ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL OF SCIENCE
ENGINEERING AND TECHNOLOGY

AN INVESTIGATION ON THE CONTRIBUTION OF GOCE SATELLITE MISSION TO REGIONAL GEOID MODELLING IN TURKEY

M.Sc. THESIS

Mustafa Serkan IŞIK

Department of Geomatics Engineering

Geomatics Engineering Programme

JUNE 2016

ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL OF SCIENCE
ENGINEERING AND TECHNOLOGY

AN INVESTIGATION ON THE CONTRIBUTION OF GOCE SATELLITE MISSION TO REGIONAL GEOID MODELLING IN TURKEY

M.Sc. THESIS

Mustafa Serkan IŞIK
(501141622)

Department of Geomatics Engineering

Geomatics Engineering Programme

Thesis Advisor: Assoc. Prof. Dr. Bihter EROL

JUNE 2016

İSTANBUL TEKNİK ÜNİVERSİTESİ ★ FEN BİLİMLERİ ENSTİTÜSÜ

GOCE UYDU MİSYONUNUN TÜRKİYE'DE BÖLGESEL GEOİT MODELLEMEYE KATKISI ÜZERİNE BİR İNCELEME

YÜKSEK LİSANS TEZİ

Mustafa Serkan IŞIK
(501141622)

Geomatik Mühendisliği Departmanı

Geomatik Mühendisliği Programı

Tez Danışmanı: Doç. Dr. Bihter EROL

HAZİRAN 2016

Mustafa Serkan IŞIK, a M.Sc. student of ITU Graduate School of ScienceEngineering and Technology 501141622 successfully defended the thesis entitled "An Investigation on the Contribution of GOCE Satellite Mission to Regional Geoid Modelling in Turkey", which he/she prepared after fulfilling the requirements specified in the associated legislations, before the jury whose signatures are below.

Thesis Advisor : **Assoc. Prof. Dr. Bihter EROL** ....
Istanbul Technical University

Jury Members : **Prof. Dr. Rahmi Nurhan ÇELİK** ....
Istanbul Technical University

Assoc. Prof. Eng. Col. Dr. Hasan YILDIZ ....
General Command of Mapping

Date of Submission : **06 May 2016**
Date of Defense : **08 June 2016**

FOREWORD

First of all, I would like to thank my supervisor Dr. Bihter Erol for her support and motivation during this thesis.

I would like to thank my dearest friend Mohsen Feizabadi for his constant help and hours of discussions we had about physical geodesy. I also want to express my gratitude to Dr. Serdar Erol. He has made available his support in a number of ways. I also want to thank my friend and office mate Mohamed Abdel Azeem Mostafa Mansour for all the good times and fun we had in ISTA Lab.

Funding provided for my studies by The Scientific and Technological Research Council of Turkey (TUBITAK) Grant No. 114Y581 and Istanbul Technical University Scientific Research Projects Unit (ITU-BAP) Project No. 38828 are gratefully acknowledged.

Last but not least, I would like to thank my family for their love and support during my studies.

June 2016

Mustafa Serkan IŞIK
Geomatics Engineer

TABLE OF CONTENTS

Page

ABBREVIATIONS

CHAMP	: CHAllenging Minisatellite Payload
DEM	: Digital Elevation Model
DIR	: Direct-wise
DTM	: Digital Terrain Model
DTU	: Danmarks Tekniske Universitet
EGM2008	: Earth Gravitational Model 2008
FFT	: Fast Fourier Transform
GGM	: Global Geopotential Model
GNSS	: Global Navigation Satellite System
GOCE	: Gravity field and steady-state Ocean Circulation Explorer
GOCO	: Gravity Observation Combination
GPS	: Global Positioning System
GRACE	: Gravity Recovery and Climate Experiment
GRS80	: Geodetic Reference System of 1980
ICGEM	: International Center for Global Gravity Field Models
IGNA2005	: Istanbul GPS Triangulation Network 2005
ITRF96	: International Terrestrial Reference Frame 1996
KTH	: Kungliga Tekniska Högskolan
LAGEOS	: LAser GEOdynamics Satellite
LSC	: Least Squares Collocation
RCR	: Remove-Compute-Restore
RMSE	: Root Mean Square Error
RTM	: Residual Terrain Modelling Effect
SEM	: Spectral Enhancement Method
SPW	: Space-wise
STD	: Standard Deviation
TC	: Terrain correction
TG	: Turkey Geoid
TG03	: Turkey Geoid 2003
TG91	: Turkey Geoid 1991
TG99-A	: Turkey Geoid 1999A
TIM	: Time-wise
TUDKA99	: Turkish National Vertical Control Network 1999
TUTGA	: Turkish National Fundamental GPS Network
UNB	: University of New Brunswick

SYMBOLS

h	:	ellipsoidal height
H	:	orthometric height
N	:	geoid height
R	:	radius of Earth
e^2	:	eccentricity of the reference ellipsoid
ϕ	:	latitude
λ	:	longitude
γ	:	normal gravity
γ_a	:	normal gravity on the equator
$S(\psi)$	:	Stokes' function
ψ	:	spherical distance
Δg	:	gravity anomaly
$d\sigma$	:	infinitesimal surface element
Δg_{red}	:	reduced gravity anomaly
Δg_{FA}	:	free-air gravity anomaly
Δg_{BA}	:	bouguer gravity anomaly
F	:	free-air correction
A_{BO}	:	bouguer plate correction
Δg_{GGM}	:	gravity anomaly from global geopotential model
Δg_H	:	terrain effect on gravity
N_{GGM}	:	long-wavelength component of geoid height
$N_{\Delta g}$	:	residual geoid height
N_{ind}	:	indirect effect on geoid height
γ	:	normal gravity
ρ	:	topographic density
R	:	mean Earth radius
G	:	gravitational constant
M	:	total mass of the Earth
$C_{lm}, S_{lm},$	:	fully normalized coefficients of global geopotential model
σ_0	:	spherical cap
$P_{lm}(cos\theta)$	:	associated legendre polynomial
$\tilde{N}$	:	approximate geoid height in KTH method
δN_{comb}^{Topo}	:	combined topographic correction
δN_{DWC}	:	downward continuation correction
δN_{comb}^{ATM}	:	combined atmospheric correction
δN_{ELL}	:	ellipsoidal correction

LIST OF TABLES

Page

LIST OF FIGURES

Page

AN INVESTIGATION ON THE CONTRIBUTION OF GOCE SATELLITE MISSION TO REGIONAL GEOID MODELLING IN TURKEY

SUMMARY

The Gravity Field and Steady-State Ocean Circulation Explorer (GOCE) satellite was launched on March 2009 and it completed its mission in November 2013. Over 30 Global Geopotential Models (GGM) have been published using data collected by GOCE with various combinations of data sets. With the contribution of dedicated gravity field satellite missions and improved methodologies, computation of precise geoid model that can replace the existing vertical datum is possible. Some countries, such as Canada, adopted geoid-based vertical datum which facilitates the use of GNSS technologies to derive orthometric heights from ellipsoidal heights precisely. Currently, there are ongoing height modernization efforts to re-define the vertical datum in Turkey. Through this modernization, methodological developments and new approaches are needed to be investigated as well as acquiring the optimum data set to determine a centimeter accuracy geoid model.

In the first part of this study, the evaluation of the performance of the recent GOCE-only and GOCE-GRACE models using four different GPS/Levelling data sets in Turkey are presented. Direct (DIR), Time-wise (TIM), Space-wise (SPW), GOCO and GOGRA models are evaluated. In order to make the models comparable with the observation dataset, spectral enhancement method is applied. In the calculation of geoid heights, high-frequency components of the gravity field are computed from EGM2008 and Residual Terrain Model (RTM). The standard deviations of the evaluated models are compared with the combined model EGM2008. As a result of the assessment of global geopotential models, the contribution of GOCE in the medium wavelength is obvious. There are significant improvements in the releases of the models. In the second part of the study, the models DIR R5, TIM R5 and GOCO05S, which shows superior agreement with the GPS/Leveling data sets, are used in the computation of regional geoid via the classical Remove-Compute-Restore (RCR) and KTH approach, also known as the Least Squares Modification of Stokes' formula (LSMS), to clarify the contribution of GOCE mission to the gravimetric geoid modeling in Turkey. The theoretical differences of the two geoid modelling methods are provided with their performance comparisons based on numerical results. The geoid models are tested using 30 GPS/Levelling benchmarks distributed through the whole Turkey, besides the local validation is carried out in the Marmara region using 81 GPS/Levelling benchmarks. In RCR method, the standard deviations of the geoid models that are computed with GOCE models show the same performance compared to that of EGM2008 at the same degree. The best results are achieved when the optimum degree of global geopotential model is used. In KTH method, the best results are achieved when the cap size $\psi_0 = 0.1^0$ and EGM2008 is used up to $L = 360$. Additionally, GOCE models performed better in the geoid modelling compared to

EGM2008 at the same degree. The standard deviation of the geoid computed with the maximum degree of the model is smaller than that of the optimum degree of the model. So it can be said that KTH method minimizes the effect of the low quality terrestrial gravity data using the high-frequency information of the global geopotential model. The best standard deviations are found as 22.4 cm and 17.6 cm (before fitting the geoid models to regional vertical datum) using RCR and KTH methods at 30 GPS/Levelling points, respectively. After the corrector surface fitting is applied at GPS/Levelling data sets using different parametric models, the performance of the models are compared to each other and Turkey Geoid (TG03). At 30 TUTGA points, 4-parameter Helmert similarity transformation model shows better results and the standard deviations are 21.2 and 16.6 cm for RCR and KTH methods respectively. The standard deviation of TG03 is 16.1 cm for the same data set. At 81 TUTGA points, the performance of RCR model is better than KTH model. However, both model cannot reach the accuracy of TG03 which is 6.3 cm. The best standard deviation is achieved when the 5^{th} degree polynomial model is used, but the cross-validation results revealed that the optimal model is the 3^{rd} degree polynomial model. At IGNA points, the performance of KTH model is slightly better than RCR model. There is no significant difference in terms of standard deviations for 3^{rd}, 4^{th} and 5^{th} degree polynomial models. Thus, 3^{rd} degree polynomial model is preferred as optimal. The validations at the test points confirms this result as well. the Both models outperform the TG03 in that region about 4 cm. The validations at İzmir points shows that the best model is 5^{th} degree polynomial model for RCR and KTH. The standard deviations for RCR and KTH are 6.9 cm and 7.2 cm at the test points. TG03 performs better than both models. In conclusion, the computed geoid models are not accurate enough to be used for transforming ellipsoidal heights to orthometric heights because of the low accuracy of the terrestrial gravity data. Additionally, in order to make more clear statements about the performance of the RCR and KTH methods in Turkey, a better set of GPS/Levelling points whose distribution represents the changes of the topography must be used.

GOCE UYDU MİSYONUNUN TÜRKİYE'DE BÖLGESEL GEOİT MODELLEMEYE KATKISI ÜZERİNE BİR İNCELEME

ÖZET

GOCE uydu gravite misyonu Kasım 2013'de görevini tamamladı. GOCE verileri kullanılarak yaklaşık 30 un üstünde global jeopotansiyel model yayımlandı. Gravite uydu misyonlarının katkılarıyla ve gelişen teorik araştırmalarla birlikte, geleneksel düşey datumun yerini alabilecek yüksek doğruklu geoidin hesaplanabilmesi günümüzde mümkündür. Bazı ülkeler, örneğin Kanada, düşey datumlarını geoide dayandırarak elipsoidal yüksekliklerin ortometrik yüksekliklere dönüştürülmesinde GNSS teknolojilerinin kullanımına olanak sağlamışlardır. Türkiye'de 1970'den beri farklı geoit modelleri hesaplanmıştır ve güncel geoit modellerinin doğrulukları yaklaşık 8 cm olarak rapor edilmiştir. Ancak, pratik uygulamalarda yükseklik dönüşümü için bu doğruluk ne yazık ki gereksinimi karşılamamaktadır. Türkiye'de yükseklik sistemi modernizasyonu çalışmaları son yıllarda hız kazanmıştır. Bu modernizasyon sürecinde yeni veri setlerinin, özellikle gravite verisinin, elde edilmesi için yapılan çalışmaların yanında metodolojik gelişmeler ve yeni teorilerin araştırılması ve Türkiye için en uygun bölgesel geoit belirleme yönteminin ortaya konulması büyük önem taşımaktadır.

Bu çalışmada, Türkiye'de "Kaldır-hesapla-yerine koy (RCR)" yöntemi ve "Stokes integralinin en küçük kareler modifikasyonu (KTH)" yöntemleri kullanılarak bölgesel geoit modelleri hesaplanmıştır. Bu kapsamda, en uygun veri setinin kullanılabilmesi için uzun dalga boylu bileşenin hesaplanmasında kullanılan farklı global jeopotansiyel modeller test edilmiştir. Test edilen modeller GOCE uydu misyonu verisi kullanılarak hesaplanan Direct (DIR), Time-wise (TIM), Space-wise (SPW), GOCO ve GOGRA modellerinin yayınlanan tüm sürümlerini içermektedir. Böylelikle GOCE uydu misyonunun Türkiye'de geoit belirlemeye olan katkısı ortaya konulşmuştur. Modeller, dört farklı GPS nivelman veri seti ile Spektral İyileştirme Yöntemi (Spectral Enhancement Method - SEM) kullanılarak mutlak anlamda karşılaştırılmıştır. Modellerin iyileştirilmesinde 2190 derece/mertebe açılıma sahip EGM2008 modeli ve SRTM sayısal yükseklik modeli ile hesaplanan Artık Arazi Modeli (Residual Terrain Model - RTM) kullanılmıştır. Karşılaştırmada kullanılan GPS nivelman verilerinden 30 adet TUTGA noktaları Türkiye genelinde dağılıma sahiptir. 81 TUTGA noktası Marmara bölgesinde 3^ox4^o alanda homojen dağılıma sahiptir. 1204 adet IGNA ve 301 adet İzJRS-2001 noktaları sırasıyla İstanbul ve İzmir'de sıklaştırma ağı amacıyla elde edilen lokal ağlara aitlerdir ve yayıldıkları alanlar Türkiye'nin tamamı ele alındığında çok küçük kalmaktadır. 30 GPS nivelman noktasında yapılan kıyaslamalar sonucunda modellerde orta dalga boyunda EGM2008'e göre iyileşme gözlenmiştir ancak iyileşmenin miktarı en fazla TIM R5 modelinde 2 cm olarak bulunmuştur. Modellerin son sürümlerinde optimum dereceler 205-235 derece arasında çıkmaktadır. 81 noktada ise orta dalga boyunda EGM2008'e göre iyileşme

miktarı tüm modeller için ortalama 4 cm'i bulmaktadır ve optimum derece 155 olarak gözlenmiştir. İstanbul ve İzmir'de yapılan kıyaslamalarda da büyük miktarda iyileşmeler görülse de sayısal sonuçlar modellerin lokal özelliklerini göstermiş olup modellerin sürümlerindeki iyileşme 30 ve 81 TUTGA noktasında açıkça görüldüğü gibi gözlenememiştir. Genel olarak DIR R5, TIM R5 ve GOCO05S modelleri GPS nivelman verisi ile en iyi uyum gösteren modeller olmuştur. Bu modeller, RCR ve KTH geoit belirleme yöntemlerinde kullanılarak EGM2008 ile hesaplanan geoit modellerine göre yaptıkları iyileştirmeler incelenmiştir. RCR yönteminde yapılan sayısal validasyonlarda Stokes integralinin hesaplanmasında integrasyon alanında kısıtlama yapılmadığında, yani integrasyon yarıçapı kullanılmadığında en iyi sonuç elde edilmştir. Bunun temel sebebi, klasik anlamda RCR yönteminde kullanılan Stokes integrali modifiye edilmemiş olup orjinal kerneli kullanır. Bu durum, integrasyon yarıçapı kullanıldığında büyük miktarda kesme hatasının oluşmasına sebep olmaktadır. GOCE modelleriyle yapılan hesaplamalarda ise elde edilen geoit doğrulukları EGM2008 ile elde edilen doğruluklarla benzer sonuçlar vermektedir ve modellerin optimum dereceleri maksimum derecelerinden daha iyi sonuç vermektedir. 30 noktada yapılan kıyaslamada sistematik hata giderilmemiş halde bulunan en iyi standart sapma 22.0 cm'dir.KTH yönteminde, optimum parametrelerin belirlenmesi amacyla EGM2008 modeli kullanılarak farklı integrasyon yarıçapları, global jeopotansiyel model derecesi ve yersel gravite verisi varyans değeri denenmiştir. Yersel gravite verisinin varyans değerindeki değişim geoit doğruluğunda anlamlı bir farka sebep olmamıştır. İntegrasyon yarıçapı $\psi_0 = 0.1^0$ ve EGM2008 modeli 360 dereceye kadar kullanıldığında en iyi standard sapma elde edilmiştir. İntegrasyon yarıçapı azaldıkça ve model derecesi arttıkça görülen iyileşme, yersel verideki hatanın geoide etkisinin global jeopotansiyel modelden gelen yüksek frekanslı bileşen ile dengelenebildiğini göstermektedir. GOCE modelleri KTH yöntemi ile geoit hesabında EGM2008 ile aynı açılım derecesi kullanıldığında daha iyi sonuç vermektedir ve modellerin maksimum dereceleri optimum derecelerinden daha düşük standart sapmaya sahiptir. 30 noktada yapılan kıyaslamada (fit öncesi) bulunan en iyi standart sapma RCR için 22.4 cm ve KTH için 17.3 cm'dir. Sonuç olarak Türkiye'de KTH yöntemi RCR yönteminden daha iyi sonuç vermektedir. GPS/Nivelman verisi ile hesaplanan geoit yükseklikleri ile hesaplanan artik geoit yüksekliklerindeki rastlantısal ve sistematik hataların etkilerinin modellenebilmesi için farklı parametrik modeller ile regresyon analizi yapılmış ve geoit modelleri üzerine düzeltmeler getirilerek tekrar doğrulukları hesaplanmıştır. 30 TUTGA noktasında yapılan analizlerde nokta sayısının yetersizliği sebebiyle en düşük parametreli modellerden 4 parametreli benzerlik dönüşümü ve 1. derece polinom kullanılmıştır. Analizin sonucunda, 4-parametreli benzerlik dönüşümü daha iyi sonuç vermiş olup RCR modelinin doğruluğu 21.2 cm ve KTH modelinin doğruluğu 16.6 cm olarak bulunmuştur. Aynı veri seti ile Türkiye Geoidi 2003 (TG03) 16.1 cm uyuşum göstermiştir. 81 TUTGA veri setinde yapılan analizlerde 5. derece polinom modeli en iyi standart sapmayı verse de, çapraz doğrulama sonuçları 3. derece polinom modelinin optimum olduğunu göstermiştir. İstanbulda IGNA veri setinde yapılan analizlerde RCR ve KTH modelleri TG03 modelinden yaklaşık 4 cm daha iyi sonuç vermiştir. KTH modelinin performansı RCR modelinden daha iyi bulunmuş olsada aradaki fark 3 mm'yi geçmemektedir. 3., 4. ve 5. derece polinom modellerinin standart sapmaları arasındaki fark çok küçüktür ve test noktalarında yapılan sonuçlarla 3. derece polinom modeli yine optimum bulunmuştur. İzmir bölgesinde yapılan analizler sonucunda, İGNA veri setinde olduğu gibi, RCR ve

KTH yöntemi arasında anlamlı bir performans farkı görülmemiştir. 5. derece polinom modelini her iki geoit modeli için en iyi sonuç vermiştir. Ancak elde edilen standart sapmalar TG03 modelinden daha kötü bulunmuştur. Sonuç olarak, hesaplanan geoit modellerinin doğrulukları, kullanılan yersel verinin düşük doğruluğu sebebiyle elipsoidal yüksekliklerden doğrudan ortometrik yükseklik türetmek için yetersizdir. Ayrıca, RCR ve KTH yöntemlerinin Türkiye'deki performansları hakkında kesin yargılarda bulunabilmesi için topoğrafyanın değişimini temsil eden, yeterli sıklıkta tüm Türkiye'ye homojen bir şekilde dağılmış GPS/nivelman veya astro-jeodezik çekül sapması veri setlerine ihtiyaç vardır.

1. INTRODUCTION

1.1 Statement of the Problem

The advancement in the field of space technologies, especially satellite-based positioning systems such as GPS, GLONASS and GALILEO, made it possible to determine the 3D position of a point on or above the surface of the Earth, accurately. The height of a point determined using these technologies refers to an analytically defined surface, i.e reference ellipsoid, meaning that the geometry of this observation, so-called ellipsoidal height, is purely mathematical. Thus, the practical usage of the ellipsoidal height derived from GNSS levelling method is limited. In order GNSS data to be applicable in height determination, ellipsoidal height must be converted to orthometric height which is a physically meaningful height measured from geoid to the surface of the Earth along the plumb line. The separation between ellipsoid and geoid respresents the geoid undulation (or geoid height)(Fig. 1.1).

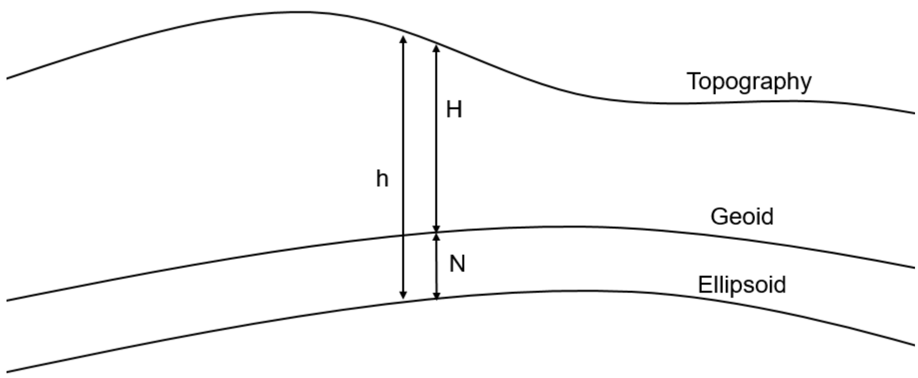

Figure 1.1. Ellipsoidal, orthometric and geoid height.

The mathematical relation between these quantities are given as:

$$h - H - N = 0 \tag{1.1}$$

In this equation, h, H and N represent ellipsoidal height, orthometric height and geoid height respectively. As mentioned above, ellipsoidal height is measured using GNSS observations. Orthometric height is measured using traditional levelling methods and refers to regional vertical datum. The conversion of ellipsoidal height to orthometric height requires a geoid model with 1-3 cm accuracy, so that the derived orthometric heights would be accurate enough to be used in engineering projects.

Canada has already transformed their vertical datum to gravimetric geoid (Canada, 2013). By this modernization not only they enable the efficient use of GPS/GNSS technologies in the height determination, but also they possess a vertical datum that is less affected from geodynamic activities.

Geoid modelling studies in Turkey began in 1970s with astro-geodetic geoids (Ayan, 1976, 1978; Gürkan, 1978). The first gravimetric geoid, on the other hand, is Turkey Geoid 1991 (TG-91)(Ayhan, 1993). With the establishment of Turkish National Fundamental GPS Network, the vertical datum studies were accelerated and after the Earthquake in 1999, TG-99A hybrid geoid model was calculated using Least Squares Collocation (LSC) and GPS/Levelling method (Ayhan et al., 2011). With the release of higher degree and order global geopotential models and methodological improvements, Turkey Geoid 2003 (TG03) was computed using remove-compute-restore method. The accuracy of TG03 was reported as $\pm 10cm$ (Kılıçoğlu et al., 2005). Turkey Geoid 2007 (TG07) was calculated using Fast Fourier Transform (FFT). EGM96 and GGM02S geopotential models were used and the accuracy of TG07 was reported as $\pm 9cm$ (Yıldız et al., 2006). After the release of Earth Gravitational Model 2008 (EGM2008), Turkey Geoid 2009 (TG09) was computed by using FFT with an accuracy of $\pm 8.4cm$ (Kılıçoğlu et al., 2011).

The geoid modelling studies in Turkey has accelerated within the height modernization efforts and re-definition of the vertical datum. Currently, the accuracy of geoid models in Turkey is not sufficient for deriving orthometric heights directly using GNSS levelling instead of traditional levelling methods. In order to achieve a precise geoid model, combination of various types of data sets such as gravimetric, GNSS-levelling, astro-geodetic, satellite altimetry and gravity data, must be used. Though there are problems with these datasets in Turkey (TUJK, 2012). On the other hand, the methodology has significant impact on geoid accuracy as well. Therefore, within the scope of the modernization of Turkish Height System, investigation of the geoid determination methods and their practical use must be conducted to find the most suitable approach for Turkey.

1.2 Thesis Objectives

Within the context of this research, the objectives of this thesis are given as follow:

- The primary objective is to evaluate the performance of recent releases of GOCE-only and GOCE-GRACE global geopotential models using spectral enhancement method to determine which models agree best with the independent data sets in Turkey.
- The second objective is to compare the "classical remove-compute-restore" and the "least squares modification of Stokes' integral with additive corrections" methods for the determination of regional gravimetric geoid model to find out which method is superior to another in Turkey.
- The third and the final objective is to investigate how much the accuracy of regional geoid can be improved with respect to current official geoid model using the available gravity data set with the contribution of recent GOCE/GRACE models.

1.3 Thesis Outline

This thesis is structured in five chapters. In Chapter I, background information about geoid, its necessity and the problems concerning with gravimetric geoid determination in Turkey is explained. In Chapter II, two geoid determination methods, Remove-Compute-Restore and Least Squares Modification of Stokes' Integral with additive corrections, used in this study are explained. In Chapter III, the creation of the optimum dataset for the geoid determination in terms of global geopotential model, terrestrial gravity data and digital terrain model are given. A special attention is made on the evaluation of the performance of the recent GOCE-only and GOCE-GRACE global geopotential models. In Chapter IV, the numerical results of the global geopotential model assessments are provided. The models which agree best with GPS/Levelling data in terms of standard deviation are used in the geoid modelling. The accuracy of the geoid models, before and after corrector surface fitting, are given in this chapter. In Chapter V, the conclusions of the numerical results based on the objectives of the thesis are provided and recommendations are made for the future studies.

2. GEOID DETERMINATION METHODS

In 1849, G. G. Stokes published his well-known formula (Eq. 2.1) for the determination of geoid height (Stokes, 1849). It is considered as the most important formula of physical geodesy since it allows the computation of geoid height using gravity data, hence it is the solution of the boundary value problem of potential theory (Heiskanen and Moritz, 1967).

$$N = \frac{R}{4\pi\gamma}\iint S(\psi)\Delta g d\sigma \tag{2.1}$$

In this formula, N represents geoid height, R is the mean Earth radius γ is the normal gravity on the reference ellipsoid, $S(\psi)$ is the Stokes' kernel with ψ being geocentric angle, Δg is the gravity anomaly and $d\sigma$ is the infinitesimal surface element of the unit sphere (Heiskanen and Moritz, 1967). Stokes' kernel is given as

$$\begin{aligned} S(\psi) = {} & \frac{1}{sin(\psi/2)} + 1 - 5cos(\psi) - 6sin(\psi/2) \\ & - 3cos(\psi)ln(sin(\psi/2) + sin^2(\psi/2)) \end{aligned} \tag{2.2}$$

Geoid determination using Stokes equation requires the integral to be applied over the whole Earth. In practice, since the whole gravity data set of the Earth cannot be obtained, the integral is limited to a spherical cap σ_0 around the computation point, thus causing a truncation at the borders of the study area.

The most common approach for the determination of gravimetric geoid is the remove-compute-restore method (Schwarz et al., 1990). It is based on removing the long wavelength and short wavelength effect from observations to compute the residual geoid heights using Stokes equation and later restoring them to obtain the final geoid undulations (Sansò and Sideris, 2013). Researchers in Royal Institute of Technology (KTH) introduced an alternative approach KTH method or Least Squares Modification of Stokes Integral (LSMS) with additive corrections (Sjöberg, 2003a) and method has been applied in some countries (Ågren, 2004; Ellmann, 2004; Kiamehr, 2006; Daras, 2008). The application of remove-compute-restore and KTH methods in Turkey, both local and regional, can be found in (Yıldız et al., 2006; Erol, 2007; Abbak, 2011; Abbak et al., 2012; Işık and Erol, 2015, 2016)

2.1 Remove-Compute-Restore

In the remove-compute-restore technique, the long wavelength and short wavelength (also called terrain effect) components are removed from terrestrial gravity anomalies to obtain reduced gravity anomalies in the "remove" step. (Eq.2.3).

$$\Delta g_{red} = \Delta g_{FA} - \Delta g_{GGM} - \Delta g_H \tag{2.3}$$

In this equation, Δg_{red} is the reduced gravity anomaly, Δg_{FA} represents the free-air gravity anomaly, Δg_{GGM} represents the long-wavelength component of the gravity anomaly from global geopotential model and Δg_H represents the short-wavelength component of the gravity anomaly.

In the "compute" step, the residual co-geoid heights are calculated using Stokes integral. After that, the long-wavelength component from global geopotential model is restored and topograhic indirect effect (N_{ind}) is added to co-geoid heights to calculate the final geoid heights (Eq. 2.4) in the "restore" step (Fig. 2.1).

$$N = N_{GGM} + N_{\Delta g} + N_{ind} \tag{2.4}$$

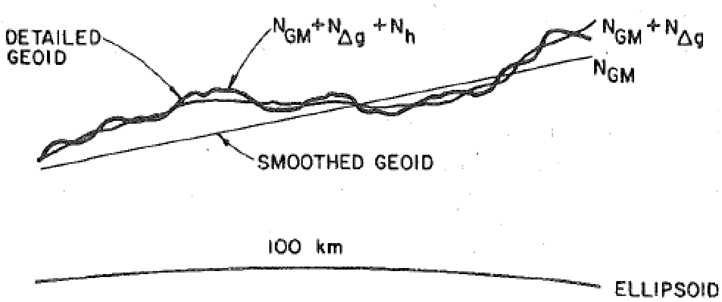

Figure 2.1. Detailed geoid heights (Schwarz et al., 1987).

In the computation of the reduced gravity anomalies, Δ_{GGM} which is calculated from global geopotential model (Eq. 2.6) is subtracted from the gridded terrestrial gravity anomalies. These reduced gravity anomalies are used to calculate residual geoid heights $N_{\Delta g}$ by using Stokes' formula,

$$N_{\Delta g} = \frac{R}{4\pi\gamma} \iint S(\psi) \Delta g_{red} d\sigma \tag{2.5}$$

The evaluation of Stokes integral, which is a convolution integral, is carried out using 1D Fast Fourier Transform on the sphere (Haagmans et al., 1993).

As mentioned above, Stokes' integral requires gravity anomalies all over the Earth. But, in practice the integral is applied in a much smaller region. That is why the long wavelength contributions of the gravity field cannot be represented properly. The long wavelength component of the geoid is computed from global geopotential model at a chosen degree and order L (Eq.2.7), that is the same degree and order selected for the computation of Δg_{GGM}.

$$\Delta g_{GGM} = \frac{GM}{r^2}\sum_{l=2}^{L}(n-1)\left(\frac{R}{r}\right)^{l}\sum_{m=0}^{l}\left(\bar{C}_{lm}\cos m\lambda + \bar{S}_{lm}\sin m\lambda\right)\bar{P}_{lm}(\cos\theta) \tag{2.6}$$

$$N_{GGM} = \frac{GM}{r\gamma}\sum_{l=2}^{L}\left(\frac{R}{r}\right)^{l}\sum_{m=0}^{l}\left(\bar{C}_{lm}\cos m\lambda + \bar{S}_{lm}\sin m\lambda\right)\bar{P}_{lm}(\cos\theta) \tag{2.7}$$

In this equation, G is the gravitational constant, M is the total mass of the Earth, $\bar{C}_{lm}$ and $\bar{S}_{lm}$ are the fully normalized coefficients of the global geopotential model and $\bar{P}_{lm}(\cos\theta)$ is the associated legendre polynomial.

The computation of terrain effects (C) on gravity (Eq. 2.8) is given as (Erol et al., 2009). Accordingly, the terrain effects on geoid undulations (N_{Topo}) are computed with 2D FFT technique.

$$C = -\Delta g H = \frac{G\rho R^2}{2}\iint_{\sigma}\frac{(H_Q - H_P)^2}{S^3}d\sigma \tag{2.8}$$

The indirect effect of the Helmert's second condensation reduction of of gravity on the geoid is calculated as

$$N_{ind} = -\frac{\pi G\rho H_Q^2}{\gamma} - \frac{G\rho R^2}{6\gamma}\iint_{\sigma}\frac{(H_Q^3 - H_P^3)}{S^3}d\sigma \tag{2.9}$$

Finally, the geoid undulations are obtained by combining the components N_{GGM}, $N_{\Delta g}$, N_{Topo} and N_{ind}.

2.2 Least Squares Modification of Stokes' Integral

The least squares modification of Stokes' Integral method is a gravimetric geoid determination technique developed at KTH (Sjöberg, 2003b). The methodology involves a least squares Stokes' kernel modification to minimize the expected global mean square error (Sjöberg, 2003c). In this technique, the Stokes' integral, truncated to a cap, is applied to the uncorrected surface gravity anomalies (Ågren et al., 2009).

2.2.1 Modification of Stokes' Integral

In the modification of Stokes' formula, long-wavelength component from GGM and terrestrial gravity data is combined to reduce the truncation error occurred.

$$\tilde{N} = \frac{R}{4\pi\gamma} \iint_{\sigma_0} S^L(\psi)\Delta g d\sigma + c \sum_{n=2}^{M} (s_n + Q_n^L)\Delta g_n^{EGM} \tag{2.10}$$

where σ_0 is the spherical cap, $S^L(\psi)$ is the modified Stokes' function, s_n are the modification parameters, Q_n^L are the truncation coefficients, Δg_n^{EGM} is the Laplace harmonics of degree n calculated from global geopotential model (Heiskanen and Moritz, 1967).

The modified Stokes' function is given as

$$S^L(\psi) = \sum_{n=2}^{\infty} \frac{2n+1}{n-1} P_n(\cos\psi) - \sum_{n=2}^{L} \frac{2n+1}{2} P_n(\cos\psi) \tag{2.11}$$

where the fist term on the right-hand side of the equation is the original Stokes function in spectral form.

Laplace harmonics of degree n can be calculated as

$$\Delta g_n^{EGM} = \frac{GM}{a^2}\left(\frac{a}{r}\right)^{n+2}(n-1)\sum_{m=-n}^{n} C_{nm}Y_{nm} \tag{2.12}$$

where a is the equatorial radius of the reference ellipsoid, r is the geocentric radius, C_{nm} are the fully normalized spherical harmonic coefficients from GGM and Y_{nm} are the fully normalized spherical harmonics.

The truncation coefficients in Eq. 2.10 are given as

$$Q_n^L = Q_n - \sum_{k=2}^{L} \frac{2k+1}{2} s_k e_{nk} \tag{2.13}$$

where Q_n are the Molodensky truncation coefficients and e_{nk} are the Paul's coefficients (Paul, 1973).

2.2.2 Additive Corrections

In the KTH method, the surface gravity anomalies are used for the computation of approximate geoid heights and then the necessary corrections for downward continuation of gravity anomalies to geoid, combined topographic and atmospheric corrections for the existence of masses and ellipsoidal correction for the spherical approximation of geoid are applied. The final geoid height in KTH schema is obtained by the formula given

$$\hat{N} = \tilde{N} + \delta N_{comb}^{Topo} + \delta N_{DWC} + \delta N_{comb}^{ATM} + \delta N_{ELL} \tag{2.14}$$

where δN_{comb}^{Topo} is the combined topographic correction, δN_{DWC} is the downward continuation correction, δN_{comb}^{ATM} is the combined atmospheric correction and δN_{ELL} is the ellipsoidal correction.

2.2.2.1 The Topographic Correction

The combined topographic effect involves both direct and indirect topographic effects and its formula is given as

$$\delta N_{comb}^{Topo} = \delta N_{dir} + \delta N_{ind} \approx -\frac{2\pi G\rho}{\gamma} H^2 \tag{2.15}$$

where the mean topographic mass density ρ is taken as 2.67 cm^3 and H represents the orthometric height (Kiamehr, 2006).

2.2.2.2 The Downward Continuation Correction

This correction is applied for the analytical continuation of gravity anomalies to the geoid. The formula purposed by Sjöberg (2003c) computes the downward continuation effect directly on the geoid, not the gravity anomaly.

$$\delta N_{DWC} = \frac{R}{4\pi\gamma} \iint_{\sigma_0} S^L(\psi)(\Delta g^* - \Delta g)\, d\sigma \tag{2.16}$$

where Δg is the gravity anomaly at the surface computation point P and $\Delta g*$ is the corresponding quantity downward continued to the geoid. The final formulas for the downward continuation are given as (Ågren, 2004)

$$\delta N_{DWC}(P) = \delta N_{DWC}^{1}(P) + \delta N_{DWC}^{L1,Far}(P) + \delta N_{DWC}^{L2}(P) \tag{2.17}$$

where

$$\delta N^{1}_{DWC}(P) = H_P \left(\frac{\Delta g(P)}{\gamma} + 3\frac{N^0_P}{r_P} - \frac{1}{2\gamma}\frac{\partial \Delta g}{\partial r}\bigg|_P H_P \right) \tag{2.18}$$

and

$$\delta N^{L1,Far}_{DWC}(P) = c \sum_{n=2}^{M} \left(s^*_n + Q^L_n \right) \left[\left(\frac{R}{r_P} \right)^{n+2} - 1 \right] \Delta g_n(P) \tag{2.19}$$

and

$$\delta N^{L2}_{DWC}(P) = \frac{c}{2\pi} \iint_{\sigma_0} S_L(\psi) \left(\frac{\partial \Delta g}{\partial r}\bigg|_P \right) (H_P - H_Q) \tag{2.20}$$

where $r_P = R + H_P$, σ_0 is a spherical cap with radius ψ centered around P and is taken the same with the one in modified Stokes' formula, H_P represents the orthometric height of the computation point, and gravity gradient in point P according to Heiskanen and Moritz (1967) is calculated as

$$\left(\frac{\partial \Delta g}{\partial r}\bigg|_P \right) = \frac{R^2}{2\pi} \iint_{\sigma_0} \frac{\Delta g_Q - \Delta g_P}{l_0^3} d\sigma_Q - \frac{2}{R}\Delta g(P) \tag{2.21}$$

where $l_0 = 2Rsin\frac{\psi_{PQ}}{2}$.

2.2.2.3 The Atmospheric Correction

The atmospheric mass outside of the surface of the geoid needs to be taken into account, since the determination of geoid using Stokes' formula requires that all masses outside the geoid must be removed. The combined atmospheric correction in KTH method is given as (Kiamehr, 2006)

$$\begin{aligned} \delta N^{ATM}_{comb}(P) = & -\frac{2\pi R\rho_0}{\gamma} \sum_{n=2}^{M} \left(\frac{2}{n-1} - s_n - Q^L_n \right) H_n(P) \\ & - \frac{2\pi R\rho_0}{\gamma} \sum_{n=M+1}^{\infty} \left(\frac{2}{n-1} - \frac{n+2}{2n+1} Q^L_n \right) H_n(P) \end{aligned} \tag{2.22}$$

where ρ_0 is the atmospheric density at sea level radius and H_n is given by the Laplace harmonics for the topographic height,

$$H_n = \sum_{m=-n}^{n} H_{nm} Y_{nm} \tag{2.23}$$

2.2.2.4 The Ellipsoidal Correction

The ellipsoidal correction is applied for the spherical approximation of geoid in the computation of Stokes integral. The formulation of ellipsoidal correction in KTH method is given in (Sjöberg, 2004a)

$$\delta N_{ELL}(P) = \frac{R}{2\gamma}\sum_{n=2}^{\infty}\left(\frac{2}{n-1} - s_n - Q_n^L\right)\left(\frac{a-R}{R}\Delta g_n^{EGM}(P) + \frac{a}{R}(\delta g_e)_n\right) \quad (2.24)$$

More simple approximation of ellipsoidal correction is given in (Sjöberg, 2004b) as

$$\delta N_{ELL}(P) \approx \psi_0[(0.12 - 0.38\cos^2\theta)\Delta g + 0.17\tilde{N}sin^2\theta] \quad (2.25)$$

3. DATA PREPARATION

In this chapter, the data set that are used in the computation of regional geoid models are introduced.

3.1 Terrestrial Gravity Data

The terrestrial gravity data used in this study is the gridded version of bouguer gravity anomalies by General Directorate of Mineral Research and Exploration (MTA). The gravity measurements done by MTA are given in Fig.3.1. The complete bouguer anomalies (Fig.3.2) originally refer to Potsdam datum. They are converted to International Gravity Standardization Net 1971 (IGSN71) datum by subtracting 14 mgal from all values. Additionally, the normal gravity values γ were calculated from GRS67 ellipsoid. In order to convert the normal gravity values from GRS67 to GRS80, $\gamma_{GRS80} - \gamma_{GRS670}$ corrections are calculated using Somigliana's closed formula (Eq.3.1) or its Chebyshev approximation (Eq.3.2). The coefficients in Eq.3.2 are given in Table 3.1.

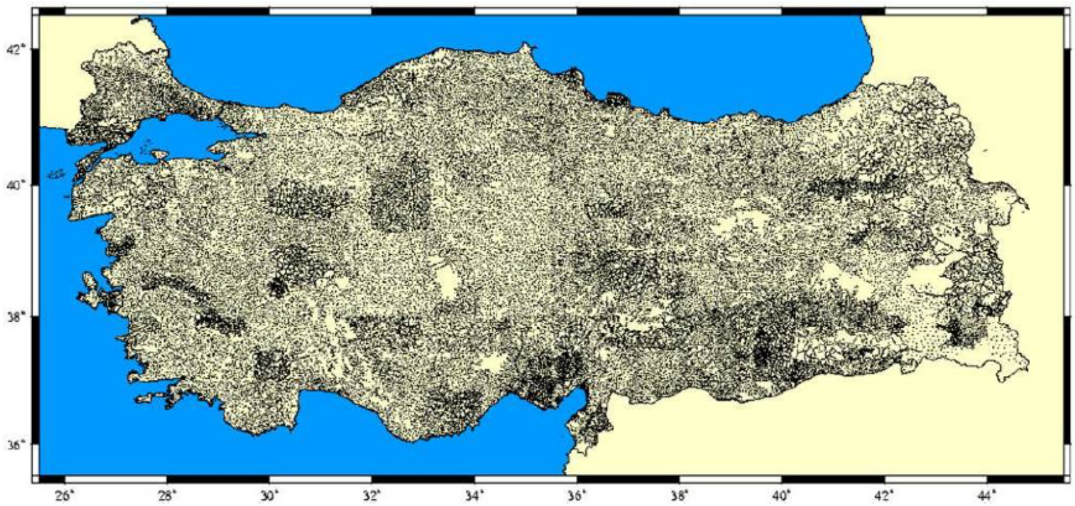

Figure 3.1. MTA gravity measurements (Simav et al., 2012)

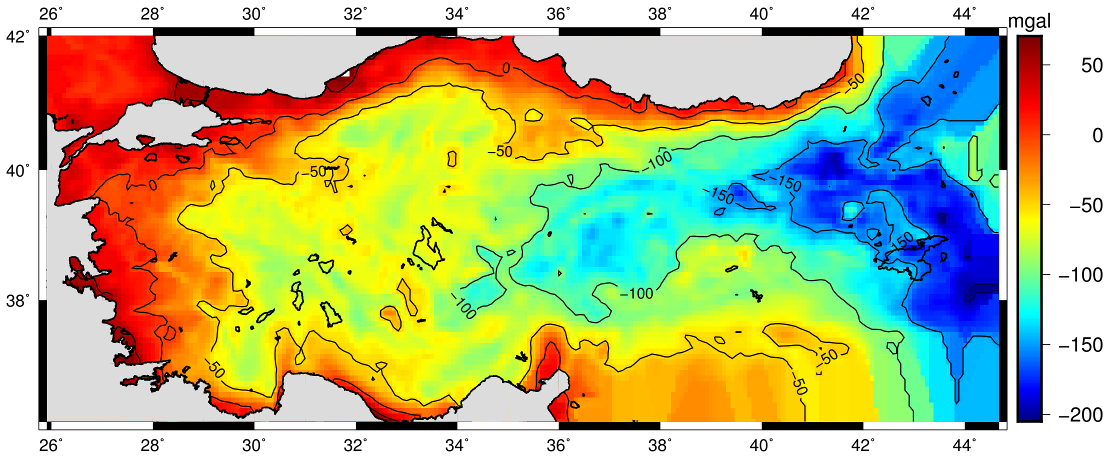

Figure 3.2. Bouger gravity anomaly map of Turkey

$$\gamma = \gamma_a \frac{1 + k \sin^2 \phi}{\sqrt{1 - e^2 \sin^2 \phi}} \tag{3.1}$$

$$\gamma = \gamma_a (1 + a \sin^2 \phi + a_1 \sin^4 \phi + a_2 \sin^6 \phi + a_3 \sin^8 \phi) \tag{3.2}$$

Table 3.1. Physical constants required to compute normal gravity using eighth-order Chebyshev approximate formula (Featherstone and Dentith, 1997).

-	GRS67	GRS80
$\gamma_a (msec^{-2})$	9.780318459	9.780326772
a	0.0052789660	0.0052790414
a_1	0.0000232725	0.0000232718
a_2	0.0000001262	0.0000001262
a_3	0.0000000007	0.0000000007

The 5' x 5' free-air gravity anomalies are derived from bouguer anomalies using a well-known Eq. (3.3) and Eq. (3.4). In the equation, Δg_{FA} and Δg_{BA} represents free-air and bouguer anomaly respectively, F is free-air correction and $2\pi k \rho H$ is bouguer plate correction where H is orthometric height (Heiskanen and Moritz, 1967).

$$\Delta g_{FA} = g_{obs} - \gamma + F \tag{3.3}$$

$$\Delta g_{BA} = g_{obs} - \gamma + F - A_{BO} + TC \tag{3.4}$$

In this equations, Δg_{FA} and Δg_{BA} represents free-air and bouguer anomaly respectively, F is free-air correction and is given as $-\frac{\partial \gamma}{\partial h} H = +0.3086H$ mgal, A_{BO} is the bouguer plate correction $A_{BO} = 2\pi k \rho H$ where H is orthometric height, and TC is the terrain correction.

The terrain correction is calculated up to Hammer J zone where the inner and outer radius of computation are 4.45 km and 6.50 km, respectively (Yen et al., 1994). The topographic density (ρ) is taken as 2.40 gr/cm^3 (Fig.3.3).

The calculated free-air gravity anomalies are given in Fig. 3.4. Since there is no ship-born gravity measurements, gravity values in water bodies are excluded from the observation data and filled with DTU13 global gravity field model (Andersen et al., 2014)(Fig. 3.5).

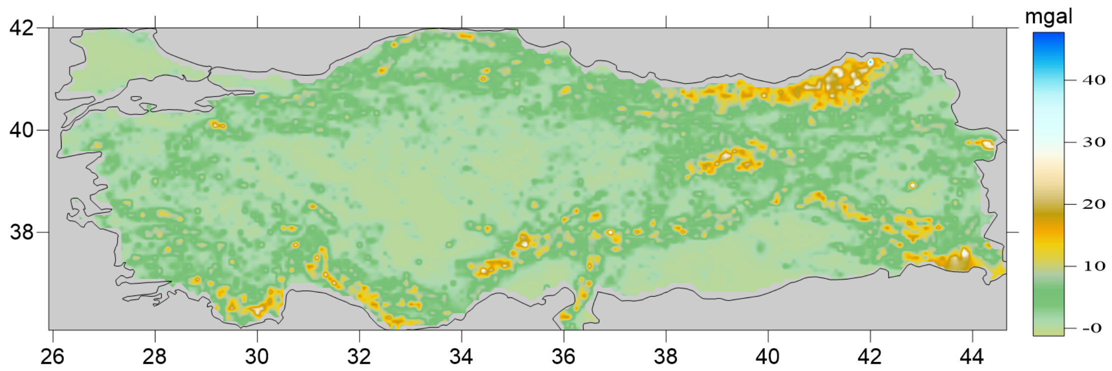

Figure 3.3. Terrain corrections in complete bouguer gravity anomalies

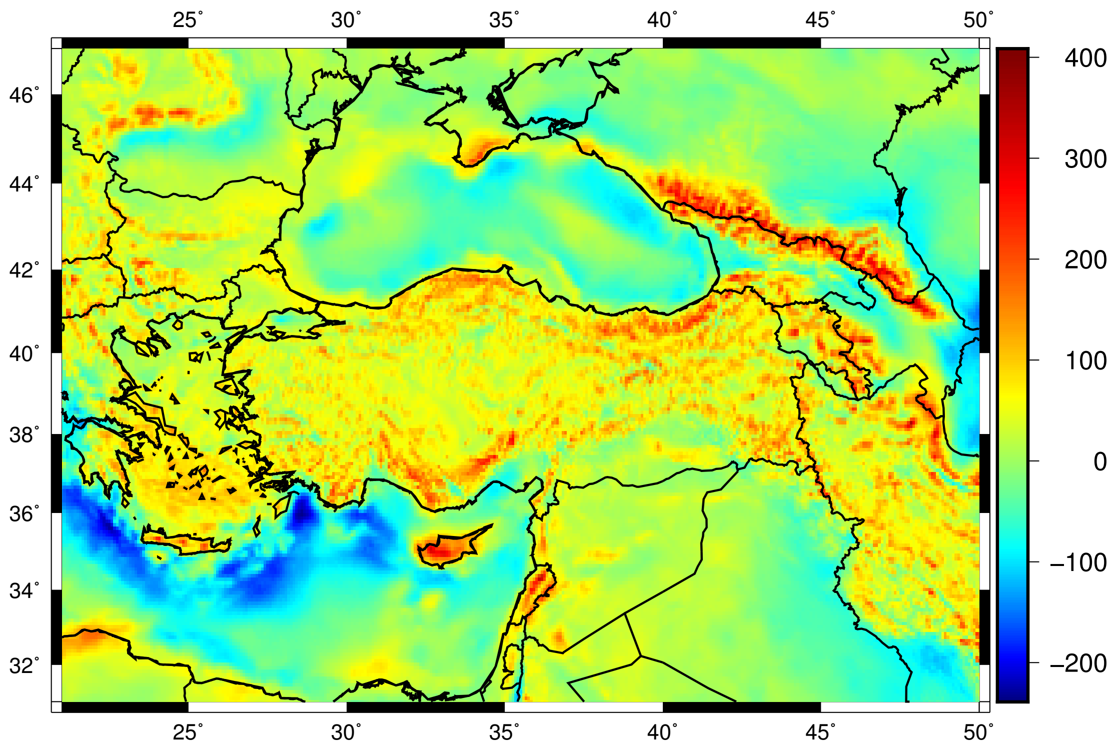

Figure 3.4. Turkey free-air gravity anomaly

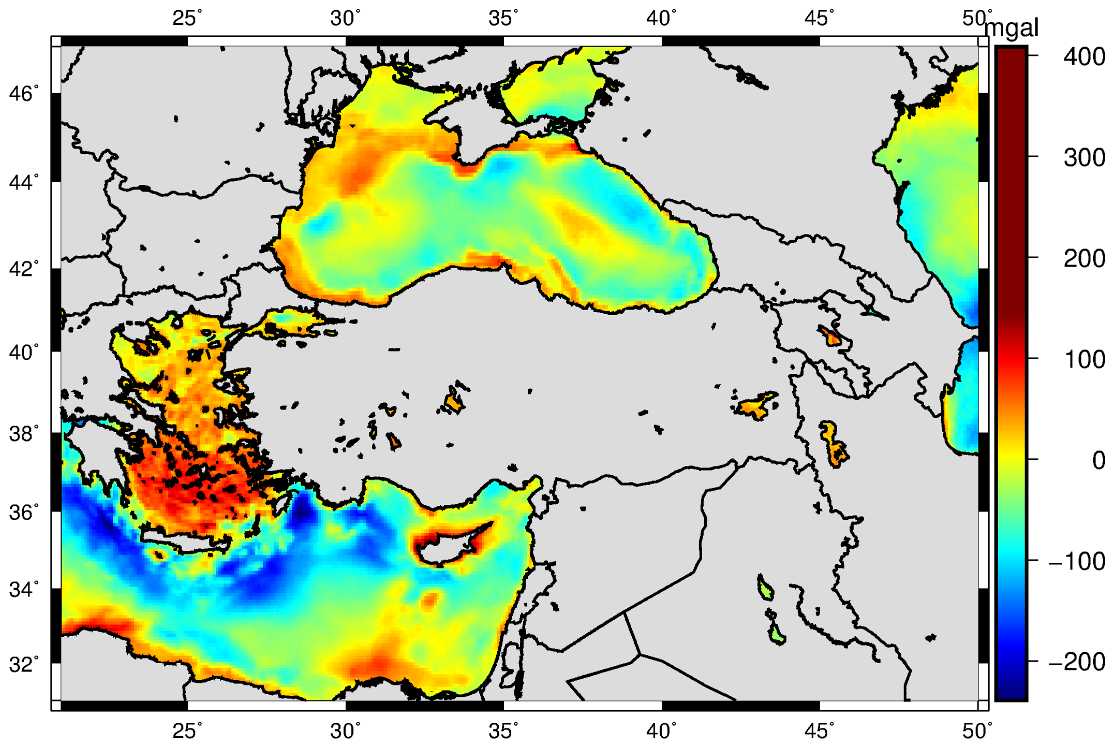

Figure 3.5. DTU13 free-air gravity data in Turkey

3.2 GNSS/Levelling Benchmarks

3.2.1 Turkish National Fundamental GPS Network (TNFGN – TUTGA)

Two GPS/Levelling data sets, 30 and 81 benchmarks belong to TUTGA network and their locations are given in Fig.3.6 and Fig. 3.7. The coordinates of the benchmarks are in ITRF96 datum and helmert orthometric heights refer to Turkish National Vertical Control Network 1999 (TUDKA99). The positional accuracy of GPS/Levelling points are reported as ±1-3 cm in horizontal and vertical components (Ayhan et al., 2002).

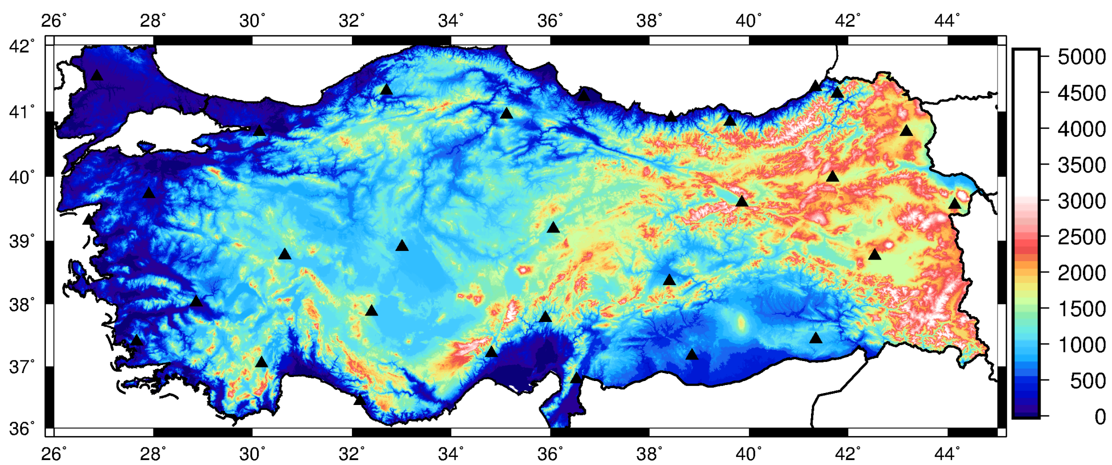

Figure 3.6. 30 TUTGA GPS/Levelling points

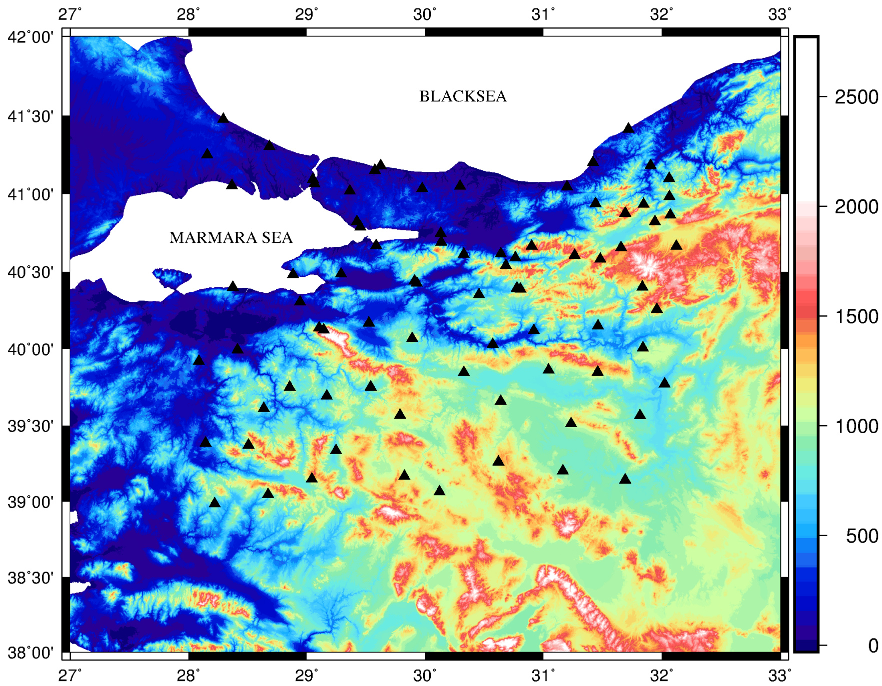

Figure 3.7. 81 TUTGA GPS/Levelling points in Marmara Region of Turkey

3.2.2 Istanbul GPS/Levelling Network (IGNA)

İstanbul is located between $40^o30'$N-$41^o30'$N latitudes and 27^o30E-$30^o00'$E longitudes. The GPS/Levelling benchmarks were collected in GPS triangulation densification network project, Istanbul GPS Triangulation Network 2005(IGNA). The first project was conducted in 1999, but after earthquakes in Gölcük (August 17, 1999) and Düzce (November 12,1999) the benchmarks were deformed and renewed between years 2005 to 2006. The density of the points in the network is 1 benchmark per 20 km^2 (Ayan et al., 2006; Erol, 2007). There are 1024 benchmarks in the network. In this study, only 200 homogeneously chosen benchmarks (Fig.3.8) are used in the validation. Besides the result of the validation remained uneffected when the whole network is used.

3.2.3 İzmir 2001 GPS/Levelling Network

İzmir is located between $38^o00'$N–$38^o36'$N latitudes and $26^o30'$E–$27^o30'$E longitudes in the west part of Turkey near Aegean Sea. The GPS/levelling benchmarks were measured in the İzmir Geodetic Reference System (İzJRS-2001) project in 2001 (Ayan et al., 2001). The density of the points in the network is 1 benchmark per 8 km^2 (Erol, 2007) (Fig. 3.9).

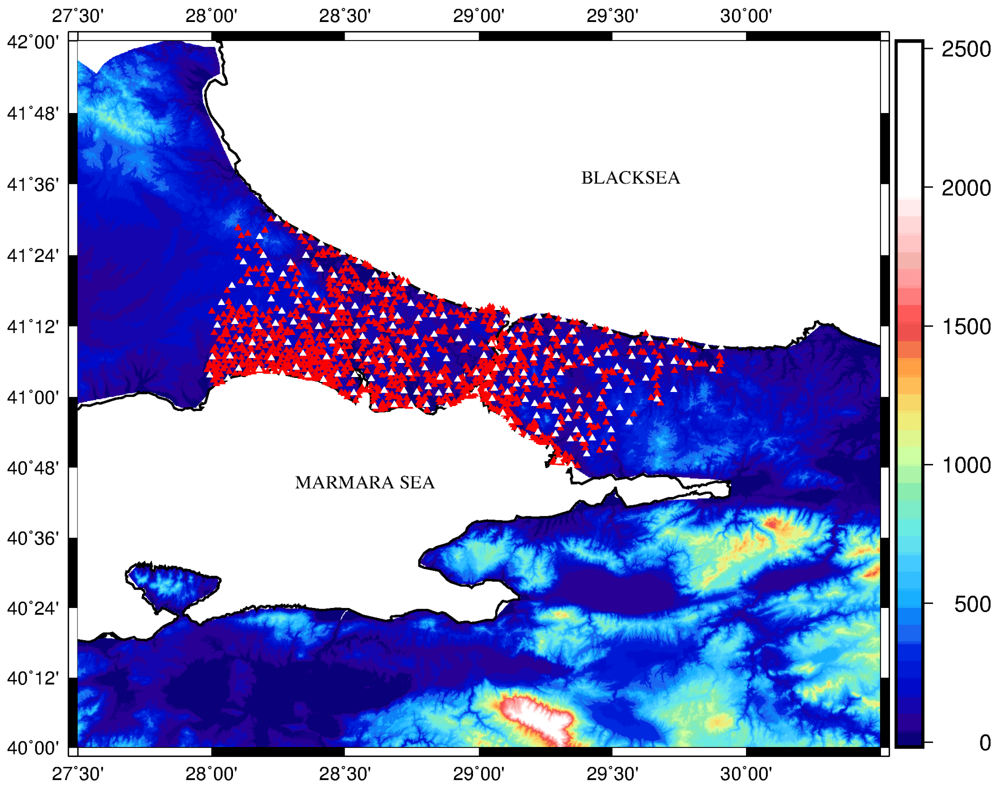

Figure 3.8. 1204 IGNA benchmarks in Marmara Region of Turkey (Red marks show the whole network, gray marks show 200 benchmarks used.)

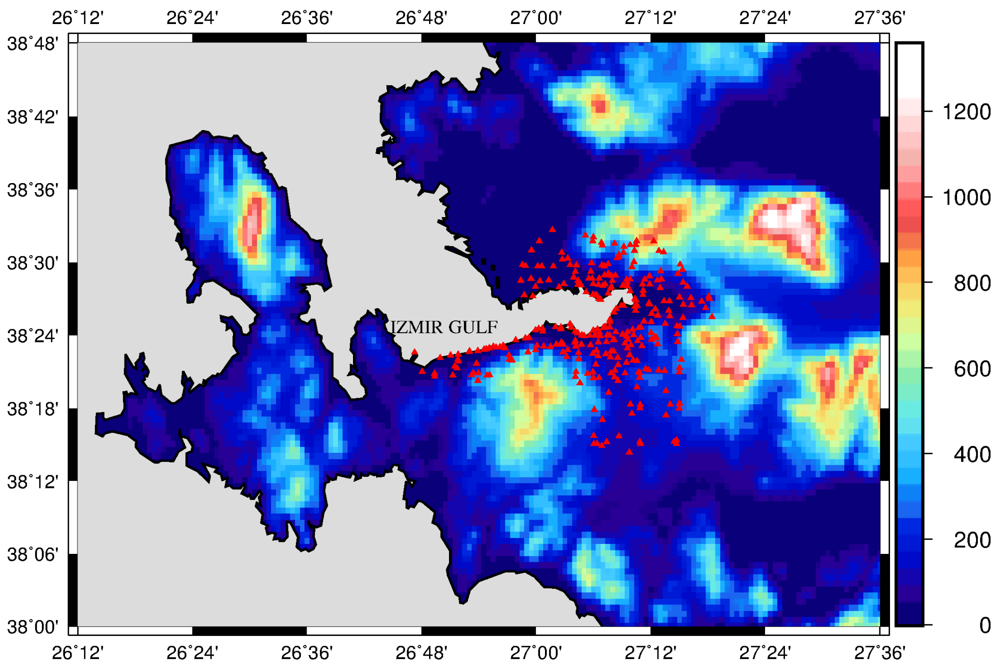

Figure 3.9. 301 İzmir benchmarks in west of Turkey

3.3 Global Geopotential Models

The Gravity Field and Steady-State Ocean Circulation Explorer (GOCE) satellite was launched on March 17th, 2009 by European Space Agency (ESA) and completed its mission on November 11th, 2013, about two and a half year more than planned. The main objectives of the GOCE mission are the determination of the geoid with 1-2 cm and gravity anomalies with 1 mGal accuracy. This corresponds to 100 km spatial resolution and spherical harmonic degree and order 200 (Pail et al., 2011; Rummel et al., 2011).

Various global geopotential models have been released using GOCE observations. Latest releases include 4 years of GOCE observations. Three different approaches, namely direct, time-wise and space-wise, are applied for the analysis of the GOCE observations and computation of satellite-only models. The methodology of these approaches are described in Pail et al. (2011).

Considering that the quality of the global geopotential models over an area directly affects the accuracy of the gravimetric geoid, it is quite important to evaluate the performance of the models to find out the best model that fits to local data sets. The assessment of the GOCE-based models were conducted with both local and global data sets by many authors (e.g. Gruber et al., 2011; Gruber, 2014; Tocho and Vergos, 2015; Vergos et al., 2015). The validation of earlier releases of the GOCE-based models in Turkey can be found in Ince et al. (2014) and Avsar et al. (2015). These studies revealed that the performance of GOCE-based gravity field models show significant improvements in the medium-frequency band.

3.3.1 Validation of GOCE-only and GOCE/GRACE Global Geopotential Models

All releases of satellite-only models that are developed based on direct, time-wise and space-wise approaches are included in the validation. For the GOCE/GRACE models, GOCO and GOGRA models are included to the assessment (Table 3.2). Aside from satellite-only models, the performance of EGM2008 combined model is compared with other models.

In order to analyze the signal powers and error content of the global geopotential models in different degrees, signal and error degree variances of the models in terms of geoid heights are calculated using Eq. 3.5 and Eq. 3.6. Additionally, the cumulative global geoid errors are computed using Eq. 3.7 In order to make the discrepancies in the signal powers more visible, the degrees below 120 are excluded from the graphics (from Fig. A.1 to Fig. A.12).

$$\sigma_n(N) = R\sqrt{\sum_{m=0}^{n}(C_{nm}^2 + S_{nm}^2)} \tag{3.5}$$

$$\sigma_{c_n}(N) = R\sqrt{\sum_{m=0}^{n}(\sigma_{C_{nm}}^2 + \sigma_{S_{nm}}^2)} \tag{3.6}$$

$$RMSE(N) = \sqrt{\sum_{n=2}^{n_{max}} \sigma_{c_n}^2} \tag{3.7}$$

where $\sigma_n(N)$, $\sigma_{c_n}(N)$ and $RMSE(N)$ are signal degree variance, error degree variance and cumulative global geoid errors respectively.

In the assessment of a global geopotential model in an absolute sense, a model and observation data must be spectrally comparable to eliminate the omission error. Though, observation data contains the full spectrum of the gravity signal, geopotential model represents only the part of the spectrum constrained by its maximum degree (Hirt et al, 2013). Therefore, one should either apply a low-pass filter to the observation data to remove the high-frequency part of the spectrum or apply the spectral enhancement (Eq.3.8) to the geopotential model by completing the high-frequency part of the spectrum using another geopotential model with higher degree/order and residual terrain model (RTM) effect (Carrion et al., 2015).

In this study, only the spectral enhancement method is applied. EGM2008 is used for completing the high degree and orders of the global geopotential models in the computation of geoid undulations and free-air gravity anomalies. Very high-frequency components of the gravity field are obtained by calculating RTM effect using TC module of GRAVSOFT program (Tscherning et al., 1992).

$$N_{res} = N_{obs} - N^{GOCE}\Big|_{2}^{n} - N^{EGM2008}\Big|_{n+1}^{2190} - N_{RTE} \tag{3.8}$$

Table 3.2. Characteristics of the global geopotential models.

Model	Max d/o	Data	Reference
GO_CONS_GFC_2_DIR_R1	240	S(Goce)	Bruinsma et al. (2010)
GO_CONS_GFC_2_DIR_R2	240	S(Goce)	Bruinsma et al. (2010)
GO_CONS_GFC_2_DIR_R3	240	S(Goce, Grace, Lageos)	Bruinsma et al. (2010)
GO_CONS_GFC_2_DIR_R4	260	S(Goce, Grace, Lageos)	Bruinsma et al. (2013)
GO_CONS_GFC_2_DIR_R5	300	S(Goce, Grace, Lageos)	Bruinsma et al. (2013)
GO_CONS_GFC_2_TIM_R1	224	S(Goce)	Pail et al. (2010a)
GO_CONS_GFC_2_TIM_R2	250	S(Goce)	Pail et al. (2011)
GO_CONS_GFC_2_TIM_R3	250	S(Goce)	Pail et al. (2011)
GO_CONS_GFC_2_TIM_R4	250	S(Goce)	Pail et al. (2011)
GO_CONS_GFC_2_TIM_R5	280	S(Goce)	Brockmann et al. (2014)
GO_CONS_GFC_2_SPW_R1	210	S(Goce)	Migliaccio et al. (2011)
GO_CONS_GFC_2_SPW_R2	240	S(Goce)	Migliaccio et al. (2011)
GO_CONS_GFC_2_SPW_R4	280	S(Goce)	Gatti et al. (2014)
GOCO01S	224	S(Goce, Grace)	Pail et al. (2010b)
GOCO02S	250	S(Goce, Grace)	Goiginger et al. (2011)
GOCO03S	250	S(Goce, Grace)	Mayer-Gürr et al. (2012)
GOCO05S	250	S(Goce, Grace)	Mayer-Guerr and the GOCO Team (2015)
GOGRA02S	230	S(Goce, Grace)	Yi et al. (2013)
GOGRA04S	230	S(Goce, Grace)	Yi et al. (2013)
EGM2008	2190	S(Grace), G, A	Pavlis et al. (2012)

S: Satellite tracking data, G: Gravity data, A: Altimetry data, LAGEOS: LAser GEOdynamics Satellites

3.4 Digital Terrain Model (DTM)

Shuttle Radar Topography Mission (SRTM) mission is a joint project of National Geospatial-Intelligence Agency (NGA), National Aeronautics and Space Administration (NASA) and National Imagery and Mapping Agency (NIMA) that aimed the collection of elevation data between the latitudes 60^oN and 56^oS (SRTM, 2000). Originally, the data is disributed as $1''$ resolution for USA and $3''$ resolution for other countries.

The study conducted by Feizabadi (2014) shows that the SRTM performs better than ACE2 and ASTER data. In this study, $30''$ resolution SRTM data that includes bathymetry globally, SRTM30Plus (Becker et al., 2009), is used (Fig. 3.10).

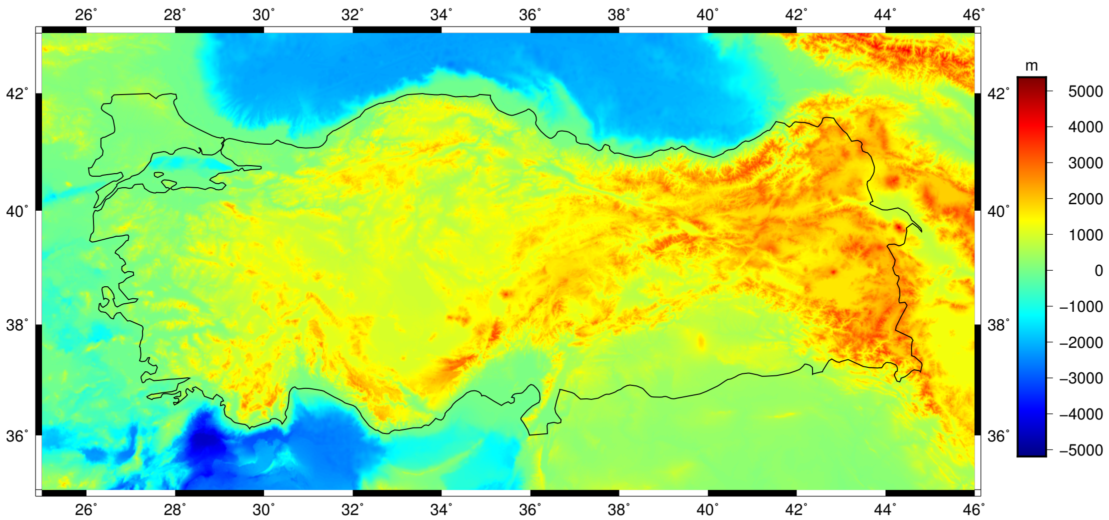

Figure 3.10. SRTM30plus data in Turkey

4. NUMERICAL EVALUATION OF GEOID HEIGHTS

In this chapter, the numerical results of the assessment of global geopotential models and the validation of computed regional geoid models using RCR and KTH methods are given.

4.1 Global Geopotential Models

In this study, the recent releases of GOCE-based global geopotential models are evaluated in Turkey. Geoid heights that are computed from global geopotential models are compared with 30 GPS/Levelling benchmarks distributed through the whole Turkey, 200 co-located GPS/Levelling benchmarks in Istanbul region, 81 GPS/Levelling benchmarks located in Marmara region and 301 GPS/Levelling benchmarks in İzmir by using spectral enhancement method.

The statistics of the absolute differences between geoid heights calculated using spectrally enhanced geopotential models and corresponding geoid heights in 30 TUTGA GPS/Levelling benchmarks are given in Table 4.1 and the standard deviations of the residual geoid heights are shown in Figures 4.1 (DIR), 4.2 (SPW), 4.3 (TIM) and 4.4 (GOCE/GRACE). In case of DIR models, there is no significant improvement over EGM2008 in terms of standard deviations of geoid height differences for 1^{st}, 2^{nd} and 3^{rd} releases. It is obvious that the quality of the coefficients between 180 and 240 are improved in 4^{th} and 5^{th} releases, but it resulted up to 1 cm improvement only in degree 220. In degree 235, TIM models shows the highest improvements in 4^{th} and 5^{th} releases with 1.3 cm and 2 cm respectively. Similar to DIR models, TIM R4 and TIM R5 are improved significantly compared to the earlier releases. The improvements in the SPW models are statistically insignificant. Since SPW R4 does not contain low-orbit GOCE SGG data and contains 26 months of GOCE data, it does not out-perform the EGM2008 between degrees 200 and 230 like DIR R5 and TIM5 do. In Figure 4.2, the enhancement in the quality of the coefficient is clearly seen in SPW R4 above 180 d/o compared to SPW R1 and SPW R2. The first three releases of

GOCO models shows the same performance and the magnitude of the improvements is 5-6 mm in degree 180. But the advancement in GOCO05S model beyond degree 200 is clearly seen and the improvement w.r.t. EGM2008 reaches 1.9 cm at degree 235 (Figure 4.4). The performance of GOGRA02S and GOGRA04S models are quite similar to GOCO05S upto degree 215. The best results for GOGRA02S and GOGRA04S are achieved in degree 220 with 1 cm and 1.7 cm respectively.

Table 4.1. Statistics of the differences between geoid heights computed from global geopotential models spectrally enhanced with EGM2008 and corresponding geoid heights obtained from 30 TUTGA points in Turkey[unit: m].

Model	Degree	Min	Max	Mean	STD	RMSE
DIR_R1	80	-0.136	0.813	0.445	0.159	0.473
DIR_R2	80	-0.132	0.819	0.446	0.159	0.474
DIR_R3	180	-0.142	0.729	0.454	0.156	0.481
DIR_R4	205	-0.134	0.674	0.435	0.152	0.461
DIR_R5	220	-0.063	0.687	0.435	0.151	0.460
TIM_R1	70	-0.122	0.792	0.444	0.156	0.470
TIM_R2	185	-0.159	0.677	0.443	0.157	0.470
TIM_R3	185	-0.171	0.707	0.448	0.159	0.475
TIM_R4	235	0.055	0.681	0.423	0.148	0.448
TIM_R5	235	-0.027	0.622	0.426	0.141	0.448
SPW_R1	80	-0.138	0.806	0.445	0.157	0.472
SPW_R2	80	-0.126	0.816	0.452	0.159	0.479
SPW_R4	80	-0.133	0.817	0.448	0.159	0.475
GOCO01S	180	-0.070	0.698	0.441	0.155	0.468
GOCO02S	180	-0.110	0.684	0.445	0.155	0.471
GOCO03S	180	-0.124	0.696	0.448	0.156	0.474
GOCO05S	235	-0.057	0.622	0.418	0.142	0.442
GOGRA02S	220	-0.013	0.719	0.457	0.151	0.481
GOGRA04S	220	-0.039	0.681	0.442	0.144	0.464
EGM2008	2190	-0.145	0.819	0.445	0.161	0.473

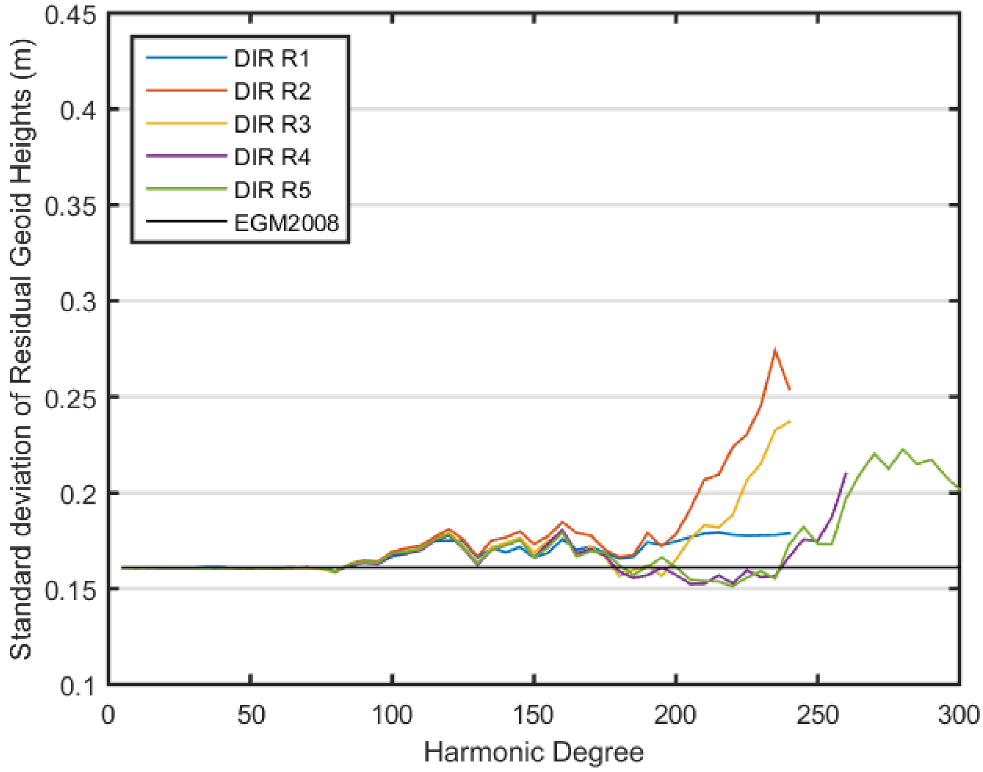

Figure 4.1. Standard deviation of geoid differences between DIR models and 30 TUTGA GPS-Levelling points.

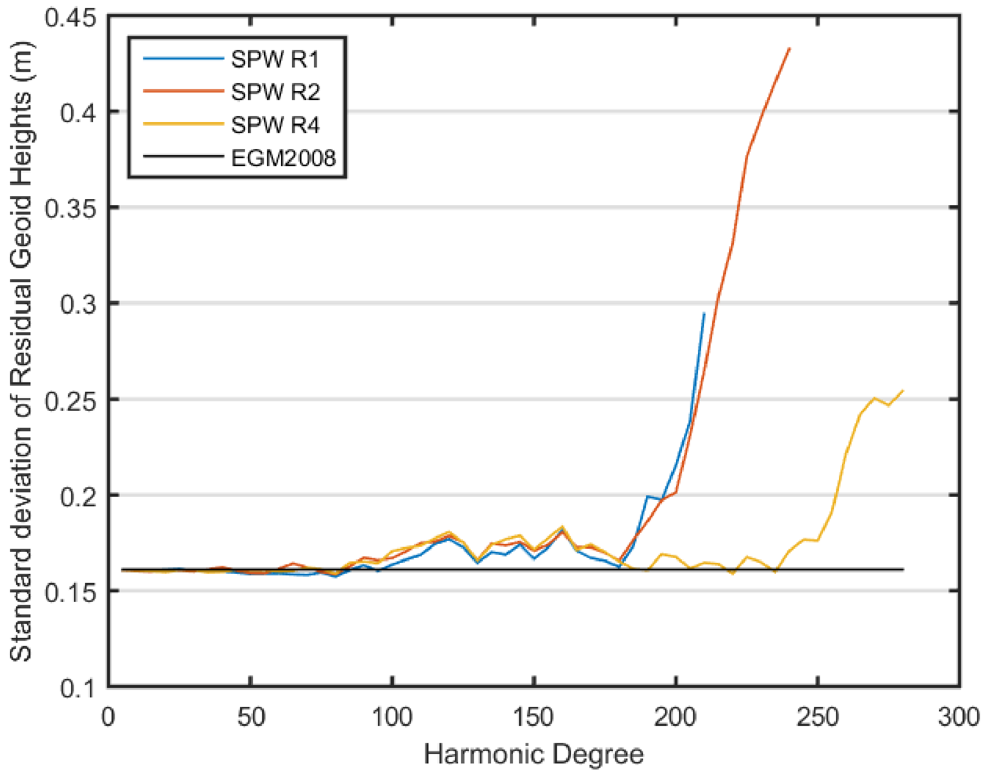

Figure 4.2. Standard deviation of geoid differences between SPW models and 30 TUTGA GPS-Levelling points.

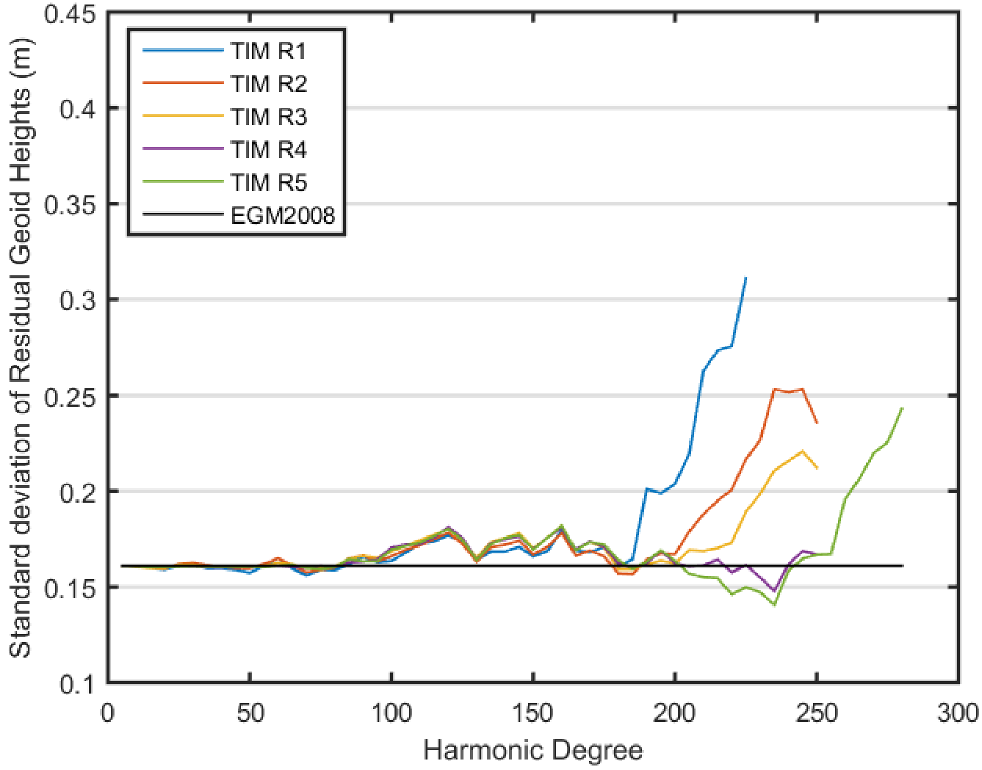

Figure 4.3. Standard deviation of geoid differences between TIM models and 30 TUTGA GPS-Levelling points.

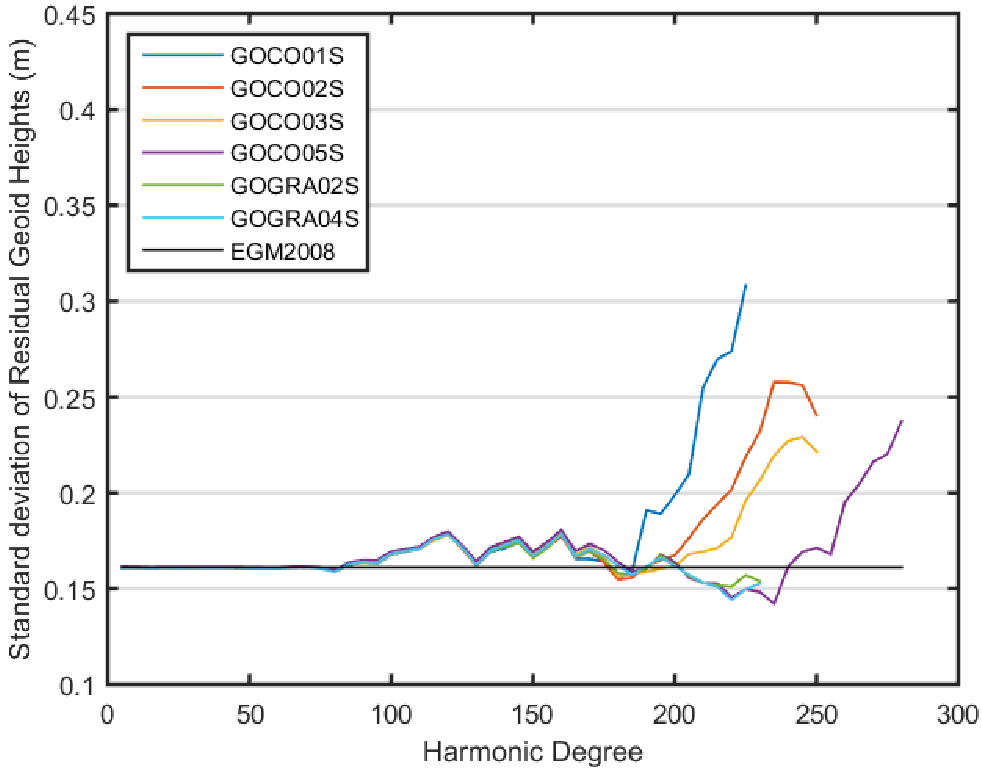

Figure 4.4. Standard deviation of geoid differences between GOGRA and GOCO models and 30 TUTGA GPS-Levelling points.

The statistics of the absolute differences between geoid heights calculated using spectrally enhanced geopotential models and corresponding geoid heights in 81 TUTGA GPS/Levelling benchmarks in Marmara region are given in Table 4.2 and the standard deviations of the residual geoid heights are shown in Figures 4.5 (DIR), 4.6 (SPW), 4.7 (TIM) and 4.8 (GOCE/GRACE). The enhancement over EGM2008 is much more clear in comparison with 81 benchmarks. The magnitude of the improvements for all models is about 4 cm. Consistently, the latest releases of the models shows the best performance in degree 155. The enhancement in the quality of the coefficient for all type of models is quite clear in the latest releases.

Table 4.2. Statistics of the differences between geoid heights computed from global geopotential models spectrally enhanced with EGM2008 and corresponding geoid heights obtained from 81 TUTGA points in Turkey[unit: m].

Model	Degree	Min	Max	Mean	STD	RMSE
DIR R1	105	0.093	0.914	0.452	0.132	0.471
DIR R2	155	0.227	0.887	0.479	0.127	0.496
DIR R3	155	0.222	0.889	0.472	0.119	0.487
DIR R4	155	0.196	0.884	0.471	0.122	0.487
DIR R5	155	0.192	0.875	0.467	0.120	0.482
TIM R1	125	0.131	0.875	0.452	0.132	0.471
TIM R2	125	0.127	0.862	0.449	0.130	0.468
TIM R3	155	0.206	0.886	0.465	0.120	0.480
TIM R4	155	0.200	0.869	0.468	0.120	0.483
TIM R5	155	0.195	0.877	0.466	0.120	0.481
SPW R1	135	0.180	0.885	0.463	0.125	0.480
SPW R2	155	0.225	0.902	0.481	0.125	0.497
SPW R4	155	0.221	0.889	0.475	0.118	0.489
GOCO01S	155	0.196	0.866	0.470	0.125	0.486
GOCO02S	155	0.198	0.868	0.469	0.125	0.486
GOCO03S	155	0.206	0.885	0.472	0.120	0.487
GOCO05S	155	0.189	0.868	0.461	0.119	0.442
GOGRA02S	155	0.199	0.872	0.469	0.121	0.485
GOGRA04S	155	0.196	0.871	0.469	0.120	0.485
EGM2008	2190	-0.002	0.947	0.454	0.157	0.480

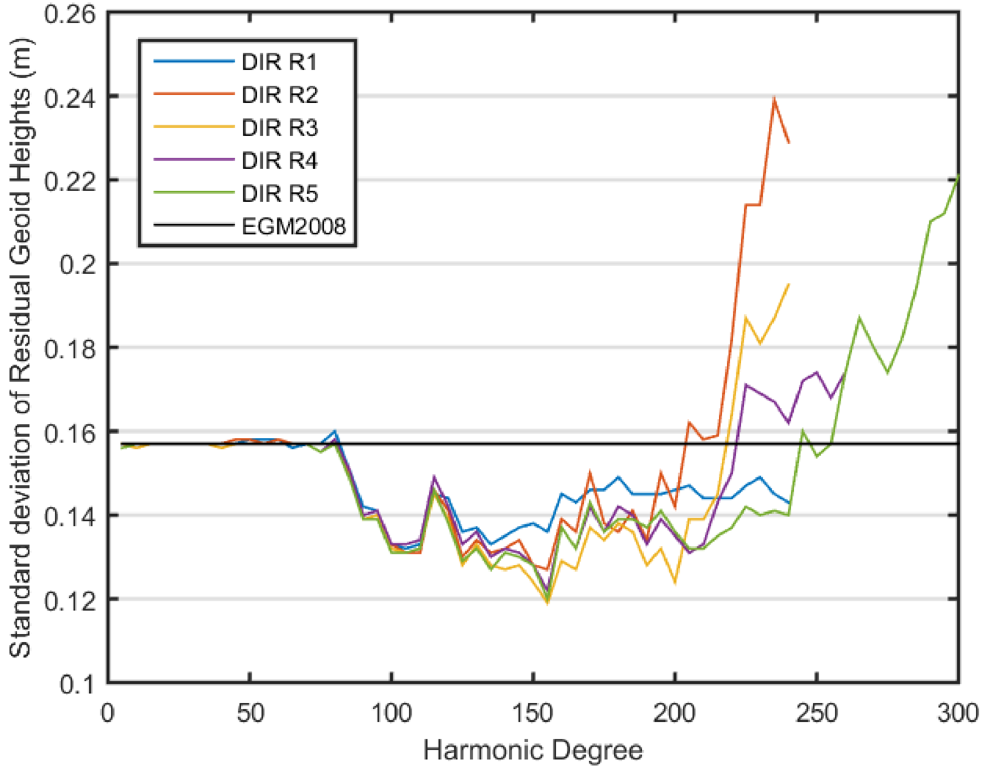

Figure 4.5. Standard deviation of geoid differences between DIR models and 81 TUTGA GPS-Levelling points.

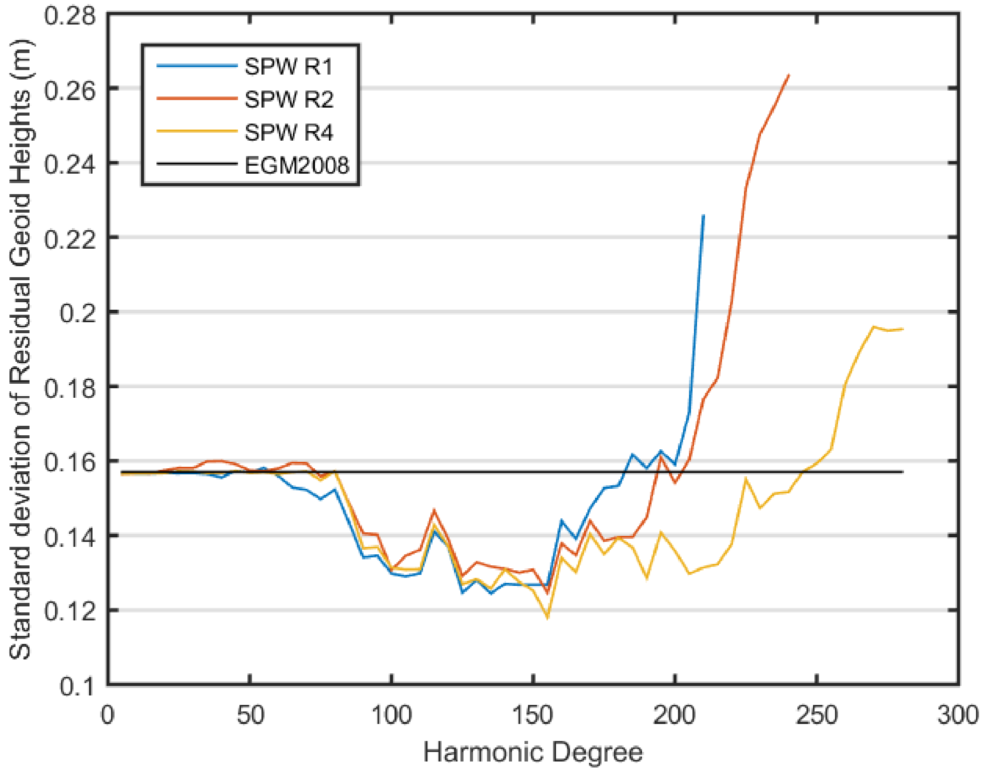

Figure 4.6. Standard deviation of geoid differences between SPW models and 81 TUTGA GPS-Levelling points.

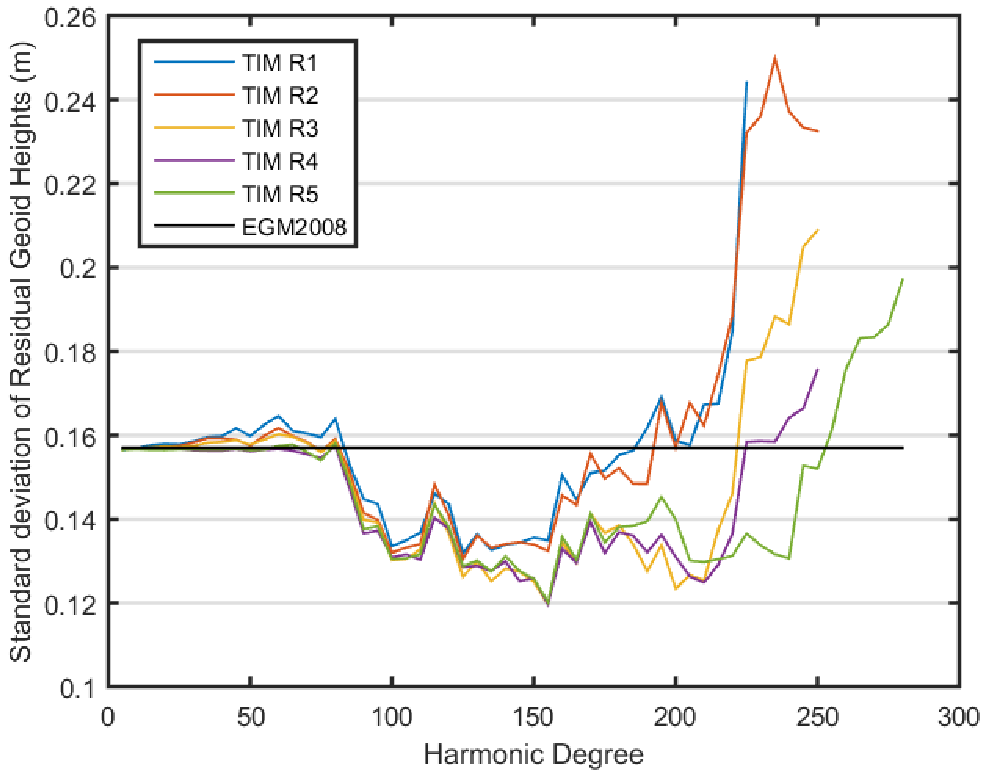

Figure 4.7. Standard deviation of geoid differences between TIM models and 81 TUTGA GPS-Levelling points.

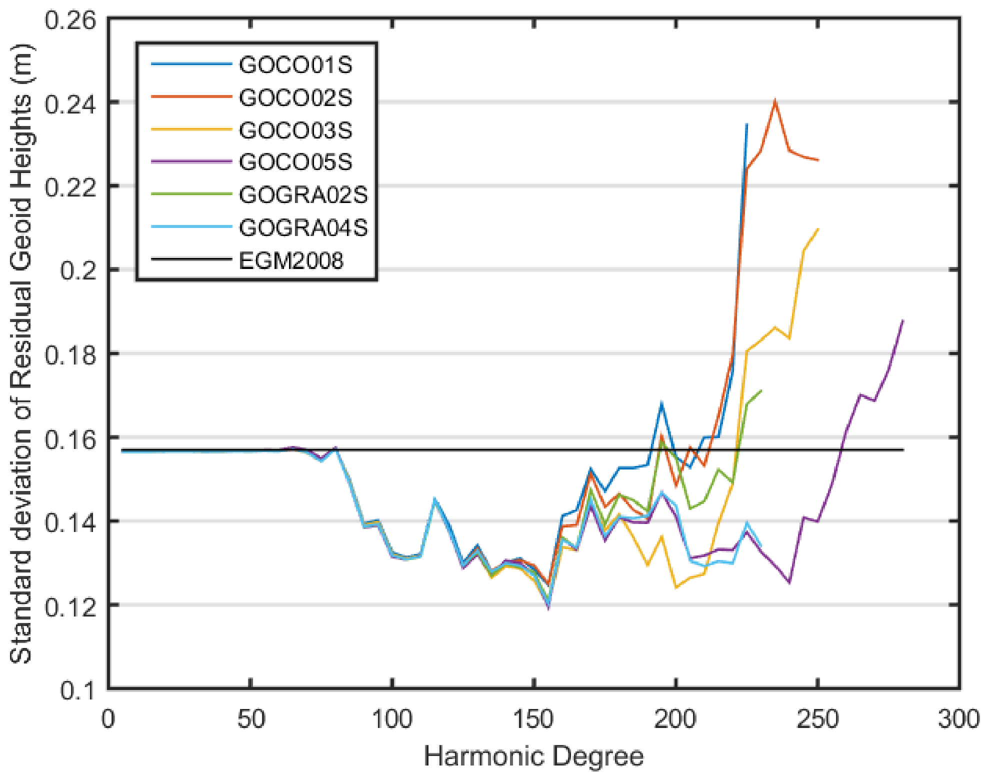

Figure 4.8. Standard deviation of geoid differences between GOGRA and GOCO models and 81 TUTGA GPS-Levelling points.

The statistics of the absolute differences between geoid heights calculated using spectrally enhanced geopotential models and corresponding geoid heights in 200 IGNA benchmarks and 301 İzmir benchmarks are given in Table 4.3 and Table 4.4 respectively. The corresponding figures that show the standard deviations can be found from Figure 4.9 to Figure 4.16. These results also show improvements over EGM2008. Especially, the comparison in İzmir region shows how well the geopotential models fit in the western part of the Turkey. However, these two test areas are rather small when the whole Turkey is considered. Thus, the validations in these regions show local performance of the geopotential models.

Table 4.3. Statistics of the differences between geoid heights computed from global geopotential models spectrally enhanced with EGM2008 and corresponding geoid heights obtained from 200 IGNA points in Istanbul region [unit: m].

Model	Degree	Min	Max	Mean	STD	RMSE
DIR_R1	235	-0.116	0.730	0.338	0.129	0.362
DIR_R2	195	-0.054	0.776	0.351	0.106	0.367
DIR_R3	220	-0.043	0.760	0.355	0.102	0.370
DIR_R4	225	-0.106	0.718	0.353	0.120	0.373
DIR_R5	280	-0.137	0.631	0.260	0.108	0.281
TIM_R1	215	0.023	0.851	0.384	0.109	0.399
TIM_R2	215	-0.009	0.780	0.332	0.103	0.347
TIM_R3	220	-0.105	0.689	0.327	0.111	0.345
TIM_R4	200	-0.124	0.724	0.335	0.123	0.357
TIM_R5	200	-0.136	0.683	0.314	0.119	0.336
SPW_R1	195	-0.194	0.638	0.263	0.128	0.293
SPW_R2	215	0.030	0.820	0.434	0.102	0.446
SPW_R4	200	-0.102	0.738	0.341	0.116	0.360
GOCO01S	215	0.006	0.836	0.384	0.107	0.399
GOCO02S	215	-0.013	0.781	0.338	0.101	0.353
GOCO03S	220	-0.098	0.696	0.327	0.112	0.346
GOCO05S	200	-0.146	0.671	0.308	0.120	0.331
GOGRA02S	200	-0.088	0.708	0.344	0.111	0.361
GOGRA04S	225	-0.145	0.655	0.312	0.118	0.334
EGM2008	2190	0.056	0.946	0.525	0.158	0.549

Table 4.4. Statistics of the differences between geoid heights computed from global geopotential models spectrally enhanced with EGM2008 and corresponding geoid heights obtained from 301 İzmir points in Istanbul region [unit: m].

Model	Degree	Min	Max	Mean	STD	RMSE
DIR R1	240	0.064	0.642	0.424	0.077	0.430
DIR R2	235	0.011	0.506	0.348	0.068	0.354
DIR R3	235	-0.001	0.456	0.297	0.067	0.305
DIR R4	235	-0.035	0.467	0.281	0.065	0.288
DIR R5	245	0.048	0.559	0.364	0.068	0.371
TIM R1	224	-0.176	0.310	0.109	0.068	0.128
TIM R2	225	-0.039	0.468	0.273	0.066	0.281
TIM R3	230	-0.086	0.421	0.250	0.067	0.259
TIM R4	235	-0.010	0.541	0.337	0.071	0.345
TIM R5	245	-0.032	0.520	0.306	0.073	0.315
SPW R1	200	0.102	0.660	0.484	0.075	0.489
SPW R2	230	-0.220	0.210	0.050	0.064	0.081
SPW R4	240	-0.018	0.539	0.326	0.073	0.334
GOCO01S	224	-0.191	0.272	0.080	0.065	0.103
GOCO02S	225	-0.049	0.455	0.258	0.066	0.266
GOCO03S	230	-0.108	0.389	0.217	0.066	0.226
GOCO05S	245	-0.026	0.533	0.320	0.073	0.328
GOGRA02S	230	0.038	0.574	0.375	0.069	0.381
GOGRA04S	230	0.033	0.563	0.365	0.068	0.371
EGM2008	2190	0.134	0.752	0.513	0.089	0.521

As a result of the assessment of global geopotential models, DIR R5, TIM R5 and GOCO05S models show the best agreement in Turkey and they are used in the regional geoid computations together with EGM2008.

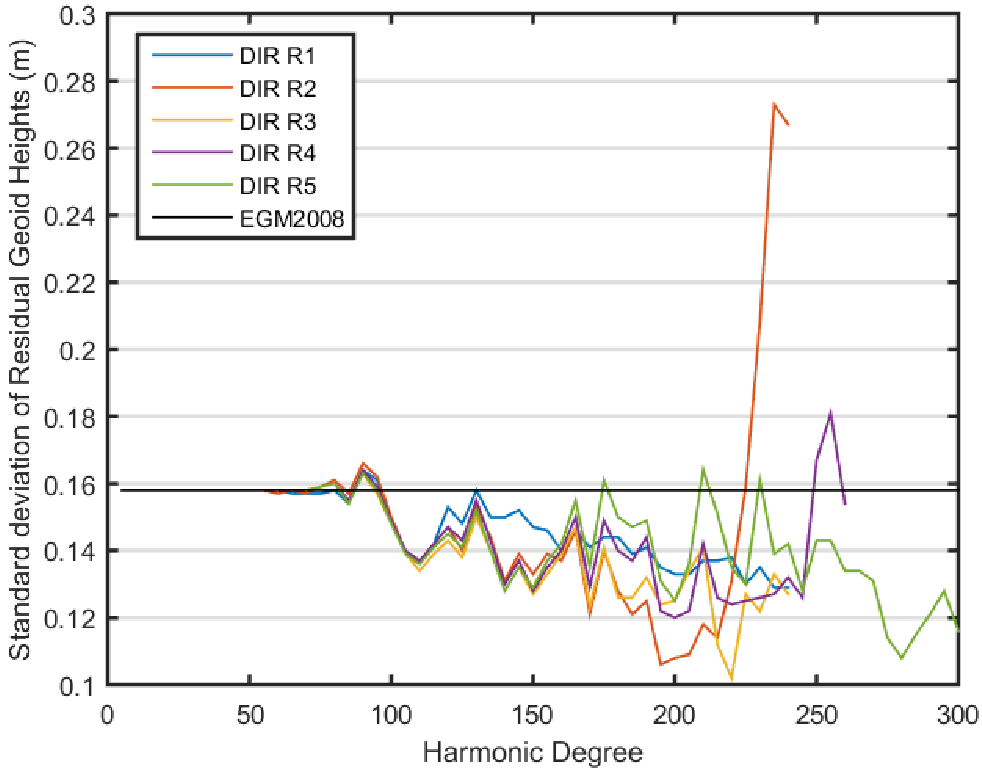

Figure 4.9. Standard deviation of geoid differences between DIR models and IGNA GPS-Levelling points.

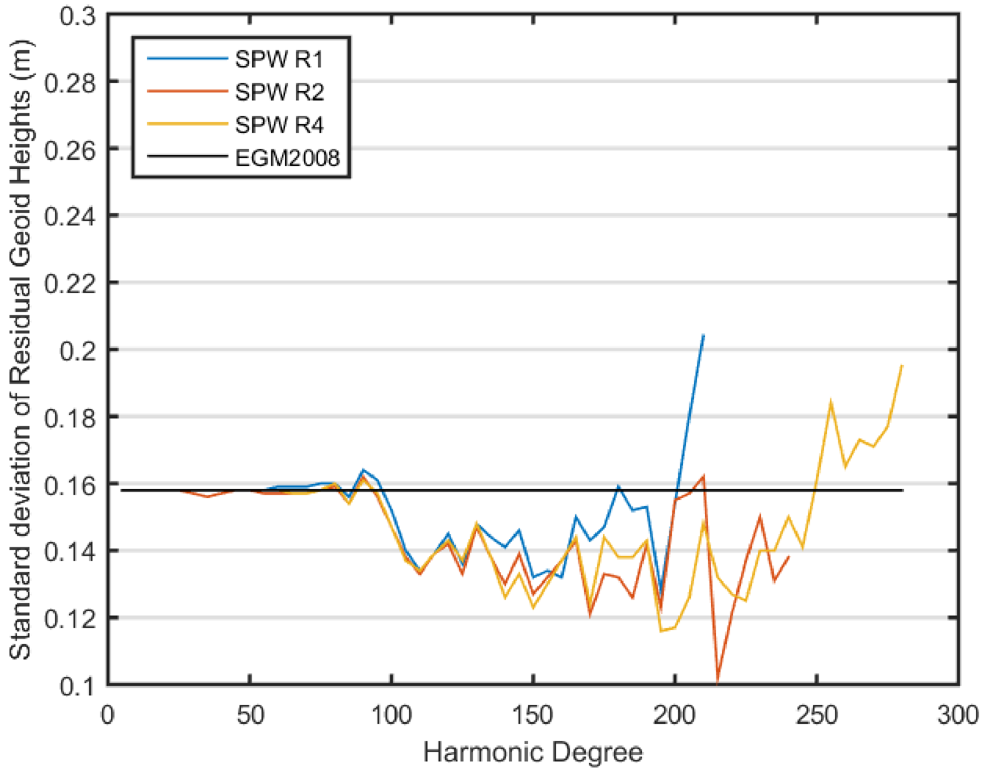

Figure 4.10. Standard deviation of geoid differences between SPW models and IGNA GPS-Levelling points.

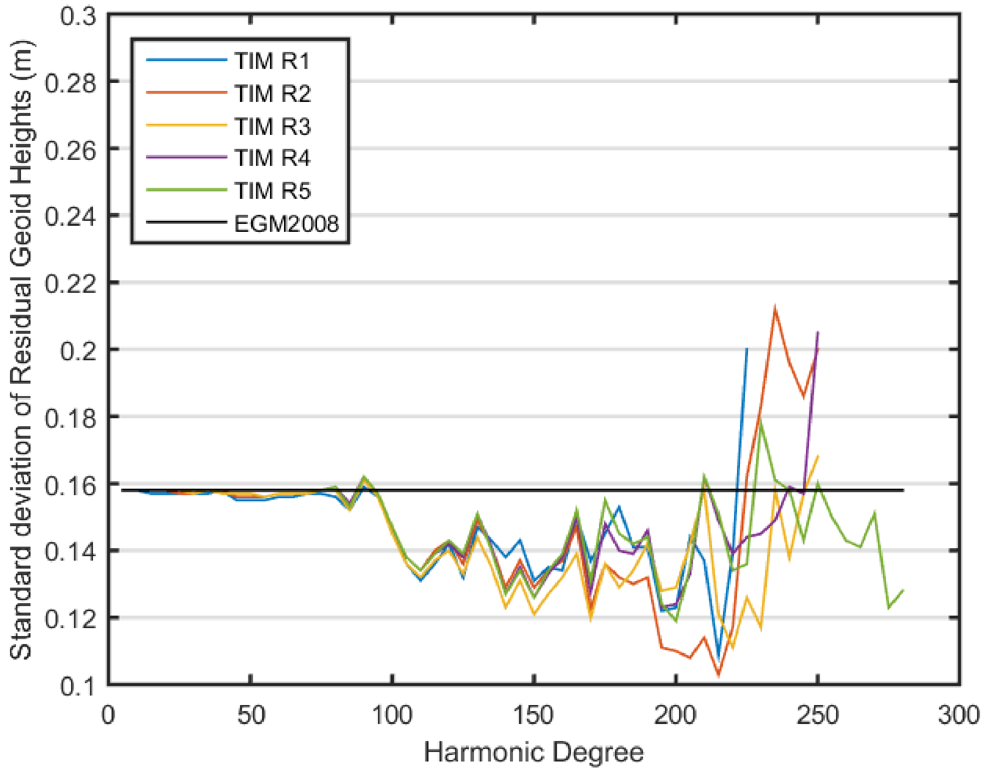

Figure 4.11. Standard deviation of geoid differences between TIM models and IGNA GPS-Levelling points.

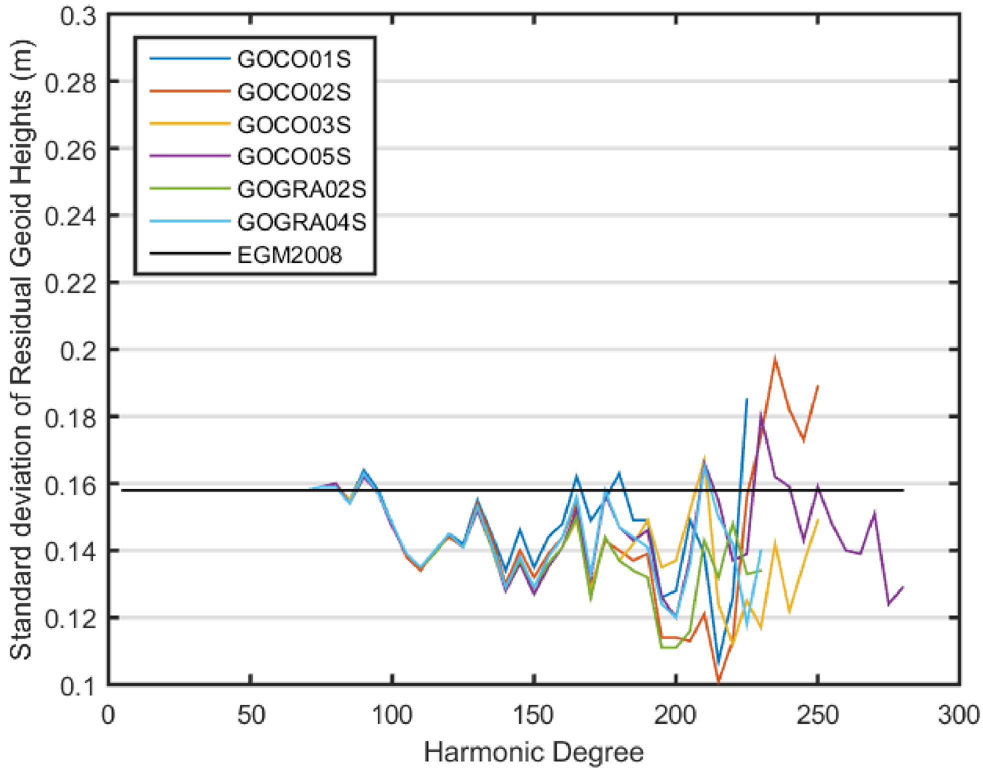

Figure 4.12. Standard deviation of geoid differences between GOGRA and GOCO models and IGNA GPS-Levelling points.

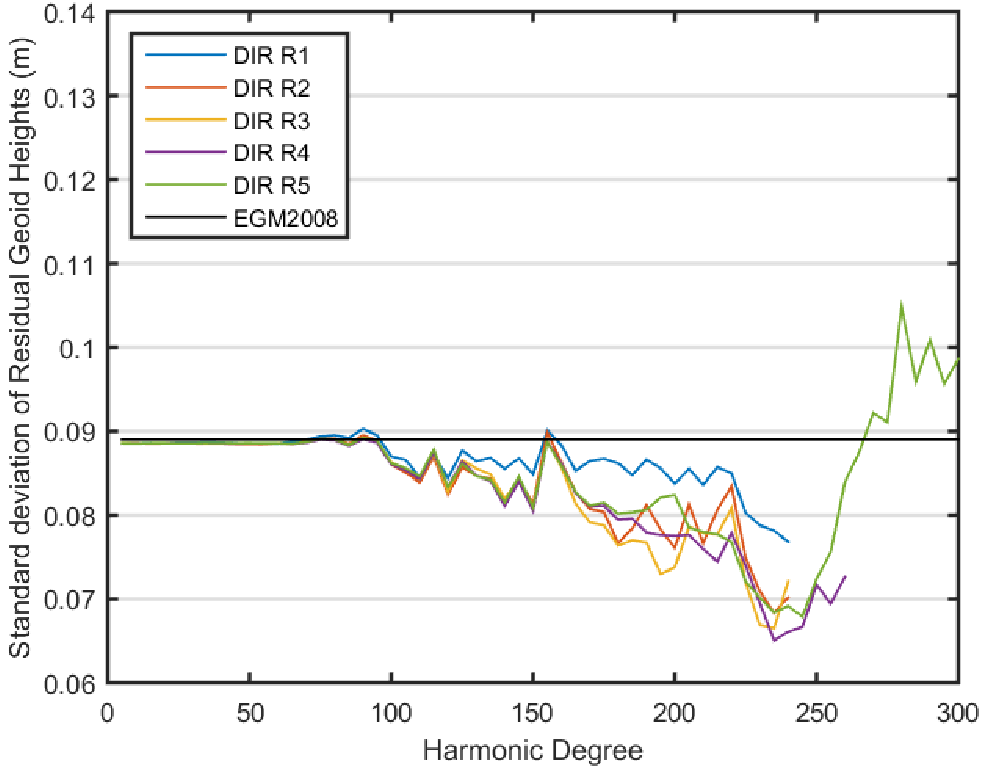

Figure 4.13. Standard deviation of geoid differences between DIR models and İzmir GPS-Levelling points.

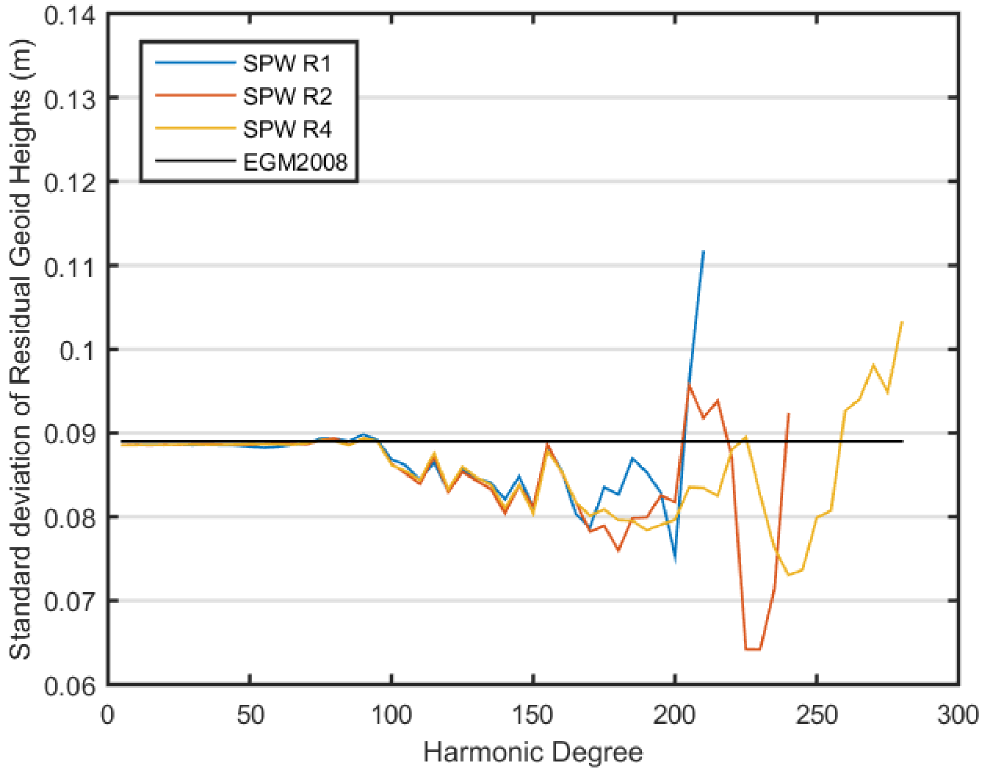

Figure 4.14. Standard deviation of geoid differences between SPW models and İzmir GPS-Levelling points.

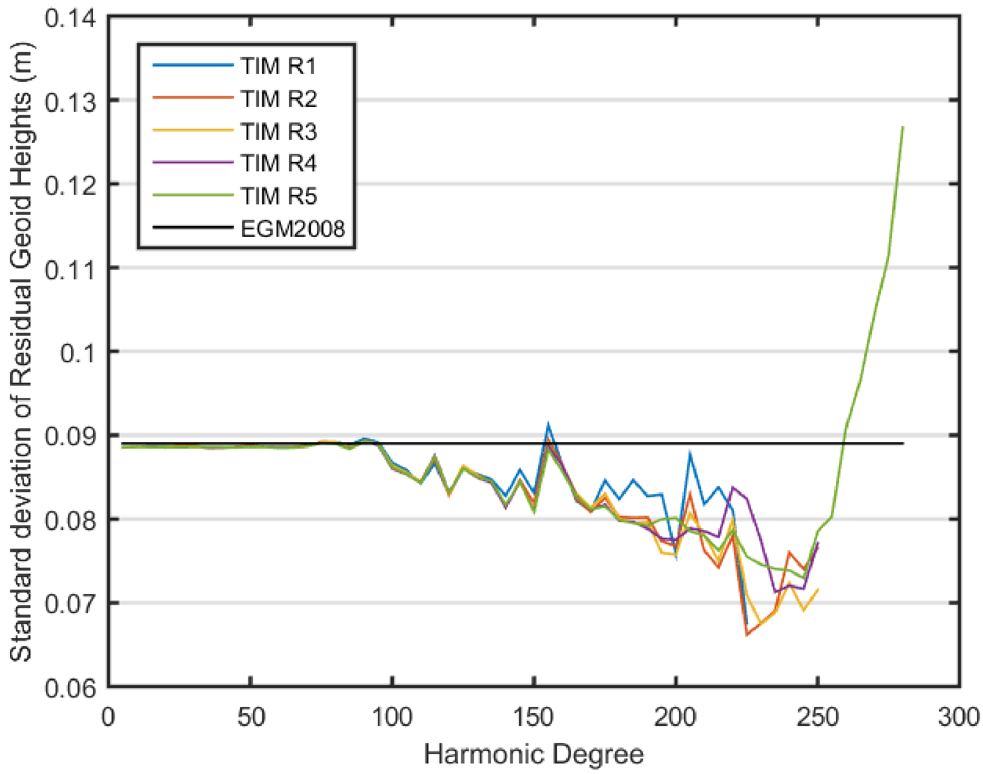

Figure 4.15. Standard deviation of geoid differences between TIM models and İzmir GPS-Levelling points.

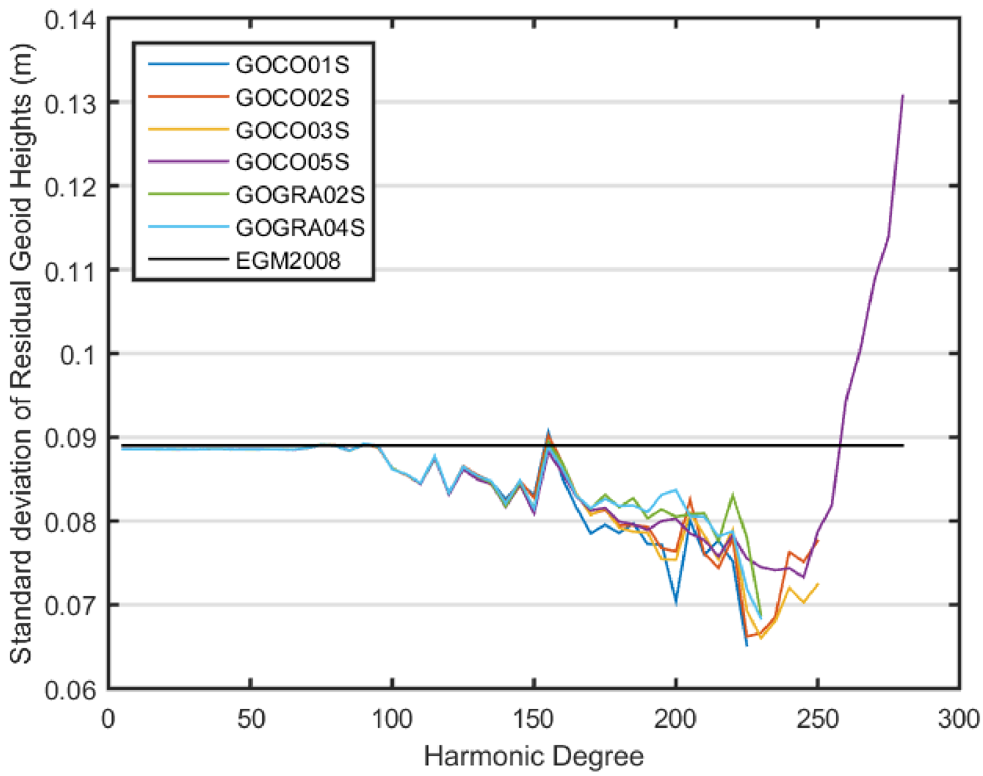

Figure 4.16. Standard deviation of geoid differences between GOGRA and GOCO models and İzmir GPS-Levelling points.

4.2 Remove-Compute-Restore

The computation of regional geoid models with RCR method is carried out with the revised version of software package developed by University of Calgary (Sideris, 1994).

The first computations are done with EGM2008 model with and without using cap size for the Stokes integral. The geoid are validated with 30 GPS/levelling data set and the standard deviations are given in Table 4.5. Since there is no modification applied for the Stokes integral to reduce the truncation error, the standard deviations are very high when the computation is limited to a cap size. The best result is obtained when the whole integration area is used for the computation (i.e no limitation) and the degree of GGM is taken as $L = 360$.

Table 4.5. Statistics of the RCR geoid models computed using EGM2008 Model [unit: m].

Model	**Degree**	$(\psi = 0.1^0)$	$(\psi = 1^0)$	No limitation
EGM2008	120	-	86.1	-
EGM2008	180	70.4	70.7	-
EGM2008	240	56.4	53.6	-
EGM2008	300	46.2	43.7	32.6
EGM2008	360	47.4	46.1	22.4

The computations are repeated with DIR R5, TIM R5 and GOCO05S models with no limit in the computation of Stokes integral using 1D FFT. For the GGM degree L, the degrees that are found best in the validation with 30 TUTGA benchmarks and 81 TUTGA benchmarks together with their maximum degrees are used. The standard deviation of geoids computed with GOCE models are compared with the geoids computed with EGM2008 model. The statistics of computed geoids can be found in Table 4.6 and 4.7.

The geoids computed using GOCE models (with the exception of DIR R5) show equivalent or worse agreement with the GPS/levelling data compared to the ones computed using EGM2008 when the same degree of GGMs is considered. In Table 4.6, geoid models are tested at 30 benchmarks and the best result are obtained when the optimum degrees are chosen w.r.t validations in 30 benchmarks. The similar situation is seen in the validation with 81 benchmarks in Table 4.7.

Overall, the best standard deviation is found as 22.0 cm with globa geopotential model EGM2008 up to $L = 360$. The components of the geoid and the final geoid heights are given from Fig. 4.17 to Fig. 4.21.

Table 4.6. Statistics of the RCR geoid model validations with 30 TUTGA Benchmarks [unit: m].

Model	Degree	Min	Max	Mean	STD	RMSE
DIR R5	155	1.697	3.200	2.108	0.312	2.131
DIR R5	220	0.744	2.091	1.188	0.261	1.216
DIR R5	300 (Nmax)	0.685	2.014	1.078	0.266	1.110
GOCO05S	155	1.700	3.200	2.109	0.312	2.131
GOCO05S	235	0.654	2.019	1.122	0.259	1.152
GOCO05S	280 (Nmax)	0.702	2.026	1.086	0.264	1.118
TIM R5	155	1.701	3.203	2.111	0.312	2.134
TIM R5	235	0.668	2.021	1.127	0.256	1.156
TIM R5	280 (Nmax)	0.710	2.030	1.103	0.261	1.134
EGM2008	155	1.640	3.143	2.061	0.310	2.084
EGM2008	280	1.584	2.857	1.977	0.249	1.993
EGM2008	300	1.169	2.561	1.830	0.326	1.859
EGM2008	360	1.461	2.595	1.829	0.224	1.843

Table 4.7. Statistics of the RCR geoid model validations with 81 TUTGA Benchmarks [unit: m].

Model	Degree	Min	Max	Mean	STD	RMSE
DIR R5	155	1.593	2.645	2.217	0.177	2.224
DIR R5	220	0.563	1.640	1.225	0.191	1.239
DIR R5	300 (Nmax)	0.458	1.560	1.130	0.200	1.148
GOCO05S	155	1.593	2.646	2.218	0.177	2.225
GOCO05S	235	0.433	1.548	1.134	0.200	1.151
GOCO05S	280 (Nmax)	0.462	1.569	1.139	0.202	1.157
TIM R5	155	1.596	2.647	2.220	0.177	2.227
TIM R5	235	0.455	1.564	1.148	0.198	1.165
TIM R5	280 (Nmax)	0.478	1.583	1.153	0.201	1.171
EGM2008	155	1.551	2.605	2.180	0.177	2.187
EGM2008	280	1.405	2.510	2.077	0.200	2.087
EGM2008	300	1.249	2.518	1.986	0.258	2.002
EGM2008	360	1.325	2.438	2.001	0.210	2.012

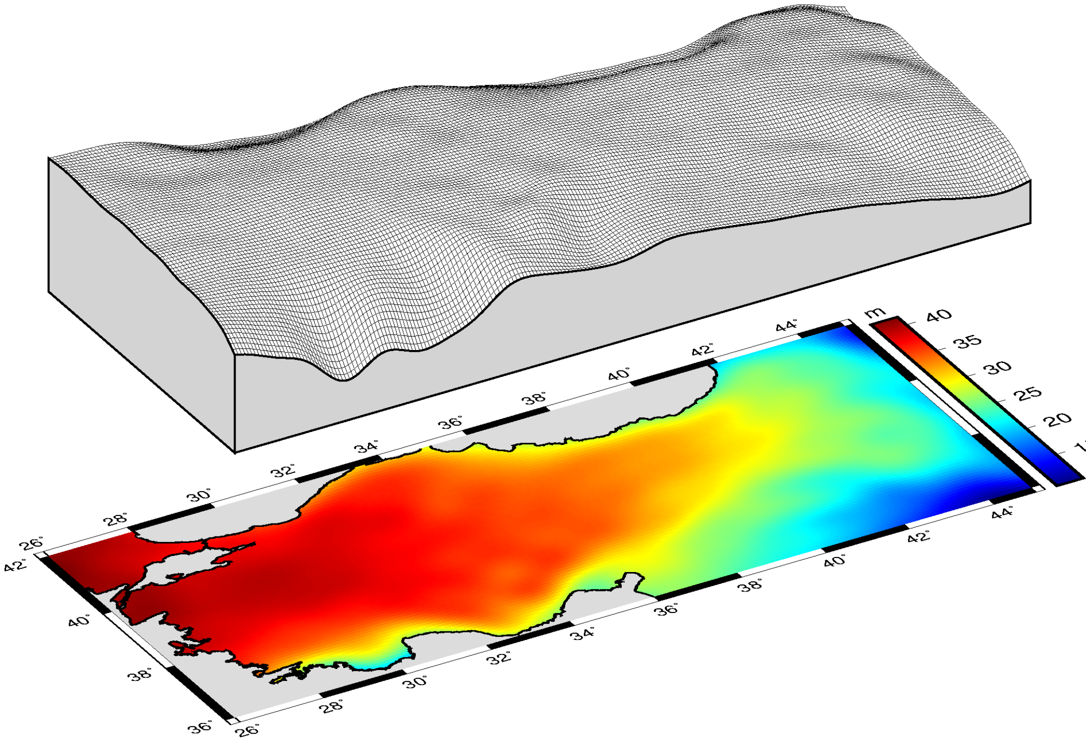

Figure 4.17. N_{GGM} component in RCR geoid.

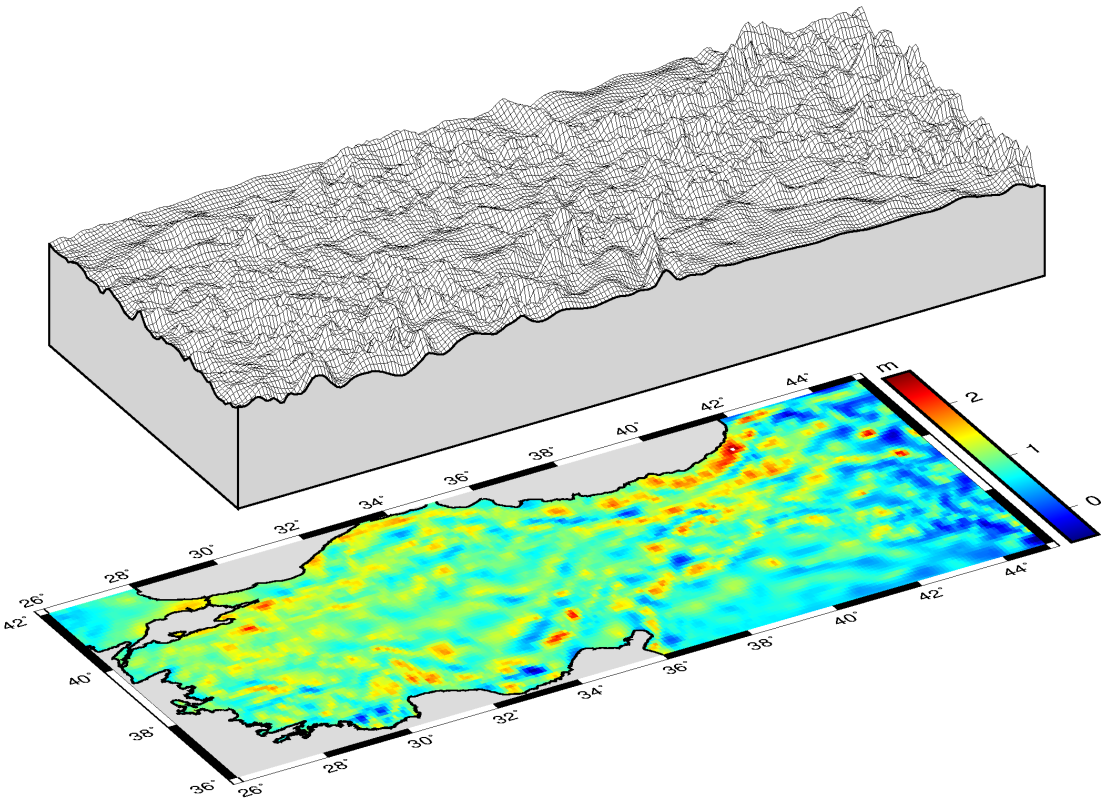

Figure 4.18. $N_{\Delta g}$ component in RCR geoid.

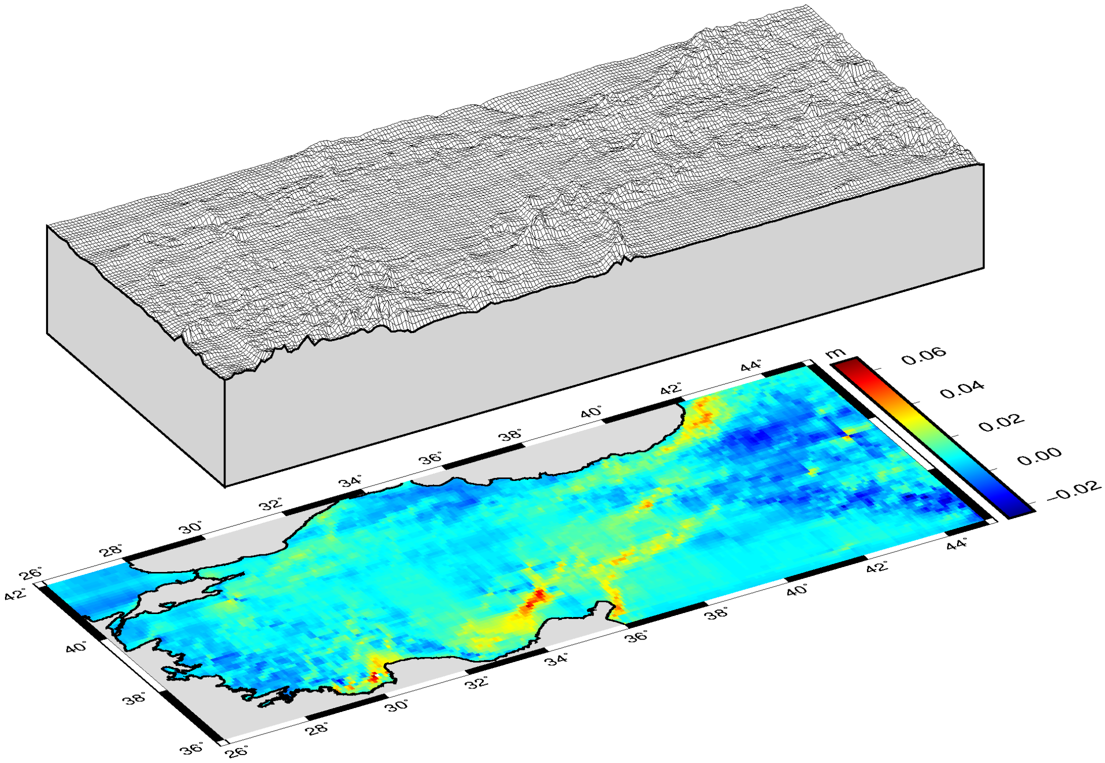

Figure 4.19. N_{Topo} component in RCR geoid.

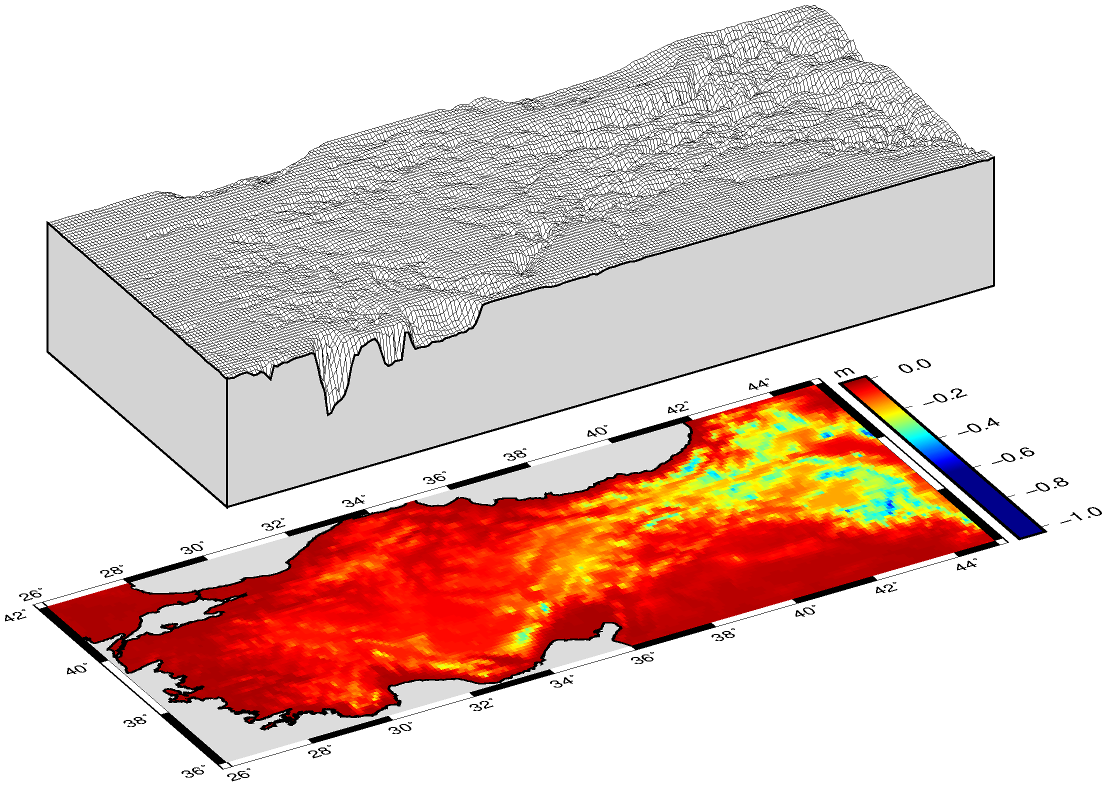

Figure 4.20. N_{ind} component in RCR geoid.

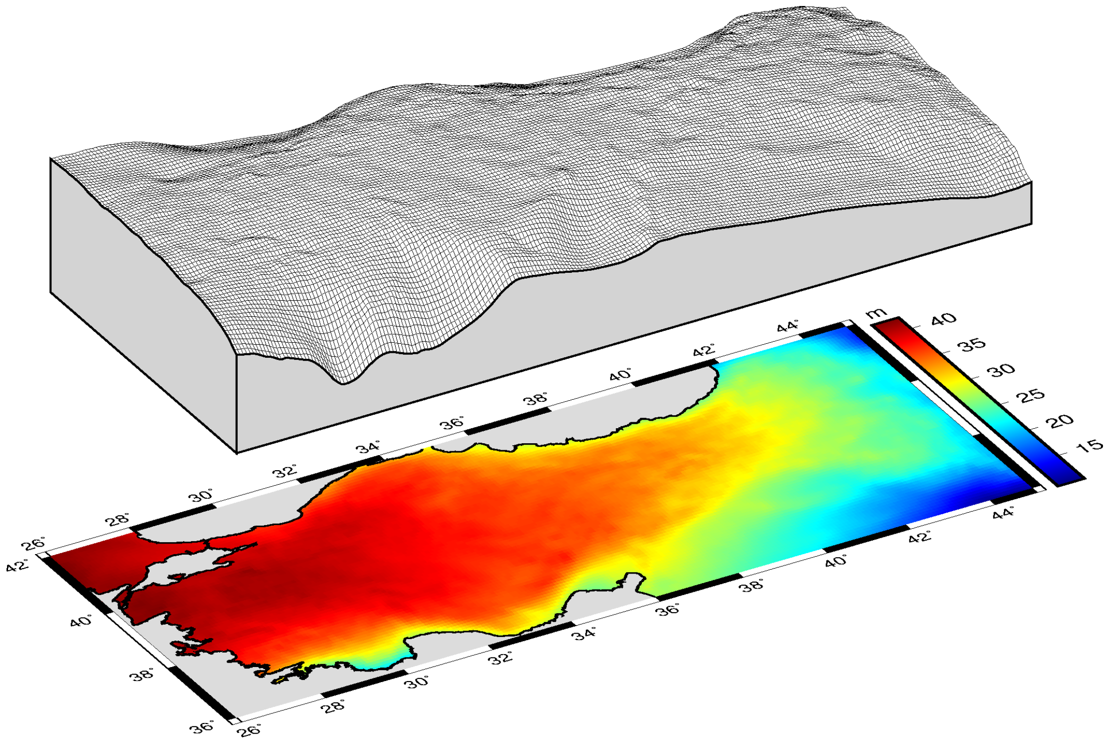

Figure 4.21. Final geoid heights in RCR geoid.

4.3 Least Squares Modification of Stokes Integral

The computation of regional geoid models with KTH method is carried out with the software package KTH Geolab Suite (Ågren, 2004; Ellmann, 2004; Kiamehr, 2006). In KTH method, the determination of optimum modification parameters depends on the maximum degree of the global geopotential model used (L), the quality of the terrestrial gravity data ($\sigma_{\Delta g}$) and the cap size choice (ψ_0). Though there is no certain way to determine these parameters, only assumptions can be made (Abbak, 2011).

In Table 4.8, the standard deviations of the geoid computed with different degree and order of EGM2008 and corresponding cap sizes used in the KTH technique are given. The validations in Table 4.8 are carried with 30 TUTGA benchmarks. The radical change in the $\psi_0 = 0.1^0$ from GGM degree 120 to 360 is quite visible (Fig.4.22). The best standard deviation is achieved when $\psi_0 = 0.1^0$ and GGM degree $L = 360^0$. Additionally, different quality of terrestrial gravity data $\sigma_{\Delta g1} = 1mgal$, $\sigma_{\Delta g2} = 5mgal$ and $\sigma_{\Delta g3} = 10mgal$ are tested with $\psi_0 = 0.1^0$ and $L = 360^0$. The change in the quality

Table 4.8. Standard deviation of KTH geoids with varying cap size and maximum degree/order of geopotential model [unit: cm].

Model-Degree/Order	($\psi = 0.1^0$)	($\psi = 0.25^0$)	($\psi = 0.5^0$)	($\psi = 1.0^0$)
EGM2008-120	68.5	31.3	21.3	23.9
EGM2008-180	43.4	23.6	19.2	22.1
EGM2008-240	32.3	19.6	19.0	23.0
EGM2008-300	24.3	18.0	19.2	23.4
EGM2008-360	17.6	17.7	20.3	23.4
Model-Degree/Order	($\psi = 2.0^0$)	($\psi = 3.0^0$)	($\psi = 4.0^0$)	($\psi = 5.0^0$)
EGM2008-120	25.4	28.3	29.4	28.9
EGM2008-180	26.1	28.5	29.4	28.4
EGM2008-240	26.3	28.6	29.3	28.1
EGM2008-300	26.3	28.7	29.3	28.0
EGM2008-360	26.4	28.8	29.2	27.9

of the gravity data makes no significant effect to the results and standard deviations remains the same (17.3 cm).

With the exception of $\psi_0 = 0.1^0$ and $\psi_0 = 0.25^0$, there is no significant change in the standard deviation while the degree of geopotential model L changes. It can be said that the drawback of the quality of the terrestrial gravity data set can be compensated with the high frequency information of global geopotential model and small cap size in KTH method. Besides, it is observed that there is correlation between the choice of cap size ψ_0 and GGM degree L. When the initial conditions changes, the optimum couple of ψ_0 and L degrades. However, this may stem from the fact that EGM2008 is a combined model, thus the terrestrial gravity data in EGM2008 causes a correlation in the results. To get more accurate results, the same analysis must be repeated with satellite-only global geopotential model.

The geopotential models that are found best in the validation are used in KTH approach. In all computation, cap size is used as $\psi_0 = 0.1^0$. For the GGM degree L, the degrees that are found best in the validation with 30 TUTGA benchmarks and 81 TUTGA benchmarks together with their maximum degrees are used. The statistics of computed geoids can be found in Table 4.9 and 4.10. When the same degree and order of GOCE models are compared with EGM2008, the regional geoids that are computed with GOCE models are superior to that of EGM2008. This supports the results of the global geopotential model validations. Unlike RCR, when the degree of GGM increases, the standard deviation of geoid decreases in the KTH method. This

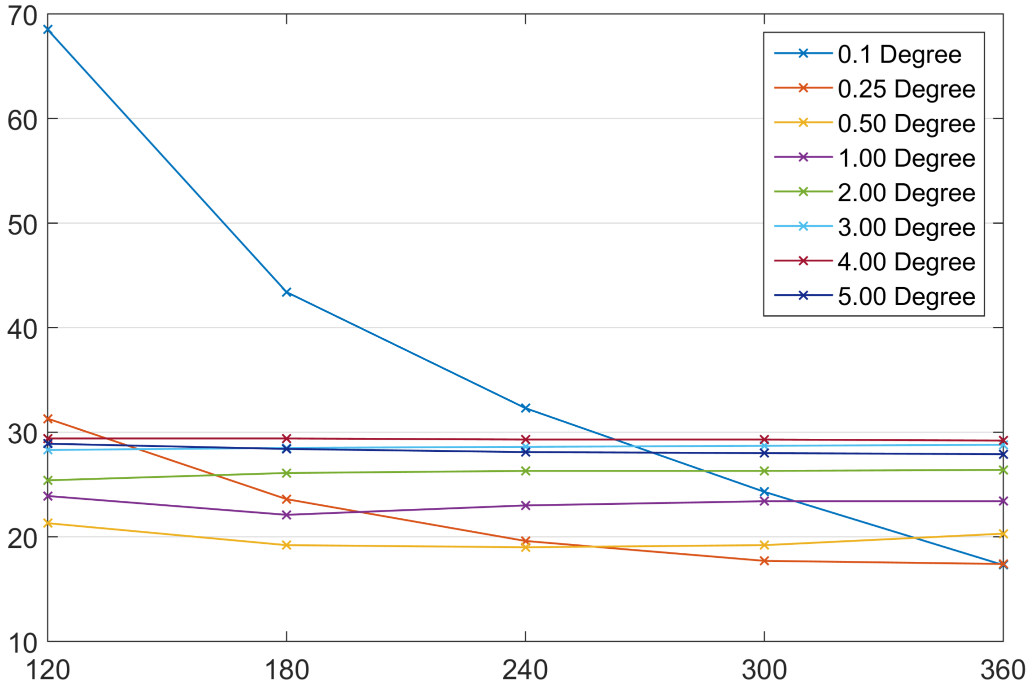

Figure 4.22. Standard deviation of KTH geoids with varying cap size and maximum degree/order of geopotential model.

also shows that the effect of low quality of the terrestrial gravity data is minimized by the higher frequency components of the GGM.

Table 4.9. Statistics of KTH geoid validations with 30 TUTGA benchmarks.

Model	Degree	Min	Max	Mean	STD	RMSE
DIR R5	155	-1.304	0.713	-0.046	0.474	0.477
DIR R5	220	-0.547	0.836	0.279	0.332	0.434
DIR R5	300 (Nmax)	0.006	0.848	0.493	0.223	0.542
GOCO05S	155	-1.312	0.708	-0.051	0.476	0.479
GOCO05S	235	-1.075	0.860	0.163	0.460	0.488
GOCO05S	280 (Nmax)	-0.350	0.784	0.350	0.266	0.439
TIM R5	155	-1.299	0.709	-0.045	0.475	0.478
TIM R5	235	-1.064	0.867	0.169	0.459	0.489
TIM R5	280 (Nmax)	-0.305	0.768	0.368	0.266	0.454
EGM2008	155	-1.390	0.705	-0.061	0.477	0.481
EGM2008	280	-0.094	1.055	0.441	0.256	0.510
EGM2008	300	-0.008	1.035	0.470	0.243	0.530
EGM2008	360	0.255	1.265	0.800	0.176	0.819

Table 4.10. Statistics of KTH geoid validations with 81 TUTGA Benchmarks.

Model	Degree	Min	Max	Mean	STD	RMSE
DIR R5	155	-1.372	0.771	-0.094	0.492	0.501
DIR R5	220	-0.432	0.713	0.254	0.275	0.374
DIR R5	300 (Nmax)	-0.043	0.889	0.477	0.228	0.529
GOCO05S	155	-1.377	0.764	-0.101	0.491	0.502
GOCO05S	235	-0.965	0.537	0.008	0.357	0.357
GOCO05S	280 (Nmax)	-0.317	0.701	0.327	0.253	0.413
TIM R5	155	-1.372	0.773	-0.095	0.491	0.501
TIM R5	235	0.538	-0.941	0.018	0.354	0.354
TIM R5	280 (Nmax)	-0.288	0.722	0.345	0.252	0.427
EGM2008	155	-1.492	0.806	-0.115	0.532	0.544
EGM2008	280	0.858	-0.243	0.398	0.276	0.484
EGM2008	300	-0.184	0.899	0.436	0.265	0.510
EGM2008	360	0.267	1.268	0.817	0.198	0.840

Overall, the best standard deviation is achieved when EGM2008 model is used up to 360 d/o and the cap size is taken as $\psi_0 = 0.1^0$. The magnitudes of each components of the KTH geoid can be found from Fig. 4.23 to Fig. 4.28.

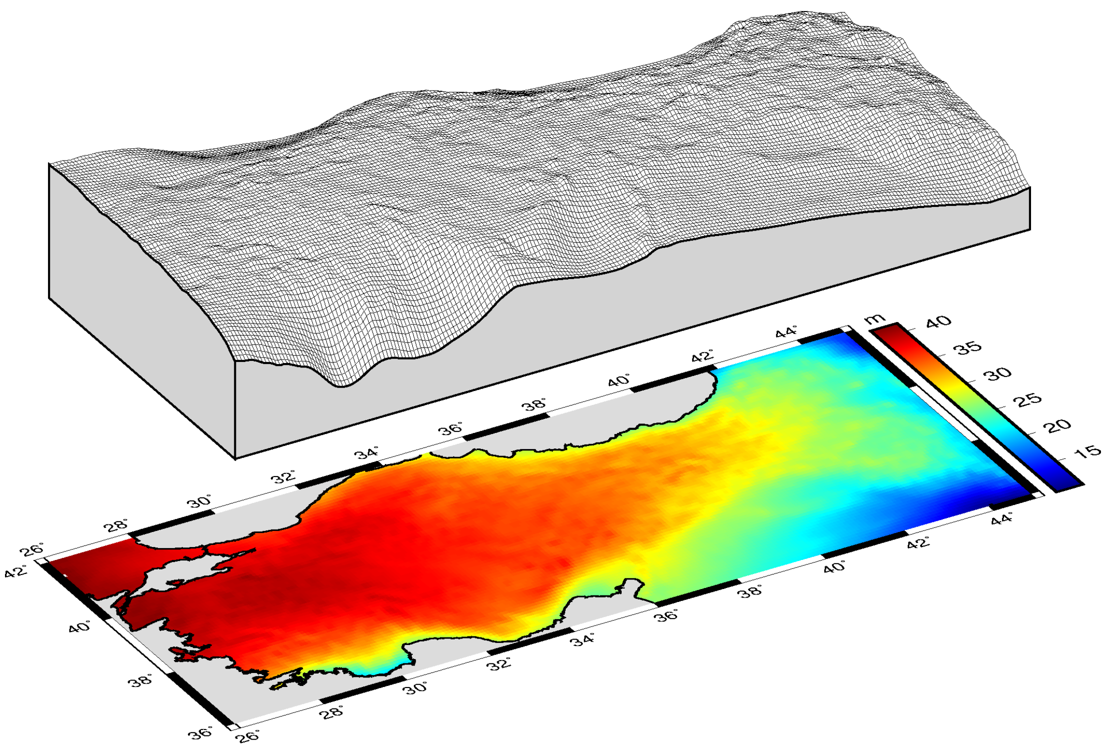

Figure 4.23. Approximate geoid heights in KTH geoid.

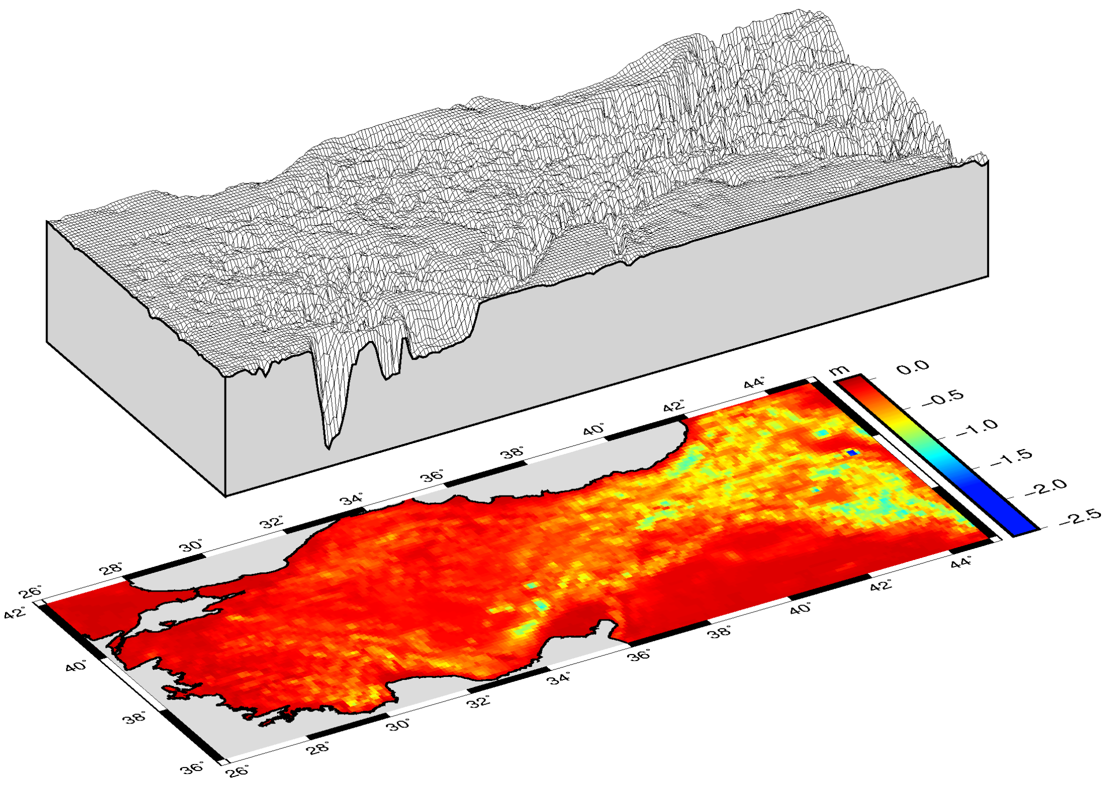

Figure 4.24. Combined topographic correction.

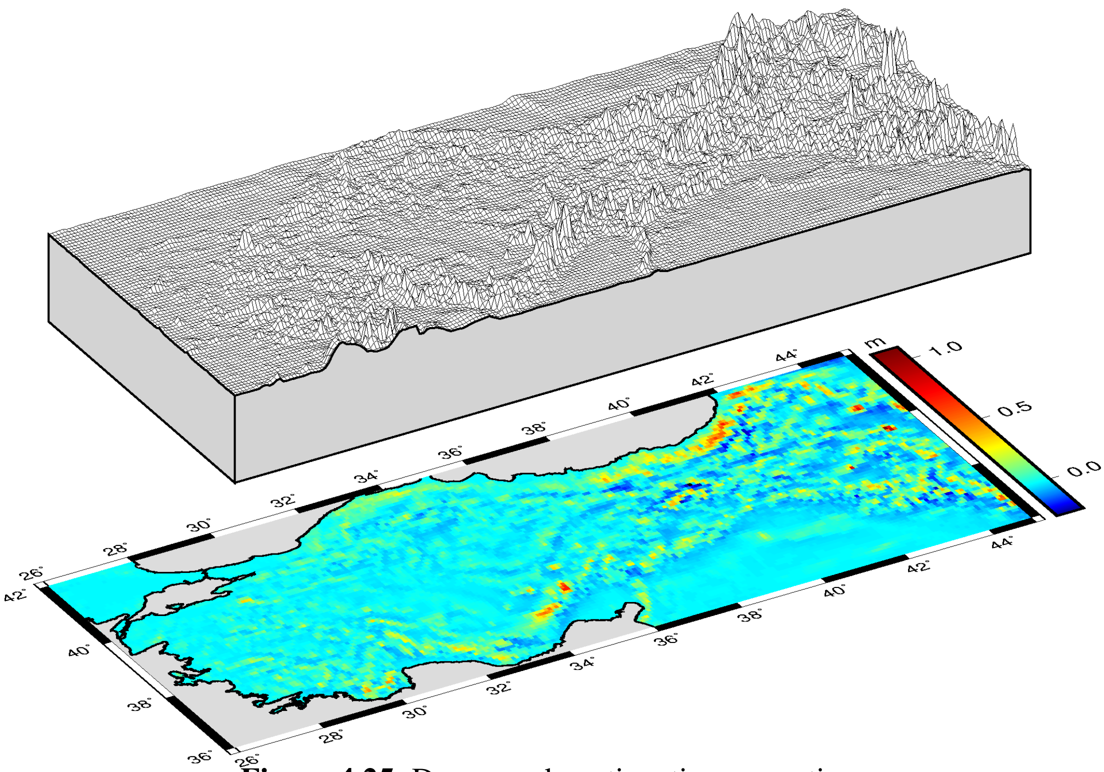

Figure 4.25. Downward continuation correction.

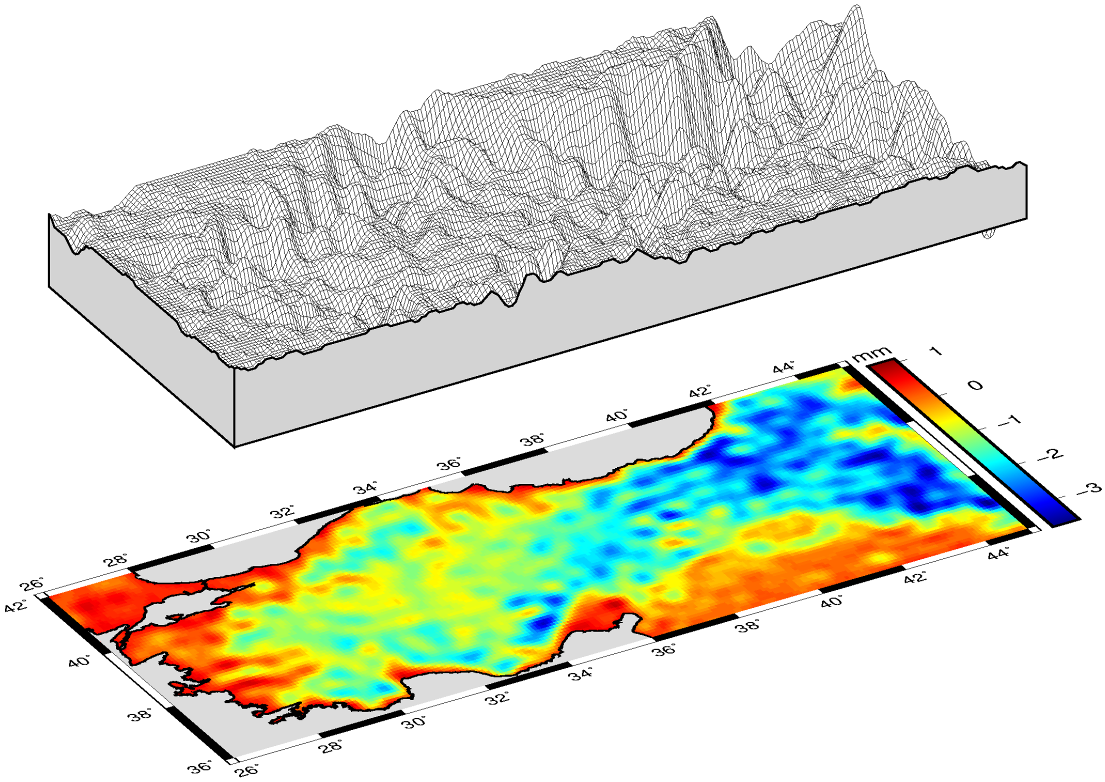

Figure 4.26. Combined atmospheric correction.

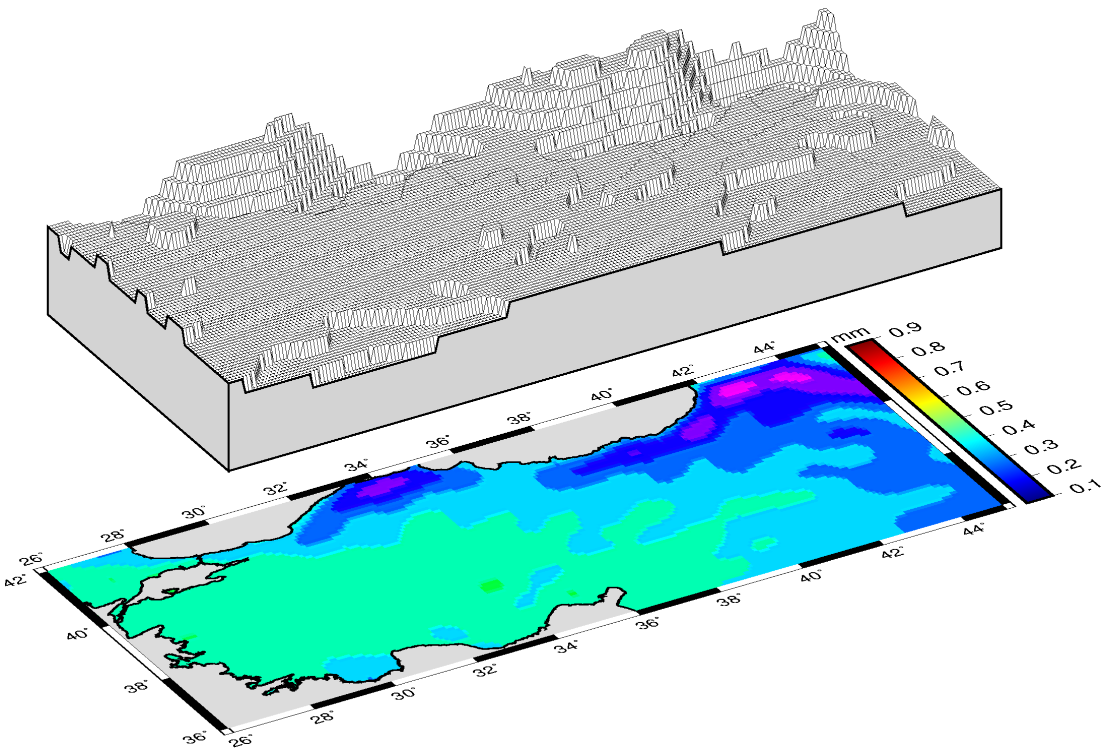

Figure 4.27. Ellipsoidal correction.

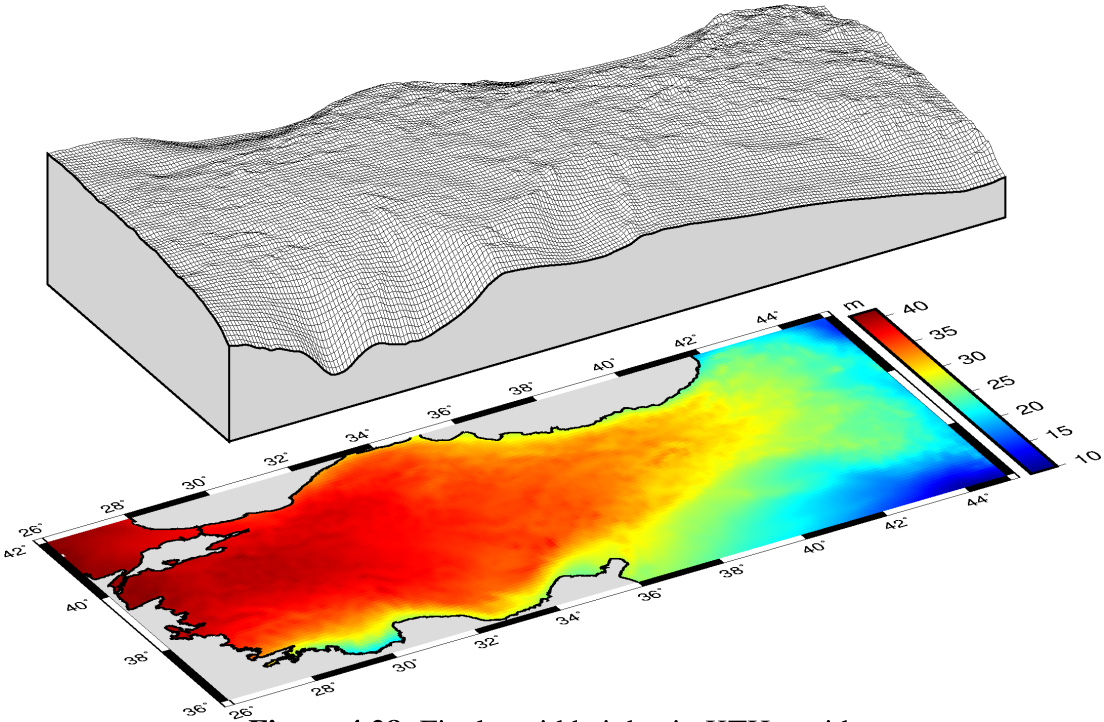

Figure 4.28. Final geoid heights in KTH geoid.

4.4 Corrector Surface Fitting

In the validation of calculated geoid models with GPS/Levelling data in an absolute sense, the use of the aforementioned mathematical relation (Eq.1.1) in practice is more complicated. Each component in this equation introduces random and systematic errors, such as datum inconsistency, distortions in the regional vertical datum, long-wavelength geoid errors and poorly modeled GPS errors (Fotopoulos, 2003).

In order to reduce the effect of these errors, different parametric models can be used to fit the computed geoid to the GPS/Levelling data using least squares adjustment.

$$\Delta N = N_{GPS} - N_{Grav} = a^T \mathbf{x} + \varepsilon \tag{4.1}$$

In this equation, a is the vector of known coefficients, $\mathbf{x}$ is the vector of unknown parameters and ε represents the random noise. The parametric models that are frequently used for corrector surface fitting are 4-parameter, 5-parameter and 7-parameter Helmert similarity transformation and polynomial regression models (Fotopoulos, 2003). The similarity transformation models are given as,

$$a_i = [1 \quad \cos\phi_i \cos\lambda_i \quad \cos\phi_i \sin\lambda_i \quad \sin\phi_i] \tag{4.2}$$

$$a_i = [1 \quad \cos\phi_i \cos\lambda_i \quad \cos\phi_i \sin\lambda_i \quad \sin\phi_i \quad \sin^2\phi_i] \tag{4.3}$$

$$\begin{aligned} a_i = [1 \quad \cos\phi_i \cos\lambda_i \quad \cos\phi_i \sin\lambda_i \quad \sin\phi_i \quad \cos\phi_i \sin\phi_i \cos\lambda_i / W_i \\ \cos\phi_i \sin\phi_i \sin\lambda_i / W_i \quad \sin^2\phi_i / W_i] \end{aligned} \tag{4.4}$$

where $W_i = \sqrt{1 - e^2 \sin^2\phi_i}$, and e^2 is the eccentricity of the reference ellipsoid.

The general formulation of the polynomial models and the equation of 5^{th} degree polynomial model are given as,

$$a_i = \sum_{l=0}^{n} \sum_{m=0}^{n} \Delta\phi_i^l \Delta\lambda_i^m \tag{4.5}$$

$$\begin{aligned} a_i^T x = x_0 &+ x_1\Delta\phi_i + x_2\Delta\lambda_i + x_3\Delta\phi_i\Delta\lambda_i + x_4\Delta\phi_i^2 + x_5\Delta\lambda_i^2 + x_6\Delta\phi_i^2\Delta\lambda_i \\ &+ x_7\Delta\phi_i\Delta\lambda_i^2 + x_8\Delta\phi_i^3 + x_9\Delta\lambda_i^3 + x_{10}\Delta\phi_i^3\Delta\lambda_i + x_{11}\Delta\phi_i^2\Delta\lambda_i^2 \\ &+ x_{12}\Delta\phi_i\Delta\lambda_i^3 + x_{13}\Delta\phi_i^4 + x_{14}\Delta\lambda_i^4 + x_{15}\Delta\phi_i^4\Delta\lambda_i + x_{16}\Delta\phi_i^3\Delta\lambda_i^2 \\ &+ x_{17}\Delta\phi_i^2\Delta\lambda_i^3 + x_{18}\Delta\phi_i\Delta\lambda_i^4 + x_{19}\Delta\phi_i^5 + x_{20}\Delta\lambda_i^5 \end{aligned} \tag{4.6}$$

where $\Delta\phi_i = \phi_i - \phi_0$, $\Delta\lambda_i = \phi_i - \lambda_0$, ϕ_0 and λ_0 are the mean values of latitude and longitude.

The statistical results of the corrector surface fitting of models to four GPS/Levelling data sets (TUTGA, IGNA and İzmir) are given in Table 4.11 - 4.18. All parametric models are compared with the TG03 which is already fitted to the vertical datum.

In the 30 TUTGA benchmarks, only 4-parameter and 1^{st} degree polynomial models are used considering that the number of points is not sufficient for fitting the geoid to regional vertical datum. Nevertheless, in order to make the geoid models comparable, corrector surface fitting is applied. It can be seen that the 4-parameter similarity transform gives better standard deviation for both models and the range of the residuals are smaller compared to 1^{st} degree polynomial model. Both model shows lower accuracy than TG03 geoid model which has 16.1 cm standard deviation.

Table 4.11. Statistics of residual geoid height of RCR geoid after fitting at 30 TUTGA Benchmarks.

Model	Min	Max	Mean	STD	RMSE
Before-Fit	1.461	2.595	1.829	0.224	1.843
4-parameter	-0.349	0.689	0.000	0.212	0.212
Cross-validation	-0.499	0.784	-0.016	0.251	0.252
Polynomial	-0.380	0.679	0.000	0.213	0.213
Cross-validation	-0.368	0.765	-0.007	0.224	0.224
TG03	-0.650	0.101	-0.107	0.161	0.193

Table 4.12. Statistics of residual geoid height of KTH geoid after fitting at 30 TUTGA Benchmarks.

Model	Min	Max	Mean	STD	RMSE
Before-Fit	0.255	1.265	0.800	0.176	0.819
4-parameter	-0.526	0.416	0.000	0.166	0.166
Cross-validation	-0.561	0.474	-0.003	0.193	0.193
Polynomial	-0.547	0.432	0.000	0.173	0.173
Cross-validation	-0.545	0.465	-0.003	0.178	0.178
TG03	-0.650	0.101	-0.107	0.161	0.193

In the 81 TUTGA benchmarks, the model all similarity tranforms and upto 5^{th} degree polynomial models are used for fitting. In the RCR and KTH models the best standard deviation is achieved for 5th degree polynomial model. However, the cross-validation results revealed that in 4th and 5th degree models the standard deviations derived from the residual geoid heights at reference points and cross-validations are quite different. Therefore, they ar e not prefered, hence the 3^{rd} degree polynomial model is found optimal.

TG03 shows quite good results compared to the RCR and KTH models. This is not surprising considering that these points are used for fitting the TG03 to regional vertical datum. The mean value of the TG03 confirms this situation as well.

Table 4.13. Statistics of residual geoid heights of RCR geoid after fitting at 81 TUTGA Benchmarks.

Model	Min	Max	Mean	STD	RMSE
Before-Fit	1.325	2.438	2.001	0.210	2.012
4-parameter	-0.577	0.329	0.000	0.141	0.141
Cross-validation	-0.596	0.341	-0.001	0.149	0.149
5-parameter	-0.551	0.338	0.008	0.140	0.140
Cross-validation	-0.551	0.338	0.007	0.140	0.140
7-parameter	-0.552	0.318	0.000	0.135	0.135
Cross-validation	-0.552	0.318	-0.001	0.135	0.135
1st deg poly	-0.562	0.435	0.000	0.166	0.166
Cross-validation	-0.676	0.437	0.000	0.213	0.213
2nd deg poly	-0.535	0.336	0.000	0.137	0.137
Cross-validation	-0.535	0.336	-0.001	0.138	0.138
3rd deg poly	-0.530	0.309	0.000	0.121	0.121
Cross-validation	-0.530	0.309	0.001	0.122	0.122
4th deg poly	-0.333	0.479	0.000	0.110	0.110
Cross-validation	-0.358	0.568	-0.001	0.139	0.139
5th deg poly	-0.300	0.345	0.000	0.096	0.096
Cross-validation	-0.330	0.459	-0.001	0.131	0.131
TG03	-0.210	0.144	-0.033	0.063	0.071

Table 4.14. Statistics of residual geoid heights of KTH geoid model after fitting at 81 TUTGA points (above) and cross-validation results (below).

Model	Min	Max	Mean	STD	RMSE
Before-Fit	0.267	1.268	0.817	0.198	0.840
4-parameter	-0.595	0.386	0.000	0.170	0.170
Cross-validation	-0.611	0.400	-0.003	0.178	0.178
5-parameter	-0.626	0.385	0.000	0.166	0.166
Cross-validation	-0.626	0.385	-0.002	0.167	0.167
7-parameter	-0.555	0.406	0.000	0.161	0.161
Cross-validation	-0.555	0.406	-0.001	0.162	0.162
1st deg poly	-0.567	0.448	0.000	0.178	0.178
Cross-validation	-0.550	0.451	-0.002	0.200	0.200
2nd deg poly	-0.570	0.395	0.000	0.163	0.163
Cross-validation	-0.570	0.395	-0.001	0.164	0.164
3rd deg poly	-0.601	0.370	0.000	0.149	0.149
Cross-validation	-0.601	0.370	0.001	0.150	0.150
4th deg poly	-0.386	0.579	0.000	0.140	0.140
Cross-validation	-0.466	0.642	-0.004	0.178	0.178
5th deg poly	-0.352	0.503	0.000	0.128	0.128
Cross-validation	-0.422	0.579	-0.008	0.178	0.178
TG03	-0.210	0.144	-0.033	0.063	0.071

The results of IGNA and İzmir data sets are much more meaningful, since both data sets have enough number of GPS/Levelling points to create independent test points which are not used in the fitting. The distribution of the model points and test points for IGNA and İzmir networks are given in Fig.4.29 and Fig.4.30

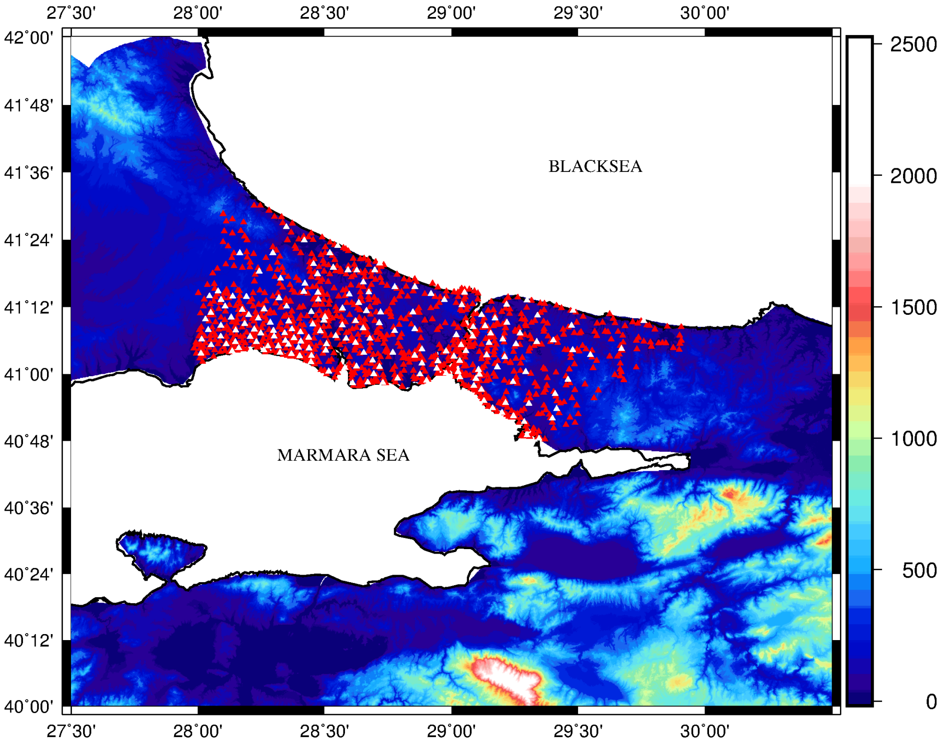

Figure 4.29. Distribution of the model (red) and test (white) points in IGNA network.

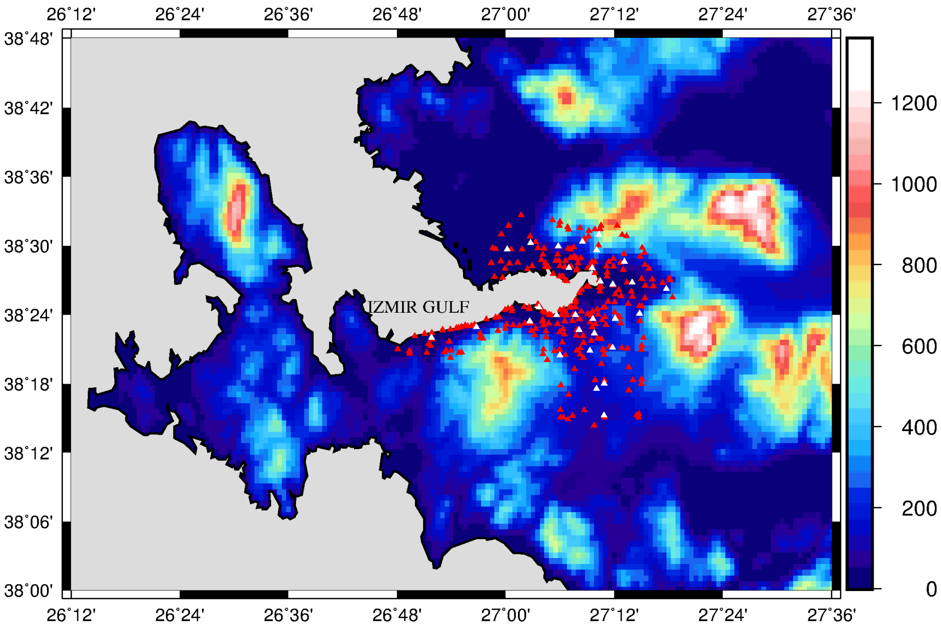

Figure 4.30. Distribution of the model (red) and test (white) in İzmir network.

The results of IGNA validations (Table 4.15 - 4.16) show that KTH model works slightly better than RCR model in İstanbul region. Both models shows an improvement over TG03 around 3-4 cm. The performance of 3^{rd}, 4^{th} and 5^{th} degree polynomials are quite close to each other. Since the differences are statistically small (only 3 mm), 3^{rd} degree polynomial is more preferable model.

Table 4.15. Statistics of residual geoid heights of RCR geoid model after fitting at IGNA model points (above) and test points (below).

Model	Min	Max	Mean	STD	RMSE
Before-Fit	1.685	2.560	2.090	0.186	2.099
4-parameter (Model)	-0.344	0.332	0.000	0.089	0.089
4-parameter (Test)	-0.252	0.284	-0.002	0.088	0.088
5-parameter (Model)	-0.324	0.338	0.000	0.079	0.079
5-parameter (Test)	-0.232	0.244	0.002	0.078	0.078
7-parameter (Model)	-0.299	0.293	0.000	0.077	0.077
7-parameter (Test)	-0.234	0.240	0.002	0.076	0.076
1^{st} deg. poly (Model)	-0.400	0.377	0.000	0.099	0.099
1^{st} deg. poly (Test)	-0.325	0.282	-0.017	0.093	0.095
2^{nd} deg. poly (Model)	-0.294	0.303	0.000	0.077	0.077
2^{nd} deg. poly (Test)	-0.316	0.197	-0.021	0.080	0.083
3^{rd} deg. poly (Model)	-0.300	0.293	0.000	0.072	0.072
3^{rd} deg. poly (Test)	-0.262	0.203	-0.018	0.075	0.077
4^{th} deg. poly (Model)	-0.286	0.289	0.000	0.070	0.070
4^{th} deg. poly (Test)	-0.313	0.163	-0.016	0.076	0.077
5^{th} deg. poly (Model)	-0.294	0.279	0.000	0.069	0.069
5^{th} deg. poly (Test)	-0.269	0.217	-0.015	0.077	0.078
TG03 (Model)	-0.300	0.325	0.005	0.107	0.107
TG03 (Test)	-0.279	0.316	-0.008	0.109	0.110

Table 4.16. Statistics of residual geoid heights of KTH geoid model after fitting at IGNA model points (above) and test points (below).

Model	Min	Max	Mean	STD	RMSE
Before-Fit	0.617	1.375	0.932	0.154	0.945
4-parameter (Model)	-0.343	0.344	0.000	0.092	0.092
4-parameter (Test)	-0.257	0.279	-0.014	0.090	0.091
5-parameter (Model)	-0.323	0.368	0.000	0.085	0.085
5-parameter (Test)	-0.232	0.266	-0.011	0.084	0.084
7-parameter (Model)	-0.283	0.297	0.001	0.072	0.072
7-parameter (Test)	-0.238	0.176	-0.005	0.075	0.075
1^{st} deg. poly (Model)	-0.356	0.339	0.000	0.092	0.092
1^{st} deg. poly (Test)	-0.274	0.280	-0.013	0.090	0.091
2^{nd} deg. poly (Model)	-0.284	0.280	0.000	0.076	0.076
2^{nd} deg. poly (Test)	-0.284	0.158	-0.010	0.079	0.080
3^{rd} deg. poly (Model)	-0.285	0.280	0.000	0.070	0.070
3^{rd} deg. poly (Test)	-0.250	0.173	-0.006	0.073	0.074
4^{th} deg. poly (Model)	-0.277	0.277	0.000	0.068	0.068
4^{th} deg. poly (Test)	-0.287	0.157	-0.005	0.074	0.074
5^{th} deg poly (Model)	-0.281	0.265	0.000	0.067	0.067
5^{th} deg. poly (Test)	-0.239	0.224	-0.004	0.074	0.074
TG03 (Model)	-0.300	0.325	0.005	0.107	0.107
TG03 (Test)	-0.279	0.316	-0.008	0.109	0.110

The statistics of validations at İzmir benchmarks (Tables 4.17-4.18) are quite similar to İstanbul region. But, the performance of TG03 in İzmir is superior than Istanbul. 5^{th} degree polynomial gives the best result for RCR and KTH geoid models. The standard deviation of RCR geoid after 5^{th} degree polynomial model in İzmir is slightly better than TG03 and KTH at model points, though the results of test points for TG03 is better than RCR and KTH model.

Table 4.17. Statistics of residual geoid heights of RCR geoid model after fitting at İzmir model points (above) and test points (below).

Model	Min	Max	Mean	STD	RMSE
Before-Fit	1.478	2.210	1.914	0.108	1.917
4-parameter (Model)	-0.383	0.285	0.000	0.096	0.096
4-parameter (Test)	-0.165	0.236	-0.014	0.085	0.086
5-parameter (Model)	-0.395	0.285	0.000	0.096	0.096
5-parameter (Test)	-0.175	0.236	-0.015	0.084	0.085
7-parameter (Model)	-0.301	0.236	0.000	0.085	0.085
7-parameter (Test)	-0.140	0.222	0.000	0.084	0.084
1^{st} deg. poly (Model)	-0.417	0.297	0.000	0.099	0.099
1^{st} deg. poly (Test)	-0.172	0.248	-0.008	0.086	0.086
2^{nd} deg. poly (Model)	-0.366	0.288	0.000	0.095	0.095
2^{nd} deg. poly (Test)	-0.164	0.239	-0.015	0.083	0.084
3^{rd} deg. poly (Model)	-0.203	0.223	0.000	0.079	0.079
3^{rd} deg. poly (Test)	-0.137	0.174	-0.020	0.082	0.084
4^{th} deg. poly (Model)	-0.147	0.198	0.000	0.062	0.062
4^{th} deg. poly (Test)	-0.143	0.139	-0.037	0.071	0.080
5^{th} deg. poly (Model)	-0.139	0.198	0.000	0.058	0.058
5^{th} deg. poly (Test)	-0.150	0.129	-0.032	0.069	0.076
TG03 (Model)	-0.370	0.083	-0.051	0.065	0.082
TG03 (Test)	-0.212	0.036	-0.052	0.052	0.073

Table 4.18. Statistics of residual geoid heights of KTH geoid model after fitting at İzmir model points (above) and test points (below).

Model	Min	Max	Mean	STD	RMSE
Before-Fit	0.461	1.217	0.879	0.128	0.888
4-parameter (Model)	-0.325	0.302	0.000	0.101	0.101
4-parameter (Test)	-0.143	0.252	-0.034	0.091	0.097
5-parameter (Model)	-0.335	0.301	0.000	0.101	0.101
5-parameter (Test)	-0.152	0.251	-0.035	0.090	0.097
7-parameter (Model)	-0.274	0.255	0.000	0.092	0.092
7-parameter (Test)	-0.143	0.243	-0.011	0.090	0.091
1^{st} deg. poly (Model)	-0.349	0.310	0.000	0.103	0.103
1^{st} deg. poly (Test)	-0.148	0.261	-0.030	0.092	0.096
2^{nd} deg. poly (Model)	-0.350	0.300	0.000	0.101	0.101
2^{nd} deg poly (Test)	-0.158	0.250	-0.034	0.090	0.096
3^{rd} deg. poly (Model)	-0.226	0.252	0.000	0.090	0.090
3^{rd} deg. poly (Test)	-0.158	0.198	-0.034	0.088	0.095
4^{th} deg. poly (Model)	-0.184	0.249	0.000	0.072	0.072
4^{th} deg. poly (Test)	-0.185	0.158	-0.051	0.077	0.093
5^{th} deg. poly (Model)	-0.188	0.226	0.000	0.066	0.066
5^{th} deg. poly (Test)	-0.174	0.149	-0.045	0.072	0.085
TG03 (Model)	-0.370	0.083	-0.051	0.065	0.082
TG03 (Test)	-0.212	0.036	-0.052	0.052	0.073

5. CONCLUSIONS AND RECOMMENDATIONS

In the final chapter, overall conclusions of the numerical studies and recommendations for the future studies for regional geoid determination in Turkey within the height system modernization efforts are given.

This study mainly investigates the regional geoid determination using remove-compute-restore and least squares modification of Stokes integral methods in Turkey. In this context, first the optimum data set for geoid determination is created in terms of terrestrial gravity data, global geopotential model and digital elevation model. In order to determine the best global geopotential model, the performances of the recent releases of GOCE-only and GOCE/GRACE models are tested at four different GPS/Levelling data sets. All releases of DIR, SPW, TIM, GOCO and GOGRA models are included in the validation. In order to make the models comparable with the GPS/Levelling data, spectral enhancement method is used. The high frequency information is calculated from EGM2008 and RTM. As a result of the assessment of global geopotential models, the following statements can be concluded:

- The quality of the GPS/levelling data sets are accurate enough for the validations. However, 200 IGNA benchmarks and 301 İzmir benchmarks cover very small regions when the whole Turkey is considered. Their validation results show the local performance of the models only. Thus, the validations at 30 and 81 TUTGA benchmarks show more reliable results, though these benchmarks cannot be characterized as the optimum data set for GGM validation as well. In order to conduct the GGM validation properly, GPS/Levelling benchmarks must be distributed homogeneously through the area where the regional geoid model is to be computed.

- The contribution of GOCE data in the medium wavelength, specifically between degrees 100 and 200, is clearly seen.

- In general, the performance of DIR, SPW, TIM, GOCO and GOGRA models are equivalent to each other. The improvement in the releases is observed from first to last releases as expected. DIR R5, TIM R5 and GOCO05S models show the best agreement with the GPS/levelling data sets.

The global geopotential models that are found best in the assessment and EGM2008 model are used in the computation of regional geoid determination using RCR and KTH methods in Turkey between 36^{o}N-42^{o}N latitudes and 26^{o}E-45^{o}E longitudes. Based on the numerical results of geoid validations at 4 GPS/Levelling data sets, the following statements can be concluded:

- Considering the quality of the terrestrial gravity data set, the expectation of the accuracy of computed geoid models is not 1-3 cm.

- In the study, the classical remove-compute-restore method is applied without the kernel modification in Stokes' integral. Consequently, when the Stokes integration is limited to a spherical cap, large truncation error occurs. When the whole area is used in the Stokes integration, the best standard deviation is obtained.

- In the KTH method, the best result is achieved when the cap size $\psi_0 = 0.1^o$ and EGM2008 up to $L = 360$ are chosen as computation parameters. This simply shows that the drawback of low quality terrestrial gravity data can be compensated with high frequency information of global geopotential model and using small cap size in KTH method.

- In RCR method, the standard deviations of geoid models computed using GOCE models are equivalent to geoid models computed using EGM2008 model as reference model. In KTH method, the performance of geoid models computed using GOCE models shows better results than the geoid models computed using EGM2008 model when the same degree and order of global geopotential model is considered.

- In RCR method, the best standard deviations are achieved when the optimum degree of GOCE GGM is used. However, in KTH method, better results are found while the degree of GGM increases. This is, as explained in the previous statement, because of the fact that the high frequency information in the GGM improves the geoid better than the low quality terrestrial gravity data.

- Overall, the best standard deviations for RCR and KTH geoids are 22.4 cm and 17.6 cm (before-fit) respectively. After the corrector surface fitting with 4-parameter similarity transform, which give better results for the data set, at 30 GPS/Levelling benchmarks the standard deviations are found as 21.2 cm for RCR geoid and 16.6 cm for KTH geoid. The Turkey Geoid 2003 (TG03) shows 16.1 cm agreement with the same data. Though, the number of the GPS/Levelling data is not suitable for fitting the geoid to regional vertical datum.

- In 81 GPS/Levelling benchmarks, the performance of RCR and KTH geoids are 21.0 cm and 19.8 cm (before-fit) respectively. After the corrector surface fitting with 3^{rd} degree polynomial model, which is found best for the data set, the standard deviations are found as 12.2 cm for RCR geoid and 14.9 cm for KTH geoid. Both geoid models perform worser than TG03 in this region.

- Local performances of the RCR and KTH geoid models in IGNA and İzmir benchmarks are quite similar to each other. In İstanbul, both geoid models can outperform TG03 in the order of 4 cm. However, the performance of TG03 is much better in İzmir. The standard deviation of RCR model is slightly better than KTH model in this region. The performance of RCR and KTH models is nearly same as the TG03.

The following statements can be recommended for the future studies:

- Stokes kernel modification must be tested in the RCR method to decrease the truncation error.

- In order to obtain geoid model with 1-3 cm accuracy, these studies must be repeated with a better quality terrestrial gravity data set which is collected for geodetic purposes.

- The statistical results over whole Turkey with 30 GPS/Levelling points cannot reveal the real performance of the model. It is important to test the geoid models using denser GPS/Levelling data set that characterizes the rough topography of

Turkey. Than, a better conclusions on how realistic these methods can model the physical properties of the topography can be made. Otherwise, there is no way to determine which methods gives better results in Turkey.

- In addition to RCR and KTH methods, Stokes-Helmert (UNB) method must be tested in Turkey.

REFERENCES

Abbak, R. A. (2011). *Global Yerpotansiyel Modellerin Spektral Yöntemlerle Değerlendirilmesi ve Jeoid Belirleme İçin Yerel Olarak İyileştirilmesi*. PhD thesis, Selçuk University.

Abbak, R. A., Erol, B., and Ustun, A. (2012). Comparison of the kth and remove–compute–restore techniques to geoid modelling in a mountainous area. *Computers & Geosciences*, 48:31–40.

Ågren, J. (2004). *Regional geoid determination methods for the era of satellite gravimetry: numerical investigations using synthetic Earth gravity models*. PhD thesis, Royal Institute of Technology.

Ågren, J., Sjöberg, L. E., and Kiamehr, R. (2009). The new gravimetric quasigeoid model KTH08 over Sweden. *Journal of Applied Geodesy*, 3(3):143–153.

Andersen, O., Knudsen, P., Kenyon, S., and Holmes, S. (2014). Global and Arctic marine gravity field from recent satellite altimetry (DTU13). In *76th EAGE Conference and Exhibition 2014*.

Avsar, N. B., Erol, B., And Kutoglu, S. H. (2015). *Evaluation of GOCE-Based Global Geopotential Models Versus EGM2008 and GPS/Levelling Data in Northwest of Turkey*, pages 1–8. Springer Berlin Heidelberg, Berlin, Heidelberg.

Ayan, T. (1976). *Astrogeodätische Geoidberechnung für das Gebiet der Türkei*. PhD thesis, Geodätisches Institut, Universität Fridericiana Karlsruhe.

Ayan, T. (1978). Türkiye geoidi. *Harita Dergisi*, 85:5–17.

Ayan, T., Deniz, R., Arslan, E., Çelik, R. N., Denli, H. H., Akyılmaz, O., Özşamlı, C., Özlüdemir, M. T., Erol, S., Erol, B. Acar, M., Mercan, H., and Tekdal, E. (2006). İstanbul gps nirengi ağı (İgna) 2005-2006 yenileme Ölçü ve değerlendirmesi. *Istanbul Technical University Report*,Volume 1.

Ayan, T., Deniz, R., Çelik, R. N., Denli, H. H., Özlüdemir, M. T., Erol, S., Erol, B., Akyılmaz, O., and Güney, C. (2001). İzmir geodetic reference system-2001. *Istanbul Technical University Report*, ITU 2000/2294.

Ayhan, M., Demir, C., Lenk, O., Kılıçoğlu, A., Aktuğ, B., Açıkgöz, M., Fırat, O., Şengün, Y., Cingöz, A., Gürdal, M., Kurt, A., Ocak, M., Türkezer, A., Yıldız, H., Bayazıt, N., Ata, M., Çağlar,

Y., and Özerkan, A. (2002). Türkiye Ulusal Temel GPS Ağı-1999 (TUTGA99A). *Harita Dergisi Özel Sayı, No:16*.

Ayhan, M. E. (1993). Geoid determination in Turkey (TG-91). *Bulletin Geodesique*, 67(1):10–22.

Becker, J., Sandwell, D., Smith, W., Braud, J., Binder, B., Depner, J., Fabre, D., Factor, J., Ingalls, S., Kim, S., et al. (2009). Global bathymetry and elevation data at 30 arc seconds resolution: SRTM30_PLUS. *Marine Geodesy*, 32(4):355–371.

Brockmann, J. M., Zehentner, N., Höck, E., Pail, R., Loth, I., Mayer-Gürr, T., and Schuh, W.-D. (2014). Egm_tim_rl05: An independent geoid with centimeter accuracy purely based on the GOCE mission. *Geophysical Research Letters*, 41(22):8089–8099.

Bruinsma, S. L., Förste, C., Abrikosov, O., Marty, J.-C., Rio, M.-H., Mulet, S., and Bonvalot, S. (2013). The new ESA satellite-only gravity field model via the direct approach. *Geophysical Research Letters*, 40(14):3607–3612.

Bruinsma, S. L., Marty, J. C., Balmino, G., Biancale, R., Förste, C., Abrikosow, O., and H., N. (2010). Goce gravity field recovery by means of the direct numerical approach. In *ESA Living Planet Symposium*. Bergen, 28 June - 02 July 2010.

Natural Resources Canada (2013). Height modernization 2013, a new Canadian vertical datum.

Carrion, D., Vergos, G., Albertella, A., Barzaghi, R., Tziavos, I. N., and Grigoriadis, V. N. (2015). Assessing the Goce models accuracy in the Mediterranean area. *Newton's Bulletin 5*, pages 63–82.

Daras, I. (2008). Determination of a gravimetric geoid model of Greece using the method of KTH. Master's thesis, Royal Institute of Technology.

Ellmann, A. (2004). *The geoid for the Baltic countries determined by the least squares modification of Stokes formula*. PhD thesis, Royal Institute of Technology.

Erol, B. (2007). *Investigations on local geoids for geodetic applications*. PhD thesis, Istanbul Technical University.

Erol, B., Sideris, M. G., and Çelik, R. N. (2009). Comparison of global geopotential models from the CHAMP and GRACE missions for regional geoid modelling in Turkey. *Studia Geophysica et Geodaetica*, 53(4):419–441.

Featherstone, W. and Dentith, M. (1997). A geodetic approach to gravity data reduction for geophysics. *Computers & Geosciences*, 23(10):1063–1070.

Feizabadi, M. (2014). Assessment of high-resolution digital terrain models and their contribution in gravimetric geoid modelling. *Istanbul Technical University, Bachelor Thesis*.

Fotopoulos, G. (2003). *An Analysis of Optimal Combination of Geoid, Orthometric and Ellipsoidal Height Data*. PhD thesis, University of Calgary.

Gatti, A., Reguzzoni, M., Migliaccio, F., and Sansò, F. (2014). Space-wise grids of gravity gradients from GOCE data at nominal satellite altitude. In *5th International GOCE User Workshop*, pages 25–28.

Goiginger, H., Hoeck, E., Rieser, D., Mayer-Guerr, T., Maier, A., Krauss, S., Pail, R., Fecher, T., Gruber, T., Brockmann, J., et al. (2011). The combined satellite-only global gravity field model GOCO02S. In *Geophys. Res. Abstr*, volume 13, page 571.

Gürkan, O. (1978). Astrojeodezik ağların deformasyonu ve Türkiye I. derece triyangülasyon ağı. *KTÜ Yer Bilimleri Fakültesi*, 104.

Gruber, T. (2014). GOCE gravity field models—overview and performance analysis. In *Presentation at the fifth international GOCE user workshop, UNESCO, Paris*, pages 25–28.

Gruber, T., Visser, P. N. A. M., Ackermann, C., and Hosse, M. (2011). Validation of goce gravity field models by means of orbit residuals and geoid comparisons. *Journal of Geodesy*, 85(11):845–860.

Haagmans, R., De Min, E., and Van Gelderen, M. (1993). Fast evaluation of convolution integrals on the sphere using 1D fft, and a comparison with existing methods for stokes' integral. *Manuscripta geodaetica*, 18:227–227.

Heiskanen, W. A. and Moritz, H. (1967). *Physical geodesy*. W. H. Freeman and Co., San Francisco.

Işık, M. S. and Erol, B. (2015). Geoid modelling in Turkey using remove-compute-restore and least squares modification of stokes' integral methods. *26th IUGG General Assembly*.

Işık, M. S. and Erol, B. (2016). Geoid determination using GOCE-based models in Turkey. In *EGU General Assembly 2016*, volume 18.

Ince, E. S., Erol, B., and Sideris, M. G. (2014). Evaluation of the GOCE-based gravity field models in Turkey. In Marti, U., editor, *Gravity, Geoid and Height Systems: Proceedings of the IAG Symposium GGHS2012, October 9-12, 2012, Venice, Italy*, pages 93–99, Cham. Springer International Publishing.

Kiamehr, R. (2006). *Precise Gravimetric Geoid Model for Iran Based on GRACE and SRTM Data and the Least-Squares Modification of Stokes' Formula: with Some Geodynamic Interpretations*. PhD thesis, Royal Institute of Technology.

Mayer-Guerr, T. and the GOCO Team (2015). The combined satellite gravity field model GOCO05S. In *EGU General Assembly Conference Abstracts*, volume 17, page 12364.

Mayer-Gürr, T., Rieser, D., Höck, E., Brockmann, J., Schuh, W., Krasbutter, I., Kusche, J., Maier, A., Krauss, S., Hausleitner, W., et al. (2012). The new combined satellite only model GOCO03S. *Abstract, GGHS2012, Venice*.

Migliaccio, F., Reguzzoni, M., Gatti, A., Sansò, F., and Herceg, M. (2011). A goce-only global gravity field model by the space-wise approach. In *4th International GOCE User Workshop*.

Pail, R., Bruinsma, S., Migliaccio, F., Förste, C., Goiginger, H., Schuh, W.-D., Höck, E., Reguzzoni, M., Brockmann, J. M., Abrikosov, O., et al. (2011). First GOCE gravity field models derived by three different approaches. *Journal of Geodesy*, 85(11):819–843.

Pail, R., Goiginger, H., Mayrhofer, R., Schuh, W.-D., Brockmann, J. M., Krasbutter, I., Höck, E., and Fecher, T. (2010a). GOCE gravity field model derived from orbit and gradiometry data applying the time-wise method. In *Proceedings of the ESA living planet symposium*, volume 28. European Space Agency Bergen, Norway.

Pail, R., Goiginger, H., Schuh, W.-D., Höck, E., Brockmann, J. M., Fecher, T., Gruber, T., Mayer-Gürr, T., Kusche, J., Jäggi, A., and Rieser, D. (2010b). Combined satellite gravity field model GOCO01S derived from GOCE and GRACE. *Geophysical Research Letters*, 37(20): L20314.

Paul, M. (1973). A method of evaluating the truncation error coefficients for geoidal height. *Bulletin Géodésique (1946-1975)*, 110(1):413–425.

Pavlis, N. K., Holmes, S. A., Kenyon, S. C., and Factor, J. K. (2012). The development and evaluation of the Earth Gravitational Model 2008 (EGM2008). *Journal of Geophysical Research: Solid Earth*, 117(B4): B04406.

Rummel, R., Yi, W., and Albertella, A. (2011). GOCE: its principles and science. In *Proceedings of the Journées*.

Sansò, F. and Sideris, M. G. (2013). *Geoid determination: theory and methods*. Springer Science & Business Media.

Schwarz, K., Sideris, M., and Forsberg, R. (1987). Orthometric heights without leveling. *Journal of Surveying Engineering*, 113(1):28–40.

Schwarz, K., Sideris, M., and Forsberg, R. (1990). The use of fft techniques in physical geodesy. *Geophysical Journal International*, 100(3):485–514.

Sideris, M. (1994). Geoid determination byfft techniques. lecture notes. *International School for the Determination and Use of the Geoid, International Geoid Service*, pages 213–272.

Simav, M., Direnç, A., Türkezer, A., Kurt, A., and Kurt, M. (2012). Türkiye yersel gravite veri kütüğü hakkında değerlendirme. In *TUJK 2012, Türkiye Yükseklik Sisteminin Modernizasyonu Çalıştayı*.

Sjöberg, L. E. (2003a). A computational scheme to model the geoid by the modified stokes formula without gravity reductions. *Journal of Geodesy*, 77(7-8):423–432.

Sjöberg, L. E. (2003b). A general model for modifying stokes' formula and its least-squares solution. *Journal of Geodesy*, 77(7-8):459–464.

Sjöberg, L. E. (2003c). A solution to the downward continuation effect on the geoid determined by stokes' formula. *Journal of Geodesy*, 77(1-2):94–100.

Sjöberg, L. E. (2004a). The ellipsoidal corrections to the topographic geoid effects. *Journal of Geodesy*, 77(12):804–808.

Sjöberg, L. E. (2004b). A spherical harmonic representation of the ellipsoidal correction to the modified Stokes formula. *Journal of Geodesy*, 78(3):180–186.

SRTM (2000). http://www2.jpl.nasa.gov/srtm.

Stokes, G. G. (1849). On the variation of gravity on the surface of the earth. *Transactions of the Cambridge Philosophical Society*, 8:672–695.

Tocho, C. and Vergos, G. (2015). Assessment of different-generation GOCE-only and GOCE/GRACE earth global gravity models over Argentina using terrestrial gravity anomalies and GPS/levelling data. *Newton's Bulletin 5*, pages 105–126.

Tscherning, C., Forsberg, R., and Knudsen, P. (1992). The Gravsoft package for geoid determination. In *Proc. 1. continental workshop on the geoid in Europe, Prague*, pages 327–334.

TUJK (2012). Türkiye Yükseklik Sisteminin Modernizasyonu Çalıştayı - Toplantı Sonuç Bildirgesi.

Vergos, G. S., Andritsanos, V. D., Grigoriadis, V. N., Pagounis, V., and Tziavos, I. N. (2015). *Evaluation of GOCE/GRACE GGMs Over Attica and Thessaloniki, Greece, and Wo Determination for Height System Unification*, pages 1–9. Springer Berlin Heidelberg, Berlin, Heidelberg.

Yen, H.-Y., Yeh, Y.-H., and Chen, C.-H. (1994). Gravity terrain corrections of Taiwan. *Terrestr. Atmos. Oceanic Sci*, 5:1–10.

Yi, W., Rummel, R., and Gruber, T. (2013). Gravity field contribution analysis of goce gravitational gradient components. *Studia Geophysica et Geodaetica*, 57(2):174–202.

Yıldız, H., Fırat, O., Simav, M., and Ünver, Y. (2006). The high resolution geoid model for Turkey, TG-07. In *1st International Symposium of the International Gravity Field Service (IGFS), Gravity Field of the Earth, Program & Abstract Book*, volume 18, page 28.

APPENDICES

APPENDIX A.1 : Signal-Error degree variances and cumulative global geoid errors of Global Geopotential Models

APPENDIX A.1

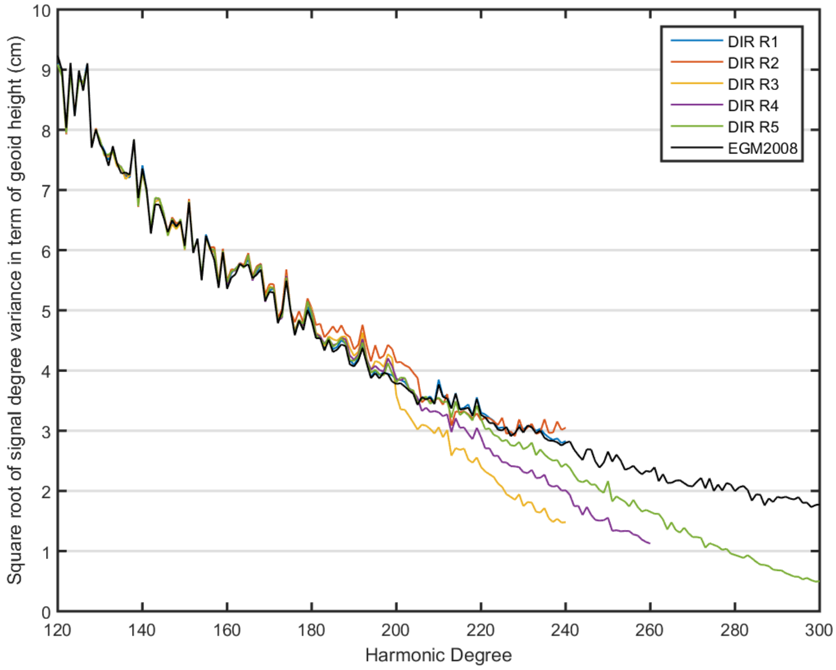

Figure A.1. Signal degree variances of DIR models in terms of geoid height (cm)

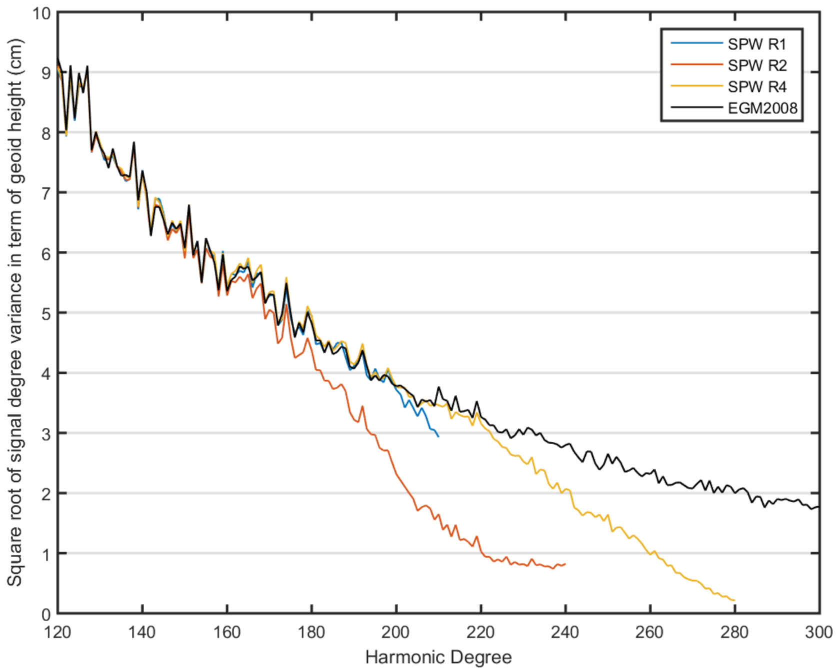

Figure A.2. Signal degree variances of SPW models in terms of geoid height (cm)

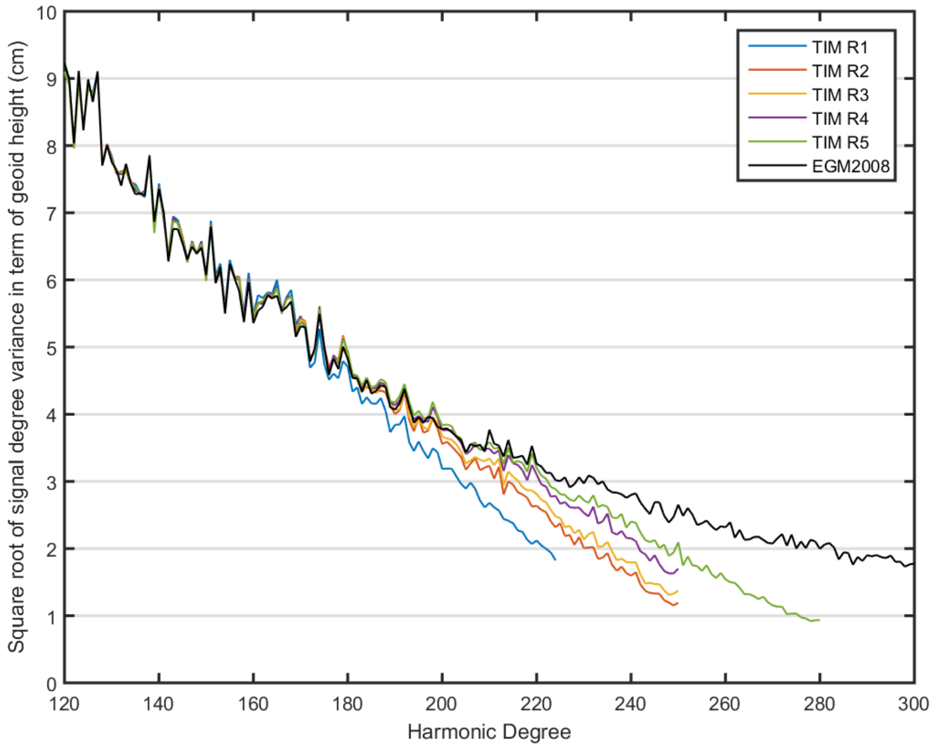

Figure A.3. Signal degree variances of TIM models in terms of geoid height (cm)

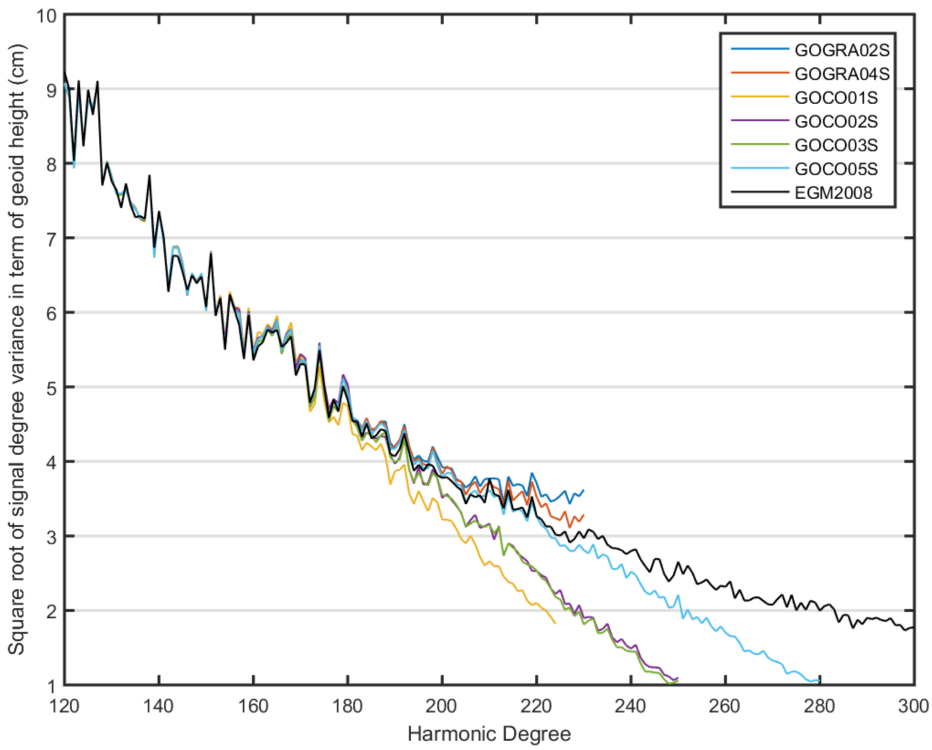

Figure A.4. Signal degree variances of GOGRA and GOCO models in terms of geoid height (cm)

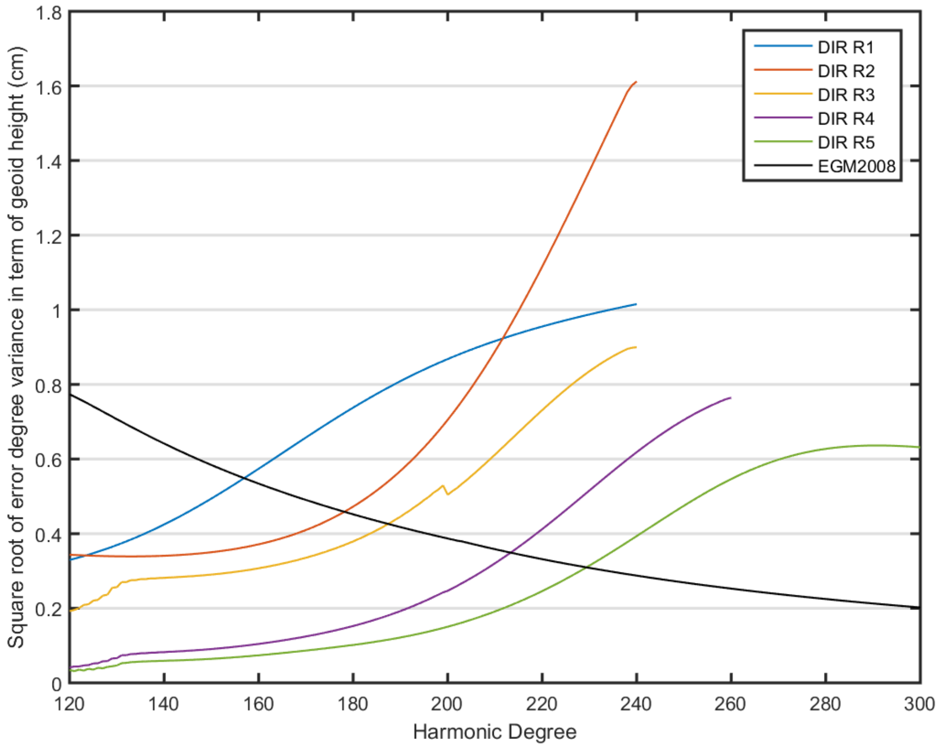

Figure A.5. Error degree variances of DIR models in terms of geoid height (cm)

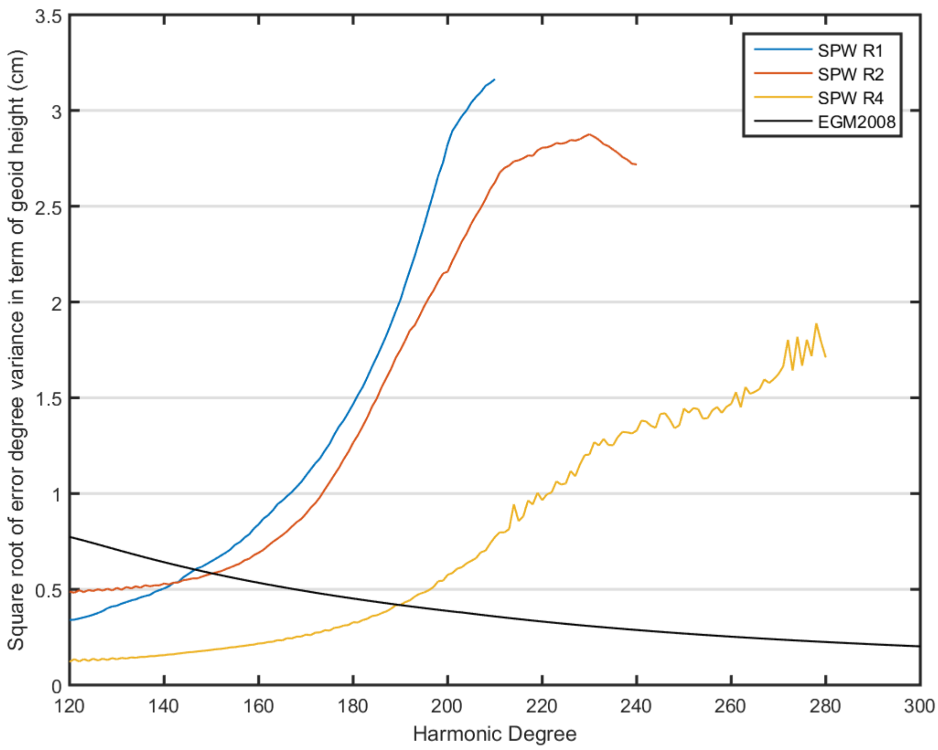

Figure A.6. Error degree variances of SPW models in terms of geoid height (cm)

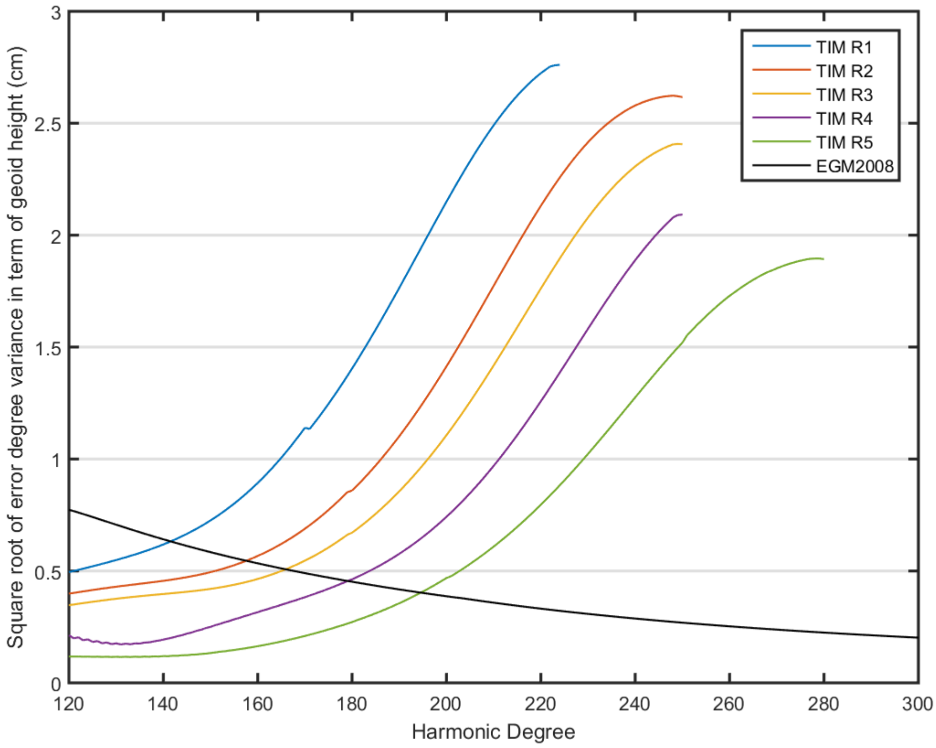

Figure A.7. Error degree variances of TIM models in terms of geoid height (cm)

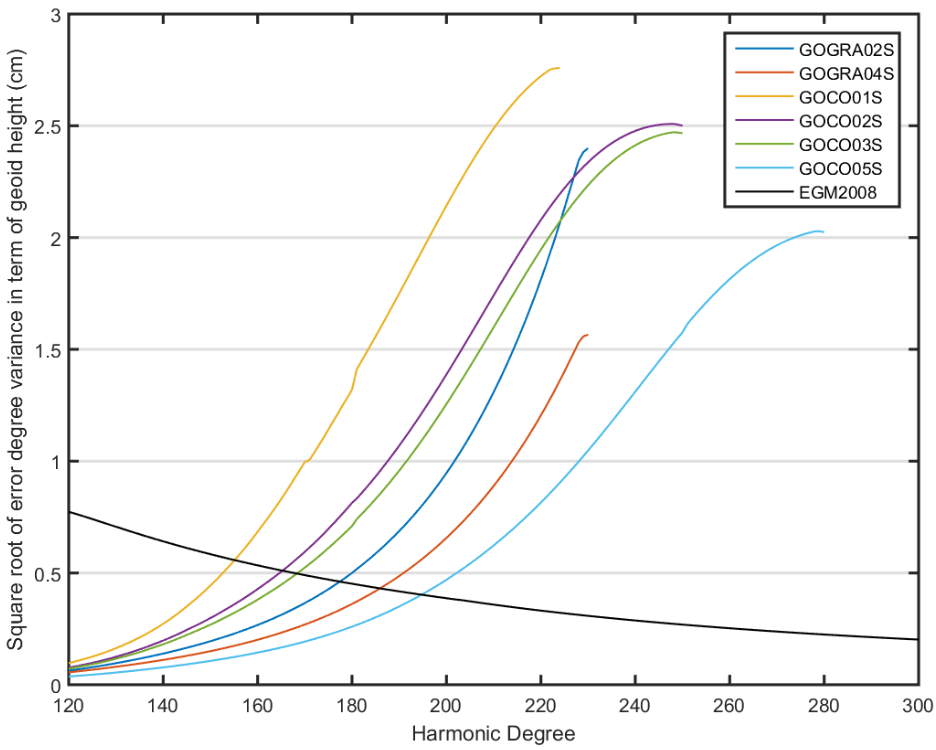

Figure A.8. Error degree variances of GOGRA and GOCO models in terms of geoid height (cm)

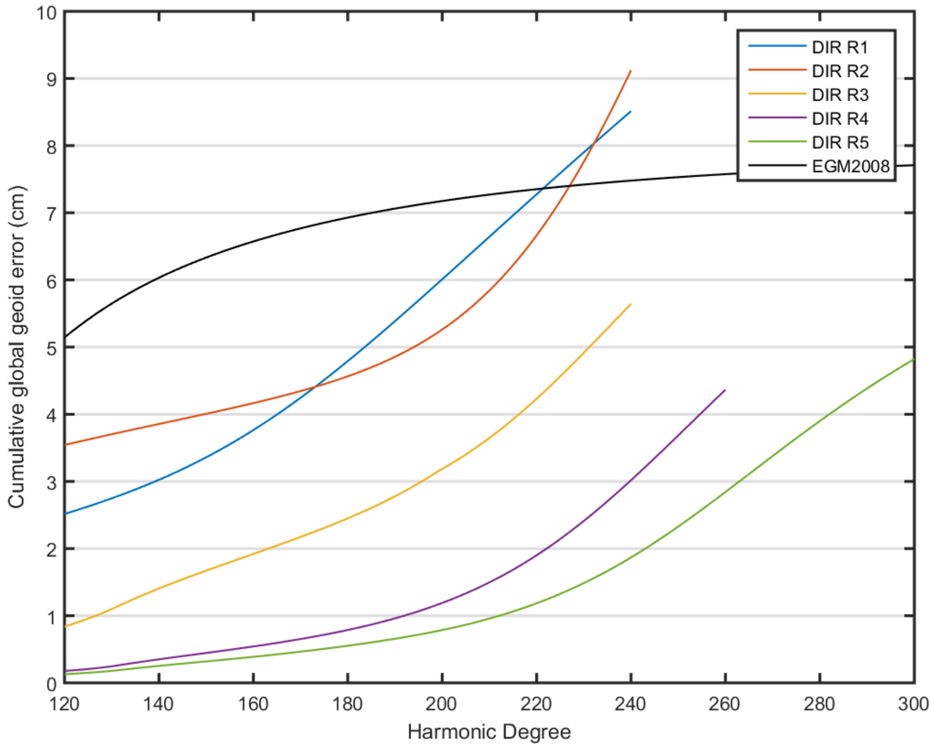

Figure A.9. Cumulative global geoid error of DIR models (cm)

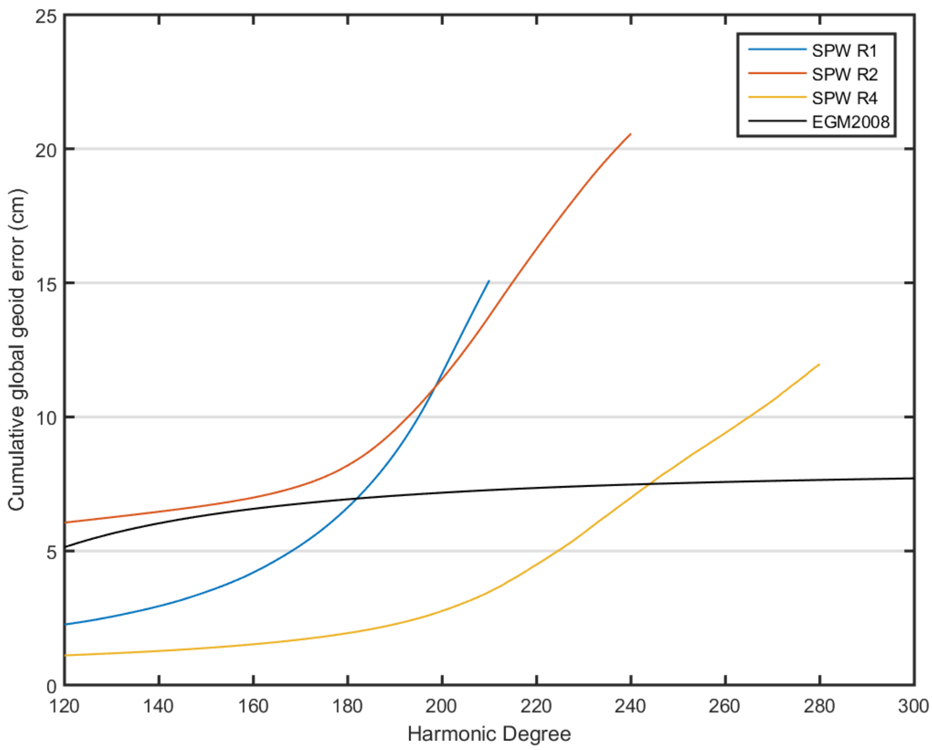

Figure A.10. Cumulative global geoid error of SPW models (cm)

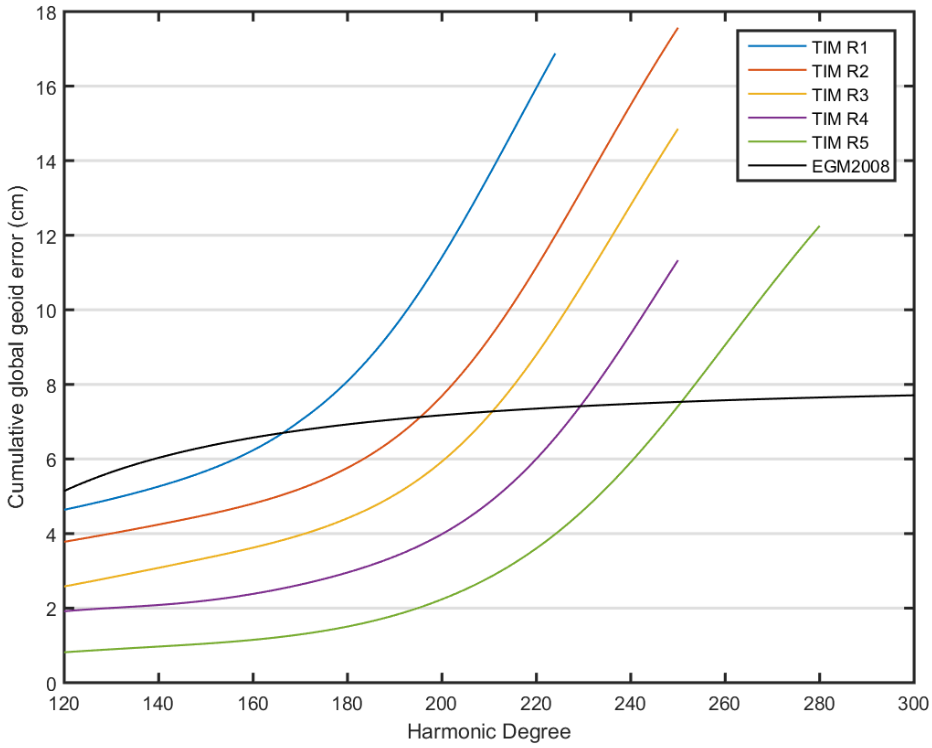

Figure A.11. Cumulative global geoid error of TIM models (cm)

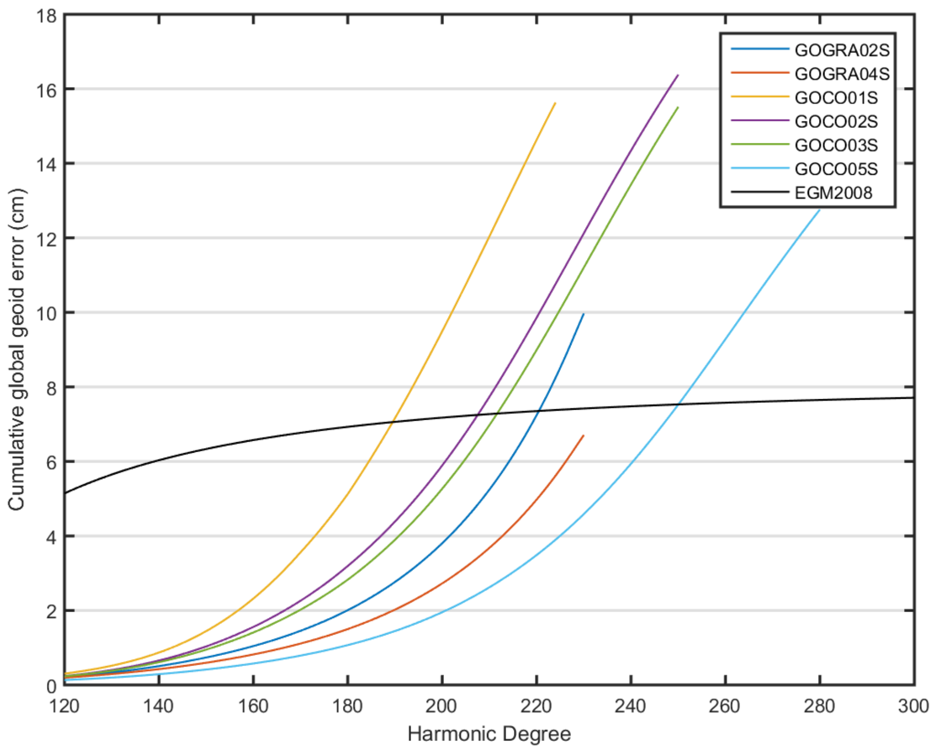

Figure A.12. Cumulative global geoid error of GOGRA and GOCO models (cm)

CURRICULUM VITAE

Name Surname: Mustafa Serkan Işık

Place and Date of Birth: Tarsus - 03.08.1992

Adress: Istanbul Technical University, Geomatics Engineering Department, 34469 Ayazaga, Istanbul, Turkey

E-Mail: isikm@itu.edu.tr

B.Sc.: 2010-2014, Istanbul Techinal University, Geomatics Engineering (100 % English Programme)

Professional Experience and Rewards:

▪ 2016/02-, Research Assistant, Istanbul Technical University (İTÜ), Civil Engineering Faculty, Geomatics Engineering Department.

▪ 2015/05-, Scholar, The Scientific and Technological Research Council of Turkey (TÜBİTAK), Türkiye'de Bölgesel Geoid Modelinin İyileştirilmesinde Yöntem Arastırması ve Güncel Uydu Gravite ve Altimetre Verilerinin Katkılarının Belirlenmesi.

▪ 2014/06-2014/08, Intern, Istanbul Technical University (İTÜ), IGS-ISTA Observation and Geodetic Laboratory.

▪ 2013/07-2013/08, Intern, NOKIA HERE MAPS.

List of Publications and Patents:

▪ **Işık, M. S.**, Erol, B. (2016) Geoid Determination Using GOCE-Based Models in Turkey. Presented at the 2016 EGU General Assembly, Session Session G4.2 - Satellite Gravimetry: Data Analysis, Results and Future Concepts, April 17th-22th, Vienna, Austria.

▪ Vergos, G., Erol, B., Natsiopoulos, D. A., Grigoriadis, V. N., **Işık, M. S.**, Tziavos, I. A. (2016) GOCE-based height system unification between Greece and Turkey. First considerations over marine and land areas. Presented at the 2016 EGU General Assembly, Session Session G4.2 - Satellite Gravimetry: Data Analysis, Results and Future Concepts, April 17th-22th, Vienna, Austria.

▪ Erol, S., **Işık, M. S.**, Erol, B. (2016) Assessments on GOCE-based Gravity Field Model Comparisons with Terrestrial Data Using Wavelet Decomposition and Spectral Enhancement Approaches. Presented at the 2016 EGU General Assembly, Session Session G4.2 - Satellite Gravimetry: Data Analysis, Results and Future Concepts, April 17th-22th, Vienna, Austria.

▪ **Işık, M. S.**, Erol, B. (2015) Geoid Modelling in Turkey Using Remove-Compute-Restore and Least Squares Modification of Stokes' Integral Methods, 26^{th} International Union of Geodesy and Geophysics (IUGG) General Assembly, Prague, 22 June - 02 July.

▪ Feizabadi, M., **Işık, M. S.**, İpbüker, C. (2015) Geomatik Mühendisliği Uygulamalarında Dönüşüm Yöntemleri, 15. Harita Bilimsel ve Teknik Kurultayı, Ankara, Mart 25-28.

▪ **Işık, M. S.**, Erol, B. (2014) Comparison of Remove-Compute-Restore and KTH Method for Gravimetric Geoid Determination in the Northwest of Turkey, XXIV International Symposium on Modern Technologies, Education and Professional Practice in Geodesy and Related Fields, Sofya, 6-7 Kasım.

▪ **Işık, M. S.**, Şenyıldız, Z., Sunar, F. (2014) Çok Zamanli Uydu Verilerinin Tarımsal Haritalamada Kullanimi: Altinova Devlet Üretme Çiftliği, 5. Uzaktan Algılama-Cbs Sempozyumu (UZAL-CBS 2014), İstanbul, Ekim 14-17.

▪ **Işık, M. S.**, Şenyıldız, Z. (2014) Use of Multi-temporal Data in Agricultural Mapping: Case Study of Altınova Manifacturing Farm, Bachelor's Degree Graduation Thesis, Istanbul Technical University