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AKIŞKAN YÜKLÜ EĞİMLİ PLAKLARIN TİTREŞİM VE FLATER ANALİZİ

YÜKSEK LİSANS TEZİ

Müh. Abdurrahman Şeref CAN

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Tez Danışmanı : Doç. Dr. Zahit MECİTOĞLU

Diğer Jüri Üyeleri : Doç. Dr. Hamza DİKEN

Doç. Dr. Temel KOTİL

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VIBRATION AND FLUTTER

ANALYSIS OF FLUID LOADED INCLINED PLATES

M.Sc. THESIS

Abdurrahman Şeref CAN, B.Sc.

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		Date	Signatu
Adviser	: Assoc. Prof. Dr. Zahit MECİTOĞLU	08.07.1996	
Members	: Assoc. Prof. Dr. Hamza DİKEN	8.8.96	
	Assoc. Prof. Dr. Temel KOTİL	08.07.1996	

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CONTENTS

NOMENCLATURE

D	bending rigidity
w	displacement in z-direction
ρ	density
h	thickness of plate
γ	poisson ratio
p_a	aerodynamic pressure
p_f	fluid pressure
r	aspect ratio of plate
λ	dynamic pressure parameter
A	aerodynamic matrix
ω	natural frequency (rad/sec)
Ω	frequency parameter
A_{mn}	unknown parameters
a,b	dimensions of plate
h_0	thickness of base fluid
h_x	thickness of fluid at x=a
h_y	thickness of fluid at y=b
h_f	thickness of fluid anywhere
V_a	airflow velocity
M	mach number
ξ,η	nondimensional parameter
m,n	number of waves
M_x , M_y	bending moments
M_{xy}	twisting moments
Q	force
E	elastic modulus
σ,ε	stress,strain

LIST OF FIGURES

SUMMARY

Flutter and divergence phenomena are defined respectively as the dynamic or static instability of an aeroelastic system , characterized by the interactions of elastic deformation and aerodynamic loads. They are aeronautically very important since the onset of flutter/divergence may quickly develop into catastrophic structural failure or undesirable limit cycle oscillations.

Fluid-loaded plates and shells have primary interest especially in the aerospace and offshore structures. Flutter of a plate in the supersonic flow (panel flutter) falls in the category of self-excited oscillations. The panel flutter differs from wing flutter in that the aerodynamic force acts only one side of the panel and that the phenomena of panel flutter is generally less catastrophic than wing flutter. Small amplitude linear structural theory indicates that there is a critical dynamic pressure above which the panel motion becomes unstable and grows exponentially with time.

On the other hand, the determination of free vibration characteristics of a fluid loaded plate is essential to obtain dynamic and aeroelastic responce of a plate subjected to dynamic loading. A vibrating plate with fluid load is under the effect of an added pressure due to the fluid motion. If the high frequency vibrations are of primary concern as in problems of noise and sound, the added pressure loading will be mainly proportional to velocity.In the problems above the effects of added mass are negligible. On the other hand, there are many instances such as vibration and flutter of fuel-loaded wing panels and hydroelastic problems of hydrofoils, in which low frequency characteristics of such elastic plates are of importance. In these cases the effect of added mass predominate.

In this study, free vibration and flutter of a fluid-loaded plate is studied numerically. The equations of motion for the plate are stated within the frame of Kirchoff's thin plate theory with temperature dependent material properties. The coupling between the vibrating plate and the fluid is of the form of an induced pressure at the interface. This pressure may be conveniently interpreted as added inertia of plate. The aerodynamic pressure is considered by the use of Piston theory of linear aerodynamics. The interaction problem among the airflow, plate and fluid is solved numerically using the Galerkin method. Vibration and flutter characteristics of the inclined fluid-loaded plate are obtained. Then, influence of geometrical properties of the plate and fluid on the vibration and flutter characteristics of the plate are studied.

Fluid load reduces the natural frequencies of the plate in all the considered cases. The spectrum of the frequencies has importance on the dynamic behavior of the plate. Due to the added mass the thickness of base fluid has tendency to reduce the natural frequencies. The aspect ratio of the plate also effect the frequency ratios for different inclinations.

This study may be extended to investigation of nonlinear behavior of fluid-loaded plate. Having obtanied thermal effects can be taken into account. The flutter analysis of fluid-loaded inclined plates with temperature effects will have importance for aerospace structures. Influence of the change of plate's curvature due to the fluid load on the vibration and flutter characteristics may be analyzed.

ÖZET

AKIŞKAN YÜKLÜ EĞİMLİ PLAKLARIN TİTREŞİM VE FLATER ANALİZİ

Flater; aerodinamik kuvvetler, elastik reaksiyonlar ve atalet kuvvetlerinin karşılıklı etkileşimleriyle oluşan sürekli salınımlardır. Olay inşaat mühendisliği yapılarında, özellikle uzay ve hava araçlarında kendini göstermektedir. Uçağın ekonomik olarak işletilebilmesi için gerekli olan hafiflik şartı, ister istemez yük altında büyük ölçüde deforme olan esnek yapılara yol açmıştır. Böyle deformasyonlar, neticede yine deformasyonlara sebeb olacak olan aerodinamik yük dağılımı neticeleriyle sonuçlanır. Bu karşılıklı geri besleme işlemi, kendi kendini tahrik eden salınımlar şeklinde beliren ve genellikle tahribatla sonuçlanan flatere yol açar. Titreşen uçak yapıları ile civardaki akım arasında enerji transferine sebeb olan bu kararsızlık ya dizaynla tamamen yok edilmeli ya da manevra zarfi dahilinde ortaya çıkmasına müsaade edilmemelidir.

Flaterin başlaması doğrudan doğruya yapının rijidliği ile ilgili olduğundan, artık uçağın rijid bir cisim olarak değil, aksine elastik bir yapı olarak ele alınması gerekir.

Ses üstü uçuşun başlamasıyla birlikte uçak yapılarında, panel flateri denilen yeni bir olayla karşılaşılmıştır. Panel, uçuş vasıtalarındaki kaplamaların iç yapı elemanlarıyla desteklenmesi sonucunda dış yüzeylerin bölündüğü ayrık elemanların her birine verilen addır.

Panel flaterin önemi havacılık tarihinde karşılaşılan bazı vakalarla vurgulanmıştır. Mesela 1950'lerin ortalarında bir avcı uçağı, flatere giren bir panele tutturulmuş bulunan bir hidrolik hattın parçalanmasından dolayı deney uçuşu sırasında kaybedilmiştir. Pilot mahalli dayanılmaz derecede gürültülü olan bir başka avcı uçağı problemin panel flaterinden kaynaklandığı anlaşıldıktan sonra problem giderilmiştir. Uzaya gönderilen roket ve diğer araçlarda da, panel flateri problem doğurmuştur.

Probleme önce lineer modeller ve viskozitesiz aerodinamikle yaklaşım yapılmıştır. Ancak son zamanlarda yapılan kontrollü deneyler ve geliştirilmiş teorilerle elde edilen çözümler, plakların dinamik aeroelastik kararlılığında yapısal non-lineerliklerin ve viskoz akım tesirlerinin önemli olduğunu göstermiştir. Yapılan çalışmalar şu şekilde sınıflandırılabilir :

(i) Deneysel Çalışmalar

Aeroelastisite sahasındaki deneysel çalışmalar iki ana gayeyle yapılmışlardır. Bunlar işe yarar teorilerin geliştirilmesinde gerekli yönlendirici tesiri yapmışlar ve mevcut teorilerin güvenilirliğinin bulunmadığı birçok alanda pratik problemler için çözümler sağlamışlardır.

Flater olayı açısından önemli olan kararsızlığın başlama noktasının belirlenmesidir. Plakların flater sınırını deneysel olarak belirlemek için bir takım teknikler uygulanmıştır. Dowell ve Voss, Fung ve Lock, Nasa'da araştırmalar yapan Langley Grubu deneysel çalışmalarla flater sınırını aramışlardır.

(ii) Teorik Çalışmalar

Flater problemi üzerinde çok geniş bir literatür oluşmakla beraber analizlerin pek çoğu kullanılan yapısal ve aerodinamik teoriye dayandırılan dört kategoriden birine yerleştirilebilir : 1) Lineer yapısal teori; sanki daimi (quasi-steady) aerodinamik teori 2) Lineer yapısal teori; tam lineerleştirilmiş (viskozitesiz potansiyel), aerodinamik teori 3) Non-Lineer yapısal teori; sanki daimi aerodinamik teori 4) Non-Lineer yapısal teori; tam lineerleştirilmiş aerodinamik teori. Lineerleştirilmiş teoriler hata paylarını da beraberlerinde getirmişlerdir

Plak çökmesini temsil etmek ve hareket denklemlerini türetmek için bir takım yaklaşımlar yapılmıştır. 1) Galerkin yöntemi ile birleştirilmiş modal açılım. 2) Sonlu fark temsiller. 3) Sonlu eleman temsiller. 4) Kesin çözümler diye bilinen değişkenlerine ayırma.

Birinci yaklaşımda yapısal deformasyon tabii veya en azından tam modların serisine açılır; verilmiş modal deformasyon için aerodinamik kuvvetler -mesela, transformasyon teknikleri kullanılarak- hesaplanır. Bu iki adımdan elde edilen sonuçlar hareket denklemlerinde kullanılarak, modal genlikler için bir diferansiyel denklem takımına ulaşmak üzere, Galerkin yöntemi uygulanır. Bu denklem takımı sayısal integrasyonla veya diğer herhangi bir sayısal teknikle çözülür.

Belli bir doğruluk derecesi için gerekli olan modların sayısı elemanların sayısından oldukça az olduğundan, Galerkin yöntemi sonlu fark yaklaşımına göre daha az serbestlik derecesi gerektirir. Buna karşılık sonlu farklar yöntemini uygulama alanı geniştir ve digital hesaplamada kolaylık sağlamaktadır.

Mühendislik problemlerinde daima bir olay ve bu olayın cereyan ettiği bir sistem vardır. Bu durumda yapılan iş, probleme teorik ve/veya deneysel yoldan yaklaşım yaparak olayı uygun şekilde modellemek ve sonra çözüme gitmektir. Burada ele alınan problemde olay, akım içinde bulunan bir plağın gazla olan etkileşimidir. Sistem iki kısımdan oluşmaktadır : Gaz ve elastik bir katı cisim. Bunların ortak özelliği sürekli ortam olmalarıdır. Parçacık hareketi için Newton'un ortaya koyduğu aksiyomların sürekli ortam için geliştirilmiş şekli ile, bünye için yapılan bazı kabuller yardımıyla elastisitenin ve aerodinamiğin temel denklemleri elde edilir.

Mühendislik uygulamaları bakımından plaklar teorisi, bugün için elastisite teorisindeki en ilginç ve en önemli konulardan biridir. Plak deyimi tanım olarak, küçük eğrilikli iki yüzeyle sınırlanan ve kalınlıkları yüzey boyutlarına kıyasla küçük olan cisimlere verilen addır. Bizim çalışmamızda Kirchoff'un plak modeli kullanılmıştır. Buna göre; plak malzemesi elastik, homojen ve izotropiktir. Başlangıçta plak düzdür, plağın kalınlığı diğer boyutlarına oranla çok küçüktür. Çökmeler de plak kalınlığına göre küçüktür. Enlemesine kesme ve normal uzamalar ihmal edilmiştir. Orta yüzeye dik olan gerilmeler ihmal edilebilir. Orta yüzeyde düzlemsel kuvvetlerin meydana getirdiği uzamalar da kaale alınmayabilir. Bu şekilde ortaya koyduğumuz teori ile plak denklemindeki non-lineer terimler de ihmal edilmiş olur. Flaterin sınırını verecek hareket denklemi, aerodinamik yaklaşım da ele alındıktan sonra ifade edilecektir.

Kullanılan aerodinamik model potansiyel akım modeli olup; viskoz tesirlerin ihmal edildiği bu teori, geniş bir ses altı-ses üstü Mach sayısı ve plak uzunluk/genişlik oranı aralığında tatminkar sonuçlar verir. Kenar tabaka tesirlerinin önemli hale geldiği ses aşımı-alçak ses üstü Mach sayılarında uygun sonuçlar elde edebilmek için geliştirilmiştir. Bu teoride ortam süreklidir, akım sürtünmesizdir, hiçbir ısı transferi yoktur. Sonlu güçte şok dalgaları yoktur ve hiçbir kütlesel kuvvet mevcut değildir.

Yapılan bu çalışmada, akışkan yüklü bir plağın titreşim ve flater analizleri nümerik olarak yapıldı. Plağın hareket denklemleri Kirchoff'un ince plak teorisi esas alınarak çıkarıldı. Akışkan tabakanın kalınlığının plak boyutlarına oranı yaklaşık 1/10 kabul edildi. Hiçbir yüzey dalgasının bulunmadığı farzedildi.Plak üzerindeki akışkanın atalet etkisi hareket denklemine ilave edildi. Aerodinamik basınç dağılımı için piston teorisi kullanıldı. Bu teorinin yüksek Mach sayıları için geliştirilmiş formu kullanıldı. Hareket denklemi nümerik olarak, Galerkin metodu kullanılarak çözüldü. Eğimli akışkan yüklü bir plağın titreşim ve flater karakteristikleri bulundu. Plak ve akışkanın geometrik özelliklerinin titreşim ve flater üzerine etkileri araştırıldı.

Bu kısa özetten ve yapı-aerodinamik kabuller hakkında bilgi verdikten sonra hareket denklemini yazabiliriz :

$$D\left[\frac{\partial^4 w}{\partial x^4}+\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right]+\rho h\frac{\partial^2 w}{\partial t^2}=p_f+p_a$$

Burada ilk terim plak denklemi, ikinci terim ise atalet terimidir. p_a aerodinamik basıncı, p_f ise plak üzerindeki akışkanın basıncını ifade eder. Plak üzerindeki akışkan tabakası, plağın farklı eksenlerde eğimlenmesinden dolayı farklı kalınlığa sahiptir. Yapılan çalışmada bu durum göz önüne alınmıştır. Bunun basınç ifadesine de bir etkisi olmuştur. Her iki basınç terimi denkleme yerleştirildikten sonra bir λ (aerodinamik basınç parametresi) tarifi yapılmıştır. Flater sınırının belirlenmesinde bu parametrenin önemi büyüktür. Plağın harmonik titreşim yaptığı kabul edilerek yerdeğiştirme ifadesindeki zaman terimi, uzaysal terimlerden ayrılmıştır. Bundan sonra boyutsuzlaştırma işlemleri yapılarak denklem çözüme hazır hale getirilmiştir.

Üzerinde akışkan yük bulunan plak, dört köşesinden basit mesnetlenmiştir. Çözüm için teklif edilen fonksiyon bu sınır şartlarını sağlamak zorundadır. Fakat plak denklemini sağlaması zorunlu değildir. Plak kuvvetlerine virtüel bir iş yaptırıldıktan sonra, teklif ettiğimiz fonksiyon buraya yerleştirilirse MxN tane lineer, cebrik denklem elde edilir, bunlar da matris formda yazılabilir. Artık burada aşağıda ifade edilen şekilde bir özdeğer problemi ortaya çıkmıştır.

$$\left[K+\lambda A-\omega^2 M\right]\left\{A_{mn}\right\}^T=0$$

Eğer aerodinamik basınç sıfır ise yukarıdaki ikinci terim ortadan kaybolur ve eşitlik bize akışkan yüklü bir plağın serbest titreşim frekanslarını verir. Aerodinamik basınç ifadesi bulunursa λ değeri yavaş yavaş artırılırken özdeğer problemi çözülür ve flater sınırı tesbit edilir.

Yapılan çözümlerde poisson oranının değeri 0.3, akışkan yoğunluğunun plak oranına oranı 0.308 olarak alınmıştır. Bir Ω frekans parametresi tariflenerek, grafiklerde bu parametre kullanılmıştır. Akışkan yüklü bir plağın serbest titreşimleri bulunmuştur. Tablo 1'de görülebileceği gibi, frekanslar küçük modlarda daha çabuk yakınsamıştır. Plağa x ve/veya y yönlerinde verilen eğim frekanslarda düşmelere neden olmuştur. Başka bir deyişle plağın üzerine konan ilave kütle, serbest titreşim frekanslarının değerini azaltmaktadır. Dalga sayısı artarken azalma oranı da yükselmektedir (fig. 4.1).

Plak eğiminin küçük frekanslarda çok fazla etkisi görülmemektedir. Bu etki yüksek frekanslarda,bu frekanslara bağlı modlardaki yüksek enerji seviyeleri yüzünden büyümektedir. (fig. 4.2).

Orijinde bulunan akışkan seviyesinin de frekans oranları üzerinde etkileri mevcuttur. Burada akışkan seviyesi yükselirken frekans oranı düşmektedir. Aynı anda plak x ekseni yönünde eğimlendirilirse frekans oranındaki azalma daha da artmaktadır (y ekseni boyunca eğim yokken).

Plak görünüş oranının frekans oranları üzerindeki etkileri fig. 4.4'den görülebilir. Büyük açıklık oranlarında frekans oranları da büyümektedir. Buradan şu etki de görülebilir : Plağın görünüş oranı küçük ise, plağa eğim vermenin önemi yok gibidir. Sadece büyük görünüş oranlarında plağa verilen eğim, frekans oranlarını azaltıcı yönde etkilere sebeb olmaktadır.

Plağın üzerindeki akışkan yükün mod şekilleri üzerinde de tesirleri vardır : Plak x ve/veya y eksenleri doğrultusunda eğimlenirse, akışkanın kütle merkezi değiştiği için konturlarda x ve/veya y eksenleri doğrultusunda kaymalar görülmektedir. Eğim her iki yönde aynı zamanda verilirse, simetrik olmayan bu yük anti-simetrik modların oluşmasına sebeb olmaktadır.

Bir önceki sayfada matris formuyla verilen denklemde K matrisi gerçek ve simetrik bir matristir. Aerodinamik yük matrisin simetrikliğini bozmaktadır. Dinamik basınç parametresi λ'nın değeri artarken frekans değerleri bir λ_{cr} değerine kadar artar ve iki özdeğer burada birleşir. λ daha da artırılırsa frekanslar kompleks ifadeler haline gelir. İşte tam bu nokta kararsızlığın başladığı yer olup flater sınırı olarak adlandırılır. Yapılan çalışmada plağın yüksüz halde kararsızlık analizi de yapılmıştır. Daha sonra akışkan yük ilave edilerek ve plağa eğim verilerek flater sınırı belirlenmiştir.

Üzerinde akışkan olmayan dört köşesinden basit mesnetlenmiş kare bir plak için, kritik dinamik basınç parametresi 512.714 olarak bulunmuştur. Bu andaki kritik özdeğer ise 1848.29'dur. Dinamik basınç parametresi ve özdeğer için tam sonuçlar 512.651 ve 1848.21'dir. Bu sonuçlardan, Galerkin Metodu ile yaptığımız çözümün tam değerlere çok yakın olduğunu görmekteyiz. Plak eğiminin flater sınırı üzerindeki etkileri fig. 4.5'de gösterilmiştir. Eğer x ekseni yönünde negatif eğim varsa, eğimlenmemiş plağın dinamik basıncından daha küçük bir basınçta flater vuku bulur. Tersine olarak x ekseni yönünde pozitif yönde plak eğimi varsa, flater düz plağa göre daha geç meydana gelmektedir. Ayrıca orijinde akışkan varsa, eğime bağlı olarak bunun da kararsızlığın başlama sınırına etkisi olmaktadır. Düz plakta bu etki söz konusu değildir. Plak x ekseni yönünde pozitif eğimlenirse, orijinde bulunan akışkan flater sınırını düşürürken; negatif yöndeki eğimlenmede orijindeki akışkanın seviyesinin artışı bu sınırı büyütmektedir. Flatere ait mod şekillerinde ise şu görülür : Dinamik basınç parametresinin küçük değerlerinde, plak üzerindeki akışkanın mod şekilleri üzerindeki etkisi fazla görülmezken, kararsızlığın başladığı sınıra yaklaştıkça akışkanın etkisi fark edilir.

Akışkan yüklü eğimli bir plağın Kirchoff Teorisi temel alınarak ve Galerkin Metodu kullanılarak titreşim ve aeroelastik kararlılığı incelendi. Yapılan titreşim ve lineer flater analizlerinde tam sonuçlara benzer değerler bulundu.

Akışkan yük tüm durumlarda tabii frekansları düşürmektedir.Yüksek modlarda frekansların azalması düşük modlara göre daha fazladır. Akışkan yük daralan bir tabii frekans spektrumuna sebeb olur. Bunun plağın dinamik davranışı üzerinde etkisi vardır. Eğim sebebiyle oluşan frekans oranı değişimleri, plak eğiminin küçük değerleri için öneme haizdir.

Orijinde bulunan akışkanın yüksekliğinin, tabii frekansları azaltma eğilimi vardır. Y ekseni boyunca eğim, frekans oranı değişimlerini x eksenine göre değiştirmez fakat oranları azaltır. Bu tip eğimin, plağın anti-simetrik mod şekilleri üzerinde önemli tesirleri mevcuttur.

Plak görünüş oranının frekans oranları üzerinde etkileri şu şekilde ifade edilebilir : Büyük görünüş oranı yüksek frekans oranlarına sebeb olur ve burada plak eğikliği frekans oranını düşürücü yönde çalışır. Küçük görünüş oranlarında plak eğiminin frekans oranlarına etkileri önemsizdir.

Mod şekillerindeki kontur çizgileri üzerindeki değişmeler ve maksimum yer değiştirme noktaları üstündeki kaymalar, eğiklik sebebiyle oluşur. Diğer bir söyleyişle, akışkanın kütle merkezinin değişimi aeroelastik kararsızlığın başlama sınırına etkir. x ekseni boyunca pozitif plak eğimi, kritik dinamik basıncın artışına neden olur; bu da kararsızlık noktasını öteler.

Bu çalışmada yapı ve aerodinamik modellerde lineer teoriler kullanıldı. Plağın hareket denklemi, sıcaklığın malzeme özelliklerini etkilediği durumda elde edildi. Çözümde ise tüm termal etkiler ihmal edildi. Bu etkiler hesaba katılarak çözüm yapılabilir. Non-lineerlikler dikkate alarak titreşim ve flater analizleri yapılabilir. Akışkan yükün plak eğriliğinin değişimi üzerine olan etkileri incelenebilir.

CHAPTER 1. INTRODUCTION

For the aircraft structure subjected to aerodynamic loading, the interactions between elastic deformation of structure and aerodynamic forces can make the structure unstable, i.e., flutter or divergence may develop. As a consequence, the onset of instability may induce disastrous structural failure or undesirable limit cycle oscillations. Fluid-loaded plates and shells have primary interest especially in the aerospace and offshore structures. Flutter of a plate in the supersonic flow (panel flutter) falls in the category of self-excited oscillations. The panel flutter differs from wing flutter in that the aerodynamic force acts only one side of the panel and that the phenomenon of the panel flutter is generally less catastrophic than wing flutter. Small amplitude linear structural theory indicates that there is a critical dynamic pressure above which the panel motion becomes unstable and grows exponentially with time.

On the other hand, the determination of free vibration characteristics of a fluid-loaded plate is essential to obtain dynamic and aeroelastic response of a plate subjected to dynamic loading. A vibrating plate with fluid load is under the effect of an added pressure due to the fluid motion. If the high frequency vibrations are of primary concern as in problems of noise and sound, the added pressure loading will be mainly proportional to velocity. In the problems above the effects of added mass are negligible. On the other hand, there are many instances, such as vibration and flutter of fuel-loaded wing panels and hydroelastic problems of hydrofoils, in which low frequency characteristics of such elastic plates are of importance. In these cases the effects of added mass predominate.

Flutter of plates have been studied by many researchers. Dowell [1] gave a comprehensive review on both linear and nonlinear panel flutter. Srinivasan and Babu [2],[3] investigated flutter of quadrilateral plates. Liao-Sun [4] and Lin-Lu performed flutter analysis composite plates and shells in supersonic flow. Flutter of composite plates in subsonic flow was studied by Lin, Lu and Tarn [5]. Recently, Nasr [6] gave a large review on the finite element analysis of aeroelasticity of plates and shells. There are some other investigations including temperature effects on structures [7].

The vibration problems of fluid-loaded plates and shells have been investigated by some authors. Eatwell and Butler have studied the response of a fluid-loaded beam stiffened plate [8]. Espinosa and Gallego-Juarez [9] have studied the forced vibrations of water-loaded circular plates. Nagaya and Takeuchi have analyzed the vibration of fluid-loaded plate with arbitrary shape[10]. Komamatsu and Matsushima [11] have made some experiments on the vibration of fluid filled hemispherical shells. Mikami and Yoshimura [12] have used the collocation method for analyzing free vibration of shells with fluid loads. The basic theories and detailed account of the developments on panel flutter can be found in Ref [13]. Mecitoglu [14] has investigated the aeroelastic behavior of a plate. Most recently, Mecitoglu and Can has studied on the Vibration and Flutter of Fluid Loaded Inclined Plates [15].

In this study, free vibration and flutter of a fluid-loaded plate with simply supported edges is studied numerically. The equations of motion for the plate are stated within the frame of Kirchoff's thin plate theory with temperature-dependent material properties. The thickness of the fluid layer is considered to be approximately one-tenth of the plate length. It is also assumed that no surface waves are present. The coupling between the vibrating plate and the fluid is of the form of an induced pressure at the interface. This pressure may be conveniently interpreted as added inertia of plate. The aerodynamic pressure is considered by the use of Piston theory of linear aerodynamics. The interaction problem among the airflow, plate and fluid is solved numerically using the Galerkin method. Vibration and flutter characteristics of the inclined fluid-loaded plate are obtained. Then influence of geometrical properties of the plate and fluid on the vibration and flutter characteristics of the plate are studied.

CHAPTER 2. GENERAL EQUATIONS

2.1 THIN PLATE THEORY

2.1.1 Coordinate System and Sign Convention

The shape of a plate is adequately defined by describing the geometry of its middle surface, which is a surface that bisects the plate thickness h at each point (fig.2.1). The small deflection plate theory, generally attributed to Kirchhoff and Love (other contributors include Bernoulli, Navier, Poisson, Saint-Venan, and Lagrange)[16], is based on the following assumptions :

1. The material of the plate is elastic, homogeneous, and isotropic linear thermo elastic; at the end of chapter, thermo-elastic parts will be neglected.
2. The plate is initially flat.
3. The thickness of the plate is small compared to its other dimensions. The smallest lateral dimension of the plate is at least ten times larger than its thickness.
4. The deflections are small compared to the plate thicknesses. A maximum deflection from one tenth to one fifth of the thickness is considered as the limit for small deflection theory. This limitation can also be stated in terms of length; i.e., the maximum deflection is less than one fiftieth of the smaller span length.
5. The slopes of the deflected middle surface are small compared to unity.
6. The deformations are such that straight lines, initially normal to the middle surface, remain straight lines and normal tothe middle surface (deformations due to transverse shear will be neglected).
7. The deflection of the plate is produced by displacement of points of the middle surface normal to its initial plane.

8. The stresses normal to the middle surface are of a negligible order of magnitude.

Many of these assumptions are familiar to the reader since they have their equivalent counterparts in elementary beam theory. Small and large scale tests have proved the validity of these assumptions. In the case of plates with bending resistance, an additional simplifying assumption can be introduced :

9. The strains in the middle surface produced by inplane forces can usually be neglected in comparison with strains due to bending (inextensional plate theory).

For rectangular plates, the use of Cartesian coordinate system is the most convenient (fig. 2.1). The external and internal forces and the deflection components u, v and w are considered positive when they point toward the positive direction of the coordinate axes X, Y and Z. In general engineering practice, positive moments produce tension in the fibers located at the bottom of structure. This sign convention is maintained for plates.

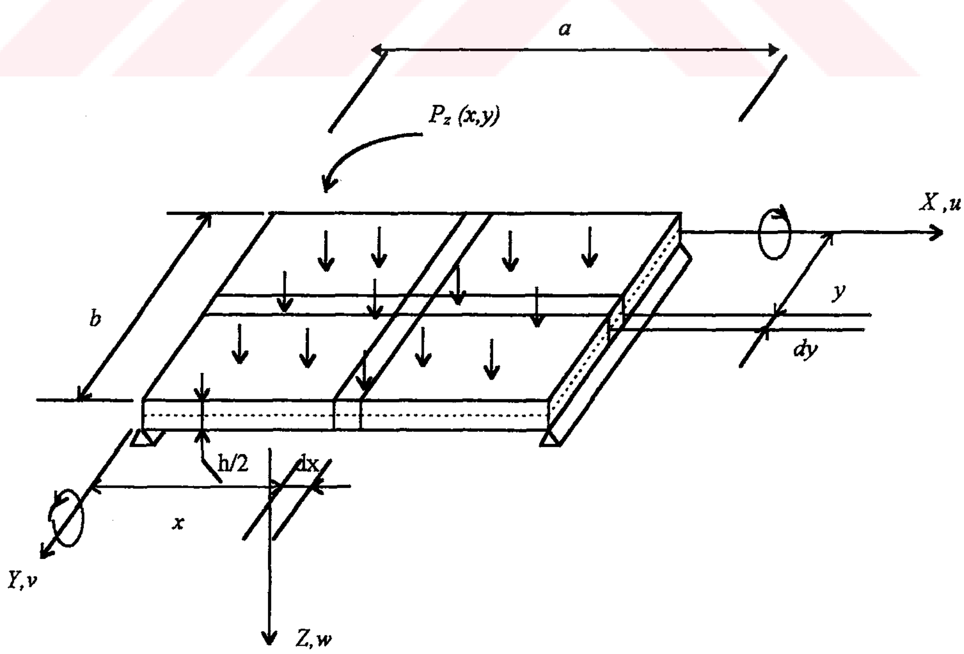


Figure 2.1 Laterally loaded rectangular plate.

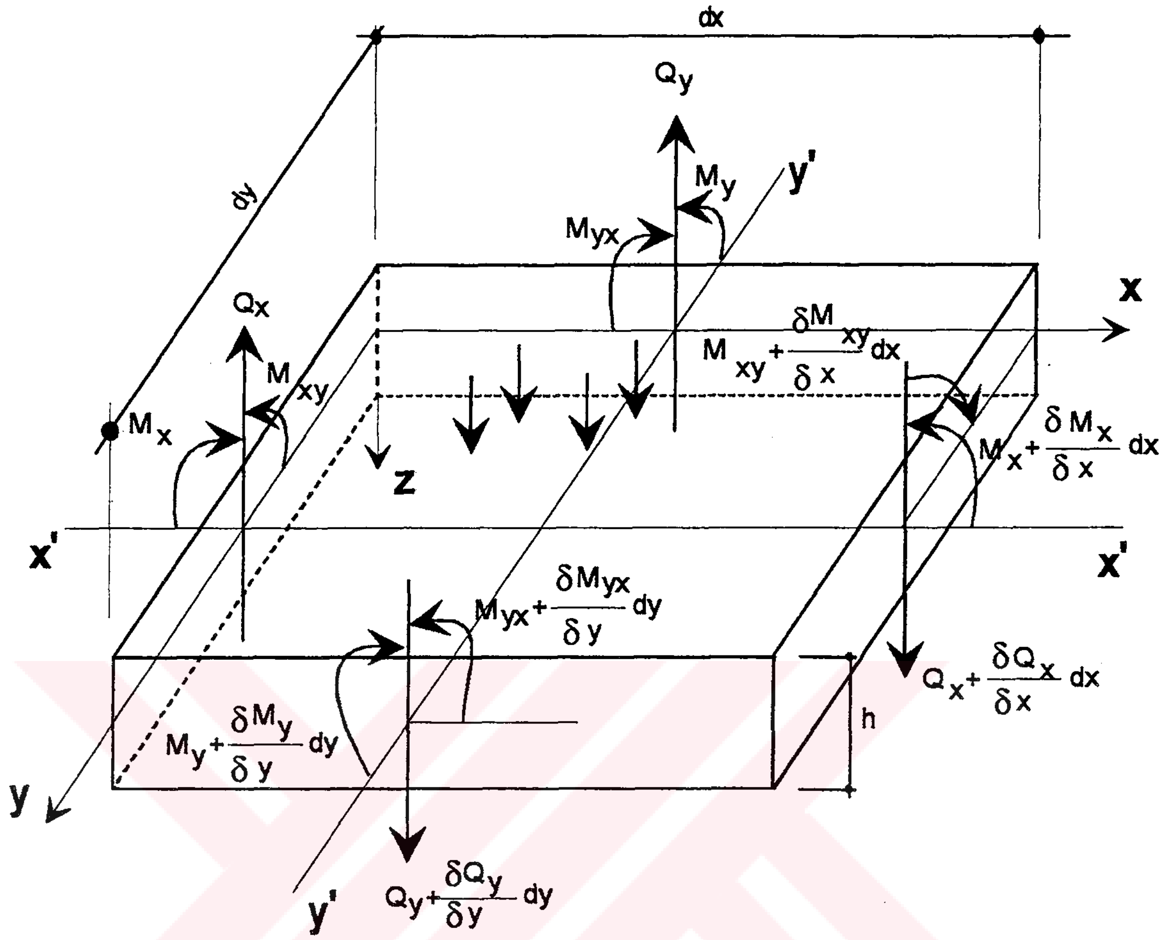


Figure 2.2 External and internal forces on the element of middle surface

Considering an elemental parallelepiped cut out of the plate, as shown in fig.2.1, we assign positive internal forces and moments to the near faces. To satisfy the equilibrium of the element, negative internal forces and moments act on its far sides. The first subscript of the internal forces indicates the direction of the surface normal pertinent to the section.

2.1.2 Equilibrium of the Plate Element

Assuming that the plate is subjected to lateral forces only, from the six fundamental equlibrium equations, the following three can be used :

$$\Sigma M_x = 0, \qquad \Sigma M_y = 0, \qquad and \qquad \Sigma P_z = 0 \qquad (2.1.1.)$$

The behavior of the plate is in many respects analogous to that of a two-dimensional gridwork of beams. Thus the external load P_x is carried by Q_x and Q_y transverse shear forces and by M_x and M_y bending moments. The significant derivation from the two-dimensional gridwork action of beams is thepresence of the twisting moments M_{xy} and M_{yx} (fig. 2.2). In the theory of plates it is customary to deal with internal forces and moments per unit length of the middle surface. To distinguish these internal forces from the above mentioned resultants, th notations q_x , q_y , q_y , m_x m_y , m_{xy} , and m_{yx} are introduced.

The procedure involved in setting up the differential equation of equilibrium is as follows :

1. Select a convenient coordinate system and draw a sketch of plate element.
2. Show all external and internal forces acting on the element.
3. Assign positive internal forces with increments (q_x+ ...,q_y+.....etc.) to the near sides.
4. Assign negative internal forces to the far sides.
5. Express the incremens by a truncated Taylor series in the form

$$q_x + dq_x = q_x + \frac{\partial q_x}{\partial x}dx, \quad m_y + dm_y = m_y + \frac{\partial m_y}{\partial y}dy, \quad etc. \qquad (2.1.2.)$$

6. Express the equilibrium of the internal and external forces acting on the element. Let us express, for instance, that the sum of the moments of all forces around the Y axis is zero (fig. 2.2). This gives

$$\left(m_x + \frac{\partial m_x}{\partial x}dx\right)dy - m_x dy + \left(m_{yx} + \frac{\partial m_{yx}}{\partial y}dy\right)dx - m_{yx}dx$$

$$-\left(q_x + \frac{\partial q_x}{\partial x}\right)dy\frac{dx}{2} - q_x dy\frac{dx}{2} = 0 \qquad (2.1.3)$$

After simplification, we neglect the term containing $1/2(\partial q_x/\partial x)(dx)^2\, dy$, since it is a small quantity of higher order. Thus equation (2.1.3) becomes

$$\frac{\partial m_x}{\partial x} dx\, dy + \frac{\partial m_{yx}}{\partial y} dy\, dx - q_x\, dx\, dy = 0 \tag{2.1.4}$$

and after division by dx dy, we obtain

$$\frac{\partial m_x}{\partial x} + \frac{\partial m_{yx}}{\partial y} = q_x \tag{2.1.5}$$

In a similar manner the sum of moments around the X axis gives,

$$\frac{\partial m_y}{\partial y} + \frac{\partial m_{xy}}{dx} = q_y \tag{2.1.6}$$

The summation of all forces in the Z direction yields the third equilibrium equation :

$$\frac{\partial q_x}{\partial x} dxdy + \frac{\partial q_y}{\partial y} dxdy + p_z\, dxdy = 0 \tag{2.1.7}$$

which, after division by dx dy , becomes

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = -p_z \tag{2.1.8}$$

Substituting eqs. (2.1.5) and (2.1.6) into (2.1.8) and observing that $m_{xy} = m_{yx}$, we obtain

$$\frac{\partial^2 m_x}{\partial x^2} + 2\frac{\partial^2 m_{xy}}{\partial x\, \partial y} + \frac{\partial^2 m_y}{\partial y^2} = -p_z(x,y) \tag{2.1.9}$$

The bending and twisting moments in eq. (2.1.9) depend on strains, and the strains are functions of the displacement components (u, v, w), as discussed before. Thus, in the next steps, relation between the internal moments and displacement components are sought.

2.1.3 Relation Between Stress, Strain, and Displacements

The stresses σ_x and σ_y are related to strains by means of the constitutive relations of an isotropic linear thermo-elastic material

$$\sigma_x = \frac{E(T)}{1-\gamma^2}\left(\varepsilon_x + \gamma\varepsilon_y - (1+\gamma)\alpha(T)T\right)$$

$$\sigma_y = \frac{E(T)}{1-\gamma^2}\left(\varepsilon_y + \gamma\,\varepsilon_x + (1+\gamma)\alpha(T)T\right) \tag{2.1.10}$$

where T is the difference from the reference temperature. The temperature variation across the thickness, T(x,z), is dependent on the conductivity of the plate material. For illustrative purposes, the temperature is assumed to vary linearly across the thickness of the plate. This assumption is valid for thin plates, and we may write

$$T(x,z) = T_1 + T_2\,x + T_3\,z + T_4\,xz \tag{2.1.11}$$

The material properties, such as Young's modulus and the thermal expansion coefficient, have a spatial variation due to temperature distribution over the plate. Therefore, the material of the plate has a kind of inhomogenity which must be considered in the constitutive relations. For purposes of demonstration, a linear variation of the material properties with the temperature is considered for the temperature ranges under consideration described by the reference [17],

$$E(T) = E_0 + E_1\,T \qquad \text{and} \qquad \alpha(T) = \alpha_0 + \alpha_1\,T \tag{2.1.12}$$

where E_0 and α_0 are Young's modulus and the thermal expansion coefficient at the zero temperature, respectively. E_1 and α_1 are constant coefficients for the considered model. The torsional moments m_{xy} and m_{yx} produce in-plane shear stress τ_{xy} and τ_{yx} which are again related to the shear strain γ by the pertinent Hookean relationship, giving

$$\tau_{xy} = G\gamma_{xy} = \frac{E(T)}{2(1+\gamma)}\gamma_{xy} = \tau_{yx} \tag{2.1.13}$$

Next, we consider the geometry of the deflected plate to express the strains in terms of the displacement coefficients. Taken a section of a constant y, as shown in fig. 2.3, we compare the section before and after deflection. Using assumptions 5 and 6, introduced earlier in this section, we express the angle of rotation of lines I-I and II-II by

$$\vartheta = -\frac{\partial w}{\partial x} \qquad and \qquad \vartheta + ... = \vartheta + \frac{\partial \vartheta}{\partial x}dx, \tag{2.1.14}$$

respectively. After the deformation the length $\overline{AB}$ of a fiber, located at z distance from the middle surface, becomes $\overline{A'B'}$ (Figure 2.3). Using the definition of strain,

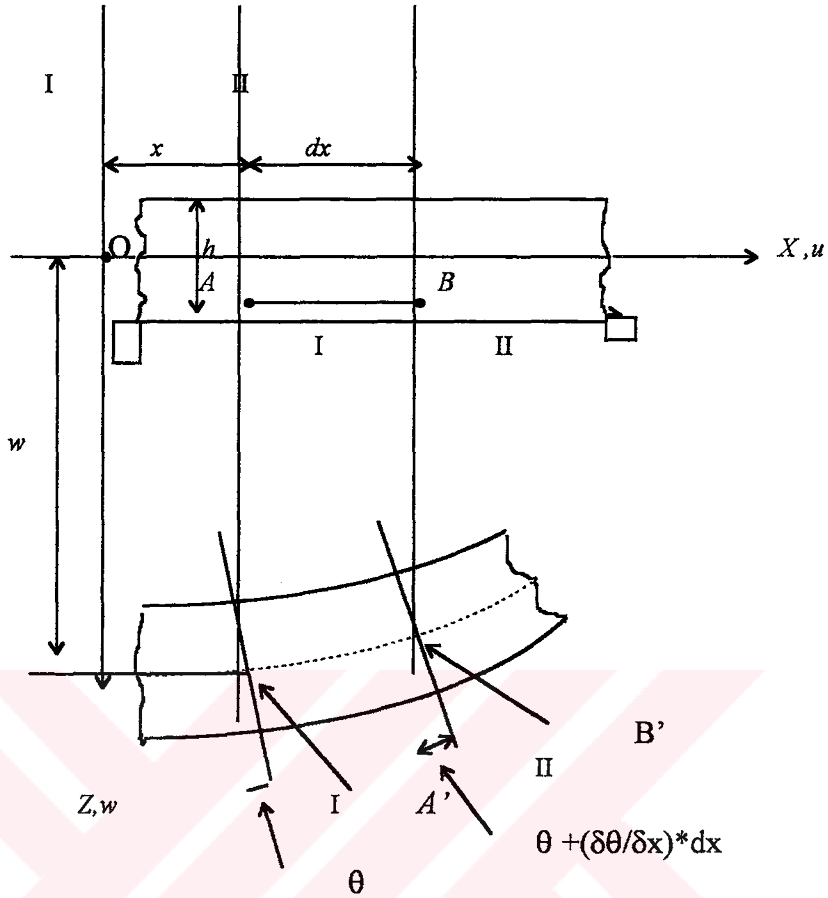
$$\varepsilon_x = \frac{\Delta dx}{dx} = \frac{\overline{A'B'} - \overline{AB}}{\overline{AB}} = \frac{[dx + z(\partial\vartheta/\partial x)dx] - dx}{dx} = z\frac{\partial\vartheta}{\partial x} \tag{2.1.15}$$

Substituting into this expression the first of the equations (2.1.14), we obtain

$$\varepsilon_x = -z\frac{\partial^2 w}{\partial x^2} \tag{2.1.16}$$

A similar reasoning yields ε_y, the strain due to normal stresses in the Y direction; thus

$$\varepsilon_y = -z\frac{\partial^2 w}{\partial y^2} \tag{2.1.17}$$



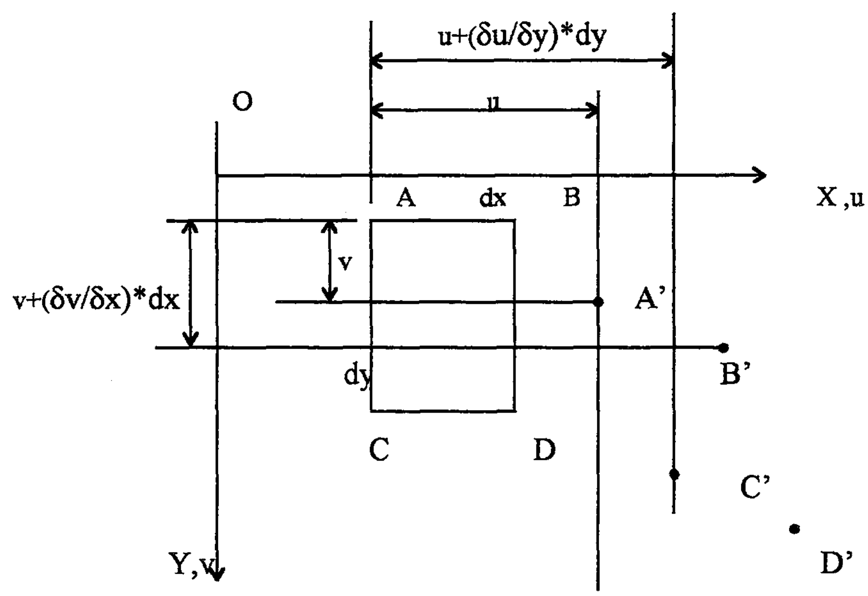


Figure 2.3 and 2.4 Section before-after deflecton and Angular distorsion

Let us now determine the angular distorsion $\gamma_{xy} = \gamma' + \gamma''$ by comparing an ABCD rectangular parallelogram (fig. 2.4), located at a constant distance z from the middle surface, by its deformed shape $A'B'C'D'$ on the deflected plate surface. From the two small triangles in Fig. 2.4 and from another equation, it is evident that

$$\gamma' = \frac{\partial v}{\partial x} \quad \text{and} \quad \gamma'' = \frac{\partial u}{\partial y} \tag{2.1.18}$$

but from Fig. 2.3

$$u = z\vartheta = -z\frac{\partial w}{\partial x} \tag{2.1.19}$$

similarly,

$$v = -z\frac{\partial w}{\partial y} \tag{2.1.20}$$

consequently,

$$\gamma_{xy} = \gamma' + \gamma'' = -2z\frac{\partial^2 w}{\partial x \partial y} \tag{2.1.21}$$

The curvature changes of the deflected middle surface are defined by

$$\kappa_x = -\frac{\partial^2 w}{\partial x^2}, \quad \kappa_y = -\frac{\partial^2 w}{\partial y^2} \quad \text{and} \quad \kappa_{xy} = -\frac{\partial^2 w}{\partial x \partial y} \tag{2.1.22}$$

2.1.4 Internal Forces Expressed in Terms of w

The stress components σ_x and σ_y produce bending moments in the plate element in a manner similar to that in elementery beam theory. Thus, by integrating of the normal stress components, the bending moments, acting on the plate element, are obtained

$$m_x = \int_{-h/2}^{h/2} \sigma_x z\,dz \qquad and \qquad m_y = \int_{-h/2}^{h/2} \sigma_y z\,dz \tag{2.1.23}$$

Similarly, the twisting moments produced by the shear stresses τ , τ_{xy} , τ_{yx} can be calculated from

$$m_{xy} = \int_{-h/2}^{h/2} \tau_{xy} z\,dz \qquad and \qquad m_{yx} = \int_{-h/2}^{h/2} \tau_{yx} z\,dz \tag{2.1.24}$$

but $\tau_{xy} = \tau_{yx} = \tau$, and therefore $m_{xy} = m_{yx}$.

If we substitute Eqs. (2.1.16) and (2.1.17) into (2.1.10), the normal stresses σ_x and σ_y are expressed in terms of the lateral deflection w. Thus, we can write

$$\begin{aligned} \sigma_x &= \frac{E(T)}{1-\gamma^2}\left(\frac{\partial^2 w}{\partial x^2} + \gamma \frac{\partial^2 w}{\partial y^2} + (1+\gamma)\alpha(T)T\right) \\ \sigma_y &= \frac{Ez}{1-\gamma^2}\left(\frac{\partial^2 w}{\partial y^2} + \gamma \frac{\partial^2 w}{\partial x^2} + (1+\gamma)\alpha(T)T\right) \end{aligned} \tag{2.1.25}$$

Integration of the Eqs. (2.1.24), after substitution of the above expressions for σ_x and σ_y gives

$$\begin{aligned} m_x &= \frac{A_2}{1-\gamma^2}\left(\kappa_x + \gamma\,\kappa_y\right) - \left(\frac{1+\gamma}{1-\gamma^2}\right)B_1 \\ m_y &= \frac{A_2}{1-\gamma^2}\left(\kappa_y + \gamma\,\kappa_x\right) - \left(\frac{1+\gamma}{1-\gamma}\right)B_1 \\ m_{xy} &= \frac{A_1}{1+\gamma}\kappa_{xy} \end{aligned} \tag{2.1.26}$$

where coefficients are defined as

$$A_1 = \frac{E_1 h^3}{12}\left(T_3 + T_4\, x\right)$$

$$A_2 = \frac{h}{12}\left(E_0 + E_1\, T_1 + E_1\, T_2\, x\right) \qquad (2.1.27)$$

$$B_1 = \left\{ \frac{\alpha_1 E_1 h^5 (T_3 + T_4 x)^3}{80} + \left[h^3\,(T_3 + T_4 x)(\alpha_0 E_0 + 2\alpha_1 E_0 T_1 + 2\alpha_0 E_1 T_1 + \right.\right.$$

$$\left.\left. 3\alpha_1 E_1 T_1^2 + 2\alpha_1 E_0\, T_2 x + 2\alpha_0 E_1 T_2 x + 6\alpha_1 E_1 T_1 T_2 x + 3\alpha_1 E_1 T_2^2 x^2)\right] / 12 \right\}$$

Since the shear strain, γ, as already mentioned, is unaffected by temperature variation, eq. (2.1.9) remains unchanged. Integration of the appropriate stress components over the plate thickness yields

$$m_x^{(T)} = m_x - m_T\,, \quad m_y^{(T)} = m_y - m_T \quad \text{and} \quad m_{xy} = m_{yx} \qquad (2.1.28)$$

where

$$m_T = \frac{E(T)\alpha(T)}{1-\gamma} \int_{-h/2}^{h/2} T(x,z)\, z\, dz \qquad (2.1.29)$$

represents the thermal equivalent bending moment. Substitution of Eq. (2.1.19) into Eq. (2.1.9), which represents the equilibrium of the infinitesimal plate element in the Z direction, results in

$$\frac{\partial^2 m_x}{\partial x^2} + 2\frac{\partial^2 m_{xy}}{\partial x\, \partial y} + \frac{\partial^2 m_y}{\partial y^2} = -p_z + \nabla^2 m_T \qquad (2.1.30)$$

In this study, p_z consists of p_a and p_f. p_f and p_a denote the pressure of fluid and airflow, respectively. The substitution of Eq. (2.1.26) into Eq. (2.1.9) yields the governing differential equation of the plate subjected to the lateral loads :

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2\,\partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p_z(x,y)}{D} + \frac{\nabla^2 m_T}{D} \tag{2.1.31}$$

This equation is a fourth-order, nonhomogeneous, partial differential equation of the elliptic type with constant coefficients, often called a nonhomogeneous biharmonic equation. Equation (2.1.31) is linear since the derivatives of w do not have exponents higher than one. The plate problem is considered solved if a suitable expression for the deflected plate surface w(x,y) is found which simultaneously satisfies the differential equation of equilibrium (2.1.31) and boundary conditions. Consequently, it can be stated that the solution of the plate problems is a specific case of a boundary value problem of mathematical physics.

2.2 AERODYNAMIC THEORY

Aerodynamic pressure may be considered as the sum of two parts, one given by the pressure fluctuations on the plate in the absence of any plate motions, e.g., due to turbulent boundary layer fluctuations, and the other due to the plate motion itself. The superposition of these two contributions to form the total aerodynamic pressure implies that the plate motion and the consequent portion of the aerodynamic pressure are sufficiently small so that the turbulent pressure fluctuations themselves are not substantially modified.

$$\Delta p = \Delta p^{M} + \Delta p^{E} \tag{2.2.1}$$

where

Δp^{M} : aerodynamic pressure due to plate motion

Δp^{E} : external aerodynamic pressure independent of plate motion, e.g. turbulent pressure fluctuations

Δp^E is taken as prescribed from experiment or separate analysis while Δp^M may be related to w in general by an expression of the form,

$$\Delta p^M(x,t) = \rho_\infty U_\infty^2 \left\{ \frac{1}{M}\left[\frac{\partial w}{\partial x} + \frac{1}{U_\infty}\frac{\partial w}{\partial t}\right] \right.$$

$$\left. + \int_0^t \int_0^a A(x-\xi, t-\tau)\left[\frac{\partial w}{\partial \xi} + \frac{1}{U_\infty}\frac{\partial w}{\partial \tau}\right] d\xi d\tau \right\} \tag{2.2.2}$$

A is a moderately complicated function which depends parametically on Mach number, $M \equiv U_\infty/a$. An appropriate non-dimensional aerodynamic time scale is tU_∞/a where a is the plate length. For harmonic motion the corresponding frequency scaling is $\omega a/U_\infty$. A very popular assumption, for obvious reasons, is to simplify equation (2.2.2) by dropping the second (integral) term. We shall explore the consequences of and justification for this simplification later. Note that the integral term describes the effects of spatial and temporal memory, i.e., the pressure at a particular point, x at a particular time, t, is influenced by the motion at all points, $0<\xi<a$, at all previous times, $0<\tau<t$. The neglect of the memory effect leads to the so-called "piston theory" approximation (2.2.2)

$$\Delta p^M = \frac{\rho_\infty U_\infty^2}{M}\left[\frac{\partial w}{\partial x} + \frac{1}{U_\infty}\frac{\partial w}{\partial t}\right]$$

$$\Delta p^M = \rho_\infty a_\infty \left[U_\infty \frac{\partial w}{\partial x} + \frac{\partial w}{\partial t}\right] \tag{2.2.3}$$

This is recognized as the relation between the motion of and pressure on a piston in a tube where the total piston velocity includes both a convection term $U_\infty(\partial w/\partial x)$, as well as the direct velocity, $\partial w/\partial t$. Here the plate is the equivalent piston and the fluid tube is perpendicular to the plate.

When M>>1 the first term is the importance one, it leads to merging frequency flutter at sufficiently large dynamic pressure.

2.3 EQUATIONS OF MOTION

The equations of motion for the plate are stated within the frame of Kirchhoff's thin plate theory. The thickness of the fluid layer is considered to be approximately one-tenth of the plate length. It is also assumed that no surface waves are present. The coupling between the vibrating plate and the fluid is of the form of an induced pressure at the interface. This pressure may be conveniently interpreted as added inertia of the plate. The aerodynamic pressure is considered by the use of Piston theory of linear aerodynamics. The fluid-loaded rectangular plate is illustrated in Figure 2.5. Due to incline form of the plate there is a variable fluid load. If we neglect the thermal effects, the equation of motion for a fluttering elastic thin plate under the fluid load given by (2.1.33) is reduced to the following equation

$$D\left[\frac{\partial^4 w}{\partial x^4}+\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right]+\rho h\frac{\partial^2 w}{\partial t^2}=p_f+p_a \tag{2.3.1}$$

The boundary conditions for the plate with simply supports at all edges could be taken as,

$$w=\frac{\partial^2 w}{\partial x^2}=0 \quad \text{at} \quad x=0,a \tag{2.3.2a}$$

and

$$w=\frac{\partial^2 w}{\partial y^2}=0 \quad \text{at} \quad y=0,b \tag{2.3.2b}$$

The motion of incompressible and inviscid fluid is irrotational and the pressure field satisfies the following Laplace's equation

$$\nabla^2 p_f = 0 \tag{2.3.3}$$

At the interface of the fluid and plate the fluid has a prescribed motion and the boundary condition for pressure field is stated as

$$\frac{\partial p_f}{\partial z} = -\rho \frac{\partial^2 w}{\partial t^2} \tag{2.3.4}$$

Here, we assume that the thickness of the fluid layer on the plate is small compared with the smallest planar dimension of the plate. The thickness of the fluid layer may have a variation in the x-direction and/or y-direction with an inclined plate as in the following equation :

$$h_f(x,y) = h_0 + (h_x - h_0)\frac{x}{a} + (h_y - h_0)\frac{y}{b} \tag{2.3.5}$$

where h_f is the thickness of the fluid layer. h_0, h_x and h_y are the thickness of the fluid layer at the origin, at the x = a, y = 0 and x = 0, y = b, respectively. If we solve the equation (2.3.3) and (2.3.4) and obtain the equation (2.3.6)

$$p_f(x,y) = -\rho h_f(x,y)\frac{\partial^2 w}{\partial t^2} \tag{2.3.6}$$

considering the effect of fluid is in the form of added mass. The aerodynamic pressure loading will be assumed to be that of steady supersonic theory:

$$p_a(x,y,t) = \frac{-\rho_a V_a^2}{\sqrt{M^2 - 1}}\frac{\partial w}{\partial x} \tag{2.3.7}$$

We can substitute equations (2.3.6) and (2.3.7) into equation (2.3.1) to obtain :

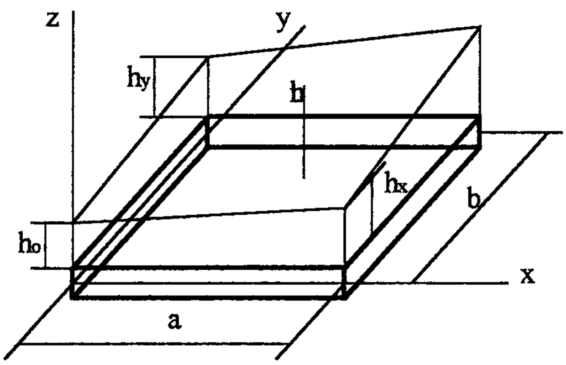


Figure 2.5 Fluid-loaded plate geometry.

$$D\left[\frac{\partial^4 w}{\partial x^4}+\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right]$$

$$+(\rho h+\rho_f h_f)\frac{\partial^2 w}{\partial t^2}+\lambda\frac{\partial w}{\partial x}=0 \qquad (2.3.8)$$

where $\lambda=\frac{\rho_a V_a^2}{\sqrt{M^2-1}}$, is aerodynamic pressure parameter.

In the equation (2.3.8) assuming that the plate is undergoing the harmonic motion, $w(x,y,t)=W(x,y)e^{i\omega t}$, and then non-dimensionializing resulting equation, it takes the form of below equation for the free vibrations of fluid-loaded plate. For the equation below, w is the frequency in rad/sec, x = x/a, h = y/b, $h_r = h_0/a$, $\Delta h_\xi=(h_x-h_0)/a$, $\Delta h_\eta=(h_y-h_0)/b$, and r = a/b, the aspect ratio of the plate.

$$\frac{D}{a^4}\left[\frac{\partial^4 W}{\partial\xi^4}+2r^2\frac{\partial^4 W}{\partial\xi^2\partial\eta^2}+r^4\frac{\partial^4 W}{\partial\eta^4}\right]$$

$$+\left[\rho h+\rho_f a(h_r+\Delta h_\xi\xi+\frac{\Delta h_\eta}{r}\eta)\right]\omega^2 W+\frac{\lambda}{a}\frac{\partial W}{\partial\xi}=0 \qquad (2.3.9)$$

CHAPTER 3. METHOD OF SOLUTION

3.1 GALERKIN'S METHOD

Galerkin's method has been used to solve problems in structural mechanics, dynamics, fluid flow, hydrodynamic stability, heat transfer, acoustics, etc. [19]. Problems governed by ordinary differential equations, partial differential equations and integral equations have been investigated via a Galerkin formulation. Steady, unseady and eigenvalue problems have proved to be equally amenable to a Galerkin treatment. Essentially, any problem for which governing equations can be written down is a candidate for a Galerkin method. The origin of the method is generally associated with a paper published by Galerkin (1915) on the elastic equilibrium of rods and thin plates. For many years, researchers have studied to apply Galerkin method to theoretical problems. The ready availability of computers during the 1960s produced a change in emphasis in the use of Galerkin method. The key features of the Galerkin method can be stated quite concisely. It will be assumed that a two-dimensional problem is governed by a linear differential equation

$$L(u) = 0 \tag{3.1.1}$$

in a domain D(x,y), with boundary conditions

$$S(u) = 0 \tag{3.1.2}$$

on ∂D, the boundary of D. The Galerkin method assumes that u can be accurately represented by an approximate solution

$$u_a = u_o(x,y) + \sum \partial\, a_j\, \phi_j(x,y) \tag{3.1.3}$$

where the ϕ_j's are known analytic functions, u_0 is introduced to satisfy the boundary conditions, and the a_j's are coefficients to be determined. Substitution of eq. (3.1.3) into eq. (3.1.1) produces a nonzero residual, R, given by

$$R(a_0 , a_1 , \ldots a_N , x , y) = L (u_a) = L (u_0) + \sum a_j L (\phi_j) \qquad (3.1.4)$$

It is convenient to define an inner product (f,g) in the following manner :

$$(f,g) = \iint fg \, dx \, dy \qquad (3.1.5)$$

In the Galerkin method the unknown coefficients, a_j in eq. (3.1..3) are obtained by solving the following system of equations :

$$(R,\phi_k) = 0 , \qquad k = 1, \ldots ,N \qquad (3.1.6)$$

Here R is just the equation residual, and the ϕ_k's are the same analytic functions that appear in eq. (3.1.3). Since this example is based on a linear differential equation, eq. (3.1.6) can be written directly as a matrix equation for the coefficients a_j as

$$\sum a_j [L(\phi_j) , \phi_k] = - [L(u_0) , \phi_k] \qquad (3.1.7)$$

Substitution of the a_j's resulting from the solution of eq. (3.1.7) into eq. (3.1.3) gives the required approximate solution u_a. The test and trial functions, $\phi_k(x)$, should be chosen from the first N functions of a complete set of functions. This is a necessary condition for convergence to the exact solution as $N \rightarrow \infty$. It is useful here to reiterate the conditions required in applying the traditional Galerkin method :

(i) The test functions wk are chosen from the same family as the trial functions, ϕ_j .

(ii) The trial and test functions must be linearly independent.

(iii) The trial and test functions should be the first N members of a complete set of functions.

(iv) The trial functions should satisfy the boundary conditions (and initial conditions if applicable) exactly. Conditions (i) defines the Galerkin method, and condition (ii) is necessary to ensure that independent equations are available to obtain the unknown coefficients a_j . Conditions (iii) and (iv) relate to the efficiency of the method. Violation of these conditions reduces the efficiency.

The governing equation (2.3.9) together with the boundary conditions (2.3.2) will be solved via Galerkin's method. The displacement function may be chosen as

$$W(\xi,\eta) = \sum_{m=1}^{M}\sum_{n=1}^{N} A_{mn} \sin(m\pi\xi)\sin(n\pi\eta) \tag{3.1.8}$$

which satisfies the boundary conditions (2.3.2). Substitution of equation (3.1.8) into equation (2.3.9) results in an error function. Application of the Galerkin method yields a set of MxN simultaneous, linear and algebraic equations to determine the frequencies and unknown parameters A_{mn}. It can be written in matrix notation as follows

$$\left[\mathbf{K} + \lambda\mathbf{A} - \omega^2\mathbf{M}\right]\{A_{mn}\}^T = 0 \tag{3.1.9}$$

If the aerodynamic pressure is zero, the second term in equation (3.1.9) will vanish and equation (3.1.9) gives the natural frequencies and modes of the fluid loaded plate. In the presence of the aerodynamic pressure,flutter boundary can be determined by the use of equation (3.1.9). To find the linear flutter boundary for a plate subjected fluid loading is to solve the eigenvalue equation (3.1.9) by gradually increasing the dynamic pressure λ .

3.2 SOLUTION OF THE EIGENVALUE PROBLEMS

After defining $K_1 = K + \lambda A$, $\{A_{mn}\}=q$ and $\omega^2 = k$,Eq. (3.1.9) can be reduced to the following form

$$[K_1 - kM]q = 0 \tag{3.2.1}$$

the solution of which yields the eigenvalues k and associated vectors q for a given value of λ. In the above formulation, K and M are symmetric matrices, whereas A is unsymmetric in nature. Also, the matrix K is positive definite in form.

Since the aerodynamic matrix A is unsymmetric, complex eigenvalues k are expected for $\lambda > 0$. When $\lambda = 0$, the eigenvalues k are real and positive. As λ is increased in value monotonically from zero, the symmetric matrix K is then perturbed by the unsymmetric matrix λA, when two of the roots approach each other and finally coalesce to the value k_{cr} for a value of $\lambda = \lambda_{cr}$. For $\lambda > \lambda_{cr}$, the two roots become a complex conjugate pair

$$k = k_R \pm i^* k_I \tag{3.2.2}$$

i* being the imaginary number. The flutter boundary corresponds to λ_{cr} in which case the flutter frequency is obtained as $\omega = i^*\sqrt{(k)}$, k being real. The flutter boundary is obtained for a complex root k when ω attains a pure imaginary value yielding the following relations for the flutter frequency :

$$\omega = i^*\sqrt{(k_R)}, \tag{3.2.3}$$

Thus, the flutter boundary may be obtained by continuing the analysis for $\lambda > \lambda_{cr}$ .The relevant analysis procedures usually compute λ_{cr} first, the dynamic pressure is then increased monotonically until equation (3.2.3) is satisfied. The computational procedure for the determination of λ_{cr} involves the solution of the eigenproblem given by equation (3.2.1) for each of the increasing values of λ until two roots coalesce. By far the major portion of the entire solution time is expended in the determination of λ_{cr} and hence the necessity of deriving an efficient solution procedure for same.

The earlier papers on supersonic panel flutter study were primarily devoted to developing the appropriate finite elements for such analysis. Reference [20] discusses the disadvantages of the conventional eigenproblem solution techniques, including the direct iterative procedure for the solution of eq. (3.2.1) and also presents details of a solution procedure adopted in that work.

CHAPTER 4. NUMERICAL RESULTS

The vibration characteristics are determined for a fluid-loaded inclined plate having simply supported edges. Poisson's ratio of the plate material is taken to be 0.3 in all numerical examples. Ratio of the fluid density to the plate density is 0.308 used in this study. The fluid considered here is a kind of jet fuel.

4.1 VIBRATION ANALYSIS

In this study, a frequency parameter is used as $\Omega = \omega a^2\sqrt{\rho h / D}$. Following geometrical para-meters are examined: *(i)* inclination of the plate *(ii)* aspect ratio of the plate and *(iii)* height of the fluid. The ratio of frequency parameter of fluid-loaded plate (Ω) to the lowest frequency parameter of unloaded plate (Ω_0) is in consideration in this numerical study.

The convergence tests show that the six terms for both directions are sufficient to obtain reliable numerical results (Table 1). Table 2 presents the lowest six natural frequencies of a square fluid-loaded plate for some inclinations in the x-direction. Figure 4.1 shows the relation between the number of waves in the x-direction and the natural frequencies for four different degrees of inclination in the same direction. The wave number in the y-direction n = 1. The geometrical properties of the inclined fluid-loaded plate are chosen as a/b = 1, b/h=50, $h_0 / a = 0, \Delta h_\eta = 0$.

Table 4.1 Convergence of natural frequencies $(\Omega = \omega a^2\sqrt{\rho h/D})$ of a fluid-loaded plate. a/b = 1, b/h = 50, $h_0 / a = 0, \Delta h_\xi = 0.08, \Delta h_\eta = 0$.

	1	2	3	4	5	6
M= 3, N= 3	15.5004	38.5915	39.1127	62.6417	76.6493	78.8239
M =4, N =4	15.5003	38.5913	39.1126	62.6409	76.6482	78.3260
M = 6,N= 6	15.5003	38.5913	39.1125	62.6404	76.6481	78.3246

Table 4.2 Variation of the natural frequencies (Ω) of a fluid-loaded plate with Δh_ξ (a/b = 1, b/h = 50, $h_0 / a = 0, \Delta h_\eta = 0$).

Δh_ξ / Mode	0.0	0.02	0.04	0.06	0.08	0.10
1 (1,1)	19.739	18.371	17.248	16.305	15.500	14.803
2 (1,2)	49.348	45.904	43.050	40.647	38.591	36.810
3 (2,1)	49.348	45.980	43.273	41.024	39.112	37.456
4 (2,2)	78.957	73.579	69.267	65.687	62.640	60.000
5 (1,3)	98.696	91.723	85.860	80.895	76.648	72.974
6 (3,1)	98.696	91.973	86.584	82.117	78.325	75.044
7 (3,2)	128.305	119.575	112.589	106.804	101.893	97.646
8 (2,3)	128.305	119.598	112.643	106.866	101.940	97.658

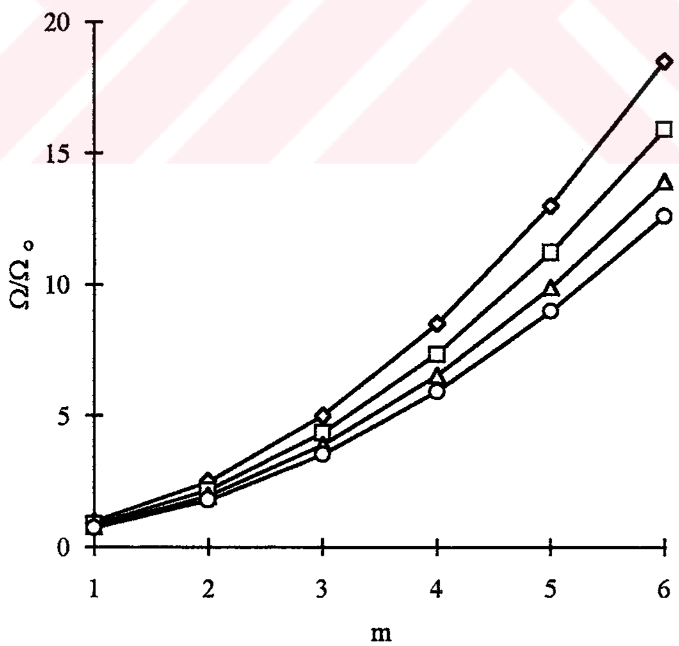


Figure 4.1 Frequency ratio vs the mode number for different inclinations. ◊ ◊ ◊, $\Delta h_\xi = 0.0$, □□□ $\Delta h_\xi = 0.04$, o o o $\Delta h_\xi = 0.08$, ▵▵▵ $\Delta h_\xi = 0.12$.

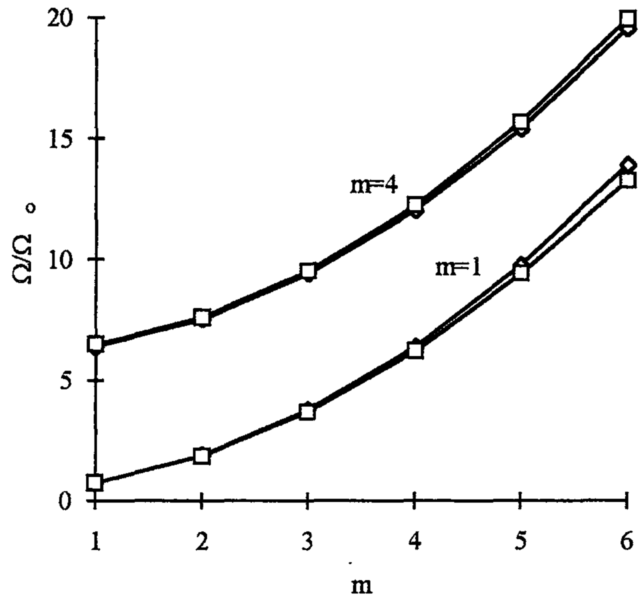


Figure 4.2. The frequency ratios of an inclined plate and an uninclined plate.

◊ ◊ ◊, $h_0/a = 0.0$, □□□ $\Delta h_\xi = 0.10$.

The effect of the fluid load which change linearly with x decreases the frequencies of plate due to the mass contribution of the fluid. The rate of decrease is gradually increased as the wave number increases. The added mass effect is much more important for the frequencies corresponding to the greater modes. The fluid load changes the frequency spectrum and hence the natural frequencies of the plate-fluid system become closer than those of the unloaded plate.

Effect of the inclination on the frequency spectrum of the fluid-loaded plate is examined comparing the frequencies of an inclined and an uninclined plate considering the same amount of fluid load (Fig. 4.2). The inclination has no significant effect on the lower frequencies of the system. The effect increases for the higher frequencies due to higher energy levels of the mode corresponding these frequencies.

A plot of frequency ratio vs Δh_ξ for three different values of h_0/a is given in Fig. 4.3. In the plot the ratio of the lowest frequency of a square plate is considered. There is no inclination in the y-direction. The fluid load has a role of added mass and hence the effect of the base fluid height causes an amount of decreasing at the natural frequencies. As seen from the Fig. 4.3, the base fluid reduces the influence of the inclination. Because the relative contribution of fluid mass due to the plate inclinations to he mass of plate-fluid system is reduced by means of the base fluid.

Figure 4.4 shows the variation of frequency ratio of plates with the inclination in the x - direction. The geometrical properties of plates are taken to be: a/b = 0.5, b/h = 50, and a/b = 2, b/h = 25. The properties of fluid load are taken to be $h_0 / a = 0, \Delta h_\eta = 0$. The frequency reducing effect of the fluid load is more significant for the plate 2 than that of the plate 0.5. This effect may be due to the fact that the inclined plate with the aspect ratio 2 has the fluid load twice of that of the other plate. Another reason for this effect can be attributed to the greatness of the natural frequencies of the plate with a/b = 2 than those of the low aspect ratio's plate.

The mode shapes of the plate under the fluid load effect are shown in Fig. 4.5 for the plate with the geometrical properties a / b = 1, b / h = 50. Following cases are considered : (a) no fluid load (b) $\Delta h_\xi = 0.24$, $\Delta h_\eta = 0$ (c) $\Delta h_\xi = 0$, $\Delta h_\eta = 0.24$ (d) $\Delta h_\xi = 0.08$, $\Delta h_\eta = 0.08$. High inclinations are taken into account to show the effect of the fluid load on the mode shapes. The height of the base fluid is taken zero at all the examples. As seen from the Fig. 4.5, the contours (node lines) and the maximum displacement points are shifted towards the downward side of the plate. This shifting is due to the movement of the fluid mass center because of the inclination of the plate. Bi-directional inclinations (d) have more effect on the mode shapes than the uni-directional inclinations (b and c). As can be seen from the Figure 4.5, the antisymmetrical modes change considerably due to the unsymmetrical fluid load.

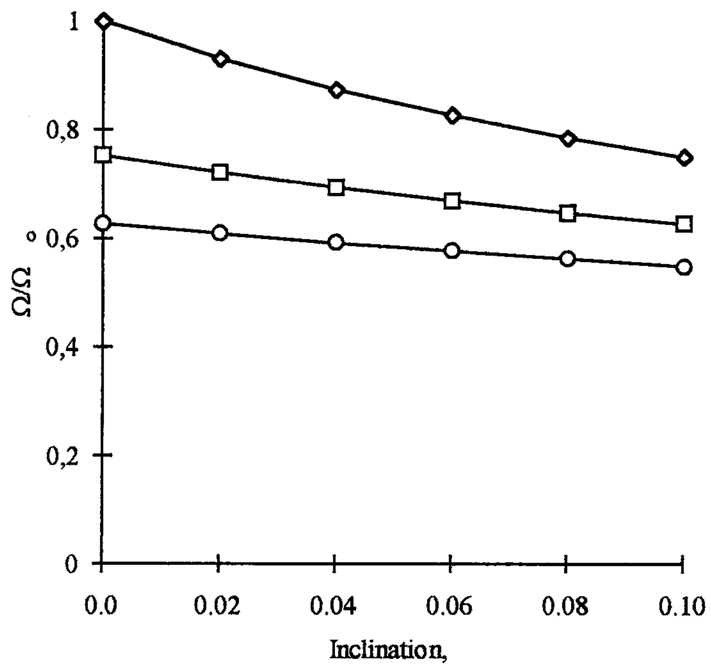


Figure 4.3 The effect of the base fluid on the frequencies of the inclined fluid-loaded plate. ◊ ◊ ◊, $h_0/a = 0.0$, □□□ $h_0/a = 0.05$, o o o $h_0/a = 0.10$.

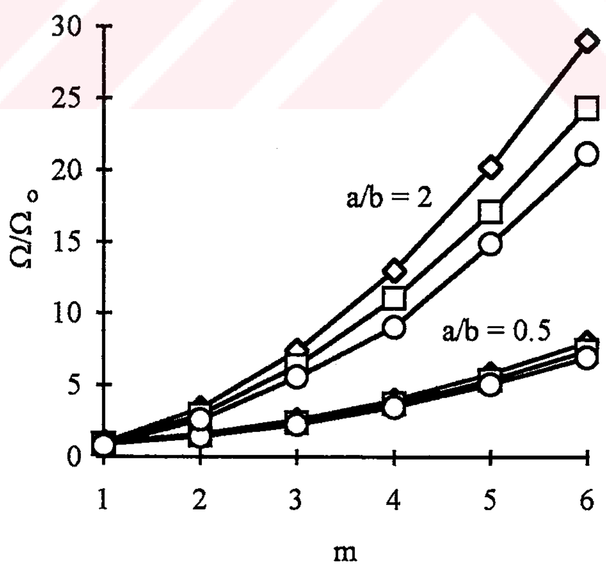


Figure 4.4. The frequency ratio vs mode number for the plates with the aspect ratios 0.5 and 2. ◊ ◊ ◊, $\Delta h_\xi = 0.0$, □□□ $\Delta h_\xi = 0.04$, o o o $\Delta h_\xi = 0.08$.

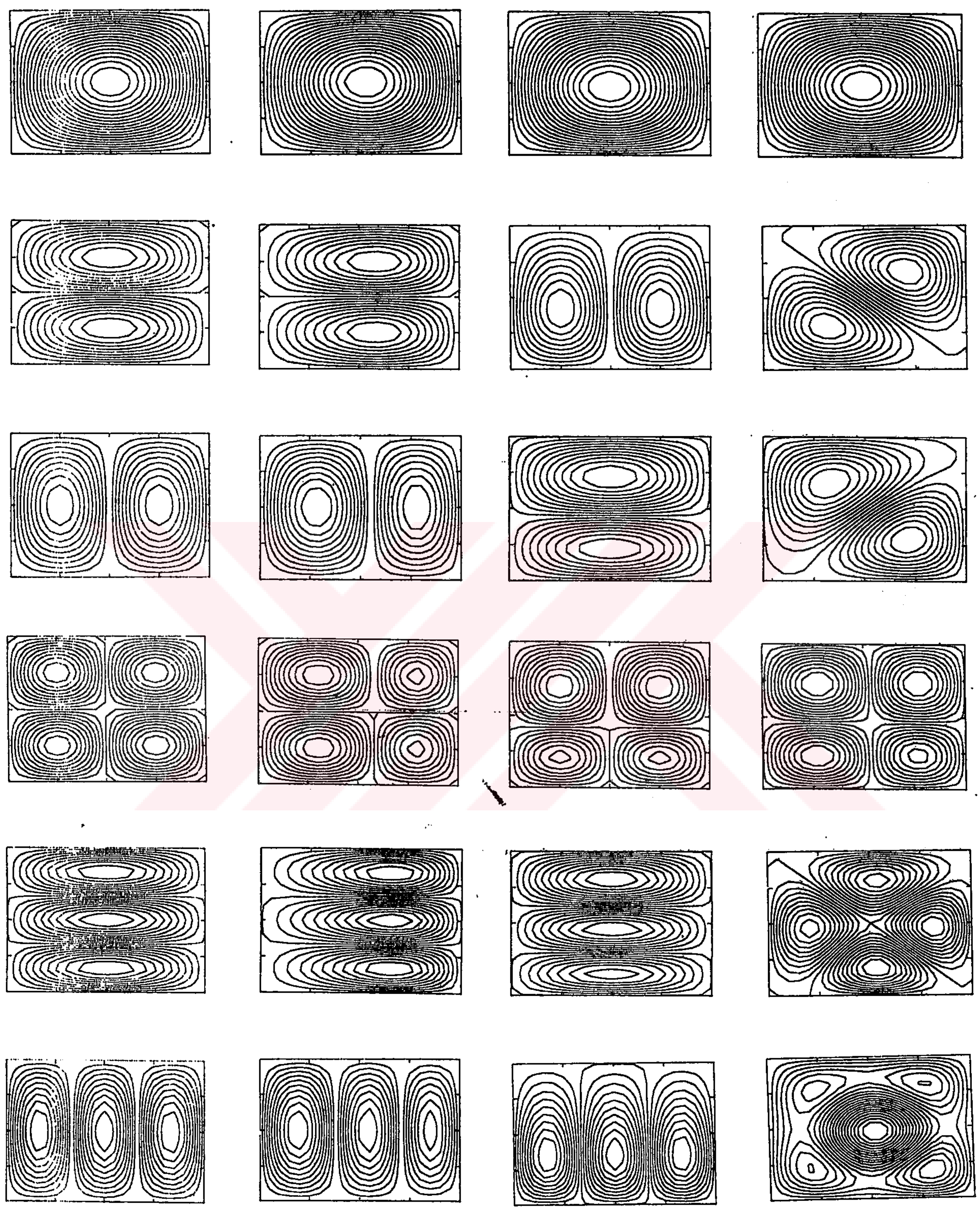


Figure 4.5 Mode Shapes under the fluid load effect

4.2 FLUTTER ANALYSIS

Let us consider the matrix $K_1 = K + \lambda A$ in equation (3.1.9). If the λ equals to zero, the K_1 matrix is real and symmetric. For the nonzero values of the dynamic pressure parameter λ, K_1 matrix is real but no more symmetrical due to the aerodynamic contribution. With the increase of the dynamic pressure parameter λ, the values of the frequencies change until they reach a value λ_{cr},where two modes coalesce. Increasing further λ, a pair of complex conjugate frequencies is obtained and in view of Equation, $w(x,y,t) = W(x,y)e^{i\omega t}$. The lowest value of λ for which the first coalescence occurs will thus determine the borderline of the stability boundary.

In this study, the flutter of a rectangular plate with various fluid loads (inclination) is analyzed. If the fluid load is not taken into account, the critical dynamic pressure of a square plate with simply supported edges is obtained as 512.714. The critical eigenvalue, $\kappa_{cr} = \omega pha^4 / D$, is obtained as 1848.29. The exact values of dynamic pressure and eigenvalue are 512.651 and 1848.21, respectively. The variation of the flutter boundary with inclination is shown in the Figure (4.6). The inclination has a considerable effect on the flutter boundary. From the Figure (4.6) if the inclination is toward to leading edge, the flutter occurs at a lower value of dynamic pressure than that of the uninclined plate. On the other hand, if the inclination of the plate is toward to the trailing edge, the flutter boundary will be higher than that of uninclined plate.

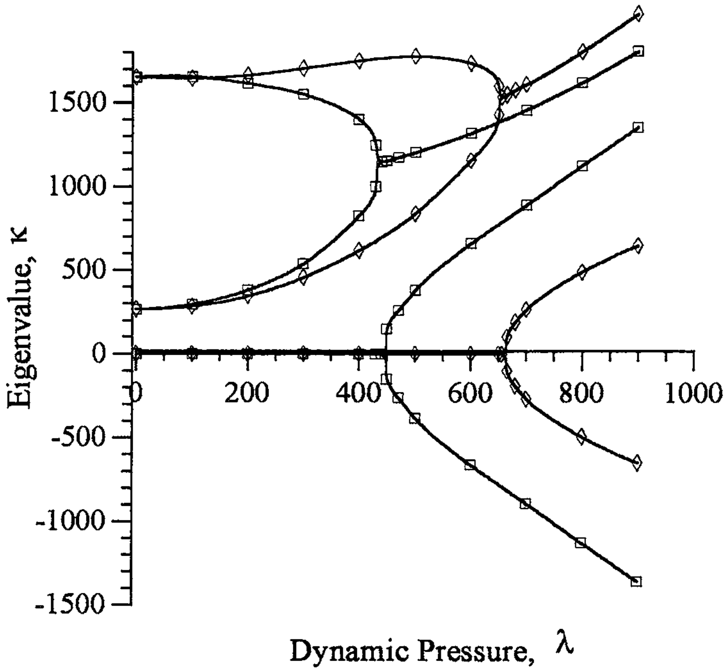


Figure 4.6. Eigenvalues vs dynamic pressures for the fluid loading. ◊ ◊ ◊ $h_0/a = 0.0$, $\Delta h_\xi = 0.06$, □□□ $h_a/a = 0.0$, $\Delta h_\xi = -0.06$.

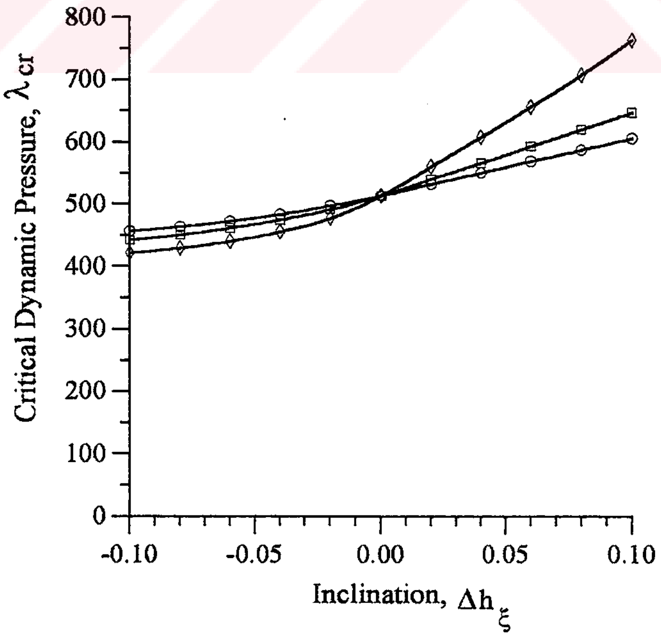


Figure 4.7 Flutter boundary vs inclination of the plate in the x-direction. ◊ ◊ ◊, $h_0/a = 0.0$, □□□ $h_0/a = 0.05$, o o o $h_0/a = 0.10$.

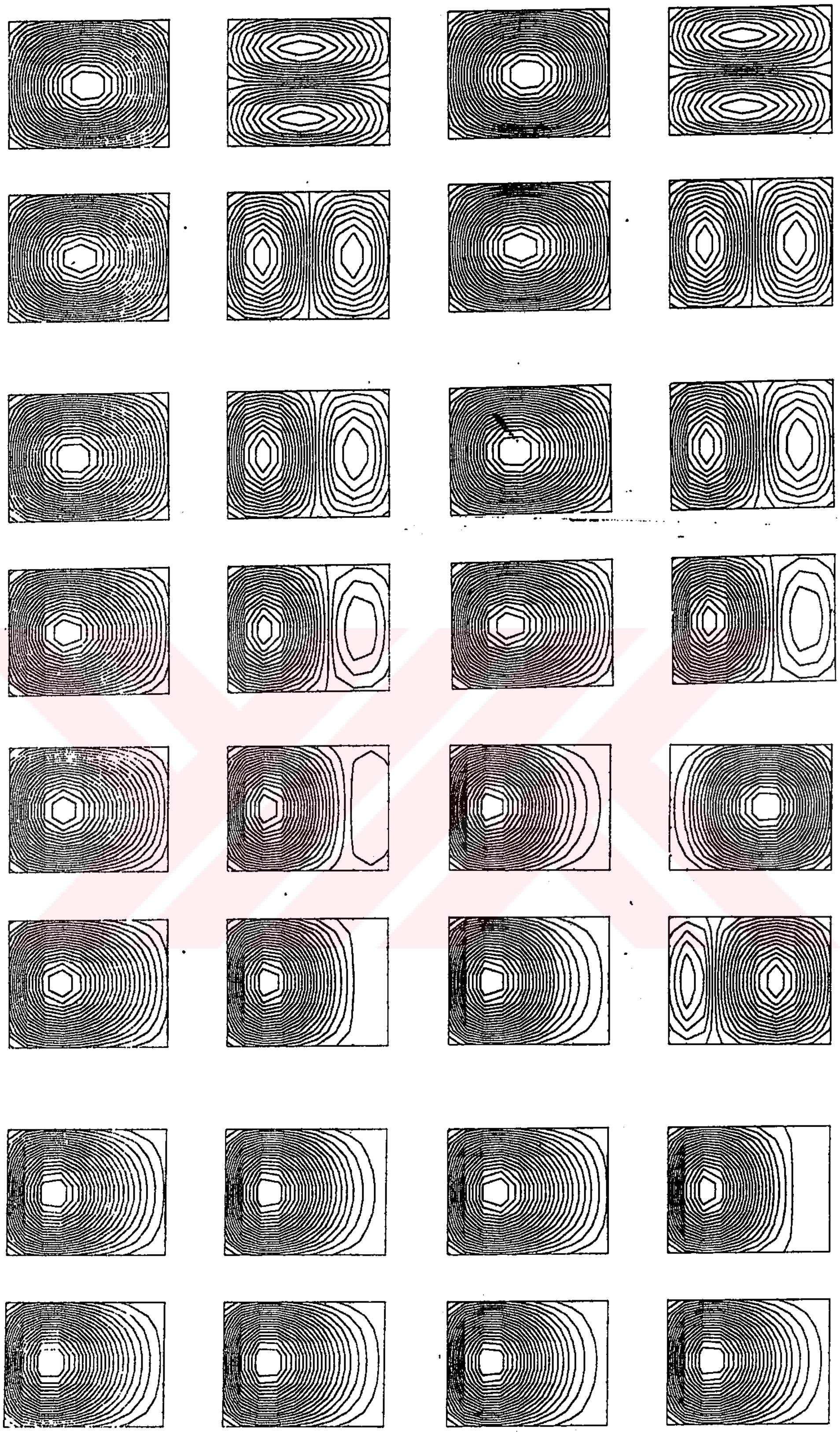


Figure 4.8 Mode Shapes of unloaded and fluid-loaded plates

4.3 CONCLUSION

The free vibration and flutter of a fluid-loaded plate are studied numerically on the basis of theory of thin elastic plates by the use of Galerkin method. A thin layer of the fluid over the vibrating plate is considered. A linear flutter analysis is also performed for the fluid-loaded plate and a good agreement is obtained with certain earlier results.

Fluid load reduces the natural frequencies of the plate in all the considered cases. Reduction of the higher natural frequencies is more dramatical than that of the lower ones. The fluid load effects results in narrowing the natural frequency spectrum. The narrowing spectrum has importance on the dynamic behavior of the plate. Frequency ratio variations due to the inclinations are significant for small value of plate inclinations.

Due to the added mass the thickness of base fluid has tendency to reduce the natural frequencies. Inclination in y-direction does not change the character of frequency ratio variations with inclinations in x-direction but it reduces frequency ratios. This type of inclination has significant effect on the antisymetrical mode shapes of the plate.

The aspect ratio of the plate has no significant effect on the character of the frequency ratio variation. The reduction of frequencies due to the fluid load is more significant for plate with high aspect ratio than for plate with lower one.

Some shifting of the node lines and the maximum displacement points are observed on the mode shapes because of the inclination. This shifting results in a change in the flutter boundary. An inclination toward to leading edge decreases the critical dynamic pressure. An inclination toward to trailing edge causes an increase of the critical dynamic pressure. The variation of the related modes due to the fluid load has the effect on the flutter boundary.

This study may be extended to the investigation of nonlinear behavior of fluid-loaded plate. Temperature-dependent material property were applied to the problem while we were setting equation of plate motion, but all thermal effects were neglected during solution. This can be added to the solution procedure. The flutter analysis of fluid-loaded inclined plates with temperature effects will have importance for aerospace structures. Influence of the change of plate's curvature due to fluid load on the vibration and flutter characteristics may be analyzed.

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APPENDIX

```
C  BU PROGRAM TAKVIYELI/TAKVIYESIZ PLAKLARIN SERBEST
C  TITRESIM ANALIZLERINDE VE DINAMIK AEROELASTIK
ANALIZLERINDE KULLANILABILIR. TAKVIYE PARÇALARI ORTOTROPIK
MALZEME YAKLASIMI ILE HESABA
C  KATILMISTIR.
C
C  NPROB :  YAPININ TIPINI BELIRTEN BIR PARAMETREDIR. PLAK
ANALIZLERI
C       IçIN NPROB = 0, BASIK KABUK ANALIZLERI IçIN NPROB = 1
GIRILIR.
C  NSTIF : YAPININ TAKVIYE DURUMUNU GOSTERIR. TAKVIYESIZ
DURUM IçIN
C        NSTIF = 0, X YONUNDE TAKVIYELI YAPI HALINDE NSTIF = 1,
C        Y YONÜNDE TAKVIYELI YAPI IçIN NSTIF = 2, HER IKI YONDE
C        TAKVIYELI YAPI IçIN NSTIF = 3 GIRILIR.
C  NODOV : BU PARAMETRE SIFIR OLARAK GIRILIRSE YALNIZCA
FRAKANSLAR
C        HESAPLANIR. BIR OLARAK GIRILIRSE HEM FREKANSLAR, HEM
C        MODLAR HESAPLANIR.
C  ENBUY : CIKISTA YALNIZCA ENBUY'A KADAR OLAN FREKANSLAR VE
ILGILI
C        MOD SEKILLLERI YANSITILIR.
C  LAMDA : DINAMIK BASINC. LAMDA = 0.0 GIRILMESI HALIDE SERBEST
TITRESIM
C        ANALIZI YAPAR.
C  YMOD  : ELASTISITE MODULU
C  POIS  : POISSON ORANI
C  DENSTY: YOGUNLUK
C  A, B  : PLAK VEYA KABUGUN ENI VE BOYU
C  THICK : PLAK KALINLIGI
C
C     IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      INTEGER NVEC(500000), YAZ
      REAL VEC(1000000)
      OPEN(5,FILE='PF1.DAT')
      OPEN(6,FILE='PF1.OUT')
C
C*** PROBLEMLE ILGILI BAZI BOYUTLARI OKU
C
      READ(5,500) NPROB, NSTIF, MS, NS, NODOV, YAZ, ENBUY
  500 FORMAT(6I5,F10.0)
      IF(NPROB.EQ.0) WRITE(6,601)
      IF(NPROB.EQ.1) WRITE(6,602)
      IF(NSTIF.EQ.0) WRITE(6,603)
      IF(NSTIF.EQ.1) WRITE(6,604)
      IF(NSTIF.EQ.2) WRITE(6,605)
      IF(NSTIF.EQ.3) WRITE(6,606)
  601 FORMAT(5X,'BU BIR PLAK PROBLEMIDIR')
```

```
 602 FORMAT(5X,'BU BIR BASIK KABUK PROBLEMIDIR')
 603 FORMAT(5X,'TAKVIYESIZ YAPI')
 604 FORMAT(5X,'X YÖNÜNDE TAKVIYELI YAPI')
 605 FORMAT(5X,'Y YÖNÜNDE TAKVIYELI YAPI')
 606 FORMAT(5X,'X VE Y YÖNLERINDE TAKVIYELI YAPI')
C                                                    SHA00170
    NMAT = MS*NS
    WRITE(6,607) NMAT
 607 FORMAT(5X,'MATRISLERIN MERTEBESI =',I3//)
    I1 = 1 + NMAT*NMAT                                       SHA00030
    I2 = I1 + NMAT*NMAT
    I3 = I2 + NMAT
    I4 = I3 + NMAT
    I5 = I4 + NMAT
    I6 = I5 + NMAT
    IZ = I6 + NMAT*NMAT
    IF (IZ .GT. 1000000) GO TO 99
    CALL PLAK(NPROB, NSTIF, MS, NS, NMAT, NODOV, YAZ,
   .   ENBUY, VEC(1), VEC(I1), VEC(I2), VEC(I3), VEC(I4), VEC(I5),
   .   VEC(I6), NVEC(1))
  99 WRITE(6,999)
 999 FORMAT('!!! GIRILEN VEKTÖR BOYUTU YETERLIDEĐIL...')
    CLOSE(5)
    CLOSE(6)
    STOP
    END
□

***SUBROUTINE 1***

    SUBROUTINE QZHES(NM,N,A,B,MATZ,Z)
C
C     IMPLICIT DOUBLE PRECISION (A-H, O-Z)
    INTEGER I,J,K,L,N,LB,L1,NM,NK1,NM1,NM2
    REAL A(NM,N),B(NM,N),Z(NM,N)
C     DOUBLE PRECISION A(NM,N),B(NM,N),Z(NM,N)
    LOGICAL MATZ
C
    IF(.NOT. MATZ) GO TO 10
C
    DO 3 I = 1, N
    DO 2 J = 1, N
    Z(I,J) = 0.0
  2 CONTINUE
    Z(I,I) = 1.0
  3 CONTINUE
C     ********** B'Y˜ šST š□GEN FORMUNA ˜ND˜RGE ***********
  10 IF (N .LE. 1) GO TO 170
```

```
      NM1 = N - 1
      DO 100 L = 1, NM1
      L1 = L + 1
      S = 0.0
C
      DO 20 I = L1, N
      S = S + ABS(B(I,L))
   20 CONTINUE
      IF (S .EQ. 0.0) GO TO 100
      S = S + ABS(B(L,L))
      R = 0.0
      DO 25 I = L, N
      B(I,L) = B(I,L) / S
      R = R + B(I,L)**2
   25 CONTINUE
      R = SIGN(SQRT(R),B(L,L))
      B(L,L) = B(L,L) + R
      RHO = R * B(L,L)
      DO 50 J = L1, N
      T = 0.0
C
      DO 30 I = L, N
      T = T + B(I,L) * B(I,J)
   30 CONTINUE
      T = -T / RHO
      DO 40 I = L, N
      B(I,J) = B(I,J) + T * B(I,L)
   40 CONTINUE
   50 CONTINUE
      DO 80 J = 1, N
      T = 0.0
      DO 60 I = L, N
      T = T + B(I,L) * A(I,J)
   60 CONTINUE
      T = -T / RHO
      DO 70 I = L, N
      A(I,J) = A(I,J) + T * B(I,L)
   70 CONTINUE
   80 CONTINUE
      B(L,L) = -S * R
      DO 90 I = L1, N
      B(I,L) = 0.0
   90 CONTINUE
  100 CONTINUE
C     ********** B'Y˜ š□GEN HALDE MUHAFAZA EDEREK, A'YI šST
HESSENBERG
C              FORMUNA ˜ND˜RGE **********
      IF (N .EQ. 2) GO TO 170
```

```
      NM2 = N - 2
      DO 160 K = 1, NM2
      NK1 = NM1 - K
C     ********** L=N-1 ADIMI ˜□˜N -1'DEN K+1'E KADAR YAP -- **********
      DO 150 LB = 1, NK1
      L = N - LB
      L1 = L + 1
C     ********** A(L+1,K) S□F□R ***********
      S = ABS(A(L,K)) + ABS(A(L1,K))
      IF (S .EQ. 0.0) GO TO 150
      U1 = A(L,K) / S
      U2 = A(L1,K) / S
      R = SIGN(SQRT(U1*U1+U2*U2),U1)
      V1 = -(U1 + R) / R
      V2 = -U2 / R
      U2 = V2 / V1
C
      DO 110 J = K,N
      T = A(L,J) + U2 * A(L1,J)
      A(L,J) = A(L,J) + T * V1
      A(L1,J) = A(L1,J) + T * V2
  110 CONTINUE
C
      A(L1,K) = 0.0
C
      DO 120 J = L, N
      T = B(L,J) + U2 * B(L1,J)
      B(L,J) = B(L,J) + T * V1
      B(L1,J) = B(L1,J) + T * V2
  120 CONTINUE
C     ********** B(L+1,L) S□F□R ***********
      S = ABS(B(L1,L1)) + ABS(B(L1,L))
      IF (S .EQ. 0.0) GO TO 150
      U1 = B(L1,L1) / S
      U2 = B(L1,L) / S
      R = SIGN(SQRT(U1*U1+U2*U2),U1)
      V1 = -(U1 + R) / R
      V2 = -U2 / R
      U2 = V2 / V1
C
      DO 130 I = 1, L1
      T = B(I,L1) + U2 * B(I,L)
      B(I,L1) = B(I,L1) + T * V1
      B(I,L) = B(I,L) + T * V2
  130 CONTINUE
C
      B(L1,L) = 0.0
C
```

```
      DO 140 I = 1, N
      T = A(I,L1) + U2 * A(I,L)
      A(I,L1) = A(I,L1) + T * V1
      A(I,L) = A(I,L) + T * V2
  140 CONTINUE
C
      IF (.NOT. MATZ) GO TO 150
C
      DO 145 I = 1, N
      T = Z(I,L1) + U2 * Z(I,L)
      Z(I,L1) = Z(I,L1) + T * V1
      Z(I,L) = Z(I,L) + T * V2
  145 CONTINUE
C
  150 CONTINUE
C
  160 CONTINUE
C
  170 RETURN
      END
□
```

SUBROUTINE 2

```
      SUBROUTINE QZIT(NM,N,A,B,EPS1,MATZ,Z,IERR)
C
C     IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      INTEGER I,J,K,L,N,EN,K1,K2,LD,LL,L1,NA,NM,ISH,ITS,KM1,LM1,
     X        ENM2,IERR,LOR1,ENORN
      REAL A(NM,N),B(NM,N),Z(NM,N)
C     DOUBLE PRECISION A(NM,N),B(NM,N),Z(NM,N)
      INTEGER MAX0,MIN0
      LOGICAL MATZ,NOTLAS
C
      IERR = 0
C
      ANORM = 0.0
      BNORM = 0.0
C
      DO 30 I = 1, N
        ANI = 0.0
        IF (I .EQ. 1) ANI = ABS(A(I,I-1))
        BNI = 0
C
        DO 20 J = 1, N
          ANI = ANI + ABS(A(I,J))
          BNI = BNI + ABS(B(I,J))
   20   CONTINUE
```

```
C
        IF (ANI .GT. ANORM) ANORM = ANI
        IF (BNI .GT. BNORM) BNORM = BNI
   30 CONTINUE
C
      IF (ANORM .EQ. 0.0) ANORM = 1.0
      IF (BNORM .EQ. 0.0) BNORM = 1.0
      EP = EPS1
      IF (EP .GT. 0.0) GO TO 50
C
      EP = 1.0
   40 EP = EP / 2.0
      IF (1.0 + EP .GT. 1.0) GO TO 40
   50 EPSA = EP * ANORM
      EPSB = EP * BNORM
C
      LOR1 = 1
      ENORN = N
      EN = N
C
   60 IF (EN .LE. 2) GO TO 1001
      IF (.NOT. MATZ) ENORN = EN
      ITS = 0
      NA = EN - 1
      ENM2 = NA - 1
   70 ISH = 2
C
      DO 80 LL = 1, EN
        LM1  = EN - LL
        L = LM1 + 1
        IF (L .EQ. 1) GO TO 95
        IF (ABS(A(L,LM1)) .LE. EPSA) GO TO 90
   80 CONTINUE
C
   90 A(L,LM1) = 0.0
      IF (L .LT. NA) GO TO 95
C
      EN = LM1
      GO TO 60
C
   95 LD = L
  100 L1 = L + 1
      B11 = B(L,L)
      IF (ABS(B11) .GT. EPSB) GO TO 120
      B(L,L) = 0.0
      S = ABS(A(L,L)) + ABS(A(L1,L))
      U1 = A(L,L) / S
      U2 = A(L1,L) / S
```

```
      R = SIGN(SQRT(U1*U1+U2*U2),U1)
      V1 = -(U1 + R) / R
      V2 = -U2 / R
      U2 = V2 / V1
C
      DO 110 J = L, ENORN
        T = A(L,J) + U2 * A(L1,J)
        A(L,J) = A(L,J) + T * V1
        A(L1,J) = A(L1,J) + T * V2
        T = B(L,J) + U2 * B(L1,J)
        B(L,J) = B(L,J) + T * V1
        B(L1,J) = B(L1,J) + T * V2
  110 CONTINUE
C
      IF (L .NE. 1) A(L,LM1) = -A(L,LM1)
      LM1 = L
      L = L1
      GO TO 90
  120 A11 = A(L,L) / B11
      A21 = A(L1,L) / B11
      IF (ISH .EQ. 1) GO TO 140
C
      IF (ITS .EQ. 50) GO TO 1000
      IF (ITS .EQ. 10) GO TO 155
C
      B22 = B(L1,L1)
      IF (ABS(B22) .LT. EPSB) B22 = EPSB
      B33 = B(NA,NA)
      IF (ABS(B33) .LT. EPSB) B33 = EPSB
      B44 = B(EN,EN)
      IF (ABS(B44) .LT. EPSB) B44 = EPSB
      A33 = A(NA,NA) / B33
      A34 = A(NA,EN) / B44
      A43 = A(EN,NA) / B33
      A44 = A(EN,EN) / B44
      B34 = B(NA,EN) / B44
      T = 0.5 * (A43 * B34 - A33 - A44)
      R = T * T + A34 * A43 - A33 * A44
      IF (R .LT. 0.0) GO TO 150
C
      ISH = 1
      R = SQRT(R)
      SH = -T + R
      S = -T - R
      IF (ABS(S-A44) .LT. ABS(SH-A44)) SH = S
C
      DO 130 LL = LD, ENM2
        L = ENM2 + LD - LL
```

```
      IF (L .EQ. LD) GO TO 140
      LM1 = L - 1
      L1 = L + 1
      T = A(L,L)
      IF (ABS(B(L,L)) .GT. EPSB) T = T - SH * B(L,L)
      IF (ABS(A(L,LM1)) .LE. ABS(T/A(L1,L)) * EPSA) GO TO 100
 130 CONTINUE
C
 140 A1 = A11 - SH
    A2 = A21
    IF (L .NE. LD) A(L,LM1) = -A(L,LM1)
    GO TO 160
C
 150 A12 = A(L,L1) / B22
    A22 = A(L1,L1) / B22
    B12 = B(L,L1) / B22
    A1 = ((A33 - A11) * (A44 - A11) - A34 * A43 + A43 * B34 * A11)
   X    / A21 + A12 - A11 * B12
    A2 = (A22 - A11) - A21 * B12 - (A33 - A11) - (A44 - A11)
   X    + A43 * B34
    A3 = A(L1+1,L1) / B22
    GO TO 160
C
 155 A1 = 0.0
    A2 = 1.0
    A3 = 1.1605
 160 ITS = ITS + 1
    IF (.NOT. MATZ) LOR1 = LD
C
    DO 260 K = L, NA
      NOTLAS = K .NE. NA .AND. ISH .EQ. 2
      K1 = K + 1
      K2 = K + 2
      KM1 = MAX0(K-1,L)
      LL = MIN0(EN,K1+ISH)
      IF (NOTLAS) GO TO 190
C
      IF (K .EQ. L) GO TO 170
      A1 = A(K,KM1)
      A2 = A(K1,KM1)
 170   S = ABS(A1) + ABS(A2)
      IF (S .EQ. 0.0) GO TO 70
      U1 = A1 / S
      U2 = A2 / S
      R = SIGN(SQRT(U1*U1+U2*U2),U1)
      V1 = -(U1 + R) / R
      V2 = -U2 / R
      U2 = V2 / V1
```

```
C
      DO 180 J = KM1, ENORN
        T = A(K,J) + U2 * A(K1,J)
        A(K,J) = A(K,J) + T * V1
        A(K1,J) = A(K1,J) + T * V2
        T = B(K,J) + U2 * B(K1,J)
        B(K,J) = B(K,J) + T * V1
        B(K1,J) = B(K1,J) + T * V2
 180  CONTINUE
C
      IF (K .NE. L) A(K1,KM1) = 0.0
      GO TO 240
C
 190  IF (K .EQ. L) GO TO 200
      A1 = A(K,KM1)
      A2 = A(K1,KM1)
      A3 = A(K2,KM1)
 200  S = ABS(A1) + ABS(A2) + ABS(A3)
      IF (S .EQ. 0.0) GO TO 260
      U1 = A1 / S
      U2 = A2 / S
      U3 = A3 / S
      R = SIGN(SQRT(U1*U1+U2*U2+U3*U3),U1)
      V1 = -(U1 + R) / R
      V2 = -U2 / R
      V3 = -U3 / R
      U2 = V2 / V1
      U3 = V3 / V1
C
      DO 210 J = KM1, ENORN
        T = A(K,J) + U2 * A(K1,J) + U3 * A(K2,J)
        A(K,J) = A(K,J) + T * V1
        A(K1,J) = A(K1,J) + T * V2
        A(K2,J) = A(K2,J) + T * V3
        T = B(K,J) + U2 * B(K1,J) + U3 * B(K2,J)
        B(K,J) = B(K,J) + T * V1
        B(K1,J) = B(K1,J) + T * V2
        B(K2,J) = B(K2,J) + T * V3
 210  CONTINUE
C
      IF (K .EQ. L) GO TO 220
      A(K1,KM1) = 0.0
      A(K2,KM1) = 0.0
C
 220  S = ABS(B(K2,K2)) + ABS(B(K2,K1)) + ABS(B(K2,K))
      IF (S .EQ. 0.0) GO TO 240
      U1 = B(K2,K2) / S
      U2 = B(K2,K1) / S
```

```
      U3 = B(K2,K) / S
      R = SIGN(SQRT(U1*U1+U2*U2+U3*U3),U1)
      V1 = -(U1 + R) / R
      V2 = -U2 / R
      V3 = -U3 / R
      U2 = V2 / V1
      U3 = V3 / V1
C
      DO 230 I = LOR1, LL
        T = A(I,K2) + U2 * A(I,K1) + U3 * A(I,K)
        A(I,K2) = A(I,K2) + T * V1
        A(I,K1) = A(I,K1) + T * V2
        T = B(I,K2) + U2 * B(I,K1) + U3 * B(I,K)
        B(I,K2) = B(I,K2) + T * V1
        B(I,K1) = B(I,K1) + T * V2
        B(I,K) = B(I,K) + T * V3
  230 CONTINUE
C
      B(K2,K) = 0.0
      B(K2,K1) = 0.0
      IF (.NOT. MATZ) GO TO 240
C
      DO 235 I = 1, N
        T = Z(I,K2) + U2 * Z(I,K1) + U3 * Z(I,K)
        Z(I,K2) = Z(I,K2) + T * V1
        Z(I,K1) = Z(I,K1) + T * V2
        Z(I,K) = Z(I,K) + T * V3
  235 CONTINUE
C
  240 S = ABS(B(K1,K1)) + ABS(B(K1,K))
      IF (S .EQ. 0.0) GO TO 260
      U1 = B(K1,K1) / S
      U2 = B(K1,K) / S
      R = SIGN(SQRT(U1*U1+U2*U2),U1)
      V1 = -(U1 + R) / R
      V2 = -U2 / R
      U2 = V2 / V1
C
      DO 250 I = LOR1, LL
        T = A(I,K1) + U2 * A(I,K)
        A(I,K1) = A(I,K1) + T * V1
        A(I,K) = A(I,K) + T * V2
        T = B(I,K1) + U2 * B(I,K)
        B(I,K1) = B(I,K1) + T * V1
        B(I,K) = B(I,K) + T * V2
  250 CONTINUE
C
      B(K1,K) = 0.0
```

```
      IF (.NOT. MATZ) GO TO 260
C
      DO 255 I = 1, N
        T = Z(I,K1) + U2 * Z(I,K)
        Z(I,K1) = Z(I,K1) + T * V1
        Z(I,K) = Z(I,K) + T * V2
 255    CONTINUE
C
 260 CONTINUE
C
    GO TO 70
C
 1000 IERR = EN
C
 1001 IF (N.GT. 1) B(N,1) = EPSB
    RETURN
    END
□
```

SUBROUTINE 3

```
    SUBROUTINE QZVAL(NM,N,A,B,ALFR,ALFI,BETA,MATZ,Z)
C
C     IMPLICIT DOUBLE PRECISION (A-H, O-Z)
    INTEGER I,J,N,EN,NA,NM,NN,ISW
    REAL A(NM,N),B(NM,N),ALFR(N),ALFI(N),BETA(N),Z(NM,N)
C     DOUBLE PRECISION
A(NM,N),B(NM,N),ALFR(N),ALFI(N),BETA(N),Z(NM,N)
    LOGICAL MATZ
C
    EPSB = B(N,1)
    ISW = 1
C
    DO 510 NN = 1, N
      EN = N + 1 - NN
      NA = EN - 1
      IF (ISW .EQ. 2) GO TO 505
      IF (EN .EQ. 1) GO TO 410
      IF (A(EN,NA) .NE. 0.0) GO TO 420
C
 410    ALFR(EN) = A(EN,EN)
      IF (B(EN,EN) .LT. 0.0) ALFR(EN) = -ALFR(EN)
      BETA(EN) = ABS(B(EN,EN))
      ALFI(EN) = 0.0
      GO TO 510
C
 420    IF (ABS(B(NA,NA)) .LE. EPSB) GO TO 455
      IF (ABS(B(EN,EN)) .GT. EPSB) GO TO 430
```

```
      A1 = A(EN,EN)
      A2 = A(EN,NA)
      BN = 0.0
      GO TO 435
  430 AN = ABS(A(NA,NA)) + ABS(A(NA,EN)) + ABS(A(EN,NA))
     X   + ABS(A(EN,EN))
      BN = ABS(B(NA,NA)) + ABS(B(NA,EN)) + ABS(B(EN,EN))
      A11 = A(NA,NA) / AN
      A12 = A(NA,EN) / AN
      A21 = A(EN,NA) / AN
      A22 = A(EN,EN) / AN
      B11 = B(NA,NA) / BN
      B12 = B(NA,EN) / BN
      B22 = B(EN,EN) / BN
      E = A11 / B11
      EI = A22 / B22
      S = A21 / (B11 * B22)
      T = (A22 - E * B22) / B22
      IF (ABS(E) .LE. ABS(EI)) GO TO 431
      E = EI
      T = (A11 - E * B11) / B11
  431 C = 0.5 * (T - S * B12)
      D = C * C + S * (A12 - E * B12)
      IF (D .LT. 0.0) GO TO 480
C
C
      E = E + (C + SIGN(SQRT(D),C))
      A11 = A11 - E * B11
      A12 = A12 - E * B12
      A22 = A22 - E * B22
      IF (ABS(A11) + ABS(A12) .LT.
     X    ABS(A21) + ABS(A22)) GO TO 432
      A1 = A12
      A2 = A11
      GO TO 435
  432 A1 = A22
      A2 = A21
C
  435 S = ABS(A1) + ABS(A2)
      U1 = A1 / S
      U2 = A2 / S
      R = SIGN(SQRT(U1*U1+U2*U2),U1)
      V1 = -(U1 + R) / R
      V2 = -U2 / R
      U2 = V2 / V1
C
      DO 440 I = 1, EN
        T = A(I,EN) + U2 * A(I,NA)
```

```
         A(I,EN) = A(I,EN) + T * V1
         A(I,NA) = A(I,NA) + T * V2
         T = B(I,EN) + U2 * B(I,NA)
         B(I,EN) = B(I,EN) + T * V1
         B(I,NA) = B(I,NA) + T * V2
  440   CONTINUE
C
       IF (.NOT. MATZ) GO TO 450
C
       DO 445 I = 1, N
         T = Z(I,EN) + U2 * Z(I,NA)
         Z(I,EN) = Z(I,EN) + T * V1
         Z(I,NA) = Z(I,NA) + T * V2
  445   CONTINUE
C
  450   IF (BN .EQ. 0.0) GO TO 475
       IF (AN .LT. ABS(E) * BN) GO TO 455
       A1 = B(NA,NA)
       A2 = B(EN,NA)
       GO TO 460
  455   A1 = A(NA,NA)
       A2 = A(EN,NA)
C
  460   S = ABS(A1) + ABS(A2)
       IF (S .EQ. 0.0) GO TO 475
       U1 = A1 / S
       U2 = A2 / S
       R = SIGN(SQRT(U1*U1+U2*U2),U1)
       V1 = -(U1 + R) / R
       V2 = -U2 / R
       U2 = V2 / V1
       DO 470 J = NA, N
         T = A(NA,J) + U2 * A(EN,J)
         A(NA,J) = A(NA,J) + T * V1
         A(EN,J) = A(EN,J) + T * V2
         T = B(NA,J) + U2 * B(EN,J)
         B(NA,J) = B(NA,J) + T * V1
         B(EN,J) = B(EN,J) + T * V2
  470   CONTINUE
C
  475   A(EN,NA) = 0.0
       B(EN,NA) = 0.0
       ALFR(NA) = A(NA,NA)
       ALFR(EN) = A(EN,EN)
       IF (B(NA,NA) .LT. 0.0) ALFR(NA) = -ALFR(NA)
       IF (B(EN,EN) .LT. 0.0) ALFR(EN) = -ALFR(EN)
       BETA(NA) = ABS(B(NA,NA))
       BETA(EN) = ABS(B(EN,EN))
```

```
      ALFI(EN) = 0.0
      ALFI(NA) = 0.0
      GO TO 505
C
 480  E = E + C
      EI = SQRT(-D)
      A11R = A11 - E * B11
      A11I = EI * B11
      A12R = A12 - E * B12
      A12I = EI * B12
      A22R = A22 - E * B22
      A22I = EI * B22
      IF (ABS(A11R) + ABS(A11I) + ABS(A12R) + ABS(A12I) .LT.
     X    ABS(A21) + ABS(A22R) + ABS(A22I)) GO TO 482
      A1 = A12R
      A1I = A12I
      A2 = -A11R
      A2I = -A11I
      GO TO 485
 482  A1 = A22R
      A1I = A22I
      A2 = -A21
      A2I = 0.0
C
 485  CZ = SQRT(A1 * A1 + A1I * A1I)
      IF (CZ .EQ. 0.0) GO TO 487
      SZR = (A1 * A2 + A1I * A2I) / CZ
      SZI = (A1 * A2I - A1I * A2) / CZ
      R = SQRT(CZ*CZ+SZR*SZR+SZI*SZI)
      CZ = CZ / R
      SZR = SZR / R
      SZI = SZI / R
      GO TO 490
 487  SZR = 1.0
      SZI = 0.0
 490  IF (AN .LT. (ABS(E) + EI) * BN) GO TO 492
      A1 = CZ * B11 + SZR * B12
      A1I = SZI * B12
      A2 = SZR * B22
      A2I = SZI * B22
      GO TO 495
 492  A1 = CZ * A11 + SZR * A12
      A1I = SZI * A12
      A2 = CZ * A21 + SZR * A22
      A2I = SZI * A22
C
 495  CQ = SQRT(A1 * A1 + A1I * A1I)
      IF (CQ .EQ. 0.0) GO TO 497
```

```
      SQR = (A1 * A2 + A1I * A2I) / CQ
      SQI = (A1 * A2I - A1I * A2) / CQ
      R = SQRT(CQ*CQ+SQR*SQR+SQI*SQI)
      CQ = CQ / R
      SQR = SQR / R
      SQI = SQI / R
      GO TO 500
 497  SQR = 1.0
      SQI = 0.0
C
 500  SSR = SQR * SZR + SQI * SZI
      SSI = SQR * SZI - SQI * SZR
      I = 1
      TR = CQ * CZ * A11 + CQ * SZR * A12 + SQR * CZ * A21
     X    + SSR * A22
      TI = CQ * SZI * A12 - SQI * CZ * A21 + SSI * A22
      DR = CQ * CZ * B11 + CQ * SZR * B12 + SSR * B22
      DI = CQ * SZI * B12 + SSI * B22
      GO TO 503
 502  I = 2
      TR = SSR * A11 - SQR * CZ *A12 - CQ *SZR * A21
     X    + CQ * CZ * A22
      TI = -SSI * A11 - SQI * CZ *A12 + CQ * SZI * A21
      DR = SSR * B11 - SQR * CZ *B12 + CQ * CZ * B22
      DI = -SSI * B11 - SQI * CZ * B12
 503  T = TI * DR - TR * DI
      J = NA
      IF (T .LT. 0.0) J = EN
      R = SQRT(DR*DR+DI*DI)
      BETA(J) = BN * R
      ALFR(J) = AN * (TR * DR + TI * DI) / R
      ALFI(J) = AN * T / R
      IF (I .EQ. 1) GO TO 502
 505  ISW = 3 - ISW
 510  CONTINUE
C
      RETURN
      END
```

CURRICULUM VITAE

Abdurrahman Seref CAN was born in Kütahya-TURKIYE on March 18, 1970. He graduated from high school in 1987. He received his B.Sc. degree from Istanbul Technical University, Aeronautical and Astronautical Engineering Faculty, in 1992. He began to MS program in Aeronautical Engineering Department in 1992. He attended a R&D program on Cockpit Design in CASA, Madrid for six months.