ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL OF SCIENCE
ENGINEERING AND TECHNOLOGY

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Ph.D. THESIS

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Department of Control and Automation Engineering

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MIMO SİSTEMLERİN KÖŞEGEN BASKINLIĞI VE KARARLILIĞI İÇİN PARAMETRE BÖLGELERİNİN BELİRLENMESİ

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To all of my family and the people who believe in the guidance of science...

FOREWORD

It is a fact that PhD is one of the most challenging steps for a researcher. During my PhD period, a lot of people contributed to this thesis in different aspects. Hereby, I would like to take the opportunity to thank all of these great people.

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As one of the research assistants of ITU Control and Automation Engineering Department for 8 years, I would like to thank all of my colleagues for their help and making my time here joyful. Especially, I would like to express gratitude to my friends (Baran Alikoç, Uğur Yıldırım, Oytun Eriş, Emre Dincel) whom I started to work more or less in the same period.

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ABBREVIATIONS

ARE : Algebraic Riccati Equation
CDD : Column Diagonal Dominance
CN : Condition Number
CRB : Complex Root Boundary
IRB : Infinite Root Boundary
LHP : Left Half Plane
LPV : Linear Parameter Varying
LTI : Linear Time Invariant
LTIPD : Linear Time Invariant Parameter Dependent
LTV : Linear Time Varying
MIMO : Multi Input Multi Output
MPC : Model Predictive Control
PD : Proportional Derivative
PI : Proportional Integral
PID : Proportional Integral Derivative
PSA : Parameter Space Approach
RDD : Row Diagonal Dominance
RGA : Relative Gain Array
RHP : Right Half Plane
RMPC : Robust Model Predictive Control
RRB : Real Root Boundary
SISO : Single Input Single Output
SVD : Singular Value Decomposition
TFM : Transfer Function Matrix
TITO : Two Input Two Output

SYMBOLS

$\mathbb{R}$	: Set of real numbers.
$\mathbb{X}$	: Set of admissible state values.
$\mathbb{U}$	: Set of admissible input values.
$\mathbb{K}$	: Set of admissible controller values.
x	: State vector.
μ_{c_i}	: Weighting factor for column diagonal dominance
μ_{r_i}	: Weighting factor for row diagonal dominance
$V(x)$	: Candidate Lyapunov function.
$A(k)$	: Closed loop system matrix.
$P(k)$	: Lyapunov matrix.
$M(k)$	: Kronecker product based matrix.
I	: Identity matrix.
D	: Pre-compensator transfer function matrix.
λ_i	: Eigenvalues of the corresponding matrix.
G_{cl}	: Closed loop transfer function.
$G(s)$	: Transfer function of the plant.
$K(s)$	: Transfer function of the controller.
u_i	: Control sign.
y_i	: System output.
Re	: Real part of the polynomial.
Im	: Imaginary part of the polynomial.
j	: Imaginary unit.
k_d	: Derivative gain for PID controller.
k_i	: Integral gain for PID controller.
k_p	: Proportional gain for PID controller.
k_{p_1}, k_{p_2}	: Diagonal static controller parameters.
α_i	: Real part of the controller.
β_i	: Imaginary part of the controller.
r	: Reference input.
s	: Complex argument for the Laplace transform.
$\times$	: Schur multiplication.
$\otimes$	: Kronecker product.
$\delta(s, q_i)$	: Parameter uncertain characteristic polynomial.
$\delta(s)$	: Characteristic polynomial.
$G(s, q_i)$	: Uncertain plant transfer function.
q_i	: Uncertain parameters.

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DETERMINATION OF PARAMETER REGIONS FOR DIAGONAL DOMINANCE AND STABILITY OF MIMO SYSTEMS

SUMMARY

Most of the industrial plants include more than one input and output variable. Compared to Single Input Single Output (SISO) systems, such systems include different structural properties. For instance, an output variable is effected by all input variables in general.

On the other hand, in terms of controller structures, researchers have focused on two main approaches for such systems, which are "centralized" and "decentralized" controllers. However, it can be proposed that decentralized controllers are preferred more in practice due to various reasons like less number of tuning parameter, possibility to apply single loop controller design methods, ease of use for operators etc. Whereas, in general, performance and efficiency of such controllers reduce when there are significant interactions between different input-output pairs in a Multi Input Multi Output (MIMO) system.

Reducing the interactions between different input-output pairs in MIMO systems is crucial in terms of decentralized controller design due to the previously mentioned reasons. Diagonal dominance which is a weaker condition compared to decoupling, is one of the approaches that can be used to reduce interactions in MIMO systems. One input variable is strongly related with one specific output variable in diagonal dominant systems. One of the main aims of this thesis is to determine controller parameter regions that achieve diagonal dominance conditions. Additionally, it is also aimed to determine stabilizing parameter spaces, since diagonal dominance does not indicate stability in general. As a result, controller parameter regions that achieve both diagonal dominance and stability conditions in closed loop are determined in this thesis as the first step of decentralized controller design.

In literature, the diagonal dominance concept has gained attraction since the pioneering studies of Rosenbrock in early 1970s. However, in the meantime most of the researchers focused on determining a specific controller parameter pair that optimizes a predetermined condition. Such a case may restrict the designer in the next steps of the design process. Additionally, the number of studies are limited that investigates the diagonal dominance characteristics of the determined controller parameters in case uncertainties or checks how the system is close to the diagonal dominance boundaries.

Two Input Two Output (TITO) systems are special subset of MIMO systems since in practice many MIMO systems can be treated as several TITO subsystems as proposed in literature. In terms of diagonal dominance, particularly, TITO systems and diagonal type controllers are discussed in detail, since it is aimed to determine necessary and sufficient conditions on diagonal dominance in terms of controller parameters. For such systems, exact conditions on the controller parameters in terms of both column and row diagonal dominance are derived at a given fixed frequency. Derived results

are also valid for finite number of frequencies and practically applicable for a given frequency range. Moreover, weighting factors are added to the original definition of diagonal dominance in order to derive controller parameter regions that achieve better diagonal dominance ratios. Necessary and sufficient conditions on diagonal type controllers are also derived for the weighted diagonal dominance problem. Lastly, critical frequencies that may possibly change the interval characteristics of static diagonal controllers for the column diagonal dominance are derived. Effectiveness of the derived results in terms of diagonal dominance are demonstrated over several case studies using Gershgorin Disc plots and diagonal dominance ratio plots.

On the other hand, a Lyapunov equation based stability mapping approach is proposed within the scope of this thesis to derive stabilizing controller parameter spaces of a given MIMO system. In the present approach, it is not necessary to calculate singular frequencies or apply frequency sweeping that most of the frequency based approaches require. From the Lyapunov point of view, positive definiteness of the Lyapunov matrix $P(k)$ is necessary and sufficient for LTI systems. However, considering the numerators and denominators of the leading principal minors it is required to solve $2n$ parametric equation in order to determine positive definiteness of $P(k)$. This number is reduced to $n+1$ at the first step. After that, Lyapunov matrix equation is reduced to the standard set of equation representation using the Kronecker products and vectorization operator. At this point, a new matrix $M(k)$ is defined over the Kronecker products and it is shown that determinant of $M(k)$ is the product of binary combinations of $A(k)$. Using the relations between the system matrix $A(k)$, Lyapunov matrix $P(k)$ and $M(k)$, it is shown that it is sufficient to solve at most 2 parametric equations which are $|M(k)| = 0$ and $|M(k)| \to \infty$. Determinant of $M(k)$ includes redundant multiplications of binary combinations of eigenvalue pairs of $A(k)$ due to the matrices $P(k)$ and Q that are used in Lyapunov formulation are symmetric. In order to eliminate the redundant multiplications and reduce the computational complexity, elimination and duplication matrices are introduced as transformation matrices.

In addition to MIMO systems, the proposed stability mapping approach is applicable to a broad range of systems, further system classes and sub problems where Lyapunov formulation is possible. In order to demonstrate these properties of the proposed approach, firstly, controller integrity problem of MIMO systems is discussed in detail. An approach is proposed to determine stabilizing controller parameter regions even in case of possible failures related with controller parameters. A benchmark case study is included and effectiveness of the proposed approach is shown over a comparative study with a currently existing approach. Additionally, discrete time systems is also discussed in detail to demonstrate the further application areas of the proposed Lyapunov equation based stability mapping approach. In this case, the structure of the Lyapunov equation varies slightly compared to the continuous time case. Another benefit of the proposed Lyapunov equation based approach is the opportunity to determine analytical expressions of stability boundaries. So that, it becomes possible to use Lyapunov equation based stability mapping approach in optimization based approaches by inserting the stability boundaries as constraints on such approaches. This case is also addressed through the robust Model Predictive Control (MPC) problem. Analytical stability boundaries which is derived in the off-line phase using the proposed stability mapping approach is inserted to the robust MPC problem formulation to achieve stability. In this way, robust MPC problem is transformed into the nominal MPC problem. The effectiveness of the proposed method is also demonstrated through a benchmark system that is frequently used in the literature.

Diagonal dominance proposes weaker conditions compared to decoupling. As a result, it becomes possible to determine controller parameter regions that achieve diagonal dominance in case of parametric uncertainties. Within the scope of this thesis, two conservative approaches which are based on triangular inequality and griding are proposed for the systems that include interval type uncertainties in Transfer Function Matrix (TFM) elements. Using these approaches diagonal dominance problem of a parametric uncertain system is transferred to the weighted diagonal dominance problem of the nominal plant. After that, previously derived results are used to determine static diagonal controller parameter regions.

Lastly, stability of parameter uncertain multivariable systems is discussed in order to determine robustly stabilizing parameter spaces. There are two main assumptions on uncertain parameters in literature. In the first assumption, there is no restriction on uncertain parameters and it is aimed to determine all uncertain parameter spaces that preserve stability of the closed loop system. In this case, proposed Lyapunov equation based stability mapping approach is directly applicable. Contrary to this approach, many methods that is currently available in the literature include the results obtained by making some assumptions on the number and the type of uncertain parameters. The validity of the Lyapunov equation based method has been demonstrated through different benchmark case studies. On the other hand, in some cases, it is assumed that upper and lower bounds of uncertain parameters are known. It is aimed to determine whether the whole polynomial family is stable in all cases where the uncertain parameters take any value between these known intervals. In some special cases, it was shown in literature that stability of finite number fixed polynomials guarantee the stability of whole uncertain polynomial family in case of SISO systems. However, the characteristic polynomial of MIMO systems includes the multiplication of free controller parameters and individual transfer functions even in the simplest cases. As a result, it can be proposed that compared to SISO systems, it is more difficult to determine the controller parameter areas that provide robust stability in such systems. In the discussed problem characteristic equation includes both uncertain parameters that have known upper and lower bounds and free controller parameters. In this thesis, an approach is presented to determine robustly stabilizing parameter spaces using the Kharitonov Theorem in accordance with the Lyapunov method by applying overbounding method on characteristic polynomial coefficients. The proposed method reduces the computational complexity significantly, since Kharitonov Theorem is used. However, it must also be noted that calculation of invariant controller parameter sub regions in terms of overbounding also introduces additional analysis steps.

As a conclusion, in this thesis, it is mainly focused on determining controller parameter regions of the diagonal type controllers that make both nominal and parametric MIMO systems diagonal dominant and stable. The results are derived through TITO systems from the standpoint of diagonal dominance, since it is aimed to determine the necessary and sufficient conditions. On the other hand, there is no restriction on the system and controller type for the proposed stability mapping approach.

MIMO SİSTEMLERİN KÖŞEGEN BASKINLIĞI VE KARARLILIĞI İÇİN PARAMETRE BÖLGELERİNİN BELİRLENMESİ

ÖZET

Endüstride karşılaşılan sistemlerin birçoğu birden fazla giriş ve çıkış değişkenine sahiptir. Bu tarz sistemler SISO sistemlerle karşılaştırıldıklarında, birçok farklı yapısal özellikleri göze çarpmaktadır. Örneğin, en genel durumda herhangi bir çıkış tüm girişlerden az veya çok etkilenir.

Diğer taraftan, kontrolör türleri açısından bakıldığında ise araştırmacılar genel olarak "merkezi" ve "merkezi olmayan" olmak üzere iki farklı kontrol yapısına odaklanmışlardır. Ancak, ayarlanacak parametre sayısının azlığı, dayanıklılık ve operatör açısından kullanım kolaylığı gibi nedenlerle merkezi olmayan kontrol yapılarının uygulamalarda daha sık tercih edildiği ileri sürülebilir. Farklı giriş çıkış çiftleri arasındaki etkileşimlerin önemli boyutlara ulaştığı durumlarda ise bu tür kontrolörlerin performansı ve etkinliği genel olarak azalır.

Yukarıda bahsedilen nedenlerden dolayı MIMO sistemlerde etkileşimlerin azaltılması özellikle merkezi olmayan kontrolör tasarımı açısından büyük bir önem arz etmektedir. MIMO sistemlerde etkileşimleri azaltmak amacıyla kullanılabilecek yöntemlerden bir tanesi de tam köşegenleştirme ile karşılaştırıldığında daha zayıf bir koşulun sağlanmasını gerektiren köşegen baskınlık kavramıdır. Köşegen baskın sistemlerde bir giriş değişkeni özel bir çıkış değişkeni ile diğer çıkışlara oranla çok daha fazla ilişkilidir. Bu nedenle, bu tezin en temel hedeflerinden bir tanesi MIMO sistemlerde köşegen baskınlık koşullarını sağlayan kontrolör parametre bölgelerinin belirlenmesidir. Buna ek olarak, en genel durumda köşegen baskınlık kararlılığı gerektirmediğinden çok değişkenli sistemleri kararlı kılan kontrolör parametrelerinin belirlenmesi de yine bu tez kapsamında amaçlanan temel hedeflerden bir diğeridir. Sonuç olarak, merkezi olmayan kontrolör tasarımına ön adım oluşturacak şekilde hem köşegen baskınlık hem de kararlılık koşullarının sağlandığı kontrolör parametre bölgelerinin belirlenmesi hedeflenmektedir.

Literatürde köşegen baskınlık kavramının önemi özellikle Rosenbrock'un 1970'lerin başındaki çalışmalarından sonra artmıştır. Ancak süreç içerisinde araştırmacıların büyük bir çoğunluğu köşegen baskınlık ile ilgili olarak belirli bir ölçütü en iyileyen kontrolör parametre çiftlerinin belirlenmesine yönelmiştir. Bu durum ise bir sonraki tasarım adımında kısıtlamalara neden olabilmektedir. Buna ek olarak, parametre belirsizliği durumunda köşegen baskınlığın korunup korunmadığı ve/veya belirlenen parametre çiftinin köşegen baskınlık sınırlarına ne kadar yakın olduğu genel olarak detaylı bir şekilde araştırılmamıştır.

Bu tez kapsamında köşegen baskınlık üzerindeki gerek ve yeter koşulların belirlenmesi hedeflendiğinden, özel olarak TITO sistemler ve köşegen yapıdaki kontrolör durumu detaylı olarak ele alınmıştır. Bu tarz sistemleri, verilen sabit bir frekans değerinde köşegen baskın kılan kontrolör parametreleri üzerindeki gerek ve yeter koşullar

belirlenmiştir. Elde edilen sonuçlar sonlu sayıdaki frekans noktası için de geçerlidir ve pratik açıdan bakıldığında verilen bir frekans aralığına da genişletilebilir durumdadır. Buna ek olarak, daha iyi baskınlık oranı sağlayan parametre bölgelerinin belirlenmesine yönelik olarak orjinal köşegen baskınlık tanımına ağırlık faktörleri eklenmiş ve bu durum için gerek ve yeter koşullar belirlenmiştir. Son olarak da statik köşegen kontrolör durumunda sütun köşegen baskınlığı için kontrolör parametre bölgelerinin yapısını değiştiren kritik frekans değerleri belirlenmiştir. Elde edilen sonuçların köşegen baskınlık açısından etkinlikleri, örnek sistemler ve farklı kontrolörler üzerinden, Gershgorin Diskleri ve köşegen baskınlık çizimleri kullanılarak gösterilmiştir.

MIMO sistemleri kapalı çevrimde kararlı kılan kontrolör parametrelere bölgelerinin belirlenmesi için ise Lyapunov eşitliği temelli bir yöntem ileri sürülmüştür. Bu yöntem sayesinde frekans tabanlı yöntemlerde karşılaşılan tekil frekansların hesaplanması ve/veya frekans taraması gibi adımlara olan ihtiyaç ortadan kaldırılmıştır. Temel Lyapunov yaklaşımı açısından bakıldığında LTI sistemler için Lyapunov matrisi olan $P(k)$'nın pozitif tanımlılığı gerek ve yeter koşuldur. Ancak, Lyapunov matrisi $P(k)$'nın pozitif tanımlılığı en genel durumda $2n$ adet parametrik eşitliğin çözümünü gerektirir. Yapılan analizle bu sayı önce $n+1$'e indirilmiştir. Ardından, Lyapunov matris eşitliği Kronecker çarpımları ve vektörizasyon operatörü kullanılarak standart forma indirgenmiş ve tanımlanan yeni $M(k)$ matrisinin determinantının tartışılan sistem için bir kararlılık sınırı oluşturduğu sistem matrisi $A(k)$, Lyapunov matrisi $P(k)$ ve Kronecker çarpımları üzerinden tanımlanan $M(k)$'nin birbirleriyle olan ilişkileri üzerinden gösterilmiştir. Dolayısıyla $M(k)$ matrisinin determinantını sıfır ve sonsuz yapan kontrolör parametrelerinin ilgili sistemin kararlılık sınırını oluşturduğu belirlenmiştir. Diğer bir deyişle, kararlılık sınırlarının belirlenmesi en fazla iki adet parametrik ifadenin çözümüne indirgenmiştir. Lyapunov formulasyonunda kullanılan $P(k)$ ve Q matrislerinin simetrikliğinden kaynaklanan $M(k)$ matrisinin determinantınındaki tekrarlanan özdeğerler ise eleminasyon ve duplikasyon matrisleri kullanılarak uygulanan dönüşümler yardımıyla ortadan kaldırılmıştır. Önerilen yöntemin literatürde var olan PSA gibi yöntemlerle ilişkisi ise sonlu ve sonsuz kök sınırları üzerinden gösterilmiştir.

Kararlı kılan kontrolör parametre bölgelerinin belirlenmesinde Lyapunov temelli bir yaklaşım kullanıldığından öne sürülen yöntem sadece MIMO sistemlerde değil Lyapunov formülasyonunun kurulabildiği çok geniş bir sistem sınıfına ve alt problemlere de uygulanabilir durumdadır. Bu durumu gösterebilmek amacıyla ilk olarak MIMO sistemlerde kontolör entegrasyonu problemi ele alınmıştır. MIMO kontrolörlerde meydana gelebilecek olası hataları göz önünde bulundururak olası hata durumlarında dahi sistemin kararlılığını garanti etmeyi amaçlayan bu probleme bir çözüm önerisi sunulmuştur. Önerilen yöntemin etkinliği literatürde var olan yaklaşımlar üzerinden karşılaştırmalı olarak gösterilmiştir. Buna ek olarak, yine önerilen Lyapunov eşitliği temelli yöntemin olası diğer kullanım alanlarını vurgulamak amacıyla ayrık zamanlı sistemlerin kararlılığı ayrıntılı olarak tartışılmıştır. Bu durumda önerilen yaklaşımın nasıl değiştiği vurgulanmıştır. Lyapunov temelli yaklaşım ile kararlılık sınırlarının analitik ifadelerinin belirlenmesi de mümkündür. Bu durum da özellikle optimizasyon temelli tasarım yöntemlerinde farklı kullanım alanları açmaktadır. Bu kapsamda dayanıklı MPC problemi detaylı olarak ele alınmıştır. Lyapunov yöntemi kullanılarak belirlenen analitik kararlılık sınırları dayanıklı MPC problem formülasyonunda kullanılarak ele alınan problem nominal MPC problemine dönüştürülmüştür. Önerilen yöntemin etkinliği literatürde sıklıkla kullanılan bir sistem üzerinden de gösterilmiştir.

Tam köşegenleştirme ile karşılaştırıldığında, köşegen baskınlık daha zayıf bir koşul olarak ortaya çıkar. Bu nedenle, parametre belirsizlikleri durumunda dahi bu koşulu sağlayan kontrolör parametrelerini belirlemek mümkün hale gelir. Bu tez kapsamında, TFM elemanlarının aralık tipi parametre belirsizliği içerdiği TITO sistemler detaylı olarak tartışılmıştır. Bu tür sistemleri parametre belirsizlikleri durumunda dahi köşegen baskın kılan statik köşegen kontrolörlerin belirlenmesi hedeflenmiştir. Bu hedef doğrultusunda üçgen eşitsizliği ve tarama yöntemlerine dayanan iki farklı konservatif yöntem önerilmiştir. Bu yaklaşımlar kullanılarak tartışılan problem ilk aşamada nominal sistemin ağırlıklandırılmış baskınlık problemine dönüştürülmüştür. Sonrasında da önceki bölümlerde elde edilen sonuçlar kullanılarak sonuca gidilmiştir.

Son olarak da belirsiz parametre içeren çok değişkenli sistemlerin kararlılığı tartışılmıştır. Bu aşamada belirsiz parametreler için literatürde kullanılan iki farklı varsayıma yer verilmiştir. İlk varsayımda belirsiz parametreler üzerinde herhangi bir kısıtlama yoktur ve sistemi kararlı kılan tüm belirsiz parametre bölgelerinin belirlenmesi hedeflenmektedir. Bu durumda önerilen Lyapunov temelli yöntem direkt olarak uygulanabilir durumdadır. Bu yöntemin aksine literatürde var olan bir çok yöntemde ise belirsiz parametre sayısı ve türü üzerinde bir takım varsayımlarda bulunularak sonuçlar elde edilmiştir. Bu tez kapsamında önerilen yöntemin doğruluğu literatürde var olan farklı örnek durumlar üzerinden gösterilmiştir. Diğer taraftan, bazı durumlarda belirsiz sistem parametrelerinin alabileceği minimum ve maksimum değerler belirlidir. İlgili parametrenin bilinen bu değerler arasında bir değer aldığı tüm durumlarda polinom ailesinin kararlı kalıp kalmadığının belirlenmesi hedeflenir. SISO sistemler için bazı özel durumlarda sonlu sayıda polinomun kararlı olmasının tüm polinom ailesinin kararlığını garanti ettiği gösterilmiştir. MIMO sistemlerde ise en basit durumlarda bile kontrolör parametrelerinin ve TFM'yi oluşturan transfer fonksiyonlarının çarpımları karakteristik polinomda görünmektedir. Tartışılan bu problemde karakteristik polinom, hem alt ve üst sınırları bilinen belirsiz parametreleri hem de serbest kontrolör parametrelerini içermektedir. Bu tez kapsamında yukarı yakınsama yaklaşımından da yararlanılarak, Kharitonov Teoremi ve önerilen Lyapunov eşitliği temelli yaklaşımla bu tarz sistemleri dayanıklı kararlı kılan kontrolör parametre bölgelerinin belirlenmesine yönelik bir yöntem önerilmiştir. Önerilen bu yöntem Kharitonov Teoremi de kullanıldığından hesaplama yükünü önemli oranda azaltmaktadır ancak değişmez kontrolör parametre bölgelerinin belirlenmesinde ek analiz adımlarını da beraberinde getirmektedir.

Özetle, bu tez kapsamında nominal ve parametre belirsiz MIMO sistemeleri hem köşegen baskın kılan hem de kararlı yapan köşegen tipteki kontrolörlerin parametre bölgelerinin belirlenmesi hedeflenmiştir. Köşegen baskınlık açısından bakıldığında gerek ve yeter koşulların belirlenmesi hedeflendiğinden TITO sistemler üzerinden sonuçlar elde edilmiştir. Diğer taraftan kararlı kılan kontrolör parametrelerinin belirlenmesinde ise herhangi bir sistem veya kontrolör kısıtı bulunmamaktadır.

1. INTRODUCTION

1.1 Motivation

Decentralized controllers are preferred in practice due to various reasons like: less number of tuning parameters, flexibility of the controller design, applicability of single loop design methods, robustness against uncertainties and ease of use for operators [1]. However, as indicated in various sources [2, 3], in general performance and efficiency of such controllers decrease in case of significant interactions between different input-output pairs. As a result, it becomes crucial to decrease the interaction in a given multivariable system as the first step of decentralized controller design. In literature various decoupling methods were proposed to fully decouple the discussed MIMO system. However, most of the proposed decoupling methods face stability and causality based realization problems and they are applicable for a limited class of multivariable systems [4, 5]. Furthermore, their robustness against disturbances and measurement noises is also questionable even in the cases where fully decoupling is possible in theory. Moreover, it is also not possible to fully decouple the MIMO system in case of parametric uncertainties.

Instead of decoupling, diagonal dominance concept that is known in literature for years can be preferred in order to reduce interactions [6]. The conditions on diagonal dominance are weaker compared to decoupling methods. As a result, this condition is applicable for a broader range of systems. However, most of the researchers were focused on determining specific controller parameter pairs using different mathematical approaches like LMIs, global optimization approaches and scaling matrices etc. [7–9]. Moreover, diagonal dominance characteristics of the derived parameter pairs were not analyzed in detail for the case of parametric uncertainties. There are limited number of studies in literature that aims to achieve diagonal dominance conditions even in case of uncertainties [10–12]. For these reasons, it is aimed within the scope of this thesis to determine necessary and sufficient conditions on diagonal type controllers for TITO systems.

On the other hand, since it is defined over magnitudes, diagonal dominance does not indicates stability in general while there are some studies in terms of diagonal dominance and stability for special cases [13–15]. Even in the simplest cases, multiplication of the controller parameters and individual transfer functions are included in MIMO systems. This also makes the problem of determining stability boundaries more difficult compared to SISO systems. As a result, it is also aimed to propose approaches in order to determine stabilizing parameter spaces of multivariable systems.

As indicated earlier, the number of studies deals with robust diagonal dominance of MIMO systems are limited [16–18]. So, one of the main motivations of this thesis is to derive results on diagonal dominance even in case of parametric uncertainties. In addition to this, parameter uncertain MIMO systems are also discussed from the stability point of view since it is common to face uncertainties due to various reasons.

1.2 Literature Survey

Three main topics of the thesis which are diagonal dominance of TITO systems, stability of multivariable systems and the case of parametric uncertainties are discussed in three main sections. In order to preserve readability of the thesis, literature surveys and previously proposed studies are presented at the beginning of each main section.

Literature survey related with the diagonal dominance of MIMO systems can be found in Section 3.1. Previous studies related with the stability and stabilizing parameter space calculation of multivariable system are presented in Section 4.1. For the case of parametric uncertainty, literature reviews are respectively included in Section 5.1 and Section 5.2 for diagonal dominance and stability.

1.3 Goal and Unique Aspect of the Thesis

Reducing the interactions in multivariable systems can be accepted as the first step of decentralized controller design, since the performance of such controllers reduce in case of significant interactions in general. For this reason, proposing new approaches to determine controller parameter spaces of diagonal type controllers that achieve diagonal dominance is the focus of this thesis. More specifically, it is aimed to determine diagonal dominance conditions at a given fixed frequency for a given TITO system. Additionally, it is also intended to determine stabilizing parameter spaces, since diagonal dominance concept does not guarantee stability in general. Exact parameter spaces for a given nominal system and conservative regions for the case of parametric uncertainties are targeted.

In the context of this thesis, necessary and sufficient conditions on diagonal type controllers are derived for TITO systems at a given fixed frequency. Using the derived results, it is also possible to obtain results in terms of diagonal dominance for a given frequency range. Moreover, weighting factors are added to the original definition of diagonal dominance in order to derive controller parameter regions that cause better diagonal dominance ratios. Furthermore, for the case of static diagonal controllers and column diagonal dominance, critical frequencies that effects the interval characteristics of controller gains are derived.

Using the Kronecker products and vectorization operator a Lyapunov equation based approach is presented to derive stabilizing controller parameters in MIMO systems. Unlike the most of the previously proposed approached in literature, present approach is independent from the type of controller and the number of free controller parameters. Moreover, in addition to the MIMO systems, by minor modifications, it is also applicable to broad range systems (discrete time, switching, descriptor systems etc.) where Lyapunov formulation is possible. A solution strategy to derive exact stability regions for the controller integrity problem is also presented using the present approach.

It is also possible to derive analytical expressions of the stability boundaries in the proposed approach. As a result, it is possible to use that approach in optimization based techniques to reduce computational load. Derived results in terms of stability are used in robust MPC problem formulation and robust MPC problem is translated to nominal MPC by introducing the stability boundaries in the problem formulation.

Diagonal dominance problem of parameter uncertain TITO systems is also discussed in the context of thesis. Two approaches are presented that depends on triangular inequality and griding in order to determine static diagonal controller parameter regions. Using these approaches diagonal dominance problem of a parameter uncertain system is translated to the weighted diagonal dominance problem over certain weighting factors. Then, previously derived results are used to obtain controller parameter regions.

On the other hand, related with the stability of parameter uncertain multivariable systems first it is shown that in addition to free controller parameters Lyapunov equation based approach is also suitable to determine the bounds of uncertain parameters. Lastly, the case of interval type uncertainties discussed. A new approach is proposed using the overbounding technique and Kharitonov Theorem in accordance with the Lyapunov Equation based stability mapping approach to determine robustly stabilizing controller parameters.

1.4 Structure of the Thesis

Firstly, the motivation and goal of the thesis are presented in Section 1 in addition to the unique aspects of the thesis and brief literature survey.

After that, preliminary information related with the currently existing approaches which are used as interaction measures like Relative Gain Array (RGA), Singular Value Decomposition (SVD), Condition Number (CN) and decoupling methods are presented in Section 2. Possible problems related with the decoupling methods are also discussed over illustrative examples in the same section.

Derived necessary and sufficient conditions in terms of diagonal dominance and weighted diagonal dominance are presented in Section 3 for the case of diagonal type controllers. Using the derived results an algorithm is proposed in order to determine controller parameter regions that achieve weighted diagonal dominance conditions. It is possible to extend the proposed algorithm for a given frequency range while it is initially proposed for fixed frequencies. Derived theoretical result in terms of diagonal dominance are demonstrated over several case studies.

Stability of MIMO systems is discussed in Section 4 from the Lyapunov equation point of view. It is shown that is possible to determine stability boundaries in terms of free controller parameters using the Lyapunov equation based stability mapping approach. Its link with currently existing approaches like PSA is also set over finite and infinite stability boundaries. In the proposed approach it is not necessary to compute singular frequencies which is required in lots of frequency based approaches. Moreover, proposed approach is independent from the controller type and the number of free controller parameters. As a result, it is applicable to a broad range of systems. Additionally, by applying minor modifications it is also possible to apply proposed approach to other type of systems like discrete time systems, switching systems by where Lyapunov formulation is possible. These further application areas are also presented in Section 4.5.

The case of parametric uncertainties are discussed for both diagonal dominance and stability problem in Section 5. Triangular inequality and griding based approaches are presented in order to determine controller parameter regions that achieve diagonal dominance even in the case of parametric uncertainties. Using the aforementioned approached the problem is first translated to the weighted diagonal dominance and previously derived results are used afterwards. In Section 5.2.1 it is shown that proposed Lyapunov equation based approach is also suitable to determine the regions of both uncertain and free control parameters. Additionally, using the Kharitonov Theorem and Lyapunov equation based stability mapping technique an approach is presented in Section 5.2.2 to determine robustly stabilizing controller parameter spaces.

Derived results in terms of both diagonal dominance and stability of multivariable systems are summarized in Section 6. Furthermore, planned future studies are also expressed in the same section.

2. INTERACTION MEASURES AND DECOUPLING METHODS

As indicated in [19,20], most of the industrial plants can be considered as multivariable systems and a specific output variable is effected by all input variables in general as it is shown in Figure 2.1 for the TITO case.

From the controller design point of view, there are two main approaches for multivariable systems which are centralized and decentralized control. In the MIMO systems where centralized controllers are applied, each manipulated variable may depend on more than one manipulated variable. Due to various reasons like the number of free control parameters complexity of the controller design increases in such cases.

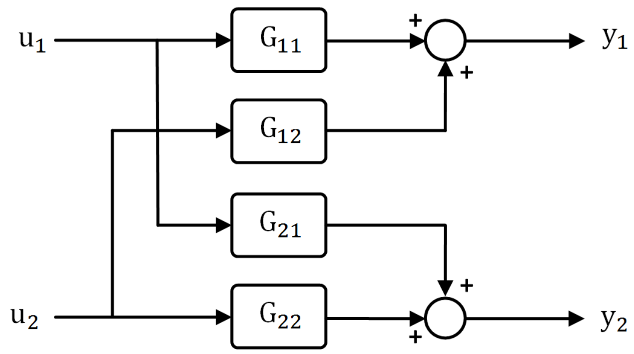


Figure 2.1 : Process interaction in a TITO system.

On the other hand, diagonal type controllers are preferred in practice due to their relatively simple structures and operability. These kind of controller are also referred as decentralized controllers in literature. As indicated in [21], these kind of controllers are easier to understand by operators, tuning parameters often localized effects and they are in general less sensitive to uncertainties. Simplified implementation, reduced computational cost are also proposed as other benefits of decentralized controllers. However, performance of such type controllers increase in general when the original multivariable system is close to the diagonal form. If the plant $G(s)$ is in the diagonal form, single loop design solution can be directly applicable. On the other hand, the performance of multivariable plant controlled by decentralized controller reduces if there are significant interactions between different input-output pairs. In such cases the level of interactions should be reduced as the first step of the design process. For this purpose, the main approaches (RGA, SVD) in order to determine interaction levels in a given MIMO system are presented within the scope of this section. Additionally, currently existing decoupling methods are also presented in Sections 2.2.1 and 2.2.2. Drawbacks of these methods are also discussed in Section 2.2.3 over illustrative examples.

2.1 Interaction Measures

Determining the level of interactions is crucial from the decentralized controller design point of view. Moreover, the methods like RGA is also suitable for pairing problem. Pairing of highly interacted input-output variables is also important for multivariable system design. In addition to RGA, SVD and CN that is also meaningful from controllability and robustness point of view are also introduced within the scope of this subsection.

2.1.1 Relative gain array

RGA is one of the first approaches that was proposed in literature to determine interaction levels in a given multivariable system. It was first proposed by Bristol in [22] and rigorous proofs were also presented in [23]. In addition to the determination of interaction levels, it is also suitable for pairing problem. When the number of inputs and outputs increases the number of possible pairings also increase significantly. For a given multivariable system that has n inputs and n outputs there are $n!$ pairing combinations. However, pairing problem is out of scope of this thesis, since it is possible to use permutation matrices to pair $i-th$ input with $i-th$ output.

An $n \times n$ dimensional linear multivariable system can be described as :

$$y(s) = G(s)u(s) \tag{2.1}$$

where $u(s)$ and $y(s)$ represent respectively input and output vectors and $G(s)$ represents the TFM of the multivariable plant. RGA of the system that is given in (2.1) can be expressed as:

$$\Lambda(G) = G_s \times (G_s^{-1})^T \tag{2.2}$$

where $\times$ denotes the Hadamart or Schur product (element by element product) and G_s denoted the steady state gain matrix of $G(s)$. Steady state gain matrix notation is common in literature, since Bristol proposed the RGA approach as a steady state interaction measure in [22]. However, RGA may be useful over the complete frequency range especially around the crossover frequencies as indicated in [24,25].

The resulting RGA matrix can be written in the following form:

$$\Lambda(G) = \begin{bmatrix} \beta_{11} & \dots & \beta_{1n} \\ \vdots & \ddots & \vdots \\ \beta_{n1} & \dots & \beta_{nn} \end{bmatrix} \tag{2.3}$$

The following properties hold for a given real R matrix:

- The sum of the elements in each row (and each column) of the RGA is equal to 1.
- Any permutation of the rows and columns of R leads to the same permutations on $\Lambda(R)$
- $\Lambda(R) = I$ if and only if R is a lower or upper triangular matrix or a diagonal matrix.
- The relationship between the RGAs of inverse of R, transpose R and transpose of the RGA is as follows: of

$$\Lambda(R^{-1}) = \Lambda(R^T) = \Lambda(R)^T \tag{2.4}$$

- The norm of the RGA is closely related to the minimized condition number. This can also be interpreted as if RGA includes larger entries then the system is ill-conditioned in general.

Detailed information related with the properties of the RGA can be found in [24].

As indicated earlier, RGA can also be used to determine appropriate input-output pairs in a multivariable system. Pairing with $\lambda_{ij} \approx 1$ should be preferred and pairing with $\lambda_{ij} < 0$ should be avoided in general.

Using the afore mentioned properties RGA of TITO system can be expressed as:

$$\Lambda(G) = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} = \begin{bmatrix} \beta_{11} & 1-\beta_{11} \\ 1-\beta_{11} & \beta_{11} \end{bmatrix} \tag{2.5}$$

where

$$\beta_{11} = \frac{1}{1 - \frac{G_{s_{12}} G_{s_{21}}}{G_{s_{11}} G_{s_{22}}}} \tag{2.6}$$

In (2.6) $G_{s_{ij}}$ represents the $j-th$ column $i-th$ row element of G_s.

RGA was originally proposed for steady state gain matrix. However, in the meantime the results were extended to all frequencies by researchers. Whereas, it is better to use RGA with other methods and metrics like SVD and CN in order to derive more insight related with the discussed multivariable system. For this reason, SVD and CN will be discussed in the next section to include more information related with the currenty existing interaction and sensitivity measures.

2.1.2 Singular value decomposition and condition number

In addition to the RGA, SVD and Condition Number (CN) are also important metrics in terms of multivariable system interactions, controllability and sensitivity. Furthermore, it is also possible to use SVD in controller design. For instance different design methods were proposed in [26] and [27] in using SVD.

Singular values of an $n \times m$ dimensional real valued [1] A matrix is defined as the eigenvalues of the matrices $A^T A$ and AA^T. Eigenvalues of both matrices are nonnegative since $A^T A$ and AA^T are non negative definite. Furthermore, as indicated in [25], when the eigenvalues of both matrices ordered in a decreasing manner, the first "min(n,m)" eigenvalues are equal to each other. This case can also be expressed as:

$$\lambda_i \left(A^T A\right) = \lambda_i \left(AA^T\right) \quad \forall i = 1,2,\ldots \min(n,m) \tag{2.7}$$

where $\lambda_i(.)$ represents the $i-th$ eigenvalue. The remaining eigenvalues (if any) should be zero. Then, the singular values of the A matrix can be defined as the square roots of these eigenvalues as:

$$\sigma_i(A) = \left(\lambda_i \left(A^T A\right)\right)^{1/2} = \left(\lambda_i \left(AA^T\right)\right)^{1/2} \quad \forall i = 1,2,\ldots \min(n,m) \tag{2.8}$$

where $\sigma_i(.)$ represents the $i-th$ singular value of A. SVD can be applied in order to determine singular values of a given matrix and SVD of a given A matrix can be expressed as:

$$A = U \Sigma V^T \tag{2.9}$$

[1] It is also possible to define singular values for complex valued matrices. In such cases transpose operator should be replaced by "Hermitian transpose" operator.

where U and V are appropriate dimensional unitary matrices ($UU^T = U^T U = I$ and $VV^T = V^T V = I$) and Σ is an $n \times m$ dimensional rectangular matrix whose diagonal entries are the singular values of A in decreasing order and all other entries are zero. So that, maximum and minimum singular values of a given $n \times m$ dimensional A matrix can be written as:

$$\overline{\sigma}(A) = \sigma_1(A), \quad \underline{\sigma}(A) = \sigma_{\min(n,m)}(A) \tag{2.10}$$

For the case of MIMO systems, there are certain advantages of SVD over the eigenvalue decomposition for analyzing gains and directionality [24]. For instance, it is possible to derive better information related with the gain of the discussed system. Moreover, direction derived from SVD are orthogonal and it is also possible to apply SVD to non-square systems. Additionally, singular values are also related with the maximum and minimum gains in a multivariable systems. This relation can be expressed as:

$$\underline{\sigma}(A) \leq \frac{\|Ab\|_2}{\|b\|_2} \leq \overline{\sigma}(A) \tag{2.11}$$

where b represents any non zero vector. Using the previously mentioned properties, condition number (CN) which is an important metric for multivariable systems can be defined as:

$$\mathrm{CN}(A) = \gamma(A) = \frac{\overline{\sigma}(A)}{\underline{\sigma}(A)} \tag{2.12}$$

is a possible measure of the difficulty of controlling the system. Larger CN values indicate such difficulties and systems with larger CN is said to be ill-conditioned (or poor-conditioned). Here it must be noted that the condition number is not invariant under scaling and using pre and post scaling units it is possible to end up with a different condition number [25]. Minimized CN can be defined as $\gamma^*(A) = \min_{D_1,D_2} \gamma(D_1 A D_2)$ where D_1 and D_2 diagonal scaling matrices. As indicated earlier, there is a relation between the RGA and minimized CN. A lower and upper bound on minimized CN can be determined using the sum norm of RGA.

A high CN can also be interpreted as that the systems is close to losing its full rank since larger CN values in general requires small value of $\underline{\sigma}(A)$ which is also undesired. Larger CN also implies the sensitivity against unstructured input uncertainties.

While RGA and CN are indicators of interactions, pairing and sensitivity for a given multivariable system they were mainly proposed and used on the analysis basis. In order to reduce the interactions between the undesired input-output pairs, different approaches were proposed in literature. In that manner, decoupling methods that aim fully diagonalize multivariable systems will be discussed in the next section.

2.2 Decoupling Methods

The methods given in subsections 2.1.1 and 2.1.2 are suitable to determine interaction levels in a given MIMO system and propose a solution strategy for the pairing problem. Additionally, SVD and CN is also meaningful from the controllability and robustness point of view.

In the case of significant interactions between different input-output pairs, each output is effected by all inputs considerably. As a result, decentralized controller design problem becomes more and more complex in such cases [28, 29]. Decentralized controllers are popular in the control engineering area due their practical advantages like less number of tuning parameters, relatively simple control structure and robustness against sensor and/or actuator failures as indicated in [30].

In case of higher interactions, it is not possible to apply previously proposed decentralized controller design methods and as a result, elimination of the such interactions can be accepted as the first step of the decentralized controller design. In order to eliminate such interaction a pre-compensator should be designed that brings the system into the diagonal form. If the diagonal form is achieved then it can be proposed that the interactions are eliminated and the system is suitable to apply single loop design solutions. Several methods were proposed in literature in order to design decoupling pre-compensators like dynamic and steady state decoupling as indicated in [21]. It is focused on conventional and inverted decoupling within the scope of this thesis since it is not possible point out the advantages and disadvantages of all the proposed approaches. Detailed information related with decoupling controller design can be found in textbooks like [3, 28].

2.2.1 Conventional decoupler

As indicated in [4], most of the decoupling methods use conventional decoupling structure that the system inputs are derived by a weighted combination of controller outputs. Block diagram of such a decoupler can be represented as it is given in

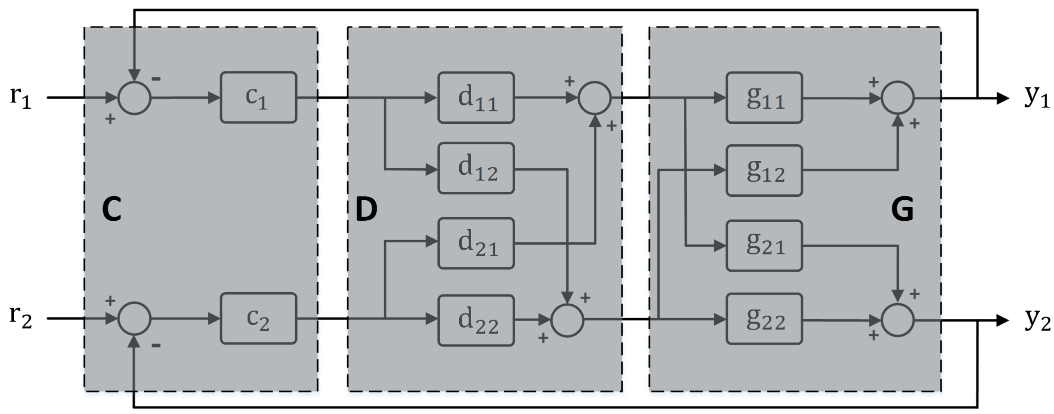


Figure 2.2 : Conventional decoupler block diagram.

Figure 2.2. In the case of conventional decoupling mainly, it is aimed to determine the pre-compensator in the form of "$D(s) = G^{-1}(s)P(s)$" where $D(s)$ represents the decoupler, $G(s)$ represents the actual multivariable system and $P(s)$ represents the desired plant.

There are two main sub categories for the case of conventional decoupler which are ideal and simplified decoupling as indicated in [31, 32]. In ideal decoupling, it is generally aimed to end up with the process transfer functions as simple as the diagonal entries of the original entries of the original TFM [4]. However, this method leads to complicated decoupler expressions in general and has realization based problems as indicated in [5].

On the other hand, simplified decoupling is widely preferred in literature. For a given TITO system that is expressed as:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{2.13}$$

using the decoupling structure given in Figure 2.2, resulting output expressions can be calculated as:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} g_{11}d_{11} + g_{12}d_{21} & g_{11}d_{12} + g_{12}d_{22} \\ g_{21}d_{11} + g_{22}d_{21} & g_{21}d_{12} + g_{22}d_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{2.14}$$

In (2.14) off diagonal terms should be zero. So that the following equations should be satisfied by the decoupling pre-compensator.

$$\begin{aligned} g_{11}d_{12} + g_{12}d_{22} &= 0 \\ g_{21}d_{11} + g_{22}d_{21} &= 0 \end{aligned} \tag{2.15}$$

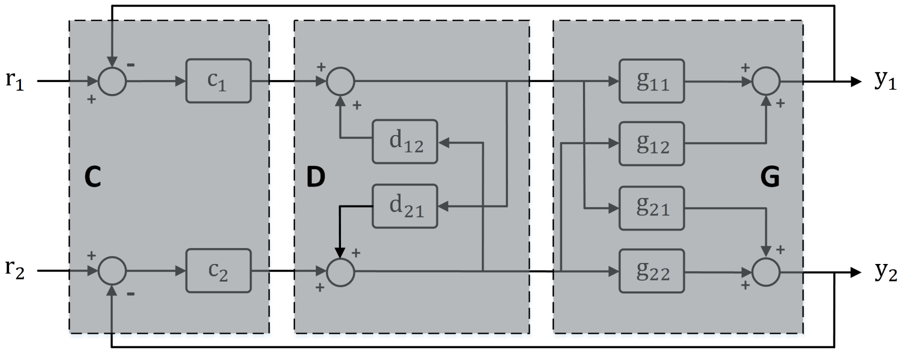


Figure 2.3 : Inverted decoupler block diagram.

By selecting the parameters d_{11}, d_{22} as $d_{11} = 1, d_{22} = 1$ simplified decoupler can be derived as:

$$D(s) = \begin{bmatrix} 1 & -\frac{g_{12}}{g_{11}} \\ -\frac{g_{21}}{g_{22}} & 1 \end{bmatrix} \tag{2.16}$$

Compared to ideal decoupling realization is relatively easier in simplified decoupling. Relative degrees of individual transfer function and right half plane zeros becomes crucial in terms of decoupler realization in such cases. On the other hand, resulting transfer function matrix is more complex in this case. The resulting transfer function matrix can be written as:

$$L(s) = \begin{bmatrix} g_{11} - \frac{g_{12}g_{21}}{g_{22}} & 0 \\ 0 & g_{22} - \frac{g_{12}g_{21}}{g_{11}} \end{bmatrix} \tag{2.17}$$

As it is seen in (2.17), diagonal entries of the resulting TFM includes the multiplication and summation of extra transfer functions. As a result, the design problem is also relatively harder in such cases.

2.2.2 Inverted decoupler

In order to overcome the complexity of the resulting transfer function matrix, a new approach was proposed in [33] that depends on a modification of the block diagram that was given in Figure 2.2. Proposed block diagram representation in the mentioned study is given in Figure 2.3.

In this approach, output equations can be written as:

$$\begin{aligned} y_1 &= g_{11}u_1 + g_{12}u_2 \\ y_2 &= g_{22}u_2 + g_{21}u_1 \end{aligned} \tag{2.18}$$

where:

$$\begin{aligned} u_1 &= c_1 + u_2 d_{12} \\ u_2 &= c_2 + u_1 d_{21} \end{aligned} \tag{2.19}$$

When equation (2.19) is inserted to (2.18) outputs can be expressed as:

$$\begin{aligned} y_1 &= g_{11}c_1 + (g_{11}d_{12} + g_{12})u_2 \\ y_2 &= g_{22}c_2 + (g_{22}d_{21} + g_{21})u_1 \end{aligned} \tag{2.20}$$

In order to eliminate the effect of u_2 on y_1 and u_1 on y_2 using the decoupler parameters corresponding coefficients should be eliminated [34]. So that, it can be proposed that the decoupler transfer functions should be selected as:

$$\begin{aligned} d_{12} &= -\frac{g_{12}}{g_{11}} \\ d_{21} &= -\frac{g_{21}}{g_{22}} \end{aligned} \tag{2.21}$$

The resulting equations that is given in (2.21) are same as (2.16). However, complexity of the resulting TFM reduced significantly due to the block diagram modifications. In addition to the above mentioned advantages of inverted decoupling, it is more sensitive to modeling errors as indicated in [35]. However, due to its practical benefits researches are trying to improve the existing approach as it is done in [4, 36, 37].

2.2.3 Possible problems and illustrative examples

Even if the original multivariable system is suitable for diagonalisation, there are some practical disadvantages of decoupling control. Firstly, it can be proposed that resulting TFM may be so sensitive against uncertainties and modeling errors [21]. Furthermore, in some cases it is required to use inverse-based decoupler that is not desirable for disturbance rejection problem. And it is also pointed out in [21], that the requirement of diagonalisation generally introduces additional right half plane zeros to the closed loop system is the system has right half plane zeros.

It was also pointed out that, in order to take into account the saturation of manipulated variables inverted decoupling approach is suitable. Whereas, it is more sensitive to modeling errors and uncertainties [38]. Using benchmark distillation column models, ideal and simplified decoupling were compared in [32, 35]. In these studies, it was concluded that simplified decoupling is more robust than ideal decoupling approach. However, in [5] robust stability and robust performance of nominally stable multivariable systems are discussed for the cases of ideal, simplified and inverted decoupling and it was concluded that it is possible to obtain same levels of robustness if the controller are tuned to obtain the same nominal performance.

In addition to the above mentioned issues, due to the realization related issues decoupling is applicable for a certain class of multivariable systems. These realization related problems can be summarized in two main topics which are causality and stability.

A system is said to be a causal system if its output depends on present and past inputs only. It is also named as strictly causal if it depends only on past inputs. This corresponds the relative degrees of numerator and denominator polynomials. In order to demonstrate such a casuality based realization problem the following TITO system is considered as an illustrative example:

$$G(s) = \begin{bmatrix} \frac{4}{(s+1)(s+2)} & \frac{2s}{s+1} \\ \\ \frac{2}{s+1} & \frac{2}{s+2} \end{bmatrix} \tag{2.22}$$

In simplified and inverted decoupling that was presented in Sections 2.2.1 and 2.2.2 it is required to use inverse based decouplers. For the case of system that is given in 2.22, the elements of decoupling controller can be determined as:

$$\begin{aligned} d_{12} &= -\frac{g_{12}}{g_{11}} = -\frac{2s(s^2+3s+2)}{4(s+1)} \\ d_{21} &= -\frac{g_{21}}{g_{22}} = -\frac{s+2}{s+1} \end{aligned} \tag{2.23}$$

As seen, from equation (2.23), it is obvious that the degree of the numerator exceeds the degree of the denominator resulting in improper transfer function.

On the other hand, RHP zeros of the individual transfer functions of TFM may lead to stability related realization problems. For instance, let us assume that it is desired to diagonalise the following multivariable system using afore mentioned methods

$$G(s) = \begin{bmatrix} \frac{17s-0.4}{s+1} & \frac{1.4s+0.4}{s+1} \\ \\ \frac{0.8s+0.2}{s+1} & \frac{0.7s-0.3}{s-1} \end{bmatrix} \tag{2.24}$$

The entries of the decoupler can be calculated as:

$$\begin{aligned} d_{12} &= -\frac{g_{12}}{g_{11}} = -\frac{1.4s+0.4}{17s-0.4} \\ d_{21} &= -\frac{g_{21}}{g_{22}} = -\frac{0.8s+0.2}{0.7s-0.3} \end{aligned} \tag{2.25}$$

From (2.25), it is clear that the resulting decoupler is unstable. Moreover, it should also be noted that RHP pole-zero cancellations may also occur, that lead to internal instability of the transfer function. This case should also be discussed in detail to be sure about the applicability of the decoupler.

The definition of decoupling proposes a strong condition on multivariable systems. As a result of this strong condition, proposed approaches face causality and stability based realization problems and they are suitable for a limited class of multivariable systems. Moreover, as indicated earlier, sensitivity against disturbances and model uncertainties are the other problems that diagonalisation methods face even if the original system is suitable for decoupling. In order to overcome such difficulties, extend the applicable class of multivariable systems diagonal dominance concept which is a weaker condition compared to decoupling will be introduced in the next section. Instead of eleminating the off diagonal terms it is aimed to limit the magnitudes of off diagonal terms and reduce the interaction between different input-output pairs in diagonal dominance concept.

3. DIAGONAL DOMINANCE

Designing efficient control structures for MIMO systems is crucial, since most of the industrial systems can be considered as MIMO systems as indicated in [19, 20]. A specific output of a given MIMO system is effected by more than one input in general. Consequently, it can be proposed that in general, enhancing the performance of a specific output may easily disturb other outputs' performance in case of significant interactions as asserted in [39].

In some specific applications, the interactions between different input-output pairs can be used to reduce the control effort. However, it is widely accepted that the controller design problem becomes more complex in case of higher interactions and single loop control solutions cannot be applied directly for these kind of systems [21, 29]. These interactions between the different input-output pairs can be accepted as one of the most challenging aspects of MIMO systems, especially for the case of decentralized controller design. It can also be proposed that these kind of controllers are preferred in practice due to the advantages like simple control structure, fewer tuning parameters, and robustness against sensor and/or actuator failures [30].

As the first step of the decentralized controller design procedure, designer should design a pre-compensator that eliminates or reduces the interactions between different input-output pairs, since it is not possible to apply single loop controller design approaches for the systems that include significant interactions [21]. Decentralized controller design for a given MIMO system is not a straight forward procedure since broad range of sytems include significant interactions.

As indicated in Section 2.2, decoupling methods can be applied if it is possible. In the case of decoupling, the effect of all off diagonal terms have to be eliminated The drawbacks and constraints of decoupling methods were pointed out in Section 2.2. In order to overcome these constraints, diagonal dominance which is a weaker condition compared to decoupling is preferred within the scope of this thesis.

Compared to decoupling, diagonal dominance can be applicable for broader range of systems. It is aimed to determine controller parameter regions that satisfy the diagonal dominance conditions. For this purpose, firstly, previous studies are pointed in this section. Then, the proposed method for deriving controller regions for both column and row diagonal dominance are discussed in detail. After that, weighting factors are inserted to the original diagonal dominance conditions. This case is discussed in detail in Section 3.3. Derived results are combined in an algorithm and effectiveness of the derived results are demonstrated over several case studies. Obtained results within the scope of this section were published in [40–46].

3.1 Previously Proposed Approaches

The importance of diagonal dominance concept in MIMO systems were first pointed in Rosenbrock's early studies. In these studies, existing frequency domain design techniques for SISO systems had been generalized to MIMO systems [13, 14, 47]. The proposed approaches mainly depend on a pre-compensator that achieves diagonal dominance as indicated in [12].

After the pioneering studies of Rosenbrock, the problem of achieving diagonal dominance has became a field of attraction for the researchers in control engineering as indicated in [48]. Various methods and techniques has been proposed using different mathematical approaches to determine pre-controllers that make multivariable system diagonal dominant. For instance, using the inverse open loop TFM, an approach to minimize the sum of squares of the magnitude of the off diagonal terms to achieve diagonal dominance was proposed by Hawkins in 1972 for single and multiple frequencies [49, 50].

In the successive years, ALIGN algorithm was proposed by MacFarlane and Kouvaritakis [51, 52] as also indicated in [7]. Hawkins's pseudodiagonalisation approach was generalized in [53] and it was asserted that diagonal dominance can be achieved for a wider range of plants compared to previous approaches. In 1979, an approach to derive diagonal dominance was also proposed by Leininger in [54]. In this method, it was aimed to achieve diagonal dominance for Nyquist array based design methods. A conjugate direction function based minimization algorithm was used to obtain diagonal dominance in a given frequency interval.

In 1981, a Perron-Frobenius Theorem based approach was proposed by Mees to achieve diagonal dominance [55]. With respect to the proposed approach, it is possible to determine a specific pre-compensator that achieves diagonal dominance if the Perron-Frobenius eigenvalue is less than 2. After that, expressing the frequency dependent magnitude change as scalar rational functions, a Perron-Frobenius eigenvector based approach was proposed by Munro in [56].

A scaling algorithm to balance the input-output pairs for a square MIMO system was proposed by Edmunds in [9]. It was aimed to maximize the geometric mean of the column diagonal dominance ratios.

In the recent years, various methods were proposed by researchers to obtain a specific controller parameter set, using different mathematical perspectives, since the diagonal dominance problem can be defined by several ways. For example, in [8], the diagonal dominance problem was defined as an optimization problem. As a result, Genetic Algorithm, which is one of the well-known global optimization methods, was used to minimize the performance function and to achieve diagonal dominance.

After that a new eigenstructure assignment based technique was proposed by Labibi et al. for the diagonal dominance of large-scale systems [57]. In this approach, after defining an equivalent descriptor system in the input-output representation, sufficient conditions for closed loop diagonal dominance were introduced. The necessity of choosing a suitable frequency was also alleviated in this method as required in the previous studies.

A frequency interpolation based approach was asserted by Nobakthi and Munro in 2004 to achieve diagonal dominance [58]. In this approach, the restrictions on the structure of each element of dynamic pre-compensators were eliminated.

On the other hand, a new LMI based method was proposed by Chughtai and Munro to reduce the effects of interaction for static and dynamic type controllers [7, 12]. It was also aimed to reduce the conservativeness by using scaling methods in the proposed approach. In 2006, a new method based on minimizing the H_2 norm of a modified system was asserted by Nobakthi and Wang [59]. Additionally, necessary conditions for static pre-compensators were given in [60].

At this point, it can be proposed that in the previously proposed approaches it was aimed to determine specific controller parameters that achieve diagonal dominance at a given certain frequency or frequency interval. As indicated earlier, different mathematical perspectives and notations were used for this purpose and to derive specific controller parameters. However, it can also be asserted that deriving whole parameter region is also meaningful considering the parametric uncertainties and disturbances that may effect the system. Additionally, determining all parameter regions that satisfy diagonal dominance conditions can create more flexible environment for the designer in terms of multivariable controller design. As a result, within the scope of this thesis study, it is aimed to determine controller parameter region(s) that achieve diagonal dominance conditions. Details of the proposed approach is presented in the next sections.

3.2 Determining Parameter Regions That Achieves Diagonal Dominance

It is important to derive significant results for the case of TITO systems which is a special subset of MIMO systems, since in practice many MIMO systems can be treated as several TITO subsystems as proposed in [61]. Therefore, it can also be proposed that it is important to derive significant results for TITO systems in terms of diagonal dominance from the practical point of view. Consequently, within the scope of this chapter it is aimed to determine conditions on diagonal type controllers for TITO systems in order to achieve diagonal dominance at a given fixed frequency.

Compared to decoupling, it can be proposed that diagonal dominance is a weaker condition, since it is not need to fully eliminate all the effects of off diagonal terms. Magnitude of all diagonal terms should be greater than or equal to the sum of the magnitudes of all off diagonal terms in the related row (or column) in order to achieve diagonal dominance. As a result, it can be asserted that a specific input variable is strongly related with a specific output variable in diagonal dominant system and other inputs have limited impact on this specific output.

A square matrix D is named as column diagonal dominant matrix in mathematics if for each column, the magnitude of the diagonal term is greater than or equal to the sum of the magnitudes of all off diagonal terms. In other words, a matrix D is column

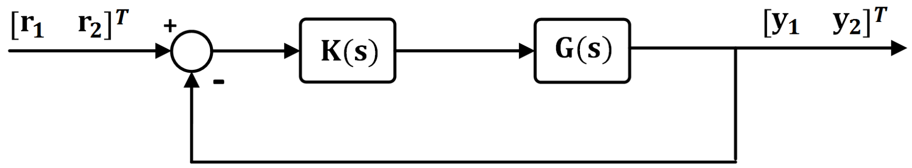


Figure 3.1 : Block diagram of the considered control system.

diagonal diagonal dominant if

$$|d_{ii}| \geq \sum_{j \neq i} |d_{ji}| \tag{3.1}$$

where d_{ji} represents the $j-th$ row and $i-th$ column term of the D matrix. Since equality is included in (3.1), it is also named as weak diagonal dominance in some sources. If the strict inequality is used in (3.1) then this case is named as strict diagonal dominance as indicated in [62]. It can be proposed that there is not any significant difference between both definitions in terms of the used algebra within the scope of this thesis. Weak inequality is used in the determination of controller parameter regions. However, it is also possible to determine easily the corresponding conditions for strict equality.

On the other hand, a matrix D is named as row diagonal dominant if for each row the magnitude of the diagonal entry is greater than or equal to the sum of the off diagonal terms at the corresponding row. As a result, a square matrix D is named as row diagonal dominant if:

$$|d_{ii}| \geq \sum_{j \neq i} |d_{ij}| \tag{3.2}$$

where d_{ij} represents the $i-th$ row and $j-th$ column term of the D matrix.

Block diagram representation of the closed loop control system that is considered in this study is given in Figure 3.1. In Figure 3.1, TITO transfer function matrix and the diagonal type controller are represented by $G(s)$ and $K(s)$ respectively. So that, considered system and controller can be expressed as:

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ \\ g_{21}(s) & g_{22}(s) \end{bmatrix} \tag{3.3}$$

$$K(s) = \begin{bmatrix} k_1(s) & 0 \\ 0 & k_2(s) \end{bmatrix} \tag{3.4}$$

In (3.3), $g_{ij}(s)$ represents the individual transfer functions from $j\ th$ input to the $i\ th$ output. Corresponding closed loop TFM can be directly determined as:

$$G_{cl} = (I+GK)^{-1}GK = \frac{1}{det\,(I+GK)}\begin{bmatrix} k_1 g_{11} + k_1 k_2\ g_d & k_2 g_{12} \\ k_1 g_{21} & k_2 g_{22} + k_1 k_2\ g_d \end{bmatrix} \tag{3.5}$$

where g_d is defined as: $g_d = g_{11}g_{22} - g_{12}g_{21}$. The controller parameters that make $det\,(I+GK) = 0$ are related with the poles of the closed loop system and as a result also related with the stability of overall multivariable system. However, from the diagonal dominance point of view, it can be proposed that the controller parameters that make this equation zero in the related frequencies makes the magnitudes of each term of TFM equal to infinity. As a result, it is not possible to compare the magnitude of elements of the closed loop system in such cases. For this reason, such controller parameter values should be excluded from the region that makes the closed loop system diagonal dominant at the related frequency.

Within the scope of this chapter, it is aimed to determine conditions on diagonal type controller parameters to achieve diagonal dominance condition for TITO system at a given fixed frequency and/or frequency interval. Column diagonal dominance and row diagonal dominance are discussed in detail respectively in Section 3.2.1 and Section 3.2.2. The case of weighting factors will also be discussed in Section 3.3 in order to derive more practically important regions. Furthermore, the stability problem will also be discussed in the next chapter in detail. At this point, it must be pointed out that the main aim of the diagonal controller is to make the closed loop system diagonal dominant and stable. Further performance criteria can be achieved considering the determined parameter regions and/or by designing a cascade controller for the diagonal dominant and stable system.

Using the magnitudes of the resulting TFM elements conditions on controller parameters are determined exactly. Derived results depend on the real and imaginary parts of individual sub-controllers. In addition to this, it also possible to get conservative results using the triangular inequality. Such approaches were presented in our previous studies [42,43,45]. Additionally, triangular inequality can also be used to derive conservative results in combination with weighting factors as it was done in our previous study [41].

3.2.1 Column diagonal dominance conditions

Considered system type (TITO systems) and the controller type (diagonal type controllers) were briefly introduced in the previous section. In this section it is aimed to give details related with the derivation of controller parameter conditions in terms of column diagonal dominance.

A matrix is named as a column diagonal dominant if it satisfies (3.1). In the specific case of TITO system it can be interpreted as the diagonal term should be greater than or equal to the of diagonal term in the corresponding column. As a result, assuming that the determinant of the closed loop system that is expressed in (3.5) is not equal to zero, conditions for column diagonal dominance can be written as:

$$|k_1 g_{11} + k_1 k_2 g_d| \geq |k_1 g_{21}| \tag{3.6}$$

$$|k_2 g_{22} + k_1 k_2 g_d| \geq |k_2 g_{12}| \tag{3.7}$$

If the given inequalities are satisfied at a given frequency than it can proposed that the considered system is column diagonal dominant at that frequency. Likewise, if both of the conditions are satisfied at a given frequency range it can be concluded that the closed loop system is column diagonal dominant at that range.

When the inequalities (3.6) and (3.7) analyzed it can be realized that controllers respectively k_1 and k_2 are shown as multiplier term on both sides of equations. As a result it is possible to simplify initial conditions as:

$$|g_{11} + k_2 g_d| \geq |g_{21}| \tag{3.8}$$

$$|g_{22} + k_1 g_d| \geq |g_{12}| \tag{3.9}$$

As indicated earlier, considering the (3.8) and (3.9) conservative results can be obtained using the triangular inequality [42]. However, exact conditions can also be derived by using the real and imaginary parts of both sides of the given inequalities. Without loss of generality, using the real and the imaginary parts of the individual transfer functions and controllers, conditions in (3.8) and (3.9) can be rewritten as:

$$(\mathrm{Re}(g_{11} + k_2 g_d))^2 + (\mathrm{Im}(g_{11} + k_2 g_d))^2 \geq (\mathrm{Re}(g_{21}))^2 + (\mathrm{Im}(g_{21}))^2 \tag{3.10}$$

$$(\mathrm{Re}(g_{22}+k_1 g_d))^2+(\mathrm{Im}(g_{22}+k_1 g_d))^2 \geq (\mathrm{Re}(g_{12}))^2+(\mathrm{Im}(g_{12}))^2 \tag{3.11}$$

At this point, it can be proposed that for a given fixed frequency left hand side of the given inequalities includes the summation of two complex numbers which is one of these numbers also formed of multiplication of two complex numbers. Consequently, the conditions given in (3.10) and (3.11) can be expressed in a more clear manner as:

$$\begin{aligned}(\mathrm{Re}(g_{11})+\mathrm{Re}(k_2)\,\mathrm{Re}(g_d)-\mathrm{Im}(k_2)\,\mathrm{Im}(g_d))^2+\\ (\mathrm{Im}(g_{11})+\mathrm{Re}(k_2)\,\mathrm{Im}(g_d)+\mathrm{Im}(k_2)\,\mathrm{Re}(g_d))^2\\ \geq (\mathrm{Re}(g_{21}))^2+(\mathrm{Im}(g_{21}))^2\end{aligned} \tag{3.12}$$

$$\begin{aligned}(\mathrm{Re}(g_{22})+\mathrm{Re}(k_1)\,\mathrm{Re}(g_d)-\mathrm{Im}(k_1)\,\mathrm{Im}(g_d))^2+\\ (\mathrm{Im}(g_{22})+\mathrm{Re}(k_1)\,\mathrm{Im}(g_d)+\mathrm{Im}(k_1)\,\mathrm{Re}(g_d))^2\\ \geq (\mathrm{Re}(g_{12}))^2+(\mathrm{Im}(g_{12}))^2\end{aligned} \tag{3.13}$$

Expanding the quadratic expressions in the left hand side of the inequalities (3.12) and (3.13) leads to the new conditions on column diagonal dominance as:

$$\begin{aligned}(\mathrm{Re}(g_{11}))^2+(\mathrm{Re}(k_2))^2\,(\mathrm{Re}(g_d))^2+(\mathrm{Im}(k_2))^2\,(\mathrm{Im}(g_d))^2+2\,\mathrm{Re}(g_{11})\,\mathrm{Re}(k_2)\,\mathrm{Re}(g_d)-\\ 2\,\mathrm{Re}(g_{11})\,\mathrm{Im}(k_2)\,\mathrm{Im}(g_d)-2\,\mathrm{Re}(k_2)\,\mathrm{Re}(g_d)\,\mathrm{Im}(k_2)\,\mathrm{Im}(g_d)+\\ (\mathrm{Im}(g_{11}))^2+(\mathrm{Re}(k_2))^2\,(\mathrm{Im}(g_d))^2+(\mathrm{Im}(k_2))^2\,(\mathrm{Re}(g_d))^2+2\,\mathrm{Im}(g_{11})\,\mathrm{Re}(k_2)\,\mathrm{Im}(g_d)+\\ 2\,\mathrm{Im}(g_{11})\,\mathrm{Im}(k_2)\,\mathrm{Re}(g_d)+2\,\mathrm{Re}(k_2)\,\mathrm{Im}(g_d)\,\mathrm{Im}(k_2)\,\mathrm{Re}(g_d)\geq\\ (\mathrm{Re}(g_{21}))^2+(\mathrm{Im}(g_{21}))^2\end{aligned} \tag{3.14}$$

$$\begin{aligned}(\mathrm{Re}(g_{22}))^2+(\mathrm{Re}(k_1))^2\,(\mathrm{Re}(g_d))^2+(\mathrm{Im}(k_1))^2\,(\mathrm{Im}(g_d))^2+2\,\mathrm{Re}(g_{22})\,\mathrm{Re}(k_1)\,\mathrm{Re}(g_d)-\\ 2\,\mathrm{Re}(g_{22})\,\mathrm{Im}(k_1)\,\mathrm{Im}(g_d)-2\,\mathrm{Re}(k_1)\,\mathrm{Re}(g_d)\,\mathrm{Im}(k_1)\,\mathrm{Im}(g_d)+\\ (\mathrm{Im}(g_{22}))^2+(\mathrm{Re}(k_1))^2\,(\mathrm{Im}(g_d))^2+(\mathrm{Im}(k_1))^2\,(\mathrm{Re}(g_d))^2+2\,\mathrm{Im}(g_{22})\,\mathrm{Re}(k_1)\,\mathrm{Im}(g_d)+\\ 2\,\mathrm{Im}(g_{22})\,\mathrm{Im}(k_1)\,\mathrm{Re}(g_d)+2\,\mathrm{Re}(k_1)\,\mathrm{Im}(g_d)\,\mathrm{Im}(k_1)\,\mathrm{Re}(g_d)\geq\\ (\mathrm{Re}(g_{12}))^2+(\mathrm{Im}(g_{12}))^2\end{aligned} \tag{3.15}$$

For the sake of simplicity, real and imaginary parts of both controller can be defined as:

$$\begin{aligned}\alpha_1=\mathrm{Re}(k_1)\quad \beta_1=\mathrm{Im}(k_1)\\ \alpha_2=\mathrm{Re}(k_2)\quad \beta_2=\mathrm{Im}(k_2)\end{aligned} \tag{3.16}$$

Using the definitions given in (3.16) the column diagonal dominance conditions that are given in (3.14) and (3.15) can be rearranged as:

$$\begin{aligned}(\mathrm{Re}(g_d))^2\alpha_2^2+2\,\mathrm{Re}(g_{11})\,\mathrm{Re}(g_d)\alpha_2+(\mathrm{Re}(g_{11}))^2+(\mathrm{Im}(g_d))^2\beta_2^2-\\ 2\,\mathrm{Re}(g_{11})\,\mathrm{Im}(g_d)\beta_2+(\mathrm{Im}(g_d))^2\alpha_2^2+2\,\mathrm{Im}(g_{11})\,\mathrm{Im}(g_d)\alpha_2+(\mathrm{Re}(g_d))^2\beta_2^2+\\ 2\,\mathrm{Im}(g_{11})\,\mathrm{Re}(g_d)\beta_2+(\mathrm{Im}(g_{11}))^2\geq(\mathrm{Re}(g_{21}))^2+(\mathrm{Im}(g_{21}))^2\end{aligned}\tag{3.17}$$

$$\begin{aligned}(\mathrm{Re}(g_d))^2\alpha_1^2+2\,\mathrm{Re}(g_{22})\,\mathrm{Re}(g_d)\alpha_1+(\mathrm{Re}(g_{22}))^2+(\mathrm{Im}(g_d))^2\beta_1^2-\\ 2\,\mathrm{Re}(g_{22})\,\mathrm{Im}(g_d)\beta_1+(\mathrm{Im}(g_d))^2\alpha_1^2+2\,\mathrm{Im}(g_{22})\,\mathrm{Im}(g_d)\alpha_1+(\mathrm{Re}(g_d))^2\beta_1^2+\\ 2\,\mathrm{Im}(g_{22})\,\mathrm{Re}(g_d)\beta_1+(\mathrm{Im}(g_{22}))^2\geq(\mathrm{Re}(g_{12}))^2+(\mathrm{Im}(g_{12}))^2\end{aligned}\tag{3.18}$$

It can be observed from (3.17) and (3.18) at most quadratic terms of real and imaginary parts of the individual controllers are included in derived conditions. As a result, it becomes possible to construct two quadratic polynomials with respect to the real and imaginary parts of controller transfer functions and rewrite the inequality as comparison of these two quadratic polynomials. For this purpose, by defining the coefficient terms as:

$$\begin{aligned}a_1&=\left((\mathrm{Re}(g_d))^2+(\mathrm{Im}(g_d))^2\right)\\ b_1&=(2\,\mathrm{Re}(g_{11})\,\mathrm{Re}(g_d)+2\,\mathrm{Im}(g_{11})\,\mathrm{Im}(g_d))\\ c_1&=(\mathrm{Re}(g_{11}))^2\\ a_2&=-\left((\mathrm{Re}(g_d))^2+(\mathrm{Im}(g_d))^2\right)\\ b_2&=-(2\,\mathrm{Im}(g_{11})\,\mathrm{Re}(g_d)-2\,\mathrm{Re}(g_{11})\,\mathrm{Im}(g_d))\\ c_2&=(\mathrm{Re}(g_{21}))^2+(\mathrm{Im}(g_{21}))^2-(\mathrm{Im}(g_{11}))^2\end{aligned}\tag{3.19}$$

$$\begin{aligned}a_3&=\left((\mathrm{Re}(g_d))^2+(\mathrm{Im}(g_d))^2\right)\\ b_3&=(2\,\mathrm{Re}(g_{22})\,\mathrm{Re}(g_d)+2\,\mathrm{Im}(g_{22})\,\mathrm{Im}(g_d))\\ c_3&=(\mathrm{Re}(g_{22}))^2\\ a_4&=-\left((\mathrm{Re}(g_d))^2+(\mathrm{Im}(g_d))^2\right)\\ b_4&=-(2\,\mathrm{Im}(g_{22})\,\mathrm{Re}(g_d)-2\,\mathrm{Re}(g_{22})\,\mathrm{Im}(g_d))\\ c_4&=(\mathrm{Re}(g_{12}))^2+(\mathrm{Im}(g_{12}))^2-(\mathrm{Im}(g_{22}))^2\end{aligned}\tag{3.20}$$

the conditions on diagonal dominance can be expressed as:

$$\begin{aligned}a_1\alpha_2^2+b_1\alpha_2+c_1&\geq a_2\beta_2^2+b_2\beta_2+c_2\\ a_3\alpha_1^2+b_3\alpha_1+c_3&\geq a_4\beta_1^2+b_4\beta_1+c_4\end{aligned}\tag{3.21}$$

Here it must be pointed out that the all coefficient terms that are given in (3.19) and (3.20) are functions of frequency. However, for a given fixed frequency all of these coefficient terms corresponds to constant values. As a result, conditions that real and imaginary parts of both controllers should satisfy in terms of column diagonal dominance be easily determined from (3.21).

For the specific case of static diagonal controllers, derived results can be easily simplified since the imaginary part of the controllers are zero for both frequencies. In this specific case conditions that should be satisfied in order to achieve column diagonal dominance can be written as:

$$\begin{aligned} a_1\alpha_2^2 + b_1\alpha_2 + (c_1 - c_2) \geq 0 \\ a_3\alpha_1^2 + b_3\alpha_1 + (c_3 - c_4) \geq 0 \end{aligned} \tag{3.22}$$

In this specific case, in addition to determining the diagonal dominance characteristics at a given fixed frequency, it becomes also possible to determine frequency intervals that the column diagonal dominance conditions can be achieved.

3.2.2 Row diagonal dominance conditions

The definition of row diagonal dominance was given in Section 3.2. Within the scope of this section, it is aimed to determine the conditions on diagonal type controllers for the TITO control systems that the block diagram representation is given in Figure 3.1. In order to be row diagonal dominant at a given fixed frequency, under the assumption that the determinant of the closed loop system is not equal to zero, the following conditions should be satisfied.

$$|k_1 g_{11} + k_1 k_2 g_d| \geq |k_2 g_{12}| \tag{3.23}$$

$$|k_2 g_{22} + k_1 k_2 g_d| \geq |k_1 g_{21}| \tag{3.24}$$

Direct simplification is not possible in this case since there is not any common multiplier in both sides of inequalities. However, it is also possible to do some algebraic manipulations. For instance left hand side of both inequalities (3.23) and (3.24) can be written written as a factor of $|k_1|$ and $|k_2|$ respectively. In this case, new variables g_{11}^*,g_{22}^*, $g_{d_{12}}^*$ and $g_{d_{21}}^*$ can be defined as:

$$g_{11}^* = \frac{g_{11}}{g_{12}}, \quad g_{22}^* = \frac{g_{22}}{g_{21}}, \quad g_{d_{12}}^* = \frac{g_d}{g_{12}}, \quad g_{d_{21}}^* = \frac{g_d}{g_{21}} \tag{3.25}$$

Using this new notation, under the assumptions $|g_{12}| \neq 0$, $|g_{21}| \neq 0$, $|k_1| \neq 0$ and $|k_2| \neq 0$, conditions on row diagonal dominance at a given fixed frequency can be rewritten as:

$$\left|g_{11}^* + k_2 g_{d_{12}}^*\right| \geq \left|\frac{k_2}{k_1}\right| \tag{3.26}$$

$$\left|g_{22}^* + k_1 g_{d_{21}}^*\right| \geq \left|\frac{k_1}{k_2}\right| \tag{3.27}$$

More or less the same methodology that was used in the previous section can be followed to derive exact conditions on the real and imaginary part of controllers in terms of row diagonal dominance. Without loss of generality, using the real and the imaginary parts of the right and left hand side of the inequalities condition on row diagonal dominance can be written as:

$$\begin{gathered}\left(\operatorname{Re}(g_{11}^*) + \alpha_2 \operatorname{Re}\left(g_{d_{12}}^*\right) - \beta_2 \operatorname{Im}\left(g_{d_{12}}^*\right)\right)^2 + \\ \left(\operatorname{Im}(g_{11}^*) + \alpha_2 \operatorname{Im}\left(g_{d_{12}}^*\right) + \beta_2 \operatorname{Re}\left(g_{d_{12}}^*\right)\right)^2 \geq \frac{\alpha_2^2 + \beta_2^2}{\alpha_1^2 + \beta_1^2}\end{gathered} \tag{3.28}$$

$$\begin{gathered}\left(\operatorname{Re}(g_{22}^*) + \alpha_1 \operatorname{Re}\left(g_{d_{21}}^*\right) - \beta_1 \operatorname{Im}\left(g_{d_{21}}^*\right)\right)^2 + \\ \left(\operatorname{Im}(g_{22}^*) + \alpha_1 \operatorname{Im}\left(g_{d_{21}}^*\right) + \beta_1 \operatorname{Re}\left(g_{d_{21}}^*\right)\right)^2 \geq \frac{\alpha_1^2 + \beta_1^2}{\alpha_2^2 + \beta_2^2}\end{gathered} \tag{3.29}$$

where α_1, α_2, β_1 and β_2 represent the real and imaginary parts of the controllers as it was given in (3.16). After expanding the quadratic expression on the left hand side of the inequalities (3.28) and (3.29), conditions that should be satisfied can be rewritten as:

$$\begin{aligned}&(\operatorname{Re}(g_{11}^*))^2 + \alpha_2^2 \left(\operatorname{Re}\left(g_{d_{12}}^*\right)\right)^2 + \beta_2^2 \left(\operatorname{Im}\left(g_{d_{12}}^*\right)\right)^2 + \\ &2\operatorname{Re}(g_{11}^*)\,\alpha_2 \operatorname{Re}\left(g_{d_{12}}^*\right) - 2\operatorname{Re}(g_{11}^*)\,\beta_2 \operatorname{Im}\left(g_{d_{12}}^*\right) - 2\alpha_2 \operatorname{Re}\left(g_{d_{12}}^*\right)\beta_2 \operatorname{Im}\left(g_{d_{12}}^*\right) + \\ &(\operatorname{Im}(g_{11}^*))^2 + \alpha_2^2 \left(\operatorname{Im}\left(g_{d_{12}}^*\right)\right)^2 + \beta_2^2 \left(\operatorname{Re}\left(g_{d_{12}}^*\right)\right)^2 + \\ &2\operatorname{Im}(g_{11}^*)\,\alpha_2 \operatorname{Im}\left(g_{d_{12}}^*\right) + 2\operatorname{Im}(g_{11}^*)\,\beta_2 \operatorname{Re}\left(g_{d_{12}}^*\right) \geq \frac{\alpha_2^2 + \beta_2^2}{\alpha_1^2 + \beta_1^2}\end{aligned} \tag{3.30}$$

$$\begin{aligned}&\left(\left(\operatorname{Re}\left(g_{d_{21}}^*\right)\right)^2 + \left(\operatorname{Im}\left(g_{d_{21}}^*\right)\right)^2\right)\alpha_1^2 + \\ &\left(2\operatorname{Re}(g_{22}^*)\operatorname{Re}\left(g_{d_{21}}^*\right) + 2\operatorname{Im}(g_{22}^*)\operatorname{Im}\left(g_{d_{21}}^*\right)\right)\alpha_1 + \\ &(\operatorname{Re}(g_{22}^*))^2 + \left(\left(\operatorname{Re}\left(g_{d_{21}}^*\right)\right)^2 + \left(\operatorname{Im}\left(g_{d_{21}}^*\right)\right)^2\right)\beta_1^2 + \\ &\left(2\operatorname{Im}(g_{22}^*)\operatorname{Re}\left(g_{d_{21}}^*\right) - 2\operatorname{Re}(g_{22}^*)\operatorname{Im}\left(g_{d_{21}}^*\right)\right)\beta_1 + \\ &(\operatorname{Im}(g_{22}^*))^2 \geq \frac{\alpha_1^2 + \beta_1^2}{\alpha_2^2 + \beta_2^2}\end{aligned} \tag{3.31}$$

In the case of column diagonal dominance there were no controller parameters on the right hand side inequalities at that step. However, real and imaginary parts of controllers are included on the right hand side of the inequalities(3.30) and (3.31). When the inequalities expressed in (3.30) and (3.31) are analysed in detail, it can be proposed that left hand side of both equations can be written as sum of two quadratic equations whose parameters are the real and the imaginary parts of the individual controllers. As a result, the conditions that should be satisfied at a given fixed frequency in terms of row diagonal dominance can be determined as:

$$\begin{aligned} a_5\alpha_2^2+b_5\alpha_2+c_5+a_6\beta_2^2+b_6\beta_2+c_6 &\geq \frac{\alpha_2^2+\beta_2^2}{\alpha_1^2+\beta_1^2} \\ a_7\alpha_1^2+b_7\alpha_1+c_7+a_8\beta_1^2+b_8\beta_1+c_8 &\geq \frac{\alpha_1^2+\beta_1^2}{\alpha_2^2+\beta_2^2} \end{aligned} \tag{3.32}$$

where the coefficient terms are:

$$\begin{aligned} a_5 &= \left(\left(\mathrm{Re}(g^*_{d_{12}})\right)^2+\left(\mathrm{Im}(g^*_{d_{12}})\right)^2\right) \\ b_5 &= \left(2\,\mathrm{Re}(g^*_{11})\,\mathrm{Re}(g^*_{d_{12}})+2\,\mathrm{Im}(g^*_{11})\,\mathrm{Im}(g^*_{d_{12}})\right) \\ c_5 &= \left(\mathrm{Re}(g^*_{11})\right)^2 \\ a_6 &= \left(\left(\mathrm{Re}(g^*_{d_{12}})\right)^2+\left(\mathrm{Im}(g^*_{d_{12}})\right)^2\right) \\ b_6 &= \left(2\,\mathrm{Im}(g^*_{11})\,\mathrm{Re}(g^*_{d_{12}})-2\,\mathrm{Re}(g^*_{11})\,\mathrm{Im}(g^*_{d_{12}})\right) \\ c_6 &= \left(\mathrm{Im}(g^*_{11})\right)^2 \end{aligned} \tag{3.33}$$

$$\begin{aligned} a_7 &= \left(\left(\mathrm{Re}(g^*_{d_{21}})\right)^2+\left(\mathrm{Im}(g^*_{d_{21}})\right)^2\right) \\ b_7 &= \left(2\,\mathrm{Re}(g^*_{22})\,\mathrm{Re}(g^*_{d_{21}})+2\,\mathrm{Im}(g^*_{22})\,\mathrm{Im}(g^*_{d_{21}})\right) \\ c_7 &= \left(\mathrm{Re}(g^*_{22})\right)^2 \\ a_8 &= \left(\left(\mathrm{Re}(g^*_{d_{21}})\right)^2+\left(\mathrm{Im}(g^*_{d_{21}})\right)^2\right) \\ b_8 &= \left(2\,\mathrm{Im}(g^*_{22})\,\mathrm{Re}(g^*_{d_{21}})-2\,\mathrm{Re}(g^*_{22})\,\mathrm{Im}(g^*_{d_{21}})\right) \\ c_8 &= \left(\mathrm{Im}(g^*_{22})\right)^2 \end{aligned} \tag{3.34}$$

In this case, as expected, it is not possible to separate the two quadratic polynomials on the different sides of the inequalities. However, left hand side of the inequalities are written as the sum of two quadratic polynomials that depend on real and imaginary parts of controllers.

Further simplifications can be achieved in the specific case of constant diagonal controllers. This kind of controllers only include real parts, so that imaginary part related terms can be eliminated. For the case of static diagonal controller, the conditions that should be satisfied in terms of row diagonal dominance can be written as:

$$\begin{aligned} a_5\alpha_2^2 + b_5\alpha_2 + (c_5 + c_6) &\geq \frac{\alpha_2^2 + \beta_2^2}{\alpha_1^2 + \beta_1^2} \\ a_7\alpha_1^2 + b_7\alpha_1 + (c_7 + c_8) &\geq \frac{\alpha_1^2 + \beta_1^2}{\alpha_2^2 + \beta_2^2} \end{aligned} \tag{3.35}$$

At this point it can be proposed that derived results in terms of both column and row diagonal dominance that are given in (3.21) and (3.32) do not include any frequency or controller type restriction. Derived results are directly applicable for all frequencies. Furthermore, it is also possible determine the conditions on real and imaginary parts of the dynamic diagonal controller. Any controller can be chosen for the diagonal entries of controller TFM. Derived results within the scope of the Section 3.2 were also expressed in our studies [41, 44, 46].

3.3 Weighted Diagonal Dominance

Derived results within the scope of the previous section are important from the theoretical point of view since by this approach it becomes possible to determine exact conditions on the real and imaginary parts of the controllers. However, in some practical applications it can be proposed that all controller gain pairs that fulfill the original diagonal dominance definition may not give practically satisfactory results. Therefore, within the scope of this section it is focused on determining the controller parameter regions that achieve better diagonal dominance ratios.

For this purpose, weighting factors are applied to both diagonal dominance conditions. Using the more or less the same approach that was also followed in the previous section it becomes possible to determine controller regions that achieve the predetermined diagonal dominance ratio(s). Since it is more focused on achieving weighted diagonal dominance, weighting factors are assumed to be constants. However, it is also possible to determine additional results using different weighting factor for different working frequencies.

Selected system and controller type is the same with the previous section(See Figure 3.1 and equations (3.3),(3.4)).

In the following equations, μ_{c_1} and μ_{c_2} are weighting factors for the first and the second column respectively. Moreover, μ_{r_1} and μ_{r_2} are weighting factors for the first and the second column. Without loss of generality all weighting factors are assumed to be equal to one or greater than one. When all of these four weighting factors are selected to be one then the conditions on column and row diagonal dominance correspond to the original case. Column and row diagonal dominance are discussed in the next two sections in terms of weighting factors. Derived results within the scope of weighting factors were also used in our previous studies [40–42].

3.3.1 Column diagonal dominance

The original column diagonal dominance condition were given in (3.8) and (3.9). Under the same assumptions weighted column diagonal dominance condition can be written as

$$|k_1 g_{11} + k_1 k_2 g_d| \geq \mu_{c_1} |k_1 g_{21}| \tag{3.36}$$

$$|k_2 g_{22} + k_1 k_2 g_d| \geq \mu_{c_2} |k_2 g_{12}| \tag{3.37}$$

where $\mu_{c_1} \geq 1$ and $\mu_{c_2} \geq 1$ are the weighting factors for the first and second column as indicated earlier. Since the added weighting factors are real numbers, conditions on weighted column diagonal dominance can be determined using the same approach that used in Section 3.2.1. After using the real and imaginary part notation on both sides of the inequalities (3.36), (3.37) and expanding the quadratic terms, the conditions on weighted column diagonal dominance at a given fixed frequency can be written as:

$$\begin{aligned} &(\mathrm{Re}(g_{11}) + \mathrm{Re}(k_2)\,\mathrm{Re}(g_d) - \mathrm{Im}(k_2)\,\mathrm{Im}(g_d))^2 + \\ &\quad (\mathrm{Im}(g_{11}) + \mathrm{Re}(k_2)\,\mathrm{Im}(g_d) + \mathrm{Im}(k_2)\,\mathrm{Re}(g_d))^2 \\ &\qquad \geq \mu_{c_1}^2 \left((\mathrm{Re}(g_{21}))^2 + (\mathrm{Im}(g_{21}))^2 \right) \end{aligned} \tag{3.38}$$

$$\begin{aligned} &(\mathrm{Re}(g_{22}) + \mathrm{Re}(k_1)\,\mathrm{Re}(g_d) - \mathrm{Im}(k_1)\,\mathrm{Im}(g_d))^2 + \\ &\quad (\mathrm{Im}(g_{22}) + \mathrm{Re}(k_1)\,\mathrm{Im}(g_d) + \mathrm{Im}(k_1)\,\mathrm{Re}(g_d))^2 \\ &\qquad \geq \mu_{c_2}^2 \left((\mathrm{Re}(g_{12}))^2 + (\mathrm{Im}(g_{12}))^2 \right) \end{aligned} \tag{3.39}$$

After that, real and imaginary parts of the controllers can be collected on the different sides of the both inequalities. At the end it can be proposed that the weighted column diagonal condition can be represented as the comparison of two 2nd order polynomials that depend on the real and imaginary part of the individual controllers. In other words the conditions that should be satisfied by the controller parameters in terms of weighted column diagonal dominance are:

$$\begin{aligned} a_{w_1}\alpha_2^2 + b_{w_1}\alpha_2 + c_{w_1} &\geq a_{w_2}\beta_2^2 + b_{w_2}\beta_2 + c_{w_2} \\ a_{w_3}\alpha_1^2 + b_{w_3}\alpha_1 + c_{w_3} &\geq a_{w_4}\beta_1^2 + b_{w_4}\beta_1 + c_{w_4} \end{aligned} \tag{3.40}$$

where the coefficient terms a_{w_i}, b_{w_i} and c_{w_i}s are as given in Appendix 1 and α_is and β_is represents the real and imaginary parts of the $i-th$ controller as it was defined in (3.16). It can be interpreted that some coefficient terms are identical with original definition. For instance a_{w_1} is equal to a_1. However, like in the case of c_{w_2} and c_{w_4} the weighting factors are also included in some of the new coefficients. For given weightings factors all the coefficient terms correspond to constant number at a given frequency while in general all of them are functions of frequency variable.

Derived result can also be simplified further for the specific case of static diagonal type controller. In this case, weighted column diagonal dominance conditions can be determined as:

$$\begin{aligned} a_{w_1}\alpha_2^2 + b_{w_1}\alpha_2 + (c_{w_1} - c_{w_2}) &\geq 0 \\ a_{w_3}\alpha_1^2 + b_{w_3}\alpha_1 + (c_{w_3} - c_{w_4}) &\geq 0 \end{aligned} \tag{3.41}$$

3.3.2 Row diagonal dominance

Under the same assumptions thaw was done in the previous sections it can be stated the conditions that should be satisfied for the weighted diagonal dominance are:

$$|k_1 g_{11} + k_1 k_2 g_d| \geq \mu_{r_1} |k_2 g_{12}| \tag{3.42}$$

$$|k_2 g_{22} + k_1 k_2 g_d| \geq \mu_{r_2} |k_1 g_{21}| \tag{3.43}$$

Using the definitions that is given in (3.16) and (3.25), pre results on weighted row diagonal dominance can be expressed as:

$$\begin{aligned} &\left(\mathrm{Re}\left(g_{11}^*\right) + \alpha_2 \mathrm{Re}\left(g_{d_{12}}^*\right) - \beta_2 \mathrm{Im}\left(g_{d_{12}}^*\right)\right)^2 + \\ &\left(\mathrm{Im}\left(g_{11}^*\right) + \alpha_2 \mathrm{Im}\left(g_{d_{12}}^*\right) + \beta_2 \mathrm{Re}\left(g_{d_{12}}^*\right)\right)^2 \geq \mu_{r_1}^2 \frac{\alpha_2^2 + \beta_2^2}{\alpha_1^2 + \beta_1^2} \end{aligned} \tag{3.44}$$

$$\begin{aligned} &\left(\mathrm{Re}\left(g_{22}^*\right) + \alpha_1 \mathrm{Re}\left(g_{d_{21}}^*\right) - \beta_1 \mathrm{Im}\left(g_{d_{21}}^*\right)\right)^2 + \\ &\left(\mathrm{Im}\left(g_{22}^*\right) + \alpha_1 \mathrm{Im}\left(g_{d_{21}}^*\right) + \beta_1 \mathrm{Re}\left(g_{d_{21}}^*\right)\right)^2 \geq \mu_{r_2}^2 \frac{\alpha_1^2 + \beta_1^2}{\alpha_2^2 + \beta_2^2} \end{aligned} \tag{3.45}$$

The same approach can be used the weighted row diagonal dominance results. In this case conditions on weighted row diagonal dominance can be determined as:

$$\begin{aligned} a_{w_5}\alpha_2^2 + b_{w_5}\alpha_2 + c_{w_5} + a_{w_6}\beta_2^2 + b_{w_6}\beta_2 + c_{w_6} &\geq \mu_{r_1}^2 \frac{\alpha_2^2+\beta_2^2}{\alpha_1^2+\beta_1^2} \\ a_{w_7}\alpha_1^2 + b_{w_7}\alpha_1 + c_{w_7} + a_{w_8}\beta_1^2 + b_{w_8}\beta_1 + c_{w_8} &\geq \mu_{r_2}^2 \frac{\alpha_1^2+\beta_1^2}{\alpha_2^2+\beta_2^2} \end{aligned} \tag{3.46}$$

Here the a_{w_i} notation is preferred in order to be compatible with Section 3.3.1. However, in this case all the coefficient terms are equal to their corresponding as given in equations (3.33), (3.34) and in Appendix A. Additionally, the conditions for the case of static diagonal controllers can be directly determined as:

$$\begin{aligned} a_{w_5}\alpha_2^2 + b_{w_5}\alpha_2 + (c_{w_5} + c_{w_6}) &\geq \mu_{r_1}^2 \frac{\alpha_2^2+\beta_2^2}{\alpha_1^2+\beta_1^2} \\ a_{w_7}\alpha_1^2 + b_{w_7}\alpha_1 + (c_{w_7} + c_{w_8}) &\geq \mu_{r_2}^2 \frac{\alpha_1^2+\beta_1^2}{\alpha_2^2+\beta_2^2} \end{aligned} \tag{3.47}$$

3.3.3 An algorithm to achieve weighted diagonal dominance

Derived results for the original diagonal dominance conditions are generalized to the case of weighted case in Section 3.3. An algorithm can be proposed to determine the controller parameter regions for closed loop TITO systems at a given fixed frequency using the theoretical results derived in this section.

The steps of the proposed algorithm in order to determine controller diagonal type controller parameter regions that satisfy weighted diagonal dominance conditions are as follows:

Step 1: Determine the weighting factors for column and row diagonal dominance as $\mu_{c_i} \geq 1$, $\mu_{r_i} \geq 1$.

Step 2: Determine the controller parameter regions that achieve weighted column diagonal dominance using the derived results given in (3.40).

Step 3: Using the inequalities that are given in (3.46) determine controller parameter spaces that satisfy weighted row diagonal dominance.

Step 4: Take the intersection of the regions obtained in Step 2 and Step 3.

Step 5: Determine the controller parameters that makes determinant of the closed loop system zero at the given frequency. If the solution intersects with the region obtained in Step 4 exclude these parameter regions.

The algorithm was proposed for a given fixed frequency but it can also be directly applied for a given frequency sequence (combination of finite fixed frequencies). Furthermore, it can also be easily expanded for a given frequency range. In such a case, after griding the related frequency range into small intervals and determining the specific frequency values, algorithm steps should be repeated for every specific frequency value. However, it can be proposed that the accuracy of the results depends on the griding step size in this case.

In the specific case of static diagonal type controllers, by replacing the equations (3.40), (3.46) respectively with (3.41), (3.47) conditions that constant controllers k_1 and k_2 should satisfy determined directly.

3.4 Determination of the Frequency Ranges

In the previous sections, it was aimed to determine the conditions that should be satisfied by controller parameter for a given fixed frequency. For the constant value of the frequency parameter all conditions are simplified significantly. However, the diagonal dominance problem can be also discussed from a different perspective. Assuming the frequency as a free parameter, for the case of static diagonal controllers it is possible to derive results on frequency intervals that the closed loop TITO system can be made diagonal dominant.

In this section, the conditions on frequency intervals for static diagonal type controllers are discussed in detail for column diagonal dominance. At this point it is aimed to answer "Is it possible to determine critical frequencies that diagonal dominance characteristic may change?" Determining the diagonal dominance characteristic of a given system at a fixed frequency is a straight forwards task using results derived in previous sections. However, this is not the case when it is aimed to determine critical frequencies.

Considered controller type within the scope of this subsection can be written as:

$$K = \begin{bmatrix} k_{p_1} & 0 \\ 0 & k_{p_2} \end{bmatrix} \tag{3.48}$$

Using the (3.22), conditions on column diagonal dominance can be rewritten as:

$$\begin{aligned} a_{f_1}(\omega)k_{p_2}^2 + b_{f_1}(\omega)k_{p_2} + c_{f_1}(\omega) \geq 0 \\ a_{f_2}(\omega)k_{p_1}^2 + b_{f_2}(\omega)k_{p_1} + c_{f_2}(\omega) \geq 0 \end{aligned} \tag{3.49}$$

Conditions that are expressed in (3.49) are quadratic function with respect to k_{p_2} and k_{p_1}. In this case, extremum points can be determined by taking the derivatives of both conditions as:

$$\begin{aligned} \frac{\partial\ a_{f_1}(\omega)k_{p_2}^2+b_{f_1}(\omega)k_{p_2}+c_{f_1}(\omega)}{\partial k_{p_2}} &= 2a_{f_1}(\omega)k_{p_2}+b_{f_1}(\omega)=0 \\ \frac{\partial\ a_{f_2}(\omega)k_{p_1}^2+b_{f_2}(\omega)k_{p_1}+c_{f_2}(\omega)}{\partial k_{p_1}} &= 2a_{f_2}(\omega)k_{p_1}+b_{f_2}(\omega)=0 \end{aligned} \tag{3.50}$$

Solutions of the corresponding equations in (3.50) can be written as:

$$\begin{aligned} k_{p_2}^* &= \frac{-b_{f_1}(\omega)}{2a_{f_1}(\omega)} \\ k_{p_1}^* &= \frac{-b_{f_2}(\omega)}{2a_{f_2}(\omega)} \end{aligned} \tag{3.51}$$

By calculating the value of the second order polynomials at the points $k_{p_2}^* = \frac{-b_{f_1}(\omega)}{2a_{f_1}(\omega)}$ and $k_{p_1}^* = \frac{-b_{f_2}(\omega)}{2a_{f_2}(\omega)}$ it can be written as:

$$\begin{aligned} a_{f_1}(\omega)\left(\frac{-b_{f_1}(\omega)}{2a_{f_1}(\omega)}\right)^2+b_{f_1}(\omega)\frac{-b_{f_1}(\omega)}{2a_{f_1}(\omega)}+c_{f_1}(\omega)=0 \\ a_{f_2}(\omega)\left(\frac{-b_{f_2}(\omega)}{2a_{f_2}(\omega)}\right)^2+b_{f_2}(\omega)\frac{-b_{f_2}(\omega)}{2a_{f_2}(\omega)}+c_{f_2}(\omega)=0 \end{aligned} \tag{3.52}$$

When the derived equations that is given in simplified it can be concluded that controller parameter ranges in terms of column diagonal dominance may change at critical frequencies. These critical frequencies are the solution of the following equations that can be expressed as:

$$4c_{f_1}(\omega)\left(a_{f_1}(\omega)\right)^2-a_{f_1}(\omega)\left(b_{f_1}(\omega)\right)^2=0 \tag{3.53}$$

$$4c_{f_2}(\omega)\left(a_{f_2}(\omega)\right)^2-a_{f_2}(\omega)\left(b_{f_2}(\omega)\right)^2=0 \tag{3.54}$$

The roots of the equation (3.53) and (3.54) are the candidate frequency values that may change the answer of our question. After determining the roots of (3.53) and (3.54) a random frequency value should be selected from every interval and the second order polynomial should be plotted in order to interpret the results. In Section 3.5.3 a case study is also included to demonstrate the derived results within the scope of this subsection for a given TITO system.

3.5 Case Studies

Within the scope of this subsection it is aimed to discuss the effectiveness of the derived results and the proposed algorithm in terms of diagonal dominance. For this purpose, two case studies are included for the static and dynamic diagonal type controllers. The considered control system block diagram in all case studies are as given in Figure 3.1.

In the first two case studies, it is focused on determining controller parameter ranges for a given frequency and/or frequency interval. Derived results for these cases are verified for the selected controller parameter pairs using the Gershgorin disc plots and diagonal dominance ratio plots. In the third and the last case study it was focused on determining the critical frequency ranges that change the structure of controller parameters in terms of column diagonal dominance.

3.5.1 Static diagonal controller

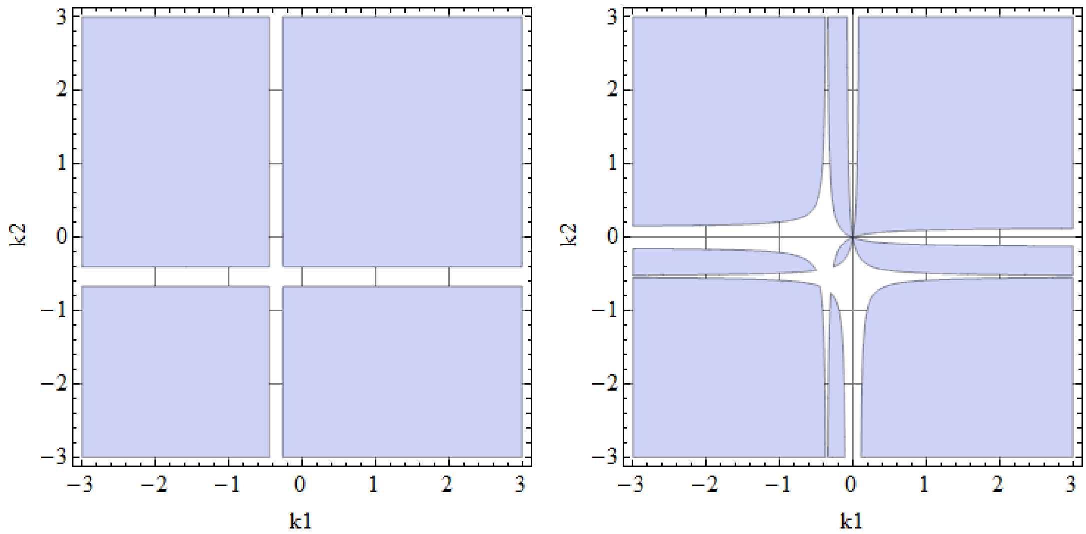
In this section it is aimed to determine controller parameter spaces that achieves standard and weighted diagonal dominance conditions at a given frequency and frequency interval. The TITO system and the controller that is discussed in this case study can be written as:

$$G(s) = \begin{bmatrix} \frac{6-2s}{(2+5s)} & \frac{1+4s}{(2+7s)(1+10s)} \\ \\ \frac{1.5}{(2+5s)} & \frac{2}{(1+10s)} \end{bmatrix} \tag{3.55}$$

$$K = \begin{bmatrix} k_{p_1} & 0 \\ 0 & k_{p_2} \end{bmatrix} \tag{3.56}$$

For this specific case the equations (3.41), (3.47) are used instead of (3.40), (3.46) for column and row diagonal dominance calculations respectively. At this point, it must also be noted that the selected system is not suitable for diagonalization methods that was introduced in the previous sections.

As the first step it is focused on the case of fixed frequency. In this case all the coefficient terms correspond to scalar numbers. All weighting factors should be selected as one to determine conditions for standard diagonal dominance definitions. For $\omega = 0$, using (3.41) the conditions that should be satisfied in terms of column



(a) Column Diagonal Dominance (b) Row Diagonal Dominance

Figure 3.2 : Controller parameter regions for diagonal dominance at $\omega = 0$.

diagoninal dominance can be expressed as:

$$\begin{aligned} 31.64k_{p_2}^2 + 33.75k_{p_2} + 8.44 \geq 0 \\ 31.64k_{p_1}^2 + 22.50k_{p_1} + 3.75 \geq 0 \end{aligned} \tag{3.57}$$

Likewise, the following conditions should be satisfied for row diagonal dominance at $\omega = 0$.

$$\begin{aligned} 126.56k_{p_2}^2 + 135k_{p_2} + 36 \geq \frac{k_{p_2}^2}{k_{p_1}^2} \\ 56.25k_{p_1}^2 + 40k_{p_1} + 7.11 \geq \frac{k_{p_1}^2}{k_{p_2}^2} \end{aligned} \tag{3.58}$$

As result, it can be proposed that the controller parameter regions that achieve column and row diagonal dominance at can be determined respectively as it is given in Figure 3.2(a) and Figure 3.2(b). If it is desired to derive controller parameter regions that achieves both conditions then the intersection area should be determined using 3.2(a) and Figure 3.2(b). By this way, it can be asserted that $k_{p_1} - k_{p_2}$ parameter regions that satisfy both standard column and row diagonal dominance at zero frequency is as given in Figure 3.3.

As it is seen from Figure 3.3 it is possible to determine controller parameters that achieve both column and diagonal dominance conditions. It can also be proposed that in some specific cases it is not possible to find controller pairs that achieve both conditions unlike the discussed case.

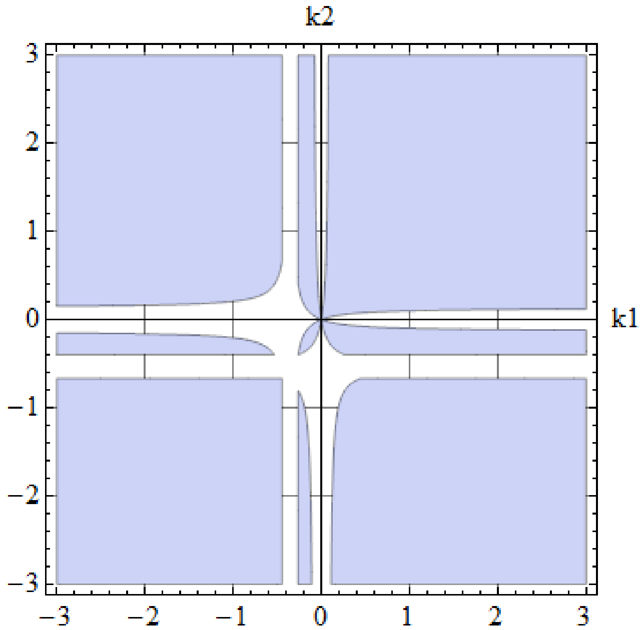


Figure 3.3 : Parameter regions that satisfy both CDD and RDD at $\omega = 0$.

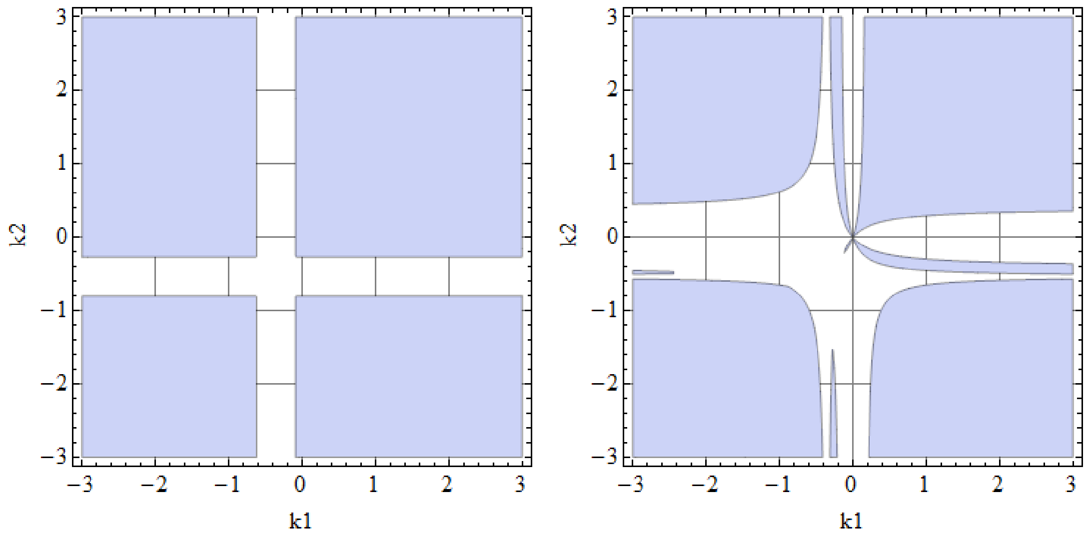
As it was discussed in Section 3.3 standard definition may not be satisfactory enough from the practical point of view. Derived results for the weighted diagonal dominance can also be applied in this case. In order to demonstrate the results for the case of weighted diagonal dominance the weighting factors are selected as $\mu_{c_1}, \mu_{r_1} = 2$ $\mu_{c_2}, \mu_{r_2} = 3$. It is aimed to determine controller parameters that make the magnitude of the diagonal entry in the first column and first row two times greater that the corresponding off diagonal element. Likewise, diagonal entry in the second column and second row should be three times greater than the corresponding off diagonal term.

Under this selection of weighting factors new conditions for weighted and row diagonal dominance can be written as:

$$\begin{aligned} 31.64k_{p_2}^2 + 33.75k_{p_2} + 6.75 \geq 0 \\ 31.64k_{p_1}^2 + 22.50k_{p_1} + 1.75 \geq 0 \end{aligned} \tag{3.59}$$

$$\begin{aligned} 126.56k_{p_2}^2 + 135k_{p_2} + 36 \geq \frac{4k_{p_2}^2}{k_{p_1}^2} \\ 56.25k_{p_1}^2 + 40k_{p_1} + 7.11 \geq \frac{9k_{p_1}^2}{k_{p_2}^2} \end{aligned} \tag{3.60}$$

In this case, using (3.59) and (3.60), static controller gain regions that satisfy weighted column and row diagonal dominance can be determined as it is given in Figure 3.4.



(a) Weighted Column Diagonal Dominance (b) Weighted Row Diagonal Dominance

Figure 3.4 : Controller parameter regions for weighted diagonal dominance at $\omega = 0$ ($\mu_{c_1}, \mu_{r_1} = 2\ \mu_{c_2}, \mu_{r_2} = 3$).

When Figure 3.4 and Figure 3.2 examined, it can be interpreted that resulting figures are similar to each other. However, it can be clearly observed that in the case of weighted diagonal dominance resulting regions are relatively narrow compared to the standard case. It can also be interpreted that some regions that included in Figure 3.2 are not included in Figure 3.4 as expected.

By taking the intersection of the derived regions that are given in Figure 3.4(a) and Figure 3.4(b) static controller gain intervals that achieve CDD and RDD at $\omega = 0$ can be determined directly as in 3.5.

For a given TITO system it can be proposed that achieving diagonal dominance at a single frequency may not be sufficient from the practical point of view. Mostly, it is desired to achieve diagonal dominance and to reduce the interactions in a specific frequency range as indicated in [42]. As indicated earlier, derived results can be easily extended for a given frequency range. In such a case, specified frequency interval have to be grided and after that steps of the proposed algorithm have to be repeated for every specific frequency. If it is desired to achieve diagonal dominance within the frequency interval $100 \geq \omega \geq 0$ then static diagonal controller parameter regions can be determined as given in Figure 3.6 for standard diagonal dominance. The parameter regions that is given in Figure 3.6 satisfy both column and row diagonal dominance for the given interval.

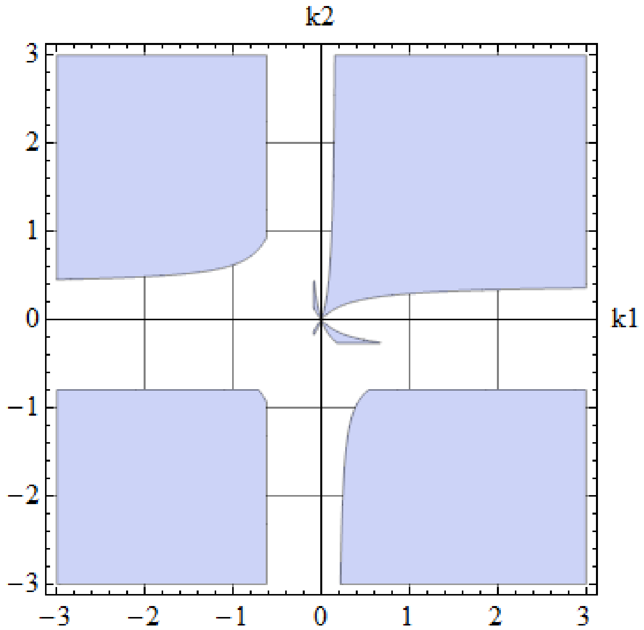


Figure 3.5 : Parameter regions that satisfy weighted CDD and RDD at $\omega = 0$ $(\mu_{c_1}, \mu_{r_1} = 2\ \mu_{c_2}, \mu_{r_2} = 3)$.

Corresponding parameter regions for the case of weighted diagonal dominance can be derived as in Figure 3.7 for $\mu_{c_1}, \mu_{r_1} = 2$ and $\mu_{c_2}, \mu_{r_2} = 3$. It can be proposed that certain amount of the parameter region is excluded in the case of weighted diagonal dominance when 3.7 and 3.6 compared.

It can be proposed that there are two main graphical methods to verify the diagonal dominance characteristics of a given MIMO system in a predetermined frequency range [7]. These graphical methods are named as Gershgorin Disc Plot (or Nyquist Arrays and Gershgorin Disc Plot) and Diagonal Dominance Ratio Plot.

In the case of Gershgorin Plot the magnitude of the sum of off diagonal terms are superimposed to the Nyquist plot of the diagonal term as a circle. In order to be a diagonal dominant system at the given frequency range resulting Gershgorin Plot should exclude the origin. Moreover, Gershgorin Discs can be plotted separately for the column and row diagonal dominance cases so that column and row diagonal dominance characteristics can be analyzed separately

One controller gain pair is selected in order to analyse the diagonal dominance of the closed loop system since it is not possible to apply the same process for all parameter values that is given in Figure 3.7. When Figure 3.7 examined it can be proposed that $k_{p_1} = 0.25, k_{p_2} = 2.5$ controller gain pair achieves the weighted CDD and RDD at

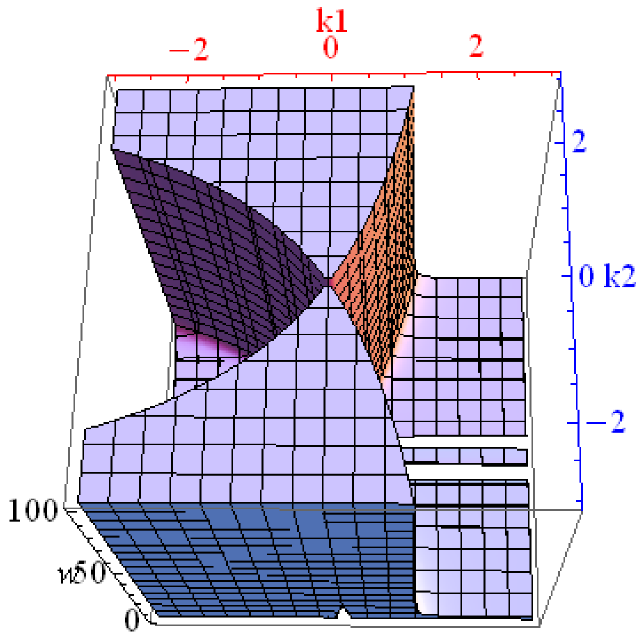


Figure 3.6 : $k_{p_1} - k_{p_2}$ regions that achieve CDD and RDD for $100 \geq \omega \geq 0$.

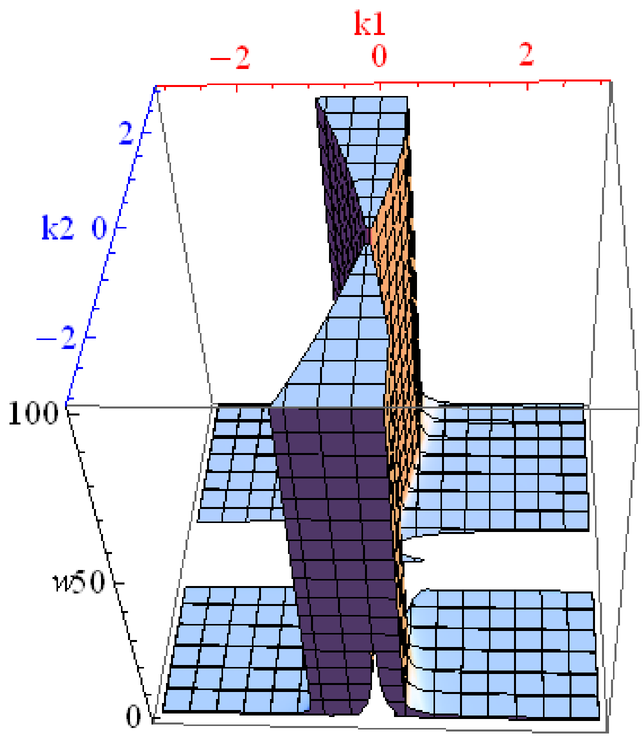
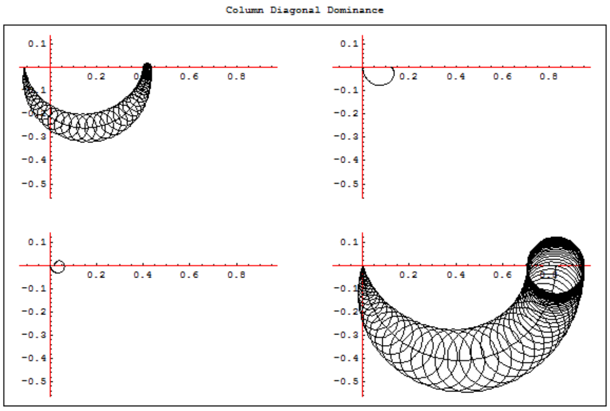
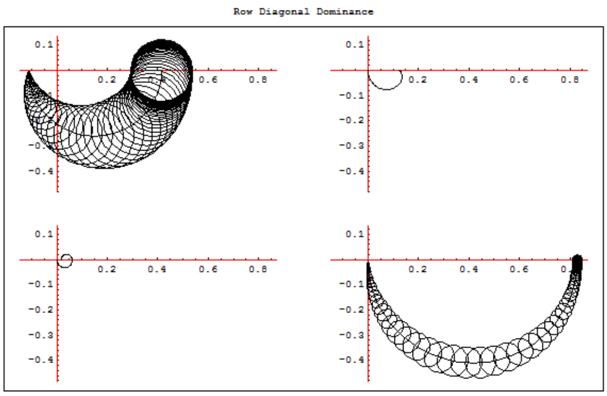


Figure 3.7 : $k_{p_1} - k_{p_2}$ regions that achieve CDD and RDD for $100 \geq \omega \geq 0$ ($\mu_{c_1}, \mu_{r_1} = 2$ and $\mu_{c_2}, \mu_{r_2} = 3$).

$100 \geq \omega \geq 0$. For the selected controller parameter pair, Gershgorin Discs can be plotted as it is given in Figure 3.8.



(a) Weighted Column Diagonal Dominance



(b) Weighted Row Diagonal Dominance

Figure 3.8 : Nyquist Arrays and Gershgorin Discs for $100 \geq \omega \geq 0$ $(k_{p_1} = 0.25, k_{p_2} = 2.5)$.

As it is seen in Figure 3.8, Gershgorin Discs exclude the origin. As a result, it can be interpreted that the given controller achieves both column and row diagonal dominance at the desired frequency interval as expected.

While Gershgorin Discs give a useful graphical approach to determine that a given specific system is diagonal dominant or not, it is sometimes hard to determine how the system is close to the bounds of diagonal dominance. In order to determine the exact dominance ratios at a given frequency range diagonal dominance ratio plots can be preferred in such cases.

The row/column diagonal dominance ratio is defined as the ratio of the sum of all off diagonal terms in a row/column, divided by the magnitude of the diagonal term at the given frequency. So that it can be interpreted that smaller diagonal dominance ratios indicate lower interaction at that frequency. For the case of standard diagonal dominance definition the ratio plots should not exceed one in order to be a diagonal dominant system. Additionally, it can also be directly proposed that the ratio plots should not exceed $\frac{1}{\mu_{c_i}}$ and $\frac{1}{\mu_{r_i}}$ in case of weighted diagonal dominance.

Diagonal dominance ratio plots for $k_{p_1} = 0.25, k_{p_2} = 2.5$ is as given in Figure 3.9 for the same frequency range. In this case diagonal dominance ratio plots indicate that designed dynamic controller fulfills the weighted diagonal dominance conditions since ratio plot of the first row and first column do not exceed 0.5 and the ratio plot of the second row and second column do not exceed 0.33.

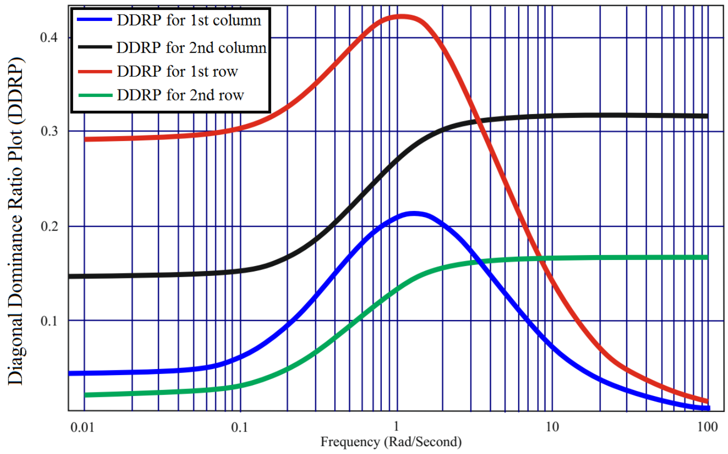


Figure 3.9 : Diagonal dominance ratio plots for $100 \geq \omega \geq 0$ ($k_{p_1} = 0.25, k_{p_2} = 2.5$).

Static controller parameter regions are derived for a given fixed frequency and frequency intervals within the scope of this case study. It is also shown that parameter regions that achieve weighted diagonal dominance can be derived directly. In order to demonstrate the diagonal characteristics of the closed loop system graphical methods like Diagonal Dominance Ratio Plots and Gershgorin Discs are applied for the selected controller parameters. All derived results within the scope of this subsection verify the effectiveness of the obtained results in terms of determining static diagonal controller parameter regions that satisfy standard and weighted diagonal dominance conditions. It must also be noted that weighting factors are chosen to be the same constant value in all frequencies in the discussed case. However, they can be easily selected as different values for different frequencies.

3.5.2 Dynamic diagonal controller

In this case study, it is aimed to show that the derived results are also applicable for dynamic diagonal type controller. The selected TITO system and dynamic controller can be written as:

$$G(s) = \begin{bmatrix} \frac{5}{1+5s} & \frac{1+0.01s}{1+8s} \\ \frac{1.5+0.03s}{1+10s} & \frac{6}{1+4s} \end{bmatrix} \tag{3.61}$$

$$K(s) = \begin{bmatrix} k_{p_1} + k_{d_1} s & 0 \\ 0 & k_{p2} + \frac{k_{i_2}}{s} \end{bmatrix} \tag{3.62}$$

The conditions that are given in (3.40), (3.46) are used to determine controller parameter regions. For the sake of simplicity and in order to be able to plot the parameter regions k_{d_1} and k_{i_2} are assumed to be $k_{d_1} = k_{i_2} = 1$. However, at this point it must be noted that this assumption was made only for the sake of simplicity. It is possible to get results for any selection of k_{d_1} and k_{i_2}.

In this case study, only weighted diagonal dominance results are presented. All the weighting factors are selected to be two ($\mu_{c_1}, \mu_{r_1}, \mu_{c_2}, \mu_{r_2} = 2$) since it is desired to determine controller parameter regions that make the magnitude of the diagonal term at least two times greater than the corresponding off diagonal terms.

Under the given assumptions $k_{p_1} - k_{p_2}$ parameter regions that satisfy both weighted CRR and RDD can be determined by using (3.40) and (3.46). The parameter regions that achieve both condition at the frequency $\omega = 10$ can be determined as it is given in 3.10.

If it is desired to achieve weighted diagonal dominance in the frequency range $100 \geq \omega \geq 0$ under the same selection of weighting factors the same process that was used in the previous section can be applied. So that, the parameter regions that achieve weighted diagonal dominance can be derived as given in Figure 3.11. It can be proposed that in this specific case the resulting parameter regions are narrowed in higher frequencies, when the Figure 3.11 investigated in detail. It can also be interpreted from the practical point of view that it becomes harder to determine controller parameters that satisfy condition in higher frequencies.

When Figure 3.11 a controller pair that achieves the diagonal dominance conditions in the desired frequency range can be selected as $k_{p_1} = 20, k_{p_2} = 40$. In this case, the Gershgorin Discs plot can be obtained as it is given in Figure 3.12. It can be proposed that in the related frequency range the effect of g_{12} is limited for the corresponding

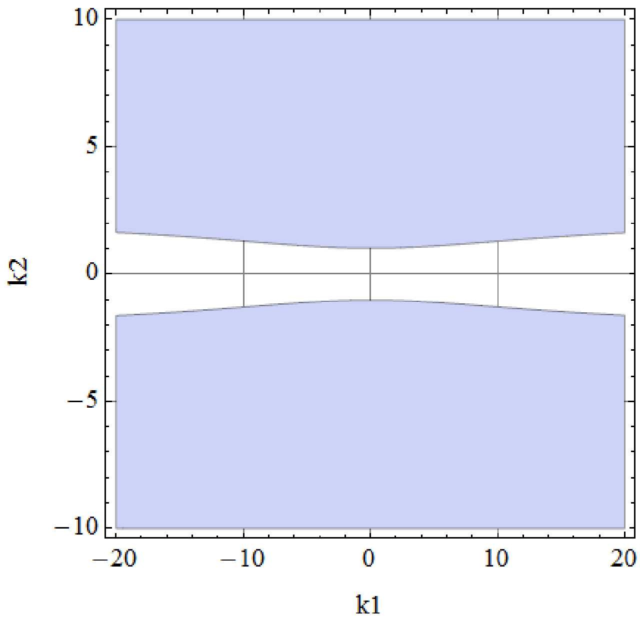


Figure 3.10 : Parameter regions that satisfy weighted CDD and RDD at $\omega = 10$ $(\mu_{c_1}, \mu_{r_1}, \mu_{c_2}, \mu_{r_2} = 2)$.

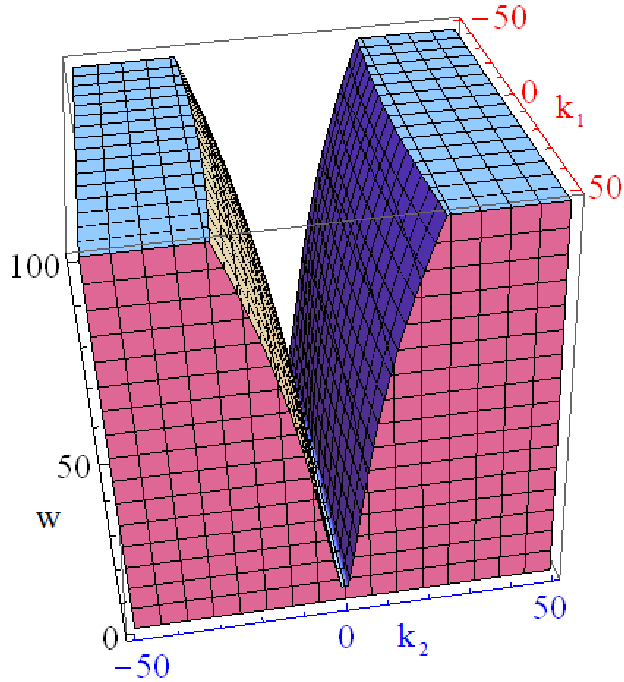
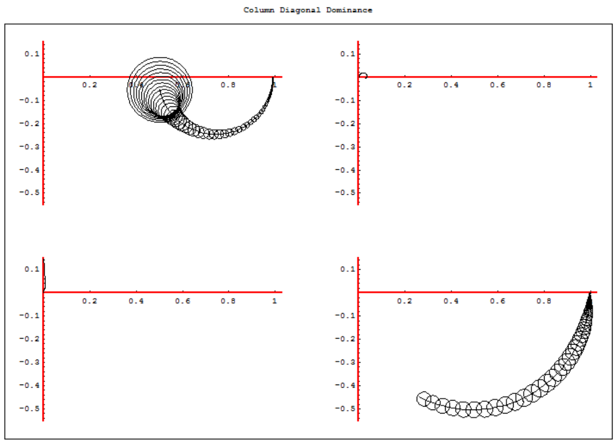
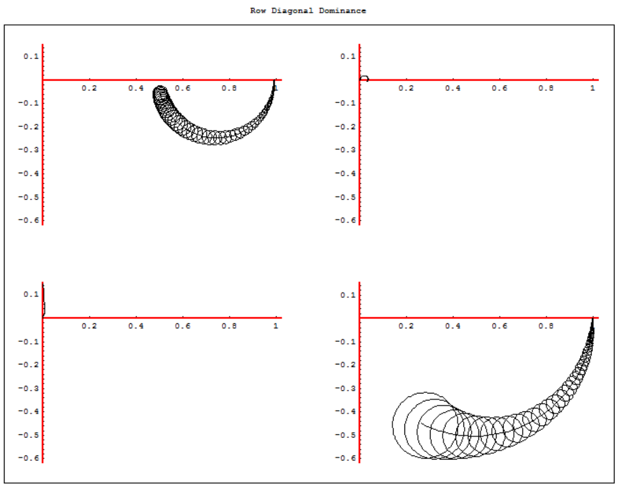


Figure 3.11 : $k_{p_1} - k_{p_2}$ regions that achieve CDD and RDD for $100 \geq \omega \geq 0$ $(\mu_{c_1}, \mu_{r_1}, \mu_{c_2}, \mu_{r_2} = 2)$.

weighted column and row diagonal dominance. However, it is not possible to propose the same comments for g_{21}. In the first sub figure of Figure 3.12(a) and in the last sub figure of Figure 3.12(b) after a certain frequency the effect of g_{21} to the corresponding diagonal term can be clearly observed. However, the origin is not included by Gershgorin Discs in any case.



(a) Weighted Column Diagonal Dominance



(b) Weighted Row Diagonal Dominance

Figure 3.12 : Gershgorin disc plot related with 2nd case study.

In order to have more insight related with the diagonal dominance characteristics of the given system diagonal dominance ratios can be plotted as it was done in the previous section. For the same selection of controller parameters, diagonal dominance ratio plots can be derived as it is given in Figure 3.13. In this case the ratio plots should not exceed 0.5 since all weighting factors are selected as 2. It can be easily recognized that all the ratio plots are less than 0.5 for the related frequency range. It can also be interpreted that two of the ratio plots (column 1 CDD AND row 2 RDD) tend to

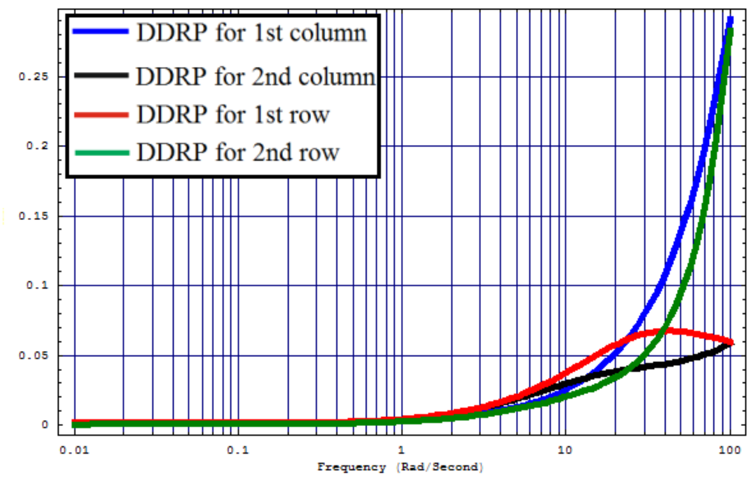


Figure 3.13 : Diagonal dominance ratio plot.

increase after a certain frequency. This is also compatible with the Gershgorin disc plot that is given in Figure 3.12(a).

A static controller have the same magnitude in all frequencies. From this point of view, it can be proposed that such controller are relatively restricted to modify the diagonal dominance characteristic of a given system. Instead of static controllers, dynamic controllers can be used to fulfill the diagonal dominance conditions in different frequencies or frequency ranges. With this aim, the case of dynamic diagonal controller case was discussed within the scope of this case study. It is shown that derived results are also applicable for dynamic diagonal type controllers. PI and PD type controller are used in the diagonal entries of the controller TFM. Derived results are also interpreted for a selected pair of controller parameter using Gershgorin Discs and diagonal dominance ratio plots. In addition to the results that are presented in this case study, triangular inquality and optimization algorithms can also be used for determining dynamic diagonal controller as it was done in our previous study [42].

3.5.3 Frequency ranges

In this case study it is aimed to demonstrate the derived results in Section 3.4 and to discuss the diagonal dominance problem from a different point of view. The TITO system and the controller type that will be used as a case study in this section are:

$$G(s) = \begin{bmatrix} \frac{(6-2s)}{(5s+2)} & \frac{(4s+1)}{(5s+3)(10s+1)} \\ \\ \frac{1.5}{(7s+1)} & \frac{2}{(s+1)(s+2)} \end{bmatrix} \tag{3.63}$$

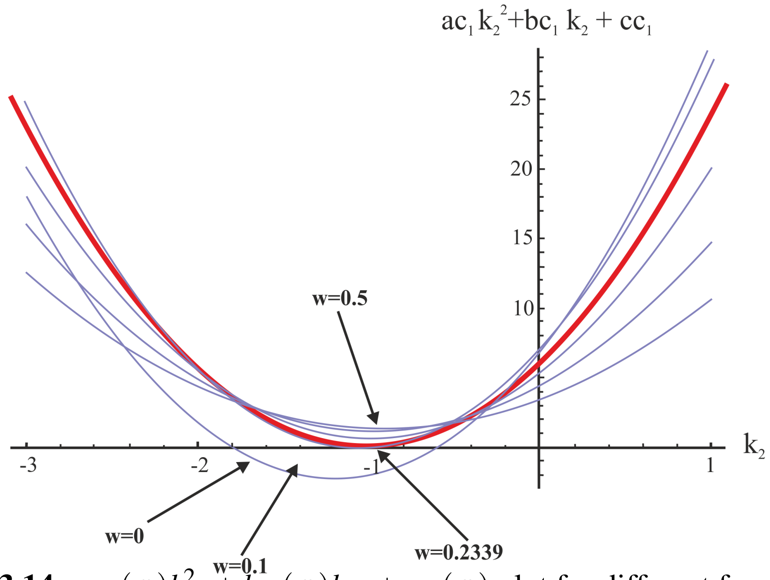


Figure 3.14 : $a_{f_1}(\omega)k_{p_2}^2 + b_{f_1}(\omega)k_{p_2} + c_{f_1}(\omega)$ plot for different frequencies.

$$K = \begin{bmatrix} k_{p_1} & 0 \\ 0 & k_{p_2} \end{bmatrix} \tag{3.64}$$

In order to analyze the column diagonal dominance firstly, the real roots of the equation (3.53) and (3.54) should be determined. For the sake of simplicity the conditions for first and second column are discussed separately.

The real roots of the equation (3.53) that will lead us to the determination of critical frequencies in terms of the CDD of the first column can be calculated as:

$$4c_{f_1}(\omega)\left(a_{f_1}(\omega)\right)^2 - a_{f_1}(\omega)\left(b_{f_1}(\omega)\right)^2 = 0 \Rightarrow \omega_1 = -0.2339 \ \omega_2 = 0.2339 \tag{3.65}$$

There is a real root at $\omega_2 = 0.2339$. This may lead to the interpretation that the range of controller parameters that satisfy the diagonal dominance condition of the first column may change at this frequency. So that this frequency can be named as critical frequency in terms of column diagonal dominance. The second order polynomial $a_{f_1}(\omega)^2 + b_{f_1}(\omega)k_{p_2} + c_{f_1}(\omega)$ is plotted for different frequency values ω $(0 \geq \omega \geq 5)$ as it is given in Figure 3.14 .

As it is observed from Figure 3.14, from $\omega = 0$ to $\omega = 2.339$ there are two k_{p_2} intervals that satisfies the diagonal dominance condition given in (3.22). For instance at $\omega = 0$ the inequality given in (3.22) is satisfied for $-\infty < k_{p_2} \leq -1.8$ and $-0.6 \leq k_{p_2} < \infty$

Same procedure can also be applied for the second column. For this purpose firstly the real roots of (3.54) should be determined. For the given system real roots of (3.54) can

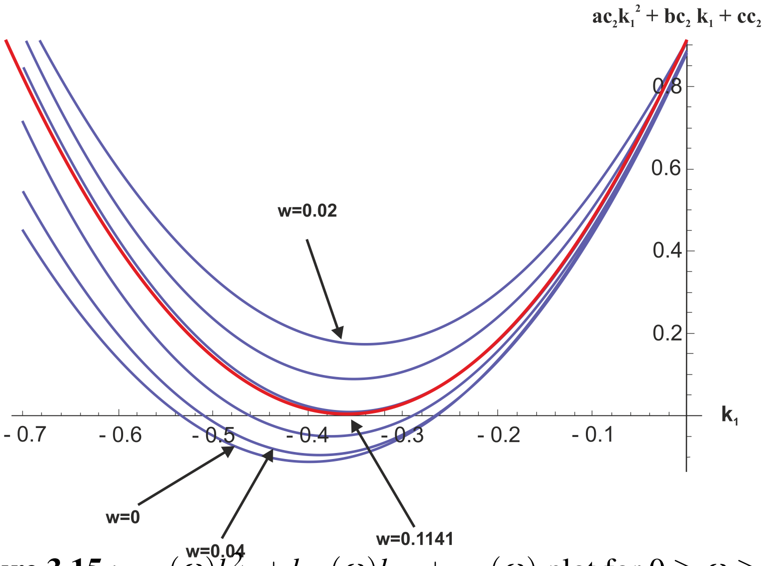


Figure 3.15 : $a_{f_2}(\omega)k_{p_1}^2 + b_{f_2}(\omega)k_{p_1} + c_{f_2}(\omega)$ plot for $0 \geq \omega \geq 0.2$.

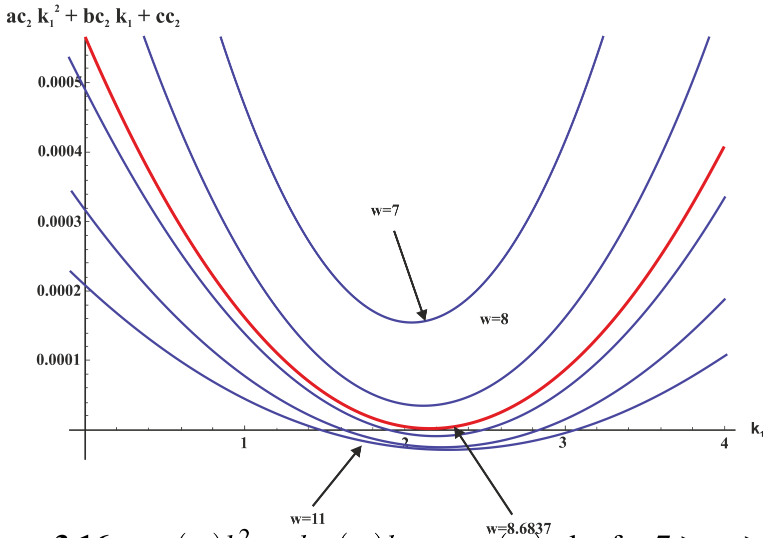


Figure 3.16 : $a_{f_2}(\omega)k_{p_1}^2 + b_{f_2}(\omega)k_{p_1} + c_{f_2}(\omega)$ plot for $7 \geq \omega \geq 11$.

be determined as:

$$\omega_1 = -8.68376 \quad \omega_2 = -0.114125 \quad \omega_3 = 0.114125 \quad \omega_4 = 8.68376 \tag{3.66}$$

There are two critical frequencies ($\omega_3 = 0.114125$ and $\omega_4 = 8.68376$) for the second column. The condition function for the second column diagonal dominance $a_{f_2}(\omega)k_{p_1}^2 + b_{f_2}(\omega)k_{p_1} + c_{f_2}(\omega)$ is plotted for different values of ω ($0 \geq \omega \geq 0.2$) as it is given in Figure 3.15. Likewise, the same function is also plotted for $7 \geq \omega \geq 11$ as given in Figure 3.16.

By interpreting the Figure 3.15 and Figure 3.16 it can be asserted that all k_{p_1} values in the frequency interval $0.11 < \omega < 8.68$ satisfies the diagonal dominance condition of

Table 3.1 : Frequency ranges and the intervals of controller parameters that satisfy column diagonal dominance conditions.

	$0 < \omega < 0.11$	$0.11 < \omega < 0.2339$	$0.2339 < \omega < 8.68$	$8.68 < \omega < \infty$
k_{p_1}	Interval	All k_{p_1} values	All k_{p_1} values	Interval
k_{p_2}	Interval	Interval	All k_{p_2} values	All k_{p_2} values

the second column. For all other frequencies there are varying parameter intervals that satisfies the diagonal dominance condition. The frequency ranges and the parameter values that satisfy the conditions are given in Table 3.1.

In this case study diagonal dominance was discussed from a different point of view compared to the previous two case studies. It was shown that it is possible to determine the critical frequencies in terms of column diagonal dominance that changes the interval characteristics of controller parameters. However, it must also be noted that derived result in terms of frequency intervals are only valid for the case of static diagonal type controllers.

4. STABILITY OF MIMO SYSTEMS

From the practical point of view, it can be proposed that the derived results in the previous section in terms of diagonal dominance are crucial for MIMO system design, especially when it is desired to design decentralized controllers for MIMO systems. Additionally, Nyquist Theorem can be generalized to MIMO systems for the case of diagonal dominant systems as indicated in the early studies of Rosenbrock. However, in general, achieving diagonal dominance does not indicate that the closed loop system is stable, since the diagonal dominance is defined over magnitude conditions only. Therefore, it is aimed to determine stability mapping conditions and stabilizing parameter spaces in this chapter.

Stability of dynamical systems is one of the core problems in control engineering. It is possible to define stability from different perspectives (asymptotic stability, exponential stability, BIBO stability etc.). However, these different definitions have no practical significance for the case of linear time invariant systems [21]. In addition to the stability, in general, the most important objective of a dynamical system is to achieve certain performance specification(s). While these two problems can be seen as two distinct problems at the first sight, it is possible to map performance criteria to the stability conditions using eigenvalue specifications and parametrization [63], [64]. As a result, it is also possible to set links between stability and performance problems. Deriving powerful approaches in terms of stability is also meaningful from this perspective.

In this chapter, it is aimed to propose a Lyapunov equation based stability mapping approach, after pointing out the previous studies in terms of LTI and MIMO system stability. Compared to the frequeny based approaches in literature, this method has several advantages like eleminating the need for frequency sweeping, decoupling at singular frequencies and controller type dependencies. Since the approach was proposed in the sense of Lyapunov, it is also applicable to the other type of systems where Lyapunov function formulation is possible (SISO systems, discrete time systems, switching systems etc.). Derived results within the scope of this section were published in [40, 46, 65–69].

4.1 Previously Proposed Approaches

In order to increase the readability, previously proposed approaches are discussed in three subsections. In the first subsection, it was aimed to point out the first major studies in terms of LTI system stability. Later on, major studies both for SISO and MIMO systems were pointed out in the second subsection. And lastly, it was mentioned about the recent studies that has been proposed in litertature in terms of MIMO stability in the last subsection.

4.1.1 First studies on stability

The first systematic studies in terms of stability of dynamical systems date back to the second half of the 19th century. It was first proposed by Maxwell in 1868 that the stability of a closed loop system could be determined by analyzing the roots of an algebraic equation [70]. As indicated in [71], in this paper, Maxwell developed the differential equations of the governor, linearized them and stated stability is related with the real parts of the characteristic equation. However, only second and third order cases were solved by him. In the meantime, a stability criteria was also developed by Vyshnegradsky independently for cubic polynomials in 1876 [72].

The general problem of stability that was pointed out by Maxwell was first solved by Routh in 1877 [73]. Necessary and sufficient stability conditions for a given system were proposed by him in terms of the coefficients of the characteristic polynomial. Years later, the results of Vyshnegradsky was used by Stodola in order to design water turbine governors. As indicated in [74], he asked Hurwitz to solve the stability problem. As a result, a general stability criterion was also developed by Hurwitz in 1895 using other methods than Routh [75, 76]. Independently developed methods of Routh and Hurwitz are today's well-known Routh-Hurwitz criterion.

While these developments were achieved in terms of linear systems, in his pioneering doctoral thesis Lyapunov, bring a different point of view to the non linear system stability. It is proposed in [77] that, at the current time he was also aware of and acknowledged Routh's approach to the stability. However, by using the work of

Poincare, series solutions of non-linear differential equations was proposed by him. However, with respect to various sources, the most important contribution of Lyapunov is to bring a different point of view to dynamical system stability. Instead of dealing with rigorious time domain solutions of differential equations, Lyapunov basically intriduced the energy like functions of state variable in his "second" or "indirect" method [78]. In this approach, it is sufficient to show negativeness of the Lyapunov function to ensure stability. His thesis was first translated to French in 1908 and this translation was reprinted as a book by Princeton University [79]. However, the contributions of Lyapunov are mainly realized by the western community after the late 1950s [77].

Detailed information on the first studies of dynamical system stability can be found on [71, 74, 77]

4.1.2 Major studies in the 20th century

Visualisation of stability region can be traced back to the pioneering study of Vyshnegradsky [80]. This study constituted the basis for the well known D-decomposition. The rigorious approach to divide the parameter space into sub regions where the number of unstable closed loop eigenvalues are invariant was first developed by Neimark [81,82]. The perspective created a strong basis for the following major contributions like [83, 84]. First counterparts of the studies of Neimark in the western community can be found in [85–87].

The first systematic studies in terms of analysing the stability of multivariable feedback systems dates back to late 1940s [88]. However, the later studies are more focused on the non interacting synthesis problem where the stability analysis were not centre of attraction [89–91]. Nyquist array based frequenc domain approached were proposed in for the systems that achieve diagonal dominance conditions in [13, 92]. More details related with the MIMO system design methods can be found in the survey of MacFarlane [93].

From the stability perspective of linear multivariable systems, it was shown in [94] that stability of these kind of systems depends on the closed loop difference matrix and the characteristic polynomials. After that, using the dynamic equation representation necessary and sufficient conditions for a given MIMO system in order to be stable are proved in [95].

The effect of loop gains on stability of MIMO systems were analyzed by Rosenbrock in [14]. Nyquist criterion (Theorem 2,3) and the diagonal dominance properties (Theorem 4) was the basis of this study. After that frequency based approaches were studied by various researchers and gerelized Nyquist stability criterion was proposed [96–99].

On the other hand, an approach to determine all stabilizing controllers for a given control system was first proposed by Youla et al. in 1976 [100]. However, the order of the controller cannot be fixed in this approach. In order to overcome this drawback, in the forthcoming years, researchers in this area more focused on to determine the stabilizing parameters spaces for a given fixed order controller as indicated in [101]. Parameter Space Approach (PSA) which was first proposed by Ackermann is one of the most important approaches that aims to determine stability regions for a given fixed type of controller [102]. For the specific case of PSA there are lots of studies that deal with the determination of stabilizing P, PI, PD and PID controllers but for the case of MIMO systems derived results for such cases are not directly applicable.

On the other hand, as indicated earlier, the studies of Lyapunov were recognized by the western researchers during the late 1950s. The case of stability of linear differential equations in terms of Lyapunov's second method was first questioned by Kalman and Bertram in 1960 [79]. Using the special solution of the Lyapunov matrix equation, the connection with the classical results of Routh and Hurwitz were establised in [103]. Then, a new way of solving Lyapunov equation was proposed in [104] by using the skew symmetric matrices. Additionally, stability conditions of a nominal system matrix were given in [105]. Moreover, in some studies special types of Lyapunov functions like $V = \sum_{i=1}^{n} |x_i|$ and $V = \max_i |x_i|$ were used to discuss the stability problem from a different point of view. This perspective is also related with the diagonal dominance characteristics [79, 106].

4.1.3 Recent studies related with MIMO system stability

It can be proposed that a continuous time MIMO system is stable if and only if all of its poles lie on the complex left half plane. The resulting characteristic polynomial can be so complex even for the basic controller types while the condition on stability is like in the case of SISO systems. For instance, multiplication of the controller parameters and multiplication of the individual transfer functions are included in the characteristic equation in the case of a TITO system that is controlled with a static diagonal controller.

In the recent years, some of the well known approaches that were initially proposed for SISO systems were generalized for the MIMO case. For instance, D-decomposition approach which is based on the regions where unstable roots remains the same in the controller parameter space, was generalized to multivarible systems by Gryazina and Polyak [80, 107].

Eigenvalue and ARE based mapping equation were presented in [63]. Moreover, mappping specifications of these equations on to parameter space was also discussed in detail in the same study. Additionally, an approach to extend the parameter space approach to MIMO systems using the mapping of design objectives for MIMO systems were presented in [108].

Additionally, PSA approach that was first proposed by Ackermann extended to the cases of MIMO and multiloop control structures in [109, 110]. A decoupling based approach were proposed in these studies. However, the generalisation approach for the systems where decoupling is not possible due system limitation is still an ongoing study.

Recently, Nyquist stability criteria was used in the PI controller design problem of MIMO systems [15]. Furthermore, a stability analysis approach was proposed by Keel and Bhattacharya for MIMO systems by using an equivalent scalar transfer function representation [111]. In that approach, a scalar transfer function is constructed from the leading principal minors of the original transfer function matrix. However, within the scope of that study only square systems are considered. There are also some studies in literature that deal with the stability problem of specific subsets of MIMO systems. For instance, a frequency domain approach is presented in [112, 113] to determine the stabilizing static diagonal controller parameters for TITO Systems. A frequency sweeping is needed in this approach to determine stabilizing parameter regions.

In most of the above mentioned approaches, frequency sweeping and griding are required, since the stability problem was defined in the frequency domain. It can be proposed that in case of interacted complex loops, lots of free parameters and higher order of the characteristic polynomial these methods become inefficient.

On the other hand, within the scope of this thesis, it is aimed to Lyapunov based approached to determine the stability boundaries of a given MIMO system. Since the formulation of Lyapunov approach is independent from the type of the controller and number free parameters, the proposed approach can be easily applied for diffent type of controllers. Additionally, the case of parameteric uncertainties can also be discussed from this point of view. This case is discussed in detail in Section 5.

4.2 Lyapunov Stability

In his pioneering doctoral thesis, Lyapunov proposed two main approaches for the stability analysis of dynamical systems. Today, these approaches are named as the "indirect" and "direct" methods (or "first" or "second" method respectively). While these approaches initially proposed for the non linear systems, in the successive years these theorems are also applied to the case of LTI and/or LTV system types.

While the indirect method of Lyapunov is out of scope of this thesis, it is also aimed to give brief information related with it for the better understanding of the whole Lyapunov Theorem.

The main idea of the Lyapunov's indirect method is to use linearization around a given working point and then solve the resulting linear equation. Local stability results can be derived by this approach.

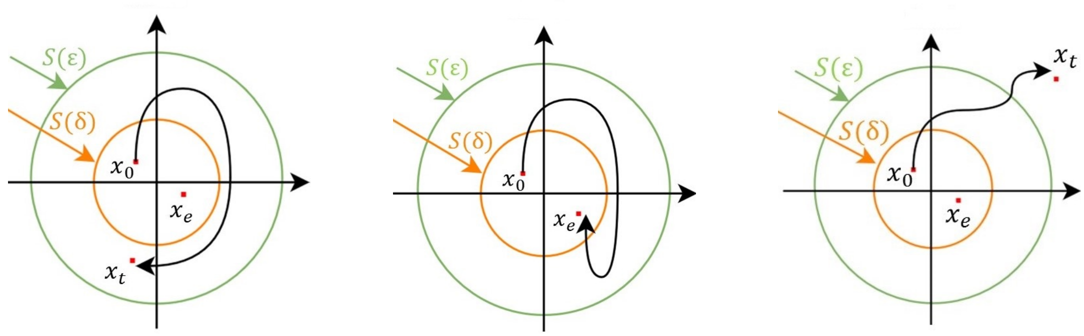
The idea of linearization around a given working point was used in the Lyapunov's indirect method and as a result, only local stability results with small stability region can be achieved.

The following nonlinear system can used to express the methodology of the direct approach

$$\dot{x} = f(x) \tag{4.1}$$

In the system described above (4.1), a state x_e is called an equilibrium state of the system if $f(x_e) = 0$.

An equilibrium point is said to be locally stable in the sense of Lyapunov, if we start from an initial condition x_0 within the region $S(\delta)$ and the trajectories of the system do not exceed the boundary of region $S(\varepsilon)$, as time increases as shown in Figure 4.1(a). The equilibrium point x_e is said to be asymptotically stable, if it is locally stable and if every solution starting within region $S(\delta)$ converges to x_e without exceeding the boundary of $S(\varepsilon)$ as time increases indefinitely as shown in Figure 4.1(b). Finally, if none of these two conditions are satisfied, then the system is said to be unstable. In other words, a system is said to be unstable, if there is a state x_0 in region $S(\delta)$ such that a trajectory starting from this state exceeds the boundary of $S(\varepsilon)$ as $t \to \infty$ as shown in Figure 4.1(c).



(a) Local stability (b) Asymptotic stability (c) Unstability

Figure 4.1 : Lyapunov stability.

Using the indirect method of Lyapunov, the following therorem can be proposed to determine the stability of that equilibrium point.

Theorem 4.1: Let $x = 0$ be an equilibrium point for the system that is given in (4.1) where f is differentiable and defined as $f : D \to \mathbb{R}^n$. Here D is a neighborhood of the origin. A is the linearized version of $f(x)$ around $x = 0$

$$A = \frac{\partial f(x)}{\partial x}|_{x=0} \tag{4.2}$$

Then, the origin is asymptotically stable, if all the eigenvalues of A have negative real parts and the origin is unstable if at least one of the eigenvalues of A has positive real part.

Since the indirect method of Lyapunov is not within the scope of this study, the proof will not be included in the thesis. However, the proof and the detailed anaysis related with the indirect method can be found in [114].

On the other hand, in the direct method of Lyapunov energy-like functions were used to determine the stability of a given system. This direct method eleminates the need for the solution of nonlinear system equation. Instead of this, it was proposed that the negative definiteness of a positive definite Lyapunov function is sufficient to achieve stability. The following theorem can be proposed to determine a sufficient conditions for the systems as it is given in (4.1).

Theorem 4.2: Let $x = 0$ be an equilibrium point for the system that is given in (4.1). Moreover, assume that $D \subset \mathbb{R}^n$ is that includes $x = 0$. Let V is a continuously differentiable function that is defined on the domain $V : D \to R$ such that:

$$V(0) = 0 \quad and \quad V(x) > 0 \quad in \quad D - \{0\} \tag{4.3}$$

Then if:

$$\dot{V}(x) \leq 0 \quad in \quad D \tag{4.4}$$

$x = 0$ is stable.

In order to prove that theorem, the procedure that is also used in Khalil's study will be followed [114]. First of all, a positive real number r can be selected from the domain $r \in (0, \varepsilon]$ where $\varepsilon > 0$. In this case, a subset of D can be constructed as follows:

$$B_r = \{x \in R^n | \, \|x\| \leq r\} \subset D \tag{4.5}$$

Now, the new variable α can be defined as: $\alpha = min_{\|x\|=r} V(x)$. Then considering the (4.3) it can be proposed that $\alpha > 0$. In this case, a new variable β can be defined on the interval $\beta \in (0, \alpha)$. For this new variable β, the set Ω_β can be defined as:

$$\Omega_\beta = \{x \in B_r | V(x) \leq \beta\} \tag{4.6}$$

In this case it can be shown by contratiction that Ω_β is in the interior region of B_r. The relationship between the regions D, B_r and Ω_β is sketched in Figure 4.2. As a result, it can be stated that the set Ω_β is a closed set by definition and additionally, it is also bounded, since it is contained by the set B_r. So that, it can be proposed that

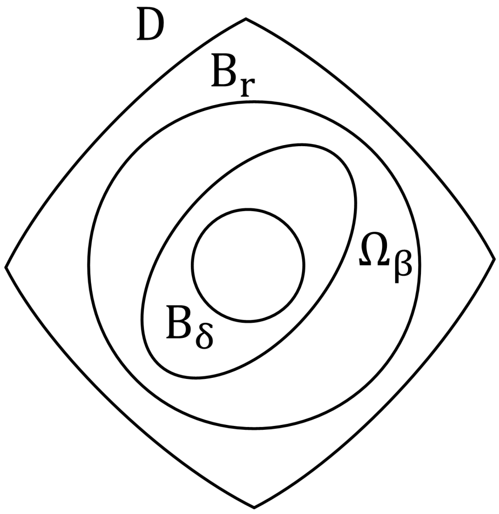


Figure 4.2 : Geometric representaion of the sets used in the proof.

any trajectory that starts in Ω_β at time $t = 0$ stays in Ω_β for all $t \geq 0$. This can also be shown using the property (4.4) as:

$$\dot{V}(x(t)) \leq 0 \Rightarrow V(x(t)) \leq V(x(0)) \leq \beta, \forall t \geq 0 \tag{4.7}$$

Using (4.7), it can be concluded that (4.1) has a unique solution for all $t \geq 0$ when $x(0) \in \Omega_\beta$. So that it can be proposed that there is $\delta > 0$ such that

$$\|x\| \leq \delta \Rightarrow V(x) < \beta \tag{4.8}$$

since $V(x)$ is continuous and $V(0) = 0$. After that it can be written as:

$$B_\delta \subset \Omega_\beta \subset B_r \tag{4.9}$$

The relation between the defined sets B_δ Ω_β and B_r that is given in (4.9), can now be used to propose that the trajectory of x lies in the set B_r . Using (4.9) it can be written as:

$$x(0) \in B_\delta \Rightarrow x(0) \in \Omega_\beta \Rightarrow x(t) \in \Omega_\beta \Rightarrow x(t) \in B_r \tag{4.10}$$

Therefore (4.10) leads to:

$$\|x(0)\| < \delta \Rightarrow \|x(t)\| < r \leq \varepsilon, \forall t \geq 0 \tag{4.11}$$

that shows the equilibrium point $x = 0$ is stable. Using the same approach and algebra, the results of the Theorem 4.2 can be extended to the cases *"asymptotically stable"* and *" globally asymptotically stable"*.

Eleminating the need for solving the nonlinear differential equations in terms of stability can be accepted as the main contribution of this theory. The Lyapunov theory initially proposed for nonlinear systems and it gives a sufficient condition on stability for these kind of systems. However, it can be proposed that it is possible to set a link between the Hurwitz stability and the Lyapunov Theory for the case of linear systems. Additionally, the results that is derived using the Lyapunov approach give the necessary and sufficient conditions for the case of LTI systems. The case of LTI systems is discussed in Section 4.2.1 and Section 4.3 in detail.

4.2.1 Lyapunov stability of LTI systems

One of the major problems that researchers deal with for nonlinear systems is the determination of the Lyapunov function. Especially for the case of non linear systems, it can be said that it can be diffiult to determine positive definite Lyapunov function that can also be named as *"Candidate Lyapunov Function"*. However, in the case of linear systems, it is generally straightforward process to propose the Lyapunov funtion.

A closed loop LTI system can be represented in the state space as:

$$\dot{x} = Ax, \quad x \in \mathbb{R}^n \tag{4.12}$$

where A is the closed loop system matrix. The case that A includes free controller parameters and/or uncertain parameter will be discussed in detail in the next sections.

For the linear system that is given in (4.12) a Lyapunov function can be directly proposed as:

$$V(x) = x^T Px \tag{4.13}$$

where P is a symmetric matrix. It can be proposed that the Lyapunov function representation that is given in (4.13) in the general quadratic form. The following proposition can be easily asserted for the positiveness of the Lyapunov function $V(x)$.

Propositon 4.1: The function $V(x)$ that is defined in (4.13) is a positive function if and only if all the eigenvalues of P is positive.

Using the symmetricity of P, this proposition can be easily proven by rewritting the P as:

$$P = U^T DU \tag{4.14}$$

where $U^T U = I$ and D is a diagonal matrix that includes the eigenvalues of the P. Then by defining a new variable y as $y = Ux$ the original Lyapunov function can be rewritten as:

$$V(x) = y^T P y = \sum_{i=1}^{n} \lambda_i |y_i|^2 \tag{4.15}$$

As a result it can be concluded that:

$$\forall x \neq 0, V(x) > 0 \Leftrightarrow \lambda_i > 0, \forall i \tag{4.16}$$

A matrix P is a positive definite matrix if and only if its all eigenvalues are positive. Combining this property with the Preposition 4.1, the matrix P that is included in (4.13) must be positive definite function for the positivity of the Lyapunov function $V(x)$.

In the second step, it must be shown that time derivate of the proposed Lyapunov function is negative.

By taking the derivative of $V(x)$, it can be written as:

$$\begin{aligned}\dot{V}(x) = \frac{\partial V}{\partial t} &= \dot{x}^T P x + x^T P \dot{x} \\ &= (Ax)^T P x + x^T P (Ax) \\ &= x^T A^T P x + x^T P A x \\ &= x^T (A^T P + PA) x\end{aligned} \tag{4.17}$$

In terms of Lyapunov, $\dot{V}(x)$ must be negative definite in order to be a stable system. Therefore, it can be directly proposed that the inequality:

$$A^T P + PA < 0 \tag{4.18}$$

should be satisfied. While (4.18) seem to be a LMI at first sight, in the case of LTI systems it is equivalent to the matrix equation

$$A^T P + PA = -Q \tag{4.19}$$

where Q is a positive definite matrix. The equation that is given in (4.19) is a special case of Sylvester and Algebraic Riccati Equations and it is named as *"Lyapunov Equation"* or *"Lyapunov Matrix Equation"* in literature.

As indicated earlier, for non linear system only sufficient results can be derived using the Lyapunov approach. However, it becomes possible to derive necessary and sufficient conditions for LTI systems. This result is stated in the following theorem.

Theorem 4.3: For the system that is given in (4.12) and for any $Q > 0$, there exist a positive definite solution of the Lyapunov equation that is given in (4.19) if and only if all the eigenvalues of the system matrix A lie in the open LHP. Additionally, the solution P is unique.

In order to preserve the readability of the thesis, the proof of Theorem 4.3 is given in Appendix A.

It can be proposed that two possible approaches can be followed to determine the stability of a systems that is given in (4.12) [115]. First way is to calculate the solution of the Lyapunov matrix equation for a given symmetric positive definite Q matrix. On the other hand, second way is to solve the matrix Lyapunov equation for a given positive definite P matrix. It can be proposed that it is meaningless to use the second way for the case of LTI systems while both approaches can be applied for the other system classes like Time Delay Systems, Switching Systems, etc. Theorem 4.3 is necessary and sufficient and it is pointed out that for any given positive definite Q there exist a unique solution P. Another result of Theorem 4.3 is that the Q matrix can be selected as simple as possible (for instance identity matrix). However, in order to preserve the generalized notation the Q matrix notation will be used with in the scope of this thesis.

4.3 Lyapunov Equation Based Stability Mapping Approach

Instead of determining the stability characteristic of a given fixed system, Lyapunov theory can also be used to determine stability boundaries for any given free parameter. In such cases the closed loop system can be rewritten as

$$\dot{x} = A(k)x, \quad x \in \mathbb{R}^n \tag{4.20}$$

where $k \in \mathbb{R}^p$ represents the controller parameters. For instance, in case of a TITO system and a static diagonal controller control parameters can be written as $k = \begin{bmatrix} k_1 & k_2 \end{bmatrix}^T$. Here, it is aimed to determine for which values of the given free parameter(s) k the closed loop system remains stable. For this purpose, considering the free parameters the Lyapunov equation can be reformulated as:

$$A(k)^T P(k) + P(k)A(k) = -Q \tag{4.21}$$

Actually, the matrix equation that is given in (4.21) is special case of Sylvester and Algebraic Riccati Equations which can be represented as

$$AX + XB = C \tag{4.22}$$

$$A^T X + XA - XBR^{-1}B^T X + Q = 0 \tag{4.23}$$

respectively. As indicated in [116], for the existence and uniqueness of the solution a Sylvester equation, A and $-B$ should not have common eigenvalues. As a result, it can be proposed that in the Lyapunov equation case, the matrices $A(k)$ and $-A(k)^T$ should not have commom eigenvalues in terms of the existence and uniqueness of the solution. Since a matrix A and its transpose A^T have the same eigenvalues, this also corresponds to the case where A have symmetric eigenvalues with respect to the imaginary axis in the Lyapunov equation. This case was also discussed as *"fictitious stability boundaries"* in [117].

The Lyapunov equation represented in (4.21) is a special matrix equation and it is possible to transform it to a set of linear equations using the Kronecker product and vectorization operator.

Kronecker product that has wide application areas in matrix calculus, matrix equations, system theory, etc. [118] is defined as:

$$A \otimes B := \begin{bmatrix} a_{ij}B \end{bmatrix} = \begin{bmatrix} a_{11}B & a_{12}B & \cdots & a_{1n}B \\ a_{21}B & a_{22}B & \cdots & a_{2n}B \\ \vdots & \vdots & & \vdots \\ a_{m1}B & a_{m2}B & \cdots & a_{mn}B \end{bmatrix} \tag{4.24}$$

On the other hand, vectorization operator vec(.) transforms an $n \times m$ matrix to an $nm \times 1$ vector by rearranging the matrix entries column after column. As a result, vectorization operation for a given $n \times m$ dimensional matrix is defined as follows:

$$\operatorname{vec}(B) := \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix} \in \mathbb{R}^{n \times m} \tag{4.25}$$

Using the Kronecker product and vectorization operator Lyapunov equation can be rewritten as:

$$\left(I \otimes A^T(k) + A^T(k) \otimes I\right) \operatorname{vec}(P(k)) = -\operatorname{vec}(Q) \tag{4.26}$$

where I is the $n \times n$ identity matrix. The equation that is given in (4.26) is in the linear set of equations representation. Defining the new $M(k)$ matrix as

$$M(k) = \left(I \otimes A^T(k) + A^T(k) \otimes I\right) \tag{4.27}$$

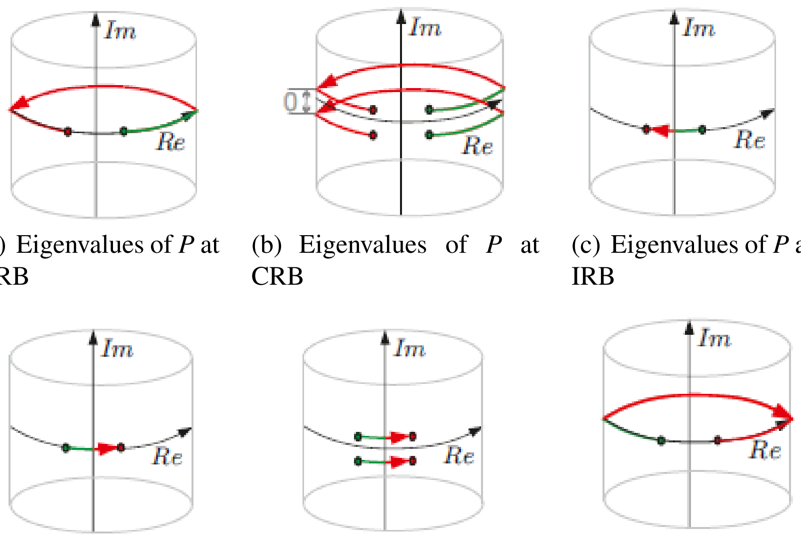
all entries of $P(k)$ can be determined from

$$\text{vec}(P(k) = M^{-1}(k)\text{vec}(-Q) \tag{4.28}$$

With respect to the Lyapunov Theorem, $P(k)$ must be a positive definite matrix for the stability. In other words, all leading proncipal minors of $P(k)$ must be positive in terms of stability. Since all entries of $P(k)$ can be determined from (4.28), it becomes possible to determine all the leading principal minors of $P(k)$. For a given stable system, at least one of the leading principal minors of $P(k)$ should change sign for instability. Considering the numerators and denominators of these leading principal minors, it can be proposed that $2n$ symbolic equations needed to be solved in order to determine stability boundaries which needs dense computational effort. However, this computational complexity can be reduced by analysing the (4.28) in detail.

All the denominator elements of the $P(k)$ matrix that is given in (4.28) are equal to the determinant of $M(k)$. As a result, the denominators of the leading principal minors of $P(k)$ only include the determinant of $M(k)$ and its increasing powers. So that, it can be proposed that it is sufficient to solve only the determinant of $M(k)$ to check the denominators of the leading principal minors of $P(k)$. The required number of equations that should be solved in order to determine stabilizing parameters are reduced to $n+1$ by this analysis. However, solving $n+1$ symbolic equations still needs a high computational effort.

In addition to the previous analysis, significant reductions on the computational complexity may occur if the relations between the $A(k)$, $P(k)$ and $M(k)$ matrices are analysed in detail. All eigenvalues of the A have negative real parts for the stability while the eigenvalues of the $P(k)$ be positive to be a positive definite matrix. As a result, it can be proposed that stability crossing boundaries for these matrices are the controller coefficients that the real parts of the eigenvalues of $A(k)$ and $P(k)$ change sign.



(a) Eigenvalues of P at RRB
(b) Eigenvalues of P at CRB
(c) Eigenvalues of P at IRB
(d) Eigenvalues of A at RRB
(e) Eigenvalues of A at CRB
(f) Eigenvalues of A at IRB

Figure 4.3 : Corresponding eigenvalue characteristics of P and A with respect to RRB, CRB and IRB.

Another important connection between the eigenvalues of $A(k)$ and the determinant of $M(k)$ can be given as [119]:

$$|M(k)| = \prod_{i=1}^{n}\prod_{j=1}^{n}(\lambda_i + \lambda_j) \tag{4.29}$$

where $\lambda_1, \ldots, \lambda_n$ are the eigenvalues of the $A(k)$.

A stable continuous time LTI system may become unstable in three different ways as indicated in the Parameter Space Approach (PSA) [83]. These stability boundaries are named as: Real Root Boundary (RRB $s = 0$), Complex Root Boundary(CRB $s = \pm j\omega$ and Infinite Root Boundary (IRB $s \rightarrow \infty$). Using these stability boundaries, it can be proposed that it is necessary and sufficient to determine the parameter values that make $|M(k)| = 0$ and $|M(k)| \rightarrow \infty$ under the condition that $A(k)$ does not have any symmetric eigenvalues with respect to the imaginary axis.

In the case of RRB or CRB ($\lambda = 0$ or $\lambda_{i,j} = \pm j\omega$ satisfy $|\lambda I - A(k)| = 0$), it can be directly concluded that $|M(k)| = 0$. And in this case, at least one of the eigenvalue of the $P(k)$ traverses over the infinity, since all elements of $P(k)$ include $|M(k)|$ in the denominator. It can also be interpreted that in this case the $|P(k)|$ goes to infinity, since denominator of the *nth* leading principle minor of $P(k)$ which is $|P(k)|$ is equal to $|M(k)|^n$. This case is also illustrated in Figure 4.3.

On the other hand, under the assumption that $A(k)$ does not have any symmetric eigenvalues with respect to the imaginary axis [1], the condition $|M(k)| = 0$ can be satisfied in two different ways. Considering the condition that is given in (4.29) either $\lambda_i = 0$ (RRB case) or $\lambda_i = -\lambda_j$ (CRB case). As a result, it can be proposed that $\lambda = 0$ or $\lambda_{i,j} = \pm j\omega$ satisfy $|\lambda I - A(k)| = 0$ if and only if $|M(k)| = 0$.

On the other hand, if at least one of the eigenvalues of $A(k)$ goes to infinity, i.e. $\lambda \to \infty$ satisfied (IRB case), then this implies that $|M(k)| \to \infty$ as well. In this, it can be stated that $|P(k)| = 0$ since $|M(k)|$ is included in the denominator of the $|P(k)|$. As a result, it can be interpreted that while at least one eigenvalue of the closed loop system traverses over the infinity at least one of the eigenvalue of $P(k)$ passes to the left half plane over the origin. This case is also illustrated in Figure 4.3.

Lastly, the case that $|M(k)| \to \infty$ should be considered. It can be directly concluded from (4.29) that $|M(k)|$ includes finite product where all of the multipliers include sum of two eigenvalue pairs. Here, it must be remembered that all of these eigenvalues are in general complex numbers and have finite magnitude, unless they go to infinity. As a result, it can be proposed that finite product of finite values can not lead to infinity and it can be concluded that at least one of the eigenvalues of $A(k)$ must go to infinity if $|M(k)| \to \infty$.

In general $|M(k)|$ can be expressed in the rational form as:

$$|M(k)| = \frac{m_{num}(k)}{m_{den}(k)} \tag{4.30}$$

where $m_{num}(k)$ and $m_{den}(k)$ are the numerator and denominator polynomials respectively. It is clear that when $|M(k)|$ is a pure polynomial then $m_{den}(k)$ can be taken as 1. The cases $|M(k)| = 0$ and $|M(k)| \to \infty$ respectively leads to $m_{num}(k) = 0$ and $m_{den}(k) = 0$. The intersection points of the IRB and RRB or IRB and CRB lead to zero by zero division and the value of $|M(k)|$ is undefined in these cases. It can be interpreted that some eigenvalues may traverse to the right or left half plane by crossing the origin or over the infinity in such cases.

[1]For the existence and the uniqueness of the solution of the Lyapunov Equation, A and $-A^T$ should not have any common eigenvalues.

At the end, considering all the further analysis that was given in this section, it can be concluded that it is sufficient to check the following two conditions

$$|M(k)| = 0 \quad and \quad |M(k)| \rightarrow \infty \tag{4.31}$$

in order to determine the stability boundaries.

The conditions that are given in (4.31) reduce the required number of equations significantly. It is shown that, in order to determine the stability boundaries of the original system that is given in (4.20), it is sufficient to solve at most 2 symbolic equations.

Solutions of these equations divide the whole parameter space into several subregions in terms of stability. A controller pair can be selected from ever subregion manually and the stability of every sub region can be determined. However, it is also possible to automatize this process by determining the intersection points of all solution functions. After that, considering the gradient of these functions a controller pair can be determined from each subregion. After that stabilizing parameter regions can be determined automatically by checking the stability of the determined controller parameter pairs.

4.3.1 Transformations to eliminate redundancy

When the $|M(k)|$ that is given in (4.29) investigated in detail it can be observed that duplicated products of eigenvalue pairs are included. For instance, both $(\lambda_1 + \lambda_2)$ and $(\lambda_2 + \lambda_1)$ are included in (4.29). However, it is sufficient to check only one of them in terms of stability.

Since both $P(k)$ and Q are symmetric matrices, these redundant multipliers can be eliminated using transformations. Any given $n \times n$ symmetric S matrix, includes only $n(n+1)/2$ unique elements.$\overline{\text{vec}}(S)$ than only includes these unique elements can be written as:

$$\overline{\text{vec}}(S) = \begin{bmatrix} S_{11} & \dots & S_{n1} & S_{22} & \dots & S_{n2} & \dots & S_{nn} \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{n(n+1)/2} \tag{4.32}$$

It can be asserted that for any given symmetric S matrix there exists a full column rank transformation $D_n \in \mathbb{R}^{n^2 \times n(n+1)/2}$ such that:

$$\text{vec}(S) = D_n \overline{\text{vec}}(S) \tag{4.33}$$

In literature, this transformation matrix D_n is named as duplication matrix and as indicated in [120] it is independent from the entries of S matrix and only depends on the dimension of S. By using the duplication matrix (4.26) can be rewritten as follows:

$$M_T(k)\overline{\text{vec}}(P(k)) = \overline{\text{vec}}(-Q) \tag{4.34}$$

where

$$M_T(k) = D_n^+ M(k) D_n \tag{4.35}$$

In (4.35), D_n^+ represents the pseudo inverse of the duplication matrix D_n. In literature the matrix D_n^+ is also named as the elimination matrix.

As a result, it can be proposed that all the unique entries of the original $P(k)$ matrix can be determined from:

$$\overline{\text{vec}}(P(k)) = M_T^{-1}(k)\overline{\text{vec}}(-Q) \tag{4.36}$$

It is also possible to set a connection between the eigenvalues of the $A(k)$ and the $|M_T(k)|$. Using the Kronecker Sum and elemination matrix properties that is obtained in [121], it can be proposed that the determinant of the new $n(n+1)/2 \times n(n+1)/2$ dimensional $M_T(k)$ matrix can be expressed as:

$$|M_T(k)| = \prod_{i=1}^{n} \prod_{j \geq i}^{n} (\lambda_i + \lambda_j) \tag{4.37}$$

Compared to (4.29), it can be concluded that redundant multiplications are eliminated in (4.37). By this further analysis now it is possible to determine the stability boundaries of a given system by calculating the determinant of a $n(n+1)/2 \times n(n+1)/2$ dimensional $M_T(k)$ instead of a $n^2 \times n^2$ dimensional $M(k)$ matrix.

4.4 Case Studies

It is aimed to include several case studies in order to demonstrate the effectiveness of the proposed Lyapunov equation based stability mapping approach. While the proposed approach is also suitable for n input n output MIMO systems, within the scope of this chapter TITO systems will be used as case studies. As indicated in [61], many industrial processes which have higher dimensions can be practically divided as several TITO subsystems for operation. Additionally, it can also be proposed that

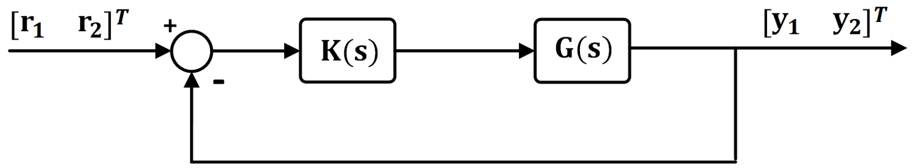


Figure 4.4 : Block diagram of the considered control system.

solutions to the TITO problems would be an effective tool to solve general case beyond TITO systems [111].

4.4.1 Case study I: Finite root boundaries

In order to visualize the derived theoretical results for the case of finite root boundaries the following TITO system and the controller type is selected as

$$G(s) = \begin{bmatrix} \frac{1}{(s+1)(s+2)} & \frac{1}{(s+2)} \\ \\ \frac{2}{(s+1)} & \frac{2}{(s+2)} \end{bmatrix} \tag{4.38}$$

$$K = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \tag{4.39}$$

where the block diagram representation of the control systems that is considered in this thesis in given in 4.4. Here, it must be point out that the selected model in (4.38) has also a practical meaning, since in practice most of the dynamical systems can be modelled as first or second order systems. Additionally, the system and controller pair that are given in (4.38) and (4.39) were selected to their relatively simpler structure in order to make it easier to visualize the theoretical results.

The corresponding characteristic equation for the given system and controller type can be determined as:

$$\delta(s) = s^3 + (2k_2+5)\,s^2 + (k_1+6k_2-2k_1k_2+8)\,s + (4+2k_1+4k_2-2k_1k_2) \tag{4.40}$$

It can be directly interpreted from (4.40) that multiplications of the controller parameters are included in the characteristic equation in contrast to SISO systems. So that, it can be proposed that in general it is more difficult to derive conditions on controller parameters in terms of stability. Corresponding closed loop system $A(k)$ matrix for the given characteristic polynomail in (4.40) can be expressed in the canonical form as:

$$
\begin{aligned}
A(k) &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \\
a_{31} &= -(4+2k_1+4k_2-2k_1k_2) \\
a_{32} &= -(k_1+6k_2-2k_1k_2+8) \\
a_{33} &= -(2k_2+5)
\end{aligned}
\tag{4.41}
$$

Using the equation (4.27), $|M(k)|$ can be determined as:

$$
|M(k)| = 16(-3(12+k_1)+6(-7+k_1)k_2+4(-3+k_1)k_2^2)^2\,(k_1(-1+k_2)-2(1+k_2))
\tag{4.42}
$$

In this case $|M(k)|$ is a polynomial expression as a result, there is only finite stability boundaries (RRB and CRB). So that, it is sufficient to determine the controller parameters that make $|M(k)| = 0$. Moreover, it can also be interpreted that duplicated multiplier of $|M(k)|$ which is $(-3(12+k_1)+6(-7+k_1)k_2+4(-3+k_1)k_2^2)^2$ can be reduced to $(-3(12+k_1)+6(-7+k_1)k_2+4(-3+k_1)k_2^2)$ by using the transformations that was introduced in Section 4.3.1.

In this specific case, the $|M(k)|$ can be expressed as product of two functions that depend on free controller parameters k_1 and k_2. And it is sufficient to find the zeros of these functions in order to determine the the zeros of the $|M(k)|$ and the stability boundaries. These functions can be written as:

$$
f_{M_1} = (-3(12+k_1)+6(-7+k_1)k_2+4(-3+k_1)k_2^2)^2
\tag{4.43}
$$

$$
f_{M_2} = (k_1(-1+k_2)-2(1+k_2))
\tag{4.44}
$$

In order to solve the symbolic equations (4.43) and (4.44) Wolfram Mathematica 10.3 software was used. The zeros of these functions can be determined as:

$$
k_2 = f_{LY_1}(k_1) = \frac{21-3k_1-\sqrt{3}\sqrt{3-6k_1+7k_1^2}}{4(-3+k_1)}
\tag{4.45}
$$

$$
k_2 = f_{LY_2}(k_1) = \frac{21-3k_1+\sqrt{3}\sqrt{3-6k_1+7k_1^2}}{4(-3+k_1)}
\tag{4.46}
$$

$$
k_2 = f_{LY_3}(k_1) = \frac{2+k_1}{-2+k_1}
\tag{4.47}
$$

The equations that are given in (4.45)-(4.47) are candidate functions for stability boundaries. In this case study these stability boundaries divide the parameter space

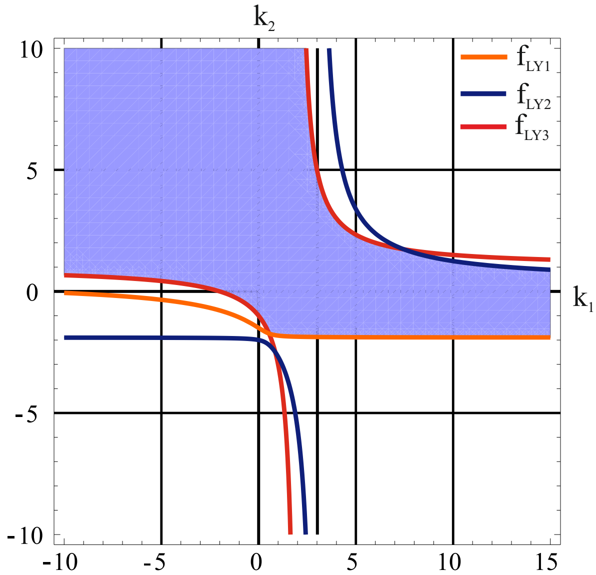


Figure 4.5 : Case study I: Stabilizing parameter space.

into 9 subregions in terms of stability. A controller parameter can be easily selected from each sub region and after that stabilizing parameter regions can be determined by checking the stability of the determined controller parameter pairs.

Stability boundaries for the given system and stabilizing parameter region that makes the closed loop system stable is given in Figure 4.5. When the stabilizing parameter space investigated, it can be interpreted that there is a large stabilizing area for the negative values of k_1 and positive values of k_2. At this point, it must be noted that this region is an open region. There is also relatively, small stabilizing area around the $k_2 = 0$ line after a specific value of k_1. Stabilizing parameter region that is given in Figure 4.5 is identical with region that can be obtained using the other methods in literature like the traditional Routh-Hurwitz approach. Additionally, frequency domain based approaches can also be applied to get the same stabilizing area. However, it must be pointed out that most of the frequeny domain based approaches are controller specific and generally it is not possible to generalize these approaches for all controller types. Calculations should be renewed for every specific controller type. However, proposed approach within the scope of this thesis is more general compared to these approaches since it is independent from the controller type and the number of free parameters. Furthermore, is also possible to apply this approach to the different system classes where Lyapunov formulation is possible. For instance by minor modifications it is possible to apply the same methodology to the discrete time systems.

There are three functions that may affect the stability boundary in this specific case. In Figure 4.5, it is seen that all of these functions have an effect on the stabilizing region. When the Figure 4.5 examined, it is also seen that there are two jumps from negative to positive infinity at $k_1 = 2$ and $k_1 = 3$. These jumps occur at the controller parameter values that make denominators of f_{LY_2} and f_{LY_3} zero. It is also expected to see a jump for f_{LY_1} at $k_1 = 3$ but for this specific point both the numerator and denominator polynomials of f_{LY_1} are zero. As a result, it can be proposed that f_{LY_1} is indefinite at this point and a jump over infinity was not observed as a result of this.

It can also be proposed that these jumps over the infinity for the stability boundaries are not necessarily affect the resulting stabilizing regions directly. For instance, for the case of f_{LY_3} the jump at $k_1 = 2$ directly affects the stabilizing region. On the other hand, for the case of f_{LY_2} the jump at $k_1 = 3$ does not affect the stability area directly.

As it was discussed in the previous section, another indicator of the stability of the closed loop system is the positive definiteness of the $P(k)$ matrix. In order to be a positive definite matrix all eigenvalues of $P(k)$ must be positive.

The resulting stabilizing parameter region can also be verified by checking the eigenvalues of the $P(k)$. For example, for the case $k_2 = 0$ the eigenvalue plot of the P matrix is given in Figure 4.6. In this case, all of the three eigenvalues must be positive for stability. n Figure 4.6, an eigenvalue of $P(k)$ jumps from negative to positive infinity at $k_1 = -2$ and the system becomes stable for $k_1 \geq -2$. This result is also compatible with the stabilizing parameter region that is given in Figure 4.5 (for $k_2 = 0$). As indicated earlier, the proposed approach can be directly applied for different type of controllers. For instance, if the controller type is switched to static upper triangular controller for the system (4.38) it is also possible to determine stabilizing parameter regions. In this case, static upper triangular controller that can be written as follows:

$$K = \begin{bmatrix} k_1 & k_2 \\ 0 & k_3 \end{bmatrix} \tag{4.48}$$

In this case, $|M|$ is also a pure polynomial expression like in the previous case and it can be expressed as follows:

$$\begin{aligned} |M| =& 16((3+2k_2)(12+k_1+8k_2) - 2(-7+k_1) \\ & (3+2k_2)k_3 - 4(-3+k_1)k_3^2)^2 \\ & (k_1(-1+k_3) - 2(1+2k_2+k_3)) \end{aligned} \tag{4.49}$$

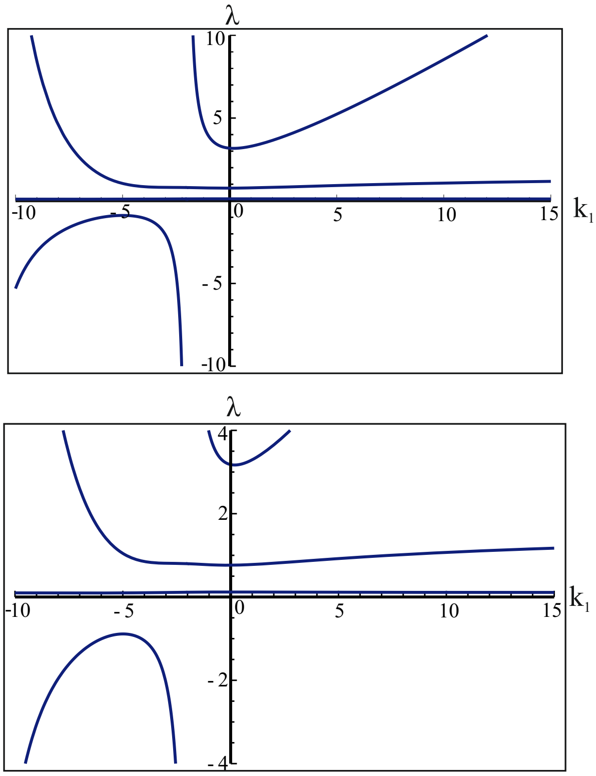


Figure 4.6 : Case study I: Eigenvalues of P for $k_2 = 0$.

Since the controller is static upper triangular, in this case, dimension of the parameter space is three. However, when (4.49) examined, it can be proposed that as in the previous case $|M(k)|$ matrix can be written as product of two functions. Stability boundaries of the closed loop system can be determined by determining the zeros of these functions.

Using these zeros, the stabilizing parameter region for the system (4.38) that is controller by (4.48) can be determined as it is given in Figure 4.7. In this case, the parameter values that make $|M(k)| = 0$ represent planes in three dimensional space. Unlike the previous case, now the intersection of these planes corresponds to curves instead of points. In this case, previous results can also be verified by selecting the k_2 as $k_2 = 0$. Then, the controller reduces to the static diagonal controller and the derived parameter region is identical with Figure 4.5.

While it was mainly discussed the parameters that leads to stability, with this approach it is also possible to make counter interpretations. For instance, if the controller parameters k_2, k_3 are selected as $k_2 = 0$, $k_3 = -2$ then it is possible to conclude that there is not any k_1 parameter value that satisfies stability condition.

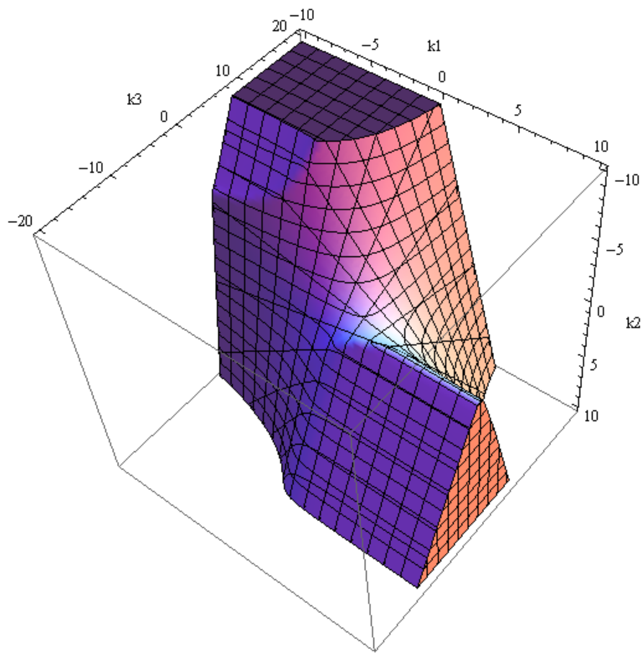


Figure 4.7 : Case study II: Stabilizing parameter space.

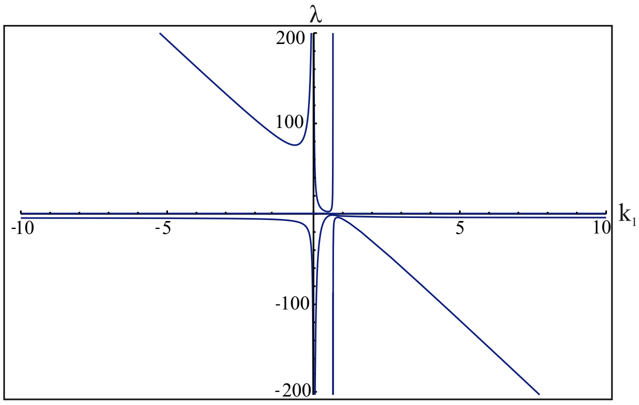


Figure 4.8 : Case study II: Eigenvalues of P for $k_2 = 0$ and $k_3 = -2$.

In addition to the Figure 4.7, this result can also be derived by plotting the eigenvalues of $P(k)$ for the selected controller parameters that is given in Figure 4.8. It can be interpreted from Figure 4.8, it is not possible to make all eigenvalues of $P(k)$ in this case.

4.4.2 Case study II: Finite and infinite root boundaries

In this case study, both finite and infinite root boundary cases are discussed. In fact, this case corresponds to the case where $|M(k)|$ is a rational expression instead of polynomial. The system and the controller type that is selected in order to visualize the effect of infinite stability boundaries written as:

$$G(s) = \begin{bmatrix} \frac{(-2s+6)}{(5s+2)} & \frac{(4s+1)}{(7s+2)(10s+1)} \\ \\ \frac{1.5}{(5s+2)} & \frac{2}{(10s+2)} \end{bmatrix} \tag{4.50}$$

$$K = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \tag{4.51}$$

When the characteristic equation of the given system is determined, it can be observed that the coefficient of the highest order term depends on controller parameters. This parameter dependency actually is the source of the infinite root boundaries. More precisely, the characteristic polynomial of the given system and the controller type can be expressed as:

$$\delta(s) = \alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s^1 + \alpha_0 \tag{4.52}$$

where $\alpha_3 = 350 - 140k_1$. Since the coefficient term depends on the controller parameter k_1, it is expected to end up with a infinite root boundary in addition to the finite root boundaries in this specific case.

In this case study, complete expression of the $|M|$ will not be given in order to ease the readability. However, it can be said that $|M|$ is a ration expression in this case and the denominator of $|M|$ is equal to $(-2.5 + k1)^5$ as expected. As a result, in addition to the parameter values that make $|M| = 0$, parameter values that $|M(k)| \to \infty$ have to be determined in terms of stability boundaries. By solving the corresponding symbolic equations, finite and infinite stability boundaries can be determined as given in Figure 4.9. As discussed in the previous section, these boundaries divide the whole parameter region into sub regions in terms of stability. By checking the stability of each region, stabilizing controller parameter space can be determined as given in Figure 4.9.

As in the previous section it is also possible to verify these results by checking the positive definiteness of $P(k)$. For instance, if the parameter k_2 is selected as zero, then the eigenvalue plot of the P matrix can be derived as given in Figure 4.10. The minimum value of the third eigenvalue is 50. As a result, in order to increase the readability of the plot, only two eigenvalue of the $P(k)$ matrix is included in Figure 4.10. It can be interpreted from Figure 4.10 that stabilizing region is $2.5 > k_1 > -0.334$, if the k_2 parameter is selected as zero. These results are compatible with the stabilizing parameter region that is Figure 4.9 as expected.

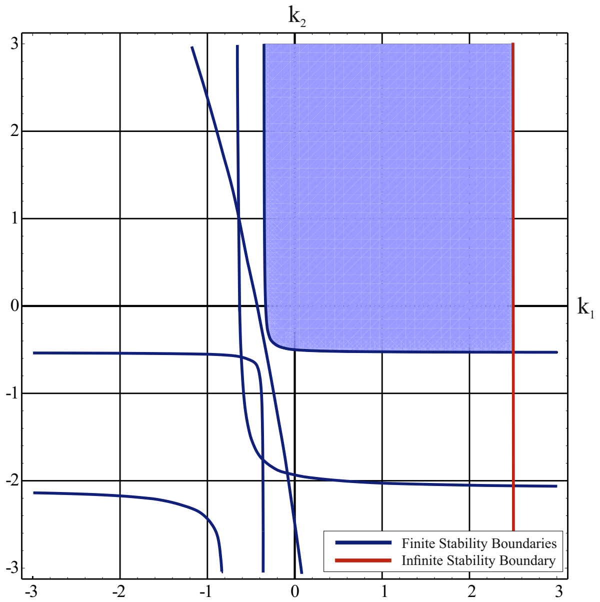


Figure 4.9 : Case study III: Stabilizing parameter space.

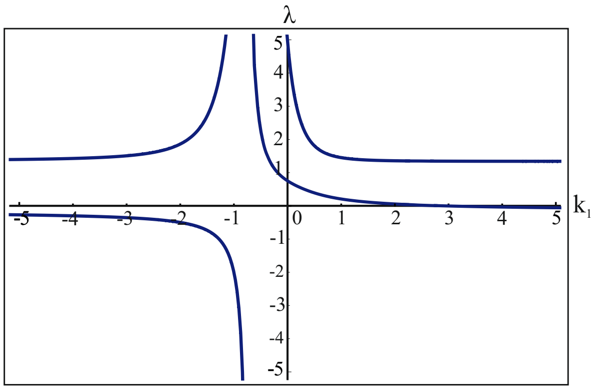


Figure 4.10 : Case study II: Eigenvalues of P for $k_2 = 0$.

4.4.3 Case study III: Stability and diagonal dominance

Compared to the previous case study, a slightly different system is discussed in this case study in order to demonstrate both stability and diagonal dominance resuts. Considered system model within the scope of this case study can be written as:

$$G(s) = \begin{bmatrix} \frac{(-2s+6)}{(5s+2)} & \frac{(4s+1)}{(7s+2)(10s+1)} \\ \\ \frac{1.5}{(5s+2)} & \frac{2}{(10s+1)} \end{bmatrix} \tag{4.53}$$

Controller type is static diagonal controller that is given in (4.51).

In this case, using the Lyapunov equation based stability mapping approach stabilizing parameter region can be determined as it is given in Figure 4.11. Using the derived diagonal dominance related results that are obtained in Section 3, it can be proposed

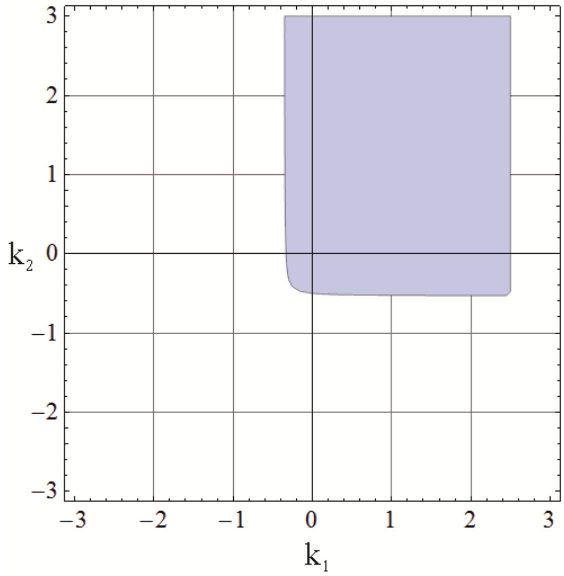


Figure 4.11 : Case study III: Stabilizing parameter region.

that controller gain regions that satisfy both of the diagonal dominance conditions at $\omega = 0$ rad/s and stability criteria can be given as in Figure 4.12.

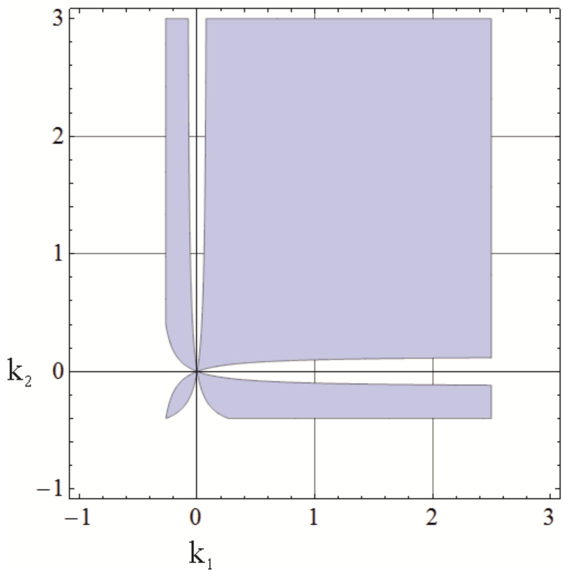


Figure 4.12 : Case study III: Parameter regions that achieve both diagonal dominance conditions at $\omega = 0$ rad/s and stability criteria.

As it was discussed in detail in Section 3, derived results in terms of diagonal dominance can also be extended for a given frequency range. Here, it was assumed that this range is 100 $rad/s \geq \omega \geq 0$ rad/s. Controller gain regions that achieve both of the diagonal dominance conditions for 100 $rad/s \geq \omega \geq 0$ rad/s and the stability criteria are given in Figure 4.13.

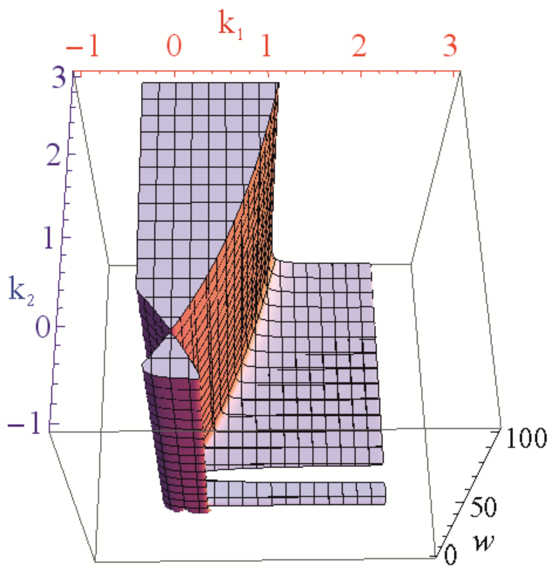


Figure 4.13 : Case study III: Parameter regions that achieve both diagonal dominance conditions for 100 $rad/s \geq \omega \geq 0$ rad/s and stability criteria.

4.5 Further Application Areas

As it was pointed out in the previous sections, Lyapunov based stability mapping approach is a powerful tool to determine the stabilizing parameter spaces. Since it has its roots on the Lyapunov Theory, it is possible to extend application areas of the proposed approach to the other type of systems where Lyapunov function formulation is possible.

Procedure that should be followed is similar in both cases. However, as in the case of discrete time systems, resulting Lyapunov equation and the M matrix formulation may vary slightly.

Within the scope of this thesis, in addtion to the MIMO systems, other application areas are also discussed. Three further application areas which are: controller integrity problem of MIMO systems, discrete time systems and robust MPC calculations were discussed in detail to demonstrate the effectiveness of the propsed stabilizing parameter space calculation approach. Derived results for these subproblems were published in [66–69].

At this point, it must be said these system types and problem formulations are not the only ones that a designer can apply the proposed Lyapunov equation based mapping strategy. For instance, this approach was applied to switching systems in [122] and to descriptor systems in [123] by other researchers.

4.5.1 Controller integrity problem

Due to their complex structures, many automated industrial systems are vulnerable to faults [124]. Here the "*fault*" term corresponds to any kind of degradation in the system that forces it to diverge far from its normal operating point. In such cases, the system performance is effected by these faults or even worse the stability can be lost. Therefore, it is meaningful, from the practical point of view, to synthesize controllers considering the possible system failures and/or parameter perturbations.

An effective way to prevent system's faults to cause instability is to consider the "*controller integrity*" in the design phase. Here, integrity refers to a property where in the presence of arbitrary failures of certain sensors or actuators, the system retains stability and acceptable performance without reconfiguring controllers. A multivariable feedback system possesses controller integrity, if it remains stable and gives acceptable performance in the presence of arbitrary failure of certain sensors or actuators without modifying controllers.

The integrity problem has drawn increasing attention due to the growing demand for reliability and availability of industrial systems, especially in the last two decades. In the meantime, different approaches were propsed from different mathematical perspectives in order to achieve the controller integrity [125–129]. Details of these approaches can be found in our previous study [67].

In the controller integrity problem, instead of only determining the stabilizing parameter regions for the nominal system, it is also required to determine the stability boundaries and stabilizing parameter regions for the subsystems resulted under predicted/possible failures. As a result, it can be proposed that in such problems it is important to use a suitable approach for the calculation of stability boundaries calculations from the computational point of view.

However, in general, it is very difficult for multivariable systems to achieve integrity unless controllers are synthesized to possess this property from the design process. Before going further into the details, firstly, it is aimed to demonstrate the importance of taking the controller integrity problem into account during the design process using an illustrative example as it is given in [129].

A motivational example :

Suppose that, the TITO system plant and the controller transfer matrix are given as follows:

$$G(s) = \begin{bmatrix} \frac{s-1}{s^2+3s+2} & \frac{s}{s^2+3s+2} \\ \frac{-6}{s^2+3s+2} & \frac{s-2}{s^2+3s+2} \end{bmatrix} \tag{4.54}$$

$$K(s) = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} = \begin{bmatrix} \frac{-s-3}{s} & \frac{-3s-1.6}{s} \\ \frac{0.6s-0.4}{s} & \frac{-0.7s-0.3}{s} \end{bmatrix} \tag{4.55}$$

The characteristic equation of the given system can be easily determined from the numerator of the determinant $|I+KG|$ to be:

$$\delta(s) = s^6 + 4.9s^5 + 28.9s^4 + 82.76s^3 + 88.74s^2 + 31.5s + 0.52 \tag{4.56}$$

It can be easily calculated that all the eigenvalues lie in the left half plane in this case. However, if k_{11} becomes zero, in other words it fails to operate because of actuator failure or any software failure related with that part of the system or controller, one of the closed loop eigenvalue jumps to the right half plane and the system becomes unstable. Closed loop system poles in this case can be calculates as:

$$\begin{aligned} s_{1,2} &= -1.2154 \pm 4.4623j \\ s_3 &= -0.5259 \\ s_4 &= -1 \\ s_5 &= -2 \\ s_6 &= 0.0568 \end{aligned}$$

As it is seen from this simple example, the controller stabilizes the nominal plant, however, it fails to stabilize the system when one of the entries of the transfer function controller matrix fails to operate. Therefore, it can be proposed that in order to achieve integrity, controllers must be designed carefully in the design phase.

4.5.1.1 Problem formulation

For the sake of simplicity, it is preferred to use TITO systems within the scope of this section. However, it must be pointed out that derived results in this section is also valid for MIMO systems. The problem formulation slightly differs in such cases but the methodology that should be followed is the same with TITO systems.

Additionally, as asserted in [111] deriving significant results for such systems would be an effective tool to solve the general case. Furthermore, it is possible to accept TITO systems as a special subset of MIMO systems. Many industrial systems that have higher dimensions can be divided into several TITO subsystems as asserted in [61].To have a better insight about the problem of controller integrity for multivariable systems, the following general TITO closed loop system structure will be discussed.

Assume a stable plant G and a diagonal controller K are represented by the following transfer functions respectively

$$G = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \quad and \quad K = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \tag{4.57}$$

The closed loop stability of the nominal system is satisfied,if the roots of the following characteristic equation are located in the left half plane. The characteristic equation can be determined as:

$$\delta(s) = 1 + k_1 g_{11} + k_2 g_{22} + k_1 k_2 (g_{11} g_{22} - g_{12} g_{21}) = 0 \tag{4.58}$$

If k_1 or k_2 fails to operate due to any failure in software or in actuators, the characteristic equation reduces to:

$$1 + k_2 g_{22} = 0 \tag{4.59}$$

$$1 + k_1 g_{11} = 0 \tag{4.60}$$

Therefore, in order to possess integrity, it is required to determine the set of k_1 and k_2 that make the zeros of the above three characteristic equations lie in the LHP. As a result, it is required to determine stabilizing parameter regions for all the subsystems that result under various predicted failures.

Furthermore, the integrity problem can also be solved, even if the type of controller is not diagonal since the proposed Lyapunov equation approach is independent from the controller type and number of free control parameters. However, in this case the dimension and the complexity of the problem increase. The number of systems that needs to be analyzed in terms of stability can easily be determined from the relation $(2^n - 1)$ where n refers to the number of controller parameters. For instance, if the type of the controller is static upper triangular controller, then the number of systems, including the nominal system, that needs to be analyzed are seven for TITO systems.

These results can be merged in an algorithm. The steps of the proposed algorithm in order to derive the parameter regions that achieve controller integrity can be stated as:

- **Step 1:** Determine parameter regions that guarantee nominal system stability.
- **Step 2:** Considering the controller structure and possible faults determine the subsystem characteristic equations.
- **Step 3:** Using the Lyapunov equation based approach determine the controller gain regions that stabilizes the subsystem characteristic equations.
- **Step 4:** Take the intersections of the regions derived in Step 1, and Step 3.

This algorithm is applied to a benchmark case study in next section.

4.5.1.2 Comparison with a benchmark example

To compare the derived results, a benchmark example is selected from literature. The system and controller structure that was used in [129] are:

$$G(s) = \begin{bmatrix} \frac{s+3}{s^2+2s+3} & \frac{s+1}{s^2+2s+3} \\ \frac{s+2}{s^2+2s+3} & \frac{s+4}{s^2+2s+3} \end{bmatrix}, \quad K(s) = \begin{bmatrix} k_1(s) & 0 \\ 0 & k_2(s) \end{bmatrix} = \begin{bmatrix} \frac{k_{p_1}s+k_{i_1}}{s} & 0 \\ 0 & \frac{k_{p_2}s+k_{i_2}}{s} \end{bmatrix} \quad (4.61)$$

A Bounded Phase Theorem based approach was proposed in that study to determine conditions on controller parameters to achieve integrity. However, in our study [67] it was shown that Lyapunov equation based approach can be used to obtain better result.

Since the problem is defined in four dimensional space that is not possible to be plotted, k_{i_1} and k_{i_2} are selected to be 2 and 3, respectively. Then, the objective is to determine the regions of the k_{p_1} and k_{p_2} parameters in term of integrity.

In the proposed controller integrity algorithm, the first step is to apply Lyapunov approach to the nominal system. In order to ease the readability of this study, the complete expression of $\mid M(k) \mid$ will not be given. The parametric solutions of

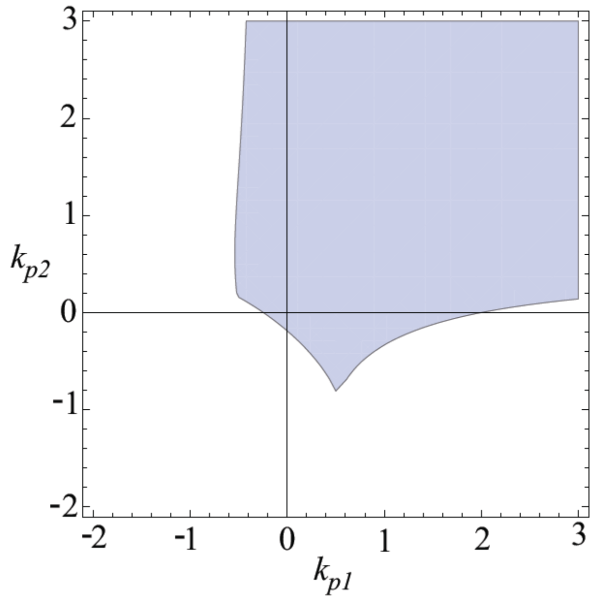


Figure 4.14 : Controller integrity: Stabilizing region for the nominal system.

$\mid M(k) \mid = 0$ are calculated to determine the finite stability boundaries for this specific case as in Fig. 4.14.

Here, it should be pointed out that Fig. 4.14 is plotted for the parameter range $k_{p_1}, k_{p_2} \in [-2, 3]$ and the parameter region in that figure is open and non-bounded.

As a second step, the number of subsystems should be determined. Since the number of free control parameters is two in our case, it can be concluded that 2 additional stability problems should be solved. Then, in the third step parameter conditions on these additional subsystems should be determined in terms of stability.

When $k_1(s)$ fails to operate, $\mid M(k) \mid$ can be written as:

$$\mid M(k) \mid = -384(k_{p_2})^2(7 + 2k_{p_2})^2 \tag{4.62}$$

In this case parametric solutions of $\mid M(k) \mid$ are $k_{p_2} = 0$ and $k_{p_2} = -3.5$. These equations divide the k_{p_2} parameter region into three regions. By checking the stability of each region, it is found that, for the system to be stable, the following condition must be satisfied:

$$k_{p_2} \geq 0 \tag{4.63}$$

On the other hand, for the case that k_2 fails to operate, the determinant of $\mid M(k) \mid$ is calculated to be:

$$\mid M(k) \mid = -48(4 + k_{p_1}(11 + 3k_{p_1}))^2 \tag{4.64}$$

By solving $| M(k) | = 0$, it can be determined that the condition:

$$k_{p_1} \geq -0.4093 \tag{4.65}$$

should be satisfied for stability.

The final step is to take the intersections of the region derived for the nominal system as it is in Fig. 4.14 and conditions 4.63 and 4.65 in order to determine the stabilizing parameter region for the system to possess integrity. This parameter regions can be demonstrated as it is given in Fig. 4.15.

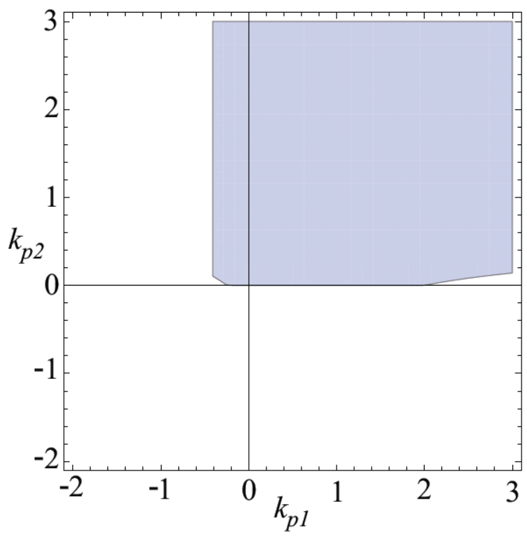


Figure 4.15 : Controller integrity: Stabilizing region for the system to possess integrity.

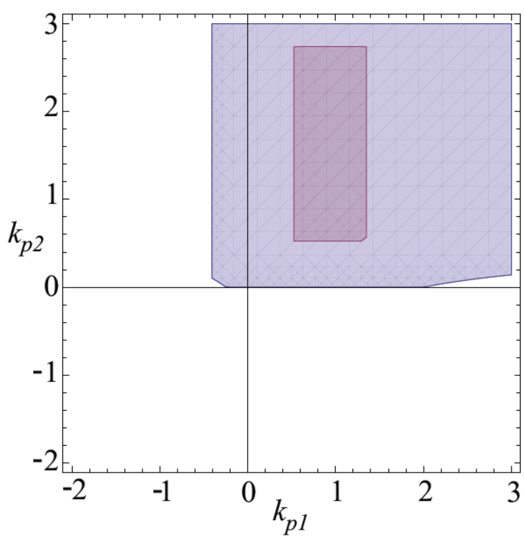


Figure 4.16 : Controller integrity: Comparison between Lyapunov approach and approach in literature.

On the other hand, for the Bounded Phase Theorem based approach that was proposed in [129], the bounds of the proportional parameters for the same conditions are:

$$0.5254 \leq k_{p_1} \leq 1.3496, \qquad 0.5254 \leq k_{p_2} \leq 2.7402 \tag{4.66}$$

By comparing the results derived using both approaches, it is found that, the results obtained by the Bounded Phase Theorem are significantly conservative. Both results are plotted in Fig. 4.16.

4.5.2 Discrete time systems

This section aims to set the link between the proposed Lyapunov equation based stability boundary mapping approach and the discrete time systems. While the time derivative of the Lyapunov function is slightly different in this case, the main methodology to determine stabilizing parameter spaces are quite similar. Derived results in this section were published in [66, 68].

It can be proposed that calculation of all controller parameter combinations that satisfy stability conditions is non trivial despite the existence of necessary and sufficient conditions for the stability of discrete time linear systems.

Various methods were proposed in literature to determine stability boundaries and stable parameter regions. The first systematic studies in that area date back to early 1960s [130]. D-decomposition method which was first proposed by Neimark in [81] was also applicable for discrete time systems as asserted in [107]. Additionally, Parameter Space Approach (PSA) based approached were also proposed in literature for the determination of stabilizing parameters. [131, 132]. The main drawbacks of these methods is the need for frequency sweeping and the need for decoupling at singular frequencies. Such needs increases the computational complexity Another drawback is that these kind of approaches are proposed for a specific type of controller. More recently reflection segments based technique was proposed by Avanessov and Nurges [133, 134]. However this method is only applicable for controllers with up to three parameters due to computational complexity.

On the other hand, using transformations, the methods developed for continuous time systems can also be used in order to analyze the stability of discrete time systems. However, computational complexity is usually increased compared to the discrete time case: E.g. for the bi quadratic transformation the order of the polynomial is doubled, which makes analysis and computation unfavorably expensive. Details of the transformation and Tchebyshev representation based approaches can be found in studies [135–137].

Instead of facing the drawbacks of the previously mentioned approaches like decoupling at singular frequencies, frequency sweeping and transformations in this chapter, it was aimed to extend the derived results in Section 4.3 to the discrete time case. Proposed approach bypasses the addressed problems and further may be used for many different control structures.

For this purpose the well-known Lyapunov conditions are reformulated for discrete time systems. Instead of checking the absolute value of every single eigenvalue a unified condition for all eigenvalues is examined, which is far more easy to carry out and greatly reduces computational complexity.

4.5.2.1 Lyapunov formulation for discrete time systems

In this section, extension of the proposed Lyapunov based stability mapping technique to LTI discrete time systems is discussed in detail. As indicated earlier, methodology is similar to the continuous time case. However, there are some differences in terms of time derivative of Lyapunov function and resulting Kronecker product based $M(k)$ matrix.

Within the scope of this thesis, the following LTI state space model of a closed loop system in discrete-time is considered:

$$x_{t+1} = A(k)x_t \quad ; \quad A(k) \in \mathbb{R}^{n\times n}, \quad x \in \mathbb{R}^n, \quad k \in \mathbb{R}^p \tag{4.67}$$

In 4.67, x is the n dimensional state vector, $A(k)$ is the system dynamics matrix, t is the time index and k represents the controller parameters. For instance, in the case of static state feedback controller for a given $n-th$ order SISO system it can be represented as, $k = [k_1, k_2, ..., k_n]^T$.

In case of LTI systems, the determination of candidate Lyapunov fuction is a straight forward process. In the specific case of LTI discrete-time systems Lyapunov function $V(x_\kappa)$ can be selected as:

$$V(x_t) = x_t^T P(k) x_t \tag{4.68}$$

In order to be a positive function for all values of x_t except $x_t = 0$, it is necessary and sufficient that the matrix P in 4.68 should be positive definite.

The discrete derivative of the candidate Lyapunov function ΔV is evaluated to be:

$$\begin{aligned}
\Delta V &= V_{t+1} - V_t \\
\Delta V &= x_{t+1}^T P(k) x_{t+1} - x_t^T P(k) x_t \\
\Delta V &= (A(k)x_t)^T P(k)(A(k)x_t) - x_t^T P x_t \\
\Delta V &= x_t^T (A^T(k)P(k)A(k) - P(k))x_t
\end{aligned} \tag{4.69}$$

In terms of Lyapunov Theory, a negative definite time derivate indicates the stability. In case of LTI systems it is both necessary and sufficient, while this condition is only a sufficient condition in case of non-linear systems. When the last row of equation 4.69 investigated, it can be easily determined that it is a quadratic function since it is multiplied by x_t^T from the left and multiplied by x_t from the right. As a result, it can be directly concluded that the inner part of 4.69 should be negative definite which means that:

$$A^T(k)P(k)A(k) - P(k) < 0 \tag{4.70}$$

While 4.70seems to be matrix inequality, for the case of LTI systems, it is possible to represent it as a matrix equality as:

$$A^T(k)P(k)A(k) - P(k) = -Q \tag{4.71}$$

where the matrix Q is any symmetric positive definite matrix. As indicated in [138], 4.71 has a unique solution for a given Q. It must also be noted that 4.71 is not in the standard linear matrix equation representation. However, it is possible to formulate this problem in linear set of equation representation using the Kronecker product and vectorization operator. Using these algebraic tools and operators, it is ossible to rewrite 4.71 as a standard linear set of equation format as follows:

$$(A^T(k) \otimes A^T(k) - I)\text{vec}(P(k)) = -\text{vec}(Q) \tag{4.72}$$

where I is a $n^2 \times n^2$ identity matrix and vec(.) represents matrix vectorization.Left hand side of the equation 4.72 can be rewritten by defining the $M(k)$ matrix as follows

$$M(k) = A^T(k) \otimes A^T(k) - I. \tag{4.73}$$

Without loss of generality, Q can be selected as $I_{n\times n}$ where n is the order of the system. All the entries of $P(k)$ matrix can be calculated using the following equation:

$$\text{vec}(P) = M(k)^{-1}\text{vec}(-Q) \tag{4.74}$$

As it was done in the continous time case, by further analyzing the relations between the stability indicators the number of equations that should be solved can be reduced. From the Lyapunov point of view, positive defineteness of the $P(k)$ indicates stability. Considering the numerators and denominators of leading principal minors of $P(k)$ as a first step, it can be propsed that $2n$ sybolic equations should be solved. However, 4.74 indicates that denominator of every single entry of $P(k)$ includes $\mid M(k) \mid$. This indicates that the denominators of the leading principal minors of $P(k)$ only include $\mid M(k) \mid$ and its increasing powers. As the first step of the analysis, this results with the conclusion that instead of $2n$ equations $n+1$ symbolic equations needed to be solved for determining the stability characteristics.

In addition to the positive definiteness of the $P(k)$ matrix, eigenvalues of the closed loop system matrix $A(k)$ is the another indicator of stability. It is a well known fact that all eigenvalues of $A(k)$ should lie inside the unit circle. As indicated earlier, by further analysis, it becomes possible to set a connection between the determinant of the Kronecker product based $M(k)$ matrix and eigenvalues of the closed loop system $A(k)$ matrix. $\mid M(k) \mid$ can be evaluated as:

$$\mid M(k) \mid = \prod_{i=1}^{n}\prod_{j=1}^{n}(\lambda_i\lambda_j - 1), \tag{4.75}$$

where λ_i,...,λ_n are the eigenvalues of $A(k)$.

For a given stable system that is represented as 4.67, corresponding $\mid M(k) \mid$ becomes equal to zero in two cases according to 4.75, either the $A(k)$ matrix has an eigenvalue equal to one or negative one i.e. $\lambda_i = 1$ or $\lambda_j = -1$, which refers to the RRB of $A(k)$. Another possibility for $A(k)$ to become unstable is the complex conjugate eigenvalues i.e. $\lambda_{i,j} = e^{\pm j\omega}$ which refers to the CRB of $A(k)$. In both cases it can be easily shown that $\mid M(k) \mid$ is equal to zero with respect to 4.75. So in this approach, it is only needed to calculate the parameter values that makes $\mid M(k) \mid = 0$ in order to determine the stability boundaries.

Considering the drawbacks of PSA and frequency domain based approaches, it can be stated that Lyapunov approach is so powerful and flexible for the calculation of stabilizing parameter spaces. First of all, the computational complexity of this method is tremendously minimized in the proposed approach. For instance, when utilizing Lyapunov′s second method for determining the stabilizing controller parameter region of a discrete-time system, it is required to solve $2n$ symbolic equations in order to check for the positive definiteness of $P(k)$ matrix. However, it is sufficient to solve only one equation by investigating the relationships between $A(k)$, $P(k)$ and $M(k)$ matrices in case of using this novel approach. Moreover, this procedure avoids the problems of frequency sweeping, decoupling at singular frequencies and discretization of the parameter space.

The Lyapunov based mapping technique for LTI discrete-time systems is applicable for systems with any controller structure. Moreover, it is independent on the number of controller parameters. This proposed approach can also be utilized to determine the stability boundaries of systems with uncertain parameters.

4.5.2.2 Calculation of stabilizing PI and PID parameters

In this subsection, the system that has been also used in [139] is selected as a benchmark system in order to be able to compare the derived results. The system and *PI* controller transfer functions are given as

$$g(z) = \frac{z+1}{z^2 - 1.5z + 0.5}, \qquad c(z) = k_p + \frac{k_i z}{z-1}, \tag{4.76}$$

where backwards difference methods have been used to represent the *PI* controller. The corresponding closed loop $A(k)$ for the system and the controller type given in 4.76 can be determined in the controllable canonical form as:

$$A(k) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 + k_p & -2 - k_i & 2.5 - k_i - k_p \end{bmatrix}. \tag{4.77}$$

For this case $|M_T(k)|$ can be calculated as

$$|M_T(k)| = -12k_i(1.5k_i - k_p + k_i k_p + 2k_p^2). \tag{4.78}$$

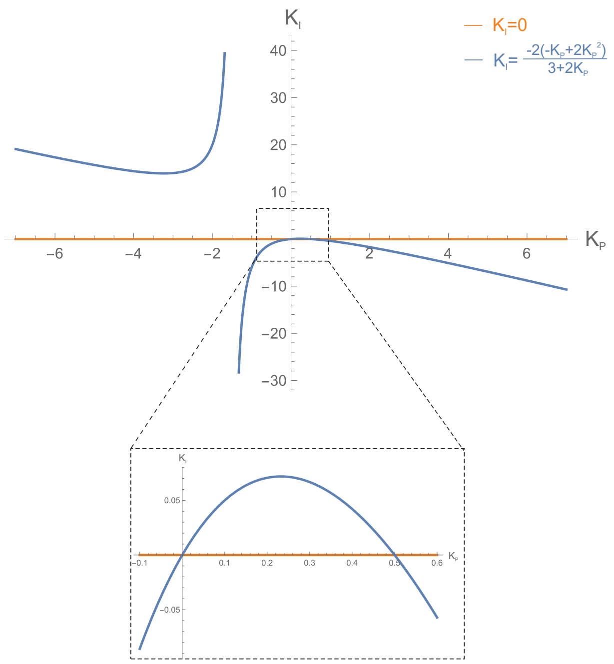


Figure 4.17 : Discrete systems: Stabilizing boundaries for *PI* controller.

The parametric solution of $|M_T(k)| = 0$ with respect to the k_i parameter leads to the following solution set:

$$
\begin{aligned}
k_i &= 0, \\
k_i &= \frac{-2(-k_p + 2k_p^2)}{3 + 2k_p}.
\end{aligned}
\tag{4.79}
$$

The solution functions which are given in 4.79, are plotten in Fig. 4.17. This two functions are the possible stability boundaries for the given system and the dimensional parameter space was divided into six subspaces in terms of stability as it is given in 4.17.

In order to determine the stability characteristics of these subregions a single controller pair can be selected from every sub region and closed loop stability of the system can be checked. By determining the intersection points of the solutions and using the gradients of these functions at that point a the procedure can be proposed to select points from every subspace and this stability checking process can be automatized. After that analysis, it is determined that only the small region which is close to the k_p axis is stable. As a result, the stabilizing parameter space for this benchmark case study is determined as it is given in Fig. 4.18.

The outer bounds of the derived region are identical with the stability region given in [139]. However, the accuracy of the results in [139] depends on the griding step

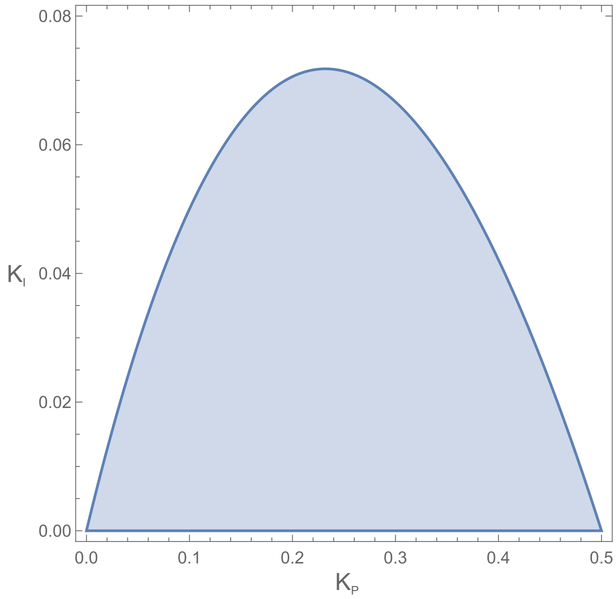


Figure 4.18 : Discrete systems: Stabilizing region for PI controller.

sizes, because of the frequency based approach and bilinear transformation that were used to determine the stabilizing parameter regions. Since the stability problem is not defined in frequency domain in our approach, there is no need for frequency sweeping.

Additionally, the approach in this study is independent from the controller type and the number of free parameters. For example, the calculation of the stability boundaries of a PID controller for the system that is given in (4.76) led to further calculations in [139]. On the other hand, in our approach nearly the same methodology and code are used to determine the stabilizing PID parameter range as it is given in Fig. 4.19.

Another advantage of the proposed approach is to derive the anaytical conditions on controller parameters in terms of stability. These conditions can be inserted to optimisation based algorithm to stay away from the parameter region. In order to discuss the benefits of that point of view robust MPC problem will be discussed in the next section in detail.

4.5.3 Robust MPC calculations

One of the most important advantages of the proposed Lyapunov equation based approach is the opportunity to determine the analytical expressions of the stability boundaries. It can be stated that this feature may lead to numerous application areas where an optimization on controller parameters are needed. sing the derived conditions for stability the parameter space that should be searched in these optimisation based

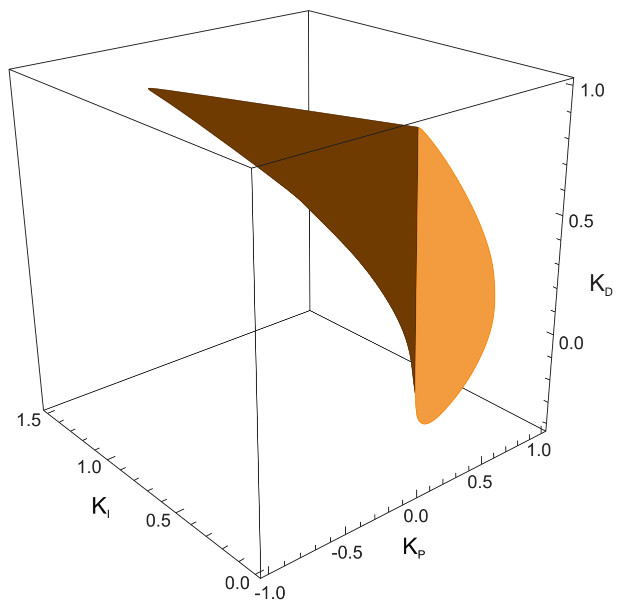


Figure 4.19 : Discrete systems: Stabilizing region for PID controller.

approaches can be significantly reduced. Within the scope of this section, the stability conditions that are derived from the proposed approach in this chapter are inserted to the robust MPC problem formulation for the first time. Details of the derived results within the scope of this section can be found in our study [69].

As indicated in various studies, Model Predictive Control (MPC) is one of the trending state of the art methods in optimal operating of plants [140, 141].

The overall control performance of MPC is highly affected by the imperfect model used for the future prediction of the plant behavior, i.e., so-called *process-model mismatch*. If the model is highly effected by the uncertain parameters, then the control performance can be significantly decreased, and, in the worst case, the closed-loop system behavior can lead to unstable response. The influence of uncertain parameters can be reduced by using *robust* MPC as indicated in [142]. However, the main limitation of practical robust MPC implementation is the complexity of the solved optimization problem in each control step. It was shown in robust MPC literature that the linear matrix inequalities can be used to solve the case of parametric uncertainties [143]. In the pioneering study of Kothare et al. [144], LMI based RMPC design were formulated in the form of semidefinite programming [145]. This approach was improved by the later studies like [146–148]. The non linear case and the output feedback RMPC cases were respectively discussed in [149] and in [150]. Lastly, in terms of the previous studies, it must also be pointed out that, another perspective in order to discuss the parametric uncertainties using the so-called *tubes* was proposed by Zeilinger et al. in [151].

The proposed approach in this section to overcome the robust MPC problem consists of two main steps. In the offline first step, considering the parametric uncertainties stabilizing controller parameter spaces and the analytical conditions related with the stabilizing regions needed to be determined. After that, in the second and the last step, nominal MPC needed to be evaluated using these conditions as constraints on controller parameters. In order to eliminate repetition, stabilizing parameter space calculation step will not be expressed in this section. For details of that step the Section 4.5.2.1 or our study [69] can be investigated. On the other hand, details of the robust MPC problem formulation is discussed in the next section.

4.5.3.1 Robust MPC design based on stabilizing parameter spaces

This section introduces design of a robust MPC design strategy in an indirect way. In other words, MPC is evaluated over the robustly stabilizing set of controller parameters $\mathbb{K}$. As a consequence, the robust stability of the closed-loop control system is achieved for slow variations of controller parameters. The main benefit of this approach is to design an optimal controller that is subject to system uncertainties in an effective way.

An indirect implementation of robust stability is ensured by the calculus of stabilizing parameters space $\mathbb{K}$ in off-line phase (for more details on stabilizing parameter space calculations for discrete time systems please see Section 4.5.2.1). Then, it becomes possible to achieve robust stability of the closed loop system using an arbitrary controller $k \in \mathbb{K}$. As the next step, in the on-line phase, it is sufficient to evaluate the parameters of $k \in \mathbb{K}$ to optimize the overall control performance. Furthermore, the constraints on control inputs u and system states x are also included in the MPC problem formulation. As a result, it can be proposed that, the optimization problem of

MPC can be formulated as:

$$\min_{K} \left(x_N^T P x_N + \sum_{t=0}^{N-1} \left(x_t^T Q_{\mathrm{x}} x_t + u_t^T Q_{\mathrm{u}} u_t \right) \right), \tag{4.80a}$$

$$\text{s.t.}: \quad x_{t+1} = A(q)\,x_t + B(q)\,u_t, \tag{4.80b}$$

$$u_t = -K_t\,x_t, \tag{4.80c}$$

$$u_t \in \mathbb{U}, \tag{4.80d}$$

$$x_t \in \mathbb{X}, \tag{4.80e}$$

$$K \in \mathbb{K}, \tag{4.80f}$$

$$x_0 = x(0), \tag{4.80g}$$

$$\forall t \geq 0, \tag{4.80h}$$

where $K \in \mathbb{K}$ is a set of robustly stabilizing parameters of the controller, and $P \succeq 0$, $Q_{\mathrm{x}} \succeq 0$, $Q_{\mathrm{u}} \succ 0$ are weighting matrices of appropriate dimensions.

In (4.80), it is aimed to minimize the quadratic objective function 4.80a subject to an uncertain system that is given in (4.80b). At the same time, the constraints on control inputs in (4.80d) and states constraints that is given in (4.80e) have to be satisfied. The control input is evaluated using linear control law in (4.80c). The precomputed conditions on robust stability are included in the MPC formulation as constraint on controller parameters as it is given in (4.80f) where $\mathbb{K}$ represents the set of robustly stabilizing parameters. The control problem in (4.80) is evaluated for given initial conditions in (4.80g).

Although, the sets $\mathbb{U}$, $\mathbb{X}$, $\mathbb{K}$ in (4.80d)–(4.80f) can be convex, the optimization problem in (4.80) is not convex, in general. The non-convex formulation originates in (4.80c), where the prediction horizon $N > 1$ introduces the multi-linear terms into (4.80b).As a result, the solution of the optimization problem in general form, (4.80) subject to $N > 1$, leads to a non-convex scenario. If the considered runtime prevents implementation of NP-hard optimization problem in (4.80), then the one-step-ahead prediction horizon, i.e., *a convex scenario*, should be considered.

Otherwise, if the limited hardware computational power prevents solving even a convex optimization problem, then an optimal time-invariant controller K can be designed for $N > 1$ in off-line phase, i.e, *fast scenario* is implemented. Note, then a time-invariant controller is implemented and the advantage of receding horizon control strategy vanishes.

It is aimed to demonstrate the benefits of the proposed approach over a benchmark example.

4.5.3.2 Case study: RMPC design for an uncertain system

In order to demonstrate the results of the proposed RMPC design strategy for the systems with the parametric uncertainties, the following benchmark system was adopted from [152]:

$$x(t+1) = \begin{bmatrix} 1 & 0 \\ q & 1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t), \tag{4.81}$$

where the uncertain parameter is given as $0.5 \leq q \leq 1.5$. For a state-feedback controller the closed-loop system matrix $A(K,q)$ has the form:

$$A(K,q) = \begin{bmatrix} 1-k_1 & -k_2 \\ q & 1 \end{bmatrix}. \tag{4.82}$$

As the first step, robustly stabilizing parameter space should be determined using the proposed Lyapunov equation based approach. As a result, the stability boundaries of the given system were determined using the parametric solution of $\mid M(K) \mid = 0$ in the off-line phase. For this case $\mid M(K) \mid$ and its roots can be expressed as:

$$\mid M(K) \mid = k_2\, q\, (k_1 - k_2\, q)^2 (4 - 2k_1 + k_2\, q), \tag{4.83}$$

$$k_2 = 0, \quad k_2 = (2k_1 - 4)/q, \quad k_2 = k_1/q. \tag{4.84}$$

Using the parametric solutions of $\mid M(K) \mid = 0$ given in (4.84) the stabilizing parameter region for the given system was determined as shown in Fig. 4.20. In this case, the intersections of stabilizing parameter spaces can be projected on the $k_1 - k_2$ plane. The stabilizing controller parameter space independent from the uncertain parameter can be determined as shown in Fig. 4.21.

The corresponding analytical conditions on controller parameters in terms of the stability of the closed loop system were expressed as:

$$0 < k_2 < 1.6 \quad \wedge \quad 1.5k_2 < k_1 < 0.25(8 + k_2). \tag{4.85}$$

For the uncertain system in (4.81) a robust MPC is designed using the derived stability conditions given in (4.85). These conditions are applied as constraints on

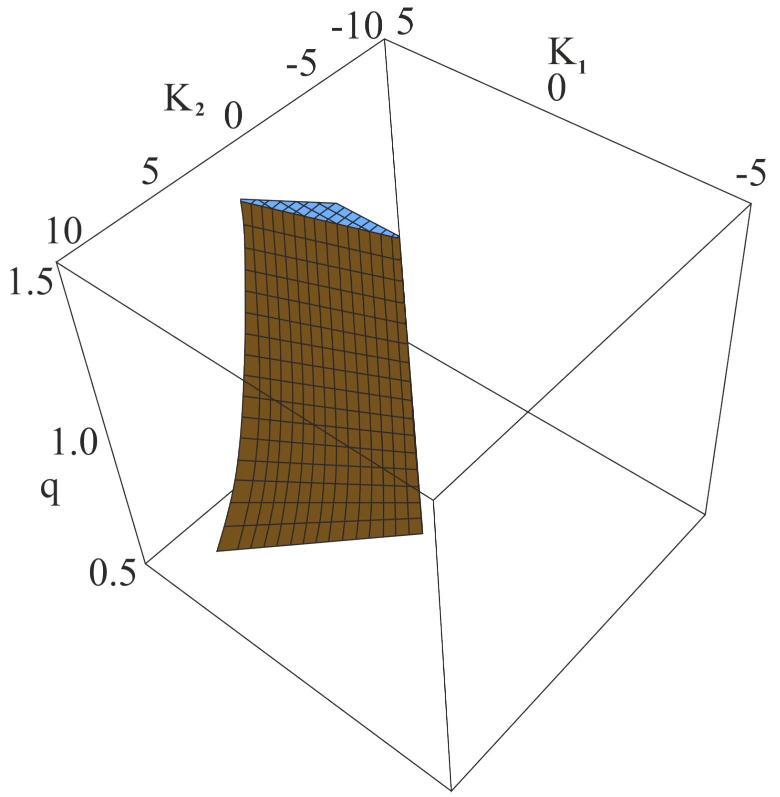


Figure 4.20 : Robust MPC: Stabilizing parameter region.

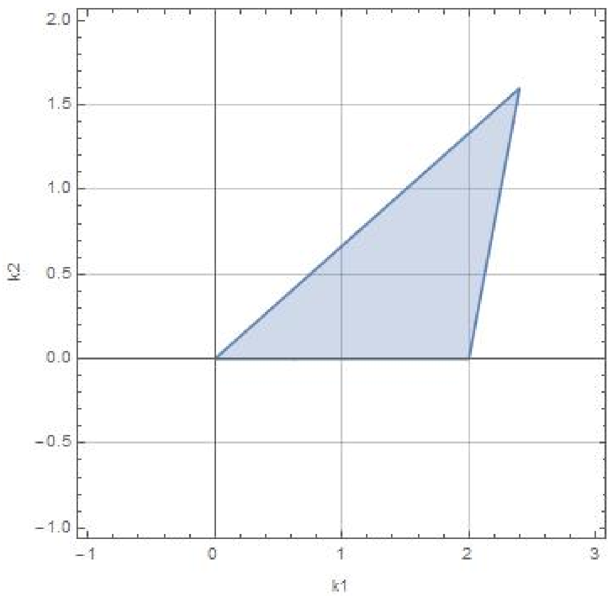


Figure 4.21 : Robust MPC: Stabilizing controller parameter space.

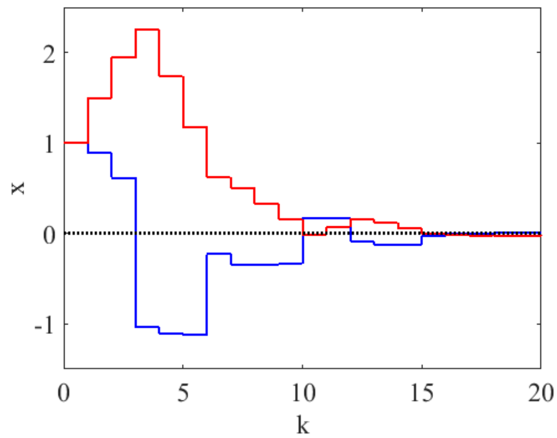
controller parameters in the MPC algorithm to ensure the optimal closed-loop control performance.

In order to simulate the closed-loop response of the given system a time-varying uncertain parameter sequence where $0.5 \leq q(t) \leq 1.5$ is considered. This uncertain parameter sequence is selected as, $q(t) \neq q(t+1)$.

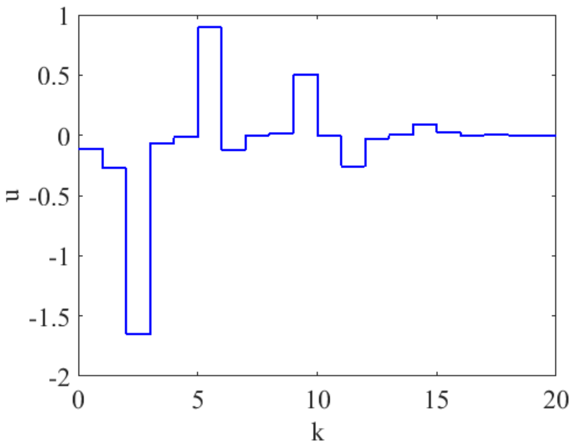
The conditions given in (4.85) correspond to a convex region. As a result, these constraints can be directly used in the proposed robust MPC design approach described in Section 4.5.3.1 to achieve an optimal control performance. Additionally, there are active constraints on control inputs: $-1.4 \leq u(k) \leq 0.6$, prediction horizon is $N = 3$, $Q_{\mathrm{x}} = \mathrm{diag}([1,1]) \times 10^3$, $Q_{\mathrm{u}} = 1 \times 10^{-3}$, for the initial conditions $x(0) = [1,1]^T$ in MPC problem (4.80).

The computed initial values of controller parameters are $K(1) = [0.0893, 0.0224]^T$ that clearly satisfy robust stability condition in (4.85). The other controllers $K(t)$ evaluated for $t > 1$ also satisfy this condition. Fig. 4.22 shows simulation results of the closed-loop control performance. As can be seen, the control trajectories of the uncertain system converge into the origin (Fig. 4.22(a)), and the associated optimal sequence of control actions is depicted in Fig. 4.22(b). Note that the trajectoris in Fig. 4.22 are generated subject to the time-varying values of uncertain parameter q.

Implementing the stability conditions into the MPC problem formulation as constraints is the main novelty of the proposed approach. By this way, it becomes possible to design RMPCs by solving nominal MPC problems. In the proposed MPC design approach, the stabilizing set of controller parameters are considered to ensure the optimal system response.



(a) Controlled states trajectories, x_1 (blue), x_2 (red), reference (black dotted).



(b) Control inputs.

Figure 4.22 : Closed-loop control performance assured by robust MPC.

5. THE CASE OF PARAMETRIC UNCERTAINTIES

In the previous sections, diagonal dominance and stability problems of multivariable systems were discussed in detail for a given nominal system. The derived results are meaningful from the controller parameter space determination point of view when there is no parametric uncertainty. However, further analysis is required in case of parametric uncertainties and in general (except limited class of uncertain systems) it is not possible to determine necessary and sufficient conditions for such systems [153].

It can be proposed that in dynamical control systems, uncertainties are unavoidable and typical sources of such uncertainties are unmodelled dynamics, effects of deliberate reduced order models, neglected nonlinearities, system-parameter variations due to torn and worn factors, equipment aging and environmental changes [154]. Ignoring such uncertainties may affect the performance, diagonal dominance characteristics and the stability of closed loop system in an adverse manner.

In dynamical control systems, uncertainties can be classified in different ways. However, in general, it can be proposed that they are classified as unstructured and parametric uncertainties. In representing the effect of neglected or unmodelled system dynamics and nonlinearities, the unstructured uncertainty may be more efficient as indicated in [154]. In a given dynamical system, these kind of uncertainties can be represented in different ways like additive perturbation, inverse additive perturbation, input and output multiplicative perturbation etc.

On the other hand, inaccurate representation of system component characteristics, torn-and-worn effects, equipment aging, the effect of environmental conditions on system parameters may lead to perturbations in most of the industrial control systems [155]. These kind of uncertainties can be represented as variations in certain parameters of the system. Such type of uncertainty representation is named as "parametric uncertainty" in literature. It is also possible to divide parametric uncertainties into subclasses like interval, multi linear, nonlinear etc. with respect to how varying parameters effect the overall system and characteristic equation. Within the scope of this thesis, it is aimed to propose solutions for parametric uncertain MIMO systems. More specifically, it was assumed that the individual transfer functions that constitute overall MIMO transfer function matrix include interval type parametric uncertainties.

In this section, first, diagonal dominance of TITO systems is discussed in detail for the case of interval type parametric uncertainties in 5.1. It is shown that it is possible to derive conservative results using the triangular inequality and weighting factors. Derived results in that subsection was published in [41]. Additionally, the stability of multivariable systems is also discussed in subsection 5.2. After pointing out the existing methods in literature in terms of parameter dependent Lyapunov functions, it is shown that proposed approach in Section 4.3 is also suitable for such problems. Lastly, Kharitonov Theorem is used in accordance with the proposed Lyapunov equation based method to determine robustly stabilizing parameter spaces for uncertain MIMO systems. Detailed analysis of the derived results related with the stability of parameter uncertain MIMO system can be found in our study [156].

5.1 Diagonal Dominance

Decoupling methods that was introduced in Section 2 is applicable for a limited class of MIMO systems. Furthermore, it becomes impossible to achieve decoupling in case of uncertainties. In such cases, diagonal dominance which is a weaker condition compared to diagonalisation can be preferred in order to reduce the interactions. However, the results derived in Section 3 are valid for the case of nominal systems and cannot be directly applied to parameter uncertain MIMO system. For this reason, it is aimed to propose an approach to determine static diagonal controller parameter regions in case of interval type parametric uncertainties. Derived results within the scope of this section was published in our previous study [41].

At this point, it must be noted that the number of studies that aims to determine robust diagonal dominance conditions are very limited in literature. While some of the existing studies aim to find specific controller values that satisfies the desired predetermined conditions, others aim to use diagonal dominance characteristic of the system to derive some conservative results on stability. For instance, fundamental dominance condition is asserted by Kontogiannis and Munro for parameter uncertain

MIMO systems in [10]. After that sufficient conditions for the robust stability of interval and affine linear MIMO uncertain systems, based on the Rosenbrock's Direct Nyquist Array is proposed by Kontogiannis and Munro [11, 157]. Furthermore, using the diagonal dominance property, a sufficient condition for robust stability of interval type MIMO systems was proposed in [158]. In [159] robust Nyquist arrays and Gershgorin bands were used for the same purpose and more recently, an H_∞ norm metric was proposed in [160] for the case of parameter uncertain MIMO systems in order to reduce the interactions. However, all of the mentioned studies aim to determine specific controller parameter pairs or to use diagonal dominance property in robust stability calculations.

The main focus of this section is to determine controller parameter regions that satisfy diagonal dominance at a given frequency and/or frequency interval. For this purpose, using the triangular inequality, it is aimed to transform the diagonal dominance problem of a given TITO system to the weighted diagonal dominance problem of the nominal TITO system.

In the first triangular inequality based approach considering the parametric uncertainties, minimum and maximum magnitude of the discussed transfer function should be determined at a given fixed frequency. This task can be accepted as a relatively simple for the case of interval type uncertainties. For example, the magnitude of a given transfer function that is given in the following form:

$$g(s) = \frac{n_0 + n_1 s + n_2 s^2 + n_3 s^3}{d_0 + d_1 s + d_2 s^2 + d_3 s^3} \tag{5.1}$$

can be directly derived as:

$$|g(s)| = \sqrt{\frac{(n_0 - n_2\ \omega^2)^2 + (n_1 \omega - n_3\ \omega^3)^2}{(d_0 - d_2\ \omega^2)^2 + (d_1 \omega - d_3\ \omega^3)^2}} \tag{5.2}$$

For the sake of simplicity, only maximization approach is discussed. However, a similar methodology can be followed to determine the minimum magnitude of the system that is given in (5.1). When (5.2) is examined, it can be observed that both the numerator and the denominator terms include two quadratic equations. In order to determine the maximum magnitude of the given transfer function, the ratio between the numerator and denominator must be maximized. So that, it can be proposed that the quadratic terms in the numerator should be maximized while the quadratic terms in denominator should be minimized in order to determine the maximum magnitude of the given transfer function at a given fixed frequency. As a result, it can be proposed that the maximum magnitude of a given transfer function as in (5.1) can be reduced to the solution of the following sub optimization problems:

$$\begin{array}{ll} \text{Max}\left[\left(n_0 - n_2\ \omega^2\right)^2\right], & \text{Max}\left[\left(n_1\omega - n_3\ \omega^3\right)^2\right] \\ \text{Min}\left[\left(d_0 - d_2\ \omega^2\right)^2\right], & \text{Min}\left[\left(d_1\omega - d_3\ \omega^3\right)^2\right] \end{array} \tag{5.3}$$

Considering the interval type parametric uncertainties on the coefficient terms ($n_i \in [n_{i_{min}} \quad n_{i_{max}}]$ and $d_i \in [d_{i_{min}} \quad d_{i_{max}}]$), the optimization problems given in (5.3) can be solved easily. Here, it must be pointed out that discussed system is a third order system. If the order of the system increases, then the number of coefficient terms that quadratic functions in (5.3) will increase. The presented methodology can also be extended to the case of multiplication of transfer functions in a conservative way. Multiplication of transfer functions that include interval type parametric uncertainties can be converted to the formulation that is given in equations (5.1)-(5.3). In that case, the upper and lower bounds of the resulting coefficient terms should be determined firstly. Then, using the previously mentioned approach, maximum and the minimum values of the resulting transfer function can be determined in the related frequency. The same approach can also be followed for the summation of two parameter uncertain transfer functions.

For the case of diagonal dominance of a given parameter uncertain MIMO system, it can be proposed that controller terms are included in the initial conditions as multipliers in general. If the related entry of the discussed transfer function matrix includes a controller parameter which is not a multiplier of all terms in that entry, then maximum and minimum magnitude values of the related element cannot be directly determined using the previously proposed approach. However, it can be proposed that it is still possible to propose conservative approaches in that case.

In this section, it is aimed to set a connection between the diagonal dominance problem of parameter uncertain system and weighted diagonal dominance problem of the nominal system. In principle, it is desired to represent the worst case coefficient term values by nominal coefficients and the weighting factors. So that, the result that was derived in Section 3.3 for the case of static diagonal controllers can be used directly for parameter uncertain systems.

When one of the static diagonal controller parameter is not the multiplier of the all elements in the related entry of the transfer function matrix, triangular inequality can be used to conservatively determine the maximum and the minimum magnitude values in the desired frequency.

In order to express the aim of this approach in a simpler way, the following condition on row diagonal dominance which is:

$$|k_1 g_{11} + k_1 k_2 g_d| \geq |k_2 g_{12}| \tag{5.4}$$

can be used as an example. It is assumed that in (5.4) all of the individual transfer functions include interval type parametric uncertainties. Here, it is aimed to represent (5.4) as:

$$|k_1|\,|g_{11_{nom}} + k_2 g_{d_{nom}}| \geq \mu_{r_1}\,|k_2|\,|g_{12_{nom}}| \tag{5.5}$$

where both transfer functions are nominal and μ_{r_1} is the corresponding weighting factor that should be determined. Considering the parametric uncertainties that g_{12} include, maximum value of $|g_{12}|$ at a given frequency can be determined directly using the previously mentioned approach. As a result the ratio of the maximum magnitue to the nominal magniute ($\text{Max}\,|g_{12}|\,/\,|g_{12_{nom}}|$) can be determined easily.

In order to determine the value of the weighting factor in (5.5), the ratio $\frac{\text{Min}|g_{11}+k_2 g_d|}{|g_{11_{nom}}+k_2 g_{d_{nom}}|}$ should be determined as a second step. In this step, triangular inequality can be used to determine the required ratio conservatively. Here, it is aimed to determine minimum magnitude of $|g_{11} + k_2 g_d|$ in the related k_2 interval. Using the triangular inequality a lower bound for $|g_{11} + k_2 g_d|$ can be expressed as:

$$|g_{11} + k_2 g_d| \geq |g_{11}| - |k_2|\,|g_d| \tag{5.6}$$

Using the aforementioned methods, maximum and minimum magnitudes of $|g_{11}|$ and $|g_d|$ can be determined for the case of parametric uncertainties at a given frequency. As a result, it can be proposed that it becomes possible to determine minimum value of $|g_{11}| - |k_2|\,|g_d|$ conservatively in the related k_2 interval. Furthermore, it is also possible to determine the ratio:

$$\frac{\text{Min}\,|g_{11} + k_2 g_d|}{|g_{11_{nom}} + k_2 g_{d_{nom}}|} \tag{5.7}$$

where $g_{11_{nom}}$ and $g_{d_{nom}}$ represent nominal transfer functions. This approach provides an analytical method. Whereas, triangular inequality depends in the direction of individual transfer function and in general results in high conservatism and it is suitable for limited class of systems.

On the other hand, numerical (and in general computational complex) approaches can be proposed in order to reduce the conservatism. From that point of view, a griding approach is also proposed in this section. In that approach, uncertain parameter first should be grided into uniformly spaced values and then the ratio (5.7) should be determined for every predetermined value of uncertain parameters. The minimum value of (5.7) is obtained by checking the each grided value of uncertain parameters. In order to obtain more accurate results, uncertain parameters should be grided as much as possible. This also increases the computational complexity of the problem, since the more number of points determined as a result of griding uncertain parameters, the more computational effort required. Using the griding approach, better results can be derived for the case of less number of uncertain parameters.

Both of the proposed approaches in this section have their own pros and cons. An analytical approach is proposed using the triangular inequality. In this approach, conservativeness depends on the direction of the magnitudes of individual transfer functions. On the other hand, griding based numerical approach requires significant computational power. However, it is still sufficient in most cases to derive practical results.

It is also shown in this section that diagonal dominance problem of parameter uncertain system can be converted to weighted diagonal dominance problem of the nominal system. As a result, previously derived results can be used to determine the static diagonal controller gain regions that achieve diagonal dominance conditions for parametric uncertain systems.

5.1.1 Case study for diagonal dominance

As discussed in the previous sections, the problem of determining static diagonal controller parameters that achieve diagonal dominance for the case of parametric uncertainties can be conservatively converted to weighted diagonal dominance problem at a given frequency. In this section, it is aimed to demonstrate the results of the griding approach over a given TITO system. The TITO system that will be

Table 5.1 : Derived weighting factors for $\omega = 0$.

μ_{r_1}	μ_{r_2}	μ_{c_1}	μ_{c_2}
1.50348	1.70897	1.65382	1.55361

considered as a case study and the structure of the controller are as follows:

$$G(s,q_i) = \begin{bmatrix} \frac{6q_1-2s}{(2+5q_2s)} & \frac{1+4q_3s}{(1+10s)(3q_4+5s)} \\ \\ \frac{1.5q_5}{(q_6+7s)} & \frac{2q_7}{(1+q_7s)(2+s)} \end{bmatrix} \tag{5.8}$$

$$K = \begin{bmatrix} k_{p_1} & 0 \\ 0 & k_{p_2} \end{bmatrix} \tag{5.9}$$

where q_i s represent the uncertain parameters and it is assumed that all uncertain parameters may vary between 0.9 and 1.1 ($0.9 \leq q_i \leq 1.1$). In that case, nominal system can be written as:

$$G(s) = \begin{bmatrix} \frac{6-2s}{(2+5s)} & \frac{1+4s}{(1+10s)(3+5s)} \\ \\ \frac{1.5}{(1+7s)} & \frac{2}{(1+s)(2+s)} \end{bmatrix} \tag{5.10}$$

In this case study, griding method that was introduced in Section 5.1 was preferred, since triangular inequality results in high conservatism for the considered system. Assuming that static diagonal controller parameters vary between 0 and 5 uncertain parameters q_i are grided in order to determine maximum and minimum magnitude values. Using these values, magnitude of the nominal magnitudes and the method introduced in the previous section the weighting factors can be derived as it is given in Table 5.1 for the zero frequency. Using the derived results in Section 3.3 and the weighting factors that is given in Table 5.1, the controller parameter region that achieves both row and column diagonal dominance conditions for the given parameter uncertain system can be determined as it is given in Figure 5.1. Firstly, maximum value of weighting factors should be determined in a given frequency range to extend the results to frequency ranges. If the desired frequency range which is aimed to achieve both of the diagonal dominance conditions is $10 \geq \omega \geq 0$, then by griding the frequency and repeating the same procedure can be followed to derive weighting factor. Using this approach weighting factors for the related frequency interval can be determined as it is given in Table 5.2.

When Table 5.1 and Table 5.2 are examined, it can be observed that all weighting factors are greater in Table 5.2 except the weighting factor of the first column. This

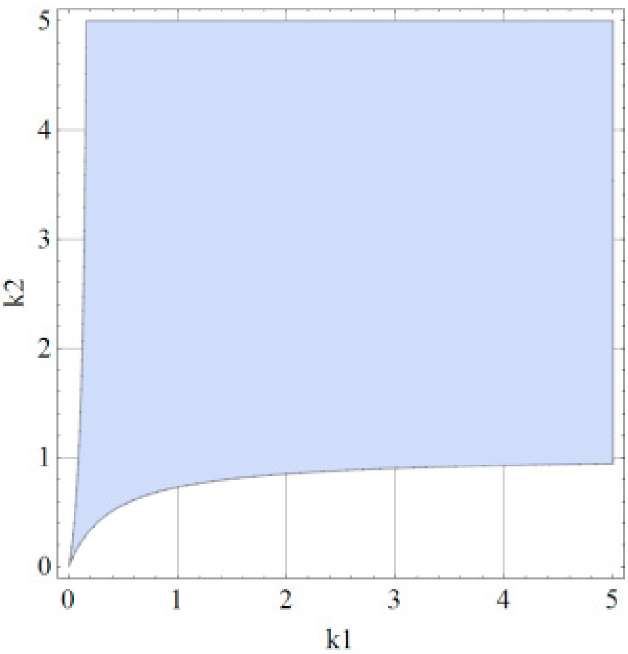


Figure 5.1 : $k_{p_1} - k_{p_2}$ region that makes the closed loop parameter uncertain system diagonal dominant at $\omega = 0$.

Table 5.2 : Derived weighting factors for $10 \geq \omega \geq 0$.

μ_{r_1}	μ_{r_2}	μ_{c_1}	μ_{c_2}
1.56375	1.77103	1.65382	1.77151

means that the greatest weighting factor derived for the first column in the frequency interval $10 \geq \omega \geq 0$ is obtained at $\omega = 0$. As a result, μ_{c_1} are same in the given two tables.

In that way, diagonal dominance problem of the parameter uncertain system is converted to the weighted diagonal dominance problem of the nominal plant. Using the weighting factors that is given in Table 5.2 and assuming that static diagonal controller parameters vary between 0 and 5 controller region that achieves robust diagonal dominance can be derived as it is given in Figure 5.2.

When Figure 5.2 is examined, it can be determined that the controller gain pair $k_{p_1} = 0.6$, $k_{p_2} = 3$ satisfy the conditions in the related frequency range. In order to analyze the diagonal dominance of the closed loop parameter uncertain system, this controller gain pair is selected. Using this controller pair, diagonal dominance ratio plots are plotted for both the nominal system and different values of uncertain parameters as it is given in Figure 5.3. Red curves in Figure 5.3 represents the diagonal dominance

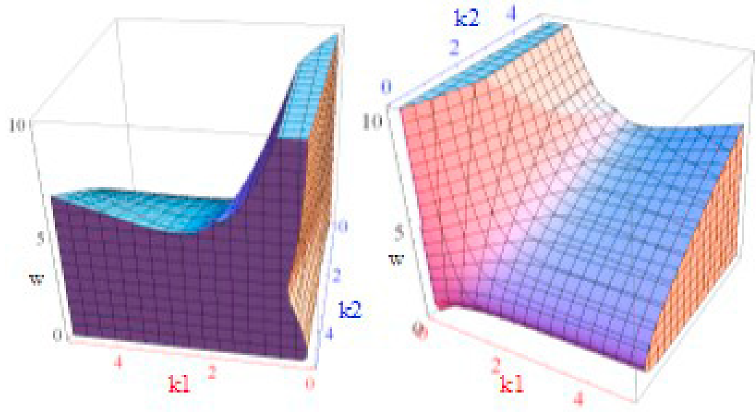


Figure 5.2 : $k_{p_1} - k_{p_2}$ region that makes the closed loop parameter uncertain system diagonal dominant at $10 \geq \omega \geq 0$.

ratio plots of the nominal system while the blue lines are derived for different values of the uncertain parameters. Each ratio plot should not exceed 1 in this case. Diagonal dominance ratio plots do not exceed the critical value even in the worst case. As a result, it can be proposed that selected controller pair achieves the diagonal dominance conditions in the related frequency range as expected.

Within the scope of this section diagonal dominance problem of interval type parameter uncertain systems is discussed from different perspectives. Two approaches that are based on triangular inequality and griding was proposed to determine static diagonal controller region in the case of parametric uncertainties. In that way, it is aimed to convert the original problem to the weighted diagonal dominance of the nominal system. Then, it is possible to use the derived results in previous sections in terms

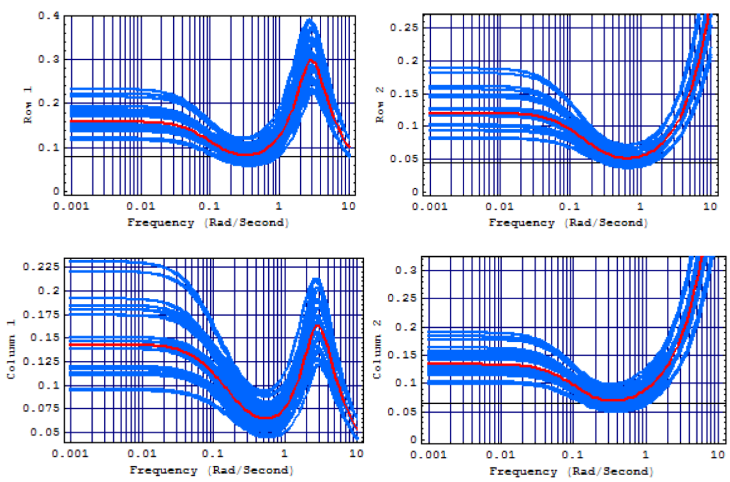


Figure 5.3 : Diagonal dominance ratio plots for parametric uncertain system for $10 \geq \omega \geq 0$.

of weighted diagonal dominance. A case study is also included to demonstrate the applicability of proposed griding based approach.

5.2 Stability of Parameter Uncertain MIMO Systems

Stability of parameter uncertain systems can be discussed from two different main perspectives. In the first one, it is assumed that the uncertainty bounds on the parameter is known and it is searched that the whole set of polynomials that lie in these intervals are stable or not. Significant progress was achieved in literature to determine the stability of such systems. For instance, in 1978 Kharitonov proved that it is necessary and sufficient to check the stability of only 4 polynomial for the case of interval type parameter uncertain polynomials [161]. Later on, this number of fixed polynomials also reduced to 3,2,1 for the systems of order 5,4,3 in [162] and results derived for discrete time systems were presented in [163]. Further results were also derived for the cases of affine linear type uncertain polynomials and multi-linear type uncertainties. Edge and Mapping Theorems were proposed for such type of uncertainties [164–166]. The stability characteristics of a given parameter uncertain polynomial family is searched in the aforementioned approach for the given known bounds on uncertain parameters. On the other hand, in some cases, it is not possible to determine strict upper and lower bounds for uncertain parameters. So that, it becomes more logical to determine the range of uncertain parameters that make the closed loop system stable (or unstable).

A method to determine the bounds of uncertain parameters is the parameter dependent Lyapunov functions. There are certain number of studies in literature that discuss the stability of parameter uncertain systems from different perspectives [167–170]. Details of the parameter dependent Lyapunov functions and the type of uncertainties considered in these studies will also be discussed in the next section. It is possible to use the proposed Lyapunov equation based stability mapping approach in order to determine the stability boundaries of uncertain parameters. However, the computational complexity increases more and more, when the number of uncertain parameters increase. In order to overcome such complexities, using the Kharitonov theorem and proposed Lyapunov equation based approach, a combined approach was proposed for the case of parametric uncertainties. Details of the proposed approach are discussed in Section 5.2.2.

5.2.1 Parameter dependent Lyapunov functions and Lyapunov equation based approach

Lyapunov theorems can be also used for the case of LPV and LTIPD systems. It is aimed to determine the range of uncertain parameters that make closed loop system stable in such cases. Various approaches were proposed in literature to determine the bounds of uncertain parameters in terms of stability [153,170–174]. It can be proposed that in literature most of the studies focus on the cases single or double parameter dependencies. For instance in [175, 176] an approach that is named as guardian maps was proposed in the form of:

$$\dot{x} = A(q)x, \quad A(q) = A_0 + qA_1 + q^2A_2 + ... + q^mA_m \tag{5.11}$$

and

$$\dot{x} = A(q_1, q_2)x, \quad A(q_1, q_2) = \sum_{i_1, i_2 = 0}^{i_1 + i_2 = m} q_1^{i_1} q_2^{i_2} A_{i_1, i_2} \tag{5.12}$$

Using the guardian map approach proposed in [175, 176] it is possible to determine necessary and sufficient conditions for the given uncertainty domains. In [177] derived results were extended for the system class that is expressed as:

$$\dot{x} = A(q_1, q_2, ...q_m)x, \quad A(q_1, q_2, ...q_m) = A_0 + \sum_{i=1}^{m} q_i A_i \tag{5.13}$$

However, in that approach it is only possible to derive sufficient conditions as indicated in [120].

The proposed approaches, that is given in [175–177] focus on determining the stability characteristic of the system for given uncertainty bounds. However, parameter dependent Lyapunov function approach can also be used to derive exact bounds of uncertain parameters. Firstly, single parameter dependency will be discussed. For this purpose let us consider the following LTI system

$$\dot{x} = A(q)x, \quad A(q) = A_0 + qA_1 \quad q \in \Phi \tag{5.14}$$

where $A_0, A_1 \in \mathbb{R}^{n \times n}$ and $\Phi \subset \mathbb{R}$. Using the Lyapunov approach it can be proposed that the following conditions should be satisfied for the system that is given in terms of stability

$$P(q) > 0 \tag{5.15}$$

$$A(q)^T P(q) + P(q)A(q) < 0 \tag{5.16}$$

As it was done in the case of nominal systems, (5.16) can be written as matrix equality as:

$$A(q)^T P(q) + P(q)A(q) = -Q(q) \tag{5.17}$$

where $Q(p) \in \mathbb{R}^{n\times n}$ is any positive definite matrix for all values of uncertain parameters. The solution $P(q)$ of (5.17) can be written as [178]:

$$P(q) = \int_0^\infty e^{tA(q)^T} Q(q) e^{tA(q)} dt \tag{5.18}$$

When $Q(q)$ is analytic in q, it can be directly concluded that $P(q)$ is also analytic in q. As a result, solution can be expressed as the sum of infinite power series as:

$$P(q) = P_0 + qP_1 + q^2 P_2 + ... = \sum_{i=0}^{\infty} q^i P_i \tag{5.19}$$

It was shown in [179] that using the uniform convergence of the integral that is given in (5.18), infinite power series can be truncated and Lyapunov matrix can be expressed in the following form:

$$P(q) = P_0 + qP_1 + q^2 P_2 + ... + q^m P_m = \sum_{i=0}^{m} q^i P_i \tag{5.20}$$

However, an upper bound for m was not proposed in [179]. Whereas, it was shown in [178] that it is necessary and sufficient to select m as:

$$m \leq \min\left\{\frac{1}{2}\left(2nr - r^2 + r\right), \left(\frac{1}{2}n(n+1) - 1\right)\right\} \tag{5.21}$$

for the stability of whole uncertain parameters $q \in \Phi$. In (5.21), r represents the rank of A_1. Using these results, it becomes possible to propose the stability range of uncertain parameter in the sense of Lyapunov. Details of the approach can be found in [178]. In order to demonstrate the result, the same system that was considered in [178] is discussed in this section. Consider the system matrix $A(q) = A_0 + qA_1$, where

$$A_0 = \begin{bmatrix} 0.7493 & -2.4358 & -1.6503 \\ -2.0590 & -3.3003 & -1.4833 \\ -1.5019 & 1.2149 & -4.8737 \end{bmatrix} \quad \text{and} \quad A_1 = \begin{bmatrix} 1.2149 & 1.6640 & -2.2091 \\ 0.7542 & -0.1501 & 0.2109 \\ 2.1990 & 0.6493 & -0.2214 \end{bmatrix} \tag{5.22}$$

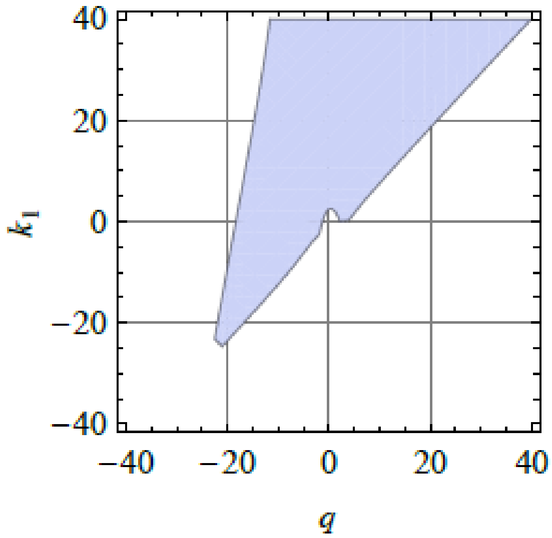


Figure 5.4 : $k_1 - q$ region that make the closed loop uncertain system stable.

Using the proposed approach [178], the exact stability domain can be calculated as: $q \in (-18.3861, -1.2729) \cup (2.1538, 3.7973)$. It is also possible to determine the same disconnected region using the Lyapunov equation based stability mapping approach that was proposed in Section 4.3. Furthermore, it is possible to determine the stabilizing parameter space, when there are more than one parameter using the proposed approach in 4.3, while the approach proposed in [178] is valid for only single parameter dependencies. For instance, if the discussed system includes one free controller parameter k_1 as:

$$A(k_1, q) = \begin{bmatrix} 0.7493 - k_1 + 1.2149q & -2.4358 + 1.6640q & -1.6503 - 2.2091q \\ -2.0590 + 0.7542q & -3.3003 - 0.1501q & -1.4833 + 0.2109q \\ -1.5019 + 2.1990q & 1.2149 + 0.6493q & -4.8737 - 0.2214q \end{bmatrix} \tag{5.23}$$

then the stabilizing parameter region can be determined as it is given in Figure 5.4.

It is also possible to use Lyapunov equation based stability mapping approach, when all the parameter are uncertain parameters. For instance, the system used in [173] as a case study can be used to verify the results. For this purpose, the following system can be considered

$$\dot{x} = (A_0 + q_1 A_1 + q_2 A_2)x \tag{5.24}$$

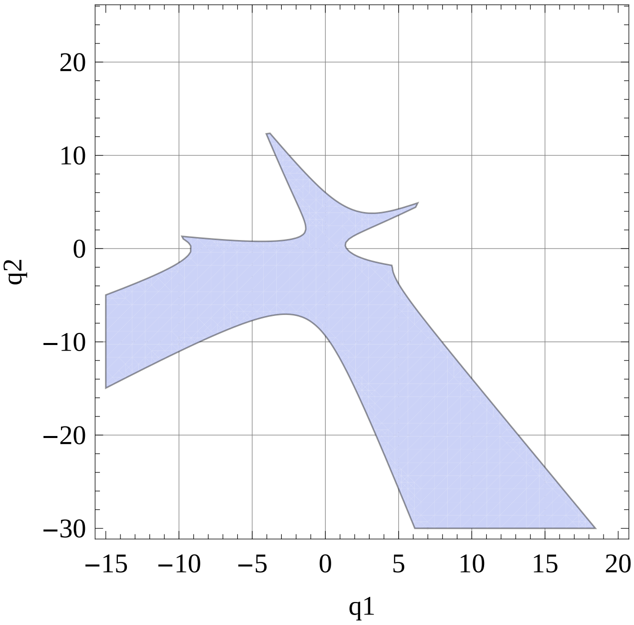


Figure 5.5 : $q_1 - q_2$ region that make the closed loop uncertain system stable.

where

$$A_0 = \begin{bmatrix} -2 & 0 & -1 \\ 0 & -3 & 0 \\ -1 & -1 & -4 \end{bmatrix}, A_1 = \begin{bmatrix} 0.916 & -0.8119 & -0.2168 \\ -0.6863 & -0.1001 & -0.4944 \\ -0.1673 & 0.7383 & -0.2912 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} 1.215 & 1.664 & -2.209 \\ 0.7542 & -0.1501 & 0.2109 \\ 2.199 & 0.6493 & -0.2214 \end{bmatrix} \tag{5.25}$$

Using the Lyapunov equation bases stability mapping approach uncertain parameter region that makes the system stable is determined as it is given in Figure 5.5. Derived region is exactly the same as given in [173]. As a result, it is shown that proposed Lyapunov equation based stability mapping approach is also suitable for the systems that include uncertain parameters. In such cases, the uncertain parameters should be treated as free parameters and then the same procedure that was proposed in Section 4.3 can be applied. For the case of known uncertainty bounds on uncertain parameter, proposed approach can still be applicable. However, in such a case, the given (or known) uncertainty range should fully intersect with the exact stabilizing parameter space in order to propose the robust stability of the given parameter uncertain system.

From the computational complexity point of view, it must be noted that complexity of the problem increases significantly, if there are too many uncertain parameters in the discussed system. As a result, further analysis is required to decrease the computational need. For this purpose, using the Kharitonov theorem a combined approach will be presented in the next section to determine controller parameter regions that make a given system robustly stable for the case of interval type uncertainties.

5.2.2 A modified Kharitonov approach for MIMO systems

In 1978 Kharitonov proposed that it is necessary and sufficient to check stability of four polynomials in order to determine the stability of interval type parameter uncertain polynomials [161]. Derived stability results in that study is significant, since it was required to check stability of infinitely many polynomials in general previously. However, the original theorem only answers that a given interval type polynomial is stable or not. It is not possible to determine directly stabilizing controller parameter region using Kharitonov Theorem.

Within the scope of this subsection, it is aimed to propose an approach to determine robustly stabilizing controller parameters using both Kharitonov Theorem and proposed Lyapunov equation based stability mapping approach. Considered subsystems (g_{ij}) will include interval type uncertainties. However, characteristic polynomial include the multiplication of uncertain parameters in general, since the multiplication of transfer functions are also included in the characteristic polynomial. Whereas, resulting uncertainties are not interval type in general. As a result, an over bounding approach is required to apply Kharitonov theorem. After applying over bounding Lyapunov equation based method can be used to determine the range of controller parameters. Derived results in this subsection are aimed to be published in [156].

An interval characteristic polynomial is the family of polynomials:

$$\delta(s,q_i) = q_0 + q_1 s + q_2 s^2 + q_3 s^3 + q_4 s^4 + ... + q_n s^n \tag{5.26}$$

where the coefficients lie within the known intervals as:

$$q_0 \in \left[q_0^-, q_0^+\right], q_1 \in \left[q_1^-, q_1^+\right], ..., q_n \in \left[q_n^-, q_n^+\right]. \tag{5.27}$$

For the case of real coefficients, Kharitonov stated that it is necessary and sufficient to test 4 specific polynomials from the uncertain polynomial family. In [161], it was proven that uncertain polynomial family is robustly stable if and only if all 4 fixed polynomials are stable. This number of polynomials were reduced to 3,2 and 1 respectively for the systems of order respectively 5,4 and 3 in [162].

The four so-called Kharitonov's polynomials can be written as:

$$\begin{aligned}
\delta^{+-}(s) &= q_0^+ + q_1^- s + q_2^- s^2 + q_3^+ s^3 + q_4^+ s^4 + ... \\
\delta^{++}(s) &= q_0^+ + q_1^+ s + q_2^- s^2 + q_3^- s^3 + q_4^+ s^4 + ... \\
\delta^{-+}(s) &= q_0^- + q_1^+ s + q_2^+ s^2 + q_3^- s^3 + q_4^- s^4 + ... \\
\delta^{--}(s) &= q_0^- + q_1^- s + q_2^+ s^2 + q_3^+ s^3 + q_4^- s^4 + ...
\end{aligned} \tag{5.28}$$

Using the Kharitonov Theorem finite number of polynomials needed to be checked in order to determine the stability of uncertain polynomial family instead of infinitely many number of poynomials.

As indicated earlier, Kharitonov Theorem considers interval type characteristic polynomial of the dynamic systems. However, for the case of parameter uncertain multivariable systems, the characteristic polynomial includes multiplication of individual transfer functions, controller and uncertain parameters in general. As a result, overbounding is required for the resulting coefficients in order to be able use Kharitonov approach. After that proposed Lyapunov equation based approach can be used to determine controller parameter regions that robustly stabilize the given multivariable system.

In general, overbounding process is relatively simple, when there is no free controller parameter. However, it becomes more difficult to apply overbounding in the case of free controller parameters. It is not possible to determine the value of the uncertain parameters. They may take any value within the specified region. However, free controller parameters will be determined by the designer at the end. This can also be interpreted as such parameters can be treated as constants. Whereas there may be certain regions for controller parameters that may effect the lower and upper bounds of the resulting coefficients. For this reason, free controller parameter were also treated like uncertain parameters at the beginning of the proposed approach and it is aimed to determine stabilizing controller parameter regions in this predetermined region. After brief analysis on resulting coefficients, controller parameter regions can also be divided into subregions. Lastly, Lyapunov equation based approach can be applied for all subregions to determined stabilizing parameter ranges in the determined regions.

Last step should be taking the intersection of the stabilizing regions determined for all Kharitonov polynomials.

The steps of the proposed approach that will be used to determine the controller parameter regions that robustly stabilize the uncertain systems can be summarized as follows:

- **Step1:** Divide the controller parameter regions into several sub regions based on the sign of each controller parameter. The number of initial regions will be 2^n where n represents number of controller parameters.
- **Step2:** If necessary, divide the resulted regions into several intervals (if the same uncertain parameter appears in the uncertain coefficient more than once with different signs).
- **Step 3:** For each region, determine the upper and lower bound of each coefficient.
- **Step 4:** Using the derive bounds in Step 3 determine the Kharitonov polynomaials for each interval.
- **Step 5:** For each region, apply Lyapunov equation based stability mapping approach to the determined Kharitonov polynomials and then take the intersections of the derived regions.
- **Step 6:** Combine the obtained parameter regions to determine the robustly stabilizing parameter region(s).

In order to express the details of the proposed approach, a case study is included in this section. Suppose that it is aimed to determine robustly stabilizing controller parameter regions for the multivariable system and controller pair that is expressed as:

$$G(s,q_i) = \begin{bmatrix} \frac{q_1}{(s+1)(s+2)} & \frac{1}{s+q_2} \\ \frac{2q_3}{s+1} & \frac{2}{(s+2)} \end{bmatrix} \tag{5.29}$$

$$K = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \tag{5.30}$$

where q_1, q_2 and q_3 are the uncertain physical parameters of the system and all of them are assumed to vary between 0.8 and 1.2 ($q_i \in [0.8, 1.2]$). These physical parameters may include stiffness, inertia or viscosity coefficients in mechanical systems, the values of resistors and capacitors in electrical circuits and aerodynamic coefficients in flight control. Moreover, this system has a practical meaning, since most of dynamic systems can be modeled as first or second order systems. If it was desired to determine the stabilizing controller parameter spaces for the given uncertain parameters, using the Lyapunov approach then uncertain parameters should also be treated as free parameter. Such a case increases the computational complexity significantly. However, using the proposed approach within the scope of this section, it becomes possible to determine robustly stabilizing controller parameter regions by solving finite number of polynomials that depends on only two controller parameters.

In case of the unity feedback characteristic equation of the discussed system can be determined as:

$$\begin{aligned}\delta_{cl}(s,q_i) = &\underbrace{(4q_2+4k_2q_2+2k_1q_1q_2+2k_1k_2q_1q_2-8k_1k_2q_3)}_{a_\circ}+\\ &\underbrace{(4+4k_2+2k_1q_1+2k_1k_2q_1+8q_2+6k_2q_2+k_1q_1q_2-8k_1k_2q_3)}_{a_1}s\\ &+\underbrace{(8+6k_2+k_1q_1+5q_2+2k_2q_2-2k_1k_2q_3)}_{a_2}s^2+\underbrace{(5+2k_2+q_2)}_{a_3}s^3+s^4\end{aligned} \tag{5.31}$$

In this case study, it is aimed to determine robustly stabilizing k_1-k_2 regions for $k_1,k_2 \in [-5,5]$. Each quadrant may effect the lower and upper bounds in a different way. As a result the following cases:

$$\begin{aligned} &k_1>0, k_2>0\\ &k_1<0, k_2>0\\ &k_1<0, k_2<0\\ &k_1>0, k_2<0 \end{aligned} \tag{5.32}$$

should be discussed separately. However, in order to improve the readability, only the case of first quadrant will be expressed in detail. Whereas the uncertain parameter a_3 is a special case since it only include one controller parameter as a sum. As a result, upper and lower bounds for this parameter is the same for all quadrants and it can be expressed as:

$$\begin{aligned} a_3^+ &= 6.2+2k_2 \Rightarrow q_2^+\\ a_3^- &= 5.8+2k_2 \Rightarrow q_2^- \end{aligned} \tag{5.33}$$

where "$^+$" represents the maximum value of the uncertain parameter and "$^-$"represents the minimum value.

Table 5.3 : Upper bounds of the uncertain coefficients a_0, a_1, a_2 and a_3 in the first quadrant.

	Parameters	Upper Bound
a_0^+	$q_1^+,\ q_2^+,\ q_3^-$	$4.8+2.88k_1+4.8k_2-3.52k_1k_2$
a_1^+	$q_1^+,\ q_2^+,\ q_3^-$	$13.6+3.84k_1+11.2k_2-4k_1k_2$
a_2^+	$q_1^+,\ q_2^+,\ q_3^-$	$14+1.2k_1+8.4k_2-1.6k_1k_2$
a_3^+	q_2^+	$6.2+2k_2$

Table 5.4 : Lower bounds of the uncertain coefficients a_0, a_1, a_2 and a_3 in the first quadrant.

	Parameters	Lower Bound
a_0^-	$q_1^-,\ q_2^-,\ q_3^+$	$3.2+1.28k_1+3.2k_2-8.32k_1k_2$
a_1^-	$q_1^-,\ q_2^-,\ q_3^+$	$10.4+2.24k_1+8.8k_2-8k_1k_2$
a_2^-	$q_1^-,\ q_2^-,\ q_3^+$	$12+0.8k_1+7.6k_2-2.4k_1k_2$
a_3^-	q_2^-	$5.8+2k_2$

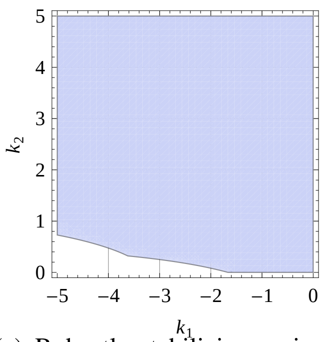
A. First Quadrant: ($k_1 \in [0,5]$ and $k_2 \in [0,5]$)

It is relatively easier to determine the upper and lower bounds of the uncertain coefficients since both k_1 and k_2 is positive in this quadrant. When the coefficients of (5.31) examined in detail, it can be observed for all uncertain parameters a_2, a_1, a_0 include uncertain parameter q_1 and q_2 with positive multipliers and q_3 with negative multiplier. As a result of this, in order to determine the maximum values of a_i's original uncertain parameters q_1 and q_2 should be maximum while q_3 should be minimum. This case and resulting coefficient terms are also expressed in Table 5.3.

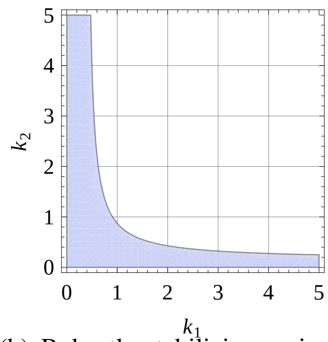
On the other hand, due to the same reasons original uncertain parameters q_1 and q_2 should be minimum while q_3 should be maximum in order to determine the minimum values of the resulting coefficients $a_0 - a_3$. Lower bounds of the resulting coefficients are given in Table 5.4.

For this quadrant Kharitonov polynomials can be written as follows:

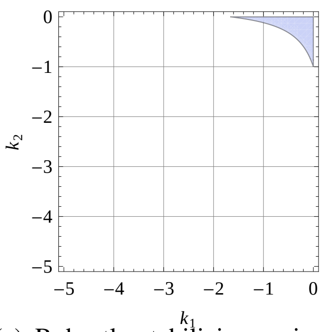
$$\begin{aligned}
\delta^{+-}(s) &= a_0^+ + a_1^- s + a_2^- s^2 + a_3^+ s^3 + s^4 \\
\delta^{++}(s) &= a_0^+ + a_1^+ s + a_2^- s^2 + a_3^- s^3 + s^4 \\
\delta^{-+}(s) &= a_0^- + a_1^+ s + a_2^+ s^2 + a_3^- s^3 + s^4 \\
\delta^{--}(s) &= a_0^- + a_1^- s + a_2^+ s^2 + a_3^+ s^3 + s^4
\end{aligned} \tag{5.34}$$



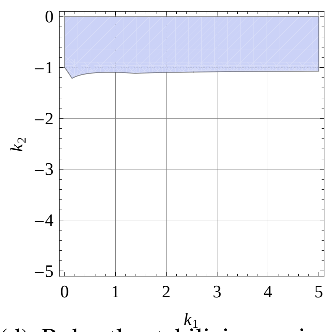
(a) Robustly stabilizing region (second quadrant).



(b) Robustly stabilizing region (first quadrant).



(c) Robustly stabilizing region (third quadrant).



(d) Robustly stabilizing region (fourth quadrant).

Figure 5.6 : $k_1 - k_2$ region that make the closed loop system robustly stable.

After that, using the Lyapunov equation based stability mapping approach stabilizing controller parameter spaces within the range $k_1, k_2 \in [0,5]$ for each polynomial can be determined separately. Then, intersection of these regions should be taken to determine the robustly stabilizing parameter space in the given controller parameter region. Robustly stabilizing controller parameter region is given in Figure 5.6(b) for the first quadrant. Using the same approach stabilizing controller parameter spaces can be determined as it is given in Figures 5.6(a),5.6(c),5.6(d).

At this point, it must be noted that further partitioning in the controller parameter regions were required for other quadrants in this case. As the last step of the proposed approach derived regions given in Figure 5.6 should be combined to illustrate the stabilizing controller parameter regions for $k_1, k_2 \in [-5,5]$. Resulting parameter region is given in Figure 5.7.

In order to verify the correctness of the derived results and to be able to compare both results griding method was preferred. In that approach, it is required to sweep over the entire range of the uncertain parameters.As a result accuracy of the derived results depends on the step size. Whereas, decreasing the step size increases the computational

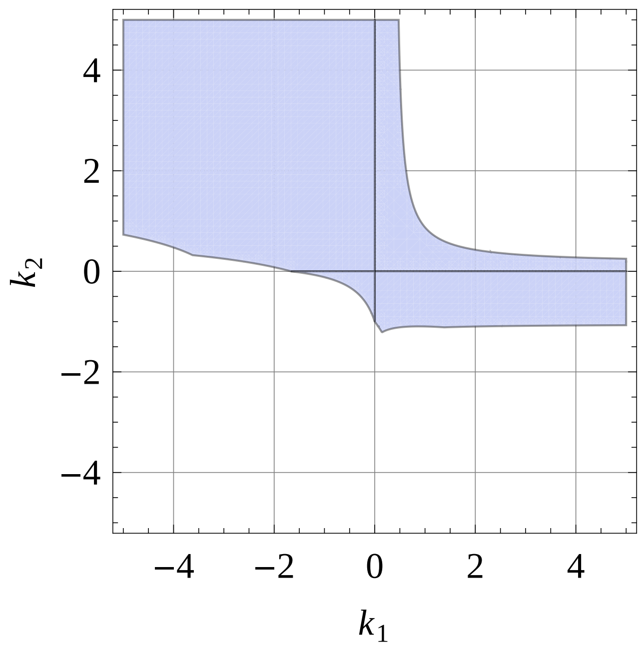


Figure 5.7 : Derived $k_1 - k_2$ region that make the closed loop uncertain system stable.

effort significantly. In that approach, for a given valued of controller parameter pair, each uncertain parameter is grided and resulting fixed coefficient characteristic polynomials are checked in terms of stability.

After that, if the system is stable for each grided value of uncertain parameters, that specific selection of controller parameters are assumed to be stabilizing parameters. Of course accuracy of the results depend on the step sizes as indicated earlier in this

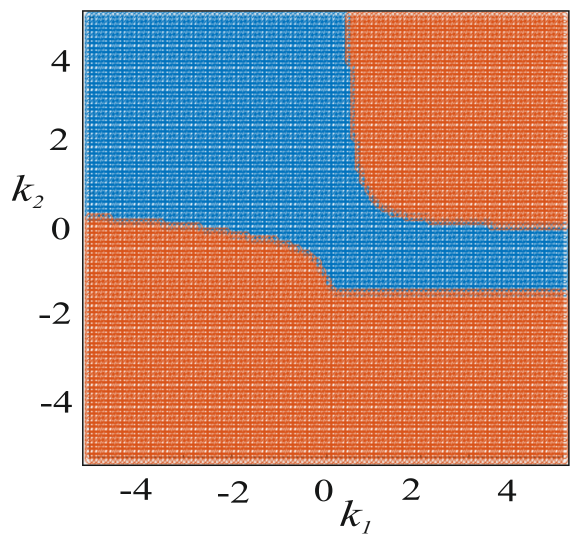


Figure 5.8 : Derived $k_1 - k_2$ region that make the closed loop uncertain system stable by griding uncertain parameters.

approach. In Figure 5.8, blue region represents the stabilizing controller parameter regions while the orange region represents the unstable parameter area.

By investigating Figures 5.7 and 5.8, it is obvious that both stable regions are so close to each other. When the number of uncertain parameters increases, after a certain point it becomes practically impossible to derive accurate results using the griding approach. On the other hand, computational complexity is significantly reduced in the proposed combined approach since Kharitonov theorem is also used. However, it must also be noted that a pre-step is required to determine the sub controller parameter intervals in that approach.

6. CONCLUSION

In this thesis, various approached are proposed to determine controller parameter spaces that achieve diagonal dominance and stability in multivariable systems. From the diagonal dominance point of view, necessary and sufficient conditions on diagonal type controllers are determined for TITO systems at a given fixed frequency. Derived results are also extended to the case of weighted diagonal dominance using weighting factors for each column and row. Furthermore, an algorithm is proposed to determine controller parameter regions that achieve weighted column and row diagonal dominance at a given frequency. It is also shown that derived result can be extended to a given frequency range. Gershgorin Discs plots and diagonal dominance ratio plots were used in order to demonstrate the derived results. Additionally, critical frequencies that may effect the interval characteristics of controller gains are derived for the case of static diagonal controllers and column diagonal dominance. It is important to determine all controller parameter regions that achieve diagonal dominance, since most of the previously proposed approaches in terms of diagonal dominance aimed to determine single controller parameter pair and did not check the fragility of diagonal dominance. The derived results in this direction also create flexibility in controller design and designer can also have more information on the controller parameter set(s). Such an information can also be meaningful in order to interpret on how the system is close to diagonal dominance boundaries.

Derived results in terms of diagonal dominance are important in order to reduce the interactions. However, it is not possible to propose that a diagonal dominant systems satisfies stability except some special cases since diagonal dominance is defined over magnitudes. For this reason, stability of multivariable systems is also discussed within the scope of this thesis. In order to overcome the difficulties that most of the frequency based approaches face, using the Kronecker product and vectorization operator a Lyapunov equation based stability mapping technique is proposed for multivariable systems. Proposed approach is independent from the number and type of the parameters. Required number of equations reduced significantly when Kronecker products and vectorization operator are used instead of checking the positive definiteness of the P matrix. Lastly, it is shown that checking at most two equations($|M(k)| = 0$ and $|M(k)| \to \infty$) is sufficient in terms of stability boundary determination. As a result, it is applicable to a broad range of multivariable systems and controller types including the decentralized controllers. In the proposed approach using elimination and duplication matrices, transformations are also introduced to reduce the computational load of the stability boundary calculations. In addition to determining the stability boundaries using a Lyapunov equation based approach, its link with currently existing approaches like PSA are also shown over finite and infinite root boundaries.

It is possible to apply the proposed stability mapping approach to broad range of systems where Lyapunov equation formulation is possible. In order to demonstrate the further application areas, controller integrity problem of multivariable systems is discussed. Exact stabilizing parameter spaces were derived for the controller integrity problem. Effectiveness of the proposed method is shown with a comparative benchmark case study. In order to demonstrate the further application areas of the proposed Lyapunov equation based approach, the case of discrete time systems is also discussed from the stabilizing parameter space point of view. In this case, resulting Lyapunov equation is slightly different. And lastly, stability of parameter uncertain MPC problem is discussed in the further application areas section. In the proposed stability mapping approach it is possible to determine analytical expressions of stability boundaries. This stability boundaries are inserted to the robust MPC design problem in order to achieve the stability of parameter uncertain MPC problems. By this way, it was shown that robust MPC design problem can be translated to nominal MPC case by inserting the robust stability boundaries in the MPC formulation.

Two approaches that are based on triangular inequality and griding are presented related with the diagonal dominance of parameter uncertain TITO systems. By this way, diagonal dominance problem of parameter uncertain system was first translated to the weighted diagonal dominance problem of the nominal plant. After that, using the previously derived results on weighted diagonal dominance it becomes possible to conservatively determine static diagonal controller parameter regions that satisfy robust diagonal dominance conditions at a given frequency or frequency range.

Stability of parameter uncertain multivariable systems is also discussed in the context of thesis. Firstly, it is shown over the benchmark case studies that proposed Lyapunov equation based stability mapping approach is also applicable in case of parametric uncertainties. However, the required computational effort increases significantly when the number of uncertain parameters increases. In order to overcome such difficulties, the case that individual transfer functions of TFM include interval type uncertainties is discussed in detail. A new approach is proposed using the overbounding technique and Kharitonov Theorem in accordance with the Lyapunov Equation based stability mapping approach to determine robustly stabilizing controller parameters. The required computational effort is decreased in the proposed method since the Kharitonov theorem is used. However, extra analysis steps are included in the intermediate steps and robustly stabilizing parameter sets were derived for a given range of controller parameters.

In addition to the derived results in terms of diagonal dominance, it is also aimed to extend the range of results to the cases of triangular and full matrix controllers as future studies. Additionally, it is also targeted to determine type of n by n systems that the existing results are directly applicable. In the general case, determining necessary and sufficient conditions on frequency variable in terms of diagonal dominance is also aimed as a future work.

From the stability point of view, proposed approach is applicable to a broad range of systems as indicated in the thesis. Some of these further application areas are also presented in the thesis. However, it is also aimed to derive significant results in other type of systems like descriptor and switching systems. Especially deriving stability results for switching systems are aimed since the results can also be used in robust MPC problem. After determining the stability boundaries using the Lyapunov equation based approach, more or less a manual method is required in the currently existing approach. However, it is also targeted to use methods such as R-functions in order to automate the derivation of the stabilizing parameter spaces as a future work.

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APPENDICES

APPENDIX A.1 : Coefficient Terms for Weighted Diagonal Dominance
APPENDIX A.2 : Proof of Theorem 4.3

APPENDIX A.1: Coefficient Terms for Weighted Diagonal Dominance

Coefficient terms for the weighted column diagonal dominance can be written as:

$$\begin{aligned}
a_{w_1} &= \left((\mathrm{Re}(g_d))^2 + (\mathrm{Im}(g_d))^2\right) \\
b_{w_1} &= (2\,\mathrm{Re}(g_{11})\,\mathrm{Re}(g_d) + 2\,\mathrm{Im}(g_{11})\,\mathrm{Im}(g_d)) \\
c_{w_1} &= (\mathrm{Re}(g_{11}))^2 \\
a_{w_2} &= -\left((\mathrm{Re}(g_d))^2 + (\mathrm{Im}(g_d))^2\right) \\
b_{w_2} &= -\,(2\,\mathrm{Im}(g_{11})\,\mathrm{Re}(g_d) - 2\,\mathrm{Re}(g_{11})\,\mathrm{Im}(g_d)) \\
c_{w_2} &= \left(\mu_{c_1}^2\,(\mathrm{Re}(g_{21}))^2 + (\mathrm{Im}(g_{21}))^2\right) - (\mathrm{Im}(g_{11}))^2
\end{aligned} \tag{A.1}$$

$$\begin{aligned}
a_{w_3} &= \left((\mathrm{Re}(g_d))^2 + (\mathrm{Im}(g_d))^2\right) \\
b_{w_3} &= (2\,\mathrm{Re}(g_{22})\,\mathrm{Re}(g_d) + 2\,\mathrm{Im}(g_{22})\,\mathrm{Im}(g_d)) \\
c_{w_3} &= (\mathrm{Re}(g_{22}))^2 \\
a_{w_4} &= -\left((\mathrm{Re}(g_d))^2 + (\mathrm{Im}(g_d))^2\right) \\
b_{w_4} &= -\,(2\,\mathrm{Im}(g_{22})\,\mathrm{Re}(g_d) - 2\,\mathrm{Re}(g_{22})\,\mathrm{Im}(g_d)) \\
c_{w_4} &= \left(\mu_{c_2}^2\,(\mathrm{Re}(g_{12}))^2 + (\mathrm{Im}(g_{12}))^2\right) - (\mathrm{Im}(g_{22}))^2
\end{aligned} \tag{A.2}$$

Coefficient terms for the weighted row diagonal dominance can be given as:

$$\begin{aligned}
a_{w_5} &= \left(\left(\mathrm{Re}(g_{d_{12}}^*)\right)^2 + \left(\mathrm{Im}(g_{d_{12}}^*)\right)^2\right) \\
b_{w_5} &= \left(2\,\mathrm{Re}(g_{11}^*)\,\mathrm{Re}(g_{d_{12}}^*) + 2\,\mathrm{Im}(g_{11}^*)\,\mathrm{Im}(g_{d_{12}}^*)\right) \\
c_{w_5} &= (\mathrm{Re}(g_{11}^*))^2 \\
a_{w_6} &= \left(\left(\mathrm{Re}(g_{d_{12}}^*)\right)^2 + \left(\mathrm{Im}(g_{d_{12}}^*)\right)^2\right) \\
b_{w_6} &= \left(2\,\mathrm{Im}(g_{11}^*)\,\mathrm{Re}(g_{d_{12}}^*) - 2\,\mathrm{Re}(g_{11}^*)\,\mathrm{Im}(g_{d_{12}}^*)\right) \\
c_{w_6} &= (\mathrm{Im}(g_{11}^*))^2
\end{aligned} \tag{A.3}$$

$$\begin{aligned}
a_{w_7} &= \left(\left(\mathrm{Re}(g_{d_{21}}^*)\right)^2 + \left(\mathrm{Im}(g_{d_{21}}^*)\right)^2\right) \\
b_{w_7} &= \left(2\,\mathrm{Re}(g_{22}^*)\,\mathrm{Re}(g_{d_{21}}^*) + 2\,\mathrm{Im}(g_{22}^*)\,\mathrm{Im}(g_{d_{21}}^*)\right) \\
c_{w_7} &= (\mathrm{Re}(g_{22}^*))^2 \\
a_{w_8} &= \left(\left(\mathrm{Re}(g_{d_{21}}^*)\right)^2 + \left(\mathrm{Im}(g_{d_{21}}^*)\right)^2\right) \\
b_{w_8} &= \left(2\,\mathrm{Im}(g_{22}^*)\,\mathrm{Re}(g_{d_{21}}^*) - 2\,\mathrm{Re}(g_{22}^*)\,\mathrm{Im}(g_{d_{21}}^*)\right) \\
c_{w_8} &= (\mathrm{Im}(g_{22}^*))^2
\end{aligned} \tag{A.4}$$

APPENDIX A.2: Proof of Theorem 4.3

Theorem 4.3: For the system that is given as:

$$\dot{x} = Ax, \quad x \in \mathbb{R}^n \tag{A.5}$$

and for any $Q > 0$, there exist a positive definite solution P of the Lyapunov equation that is given as

$$A^T P + PA = -Q \tag{A.6}$$

if and only if all the eigenvalues of the system matrix A lie in the open LHP. Additionally the solution P is unique.

Proof: If there is postive definite solution P of (A.6) then it is clear that $V(x) = x^T PX$ is a Lyapunov function since $V(x) > 0$ for all $x \neq 0$ and $V(x) = 0$ for $x = 0$. And since Lyapunov equation has a positive definite solution it can be concluded that $\dot{V}(x) < 0$ for all $x \neq 0$ and the given system is globally asympototically stable. Since the system is globally asympototically stable then all the eigenvalues of A should lie on the open LHP.

In order to prove the other way, let us assume that all eigenvalues of A lie on the open LHP. In this case for any given $Q > 0$ the P can be defined as:

$$P = \int_0^\infty e^{tA^T} Q e^{tA} dt \tag{A.7}$$

So that (A.6) can be written as:

$$\begin{aligned} A^T P + PA &= A^T \int_0^\infty e^{tA^T} Q e^{tA} dt + \int_0^\infty e^{tA^T} Q e^{tA} dt\, A \\ &= \int_0^\infty A^T e^{tA^T} Q e^{tA} dt + \int_0^\infty e^{tA^T} Q e^{tA} A dt \\ &= \int_0^\infty \frac{\mathrm{d}}{\mathrm{d}t} \left[e^{tA^T} Q e^{tA} \right] dt \end{aligned} \tag{A.8}$$

Using the derived results in (A.8), Lyapunov equation can be expressed as:

$$A^T P + PA = e^{tA^T} Q e^{tA} |_0^\infty = 0 - Q = -Q \tag{A.9}$$

so this proves that defined P satisfies the Lyapunov equation. As the second step positive definiteness of P must be proven. For any positive definite matrix P, $x^T Px$ should be greater than zero for every $n \neq 0$. Additionaly, $x^T Px = 0$ if $x = 0$. In order to prove the positive definiteness of P, $x^T Px$ can be expressed as:

$$\begin{aligned} x^T Px &= \int_0^\infty x^T e^{tA^T} Q e^{tA} x dt \\ &= \int_0^\infty (x^T e^{tA^T} Q^{1/2})(Q^{1/2} e^{tA} x) dt \\ &= \int_0^\infty \left\| Q^{1/2} e^{tA} x \right\|^2 dt \geq 0 \end{aligned} \tag{A.10}$$

where $Q^{1/2}$ denotes the square root of Q. The case of zero equality can be shown as:

$$x^T Px = 0 \quad \Rightarrow \quad Q^{1/2} e^{tA} x = 0 \quad \Rightarrow \quad x = 0 \tag{A.11}$$

The equations (A.10) and (A.11) proves the positive definiteness of P.

Lastly, uniquenes of P should be proven. In order to prove this contradiction method can be followed. In order to prove that let us assume that P_2 is another solution of the Lyapunov equation in addition to the P. For a given P_2 it can be written as:

$$\int_0^\infty \frac{\mathrm{d}}{\mathrm{d}t}\left[e^{tA^T}P_2e^{tA}\right]dt = e^{tA^T}P_2e^{tA}|_0^\infty = 0 - P_2 = -P_2 \tag{A.12}$$

Using (A.12) it can be written as:

$$\begin{aligned} P_2 &= -\int_0^\infty \frac{\mathrm{d}}{\mathrm{d}t}\left[e^{tA^T}P_2e^{tA}\right]dt \\ &= -\int_0^\infty e^{tA^T}\left(A^TP_2 + P_2A\right)e^{tA}dt \end{aligned} \tag{A.13}$$

Since it is assumed that P_2 is the solution of the Lyapunov equation ($A^TP_2 + P_2A = -Q$) it can be concluded that:

$$P_2 = \int_0^\infty e^{tA^T}Qe^{tA}dt = P \tag{A.14}$$

This completes the proof.

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