İSTANBUL TECHNICAL UNIVERSITY ★ INSTITUTE OF SCIENCE AND TECHNOLOGY

KINEMATICAL AND DYNAMICAL MODELLING AND DESIGN OF A MULTIMODAL MOBILE ROBOT

M.Sc. Thesis by
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Department : Mechanical Engineering
Programme: System Dynamics and Control

JUNE 2008

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JUNE 2008

<u>**İSTANBUL TEKNİK ÜNİVERSİTESİ ★ FEN BİLİMLERİ ENSTİTÜSÜ**</u>

ÇOK MODLU BİR MOBİL ROBOTUN KİNEMATİK VE DİNAMİK MODELLENMESİ VE TASARIMI

YÜKSEK LİSANS TEZİ

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ABBREVIATIONS

CAD	: Computer Aided Design
FEM	: Finite Element Method
DC	: Direct Current
AL	: Aluminum
ICR	: Instantaneous Center of Rotation

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LIST of SYMBOLS

α	**:**	Angle between chassis and articulations
β	**:**	Angle of an Articulation with respect to tha Chassis
ξ	**:**	Position of the Robot
R	:	Transformation Matrix
θ	:	Orientation of the Robot
v	:	Velocity of the Wheel
φ	:	Angular Displacement of the Wheel

ÇOK MODLU BİR MOBİL ROBOTUN TASARIMI, KİNEMATİK VE DİNAMİK MODELLENMESİ

ÖZET

Mobil robotlar günümüzde birçok uygulama alanında kullanılmaya başlanmıştır. Çok çeşitli mobil robot uygulamaları çeşitli görevleri yerine getirmek üzere tasarlanmıştır. Doğal afetler ve askeri uygulamalar mobil robotların kullanıldığı uygulama alanalarından sadece birkaçıdır. Bu tür uygulamalarda bir robotun hareket kabiliyeti büyük önem taşımaktadır. Sınırlı çalışma alanı ve engebeli arazilerde oluşabilecek kararsızlık koşulları esnek bir tasarımın oluşturulmasını gerektirir. Bu amaçla bu tür bir mobil robotun tasarlanması ve modellenmesine karar verilmiştir. Çalışma gereksinimlerini karşılayacak yenilikçi bir tasarım yapılmış ve bu tasarımın yapısal, kinematik ve dinamik analizleri gerçekleştirilmiştir. Bu analizler enerji gereksinimlerini minimuma indirecek komponentlerin seçilmesine imkan vermiştir. Robotun prototipinin üretilmesi ve imalat esnasında mümkün olduğunca zorluklarla karşılaşmamak için ticari olarak hazır bulunan bileşenler seçilmiş, optimum ve modüler bir tasarım gerçekleştirilmiştir.

KINEMATICAL AND DYNAMICAL MODELLING AND DESGIN OF MULTIMODAL MOBILE ROBOT

SUMMARY

Mobile robots have started to play a geat role in many applications. Variety of applications are available in order to perform some special tasks. Natural deseases and military applicatins are one of the leading areas that mobile robots are inevitable to be used. In these applications locomotion capabilities of a mobile robot is considered being the most important facilities. Restricted working space and difficulties in stability conditions in rough terrain require a mobile robot to have as much flexibility as it can have. To this end, design and modeling of this kind of multimodal mobil robot is decided to be produced. An innovative design have been prepared to meet the design necessities considering the structural, dynamical and kinematical analysis of the mobile robot. These analysis lead us to choose appropriate system componenets that minimize the energy requirements. In order for manufacturing of this mobile robot to be managed without facing any difficulties, comercially available components have been chosen and optimum and modular design has been accomplished.

1. INTRODUCTION

If you ask a person something about robotic, the first responce you are going to have would probably be an explanation that a robot is a mecahnical device behaves like a human. It is not surprising that people in real life give such a reponce. The movies made so far have a great and influential role in this subject. During the time we watch these movies, robots had been working at factories in a variety of jobs. Many examples can be given such as painting of cars, welding of mechanical parts again in car factories, or in extremely dangereous chemical applications. The word *robot* introduced by Czech playwright Karel Capek : robots are machines which resemble people but work tirelessly.

Using robots in real life applications brought some advantages to manufacturers. Robots that is composed of arms and linkages is called robotic manipulators. And this kind of robotic manipulators have been extensively used in industrial applications in order to be able to perform such duties that is not phsically possible for people to manage.

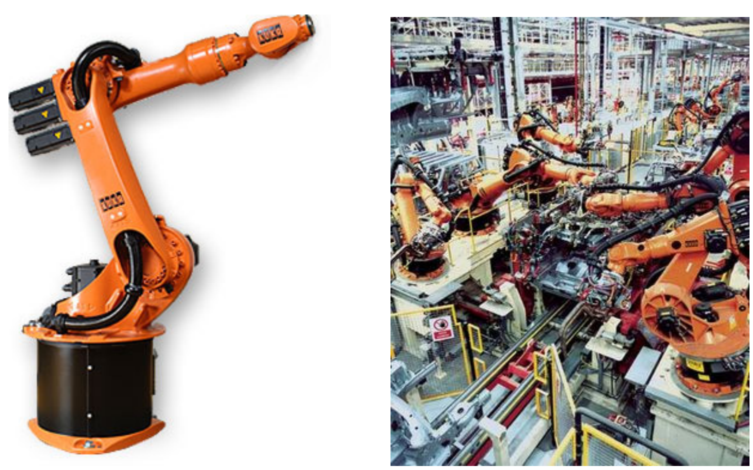


Figure 1.1: Industrial Robotic Manipulators

Think of a production area in which variety of gases present which may cause vital health problems on people, it is inevitable to use robotic manipulators in such areas. And also accurate positioning capabilities of robot manipulators enable manufacturers to increase the quality of their products. And low cost operation lead robots to take place of people in industry.

Although robotic manipulators have a great success in industrial applications, they have some disadvantages due to the fact that they can not change their position and should move only in a position assambled to a fixed frame.

While a robot placed to a fixed frame has only a restricted working space, a mobile robot can move freely in a manufacturing area and use their tools in order to perform some special tasks. Actually mobile robots have wide range of application areas and they can be used in some special purposes such as bomb destruction for military purposes, underground applications or a life-saving applications in natural diseases.

1.1 Definitions of Robots

There are different types of definitions in literature about what makes a machine be called robot. One can say that a machine should have some properties to be considered robot such as the ability of sensing, intelligence and mobility. Some definitions are the following :

- A mechanical device that sometimes resembles a human and is capable of performing a variety of often complex human tasks on command or by being programmed in advance.
- A machine or device that operates automatically or by remote control

According to some robotic associations a robot is considered as follows :

- It possesses some form of mobility
- It can be programmed to accomplish a large variety of tasks
- After being programmed, it operates automatically.

Although there is not a precise definiton as shown the explanatios above, qualificaitons of robots may be arranged as the following :

- Sensing and perception - get information from its surroundings
- Carry out different tasks - locomotion, manipulation, physically movo objects
- Re-programmable - perform different things
- Behavior and communication - function autonomously and interact with human beings

1.2 Classifications of Robots

In order to make a classification of robots taking previous statements into consideration robots may be grouped as the following:

- Manipulators - robotic arms most commonly used in industrial applications
- Mobile robots - unmanned vehicles capable of locomotion as well as performing tasks
- Hybrid robots - mobile robots with manipulators

1.3 Manipulators

Robot manipulators are the most commonly used robots in industrial applications. And they constitute arms attached to each other by means of joints.

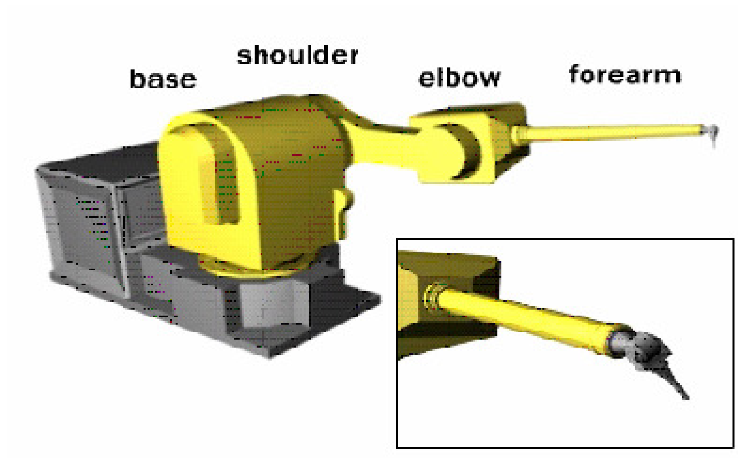


Figure 1.2: Robotic Manipulator

Basic configuration of a robot manipulator is shown in the figure above. Applying kinematical and dynamical analysis, a manipulator can be used to perform some tasks. In order to be able to obtain kinematical equations and positions of links of manipulators reference frames are attached to joints. And as a matter of convention, standard names are assigned to joint reference frames.

1.4 Mobile Robots

Mobile robots have a major advantage over manipulators, the capability of locomotion in environment. This enables mobile robots to be used for various applications including outdoor and indoor tasks. Locomotion capability of a mobile robot can be defined through the mechanisms designed to move it in different positions. Actually, many researchers have an inspiration on biological anatomy of real creatures to design their own mobile robots. Mobile robots can also be classified by the method of locomotion as:

- Legged robots
- Wheeled robots
- Tracked robots

Considering the three major locomotion methods above, variety of robots can be designed using combinations of the three.

The mobile robot which is the main subject of this thesis has been designed using leg-track-wheel locomotion architecture.

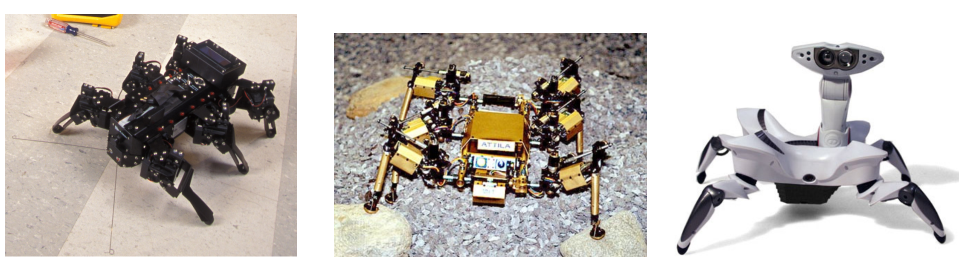


Figure 1.3 : Legged Mobile Robots

Some configurations for legged mobile robots can be seen in figure 1.3. The major problem for the legged robots is the dynamic stability of the robot itself. If the number of legs decreases, it will be difficult to stabilize the system.

Recent technological developments enabled scientists to work on two-legged robots which are also called biped or humanoid robots.

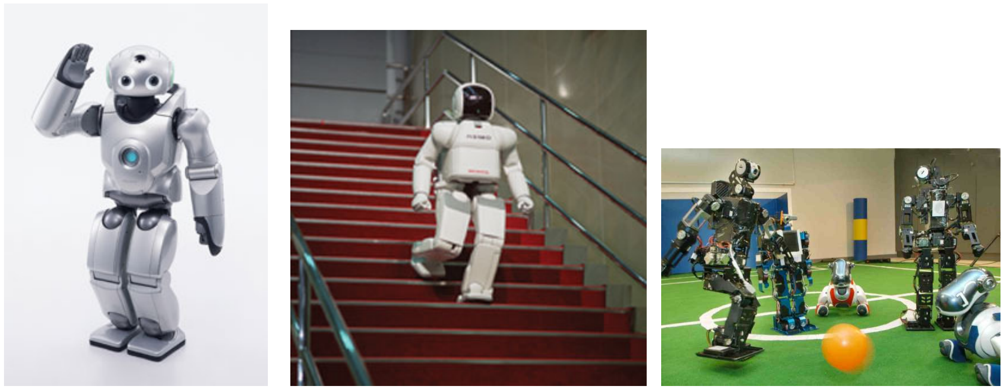


Figure 1.4 : Humanoid Biped Robots

Some examples of humaoid robots can be seen in figure 1.4. Honda with Asimo and Sony with Qrio lead the desing and research of biped walking humanoid robots.

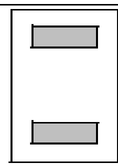
Recent researches shows that walking robots is going to play a great role in daily life of people in the next few years. But the interest in this thesis is mainly on tracked robots. It has been aimed to design a mobile robot that has more alternative locomotion modes than robots designed so far and can smoothly move in rough terrain.

Before explaininig the design stages of the mobile robot, it will be useful to give some information about wheeled robots.

Wheeled robots that operate on eart can be classifed according to the number and arrangement of the wheels.

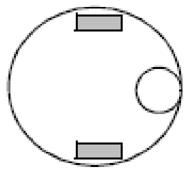
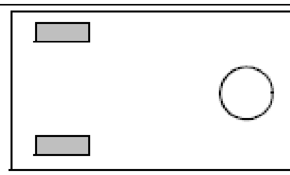
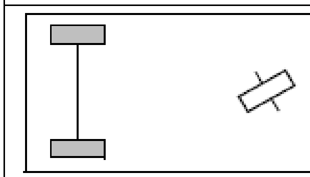
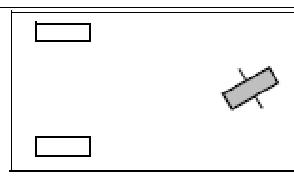
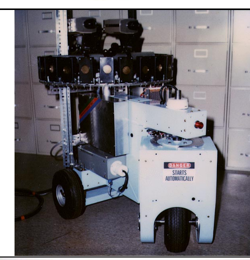
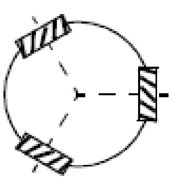
Two wheeled configurations can be seen in table 1.1. As a current example for these type of mobile robots can be given as the Ginger that has been started to use in airports.

Table 1.1: Two Wheeled Configurations for Mobile Robots

Number of wheels	Arrangement	Description	Typical applications
2		One steering wheel in the front, one traction wheel in the rear	
		Two-wheel differential drive with the center of mass below the axle	

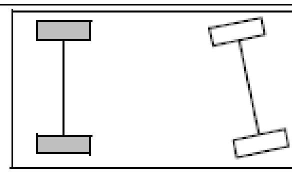
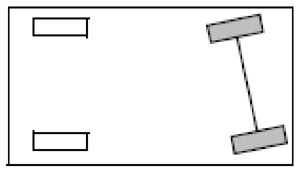
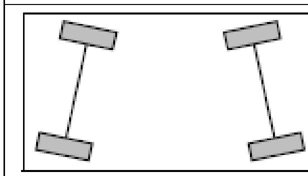
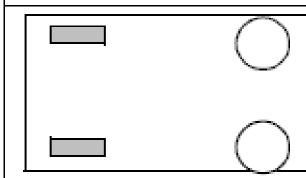
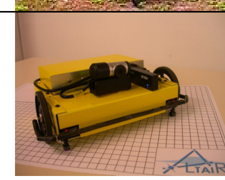
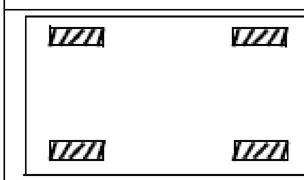
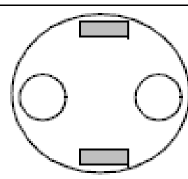
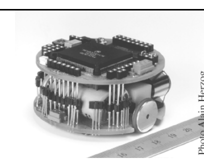
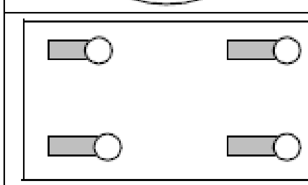
Some three wheeled configurations are available for the use in many areas. Some examples can be seen in table 1.2.

Table 1.2: Three Wheeled Configurations for Mobile Robots

Number of Wheels	Arrangement	Description	Typical Applications
3		Two wheel center differential drive with a third point of contact	
		Two independently driven wheels in the rear and one un-powered omni-directional drive in the front	
		Two connected traction wheels in the rear and one steered free wheel in the front	
		Two free wheels in the rear and one steered traction wheel in the front	
		Three motorized Swedish or spherical wheels arranged in a triangle which makes omni-directional movement is possible	

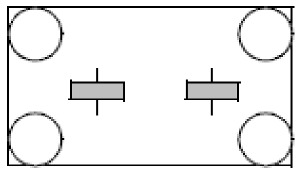
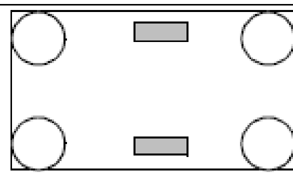
In many mobile robot applications four wheeled mechanisms are encountered and also our mobile robot can be considered as four wheeled mobile robot. Some kind of applications is shown in table 1.3.

Table 1.3: Four Wheeled Configurations for Mobile Robots

Number of Wheels	Arrangement	Description	Typical Applications
4		Two motorized wheels in the rear, and two steered wheels in the front	
		Two motorized and steered wheel in the front and two free wheels in the rear	
		Four steered and motorized wheels	
		Two traction wheels in the rear and two omni-directional wheels in the front	
		Four omni-directional wheels	
		Two wheel differential drive with additional two contact points	
		Four motorized and steered castor wheel	

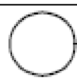
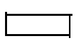
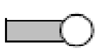
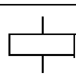
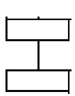
With the combinations of two, three and four wheeled applications six or more wheels can be attached to a mobile robot with respect to the aim of the design. And some of the application types are shown in table 1.4.

Table 1.4: Six Wheeled Configurations for Mobile Robots

Number of Wheels	Arrangement	Description	Typical Applications
6		Two motorized and steered wheels at the center and one omni-directional wheels at each corner	First
		Two traction wheel in center and one omni-directional wheel at each corner	Terregator (Carnegie Melon University)

In literature standard icons are given to the wheels for simplicity. These icons which are shown in table 1.5 can be used in any design and representation.

Table 1.5: Icons Used for Wheels with Standard Representation

Icons for each wheel type is as the following	
	Un-powered omni-directional wheel (spherical, castor, Swedish)
	Motorized Swedish wheel
	Un-powered standard wheel
	Motorized standard wheel
	Motorized and steered castor wheel
	Steered standard wheel
	Connected wheels

1.5 Omni-direction and Maneuverability

Some robots can move at any time in any direction along the ground regardless of the orientation. This kind of robots is called omni-directional. This level of maneuverability can be obtained only if the wheels of the robot can move in more than one direction. Omni-directional robots generally employ Swedish or castor wheels which are powered. Maneuverability brings a robot many advantages in variety of applications. So in our design we wanted to obtain a full maneuverability by enabling motion around the vertical axis of each articulation at the corners attachment points of the chassis.

1. DESIGN OF MULTI-MODAL MOBILE ROBOT

Design of a multimodal mobile robot is a multidisciplinary project where special care should be considered. In this design, a new type of mobile mechanism robot has been proposed excluding electronics. Mechanical design and kinematical and dynamical analysis are included in different chapters. In the following section design stages and the mechanism of the mobile robot will be introduced.

2.1 Design and Main Components of Multimodal Mobile Robot

The mobile robot is designed so as to move in rough terrain with multimodal motion capabilities. Four articulations attached to the chassis with tree degrees of freedom each. Articulations can turn around an axis vertical to the plane of robot chassis by means of a motor and planet gear application mounted to the chassis and a worm gear is used in front of the planet gear in order to turn the direction of motion ninety degrees.

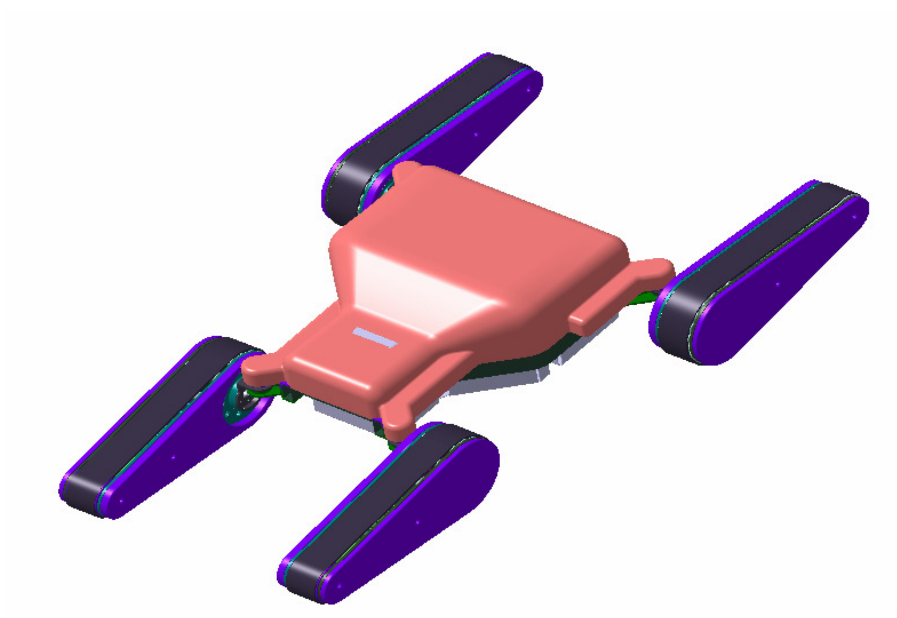


Figure 2.1: Multi-modal Mobile Robot

Compact form of the multimodal mobile robot is shown in figure 2.1. Four belts have been used to drive the robot in the ground. Each belt is driven by a DC motor and a harmonic drive assembly inside the wheel placed in the front side of each articulation. And covers have been designed to give the mobile robot a compact form and to prevent each part from the dust in environment.

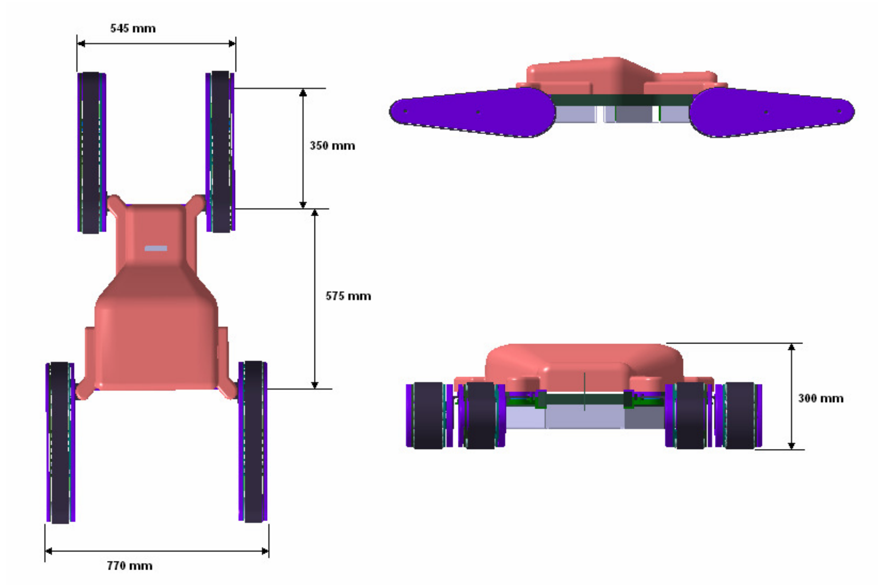


Figure 2.2: Main Dimensions of the Robot

Main dimensions of the mobile robot can be seen in figure 2.1. Dimensions of the mobile robot have not been defined at the beginning of this design process. Some requirements are determined and dimensions are automatically chosen so that the components such as motors can be replaced inside the body. It is assumed that this mobile robot may contain a camera and a manipulator placed on the chassis for a further design and the weight of these components may reach up to 30 kilogram. Maximum moments at the joints are obtained by a dynamical analysis assuming the articulations are going to lift the body with a constant velocity. Details of this process will be explained in the sections related to dynamical analysis.

The components that the mobile robot contains are going to be mentioned in the subsequent sections.

2.1.1 Components of the Articulation

An articulation is composed of motors, gears and framework in order for the parts of the articulation to be held in the right position.

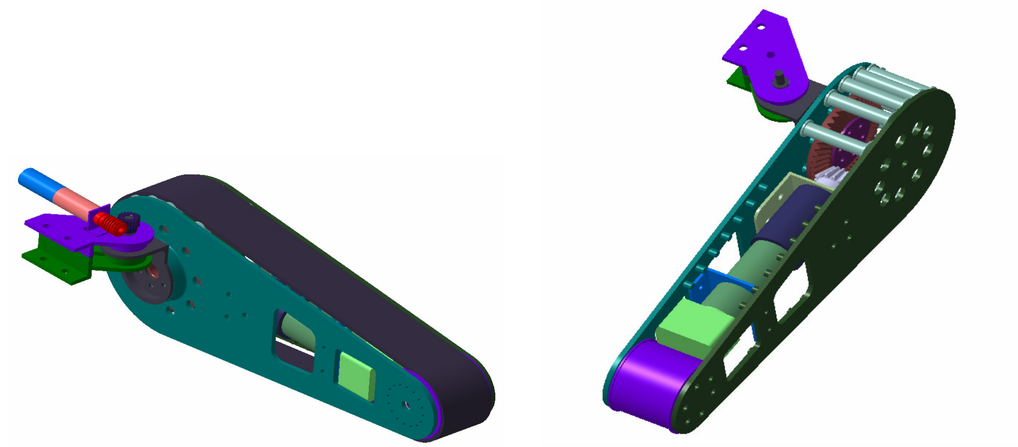


Figure 2.3: An Articulation of the Robot

Main components and framework is shown in the figure 2.2. An articulation may be called leg. Since the leg can turn around an axis horizontal to the ground. So the chassis can move up and that gives the robot one extra degree of freedom.

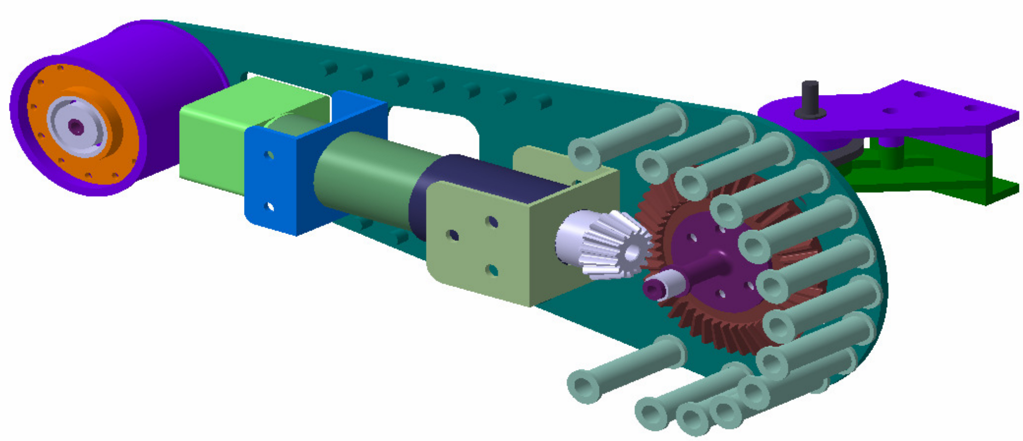


Figure 2.4: Perspective View of an Articulation

Assembly of the leg model is shown in the figure 2.3. When the motor inside the leg drives the bevel gear and so the legs gives robot a motion through the upper side of the ground. This enables the robot to stand on the legs.

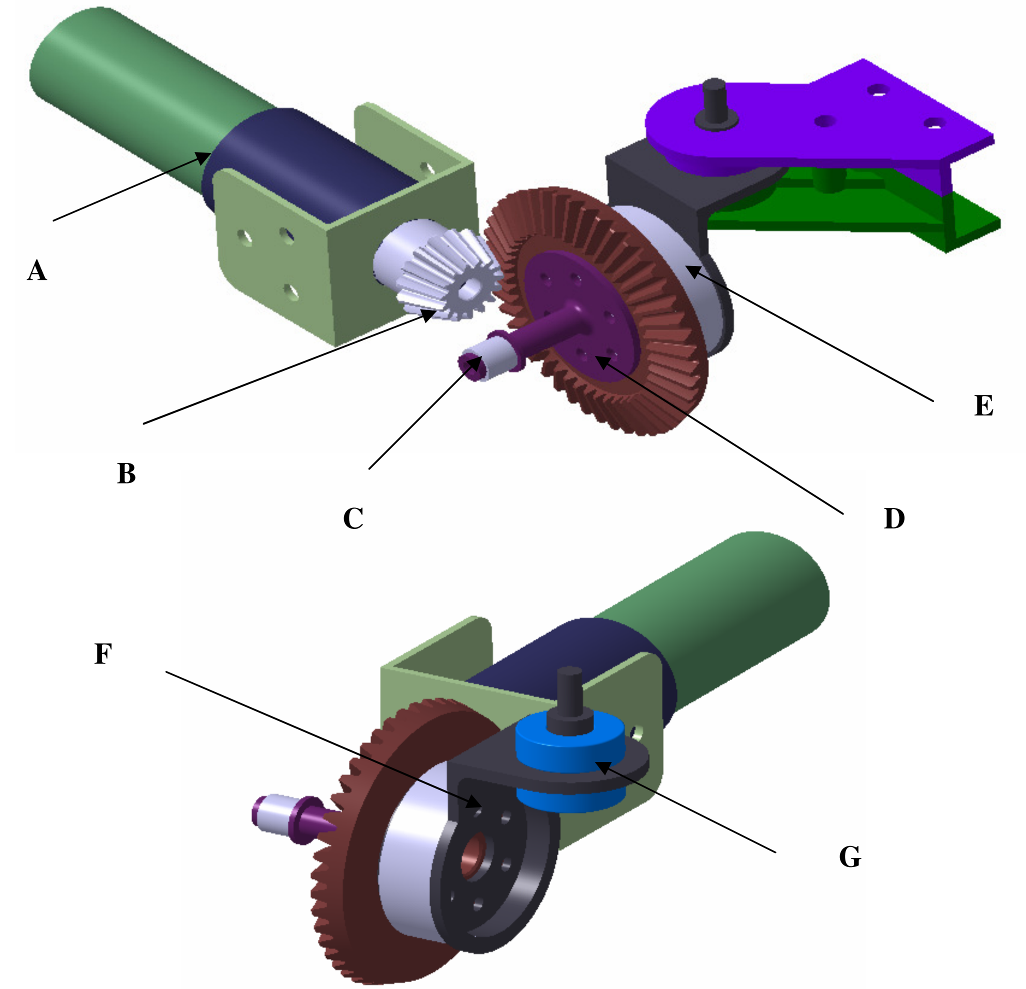


Figure 2.5: Components inside the Articulation

Driven gear of the bevel gear assembly and the middle shaft are connected to the part ‘A’ which has a roller bearing to support the motion of the first cover. And the middle shaft has a roller bearing for the same purpose for the second cover. And two roller bearings are attached to the part which connects the leg to the chassis. These roller bearings react to axial and radial forces and aligned in opposite direction in order to be able to support the motion of the robot in all directions with minimum friction forces.

Components inside the articulation is as the following

A - DC motor and planet gear assembly

B - Bevel gear mechanism

C - Radial roller bearing

D - Middle shaft to connect the covers

E - Radial roller bearing

F - Elbow

G - Conical roller bearings

2.1.2 Motor Wheel

Construction of such a mobile robot is a time-consuming and a multidisciplinary work to perform. Many difficulties may arise during design stages. The main problem confronting us is the placement of the motors inside the leg in the right position so that the robot can easily perform the motion required to have multimodal locomotion capability. For this purpose the belt is driven by a motor wheel assembly which comprises a frameless DC motor and a harmonic drive mechanism.

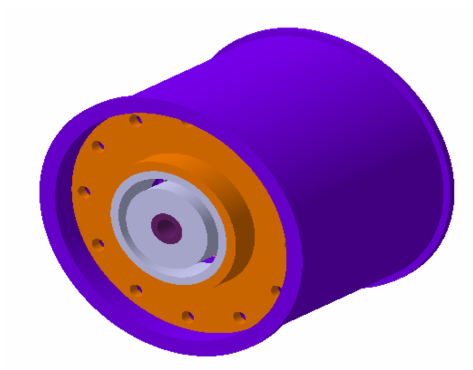


Figure 2.6: Motor Wheel Assembly

This assembly may be called motor wheel since the mechanism includes a DC motor. Compact form of the motor wheel assembly is shown in figure 2.6.

In order to be able to explain which components are available inside the motor wheel, cross-sectional view of the assembly has been obtained from Catia V5.

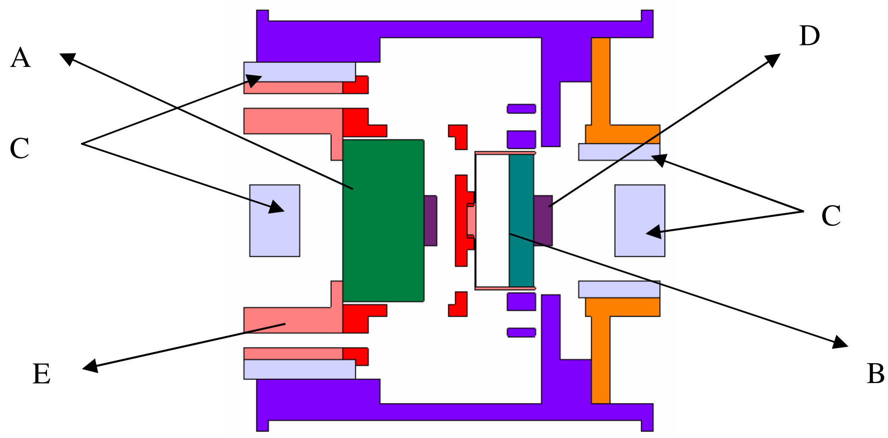


Figure 2.7: Cross-sectional View of the Motor Wheel Assembly

A	-	DC Motor
B	-	Harmonic Drive
C	-	Roller Bearings
D	-	Input Shaft
E	-	Motor Holding Frame

Section view of the motor wheel is shown in the figure. Rotor of the frameless DC motor is connected to the wave generator. Stator of the motor and flex-spline of the harmonic drive are attached to the cover through a mechanical component designed. And the circular spline is connected to the wheel through bolts. Roller bearings shown in the figure support the motion of the shaft and the wheel itself. The important thing in such a design process is to manufacture the components precisely so that the rotor and stator of the motor stand in right position such that they do not crash each other. And again the harmonic drive assembly requires precise mounting procedure.

Components of the motor wheel assembly can be seen in figure 2.8. As shown in the figure this is a complex mechanism and hard to produce. But if the production of this mechanism can be managed it can be used in many areas in robotic for further designs.

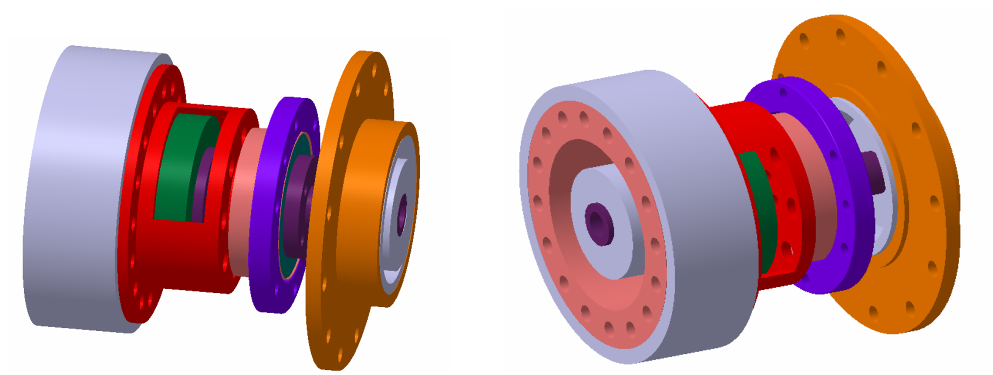


Figure 2.8: Components of Motor Wheel Assembly

There are different types of harmonic drive mechanisms. A flat type harmonic drive has been chosen due to the fact that the availability of place is restricted. The main structure of this harmonic drive assembly can be seen in figure 2.8.

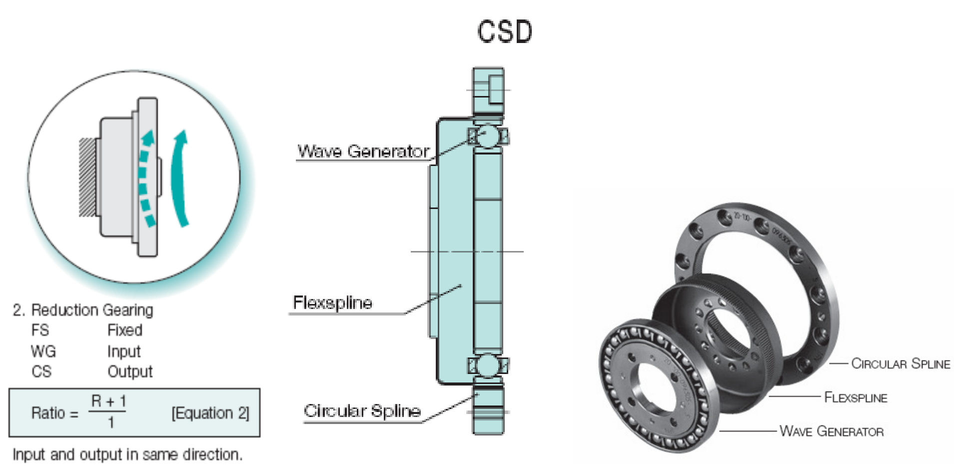


Figure 2.9: Harmonic Drive Assembly

Reduction ratio and rotational direction of the motor with respect to the output direction of the motion of harmonic drive differ with assembly position. In figure 2.8 above, reduction ratio and direction of the motion is shown as used in our design.

2.1.3 Chassis

Many different types of robot chassis design are available for different purposes. Mobile robot chassis should contain the whole equipment including electronics, sensors, batteries, motors and it should be easy to manufacture. For these purposes, a framework with standard steel profiles available in the market for any application purposes has been designed. So a light and strong structure could be obtained. And sheet metals are attached to the chassis by means of screws in order to have space enough for the equipment to be arranged.

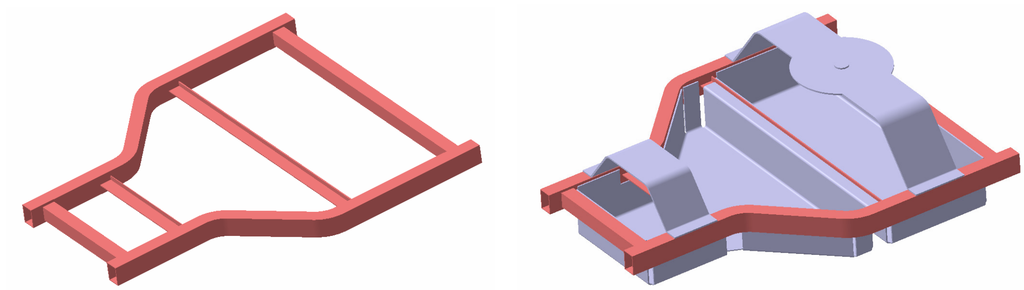


Figure 2.10: Mobile Robot Chassis

The design of the mobile robot chassis is shown in figure 2.9. Two types of manufacturing processes can be considered to build this structure. Welding of the profiles each other or hinges with bolt screws are chosen as two alternative production approaches. Sheet metal production is an easy way to obtain a compact design. Sheet metals have been designed using Catia V5 generative sheet metal design tool. And finally two hard sheet metals for a camera at the rear of the robot and a robot manipulator in the front which may be considered for a further design have been attached to the chassis.

Motors and related driving mechanisms for the robot to be able to move in an axis vertical to the ground are mounted to the chassis again by using bolts. And assembly of these components to the chassis is shown in figure 2.10.

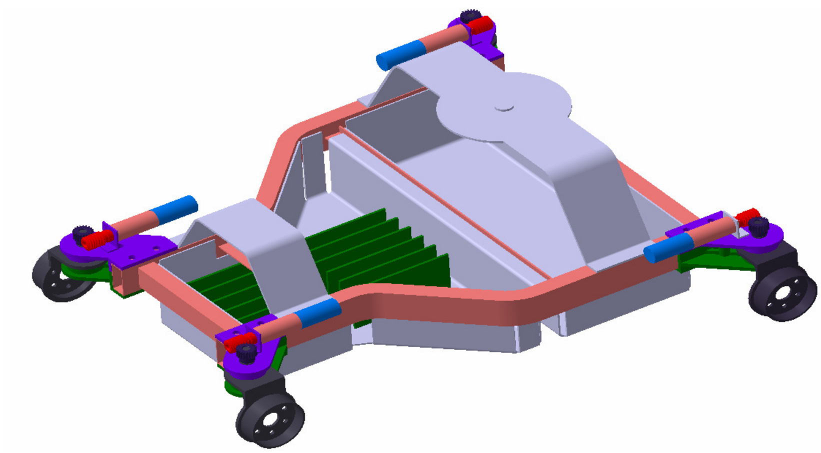


Figure 2.11: Attachment of the Articulations to the Chassis

As a final mechanical design of the mobile robot ends with the cover made of plastics in order to have a compact design and to prevent the parts from dust and any other environmental affects. Production of these plastic parts can easily be managed by layered manufacturing process. And this process can be thought as a prototyping process. Plastic components designed for this mobile robot are shown in figure 2.11.

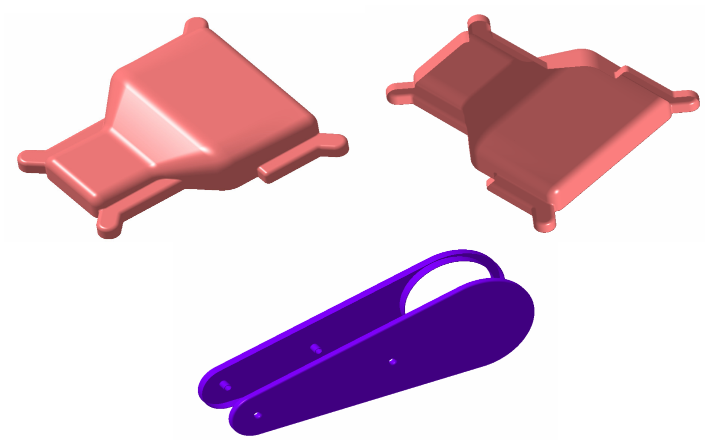


Figure 2.12: Plastic Components of the Mobile Robot

2.2 Electrical And Mechanical Components Required For Design

In the design of this multimodal mobile robot some mechanical and electrical components such as DC motors, roller bearings and accumulators have been used. In this section, these parts are going to be grouped.

Table 2.1: DC Motor Properties

Model	**Power Output**	**Nominal speed**	**Input Voltage**	**Nominal Torque**
Maxon EC 45 No : 136207	250 Watt	4520 rpm	24 V	310 mNm
Kollmorgen No : QT-1204	57 Watt	43000 rpm	24 V	78 mNm
Maxon EC 22 No : 200863	20 Watt	20500 rpm	24 V	14 mNm

Table 2.2: Reducers

Model	Max. Continuous Torque	Reduction Ration
Maxon GP52C No:223093	30Nm	81/1
Maxon GP22C No:144010	2Nm	1249/1

3. STRUCTURAL ANALYSIS OF THE COMPONENTS OF ROBOT

Every component of machine parts is exposed to different types of loading. So every part should be designed to resist some kind of dynamic, static or heat loads. In this section it is going to be checked whether our design is satisfactory for structural considerations.

3.1 An Overview of the Finite Element Procedure

Different approaches are used to compute the response of a system to applied loading and boundary conditions. Complex geometrical shapes of machine parts make it impossible to find the exact solution of the problem. Some approximations should be made in order to obtain satisfying results. The finite element method is a powerful tool for solving the differential equations of systems. In the finite element method complex geometrical shape of the model is represented with sub-domains called elements. And the equations are solved for each element considering the relation of the adjacent elements.

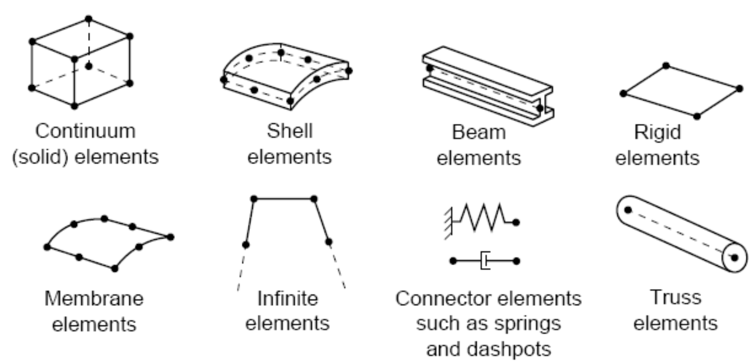


Figure 3.1: Element Types Used To Simulate Models

Some kind of elements used for analysis purposes are shown in figure 3.1. These elements connected each other with nodes represent the continuous domain of the model.

Since finite element is an approximation method itself there exists a question whether how the results are approximated to the exact results. There are three kinds of errors in finite element analysis procedure. We can explain these as:

- Geometrical errors - Discretization of the geometry with elements is not exact for complex geometrical parts
- Round-Off errors - Representation of the number in computers with digits
- Solution Approximation - Finite element method is an approximate method where the exact solution is approximated by combination of polynomials

3.2 Software Packages Used For Finite Element Analysis

Different software packages are available for finite element analysis. Some of them have been used for analyzing purposes. These packages are listed as

- HyperMesh - Pre and Post processor
- Abaqus v6.7 - Finite Element Solver
- Abaqus CAE - Post-processor
- OptiStruct - Optimization Solver

3.3 Element Types Used In Analysis

Three types of elements have been used in the analysis. There are different types of elements that can be used to model a system. It differs in the way of your choice to model the system. Behavior of the models has been simulated using solid, shell and rigid elements.

Abaqus offers some type of solid elements and the physic of these solid elements is shown is figure 3.2.

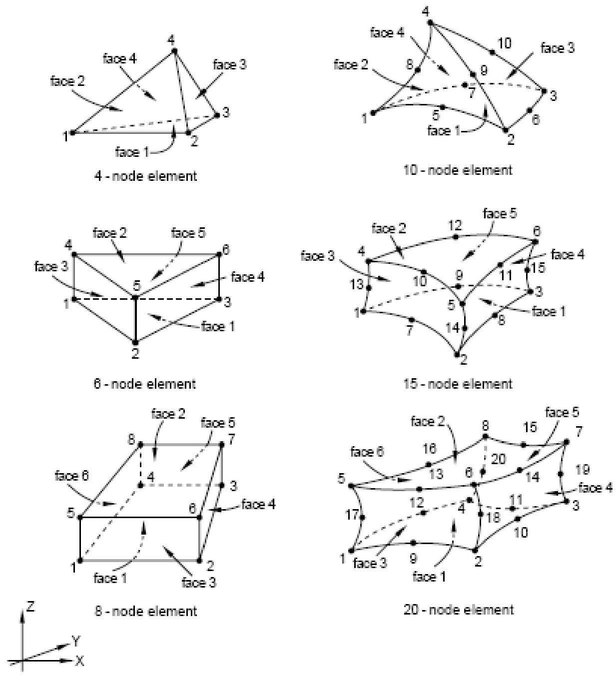


Figure 3.2: Solid Elements Used For Analysis

Solid element types are different in geometry and interpolation function used for formulation. For complex parts tetrahedral elements are used to model the system since it can be hard to use hexahedral elements. And in formulation of finite element method solution is approximated by linear or quadratic interpolation functions. In Abaqus, quadratic elements have twice as many nodes as the linear elements. In figure 3.2 quadratic elements is shown in the right column. Although quadratic elements give better results than the linear elements, computational costs increase. And this brings a trade-off to the engineer. Linear elements have been used in the analysis of the components of the mobile robot.

If the thickness of a part is less then the other geometrical measurements then it will be better to use shell elements to represent the model. Using shell elements give better results since the formulation of finite elements is different than solid elements.

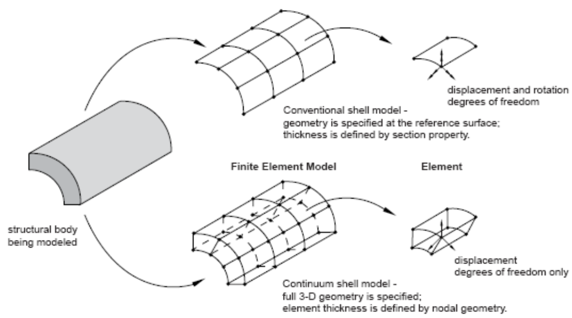


Figure 3.3: Representation of a Model with Shell Elements

Figure 3.3 shows how shell elements are used to model the system. In order to obtain shell elements from CAD data middle surface of the part is extracted by some methods available and then this surface is meshed with shell elements.

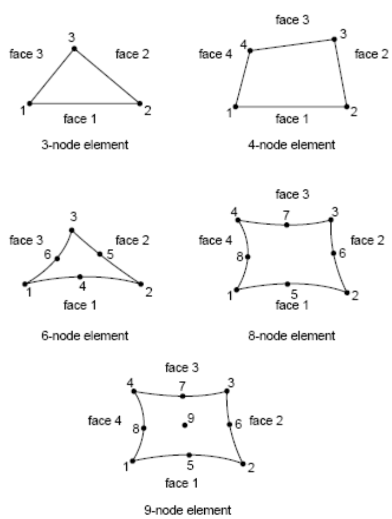


Figure 3.4: Shell Elements Used For Analysis

In Abaqus, shell elements are available as solid elements in triangular and quadratic form. Types of elements to use, linear or quadratic, can be chosen with respect to the engineering approach.

3.4 Main Steps of an Analysis Process

There are some general purpose finite element software packages in order to compute the response of the machine parts exposed to different types of boundary conditions. These packages are used extensively in industrial applications. The finite element analysis of systems has three major steps in these software packages or self-written codes. They can be arranged as the following

- Pre-processing
- Computation of the Response
- Post-processing

Finite element models of the parts should be obtained. For this purpose CAD data is obtained in various types of geometrical formats such as iges, stp, stl. Then this data is imported to any modeling software and element mesh is obtained employing some special procedures. Material properties (may be different for each part) and boundary conditions are applied to the related elements and nodes. This process is called pre-processing.

In the computation process finite element model of the system with necessary properties as explained above is imported to a solver program. Then analysis procedure is run and pre-defined results such as von-misses stress data, deformations of the model or time-history data are obtained.

The last step includes interpreting the results. It can be determined that the model prepared is acceptable for the use considering the analysis results obtained in the previous step. Stress data, deformation or any defined result can be visualized using again some software packages. This process is called post-processing. For post-processing purposes some special programs are available. Although finite element software includes an embedded post-processor one can use more capable programs designed only for post-processing purposes.

3.5 Finite Element Analysis of Components of the Mobile Robot

Modeling steps and analysis results obtained form the finite element analysis are going to be explained.

3.5.1 Material Properties

AL 7075 aluminum has been chosen for the most parts as the material. Since aluminum is a soft material it is easier to process. And for the chassis, standard profiles have been chosen and the material of the profiles is steel as standard. Mechanical properties of these two materials are given in table 3.1 as:

Table 3.1: Material Properties

	Aluminum	**Steel**
Density	2.81E-6 kg/m^3	7.86E-6 kg/m^3
Modulus of Elasticity	71.7 GPa	210GPa
Poisson's Ratio	0.33	0.3
Yield Strength	96.5 Mpa	315 MPa

3.5.2 Structural Analysis of Leg Frame Assembly

In the analysis procedure of leg frame, plates are modeled as shell elements and kinematical coupling elements which are also called rigid elements are used to simulate the bolt behavior. Bolt stiffness has not been included in the models. Finite element model prepared in Hyper Mesh is shown in figure 3.5

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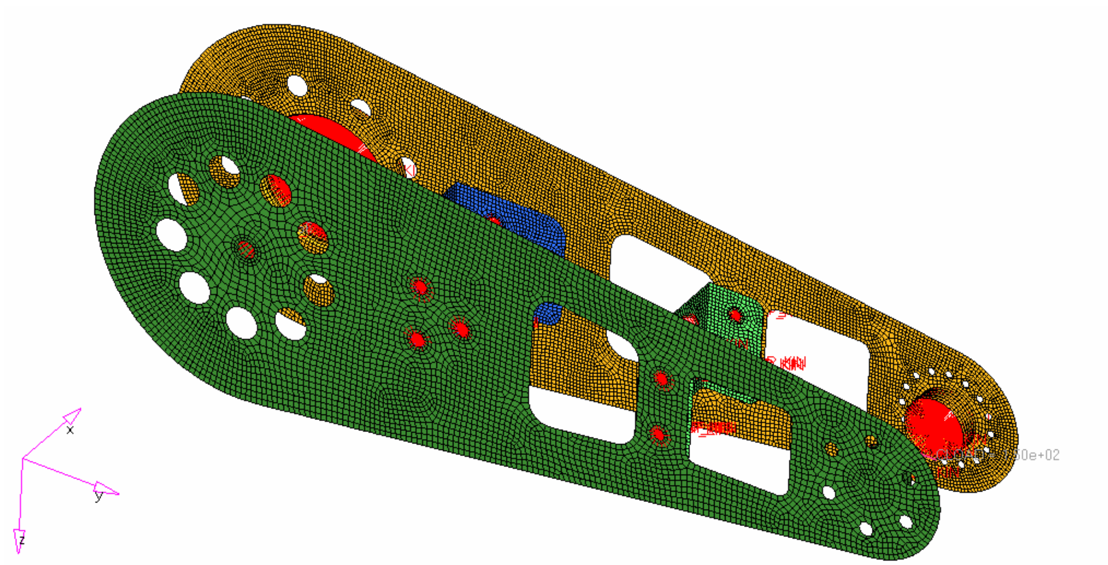


Figure 3.5: Finite Element Model of the Cover Frame

In the analysis of the model forces and boundary conditions should be defined to obtain considerable results. For this purpose some assumptions have been made. This mobile robot has been designed at first such that it can carry 40 kg load on the chassis. So, each leg is assumed to resist 10 kg load on the joints. But, 15 kg which produce 150N force has been chosen for the safety factors.

And two analysis types have been employed for the leg frame assembly. In the first one, leg has been assumed to stand in vertical position so that 150N force is excited through negative y axis as in the figure 3.5. And the nodes related to roller bearings at the joint are fixed in all degree of freedom.

Boundary conditions can be seen in figure 3.6. Here the nodes related to roller bearings in the front are coupled to each other by kinematic coupling elements.

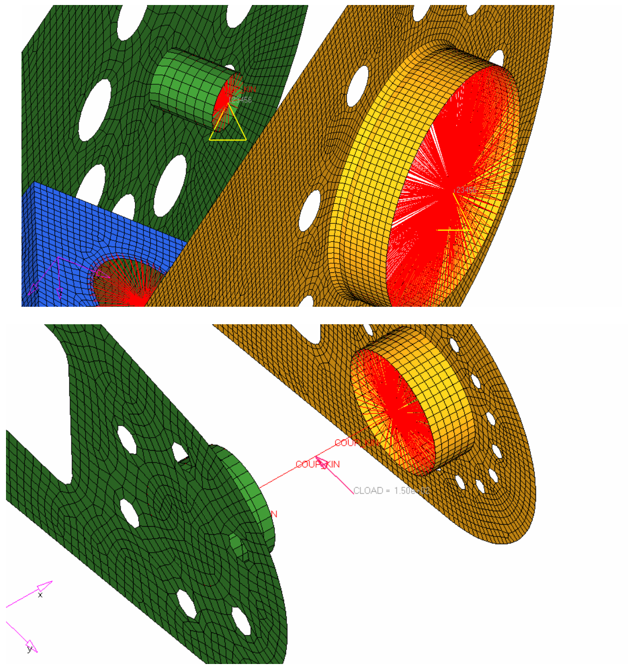


Figure 3.6: Boundary Conditions for the First Analysis

The results can be shown in figure 3.7 that maximum von-misses stresses do not exceed the yield limit of the material. And maximum stress is 0.9 MPa which can lead a design optimization process.

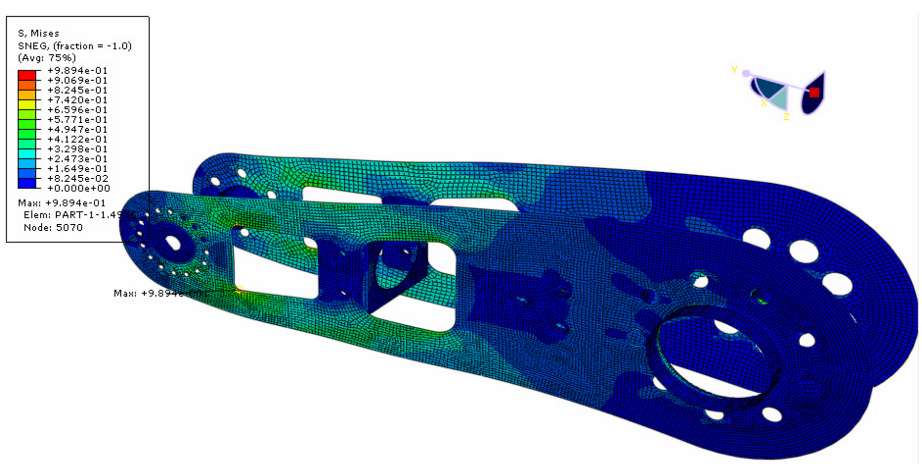


Figure 3.7: Von-Misses Stress Distribution on the Leg Frame

And also maximum deformation is obtained as 4.13E-3 mm which is acceptable and not much important such a design. Deformations are plotted in figure 3.8.

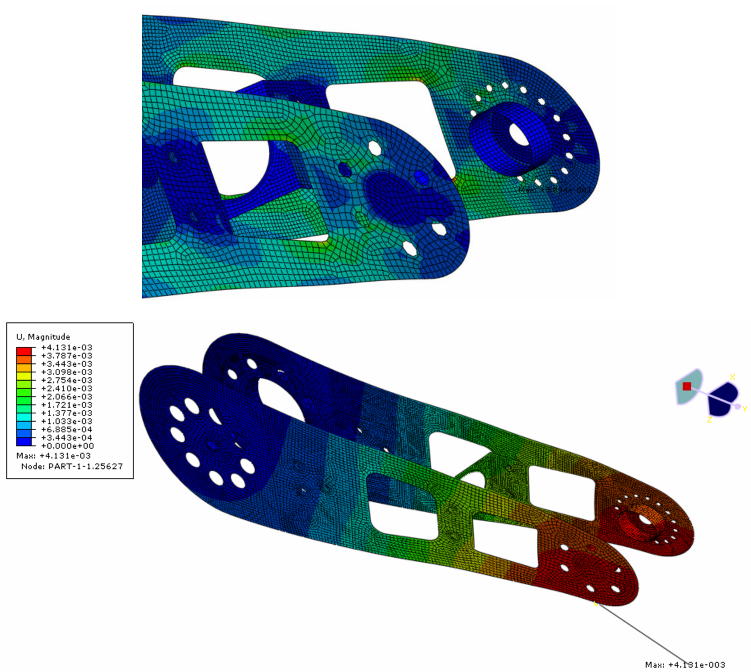


Figure 3.8: Displacement of Leg Frame

In the second approach, it has been assumed that the legs stand in horizontal direction and load and boundary conditions are applied as follows:

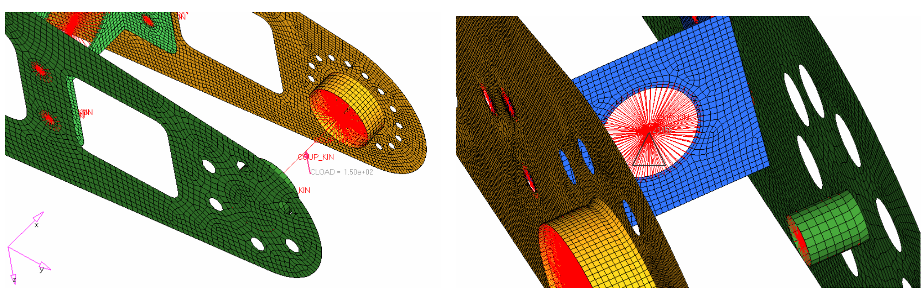


Figure 3.9: Boundary Conditions for the Second Analysis

Then total stress is obtained as 78MPa and it is below the yield strength of the material. Stress distribution is shown in figure 3.10.

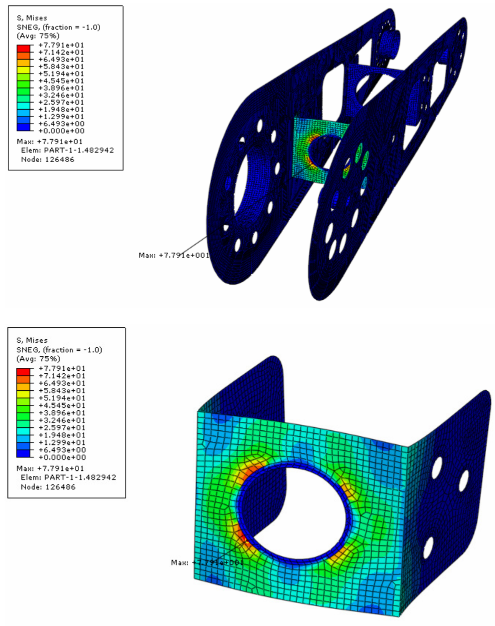


Figure 3.10: Von-Misses Stress Distribution on Motor Holder

Von-misses stress increases at some local areas of the model. Here it has been assumed that the motor which gives the motion to the joint in vertical direction is directly connected to the edges of the motor holder as shown in figure. So boundary conditions have been applied at the nodes attached to this edge. And the local stress increased at this area. But in real application motor is going to be attached to the holder with bolts and related holes will exist in the model of the motor holder. This is going to reduce the stress distribution in a level.

Thus, the maximum stress which is obtained as 78MPa is not an exact value. But it is not required to make a new analysis for this part. Because it is known from the preceding experiments that von-misses stress will not exceed the yield limit. So this design can be accepted as satisfactory for structural considerations.

3.5.3 Structural Analysis of Rotational Joint Part

Finite element model of the rotational joint part is shown in the figure. Tetrahedral elements have been used to model this part. And 150N force has been applied at the upper side of the part through the negative z direction. Holes for bolts are fixed at each node. These types of boundary conditions have been assumed to be the worst case.

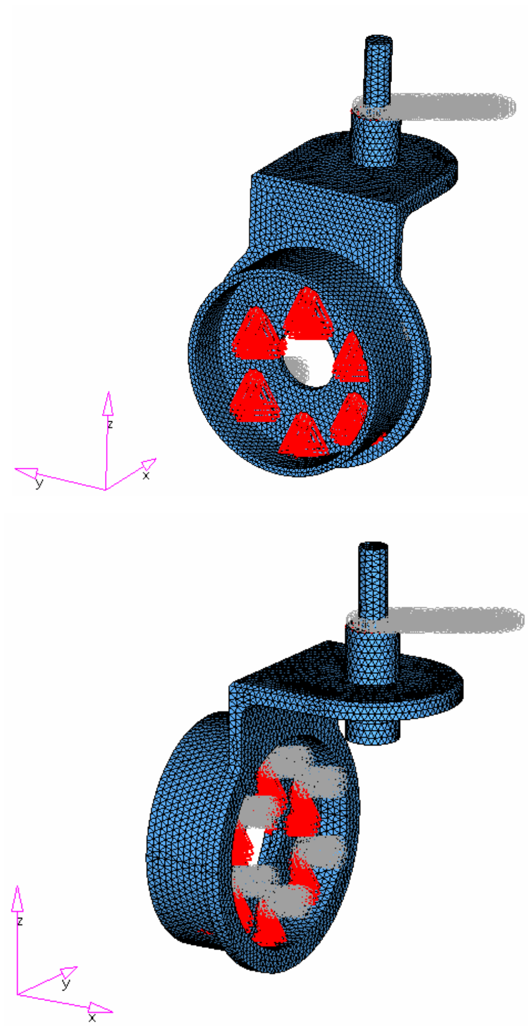


Figure 3.11: Finite Element Model of the Rotational Joint

Actual boundary conditions may be different from the ones shown in figure 3.11. Since it is very hard for a mobile robot to determine the exact boundary conditions these assumptions have been accepted as the right one.

Maximum stress is obtained as 26.14MPa for rotational joint as shown in figure 3.12. This result satisfies the structural requirements. Again a local stress most probably induced by an element that has not meet the requirements such as, aspect ratio, warpage occured at the corner point of the model.

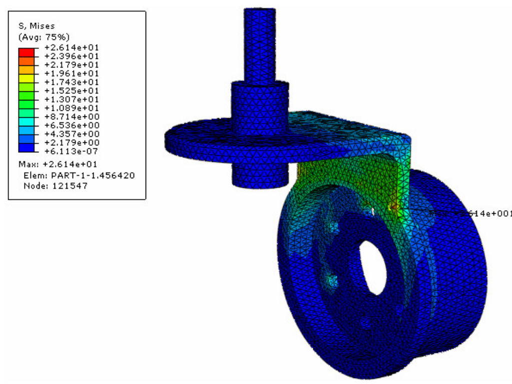


Figure 3.12: Von-Misses Stress Distribution in Rotational Joint

Maximum deformation is 0.21 mm at the upper side of the part as shown in figure 3.13. Here, deformations in the model have not been interested.

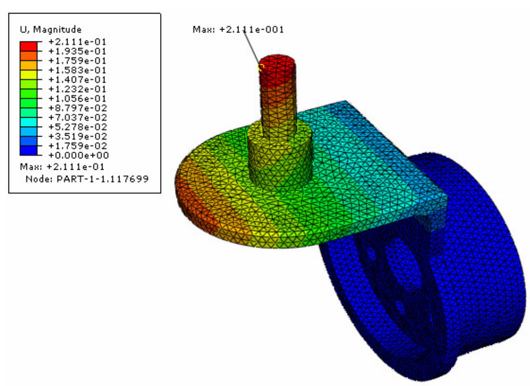


Figure 3.13: Deformations in Rotational Joint

Since materials which have high modulus of elasticity have been used, very high deformations are not expected.

3.5.4 Structural Analysis of the Middle Shaft

Finite element model of the middle shaft is shown in figure 31.4. Model is meshed using first order tetrahedral elements. This part is attached to the gear with bolts from the holes upon the model.

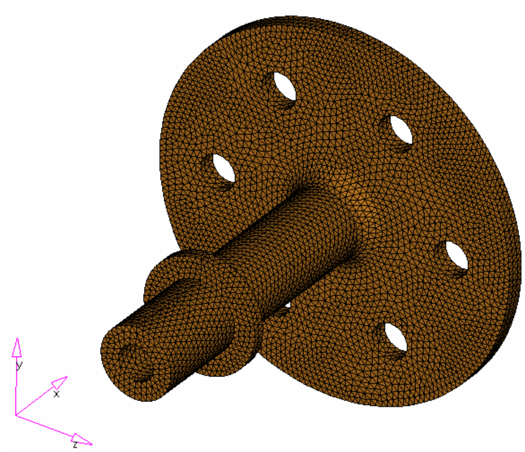


Figure 3.14: Finite Element Model of the Middle Shaft

Boundary conditions have been applied at the holes related to bolts. And 15N force has been distributed in z direction through the nodes attached to the half circle representing the roller bearing. Some assumptions have been made to determine the boundary conditions. This part may be exposed to this kind of forces as shown in figure 3.15 during actual running of the mobile robot.

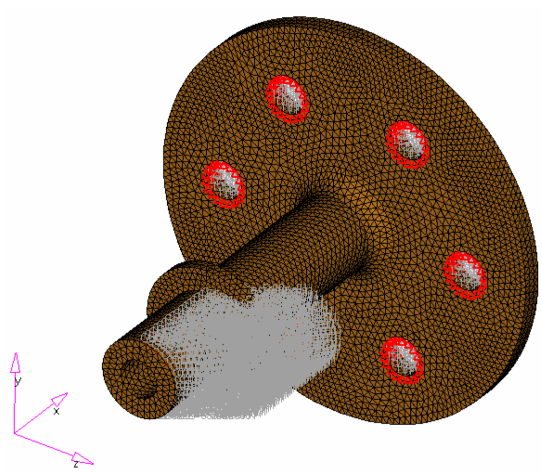


Figure 3.15: Boundary Conditions for Middle Shaft

Maximum von-misses stresses are shown in figure 3.16 as 57.66 MPa which is an acceptable value.

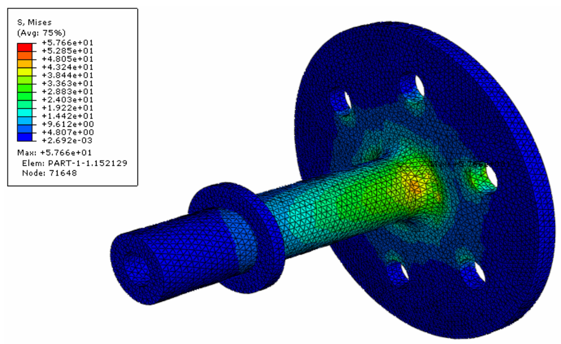


Figure 3.16: Von-Misses Stresses in Middle Shaft

And maximum deformation at the end point of the model has been obtained as 0.14mm.

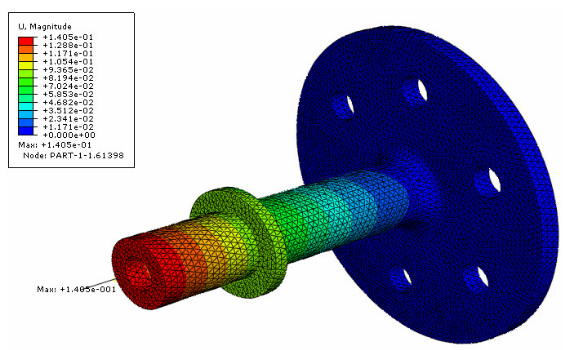


Figure 3.17: Deformations in Middle Shaft

Figure 3.16 shows the deformations in the model of middle shaft. In real conditions this value may not be reached since the cover has not been directly connected to this part. The cover also connected to the motor holder by bolts and this creates a connection between the other parts.

3.5.5 Structural Analysis of the Elbow

Modeling procedure for this part is the same as the parts prepared before. Finite element model has been prepared considering the two parts as a whole as shown in figure 3.17.

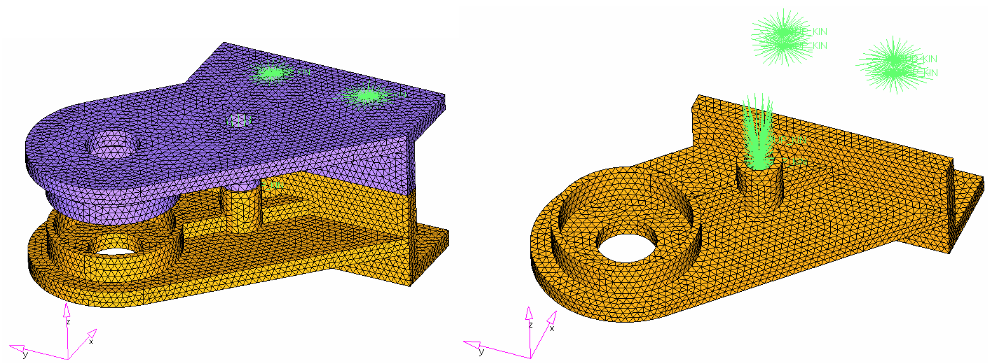


Figure 3.18: Finite Element Model of the Elbow

Here two parts have been attached to each other by a bolt. And the associated nodes of two separate parts have been coupled by rigid elements. And the nodes around the bolt holes joining these two parts to the chassis are fixed as boundary conditions. Again 150N force has been distributed among the nodes inside the roller bearing of the part in positive z direction as shown in figure 3.17.

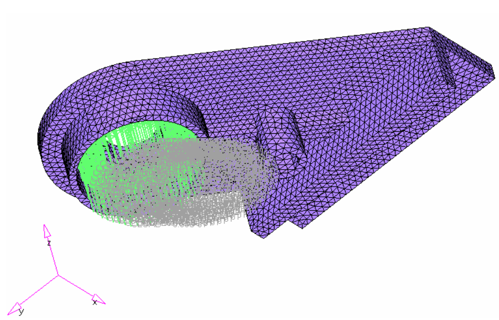


Figure 3.19: Force Direction for the Elbow

Assuming the roller bearings resist the force under real condition, this force has been applied inside the bearings prepared for roller bearings as shown in figure 3.18.

Stress results can be seen in figure 3.19 and maximum stress has been obtained as 26.45MPa at a local position as expected. But here the local stress distribution is the real one since the constraints are applied to enable this kind of distribution.

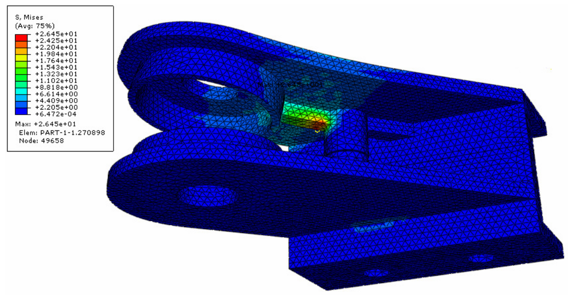


Figure 3.20: Von-Misses Stresses in Elbow

Maximum deformation has been obtained as 0.056 mm and plotted in figure 3.20. These results are acceptable but it is shown that the stress concentrated at a local area. This means that more material has been used in designed and an optimization process may be applied.

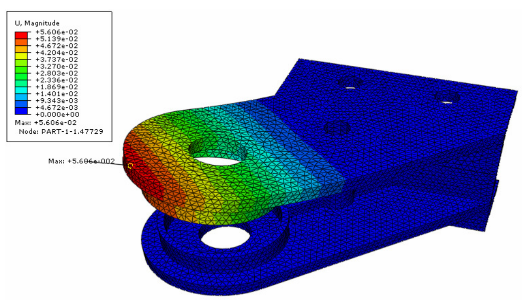


Figure 3.21: Deformations on Elbow

An optimization process will be quite efficient if we determine the exact boundary conditions and load for the model. But in every condition this model is shown to have more material than required. A topology optimization has been employed and will be explained in the next sections.

3.5.6 Structural Analysis of the Chassis

The last static finite element analysis has been applied for modeling of the chassis. The model of the chassis has been prepared using first order shell elements. And boundary conditions have been applied at the end areas where articulations are attached to the chassis. Finite element model of the chassis can be seen in figure 3.21.

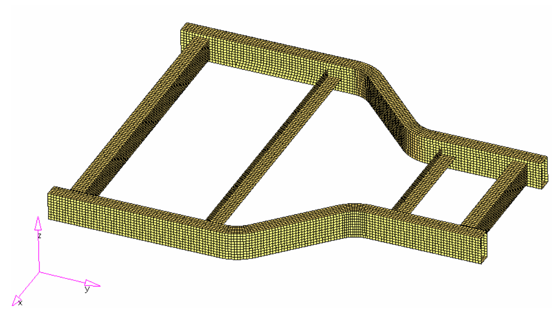


Figure 3.22: Finite Element Model of the Chassis

250N, 50N and 150N forces have been applied on the profiles in order from right to left as shown in yellow in the figure 3.22.

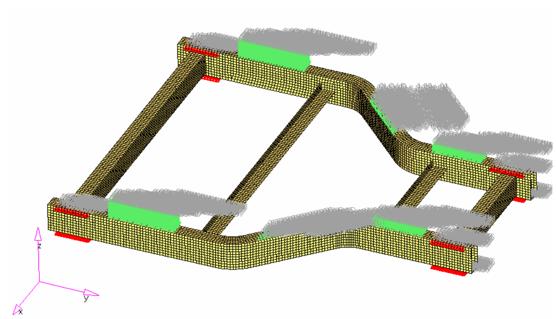


Figure 3.23: Boundary Conditions Applied To the Chassis

The chassis of the mobile robot is made of steel and is thought to be strong enough to the loads that may be applied onto it.

Stress distribution is shown in figure 3.24 and maximum stress has been obtained as 5.21 MPa which is below the yield strength of the material. Again an optimization procedure may be performed for this part. An optimization procedure has not been performed for this part. Because this mobile robot will be the first prototype and many problems are expected to occur.

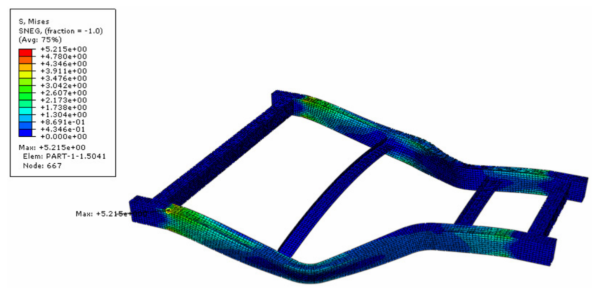


Figure 3.24: Von-Misses Stresses in the Chassis

Maximum deformation is 0.00695 mm in the middle of the chassis. Results are shown to be satisfactory for operation.

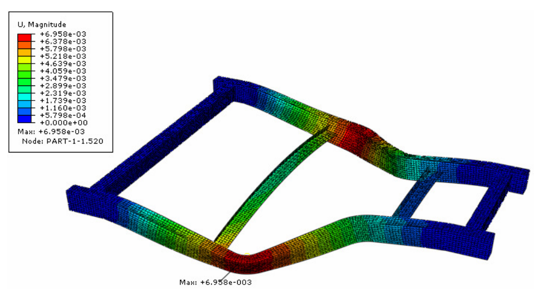


Figure 3.25: Deformation in the Chassis

Deformations can be seen in figure 3.25 and the results are satisfactory. If this part can be considered as a beam element, deformations resulting from these boundary conditions and the applied forces will be the same as shown in figure. If 1N force is applied at the center of the chassis and the stiffness is computed, stresses approximately for any values of forces behind the elastic limit can be calculated.

3.5.7 Design Optimization of the Elbow

It has been observed in the preceding analysis of the elbow that the stress level occurs over this part is quite low excluding some local areas. This means that the volume of the part is much more than required to react to the forces without any overstress. So a design optimization procedure might be a good choice in order to reduce the amount of material used so that the weight of the part decreases to an acceptable value and the use of less material is sufficient than used for the first prototype. So, the model has been prepared as in figure 3.26.

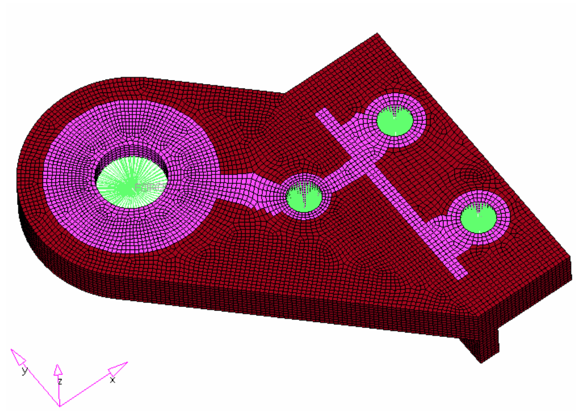


Figure 3.26: Model of Elbow for Optimization

In this model, some of the area is shown as pink and some of it is shown in red. The red area represents the part of the model that will be used for optimization. The pink area is non-design section of the model and this part remains same in volume during the optimization process.

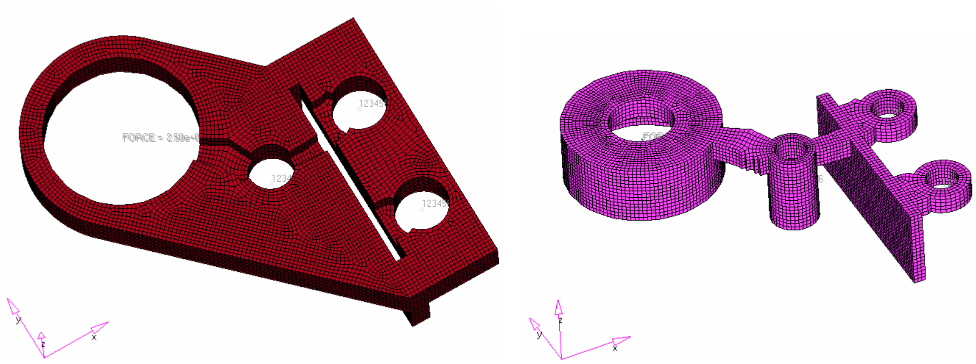


Figure 3.27: Separation of Design and Non-design Areas

The force acting on the part is assumed to be 250N which can occur as an impact. This force has been applied through y and z directions within two separate solution step. And optimization algorithm includes both steps when computing the results. It has been aimed to reduce the volume of the model and this type of optimization procedure is called topology optimization. The criteria for optimization are that the stress value over the model must not exceed 50MPa.

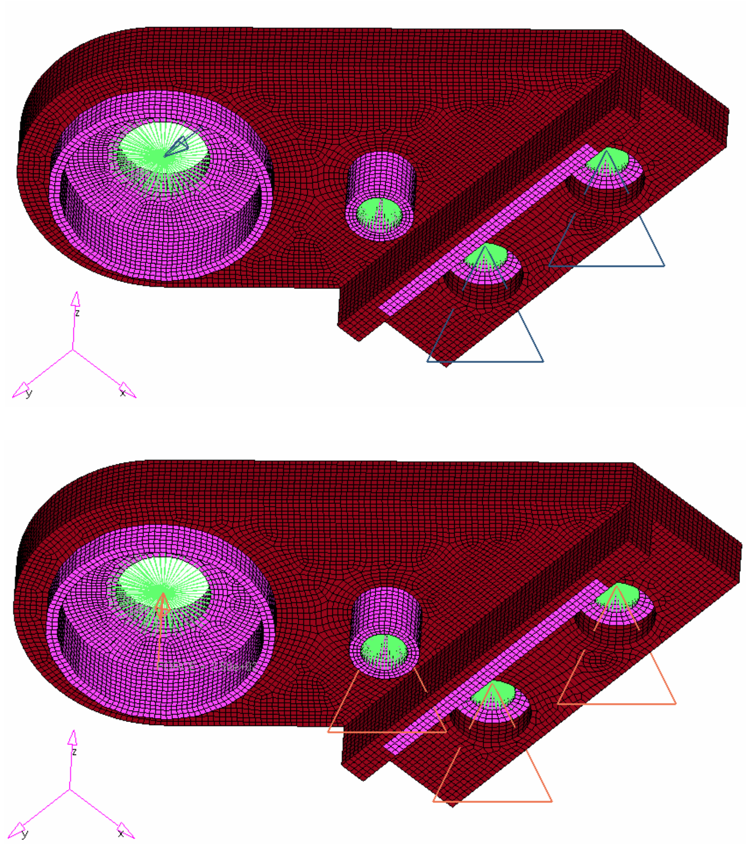


Figure 3.28: Boundary Conditions for Two Analysis Steps

Boundary conditions and forces have been applied as in figure 3.28 for two separate solution step.

The optimum design has been obtained as the following figure.

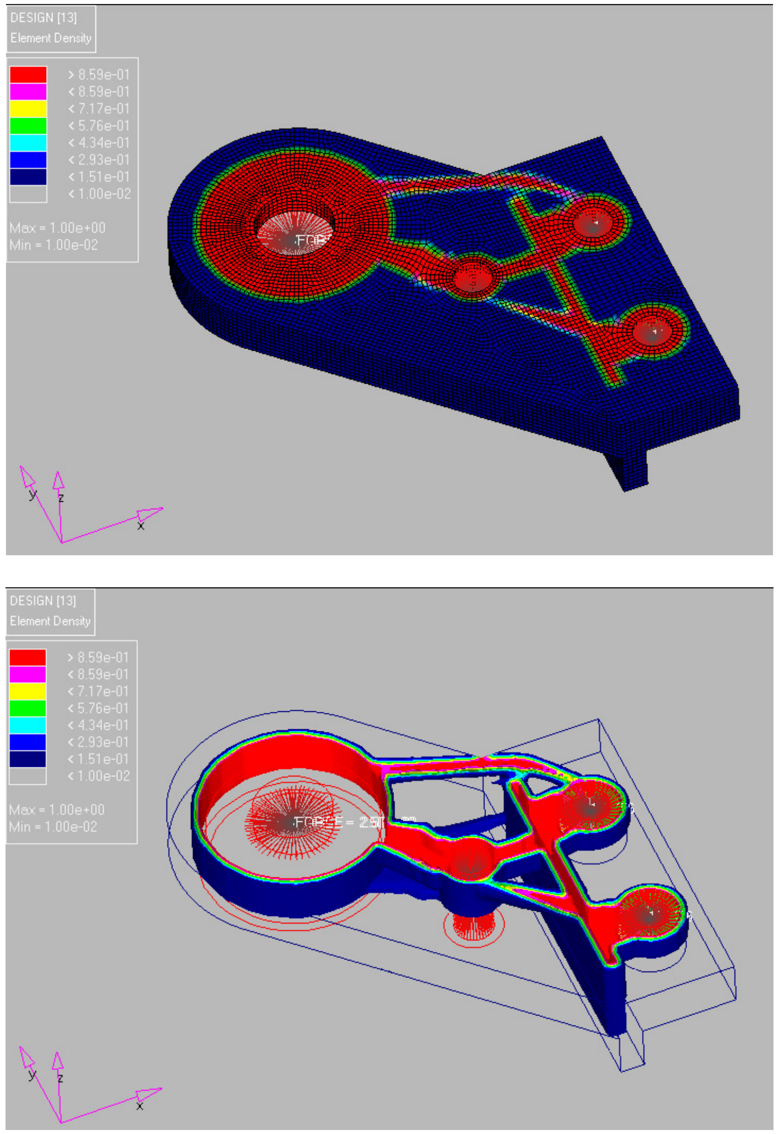


Figure 3.29: Optimum Design of the Elbow

This figure shows the optimum design that has as much material as required to resist the maximum forces acting on the part. Optimization algorithm has computed the figure of this model considering both the two solution steps.

In the figures below, von misses stress plot is shown before and after the optimization procedure. Picture on the left shows the contour for the non-design part and the right one show the results for the optimum part.

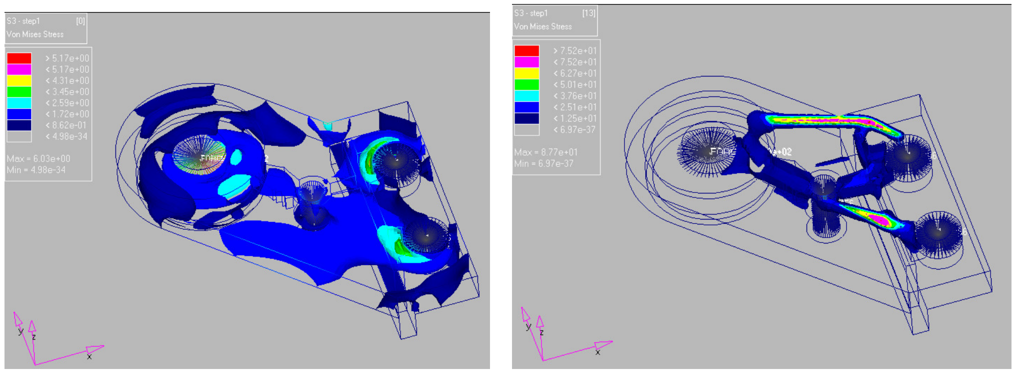


Figure 3.30: Stress Distribution for the First Step

Maximum von-misses stress has been observed as 6MPa for non-designed part for the first step. For optimum designed part it has also been obtained as 87.7MPa which is expected to occur due to the reduction of the material.

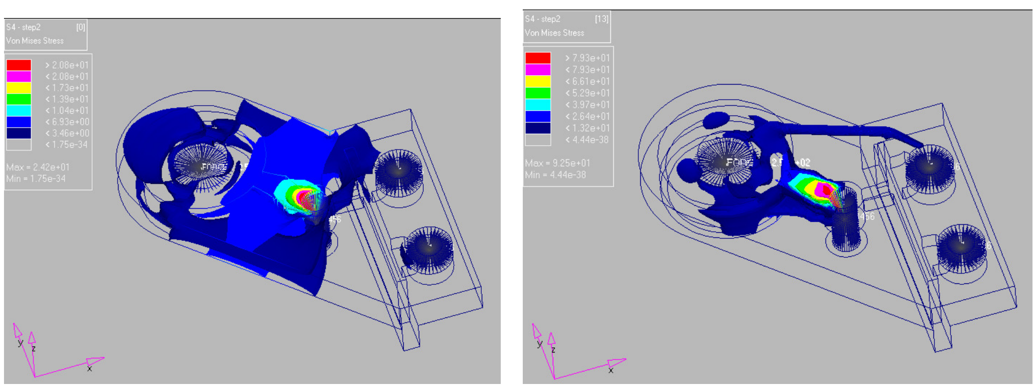


Figure 3.31: Stress Distribution for the Second Step

For the second step, maximum von-misses stress has been observed as 24MPa for non-designed part. For the optimum designed part it has also been obtained as 92.5MPa which is also expected to occur due to the reduction of the material.

A new design has been prepared using Catia V5 considering the optimum volume of the part. Using the same forces and boundary conditions two analysis has been performed that simulate the behavior of the system under these conditions.

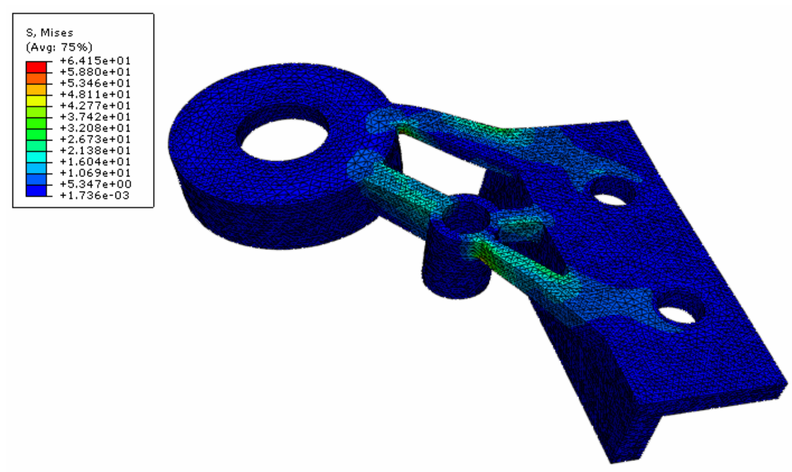


Figure 3.32: Results of New Design for the First Step

Maximum von-misses stress observed over the model in the first step is 64MPa which is approximate to the value obtained in the optimization process. Due to the geometry of this new design, it is not exactly equal to 87.7 MPa.

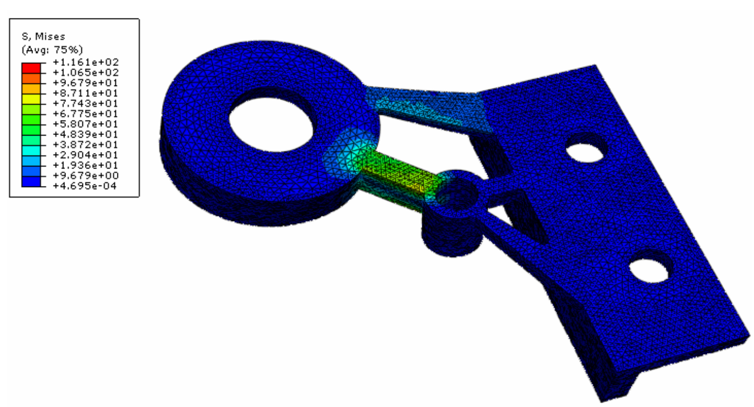


Figure 3.33: Results of New Design for the Second Step

Maximum von-misses stress observed over the model is 116MPa which is approximate to the value obtained in the optimization process. Due to the geometry of this new design, it is not exactly equal to 92.5MPa.

4. KINEMATICS OF THE MOBILE ROBOT

In this section kinematical model of the mobile robot will be prepared and locomotion modes will be explained. Obtaining the kinematical model of the full system is such a complex work. Including all joint velocities in three dimensions makes it very difficult to obtain a correct kinematical model.

4.1 Obtaining Kinematical Model

A simplified version of the kinematical model of the system is going to be prepared. The mobile robot has four legs and tracks. These tracks can be modeled as standard wheels. But the attachment type of these wheels to the chassis is somewhat different from the construction of available robots.

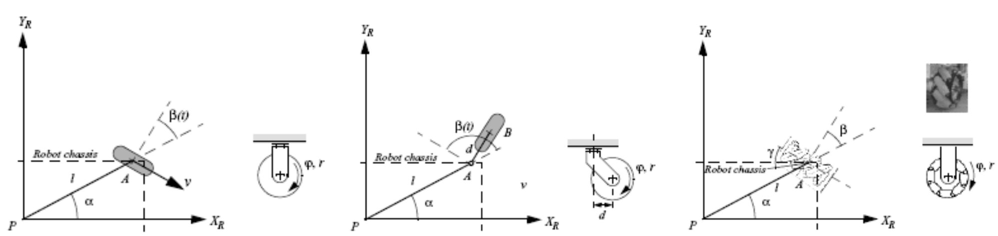


Figure 4.1: Attachment of the Wheels to the Chassis

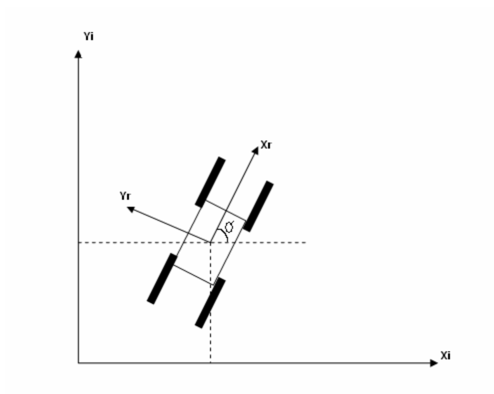
Types of three mainly used wheels are shown in the figure above. These figures represent the position and the motion capability of the wheels with respect to the robot chassis. In this design, different type of wheel attachment has been achieved and the wheel constraints will be different than those shown in figure 4.1.

It is assumed here that the mobile robot moves in the ground formed as plane. Obstacles in the third dimension will not be included and the standard wheeled mobile robot kinematics will be obtained.

Mobile robot position and global and local reference frames can be shown in figure 4.2. Well kwon relationship for two reference frames can be expressed as:

$$\dot{\xi}_R = R(\theta)\dot{\xi}_I \tag{4.1}$$

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4.2}$$



(4.1)

Figure 4.2: Position of the Mobile Robot in Global Reference Frame

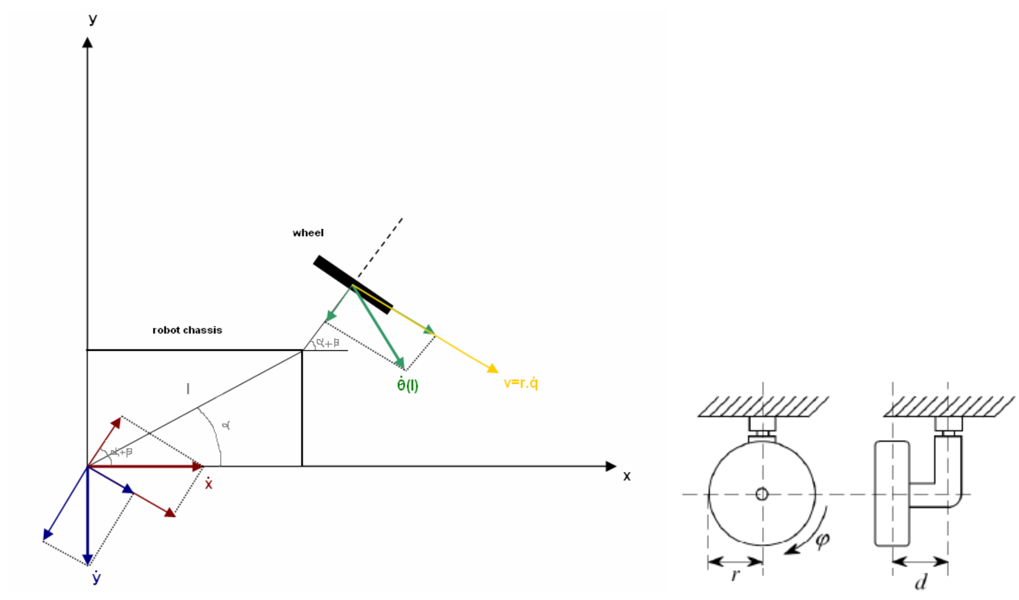


Figure 4.3: Kinematical Constraints of the Wheel

In figure 4.3 the kinematical constraints of the wheel have been arranged and the velocities have been obtained in a constrained form. Robot position can be defined as

$$[\sin(\alpha+\beta) \quad -\cos(\alpha+\beta) \quad d+l\cos\beta]R(\theta)\dot{\xi}_I - r\dot{\varphi} + d\dot{\beta} = 0 \tag{4.3}$$

$$[\cos(\alpha+\beta) \quad \sin(\alpha+\beta) \quad l\sin\beta]R(\theta)\dot{\xi}_I = 0 \tag{4.4}$$

Equations 4.3 and 4.4 represent the rolling and sliding constraints of the wheel. Four sliding constraints can be collected in the following matrix form.

$$C(\beta_1, \quad \beta_2, \quad \beta_3, \quad \beta_4)R(\theta)\dot{\xi}_I = 0$$

4.2 Locomotion Modes of the Mobile Robot

The main aim of this thesis is to obtain a multi-modal mobile robot that can easily move on rough terrain and has multi-modal locomotion capabilities. A wheeled mobile robot can move if each wheel plain axis intersect at the same point. This mobile robot with 12 degrees of freedom has different types of locomotion modes as shown in figure 4.4.

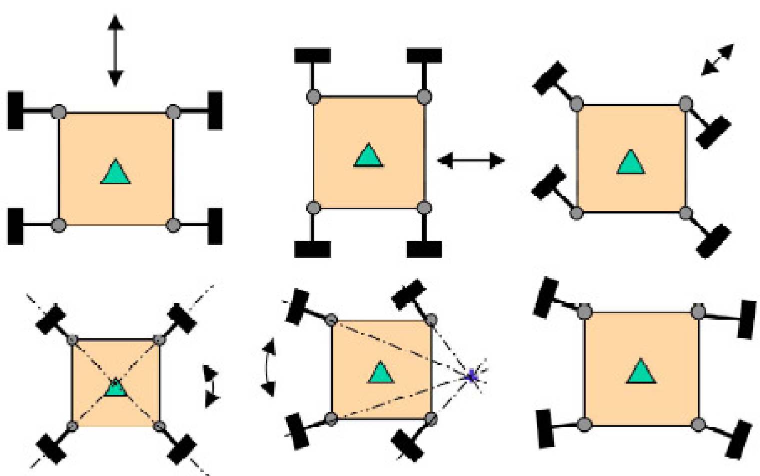


Figure 4.4: Locomotion Modes of Mobile Robot

Using these locomotion modes mobile robot can move in many areas even the motion space is restricted by many obstacles. This capability of our mobile robot brings great advantages in special applications.

For space applications this type of robot can be more suitable than the others designed so far. Stability problems can be regarded since the design of the robot is considered to give the robot a stable frame structure.

Motion of the mobile robot can be managed by different motion algorithms. Differential steering may be one of the control methods. In figure 4.5 some motion capabilities and the navigation of mobile robot can be seen.

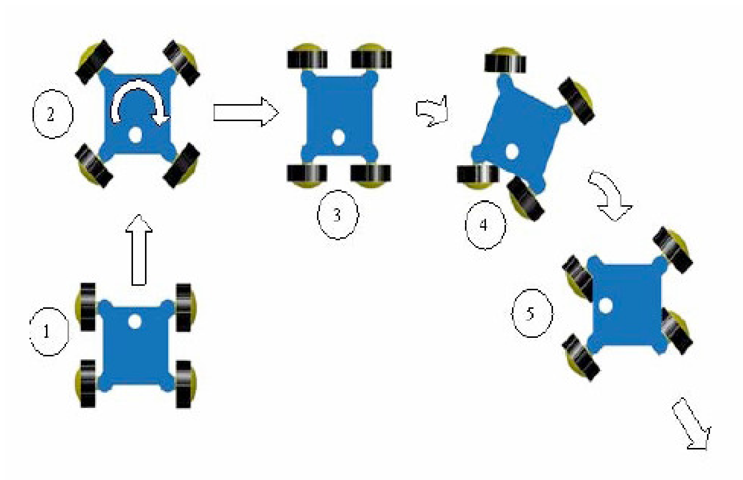


Figure 4.5: Multi-directional Capabilities of the Mobile Robot

5. BRIEF INTRODUCTION TO ANALYTICAL DYNAMICS

Dynamical modelling of a mechanical system is one of the most important tasks an engineer should implement. There are some basic and powerful technics to obtain a dynamic model. Most fundemental and well-known technic is based on the Newton's laws of motion. The equations of motions are expressed in terms of physical coordinates and forces and both quantities are represented by vectors. For this reason, Newtonian mechanics is often refered to as vector mechanics. Newtonian mechanics requires a free body diagram for each of the masses in the system and includes reaction forces and ineracting forces. Application of newtonian mechanics for comlex systems is extremely challenging and almost impossible and useless to prepare a general purpose computer programs.

A different approach referred to as analytical dynamics are based on the total kinetic and potential energy of the system and much more powerful than Newtonian mechanics. Equations of motions in this approach are formulated in terms of two scalar functions and an initesimal expression, the virtual work performed by the nonconservative forces. Generalized coordinates and generealized forces with any special system of coordinates are used to prepare the model of the system. In this approach a powerful technic called Lagrange equations are used to obtain dynamic model of systems. This method is also called Lagrangian dynamics. To obtain a dynamic model of a system, one approach is to represent the dependent coordinates in terms of independent coordinates. This enables to model the system with minimum system of equations that is as the same number as the system degrees of freedom. The second and much general approach is to use generalized coordinates to represent the position and orientation of each partical of the system. In this approach, dynamic model is obtained in terms of generalized forces and coordinates. And system of equations are solved simultaneously with the constraint equations which are nonlinear system of equations. The number of equations is the same as the generalized coordinates used to model the system. This kind of formulation leads to the preparation of a general purpose computer program such as ADAMS.

Since a computer algorithm does not recognize which coordinates are independant or not, system is represented by dynamic equations written in terms of generalized coordinates and the relation between each part is computed with constraint equations.

5.1 Generalized Coordinates

In formulating dynamical system, one of the possibilities is to use the physical coordinates which may not always be independent. As an example, consider a dummbell in figure 5.1 consisting of two masses connected by a massless rigid bar of length L. Assuming a planar motion, we can define the motion by the porsition vector $\mathbf{r}_1$ and $\mathbf{r}_2$. These vectors involve for coordinates x_1, x_2, y_1 and y_2.

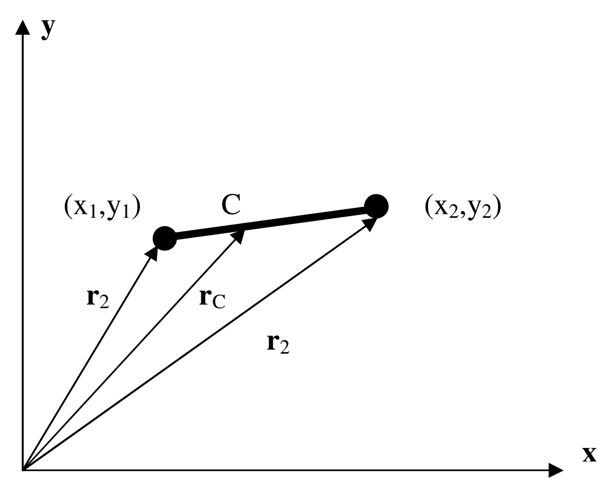


Figure 5.1: Coordinate of a Particle

But these four coordinates are related by the equation

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 = L^2 \tag{5.1}$$

which represents a constraint equation. Since each term in this equation is represented in terms of the remaining three, system degrees of freedom is equal to three which means only three coordinates are independent. If this four coordinates are chosen, the problem is formulated in terms of these coordinates and the constrained equation. A better choise of coordinates eliminates this constraint equation.

In planar motion only three coordinates, which are x_C, y_C and θ, are necessary to define the position of the particle. Here x_C and y_C are the componets of $\mathbf{r}_C$ and θ represents the orientaiton of the particle with respect to global coordinate system.

These random chosen coordinates are called *generalized coordinates.* Any set of generalized coordinates can be used to formulate the equations of motion. In many multibody computer programs generalized coordinates are used for the sake of generality. Formulation of the equations of motion is prepared for each part using related generalized coordinates and constraint equations between the components are solved simultaneously to include the kinematic relations between each part.

5.2 Priciple of Virtual Work

Principles of virtual work is a tool for transition form Newtonian mechanics to Lagrangian mechanics. It is based on variational calculus and the first variation. Definition of virtual displacements and generalized forces is important in the application of the principle of virtual work.

5.2.1 Virtual Displacements

Virtual displacement is defined to be an infinitesimal change of the position of a point on the body. Position vector of an arbitrary point is given by the equation :

$$\mathbf{r}_P^i = \mathbf{R}^i + \mathbf{A}^i \bar{\mathbf{u}}_P^i \qquad \textbf{(5.2)}$$

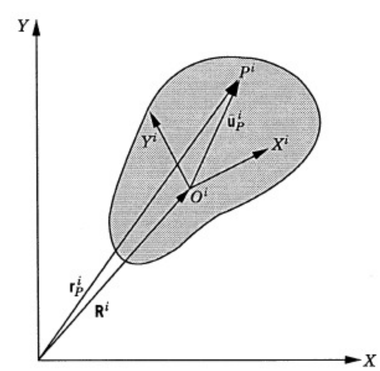


Figure 5.2: Position Representation of a Mass

$\mathbf{R}^{i}$ is the position vector of the reference point, $\bar{\mathbf{u}}_{P}^{i}$ is the position vector of point P^{i} with respect to the reference point O^{i} and $\mathbf{A}^{i}$ is the transformation matrix given by

$$\mathbf{A}^{i} = \begin{bmatrix} \cos\theta^{i} & -\sin\theta^{i} \\ \sin\theta^{i} & \cos\theta^{i} \end{bmatrix} \quad \textbf{(5.3)}$$

In equation 5.3, θ is the orientation of the body. Virtual change in the position vector of point P^{i} is ddenoted as $\delta\mathbf{r}_{P}^{i}$ and is given by the equation

$$\delta\mathbf{r}_{P}^{i} = \delta\mathbf{R}^{i} + \delta(\mathbf{A}^{i}\bar{\mathbf{u}}_{P}^{i}) \quad \textbf{(5.4)}$$

and this equation can be written as

$$\delta\mathbf{r}_{P}^{i} = \delta\mathbf{R}^{i} + \mathbf{A}_{\theta}^{i}\bar{\mathbf{u}}_{P}^{i}\,\delta\theta^{i} \quad \textbf{(5.5)}$$

where

$$\mathbf{A}_{\theta}^{i} = \frac{\partial\mathbf{A}^{i}}{\partial\theta^{i}} = \begin{bmatrix} -\sin\theta^{i} & -\cos\theta^{i} \\ \cos\theta^{i} & -\sin\theta^{i} \end{bmatrix} \quad \textbf{(5.6)}$$

5.2.2 Virtual Work and Generalized Forces

Virtual work of a force vector is defined to be the dot product of the force vector and the vector of the virtual change of the position vector of the point of application of the force. Both vectors must be defined in the same coordinate system. Virtual work of a moment is also defined to be the product of the moment and the angular orientation of the body.

Figure shows the moment M^{i} and the force $\boldsymbol{F}^{i}$ acting upon the body. And the point of application is denoted as P^{i}. Virtual work of these forces is given by

$$\delta W^{i} = \boldsymbol{F}^{i^{T}} \delta \boldsymbol{r}_{P}^{i} + M^{i} \delta\theta \quad \textbf{(5.7)}$$

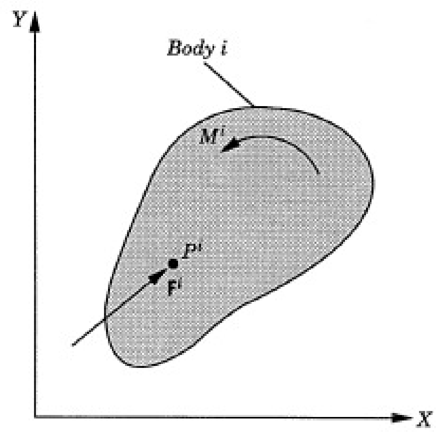


Figure 5.3: Forces Acting on a Particle

The position vector of an arbitrary point on a rigid body can be expressed in terms of the position vector of the reference point and the angular orientation of the body. This position vector is given as

$$\mathbf{r}_{P}^{i} = \mathbf{R}^{i} + \mathbf{A}^{i} \bar{\mathbf{u}}_{P}^{i} \quad \textbf{(5.8)}$$

Using the virtual displacements of the position vector and the forces acting on this point of application, virtual work of these forces can be written as

$$\delta W^{i} = \boldsymbol{F}^{i^{T}} (\delta \mathbf{R}^{i} + \mathbf{A}_{\theta}^{i} \bar{\mathbf{u}}_{P}^{i} \delta\theta^{i}) + M^{i} \delta\theta \quad \textbf{(5.9)}$$

$$\delta W^{i} = \boldsymbol{F}^{i^{T}} \delta \mathbf{R}^{i} + (\boldsymbol{F}^{i^{T}} \mathbf{A}_{\theta}^{i} \bar{\mathbf{u}}_{P}^{i} + M^{i}) \delta\theta^{i} \quad \textbf{(5.10)}$$

In this equation, $\boldsymbol{F}^{i}$ is the *generalized force* associated with the coordinates of reference point and $(\boldsymbol{F}^{i^{T}} \mathbf{A}_{\theta}^{i} \bar{\mathbf{u}}_{P}^{i} + M^{i})$ is the *generalized force* associated with the rotation of the body.

And the statement of the principle of virtual work is that the work performed by the applied forces through infinitesimal virtual displacement is equal to zero if the system is in static equilibrium.

5.3 d'Alembert Principle

The priciple of virtual work is concerned with the static equilibrium of the systems. However, the virtual work priciple can be extended to dynamics in which form it is known as d'Alembert's priciple. This principle includes the work performed by the inertia forces of the parts. And for a rigid body, equations of motions are given as

$$\mathbf{F}^{i} - m^{i}\mathbf{a}^{i} = 0 \tag{5.11}$$

$$M^{i} - J^{i}\ddot{\theta}^{i} = 0 \tag{5.12}$$

So the virtual work performed by these forces is given as

$$(\mathbf{F}^{i} - m^{i}\mathbf{a}^{i})^{T}\delta\boldsymbol{R} + (M^{i} - J^{i}\ddot{\theta}^{i})\delta\theta = 0 \tag{5.13}$$

This principle can be used to obtain the equations of motions of multibody systems. In order to obtain the equation, dependent coordinates are represented by the independent coordinates and the work performed by each force are computed. Knowing that $\delta\boldsymbol{R}$ and $\delta\theta$ is equal to zero, coefficients of this virtual statements are also equal to zero and the equations obtained from this calculation gives the equation of motions of the system.

5.4 Lagrange's Equations

To obtain the dynamic equations of motion of a system, a powerful and easy to implement technic is derived by Lagrange. The prrincipal of virtual work allows us to formulate the dynamic equations using any set of generalized coordinates.

According to Lagrange's formulation, generalized inertia forces of n_b rigid bodies can be given as

$$\mathbf{Q}_i = \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\mathbf{q}}_i}\right)^T - \left(\frac{\partial T}{\partial \mathbf{q}_i}\right)^T \quad \textbf{(5.14)}$$

T is the system total kinetic energy, obtained using independent generalized coordinates. **q** is the vector generalized coordinates associated with body i.

Using the principle of virtual work and d'Alembert principle, system equations of motion can be given as

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \left(\frac{\partial T}{\partial q_j}\right) = Q_j \qquad j = 1,2,3,\ldots,n \quad \textbf{(5.15)}$$

Here $q_j, j = 1,2,\ldots,n$ are the independent coordinates of the system degrees of freedom and Q_j is the generalized applied force associated with the independent coordinate q_j. This equation is called *Lagrange's equations of motion*.

5.5 Hamiltonian Formulation

The forces acting on a mechanical system can be classified as conservative and nonconservative forces. Vector of generalized forces acting on a multibody system can be written as

$$\mathbf{Q}_e = \mathbf{Q}_{nc} + \mathbf{Q}_{co} \quad \textbf{(5.16)}$$

$\mathbf{Q}_{co}$ and $\mathbf{Q}_{nc}$ are the vector of conservative and nonconservative forces. And conservative forces can be derived from a potential function V as

$$\mathbf{Q}_{co} = -\left(\frac{\partial V}{\partial \mathbf{q}}\right) \quad \textbf{(5.17)}$$

$\mathbf{q} = [q_1 .... q_n]^T$ is the vector of generalized coordinates. This equation gives

$$\mathbf{Q}_e = -\left(\frac{\partial V}{\partial \mathbf{q}}\right) + \mathbf{Q}_{nc} \tag{5.18}$$

and

$$\mathbf{Q}_e = \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\mathbf{q}}}\right)^T - \left(\frac{\partial T}{\partial \mathbf{q}}\right)^T \tag{5.19}$$

Knowing that the potential function V does not depend on the generalized velocities, system equations of motion can be written as

$$\frac{d}{dt}\left(\frac{\partial (T - V)}{\partial \dot{\mathbf{q}}}\right)^T - \left(\frac{\partial (T - V)}{\partial \mathbf{q}}\right)^T = \mathbf{Q}_{nc} \tag{5.20}$$

Here L=T-V is called the Lagrangian.

5.6 Lagrange Multipliers

In a multibody system, if the dependent coordinates are defined in terms of independent ones and write the system of equations in terms of independent coordinates, joint reaction forces are automatically eliminated from the equation. A second approach is to include the work done by reaction forces in the equations of motion. So, the generalized forces induced by reaction forces of the system have to be defined. This approach is useful and widely used in computational dynamics since the constraint forces are defined by a set of lagrange multipliers and this enables us to write a general purpose computer algorithm regardless of the structure of the system.

Consider a planar system that consists of two rigid bodies. These two bodies are connected at point P by a revolute joint.

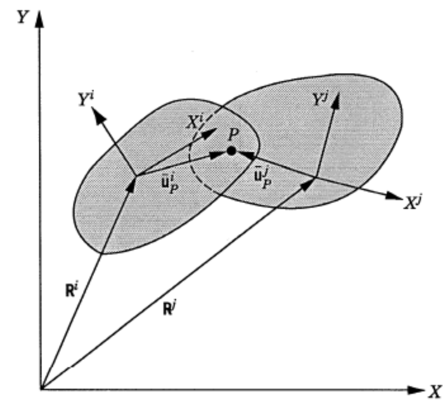


Figure 5.4: Constraint Between Two Bodies

Constraint equations for a planar revolute joint is given as

$$\mathbf{R}_i + \mathbf{A}_i \bar{\mathbf{u}}_P^i - \mathbf{R}_j + \mathbf{A}_j \bar{\mathbf{u}}_P^j = 0 \qquad \textbf{(5.21)}$$

$$\theta_i - \theta_j = 0 \qquad \textbf{(5.22)}$$

Here $\mathbf{R}_i$ and $\mathbf{R}_j$ are the global position vectors of the origins of the coordinate systems of bodies i and j. $\mathbf{A}_i$ and $\mathbf{A}_j$ are the transformation matrices of the coordinate systems of body i and body j to the global coordinate system. $\bar{\mathbf{u}}_P^i$ and $\bar{\mathbf{u}}_P^j$ are the local position vectors of point P with respect to the reference points of body i and j. θ_i and θ_j are the angular orientations of body i and body j. Constraint equations can be wrritten in a matrix form as

$$C = \begin{bmatrix} \mathbf{R}_i + \mathbf{A}_i \bar{\mathbf{u}}_P^i - \mathbf{R}_j + \mathbf{A}_j \bar{\mathbf{u}}_P^j \\ \theta_i - \theta_j \end{bmatrix} = 0 \qquad \textbf{(5.23)}$$

Constraint jacobian matrix of these constraint equations is given in a partitioned form as

$$\mathbf{C}_{\mathbf{q}^{i}} = \begin{bmatrix} \mathrm{I} & \mathbf{A}_{\theta}^{i}\bar{\mathbf{u}}_{P}^{i} \\ 0 & 1 \end{bmatrix} \tag{5.24}$$

$$\mathbf{C}_{\mathbf{q}^{j}} = \begin{bmatrix} \mathrm{I} & \mathbf{A}_{\theta}^{j}\bar{\mathbf{u}}_{P}^{j} \\ 0 & 1 \end{bmatrix} \tag{5.25}$$

Here $\mathbf{A}_{\theta}^{i}$ and $\mathbf{A}_{\theta}^{j}$ are the partial derivatives of the transformation matrices $\mathbf{A}^{i}$ and $\mathbf{A}^{j}$ with respect to θ_{i} and θ_{j}.

5.6.1 Equipollent System of Forces

Forces acting on a random point of a body can be replaced by the forces acting on the coordinate systems of bodies. This reperesentation does not change the dynamics of the system and the work done by two different type of forces is the same. This system of forces is called equipollent system of forces. This representation can be considered as the generalization of the forces acting on the body.
If we let $\boldsymbol{\lambda}$ be the vector

$$\boldsymbol{\lambda} = \begin{bmatrix} \mathbf{F} \\ \mathrm{M} \end{bmatrix} \tag{5.26}$$

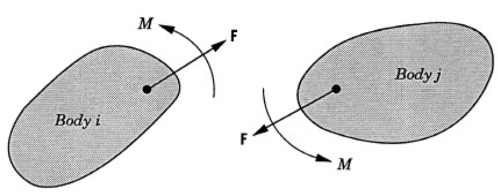


Figure 5.5: Constraint Forces

The reaction forces acting on body i and j which are equal in magnitude and opposite in direction can be expressed in a vector form as

$$\mathbf{F}^i = -\boldsymbol{\lambda} = \begin{bmatrix} \mathbf{F} \\ \mathrm{M} \end{bmatrix} \quad \text{and} \quad \mathbf{F}^j = \boldsymbol{\lambda} = -\begin{bmatrix} \mathbf{F} \\ \mathrm{M} \end{bmatrix} \tag{5.27}$$

Equipollent system of forces defined at the origins of the coordinate systems of two bodies is given as

$$\mathbf{Q}_c^i = \begin{bmatrix} \mathbf{F} \\ \mathrm{M} + \bar{\mathbf{u}}_P^{i\,T} \mathbf{A}_\theta^{i\,T} \mathbf{F} \end{bmatrix} \tag{5.28}$$

$$\mathbf{Q}_c^j = \begin{bmatrix} \mathbf{F} \\ \mathrm{M} + \bar{\mathbf{u}}_P^{j\,T} \mathbf{A}_\theta^{j\,T} \mathbf{F} \end{bmatrix} \tag{5.29}$$

This equation is the same as generalized forces explained in the preceeding sections. In this case, these equations are called generalized reaction forces.

5.6.2 Definition of Lagrange Multipliers

Generalized reaction forces can be written as

$$\mathbf{Q}_c^i = \begin{bmatrix} \mathbf{I} & 0 \\ \bar{\mathbf{u}}_P^{i\,T} \mathbf{A}_\theta^{i\,T} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{F} \\ \mathrm{M} \end{bmatrix} \tag{5.30}$$

$$\mathbf{Q}_c^j = \begin{bmatrix} \mathbf{I} & 0 \\ \bar{\mathbf{u}}_P^{j\,T} \mathbf{A}_\theta^{j\,T} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{F} \\ \mathrm{M} \end{bmatrix} \tag{5.31}$$

If we compare the square matrices in these equations, we obtain the equations below

$$\mathbf{Q}_c^i = -\mathbf{C}_{\mathbf{q}^i}^T \boldsymbol{\lambda} \tag{5.32}$$

$$\mathbf{Q}_c^j = -\mathbf{C}_{\mathbf{q}^j}^T \boldsymbol{\lambda} \tag{5.33}$$

Here $\boldsymbol{\lambda}$ is called the Lagrange multiplier.

5.7 Generalized System of Equations

From the equations and definitions below, a generalized equation that can be used to formulate the dynamic equations of motion of any mechanical system can be obtained. This equation can be written as

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{q}}}\right)^{T} - \left(\frac{\partial L}{\partial \mathbf{q}}\right)^{T} + \boldsymbol{\lambda}\frac{\partial \mathbf{Q}}{\partial \mathbf{q}} - \mathbf{Q}_{nc} = 0 \qquad \textbf{(5.34)}$$

Another representation of this equation can be written as

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \left(\frac{\partial L}{\partial q_i}\right) + \sum_{j=1}^{m}\lambda_j \frac{\partial Q_j}{\partial q_i} - \sum_{k=1}^{na} F_k \frac{\partial r_k}{\partial q_i} = 0 \qquad i = 1,2,\ldots\ldots n \qquad \textbf{(5.35)}$$

$L = T - V =$ Lagrangian

$Q =$ Constraint equations

$r =$ Application point of force

$q =$ Generalized coordinates

$F =$ Externally applied force

$\lambda =$ Lagrange multipliers

Using this equation, a set of second order differential equations is obtained. Solving these differential equations with the nonlinear algebric constraint equations, dynamic behaviour of a mechanical system is simualted.

Constraint equations for different types of joint constraints are available in literature, and one can use these equations and extend it to the tree dimensional case in order to obtain the dynamic equations of motion of a system in space.

6. DYNAMICAL MODELING OF THE MOBILE ROBOT

It is one of the most important tasks to simulate the dynamic behavior of a mechanism designed. Dynamical simulation enables engineers to be able to see how a mechanical system reacts under the forces and moments acting on the mechanism. Another thing that dynamical simulation offers is to compute the reaction forces at each joint which may use as the next step to compute the mechanical behavior of the system such as stress calculation, deformation on each component. So a design optimization procedure may be employed to have an optimized component.

And some of the mechanical parts such as connecting rods and pistons run in operation conditions where very high speeds exist. In such machines as internal combustion engines it becomes very important to consider the forces induced by inertia affects of components. Actually in internal combustions engines which run according to otto cycle forces acting on the parts of the engine reach to their maximum value at the maximum operating speed. Thus, the stress distribution and fatigue life of the components are calculated according to maximum forces which occur at maximum speed.

Dynamical simulation combined with finite element analysis enables computation of the behavior of components simultaneously considering the elastic behavior of the system.

6.1 Capabilities and Structure of ADAMS

Some software packages are used extensively for dynamical simulation of mechanisms. ADAMS is one of the most powerful general purpose software used for this purpose. RecurDyn is another one which uses recursive dynamic algorithms to simulate the system behavior. And special purpose programs used for example for crankshaft dynamics, engine valve-train are available. The ADAMS has been used to simulate the behavior of the mobile robot.

ADAMS has sub-modules for special machines and we can sort these as:

- Adams/View
- Adams/Car
- Adams/Aircraft
- Adams/Rail
- Adams/Driveline
- Adams/Engine
- Adams/Flex

Some others are available and all the capability of ADAMS is not mentioned here. In these sub-modules, Adams/View has been used in the analysis since it is a general purpose code written by Adams. Others can be used according to their special application areas.

An option of Adams is the interaction with the finite element software such as Ansys or Abaqus. As mentioned above flexible multi-body dynamic simulation can be performed using the two programs simultaneously. Although Adams/Flex enables to include the elastic behavior of some simple parts it is inevitable to use a finite element code for components which have complex geometrical properties.

6.2 Adams Model of the Mobile Robot

Adams has been used for two purposes one of which is to obtain the dynamic model to compute the torque produced at the joints. This helps us choose DC motors with correct power that compensate this torque requirement. If a dynamic simulation had not been used, necessary power to supply motion to the driven joints could not exactly be determined. And the mobile robot could not have made the articulation turn and lift the robot onto its legs.

A model at the beginning of this analysis process has been prepared in order to be able to choose right motor powers.

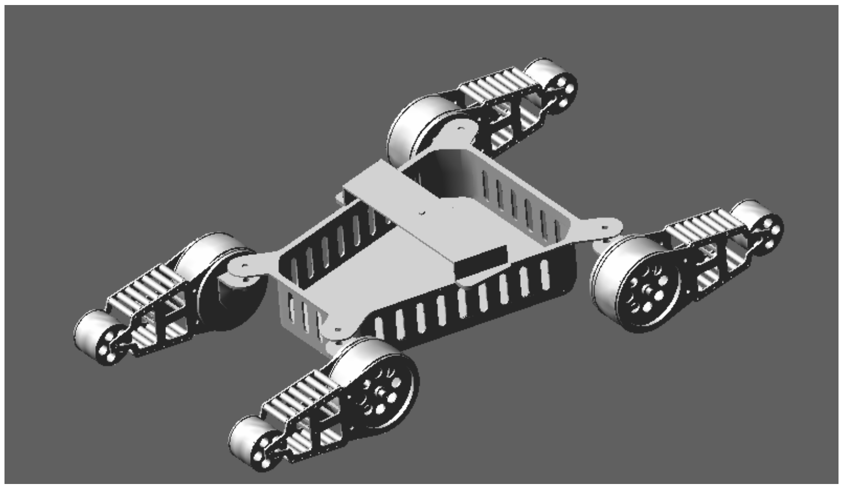


Figure 6.1: Adams Model of First Design

In this model, joint forces (torques) have been measured to determine the power of the motors that supply this torque. Joint motions have been applied in order for the articulations approach to 90 degrees in two seconds.

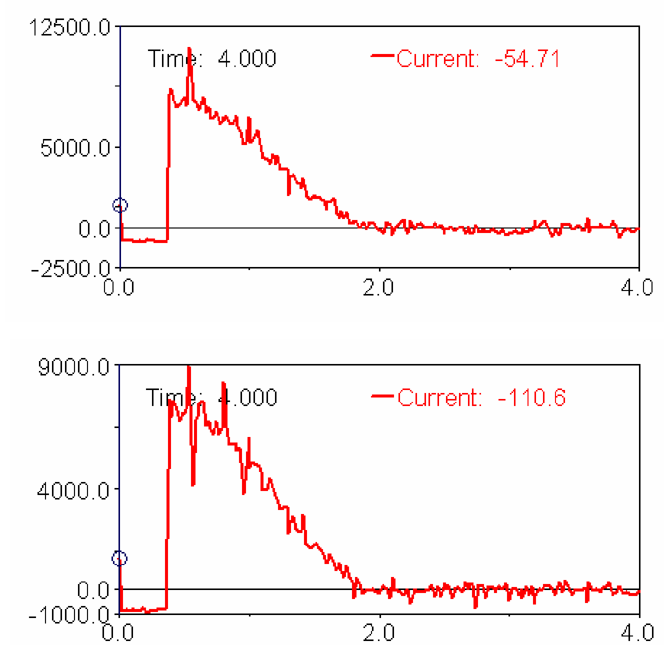


Figure 6.2: Torque Output for Lifting

In this model torque output has been obtained as 12.5 and 9 Nm in the joints that used to lift the robot up. So, the motor powers have been chosen to compensate this power requirement.

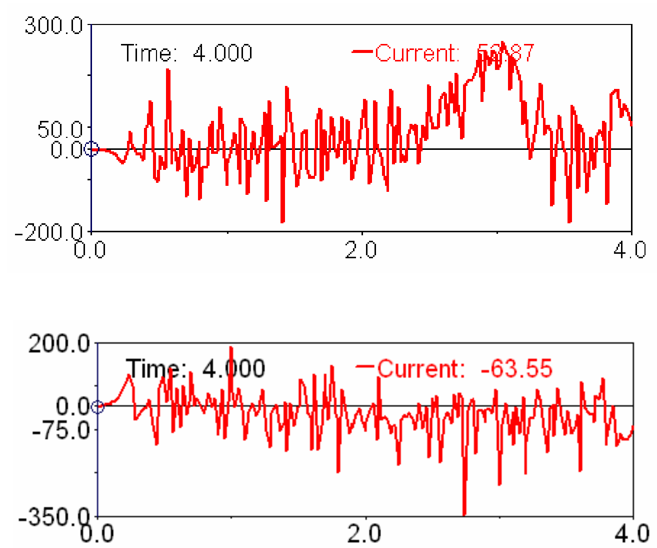


Figure 6.3: Torque Output for Vertical Axis

Torque level in order to hold the articulations parallel to each other is shown in figure 6.3. 1Nm has been assumed to occur at these joints.

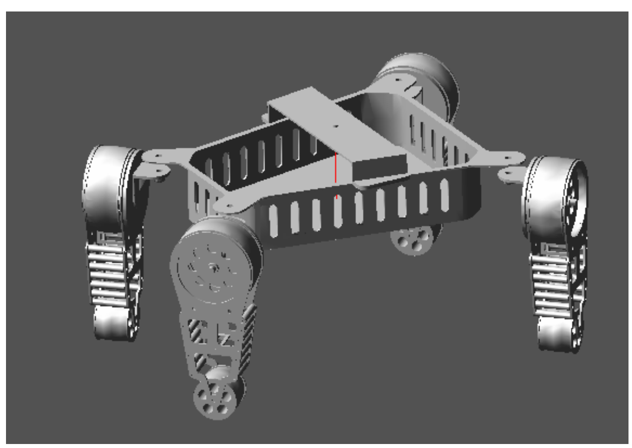


Figure 6.4: Simulated Model

Figure 6.4 shows the position of the robot after simulation. Any kind of motion is available in Adams to move the parts giving the desired motion.

Full body simulation has been performed after completing the design of the mobile robot. Adams model of the last version of mobile robot is shown in figure 6.5.

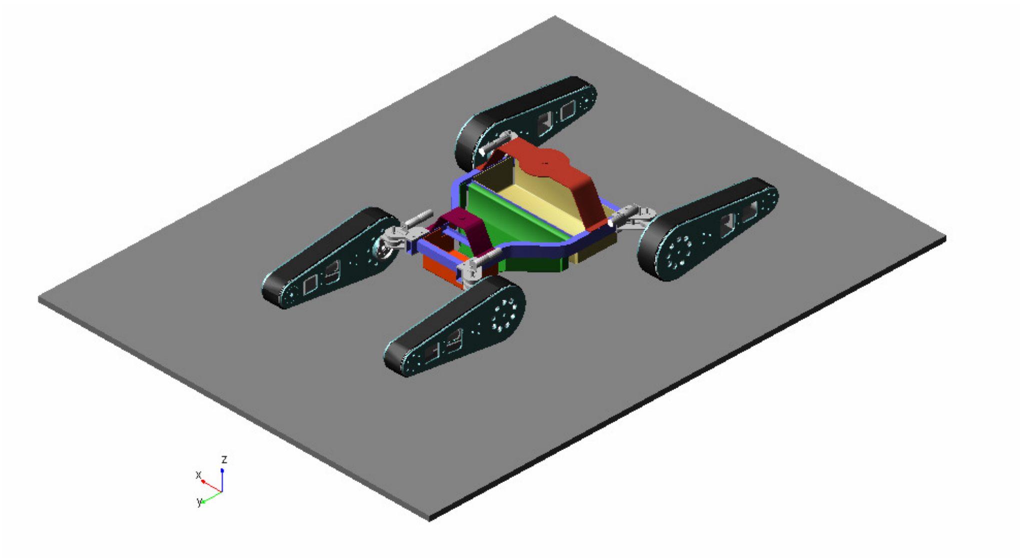


Figure 6.5: Adams Model of the Mobile Robot

The model has been directly prepared from CAD data obtained form Catia. But some intermediate steps required importing the CAD data to Adams/View interface. Catia V5 supports *.iges file format and this format has been changed into parasolid file using SolidWorks. CAD data obtained in *.igs format has been imported to the SolidWorks and then parasolid file has been exported from here. This enabled to have each part separately in Adams/View. If the *.iges file is directly imported form Catia V5 into Adams, every part is considered as a whole and this will not help us set up the dynamic model of the mobile robot.

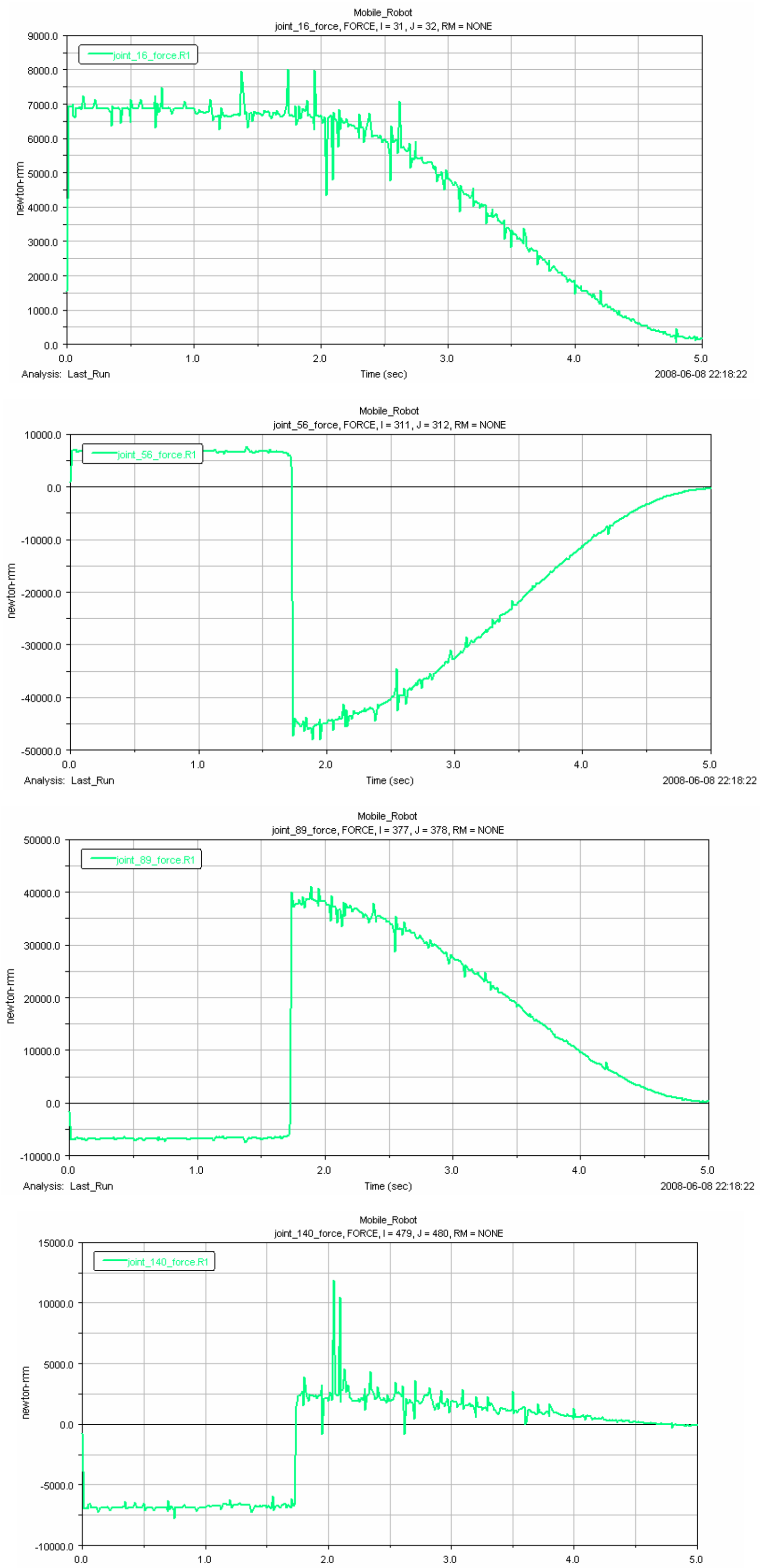


Figure 6.6: Torque Produced at the Lifting Joints

Figure 6.6 shows the torque level in order to be able to lift the legs up with a specified velocity. In this analysis no motion has been applied at the first one second in order for the model to reach to the static equilibrium conditions. At the end of one second, lifting joints have been applied a motion so that the robot's legs reach to vertical position in four seconds. Although second and third pictures in figure 6.6 are symmetric, the first and last pictures are not related to each other. This is due to the fact that two arms do not touch to the ground exactly. Thus in the second and third pictures, torque is observed as 45Nm. It may be concluded from this result that this value should be divided to two in order to obtain the required torque in each lifting joint. So, the maximum torque has been assumed as 25Nm that the motors selected satisfy this requirement.

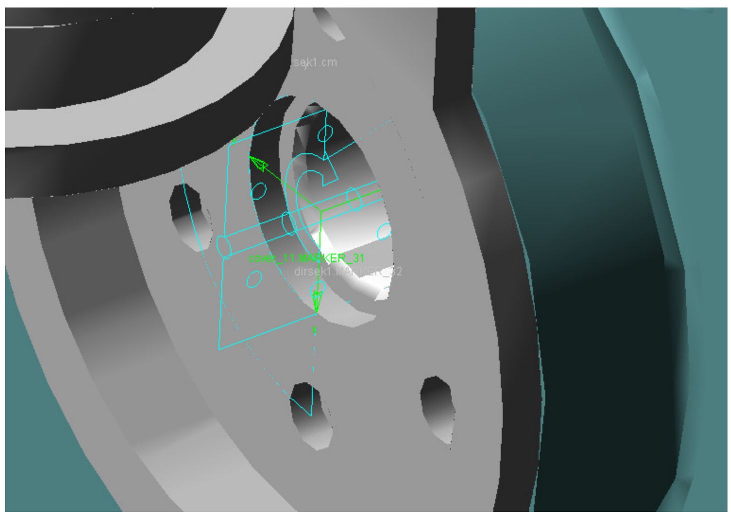


Figure 6.7: Lifting Joint One of Four

Figure 6.7 shows the position and location of the joint where measurements have been taken.

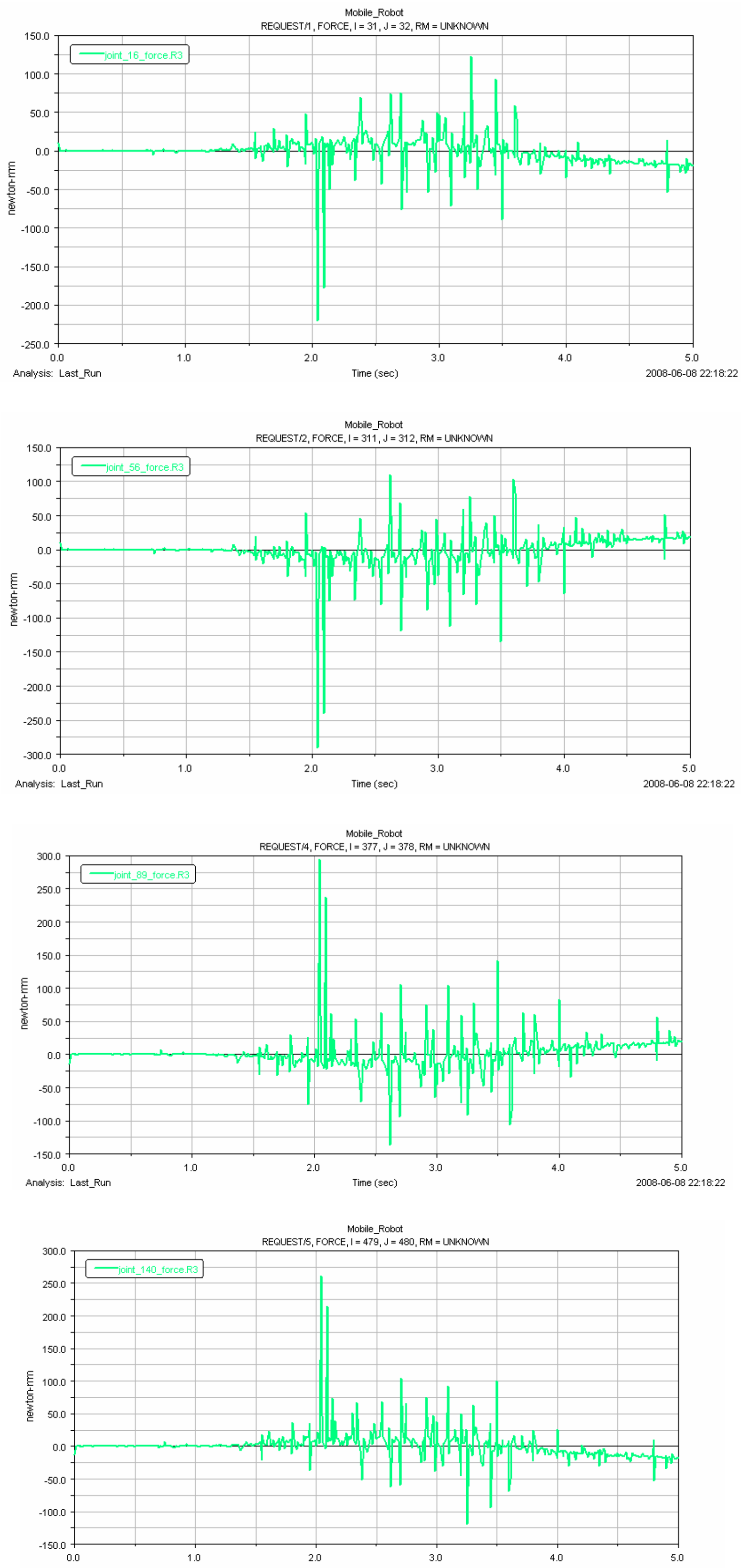


Figure 6.8: Torque Level Obtained at the Vertical Axis Joints

Figure 6.8 shows torque level in order to hold the articulations parallel to each other. Maximum results are shown as 0.3 Nm and these results are approximate to the results obtained from the first design analysis.

It can be seen in figure 6.9 where vertical axis joints stand and this figure explains what this measurement means.

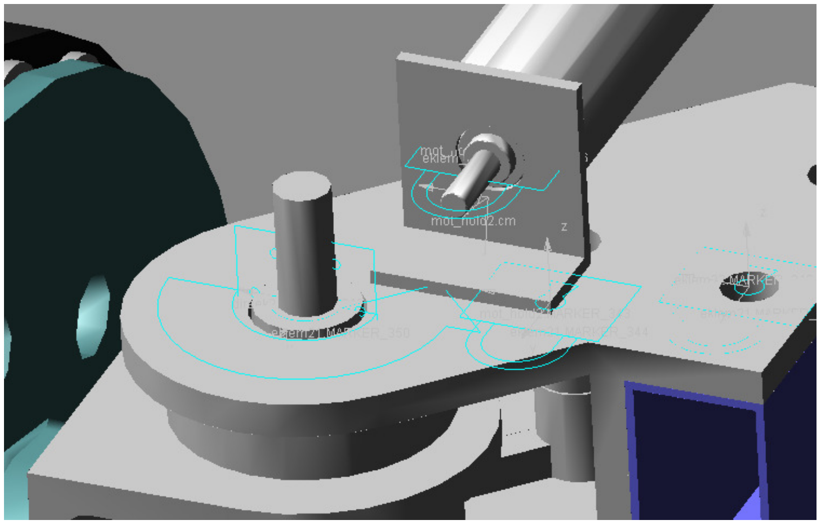


Figure 6.9: Vertical Axis Joints

From the figures above considering the maximum level that torques reaches and this allows us to choose the powers that will be used to drive the mobile robot.

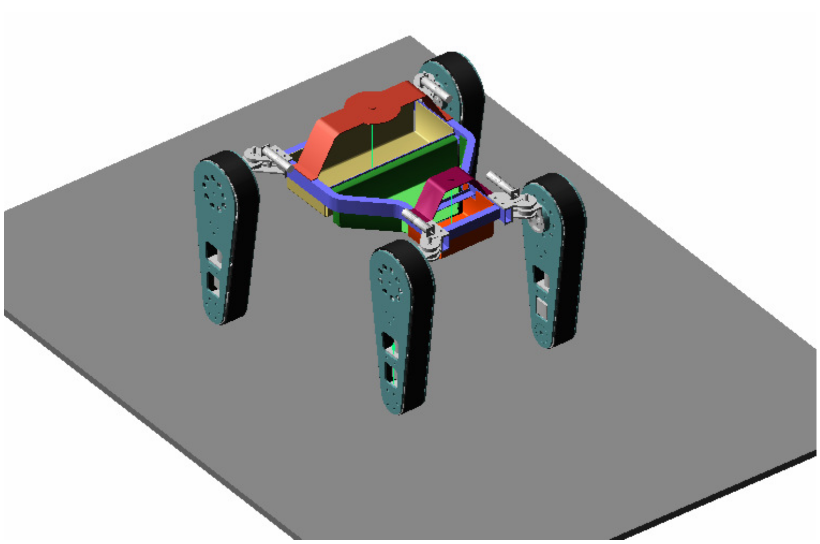


Figure 6.10: Position of the Mobile Robot at Simulation

At the final figure, the position that is specified for this mobile robot to reach can be seen. Measurements have been performed giving the robot this type of motion.

6.3 Simulink Model of the Mobile Robot

Adams has an interface with Matlab - Simulink in order to enable the engineer to design control algorithms and perform simulation mechanical system and controller simulation simultaneously.

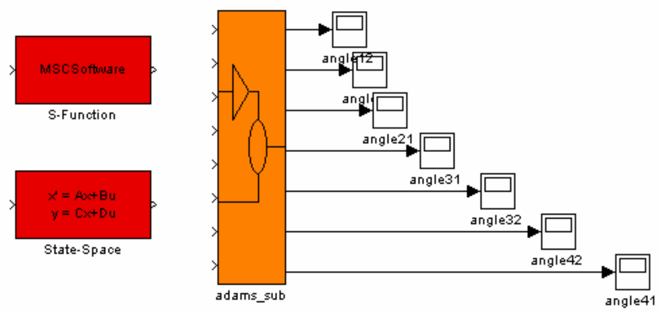


Figure 6.11: Simulink Block of the Model

Figure 6.11 shows the simulink model of this mobile robot. Eight torque inputs and eight angles related to joints where torque has been applied have been defined. This model includes a nonlinear and a linear model. Orange one represents the nonlinear model. This model can be used to simulate the behavior of the system with a controller, since the model includes nonlinearities that are not possible to linearize at equilibrium position.

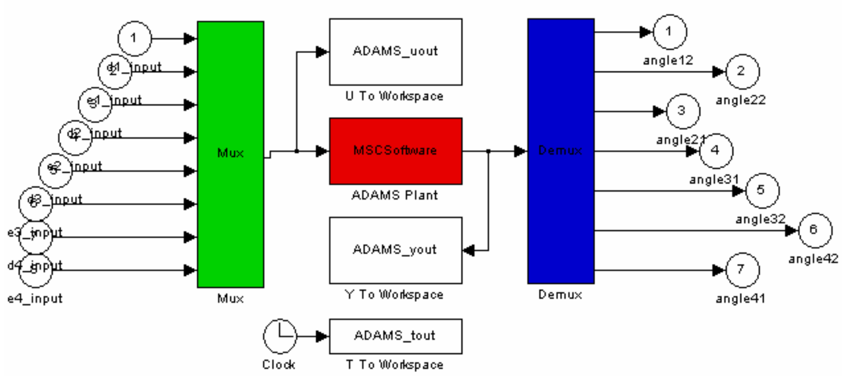


Figure 6.12: Simulink Block of Nonlinear Model

7. CONCLUSSION

In this thesis a multi modal mobile robot has been designed and structural, kinematical and dynamical analysis has been performed. In the design of the robot many affects are considered not to encounter with important problems that can accour in the manifacturing stage. Since this is the first prototype and there is not preceding experiment it is inevitable not to face with a kind of problems.

The first step after this thesis complete should be the production of this mobile robot. Considering the problems that may occur, this design should be enhanced and an optimum design must be obtained. Dynamical model of this mobile robot has been obtained using Adams. Next step of dynamical modelling can be to write the system of equations related to this mobile robot and solve the equations using any programming languages such as Matlab. Also control of such a mobile robot is an important procedure that should be considered. Simulink model of the mobile robot has been prepared for further use in the thesis. A control algorithm can be designed so that the robot move under desired conditions. Relations between Simulink and Adams can be used to determine the affect of the control parameters on the motion of mobile robot.

The first subject would be the production of the mobile robot desinged. Prodcution allows us to proceed with the completion of other works that defined above.

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A. Constraint Dynamic Formulation of a Simple Mechanism

Here, constraint dynamic formulation is prepared with respect to the generalized Lagrangian dynamic equations. This type formulation is used in ADAMS or any type of computer algorithms in order to calculate the behavior of the mechanical system. Here a simple mechanism has been considered to basically obtain the equations of motion.

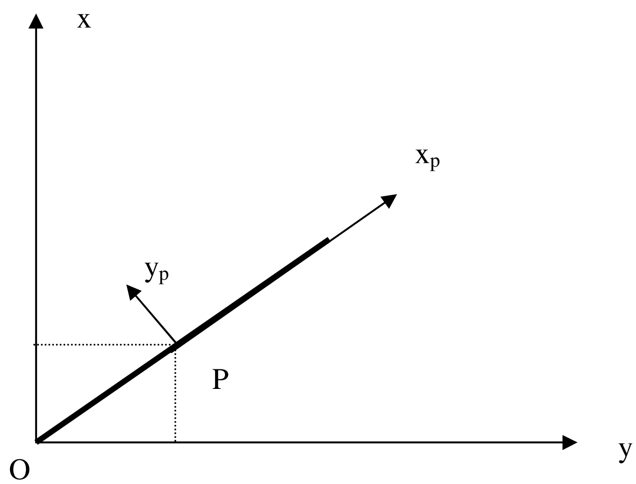

Figure A.1 : Simple Mechanism

Generalized Lagrange equations are given as explaned in the preceeding section :

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\left(\frac{\partial L}{\partial q_i}\right)+\sum_{j=1}^{m}\lambda_j \frac{\partial Q_j}{\partial q_i}-\sum_{k=1}^{na}F_k \frac{\partial r_k}{\partial q_i}=0 \qquad \textbf{(A.1)}$$

Generalized coordinates of the center of mass of the partical are given as R_x, R_y and θ. Kinetic energy of the system is written as:

$$T=\frac{1}{2}\left(m\dot{R}_x^2+m\dot{R}_y^2+I\theta^2\right) \qquad \textbf{(A.2)}$$

Potential Energy is also given as:

$$V = -mgR_y \quad \textbf{(A.3)}$$

and Lagrangian is defined as

$$L = T - V = \frac{1}{2}\left(m\dot{R}_x^2 + m\dot{R}_y^2 + I\dot{\theta}^2 \right) + mgR_y \quad \textbf{(A.4)}$$

$$\frac{\partial L}{\partial \dot{R}_x} = m\dot{R}_x \ , \ \frac{\partial L}{\partial \dot{R}_y} = m\dot{R}_y \ \text{and} \ \frac{\partial L}{\partial \dot{\theta}} = I\dot{\theta} \quad \textbf{(A.5)}$$

Equation A.5 represent the first derivative of Lagrangian with respect to each generalized velocities.

If a part is attached to the ground with a revolute joint, constraint equations are given as in equation A.6. Other types of constraint equations can be found in literature also for spatial dynamic modelling.

$$\mathbf{R} + \mathbf{A}\bar{\mathbf{u}} = 0 \quad \textbf{(A.6)}$$

this equation can be written for this mechanism as

$$\begin{bmatrix} R_x \\ R_y \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} -L \\ 0 \end{bmatrix} = 0 \quad \textbf{(A.7)}$$

this equation can be arranged as

$$R_x - L\cos(\theta) = 0 \quad \textbf{(A.8)}$$

$$R_y - L\sin(\theta) = 0 \quad \textbf{(A.9)}$$

So, the constraint partials are given as

$$\phi_q = \begin{bmatrix} 1 & 0 & -L\sin\theta \\ 0 & 1 & -L\cos\theta \end{bmatrix} \quad \textbf{(A.10)}$$

Writing Lagrange equations in terms of each generalized coordinate gives

$$m\ddot{R}_x + \lambda_1 = 0 \quad \textbf{(A.11)}$$

$$m\ddot{R}_y + \lambda_2 - mg = 0 \quad \textbf{(A.12)}$$

$$I\ddot{\theta} - \lambda_1 L \sin\theta - \lambda_2 L \cos\theta = 0 \quad \textbf{(A.13)}$$

Solution of this equations with the constraint equations

$$R_x - L\cos(\theta) = 0 \quad \textbf{(A.14)}$$

$$R_y - L\sin(\theta) = 0 \quad \textbf{(A.15)}$$

gives the system equations of motion. In order to solve these equations second order terms are represented in terms of first order ones. And Newton-Raphson algorithm is used to solve the nonliear algebric constraint equations. Any iterative method such as Runga-Kutta can be used for simulation.

CURRICULUM VITAE

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