ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL

MODEL PREDICTIVE CONTROL BASED COOPERATIVE PURSUIT EVASION FOR UAV

M.Sc. THESIS

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Department of Aeronautical and Astronautical Engineering

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İSTANBUL TEKNİK ÜNİVERSİTESİ ★ LİSANSÜSTÜ EĞİTİM ENSTİTÜSÜ

MODEL ÖNGÖRÜ GÜDÜM TABANLI SÜRÜ İNSANSIZ HAVA ARAÇLARI ARASINDAKİ ANGAJMAN

YÜKSEK LİSANS TEZİ

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Uçak ve Uzay Mühendisliği Anabilim Dalı

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ŞUBAT 2022

Mustafa Berkay AKBIYIK, a M.Sc. student of ITU Graduate School student ID 511181131 successfully defended the thesis entitled "MODEL PREDICTIVE CONTROL BASED COOPERATIVE PURSUIT EVASION FOR UAV", which he prepared after fulfilling the requirements specified in the associated legislations, before the jury whose signatures are below.

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To my family

FOREWORD

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February 2022 Mustafa Berkay AKBIYIK

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ABBREVIATIONS

UAV	**:**	Unmanned Aerial Vehicle
MPC	**:**	Model Predictive Controller
TB2	**:**	Tactic Block 2

SYMBOLS

$\mathscr{I}$	**:** Inertial Frame of Vehicles
$\mathscr{B}_a$	**:** Body Frame of Vehicles
x, y, z	**:** North, East, Down Positions
ϕ, θ, ψ	**:** Roll, Pitch, Yaw Angles
P	**:** Position Vector
V	**:** Velocity Vector
ω	**:** Angular Rates
t	**:** Time
S	**:** Score of the Game

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MODEL PREDICTIVE CONTROL BASED COOPERATIVE PURSUIT EVASION FOR UAV

SUMMARY

This thesis proposes game theoretically model predictive control based guidance approach for pursuit-evasion problem of uav's. The main idea is that guided swarm uavs pursue towards to adversary uav which evade to survive as long as possible. Game theretical approach of pursuit-evasion is based on designing the cost functions for each pursuer to converge adversary evader. Proposed approach is examined as decentralized. Therefore, each pursuer can be able to handle its mission independently without being affected by the other pursuer.

Main contribution is the formulation of swarm pursuit-evasion problem as the game theoretical which can enable to develop optimization-based algorithms that bring superior strategies to pursuers for one-to-one, two-to-one scenarios during the air combat.

This work proposes an algorithm to enhance applicability of the game theoretic non-convex model predictive control problems on real-systems that have nonlinear controland state constraints.

Proposed algorithm provide model predictive control based guidance system which orientate the pursuers according to the evaders dynamics and positions. Nonlinear constraints are convexified along the finite-horizon time without loss of generalityin successive linearizations. After discretization of dynamics, sub-optimal convex problem can be applied in model predictiveconcept for time-critical scenarios such as collaborative pursuit-evasion of aerial vehicles.

MODEL ÖNGÖRÜ GÜDÜM TABANLI SÜRÜ İNSANSIZ HAVA ARAÇLARI ARASINDAKİ ANGAJMAN

ÖZET

Doğadaki birçok canlı teknolojik sistemlere ilham vermiştir. Uçak tasarımlarının kuşlardan esinlenmesi, şahinlerin başlarını çeşitli bozuntulara karşı stabil şekilde koruyabilmeleri buna güzel birer örnektir. Arılar, bakteriler, balıklar, kuşlarda görülen diğer dikkat çekici bir özellik ise sürü davranışıdır. Arılar sürü bir şekilde işbirlik sayesinde polen toplayarak bal yaparlar. Bakteriler keza yine sürü davranışı sergileyerek çeşitli besinlerin fermetesinde katkı sağlarlar. Sürü davranışı dikkatli bir şekilde incelendiğinde teker teker yapılan işlerin işbirliği ile topluca yapılarak daha verimli şekilde işlerin sonuçlanmasına katkıda bulunduğu görülmüştür. Bu davranış modelinin sadece doğa da değil teknolojik gelişmelerde de etkisi görülmektedir. Yüzlerce küçük drone'lar ile görsel ışıklı etkinliklerin yapılması, binlerce uydu ile tüm dünyaya internetin ulaştırılması, yine çoklu küçük robotlar ile büyük bir alanın temizlenmesi. Havacılıkta da yakın zaman da bunun esameleri gözükmüştür. Baykar TB2'lerin hava muharebe saldırısında sürü şeklinde kullanılması bunun güzel örneklerinden bir tanesidir. Ancak tüm bu gelişmelere rağmen, hava savunma sistemlerinin çoklu veya toplu hava saldırılarına karşı halen maliyet anlamında efektif bir çözüm getiremediği görülmektedir. Hava savunma sistemleri maliyet anlamında çok efektif olmayan ve her fırlattığı roket başına ciddi bir maddi götürüsünün olduğu aşikardır. Sürü hava ataklarına karşı savunma zaafiyetinin sağlayıp sağlayamadığı ise halen büyük bir soru işaretidir. Bir hava sahasına gönderilebilecek yüzlerce belki de binlerce hava aracı veya mini drone'lar ile radar sistemlerinin rahatlıkla şaşırtılarak bir savunma zaafiyeti oluşturacağı kolaylıkla öngörülebilir. Bu tez hava savunmasının sürü mini drone veya hava araçları ile yapılabilebileceğini gösteren çalışmaları sunmaktadır. Yapılan araştırmalar sonucunda bu şekilde geçmişte bir çalışmanın örneğine rastlanmamıştır. Hava savunmasına girecek olan hava araçlarının sürü insansız hava araçları ile otonom şekilde takip edilmesi ve hava da imhasını mümkün kılmasını amaçlamaktadır bu çalışma. Hava savunmasına giriş yapan insansız hava aracının belli bir bölgeyi vuracağı varsayımı üzerinden bu bölgenin belirli bir alan içerisindeki hava mücadelesi üzerinden hava araçları ile korunması planlanmaktadır. Düşman hava aracının otonom şekilde takip edilip imha edilmesinde pilotların davranışından esinlenilmiştir. Usta pilotlar hava muharebesinde bire bir veya ikiye bir gibi hava hava saldırılarında karşı uçağı avlamak veya takip etmek istediklerinde hava da diğer uçağa karşı bir üstünlük kazanmak istediğinde kaşı hava aracın anlık hava daki konumundan ziyade belli bir süre sonra varmasını düşündüğü bölgeyi tahmin ederek ona göre kendi hava aracını o bölgeye doğru yönlendirmektedir. Böylelik takip eden uçağa bu hava muharebesinde avantaj sağladığı görülmektedir. Burada kaçan hava aracının da yönelim davranışı bu anlamda önem arz etmektedir. Buna yönelik çalışmaların DARPA'nın çeşitli

projelerinde halen çözülmeye çalışıldığı bilinmektedir. Bu çalışma, belli bir bölge içinde savunulacak yerin hava savunmasının sürü hava araçları ile efektik olarak yapılmasını sağlayacak sistemin tasarım ve modelleme çalışmalarını içermektedir. Hava sahasına sızdığı düşünülen rakip hava araçlarının, dost hava araçları ile sürü şekilde takip edilirken kaçmasının modellenmesi, güdüm edilecek hava sistemlerinin doğruluğu ve sağlamlığı açısından büyük önem arz etmektedir. Rakibin stratejisi öngörülebilir çerçeve de tahmin edilebilirse takip edecek sistemlerin güdüm maliyet ve efektifliği de o oranda artacaktır. Bu nedenle takip-kaçma problemine oyun teorisi ile yaklaşılarak bu durumun ele alınması düşünülmüştür. Böylelikle kaçmaya çalışan hava aracının yönelim stratejisinin ele alınarak takip edecek hava sistemlerinin güdümü planlanmıştır. Oyun teorisi ile karşı hava aracın ve güdüm edilen sistemin yönelimlerinin modellenip tahmin edilerek güdüm edilmesinde model öngörülü kontrol mantığı düşünülmüştür. Model öngörülü kontrol diğer kontrol methodlarına göre gelecek zaman yönelimlerini de hesaba katıp kontrol etmesini imkan sağlaması ile bu problemin çözümünde daha uygun olduğu görülmüştür. Bu methoda göre bir maliyet fonksiyonu ve çeşitli kısıtlamalar ile her zaman adımında gelecek zaman adımları da dahil edilerek bir optimizasyon hesaplaması yapılmaktadır. İlk aşamada bire-bir hava mücadelesi oyun teorisi üzerinden takip-kaçma'sı modellenmiştir. Rakip hava aracı çeşitli kısıtlar ile içerisindeki standard bir kontrol sistemi ile güdüm edilirken, güdüm edilen dost hava aracı lineer olmayan model öngörülü kontrol yöntemi ile güdüm edilmiştir. Lineer olmayan model öngörülü kontrol methodu hava aracına olan mesafesi gibi çeşitli parametreler üzerinden oluşturulan maliyet fonksiyonu ve kısıtlar ile karşı rakibin gelecek adımlarının oyun teorisi üzerinden model öngörülü kontrol methodu ile optimizasyon hesaplaması sonucunda güdüm edilerek bir tasarım oluşturulmuştur. Lineer olmayan optimizasyon algoritmalarının gömülü elektronik işlemcilerindeki hesaplama kısıtlamasından ötürü bir ileri aşama olarak konveks olmayan maliyet fonksiyonu ve kısıtlamaların konveksleştirilerek konveks optimizasyon problemine dönüştürülmesine karar verilmiştir. Konveks optimizasyon algoritmalarının işlemcilerde hesaplamaları mili saniyelere düştüğünden dolayı sunulan çözümün ileri de gerçeklenmesine zemin hazırlamıştır. Böylelik güdüm edilen hava aracının anlık olarak üzerindeki gömülü sistem üzerinden otonom şekilde güdüm edilerek takip edilmesine imkan sağlanmıştır. Yapılan araştırmalarda, geçmişte ve şuan da bu konuda yapılan çalışmalar da görülmektedir ki henüz bu tarz amaçla güdüm edilmek istenen sistemlerin çevrimiçi çalışmasına imkan sağlayacak bir sonuç görülmemiştir.

1. INTRODUCTION

Many living things in nature have inspired technological systems. Aircraft designs being inspired by birds and hawks' ability to stably protect their heads against various disturbances are good examples case in point. Another remarkable feature seen in bees, bacteria, fish and birds is swarm behavior. Bees make honey by collecting pollen by swarm cooperative behavior. Bacteria also contribute to the fermentation of various foods by exhibiting herd behavior. When the herd behavior is carefully examined, it is seen that the works done one by one contribute to the conclusion of the works in a more efficient way by doing them collectively. The effect of swarm behavior model is seen not only in nature but also in technological developments. Making visual light events with hundreds of small drones, delivering the internet to the whole world with thousands of satellites, and cleaning a large area with multiple small robots. In aviation, the effects of swarm technology have been seen recently. The use of Baykar TB2s as a swarm in an air combat attack is one of the good example case in point. However, despite all these developments, it is seen that air defense systems still cannot provide an effective solution in terms of cost against multiple or collective air attacks. It is obvious that air defense systems are not very effective in terms of cost and there is a serious financial cost per rocket launched. It is still a big question mark whether it can provide defense weakness against swarm air attacks. It can be easily predicted that hundreds, maybe thousands, of aircraft or mini-drones and radar systems that can be sent to an airspace will easily be confused and create a defense vulnerability. This thesis proposed studies showing that air defense can be done with swarm mini drones or aerial vehicles by controlling collaboratively. There is not any example of such a efficient study in terms of applicability on onboard-systems is not found in literature [1]. This study aims to enable guide autonomously swarm uavs to pursuit the adversary unmanned aerial vehicles that will be threat to air defense. Based on the assumption that an unmanned aerial vehicle which entering the air defense will hit a certain area, it is planned to protect base area with swarm unmanned aerial vehicles through the air combat as pursuit-evasion within a certain volume. Inspired by the behavior of pilots in autonomously tracking and destroying enemy aircraft. When master pilots want to hunt or follow the opposing aircraft in air combat, such as one-on-one or two-on-one, when they want to gain an advantage over the other aircraft in the air, they predict the area that they think will arrive after a certain period of time, rather than the aircraft's instantaneous position in the air and directs the vehicle towards that area. Thus, it is seen that it provides an advantage to the pursuer aircraft in the air combat. The orientation behavior of the evader aircraft is also important in that sense.

This study presents the design and modelling studies of the system that will enable the air defense of the base area to be defended effectively guided by swarm unmanned aerial vehicles in a certain region. Modelling the evader of adversary aircraft, which are assumed to infiltrated the airspace, while being pursue by friendly autonomous collaboratively controlled unmanned aerial vehicles, is of great importance in terms of the accuracy and robustness of the air systems to be guided. If the opponent's strategy can be predicted within a predictable framework, the cost and effectiveness of the pursuer systems will increase accordingly. For this reason, it is thought to deal with this situation by approaching the pursuit-evasion problem with game theory. Therefore, the guidance of the air systems to pursue is planned by considering the orientation strategy of the adversary unmanned aerial vehicle trying to escape. With game theory, the orientations of the adversary uav and the guided system are estimated by some assumptions. Thus, model predictive control method is considered in guiding the friendly uav by predicting counter strategies. Model predictive control has been found to be more appropriate in solving the problem, as it allows control by taking into account receding horizon of adversary orientations compared to other control methods. According to MPC, calculation of optimization is made by receding horizon in each time step with a cost function and various constraints. In the first stage, pursuit-evasion problem is modelled as one-to-one aerial combat scenario with game theory. While the evader UAV is guided with a standard control system with various constraints, the guided pursuer UAV is guided with the nonlinear model predictive control method. A design is created by contituted the cost function and constraints created on various parameters such as the nonlinear model predictive control method, the distance to the aircraft, and the optimization calculation of the receding horizon of the opposing uav over the game theory with the model predictive control method. Due to the computational constraints of nonlinear optimization algorithms in the embedded systems, it is not usable for real-time applications.

This thesis proposes that transforming the non-convex cost function and constraints into a convex optimization problem as a further step. Since the calculations of convex optimization algorithms in processors are reduced to milliseconds, it allows the way for further verifying of the presented solution. In this way, it is possible to calculate local optimal solution for the guided pursuer uav autonomously on the embedded system instantly.

1.1 Related Works

There are large number of examples in aerospace applications such as non-convex problems [1], [2].As in [1] and [1], non-convexities results from minimum thrust and dynamic relative position. In addition, receding-horizon control also takes up time for finding first control input due to prediction of future actions on each step. Therefore, there is needs to be taken low-demands calculation time solutions. In some cases, linearized models are used to control the system over receding-horizon. [3] offers controlling trajectory tracking on real-system by using linear model predictive control. Also,as suggested by [4], it proposed efficient model predictive to controls the vehicle along a rectangular trajectory. Even though the time response of linear MPC is satisfactory for airborne computational power, it can not offers effective solution in terms of tracking time of trajectory. Because of linearized on a local point, vehicle have been controlled on limited constraints that does not satisfy agility. There are various algorithms makes complexity of problems efficient for computational power. To relaxing non-convex problems, lossless convexification is proposed as in [5], [6], [7], [8]. In [5],lossless convexification is implemented to powered descent guidance problem for minimizing usage of fuel. To do this,non-convex constraints are modified with slack variables. In addition, the main goal of [5] has been extended to minimum-landing-error with some modifications such that Figure 1.1. Pursuit-Evasion of Aerial Vehicles on air space. Red dashed line represents trajectory of evaders

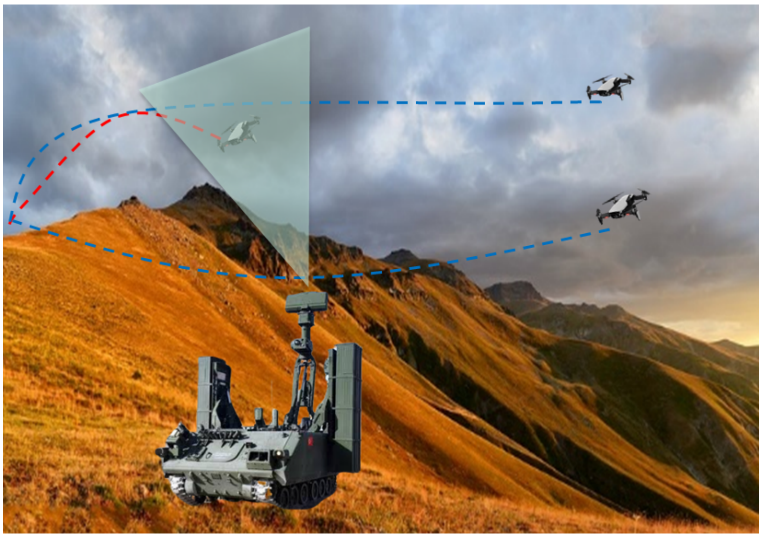

Figure 1.1 : Illustration of Pursuit-Evasion

and blue dashed lines represents trajectories of pursuers specifying cost function as a relative position in [6]. Accuracy of lossless convexification have been provided with successive convexifation that linearize the model iteratively to eliminate nonlinearites such as drag, mass depletion in [9]. Successive convexification offers to enhance applicability of non-convex control problems on real-system as in [10]. In [10], path planning of the vehicles are handled with convex optimization real-time on-boards.

1.2 Main Contributions

In this thesis, main contribution is to handle to eliminate non-convexity terms without loss of generality for game theoretic model predictive control problems such that cooperative pursuit-evasion of aerial vehicles as illustrated in Figure 1.1. Basic idea is to convexify non-convex constraints and successively linearized dynamics transformed into the sub-optimal problem over the receding-horizon time. Sub-optimal problems are solved by Second-Order Cone Programming as in [11]. Successively convexified optimal control is formulated as differential-game and impelemented on pursuit-evasion of cooperative aerial vehicles.

1.3 Organization

The structure of the remaining paper is organized as follows: Section 2 introduces model predictive control and presents the differential game theory. In Section 3, problem formulation of pursuit-evasion and controller designing are explained for the aerial vehicles has been shown. Section 4 declared configurations of simulation setup. In Section 5 shows the simulation results for the one-to-one scenario. Section 6 introduces the convexification of non-convex game theory problem.

2. BACKGROUND

Proposed definitions and baseline algorithms are based on [12], [13] in Section 2. These concepts are examined for construction of the problem

2.1 Dynamics of Unmanned Aerial Vehicle

Let us define $\mathscr{I} = \{x_i, y_i, z_i\}$ as the inertial frame of vehicles, $\mathscr{B}_a = \{x_b, y_b, z_b\}$ as the body frame of pursuer and evader.

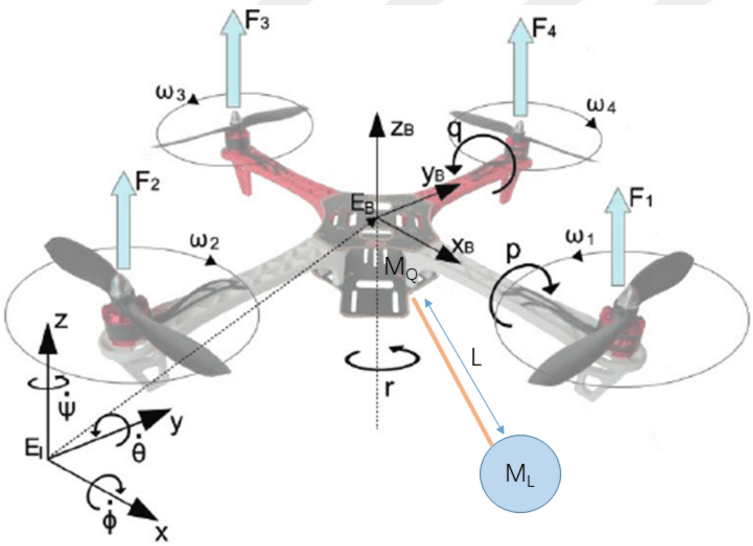

Figure 2.1 : Quadrotor

Now, let $P_a = [x_a \ y_a \ z_a]^T$ and $\mathbf{\Phi}_a = [\phi_a \ \theta_a \ \psi_a]^T$ describe position vector in $\mathscr{I}$ and attitude of UAVs using euler angles, respectively. UAV model can be written using kinematic and Newton-Euler equations as below:

$$\begin{aligned} \dot{P}^a_{\mathscr{I}} &= V^a_{\mathscr{I}}, \\ \dot{\mathbf{\Phi}}_a &= E(\mathbf{\Phi}_a)^{-1} \cdot \boldsymbol{\omega}_{\mathscr{B}_a}, \\ \dot{V}^a_{\mathscr{I}} &= m^{-1} \cdot R_{\mathscr{I}\mathscr{B}}(\mathbf{\Phi}_a) \cdot (f_e + f_a) - g, \\ \dot{\boldsymbol{\omega}}_{\mathscr{B}_a} &= I^{-1} \cdot (-\boldsymbol{\omega}_{\mathscr{B}_a} \times (I \cdot \boldsymbol{\omega}_{\mathscr{B}_a}) + \tau_e + \tau_a) \end{aligned} \tag{2.1}$$

where m is the mass and I is the inertia matrix, whom diagonal elements (i.e., I_{xx}, I_{yy}, I_{zz}) are the principal moments of inertia.

$V_{\mathscr{I}}^{a} = [u_a, v_a, w_a]^T$ and $\boldsymbol{\omega}_{\mathscr{B}_a} = [p_a, q_a, r_a]^T$ are the linear and angular velocity vectors expressed in $\mathscr{I}$ and $\mathscr{B}_a$ respectively, while $R_{\mathscr{I}\mathscr{B}}(\boldsymbol{\Phi})$ and $E(\boldsymbol{\Phi})$ stand for the known rotation matrix and its Jacobian respectively, with regards to the Euler ZYX angles. In addition, $f_e = [0, 0, f_{ez}]$ is the external force and $\tau_e = [\tau_{ex}, \tau_{ey}, \tau_{ez}]$ is the external torque vector caused by the rotation of the propellers, and $f_a = [f_{ax},_{ay},_{az}] = -diag(d_u, d_v, d_w) \cdot R_{\mathscr{I}\mathscr{B}}(\boldsymbol{\Phi})$ and similarly $\tau_a = [\tau_{ax}, \tau_{ay}, \tau_{az}] = -diag(d_p, d_q, d_r) \cdot \boldsymbol{\omega}_{\mathscr{B}_a}$ are the force and torque produced by the aerodynamic drag in which $d_u, d_v, d_w, d_p, d_q, d_r$ are positive drag constant coefficients, while g is the gravitational acceleration vector expressed in $\mathscr{I}$

2.2 Model Predictive Control

To execute high-level target pursuit and evade planning problem, use of model predictive control provide to exploit lookahead during the finite-horizon time as in Figure 2.2 . With MPC, online optimization and optimal control techniques finds the required inputs according to the finite-horizon cost values. Also, the other advantages are that some physical and game limitations can be imported as constraints in the optimization section of MPC.

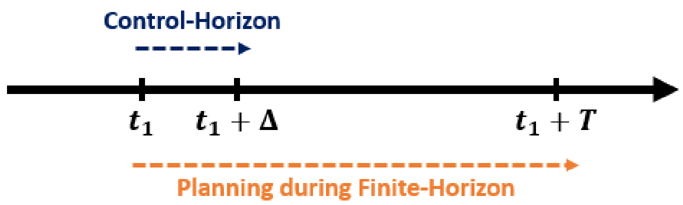

Figure 2.2 : Logic of Receding-Horizon

2.3 Game Theory - Differential Game

In this section, game theory framework is formulated, in order to design model predictive controller for pursuit-evasion game. Dynamical systems of two vehicles can be written as :

$$\dot{\boldsymbol{\xi}}_p = f_p(\boldsymbol{\xi}_p, \boldsymbol{u}_p, t), \ \dot{\boldsymbol{\xi}}_e = f_e(\boldsymbol{\xi}_e, \boldsymbol{u}_e, t) \tag{2.2}$$

where the first equation is the pursuer's model (UAV), in which $\boldsymbol{\xi}_p = [P^p_{\mathscr{I}}, \boldsymbol{\Phi}_p, V^p_{\mathscr{I}}, \boldsymbol{\omega}_{\mathscr{B}_p}]$ is state's vector, $u_p = [T_{rot}, \Phi_{des}, \Theta_{des}, r_{des}]$ is input vector and f_p is its differential equation of pursuer, and the second describe the evader's model (UAV), in which (UAV) $\boldsymbol{\xi}_e = [P^e_{\mathscr{I}}, \boldsymbol{\Phi}_e, V^e_{\mathscr{I}}, \boldsymbol{\omega}_{\mathscr{B}_e}]$ is it's state vector, $u_e = [T_{rot}, \Phi_{des}, \Theta_{des}, r_{des}]$ is input vector and f_e is its differential equation of evader. Finally, game dynamics defined as:

$$\dot{\boldsymbol{e}} = f_e(\boldsymbol{e}, \boldsymbol{u}_p, \boldsymbol{u}_e, t) \tag{2.3}$$

where:

$$e = \begin{bmatrix} x_e \\ y_e \\ z_e \end{bmatrix} = \begin{bmatrix} x_p - x_e \\ y_p - y_e \\ z_p - z_e \end{bmatrix} \tag{2.4}$$

x_e, y_e, z_e are position errors between UAV in 3-dimensional space.
Objective functions of agents described as:

$$\begin{aligned} \mathscr{J}(\boldsymbol{u}_p^*, \boldsymbol{u}_e^*) &= \min_{\boldsymbol{u}_a \in \mathscr{U}} \boldsymbol{J}_i(\boldsymbol{u}_p, \boldsymbol{u}_e^*) = \max_{\boldsymbol{u}_p \in \mathscr{V}} \boldsymbol{J}_i(\boldsymbol{u}_p^*, \boldsymbol{u}_e) \\ &= \min_{\boldsymbol{u}_p \in \mathscr{U}} \max_{\boldsymbol{u}_e \in \mathscr{V}} \mathscr{J}(\boldsymbol{u}_p, \boldsymbol{u}_e) \\ &= \max_{\boldsymbol{u}_e \in \mathscr{V}} \min_{\boldsymbol{u}_p \in \mathscr{U}} \mathscr{J}(\boldsymbol{u}_p, \boldsymbol{u}_e) \end{aligned} \tag{2.5}$$

where U,V are admissible sets for $\boldsymbol{u}_p$ and $\boldsymbol{u}_e$ and if $\boldsymbol{u}_p^*$ and $\boldsymbol{u}_e^*$ (optimal input vectors) can be found,

$$\boldsymbol{S}_g = \mathscr{J}(\boldsymbol{u}_p^*, \boldsymbol{u}_e^*) \tag{2.6}$$

which is value of the game

3. CONTROL DESIGN

3.1 Model Predictive Controller in Game Theory

The main aim of this work is to design Model Predictive Controller for each player. Purpose of pursier's controller is minimize the position error with evader's and the aim of the evader's controllere is maximize the distance between pursuer and itself. The main advantages of this selection are two:

- Model based controller gives to each player optimal strategy by predicting future states
- Using the predictions of either his, either his opponent's states over a finite amount of time, each agent can determine or even alternate his strategy for his personal interest.

Considering fixed prediction horizon N, proposed optimization problem can be defined as:

$$\min_{\boldsymbol{U}_p} \max_{\boldsymbol{U}_e} \mathscr{J}(\boldsymbol{E}, \boldsymbol{U}_p, \boldsymbol{U}_e) \tag{3.1}$$

where $\boldsymbol{E} = [e^T_{(0)}, e^T_{(1)}, ..., e^T_{(N)}]$ is the state trajectory and $\boldsymbol{U}_i = [u^T_{i(1)}, u^T_{i(2)}, ..., u^T_{i(N-1)}]$, $i \in \{p, e\}$ are the input trajectories for each agent of the game through the given time horizon N. As for objective function $\mathscr{J}$ for each agent defined as:

$$J_i(\boldsymbol{E}, \boldsymbol{U}_p, \boldsymbol{U}_e) = \sum_{k=0}^{N} J^i_s(\boldsymbol{e}_k, \boldsymbol{u}_{p(k)}, \boldsymbol{u}_{e(k)}) \tag{3.2}$$

J^i_s denotes running cost of objective function and can be written as follows:

$$J^i_s = \sum_{k=0}^{N} \boldsymbol{e}^T_{(k)} \cdot Q \cdot \boldsymbol{e}_{(k)} + \sum_{k=1}^{N-1} \Delta\boldsymbol{u}^T_{i(k)} \cdot R_i \cdot \Delta\boldsymbol{u}_{i(k)},\ i \in \{p, e\} \tag{3.3}$$

where Q, R_i are positive definite matrices, which impose the selected specifications in states and inputs, respectively, while Δu_i is the difference between two inputs in succession.

For the pursuer to solve the optimal game problem using must be known, and the equation (7), the best inputs $\boldsymbol{v}_e^*$ then proceed to minimization of its objective function to determine its best control performance $\boldsymbol{u}_p^*$. Because of the antagonistic character of the game the pursuer must act individually and predict the expected inputs $\boldsymbol{v}_e^*$ maximizing the objective function. In such a way, it predicts its opponent's best response, which corresponds in its worst case scenario, and after that it decides its actual move. This procedure can actually keep repeating, adding levels of thinking to the pursuing agent before deciding its final input trajectory. As a result, the proposed pursuer UAV controller is described in Algorithm 1, while using the same procedure, the proposed evader UAV controller is described in Algorithm 2.

Algorithm 1 Pursuer UAV Controller

1: **procedure** SOLVE $\min\max \mathscr{J}(\boldsymbol{E}, \boldsymbol{U}_p, \boldsymbol{U}_e)$
2: Solve $\max J_e(\boldsymbol{E}, \boldsymbol{U}_p^*, \boldsymbol{U}_e)$ to determine $\boldsymbol{U}_e^*$
3: Then solve $\min J_p(\boldsymbol{E}, \boldsymbol{U}_p, \boldsymbol{U}_e^*)$ to determine $\boldsymbol{U}_p^*$
4: Repeat steps 2 and 3 k times, where k is level of thinking
5: Set final $\boldsymbol{U}_p^*$ as the optimal input trajectory
6: **end procedure**
7: Set $\boldsymbol{u}_{p(1)}$ as the current UAV input vector

Algorithm 2 Evader UAV Controller

1: **procedure** SOLVE $\max\min \mathscr{J}(\boldsymbol{E}, \boldsymbol{U}_p, \boldsymbol{U}_e)$
2: Solve $\min J_p(\boldsymbol{E}, \boldsymbol{U}_p, \boldsymbol{U}_e^*)$ to determine $\boldsymbol{U}_p^*$
3: Then solve $\max J_e(\boldsymbol{E}, \boldsymbol{U}_p^*, \boldsymbol{U}_e)$ to determine $\boldsymbol{U}_e^*$
4: Repeat steps 2 and 3 k times, where k is level of thinking
5: Set final $\boldsymbol{U}_e^*$ as the optimal input trajectory
6: **end procedure**
7: Set $\boldsymbol{u}_{e(1)}$ as the current UAV input vector

4. SIMULATION

4.1 Game Setup

In order to see efficacy of MPC controllers, UAVs start pursuit-evasion game in certain constrained area. Game-scoring environment was designed with the aim to test and evaluate our different scenarios using a static perspective. The game-scoring is defined as follows:

Every game consists of five sets and in each set the winner is either the first who has two game-points difference or the first who reaches three game-points. The final winner is the one who has three or more sets in the end of the game. The positioning of the two vehicles in the start of every set is randomly chosen, as long as the two antagonistic vehicles are inside PEG hall. The status of the game is determined by the sign of the norm $\mathscr{D}$:

$$\mathscr{D} : \sqrt{(x_p - x_e)^2 + (y_p - y_e)^2 + (z_p - z_e)^2} \tag{4.1}$$

$$If = \begin{Bmatrix} \mathscr{D} < 1, \ Pursuer\ wins \\ \mathscr{D} > 1, \ Evader\ wins \end{Bmatrix}$$

This norm is calculated in every finite amount of time T_s and depending on the condition of this norm, a point is given to one vehicle. The first that collects one-hundred time-points, which also corresponds to the time someone wins, a game-point is given to that vehicle.

Evader can win with two ways:

- Collect one-hunder points
- Avoid from Pursuer with limited time

5. SIMULATION RESULTS

In this section, we shared simulation results of pursuit-evasion game in constrained 3-D space. Differential Game is executed more than 30 at different initial conditions by Monte Carlo simulation as in 5.1.

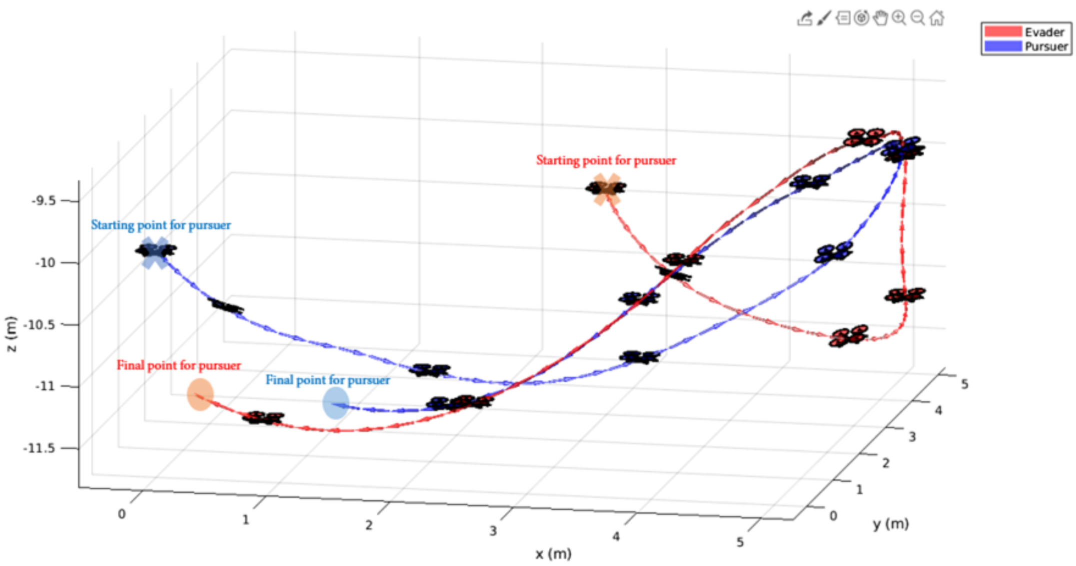

Figure 5.1 : Illustration Simulation of One-to-One Scenario

Each game is scored by pursuit-evasion rules. Depending on the starting initial conditions, some games are won by the evader, while the pursuer won some games. In some games. According to the results in the Monte Carlo simulations, there is no wins in some games.

6. CONVEXIFICATION

In this section, we describe reformulation of minimum pursuit-evasion optimizing problem consist of minimizing the total tracking error from evader to pursuer subject to non-convex thrust-magnitude constraint and vehicle's angle constraints, while guarantee to be inside limited engagement space.

6.1 Problem Formulation

Main problem present a fixed-final-time optimal control problem that performs an pursuit-evasion game. Therefore, we use t to denote t, also subscripts i, c and f represent respectively initial, intermediate and final points. We used 3-degree-of-freedom $(3-DoF)$ translational dynamics of the vehicle's for this problem. Position, velocity and commanded reference input as $p \in \mathbb{R}^3$, $v \in \mathbb{R}^3$ and $u \in \mathbb{R}^3$, respectively.

Problem 1

$$\underset{\boldsymbol{u(t)},\Gamma(t)}{\text{minimize}}\,\underset{\boldsymbol{u(t)},\Gamma(t)}{\text{maximize}}\,\|\boldsymbol{r}(t)\|^2$$

subject to:

$$\boldsymbol{r}(0) = \boldsymbol{r}_i \quad \boldsymbol{v}(0) = \boldsymbol{v}_i \quad \boldsymbol{u}(0) = g\boldsymbol{e}_3$$

$$\dot{\boldsymbol{r}}(t) = \boldsymbol{v}(t)$$
$$\dot{\boldsymbol{v}}(t) = \boldsymbol{u}(t) - g\boldsymbol{e}_3$$

$$\|\boldsymbol{u}^2(t)\|_2 \leq \Gamma(t)$$
$$0 < u_{min} \leq \Gamma(t) \leq u_{max}$$
$$\Gamma(t)\cos(\theta_{max}) \leq \boldsymbol{e}_3^T\boldsymbol{u}(t)$$

$$x_{min} \leq \boldsymbol{e}_1^T\boldsymbol{x}(t) \leq x_{max}$$
$$y_{min} \leq \boldsymbol{e}_2^T\boldsymbol{y}(t) \leq x_{max}$$
$$z_{min} \leq \boldsymbol{e}_3^T\boldsymbol{z}(t) \leq z_{max}$$

This formulation is used to solve receding horizon optimization problem to control higher order physical dynamic system. Simply stated, lossless convexification guarantees that if a feasible solution exists, the optimal solution to the (convex) relaxed problem recovers the optimal solution of the original non-convex problem. We change the input with slack variable as $\Gamma(t)$, because commanded input to vehicle contain highly non-linearity due to thrust vector. $\|\boldsymbol{r}(t)\|$ represent norm of position errors between pursuer and evader. Our objective allows us to use lossless convexification and ensures that less consuming time for performing of PEG.

Problem 2

$$\underset{\boldsymbol{u(t)},\Gamma(t)}{\text{minimize}} \|\boldsymbol{r}_e(t)\|$$

subject to:

$$\boldsymbol{r}(0) = \boldsymbol{r}_i \quad \boldsymbol{v}(0) = \boldsymbol{v}_i \quad \boldsymbol{u}(0) = g\boldsymbol{e}_3$$

$$\boldsymbol{r}_{k+1} = \boldsymbol{r}_k + \tfrac{\Delta t}{2}(\dot{\boldsymbol{r}}_k + \dot{\boldsymbol{r}}_{k+1}) + \tfrac{\Delta t^2}{12}(\boldsymbol{u}_{k+1} - \boldsymbol{u}_k)$$

$$\dot{\boldsymbol{r}}_{k+1} = \dot{\boldsymbol{r}}_k + \tfrac{\Delta t}{2}(\boldsymbol{u}_k + \boldsymbol{u}_{k+1}) + g\Delta t$$

$$\|\boldsymbol{u}(t)\|_2 \leq \Gamma(t)$$

$$0 < u_{min} \leq \Gamma(t) \leq u_{max}$$

$$\Gamma(t)\cos(\theta_{max}) \leq \boldsymbol{e}_3^T \boldsymbol{u}(t)$$

$$x_{min} \leq \boldsymbol{e}_1^T \boldsymbol{x}(t) \leq x_{max}$$

$$y_{min} \leq \boldsymbol{e}_2^T \boldsymbol{y}(t) \leq x_{max}$$

$$z_{min} \leq \boldsymbol{e}_3^T \boldsymbol{z}(t) \leq z_{max}$$

Problem 1 is discritized for onboard calculations as in Problem 2. The remainder of our solution strategy consists of solving Problem 2 in accordance to the steps outlined in Algorithm 1.

Algorithm 1

$\boldsymbol{Step 1}$:Select a final time, $t_f > 0$

$\boldsymbol{Step 2}$:Solve the relaxed minimum-pursuit-evasion optimizing problem

Problem 2 using t_f for $\{u^*(\cdot), \Gamma^*(\cdot)\}$ with corresponding trajectory

$\boldsymbol{Step 3}$: Return $\{u^*(\cdot)\}$

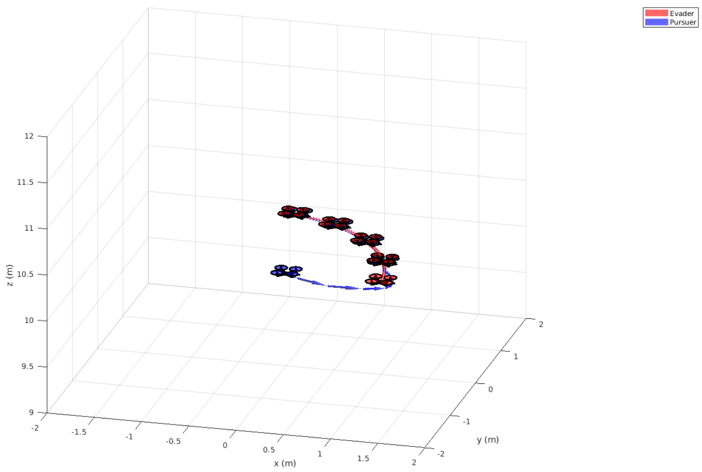

Figure 6.1 : Illustration Convexified Simulation of One-to-One Scenario

Tracking Optimal Time	t^*
Nonlinear Optimization	61.46 s
Convex Optimization	37.12 s

Difference of calculation time for nonlinear and convex optimization algorithms can be seen as remarkable in terms of onboard solutions.

6.2 Successive Relaxed Obstacle Avoidance

In this section, successive relaxed obstacle avoidance optimization formulation is introduced as in Problem 3. Therefore, relaxed obstacle avoidance allows pursuers to move as collaboratively without collisions with each other just in seconds time due to transformed convexified optimization calculation problem.

Problem 3

For all $j \in J$ and for $t \in [0, t_f]$

$$\nu_j \geq 0$$

$$H_j \succeq 0$$

$$\Delta \boldsymbol{r}^{k,j}(t) \triangleq (\boldsymbol{r}^{k-1}(t) - \boldsymbol{p}_j)$$

$$\delta \boldsymbol{r}^k(t) \triangleq \boldsymbol{r}^k(t) - \boldsymbol{r}^{k-1}(t)$$

$$\xi^{k,j}(t) \triangleq \|H_j \Delta \boldsymbol{r}^{k,j}(t)\|_2$$

$$\zeta^{k,j}(t) \triangleq \frac{t^2}{\|H_j \Delta \boldsymbol{r}^{k,j}(t)\|_2}$$

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APPENDICES

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