	746290	25.8
$[0/45]_{10}$	691296	730270	5.3
$[0/60]_{10}$	687708	697660	1.4
$[0/75]_{10}$	679154	672740	1.0
$[0/90]_{10}$	685836	663430	3.4

given previously. As the stacking sequence of $[\theta^\circ/0^\circ/0^\circ]$, $[0^\circ/\theta^\circ/0^\circ]$, and $[0^\circ/0^\circ/\theta^\circ]$ are analyzed for different values of θ . The results of these analyses are given in Figure 4.28. It can be observed that the fiber orientation of the outer lamina is the most effective one. To be sure from that result, 0° constant angle is changed with 45° and 90° . The results are given in the Figure 4.29 and Figure 4.30, respectively. 90° constant angle has the similar conclusion with 0° while the result of 45° does not exactly have similar pattern.

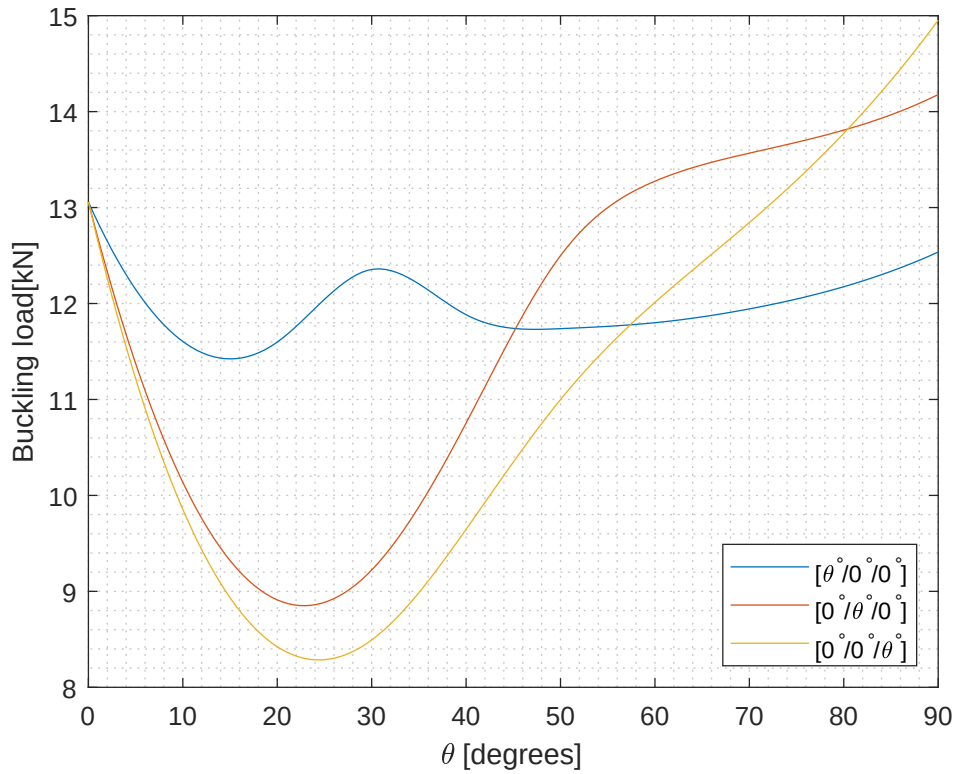


Figure 4.28 : Change in buckling load with fiber orientation of inner, middle, and outer lamina with 0° constant angle.

Furthermore, some analyses are made to see how changing fiber orientation of one layer affects the buckling load for different number of plies. For this purpose, the analyses are repeated with the stacking sequence of $[0_n/30]$ and $[0_{n+1}]$. The effect of number of plies is observed with these analysis. The analysis results are given in Table 4.6 and the % differences between stacking sequence $[0_n/30]$ and $[0_{n+1}]$ are plotted in Figure 4.31. If the fifth column of Table 4.6 is plotted as in Figure 4.32, it can be seen that changing angle of one layer can increase the buckling load about twice for the low number of plies, but this effect diminishes with more layers as expected.

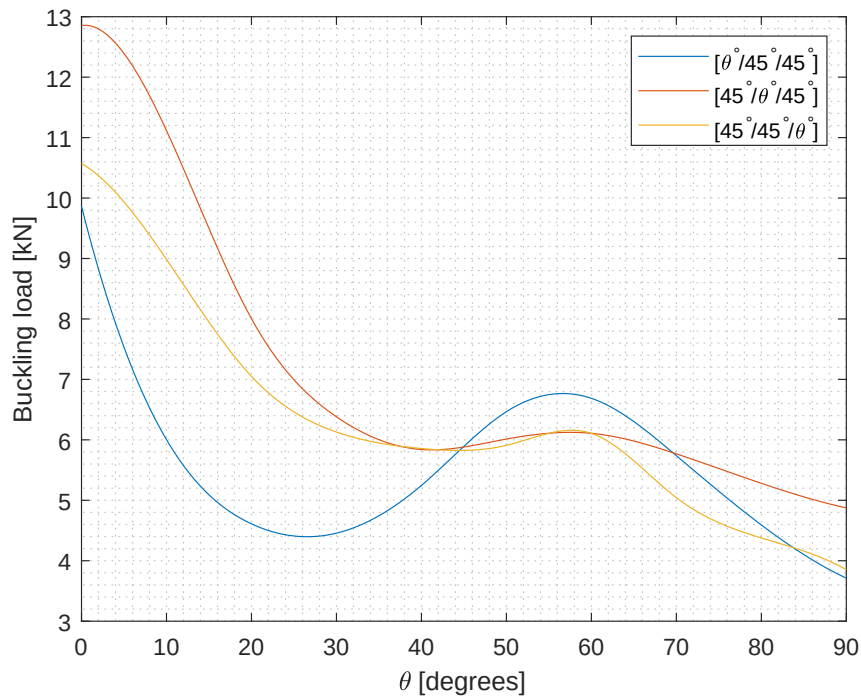


Figure 4.29 : Change in buckling load with fiber orientation of inner, middle, and outer lamina with 45° constant angle.

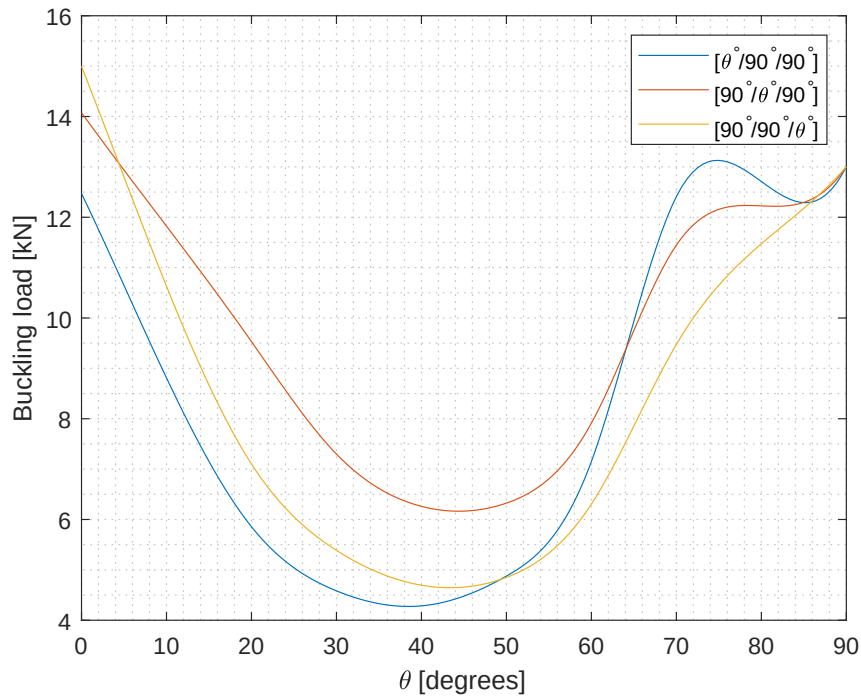


Figure 4.30 : Change in buckling load with fiber orientation of inner, middle, and outer lamina with 90° constant angle.

When a composite structure is designed, certain stacking sequence rules/recommendations should be considered and some of them are listed below [35], [36].

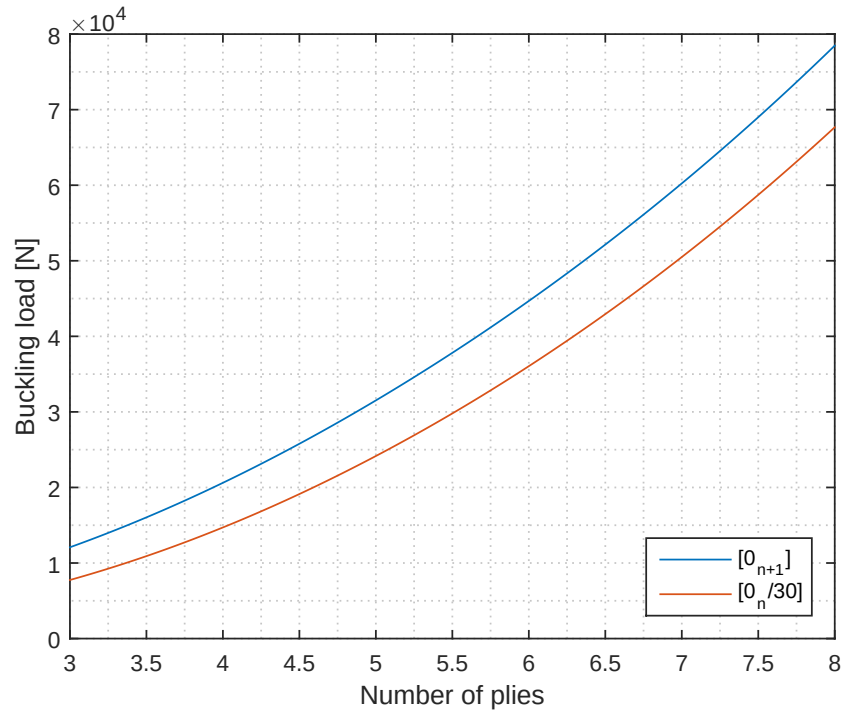


Figure 4.31 : Buckling load of $[0_n/30]$ and $[0_{n+1}]$

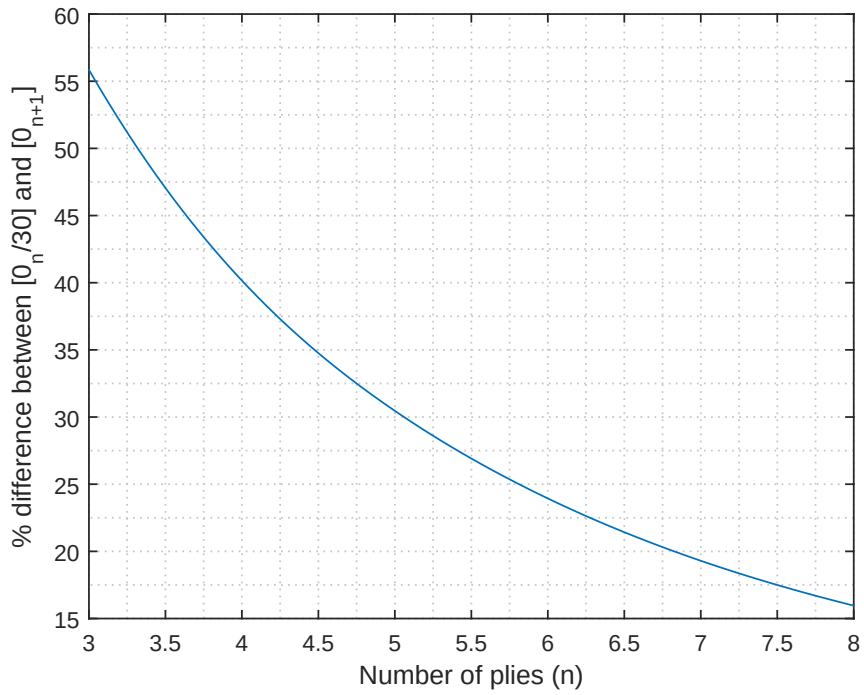


Figure 4.32 : % difference between the buckling load of stacking sequence $[0_n/30]$ and $[0_{n+1}]$.

Table 4.6 : Critical buckling load for stacking sequence $[0_n/30]$ and $[0_{n+1}]$.

number of plies(n+1)	n	$[0_n/30]$	$[0_{n+1}]$	% difference
3	2	7753	12083	55.85
4	3	14707	20614	40.16
5	4	24160	31520	30.46
6	5	36052	44682	23.94
7	6	50493	60240	19.30
8	7	67673	78474	15.96

- *Symmetric*: the sequences of the plies are symmetric about mid axis of the thickness and symmetric stacking sequence provides uncoupling of the membrane and bending behavior.
- *Balanced*: plies other than 0° and 90° should be in pair of $\pm\theta$ in both above and below of the mid-surface and it provides uncoupling of the membrane and bending behavior.
- $\pm 45^\circ$ *Outer Plies*: To increase damage tolerance, the outer ply should be $\pm 45^\circ$.
- *Ten Percentage Rule*: Minimum 10% of the plies should be in $\pm 45^\circ$, 90° , and 0° . It provides minimal in-plane stiffness in all fiber direction.
- *Contiguity*: the standard thickness of a lamina is 0.127 mm and the sequential plies with the same orientation angle should not exceed the four layer. If the plies are thicker, then it is recommended not to exceed two.
- *Damtol*: the topmost and nethermost ply should not be in the direction of the 0° .
- *Disorientation*: the difference between the angles which are sequential should not exceed 45° .

After studying the effect of angles on buckling loads, some stacking sequences are created together with those composite design rules. The created stacking sequences obey the rules as the percentage rule, contiguity, disorientation, damtol, and $\pm 45^\circ$ outer plies. In addition, 10 stacking sequences are grouped as symmetrical-unbalanced (stacking sequence label: 1,2,3,4), symmetrical-balanced (stacking sequence label: 5,6,7), and unsymmetrical-balanced (stacking sequence label: 8,9,10) and they are listed in Table 4.7.

Table 4.7 : Chosen stacking sequences.

Symmetric Unbalanced			Symmetric Balanced				Unsymmetric Balanced		
1	2	3	4	5	6	7	8	9	10
45	45	45	45	-45	45	45	45	-45	45
30	90	45	0	-45	60	90	90	-30	60
15	-45	90	-45	0	45	60	-45	0	45
0	0	90	90	0	30	45	0	0	30
0	45	45	45	45	0	30	45	30	0
-15	90	0	45	45	-30	0	90	45	-30
-30	-45	0	0	90	-45	-30	-45	90	-45
-45	0	-45	-45	-45	-60	-45	0	45	-60
-60	45	0	90	90	-45	-60	45	30	-45
90	0	-45	-45	45	90	-45	0	0	90
90	0	-45	-45	45	90	-45	0	0	90
-60	45	0	90	90	-45	-60	-45	-30	45
-45	0	-45	-45	-45	-60	-45	90	-45	60
-30	-45	0	0	90	-45	-30	45	90	45
-15	90	0	45	45	-30	0	0	-45	30
0	45	45	45	45	0	30	-45	-30	0
0	0	90	90	0	30	45	90	0	-30
15	-45	90	-45	0	45	60	45	0	-45
30	90	45	0	-45	60	90	0	30	-60
45	45	45	45	-45	45	45	-45	45	-45

The corresponding buckling load of the ply orientations are given in Table 4.8 which are calculated with linear buckling analysis, quasi static analysis and analytical methods. For the analytical calculation, a MATLAB code is written and given in Appendix B. It is seen from Table 4.8, that stacking sequence may increase the buckling load more than twice.

Table 4.8 : Critical buckling load for chosen stacking sequence.

Sequence Number	Linear Analysis [kN]	Quasi Static [kN]	Analytical [kN]	deviation1 linear analysis	deviation2 quasi static
1	751	867	725	3.59	19.58
2	1037	1037	1040	0.29	0.29
3	857	884	783	9.45	12.90
4	1103	1132	1019	8.24	11.09
5	984	1082	869	13.23	24.51
6	859	936	793	8.32	18.03
7	796	852	693	14.86	22.94
8	1091	1100	1102	1.0	0.18
9	1144	1097	1021	12.05	7.44
10	1000	996	919	8.81	8.38

Even though the same angles are used, their sequences has huge impact on the responses. To see this, stacking sequences 2,3,4,5 are investigated. These four stacking

sequences have the same number of 45° , 0° , -45° , 90° . However, buckling loads vary from each other by as much as 30%. The same situation also occur for stacking sequences 6,7,10. Also, these three stacking sequences allow us to compare the symmetry property of the composite. Although sequences 6 and 7 are symmetrical and balanced, their critical buckling loads are unexpectedly lower than sequence 10 which is unsymmetrical and balanced. Moreover, the lowest buckling load is seen at the stacking sequence which has a high variety of the orientation angles.

4.5.1.2 Effect of material and imperfection

To understand, how the material properties affect buckling resistance, some of the materials such as aluminum, steel, unidirectional composite, and woven composite are compared. Both woven and unidirectional composites are modeled with the stacking sequence of $[45/0/-45/90/45/45/0/-45/90/-45]_s$. The material properties are given in Table 4.9 for composites and Table 4.10 for metals.

Table 4.9 : Properties of composites.

Material	E_{11} [MPa]	E_{22} [MPa]	ν_{12}	ρ [kg/m ³]	G_{12} [MPa]
UD	148000	9650	0.30	1500	4550
Woven	52000	52000	0.302	1320	2350

Table 4.10 : Properties of metals.

Material name	E[MPa]	ν	ρ [kg/m ³]
Aluminum	70000	0.30	2700
Steel	200000	0.30	7800

Buckling results of analyses for these materials are given in Table 4.11. According to results, it can be said that metals have higher buckling load. However, they are heavier than the composites. Therefore, the comparison of buckling load to mass ratios is more suitable and it is lower for the composite materials (Table 4.11). Therefore, the composites are preferable in applications where weight is an important design parameter.

After the materials are compared with the same thickness, the thickness of the cylinders are adjusted so that each cylinder with different materials would have the same mass of 3kg. Their new thickness values and their buckling loads are given in Table 4.12. It is obvious that the unidirectional composites are most advantageous.

Table 4.11 : Comparison of buckling load of different materials with same thickness.

Material name	Mass[kg]	Buckling load[kN]	Ratio of buckling load to mass[kN/kg]
Unidirectional	2.99	1248	417
Woven	2.63	867	328
Aluminum	5.39	1692	314
Steel	15.7	4835	307

Table 4.12 : Comparison of buckling load with same mass.

Material name	Thickness[mm]	Buckling load [kN]
Unidirectional	2.54	1248
Woven	2.88	1106
Aluminum	1.41	524
Steel	0.48	176

Moreover, imperfection sensitivity of different materials is investigated. For all these materials, the same thickness value is used and the buckling loads are presented in Table 4.13. These values are plotted in Figure 4.33.

Table 4.13 : Buckling load [kN] of different materials with same thickness for different imperfection amount.

Material name	0	0.25	0.5	0.75	1
Unidirectional	1248	865	863	768	697
Woven	867	609	509	447	405
Aluminum	1692	1138	953	860	795
Steel	4835	3250	2727	2457	2268

The models with same mass and different thickness are also used for further investigation of the relation between material and imperfection. In this extent, the results are presented in Table 4.14 and Figure 4.34.

Table 4.14 : Buckling load [kN] of different materials with same mass for different imperfection amount.

Material name	0	0.25	0.5	0.75	1
Unidirectional	1248	865	863	768	697
Woven	1106	869	733	650	592
Aluminum	524	320	267	254	256
Steel	176	88	74	66	65

Results for same thickness show that changing materials has no influence on the imperfection sensitivity. However, the models with the same mass show the different imperfection sensitivity behavior.

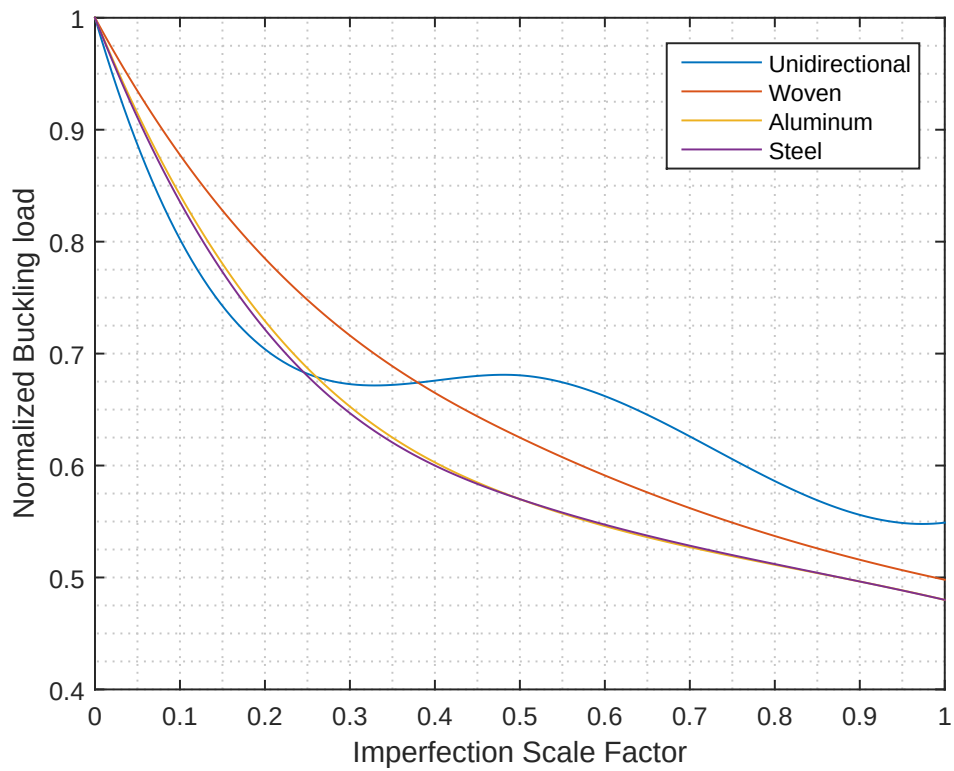


Figure 4.33 : Effect of imperfection on different materials with same thickness.

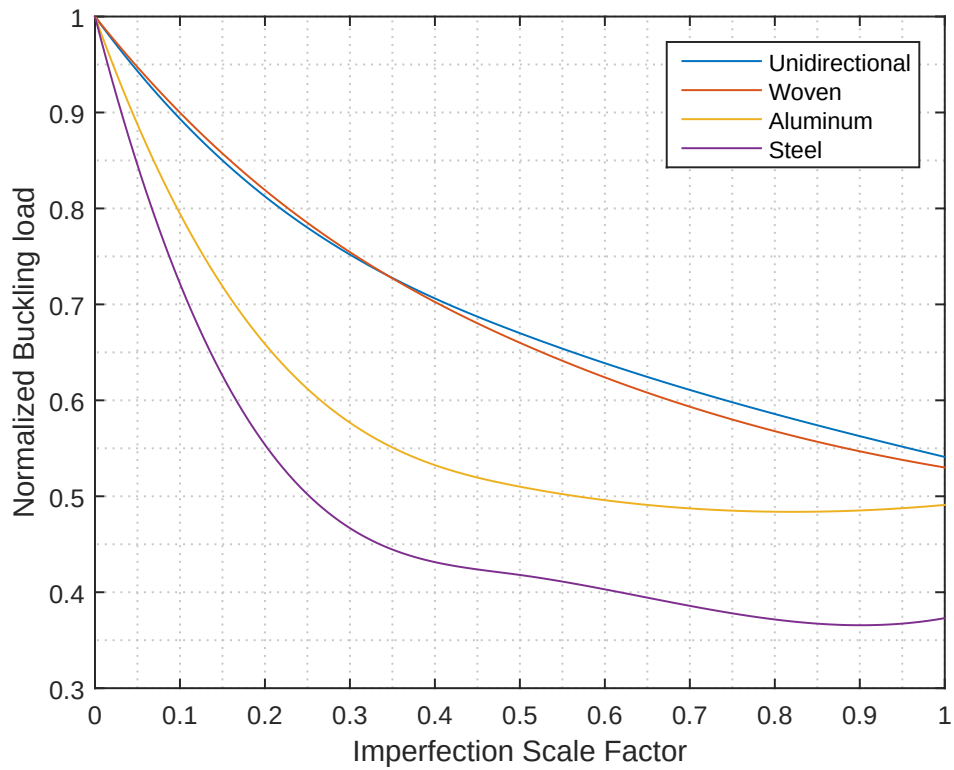


Figure 4.34 : Effect of imperfection on different materials with same mass.

4.5.2 Effect of length to diameter ratio

To determine how the critical buckling load is affected by length to diameter ratios (L/D), linear buckling analysis is run for three different stacking sequences given in Table 4.15. For this purpose, cylindrical shell with $L/D = 1, 1.5, 2$ are studied.

Table 4.15 : Composite Stacking Sequence.

Composite Types	Stacking Sequence
1	$[0/90]_{10}$
2	$[45/90/60/45/30/0/-30/-45/-60/-45]_s$
3	$[0/45/90/-45/0/0/-45/90/45/0]_s$

Results are given in Table 4.16. Buckling loads are also normalized with respect to $L/D = 1$ and are illustrated in Figure 4.35.

Table 4.16 : Comparison of buckling load of different L/D .

Length mm	Diameter [mm]	L/D	Stacking 1 [kN]	Stacking 2 [kN]	Stacking 3 [kN]
500	500	1	663	805	1064
750	500	1.5	653	800	1057
1000	500	2	650	799	1055

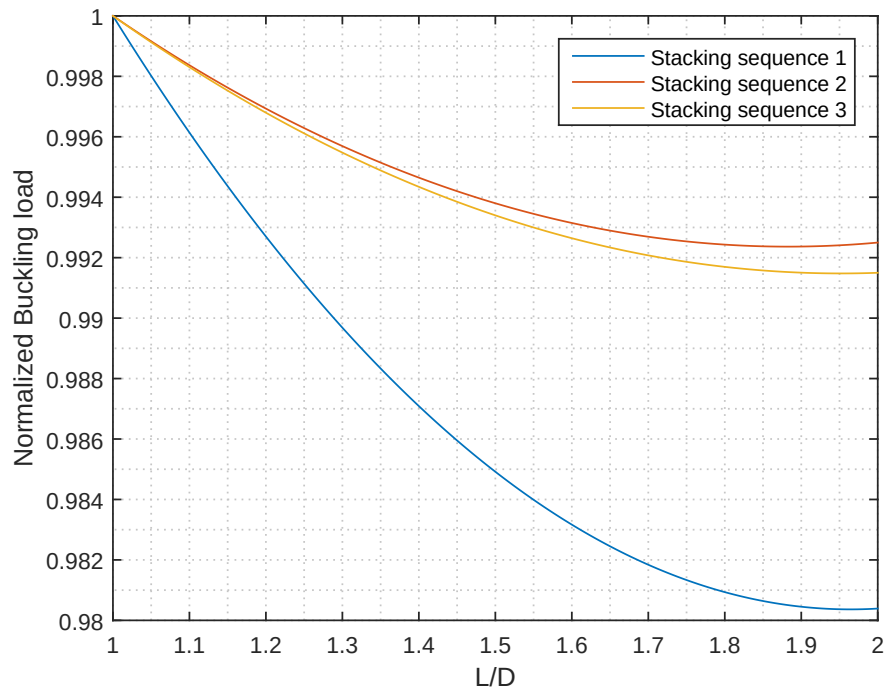


Figure 4.35 : Effect of length to diameter ratio on critical buckling load.

The buckling load does not change much with length to diameter ratio. Therefore, the normalized buckling load is close to 1. Buckling load is influenced by stacking sequence 1, 2, and 3 are very small. Therefore, it can be assumed that buckling loads are almost constant for different ratios of L/D for the same stacking sequences.

4.5.2.1 Effect of thickness

According to the basic formula of the critical buckling load for the isotropic material, the buckling load varies with the square of the thickness. However, estimation of the critical load value for orthotropic materials is not easy task as that of the isotropic materials. The analyses are made here to see whether the buckling load varies with the square of the thickness or not for the composite materials. For this aim, 2.54 mm, 3.81 mm, 5.08 mm, 6.35 mm and 7.62 mm thickness are used for three models which have the stacking sequence shown in Table 4.15.

Table 4.17 : Comparison of buckling load of different thickness.

t mm	Stacking 1 [kN]	Stacking 2 [kN]	Stacking 3 [kN]
2.54	663	805	1064
3.81	1487	1782	2391
5.08	2648	3152	4233
6.35	4090	4899	6552
7.62	5914	7003	9380

The buckling loads are obtained for these thicknesses are presented in Table 4.17 and these values are normalized by dividing the buckling load of 2.54 mm thickness value. The normalized buckling loads for different thickness values are shown in Figure 4.36. It is observed that the critical load values vary with the almost square of the thickness. It is plotted for three stacking sequences, and all of them show similar increment as shown in Figure 4.36.

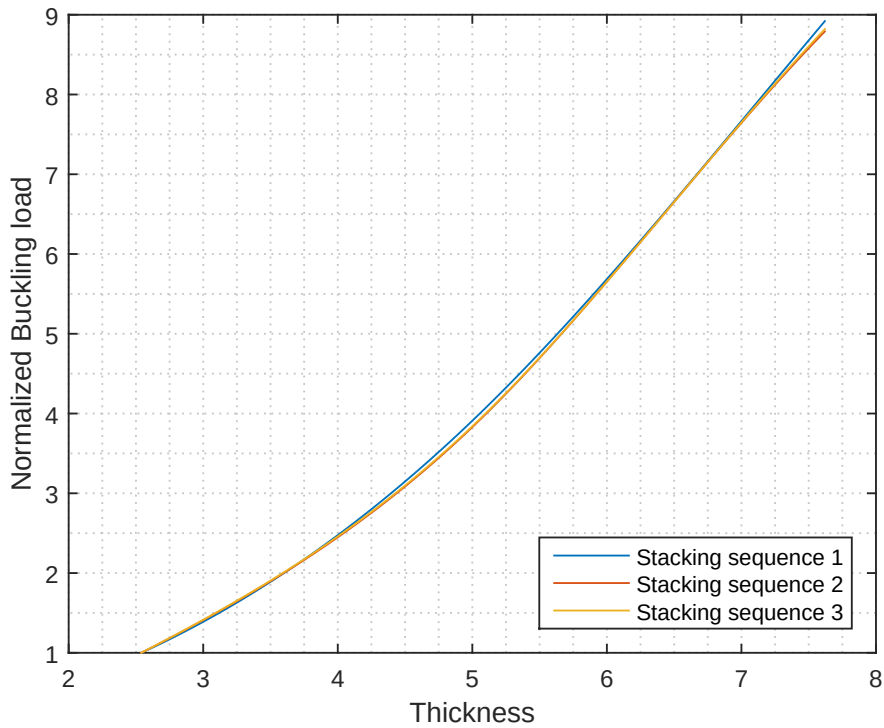


Figure 4.36 : Effect of thickness on critical buckling load.

4.5.3 Effect of some parameters on dynamic Buckling

4.5.3.1 Effect of length to diameter Ratio

Three different value of $L/D=1, 1.5, 2$ for the given stacking sequences in Table 4.15 are studied. The loading duration of 10 ms is considered in this section. The critical dynamic load values are shown in Table 4.18. The normalized values of dynamic buckling load are given in Figure 4.37. The effect of the length to diameter ratio on dynamic buckling load is higher than on the static buckling loads. It can be seen by comparing Figure 4.37 and Figure 4.35.

Table 4.18 : Comparison of dynamic buckling load of different L/D .

Length mm	Diameter [mm]	L/D	Stacking 1 [kN]	Stacking 2 [kN]	Stacking 3 [kN]
500	500	1	607	577	692
750	500	1.5	535	532	642
1000	500	2	487	482	567

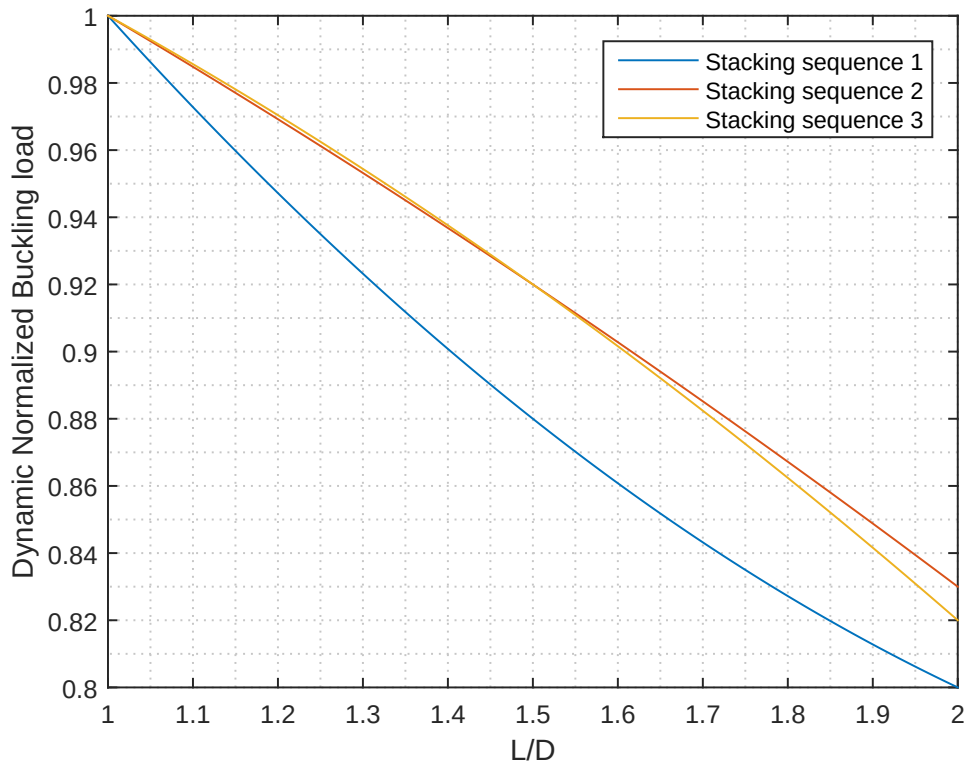


Figure 4.37 : Effect of length to diameter ratio on dynamic critical buckling load.

4.5.3.2 Effect of thickness

The effect of thickness for the static loading was investigated in the previous section. In this section, how the thickness of the shell affects the dynamic critical load is examined. The same stacking sequences in Table 4.15 are used and analyses are again made for the thickness of 2.54 mm, 3.81 mm, 5.04 mm, 6.25 mm and 7.62 mm. The results are given in Table 4.19 and the normalized values of them are presented in Figure 4.38. As it is seen, the influence of the thickness on dynamic buckling load does not depend on the stacking sequence. Buckling loads increase with the thickness for all stacking sequences.

Table 4.19 : Comparison of dynamic buckling load of different thickness.

t mm	Stacking 1 [kN]	Stacking 2 [kN]	Stacking 3 [kN]
2.54	607	577	694
3.81	1337	1445	1627
5.08	2427	2675	3102
6.35	3767	4000	4412
7.62	5552	5950	6667

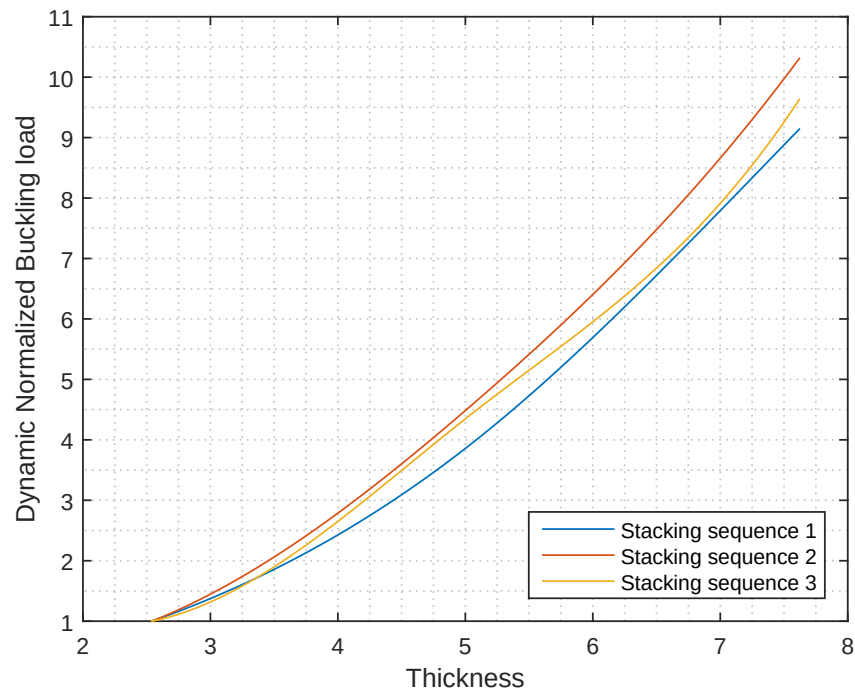


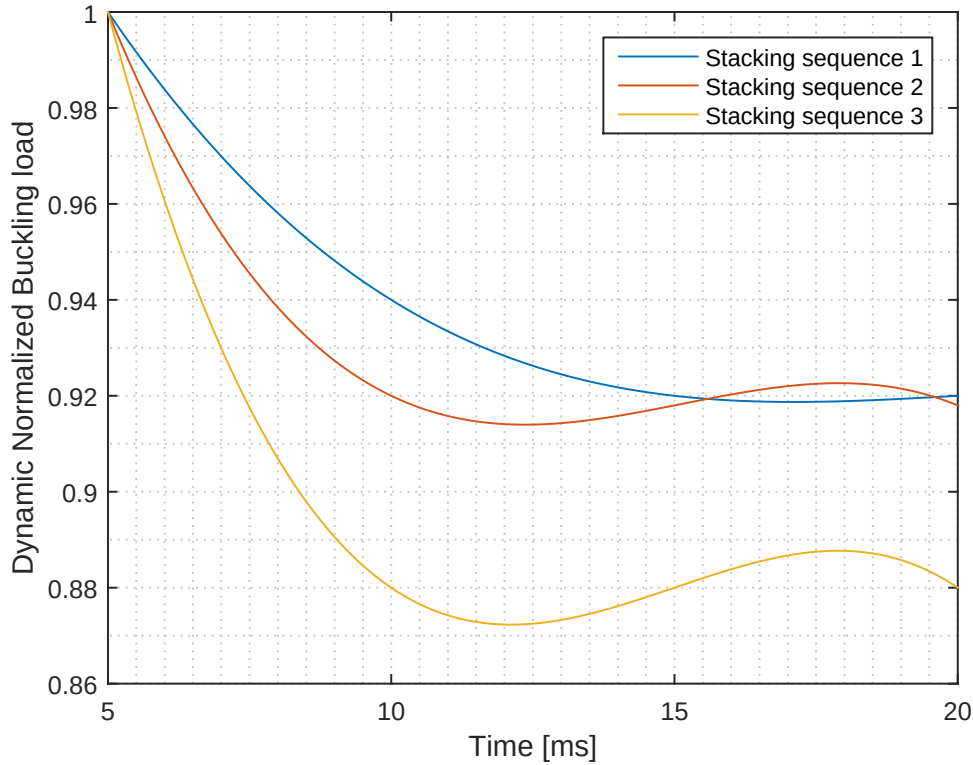
Figure 4.38 : Effect of thickness on critical dynamic buckling load.

4.5.3.3 Effect of loading duration

The investigation of how loading duration affects the critical dynamic buckling load is made in this section. To investigate the influence of the load duration, dynamic buckling load values are obtained from the FEM analyses. Loading times of 5, 10, 15, 20 ms are used for the stacking sequences in Table 4.15. Buckling loads for these durations are given in Table 4.20 and the normalized values of them are shown in Figure 4.39. In this figure, normalization is made by dividing the current dynamic buckling load to the dynamic buckling load value of 5ms. From these results, it is seen that the critical dynamic buckling loads decrease suddenly when the loading time increases from 5 ms to 10 ms. Although this behavior is the same for all stacking sequences, the dynamic buckling load of stacking sequence 3 decreases faster than that of the other stacking sequences. However, they stay almost constant after 10 ms for all cases. The longer duration of the loading, the closer it gets to the static behavior. However, when the comparison of the static buckling loads which is given in previous sections and dynamic loads are made, the static buckling is higher. The reason of that is the loading profile is not same. The static buckling has ramp profile while the step load is applied to the dynamic buckling analyses.

Table 4.20 : Comparison of dynamic buckling load of different loading duration.

time s	Stacking 1 [kN]	Stacking 2 [kN]	Stacking 3 [kN]
5	642	628	792
10	607	577	694
15	597	577	694
20	592	577	694

**Figure 4.39** : Effect of Loading Duration on Critical Dynamic Buckling Load.

4.5.3.4 Effect of loading shape

Effects of the loading shape on the dynamic buckling load are investigated in this section. For this, rectangular and sinusoidal load shapes are used. Amplitudes of those two loadings are shown in Figure 4.40. The three stacking sequences from previous sections are used.

Table 4.21 : Comparison of dynamic buckling load of different load shape.

Load Shape	Stacking 1 [kN]	Stacking 2 [kN]	Stacking 3 [kN]
Rectangular	607	577	694
Sinusodial	815	872	1227

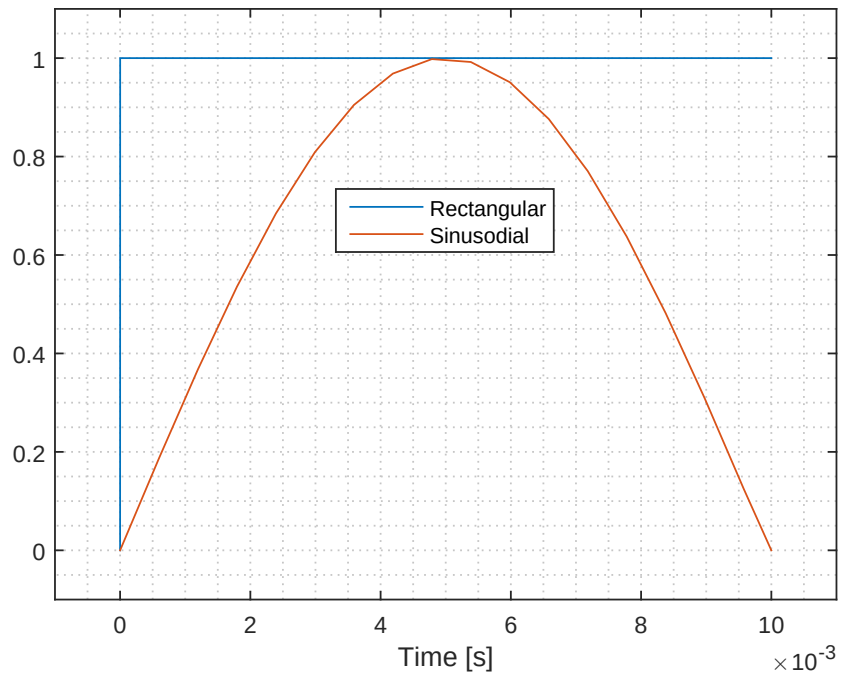


Figure 4.40 : Step load amplitude and sine load amplitude.

Comparison of the critical dynamic buckling load is presented in Table 4.21. As it is seen, the rectangular pulse shape has lower buckling load. The differences between the critical buckling load values for the rectangular and sinusoidal shapes are different for the different stacking sequences. The dynamic buckling load is increased about 35% for the stacking sequence 1 however, 51% and 77% for stacking sequence 2 and 3, respectively.



5. IMPERFECTION SENSITIVITY

In the previous chapter, the effect of initial imperfection was not considered. However, initial imperfections may reduce the critical buckling load which is calculated by the finite element method. The modification in the critical buckling load with the imperfection depends on the imperfection level and the sensitivity of the structure. In this chapter, how the imperfection is included in the FEA and how imperfection sensitivity is changed by the properties of the structure are examined for both static and dynamic loading cases.

5.1 Addition of Imperfection on the Finite Element Analysis

Imperfection can be added as small perturbation load or initial geometric imperfection. When imperfection is defined in base state, it provides the structure to be buckled in the desired region and the shape. Generally, initial geometric imperfections are defined by the perturbation of a few mode shapes which are obtained by linear eigenvalue analysis with some scale factors. The amplitude of the imperfection should be realistic. For example, the amount of the imperfection can be chosen as manufacturing tolerances. However, if there is no information about the imperfect shape or tolerance, generally a few percentage of the shell thickness is chosen as an amplitude. Figure 5.1 shows the addition procedure of the imperfection to the nonlinear analysis with Abaqus. To request .fil file, the command below should be added before the *END command.

```
*NODE FIL
```

```
U,
```

After the eigenvalue analysis is completed. Then, nonlinear analysis model is constructed and the input file is written. To add the imperfection, the command below should be added. The row after the imperfection line shows the mode number and the scale factor of corresponding mode.

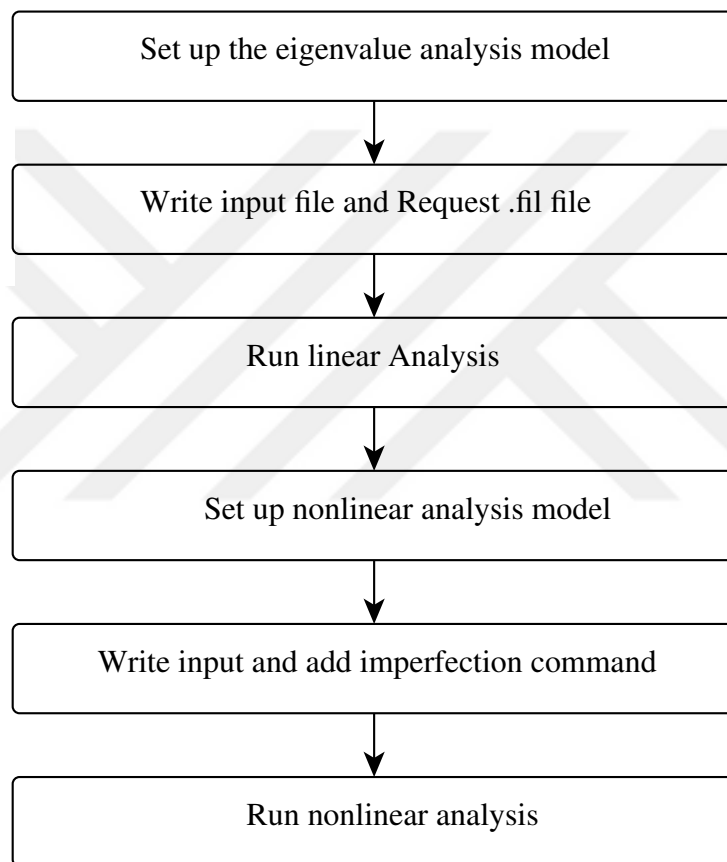


Figure 5.1 : Imperfection addition procedure to ABAQUS model.

*IMPERFECTION, FILE=<eigenvalue analysis name>, STEP=1

1,<scale factor>

2,<scale factor>

The nonlinear analysis step can be static analysis, Riks analysis, or explicit analysis with the assumption of quasi-static. Another method to add the imperfection with measured data which is in the form of node number and coordinate data. In this case, the imperfection can be added with the command below.

*IMPERFECTION, SYSTEM=<C or S>, INPUT=<file name>

node number, U1, U2, U3

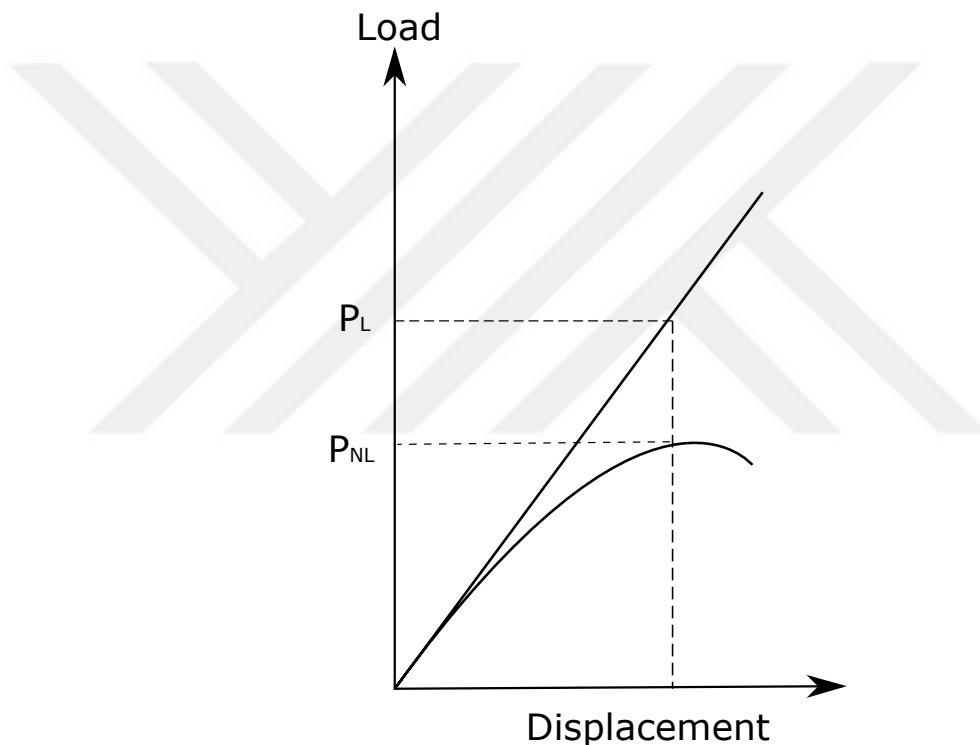


Figure 5.2 : Linear and nonlinear curve.

Another alternative method is preloading the structure which is easier than defining imperfection. In this way, the nonlinear behavior of the structure can capture more realistic than the linear analysis. As it is shown in Figure 5.2, the eigenvalue analysis follows a linear path and the estimated buckling load value is P_L . However, if the structure is imperfection sensitive the realistic path should be a nonlinear curve as shown in Figure 5.2. In this case, the buckle load is P_{NL} . Therefore, one of the clues that the structure is susceptible to imperfection is a huge difference between with and without preload. Another one may be closely spaced eigenvalues.

5.2 Effect of Imperfection on Static Buckling

Analyses are implemented for the laminated composite cylindrical-shell type structures with the diameter of 500 mm and 20 layers with the total thickness of 2.54 mm. In modelling, the reference points are assigned to the two ends of the cylinder and the reference points are connected to the ends of the cylinder nodes with rigid links. The load is applied on one of the reference points and the fixed boundary condition is applied to the other end reference point. In Figure 5.3, finite element analysis model is shown. Mechanical properties of the lamina are given in Table 5.1.

Table 5.1 : Material properties.

E_1	E_2	G_{12}	G_{13}	G_{23}	ν_{12}
127.5 GPa	9 GPa	5.7 GPa	2.7 GPa	5.7 GPa	0.25

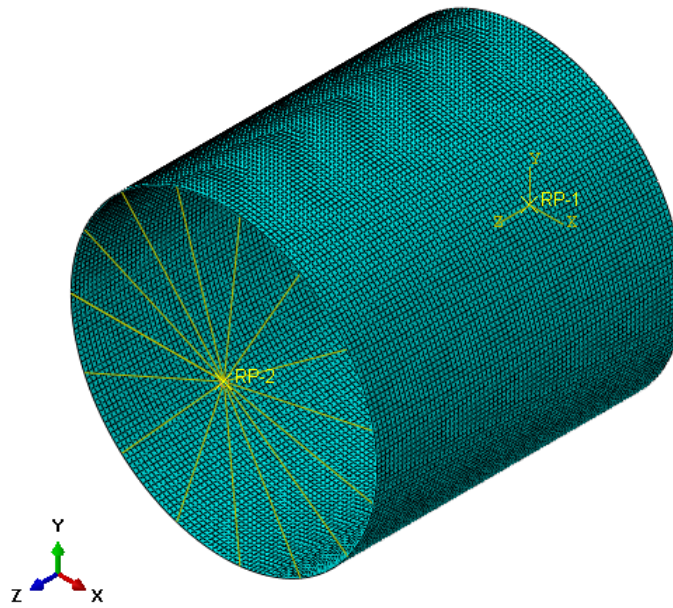


Figure 5.3 : Finite element analysis model.

Simulations with various parameters such as length to diameter ratio, stacking sequence and shape of the imperfection are conducted for better understanding of the effects of the imperfection amplitudes on the buckling loads.

In this parametric study, first, various slenderness ratios of $L/D = 1, 1.5$, and 2 are considered. Second, influence of stacking sequence in Table 5.3 is investigated for $L/D = 1$. Third, effect of imperfection shape on the imperfection sensitivity is studied for $L/D = 1$ and the second stacking sequence. Two imperfection shapes that are called as axisymmetric and helical are used.

First, the linear eigenvalue analyses are implemented, and then imperfection definitions are made with dynamic-explicit step definition with the assumption of quasi-static analysis. For the quasi-static analysis, loading time is chosen much longer than the the first natural period. The kinetic energy level is also checked to be sure that the inertial effects are negligibly small. The imperfection is defined by means of the eigenmode shapes with imperfection scale factor (α), which is the scaled shape of the nodal displacements of eigenmode. Imperfection is selected as 10% to 100% of the thickness of the shell structure.

Since the usage of the second order elements within the Abaqus/Explicit solver is not allowed, a four-node first order reduced quadrilateral shell element S4R with the size of 7 mm is used. Two reference points are assigned to the mid point of cylinder ends. These reference points are connected to the cylindrical shell with kinematic couplings which means that all nodes of the edge of the cylinder move all together. One of the reference points is displaced with a speed of 200 mm/s. As the boundary condition, all degrees of freedom are constrained at other reference point.

5.2.1 Effect of length to diameter ratio under static load

In this section, the effect of the slenderness ratio (L/D) on the buckling load is investigated for various imperfection scale factors. The analyses are run for $L/D=1$, 1.5, 2. The static critical load values are given in Table 5.2. The normalized values are found from the ratio of the loads of the imperfect model to the linear eigenvalue results of the prefect model. The sensitivity behavior is evaluated with those normalized values with as shown in Figure 5.4.

As it can be seen from Figure 5.4, the normalized buckling load decreases exponentially with the increase of the imperfection scale factor. The sensitivity increases with the slenderness ratio because the higher the slenderness ratio the lower the buckling loads [37]. However, this increase is higher between $L/D=1$ to 1.5 than between $L/D=1.5$ to 2.

Table 5.2 : Critical static buckling load value [kN] for different length to diameter ratio.

Imperfection Scale Factor[%]	L/D=1	L/D=1.5	L/D=2
10	646	627	630
20	547	535	530
30	514	503	483
40	499	470	441
50	481	429	404
60	466	397	380
70	443	372	358
80	430	355	342
90	425	346	318
100	422	346	318

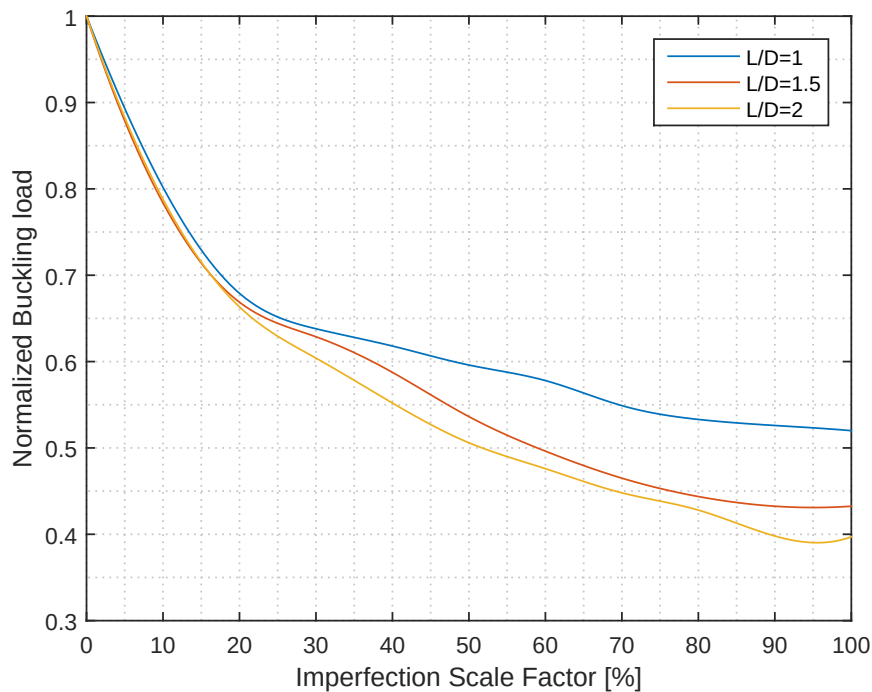


Figure 5.4 : Comparison of sensitivity of different length to diameter ratio.

5.2.2 Effect of stacking sequence under static Load

In this section, effects of stacking sequences on the buckling load are investigated. Three different stacking sequences given in Table 5.3 are used. As pointed previous section, the linear eigenvalue analyses with perfect models are performed, and critical buckling loads are determined as 676 kN, 806 kN, and 1084 kN for stacking sequence 1, 2 and 3, respectively. The critical load values of the structure are identified from the points at which the slopes of the load-axial displacement curves change. The load

displacement curves for different imperfection amplitude scale factors are shown in Figure 5.5, Figure 5.6, and Figure 5.7 for the first, second, and third stacking sequence of Table 5.3.

Table 5.3 : Composite stacking sequence.

Composite Types	Stacking Sequence
1	$[0/90]_{10}$
2	$[45/90/60/45/30/0/-30/-45/-60/-45]_s$
3	$[0/45/90/-45/0/0/-45/90/45/0]_s$

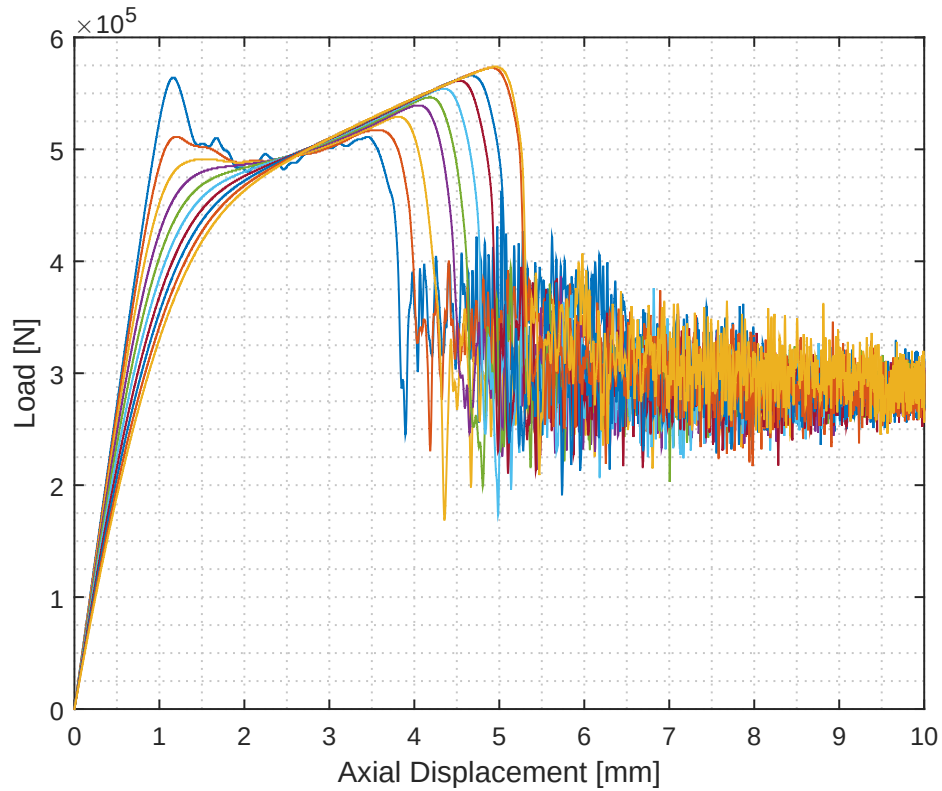


Figure 5.5 : Load displacement curve for first stacking sequence with different imperfection amplitudes.

The buckling loads of those three stacking sequences are given in Table 5.4 for different imperfection scale factors. The buckling load values are normalized with the eigenvalues obtained from the linear analysis. The normalized buckling loads vs. the imperfection scale factors for the different stacking sequences are given in Figure 5.8. It is obvious that the stacking sequence has a high influence on the normalized buckling load. The first stacking sequence (cross-ply) gives higher normalized buckling loads than other two sequences for all imperfection scale factors. Reduction in the normalized buckling loads ranges from 83% to 66% for the first stacking sequence and from 80-82% to 52% for the second and third sequences [37].

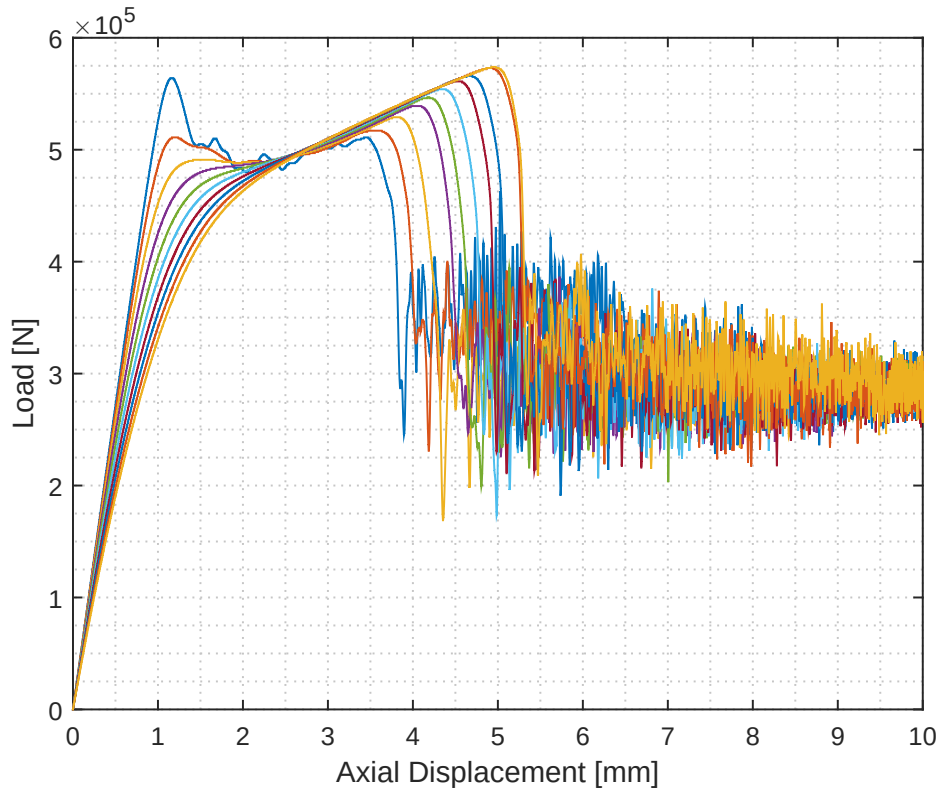


Figure 5.6 : Load displacement curve for second stacking sequence with different imperfection amplitudes.

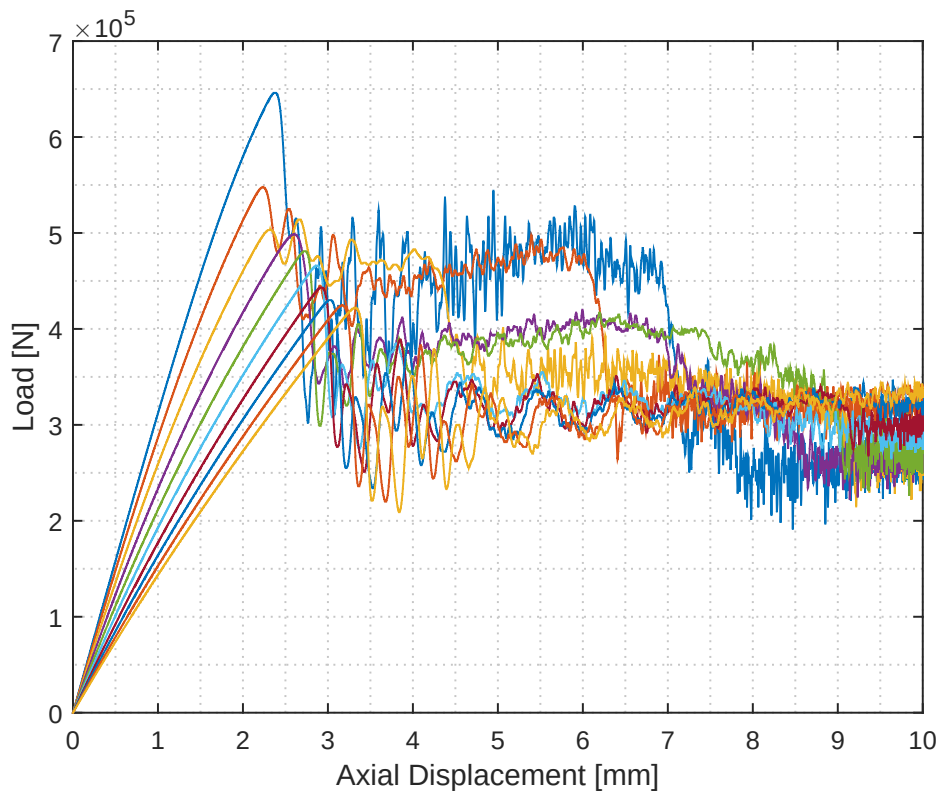
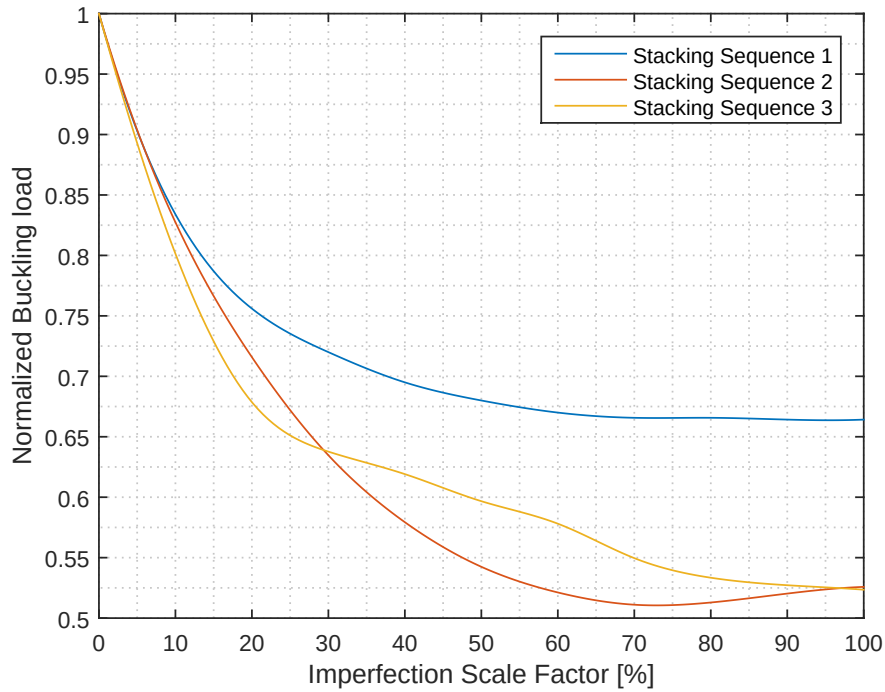


Figure 5.7 : Load displacement curve for third stacking sequence with different imperfection amplitudes.

Table 5.4 : Critical static buckling load value [kN] for different stacking sequence.

Imperfection Scale Factor[%]	1 _{st}	2 _{nd}	3 _{rd}
10	564	646	897
20	511	547	776
30	487	514	688
40	470	499	628
50	460	481	588
60	453	466	565
70	450	443	554
80	450	430	556
90	449	425	564
100	449	422	570

**Figure 5.8** : Comparison of sensitivity of different types of the composites.

5.2.3 Effect of shape of imperfection under static Load

In this section, the effect of imperfection shapes on the critical buckling loads is considered. For this purpose, two different initial imperfection shapes are chosen and the FEM analyses are performed on those two initial imperfection shapes with the imperfection scale factor from 10% to %100 of the thickness. Here, the cylindrical shell has the same length to diameter ratio of one ($L/D=1$) and the stacking sequence of $[45/90/60/45/30/0/-30/-45/-60/-45]_s$ which is the second stacking sequence in Table 5.3. The first initial imperfection shape is the helical shape which has one

wave along the length and 7 waves in the circumferential direction. Another initial imperfection shape is the axisymmetric shape which has only 8 waves along the length. Those two mode shapes shown in Figure 5.9 are for defining imperfection definitions with different imperfection scale factors.

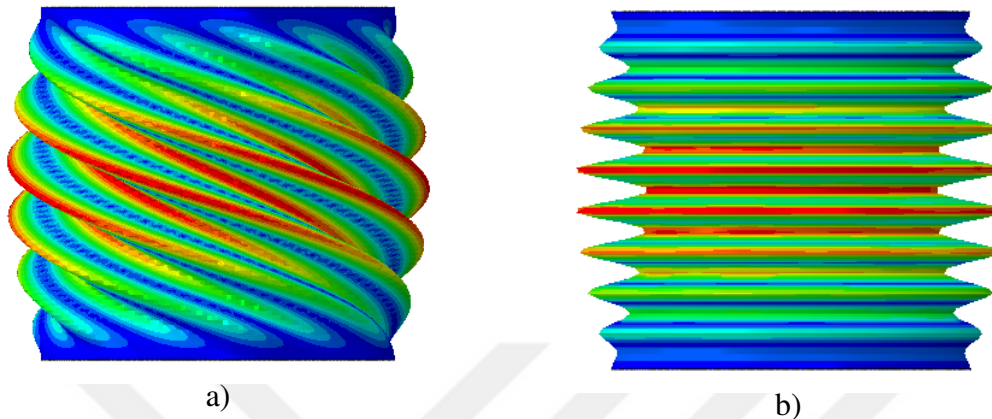


Figure 5.9 : Shapes for imperfection shape definition (a) helical (b) axisymmetric.

The critical static buckling load values for the different scale factors are given in Table 5.5. Those values are normalized with the critical load value for perfect model which is obtained from the linear eigenvalue analyses. Comparison of sensitivities to the imperfection for those initial geometric imperfection shapes is given in Table 5.10. It is found that reduction of the buckling loads with increasing value of imperfection scale factor is higher for the helical imperfection shape for low values of the imperfection scale factor. However, when the imperfection scale factor is more than the one half of the thickness, then the helical imperfection shape becomes more effective [37].

Table 5.5 : Critical static buckling load value [kN] for different imperfection shape.

Imperfection Scale Factor[%]	Helical	Axisymmetric
10	757	646
20	648	547
30	580	514
40	526	499
50	482	481
60	456	466
70	429	443
80	406	430
90	392	425
100	383	422

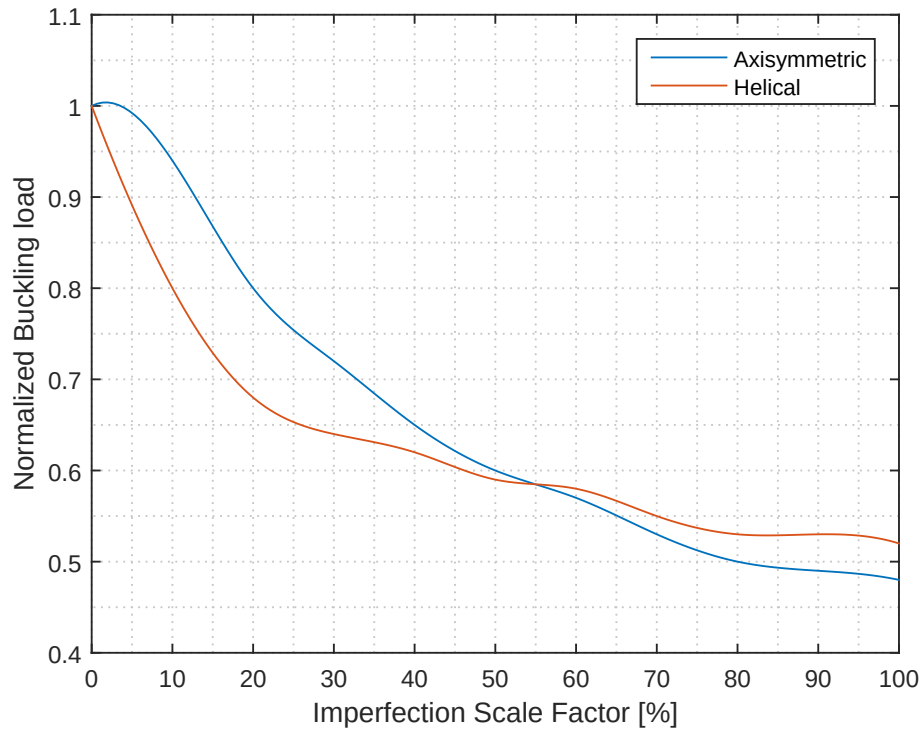


Figure 5.10 : Comparison of sensitivity of different imperfection shape.

5.3 Effect of Imperfection on Dynamic Buckling

The same model in the previous section is used to study the effect of the geometric imperfection on the dynamic buckling load. This model has the stacking sequence of $[45/90/60/45/30/0/-30/-45/-60/-45]_s$ which is second stacking sequence in Table 5.3. The effects of the same parameters which are investigated in static buckling are now investigated in dynamic buckling. These parameters are the length to diameter ratios, the stacking sequences, and the imperfection shapes. The imperfection scale factor is again varied from 10% to 100% of the thickness of the shell. The loading time is 10ms. As it is mentioned in the previous chapter, to obtain the critical dynamic buckling load, the step loads with different amplitudes are applied to the structure. Then, the axial displacement-time graphs are drawn for the different applied loads. The sudden increase in the curve indicates that the structure is dynamically buckled as in the Budiansky-Roth criteria. As an example, the axial displacement-time graph for the model with the imperfection 10% of the thickness is given for the load value below and above of the critical load value in Figure 5.11. Therefore, the critical load value should be between 495 kN and 498 kN. Thus, 497 kN can be pointed as the dynamic

buckling load value. In the other analyses, when the difference of upper and lower bounds is 5 kN, then the critical dynamic load value is chosen as mid point of those two values.

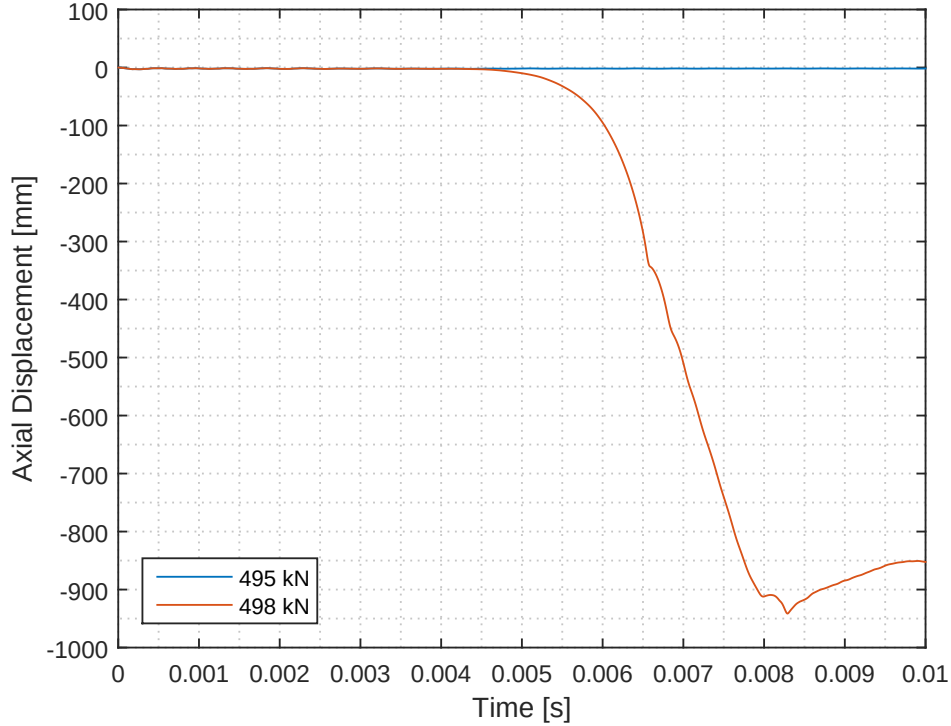


Figure 5.11 : Displacement time graph for 495 kN and 498 kN.

5.3.1 Effect of length to diameter ratio under dynamic load

To be able to understand the imperfection sensitivity behavior under dynamic load for the models with different L/D ratios, the same previous procedure is applied for the different imperfection scale factor.

The critical load values of the imperfect model with different amplitudes and slenderness ratios are given in Table 5.6, and the normalized dynamic buckling loads are given in Figure 5.12. The normalization is again made by the dividing the critical dynamic buckling load of imperfect model to the critical dynamic buckling load value of the perfect model.

The normalized buckling load results show that the sensitivity is almost the same for the different length to diameter ratios. However, the critical dynamic buckling load decreases with the increased imperfection as expected. However, this decrease does not depend on the slenderness ratio.

Table 5.6 : Critical dynamic buckling load value [kN] for the different length to diameter ratio.

Imperfection Scale Factor[%]	L/D=1	L/D=1.5	L/D=2
0	577	532	482
10	497	437	427
20	438	397	372
30	357	347	332
40	323	317	292
50	307	287	272
60	297	272	267
70	272	257	247
80	257	242	237
90	242	227	222
100	227	217	207

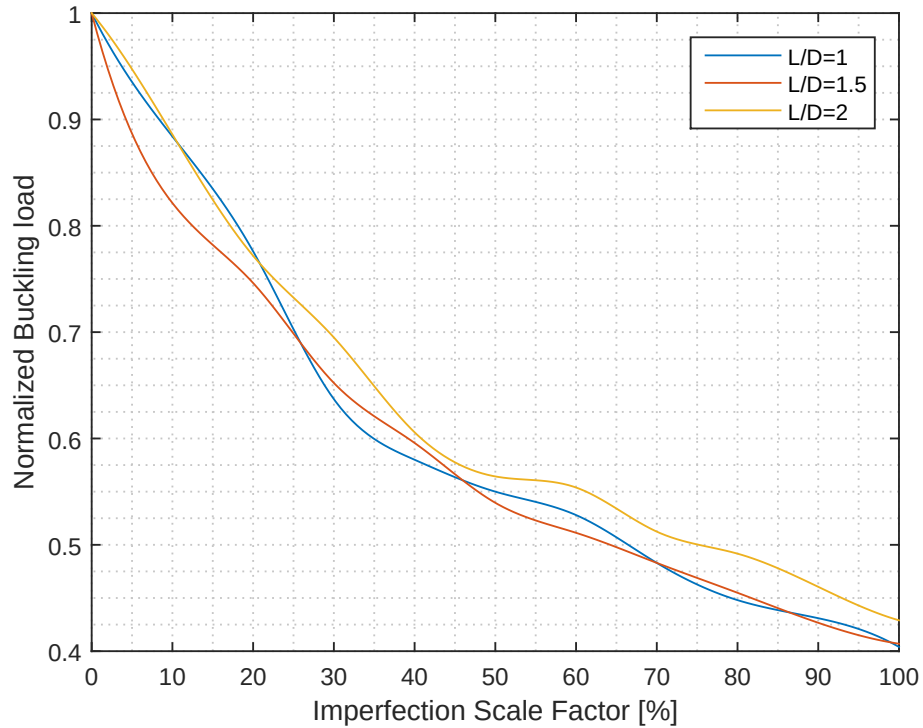


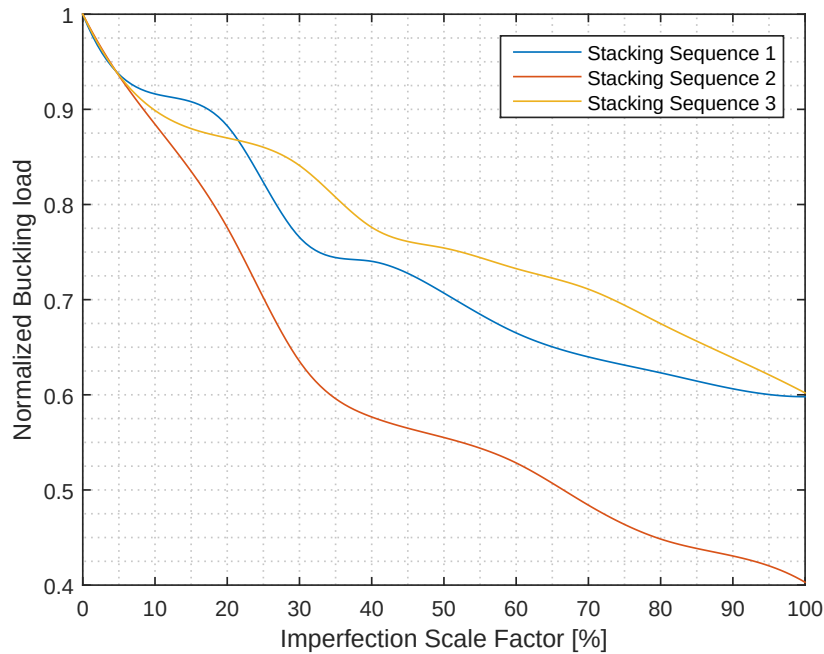
Figure 5.12 : Critical normalized dynamic buckling load value for different length to diameter ratio.

5.3.2 Effect of the stacking sequence under dynamic load

The same stacking sequences of Table 5.3 are used. The results are given in Table 5.7. The critical dynamic buckling loads are obtained as 597 kN, 567 kN, and 697 kN for the perfect model of the stacking sequence 1, 2, and 3 respectively.

Table 5.7 : Critical dynamic buckling load value [kN] for different stacking sequence.

Imperfection Scale Factor[%]	1 st	2 nd	3 rd
0	607	577	692
10	547	497	622
20	527	438	602
30	452	357	582
40	442	323	537
50	422	307	522
60	412	297	507
70	382	272	492
80	377	257	467
90	362	242	442
100	357	227	417

**Figure 5.13** : Critical normalized dynamic buckling loads for the different stacking sequences.

The normalized load values are presented in Figure 5.13. It can be seen that the most sensitive to the imperfection is the second stacking sequence, and the third one is the less sensitive stacking sequence. It shows that the tolerance to the imperfection of the structure may be increased by changing the stacking sequences.

5.3.3 Effect of imperfection shape under dynamic load

In this section, the effect of the imperfection shapes on the critical dynamic buckling loads is investigated. As the imperfection shapes, the same mode shapes of the linear buckling analysis results in Figure 5.9.

The critical dynamic loads for the different amplitudes of the helical and axisymmetric imperfection are given in Table 5.8. To recognize, the sensitivity on dynamic buckling, the load given in Table 5.8 is normalized with the critical dynamic load of the perfect model, and those values are presented in Figure 5.14.

Table 5.8 : Critical dynamic buckling load value [kN] for different imperfection shape.

Imperfection Scale Factor[%]	Helical	Axisymmetric
0	577	562
10	497	527
20	438	502
30	357	487
40	323	457
50	307	422
60	297	402
70	272	407
80	2527	402
90	242	387
100	227	367

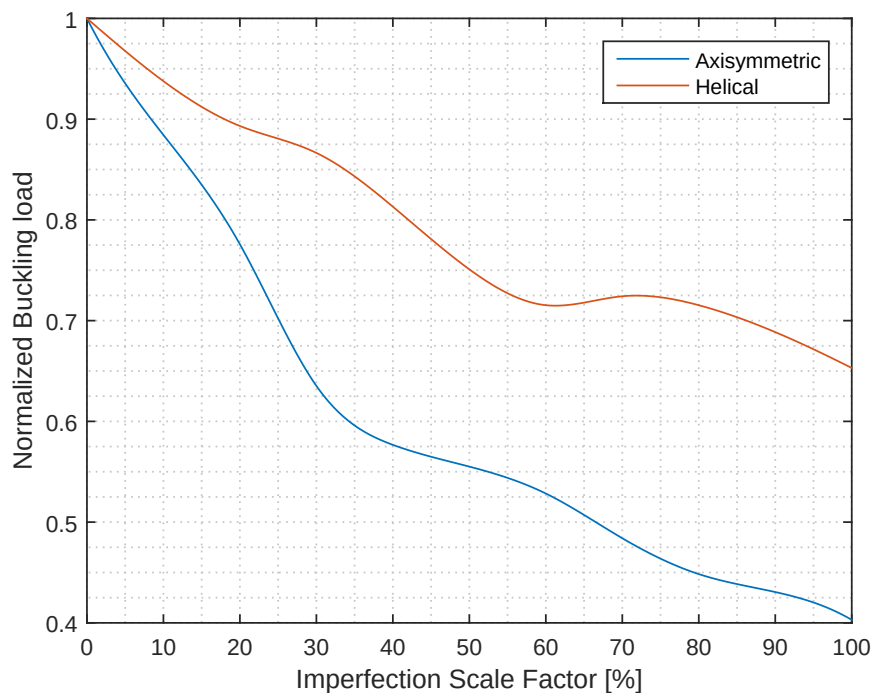


Figure 5.14 : Critical normalized dynamic buckling load value for different imperfection shape.

5.4 Comparison of the Imperfection Sensitivities of Static and Dynamic Loading

In previous section, the sensitivity of the buckling loads of cylindrical shells to several parameters was investigated. The same parameters were used for both static and dynamic loadings. Those three parameters were the length to diameter ratios, the stacking sequences, and the imperfection shapes. In this section, the results of the static and dynamic loadings are compared.

The comparisons of the analysis results of those three parameters for the static and dynamic buckling are presented in Figure 5.15, Figure 5.16, and Figure 5.17.

As it is seen in Figure 5.15, buckling versus imperfection has similar trends for the three length to diameter ratios of the shell for both static and dynamic loadings. The change in sensitivity for the static buckling is higher than the dynamic loading. However, differences are not very big.

In Figure 5.16, the changes in sensitivities for the chosen three stacking sequences for static and dynamic buckling are shown. For the first and second stacking sequences, the sensitivities of the imperfection are similar for static and dynamic buckling. However, the imperfection sensitivities for third stacking sequences are different and it can be seen here that the increase in imperfections affects the static buckling loads more than than dynamic buckling loads.

Finally, the imperfection shape is taken as the last parameter, and the analyses results are given in Figure 5.17. For the helical imperfection shape which is shown in Figure 5.9, the sensitivity to the imperfection is higher for static buckling. However, when the amount of the imperfection is increased, the dynamic loading is affected more. For the axisymmetric imperfection shape which is shown in Figure 5.9, it is seen that the static buckling load decreases faster than the dynamic buckling load with the increase of the imperfection.

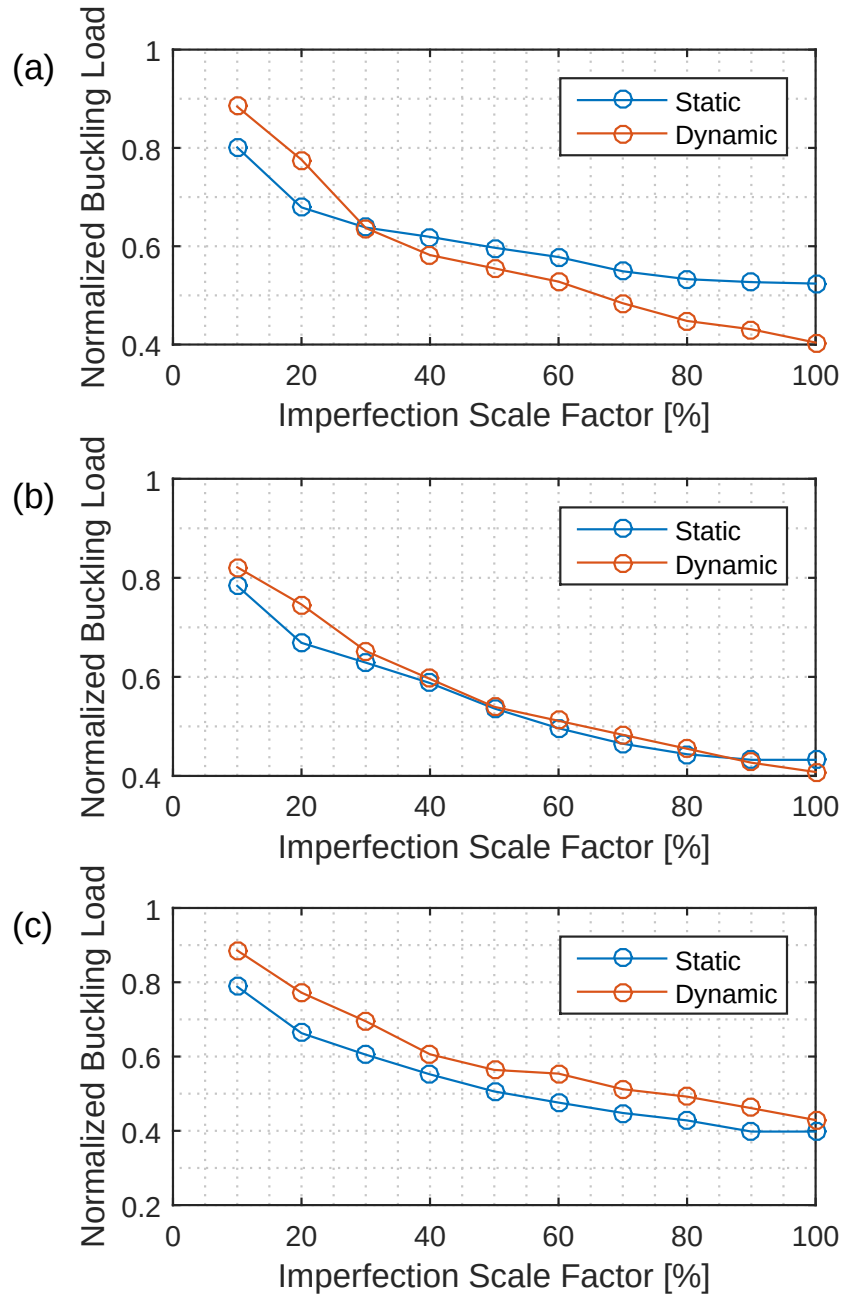


Figure 5.15 : Critical normalized static and dynamic buckling load value for different length to diameter ratio a) $L/D = 1$, b) $L/D = 1.5$, c) $L/D = 2$.

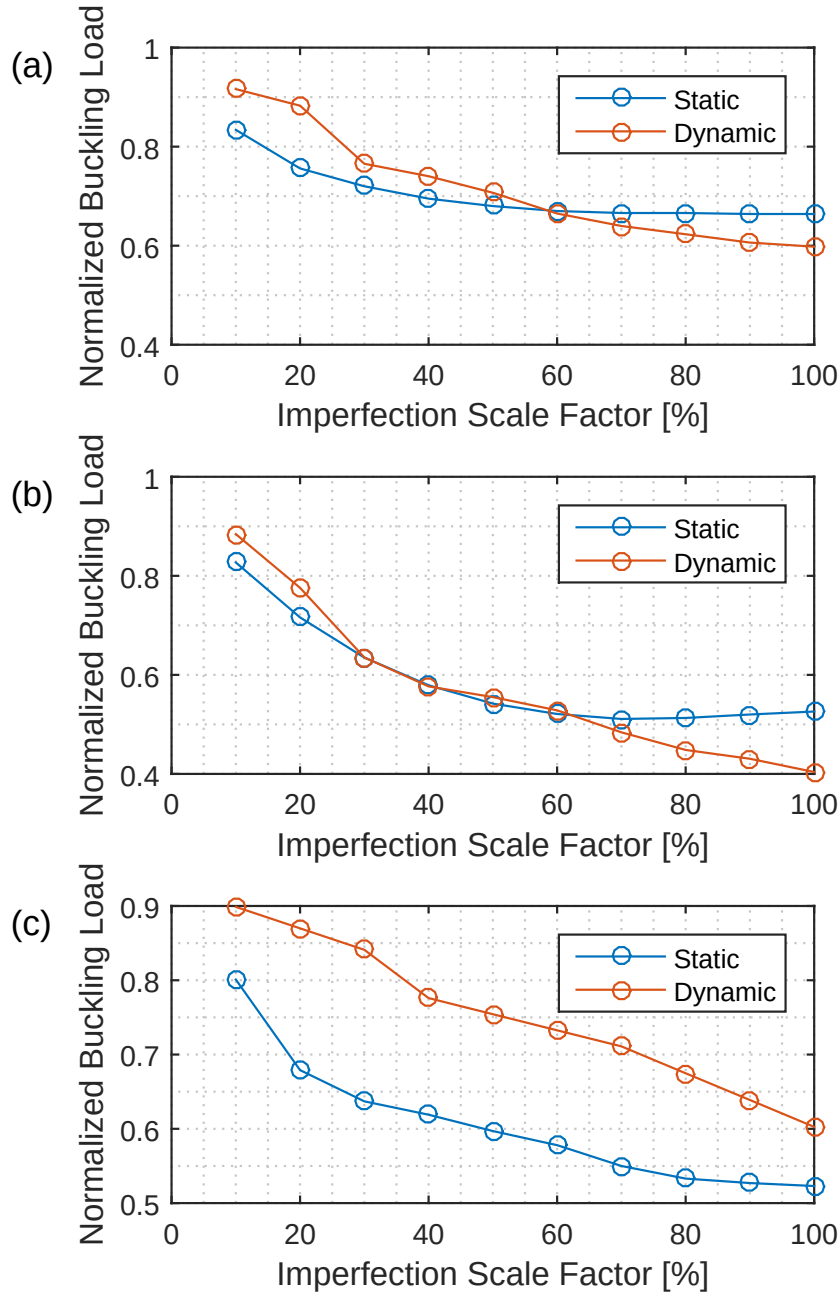


Figure 5.16 : Critical normalized static and dynamic buckling load value for different stacking sequence a)1st, b)2nd, c)3rd.

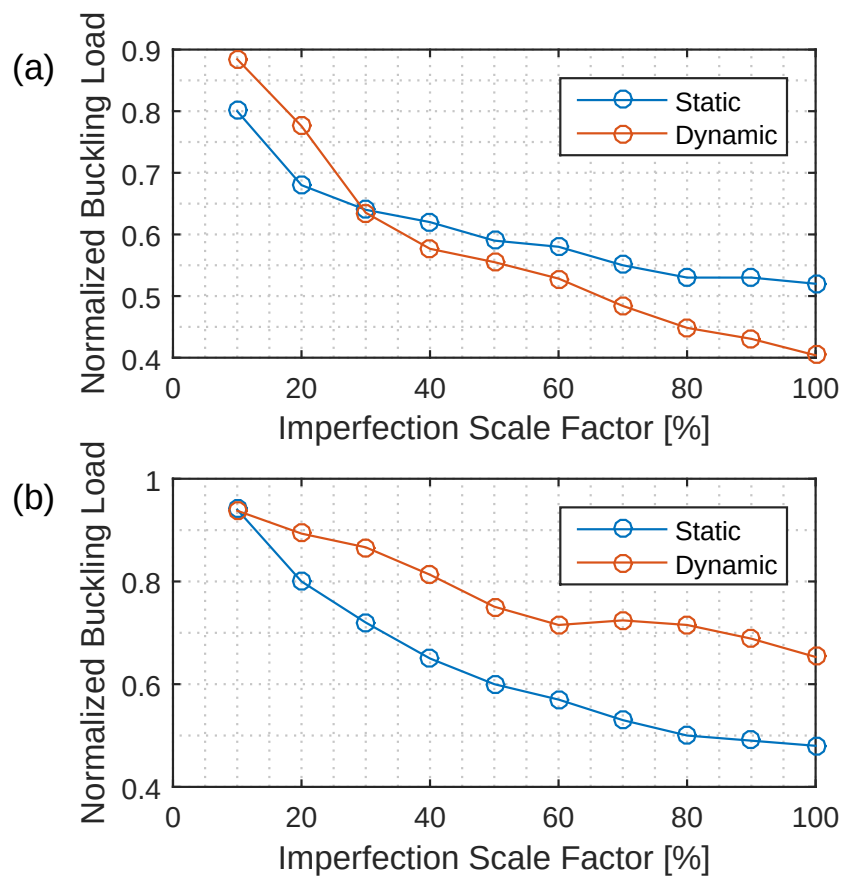


Figure 5.17 : Critical normalized static and dynamic buckling load value for different imperfection shape a)Helical,b)Axisymmetric.



6. CONCLUSIONS AND RECOMMENDATIONS

In this thesis, the buckling behavior of the composite cylindrical shells under axial load was investigated for both static and dynamic loadings. First, the analytical background of the buckling phenomena was given. A MATLAB code was also written to calculate the critical static buckling load for the composite materials and it was given in Appendix B. For the dynamic buckling analysis, some of the dynamic buckling criteria in literature were given and Budiansky - Roth criterion was used to determine the dynamic buckling load. Then, the FEA methods of the calculating static and dynamic buckling analyses were discussed and validation studies for those buckling analysis methods were performed.

For the static buckling, three analysis methods namely eigenvalue analysis, Riks analysis and quasi-static analysis were discussed. As mentioned before, the linear analysis is the first step to determine the critical load. Results of this type of analysis can be used as a first guess or the results of the analyses of different parameters can be compared. Also, the eigenvector which shows the buckling pattern for the corresponding buckling load can be used as imperfection shape. To see the post-buckling behavior, there is a need for another analysis method such as Riks analysis which is able to follow nonlinear path. However, when the imperfection is not added to the analysis, it may cause divergence due to the sharp point of the load-displacement curve. Adding the imperfection provides smooth curve and Riks method will be able to follow the nonlinear curve. Another method is the quasi-static analysis which uses dynamic solver that includes inertial effects. However, choosing suitable time and mass scaling provides negligible inertial effects. One of the advantages of this analysis type is that the divergence problem is not easily seen. However, the results should be evaluated carefully.

Although Riks and quasi-static buckling analyses give more realistic results by adding imperfections, the linear eigenvalue analysis can be used for the comparison studies. For that reason, the effects of some parameters were investigated with the linear

eigenvalue analysis. When the design reaches a certain level, then the other methods of buckling analysis can be changed and some details can be added such as imperfection.

The explicit solver was used to determine the dynamic buckling load with the help of some criteria. The criteria of Volmir, Budiansky-Roth, Hoff-Bruce, and Similes were discussed in this study. Budiansky-Roth criterion were used to determine dynamic buckling load with the explicit analysis method. To remind Budiansky-Roth criterion, the displacement-time graph were drawn for consecutive load values, then the critical load was the load value in which the displacement value suddenly increases.

The investigations were made for different parameters for the static and dynamic loading.

For the static loading, effects of some parameters which were stacking sequence of composite layers, length to diameter ratio, thickness of the plies and different materials were investigated.

- Stacking sequence of the composite layers affects the buckling load. Changing angle of one ply is not enough for obtaining the optimum stacking sequence and it should be considered as a whole. Also, there are some stacking rules that should be obeyed as a rule of thumb. Some of them are given and ten stacking sequences are constituted to comply with these rules.
- After the investigation of stacking sequences, effect of the changing material is investigated. In this extent, unidirectional and woven composite plies, aluminum and steel material properties are used for the same thickness and the same mass. As a result, if the thicknesses are the same, the buckling loads are close even if the imperfection is added. But, if the mass of the structures are equalized with changing the thicknesses, the highest buckling load is gained with unidirectional material. It means that ratio of the buckling load to mass is highest for unidirectional composite.
- Effect of length to diameter ratio and thickness is investigated. It is obtained that influence of changing the slenderness (length to diameter ratio) is small while the thickness effect is important.

For the dynamic buckling, the chosen parameters were length to diameter ratio, thickness of the plies, loading duration and loading profile shape.

- The effect of the slenderness on the dynamic buckling load is more than the static one and the influence of thickness is similar with static buckling.
- The loading time has a great influence on dynamic buckling load. Shorter loading duration, higher dynamic buckling load. Also, the loading type is investigated with sinusoidal and rectangular loading shape. The difference obtained from that analysis is considerable and the difference for various stacking sequences changes from %34 to %76.
- In terms of the parametric studies for the dynamic buckling analysis, that study fulfill a need in literature. However, there is still a need for further evaluation/study of dynamic buckling load determination.

Since the buckling is a phenomenon which is influenced by the imperfection, how the imperfection sensitivity changes with length to diameter ratio, stacking sequence, and shape of the imperfection was examined for both static and dynamic buckling. The imperfection sensitivity is one of the main focus points of the thesis. Also, the comparison was made for those parameters in terms of the loading types.

- The results show that the static buckling load is always less than the dynamic one.
- However, when the imperfection is added, that situation may be changed and the amount of the imperfection is important.
- Thus, using measured imperfection or the production tolerances can be important to obtain more realistic results.

For the future work, dynamic buckling can be investigated further with additional parameters. Moreover, the experimental study can be a suitable for further work.

Some parts of the thesis are presented with the title of “An Imperfection Sensitivity Analysis on Buckling of Laminated Composite Shells” in *2021 IEEE 4th International Conference on Nanoscience and Technology (ICNST)* on 27.06.2021.



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APPENDICES

APPENDIX A: Differential Operators

APPENDIX B: Analytical Calculation of Static Buckling Load





APPENDIX A

$$\Theta_n = \begin{bmatrix} -\frac{\partial}{\partial x} & 0 & -\frac{\partial}{R\partial\theta} & 0 & 0 & 0 \\ 0 & -\frac{\partial}{R\partial\theta} & -\frac{\partial}{\partial x} & 0 & -\frac{\partial}{R^2\partial\theta} & -\frac{\partial}{R\partial x} \\ 0 & 1/R & 0 & -\frac{\partial^2}{\partial x^2} & -\frac{\partial}{R^2\partial\theta^2} & -2\frac{\partial^2}{R\partial x\partial\theta} \end{bmatrix} \quad (\text{A.1.1})$$

$$\Theta_\theta = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{R\partial\theta} & \frac{1}{R} \\ \frac{\partial}{R\partial\theta} & \frac{\partial}{\partial x} & 0 \\ 0 & 0 & -\frac{\partial^2}{\partial x^2} \\ 0 & \frac{\partial}{R^2\partial\theta} & -\frac{\partial^2}{R^2\partial\theta^2} \\ 0 & 2\frac{\partial}{R\partial x} & -2\frac{\partial^2}{R\partial x\partial\theta} \end{bmatrix} \quad (\text{A.1.2})$$

$$\Theta_{Nx} = \begin{bmatrix} 0 & -\frac{\partial^2}{R\partial x\partial\theta} & -\frac{\partial}{R\partial x} \\ 0 & -\frac{\partial^2}{\partial x^2} & 0 \\ 0 & 0 & -\frac{\partial^2}{\partial x^2} \end{bmatrix} \quad (\text{A.1.3})$$

$$\Theta_{Ny} = \begin{bmatrix} 0 & \frac{\partial^2}{R\partial x\partial\theta} & \frac{\partial}{R\partial x} \\ -\frac{\partial^2}{R\partial x\partial\theta} & 0 & 0 \\ \frac{\partial}{R\partial x} & \frac{\partial}{R^2\partial\theta} & -\frac{\partial^2}{R^2\partial\theta^2} \end{bmatrix} \quad (\text{A.1.4})$$

$$\Theta_{Nxy} = \begin{bmatrix} -\frac{\partial^2}{R\partial x\partial\theta} & \frac{\partial^2}{R\partial x^2} & 0 \\ 0 & -2\frac{\partial^2}{R\partial x\partial\theta} & -2\frac{\partial}{R\partial x} \\ 0 & 2\frac{\partial}{R\partial x} & -2\frac{\partial^2}{\partial x\partial\theta} \end{bmatrix} \quad (\text{A.1.5})$$

$$\Theta_{Mx} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{\partial^2}{R\partial x^2} & 0 \\ 0 & -2\frac{\partial^3}{R\partial x^2\partial\theta} & -\frac{\partial^2}{R\partial x^2} \end{bmatrix} \quad (\text{A.1.6})$$

$$\Theta_{Mx} = \begin{bmatrix} 0 & 0 & 0 \\ -\frac{\partial^2}{R^2\partial x\partial\theta} & 0 & 0 \\ -\frac{\partial^3}{R^2\partial x\partial\theta^2} & \frac{\partial^3}{R\partial x^2\partial\theta} & \frac{\partial^2}{R\partial x^2} \end{bmatrix} \quad (\text{A.1.7})$$

$$\Theta_{My} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -2\frac{\partial^2}{R^2\partial x\partial\theta} & -2\frac{\partial}{R^2\partial x} \\ -\frac{\partial^3}{R\partial x^2\partial\theta} & \left(-2\frac{\partial^3}{R^2\partial x\partial\theta^2} + \frac{\partial^3}{\partial x^3}\right) & -2\frac{\partial^2}{R^2\partial x\partial\theta} \end{bmatrix} \quad (\text{A.1.8})$$

$$\Theta_q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{\partial}{\partial x} & \frac{\partial}{R\partial\theta} & \frac{1}{R} \end{bmatrix} \quad (\text{A.1.9})$$



APPENDIX B

Matlab code to calculate the static buckling load for the laminate composite is given in this section. Stacking sequence, thickness of a lamina, and radius of the cylinder are inputted. Also, material properties should be written in 'matInput-2.txt' file with the sequence in the code. The parameters that scan α , β , c_1 , and c_2 in Equation 2.12 values should be given in 'param2Input.txt'. From the code, the critical load value is given with the variable 'criticalLoad'.

```
breaklines
1  clc
2  clear all
3  close all
4  %% Inputs
5  stackSequence=input('Please Enter Stacking Sequence >> ');
6  n=size(stackSequence);
7  n=n(2);
8  tLayer=input('Please Enter Thickness of Lamina [m] >> ');
9  radiusCylinder=input('Please Enter Radius of Cylinder in [m] >> ');
10 %% Stiffness Matrices
11 [A_sk,B_sk,D_sk,q,h,Q_bar,noLayers,ULTsig_T_1,ULTsig_C_1,...
12     ULTsig_T_2,ULTsig_C_2,ULTTau_12]=skinCoeffs_2(stackSequence,tLayer);
13 %% Critical Buckling Load
14 [criticalLoad,min_params,lambdaCritical]=...
15     bucklingLoad_v2(radiusCylinder,A_sk,B_sk,D_sk);

breaklines
1  function [A_sk, B_sk, D_sk,q,h,Q_bar,noLayers, ULTsig_T_1,ULTsig_C_1...
2         ,ULTsig_T_2,ULTsig_C_2,ULTTau_12]=...
3         skinCoeffs_2(stackSequence,tLayer)
4  noLayers=length(stackSequence);
5
6  %% Material Properties
7  mat=dlmread('matInput-2.txt');
8  E1=mat(1);
9  E2=mat(2);
10 E3=mat(3);
11 n21=mat(4);
12 n31=mat(5);
13 n23=mat(6);
14 G12=mat(7);
15 G13=mat(8);
16 G23=mat(9);
17 ULTsig_T_1=mat(10);
18 ULTsig_C_1=mat(11);
19 ULTsig_T_2=mat(12);
20 ULTsig_C_2=mat(13);
21 ULTTau_12=mat(14);
```

```

22 %% Main Loop
23 h=zeros(noLayers+1);
24 h(noLayers+1)=noLayers*tLayer/2;
25 stiffnessLayer=zeros(3*noLayers,3);
26
27 %% Compliance Matrix Elements
28 S = [1/E1      -n21/E1      -n31/E1      0      0      0;...
29      -n21/E1      1/E2      -n23/E2      0      0      0;...
30      -n31/E1      -n23/E2      1/E3      0      0      0;...
31      0      0      0      1/G23      0      0;...
32      0      0      0      0      1/G13      0;...
33      0      0      0      0      0      1/G12];
34
35 %% Stiffness Matrix Elements
36 C = inv(S);
37
38 %% Reduced Stiffness
39 Q=[C(1,1)-(C(1,3))^2/C(3,3)  C(1,2)-C(1,3)*C(2,3)/C(3,3)  0;...
40   C(1,2)-C(1,3)*C(2,3)/C(3,3)  C(2,2)-(C(2,3))^2/C(3,3)  0;...
41   0      0      C(6,6)];
42
43 for k = 1 : noLayers
44
45     %% Transformation Matrix
46     m = cosd (stackSequence(k));
47     n = sind (stackSequence(k));
48
49     T = [m^2      n^2      2*m*n;...
50          n^2      m^2      -2*m*n;...
51          -m*n      m*n      m^2-n^2];
52
53     %% Transformed Stiffness Matrix
54     q=zeros(3,3);
55     q(1:2,1:2)=Q(1:2,1:2);
56     q(3,3)=2*Q(3,3);
57     q=T\q*T;
58     for i = 1:3
59         q(i,3)=q(i,3)/2;
60     end
61     stiffnessLayer(3*k-2:3*k,1:3)=q(1:3,1:3);
62     h(k)=(k-noLayers/2-1)*tLayer;
63     Q_bar(:, :, k)=q(:, :);
64 end
65
66 %% to Calculate Stiffness Coeffs. A B and D Matrices
67 A_sk=zeros(3,3);
68 B_sk=zeros(3,3);
69 D_sk=zeros(3,3);
70
71 for i=1:3
72     for j=1:3
73         q(1:3,1:3)=stiffnessLayer(1:3,1:3);
74         A_sk(i,j) = q(i,j) * (h(2) - h(1));
75         B_sk(i,j) = 1/2*(q(i,j) * (h(2)^2 - h(1)^2));

```

```

76         D_sk(i,j) = 1/3*(q(i,j) * (h(2)^3 - h(1)^3));
77
78         for k = 2 : noLayers
79             q(1:3,1:3) = stiffnessLayer(3*k-2:3*k,1:3);
80             A_sk(i,j) = q(i,j) * (h(k+1) - h(k)) + A_sk(i,j);
81             B_sk(i,j) = 1/2*(q(i,j) * (h(k+1)^2 - h(k)^2)) + B_sk(i,j);
82             D_sk(i,j) = 1/3*(q(i,j) * (h(k+1)^3 - h(k)^3)) + D_sk(i,j);
83         end
84     end
85 end
86
87 end

```

```

breaklines
1 function [criticalLoad,min_params,lambdaCritical]=...
2     bucklingLoad_v2(radiusCylinder,A_sk,B_sk,D_sk)
3 %%
4 param=dlmread('param2Input.txt');
5 alpha_lb=param(1,1); alpha_ub=param(1,2); Na=param(1,3);
6 beta_lb=param(2,1); beta_ub=param(2,2); Nb=param(2,3);
7 c_lb=param(3,1); c_ub=param(3,2); Nc=param(3,3);
8 iter=param(4,1);
9
10 %%
11 Mo=[A_sk(1,1) A_sk(1,2) 0 B_sk(1,1) B_sk(1,2) 0;...
12     A_sk(1,2) A_sk(2,2) 0 B_sk(1,2) B_sk(2,2) 0;...
13     0 0 A_sk(3,3) 0 0 B_sk(3,3);...
14     B_sk(1,2) B_sk(1,2) 0 D_sk(1,1) D_sk(1,2) 0;...
15     B_sk(1,2) B_sk(2,2) 0 D_sk(1,2) D_sk(2,2) 0;...
16     0 0 B_sk(3,3) 0 0 D_sk(3,3)];
17
18 Mn=[A_sk B_sk;...
19     B_sk D_sk];
20
21 Mn=Mn-Mo;
22 M=[Mo Mn;Mn Mo];
23 %
24 J=[0 0 0;0 0 0;0 0 1];
25 %% Main Loop
26 lambdaCritical(1)=9e20;
27 alpha_step=(alpha_ub-alpha_lb)/Na;
28 beta_step=(beta_ub-beta_lb)/Nb;
29 c_step=(c_ub-c_lb)/Nc;
30
31 for i=1:iter
32     idx=1;
33     for alpha=alpha_lb:alpha_step:alpha_ub
34         for beta=beta_lb:beta_step:beta_ub
35             for c1=c_lb:c_step:c_ub
36                 for c2=c_lb:c_step:c_ub
37                     O=[-alpha 0 beta 0 0 0;...
38                         0 -beta alpha 0 0 0;...
39                         0 1/radiusCylinder 0 alpha^2+beta^2*c2^2 ...
40                         beta^2+alpha^2*c1^2 -2*alpha*beta*(1+c1*c2)];

```

```

41
42     L=[beta*c2 0 -alpha*c1 0 0 0;...
43         0 alpha*c1 -beta*c2 0 0 0;...
44         0 0 0 -2*alpha*beta*c2 -2*alpha*beta*c1...
45         2*(c1*alpha^2+c2*beta^2)];
46
47     OL=[0 L;L 0];
48     G=OL*M*transpose(OL);
49
50     Nx0=1;
51     Phi1=Nx0*(alpha^2+beta^2*c2^2);
52     Phi2=-2*alpha*beta*(Nx0*c2);
53
54     Phi=[Phi1*J Phi2*J;...
55         Phi2*J Phi1*J];
56
57     %% Eigenvalue Problem
58     lambda=eig(G,Phi);
59     if lambda(2) < lambda(3)
60         lambda_p(idx)=lambda(2);
61         var(idx,:)=[alpha, beta, c1, c2];
62         idx=idx+1;
63     else
64         lambda_p(idx)=lambda(3);
65         var(idx,:)=[alpha, beta, c1, c2];
66         idx=idx+1;
67     end
68 end
69 end
70 end
71 end
72     lambda_p(lambda_p<=0)=[lambdaCritical(1)];
73     [lambdaCritical(i+1),minIdx]=min(lambda_p);
74     min_params=var(minIdx,:);
75 end
76
77 criticalLoad=2*pi*radiusCylinder*min(lambdaCritical);
78 end

```

CURRICULUM VITAE

Name Surname : Cansu Özgen

EDUCATION :

- **B.Sc.** : 2018, Istanbul Technical University, Faculty of Aeronautics and Astronautics, Astronautical Engineering
- **M.Sc.** : 2022, Istanbul Technical University, Faculty of Aeronautics and Astronautics, Astronautical Engineering

PROFESSIONAL EXPERIENCE AND REWARDS:

- 2018-Present Roketsan A.Ş.

PUBLICATIONS, PRESENTATIONS AND PATENTS ON THE THESIS:

- **Özgen, C. and Doğan, V.Z.** 2021. An Imperfection Sensitivity Analysis on Buckling of Laminated Composite Cylindrical Shells. *2021 IEEE 4th International Conference on Nanoscience and Technology (ICNST)*, June 26–28, 2021 Chengdu, China