EVALUATION OF ANISOTROPIC CONSTITUTIVE MODELS CONSIDERING NON-ASSOCIATED FLOW RULE

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ABSTRACT

The non-associated flow rule (Non AFR) approach neglects the constraint of equality of plastic potential and yield function enforced by the associated flow rule (AFR) assumption and can be an alternative but efficient approach to improve the accuracy of anisotropic yield functions. Accordingly, the yield function and plastic potential function are assumed to be independent and used to describe the elastic limit and plastic strain rate direction, respectively. The main objective of this paper is to develop a generalized finite element formulation of stress integration method for non-quadratic yield functions and potentials with mixed non- linear hardening under non-associated flow rule. Different approaches to analyze the anisotropic behavior of sheet materials are compared in this paper.

ÖZET

Birleşik olmayan akma kuralı (Non AFR) yaklaşımı, birleşik akma kuralı (AFR) varsayımı tarafından uygulanan plastik potansiyel ve akma fonksiyonu eşitliği kısıtını ihmal eder ve anizotropik akma fonksiyonlarının doğruluğunu iyileştirmek için alternatif ama etkili bir yaklaşım olabilir. Buna göre akma fonksiyonu ve plastik potansiyel fonksiyonu bağımsız olarak varsayılır ve sırasıyla elastik limit ve plastik şekil değiştirme yönünü tanımlamak için kullanılır. Bu araştırmanın temel amacı, kuadratik olmayan akma fonksiyonları için gerilme entegrasyon yönteminin genelleştirilmiş bir sonlu elemanlar formülasyonunu ve birleşik olmayan akma kuralı altında karışık doğrusal olmayan pekleşme özelliğine sahip potansiyelleri geliştirmektir. Bu çalışmada malzemelerinin anizotropik davranışını analiz etmek için farklı yaklaşımlar karşılaştırılmıştır.

INTRODUCTION

For decades many researchers have focused on the anisotropic constitutive models of metallic sheets which are all rolled products, whereas rare anisotropic models are adopted in hot rolling simulations. Conventional plasticity models assume that shapes of yield surfaces are remain fixed throughout plastic deformation. The foundation of most anisotropic constitutive models has been based on the associated flow rule (AFR) hypothesis which states that the flow rule is associated with the yield criterion [1,2]. In other words, the yield function is also the potential for plastic strain rate in AFR based models and most of the proposed yield functions are coupled with AFR [3,4]. On the other hand, various studies described the difficulty of the AFR concept in dealing with highly anisotropic materials. Phillips et al. [5] tested thin-walled aluminum tubes and a complex distortional forms of yield surface was obtained with a corner form in the front and a flat zone. Accordingly, starting from Hill's [6] quadratic anisotropy model, which has been widely used for analysis of orthotropic metals, various yield functions have been proposed to describe the anisotropy of metallic sheets. The non-associated flow rule approach neglects the constraint of equality of plastic potential and yield function enforced by the AFR assumption and can be an alternative but efficient approach to improve the accuracy of anisotropic yield functions. Accordingly, the yield function and plastic potential function are assumed to be independent and used to describe the elastic limit and plastic strain rate direction, respectively. The main objective of this paper is to develop a generalized finite element formulation of stress integration method for non-quadratic yield functions and potentials with mixed non-linear hardening under non-associated flow rule.

CONSTITUTIVE MODEL

In the non-associated flow rule, the yield criterion to define anisotropic yield stresses is described by

$$\varphi^p\left(\boldsymbol{\sigma}, q_{iso}^p, \boldsymbol{\alpha}\right) = f\left(\boldsymbol{\sigma}-\boldsymbol{\alpha}\right) - q_{iso}^p(\xi^p) \tag{1}$$

$$d\boldsymbol{\varepsilon}^p = d\bar{\xi}^p \frac{\partial g}{\partial\left(\boldsymbol{\sigma}-\boldsymbol{\alpha}\right)} \tag{2}$$

$$dw_{iso} = \left(\boldsymbol{\sigma}-\boldsymbol{\alpha}\right) : d\boldsymbol{\varepsilon}^p = q_{iso}^p d\xi^p = q_{pot} d\bar{\xi}^p \tag{3}$$

Where q_{pot} is the quantity to describe the size of the plastic potential, which is defined by the following potential criterion as similarly done for the stress in the yield criterion.

$$g\left(\boldsymbol{\sigma}-\boldsymbol{\alpha}\right) - q_{pot}\left(\bar{\xi}^p\right) = 0 \tag{4}$$

$$\frac{d\xi^p}{d\bar{\xi}^p} = \frac{q_{pot}}{q_{iso}^p} = \frac{g\left(\boldsymbol{\sigma}-\boldsymbol{\alpha}\right)}{f\left(\boldsymbol{\sigma}-\boldsymbol{\alpha}\right)} \tag{5}$$

Note that $\boldsymbol{\alpha}$ is the same back stress shared by both yield and potential function.

The linear elastic constitutive law for the Jaumann (or co-rotational) stress rate of the Cauchy stress is

$$d\boldsymbol{\sigma} = \mathbf{C}^e d\boldsymbol{\varepsilon}^e = \mathbf{C}^e\left(d\boldsymbol{\varepsilon} - d\boldsymbol{\varepsilon}^p\right) \tag{6}$$

Here, $C^e, d\varepsilon^e, d\varepsilon^p$ are the elastic stffness tensor, the elastic strain increment and the plastic strain increment, respectively. Putting Eq. **Hata! Başvuru kaynağı bulunamadı.**into Eq. (6) then,

$$d\boldsymbol{\sigma} = \mathbf{C}^e\left(d\boldsymbol{\varepsilon} - \frac{\partial g}{\partial\left(\boldsymbol{\sigma}-\boldsymbol{\alpha}\right)} \frac{f\left(\boldsymbol{\sigma}-\boldsymbol{\alpha}\right)}{g\left(\boldsymbol{\sigma}-\boldsymbol{\alpha}\right)} d\xi^p\right) \tag{7}$$

Yield criterion described in Eq.(1) results in by using consistency condition

$$
\begin{aligned}
&\frac{\partial f}{\partial(\boldsymbol{\sigma}-\boldsymbol{\alpha})}:d(\boldsymbol{\sigma}-\boldsymbol{\alpha})-\frac{\partial q_{iso}}{\partial \xi^{p}}d\xi^{p}= \\
&=\frac{\partial f}{\partial(\boldsymbol{\sigma}-\boldsymbol{\alpha})}:\left[\mathbf{C}^{e}\left(d\boldsymbol{\varepsilon}-\frac{f(\boldsymbol{\sigma}-\boldsymbol{\alpha})}{g(\boldsymbol{\sigma}-\boldsymbol{\alpha})}d\xi^{p}\frac{\partial g}{\partial(\boldsymbol{\sigma}-\boldsymbol{\alpha})}\right)-\frac{\partial \boldsymbol{\alpha}}{\partial \xi^{p}}d\xi^{p}\right]-\frac{\partial q_{iso}}{\partial \xi^{p}}d\xi^{p}=0
\end{aligned} \tag{8}
$$

FINITE ELEMENT FORMULATION

Firstly, we should define the variational formulation, which is based on Hellinger-Reissner potential by putting aside the hardening effect to simply notation, the latter can be defined as:

$$
\begin{aligned}
\Pi(\boldsymbol{\sigma},\mathbf{u})&=\int_{\Omega}\psi^{e}(\boldsymbol{\varepsilon}^{e})+\psi^{d}(\boldsymbol{\varepsilon}^{d},\mathbf{D})d\Omega-\int_{\Gamma}\bar{\mathbf{t}}\mathbf{u}d\Gamma-\int_{\Omega}\mathbf{f}_{v}\mathbf{u}d\Omega \\
&=\int_{\Omega}\left(-\chi^{e}(\boldsymbol{\sigma})-\chi^{d}(\boldsymbol{\sigma},\mathbf{D})+\boldsymbol{\sigma}:(\nabla^{s}\mathbf{u}-\boldsymbol{\varepsilon}^{p})\right)d\Omega-\int_{\Gamma}\bar{\mathbf{t}}\mathbf{u}d\Gamma-\int_{\Omega}\mathbf{f}_{v}\mathbf{u}d\Omega
\end{aligned} \tag{9}
$$

At equilibrium, the minimal free energy condition is valid for the potential stationarity. We subsequently obtain the two following equations:

$$
\begin{aligned}
\frac{\partial \Pi}{\partial \mathbf{u}}\delta\mathbf{u}&=\int_{\Omega}\left(\boldsymbol{\sigma}\nabla^{s}\delta\mathbf{u}\right)d\Omega-\int_{\Gamma}\left(\bar{\mathbf{t}}\delta\mathbf{u}\right)d\Gamma-\int_{\Omega}\left(\mathbf{f}_{v}\delta\mathbf{u}\right)d\Omega=0 \\
\frac{\partial \Pi}{\partial \boldsymbol{\sigma}}\delta\boldsymbol{\sigma}&=\int_{\Omega}\left\{-\frac{\partial \chi^{e}}{\partial \boldsymbol{\sigma}}-\frac{\partial \chi^{d}}{\partial \boldsymbol{\sigma}}+\nabla^{s}\mathbf{u}-\boldsymbol{\varepsilon}^{p}\right\}\delta\boldsymbol{\sigma}d\Omega=0
\end{aligned} \tag{10}
$$

NUMERICAL ANALYSIS

Comparison of predicted and experimental T/C curves is plotted in Fig. 1. It is seen that the kinematic and isotropic hardening model result respectively in under- and over-estimation of the stress upon load reversal. The overestimation of the hardening by using isotropic hardening is simply due to missing the Bauschinger effect.

Table 1. AA5754-O material properties

Parameter	Unit	Isotropic (Swift)	Kinematic (two-term Chaboche)	Mixed hardening (Zang)
σ_0	MPa	94.8	94.8	94.8
k	MPa	452.6	–	–
n	–	0.34	–	–
$\bar{\varepsilon}_0^p$	–	0.01	–	
Q	MPa	–	–	126.4
b	–	–	–	16.1
C_1	MPa	–	1997.3	4665.3
γ	–	–	23	212
C_2	MPa	–	409.6	204.8

As seen in Fig. 1 the under- and overestimation due to respectively kinematic and isotropic hardening is increased at higher pre-strains. It is also noticed that the permanent softening effect is more pronounced at higher pre-strains. As opposed to the isotropic and kinematic hardening models, application of Zang's mixed hardening model leads to stress versus plastic strain curves that are very close to the experimental ones. It must be noted that all the simulations for AA5754-O material are based on non-AFR Hill 1948.

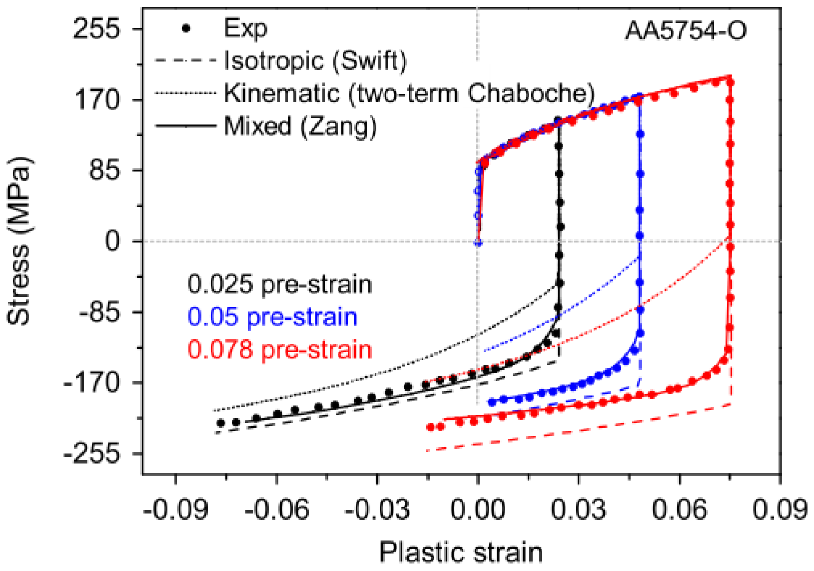

Figure 1. Comparison of experimental and simulated T/C hardening curves at different pre-strains for AA5754-O using various hardening models and non-AFR Hill 1948 anisotropic model.

RESULTS

We developed associated and non-associated flow rules of Hill1948 with a mixed hardening of Zang et al. The AFR and non-AFR based Hill 1948 with uniaxial yield stresses are investigated. An excellent accuracy in prediction of directional yield stresses was achieved by the non-AFR. A slight improvement compared to AFR Hill 1948 was observed by using its non-AFR counterpart.

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