





**ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL OF SCIENCE**  
**ENGINEERING AND TECHNOLOGY**

**AN INTERNAL MODEL CONTROL BASED TUNING METHOD FOR SINGLE  
INPUT FUZZY PID CONTROLLER**

**M.Sc THESIS**

**Arda VAR**

**Control and Automation Engineering Department**

**Control and Automation Engineering Master Program**

**DECEMBER 2016**



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**İSTANBUL TEKNİK ÜNİVERSİTESİ ★ FEN BİLİMLERİ ENSTİTÜSÜ**

**TEK GİRİŞLİ BULANIK PID KONTROLÖRLER İÇİN İÇ MODEL KONTROL  
TABANLI AYARLAMA YÖNTEMİ**

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## ABBREVIATIONS

<b>1-D</b>	:	One Dimensional
<b>2-D</b>	:	Two Dimensional
<b>ACA</b>	:	Agressive Control Action
<b>CAs</b>	:	Control Actions
<b>CFPID</b>	:	Conventional Fuzzy Proportional-Integral-Derivative
<b>CFLC</b>	:	Conventional Fuzzy Logic Controller
<b>CPID</b>	:	Conventional Proportional-Integral-Derivative
<b>FPID</b>	:	Fuzzy Proportional-Integral-Derivative
<b>FLC</b>	:	Fuzzy Logic Controller
<b>FOPDT</b>	:	First-Order Plus Delay Time
<b>IAE</b>	:	Integral Absolute Error
<b>IMC</b>	:	Internal Model Control
<b>ITAE</b>	:	Integral Time Absolute Error
<b>L-FLC</b>	:	Linear Fuzzy Logic Controller
<b>LCA</b>	:	Linear Control Action
<b>MFs</b>	:	Membership Functions
<b>OS</b>	:	Overshoot Value
<b>PD</b>	:	Proportional-Integral
<b>PI</b>	:	Proportional-Derivative
<b>SCA</b>	:	Smooth Control Action
<b>SD</b>	:	Signed Distance
<b>SDM</b>	:	Signed Distance Method
<b>SFLC</b>	:	Single Input Fuzzy Logic Control
<b>SFPID</b>	:	Single Input Fuzzy Proportional-Integral-Derivative
<b>SFs</b>	:	Scaling Factors
<b>TV</b>	:	Total Variation of The Control Signal





## SYMBOLS

$\mu_i$	:	Membership function of i
$Rule_i$	:	i. rule of rule table.
$e$	:	Error variable used as input
$\dot{e}$	:	Change rate of error used as input
$K_e$	:	Input scaling factor used for error
$K_d$	:	Input scaling factor used for change rate of error
$K_f$	:	Gain of single input fuzzy logic controller
$K_c$	:	Gain of proportional-derivative controller
$\alpha$	:	Output scaling factor
$\beta$	:	Output scaling factor used at integral part of the controller
$E_i$	:	i. membership function of input error variable
$R_i$	:	i. membership function of input change rate of error
$U_{ij}$	:	Linguistic value of the output of the fuzzy logic controller
$T_i$	:	Integral time constant
$T_d$	:	Derivative time constant
$G_x(s)$	:	Transfer function of system x
$u_{flc}$	:	Output of the fuzzy controller block
$P$	:	Plant that controller applied
$\tilde{P}$	:	Nominal model of plant that controller applied
$\tilde{P}_-(s)$	:	Minimum phase part of the plant model
$\tilde{P}_+(s)$	:	Part of model that contain all time delays and right-half-plane zeroes
$\delta(s)$	:	Nonlinear term without an explicit analytical expression
$\gamma$	:	Nonlinear time-varying parameter
$s_l$	:	Main diagonal line
$\lambda$	:	Slope magnitude of the main diagonal line
$d_s$	:	Signed distance
$LSD_i$	:	i. antecedent membership function of single input fuzzy controller
$LU_i$	:	i. consequent membership function of single input fuzzy controller
$f_i$	:	i. firing strength
$u^N$	:	Nonlinear compensation term
$u_{ij}$	:	Consequent MF for i. and j. antecedent membership functions.
$r$	:	Desired set point that is applied to the controller
$d$	:	Disturbance value that applied to the controller
$ u $	:	Magnitude of the control input
$f(s)$	:	Low pass filter with steady-state gain
$t_c$	:	Closed loop time constant

$T_s$  : Settling time

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# **AN INTERNAL MODEL CONTROL BASED DESIGN METHOD FOR SINGLE INPUT FUZZY PID CONTROLLERS**

## **SUMMARY**

The first goal of control engineering is to analyze and process the knowledge of how a process can be controlled. However, with this structured control mechanism, reliable and accurate results can be obtained on processes that require high performance.

Fuzzy control is a practical alternative for a variety of challenging control applications since it provides a convenient method for constructing nonlinear controllers via the use of heuristic information. Regardless of where the heuristic control knowledge comes from, fuzzy control provides a user-friendly formalism for representing and implementing the ideas we have about how to achieve high-performance control.

Conventional proportional-integral-derivative (CPID) controllers may not perform well for the complex process, such as the high-order and time delay systems. Under this complex environment, it is well-known that the fuzzy controller can have a better performance due to its inherent robustness. Thus, over the past three decades, fuzzy controllers, especially, fuzzy proportional-integral-derivative (FPID) controllers have been widely used for industrial processes. Because their heuristic natures associated with simplicity and effectiveness for both linear and nonlinear systems.

Even though industry shows growing interest in the applications of FPID, the tuning mechanism of scaling factors (SFs) and the stability analysis are still difficult tasks due to the complexity of the nonlinear control surface that is generated by FPID controllers.

In literature, several studies have been presented where the design of the two input FPID (CFPID) controllers have been investigated. Because of the number of the design parameters of CFPID is relatively big, the systematic design is still a challenging problem. Besides of that the majority of the research focus on the two-input structures, it has been shown in various works that single input FPID (SFPID) provide greater flexibility and better functional properties.

In the SFPID controller structure a new called “signed distance” variable is used as input signal. In this way, the number of rules and membership functions (MFs) is less than CFPID while both of the controllers have almost same control performance as showed in this thesis. This feature brings design simplicity to SFPID controller.

There are several design methods in literature for tuning design parameters of controller. One of the most common used is internal model control based tuning method. Having a single parameter dependent to the close-loop time constant is effective for commonly used.

In this thesis, firstly, a brief information will be given about CPID controller and internal model control (IMC) based tuning method. Then, the CFPID structure will be analyzed in detail. By extraction the rules and MFs, the relationship between input-output of controller will be shown.

Present the an analytical tuning method based on IMC to tune SFPID controllers will proposed in this thesis. First, reduction process from CFPID structure to SFPID structure will be explained. By examining the relationship between input-output and MFs, in order to design the desired control structure, the design rule template will be presented about how to set the membership function parameters.

As a result of all the work done, it was said that, SFPID controller can be expressed as a linear proportional-integral-derivative (PID) controller plus a nonlinear compensation item. So, it is acceptable the IMC based tuning method is applicable to SFPID controller and the formulations for tuning process will be presented. Finally, by simulation studies it will be shown that the SFPID is capable to improve the transient state performance while providing an identical disturbance rejection performance of the CFPID structure.



## TEK GİRİŞLİ BULANIK PID KONTROLÖRLER İÇİN İÇ MODEL KONTROL TABANLI TASARIM YÖNTEMİ

### ÖZET

Kontrol mühendisliğinin birinci amacı, bir sürecin nasıl kontrol edilebileceği bilgisini analiz edip işleyebilmektir. Ancak bu şekilde yapılandırılan kontrol mekanizmalarıyla yüksek performans gerektiren süreçler üzerinde güvenilir ve doğru sonuçlar elde edilebilir.

Bulanık kontrolörler, tasarım aşamasında sezgisel bilgilerin kullanılmasıyla linear olmayan sistemlerin kontrolü için kullanışlı bir yöntem sunmaktadırlar. Bundan dolayı zorlu kontrol uygulamaları için pratik bir alternatif olmuşlardır. Bu sezgisel bilgiler kontrol edilmek istenen sistemi uygun bir şekilde ifade etmesinin dışında nasıl elde edildiğine bakılmaksızın, yüksek-performanslı kontrolün nasıl sağlanacağı konusunda sahip olduğumuz fikirleri temsil etmektedir.

Gelişen teknolojiyle birlikte endüstride kontrol edilme gerekliliğini doğuran sistemler giderek daha karmaşık sistemlere dönüşmüştür. Bu tarz yüksek dereceli, zaman gecikmeli ya da lineer olmayan sistemlerde CPID kontrollörler istenilen başarıyı elde edememektedir. Bu tarz karmaşık sistemlerde bulanık kontrolörler ile, özellikle FPID kontrollörleri ile, daha sağlam bir yapıya sahip olmalarından dolayı daha iyi sonuçlar elde edildiği bilinmektedir. Bu yüzden FPID kontrollörleri yüksek dereceli lineer ya da lineer olmayan endüstriyel süreçlerde yaygın olarak kullanılmaktadırlar.

FPID kontrollörlerinin tek girişli, iki girişli ve üç girişli olmak üzere çeşitleri bulunmaktadır. Hata ve hatadaki değişimi giriş olarak kullanan iki girişli kontrollör yapısı, araştırmalarda ve uygulamalarda en çok kullanılan FPID kontrollör yapısıdır. Üç girişli FPID kontrollörleri çok sayıda kural ve parametre ayarlaması gerektirdiklerinden çok tercih edilmemektedir. SFPID kontrollörlerin türevsel evrede daha fazla bilgi kaçırılabilmesi düşünülse de birçok çalışmada diğer FPID yapılarına göre daha kullanışlı olduğu ifade edilmiştir.

FPID kontrolör tasarım parametreleri iki başlık altında toplanmıştır. Bunlar bulanık arayüzün giriş-çıkış değişkenlerini, dilsel setleri, üyelik fonksiyonlarını, bulanıklaştırma ve durulaştırma mekanizmalarını kapsayan yapısal parametreler ve giriş-çıkış ölçekleme katsayılarını kapsayan ayar parametreleridir.

Günümüzde bile kontrolör tasarım parametrelerinin ayarlanması zor ve mühendislik bilgisi gerektiren işlerdir. Literatürde, CFPID kontrolör için sistematik tasarımının, sahip oldukları parametre sayısının nispeten büyük olmasından dolayı, halen büyük bir problem olduğunu gösteren çalışmalar bulunmaktadır. Aynı şekilde, her geçen gün FPID kontrolör yapılarının endüstri uygulamalarında kullanımı yaygınlaşmakta olsa da ölçekleme katsayılarının ayarlanması ve yapısal parametrelerinin elde edilmesi problem olmayı sürdürmektedir. Sahip oldukları kontrol yüzeylerinin karmaşık yapısından bu duruma neden olduğu söylenebilir.

Yapılan çalışmaların çoğunlukla CFPID kontrolör üzerinde yoğunlaşmasına rağmen, SFPID kontrolörlerinin çok daha esneklik sunduğu ve daha işlevsellik getirdiği savunulmaktadır.

SFPID yapısı, iki boyutlu çarpık-simetrik kural tablosu kullanan geleneksel iki girişli FPID kontrolör yapısı göz önünde bulundurularak türetilmiştir. CFPID kontrolörü kural tablosunun bu özelliği, kural tablosunun ana köşegenine bir mesafe tanımlaması yapılabilmesine imkân vermiştir.

Kontrol teorisinde, elde edilen sistem modelinin derecesinin artmasıyla birlikte, modelin sistemi daha az hatayla yansıttığı bilinmektedir. Ancak yüksek dereceli sistemler için kontrolör tasarımının da çok daha zor olduğu unutulmamalıdır. Aynı şekilde kontrolör bünyesinde ne kadar fazla tasarım parametresi bulunursa sistemi daha iyi analiz edeceği, daha hassas karar vereceği ve daha iyi performans elde edileceğinin düşünülmesi yanlış olmayacaktır. FPID kontrolörlerinde kural ve üyelik fonksiyon sayılarının artması giriş sinyalinin daha iyi değerlendirilmesine ve data kayıplarının en aza indirgenmesine olanak sunmaktadır. Ancak aynı zamanda fazla sayıda parametre ve kural sayısı, kontrolör yapısını karmaşıktırmakta, tasarımı zorlaştırmakta ve yapının daha yavaş çalışmasına neden olabilmektedir. Bu durumlarda yapılan ufak hatalar sistemin performansını büyük ölçüde düşürebilmekte, hatta sistemi kararsızlığa götürebilmektedir. Bu yüzden tasarım sırasında uygun kontrolör tipinin seçilmesi, üyelik fonksiyonlarının ve dilsel terimlerin doğru tanımlanması büyük önem taşımaktadır. Görülmektedir ki fazla sayıda kural oluşturmak en mantıklı çözüm değildir ve tüm bunlar düşünüldüğünde kontrol mühendisinin en optimize seçimleri yapması gerekmektedir.

Bu çalışmada da kullanılan SFPID kontrolör yapısında, hata ve hatadaki değişim yerine “signed distance” (SD) diye adlandırılan yeni bir giriş değişkeni kullanılmaktadır. Buna bağlı olarak kullanılacak kural ve tasarım parametreleri sayısı CFPID kontrolörlerine kıyasla daha azdır. Yapılan araştırmalarda söz konusu bu durum SFPID kontrollörün tasarım kolaylığı olarak değerlendirilirken, bu tez çalışmasında da sunulmuş olduğu gibi performans olarak neredeyse CFPID kontrollörü ile eşdeğer olduğu ifade edilmiştir.

Bulanık PID kontrolör parametre ayarlaması için birçok yöntem türetilmiştir. Bu yöntemlerden en yaygın olarak kullanılanlarından biri de IMC tabanlı ayarlama yöntemidir. IMC tabanlı ayarlama yönteminin kapalı çevrim zaman sabitine bağlı tek bir ayarlama parametresine sahip olması, yaygın kullanılmasında etkili olmuştur. IMC tabanlı ayarlama yönteminde, kontrol edilecek sistem birinci dereceden ölü zamanlı bir model olarak tanımlanmakta ve ayarlama parametreleri elde edilmektedir.

Bu çalışmada öncelikle FPID kontrolörlerinin temel taşını oluşturan CPID kontrolörlerine kısaca değinilecektir. Tez kapsamında kullanılacak olan IMC tabanlı ayarlama yönteminin çıkarımı ve nasıl uygulandığı gösterilecektir. Ardından en yaygın kullanılan FPID kontrolör tipi olan CFPID kontrolör yapıları ayrıntılı bir şekilde incelenecek, kural tablosunun ve üyelik fonksiyonlarının çıkarımı elde edilerek, kontrolör giriş-çıkış arasında nasıl bir ilişki olduğu gösterilecektir. IMC tabanlı ayarlama yöntemi kullanılarak tasarım parametrelerinin nasıl elde edilebileceği anlatılacaktır.

Bu çalışmada SFPID kontrolörleri için IMC tabanlı ayarlama yönetimin kullanılabilirliğini göstermek ve bu bağlamda gerekli analitik çıkarımları sunmak amaçlanmıştır. Öncesinde CFPID yapısından SFPID yapısına indirgeme işlemi ayrıntılı bir şekilde açıklanacaktır. SFPID kontrolör giriş çıkış ilişkisi formülize

edilerek diğ er kontrol r yapıları ile karřılařtırılacaktır. T m bulanık kontrol rlerde olduėu gibi SFPID kontrol rlerinde de  yelik fonksiyonları ve kontrol r  ıkıřı arasında doėrudan bir iliřki mevcuttur. Bu iliřki incelenecek ve istenilen kontrol yapısının tasarlanabilmesi i in  yelik fonksiyon parametrelerinin nasıl ayarlanması gerektiėi hakkında tasarım kuralları sunulacaktır.

Yapılan  ıkarımlar sonucunda SFPID kontrol r giriř- ıkıř iliřkisinin CFPID yapısına benzer olduėu, hatta CPID kontrol r ve lineer olmayan bir kompanzasyon teriminin kombinasyonu olarak ifade edilebileceėine dikkat  ekilecektir. Bu bilgiler iřıėında CFPID de yaygın olarak kullanılan IMC tabanlı ayarlama y nteminin SFPID yapısında da kullanılabileceėi kabul edilecek ve parametrelerin elde edilmesinde kullanılacak form ller sunulacaktır. Son olarak IMC tabanlı ayarlama y ntemi i in tezde sunulan  ıkarımlar kullanılarak yapılan benzetim  alıřmaları ile SFPID kontrol r n, CFPID kontrol r bozucu s n mleme performansını korurken, ge ici hal performansını arttırdıėı g r lecektir.



## 1. INTRODUCTION

### 1.1 Fuzzy Logic Controller

Although high number of alternative are suggested many industrial control system use PID controller. It is known that the utilization rate of PID controller in industry is about %90. Some reasons for this can be expressed like below:

- PID controller is robust.
- PID controller design is easy.
- PID and parameters of system response are associated. PID controller has just three parameters. So, the designers have extensive knowledge about on the effects of these parameters and advantages against each other on system response.

Although the CPID controllers are commonly used, this type controllers are not suitable against all control problems. For example, complex, time varying and time delay systems. If a system is complex as cannot be expressed with analytic models, then the controlling the system with classical approaches is impossible. In this case the fuzzy control can be suggested as a solution. The control with fuzzy logic (FL) can transform successfully the knowledge of experts and the control methods to numerical algorithms [1-4].

Although CPID controller has easy implementation and produces linear control signal, in general the FPID controller provides nonlinear parameters for nonlinear control system. The FPID controllers are easily implemented to many linear and nonlinear system by the agency of transformation of the fuzzy rules to nonlinear parameters. Although demand of FPID controller applications is increased, number of standard and systematic adjusting methods for FPID controller is less than the classic PID controller. So, it is suggested that parameters of the FPID controller should be adapted in two part for a better control performance. Thus, SFs of the FPID controller are adapted at the lower part adjustment. At the higher part adjustment database

parameters (the membership parameters, fuzzy rules and fuzzy rules weights) of the controller are adapted in order to using nonlinear features of the fuzzy PID much better [1-13].

### **1.1.1 Philosophy**

The ability of a human being to find solutions for particular problematic situations is called “human intelligence”. Humans have the ability to process a large amount of information and make effective decisions, although neither input information nor consequent actions are precisely defined. Human thinking and decision making mechanisms represent a perfect model, which scientists and engineers attempt to imitate and transform into practical solutions of diverse technical and nontechnical problems [14].

Fuzzy control is a methodology of intelligent control that mimics human thinking and reacting by using a multivalent fuzzy logic and elements of artificial intelligence. The word “fuzzy” is used here to describe terms that are not-well known or not clear enough [15].

### **1.1.2 History**

The fuzzy set theory was suggested by Zadeh [16]. After that the fuzzy control theory was first described in study of Bellman and Zadeh [17]. In the study, the linguistic variables were suggested to describe the human knowledge as “if-then” rules. In the last 30 years, the fuzzy control applications were developed to include various industrial control applications and important research. In 1974 Mamdani used the inference mechanism, which was suggested by Bellman and Zadeh, to control a dynamic system [18]. After that only one year later, Mamdani and Assilian improved first FL controller (FLC) and used it to control a steam engine [19]. But, due to that algorithm was highly depended on knowledge of experts, Mac Vicar-Whelan suggested rules in order to remove that disadvantage [20].

In fuzzy controller min-max inference mechanism, which was suggested by Mamdani (1974) and product-sum inference mechanism, which suggested by Mizumoto (1996) are used mostly. The defuzzification is a time-consuming process. Therefore, Takagi and Sugeno suggested to use real numbers instead of fuzzy sets in final section of fuzzy

control rules [21]. The using this proposal with the product-sum inference mechanism gives very good results and produces an easy algorithm.

Qiao and Mizumoto suggested a fuzzy controller structure called as without self-regulation which similar as conventional PID controller [7]. According to this controller, the control signal is produced by inference mechanism and data base. At the beginning of the design all parameters of the controller are adjusted and the system response of controller is obtained.

### **1.1.3 Importance of Fuzzy Logic Controller**

While attempting to describe a system, simple or not, one must face the fact that all possible event in the system cannot be indentified. Incomplete knowledge of event and unpredictable frequency of their occurrences impose the usage of approximate modeling of systems. In control system theory there are excellent tools for approximate modeling of systems. For the systems, which can be well-described with a linear second-order model there are number of procedures for design of PI and PID controllers, while for the system modeled with high-order linear models one can use [14].

The problem arises when the model of a system is unknown or when it is known but so complex that the design of a controller by using classic analytical meyhods would be totally impractical. There are also situations when the model of a system is highly nonlinear and where variations of parameters and rates of parameter changes may be extremely high. Some of these situations can be solved by using adaptive control methods, but their basic mathematical apparatus is rather complex and very often ends in a large number of computing iterations.

A special class of control problems is control of highly nonlinear processes that are exposed to strong influence of external disturbances. Such systems are controlled by operators using their years-long experience and knowledge about static and dynamic characteristic of the system. The operator's experience is connected to monitoring of relevant process variables and depending on their states and deviations from reference values, operators decide where, how and how much the need to act on the process to achieve a given control goal. In other words, they execute their "program" or "control algorithm" according to their experience and by applying the following typical pattern of decision making

IF such states of process variables are THEN such control action are needed.

The main problem of control designer is confronted with is how to find a formal way to convert the knowledge and experience of a system operator into a well-designed control algorithm. By using fuzzy logic, linguistic expressions in antecedent and consequent parts of IF-THEN rules describing the operator's actions can be eggicaciously converted into a fully-structured control algorithm.

Due to the fact that a fuzzy algorithm has the characteristics of a universal approximator, a designer is able to model an unknown process with a set of IF-THEN fuzzy rules [22-23]. Application of fuzzy logic is not limited only to systems difficult for modeling. By application of fuzzy logic on systems with known but complex mathematical models, the time needed for controller design and for practical application can be shortened.

## **1.2 Purpose of Thesis**

In control theory, one of the topics as important as modeling the system is tuning the design parameters correctly. There is a lot of parameter tuning method in literature. Although, there are many accepted researches for that purpose, especially in the high-number parameter requirement systems , how to apply the tuning method is complicated.

In FL controllers, definition of antecedent and consequent MFs and proper selection of linguistic variables require experience and engineering knowledge. Incorrect parameter settings or assignment of linguistic variables cause low performance or unstable controller structures. Also, increasing the number of rules could be cause delays in the controller and wrong decision making.

In controller design, it is a desired feature that low number of tuning parameters and rules to reduce design complexity and time required. But, when considered the data losses and accurate analysis of reference signal, using the small size rule table or low number of parameters do not mean more efficient controller [14].

It is known that, SFPID controllers have better or same performance than CFPID controllers while require less design parameter and rule. Purpose of these thesis is applying the commanly used IMC based tuning method to the SFPID controller and presenting the required formulation for this process to literature. At the same time,



one-input controller structure will be examined in detail and investigated that how flexible structure can be used more efficiently.



## 2. FUZZY LOGIC SYSTEMS

### 2.1 Introduction

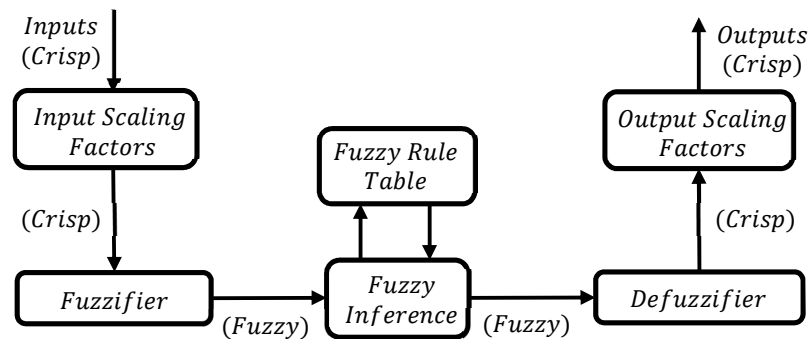
FL is a mathematical approximation which based on fuzzy set theory instead of classical set theory. Namely, it is based on degrees of truth rather than usual true or false. The idea of FL was first purposed by Lotfi Zadeh in 1965 [16].

Natural language is not easily translated into the terms of 0 and 1. In the Zadeh's study, it was said that the classical set theory is inadequate and the use of linguistic expressions as needed. So that, the human logic can be modeled better.

Fundamentals of FL is described as follows by Lotfi Zadeh.

- In the FL, the truth is between  $[0, 1]$  instead of "0" or "1".
- In the FL, the linguistic modifiers and linguistic expressions are used.
- Some rules are used for the fuzzy inference. These rules, which are identified by linguistic expressions, are called fuzzy rules.
- Every logical system can be expressed with FL.
- The FL is very convenient for systems, which have complex mathematical model.

In general a FL system is shown in Figure 2.1.



**Figure 2.1 :** Structure of a fuzzy logic system.

A FL system occurs by input SFs, fuzzifier, fuzzy inference system, fuzzy rule table, defuzzifier and output SFs.

In design of the FL system, firstly, the crisp input values are determined. These input values can be in any range and must be meaningful for FL system. Thus, input values are multiplied by input SFs. So, the input values are synchronized with the interval that is using for inputs of FL system. The fuzzifier converts the crisp input values to fuzzy input values by using the described MFs. After that, these fuzzy input values are translated one fuzzy output value by using fuzzy inference system and fuzzy rule table. The obtained fuzzy output value is converted to crisp value with defuzzifier. Because the fuzzy output value is not a valid variable for apply to process. Finally, the crisp value is multiplied by output SFs for synchronized to interval that is suitable for the process [24].

## 2.2 Sets

### 2.2.1 Elements of sets

A universal set  $X$  is defined in the universe of discourse and it includes all possible elements related with the given problem. If a set  $A$  is defined in the universal set  $X$ , the following relationships can be written [25].

$$A \subseteq X \quad (2.1)$$

In this case, a set  $A$  is included in the universal set  $X$ . If  $A$  is not included in  $X$ , this relationship is represented as follows:

$$A \not\subseteq X \quad (2.2)$$

If an element  $x$  included in the set  $A$ , this element is called as a member of the set and the following notation is used.

$$x \in A \quad (2.3)$$

In general, a set is represented by enumerating its elements. For example, elements  $a_1, a_2, \dots, a_n$  are the elements of set  $A$ , it is represented as follows:

$$A = \{a_1, a_2, \dots, a_n\} \quad (2.4)$$

Another representing method of sets is given by specifying the conditions of elements. For example, if the elements of set  $B$  should satisfy the conditions  $P_1, P_2, \dots, P_n$ , then the set  $B$  is defined by the following:

$$B = \{b | b \text{ sat, sf, ed } p_1, p_2, \dots, p_n\} \quad (2.5)$$

### 2.2.2 Relation between sets

If all elements in set  $A$  are also elements of set  $B$ ,  $A$  is a subset of  $B$ .

$$A \subseteq B \text{ iff (if only if) } x \in A \Rightarrow x \in B \quad (2.6)$$

The symbol  $\Rightarrow$  means implication. If the following relation is satisfied

$$A \subseteq B \quad \text{and} \quad B \subseteq A \quad (2.7)$$

$A$  and  $B$  have the same elements and thus they are the same sets. This relation is denoted by

$$A = B \quad (2.8)$$

### 2.2.3 Membership

Membership can represent whether an element  $x$  is involved in a set  $A$  or not. For a set  $A$ , a membership  $\mu_A$  can be defined as follows:

$$\mu_A(x) = \begin{cases} 1 & \text{if and only if } x \in A \\ 0 & \text{if and only if } x \notin A \end{cases} \quad (2.9)$$

## 2.3 Operation Sets

### 2.3.1 Complement

The relative complement set of set  $A$  to set  $B$  consists of the elements which are in  $B$  but not in  $A$ . The compliment set can be defined by the following formula.

$$B - A = \{x | x \in B, x \notin A\} \quad (2.10)$$

If the set  $B$  is the universal set  $X$ , then this kind of compliment is an absolute complement set  $\bar{A}$ . That is

$$\bar{A} = X - A \quad (2.11)$$

The complement of an empty set is the universal set and vice versa.

$$\bar{\emptyset} = X \quad \text{and} \quad \bar{X} = \emptyset \quad (2.12)$$

### 2.3.2 Union

The union of sets  $A$  and  $B$  is defined by the collection of whole elements of  $A$  and  $B$ .

$$A \cup B = \{x | x \in B, x \in A\} \quad (2.13)$$

The union of certain set  $A$  and universal set  $X$  is reduced to the universal set.

$$A \cup X = X \quad (2.14)$$

The union of certain set  $A$  and empty set  $\emptyset$  is  $A$ .

$$A \cup \emptyset = A \quad (2.15)$$

The union of set  $A$  and its complement set is the universal set.

$$A \cup \bar{A} = X \quad (2.16)$$

### 2.3.3 Intersection

The intersection  $A \cap B$  consists of whose elements are commonly included in both sets  $A$  and  $B$ .

$$A \cap B = \{x | x \in B \text{ and } x \in A\} \quad (2.17)$$

The intersection between set  $A$  and universal set  $X$  is  $A$ .

$$A \cap X = A \quad (2.18)$$

The intersection between set  $A$  and empty set is empty set.

$$A \cap \emptyset = \emptyset \quad (2.19)$$

The intersection between set  $A$  and its complement is all the time empty set.

$$A \cap \bar{A} = \emptyset \quad (2.20)$$

When two sets  $A$  and  $B$  have nothing in common, the relation is called as disjoint. Namely, it is when the intersection of  $A$  and  $B$  is empty set.

$$A \cap B = \emptyset \quad (2.21)$$

## 2.4 Fuzzy Sets

### 2.4.1 Expression for fuzzy set

Membership function  $\mu_A$  in crisp set maps whole members in universe set  $X$  to set  $\{0, 1\}$ .

$$\mu_A(x): X \rightarrow \{0, 1\} \quad (2.22)$$

In fuzzy sets, each elements is mapped to  $[0, 1]$  by membership function.

$$\mu_A(x): X \rightarrow [0, 1] \quad (2.23)$$

In the above equation,  $\mu_A$  represent the belonging degree of  $x$  to fuzzy set  $A$ .

### 2.4.2 Operator in fuzzy sets

There are some operators in the fuzzy expression such as negation, conjunction, disjunction, implication [25]. The operators are defined as follows for  $a, b \in [0, 1]$ .

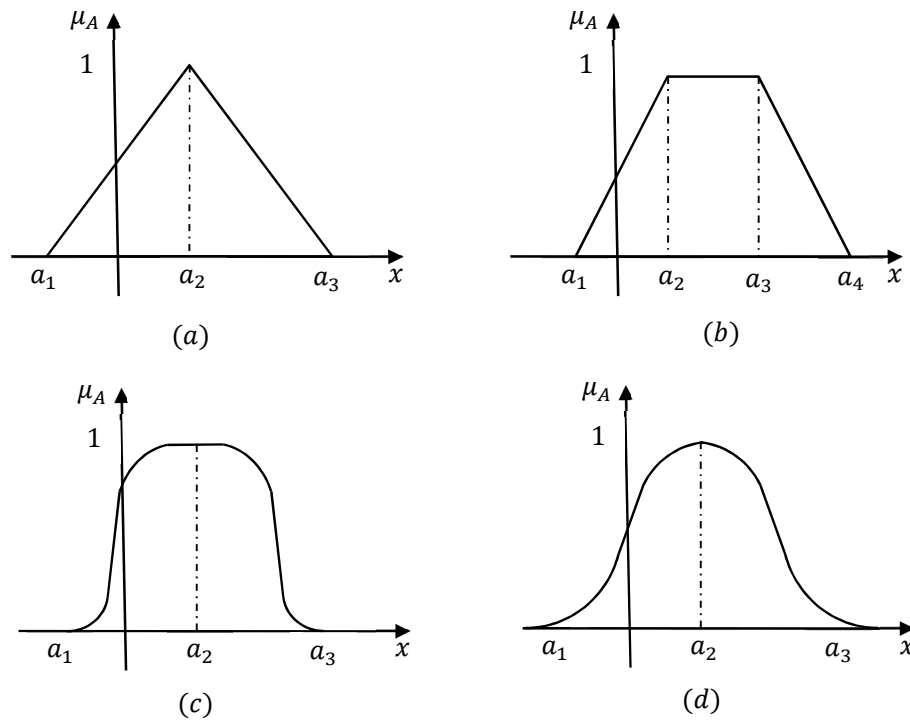
$$\begin{aligned} \text{Negation} \quad & \bar{a} = 1 - a \\ \text{Conjunction} \quad & a \wedge b = \min(a, b) \\ \text{Disjunction} \quad & a \vee b = \max(a, b) \\ \text{Implication} \quad & a \rightarrow b = \min(1, 1 + b - a) \end{aligned} \quad (2.24)$$

## 2.5 Linguistic Variable

### 2.5.1 Membership functions

In crisp set theory, an element belongs to a set or does not. But in FL set theory, the belonging of an element is graded. The crisp sets are defined with characteristic functions while the fuzzy sets are defined with MFs. So, a fuzzy set, defined on  $X$ , is described with membership function  $\mu_A$ .

In literature, according to controlled system there are four types of MFs are used often: Triangular, bell shape, Gaussian and trapezoidal as shown in Figure 2.2 [24].

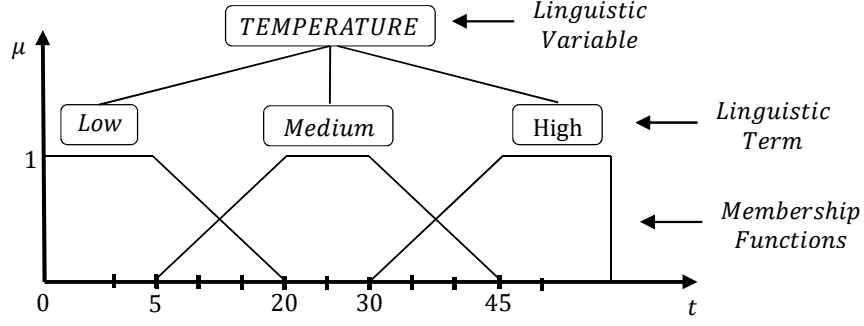


**Figure 2.2 :** Types of membership functions (a) Triangular, (b) Trapezoidal, (c) Bell shape, (d) Gaussian.

### 2.5.2 Definition of linguistic variable

In general, a variable takes number as its value. If the variable takes linguistic terms, it is called “linguistic variable”. The values of these linguistic variables are can be “young”, “hot” and “very young”. For example, a temperature linguistic variable is given in Figure 2.3.





**Figure 2.3 :** “Temperature” linguistic variable and linguistic terms.

In Figure 2.3, the temperature has three linguistic term defined as “low”, “medium” and “high”. Each linguistic term has own membership function. This MFs are shown as below.

$$\mu_{Low} = \begin{cases} 1, & 0 < t < 5 \\ \frac{20-t}{15}, & 5 < t < 20 \end{cases} \quad (2.25)$$

$$\mu_{Medium} = \begin{cases} 1, & 20 < t < 30 \\ \frac{t-5}{15}, & 5 < t < 20 \\ \frac{40-t}{15}, & 30 < t < 45 \end{cases} \quad (2.26)$$

$$\mu_{High} = \begin{cases} 1, & 45 < t \\ \frac{t-30}{15}, & 30 < t < 45 \end{cases} \quad (2.27)$$

### 2.5.3 Linguistic modifiers

Suppose that, set  $A$  is a fuzzy set and  $\mu_A$  is the membership function belongs to set  $A$ . Also  $m$  is a linguistic modifier (more, less) belongs to  $\mu_A$ . Thus,  $mA$  defines the qualified fuzzy set and  $\mu_{mA}$  defines the membership function of the qualified fuzzy set [24]. Some linguistic modifiers, which are used often, and MFs of these linguistic modifiers are given as below.

$$\begin{aligned}
\text{Not: } \mu_{\text{not}A} &= 1 - \mu_A \\
\text{More: } \mu_{\text{more}A} &= (\mu_A)^2 \\
\text{More or less: } \mu_{\text{mol}A} &= (\mu_A)^{1/2}
\end{aligned} \tag{2.28}$$

## 2.6 Fuzzification

Fuzzification is the conversion process of the crisp values of the inputs of fuzzifier to fuzzy values by using the MFs. Fuzzification is performed by fuzzifier.

## 2.7 Fuzzy Rule Structure

Fuzzy rule table occurs by rules which have “if-then” structure. Fuzzy rules relate the input and output of fuzzy system. A rule form of a two input-one output fuzzy system is given below.

$$\text{Rule}_k: \text{If } x \text{ is } A \text{ and } y \text{ is } B, \quad \text{Then } z \text{ is } C \text{ (with } w\text{)}.$$

Where, x and y are the input variables and z is the output variables of the fuzzy system. The w is defines the weight of that fuzzy rule and in general takes values in interval of [0, 1]. In given rule form “A” and “B” define the MFs belong to input variables and “C” defines the membership function belongs to output variable.

## 2.8 Fuzzy Inference

The fuzzy inference system is the most important unit of the fuzzy system and simulates the human decision-making and suggestion. The inference system obtain the firing values of each fuzzy rule in a certain time interval and by using fuzzy rules calculates the outputs.

The most known three fuzzy inference system are given below.

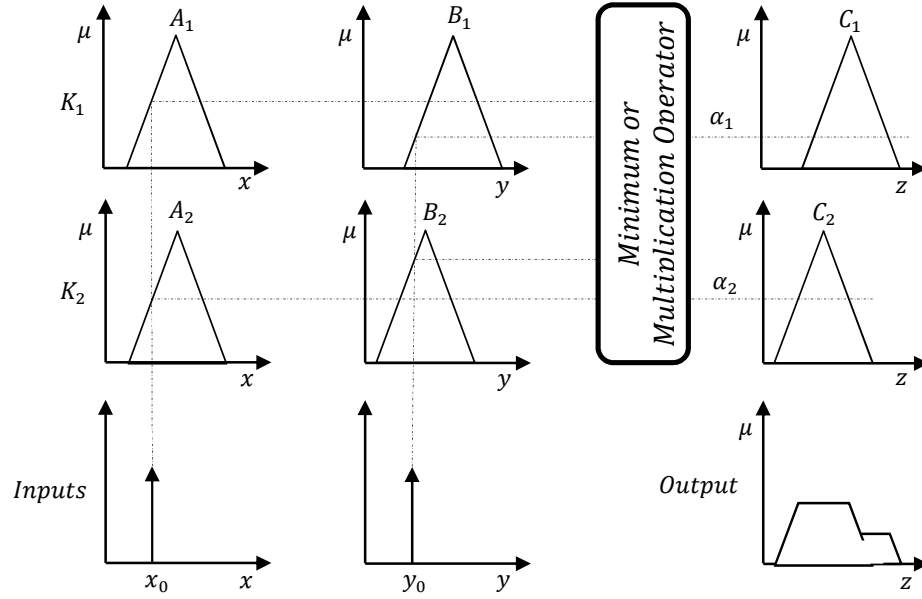
### 2.8.1 Fuzzy inference for Mamdani type fuzzy logic systems

In this type fuzzy inference system, fuzzy sets are used as output of the fuzzy rule table. Thus, rules of the system, which has a two inputs and one output can be given as below.

*Rule<sub>1</sub>: If  $x$  is  $A_1$  and  $y$  is  $B_1$ , Then  $z$  is  $C_1$ .*

*Rule<sub>2</sub>: If  $x$  is  $A_2$  and  $y$  is  $B_2$ , Then  $z$  is  $C_2$ .*

Where  $x$  and  $y$  are the input variables,  $z$  is the output variables,  $A$ ,  $B$  and  $C$  are the fuzzy sets of input and output variables, respectively. In Figure 2.4, the fuzzy inference system for Mamdani type fuzzy structure is given.



**Figure 2.4 :** Fuzzy inference for Mamdani type structure with two rule.

In certain time, the each fuzzy rules are fired by input values and for fired fuzzy rule the membership function values of each input value are calculated. The total firing strength ( $\alpha_i$ ) for each fuzzy rule is calculated by applying the max or min operator to obtained membership function values. The output membership function is subjected to multiplication or min logic operator with obtained total firing strength. Thus, the upper part of fuzzy set by  $\alpha_i$  is cropped. By combining the cropped fuzzy sets, which obtained by each fuzzy rules, the fuzzy set is obtained, that will be applying to the defuzzifier unit.

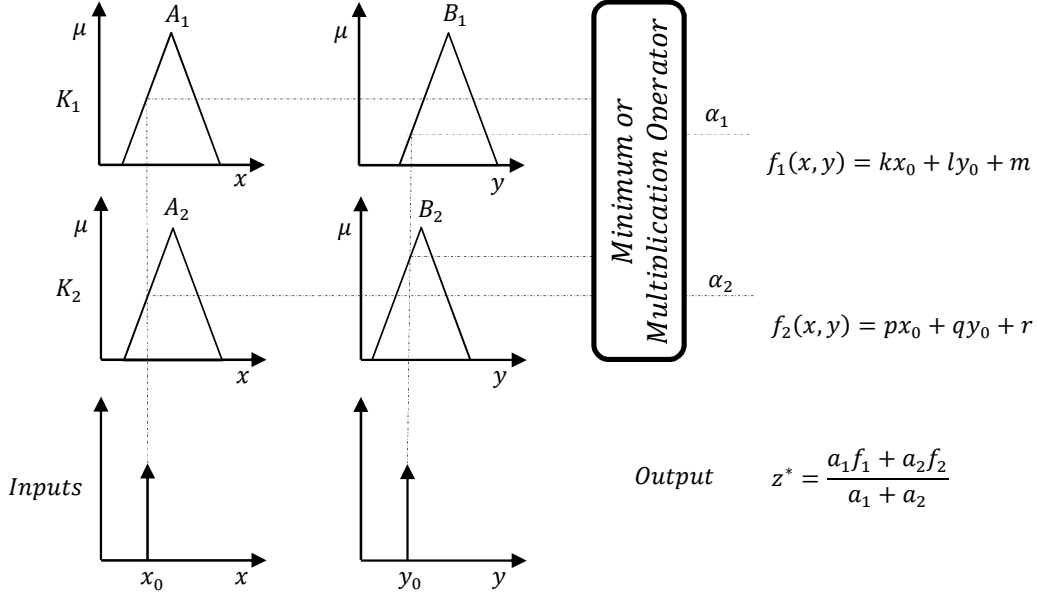
### 2.8.2 Fuzzy inference for Takagi-Sugeno type fuzzy logic systems

In this type fuzzy inference system, crisp functions are used as output of the fuzzy rule table. Thus, rules of the system, which has a two inputs and one output can be given as below.

*Rule<sub>1</sub>: If  $x$  is  $A_1$  and  $y$  is  $B_1$ , Then  $z = f_1(x, y)$ .*

*Rule<sub>2</sub>: If  $x$  is  $A_2$  and  $y$  is  $B_2$ , Then  $z = f_2(x, y)$ .*

In Figure 2.5, the fuzzy inference system for Takagi-Sugeno type fuzzy structure is given.



**Figure 2.5 :** Fuzzy inference for Takagi-Sugeno type structure.

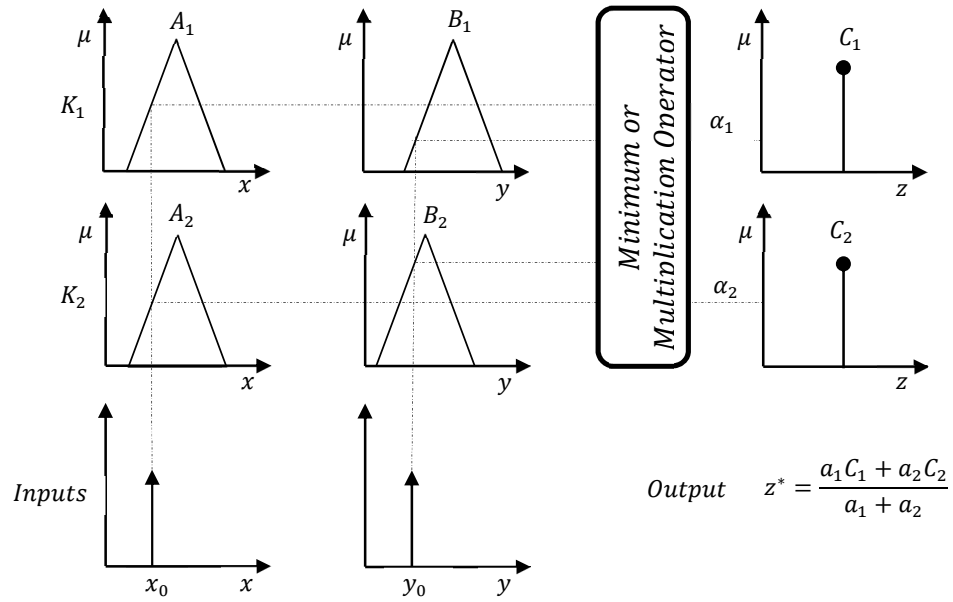
### 2.8.3 Fuzzy inference structure with Singleton consequents

In this type fuzzy inference system, crisp constant values are used as output of the fuzzy rule table. This type fuzzy structure is a special format of Mamdani and Takagi-Sugeno type fuzzy structures. Thus, rules of the system, which has a two inputs and one output can be given as below.

*Rule<sub>1</sub>: If  $x$  is  $A_1$  and  $y$  is  $B_1$ , Then  $z = C_1$ .*

*Rule<sub>2</sub>: If  $x$  is  $A_2$  and  $y$  is  $B_2$ , Then  $z = C_2$ .*

In Figure 2.6, the fuzzy inference system for singleton type fuzzy structure is given.



**Figure 2.6 :** Fuzzy inference structure with singleton consequent.

## 2.9 Defuzzification

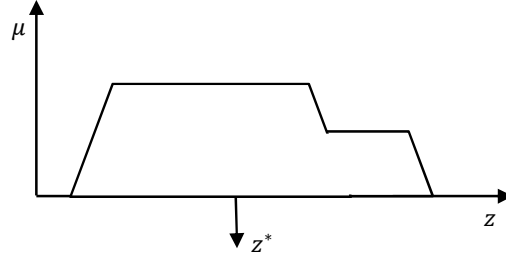
The output of the fuzzy inference system is a fuzzy set. The conversion output value of the fuzzy inference system to a crisp value is called defuzzification and the unit where defuzzification is performed is called defuzzifier. The most used methods for defuzzification are center of gravity, weight average and maximum defuzzification methods.

### 2.9.1 Center of gravity defuzzification method

In this defuzzification method, center of gravity of the areas, which obtained by using fuzzy inference system is calculated. The applying center of gravity method to fuzzy set that obtained in Figure 2.4 is shown in Figure 2.7.

The formula of the defuzzification is given below.

$$z^* = \frac{\int \mu_C z dz}{\int \mu_C dz} \quad (2.29)$$



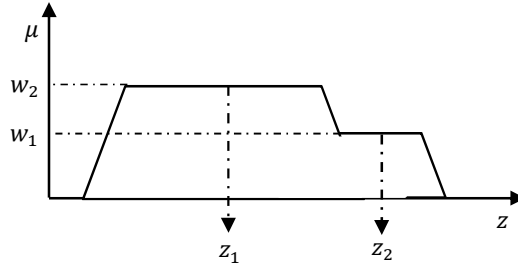
**Figure 2.7 :** Center of gravity defuzzification method.

### 2.9.2 Weight average defuzzification method

In this method, the defuzzification is performed by used all of the fuzzy values and membership function values. By weighting the each membership function with its own maximum and using given formula, the output value is calculated.

$$z^* = \frac{\sum \mu_c z}{\sum \mu_c} \quad (2.30)$$

The calculating of the output value by using the weight average method is shown in Figure 2.8.



**Figure 2.8 :** Weight average defuzzification method.

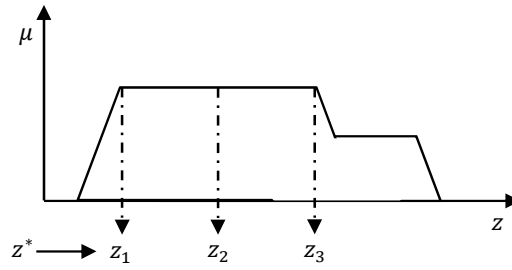
For calculating of the output crisp value, the given formula in Eq.2.31 is used.

$$z^* = \frac{w_1 z_1 + w_2 z_2}{w_1 + w_2} \quad (2.31)$$

### 2.9.3 Maximum defuzzification method

In these method, interval of maximum value of obtained fuzzy set is determined. Then the defuzzification is performed with three different method. In first method, the first value of the determined interval, in second method, the median of the determined interval and the third method, the last value of the determined interval is assigned as

the crisp value. The maximum defuzzification method and the calculation of the output value are showed in Figure 2.9.



**Figure 2.9 :** Maximum defuzzification method.





### 3. CONVENTIONAL PID AND FUZZY PID CONTROLLER

#### 3.1 Conventional PID Controller Structure

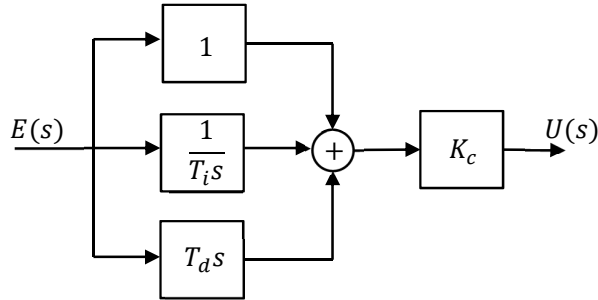
The combination of the proportional, integral and derivative control modes are called as PID controllers. There are many variations of PID control used in practice. Most common forms are parallel and series forms of PID control [26].

##### 3.1.1 Parallel form of PID control

Derivative action can be combined with proportional and integral actions by having each of the modes, operate in parallel [27]. The parallel form of the PID control algorithm is given by

$$G_{Parallel}(s) = \frac{U(s)}{E(s)} = K_c \left[ 1 + \frac{1}{T_i s} + T_d s \right] \quad (3.1)$$

Figure 3.1 illustrates that this controller can be viewed as three separate elements operating in parallel on  $E(s)$ .

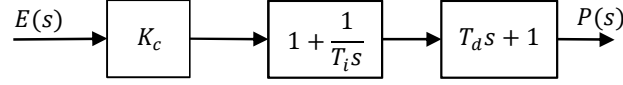


**Figure 3.1 :** Block diagram of the parallel form of PID control.

##### 3.1.2 Series form of PID control

A PI element and a PD element operated in series. The series form of PID control is shown in Figure 3.2 [27]. The series form of the PID control algorithm is given by

$$G_{series}(s) = \frac{U(s)}{E(s)} = K_c \left(1 + \frac{1}{T_i s}\right) (T_d s + 1) \quad (3.2)$$



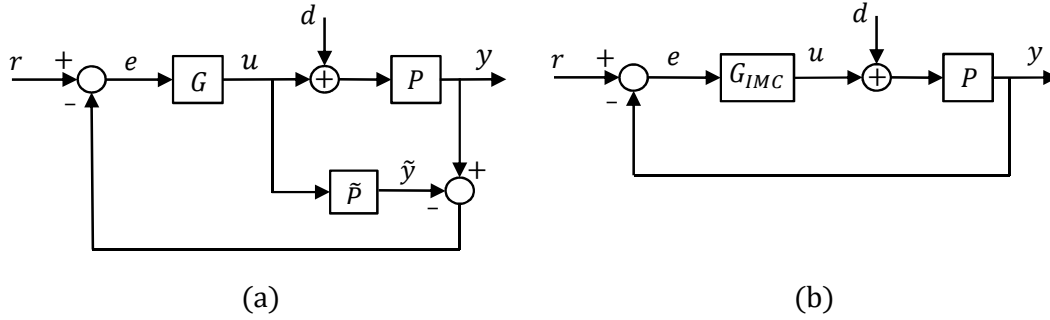
**Figure 3.2 :** Block diagram of the series form of PID control.

In this study, series form of PID control is used.

### 3.1.3 PID design strategy

#### 3.1.3.1 Principle of internal model control

A comprehensive model-based design method, Internal Model Control (IMC), was developed by Morari *et.al* [28 – 29]. The IMC method is based on an assumed process model and leads to analytical expressions for the controller setting. The main idea is simplified block diagram shown in Figure 3.3 [26].



**Figure 3.3 :** Illustration of the (a) IMC scheme, (b) IMC equivalent feedback control system.

Where  $P$  is the plant,  $\tilde{P}$  is a nominal model of the plant,  $G$  is a controller,  $r$  and  $d$  are the set point and the disturbance,  $y$  and  $\tilde{y}$  are the outputs of the plant and it's nominal model, respectively.

The IMC structure is equivalent to the classic single-loop feedback controller shown in Figure 3.3 b, if the  $G_{IMC}$  is given by [3];

$$G_{IMC} = \frac{G(s)}{1 - G(s)\tilde{P}(s)} \quad (3.3)$$

and

$$G(s) = \frac{1}{\tilde{P}_-(s)} f(s) \quad (3.4)$$

where  $P(s) = \tilde{P}_-(s)\tilde{P}_+(s)$ . Here,  $\tilde{P}_-(s)$  is the minimum phase part of the plant model and  $\tilde{P}_+(s)$  contains all time delays and right-half-plane zeroes.  $f(s)$  is a low-pass filter with steady-state gain of one, given as

$$f(s) = \frac{1}{(1 + t_c s)^n} \quad (3.5)$$

where  $t_c$  is represented as closed-loop time constant and  $n$  is a positive integer to be determined.

Suppose that an industrial process can be modeled by a first order plus delay time (FOPDT) structure as follows:

$$\tilde{P}(s) = \frac{K}{Ts + 1} e^{-Ls} \quad (3.6)$$

where  $K$ ,  $T$  and  $L$  are the steady state gain, the time constant and the time delay, respectively.

By the applying first-order Pade approximation, the delay time is defined as follows:

$$e^{-Ls} = \frac{1 - \frac{L}{2}s}{1 + \frac{L}{2}s} \quad (3.7)$$

Therefore, the  $\tilde{P}(s)$  can be factorized as  $\tilde{P}(s) = \tilde{P}_+(s)\tilde{P}_-(s)$ , where  $\tilde{P}_+(s) = 1 - Ls/2$  and

$$\tilde{P}_-(s) = \frac{K}{(Ts + 1)(1 + \frac{L}{2}s)} \quad (3.8)$$

Substituting  $\tilde{P}_-(s)$  into definition of  $G_{IMC}(s)$  and setting  $n = 1$  in filter

$$G_{IMC}(s) = \frac{(Ts + 1) \left(1 + \frac{L}{2}s\right)}{K \left(\frac{L}{2} + t_c\right)s} \quad (3.9)$$

### 3.1.3.2 Internal model control based tuning for PID controller

Transfer function of PD-PI control  $G_{Series}(s)$  can be written as

$$G_{Series}(s) = K_c \left(1 + \frac{1}{T_i s}\right) (T_d s + 1) \quad (3.10)$$

$$G_{Series}(s) = K_c \frac{(1 + T_i s)(1 + T_d s)}{T_i s}$$

If  $G_{Series}(s)$  and  $G_{IMC}(s)$  are equalized, the IMC-base tuning for PD-PI control can be simplified as follows:

$$K_c \frac{(1 + T_i s)(1 + T_d s)}{T_i s} = \frac{(1 + Ts) \left(1 + \frac{L}{2}s\right)}{K \left(\frac{L}{2} + t_c\right)s} \quad (3.11)$$

In terms of Eq. 3.11, parameters  $K_c$ ,  $T_i$  and  $T_d$  can be given as follows:

$$T_i = T \quad T_d = \frac{L}{2} \quad \text{or} \quad T_i = \frac{L}{2} \quad T_d = T \quad (3.12)$$

$$K_c = \frac{T_i}{K \left(\frac{L}{2} + t_c\right)}$$

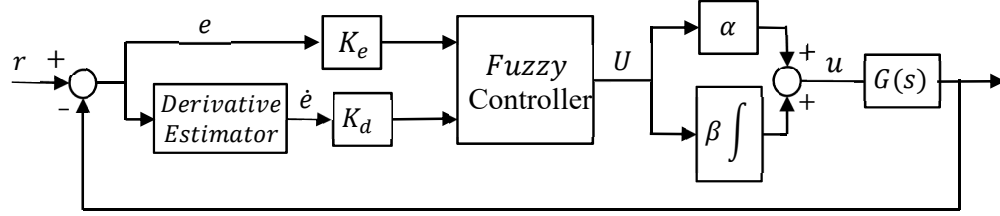
A small value of  $T_d$  gives fast response. To improve the rise time, the value of  $T_d$  should be small [3]. Therefore, the two parameters  $T_d$  and  $T_i$  can be determined as

$$T_d = \min\left(T, \frac{L}{2}\right) \quad T_i = \max\left(T, \frac{L}{2}\right) \quad (3.13)$$

### 3.2 Conventional Fuzzy PID controller structure

#### 3.2.1 General structure

Fuzzy PID controller that retains the characteristic similar to the conventional PID controller, was proposed by Qiao and Mizumoto [7]. Structure of fuzzy PID occurs by connecting a PI device with the basic fuzzy controller serially as shown in Figure 3.4.



**Figure 3.4 :** Closed-loop control structure for fuzzy PID.

Here,  $K_e$  and  $K_d$  are the input SFs and  $\alpha$  and  $\beta$  are the output SFs. Error  $e$  and change rate of error  $\dot{e}$  are the inputs and the control signal  $u$  is the output of the fuzzy PID controller.  $U$  is the output of the fuzzy controller block [30]. The universes of discourses of  $e$  and  $\dot{e}$  are  $e \in \mathcal{R}$  and  $\dot{e} \in \mathcal{R}$ . If linguistic values of  $e$  and  $\dot{e}$  are denoted as  $E_i (i \in I = [-m, \dots, -1, 0, 1, \dots, m])$  and  $R_j (j \in J = [-n, \dots, -1, 0, 1, \dots, n])$  respectively, the fuzzy control rules are given as in the form of

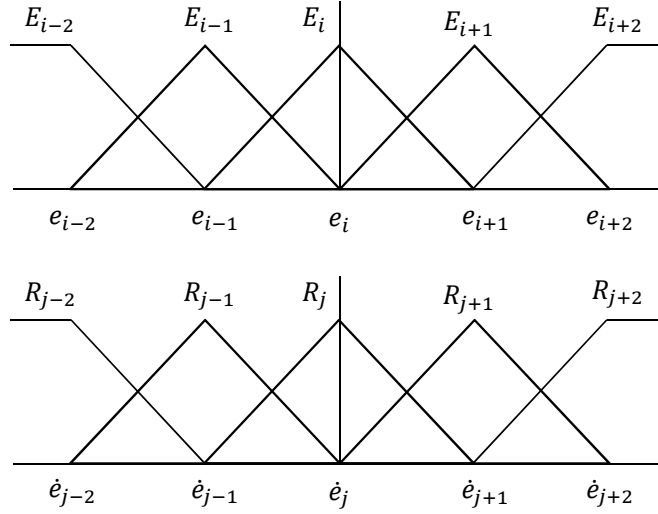
$$\text{If } e \text{ is } E_i \text{ and } \dot{e} \text{ is } R_j, \quad \text{Then } U \text{ is } u_{ij}$$

Suppose that the MFs of  $E_i$  and  $R_j$  are  $E_i(e)$  and  $R_j(\dot{e})$ . In Mizumoto's study (1996), the triangular MFs were used for each fuzzy linguistic value of the error  $e$  and the change of error  $\dot{e}$  and output as shown in Figure 3.5.

The support sets of  $E_i$  are equal to  $[e_{i-1}, e_{i+1}]$  and those of  $R_j$  are equal to  $[\dot{e}_{j-1}, \dot{e}_{j+1}]$ .

$$E_i(e) = \frac{e_{i+1} - e}{e_{i+1} - e_i}, \quad E_{i+1}(e) = \frac{e - e_i}{e_{i+1} - e_i}, \quad (3.14)$$

$$E_k(e) = 0 \quad (k \neq (i, i+1) \in I)$$



**Figure 3.5 :** The antecedent membership functions.

for  $e \in [e_i, e_{i+1}]$

$$R_j(\dot{e}) = \frac{\dot{e}_{i+1} - \dot{e}}{\dot{e}_{i+1} - \dot{e}_i}, \quad R_{j+1}(e) = \frac{\dot{e} - \dot{e}_i}{\dot{e}_{i+1} - \dot{e}_i}, \quad (3.15)$$

$$R_t(\dot{e}) = 0 \quad (t \neq (j, j+1) \in J)$$

With the linguistic values of  $e$  and  $\dot{e}$ , the rule table can be constructed as shown in Table 3.1.

Applying the center of gravity method for defuzzification, the real output of  $U$  is given by

$$U = \frac{\sum_{k,t} E_k(e) R_t(\dot{e}) U_{kt}}{\sum_{k,t} E_k(e) R_t(\dot{e})} \quad (3.16)$$

Here,  $U_{kt}$  ( $k \in I, t \in J$ ) is the linguistic value of the output of the FLC. Under the above descriptions, at any time only two neighborhood MFs can have non zero degrees for each  $e$  and  $\dot{e}$ . This means that at most 4 rules are fired at the same time [7].

For instance, if the inputs of the fuzzy controller are located in zone  $S$ , which is described in  $S = [e_i, e_{i+1}] \times [\dot{e}_j, \dot{e}_{j+1}]$ ,

**Table 3.1** : Rule base for the conventional FPID.

$\dot{e} \setminus e$	$E_{i-2}$	$E_{i-1}$	$E_i$	$E_{i+1}$	$E_{i+2}$
$R_{j-2}$	$U_{(i-2)(j-2)}$	$U_{(i-1)(j-2)}$	$U_{i(j-2)}$	$U_{(i+1)(j-2)}$	$U_{(i+2)(j-2)}$
$R_{j-1}$	$U_{(i-2)(j-1)}$	$U_{(i-1)(j-1)}$	$U_{i(j-1)}$	$U_{(i+1)(j-1)}$	$U_{(i+2)(j-1)}$
$R_j$	$U_{(i-2)j}$	$U_{(i-1)j}$	$U_{ij}$	$U_{(i+1)j}$	$U_{(i+2)j}$
$R_{j+1}$	$U_{(i-2)(j+1)}$	$U_{(i-1)(j+1)}$	$U_{i(j+1)}$	$U_{(i+1)(j+1)}$	$U_{(i+2)(j+1)}$
$R_{j+2}$	$U_{(i-2)(j+2)}$	$U_{(i-1)(j+2)}$	$U_{i(j+2)}$	$U_{(i+1)(j+2)}$	$U_{(i+2)(j+2)}$

the MFs and the output  $u_{flc}$  can be defined as follows

$$\begin{aligned} E_k(e) &= 0 \quad (k \neq (i, i+1) \in I) \\ R_t(\dot{e}) &= 0 \quad (t \neq (j, j+1) \in J) \end{aligned} \quad (3.17)$$

$$U = \frac{\sum_{k=(i,i+1)} E_k(e) R_t(\dot{e}) U_{kt}}{\sum_{k=(i,i+1)} E_k(e) R_t(\dot{e})} \quad (3.18)$$

$$\begin{aligned} U &= \left[ u_{ij} - \frac{u_{(i+1)j} - u_{ij}}{e_{i+1} - e_i} e_i - \frac{u_{i(j+1)} - u_{ij}}{\dot{e}_{j+1} - \dot{e}_j} \dot{e}_j \right] + \frac{u_{(i+1)j} - u_{ij}}{e_{i+1} - e_i} e \\ &\quad + \frac{u_{i(j+1)} - u_{ij}}{\dot{e}_{j+1} - \dot{e}_j} \dot{e} \end{aligned} \quad (3.19)$$

$$U = A + Pe + D\dot{e} \quad (3.20)$$

where

$$\begin{aligned} A &= u_{ij} - \frac{u_{(i+1)j} - u_{ij}}{e_{i+1} - e_i} e_i - \frac{u_{i(j+1)} - u_{ij}}{\dot{e}_{j+1} - \dot{e}_j} \dot{e}_j \\ P &= \frac{u_{(i+1)j} - u_{ij}}{e_{i+1} - e_i} \\ D &= \frac{u_{i(j+1)} - u_{ij}}{\dot{e}_{j+1} - \dot{e}_j} \end{aligned} \quad (3.21)$$

Here, Eq. 3.21 is the relation between the input and the output variables of FLC for the fuzzy controllers with product-sum inference method, the center of gravity defuzzification method and triangular MFs for the inputs and a crisp output. As it is seen in Figure 4.4, the output of the fuzzy PID controller  $u$  is given by

$$u = \alpha u_{flc} + \beta \int u_{flc} dt \quad (3.22)$$

Therefore, from Eq. (3.20) and Eq. (3.22), the controller output is obtained as

$$u = \alpha A + \beta A t + (\alpha K_e P + \beta K_d D)e + \beta K_e P \int e dt + \alpha K_d D \dot{e} \quad (3.23)$$

Thus, the equivalent control components of the FPID controller are obtained as follows [7]:

- Proportional gain:  $\alpha K_e P + \beta K_d D$
- Integral gain:  $\beta K_e P$
- Derivative gain:  $\alpha K_d D$

### 3.2.2 Fuzzy PID design strategy

#### 3.2.2.1 The analytical structure of the fuzzy PID controller

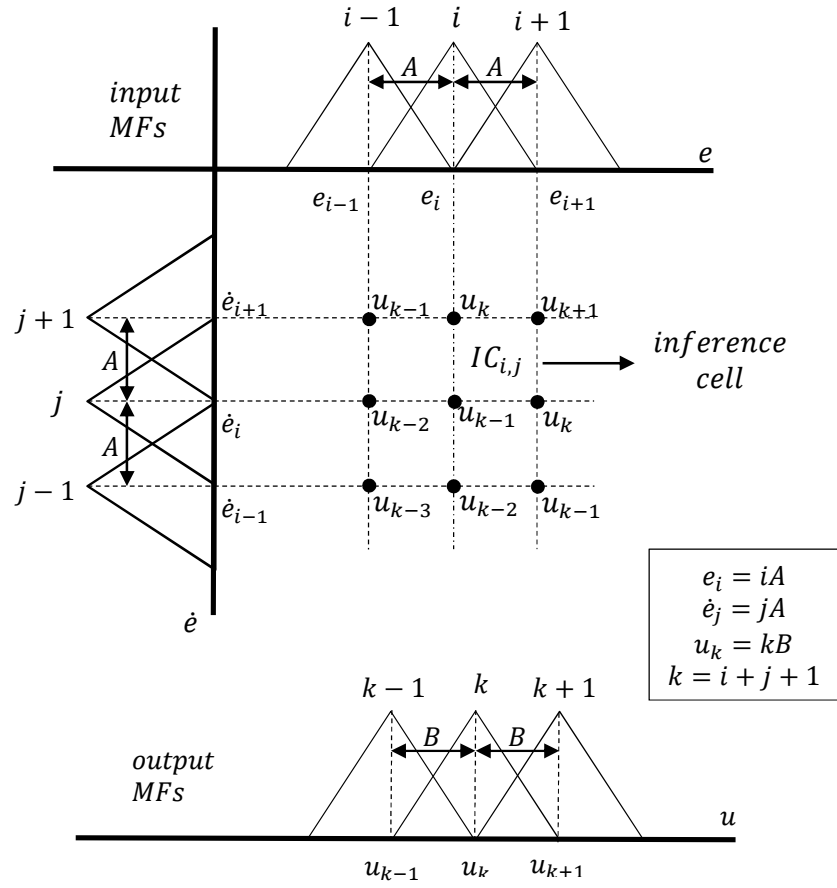
Assume that input-output MFs are chosen as triangular shapes shown in Figure 3.6. All input MFs have an equal spread  $2A$  while all output MFs have an equal spread  $2B$ .

##### Decomposing rule base into inference cells

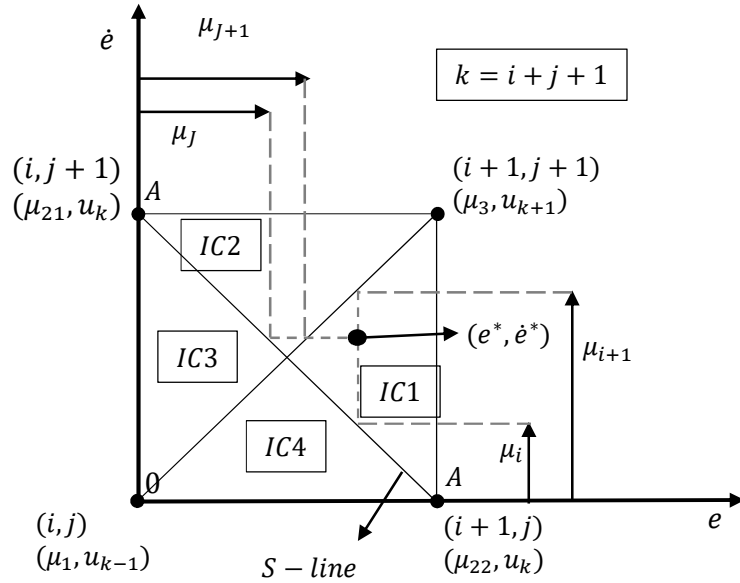
The rule base can be divided into many square blocks as shown in Figure 3.6. Output rules are matched with four corners. All fuzzy inference operations can be calculated on these blocks, which are called inference cells [31].

To maintain generality, the  $IC_{i,j}$  is chosen for analysis. Two MFs from  $E$  ( $i$ th and  $i + 1$  th) and  $R$  ( $j$ th and  $j + 1$  th) are folded together to form the  $IC$ . Two line from MFs divide the  $IC$  into four regions ( $IC1 - IC4$ ) as shown in Figure 3.7. One of the diagonal lines is called the  $S$ -line on which all the points have the same distance to the ( $S = 0$ ). The output from the inference is shown on the relevant corner and indexed by the sequence number of input membership function.





**Figure 3.6 :** Decomposition of the rule base into inference cells.



**Figure 3.7 :** Functional composition of the inference cell  $IC_{ij}$ .

The absolute position of  $IC_{ij}$  in the rule base is from  $[iA, jA]$  to  $[(i + 1)A, (j + 1)A]$ . Its relative position in the  $IC$  plane is from  $[0, 0]$  to  $[A, A]$ . The input data to the rule base can always be mapped into the relative input data  $(e^*, \dot{e}^*)$  by the following mapping formula

$$\begin{aligned} e &= iA + e^* \quad (i = \dots, -1, 0, 1, \dots) \\ \dot{e} &= jA + \dot{e}^* \quad (j = \dots, -1, 0, 1, \dots) \end{aligned} \quad (3.24)$$

### Fuzzy inference operation

While the creating mathematic model of FPID, Mamdani's max-min method has been used as the inference method.

Fuzzification: The grades for  $e^*$  are  $\mu_i$  and  $\mu_{i+1}$ , the grades for  $\dot{e}^*$  are  $\mu_j$  and  $\mu_{j+1}$ . As there always exists  $\mu_i + \mu_{i+1} = 1$  and  $\mu_j + \mu_{j+1} = 1$ . Therefore, all the fuzzified values of the input can be obtain as follows [31].

$$\begin{aligned} \mu_i &= 1 - \frac{e^*}{A} & \mu_{i+1} &= \frac{e^*}{A} \\ \mu_j &= 1 - \frac{\dot{e}^*}{A} & \mu_{j+1} &= \frac{\dot{e}^*}{A} \end{aligned} \quad (3.25)$$

Inference Operation: Rule form “If  $e$  is  $E_i$  and  $\dot{e}$  is  $R_j$  than the output is  $u_{k-1}$ ” follows the linear operation indicated by the index formula

$$k = i + j + 1 \quad (3.26)$$

The output rule  $u_k$  can be obtained from the output MFs in Figure 3.6 as

$$u_k = kB \quad (3.27)$$

The minimum operation is used to get the output grade as shown in Eq. 3.28. Their positions on the  $(IC_{ij})$  are shown in Figure 3.7.

$$\begin{aligned} \mu_1 &= \min(\mu_i, \mu_j) \quad \text{for output } u_{k-1} \\ \mu_{21} &= \min(\mu_i, \mu_{j+1}) \quad \text{for output } u_k \\ \mu_{22} &= \min(\mu_{i+1}, \mu_j) \quad \text{for output } u_k \end{aligned} \quad (3.28)$$

$$\mu_3 = \min(\mu_{i+1}, \mu_{j+1}) \quad \text{for output } u_{k+1}$$

The maximum operation is used to obtain the maximum effect of two outputs  $u_k$  on the S-line

$$\mu_2 = \max(\mu_{21}, \mu_{22}) \quad (3.29)$$

Defuzzification: Center of gravity method is used for defuzzification.

$$U = \frac{\sum_{l=1}^3 \mu_l u_{k+l-2}}{\sum_{l=1}^3 \mu_l} \quad (3.30)$$

### Mathematical description

Different inputs may fall in the different regions ( $IC1 - IC4$ ), which cause different results from the max-min operation. All possible results are summarized in Table 3.2 [31].

**Table 3.2 :** Results from max-min operation.

<i>Region</i>	$\mu_1$	$\mu_2$	$\mu_3$
<i>IC1</i>	$\mu_i$	$\mu_j$	$\mu_{j+1}$
<i>IC2</i>	$\mu_j$	$\mu_i$	$\mu_{i+1}$
<i>IC3</i>	$\mu_j$	$\mu_{j+1}$	$\mu_{i+1}$
<i>IC4</i>	$\mu_i$	$\mu_{i+1}$	$\mu_{j+1}$

for region *IC1*, from Table 3.2 and Eq. 3.30

$$\begin{aligned}
 \sum_{l=1}^3 \mu_l &= \mu_i + \mu_j + \mu_{j+1} \\
 &= \mu_i + 1 = 2 - \frac{e^*}{A} \\
 &= \gamma_1^{-1}
 \end{aligned} \quad (3.31)$$

from Table 3.2 and Eq. 3.31

$$\begin{aligned}
\sum_{l=1}^3 \mu_l u_{k+1-l} &= \mu_i u_{k-1} + \mu_j u_k + \mu_{j+1} u_{k+1} \quad (3.32) \\
&= \left(1 - \frac{e^*}{A}\right)(k-1)B + \left(1 - \frac{\dot{e}^*}{A}\right)kB + \left(\frac{\dot{e}^*}{A}\right)(k+1)B \\
&= \left(1 - \frac{e^*}{A}\right)(k-1)B + \left(1 - \frac{\dot{e}^*}{A}\right)kB + \left(\frac{\dot{e}^*}{A}\right)(k+1)B \\
&= \frac{B[(k-1)A + e^* + \dot{e}^*]}{A} + kB\left(1 - \frac{e^*}{A}\right) \\
\sum_{l=1}^3 \mu_l u_{k+1-l} &= \frac{B}{A}S + kB(\gamma_1^{-1} - 1)
\end{aligned}$$

where  $S = E + R = (k-1)A + e^* + \dot{e}^*$ . From Eq. 3.30, Eq. 3.31 and Eq. 3.33, the output of fuzzy control block in region *IC1* can be obtained

$$\begin{aligned}
U_1 &= \frac{\sum_{l=1}^3 \mu_l u_{k+1-l}}{\sum_{l=1}^3 \mu_l} = \frac{\frac{B}{A}S + kB(\gamma_1^{-1} - 1)}{\gamma_1^{-1}} \quad (3.32) \\
U_1 &= \frac{B}{A}\gamma_1 S + kB(1 - \gamma_1) \\
U_1 &= kB + \frac{B}{A}\gamma_1(S - kA)
\end{aligned}$$

The output of fuzzy block is obtained for only in region *IC1*. By using the same procedure in other regions, all the possible outputs of fuzzy block can be obtain as follows.

$$U_1 = kB + \frac{B}{A}\gamma_l(S - kA) \quad (l = 1, 2, 3, 4) \quad (3.33)$$

where

$$\begin{aligned}
\gamma_1 &= (1 + \mu_i)^{-1} \\
\gamma_2 &= (1 + \mu_j)^{-1}
\end{aligned} \quad (3.34)$$

$$\gamma_3 = (1 + \mu_{i+1})^{-1}$$

$$\gamma_4 = (1 + \mu_{j+1})^{-1}$$

$$S = E + R = (k - 1)A + e^* + \dot{e}^*$$

$$k = i + j + 1$$

The output of FPID controller  $u$  is described as follows:

$$u = u_{frc}(\alpha + \beta \frac{1}{\rho}) \quad (3.35)$$

where

$$U = kB(1 - \gamma) + \frac{B}{A}\gamma S \quad (3.36)$$

where  $1/\rho = \int dt$ ,  $S = K_e e + K_d \dot{e} = E + R$ ,  $K_e = 1$ ,  $\gamma$  is a nonlinear time-varying parameter ( $2/3 \leq \gamma \leq 1$ ) [3]. By substituting  $k = i + j + 1$ , where  $i = (e - e^*)/A$  and  $j = (\dot{e} - \dot{e}^*)/A$  into the Eq. 3.37

$$u = \frac{B}{A}(S + \delta)(\alpha + \beta \frac{1}{\rho}) \quad (3.37)$$

where

$$\delta = (1 - \gamma)(A - e^* - \dot{e}^*) = (1 - \gamma)\sigma \quad (3.38)$$

$$\sigma = A - e^* - \dot{e}^* \quad (3.39)$$

Using  $S = E + R = K_e(e + K_1 \dot{e})$  and letting  $K_2 = \alpha/\beta$ , Eq. 3.39 becomes

$$\begin{aligned} u = & K_e \beta \frac{B}{A} (K_1 + K_2) \left( e + \frac{1}{K_1 + K_2} \int e + \frac{K_1 K_2}{K_1 + K_2} \dot{e} \right) \\ & + \beta \frac{B}{A} \left( K_2 \delta + \int \delta \right) \end{aligned} \quad (3.40)$$

$$u = u^{PID} + u^N$$

where

$$u^{PID} = K_e \beta \frac{B}{A} (K_1 + K_2) \left( e + \frac{1}{K_1 + K_2} \int e + \frac{K_1 K_2}{K_1 + K_2} \dot{e} \right) \quad (3.41)$$

$$u^N = \beta \frac{B}{A} \left( K_2 \delta + \int \delta dt \right)$$

If Laplace transform is taken of Eq. 3.42

$$U^{PID}(s) = K_e \beta \frac{B}{A} (K_1 + K_2) \left( 1 + \frac{1}{(K_1 + K_2)s} + \frac{K_1 K_2}{K_1 + K_2} s \right) E(s) \quad (3.42)$$

$$U^{PID}(s) = K_e \beta \frac{B}{A} \frac{(1 + K_1 s)(1 + K_2 s)}{s} E(s)$$

$$U^N(s) = \beta \frac{B}{A} \left( K_2 + \frac{1}{s} \right) \delta(s)$$

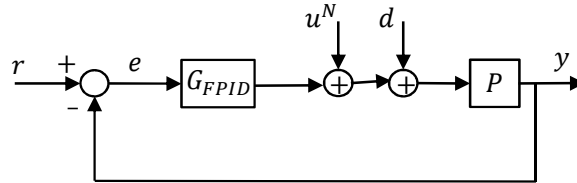
With  $\delta(s)$  is a nonlinear term without an explicit analytical expression can be rewritten as a transfer functions form.

$$G_{FPID}(s) = \frac{U^{PID}(s)}{E(s)} = K_e \beta \frac{B}{A} \frac{(1 + K_1 s)(1 + K_2 s)}{s} \quad (3.44)$$

It can be obviously said, the fuzzy PID control can be considered as a conventional PID with a nonlinear compensation [3].

### 3.2.2.2 Internal model control based tuning for the conventional fuzzy PID controller

If nonlinear compensation  $u^N$  is considered as a process disturbance, which is shown in Figure 3.8, and set  $G_{FPID}(s) = G_{IMC}(s)$ , the IMC-based tuning for fuzzy PID controllers can be simplified.



**Figure 3.8 :** Equivalent format of FPID control in the closed loop.

In section 3.1, the FOPDT model of an industrial process and the transfer function of IMC, which obtained by FOPDT, were given as follows:

$$\tilde{P}(s) = \frac{K}{Ts + 1} e^{-Ls} \quad (3.43)$$

$$G_{IMC}(s) = \frac{(Ts + 1) \left(1 + \frac{L}{2}s\right)}{K \left(\frac{L}{2} + t_c\right)s} \quad (3.44)$$

By comparing  $G_{IMC}(s)$  with  $G_{FPID}(s)$ , parameters of fuzzy PID can be calculated [3].  $G_{FPID}(s)$  was obtained in previous section.

$$K_e \beta \frac{B}{A} \frac{(1 + K_1 s)(1 + K_2 s)}{s} = \frac{(1 + Ts) \left(1 + \frac{L}{2}s\right)}{K \left(\frac{L}{2} + t_c\right)s} \quad (3.45)$$

In terms of Eq. 3.47, parameters  $K_1$ ,  $K_2$  and  $\beta$  can be given as follows.

$$\beta = \frac{A}{B} \frac{1}{K_e K \left(\frac{L}{2} + t_c\right)} \quad (3.46)$$

$$K_1 = T \quad K_2 = \frac{L}{2} \quad \text{or} \quad K_1 = \frac{L}{2} \quad K_2 = T$$

The bandwidth of the fuzzy PID at the  $k$ th level can be controlled by adjusting  $K_1$ . A small value of  $K_1$  gives wide bandwidth and fast response, otherwise it gives a low bandwidth and sluggish response. To improve the rise time, the value of  $K_1$  should be small [3]. Therefore, the two parameters  $K_1$  and  $K_2$  can be determined as

$$K_1 = \min\left(T, \frac{L}{2}\right) \quad K_2 = \max\left(T, \frac{L}{2}\right) \quad (3.47)$$



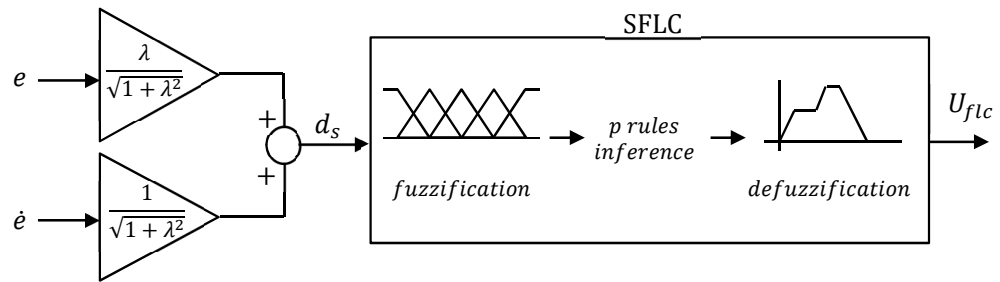


#### 4. SINGLE INPUT FUZZY PID CONTROLLER STRUCTURE

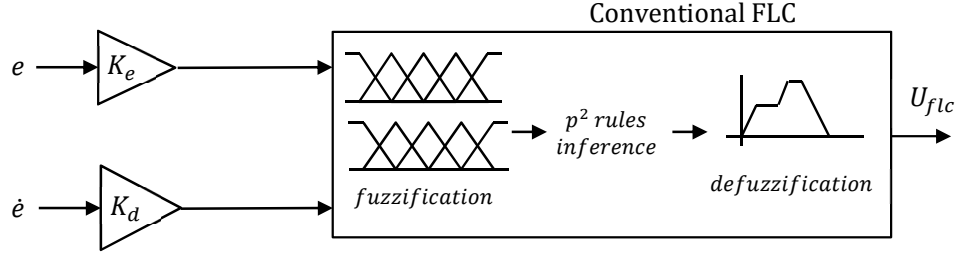
In case of high complex order plants, the design of an FLC is very difficult due to increased number of fuzzy control rules as well as tuning parameters. Therefore, it is necessary to design an FLC that has a simple control structure. The simplification converts two inputs of FLC to a single input called as signed distance. For this conversion “signed distance method (SDM)” is used. SDM was suggested by Choi et al [12].

The main difference of SFPID from CFPID is the number of inputs. In SFPID structure input of the fuzzy block is  $d_s$ , as shown in Figure 4.1. Only one input scaling factor ( $\lambda$ ) is used while applying SDM. On the other hand, observing Figure 4.2, in CFPID structure the variables  $K_e$  and  $K_d$  are the input SFs for inputs  $e$  and  $\dot{e}$ , respectively.

One of the important features of the SFPID is the significant reduction in the number of rules. For a CFPID with  $p$  memberships, the number of rules will be  $p^2$  whereas an equivalent SFPID requires only  $p$  rules. Therefore it is possible that producing an effective control surface without complex computations and convert the nonlinear control surface to piecewise linear. But, it should be noted that this process will not dispossess SFPID controller from the nonlinear properties that have made it a robust controller [12, 32-33].



**Figure 4.1 :** Single input FLC structure.



**Figure 4.2 :** Conventional FLC structure.

In summary, it can be listed as below[32, 34-37];

- Single input FLC (SFLC) requires only one input instead of two.
- The control rule table for SFLC is constructed on a 1-D space.
- The number of tuning parameters is greatly decreased in SFLC structure.
- As a result, the SFLC provides a simpler method for design of FLC's.

#### 4.1 The Internal Structure of the SFPID Controller

As described, a FPID rule structure was described as follows

$$Rule_k: \text{If } E \text{ is } A_{1j} \text{ and } \dot{E} \text{ is } A_{2j}, \quad \text{Then } U \text{ is } C_{ij} \quad i, j = 1, 2, 3, 4, 5$$

Where  $A_{1j}$  and  $A_{2j}$  are the actecedent MFs for  $E$  and  $\dot{E}$ , respectively, and  $C_{ij}$  is the consequent MF. The antecedent MFs are defined with 50% overlappig triungular fuzzy sets which are denoted  $NB$  (Negative Big),  $NM$  (Negative Medium),  $Z$  (Zero),  $PM$  (Positive Medium),  $PB$  (Positive Big). The consequent MFs are defined with singleton consequent and are defined as  $NB = -1$ ,  $NM = -0.5$ ,  $Z = 0$ ,  $PM = 0.5$  and  $PB = 1$ .

Thus, a two-dimensional (2-D) rule table can be constructed depending on  $E$  and  $\dot{E}$  as shown in Table 4.1. Here, if there are  $p$  MFs for both of input and output, the number of control rules is  $p^2$ .

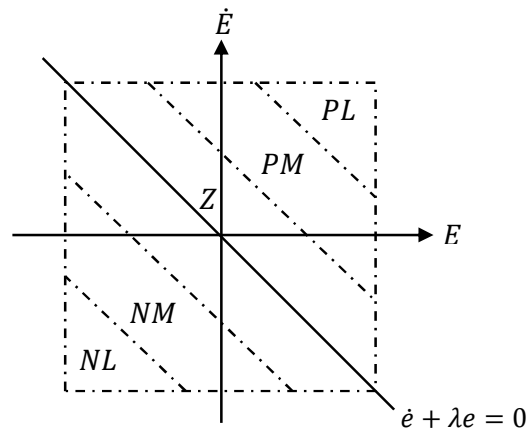
**Table 4.1 :** Diagonal rule table of the conventional FPID controller

$\dot{E}/E$	$NB$	$NM$	$Z$	$PM$	$PB$
$PB$	$Z$	$PM$	$PM$	$PB$	$PB$
$PM$	$NM$	$Z$	$PM$	$PM$	$PB$
$Z$	$NM$	$NM$	$Z$	$PM$	$PM$
$NM$	$NB$	$NM$	$NM$	$Z$	$PM$
$NB$	$NB$	$NB$	$NM$	$NM$	$Z$

*Main Diagonal  
Line*

The rule tables, which are similar to this structure have the same output in a diagonal direction. Additionally, the magnitude of the control input  $|u|$  is approximately proportional to the distance from the main diagonal line. Instead of using two-variable input sets  $(E, \dot{E})$ , it is possible to obtain the corresponding output, using a single variable input only. The significance of the reduction was first realized by Choi *et al.* and is known as the signed distance method.

Rule table has a skew-symmetric property, namely  $C_{ij} = -C_{ji}$ . If the quantization level of the independent variables is halved, then the boundaries of the control regions become staircase shapes with double number of stairs and half pitches. Afterwards, the control law describes the multilevel relay controller with five bands as shown in Figure 4.3. The absolute magnitude of the control input is proportional to the distance  $d$  from the main diagonal line [12].

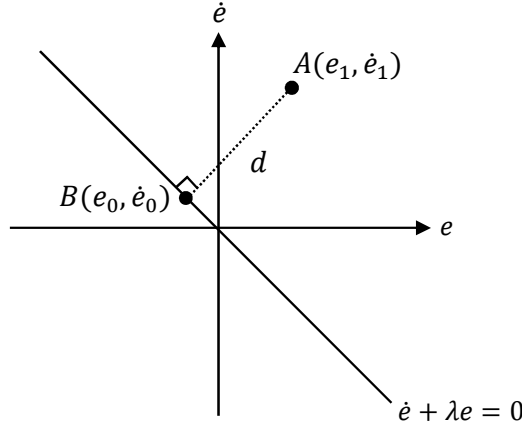


**Figure 4.3 :** Rule table with the infinitesimal quantization levels.

Main diagonal line can be represented as a straight line called the switching line as follows:

$$s_l : \dot{e} + \lambda e = 0$$

Variable  $\lambda$  is the slope magnitude of the main diagonal line. Let  $B(e_0, \dot{e}_0)$  be the intersection point of the switching line and the line perpendicular to the switching line from a known operating point  $A(e_1, \dot{e}_1)$  as illustrated in Figure 4.4.



**Figure 4.4 :** Derivation of the signed distance.

The distance between the points  $A(e_1, \dot{e}_1)$  and  $B(e_0, \dot{e}_0)$  can be expressed as

$$d_1 = \sqrt{(e_0 - e_1)^2 + (\dot{e}_0 - \dot{e}_1)^2} = \frac{|\dot{e}_1 + \lambda e_1|}{\sqrt{1 + \lambda^2}} \quad (4.1)$$

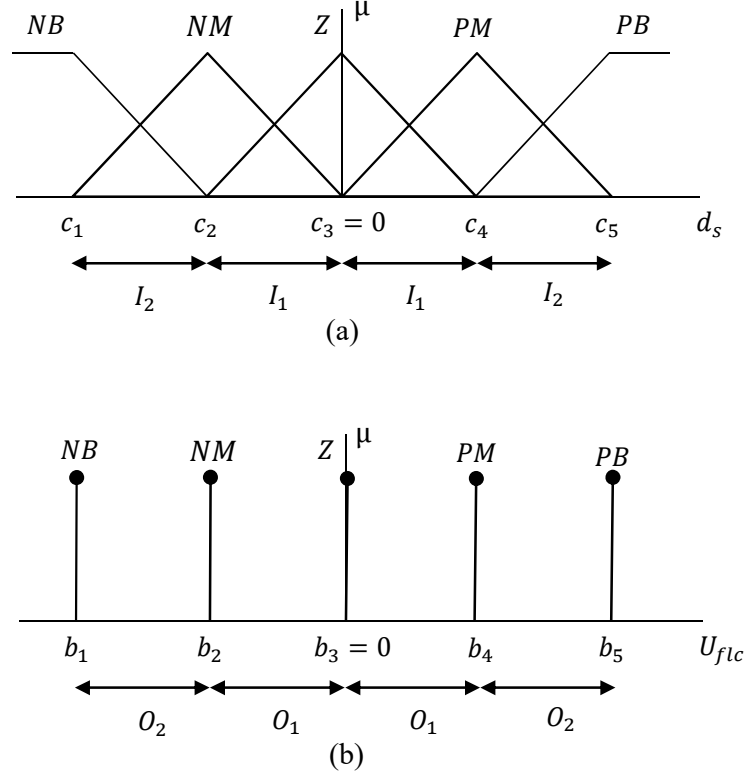
By the generalization of Eq. 4.1, signed distance can be rewritten as follows:

$$d_s = \frac{\dot{e} + \lambda e}{\sqrt{1 + \lambda^2}} \quad (4.2)$$

Hence, the control action can be determined by  $d_s$  only. The rule table can be reconstructed on the one-dimensional (1-D) space with this derived distance  $d_s$  instead of the 2-D. So, it can be called SFLC and the new 1-D rule base can be simply generated from 2-D rule base as shown in Table 4.2 [12]. The rule form for the SFLC is given as follows:

$$\text{Rule}_k: \text{If } d_s \text{ is } A_k, \text{ Then } U_{flc} \text{ is } b_k \quad k \in 1, 2, 3, \dots, 5$$

Where  $A_k$  is the antecedent MFs (AMFs) and is defined with its center ( $c_k$ ) and  $b_k$  is the onsequent MFs (CMFs) as shown in Figure 4.5 a and Figure 4.5 b, respectively. Here  $I_1$  and  $I_2$  represent the spread of the AMFs while  $O_1$  and  $O_2$  are the CMFs. The MFs are denoted with  $NB, NM, Z, PM$  and  $PB$ .



**Figure 4.5 :** Illustration of the (a) antecedent and (b) consequent MFs.

**Table 4.2 :** The rule table of the SFPID controller

$d_s$	$NB$	$NM$	$Z$	$PM$	$PB$
$U_{flc}$	$NB$	$NM$	$Z$	$PM$	$PB$

For an input  $d_s \in [c_k, c_{k+1}]$ , the MFs can be defined as

$$A_k(d_s) = \frac{c_{k+1} - d_s}{c_{k+1} - c_k}, \quad A_{k+1}(d_s) = \frac{d_s - c_k}{c_{k+1} - c_k}, \quad (4.3)$$

By using the observation value of  $d_s$ , the firing strength for the  $k^{th}$  rule is calculated as follows [7]:

$$f_k = A_k(d_s) \quad (4.4)$$

Then, by applying the product-sum inference method to defuzzification, the real output of the FLC can be obtained as follows [7]:

$$U = \frac{\sum_k f_k b_k}{\sum_k f_k} \quad (4.5)$$

Combining Eq. 4.4 with Eq. 4.5

$$U_{flc} = \frac{\sum_k A_k(d_s) b_k}{\sum_k A_k(d_s)} \quad (4.6)$$

Obviously, by definition of the membership function and by constructed 1-D rule base,  $U_{flc}$  can be rewritten as

$$U_{flc} = \frac{b_{k+1} - b_k}{c_{k+1} - c_k} d_s - \frac{b_{k+1} c_k - b_k c_{k+1}}{c_{k+1} - c_k} \quad (4.7)$$

Eq. 4.7 can be rewritten as follows

$$U_{flc} = K_f d_s - T_{OF} \quad (4.8)$$

Here,  $K_f$  and  $T_{OF}$  are the gain and offset values generated from the fuzzy mapping and are defined as follows:

$$K_f = \frac{b_{k+1} - b_k}{c_{k+1} - c_k}, \quad T_{OF} = \frac{b_{k+1} c_k - b_k c_{k+1}}{c_{k+1} - c_k} \quad (4.9)$$

For the handled SFLC structure, the  $K_f$  and  $T_{OF}$  are derived for  $d_s \in [c_1, c_5]$  and are presented in Table 4.3.

**Table 4.3 :** The gain and offset values of the SFPID

<i>Region</i>	<i>K<sub>f</sub></i>	<i>T<sub>OF</sub></i>
$d_s \in [c_1, c_2]$	$(b_2 - b_1)/(c_2 - c_1)$	$(b_2 c_1 - b_1 c_2)/(c_2 - c_1)$
$d_s \in [c_2, c_3]$	$b_2/c_2$	0
$d_s \in [c_3, c_4]$	$b_4/c_4$	0
$d_s \in [c_4, c_5]$	$(b_5 - b_4)/(c_5 - c_4)$	$(b_5 c_4 - b_4 c_5)/(c_5 - c_4)$

## 4.2 General Structure

SFPID structure occurs with three sections. First section, where the signed distance method applied, second section is the FLC block and the last section is the proportional-integral (PI) controller block as shown in Figure 4.5.

In SDM block shown in Figure 4.6,  $e$  and  $\dot{e}$  are reduced into one parameter.  $d_s$  was defined in Eq. 4.2. Transfer function of the SDM block is [12]

$$\frac{D_s(s)}{E(s)} = \frac{\lambda + s}{\sqrt{1 + \lambda^2}} \quad (4.10)$$

When considering the PD controller transfer function [38], SDM block is similar to proportional-derivative (PD) controller. Eq. 4.10 has gain and derivative time constant like a PD controller.

$$K_c(1 + T_d s) = \frac{\lambda}{\sqrt{1 + \lambda^2}} \left(1 + \frac{1}{\lambda} s\right) \quad (4.11)$$

where  $K_c$  and  $T_d$  are the gain and time constant of PD controller, respectively. Thus, it can be said that SDM block acts like PD controller with gain and time constant as below.

$$K_c = \frac{\lambda}{\sqrt{1 + \lambda^2}} \quad T_d = \frac{1}{\lambda} \quad (4.12)$$

Output of SFLC is defined previous section. By substituting definition of  $d_s$  into the Eq. 4.7.

$$U_{flc} = \frac{\lambda b_{k+1} - \lambda b_k}{(c_{k+1} - c_k)\sqrt{1 + \lambda^2}} e + \frac{b_{k+1} - b_k}{(c_{k+1} - c_k)\sqrt{1 + \lambda^2}} \dot{e} + \frac{b_{k+1}c_k - b_k c_{k+1}}{c_k - c_{k+1}} \quad (4.13)$$

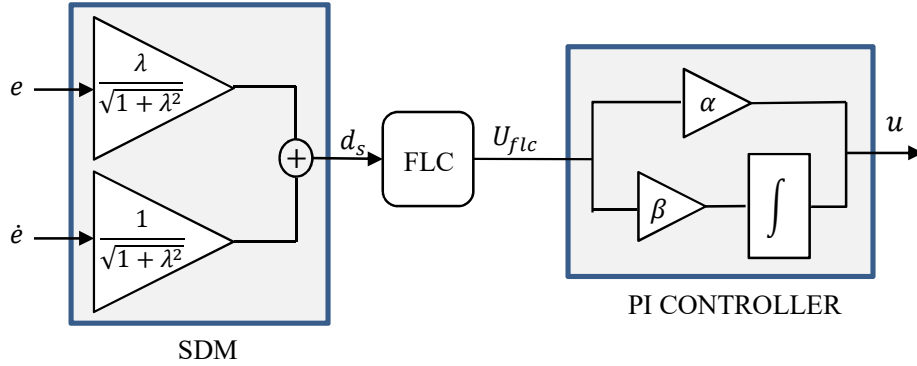
$$U_{flc} = A + P e + D \dot{e} \quad (4.14)$$

where

$$\begin{aligned}
P &= \frac{b_{k+1}\lambda - b_k\lambda}{(c_{k+1} - c_k)\sqrt{1 + \lambda^2}} \\
D &= \frac{b_{k+1} - b_k}{(c_{k+1} - c_k)\sqrt{1 + \lambda^2}} \\
A &= \frac{b_{k+1}c_k - b_k c_{k+1}}{c_k - c_{k+1}}
\end{aligned} \tag{4.15}$$

As it is seen in Figure 4.6, the output of the SFPID controller  $u$  is given by

$$\begin{aligned}
u &= \alpha u_{flc} + \beta \int U_{flc} dt \\
u &= \alpha(A + Pe + D\dot{e}) + \beta \int (A + Pe + D\dot{e}) dt \\
u &= \alpha A + \beta A t + (\alpha P + \beta D)e + \alpha D\dot{e} + \beta P \int e dt
\end{aligned} \tag{4.16}$$



**Figure 4.6 :** General structure of the SFPID.

Thus, the equivalent control components of the SFPID controller are obtained as follows:

- Proportional gain:  $\alpha P + \beta D$
- Integral gain:  $\beta P$
- Derivative gain:  $\alpha D$

In section 3.2, the equivalent control components of the FPID controller were obtained as follows:



- Proportional gain:  $\alpha K_e P + \beta K_d D$
- Integral gain:  $\beta K_e P$
- Derivative gain:  $\alpha K_d D$

So, it can be said that the structure of SFPID is similar to CFPID structure. Both can be expressed with similar proportional, integral and derivative components [39].

### 4.3 Gain Analysis of Single Input Fuzzy Controller

As shown in the previous section, the gain of SFLC depends on parameters of antecedent and consequent MF parameters. In this section, we will analyze the gain variation.

If MF parameters are set and fixed as  $c_k = b_k$  for  $\forall k$ , then the SFLC will be linear ( $K_k = 1$  for  $\forall k$ ) and thus a linear FLC (L-FLC) will be constructed. Thus, in comparison to its linear counterpart, we can easily derive when the FLC act aggressively or smoothly. In Table 4.4, we tabulated the possible control actions (CAs) which can be generated by SFLC [39]. It can be concluded that:

- While keeping the spreads  $O_1$  and  $O_2$  fixed, increasing the spread  $I_1$  ( $c_4$ ) (thus decreasing  $I_2 = c_5 - c_4$ ) will result with a smooth CA (SCA). Conversely, decreasing the spread  $I_1$  will result with an aggressive CA (ACA).
- While keeping the spread  $I_1$  and  $I_2$  fixed, decreasing the spread  $O_1$  ( $b_4$ ) (thus increasing  $O_2 = b_5 - b_4$ ) will result more SCA. Conversely, increasing  $O_1$  will result with an ACA.
- Increasing/decreasing the spread  $I_1$  and  $O_1$  (thus decreasing/increasing  $I_2$  and  $O_2$ ) simultaneously with the same ratio will result with a linear CA (LCA).

It can be concluded that it is possible to generate ACA and SCA by tuning the spreads of the AMFs and CMFs. Thus, to show the difference of the FLC generated by tuning the  $I_1$  (FLC-I1) and by tuning  $O_1$  (FLC-O1), we have examined the effects of various MF parameter settings on the output of the FLC and gain variations which are illustrated in Figure 4.7. The corresponding parameter settings are tabulated in Table 4.5. The presented results coincide with derivations presented in Table 4.4 and clearly illustrate the design flexibility of the SFLC structure. Moreover, to clearly distinguish the fundamental differences of the FLC-I1 and FLC-O1, we have presented in Figure

4.7 c the results of the FLC-I1-3 and FLC-O1-3 which result with the same  $K_f$  value for  $d_s \in [c_2, c_4]$ . It can be concluded that by tuning the spread of MFs, it is possible to determine the interval where the FLC should result with a constant gain.

**Table 4.4 :** Control actions of fuzzy logic controller

CA	Region			
	$d_s \in [c_1, c_2]$	$d_s \in [c_2, c_3]$	$d_s \in [c_3, c_4]$	$d_s \in [c_4, c_5]$
ACA	$\frac{(b_2-b_1)}{(c_2-c_1)} < 1$	$\frac{b_2}{c_2} > 1$	$\frac{b_4}{c_4} > 1$	$\frac{(b_5-b_4)}{(c_5-c_4)} < 1$
SCA	$\frac{(b_2-b_1)}{(c_2-c_1)} > 1$	$\frac{b_2}{c_2} < 1$	$\frac{b_4}{c_4} < 1$	$\frac{(b_5-b_4)}{(c_5-c_4)} > 1$
LCA	$\frac{(b_2-b_1)}{(c_2-c_1)} = 1$	$\frac{b_2}{c_2} = 1$	$\frac{b_4}{c_4} = 1$	$\frac{(b_5-b_4)}{(c_5-c_4)} = 1$

#### 4.4 The Design of The Single Input Fuzzy PID Controller

##### 4.4.1 The output formulation

The output of SFPID controller  $u$  is described in section 4.2 as follow.

$$u = \alpha A + \beta A t + (\alpha P + \beta D)e + \alpha D \dot{e} + \beta P \int e dt \quad (4.17)$$

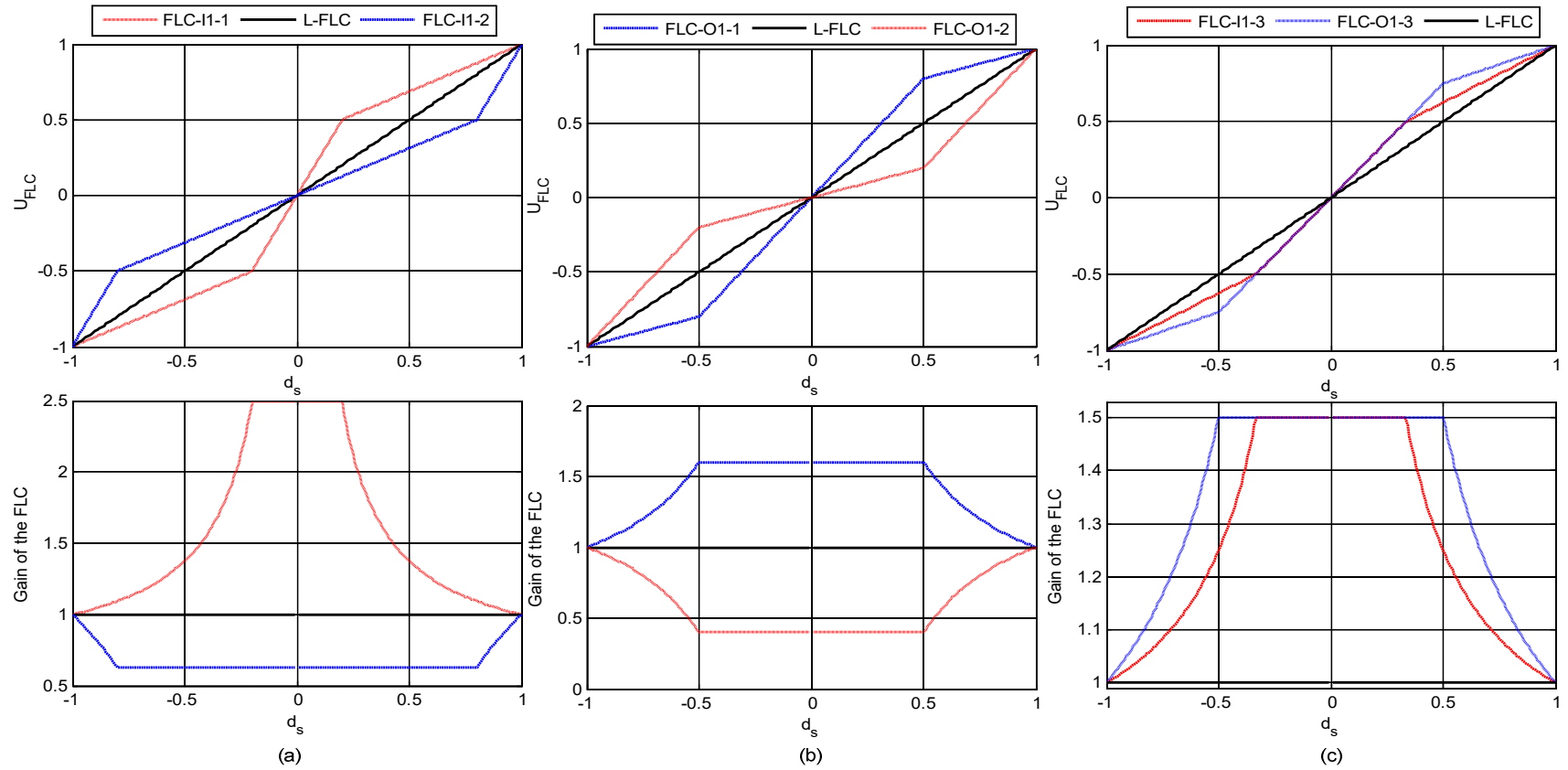
where

$$\begin{aligned}
 P &= \frac{b_{k+1}\lambda - b_k\lambda}{(c_{k+1} - c_k)\sqrt{1 + \lambda^2}} \\
 D &= \frac{b_{k+1} - b_k}{(c_{k+1} - c_k)\sqrt{1 + \lambda^2}} \\
 A &= \frac{b_{k+1}c_k - b_k c_{k+1}}{c_k - c_{k+1}}
 \end{aligned} \quad (4.18)$$

Now, in order to explicitly derive the output formulation of the SFPID, suppose that  $\int dt = 1/\rho$  and  $\gamma = \alpha/\beta$

**Table 4.5** : Parameter settings of the FLCs

FLC	Antecedent MF Parameters					Consequent MF Parameters					Spreads of the AMF		Spreads of CMS	
	$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	$I_1$	$I_2$	$O_1$	$O_2$
L-FLC	-1	-0.5	0	0.5	1	-1	-0.5	0	0.5	1	0.5	0.5	0.5	0.5
FLC-I1-1	-1	-0.2	0	0.2	1	-1	-0.5	0	0.5	1	0.2	0.8	0.5	0.5
FLC-I1-2	-1	-0.8	0	0.8	1	-1	-0.5	0	0.5	1	0.8	0.2	0.5	0.5
FLC-O1-1	-1	-0.5	0	0.5	1	-1	-0.8	0	0.8	1	0.5	0.5	0.8	0.2
FLC-O1-2	-1	-0.5	0	0.5	1	-1	-0.2	0	0.2	1	0.5	0.5	0.2	0.8
FLC-I1-3	-1	-0.33	0	0.33	1	-1	-0.5	0	0.5	1	0.33	0.67	0.5	0.5
FLC-O1-3	-1	-0.5	0	0.5	1	-1	-0.75	0	0.75	1	0.5	0.5	0.75	0.25



**Figure 4.7 :** Illustration of the outputs and gain variations of the (a) FLC-I1-1 and FLC-I1-2 (b) FLC-O1-1 and FLC-O1-2 (c) FLC-I1-3 and FLC-O1-3

$$\begin{aligned}
u &= \beta \left( \frac{b_k - b_{k+1}}{(c_k - c_{k+1})\sqrt{1 + \lambda^2}} (\lambda e + \dot{e}) + \frac{b_{k+1}c_k - b_k c_{k+1}}{c_k - c_{k+1}} \right) \left( \frac{1}{p} + \gamma \right) \\
u &= \beta \left( \frac{b_k - b_{k+1}}{(c_k - c_{k+1})\sqrt{1 + \lambda^2}} (\lambda e + \dot{e}) \right) \left( \frac{1}{p} + \gamma \right) \\
&\quad + \beta \left( \frac{b_{k+1}c_k - b_k c_{k+1}}{c_k - c_{k+1}} \right) \left( \frac{1}{p} + \gamma \right) \\
u &= u_1 + u_2
\end{aligned} \tag{4.19}$$

where

$$u_1 = \beta \left( \frac{b_k - b_{k+1}}{(c_k - c_{k+1})\sqrt{1 + \lambda^2}} (\lambda e + \dot{e}) \right) \left( \frac{1}{p} + \gamma \right) \tag{4.20}$$

$$u_2 = \beta \left( \frac{b_{k+1}c_k - b_k c_{k+1}}{c_k - c_{k+1}} \right) \left( \frac{1}{p} + \gamma \right) \tag{4.21}$$

Here,  $u_2$  can be seen as compensation term. On the other hand, it is possible to derive a transfer function from the control law  $u_1$  which is given in Eq. 4.20. Thus, by employing the Laplace transform, it can be obtained

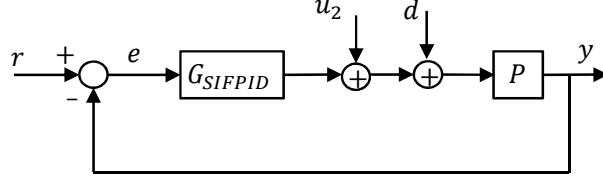
$$U_1(s) = \lambda \beta \left( \frac{b_k - b_{k+1}}{(c_k - c_{k+1})\sqrt{1 + \lambda^2}} \right) \left( \frac{\left(1 + \frac{1}{\lambda}s\right)(1 + \gamma s)}{s} \right) E(s) \tag{4.22}$$

Eq. 4.22 can be rewritten as a transfer function form.

$$\frac{U_1(s)}{E(s)} = \lambda \beta \left( \frac{b_k - b_{k+1}}{(c_k - c_{k+1})\sqrt{1 + \lambda^2}} \right) \left( \frac{\left(1 + \frac{1}{\lambda}s\right)(1 + \gamma s)}{s} \right) \tag{4.23}$$

$$\frac{U_1(s)}{E(s)} = G_{SF PID}(s) \tag{4.24}$$

In the same way with expression of the nonlinear term as a nonlinear compensation in mathematical description of FPID [3],  $u_2$  term can be considered as a compensation as shown in Figure 4.8. So, as it was said that the FPID controller can be expressed as a CPID, SFPID can be considered as a CPID with a compensation [39].



**Figure 4.84 :** Equivalent format of SFPID control in the closed loop.

#### 4.4.2 Internal model control based tuning for single input fuzzy PID controller

Assume that the design is accomplished under the assumption that  $d_s \in [c_2, c_4]$ , thus the compensation term  $u_2$  will be always zero.

If set  $G_{SFPID}(s) = G_{IMC}(s)$ , the IMC-based tuning for SFPID controllers can be simplified. By comparing Eq. 4.23 with Eq. 3.9, parameters of SFPID can be written as follows:

$$\lambda\beta \left( \frac{b_k - b_{k+1}}{(c_k - c_{k+1})\sqrt{1 + \lambda^2}} \right) \left( \frac{\left(1 + \frac{1}{\lambda}s\right)(1 + \gamma s)}{s} \right) = \frac{(1 + Ts) \left(1 + \frac{L}{2}s\right)}{K \left(\frac{L}{2} + t_c\right)s} \quad (4.25)$$

In terms of Eq. 4.25, parameters  $\lambda$ ,  $\gamma$  and  $\beta$  can be given as follows:

$$\beta = \frac{1}{\lambda K \left( \frac{b_k - b_{k+1}}{(c_k - c_{k+1})\sqrt{1 + \lambda^2}} \right) \left( \frac{L}{2} + t_c \right)} \quad (4.26)$$

$$\gamma = T \quad \frac{1}{\lambda} = \frac{L}{2} \quad \text{or} \quad \gamma = \frac{L}{2} \quad \frac{1}{\lambda} = T$$

And  $\alpha = \beta\gamma$  is formula of the output scaling factor  $\alpha$ .

The bandwidth of the fuzzy PID at the  $k$ th level can be controlled by adjusting  $\lambda$ . A large value of  $\lambda$  gives wide bandwidth and fast response. To improve the rise time, the value of  $\lambda$  should be large. Therefore, the two parameters  $\gamma$  and  $\lambda$  can be determined as

$$\gamma = \max\left(T, \frac{L}{2}\right) \quad \lambda = 1/\min\left(T, \frac{L}{2}\right) \quad (4.27)$$

It can be concluded that the SFPID structure gives the opportunity to designer to tune the SFs of the SFPID by employing the well-known IMC based PID strategy.

#### 4.5 Simulation Results

In this section, the simulation studies are presented where the performance of two SFPID structure which have different spreads of MFs are compared with a CFPD. In the simulation studies, all controllers are implemented as the discrete-time versions obtained with the bilinear transform with the sampling time  $T_s=0.01s$ . The simulations were performed on a personal computer with an Intel Core i7-4700MQ 2.40 GHz processor, 8 GB RAM, and software package MATLAB/Simulink 2014b.

The controller structures are evaluated on the following nonlinear system.

$$\frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} + 0.25y^2 = u(t - L) \quad (4.28)$$

where  $L$  represents the time delay and is taken as 0.5. In order to accomplish the IMC based design, the FOPDT model of the system is found as follows:

$$G_{FOPDT}(s) = \frac{4.0087}{1 + 2.0944s} e^{-1.86s} \quad (4.29)$$

In order to make a fair comparison, the performances of the controllers are evaluated with respect to the Settling Time ( $T_s$ ), Overshoot ( $OS\%$ ) and the IAE performance measures.

Two SFPID control structure which have deifferent sizes of MFs spread are designed. First one results a steady state gain  $K_f = 0.4$  (SFPID-1) and second one results a steady state gain  $K_f = 1.9$  (SFPID-2). The parameters of the AMF and CMF of the SFPID controllers are tabulated in Table 4.7. Then, the proposed IMC based design strategy in this thesis employed to obtain SFs of the SFPID-1 and SFPID-2 controllers. In order to make a fair comparison the time constant  $\lambda$  is set and fixed to 1.078 for both SFPID structures. The calculated SFs for SFPID-1 and SFPID-2 are found as  $\alpha_{SFPID-1} =$

0.9022,  $\beta_{SFPID-1} = 0.4307$  and  $\alpha_{SFPID-2} = 0.1899$ ,  $\beta_{SFPID-2} = 0.0907$ , respectively.

The CFPID controller is constructed with a symmetrical 3x3 rule base where 50% overlapping triangular MFs and crisp consequents are employed. It is worth to mention that, in comparison to the, both SFPID controllers are constructed only from 5 rules while the CFPID structure from 9 rules, i.e. in total 4 more rules and thus more design parameters. The SFs of the CFPID are found via the IMC based design method. For the same the time constant value  $\lambda = 1.078$ , The SFs of the CFPID are calculated as  $K_e = 4$ ,  $K_{de} = 3.709$ ,  $\alpha_{CFPID} = 0.0661$  and  $\beta_{CFPID} = 0.0316$ .

The performances of the SFPID and CFPID controllers are illustrated in Figure 4.9 for a reference signal with a magnitude of 0.25. The calculated performance measures are tabulated in Table 4.6. It can be clearly observed that, in comparison to the SFPID-1, the SFPID-2 was able to reduce the  $OS\%$  value almost by 29%. On the other hand, the SFPID-1 structure resulted with a faster convergence speed. It can be also observed that, in comparison to the CFPID, the SFPID-2 was able to reduce the  $OS\%$  value almost by 14%. Moreover, we have also employed an input disturbance at the 40th second with a magnitude of “0.03”. It can be clearly observed that both the CFPID and the SFPID resulted with almost an identical disturbance rejection performance. However, it is worth to remind that the transient performance of the SFPID-2 is better with respect to the performance values.

It can be concluded that by only tuning the spread of the SFPID structure, the control system designer has the opportunity to enhance the transient state performance while the preserving the satisfactory disturbance rejection performance of the CFPID.

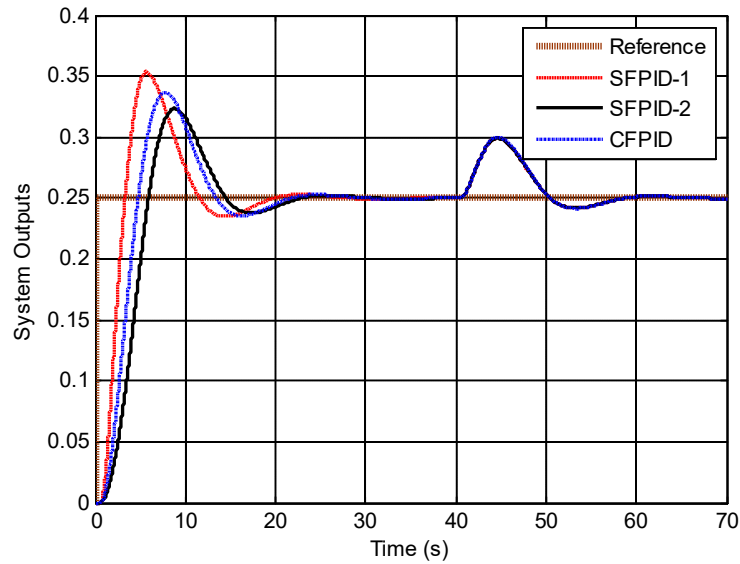
**Table 4.6** Performance measures

	Transient State Performance			Disturbance Rejection Performance
	$T_s(s)$	$OS\%$	$IAE$	$IAE$
<i>SFPID-1</i>	18.088	41.62	1.074	0.320
<i>SFPID-2</i>	20.706	29.45	1.368	0.320
<i>CFPID</i>	20.001	34.59	1.248	0.328

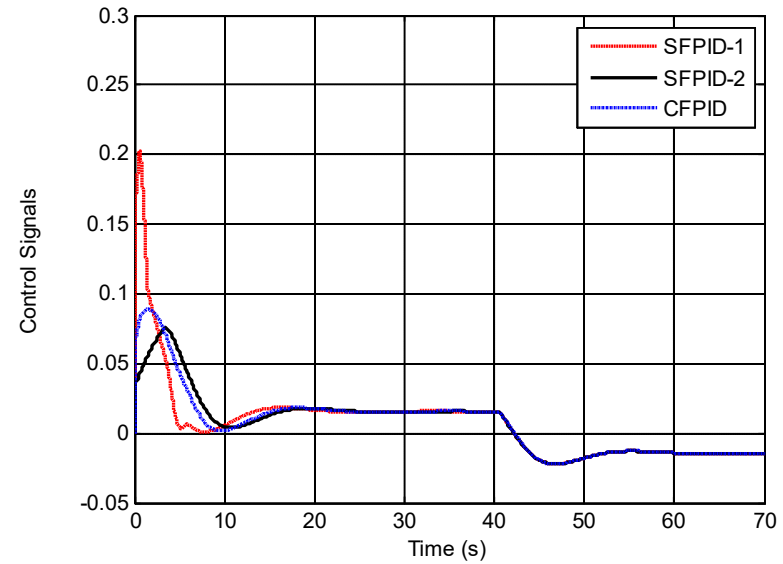


**Table 4.7 :** Parameter settings of the SFPID and CFPID structures

FLC	Antecedent MF Parameters					Consequent MF Parameters					<i>Spreads of the AMF</i>		<i>Spreads of the CMF</i>		
	$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$I_1$	$I_2$	$O_1$	$O_2$	
SFPID-1	-1	-0.4	0	0.4	1	-1	-0.16	0	0.16	1	0.4	0.6	0.16	0.84	
SFPID-2	-1	-0.4	0	0.4	1	-1	-0.76	0	0.76	1	0.4	0.6	0.76	0.24	
CFPID	$E$	-1	0	1	-	-	-1	0	1	-	-	1	1	1	1
	$\dot{E}$	-1	0	1	-	-	-1	0	1	-	-	1	1	1	1



(a)



(b)

**Figure 4.9 :** Illustration of the (a) system outputs and (b) control signals.



## 5. CONCLUSIONS AND RECOMMENDATIONS

Conventional control structures are inadequate while operating in nonlinear and high-order systems. The fuzzy controller structures, which allow the use of intuitive data in the design phase, provide higher performance control in such complex systems. Therefore, every day, fuzzy control structures take place of conventional control structures.

The selection of the appropriate controller for the system during the design as well as the correct design of the design parameters is one of the main factors directly affecting system performance. Although many parameters tuning methods have been studied for this, correct tuning of the parameters continues to be a problem. In fuzzy controller structures, the correct evaluation of linguistic terms, ie, input and output signals, is directly related to the proper selection of antecedent and consequent membership functions used in the design of the fuzzy interface. High degree of engineering experience is required at this stage of design in order to avoid data loss and to make accurate evaluation. Although various studies have been done in the regard, there is no established rule yet, the design is left to the engineer's skill.

In Chapter 2, fuzzy logic terms and expressions are given. Membership functions, fuzzification and defuzzification methods that form the basis of the fuzzy interface, are briefly mentioned.

In Chapter 3, general information about PID and CFPID controller structure is given. It is pointed out that CFPID resembles the structure of PID. Internal structure of the commonly used IMC based tuning method is described and it is applied on controllers.

In this thesis, the structure of SFPID controller is examined and the effect of membership functions on the input-output relationship is analyzed. As a result of this analysis, how the membership functions should have a distribution for the desired control structure is explained in detail in Section 4.3 and a rule template for future work is presented. The control structure is left to the designer to present linear,

aggressive or smooth control. How the relationship between antecedent and consequent MFs should be defined in order to able to obtain the selected control structure or to provide the desired gain value in the desired working range and it is shown in Figure 4.7. These definitions are derived using 5 antecedent and 5 consequent MFs. According to this;

- a) If a linear controller structure is to be selected, the MFs must be selected in a uniform distribution and the changes in the membership function distributions in the same ratio will not affect linearly.
- b) By removing the antecedent membership function distributions from the center, the aggressive controller structure will be obtained and the work area where stable gain is obtained will be narrowed. Antecedent membership function distributions have an effect on gain of the controller and fixed gain working range.
- c) The smooth controller structure will be obtained by removing the consequent membership function distributions from the center. However, consequent MFs alone do not have an effect on the working range where stable gain are obtained.

Another result obtained during the study is that it is possible to control in which working area the desired gain value can be obtained. In other words, by maintaining the ratio between the distributions of MFs, removing the distributions from the center or bringing them closer to the center, it will lead to enlargement or contraction of the field of study where the desired value of gain will be obtained.

The main purpose of this thesis is to prove the applicability of the IMC based tuning method to the SFPID controller. For this, it has been shown that the SFPID controller can be expressed as a combination of the CFPID and a compensation term by following the method in which Duan applies the IMC based tuning method to the CFPID controller. The necessary equations are presented taking into account the steady state conditions.

In the simulations studies, CFPID and two SFPID controllers which have different distribution MFs were compared. As a result, all controllers gave equivalent damping response to the disturbance signal. The aggressive controller responded faster while the smooth controller has less overshoot value than CFPID. Considering the obtained

results, it seems that the SFPID controller performs the same performance as the CFPID controller, although it has fewer rules and design parameters. Obtaining the different controller structures by only changing the antecedent and consequent membership function parameters shows the flexibility of the SFPID controller structure. It also allows the use of this flexible structure with less rules and linguistic terms than other fuzzy controller structures.

In future studies, the correctness of the suggested deductions in this thesis can be tested on real-time systems. It is also suggested to test different types of MFs and distribution variations to compare the obtained CSs with the antecedent and consequent membership function distributions presented.



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### PUBLICATIONS, PRESENTATIONS AND PATENTS ON THE THESIS:

- **Var, A., Kumbasar, T., & Yesil, E.** (2015, August). An Internal Model Control based design method for Single input Fuzzy PID controllers. In Fuzzy Systems (FUZZ-IEEE), 2015 IEEE International Conference on (pp. 1-7). IEEE.

### OTHER PUBLICATIONS, PRESENTATIONS AND PATENTS: