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ISTANBUL TECHNICAL UNIVERSITY ★ ISTITUTE OF SCIENCE AND TECHNOLOGY

AIRCRAFT MOTION CONTROL:

INVERSE SIMULATION

M.S. THESIS

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PREFACE

For several years nations has been trying to gain both technological and social advantage on the other nations. Meanwhile, the most investment has been made to defence sectors, causing this sector tı be the leading and guiding other areas. Thecnology trasferred from this field to our daily lifes has gain access and become a necessity. One of these fields is control of a moving body. Wether it is a land, sea or air vehicle, computers always involved in, from design to analysis stage and numerical methods developed as to make the machines come to alive. Simulations are of this kind of area that a researcher can predict the results by making abstractions of real life problems. In automatic control concept, guiding un manned aircrafts, designing auto pilots, and guided missiles inverse simulation has practical usage.

If you are asking the question "what will happen if I change or set up the system in a particular way? ", the answer comes from the simulation runs.. Inverse simulation as the name implies is the opposite: " Given set of results, inverse simulation tries to answer " what would be the cause of this results ? ".

In this thesis development of inverse simulation has been discussed for aircrafts. Both arithmetical and numerical methods are presented to solve the problem and close to real life maneuvering aircraft example has been given.

I would like to thank Mr. Ramazan Tasaltın, Ph.D, for his guidance and support.

Istanbul, April 1995 Alpaslan Menevse

CONTENTS

NOMENCLATURE

g	=	gravitational acceleration
I_X, I_Y, I_Z, I_{ZX}	=	moments of inertia and product of inertia
L, M, N	=	components of total external moment=
m	=	airplane mass
P, Q, R	=	angular velocity components
U, V, W	=	velocity components
V_T	=	tangent velocity
X, Y, Z	=	components of total external force except gravity
x, y, z	=	position components
X_U, Y_V, L_β, etc.	=	dimensional stability derivatives
α, β, φ	=	angle of attack, sideslip angle, bank angle
δa, δe, δr, δT	=	control surfaces deflections from trim condition (aileron, elevator, rudder and power)
ψ, θ	=	flight attitude angles (heading and pitch angles)
ψ_W, θ_W	=	flight-path angles
ξ	=	disturbance

Abbrevations

AFCS	:	Automatic Flight Control System
FBW	:	Fly by wire
RVDT	:	Rotational Variable Differential Transformers
LVDT	:	Linear Variable Differential Transformers
DFC	:	Direct Force Control
DL	:	Direct Lift
DSF	:	Direct Side Force
BVR	:	Beyond Visual Range

SUMMARY

Combat aircraft simulators has been widely used in training pilots. The possibility of implementing different scenarios without additional cost, makes the simulators attractive to military use. The scenarios have been set up in order to maintain situational awareness of the crew in highly complex environment. A modern pilot should have the ability to react instantly to changes in enemy behaviour and the quality of a simulator has been measured by its resembling real life problems. At this point, modeling enemy forces plays major role in achieving this goal.

EUCLID CPA 11.3 multinational aircraft simulator development project is devoted to implement artificial agents in simulators as an enemy forces that can decide which maneuver to accomplish in different flight conditions. Therefore, understanding of flight conditions, air combat tactics, decision making were considered as functional requirements in this project. Maneuver implementation was one of the sub-function of pilot decision making module.

In this thesis general inverse simulation method is introduced and designed to answer the question of " What kind of control should be applied in order to achieve desired maneuver or trajectory. Examples of such maneuvers are presented. Real time numerical solution package has been considered to be embedded in an artificial pilot decision making steps, to form a hybrid expert system. General flight principles, such as stability, maneuver types and flight control basics are also reviwed to form an introductory knowledge base.

UÇAK HAREKET KONTROLÜNDE TERS SİMÜLASYON

ÖZET

Pilot eğitiminde önemli bir yer tutan savaş uçakları simülatörlerinin geliştirilmesi son yıllarda büyük önem kazanmıştır. Farklı senaryoların kolayca uygulanabilmesi, maliyet açısından gerçek uçuşlara kıyasla çok daha ucuz olması ve yeni programlama tekniklerinin uygulamaya konması bu önemi daha da arttırmıştır.

Yapay zeka alanındaki gelişmeler neticesinde numerik yöntemler ile akıllı sistemler bütünleştirilmiş ve hibrid (hybrid) sistemler elde edilmiştir. Bu tezde yapay bir pilotun yörüngeye karar verdikten sonra, belirlenen manevrayı nasıl yapabileceği ters problem yöntemiyle incelenmiştir. Ters problemlerde kullanılan türev alma yöntemi ile integral yöntemi karşılaştırılmış avantaj ve dezavantajları belirtilmiştir.

Bir pilotun almış olduğu eğitim neticesinde uçuş sırasında doğal olarak akıldan yapmış olduğu ters problem çözümünün anlaşılabilmesi öncelikle uçuş şartları, kararlılık, manevra tipleri gibi temel kavramları bilinmesini gerektirmektedir. Bu nedenle tezin başlangıç bölümünde bu tür temel kavramlara yer verilmiştir. Tez içerisinde incelenen yöntemler sayısal çözümler olup, yapay zeka yöntemleri yada savaş senaryoları dahil edilmemiştir.

Daha detaya inmeden önce bazı tanımların yapılmasında fayda vardır. Bir kütlenin durgun yada sabit lineer ve açısal momentlere sahip olması haline, o kütlenin denge hali (equilibrium) denir. Uçağın denge hali ikinci tip olan momentlerinin değişmemek kaydıyla hareket halinde bulunmasıdır. Aerodiamik kuvvetlerin bileşkelerinin uçağın ağırlığını dengeleme koşulu bulunduğundan, denge halinde herhangi bir dönme etkisi bulunmaz. Bir uçağın hızının farklılaşmasına ve/veya yönünün değişmesine engel olan karakteristiği o uçağın kararlılığını oluşturur.

Uçak dinamiğinde iki tür kararsızlık mevcuttur. Bunlardan birincisi statik kararsızlık diğeri ise dinamik yani uçağın denge konumu etrafında salınması durumudur. Uçağın yörüngesini değiştirmek için uygulanan etkiye ise kontrol adı verilir. Bir sistemin kontrolünün iki tür fonksiyonu vardır. Birincisi uçağın denge durumunu sabitleştirmek yada değiştirmektir. Bir denge durumundan diğerine geçişteki dinamikler daha çok kararlılık araştırmalarının konusu olup burada incelenmeyecektir. Kontrolün ikinci fonksiyonu ise dengesizlik (unequilibrium) denilen ivmeli hareketler üretmek olup, uçak manevraları bu konuya girmektedir. Uçağın bir manevra halinden diğer bir manevra haline geçişi uçak kontrolünün ana araştırma

konusu olmaktadır.

Otomatik uçuş kontrol sistemleri bilgisayar yardımı ile sürekli düzeltmelerde bulunarak uçağın istenilen doğrultuda hareket etmesini sağlarlar. Her uçakta üç boyutlu mekanda gerekli hareketi sağlayacak çeşitli şekillerde kontrol yüzeyleri mevcuttur. Temel olarak üç tip olan bu yüzeyler kanatçık (aileron), irtifa dümeni (elevator), istikamet dümeni (rudder) olarak adlandırılırlar. Motorlarda üretilen itki gücü (thrust) ise dördüncü kontrol olarak düşünülür. Bununla birlikte savaş uçaklarında manevra kabiliyetini artırıcı ek unsurlarda bulunabilmektedir. Bunlar yatay ve dikey kanardlar, spoiler değişken kenarlı kanatlar ve son zamanlarda uygulamaya konulan motor sonrası kontrol yüzeyleridir.

Bir kontrol yüzeyinin hareket ettirilmesi genellikle birden fazla eksene etki ederek istenilenden farklı hareketler doğurur. Birden fazla kontrol yüzeyinin kullanılması ile hareket değişkenleri arasında birliktelik (coupling) ve etkileşim (interaction) oluşturur. Bu fiziksel sınırlamalar özel teknikler kullanmadan otomatik uçuş kontrol sistemlerini analiz etmeyi imkansız kılar. Bu metodlardan biriside ters (geri) problem olup uçuş dinamik ve konrolünü anlamamızda bize yardımcı olur.

TERS (INVERSE) PROBLEMLER

Bir uçağın dinamik denklemleri iki tür değişkeni (giriş ve çıkış) ve bir grup sabiti birbirine bağlayan sistemlerdir. Genel (forward) problemler giriş ve sabitler bilindiğinde çıkışı hesaplamayı amaçlar. Ancak, bununla birlikte problemin iki farklı olasılığı daha mevcuttur. Bunlardan birisi çıkış ve sabitlerin bilinmesi ile girişin bulunması, diğeri ise giriş ve çıkışın bilinmesi ile sabitlerin bulunmasıdır. Bu iki türde ters (inverse) problemler dediğimiz alana girmekte olup her ikisininde uygulamada önemi vardır.

Genel olarak bir sistem denklemi aşağıdaki şekilde verilir:

$$\bar{x}_0 = G(s)\bar{x}_i$$

Bu üç fonksiyondan herhangi biri $x_0(t)$, $x_i(t)$ veya G(s) bilinmiyor olabilir ve herhangi ikisi verildiğinde diğer üçüncüsü bulunabilir.

Giriş fonksiyonu $x_i(t)$ 'nin bulunması "Uçağın yapmış olduğu hareket verildiğinde, bu harekete sebep olan pilot davranışı yada güdüm sinyali nedir?" sorusunu kendimize sormamızın neticesidir. Bu tür sorular kontrol dizaynı, manevra yapan cisimlerin analizi ve önceden programlanmış uçuşlarda kullanım alanı bulmaktadır. Bilhassa son zamanlarda roket güdümlerinin geliştirme çalışmaları ters problemlerin önemini daha da arttırmıştır. Uçaklarda mevcut olan kontrol yüzeyleri sayesinde bir araç altı dereceden serbestiye sahiptir. Genel olarak uçaklarda irtifa dümeni, istikamet dümeni ve kanatçık sırayla hücum açısı α, yana kayma açısı β ve yatma açısı φ' yi kontrol eder. Bu nedenle (α, β, φ) açıları temel alınarak ters problem yöntemi geliştirilir.

Bir uçağın uçuş güzergahını belirlemek, uçağın zamana bağlı x(t), y(t), z(t) pozisyonlarını öngörmekle mümkündür. Bu değişiklikler verildiği taktirde zamana göre türevleri alınıp

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = V_T \begin{bmatrix} \cos\theta_w \cos\psi_w \\ \cos\theta_w \sin\psi_w \\ -\sin\theta_w \end{bmatrix}$$

denklem takımı çözülerek teğetsel hız $V_T(t)$ ve patika açıları $\psi_W(t)$, $\varphi_W(t)$ elde edilir. Daha sonra

$$\sin\theta_w = \cos\beta\cos\alpha\sin\theta - (\sin\beta\sin\varphi + \cos\beta\sin\alpha\cos\varphi)\cos\theta$$

$$\sin(\psi_w - \psi) = \frac{\sin\beta\cos\varphi - \cos\beta\sin\alpha\sin\varphi}{\cos\theta_w}$$

çözülerek sistem zaman t, $\alpha(t)$, $\beta(t)$, $\varphi(t)$' ye bağlı olarak

$$\psi = \psi(t, \alpha(t), \beta(t), \varphi(t))$$

$$\theta = \theta(t, \alpha(t), \beta(t), \varphi(t))$$

eşitlikleri bulunur. Açısal hızlar, P, Q, R ise aşağıdaki şekilde yazılabilir.

$$P = P(t, \alpha(t), \beta(t), \varphi(t), \dot{\alpha}(t), \dot{\beta}(t), \dot{\varphi}(t))$$

$$Q = Q(t, \alpha(t), \beta(t), \varphi(t), \dot{\alpha}(t), \dot{\beta}(t), \dot{\varphi}(t))$$

$$R = R(t, \alpha(t), \beta(t), \varphi(t), \dot{\alpha}(t), \dot{\beta}(t), \dot{\varphi}(t))$$

Bu sadeleşmeden sonra hız değişkenleri

$$U = U(t, \alpha(t), \beta(t))$$

$$V = V(t, \beta(t))$$

$$W = W(t, \alpha(t), \beta(t))$$

durumuna gelerek sadece $\alpha(t)$, $\beta(t)$ cinsinde yazılır. Sayısal çözüm yöntemleri kullanılarak önce basitleştirilmiş $\alpha(t)$, $\beta(t)$ daha sonra gerçek değerler saptanır.

$\alpha(t)$, $\beta(t)$ bulunduktan sonra kuvvet denklemlerinde yerlerine konarak $\psi(t)$, $\varphi(t)$ elde edilir. Bu değişkenler moment denklemlerinde yerlerine yerleştirilerek δa, δe, δr, bulunur. Difransiyel denklemler Newton orta farklılıklar (central difference methods) metodu kullanılarak hesaplanır.

3.,4. ve 5. bölümlerdeki şekillerden de görüleceği üzere temel çözümler ve esas çözümler arasında farklılıklar vardır. Bunun nedeni kuvvet denklemlerindeki etkisi ihmal edilen $Y_{\delta r}\delta r$ kuvvet bileşenidir. İstikamet dümeni (rudder) ve yana kayma arasında bulunan doğal mekanizmadan görüleceği üzere, $Y_{\delta r}\delta r$ kuvvet bileşeni genellikle β-yanal kuvveti $Y_V v$'ye ters istikamettedir. Bu nedenle toplam yan kuvvetlerdeki azalma, β-yana kayma açısınının esas çözümde temel çözüme kıyasla daha büyük çıkmasına neden olmaktadır. β daki bu artış irtifa dümeni (elevator) da dahil olmak üzere her üç kontrol yüzeyini de etkilemektedir. Aynı neticeyi $Z_{\delta e}\delta e$ bileşeninde de görmekteyiz. Fakat $Z_{\delta e}\delta e/Z_W$ oranı $Y_{\delta r}\delta r/Y_V v$'ye kıyasla daha küçük olduğundan $Z_{\delta e}\delta e$'nin etkisi çok daha küçük olmaktadır. Kullanılan örneklerde manevra neticesinde ortaya çıkan istikamet dümeni kullanımı oldukça fazla gözükmektedir. Ancak bilhassa kordineli manevralarda istikamet dümenini (rudder) ikincil olarak kullanmaktadırlar. Bu nedenle diğer manevralarda temel-esas çözüm farkı bu örneklerde görüldüğünden daha az olmaktadır.

Bilhassa 360° lik manevra bütün kontrol yüzeylerini kullanması açısından iyi bir örnek teşkil etmektedir. Bilhassa istikamet dümeninin β-yanal kuvvetini oluşturmak için kullanımı önem teşkil etmektedir. Açıkça bunun sebebi, kaldırma kuvvetinin tek başına yerçekimi kuvvetine karşı gelemeyişidir. Bu nedenle uçağı düzgün bir hatta döndürebilmek için kaldırma kuvvetine dik olan yan kuvvet gerekmektedir. Sonuç olarak şekil 1.1 den görüleceği üzere uçağın burnunun yukarı, aşağı ve sağa ve sola doğru hareket ettirilmesi gerekmektedir. Manevra bu yönüyle normal kanatçık yuvarlanmasından (aileron roll) ayrılır. Kontrol bakımından insan pilotlor için oldukça zor bir manevra olan 360 derecelik doğru hat yuvarlanmasının incelenmesi ise ayrı bir araştırma konusudur. Tez sonunda karşılaştırma amacıyla Hess [7] tarafından belirlenen aynı manevranın F-16 için yapılmış çözümleri verilmiştir. Karşılaştırmadan anlaşılacağı üzere aynı manevra kontrolünün farklı uçaklarda nekadar değişik olabileceğinin bir kanıtıdır. Tezin son bölümü türevsel ve entegre yöntemleriyle geliştirlen metodların karşılaştırılmasına ayrılmış avantaj ve dezavantajları belirtilmiştir.

CHAPTER 1

INTRODUCTION

1.1. AIRCRAFT CONTROL

The value of a vehicle to its user mainly depends on how effectively it can be made to travel in the predetermined period of time, between its departure and destination points. The path between this points is considered precisely controllable, allowing user to choose the optimal one. This is where most of the solution methods developed for the problems arising from aerodynamics, mechanics, electronics etc. fields.

Choosing the optimal combination of all, is the most important engineering problem. In this way, real world simulations is made possible by manufacturing multi processor super computers handling huge amount of data.

Any type of a vehicle in motion can be characterized by its velocity vector. The path of the vehicle is obtained using integration methods with respect to time. General representation of a motion is given by

$$\dot{x} = f(x,u,\xi,t)$$

Where $\dot{x}$ is velocity affected by the position x, by any disturbance, ξ by time t and by control u. Meaning that the path of any vehicle can be controlled vary widely in the absence of physical constraints.

Aircraft which is the interest of this paper differs from other land vehicles, having six degrees of freedom three associated with angular motion about aircraft's centre of gravity and three associated with the translation of the centre of gravity. Because of this very high degree of freedom of motion, aircraft control problems are usually more complicated than those of other vehicles. Before going into the detail some definitions is required to make the problems more clear.

A body is in equilibrium when it is at rest or in uniform motion (i.e. has constant linear and angular moments.) The equilibrium of an aircraft in flight is of the second type: that is, uniform motion. Because the aerodynamic forces are dependent on the angular orientation of the aircraft relative to its flight path, and because the resultant of them must exactly balance its weight, the equilibrium state is without rotation; (i.e. it is a motion of rectilinear translation.) The qualities of an aircraft which tend to make it resist any change of its velocity vector, either in its direction or its magnitude or in both, are what constitutes its stability. Two kinds of instability are of interest in aircraft dynamics.

The first one called static instability, the body departs continuously from its equilibrium condition. The second, called dynamic instability, is a more complicated phenomenon in which the body oscillates about its equilibrium condition with increasing amplitude.

Affecting the change of the path of an aircraft is called control. The primary functions of an control system are two kinds. The first is to fix or change the equilibrium condition (speed or angle of climb). The dynamics of the transition from one equilibrium state to another which is not dealt in this paper are of interest of stability researches. The second function of the control is to produce nonequilibrium or accelerated motions:that is, maneuvers. These may be steady states or they may be transient states. Investigations of the transition from one maneuvering steady state to another, form the subject matter of aircraft control. Automatic flight control system provides continuous corrections for an aircraft in order to fly a straight and level course with the aid of a computer.

Every aircraft has some kind of control surfaces to generate the necessary forces and moments causing the production of required acceleration for steering along its three-dimensional flight path. A conventional aircraft has three kinds of control surfaces that is rudder, elevator and aileron. The thrust produced by engines is considered forth control. However, combat aircrafts have some more additional control surfaces to increase the maneuverability of a vehicle by means of supplying extra forces and moments. These are horizontal and vertical canards, spoilers, variable cambered wings, reaction jets.

The use of a single control surface always produces other motion as well as the intended one. Using more than one control surface simultaneously causes considerable coupling and interaction between motion variables. These physical constraints make automatic flight control subject very difficult to analyze without employing special techniques. Inverse problem is one of these methods allowing us the understand the flight dynamics and control.

1.2. FLIGHT CONTROL SYSTEMS

Every aircraft contains motion sensors which provide measures of

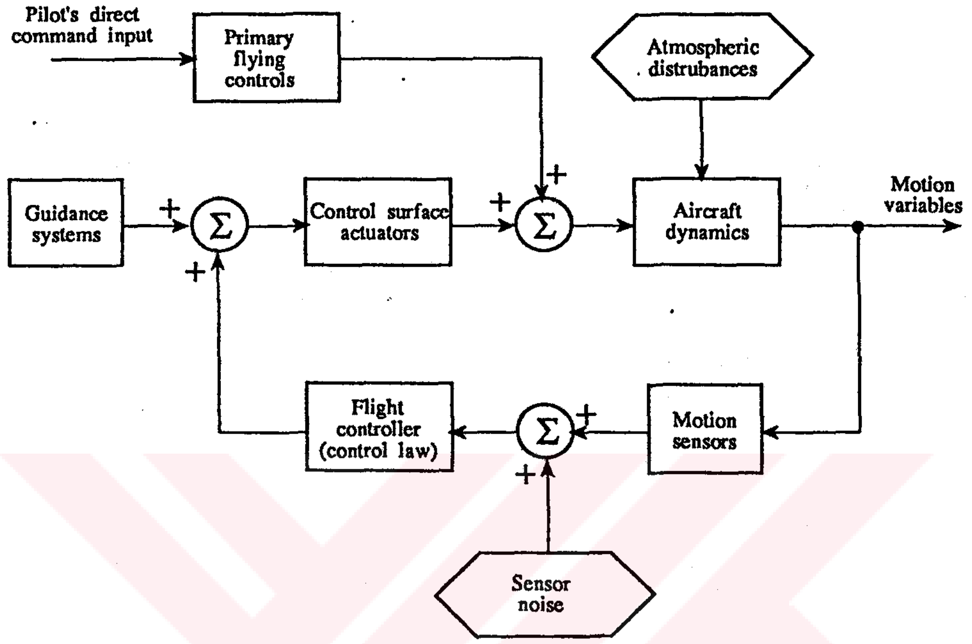


Figure 1.1 Automatic Flight Control System

changes in motion variables. These sensors contribute to feed back signals for the Automatic Flight Control Systems (AFCS). With the aid of a visual display a pilot can beware of the state of an aircraft.

General structure of an AFCS can be represented by a block diagram as shown Figure 1.1.

The duty of a controller is to compare the commanded motion with the measured motion, and if any difference exists, to generate necessary signals for actuators to produce control surface deflection. This force and moment exertion continues until measured and commanded motion are in correspondence.

1.3 INVERSE PROBLEMS

If we are given the system equation

$$\bar{x}_0 = G(s)\bar{x}_i$$

then any one of the three functions $x_0(t)$, $x_i(t)$, and G(s) may be the unknown, and can be found if other two are given.

The problem of finding $x_i(t)$ (i.e. the input) arises when we seek answers to questions of the kind: "Given the airplane motion, what is the pilot action, or guidance signal, required to produce it?" Questions of this kind are relevant to problems of control design and maneuvering loads, and may have application to preprogrammed flight plans. The mathematical problem that results is generally simpler than those of stability and response. The equations to be solved are sometimes algebraic, and sometimes differential and it is frequently possible to retain nonlinear terms without much extra diffuculty. It has been applied to several problems of maneuvering flight, notably that of determining the horizontal-tail loads in pull-up maneuvers.

The second inverse problem, that of finding the system transfer function G(s), is central to the determination of stability derivatives from flight tests. Here the relevant features of the input and output are known (i.e. measured), and it is the transfer function that must be calculated. The analytical techniques used must be adapted to deal with the effects of errors in measurement, and with both transient and steady-state periodic motions. Consideration must also be given to the form of the transfer function used; it must be assumed a priori for any given analysis and adjusted subsequently if necessary. Finally, if the maximum of information is to be deduced, the technique should be capable of producing the individual stability derivatives, not merely the coefficients in G(s). The flight-test methods of finding dynamic stability derivetives appears to have been used widely at several Aeronautical Laboratories.

Problems involving AFCSs are generally related to events which do not persist, that is the dynamics situation being considered rarely lasts more than a few minutes. Consequently,a convenient inertial reference frame is a tropocentric coordinate systems i.e. one whose origin is regarded as being fixed at the centre of the Earth: the Earth axis system.

It is used primarily as a reference system to express gravitational effects,altitude, horizontal distance and the orientation of the aircraft.The axis X_E, is chosen to point north, the axis,Y_E, then pointing east with the orthogonal triad being completed when the axis, Z_E, points down.

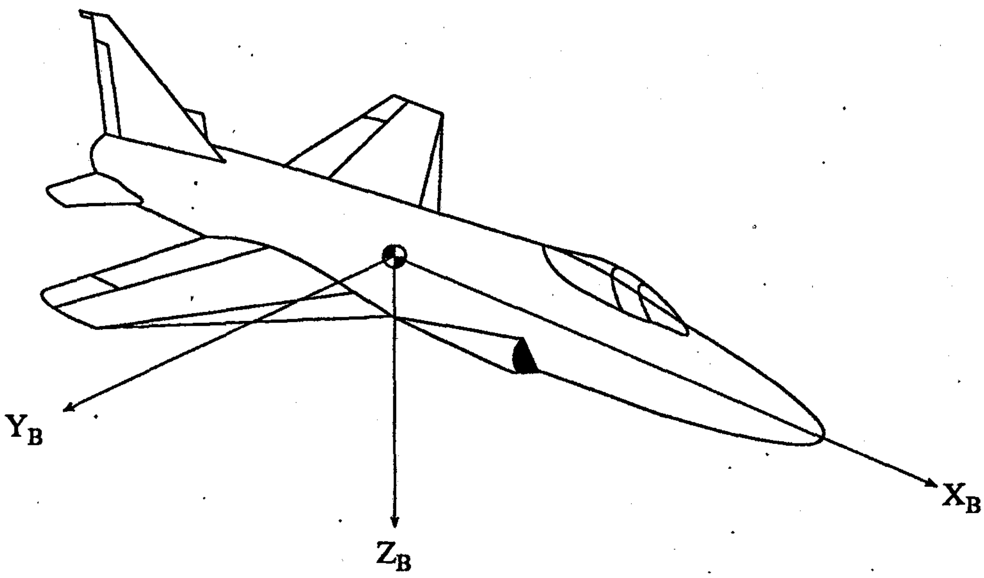


Figure 1.2 Body axis system

1.4. FLIGHT AXIS (COORDINATE) SYSTEM

The choice of axis system governs the form taken by the equations of motion. However, only body-fixed axis systems, i.e.only systems whose origins are located identically at an aircraft's centre of gravity. For such system, the X_B, points forward out of the aircraft; the axis Y_B, points out through the starboard (right) wing, and the axis, Z_B points down.

By using a system of axes fixed in the aircraft the inertia terms, which appear in the aerodynamics forces and moments depend only upon the angles, alfa and beta, which orient the total velocity vector,V_T, in relation to the axis, X_B. (Fig 1.2-1.3)

The angular orientation of the body axis system with respect to the Earth axis system depends strictly upon the orientation sequence. This

sequence of rotations is customarily taken as follows:

1. Rotate the Earth axes, X_E, Y_E and Z_E,through some azimuthal angle, ψ , about the axis, Z_E, to reach some inter mediate axes X_1, Y_1, Z_1.

2. Next, rotate these axes X_1, Y_1, Z_1 through some angle of elevation Θ, about the axis Y_1 to reach second, intermediate set of axes,

X_2, Y_2, and Z_2.

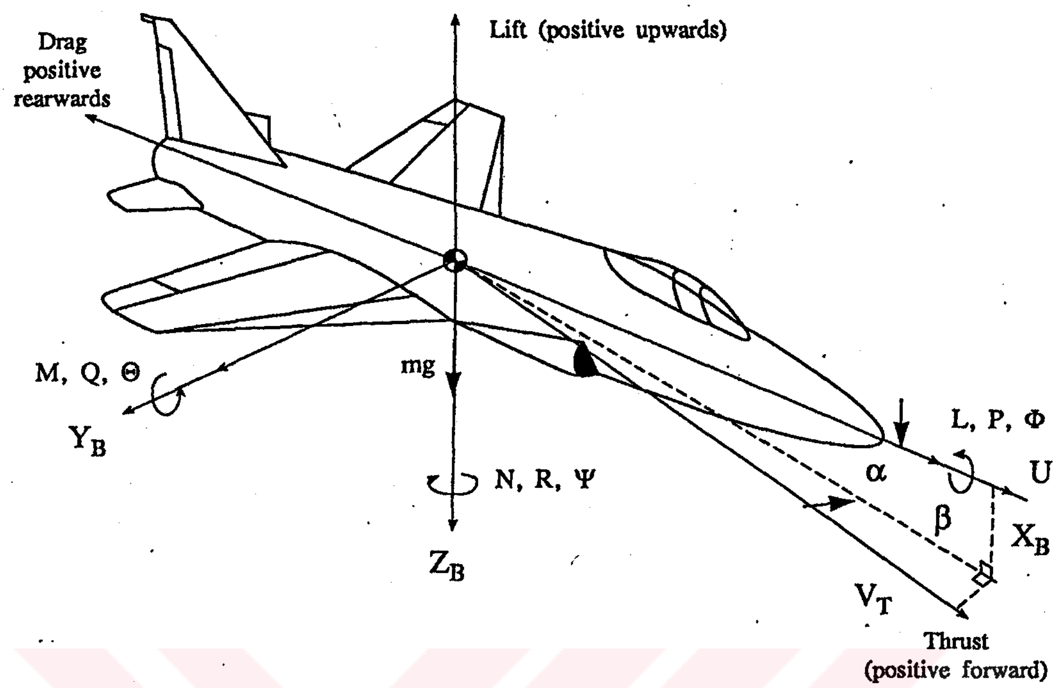


Figure 1.3 The wind axis system

3. Finally, the axes X_2, Y_2, Z_2 are rotated through an angle of bank Φ, about the axis, X_2 to reach the body axes X_B, Y_B and Z_B.

Other axis systems are: the stability axis system, the principal axis system and the wind axis system.

In the stability axis system the axis X_S is chosen to coincide with the velocity vector, V_T,at the start of the motion. There fore, between the X-axis of the stability system and the X-axis of the body axis system, there is a trimmed angle of attack α_0. The equations of motion derived by using this axis system are a special subset of the set derived by using the body axis system.

The principal axis system axes is specially chosen to coincide with the principal axes of the aircraft. The convenience of this system resides in the fact that in the equations of motion, all the product of inertia terms are zero, which greatly simplifies the equations. The wind axis system is oriented with respect to the aircraft's flight path, time-varying terms which corresponds to the moments and cross-products of inertia appear in the equations of motion.

CHAPTER 2

AIRCRAFT STABILITY AND CONTROL

2.1. AIRCRAFT STABILITY

Stability is the property of a system whereby the latter returns to a state of equilibrium after it has been displaced from a state of rest or a state of uniform motion. In applying this definition to an aircraft, it can

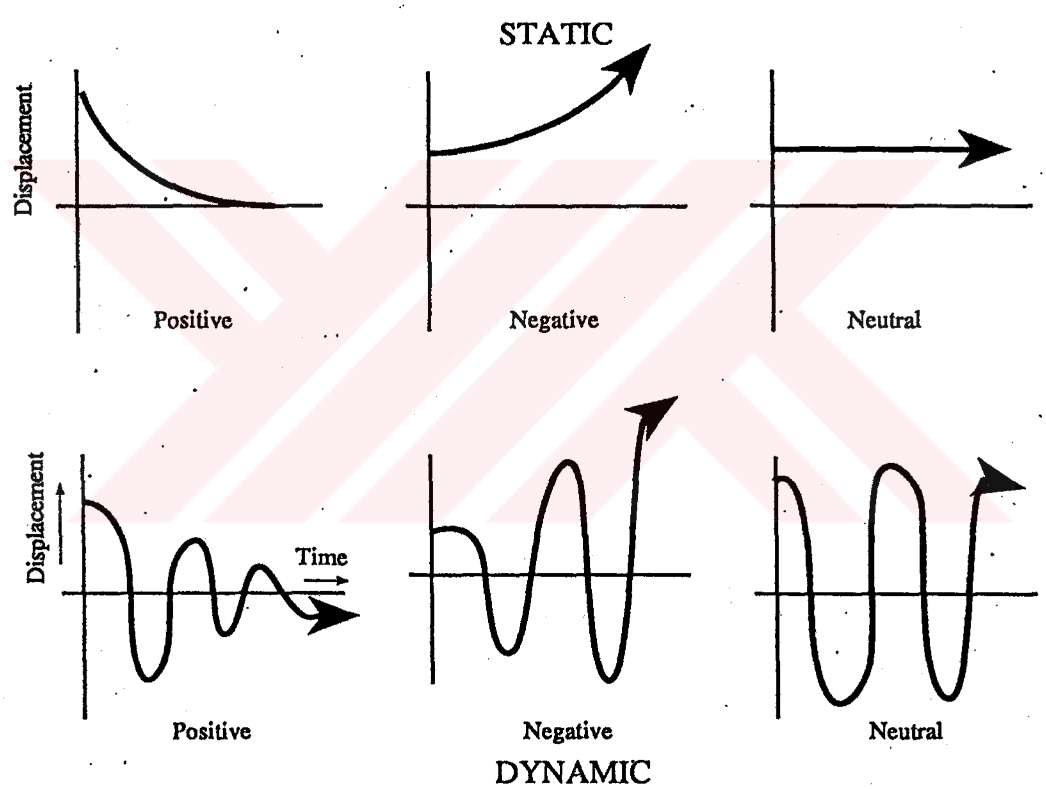


Figure 2.1 Stability types

be stated therefore, that following a displacement from original steady flight path, an aircraft has stability if it returns to that path without movements of its flight control surfaces having to be applied. In practice however, there are two types of stability to consider: static stability and dynamic stability. (fig 2.1). Static stability refers to the immediate reaction of the aircraft and its tendency to return to equilibrium after displacement, while dynamic stability refers to the subsequent long-term reaction which is an oscillatory nature about a neutral or equilibrium position. It is usual to classify both types of stability according to the nature of an aircraft's response to displacements from its original steady flight path: thus, stability is positive when, subsequent to the displacement, the forces and moments acting on the aircraft return it to its original steady flight path; neutral, if the forces and moments cause the aircraft to take up a new flight path of constant relationship to the original; and negative, if the aircraft is caused to diverge from the original steady flight path (an unstable condition). Static stability is a prerequisite for dynamic stability, although the converse is not true; it is possible to have a system which is statically stable, but dynamically unstable.

The displacements of an aircraft which, for example, result from an air disturbance, or by the operation of its flight control system, can take place in any one of three planes; known as the pitching, yawing and rolling planes. This also applies to the aircraft's stabilizing motions in response to the displacements. The planes are not constant relative to the earth but, as indicated in fig 1.3, they are always constant relative to the three body axes passing through the center of gravity of the aircraft. Both forms of stability relate to the three axes in the following manner: longitudinal stability about the lateral axis, directional stability about the normal axis, and lateral stability about the longitudinal axis. In addition to the forces and moments set up by any displacement, forces are also set up as a result of the velocities of motion. These forces are a necessary contribution to the stability of an aircraft, and provide what is termed aerodynamic damping so that motions may be limited or eventually

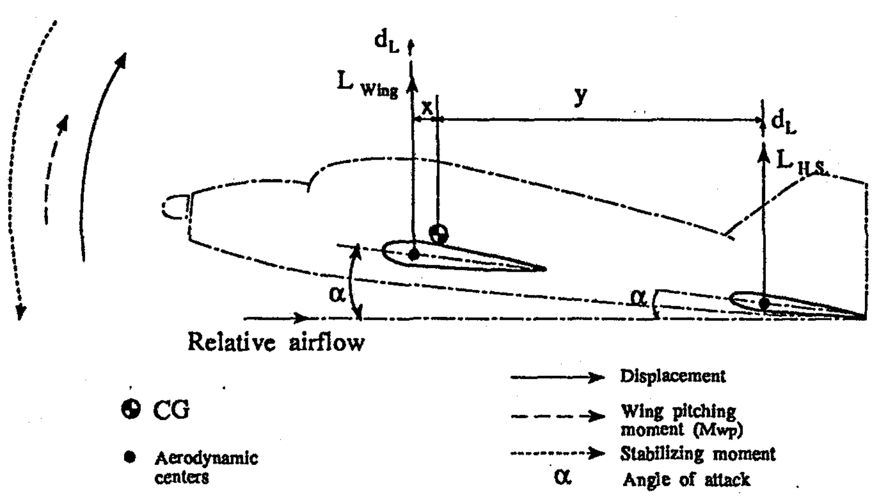


Figure 2.2 Statically unstable design

eliminated. The damping in roll, for instance, is the rolling moment due to angular velocity in roll and, since it acts in the opposite sense to the rolling velocity achieved by deflection of the ailerons, the velocity is limited. Damping also applies to pitch and yaw displacements. When this natural form of damping cannot be obtained, it must be furnished by artificial means, e.g. electrically-controlled yaw dampers.

2.2. LONGITUDINAL STATIC STABILITY

When an aircraft has a tendency to return to a trimmed angle of attack position following a displacement, it is said to have positive static longitudinal stability; it thus refers to motion in the pitching plane, and is influenced largely by the design of the horizontal stabilizer, and on the position of the aircraft's center of gravity under the appropriate flight and load conditions.

The horizontal stabilizer together with the elevators in the neutral or streamlined position form an aerofoil which produces lift at varying angles of attack, the lift in turn producing either an upward or downward restoring moment to balance wing pitching moments about the aircraft's center of gravity. The lift and restoring moments produced are governed by such factors as the area and planform of the stabilizer, the distance of its aerodynamic center from the center of gravity, i.e. the moment arm, and also by the effects of airflow downwash from the wings. When the elevators are maintained in the neutral streamlined position, static stability is referred to as stick-fixed stability, as opposed to stick-free stability which refers to the condition in which the elevators are allowed to float in the airflow, i.e. 'hands-off ' flight condition.

Assuming that in the stick-fixed position the aircraft is displaced nose up, the angle of attack of the wings and, therefore, the lift is produced, will be temporarily increased by an amount dL, resulting in an increase of the wing pitching moment about the aircraft's centre of gravity. Thus, if the aerodynamic centre is forward of the centre of gravity giving a moment arm of length x as shown in fig. 2.2 the wing pitching moment (Mwp) is increased by the amount d1x, the nose-up displacement lowers the horizontal stabilizer, then its angle of attack and corresponding lift force will also be increased, but as the position of the aerodynamic centre with respect to the aircraft's centre of gravity provides the longer moment arm, stabilizer lift force produces a stabilizing nose-down moment. When

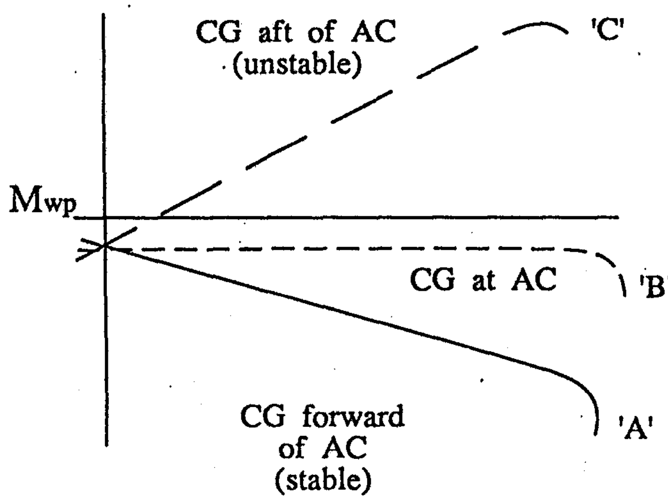


Figure 2.3 C.G. v.s. Stability

the aerodynamic centre of the wings is to the rear of the centre of gravity (fig.2.2) the increase in Mwp will be stabilizing in its effect so that in conjunction with that produced by the horizontal stabilizer a greater restoring moment is provided.

For a given weight in level flight there is one speed and angle of attack at which an aircraft is in equilibrium, i.e. tail moments equal to wing moments. The speed and angle of attack depend upon the difference in rigging incidence between the chord lines of the wing and horizontal stabilizer; a difference known as the longitudinal dihedral angle. The angle of attack at which equilibrium is obtained is called the trim point [1].

It is thus apparent from fig. 2.2 that the ratio of the wing moment, to stabilizer moment, and therefore the degree of longitudinal stability, is affected by the relative positions of both aerodynamic centers, and of centre of gravity. An indication of this is given in fig. 2.3, which is a graphical representation of the conditions appropriate to the wing of an aircraft. Since stability is evidenced by the development of restoring moments, for the wing to contribute to positive static longitudinal stability the aircraft's centre of gravity must be forward of the aerodynamic center. In this case, the wing contribution is a stable one, and curve of Mwp to lift coefficient (C_L) would have a negative slope (curve 'A'). If the centre of gravity were located at the aerodynamic centre, all changes of lift would take place at the centre of gravity and so the wing contribution would be neutral (curve 'B'). An unstable contribution would be made

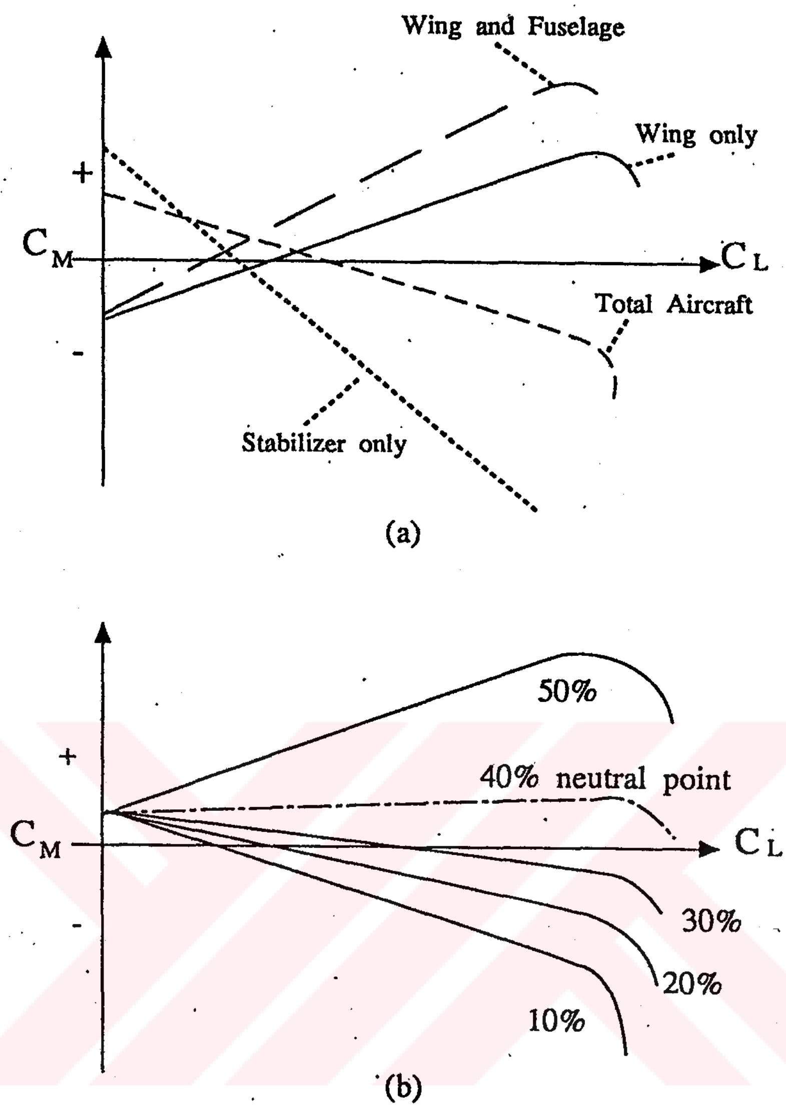


Figure 2.4 C_L v.s. Stability

with the centre of gravity to the rear of the aerodynamic centre, and the Mwp/Cl curve would then have a positive slope (curve'C').

In addition to the wings and horizontal stabilizer, other major components of an aircraft such as the fuselage and engine nozells can also influence the degree of longitudinal stability since, under varying angles of attack, the conditions of airflow and pressure distribution will produce individual pitching moments which can be either stabilizing or destabilizing in their influence. In plotting the total pitching moments against C_L, and the contribution of the major components to stability, curves similar to those shown in fig. 2.4 a are obtained (it is assumed in this example that

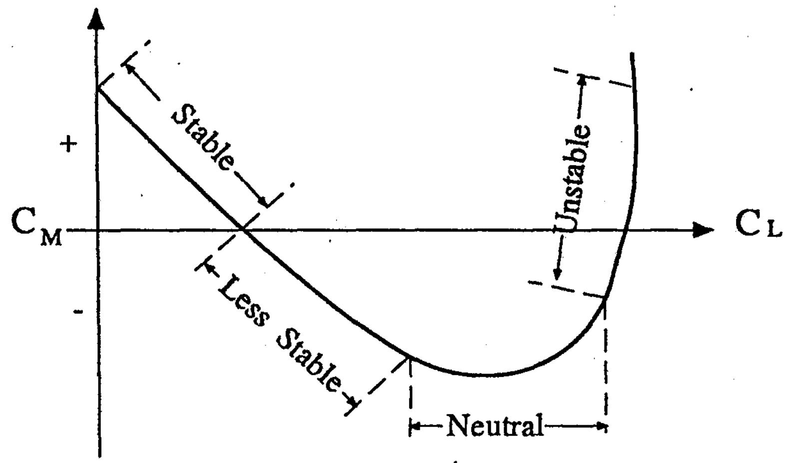
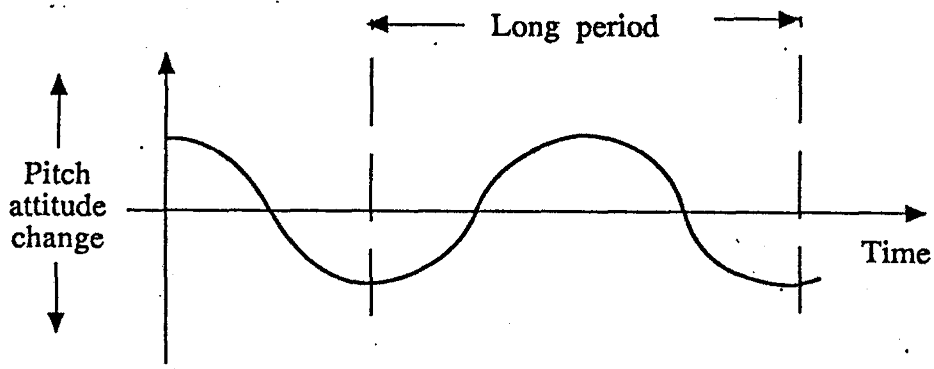


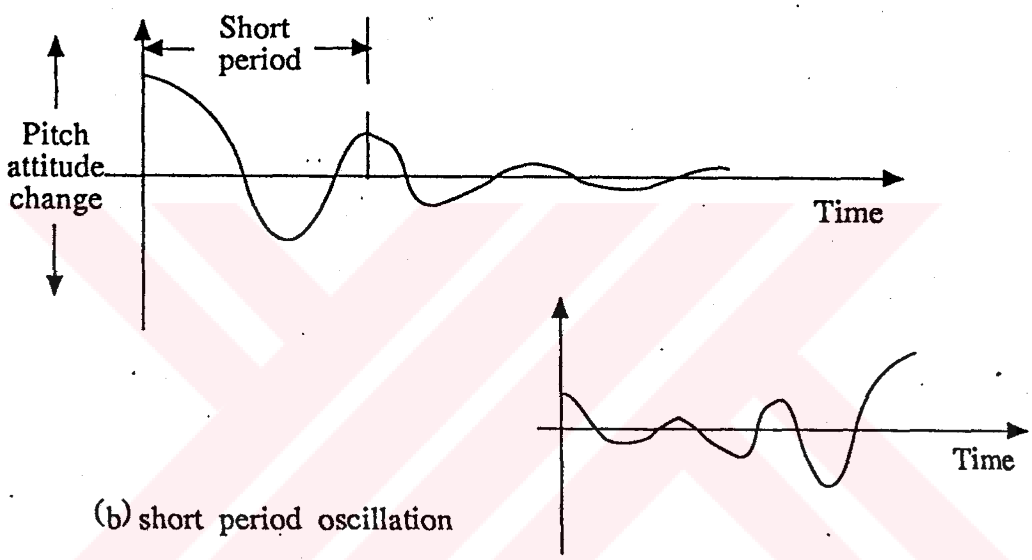
Figure 2.5

the centre of gravity is at 30% of the mean aerodynamic chord). The contribution of the wing alone is destabilizing as indicated by the positive slope of the curve, an effect which is further increased by the fuselage contribution. The large negative slope of the curve of the horizontal stabilizer contribution indicates its highly stabilizing effect, which must be sufficient for the complete aircraft to exhibit positive static stability at the anticipated locations of the centre of gravity. The typical effect of varying locations of the centre of gravity on static stability is indicated in fig. 2.4b. As the centre of gravity moves rearward, the static stability decreases, then becomes neutral, and finally results in an unstable condition. The centre of gravity location which produces zero slope and neutral static stability is referred to as the neutral point. The distance of the centre of gravity at any time from the neutral point is known as the static margin and is an indication of the degree of longitudinal stability. Noticeable changes in static stability can occur at varying values of C_L, particularly when power effects contribute largely to stability, or when significant changes in downwash at the horizontal stabilizer occur. Such changes are illustrated in fig. 2.5. At low values of C_L, the slope of the curve indicates good positive stability, but this gradually starts decreasing with increasing C_L. With continued increase in C_L the slope becomes zero indicating that neutral stability exists. Eventually the slope becomes rapidly positive indicating an unstable 'pitch-up'condition.

If the elevators are allowed to float free, they may have a tendency to float or streamline relative to the airflow, when the angle of attack of



(a) phugoid



(b) short period oscillation

Figure 2.5 Dynamic stability modes

the horizontal stabilizer is changed. Thus, if the angle of attack is increased and the elevators tend to float up, the change in lift produced by the horizontal stabilizers less than if the elevator remain fixed; stick-free stability of an aircraft is, therefore, usually less that the stick-fixed stability.Elevators must therefore be properly balanced to reduce floating and so minimize the difference between stick-free stability. In the case of fully powered control systems actuated by irreversible mechanisms, the elevators are not free to float and so there is no difference between stick-fixed and stick-free stability [2].

2.3. LONGITUDINAL DYNAMIC STABILITY

Longitudinal dynamic stability consists of three basic modes of oscillation, and these are illustrated in fig. 2.6, The first mode (fig. 2.6a) is of very long period and is referred to as a phugoid which involves noticeable variations in pitch attitude, altitude and airspeed, The period of oscillation is quite large, and may be counteracted by very small displacements of the elevator control system. The pitching rate is low, and as also only negligible changes in angle of attack take place, damping of the phugoid is weak and possibly negative.

The second mode (fig. 2.6b) is a relatively short period motion that can be assumed to take place with negligible changes in velocity. During the oscillation the aircraft is restored to equilibrium by the static stability, and the amplitude of oscillation decreased by pitch damping. If the aircraft has stick-fixed static stability, the pitch damping contributed by the horizontal stabilizer will usually assume sufficient dynamic stability for the short period oscillation. The second mode, stick-free, has the possibility of weak damping or unstable oscillations, and for this reason elevators

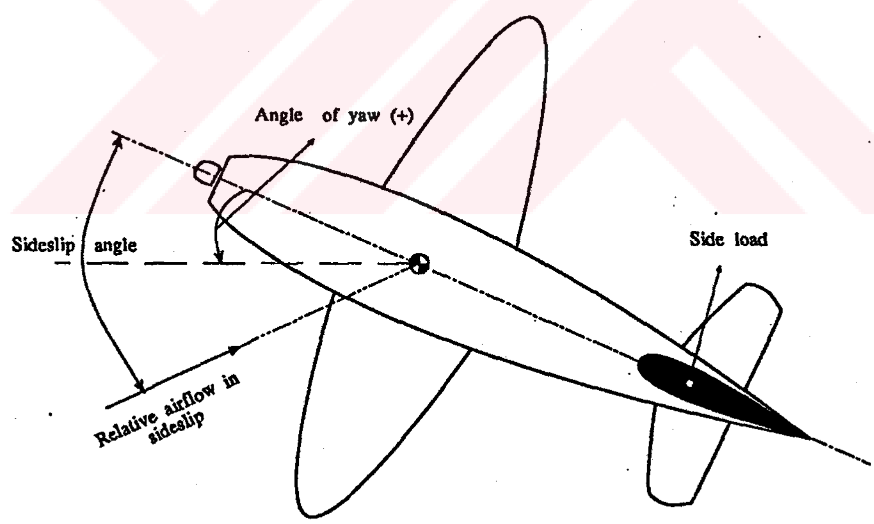


Figure 2.7 Directional stability

must be statically balanced about their hinge line, and aerodynamic control must be within certain limits. If instability were to exist in the second mode, 'porpoising' of the aircraft would result, and because of the short period of oscillation the amplitude can reach dangerous proportions with the possibility of structural damage resulting from the severe flight loads imposed.

The third mode occurs in the stick-free case, and is usually a very-short period oscillation. The motion is essentially one whereby the elevators flap about the hinge line and in most cases the oscillation has very heavy damping.

2.4. DIRECTIONAL STABILITY

Directional stability involves the development of yawing moments which will oppose displacements about the aircraft's vertical axis and so restore it to equilibrium. Unlike longitudinal stability, however, directional stability is not independent in its influence on the behavior of an aircraft, because as a result of what is termed aerodynamic coupling effect, yaw displacements and moments also producing roll displacements and moments also producing roll displacements and moments about the longitudinal axis. Thus, directional motions have an influence on lateral motions and vice versa, the motions involved in each case being yawing, rolling, sideslipping or any combination of these.

As far as yawing displacement, forces, and moments only, are concerned conditions are, in fact, analogous to those relating to longitudinal stability but, whereas in the latter case a horizontal stabilizer has the greatest influence, directional stability is influenced by a vertical stabilizer. This may be seen from fig.2.7.

Assuming that with the rudder in neutral position the aircraft is yawed to starboard by a disturbance (fig. 2.7), the vertical stabilizer will be at some angle of attack with respect to the airflow and a corresponding side force (lift) will be produced. Since the position of the aerodynamic center of the stabilizer with respect to the aircraft's center of gravity provides the longer moment arm, a stabilizing yawing moment to port is created and equilibrium is restored. In addition to the stabilizer moment arm, other factors affecting the size of the stabilizing moment are the area of the stabilizer, its aerofoil section, angle of attack, aspect ratio and sweepback. As in the case of longitudinal stability other major components of the aircraft can also influence the degree of directional stability, notably the fuselage and engine nozells.

When an aircraft is at some angle of yaw, its longitudinal axis is considered as being displaced from a reference azimuth, and by convention a displacement from this azimuth to starboard constitutes a positive angle of yaw, while a displacement to port constitutes a negative angle of yaw. In the yawed condition, and ignoring aerodynamic cross-coupling, the aircraft is maintaining a forward flight path so that, alternatively, the aircraft can be described as being in a condition of sideslip; thus from the example shown in fig. 2.7 and by convention, an aircraft yawed to starboard is sideslipping to port at a negative angle. The angle of sideslip, therefore, is minus the angle of yaw, and since it relates to the displacement of the aircraft's longitudinal axis from the relative airflow rather than a reference azimuth, it becomes a primary reference in directional stability considerations.

When an aircraft is subject to a sideslip angle (relative airflow coming from starboard in the case illustrated) static directional stability will be evident if a positive yawing moment coefficient results. Thus, a yawing moment to starboard would be created which tends to ' weathercock' the aircraft into the relative airflow. This is indicated by the positive slope of curve. If there is zero slope there is of course no tendency to return to equilibrium and so static directional stability is neutral. When the curve has negative slope the yawing moments developed by sideslip tend to diverge, thereby increasing sideslip such that the aircraft would be directionally unstable.

The instantaneous slope of the curve depicting yawing moment coefficient/sideslip angle will indicate the static directional stability. At small angles of sideslip, a strong positive slope depicts strong directional stability. Large angles produce zero slope and neutral stability; if the sideslip is very high the slope would indicate instability.

2.5. LATERAL STATIC STABILITY

An aircraft has lateral stability if, following a displacement about the longitudinal axis(called a roll displacement), a rolling moment is produced which will oppose the displacement and return the aircraft to a wings-level condition. In practice however, and because of aerodynamic coupling, rolling moments can also set up yawing or sideslip motions so that the opposing of lateral displacements is not so simple as it seems.

When an aircraft experiences a roll displacement, the effective angle of attack of the down-going wing becomes greater than that of the up-going wing resulting in the appropriate changes in the lift produced. These changes produce a rolling moment which although opposing the initial roll displacement will do no more that provide a damping effect proportional to the rate of displacement. In other words, the aircraft would possess neutral static stability and so would remain in the rolled or banked position. However, the aircraft also experiences a sideslipping motion which is caused by the inclination of the lift vectors at the roll or bank angle. This motion, in turn, causes the airflow to exert forces on the different parts of the aircraft, and it is the rolling moment induced by sideslipping which establishes static stability reaction and return of the aircraft to a wings-level condition. When the aircraft is subject to a positive sideslip angle, i.e. it sideslips to starboard, positive static stability will be evident if a negative rolling moment to port is produced; the curve will have a negative slope. If the slope of the curve is zero, lateral stability is neutral, while a positive slope indicates lateral instability.

The overall value of the lateral static stability will depend on the effects contributed in varying magnitudes, by each different part of the aircraft, these in turn depending on the configuration of the aircraft and on the condition of flight. The principal contributions to overall lateral static stability are as follows.

1. The dihedral angle or upward setting of the wings relative to the horizontal. Dihedral angle is one of the most important contributions to lateral stability which, for this reason, is often referred to as dihedral effect.

2. The angle at which the wings are swept back relative to the longitudinal axis. Swept-back wings are a characteristic of many types of high-performance aircraft, producing additional lateral stability which has a greater effect in sideslip at low speeds. In some types of swept-wing aircraft, it may be necessary for stability to be reduced at low speeds, and this is done by setting the wings downwards relative to the horizontal, a setting referred to as anhedral.

3. The vertical location of the wings with respect to the fuselage. In a sideslip, air flows spanwise over the aircraft and causes changes in the effective angle of attack of the wings, such that in the case of a high-wing aircraft, the rolling moment produced will be a stabilizing one, and in the case of a low-wing aircraft a de-stabilizing rolling moment will be

produced. There is a zero effect on lateral stability of an aircraft with wings in the mid-position.

4. The keel surface. The side load produced in a sideslip acts on an aircraft's fuselage and on the vertical stabilizer (fin) which together form the keel surface. This side load produces a rolling moment that is generally stabilizing, but to a smaller degree than the moments produced in other ways.

5. The flaps. When flaps are lowered, they alter the spanwise distribution of pressure and lift, and since they are usually located at the inboard sections of wings, the overall centre of lift is located closer to the fuselage centre line, i.e. the moment arm is reduced. Therefore, any changes of lift resulting from sideslip produce smaller rolling moments thereby reducing the overall lateral stability [5].

2.6. DYNAMIC LATERAL STABILITY

The relative effect of the combined rolling,yawing and sideslip motions produced by aerodynamic coupling,determine the lateral dynamic stability of an aircraft. If the stability characteristics are not satisfactory the complex interaction of the motions will produce three possible forms of dynamic instability:(i) directional divergence, (ii) spiral divergence, and (iii) an oscillatory mode termed Dutch roll.

2.6.1. DIRECTIONAL DIVERGENCE

This form of instability is a simple divergence in yaw which may occur if the aircraft is statically unstable about the vertical axis; thus, if the aircraft is flying straight and level and it experiences a small displacement in yaw to port, say,the result will be a yawing moment in the same direction thereby increasing the displacement. In addition, a side force will act on the aircraft in the yawed attitude, so that it will curve away from its original flight path. If the aircraft has lateral static stability directional divergence will occur without any significant degree of bank angle, and the aircraft would still fly a curved path with a very large amount of sideslip.

2.6.2. SPIRAL DIVERGENCE

This form of instability exists when the directional static stability is very large compared with lateral stability. Assuming once again that a yaw

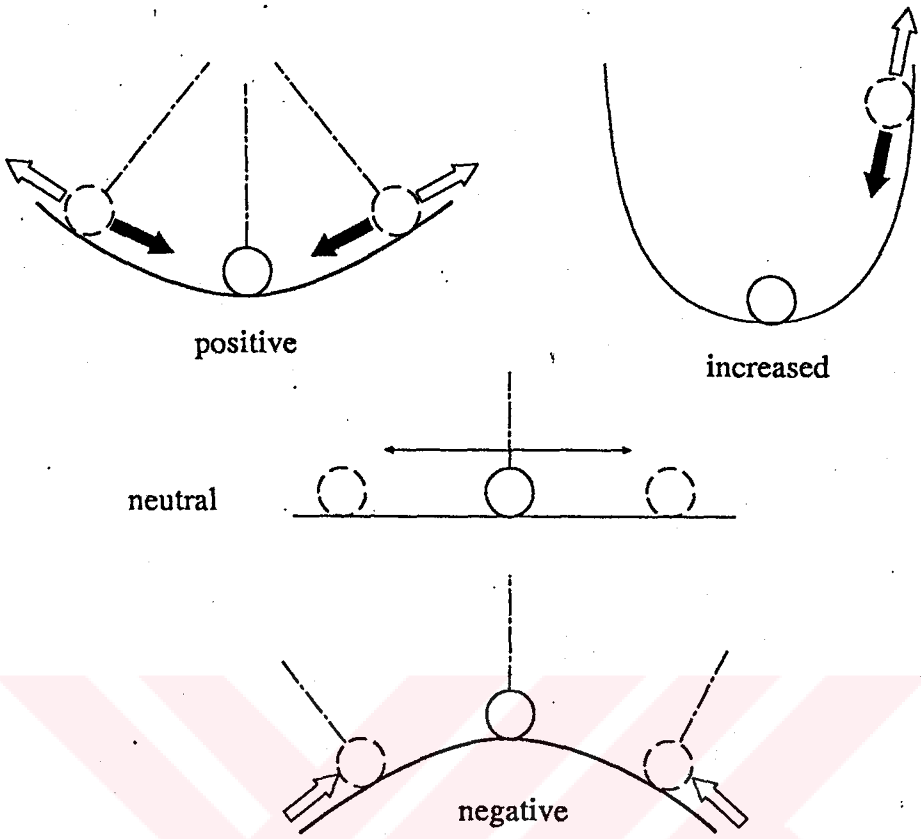


Figure 2.8 Stability v.s. Controllability

displacement to port is experienced, because of the greater directional stability the yaw would be quickly eliminated by a stabilizing yaw moment set up by the keel surface. A rolling moment to port would also have been set up by the yaw displacement and if it were strong enough to overcome the restoring moment due to lateral stability, and to the damping-in-yaw effect,the angle of bank would increase and cause the aircraft nose to drop into the direction of yawing. The aircraft then begins a nose spiral which gradually increases to a spiral dive.

2.6.3 DUTCH ROLL

This is an oscillatory mode of instability which may occur if the aircraft has positive directional static stability but not so much, in relation to the lateral stability, as may lead to spiral divergence. Dutch Roll is commonly found to a varying degree in combinations of high wing loading, sweepback, and high altitude, and where weight is distributed towards wing tips, e.g. engines mounted in pods under the wings.

Assuming yet again that the aircraft is yawed to port, it will roll in the same direction. The directional stability will then begin to reduce the yaw to the extent that the aircraft will overswing and start a yaw, and a roll, to starboard. Thus, every period of the continuing oscillations in yaw acts in such a manner as to cause further displacement in roll, the resulting motion being a combination of rolling and yawing oscillations which have the same frequency, but are out of phase with each other.

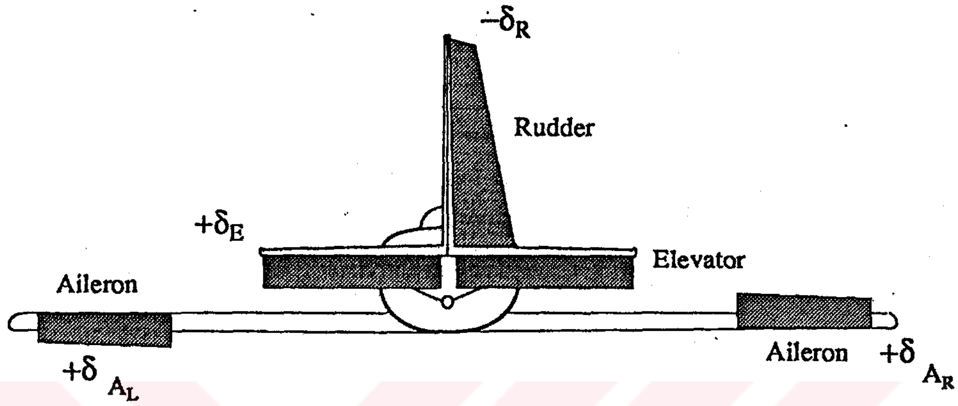


Figure 2.9 Primary flight ontrols

2.7. CONTROLLABILITY

In order that an aircraft may fullfill its intended operational role it must have, in addition to the varying degrees of stability, the ability to respond to requirements for maneuvering and trimming about its three axes, so that all desired flying attitudes can be achieved and equilibrium be established; in other words, it must satisfy controllability condition.

Controllability is a different problem from stability in that it requires aerodynamic forces and moments to be produced about the three axes of the aircraft, such forces always opposing the natural stability restoring moments and causing the aircraft to deviate from an equilibrium condition. There is, therefore, a clear relationship between the two which may be illustrated by the analogy of a ball placed on various surfaces (fig.2.8). Diagram represents the condition of positive static stability and, as any displacement in this condition will always be opposed by a tendency of the ball to return to equilibrium. If, however, it is required to control the ball, and so maintain it in the displaced position, a balancing force must be applied in the direction of displacement. When stability is increased a greater balancing force is required to control the ball to the same displaced position. A large degree of stability therefore tends to make for less controllability, so that for aircraft it can be stated that the upper limits of its stability are set by the lower limits of controllability.

In a neutrally stable condition there is no tendency for the ball to return to equilibrium from a displaced position, and since a new point of equilibrium is always obtained, then no control balancing force is required to maintain the displacement. As the static stability approaches zero, controllability increases to infinity and the only resistance to a displacement would result from damping effects, e.g. the viscosity of air is a damping factor which is proportional to the speed of the displaced body. Thus for an aircraft it can be stated that the lower limits of its stability may be set by the upper limits of controllability.

The effect of negative static stability, i.e. instability, is shown in fig.2.8. If the ball is displaced from equilibrium it will tend to continue in the displaced direction and in order to control its displacement a balancing force must, in this case, be applied in a direction opposite to the displacement. In applying this reversed from of controllability to an aircraft it would mean that the pilot, in attempting to maintain a state of equilibrium, would also be providing the stability. It will be apparent from the foregoing that for an aircraft, proper balance must be achieved between stability and controllability, the latter being provided by means of a primary flight control system, and a secondary 'trimming' system.

2.8. PRIMARY FLIGHT CONTROLS

In its basic form, a primary flight control system consists of movable control surfaces connected by cables and rods to cockpit controls which are directly operated by the pilot. The surfaces are aerodynamically balanced to reduce the pilot's physical effort in controlling the aircraft. In high performance aircraft, the mechanical sections of systems also include powered-actuators.

Conventionally there are three sets of control surfaces, and these are situated at the extremities of the wings and stabilizer units (see fig. 2.9) so as to obtain the largest possible controlling moments, consistent with stability, about the three principal axes and centre of gravity. Movement of a control surface causes a change in the aerodynamic profile and therefore a change in the forces produced. The pressures a acting through the centre of pressure of the control surface, produce a hinge moment which tries to return the surface to its neutral or 'fired' position. The size of the hinge moment is given by the product of the force on the control surface and its distance x from the hinge point. In order to maintain the surface in its deflected position, the hinge moment is balanced by a control force applied to the control system either manually or automatically.

It is desirable that each set of control surfaces should produce a moment only about the corresponding axis. In practice, however, the cross-coupling effects, which arise from interaction between directional and lateral stability, apply equally to the flight control system; e.g. a yawing moment in addition to a rolling moment is produced when the ailerons are deflected.

2.8.1. AILERONS

Ailerons provide lateral control, or roll displacements, about the longitudinal axis, the rolling moment produced being opposed by aerodynamic damping in roll. When the two are in balance the aircraft attains a steady rate of roll; ailerons are therefore essentially rate control devices. Ailerons are, in most cases, operated by a control 'wheel' pivoted on a control column. They are always connected so that in response to instinctive movements of the control wheel by the pilot, they move in opposite directions thereby assisting each other in producing a roll displacement. Thus, when the control wheel is turned to the left, the left wing aileron is raised and the wing lifting force is decreased, while the aileron of the right wing is lowered causing the lifting force of that wing to increase and thereby initiate a roll displacement to the left. A similar but opposite effect occurs when the control wheel is turned to the right.

2.8.2. ELEVATORS

Elevators provide longitudinal or pitch control about the lateral axis, and they also assist the horizontal stabilizer to maintain longitudinal stability. Elevators are usually in two separate halves and are normally mounted on a common hinge line at the rear of the horizontal stabilizer. They are connected to the control column, which can be moved backwards and forwards.

When the column is moved backwards, the elevators are raised, thereby decreasing the lift of the horizontal stabilizer so that the aircraft is displaced by a pitching moment about the lateral axis into a nose-up, or climbing attitude. Forward movement of the control column lowers the elevators to increase the lift of the horizontal stabilizer and so the pitching moment causes the aircraft to assume a nose-down or descending attitude. Pitch displacements are opposed by aerodynamic damping in pitch and by the longitudinal stability (see page 13) and as the response to elevator deflection is a steady change of attitude, elevators are essentially displacement control devices. It will also be noted that control column and control wheel movements are independent of each other so that lateral and longitudinal displacements can be obtained either separately or in combination.

2.8.3. RUDDER

This surface provides yawing moments or directional control about the normal axis of the aircraft, such control being opposed by damping in yaw, and by the directional stability. The rudder is operated in response to instinctive movements by the pilot of a foot-operated rudder bar or, more usually, of a pair of rudder pedals. Thus, if the left pedal is pushed forward the rudder is turned to the left and the force produced on the vertical stabilizer sets up a yawing moment which displaces the aircraft's nose to the left. A corresponding displacement to the right is set up when the right rudder pedal is pushed forward. The response to rudder deflection is a steady state of change of angle of attack on the keel surfaces and so, like the elevators, the rudder is a displacement control device.

2.9. EFFECTIVENESS OF CONTROLS

The effectiveness of a control system, i.e. the moment produced for a given control surface deflection, depends on the magnitude of the force produced by the control surface and also on the moment arm, i.e. distance from the centre of gravity. The aerodynamic forces acting on an aircraft depend, among other important factors, on the airstream velocity, so the effectiveness of flight controls varies accordingly. At low speeds, large control surface movements are needed, while smaller ones are necessary at high speeds. Movement of a control surface alone may also alter control effectiveness, since the loads set up tend to twist and bend the aircraft's structure which has a certain amount of inherent flexibility.

In many types of large aircraft it is usual for control surfaces to be arranged in pairs, i.e. an inboard and an outboard aileron on each wing, an inboard and an outboard elevator, and an upper and lower rudder which illustrates the arrangements in the McDonnell Douglas DC-10. The reasons for such arrangements are to ensure control effectiveness at both low and high speeds, particularly where lateral control is concerned, and also to ensure control in the event of any failures in the system. At low speeds, only the outboard ailerons provide lateral control, while at high speeds they are locked and lateral control is taken over by the inboard ailerons so that the deflecting forces act closer to the longitudinal axis thereby reducing wing twisting. The duplication of elevators and rudder is done primarily as a safety precaution. Each elevator and rudder is operated by an independent control system so that should a failure of a system occur one control surface of a pair can still be effective.

It will also be noted that the rudder sections are in forward and aft pairs. The forward sections are actuated by the rudder pedals and associated hydraulic system, while the aft sections are hinged to the forward sections and are mechanically connected by pushrods to the vertical stabilizer. This arrangement provides for proportional displacements of the forward and aft sections and thereby an overall increase in rudder efficiency [4].

2.10. CONTROL GEARING

Control gearing refers to the relationship between movements of the pilot's controls and the displacements of the corresponding control surfaces, and in the design of the flight control systems of any one type of aircraft, this relationship is a variable one so as to produce the required handling characteristics in all conditions of flight. The variation, or gear change ratio to be more precise, is effected through the mechanical linkages in the control circuits in such a way what when the pilot's controls are moved a certain extent around their neutral position, a relatively small displacement of the control surfaces will be produced; the same extent of movement near the extremities of control movement range on the other hand, will produce a much larger displacement of the control surfaces.

The control gearing can also be varied at different airspeeds by means of a dynamic, pressure sensing gear change unit. In this case, the pressure/speed signals are fed to 'ratio changers' in hydraulic power actuators which operate so as to reduce control surface displacements are airspeed increases. This method is, for example, applied to the rudder control system of the Boeing 747.

2.11. COMBINED CONTROLS

In certain types of aircraft, the primary flight control system is arranged so that one type of control surface may combine its function with that of another; e.g. on a delta wing aircraft such as Concorde, a control surface at each trailing edge can perform the function of both ailerons and elevators; such a control surface is called an elevon. When the control column is moved either backwards or forwards both surfaces move together in the manner of elevators, but when the control wheel is turned, one elevon is raised and the other lowered as in the case of conventional ailerons. The interconnection between the two control systems is such that the surfaces can be deflected simultaneously to produce combined pitching and rolling moments.

Another example of combined control is the one applied to some light aircraft having a 'V' or 'butterfly' tail. In this case, the control surfaces operate as either a rudder or as elevators, and for obvious reasons, they are known as ruddervators. They are connected to the control column and are moved up or down to produce pitching moments as in the case of conventional elevators. They are also connected to the rudder pedals, so that they move equal amounts in opposite directions to produce the required yawing moment. The control column and rudder pedal system are connected to the surfaces through a differential linkage or gearing arrangement, so that combined pitching and yawing moments can be obtained.

In some aircraft, elevators are dispensed with and they are substituted with a movable horizontal stabilizer. Thus when the control column is moved the angle of attack of the stabilizer is varied such that a negative angle produces a nose-up attitude, and a positive angle produces a nose-down attitude. Such a stabilizer is known as a stabilator.

2.12. POWERED FLIGHT CONTROLS

Powered flight controls are employed in high-performance aircraft, and are generally of two main types power-assisted and power-operated. The choice of either system for a particular type of aircraft is governed by the forces required to overcome the aerodynamic loads acting on the flight control surfaces. In basic form, however, both systems are similar in that a hydraulically-operated servo-control unit, consisting of a control valve and an actuating jack, is connected between the pilot's controls and relevant control surfaces.

In a power-assisted system, the pilot's control is connected to the control surface, e.g. control column to elevators, via a control lever. When the pilot moves the control column to initiate a climb say, the control lever pivots about point 'X', and accordingly commences moving the elevators up. At the same time, the control valve pistons are displaced and this allows oil from the hydraulic system to flow to the left-hand side of the actuating jack piston, the rod of which is secured to the aircraft's structure. The reaction of the pressure exerted on the piston causes the whole servo-unit, and control lever, to move to the left, and because of the greater control effort produced the pilot is assisted in making further upward movement of the elevators.

In a power-operated system the pilot's control is connected to the control lever only, while the servo-unit is directly connected to the flight control surface. Thus, in the example considered, the effort required by the pilot to move the control column is simply that needed to move the control lever and control valve piston. It does not vary with the effort required to move the control surface which is supplied solely by servo-unit hydraulic power. Since no forces are transmitted back to the pilot he has no 'feel' of the aerodynamic loads acting on the control surfaces. It is necessary therefore, to incorporate an 'artificial feel' device at a point between the pilot's controls, and their connection to the servo-unit control lever.

A commonly used system for providing artificial feel, particularly in elevator and horizontal stabilizer control systems, is the one known as 'q'feel. In this system, the feel force varies with the dynamic pressure of the air, the pressure being sensed by a pitot-static capsule or bellows type sensing element connected in the hydraulic powered controls, such that it monitors hydraulic pressure, and produces control forces dependent on the amount of the control movement and forward speed of the aircraft.

2.13. 'FLY-BY-WIRE' SYSTEM

Another system which may be considered under the heading of powered flight controls, is the one referred to as a 'fly-by-wire' (FBW) control system. Although not new in concept, complete re-development of the system was seen to be necessary in recent years, as a means of controlling some highly sophisticated types of aircraft coming into service. The problem associated with such aircraft has been one of designing conventional forms of mechanical linkage to suit the complex flight control systems adopted. Thus, an FBW system is one in which wires carrying electrical signals from the pilot's controls, replace mechanical linkages entirely. In operation, movements of the control column and rudder pedals, and the forces exerted by the pilot, are measured by electrical transducers, and the signals produced are then amplified and relayed to operate the hydraulic actuator units which are directly connected to the flight control surfaces.

In some current types of aircraft, the application of the FBW principle is limited to the control of only certain of the flight control surfaces; for example, wing spoiler panels in the case of the Boeing 767.

For lateral control, the deployment of the panels is initiated by movement of the pilot's control wheels to the left or right as appropriate. This movement operates position transducers, in the form of rotary variable differential transformers (RVDTs) via mechanical gear drive from the control wheels. The RVDTs produce command voltage signals proportional to control wheel position and these signals are fed into a spoiler control module for processing and channel selection.

The spoiler control module output signals are then supplied to a solenoid valve forming an integral part of a hydraulic power control actuator. The valve directs hydraulic fluid under pressure to one or other side of the actuator piston which then raises or lowers the spoiler panel connected to the piston rod. The actuator is mounted so that it pivots to allow for the required angular movement of the spoiler panel. As the actuator piston rod moves, it also actuates a position transducer of the linear variable differential transformer (LVDT) type, and this produces a voltage feedback signal proportional to spoiler panel position. When the feedback signal equals the command signal, a 'null' condition is reached and spoiler panel movement stops.

Deployment of spoiler panels for the purpose of acting as speed brakes is initiated by movement of a speed brake lever. The lever operates an LVDT type of transducer which produces a command voltage signal for processing by the signal control module. The output signal operates the actuator in the same way as for lateral control except that the spoiler panels are deployed to their fullest extent. 'Nulling' of the command signal is also produced in the same way.

Lateral control and speed brake signals are mixed in the signal control module to provide the proper ratio of simultaneous operation.

As a further advance in the 'fly-by-wire' (FBW) concept, systems utilizing fibre-optic cables for conveying flight command signals have also now been developed. The principal advantage of this method is its immunity to electromagnetic interference and the consequent elimination of heavy shielding required to protect the more conventional 'signal wires'.

In a fibre-optic cable system, signals are transmitted in the form of light through a number of glass fibers, and where applications to aircraft are concerned, this has given rise to the term 'fly-by-light'. In relation to currently developed systems, however, the term is a misnomer because in these systems, light transmission applies only to command signalling and not to signal processing which is performed electronically within control system computers. An attraction of an FBW system is the ability to replace the conventional control wheel/column with a small side stick or side arm controller. Apart from size and location and lack of movement, it acts in the same way as a normal cockpit control [4] [5].

2.14. MANEUVERING AND FORCES AFFECTING AN AIRCRAFT

The displacements resulting from the various movements of the flight control surfaces are those intentionally set up by the pilot in order to manoeuvre his aircraft into required flight attitudes. Such attitudes are: straight and level, climbing, descending, rolling, turning and a combination of these, e.g. a climbing turn. There are four principal forces affecting an aircraft in flight and the directions in which they act. Lift, as stated at the beginning of this chapter, acts at right-angles to the direction of the airflow from the centre of pressure, the position of which can vary with changing angle of attack. Weight acts vertically downward through the centre of gravity which can also vary in position with changing load conditions. Thrust is the forward propulsive force produced by either a turbine engine or a propeller to overcome the opposing total drag force.

2.14.1. STRAIGHT AND LEVEL FLIGHT

To be in equilibrium in a straight and level flight attitude at constant speed, lift force must equal to weight, and thrust force must equal drag, and it is arranged that these forces act from points which are not coincident, thereby producing couples which give rise to pitching moments. For example, because the relative positions of the centre of pressure and centre of gravity can vary during flight, the lift and weight forces produce couples which cause either a nose-up or a nose-down pitching moment. Similarly, pitching moments result from the displacement couples of thrust and drag. Ideally, the moments arising from these two couples should balance each other, and by design it is usual for a nose-down moment due to the lift/weight couple, to be balanced by a nose-up moment due to the thrust/drag couple. If the thrust is then decreased either by a deliberate reduction in engine power or by an engine failure, the lift/weight couple will overcome the reduced thrust/drag couple and cause a nose-down moment thereby putting the aircraft into a nose-down attitude.

2.14.2. CLIMBING AND DESCENDING

These are pitch attitude maneuver which are set up by upward and downward movements respectively of the elevators. In the case of stabilators referred to earlier, the horizontal stabilizer is deflected to produce a negative angle of attack for the setting up of a climbing attitude, and a more positive angle of attack for a descending attitude. As an example of how the forces act we may consider the case of an upward deflection of elevators to produce a climbing attitude from straight and level flight. When the elevators are deflected a pitching moment is produced to rotate the aircraft about the centre of gravity causing the lift vector to be inclined, and thereby constitute an accelerating force at right angles to the direction of flight causing the aircraft to initially follow a curvilinear path. The lift force in a climb is normally less than the weight, a component of which acts in the direction of drag; therefore, more power, i.e.greater thrust is required to lift the aircraft at a vertical speed, otherwise known as the rate of climb. At the required rate and angle of climb the elevators are returned to their neutral position, and the aircraft will fly along a path tangential to the original curve.

2.14.3. ROLLING AND TURNING

Rolling of an aircraft takes place about the longitudinal axis, and is initiated by deflecting the ailerons in the required direction. As the effective angle of attack of each wing is thereby changed, then the down-going wing produces a greater lift than the up-going wing and so a rolling moment is established. The total lift vector rotates through the same angle as the aircraft and gives rise to two components, one vertical and equal to *Lcos*q and the other horizontal and equal to *Lsin*q. The latter component acts through the centre of gravity, and it establishes an inward or centripetal force, thereby causing the aircraft to accelerate into a curvilinear flight path; thus displacing an aircraft into a rolled attitude also results in turning.

For any given airspeed and turn radius, however, turning of this nature would only occur at one correct angle of bank. At any other angle, and because of a cross-coupling response, a yawing moment is produced which opposes the rolling moment. The yawing moment arises in this case, because the lift vector of the down-going wing is inclined forwards, while that on the up-going wing is inclined rearward. Further more, the up-going wing, being further from the centre of the turn, has the greater angular velocity, and so it sustains more drag. The ailerons themselves, when in their deflected positions, can also create what is termed adverse aileron yaw. When an aileron is lowered, it moves into a region of low pressure. Thus, with drag greater on one aileron than on the other, the effect is to yaw an aircraft in the direction opposite to that intended. This effect is reduced as much as possible by operating the ailerons differentially, i.e. when control is applied the 'up-going' aileron moves through a greater angle than the 'down-going' one.

A yawing moment also results in sideslip and this must be prevented if the aircraft is to be manoeuvred into a steady turn at any speed and roll angle; in other words, the turn must be a co-ordinated one. If an aircraft has a well-designed aileron system, a co-ordinated turn can be achieved by aileron deflection alone. As is more often the case, however, the ailerons must be assisted by deflection of the rudder in the direction of its turn, thereby creating a side force on the vertical stabilizer to overcome adverse aileron yaw.

In the next chapter inverse simulation method will be introduced. As its name implies, maneuvers just described constitutes the input for the inverse problem while the control surface deflections becomes the output. Such a relation is established in the framework of aerodynamic forces and moments and treating α, β, and φ as key variables.

CHAPTER 3
AIRCRAFT MOTION CONTROL BY INVERSE PROBLEM

3.1. INVERSE PROBLEM (DIFFERENTIATION METHOD)

In the ordinary problem, if the controls (primarily, three control surfaces and power) and the initial values of the state variables are given, the flight trajectory and the flight attitude can be determined in principle. In the inverse problem, however, if both flight trajectory (three degrees of freedom) and flight attitude (three degrees of freedom) are given, there will be no no solution. If either flight trajectory or flight attitude alone is given, thre will be many solutions in general. This is bacause the degree of freedom of the controlled motion for a conventional airplane is between three and four.

It seemed that the way of flying and the way of piloting have been settled. A particular way of flying, whether conventional or not, depends on the control system. Therefore, consideration must be taken of these elements of the airplane when we treat the inverse problem generally. The control surfaces of an airplane usually consists of the aileron, elevator, and rudder. They generate the moments about the X-, Y-, and Z-axes, respectively, and become the inputs of the moment equation of each axis. They are quite similar to each other except for acting on different axes. But they are not analogous in piloting the airplane. For example, the pitch angle is controlled by the elevator, but the yaw (heading) control is not performed, in the ordinary way, by the rudder. Because of the gap between the mathematical model and the piloting use of control surfaces, the inverse problem can not be handled by the mathematical treatment alone. The elevator, rudder, and aileron are regarded as the controls of the angle of attack α , the side slip angle β, and the bank angle φ respectively. Therefore these angles (α, β, φ) are chosen as key variables,

$$m\left\{\begin{bmatrix}\dot{U}\\ \dot{V}\\ \dot{W}\end{bmatrix}+\begin{bmatrix}0 & -R & Q\\ R & 0 & -P\\ -Q & P & 0\end{bmatrix}\begin{bmatrix}U\\ V\\ W\end{bmatrix}\right\}=mg\begin{bmatrix}-\sin\theta\\ \cos\theta\sin\varphi\\ \cos\theta\cos\varphi\end{bmatrix}+\begin{bmatrix}X\\ Y\\ Z\end{bmatrix} \tag{3.1}$$

$$\begin{bmatrix} I_X & 0 & -I_{ZX} \\ 0 & I_Y & 0 \\ -I_{ZX} & 0 & I_Z \end{bmatrix}\begin{bmatrix} \dot{P} \\ \dot{Q} \\ \dot{R} \end{bmatrix}+\begin{bmatrix} 0 & -R & Q \\ R & 0 & -P \\ -Q & P & 0 \end{bmatrix}\times\begin{bmatrix} I_X & 0 & -I_{ZX} \\ 0 & I_Y & 0 \\ -I_{ZX} & 0 & I_Z \end{bmatrix}\begin{bmatrix} P \\ Q \\ R \end{bmatrix}=\begin{bmatrix} L \\ M \\ N \end{bmatrix} \quad (3.2)$$

and an inverse problem is developed in a general way [6].

3.2. EQUATIONS OF MOTION

The equations of motion of an airplane in body axes are given by:where the airplane is assumed to be a rigid body and has bilateral symmetry. The force equation [Eq. (3.1)] and the moment equation [Eq. (3.2)] are first-order differential equations with regard to velocity components (U, V, W) and the angular velocity components (P, Q, R) are related to position components rates $(\dot{x},\dot{y},\dot{z})$ and eulerangle rates $(\dot{\psi},\dot{\theta},\dot{\varphi})$ as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}=\begin{bmatrix} \cos\psi\cos\theta & \cos\psi\sin\theta\sin\varphi - \sin\psi\cos\varphi & \cos\psi\sin\theta\cos\varphi + \sin\psi\sin\varphi \\ \sin\psi\cos\theta & \sin\psi\sin\theta\sin\varphi + \cos\psi\cos\varphi & \sin\psi\sin\theta\cos\varphi - \cos\psi\sin\varphi \\ -\sin\theta & \cos\theta\sin\varphi & \cos\theta\cos\varphi \end{bmatrix}\begin{bmatrix} U \\ V \\ W \end{bmatrix} \quad (3.3)$$

$$\begin{bmatrix} \dot{\psi} \\ \dot{\theta} \\ \dot{\varphi} \end{bmatrix}=\begin{bmatrix} 0 & \sin\varphi/\cos\theta & \cos\varphi/\cos\theta \\ 0 & \cos\varphi & -\sin\varphi \\ 1 & \sin\varphi\tan\theta & \cos\varphi\tan\theta \end{bmatrix}\begin{bmatrix} P \\ Q \\ R \end{bmatrix} \quad (3.4)$$

These kinematic equations are added to Eqs. (3.1) and (3.2) to make state equations. The components (X, Y, Z) and (L, M, N) shown in the right-hand side of Eqs. (3.1) and (3.2) denote the sum of external forces (except the gravity force) and moments acting on the air plane, respectively. They are mainly produced aerodynamically and are considered as the functions of (U, V, W), (P, Q, R), (δa, δe, δr, δT) and their derivatives.The state variables, which describe the motion of the airplane, are (U, V, W), (x, y, z), (P, Q, R), and (ψ, θ, φ). But using angle of attack α, side-slip angle b, and tangent velocity V_T. (U, V, W) can be written as

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix}=V_T\begin{bmatrix} \cos\beta\cos\alpha \\ \sin\beta \\ \cos\beta\sin\alpha \end{bmatrix} \quad (3.5)$$

Also using path angles (ψ_W, φ_W) and V_T, the position rates $(\dot{x}, \dot{y}, \dot{z})$ are written as

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = V_T \begin{bmatrix} \cos\theta_w \cos\psi_w \\ \cos\theta_w \sin\psi_w \\ -\sin\theta_w \end{bmatrix} \tag{3.6}$$

which is simpler then Eq. (3.3). The path angles should be distinguished from the attitude angles (ψ, θ). Note that the path angles (ψ_W, φ_W) are the tfw of the Euler angles of the wind axes (ψ_W, θ_W, φ_W) provided that the atmosphere is at rest. It also should be noted that φ_W and φ are not the same. For example, by the definition of the Euler angles the relationship between φ_W and φ is

$$\tan\varphi_w = \frac{\sin\beta(\cos\alpha\sin\theta - \sin\alpha\cos\theta\cos\varphi) + \cos\beta\cos\theta\sin\varphi}{\sin\alpha\sin\theta + \cos\alpha\cos\theta\cos\varphi} \tag{3.6.1}$$

3.3. ATTITUDE ANGLES AND PATH ANGLES

Let (ζ, η, ξ) be the Euler angles defined in flight dynamics, and R(ζ, η, ξ) be the rotation given by the Euler angles. Since the rotation can be

$$R(\zeta,\eta,\xi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\xi & \sin\xi \\ 0 & -\sin\xi & \cos\xi \end{bmatrix} \begin{bmatrix} \cos\eta & 0 & -\sin\eta \\ 0 & 1 & 0 \\ \sin\eta & 0 & \cos\eta \end{bmatrix} \times \begin{bmatrix} \cos\zeta & \sin\zeta & 0 \\ -\sin\zeta & \cos\zeta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.7}$$

expressed by the means of orthogonal matrices, we have (3.7) Also, let (ψ, θ, φ) and (ψ_W, θ_W, φ_W) be the Euler agles for the body axes anr wind axes respectively. The body axes and wind axes are related by the angle of attack α and the sideslip angle β, and the rotation from the wind axes to the body axes can be shown by R($-\beta$, α, 0) Then, the relation between both Euler angles is

$$R(-\beta,\alpha,0)\cdot R(\psi_w,\theta_w,\varphi_w) = R(\psi,\theta,\varphi) \tag{3.8}$$

From this equation the Euler angles of the body axes can be obtained in terms of the Euler angles of the wind axes and the aerodynamic angles α, β, and vice versa. For instance, the heading angle is shown below

$$\tan\psi = \tan\psi(\psi_w,\theta_w,\varphi_w;\alpha,\beta)$$
$$= \begin{bmatrix} \cos\beta\cos\alpha\sin\psi_w\cos\theta_w - \sin\beta\cos\alpha(\sin\psi_w\sin\theta_w\sin\varphi_w \\ +\cos\psi_w\cos\varphi_w) - \sin\alpha(\sin\psi_w\sin\theta_w\cos\varphi_w - \cos\psi_w\sin\varphi_w) \end{bmatrix}$$
$$+ \begin{bmatrix} \cos\beta\cos\alpha\cos\psi_w\cos\theta_w - \sin\beta\cos\alpha(\cos\psi_w\sin\theta_w\sin\varphi_w \\ -\sin\psi_w\cos\varphi_w) - \sin\alpha(\cos\psi_w\sin\theta_w\cos\varphi_w + \sin\psi_w\sin\varphi_w) \end{bmatrix} \quad (3.8.1)$$

However, even though the flight path is given, the angle φ_w can not be determined. Further, the the force equation [Eq. (3.1)] does not contain φ. Therefore, what is required are relations which do not contain φ_w, i.e.,

$$\psi = \psi(\psi_w,\theta_w;\alpha,\beta,\varphi) \quad (9a)$$

$$\theta = \theta(\psi_w,\theta_w;\alpha,\beta,\varphi) \quad (9b)$$

If so, it can be said that the heading and pitch are determined by the flight path and the triplet (α, β, φ).For this purpose,

$$\begin{bmatrix} \cos\psi_w\cos\theta_w & \sin\psi_w\cos\theta_w & -\sin\theta_w \\ \cos\psi_w\sin\theta_w\sin\varphi_w & \sin\psi_w\sin\theta_w\sin\varphi_w & \cos\theta_w\sin\varphi_w \\ -\sin\psi_w\cos\varphi_w & +\cos\psi_w\cos\varphi_w & \\ \cos\psi_w\sin\theta_w\cos & \sin\psi_w\sin\theta_w\cos\varphi_w & \cos\theta_w\cos\varphi_w \\ +\sin\psi_w\sin\varphi_w & -\cos\psi_w\sin\varphi_w & \end{bmatrix}$$
$$= \begin{bmatrix} \cos\beta\cos\alpha & \sin\beta & \cos\beta\sin\alpha \\ -\sin\beta\cos\alpha & \cos\beta & -\sin\beta\sin\alpha \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \quad (3.10)$$
$$\times \begin{bmatrix} \cos\psi\cos\theta & \sin\psi\cos\theta & -\sin\theta \\ \cos\psi\sin\theta\sin\varphi & \sin\psi\sin\theta\sin\varphi & \cos\theta\sin\varphi \\ -\sin\psi\cos\varphi & +\cos\psi\cos\varphi & \\ \cos\psi\sin\theta\cos\varphi & \sin\psi\sin\theta\cos\varphi & \cos\theta\cos\varphi \\ +\sin\psi\sin\varphi & -\cos\psi\sin\varphi & \end{bmatrix}$$

multiplying Eq. (3.8) by $R^{-1}(-\beta, \alpha, 0)$ from the left we have the matrix form Eq(3.10) From this, we obtain immediately (3.11). Noting the elements, which do not contain φ_w, of the left-hand side matrix in Eq. (3.10), and after some calculations, we also get Eq(3.11) and Eq(3.12).

$$\sin\theta_w = \cos\beta\cos\alpha\sin\theta - (\sin\beta\sin\varphi + \cos\beta\sin\alpha\cos\varphi)\cos\theta \quad (3.11)$$

$$sin(\psi_W - \psi) = \frac{sin\,\beta\,cos\,\varphi - cos\,\beta\,sin\,\alpha\,sin\,\varphi}{cos\,\theta_W} \tag{3.12}$$

When path angles (ψ_W, θ_W) and (α, β, φ) are given, the attitude angles (ψ, θ) can be calculated from Eqs. (3.11) and (3.12) without approximation.

3.4. GENERAL APPROACH

Although there are many ways to prescribe flight maneuvers, the most natural one is to give the flight path and the velocity on it. Therefore, an approach to this case is mainly considered. Other cases, for example, when the flight attitude is given, can be considered almost in the same way, and the details will be presented in the last section in this chapter.

3.4.1 VARIABLES TRANSFORMATION

To give the flight trajectory is to give the position which is the function of time, x(t), y(t), z(t). When they are given, by differentiating them with respect to time and solving Eq.(6) inversely, tangent velocity $V_T(t)$ and path angles $\psi_W(t)$, $\theta_W(t)$ are obtained. Or, more practically, $V_T(t)$, $\psi_W(t)$, $\theta_W(t)$ are given directly. If ψ_W and θ_W are known as khe functions of time, then from Eq. (3.11) and (3.12) we can get heading ψ and pitch θ as functions of time t and unknown variables $\alpha(t)$, $\beta(t)$, $\varphi(t)$, i.e.,

$$\psi = \psi(t, \alpha(t), \beta(t), \varphi(t)) \tag{3.13a}$$

$$\theta = \theta(t, \alpha(t), \beta(t), \varphi(t)) \tag{3.13b}$$

Angular velocities P, Q, R are expressed by $\theta, \varphi, \dot{\psi}, \dot{\theta}, \dot{\varphi}$ as shown in Eq. (4). Further, φ, θ are expressed in Eqs. (3.13). Therefore, the angular velocities can be written in the form:

$$P = P(t, \alpha(t), \beta(t), \varphi(t), \dot{\alpha}(t), \dot{\beta}(t), \dot{\varphi}(t)) \tag{3.14a}$$

$$Q = Q(t, \alpha(t), \beta(t), \varphi(t), \dot{\alpha}(t), \dot{\beta}(t), \dot{\varphi}(t)) \tag{3.14b}$$

$$R = R(t, \alpha(t), \beta(t), \varphi(t), \dot{\alpha}(t), \dot{\beta}(t), \dot{\varphi}(t)) \tag{3.14c}$$

Velocities U, V, W are rewritten by V_T, α, β from Eq (3.5). Since the trajectory has been given, $V_T(t)$ is known as the function of time. Accordingly, the velocities simply become the functions of unknown variables $\alpha(t)$ and $\beta(t)$:

$$U = U(t, \alpha(t), \beta(t)) \tag{3.15a}$$

$$V = V(t, \beta(t)) \tag{3.15a}$$

$$W = W(t, \alpha(t), \beta(t)) \tag{3.15a}$$

To sum up, by the trajectory [x(t), y(t), z(t)] being given, all the remaining state variables (U, V, W), (P, Q, R), and (ψ, θ, φ) can be obtained as the functions which contain only α, β, φ as unknown variables.

3.4.2. FORCE EQUATIONS

The trajectory has been given. It is determined by the force equations what should first be controlled to fly the airplane along this trajectory. The second term of the right-hand side of Eq. (3.1), (X, Y, Z) are, as stated previously, considerid as the functions of (U, V, W), (P, Q, R) and (δa, δe, δr, δT). Therefore, if (X, Y, Z) is expressed in terms of these variables, then by using Eqs. (3.13), (3.14) and (3.15), Eq (3.1) will become differential equations which contain unknown variables (α, β, φ) and unknown controls (δa, δe, δr, δT). In short, Eq. (3.1) can be written as follows:

$$F_X(t; \alpha, \beta, \varphi, \delta a, \delta e, \delta r, \delta T) = 0 \tag{3.16a}$$

$$F_Y(t; \alpha, \beta, \varphi, \delta a, \delta e, \delta r, \delta T) = 0 \tag{3.16b}$$

$$F_Z(t; \alpha, \beta, \varphi, \delta a, \delta e, \delta r, \delta T) = 0 \tag{3.16c}$$

where F may include also derivatives of the variables. If (α, β, φ) and (δa, δe, δr, δT) satisfy Eqs. (3.16), the airplane can fly along the trajectory prescribed first. But because α, β, φ and δa, δe, δr are not independent, the moment equation [Eq. (3.2)] needs to be added which indicates dependency. However, in practice, the control surfaces (δa, δe, δr) are considered to play little role in the force equations and only δT works in Eq. (3.16a). Accordingly, if the three controls in Eqs. (3.16) are neglected, then Eq.(3.2) need not be added, and Eqs. (3.16) are approximated as

$$F_x(t;\alpha,\beta,\varphi;\delta T)=0 \tag{3.17a}$$

$$F_Y(t;\alpha,\beta,\varphi)=0 \tag{3.17b}$$

$$F_z(t;\alpha,\beta,\varphi)=0 \tag{3.17c}$$

As a result, we can continue to discuss the problem by only these force equations for the present.

Since the above three simultaneous equations contain for unknown variables α, β, φ and δT, unique solutions are not likely to be obtained. That is, when only flight path is given, there is the degree of freedom of [α(t), β(t), φ(t)]. This degree of freedom of (α, β, φ) becomes that of attitude angles (ψ, θ) from Eq. (3.9). If, from the first, attitude angles (ψ, θ, φ) are given arbitrarly in addition to flight path angles (ψ_W, θ_W) the aerodynamic angles (α, β,) can be determined by solving Eq. (3.9) [the simultaneous equations of (α, β,)]. But because of arbitrariness these (α, β, φ) obtained do not satisfy Eq. (3.17) in general: the airplane can not fly along the flight path given first. In this respect, we can not give both flight path and flight attitude at the same time in a conventional sense.

3.4.3. CONSTRAINTS ON (α, β, φ)

To determine (α, β, φ) or flight attitude when flight trajectory is given, one more constraint equation must be added with regard to these variables. As previously mentioned, the control of α, β, φ is closely connected with the control system of the airplane. Therefore, the considerations on the control system and the way of piloting should be made before the triplet (α, β, φ) is determined. In the following, some cases will be considered as examples.

In ordinary cases the airplane is flown by maintaining sideslip angle or side force (the total force component along Y-axis except gravity) zero, as is well known, which is a basis of piloting and called "coordinated". In this case, β(t) = 0 is the additional constrained equation, and the remaining variables α(t), φ(t) and attitude angles (ψ(t), θ(t)) can be determined uniquely (except inverted flights). This flight attitude is the most natural.

In the other cases, i.e., where the flight is not coordinated, two cases arise. In one case the flight is actively uncoordinated and in the other case passively. As an example of the former case, if the flight attitude is required to change to a certain degree from the natural one mentioned above, then, of course $\beta(t) = 0$ can not be satisfied, and (α, β, φ) must be determinedso that they can meet this requirement (which is to be the additional coondition). For practical examples, there are "aerobatics": in which the flight attitude is often unnatural for flight path. Also there is a cross-wind landing method called "wing low" in which, in order to fly the airplane straight on the final approach with its heading ψ maintaining the runway azimuth, the sideslip angle β must be made owing to the existence of crosswind. But then, if the wings are level, the airplane begins to turn due to the β - sideforce and flies off the approach. Therefore, the bank angle φ also must be made in the direction of crosswind.

The latter is the case where active control of β is not done. For examples there are "two-control turn in which elevator and either aileron or rudder are used to control α and φ, and the case in which rudder coordinations are not well performed. In these cases there are only two actual inputs in three moment equations. Therefore, even if (α, β, φ) are determined from the force equations by an arbitrary condition and substituted into the moment equations, there do not generally exist the two conntrols δe and δa (or, δe and δr) satisfying thse equations. This means that the conditions determining (α, β, φ) are the moment equations themselves. Indeed, if the moment equations are added to the force equations, the number of equations agree with that unknown variables.

Another unconventional case is the one where an airplane has direct foce control (DFC). Since diract-lift (DL), δD_L and direct side-force (DSF), δD_{SF} enter the force equations explicitly unlike the conventional three controls, the force equations are changed essentially:

$$F_X(t;\alpha,\beta,\varphi;\delta T) = 0 \tag{3.18a}$$

$$F_Y(t;\alpha,\beta,\varphi;\delta D_{SF}) = 0 \tag{3.18b}$$

$$F_Z(t;\alpha,\beta,\varphi;\delta D_L) = 0 \tag{3.18c}$$

Thus the number of unknown variables becomes six, while the number of equations remains three. Accordingly, even though (α, β, φ) are prescribed quite arbitrarily, there still remain three variables δT, δD_L and δD_{SF} in Eqs. (3.18). Therefore, if δT, δD_L and δD_{SF} have no limitations of power, and also if α and β have no limitations of magnitude, the triplet (α,

β, φ) can be chosen as we like, and so is the flight attitude. This is a well-known advantage of DFC.

3.4.4. MOMENT EQUATIONS

After the trajectory is given, the moment equations are also approximated as follows:

$$M_x(t;\alpha,\beta,\varphi;\delta a,\delta r)=0 \tag{3.19a}$$

$$M_Y(t;\alpha,\beta,\varphi;\delta e)=0 \tag{3.19b}$$

$$M_z(t;\alpha,\beta,\varphi;\delta r,\delta a)=0 \tag{3.19c}$$

In the previous sections (α, β, φ; δT) were treated as inputs to the force equations, and (α, β, φ; δT) obtained by solving those equations were consirdered as the solutions of the inverse problem. Actually, the attitude angles can be obtained from (α, β, φ) by using Eqs.(3.11) and (3.12), and these (α, β, φ) are themselves not so inappropriate for the index of piloting. In these points the inverse problem was considered and solved without the moment equations. However, to get either deflections of control surfaces or more exact solutions, the moment equetions are required. In the former case, by substituting [$\alpha(t)$, $\beta(t)$, $\varphi(t)$] obtained into moment equations and solving them with regard to the three controls, the control surface deflections $\delta a(t)$, $\delta e(t)$, and $\delta r(t)$ are obtained. In the latter case, the force and moment equations must be solved simultaneously. Although the number of equations and variables increase, the problem can be considered in the similar way.

3.4.5. ATTITUDE GIVEN CASE

In general, it is impossible to prescribe $\varphi(t)$ rigidly together with ($\psi(t)$, $\theta(t)$) for a conventional airplane. This is because the bank angle $\varphi(t)$ is not independent of the variations of the heading and pitch angles [$\psi(t)$, $\theta(t)$)]. It should rather be considered that $\varphi(t)$ is one of the controls of $\psi(t)$ and $\theta(t)$, and is determined later. Therefore, only $\psi(t)$ and $\theta(t)$ are given at the start. This case is easier than the case of flight path being given in the point that $\psi(t)$ and $\theta(t)$ are given directly as the functions of time[see Eq.(3.13)]. In a way similar to Eq. (3.17), the force equation [Eq. (1)] become Eq(3.20). Thus, the number of unknown variables becomes five. By the same argument made before, two more constraint equations can be added to get unique solutions. Therefore, it seems here that $\varphi(t)$ can also

$$F_X(t;V_T,\alpha,\beta,\varphi;\delta T)=0 \tag{3.20a}$$

$$F_Y(t;V_T,\alpha,\beta,\varphi)=0 \tag{3.20b}$$

$$F_Z(t;V_T,\alpha,\beta,\varphi)=0 \tag{3.20c}$$

be prescribed. But, if this is done, it will affect the solutions of other variables, and they may be unrealizable in effect, e.g., the magnitude of the sideslip angle $\beta(t)$ may be extremely large in order to produce sideforce, or the lift may be inverted.

In usual cases, the tangent velocity $V_T(t)$ and the condition of the coordination $\beta(t) = 0$ are given here. Then the force equations [Eq.(3.20)] are

$$F_X(t;\alpha,\varphi;\delta T)=0 \tag{3.21a}$$

$$F_Y(t;\alpha,\varphi)=0 \tag{3.21b}$$

$$F_Z(t;\alpha,\varphi)=0 \tag{3.21c}$$

By solving the Eqs. (3.21) simultaneously the angle of attack $\alpha(t)$, the bank angle $\varphi(t)$, and the power $\delta T(t)$ are obtained as the solutions of this inverse problem. The succeding procedures and the considerations about unusual cases are the same as before. The path angles $\theta_W(t)$ and $\psi_W(t)$ can be obtained from Eq. (3.11) and the following equation [from Eq.(3.10)]:

$$\tan\psi_W = \tan\psi_W(\psi,\theta,\varphi;\alpha,\beta)$$
$$= \frac{\begin{bmatrix}\cos\beta\cos\alpha\sin\psi\cos\theta+\sin\beta(\sin\psi\sin\theta\sin\varphi+\cos\psi\cos\varphi)\\+\cos\beta\sin\alpha(\sin\psi\sin\theta\cos\varphi-\cos\psi\sin\varphi)\end{bmatrix}}{\begin{bmatrix}\cos\beta\cos\alpha\cos\psi\cos\theta+\sin\beta(\cos\psi\sin\theta\sin\varphi-\sin\psi\cos\varphi)\\+\cos\beta\sin\alpha(\cos\psi\sin\theta\cos\varphi+\sin\psi\sin\varphi)\end{bmatrix}} \tag{3.22}$$

In the present approach, one of the methods of considering the problem is counting the numbers of equations and unknown variables, which is well-known. It may seem that this is not strict from the mathematical viewpoint, but because of the nonlinearity of the simultaneous equations, there is nothing else that is of practical use for examining the existance and uniqueness of the solution. Therefore, physical conditions should be taken into account, together with this method, for each individual case where the form of the equation is clearly decided.

CHAPTER 4

NUMERICAL EXAMPLES

4.1. BASIC SOLUTION FOR 360° ROLL

An interesting flight maneuver is now considered, and a numerical example of the inverse problem is illustrated. That is, the triplet (α, β, φ), flight attitude, and the controls of this flight are calculated by the procedure shown in the preceding section. Suppose one maneuver in which an airplane is flown along a straight and level flight path with a 360-deg continuous rolling. Such a maneuver looks like actual aerobatics, what is called aileron rolls or slow rolls, but it is an imaginary one in the point that it is supposed that the flight path must be exactly straight. Therefore, it is expected that the control of this maneuver differs from that of those maneuvers. The flight attitude and the three controls of this maneuver depend on the variations of α and β taken with that of φ along the trajectory. First of all, the flight trajectory is given as follows

$$[V_T(t), \psi_w(t), \theta_w(t)] \equiv (V_T, 0, 0), \qquad V_T = Const. \tag{4.1}$$

Since this is a nonaccelerated flight, the acceleration terms, i.e., the left-hand sides of Eq.(3.1) become zero. Thus, assuming linear aerodynamic forces and using dimensional stability derivatives, Eq (3.1) become

$$-g\sin\theta + X_U u + X_w w + X_{\delta T} = 0 \tag{4.2a}$$

$$g\cos\theta\sin\varphi + Y_V = 0 \tag{4.2b}$$

$$g(\cos\theta\cos\varphi - 1) + Z_U u + Z_w w = 0 \tag{4.2c}$$

where u,v,w are perturbed quantities of U, V, W from the trim condition at the starting point:

$$(U_0, V_0, W_0) = (V_T, 0, 0), \qquad (\alpha_0, \beta_0) = (0,0),$$
$$(\psi_0, \theta_0, \varphi_0) = (0,0,0) \tag{4.2d}$$

In order to express ψ, θ in terms of α, β, φ, Eqs.(3.11-12) can be applied. Since $\psi_W(t) = 0$ and $\theta_W(t) = 0$ from Eqs.(4.1), they are reduced to

$$tan\,\theta = \frac{sin\,\beta\,sin\,\varphi + cos\,\beta\,sin\,\alpha\,cos\,\varphi}{cos\,\beta\,cos\,\alpha} \tag{4.3a}$$

$$sin\,\psi = cos\,\beta\,sin\,\alpha\,sin\,\varphi - sin\,\beta\,cos\,\varphi \tag{4.3b}$$

Substituting Eq. (3.25a) and Eq. (3.5) into Eqs. (3.24), and setting

$$\theta(\alpha,\beta,\varphi) = tan^{-1}\frac{sin\,\beta\,sin\,\varphi + cos\,\beta\,sin\,\alpha\,cos\,\varphi}{cos\,\beta\,cos\,\alpha} \tag{4.3c}$$

Eqs. (3.24) become

$$\begin{aligned} -g\sin\{\theta(\alpha,\beta,\varphi)\} + X_U V_T(\cos\alpha\cos\beta - 1) \\ + X_W V_T \cos\beta\sin\alpha + X_{\delta T}\delta T = 0 \end{aligned} \tag{4.4a}$$

$$g\cos\{\theta(\alpha,\beta,\varphi)\}\sin\varphi + Y_v V_T \sin\beta = 0 \tag{4.4b}$$

$$\begin{aligned} g\left[\cos\{\theta(\alpha,\beta,\varphi)\}\cos\varphi - 1\right] + Z_U V_T(\cos\alpha\cos\beta - 1) \\ + Z_w V_T \cos\beta\sin\alpha = 0 \end{aligned} \tag{4.4c}$$

which corresponds to Eqs. (3.17).

Since δT is contained in only Eq. (4.4a) the triplet (α, β, φ) can be determined by Eqs. (4.4b) and (4.4c) will be considered hereafter. To prescribe the rolling maneuver or to give one more constraint as stated before, the variation of the bank angle with time, which should have suitable properties, is given by Etkin[1]

$$\varphi(t) = \frac{2\pi}{16}\left\{cos\,3\pi\left(\frac{t}{T}\right) - 9\,cos\,\pi\left(\frac{t}{T}\right) + 8\right\} \tag{4.5}$$

where T is the time taken for a 360-deg roll. Figure 3.1 shows this function for the case T = 6 s. If this time function is substituted into Eqs. (4.4b) and (4.4c), they become simultaneous equations with regard to $\alpha(t)$ and $\beta(t)$. Therefore, they can be solved to get $\alpha(t)$ and $\beta(t)$ as functions of time. But because of nonlinearity, it is not so easy to do even by numerical method.

To do this, first Eqs. (4.4b) and (4.4c) are rewritten in the form $\beta = f(\alpha, \beta)$ and $\alpha = g(\alpha, \beta)$, where β and α in the left-hand sides are solved with respect to β–sideforce term ($Y_V v$) and lift increment term ($Z_W w$), respectively. then, the succesive iteration method is used to get $\alpha(t_i)$ and $\beta(t_i)$. At each moment t_i at 0.1-s. intervals in the rolling maneuver. The first approximation of each iteration are taken to $\beta_0(t_i)$ = $-g\sin\varphi(t_i)/(Y_V V_T)$, $\alpha_0(t_i)$ = $g(1-\cos\varphi(t_i)/(Z_W V_T)$, which are from the simplified forms of Eqs. (4.4b) and (4.4c).

The numerical solutions of $\alpha(t)$ and $\beta(t)$ are shown in Fig.2-3. The attitude angles $\psi(t)$ and $\theta(t)$ are also obtained from these solutions by Eqs. (4.3a) and (4.3b). Lastly, the moment equation [Eq. (3.2)] is used to get the deflections of three control surfaces. Using stability derivatives as before, Eq.(3.2) yields

$$L_{\delta a}\delta a + L_{\delta r}\delta r = \dot{P} - (I_{ZX}/I_X)(\dot{R} + PQ) \tag{4.6a}$$
$$+[(I_Z - I_Y)/I_X]QR - L_\beta \beta - L_p P - L_r R$$

$$M_{\delta e}\delta e = \dot{Q} + [(I_X - I_Z/I_Y)]RP - (I_{ZX}/I_Y) \tag{4.6b}$$
$$\times(R^2 - P^2) - M_u u - M_w w - M_{\dot{w}}\dot{w} - M_q Q - M_{\delta T}\delta T$$

$$N_{\delta a}\delta a + N_{\delta r}\delta r = \dot{R} - (I_{ZX}/I_Z)(\dot{P} - QR) \tag{4.6c}$$
$$+[(I_Y - I_X)/I_Z]PQ - N_\beta \beta - N_p P - N_r R$$

All the state variables and their derivatives which appear in the right-hand sides of Eqs. (4.6) can be, as mentioned previously, rewritten by (α, β, φ) and their derivatives. By using [$\alpha(t)$, $\beta(t)$, $\varphi(t)$] and [$\psi(t)$, $\theta(t)$] obtained in the prior section and by numerical differentiations (the central differences are used, i.e., $\dot{x}_i = (x_{i+1} - x_{i-1})/2\Delta t$), the numerical values of the right-hand sides of Eqs. (4.6) are all obtained. Then Eqs. (4.6) become linear equations with regard to the controls δa, δe, and δr, so they can be solved easily. The attitude angles and the deflections of the three control surfacesare shown in Figs. 4 and 5 in the next section as well.

The airplane and its aerodynamic data are those of A4D with the configuration of 15,000 ft, 634 ft/s (375kt) and is presented in McRuer et. al.[4]. No limits are set to the magnitude of the angles α, β and δa, δe, δr

in advance. The aerodyamic forces and moments used here are simple linear combinations by the stability derivatives. If more complicated but accurate aerodynamics are available, they can also be used, and there will be no problem on calculation procedure.

4.2. EXACT SOLUTION

Until now the assumption has been made that the the control surfaces generate only moments not forces. Although the forces generated by the control surfaces are negligible compared with the main aerodynamic forces of each axis, lift $Z_w w$ and β-sideforce $Y_v v$, they must be taken into account in the strict sense. Now In order to rewrite the force equations as precisely as possible, the force terms which were ommited in the preceding calculation are added. Then, the force equations [Eq. (4.2)] become

$$-g\sin\theta + X_u u + X_w w + X_{\delta T}\delta T + X_{\delta e}\delta e = 0 \tag{4.7a}$$

$$g\cos\theta\sin\varphi + Y_v v + Y_{\delta r}\delta r + Y_{\delta a}\delta a = 0 \tag{4.7b}$$

$$g(\cos\theta\cos\varphi - 1) + Z_u u + Z_w \dot{w} + Z_{\delta e}\delta e = 0 \tag{4.7c}$$

These equations correspond to Eq. (3.16). Since Eqs. (4.7b) and (4.7c) include also unknown controls δa, δe, δr, they cannot be regarded as simultaneous (differential) equations of α and β as before. However, the values of δa(t), δe(t) and δr(t) have been obtained numerically in the basic solutions. Therefore, when they are substituded into Eqs. (4.7b) and (4.7c),

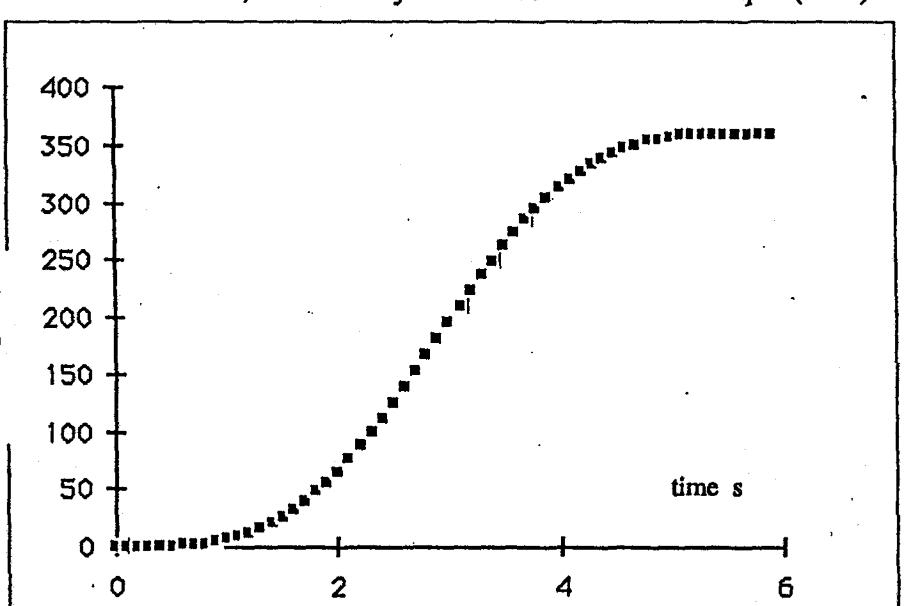


Figure 3.1 Bank angle v.s. time

these become simultaneous equations of $\alpha(t)$ and $\beta(t)$ again. Accordingly,

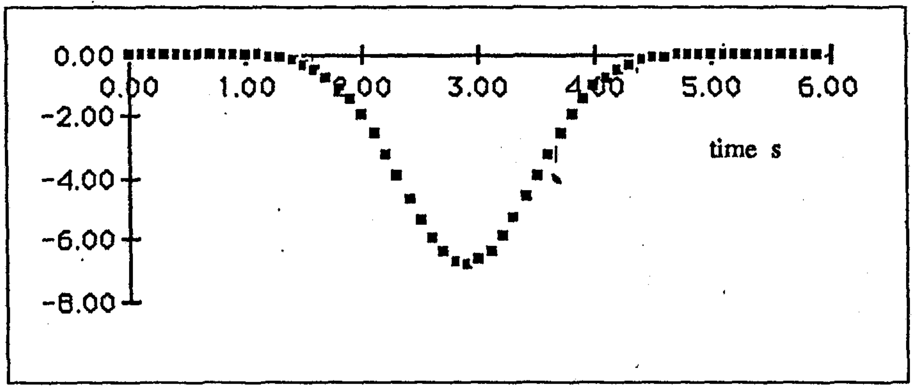


Figure 4.2 Angle of attack v.s. time (Basic)

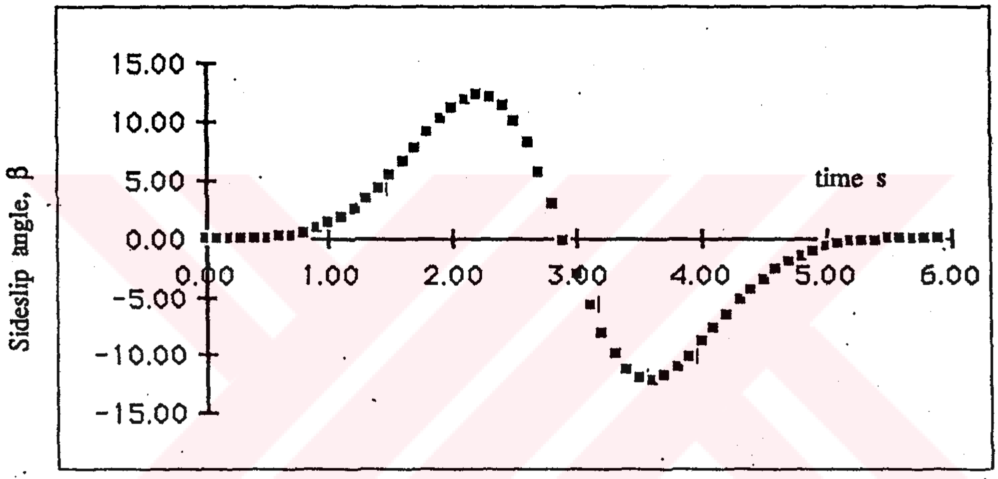


Figure 4.3 Sideslip angle v.s. time (Basic)

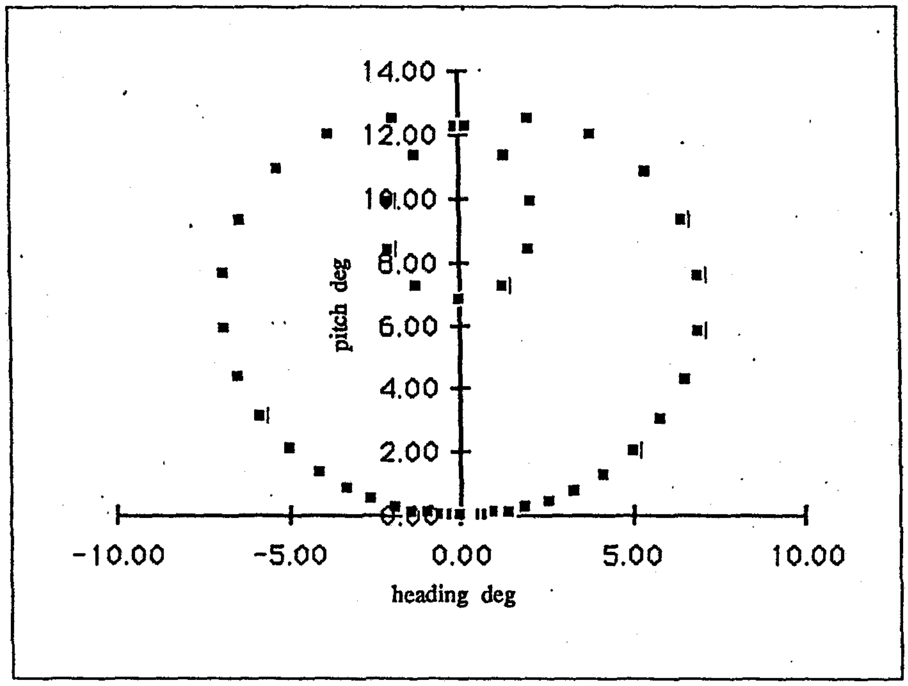


Figure 4.4 Heading and pitch attitude (Basic)

$\alpha(t)$ and $\beta(t)$ can be calculated by using the succesive iteration as before. The first aproximations this time are the basic solutions. Thus, the solutions $\alpha(t)$ and $\beta(t)$ obtained here are more exact than the basic solutions.

Then, following the procedures of the preceeding section, $[\psi(t), \theta(t)]$ and $[\delta a(t), \delta e(t), \delta r(t)]$ can be re-obtained. These are thus more exact than the basic solutions. Furthermore, if these $\delta a(t)$, $\delta e(t)$ and $\delta r(t)$ are substituted into Eqs. (4.7) again and Eqs. (4.7) are recalculated, the solutions $[\alpha(t), \beta(t)]$ and, accordingly, $[\psi(t), \theta(t)]$ will be still more exact.

Here, it can be seen that by repeating these procedures between the force equations [Eqs. (4.7)] and moment equations [Eqs. (4.6)], the

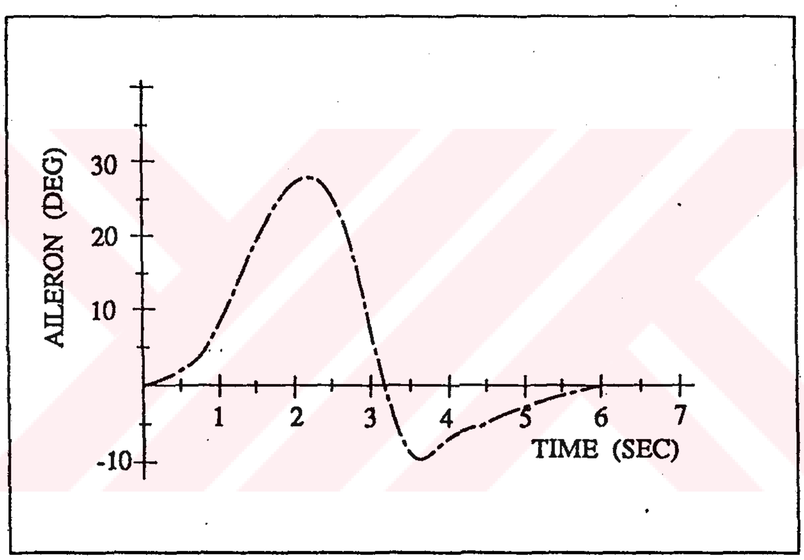


Figure 4.5.a Aileron deflection (Basic)

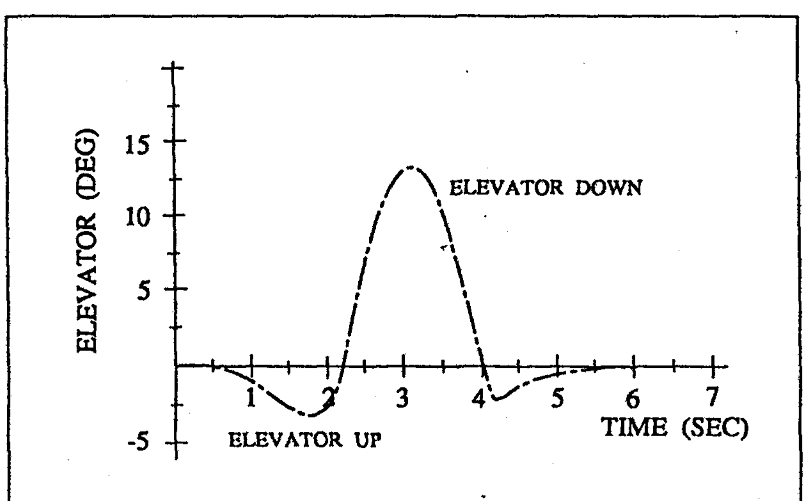


Figure 4.5.b Elevator deflection (Basic)

solutions obtained

will get more and more exact before finally converging. The exact solutions here have this sense, and they are nothing but the solutions by solving the force equations and the moment equations simultaneously by means of a numerical method. Chart 3.1-3.2 is a simplified block diagram describing these procedures. The exact solutuons [$\alpha(t)$, $\beta(t)$], [$\psi(t)$, $\theta(t)$]

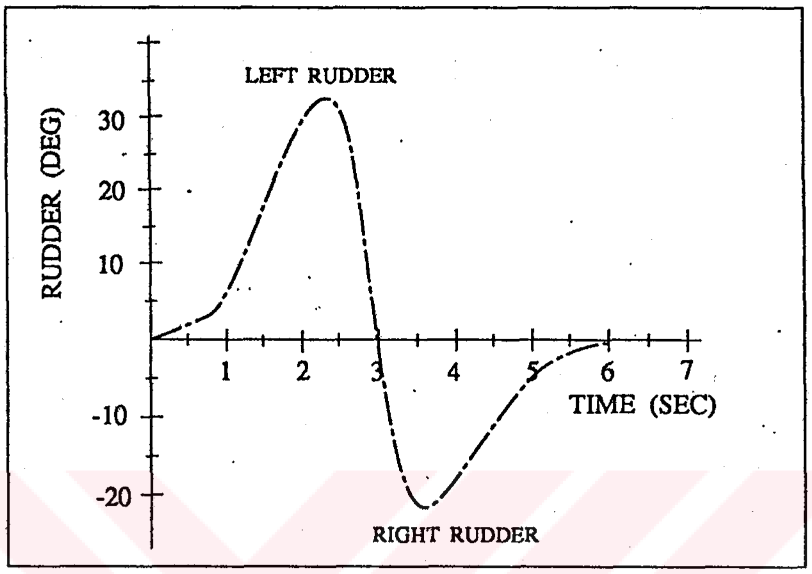


Figure 4.5.c Rudder deflection (Basic)

and [$\delta a(t)$, $\delta e(t)$, $\delta r(t)$] are shown together with the basic solutions in Figures 4.2, 4.3 and 3.4.

4.3. DISCUSSION OF NUMERICAL RESULT

As shown in Figs. 4.6 and 4.7, the exact solutions differ considerably from the basic solutions. This is due to the assumption made when calculating the basic solutions, due to the ommited force terms in the force equations, particularly $Y_{\delta r}\delta r$. As is seen from the conventional mechanism between the rudder and the sideslip, this force $Y_{\delta r}\delta r$ is usually in the opposite direction to the β-sideforce $Y_V v$. Owing to this decrease of total side force, the magnitude of β calculated as the exact solution becomes larger than that of the basic solution. The increase of three control surface deflections are affected by this increase of β (even elevator). It seems that it is the same with $Z_{\delta e}\delta e$. But, because the ratio $Z_{\delta e}\delta e/Z_W w$ is very small in comparison with $Y_{\delta r}\delta r/Y_V v$, there is almost no influence of $Z_{\delta e}\delta e$ on the exact solutions except slight changes of the magnitude of α and attitude angles due to that. The present flight example is a special case in that rudder is used great deal. However, in most flight maneuvers, especially in

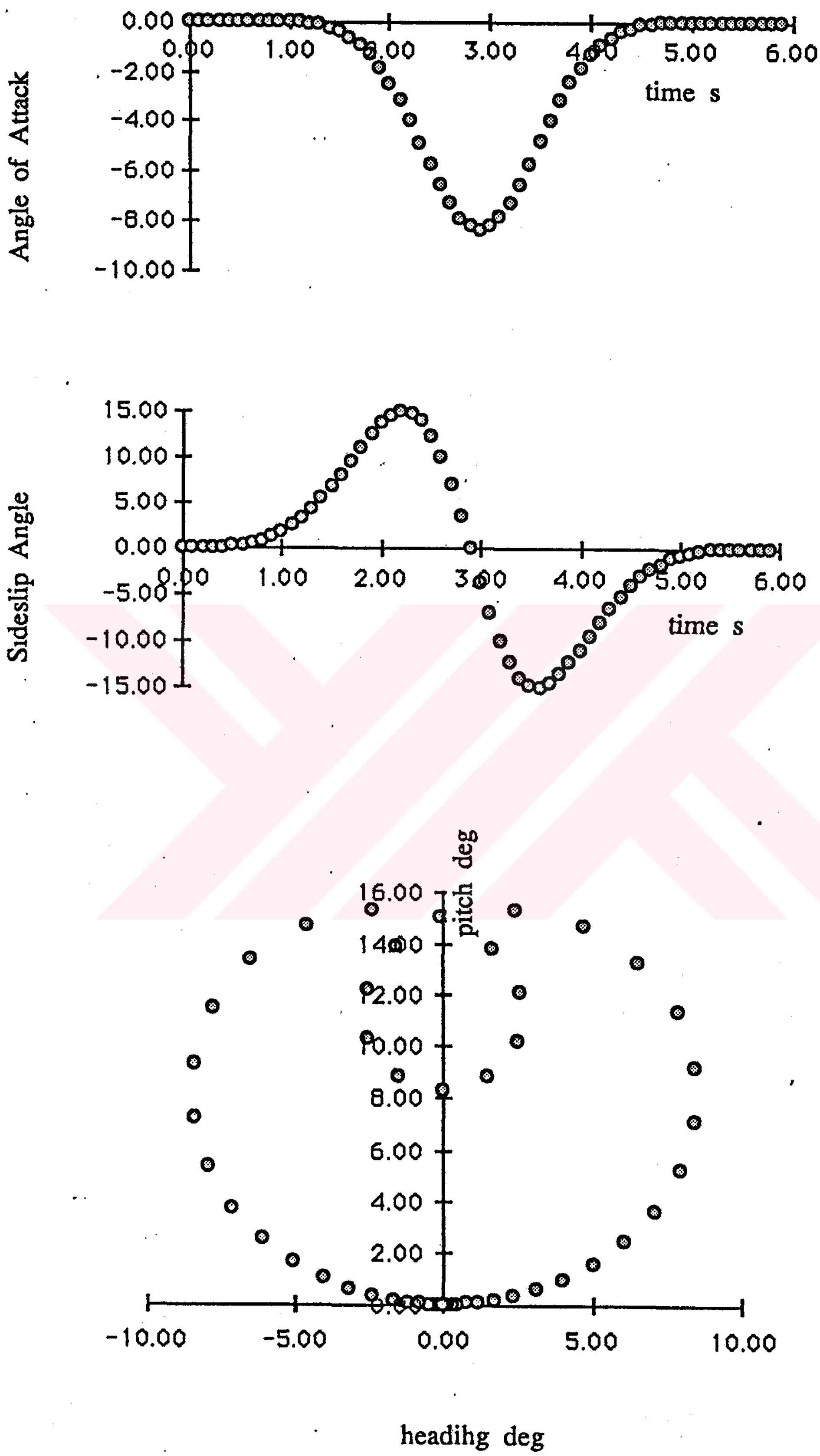
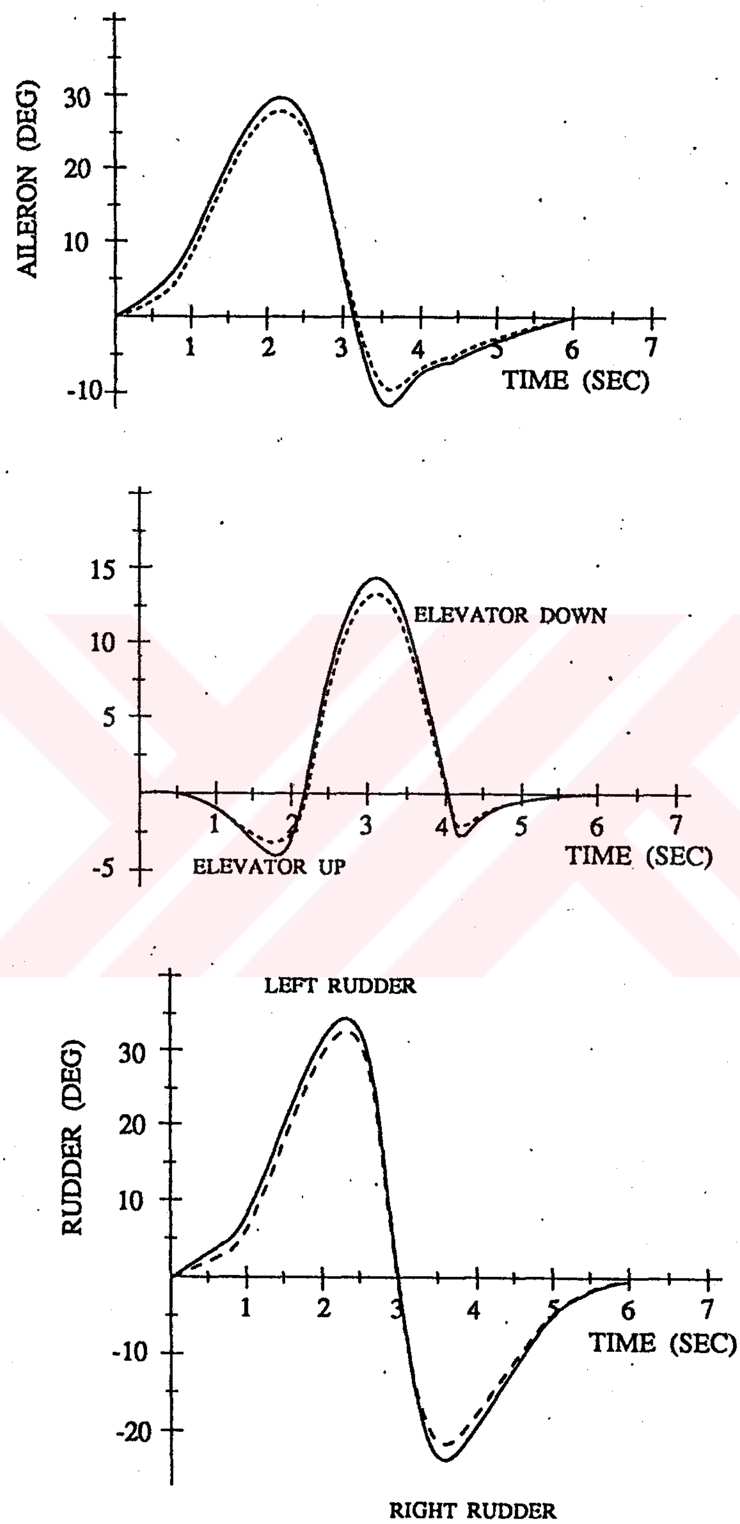


Figure 4.6 Exact solutions of 360° roll

Comparison of exact and basic solutions



coordinated flights, the rudder is an auxilary control and not used so much. Therefore, in general, the basic solutions are not as rough as this example. It is of interest here to discuss the maneuver in the numerical example. Even though it seemed that the aileron control as the main in this maneuver. The three controls all have important parts as shown in Fig. 4.7. In particular, as mentioned above, it is a distinct point of this type of maneuver that the rudder is largely used to produce the β-sideforce. Obviously, this is because the lift can not oppose gravity by itself, and the sideforce, normal to the lift, is required to fly the airrplane along the straight trajectory with rolling. As the result, the conventional airplane's nose must be moved up and down and right and left during this maneuver as shown in Fig.4. Of course, this maneuver is not coordinated. In actual aileron rolls, on the other hand, the coordination has weight rather than the straight trajectory. Therefore, as was expected, the controls of this maneuver are quite different from ordinary aileron rolls.

4.4. CONCLUDING REMARKS FOR 360° ROLL

As is well known, to fly an airplane along a desired flight path, the lift, (sometimes) sideforce, bank and power must be controlled at each moment so that the sum of the external forces can cause the acceleration determined by the flight trajectory. Therefore, for the steering control of the airplane, the triplet (α, β, φ) are fundemental variables which are closely connected with the lift, sideforce and bank, respectively, and are controlled through the moments of the three control surfaces (indirect force control). From this simple point of view, an inverse problem of the airplane general motion has been developed. 3.30

The treatment of (α, β, φ) as key variables makes the interpretation of the complicated maneuver much easier. By the strict distinction between the attitude angles and the path angles, and because of the consideration of the degree of freedom of (α, β, φ), this approach is applicable not only to conventional maneuvers but also to new or unusual maneuvers.

The general approach has been derived from the fundemental idea of the motion and controlof an airplane. The basic solutions in the numerical example were presented as an example of this general approach. However, there may be cases where the basic solutions are not enough to be exact in actual calculations. For those cases, the exact solutions based on the basic solutions were also presented.

4.5. BANK-TO-BANK ROLLING ABOUT X-AXIS

A sinusoidal desired bank command was used in this maneuver [7]. Here the desired or constrained output variables are

$$V_d(kT) = 87.9 \text{ m/s} \tag{3.31}$$

$$\theta_d(kT) = \alpha[(k-1)T]$$

$$\psi_d(kT) = 0$$

$$\varphi_d(kT) = \phi_M \cdot \sin[\Omega_r (kT)]$$

where ϕ_M is the amplitude of the roll angle (10 deg for α_{trim} = 24 deg) and Ω_r = 0.1 Hz. $\theta = \alpha$ is the condition for maintaining level flight. The angle of attack just listed corresponds to the initial trim angle of attack for the constrained velocity in (eq.3.30). This angle of attack is not constrained and varies throughout the maneuver. Inverse simulation results are shown in figure 4.8. (3.32)

4.6. COORDINATED TURN

Here the coordinated turn is defined in terms of desired bank angle time histories with zero sideslip, much as a pilot would define it. The desired or constrained bank angle is given by

$$\varphi_d(kT) = \begin{cases} \dfrac{\varphi_s}{16\cdot 57.3}[f_1(kT)]rad & for\, 0 \le kT \le 5s \\ \dfrac{\varphi_s}{57.3} rad & for\, 5 \le kT \le 15s \\ \dfrac{\varphi_s}{16\cdot 57.3}[f_2(kT)]rad & for\, 15 \le kT \le 20s \end{cases}$$

where

$$f_1(kT) = \left[\cos 3\pi\left(\frac{kT}{5}\right) - 9\cos\pi\left(\frac{kT}{5}\right) + 8\right]$$

$$f_2(kT) = \left\{1 - \frac{1}{16}\left[\cos 3\pi\left(\frac{(kT-15)}{5}\right) - 9\cos\pi\left(\frac{(kT-15)}{5}\right) + 8\right]\right\}$$

where φ_s = 60 deg for α_{trim} = 7.9 deg. The remaining three constraint equations are

$$V_d(kT) = 152 \text{ m/s}$$

$$\theta_d(kT) = \theta_{trim}$$

$\beta_d(kT) = 0$

the results are shown in figure 4.9. This maneuver is the basic turn maneuver and different turn radius can be achievable by changing the bank angle limit. That is, turns with different hardness are possible to obtain with same formula. As can be seen from the graphs of the solutions, rudder usage is decreased substantially which is the main characteristics of a coordinated flight.

CHAPTER 5
INTEGRATION INVERSE METHOD

5.1. OVERVIEW

Inverse simulation problems usually divided into three categories, depending upon whether the number of outputs are greater then, equal to, or less than the number of inputs. In the first category, it would appear that there is no way of developing general solution technique, therefore, other two cases are of importance, second as nominal and third as redundant problems types [7], [8].

The input-output relationship for a nonlinear dynamic system can be represented as a transformation rom a vector u(kT) in some input space to a vector y(kT) in an output space, i.e.,

$$y(kT) = G[u(kT)] \tag{5.1}$$

where G represents the mapping function and T the discretization interval, which is typically an order of magnitude larger than the integration interval used in simulation algorithm itself. Considering the output as desired trajectory $y_D(kT)$, Eq (5.1) can be rewritten as

$$G[u(kT)] - y_D(kT) = 0 \tag{5.2}$$

Newton's method is generally recognized as the most powerful algorithm for solving systems shown in Eq.(5.2). As an iterative prcedure, the solution can be obtained defining an error vector F_E, as

$$F_E[u(kT)] = G[u(kT)] - y_D(kT) \tag{5.3}$$

and finding succesively better estimates of u(kT) that allow $F_E \to 0$, as n, the iteration index, increass. For the system of Eq. (5.2), Newton's method becomes

$$u_{n+1}(kT) = u_n(kT) - (J\{G[u_n(kT)]\})^{-1} \, . \, F_E[u_n(kT)] \tag{5.4}$$

where n is the iteration index, $F_E[\ ]$ defines the error vector between the actual and desired output at time kT, and J[ ] is the jacobian matrix defined as (5.5)

$$J\{G[u_n(kT)]\} = Y_U \tag{5.6}$$

where ny is the number of outputs and

$$Y_U|_{ij} = \partial y_i(kT) \,/\, \partial u_j(kT) \qquad i,j = 1{:}ny$$

The partial derivatives in Eq.(5.6) cannot, in general, be determined analytically. In the algorithm shown here, the partial derivatives are approximated as

$$\partial y_i(kT) \,/\, \partial u_j(kT) \approx [y_i(u_j + \Delta u_j)|_{(k+1)T} - y_i(u_j)_{(k+1)T}] \,/\, \Delta u_j \tag{5.7}$$

where Δu_j represents a perturbation in uj and be created as some fixed percentage of u_j. Although Eq.(5.7) is an obvious example of numerical differentiation, the fact that it is a differentiation with respect to control inputs rather than time, meaning that the derivative can be determined from simulation with an accuracy that does not comprimise the solution of the inverse simulation problem.

The entire inverse simulation process begins with the definition of a desired output vector y_D over a specified time interval. The vector thus defined, must be in a reachable domain. i.e. it must be able to be generated by phisically realizable conttrol inputs. typically, the trajecory in question begins from a trimmed flight ondition. It is not necessary to know the values of the control inputs in the trim condition, instead the inverse simulation algorithm itself, can determine these. The simulation process is thus onsist of a normal forward simulation algorithm with an integration step size $\Delta t \ll T$ in which the control inputs are changed, in step fashion, every T seconds on the basis of a solution of Eq.(5.4).

5.2. COMPARISON OF DIFFERENTIATION AND INTEGRATION INVERSE METHODS

What has been termed the differentiation inverse method refers (5.8a) to techniques presented in chapter 3, whereas the integration inverse (5.8b) method referrs to that proposed in chapter 5. a very simple formulation allows a brief comparison of the two procedures. Let the linear vehicle model as follows:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$

let nx be the number of states, nu the number of inputs, and ny the number of outputs. It is desired to determine u(t) for $t_0 \leq t \leq t_1$ such that y(t) = y_D(t) for $t_0 \leq t \leq t_1$.

5.2.1 DIFFERENTIATION INVERSE METHOD (5.9)

Considering the ordinary case in which nx = nu = ny. One begins by differentiating y_D and then finding $\dot{x}(t) = C^{-1}y_n(t)$. Substituting the resulting $\dot{x}(t)$ into Eq.(5.8) and rearranging yields:

$$u(t) = B^{-1}[C^{-1}y_D(t) - Ax(t)]$$

The solution of Eq. (5.9) is undertaken at each time step. If other casessuch as nx > nu, and nx > ny. Obvious complications arise as the differentiated output equation can no longer provide the derivatives of all of the state variables needed in solving Eq. (5.8a) for u(t). The necessity of calculating the derivatives of these intermediate state variables in iterative fashion at each time step using numerical differentiation is the primary disadvantage of the differentiation approach. The inverse solution was found to be very sensitive to initial value chosen.

5.2.2. INTEGRATION INVERSE METHOD

The integration inverse method is summarized in the beginning of chapter 5. These equations and the associated inverse simulation algorithim outlined in Fig.1 provide a genealized approach to inverse simulation. The same condtion, ny <= nu still applies as in differentiation case.

As with any formulation involving Newton's method, a problem that can arise with the integration inverse method concerns multiple solutions of Eq.(5.2). For example, considering the case where a solution of Eq.(5.2) lies near a relative minimum of F_E[u(kT)]. It is possible that another solution may exist in the neighborhood of the minimum and that the solution of Eq.(5.2) by way of Eq.(5.4) may alternate between these solutions for succesive time steps in the simulation. It is stated that this problem appeared to arise in most of the maneuvers analyzed. Therefore, input filtering becomes necessary causing real time solutions impossible.

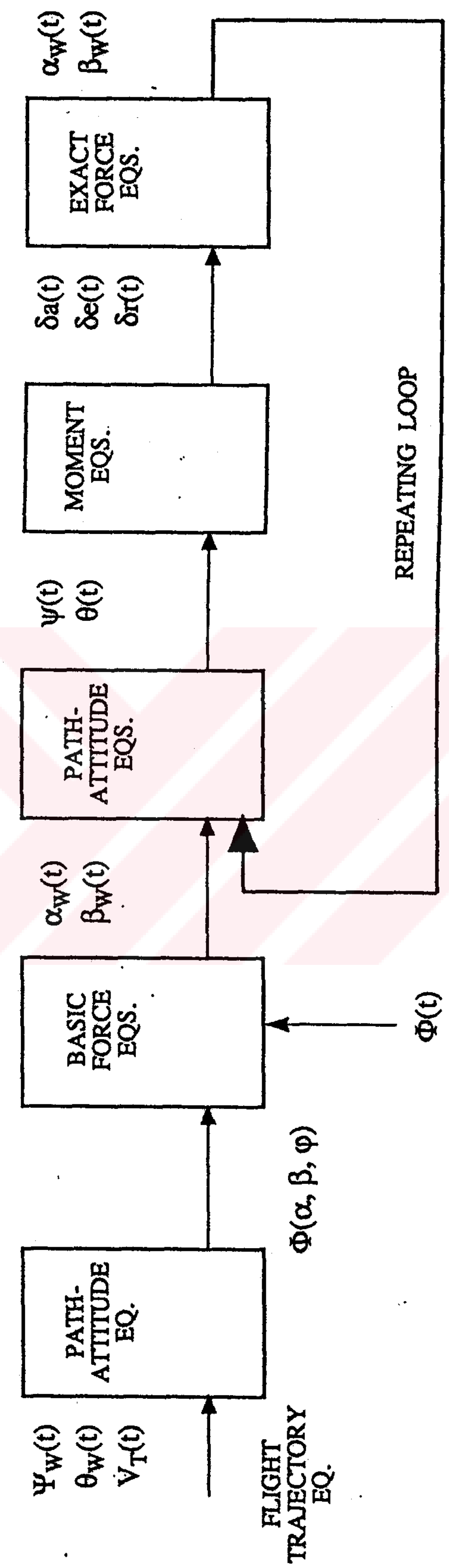
5.3. INPUT FILTERING

In some of the inverse solutions, the control inputs obtained from the simulation exhibited low-amplitude, high-frequency oscilations superimposed on the low-frequency waveform It is hypothesized that this phenomenon may be caused by the multiple solution problem just discussed. These oscilation were filtered by the vehicle dynamics and had some impact on the sdlution quality. As proposed by Hess [7], these oscilations were removed by using fifth order, low-pass digital filter in the simulation process. The filter has been designed had a cutoff frequency of 10 rad/s. The control inputs were passed through the filter twice (forward and backward) to avoid time shifting in the output data.

5.4. CONCLUSION

In this thesis general inverse problem is introduced and numerical solutions given for three different maneuvers. The differences between integration and differentiation method are reviewed. Differentiation method is chosen for its real time solution advantage so that it can be applicable in aircraft simulators. The results showed that by establishing an interpolation method between basic and exct solutions, it is possible to obtain almost exact solutions directly from basic ones. Using artificial intelligence techniques it is also possible to implement a learning mchanism with self reasoning that will allow more complicated maneuvers.

Chart 1



FLIGHT TRAJECTORY EQ.
$\Psi_W(t)$
$\theta_W(t)$
$V_T(t)$
PATH-ATTITUDE EQ.
$\Phi(\alpha, \beta, \varphi)$
BASIC FORCE EQS.
$\Phi(t)$
$\alpha_W(t)$
$\beta_W(t)$
PATH-ATTITUDE EQS.
$\psi(t)$
$\theta(t)$
MOMENT EQS.
$\delta a(t)$
$\delta e(t)$
$\delta r(t)$
EXACT FORCE EQS.
$\alpha_W(t)$
$\beta_W(t)$
REPEATING LOOP

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BIBLIOGRAPHY

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