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STOCHASTIC COMPUTATIONAL METHOD FOR EVALUATION OF BEHAVIOR OF RANDOM ELASTIC MEDIA

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ABSTRACT

In order to use an effective meso-scale model as a basis for deterministic large-scale structural analysis and stochastic simulation, it is important to evaluate the sensitivity of these meso-scale models. Particularly, the way plastic behavior is described at the meso-scale level must be considered. In the study, a single analytical criterion model was developed and tested against various elastoplastic meso-scale models for accuracy of the criterion. Present research will be an extension of the single content model. The problem is to examine the effect of meso-scale models to capture the interaction between flow fields in a multi-material material. The micro-structures are investigated taking into account the plasticity of the contents and the bonding of these plastic regions under increasing load. In this work, the main purpose is to examine flow patterns. The moving window generalized cell method is a technique used to perform meso-scale homogenization.

ÖZET

Meso-ölçek homojenizasyon, kompozit mikroyapı davranışında bölgesel değişimin bazı seviyelerinin hesaplanması için etkili bir yoldur. Bu çalışmada, hareketli pencere genelleştirilmiş hücre yöntemi(GHY) (moving-window generalized method of cell-GMC) kullanılarak yerel iki-fazlı mikroyapının homojenizasyon teknikler araştırılacaktır. Elastik ve plastik malzeme davranışları anizotropik gerilme-şekil değiştirme eğrilerini oluştan GHY kullanarak incelenecektir. Optimizasyon prosedürü genelleştirilmiş hücre yöntemi ile üretilen veriler için en uygun olan Hill akma kriteri parametrelerini tanımlamak üzere kullanılacaktır.. GHY sonuçlarına dayanan iki tam plastik model geliştirilecek: Bunlara literatürde alt hücre başlangıç akma modeli (the subcell initial yield model) ve matris ortalama akma modeli (the matrix average yield model) adı verilir.

INTRODUCTION

For decades many researchers have focused on the anisotropic constitutive models of metallic sheets which are all rolled products, whereas rare anisotropic models are adopted in hot rolling simulations. Conventional plasticity models assume that shapes of yield surfaces are remain fixed throughout plastic deformation. The foundation of most anisotropic constitutive models has been based on the associated flow rule (AFR Stochastic mechanical behavior of random media is a relevant research topic in a wide variety of applied mechanics fields, such as composite materials, geotechnical engineering and biomechanics. A common microstructure, which can be characterized only statistically. Engineering analyses typically focus on global response of random media; however, increasing attention is given to the local fluctuations of critical variables within a random medium, including local stress and strain fields, which are more readily associated with critical failure behavior. With respect to the macroscopic response of heterogeneous media in a deterministic sense, a large body of literature exists on approximations of effective properties based on periodic homogenization theory [1,2], particularly in the field of composite materials [3,4]. Parallel research efforts that attempt to account for spatial randomness of the material microstructure are based on specific models, such as inclusion based estimates, the self-consistent method, the Mori-Tanaka method, Hashin-Shtrikman bounds, and Beran-Milton bounds, etc. These statistics-based bounding techniques take the lower order statistics into account through variational principles, while they neglect the detailed probabilistic information that may have a significant impact on local behavior of random media. By connecting deterministic homogenization with stochastic analysis, stochastic homogenization problems emerge, where the analogue of deterministic periodicity becomes statistical homogeneity and ergodicity [5]. For stochastic homogenization of random media, few computational methods [6,7] have been developed beyond the conventional niche of the Monte Carlo method. There are even fewer schemes available for stochastic evaluation of local behavior. Effective computational approaches are therefore desirable to enable derivation of the statistics describing the local response of random media (e.g. stress and strain), based on the given statistics describing the random constitutive properties. A major advantage to this model is that it can be applied to evaluate statistics describing the random fluctuations in local stress and strain fields.

PROBLEM STATEMENT

Let $\omega \in \Omega$ denote a realization of a random medium, sampled from probability space Ω . This random medium occupies a region D in R^3 with the random stiffness tensor C(x,ω) described by a homogeneous stochastic tensor field. The coordinates y in the fine scale of microstructure may be defined as $\frac{x}{\varepsilon}$, where ε is a scale parameter representing the ratio between the real length of a unit vector in the microscopic coordinates *y* and macroscopic coordinates *x*. If we select a large enough unit cell Y, then we can assume that the boundaries of the unit cell behave εY -periodically. The constitutive properties can then be described by $\mathbf{C}(y,\omega) \equiv \mathbf{C}\left(\frac{x}{\varepsilon},\omega\right) \equiv \mathbf{C}^{\varepsilon}\left(x,\omega\right)$. According to the theory of stochastic processes, an appropriate length scale describing microscopic random fields is the correlation length *l*, defined as

$$l = \left[\int_0^\infty \rho(\tau)d\tau\right]^{1/d} \tag{1}$$

where q(s) is the correlation function describing the random tensor field. $\mathbf{C}(y,\omega) \equiv \mathbf{C}^{\varepsilon}\left(x,\omega\right)$ and *d* is the number of spatial dimensions of the random field.

Without loss of generality, we study elasticity problems with the following governing stochastic elliptic equations, constitutive law and boundary conditions:

$$\partial_j \sigma_{ij}^e\left(x,\omega\right) + f_i(x) = 0 \quad in\ D \tag{2}$$

$$\sigma_{ij}^{\varepsilon}(x,\omega) = C_{ijkl}(\mathbf{y},\omega) e_{kl}^{\varepsilon}(x,\omega) \quad in\ D \tag{3}$$

$$u_i^{\varepsilon}(.,\omega) = \overline{u}_i \quad on\ \partial_1 D \tag{4}$$

$$\sigma_{ij}^{\varepsilon}(\cdot,\omega) n_j = \overline{t}_i \quad on\ \partial_2 D \tag{5}$$

where $u_i^{\varepsilon}, \sigma_{ij}^{\varepsilon}, e_{kl}^{\varepsilon}$ and f_i are the displacement vector, the stress tensor, the strain tensor and the body force vector, respectively. In (4) and (5), $\partial_1 D$ and $\partial_2 D$ denote the boundaries of the domain *D* corresponding to displacement and stress boundary conditions, respectively. Note that fi, u_i and $\overline{t_i}$ are macroscopic quantities, independent of ε. The fourth order random tensor field C_{ijkl} $(\mathbf{y},\omega)$ satisfies symmetry and positive-definiteness. The strain–displacement relation describing $e_{kl}(y,\omega) \equiv e_{kl}\left(\frac{x}{\varepsilon},\omega\right) \equiv e_{kl}^{\varepsilon}(x,\omega)$ is

$$e_{kl}^{\varepsilon}(x,\omega) = \tfrac{1}{2}\left(\partial_l^x u_k^{\varepsilon}(x,\omega) + \partial_k^x u_l^{\varepsilon}(x,\omega)\right) + \tfrac{1}{2}\tfrac{1}{\varepsilon}\left(\partial_l^x u_k^{\varepsilon}(x,\omega) + \partial_k^x u_l^{\varepsilon}(x,\omega)\right) \tag{6}$$

where $\partial_l^x = \partial / \partial x_l, \partial_l^y = \partial / \partial y_l$. Applying the following asymptotic expansions of u_i^{ε} and $\sigma_{ij}^{\varepsilon}$ about scale parameter e, series expressions for displacement and stress are found to be

$$u_i^{\varepsilon}(x,\omega) = u_i^{(0)}(x,\omega) + \varepsilon u_i^{(1)}(x,y,\omega) + \varepsilon^2 u_i^{(2)}(x,y,\omega) + \cdots \tag{7}$$

$$\sigma_{ij}^{\varepsilon}(x,\omega) = \varepsilon^{-1}\sigma_{ij}^{(-1)}(x,y,\omega) + \sigma_{ij}^{(0)}(x,y,\omega) + \varepsilon\sigma_{ij}^{(1)}(x,y,\omega) + \cdots \tag{8}$$

Substituting (8) into (2), (5)–(7) into (3) and (4), and equating powers of e [1,2], we obtain the stochastic PDE's.

$$\partial_j^y\left[C_{ijmn}(y,\omega)e_{mn}^y(y,\omega)\right] = -\partial_j^y\left[C_{ijkl}(y,\omega)e_{kl}^x(x,\omega)\right] \tag{9}$$

To solve local problem (9) in a unit cell with practical homogenization techniques, unit global strain and periodic boundary conditions are prescribed in [8]. Note the local strain $e_{mn}^y(y,\omega)$ depends on the global strain $e_{kl}^y(x,\omega)$ that is defined by

$$e_{kl}^x(x,\omega) = \tfrac{1}{2}\left[\partial_l^x u_k^{(0)}(x,\omega) + \partial_k^x u_l^{(0)}(x,\omega)\right] \tag{10}$$

Based on the relationship between local strain and global strain defined by (9) it is straightforward to derive an expression for the local stress

$$\sigma_{ij}^y(y,\omega) = C_{ijkl}(y,\omega)e_{kl}^x(x,\omega) + C_{ijmn}(y,\omega)e_{mn}^y(y,\omega) \tag{11}$$

NUMERICAL ANALYSIS

Studying the response of composites under simple shear loading is necessary to build up the composite yield function. Two previously introduced unit cell containing rectangular and circular fibers are studied. The study excludes the out-of-plane shear loadings. Fig.1 shows both the conventional and higher order boundary conditions of simple shear imposed on the unit cell with rectangular fibers. Plastic strain is set to be zero at the fiber matrix interfaces due to plastic flow suppression.

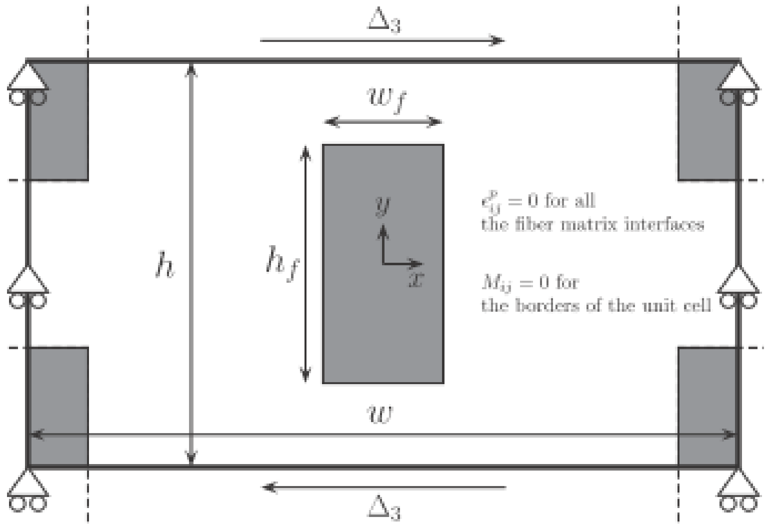

Figure.1 Boundary conditions and geometry of the unit cell of composite with rect-angular fibers under simple shear loading

Fig.2 shows the response of the unit cell with rectangular fibers under simple shear loading until $E_{12}=3.25\gamma_y$, where $\gamma_y=\tau_y/G_m$ for different value of the material length scale while V_f = 0.2. The macroscopic elastic shear modulus, $C_{12}=1.24G_m$, is higher than the matrix shear modulus and it seems to be unaffected by the material length scale.

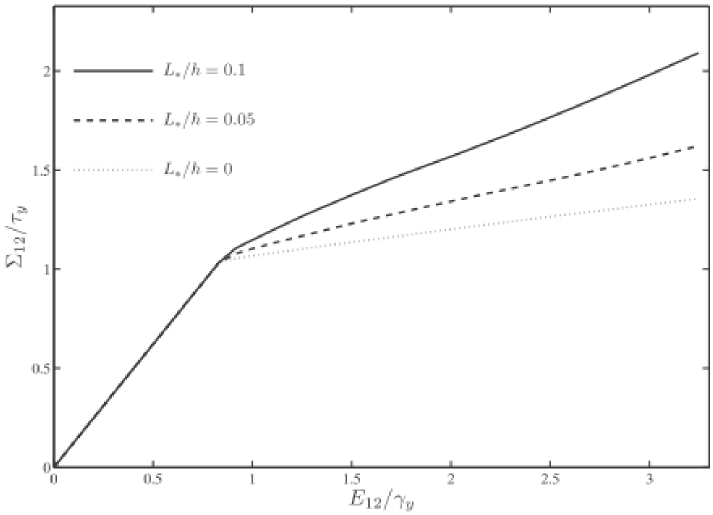

Figure.2 Stress strain response of the unit cell with rectangular fiber under simple shear loading

RESULTS

In this paper a stochastic computational model is constructed to assess global effective properties and local probabilistic behavior of random media, based on stochastic decomposition of random fields and a fast iterative numerical scheme for heterogeneous materials. Analogous with representative volume element (RVE) used in deterministic homogenization problems, a stochastic representative volume element (SRVE) is introduced as a cornerstone of the computational model. With an ensemble average scheme devised for the stochastic homogenization formulation, a closely approximated upper bound of effective properties can be derived based on solution of auxiliary stochastic equations. Meanwhile, local fluctuations can also be obtained by the relationship between global and local variables. While SRVE size is heuristically chosen in this study, we note that a rigorous error study defining the appropriate SRVE size relative to correlation length is of great interest for future research on stochastic homogenization problems.

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