

ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL

THERMAL BUCKLING ANALYSIS OF VARIABLE ANGLE TOW PLATES



M.Sc. THESIS

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Aeronautics and Astronautics Engineering Programme

JANUARY 2023

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İSTANBUL TEKNİK ÜNİVERSİTESİ ★ LİSANSÜSTÜ EĞİTİM ENSTİTÜSÜ

**DEĞİŞKEN AÇILI KOMPOZİT PLAKLARIN TERMAL BURKULMA
ANALİZİ**

YÜKSEK LİSANS TEZİ

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Date of Defense : 20 January 2023





To my spouse and family,



FOREWORD

I would like to thank my dear wife Sena for her unconditional and endless help as well as her love. I would also like to express my gratitude to my esteemed thesis advisor, Demet Balkan, for her highly motivated assistance. Finally, I am grateful to my family for supporting me no matter the circumstances.

January 2023

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ABBREVIATIONS

[A-B-D]	: Composite Stiffness Matrices
BC	: Boundary Condition
CFD	: Computational Fluid Dynamics
CLPT	: Classical Laminated Plate Theory
CPET	: Classical Plane Elasticity Theory
CPT	: Classical Plate Theory
DOF	: Degrees Of Freedom
FEM	: Finite Element Method
FSDT	: First order Shear Deformation Theory
HSDT	: High order Shear Deformation Theory
SS	: Simply Supported
VAT	: Variable Angle Tow
ULT	: Ultimate
UD	: Unidirectional





SYMBOLS

E	: Elasticity Modulus
G	: Shear Modulus
R	: Yield Stress
ρ	: Density
ν	: Poisson Ratio
$\sigma_1, \sigma_2, \sigma_3$	: Principal Normal Stresses
$\tau_{23}, \tau_{31}, \tau_{12}$	: Principal Shear Stresses
$\epsilon_1, \epsilon_2, \epsilon_3$	: Principal Normal Strains
$\gamma_{23}, \gamma_{31}, \gamma_{12}$	: Principal Shear Strains
w	: Out-of-Plane Displacement
ξ, η	: Natural Coordinates
J	: Jacobian Operator



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THERMAL BUCKLING ANALYSIS OF VARIABLE ANGLE TOW PLATES

SUMMARY

Many industrial vehicles, especially aviation sector, are exposed to thermal loads due to high temperature changes. In these structures, which are designed with the principle of lightness and strength, composite materials are frequently preferred due to their outstanding mechanical properties. Composite materials are usually formed into plates for ease of production and intended use. These plates can face many loading and failure. For instance, composite plates may show unstable behavior and buckle under thermal loads. Scientists and engineers who want to keep composite structures away from this failure have started to use variable angle composites in this field, based on the parameters affecting buckling. Here the idea of angle propagation is dominant, which depends on the orientation angle of the edge and center part of the plate, rather than having the same angle in the same layer throughout the plate. Thus, the material will be used effectively and maximum buckling strength will be provided with an optimized angle orientation and minimum material usage. Many tests and calculations have been made for this. However, for a specific design of a variable angle composite, material design, calculations are not fast enough. One of the main reasons for this is that package programs using the finite element method are only suitable for modeling straight-angle composites and are not suitable for performing calculations for variable-angle composites. From this point of view, it is focused on creating a code in which the necessary calculations will be made without the need to use package programs.

This study focuses on a code that can calculate the critical buckling temperature of variable-angle composite plates. This code requires geometric dimensions, mechanical and thermal properties of the material, boundary conditions, and the desired number of elements as input. After the inputs are provided, the code; using the finite element theory, the desired number of nodes and elements is formed to include shape functions. The code associates the geometric properties of these elements with the material properties and makes them suitable for calculations. It determines the different orientations in the in-plane position with the help of the function containing the edge and center angles of the plate. In this step, based on the Classical Lamination Theory, the material matrices, transformation matrices, etc. are calculated for A, B, D stiffness matrices and thermal loads.

After calculating the A, B, D matrices of the composite, the strain-displacement matrices per unit length are generated based on the finite element method. As a result of processing these matrices with the appropriate A, B, D matrices, the stiffness matrix of the plate is obtained. The calculated stiffness matrix is one of the two basic matrices that decides whether the plate will buckle or not. A similar procedure was followed for the geometric stiffness matrix, which is another stiffness matrix required for the buckling analysis. In the geometric stiffness matrix, unlike the material stiffness matrix, the initial stress matrix is used instead of the material matrix. Thus, the effects of stress were tried to be added to the structure.

The eigen value calculation is required for the estimation of the buckling failure that occurs as a result of the relations of the two stiffness matrices formed with each other. This factor is obtained by the code as a result of the necessary matrix operations, taking into account the boundary conditions. The eigenvalue is calculated as much as the unconstrained total degrees of freedom. It is generally sufficient to observe the 10 most critical of them. Since a buckling analysis is performed under thermal load, the output of this analysis will be the critical buckling temperature. This value indicates how far the structure is from buckling. By considering this value, the people responsible for the design can determine what the working environmental conditions should be by ensuring that the structure stays away from the unsafe region.

This code was tried to be verified with some studies in the literature (Duran, Babu), analytical formulas, and finite element model created using MSC Patran application. It was built in order from the basics to the details. The results of the validation analyses taken with low error rates in different categories are presented in tabular form. Thus, the code has become suitable for use in thermal buckling analysis of variable angle composites. Thus, the code has reached the capability to determine whether the structure is safe or under which conditions it will be safe in designs that are created with variable angle composites and need to control buckling under temperature.

It has been seen with the help of the code that the variable angle composite modeled using an example material, geometry and orientation reaches temperatures higher than the highest critical buckling temperature of a straight composite with the same properties. Thus, calculating the buckling factors of variable-angle composites, which makes straight-angle composites more effective in buckling, which is the purpose of preparing the code, has been achieved. Some studies that can be added to the code for future studies are also specified in the code.

DEĞİŞKEN AÇILI KOMPOZİT PLAKLARIN TERMAL BURKULMA ANALİZİ

ÖZET

Havacılık sektörü başta olmak üzere birçok endüstriyel araç yüksek sıcaklık değişimlerinden kaynaklı olarak termal yüklere maruz kalmaktadır. Hafiflik ve mukavimlik prensibiyle tasarlanan bu yapılarda öne çıkan mekanik özelliklerinden dolayı sık sık kompozit malzemeler tercih edilmektedir. Kompozit malzemeler üretim kolaylığı ve kullanım amacı için genellikle plak şekline getirilirler. Bu plaklar pek çok yükleme tipi ve hasar ile karşı karşıya kalabilirler. Örneğin kompozit plaklar termal yükler altında kararsız davranış gösterip burkulmaya maruz kalabilir. Özellikle kompozit yapıları bu hasar durumundan uzak tutmak isteyen bilim insanları ve mühendisler burkulmayı etkileyen parametrelerden yola çıkarak değişken açılı kompozitleri bu alanda kullanmaya başlamışlardır. Burada plak boyunca aynı tabakada aynı açiya sahip olmaksızın plağın kenar ve orta kısmının yönelim açısına bağlı olan bir açı yayılımı fikri öne çıkmıştır. Böylelikle malzeme daha efektif olarak kullanılacak, optimize edilmiş bir açı yayılımı sayesinde minimum malzeme kullanımı ile maksimum burkulma mukavimliği sağlanacaktır. Bunun için birçok test yapılmış ve hesaplamalar gerçekleştirilmiştir. Ancak paket programların kabiliyetleri ve kullanımın çok yaygın olmaması sebebiyle belirli bir açı ve malzeme tasarımı için hesaplamalar yeterince hızlı gerçekleşmemektedir. Bunun başlıca sebeplerinden biri sonlu elemanlar yöntemini kullanan paket programların sadece düz açılı kompozitler için modelleme yapılmasına uygun olup değişken açılı kompozitler için hesaplamalar gerçekleştirmeye uygun değildir. Buradan yola çıkarak paket programları kullanmaya gerek kalmadan gerekli hesaplamaların yapılacağı bir kod oluşturulması üzerinde durulmuştur.

Bu çalışmada değişken açılı kompozit plakların kritik burkulma sıcaklığını hesaplayabilecek bir kod üzerine odaklanılmıştır. Bu kod girdi olarak geometrik ölçüler, malzemenin mekanik ve termal özellikleri, plak için geometrik sınır koşulu ile istenen eleman sayısına ihtiyaç duymaktadır. Girdiler sağlandıktan sonra kod; sonlu elemanlar teorisini kullanarak istenen sayıda düğüm noktası ve elemanı şekil fonksiyonlarını kapsayacak şekilde oluşturup geometriyi kapsayacak bir sonlu eleman modeli hazırlar. Kod bu elemanların geometrik özellikleri ile malzeme özelliklerini ilişkilendirerek hesaplamalara uygun hale getirir. Plağın kenar ve orta açılarını içeren literatürden elde edilmiş fiber açısı fonksiyonunun yardımı ile düzlem içi konumundaki farklı oryantasyonları belirler. Burada adım ölçüsü olarak elemanların düğüm noktaları istasyon olarak kullanılmıştır. Bu adımda Klasik Laminasyon Teorisine dayanarak A,B,D direngenlik matrisleri ve termal noktasal yükleri hesaplamak için gerekli olan malzeme matrislerini, dönüşüm matrislerini, açılmal matrisleri vb. hesaplar.

Kompozitin A,B,D matrislerinin hesaplanmasının ardından sonlu elemanlar yöntemine dayanarak birim boy başına uzama-deplasman matrisleri oluşturulur. Bu matrislerin uygun A,B,D matrisiyle işleme girmesi sonucu (örneğin eğilme hareketi için D matrisi, membran hareketler için A matrisinin kullanılması) plağın direngenlik matrisi elde edilir. Bu matris işlemleri esnasında gerekli gauss noktaları ve şekil fonksiyonları elemanların lokal konumlarının global konumlarıyla aralarındaki ilişkilerini açıklamaya çalışır. Aynı zamanda gauss ağırlıkları da düğüm noktalarının

bir eleman içindeki ilişkilerini açıklar. Hesaplanan direngenlik matrisi plağın burkulmaya uğrayıp uğramayacağına karar verdiren temel iki matristen biridir. Burkulma analizi için gerekli bir diğer direngenlik matrisi olan geometrik direngenlik matrisi için de benzer bir yol izlenilmiştir. Matris hesapları sırasında yine gerekli gauss lokasyonları, şekil fonksiyonları, gauss ağırlıkları gibi ara işlemler kullanılıp sonucun iyileştirilmesine (integral hesabının daha doğru yapılmasına) çalışılmıştır. Geometrik direngenlik matrisinde materyal direngenlik matrisinden farklı olarak malzeme özellikleri yerine ilk gerilim matrisi kullanılmıştır. Bu gerilim matrisi sıcaklık farkında kaynaklı olarak malzemenin termal katsayılarına bağlı olacak şekilde oluşan termal yüklerin oluşturduğu bir matristir. B birim boy başına uzama-deplasman matrisi yerine ise G geometrik birim boy başına uzama-deplasman matrisi kullanılmıştır. Böylelikle yapıya gerilmenin etkileri de eklenmeye çalışılmıştır.

Oluşturulan iki direngenlik matrisinin birbirleriyle olan ilişkilerinin sonucu olarak meydana gelen burkulma hasarının tahmini için yük faktörü hesabı gereklidir. Bu faktör sınır şartları da göz önünde bulundurularak yapılan gerekli matris işlemlerinin sonucu olarak kod tarafından elde edilmektedir. Burada temel hesap geometrik direngenlik matrisinin malzeme direngenlik matrisine olan oranla bir determinant işlemine tabii tutulup aradaki kat sayının öz değer olarak atanmasıdır. Sınırlandırılmayan toplam serbestlik derecesi kadar öz değer hesaplanmaktadır. Bu yüzden çıkan öz değer sayısı serbestlik derecesi, sınır koşulu ve düğüm sayısıyla doğrudan ilintilidir. Çalışmalarda en kritik olan öz değer gözetilmektedir fakat daha kapsamlı bir analiz için özdeğerlerin en kritik 10 tanesini gözlemlemek genel olarak yeterli olmaktadır. Termal yük altında bir burkulma analizi yapıldığı için bu analizin çıktısı kritik burkulma sıcaklığı olacaktır. Bu değer yapının burkulmaya ne kadar uzak olduğunu gösterir. Tasarımdan sorumlu kişiler bu değeri göz önünde bulundurarak yapının güvensiz alandan uzak durmasını sağlayarak çalışma koşullarının neler olması gerektiğini belirleyebilirler.

Oluşturulan bu kod literatürde bulunan bazı çalışmalar (Duran, Babu), analitik formüller ve MSC Patran uygulaması kullanılarak oluşturulan sonlu elemanlar modeli ile doğrulanmaya çalışılmıştır. Sırası ile temelden detaya gidilecek şekilde yapılmıştır. Açıklamak istenirse; statik bir yükleme altındaki izotropik malzemenin doğrulanması, termal yükler altında ve farklı a/h (genişlik/kalınlık) oranlarına sahip çarpaz, açılı ve karma kompozitlerin doğrulanması, son olarak da değişken açılı kompozitlerin doğrulanması olarak sıralanabilir. İzotropik malzemeler Patran uygulaması üzerinden ve analitik formüllerle doğrulanmıştır. Çarpaz tabakalı ve açılı tabakalı kompozitler Patran programından FEM modellemesi ve Babunun çalışmalarıyla doğrulanmıştır. Karma-kompleks tabakalı kompozitler Patran programından FEM modeli ve Duranın çalışmaları ile doğrulanmıştır. Değişken açılı kompozitlerin doğrulanmasında ise FEM programlarında modelleme yapılması için bir kod daha yazılarak programlarda değişken açılı kompozit modellemesi yapılmasına olanak sağlanmıştır. Burada da hem FEM programları hem de Duranın çalışmaları kullanılmıştır. Farklı kategorilerde düşük hata oranlarıyla alınan doğrulama analizlerinin sonuçları tablo şeklinde sunulmuştur. Böylelikle kod, değişken açılı kompozitlerin termal burkulma analizinde kullanmaya uygun hale gelmiştir. Böylelikle kod, değişken açılı kompozitlerle oluşturulmuş ve sıcaklık altındaki burkulmanın kontrol edilmesine gerek duyulan tasarımlarda yapının güvende olup olmadığını veya hangi şartlar altında güvende olacağını belirleyebilecek kapasiteye ulaşmıştır.

Örnek bir malzeme, geometri ve oryantasyon kullanılarak modellenen değişken açılı kompozitin aynı özelliklere sahip düz bir kompozitin en yüksek kritik burkulma

sıcaklığından daha yüksek sıcaklıklara ulaştığı kodun yardımı ile görülmüştür. Böylelikle kodun hazırlanış amacı olan düz açılı kompozitleri burkulma konusunda daha efektif hale getiren değişken açılı kompozitlerin burkulma faktörlerini hesaplama hedefine ulaşılmıştır. Gelecek çalışmalar için koda eklenebilecek bazı çalışmalar da kod içerisinde belirtilmiştir.





1. INTRODUCTION

Aircraft or spacecraft are exposed to temperature changes for various environmental reasons. As an example, the travels of high-speed aircraft in the atmosphere cause its temperature rises due to a phenomenon called aerodynamic heating [23]. Restrained from in-plane expansion, when geometrically perfect plates are heated slowly and uniformly, they usually develop compressive stresses and then buckle at a given temperature [24]. This temperature value is called the critical thermal buckling temperature and engineers have to consider this value in their designs, especially for aircraft operating at high Mach numbers. Some of the parts that are subject to thermal buckling are composite plates.

Composite materials have been used in many different areas from the past to the present. The main factors that make composite materials preferred and that are tried to be developed further and some properties which cause disadvantages are as in Table 1.1.

Table 1.1 Properties of composites [5].

Advantages	Disadvantages
Low specific weight (ρ)	Brittle
High specific stiffness (E/ρ)	Limited shelf lives
High specific strength (R/ρ)	Hard to storage
High resistance	Difficult to process material
Low thermal conductivity	Orientation is important

Examples of early uses of the composite material were the creation of plywood by gluing strips of wood at different angles, building houses with plaster including mud/straw and wood/clay, and making archery bows by using bamboo, silk, cattle tendons, horns, and pine resin [3]. In the sectoral sense, composite materials are generally used in the shown areas in Table 1.2.

Table 1.2 Industry Applications of composites [39].

Main Applications	
Aerospace and aviation	Military equipment
Automotive industry	Marine structures

In order to illustrate, some composite applications which are installed for Boeing 777-200 are showed in Figure 1.1.

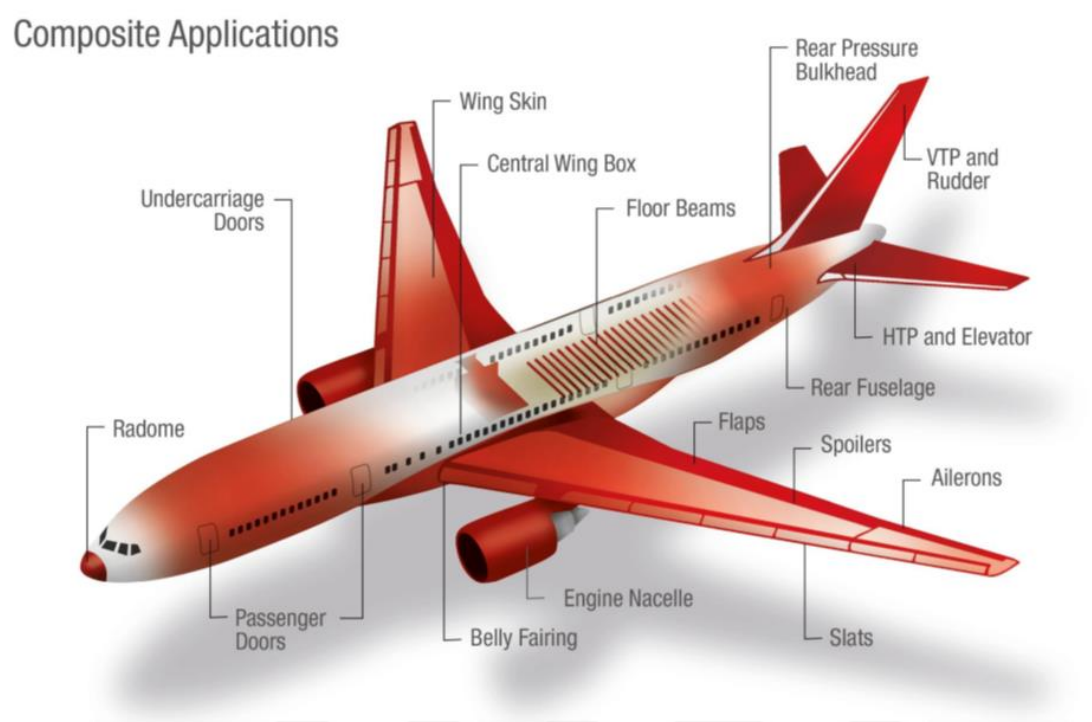


Figure 1.1 Composite Application of Boeing 777-200 [7].

As can be seen in the Figure 1.2, the percentage of composite materials used in aircraft has started to increase, especially as production and repair techniques have improved.

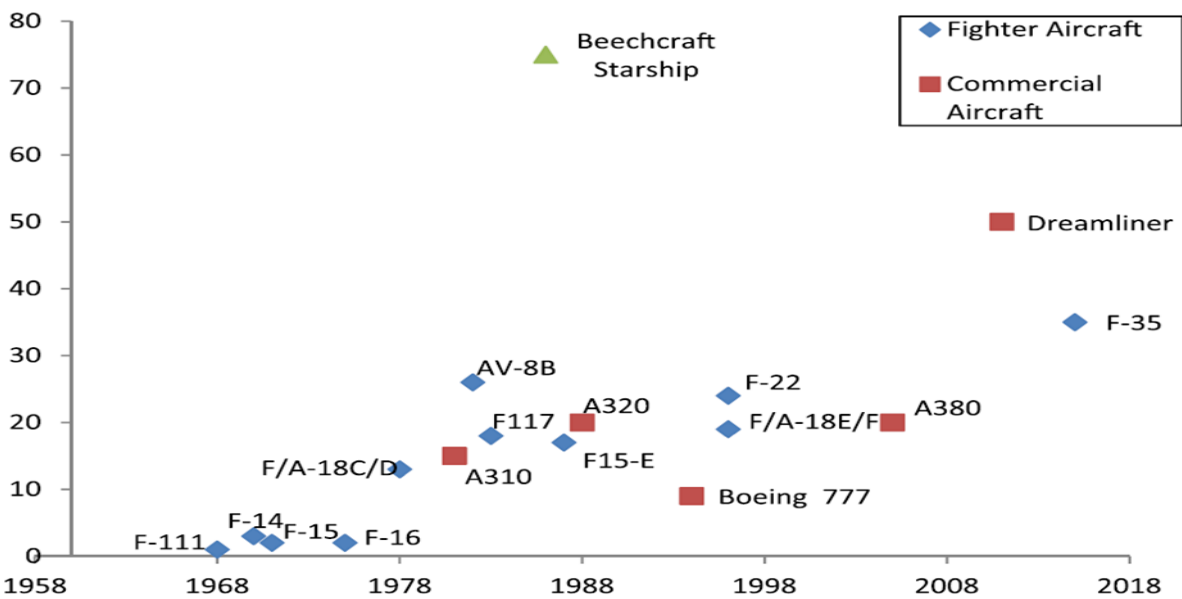


Figure 1.2 Time-varying percentage of aircraft mass composed of composites [45].

One of the disadvantages of composite material is that the direction is very important as mentioned before. While the mechanical values in the main direction are very good in classical fibers, the values in the lateral direction are not effective. As seen in the table below, while the longitudinal mechanical properties of graphite epoxy, which is a very common composite material, are very strong, those in the transverse direction are relatively weak. This situation can be examined in Table 1.3 over the properties of graphite epoxy, which is a very common composite material.

Table 1.3 Mechanical Properties of Graphite Epoxy [28].

Property	Symbol	Units	Graphite/Epoxy
Fiber Volume Fraction	V_f		0.70
Longitudinal Elastic Modulus	E_1	GPa	181
Transverse Elastic Modulus	E_2	GPa	10.30
Ultimate Longitudinal Tensile Strength	$(\sigma_1^T)_{ult}$	MPa	1500
Ultimate Transverse Tensile Strength	$(\sigma_2^T)_{ult}$	MPa	40
Ultimate Longitudinal Compressive Strength	$(\sigma_1^C)_{ult}$	MPa	1500
Ultimate Transverse Compressive Strength	$(\sigma_2^C)_{ult}$	MPa	246

Scientists and manufacturers have sought some ways to overcome this orientation problem and have found variable angle tow (VAT) as a solution.

1.1 Purpose of Thesis

VATs are based on a principle that the fibers are placed in different orientations in the plane of each ply. The main purpose here is to achieve the optimum level by obtaining variable stiffness with variable angle tows. In this way, buckling resistance and resistance to other loads can also be improved [55].

In this study, it will be discussed what the response of different types of variable angle composites will be to the thermal buckling condition. The main scope will be developing a code that models the material and finds critical thermal buckling temperature at simply support boundary conditions, validation will be done with the studies in the literature, and the finite element method models.

By the help of this study, the working conditions of composite VAT plates in thermal situations can be easily concerned and analyzed.

1.2 Literature Review

In this section, previous studies will be examined, different details of these studies will be presented and the necessary parts will be utilized.

Sun and Hsu are one of the milestones in this regard. In their study in 1989, the classical lamination theory and shear deformation theory were used and compared. A simply supported cross-ply rectangular plate thermal stability (buckling) analysis was done under a uniform temperature distribution [47].

Gurdal and Olmedo tried to create a material model in 1992 [20]. In the study, the orientation is tried to be a function of x-axis. As boundary conditions certain angles are defined for the start and end edges of x-axis of the plate. Equilibrium equations for the numerical solution were used and solved with Hermitian. In closed form, solutions were taken with boundary conditions for 3 different cases. Case 1 conditions are; a plate with free transverse edges, loaded by uniform uniaxial end shortening along the x direction. Case 2 conditions are; when the v displacement of the transverse edges, ($y = \pm b/2$), is fixed. Case 3 conditions are; Variable stiffness plate is loaded under the boundary conditions for the case I, the transverse edges do not remain straight.

Another study by Gurdal and Olmedo, includes a variable stiffness concept with a variation of tow orientations. Their boundary conditions are the same as the previous study. Variable stiffened plates were analyzed both using a numerical model and using closed-form solutions for the elastic response [21].

In the study of Matsunaga, higher-order deformation theory is used for the thermal buckling of both sandwich plates and cross-ply laminated composite. The boundary condition is preferred as the simply supported case. In order to prevent overestimation of critical temperatures, prebuckling deformation is neglected [33].

The study of Gurdal, Tatting, and Wu includes a path of fiber that has a function of both x and y. Classical lamination theory is used to get stiffness matrix. In-plane and buckling analysis are done, the Rayleigh-Ritz method is used for the eigenvalue. Pre-buckling analysis includes 2 cases; fiber orientation is a function of only x or only y. The simply supported plates are analyzed for these 2 cases, and the results are compared with the straight fibers [22].

Setoodeh, Abdalla, IJsselmuiden, and Gurdal studied on the design of composite plates which handle maximum buckling load. In order to get a critical load, first-order Taylor series expansion is used. The reciprocal analysis is done. For efficient optimization of buckling problems, sensitivity analysis is performed to know the sensitivity of the buckling load parameter to changes in design variables. In order to update the design as an optimization, the eigenvalue system is put in the first-order Taylor expansion [42].

Zhao, Lee, and Liew studied functionally graded plates' thermal and mechanical buckling analysis. In this study, material properties are not constant through the thickness, and effective material properties are changing with a power law. First-order shear deformation theory is used with classical lamination theory. After the stiffness matrices are calculated, the Ritz minimization procedure helps to get the eigenvalue. In addition doing analyze on solid plates, some holed plate analysis is done and the relation of hole size between critical load is shown [57].

Scientists from Taiwan; Shiau, Kuo, and Chen, also studied thermal buckling behavior. The study includes material modeling with classical lamination theory. Total strain energy is used to assemble stiffness matrices to get eigenvalue under simple supported edges. Different stack-ups are used as cross-ply and angle-ply laminates. Some aspect ratio changes are included with the same boundary condition. As a validation, the articles in the literature are used; for instance, the study of Matsunaga. Optimal critical thermal buckling temperature is determined from solution as a function of material, aspect ratio, and stacking sequence [44].

Groh and Weaver, from Great Britain, studied on buckling of variable angle tows with variable thickness including transverse shear effects. Three-dimensional plates with varying thicknesses are used. Sheared tow thickness is assumed as a function of shear angle and unsheared tow thickness. The reason for their demand for computing shear effects is, composite laminated more affected by shear effects than isotropic materials. It is preferred to reduce laminates into one laminate, it is assumed that firstly the plate is single layer non-symmetric flat, then it behaves like a curved plate with different thickness variations. Elliptic reference surface is used for design and analysis. Donnel's nonlinear theory is used for these shallow shells. The differential quadrature method is used for eigenvalue analysis. For validation different shell types are used with FEM and numerical solution [19].

Scientists from University of Michigan; Duran, Fasanella, Sundararaghavan, and Waas studied a variable angle composite square plate's thermal buckling. Since the geometry is square, their research is not dependent on the aspect ratio. Similarly to other studies, fiber angle is a function of position. The stacking sequence is restrained to be symmetric and balanced for this study. Simply supported square plates are used for analysis. Buckling analysis is done with out-of-plane equations. Results are found numerically, and the validation of this calculation is done with straight fibers. After that, the results are tried to be optimized with orientation changes. Finally, different material models are also tried [13].

Ghomshei and Mahmoudi studied thermal buckling with nonuniform temperature distribution. As a solution method, the differential quadrature approach is preferred. Firstly in-plane forces are found, then buckling analysis is done. The results found by the differential quadrature approach are supported by FEM results [16].

Choudhary and Mahato studied the thermal buckling of laminated composite plates with an elliptical cutout. The study aims to show the shape effects of cutouts on the thermal buckling of composite plates. The study is supported by FEM models. Variables are specified as cutout size, position, and orientation. The laminates are created with the following orientations; $[0/90]_{2s}$, $[15/-75]_{2s}$, $[30/-60]_{2s}$, and $[45/-45]_{2s}$. It has been seen that critical temperature is increasing with the increasing orientation value of cutouts for $[0/90]_{2s}$, $[45/-45]_{2s}$. There are different maximum and minimum locations for other laminates. The study gives the critical temperature for different laminates and cutouts as graphics [10].

In the study of Yahea and Majeed, they investigated the thermal buckling of composite plates under simply support boundary condition with simple four variable plate theory. No shear factor is added to the theory, and Navier's double trigonometric sequence is preferred as a solution method. The results are compared with the other theories' results. Similar results are obtained with small differences. The advantage of the theory is being simple with four unknowns. For instance, even first-order shear deformation theory has five unknowns with a longer solution time [56].

In 2008, Abu studied the finite element method for the thermal buckling analysis of anisotropic plates. Eight layers as angle ply laminate is preferred. Three type of boundary conditions which are simply supported clamped and hinged are searched.

The validation of the model is started with the simply supported isotropic plates. After that, convergence tests are set for hinged angle-ply composites and clamped cross ply composites. It has been seen that 8x8 mesh number is converged. Effects of layer number, anisotropy level, angle of orientation, boundary condition, plate thickness, and aspect ratio are searched [2].

In the study of Meyers and Hyer, they investigate the thermal buckling and post-buckling of symmetrical composite plates. The research covered fixed and sliding boundary conditions. Rayleigh-Ritz formulation is preferred to do numerical analysis. In addition to unit temperature change analysis, linearly varying temperature analysis is also implemented in the search. Under an imperfection presence such as initial flatness is investigated. The oriented versions of common layups are considered. It has been seen that the skewness will decrease the buckling response of the plate [34].

Murphy and Ferreira studied the thermal buckling of rectangular plates. The energy models are considered in this study, and the clamped plate is used. Measuring the critical temperature is focussed. It has been distinguished that the measurement of critical temperature is a complex process because of the geometric imperfections of structures. In order to make an easy measurement, a saddle node bifurcation is preferred. Von Karman's theory determined the temperature ratio between the measurement points as a function of imperfection size. In this way, measurement becomes easy and accurate. The study is validated with experiments [38].

Thornton, who is one of the earliest researchers of thermal buckling, described the metallic and composite plate behaviour for elastic thermal buckling. Shallow shells and curved plates are also considered in the study. Some analytical formulas for thermal buckling are tried to set by using heat transfer, plate theory, and test data. Thornton has provided many simple formulations in order to utilize them in thermal buckling. After the normal plates, shells and cylindrical shells are also investigated by him. The geometry and topology for different shapes are implemented in to study [51].

Verma and Singh studied the thermal buckling of composite plates with random selections of geometry and material. High-order shear deformation theories are used in the study. Uncertain material and geometric properties were modeled as principal random variables. By using these properties, they investigate the effects of modulus ratio, orientation, aspect ratio, and boundary condition. It has been seen that even small

variations have huge effects on critical buckling temperature. In order to validate the results of the study, Monte Carlo simulation and literature survey are used. This theory is used for probability studies. Since there are many variations to set a structure, the theory provides to calculate probability distributions [52].

NASA engineers Gossard, Seide, and Roberts studied the thermal buckling of plates in 1952. In their study, large deflection plate theory is used in order to obtain deformations of plates as an approximated method. Their boundary condition was simply supported and temperature distribution is ‘tentlike’ on the plate. Under the deflections and temperature with a range, they reached similar test results to the study [18].



2. MODELLING of COMPOSITE PLATE

2.1 Stacking, Lamina, and Laminate

Kaw's book and many different sources mentioned, in order to perform a mechanical analysis of the laminate, it is necessary to establish the foundation of the lamina first [28]. Lamina is a very thin composite layer (around 0.125mm) and laminates are formed by combining these laminae in the direction of thickness as can be seen in Figure 2.1.

Laminae are composed of fibers and matrices. When examining these structures, the properties of a part corresponding to any of the fibers or matrices may not be the same as another part. Therefore, the analysis of laminae is very difficult and homogeneity should be assumed during the macro mechanical analysis of lamina.

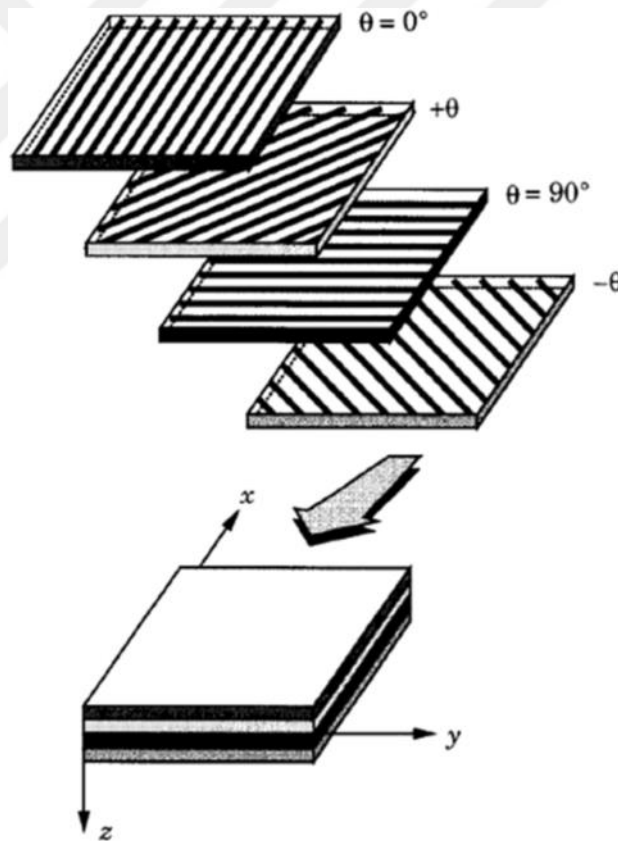


Figure 2.1 Stacking of lamina for laminate [50].

A special type of laminate, a cross-ply laminate, is an arbitrary stack of laminae that has only 0° or 90° oriented fibers and there is no limit to symmetry [46]. Another special laminate, angle-ply laminates, is a type that has an arbitrary number of layers (n) where each layer has the same thickness and material. The laminate consisting of

laminae with varying fiber orientations of $+\theta$ and $-\theta$ can be symmetrical or antisymmetrical. If all of the laminae have the same orientation, this type of laminate is a unidirectional laminate. All these three types of stacking can be seen in Figure 2.2.

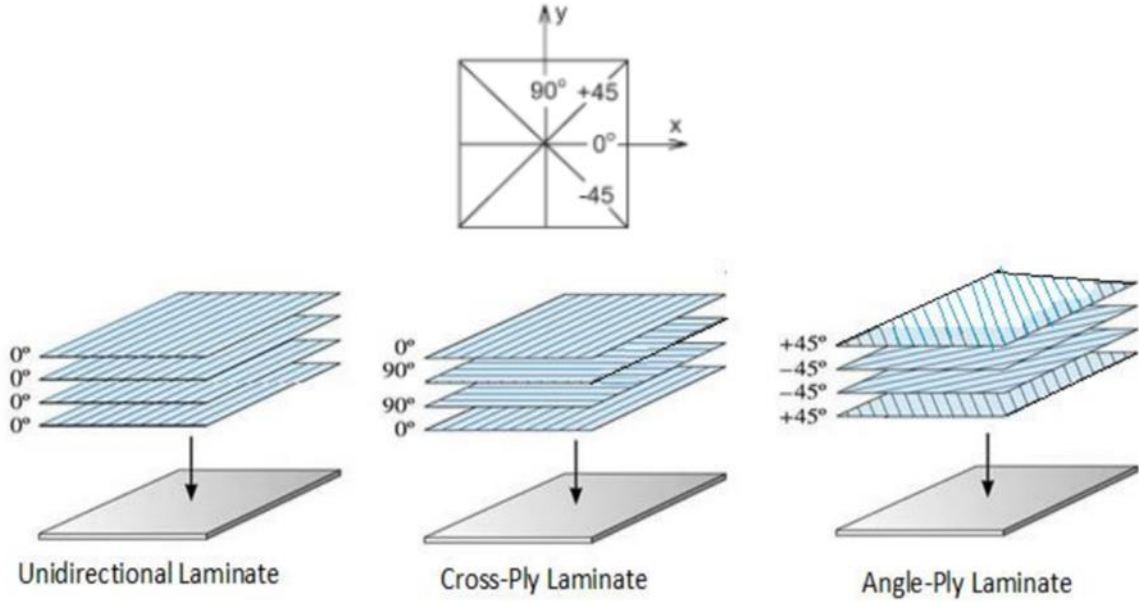


Figure 2.2 Symmetric Example of Special Laminates [12].

2.2 Classical Lamination Theory

At this stage, the type of material should be determined and its mechanical analysis should be started. Basically, material types are divided into 5 according to their mechanical properties. These can be summarized as anisotropic, monoclinic, orthotropic, transversely isotropic, and isotropic materials. The type of material changes the number of independent elastic constants and changes the stiffness matrix of the material [28]. In equation (2.1), the C matrix is the rigidity matrix which has 36 dependents.

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix} \quad (2.1)$$

If we take inverse of equation (2.1), it will give the compliance matrix S in equation (2.2).

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{bmatrix} \quad (2.2)$$

As seen, compliance matrix is directly the inverse of rigidity matrix (2.3).

$$[S] = [C^{-1}] \quad (2.3)$$

If material types are examined in detail; the stress-strain relationship shows how many elastic dependent will be get.

$$\sigma_i = \sum_{j=1}^6 C_{ij} \varepsilon_j \quad i = 1 \dots 6 \quad (2.4)$$

$$\begin{aligned} \sigma_4 &= \tau_{23} & \sigma_5 &= \tau_{31} & \sigma_6 &= \tau_{12} \\ \varepsilon_4 &= \gamma_{23} & \varepsilon_5 &= \gamma_{31} & \varepsilon_6 &= \gamma_{12} \end{aligned}$$

The energy for a unit volume is;

$$W = \frac{1}{2} \sum_{i=1}^6 \sum_{j=1}^6 C_{ij} \varepsilon_j \varepsilon_i \quad (2.5)$$

In the equation (2.5), Hooke's law is applied. If the partial derivative is taken, the rigidity parts can be found.

$$\begin{aligned} \frac{\partial^2 W}{\partial \varepsilon_i \partial \varepsilon_j} &= C_{ij} \\ \frac{\partial^2 W}{\partial \varepsilon_j \partial \varepsilon_i} &= C_{ji} \end{aligned} \quad (2.6)$$

Since the arrangements are not important,

$$C_{ij} = C_{ji} \quad (2.7)$$

Equation (2.7) shows that the rigidity and compliance matrix do have not 36 dependents anymore. Now, different material types' matrix dependents can be evaluated easily.

Anisotropic materials will have 21 dependent values instead of 36.

Monoclinic materials will have 13 dependent values.

Orthotropic materials will have 9 dependent values from equation (2.8).

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \quad (2.8)$$

Finally, transversely isotropic and isotropic materials will have 5 and 2 dependents, respectively. If Hooke's law is applied to orthotropic materials, material mechanical properties can be found.

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11}\sigma_1 \\ S_{12}\sigma_1 \\ S_{13}\sigma_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (2.9)$$

Hooke's law and Poisson ratio were;

$$E = \frac{\sigma}{\varepsilon} \quad \text{and} \quad \nu = -\frac{\varepsilon_{lateral}}{\varepsilon_{longitudinal}} \quad (2.10)$$

Therefore, compliance matrix for orthotropic composite materials will be as equation;

$$[S] = \begin{bmatrix} \frac{1}{E_1} & \frac{-\nu_{12}}{E_1} & \frac{-\nu_{13}}{E_1} & 0 & 0 & 0 \\ \frac{-\nu_{21}}{E_1} & \frac{1}{E_1} & \frac{-\nu_{23}}{E_1} & 0 & 0 & 0 \\ \frac{-\nu_{31}}{E_1} & \frac{-\nu_{32}}{E_1} & \frac{1}{E_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \quad (2.11)$$

2.2.1 Hooke law for composites

A plane can be considered to be under plane stress if the plate is thin enough and has no out-of-plane loads. This condition expresses the equality of equation (2.12).

$$\sigma_3 = 0 \quad \tau_{31} = 0 \quad \tau_{23} = 0 \quad (2.12)$$

If equation (2.12) is applied to Hooke's law;

$$\begin{aligned} \varepsilon_3 &= S_{13}\sigma_1 + S_{23}\sigma_2 \\ \gamma_{31} &= 0 \quad \gamma_{23} = 0 \end{aligned} \quad (2.13)$$

Now Hooke's law can be applied to uni-multi directional composites which are illustrated at Figure 2.3.

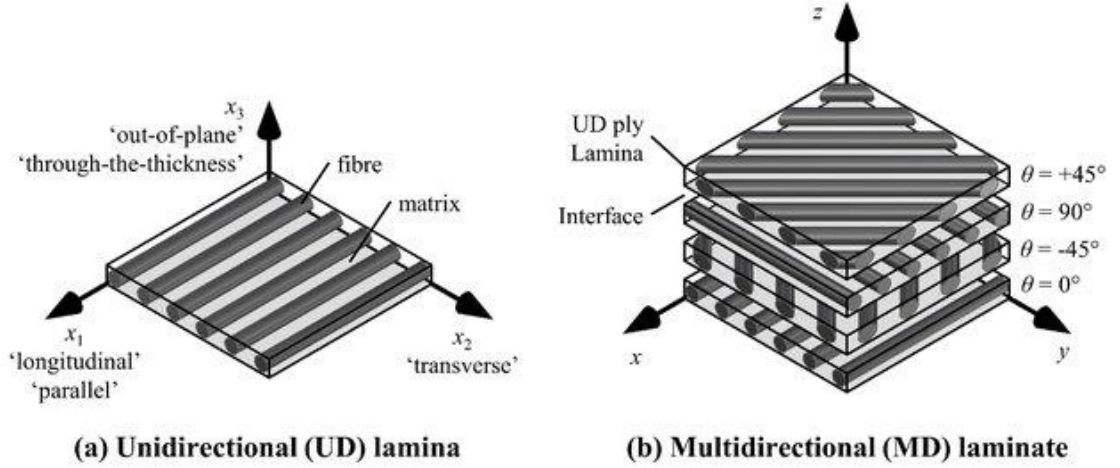


Figure 2.3 Directions of Composites [29].

2.2.1.1 Hooke law for unidirectional composites

As a result; Hooke's law for unidirectional (UD) laminas can be summarized as equation (2.14).

$$\begin{aligned} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} &= \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} \\ \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} &= \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \end{aligned} \quad (2.14)$$

Q_{ij} are reduced rigidity coefficients which have formula as equation (2.15).

$$\begin{aligned}
Q_{11} &= \frac{S_{22}}{S_{11}S_{22} - S_{12}^2} \\
Q_{12} &= \frac{-S_{12}}{S_{11}S_{22} - S_{12}^2} \\
Q_{22} &= \frac{S_{11}}{S_{11}S_{22} - S_{12}^2} \\
Q_{66} &= \frac{1}{S_{66}}
\end{aligned} \tag{2.15}$$

Q_{ij} can be also found by applying tension stresses and shear stress about one of the lateral and the longitudinal directions, respectively, of a composite plate. By this way, S_{ij} and Q_{ij} can be written as a function of the material's mechanical properties.

$$\begin{aligned}
S_{11} &= \frac{1}{E_1} \\
S_{12} &= \frac{-v_{12}}{E_1} \\
S_{22} &= \frac{1}{E_2} \\
S_{66} &= \frac{1}{G_{12}}
\end{aligned} \tag{2.16}$$

and

$$\begin{aligned}
Q_{11} &= \frac{E_1}{1 - v_{21}v_{12}} \\
Q_{12} &= \frac{v_{12}E_2}{1 - v_{21}v_{12}} \\
Q_{22} &= \frac{E_2}{1 - v_{21}v_{12}} \\
Q_{66} &= G_{12}
\end{aligned} \tag{2.17}$$

Finally, unidirectional strains etc. can be found under a certain principal axis loadings by the helps of equations (2.16), (2.17), and (2.14) respectively.

2.2.1.2 Hooke law for multidirectional composites

The procedure of multidirectional laminates is similar to unidirectional laminas. The main difference is after the calculations of S_{ij} and Q_{ij} for each unidirectional lamina, we need to apply a transformation matrix to them for the orientations which laminas have. This transformation matrix is given in equation (2.18).

$$[T] = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} \text{ where } \begin{matrix} c = \cos(\theta) \\ s = \sin(\theta) \end{matrix} \quad (2.18)$$

Therefore, the relation between local and global stresses can be done with T matrix as in the equation (2.19).

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = [T]^{-1} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} \quad (2.19)$$

Local and global strain relations are similar, however shear strains have different coefficient. Thus, a coefficient matrix which is the Reuter matrix is used to arrange the calculations.

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12}/2 \end{bmatrix} = [T] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy}/2 \end{bmatrix}$$

and arrangements,

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = [R][T][R]^{-1} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \quad (2.20)$$

where Reuter matrix is

$$[R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Finally, local stress and local strain relation (Hooke law) can be summarized as equation (2.21).

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = [T]^{-1}[Q][R][T][R]^{-1} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \quad (2.21)$$

If the all multiplier matrices are shortened to express easily;

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = [\bar{Q}] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \quad (2.22)$$

The $[\bar{Q}]$ at the equation (2.22) is named as induced rigidity matrix. This matrix has 6 components which are the functions of orientation angle and rigidity components. The open version of matrix components can be seen in equation (2.24).

$$[\bar{Q}] = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \quad (2.23)$$

$$\begin{aligned} \bar{Q}_{11} &= Q_{11}c^4 + Q_{22}s^4 + 2(Q_{12} + 2Q_{66})s^2c^2 \\ \bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})s^2c^2 + Q_{12}(s^4 + c^4) \\ \bar{Q}_{22} &= Q_{11}s^4 + Q_{22}c^4 + 2(Q_{12} + 2Q_{66})s^2c^2 \\ \bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})sc^3 - (Q_{22} - Q_{12} - 2Q_{66})s^3c \\ \bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})s^3c - (Q_{22} - Q_{12} - 2Q_{66})sc^3 \\ \bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})s^2c^2 + Q_{66}(s^4 + c^4) \end{aligned} \quad (2.24)$$

The components of equation (2.24) can be calculated with the help of equation (2.17) and trigonometric values of orientation angles.

As similar to equation (2.14), there is an inverse matrix relation between equation (2.22) and equation (2.25).

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = [\bar{S}] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \quad (2.25)$$

The relation of local and global axes of of a lamina can be seen at the Figure 2.4. x-y is the local axes, when 1-2 is the global axes .

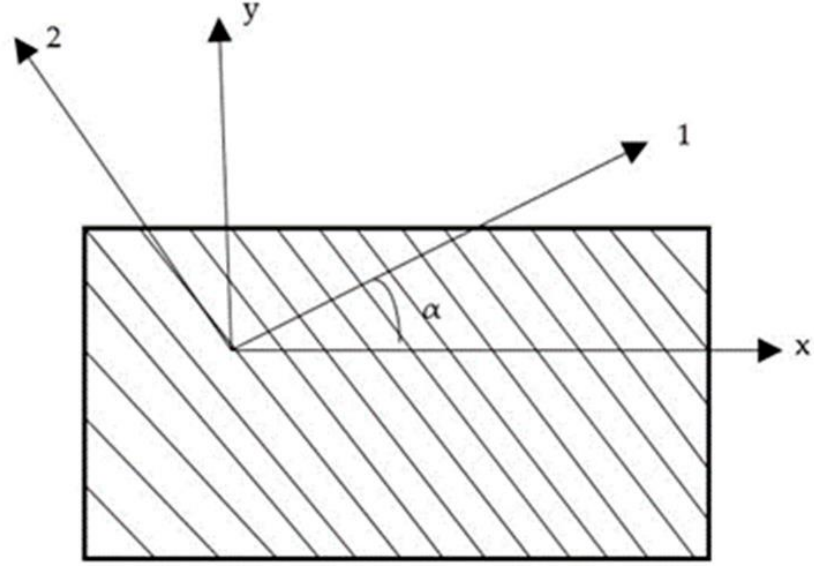


Figure 2.4 Axis relation of a lamina [25].

After the calculation of equation (2.23) for each ply, it is necessary to assemble the rigidity values of each lamina to get a unique stiffness matrix for laminate. This process is defined as the assembly of A-B-D matrices. The explanation of these matrices is; A is the extensional stiffness matrix and it gives the relation of midplane normal and shear strain between normal and shear force resultants. B matrix is the bending extensional coupling stiffness matrix, and it gives the bending curvature-force resultant relation and midplane stress-moment resultant relation. Lastly, D is the bending stiffness matrix, it gives the relation of bending curvature and moment resultant [1].

These matrices are the functions of lamina thickness and height, and induced rigidity matrix $[\bar{Q}]$ as seen in the equation (2.26).

$$\begin{aligned}
 A_{ij} &= \sum_{k=1}^n [(\bar{Q}_{ij})]_k (h_k - h_{k-1}) \\
 B_{ij} &= \frac{1}{2} \sum_{k=1}^n [(\bar{Q}_{ij})]_k (h_k^2 - h_{k-1}^2) \\
 D_{ij} &= \frac{1}{3} \sum_{k=1}^n [(\bar{Q}_{ij})]_k (h_k^3 - h_{k-1}^3)
 \end{aligned} \tag{2.26}$$

$i=1,2,6 \text{ and } j=1,2,6$

The height relation for the equation (2.26) can be seen at Figure 2.5.

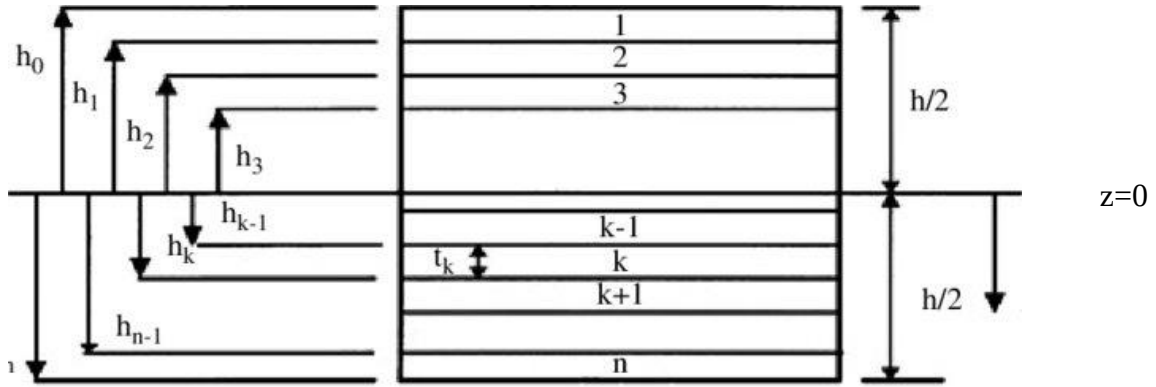


Figure 2.5 Ply locations in laminate [14].

There are some general conveniences when calculating these matrices and these can be seen in the Table 2.1.

Table 2.1 Special situations for ABD matrices [1].

Stacking Type		A	B	D
Unidirectional	[0] or [90]	$A_{16} = A_{26} = 0$	$B_{ij} = 0$	$D_{16} = D_{26} = 0$
	[θ]	-	$B_{ij} = 0$	-
Balanced		$A_{16} = A_{26} = 0$	-	-
Cross-Ply	Symmetric	$A_{16} = A_{26} = 0$	$B_{ij} = 0$	$D_{16} = D_{26} = 0$
	Asymmetric	$A_{16} = A_{26} = 0$	$B_{22} = -B_{11} \neq 0$	$D_{16} = D_{26} = 0$
Angle Ply	Symmetric	$A_{16} = A_{26} = 0$	$B_{ij} = 0$	-
	Asymmetric	$A_{16} = A_{26} = 0$	$B_{16} \neq 0 \ B_{26} \neq 0$	$D_{16} = D_{26} = 0$

After the calculation of ABD matrices, strains and curvatures under the applied force and moments can be found as equation (2.27).

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \quad (2.27)$$

The midplane strains and curvatures can be obtained by the reverse process of the equation (2.27). These values are valid for $z=0$ in Figure 2.5. In order to find strains for any height level in the laminate, we need to multiply the curvature by height level and sum it with strain as in equation (2.28).

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + z \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \quad (2.28)$$

Since strains at any thickness level are calculated, stress can be also calculated by using equation (2.22). The principal or global values of strains and stresses can be calculated by equations (2.19) and (2.20).

The strains and stresses are calculated for every h level of laminate, now the effects of hygrothermal states on force and moment components will be emphasized. These effects can be classified into two groups. The first part is the main focus in this study, temperature change effects. The second part is the moisture effect. These hygrothermal states mainly affect strains, and these will cause stress. Simply, all parts of strain can be summarized as in equation (2.29).

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \varepsilon_x^M \\ \varepsilon_y^M \\ \gamma_{xy}^M \end{bmatrix} + \begin{bmatrix} \varepsilon_x^T \\ \varepsilon_y^T \\ \gamma_{xy}^T \end{bmatrix} + \begin{bmatrix} \varepsilon_x^C \\ \varepsilon_y^C \\ \gamma_{xy}^C \end{bmatrix} \quad (2.29)$$

M: mechanical

T: Thermal (temperature change)

C: Moisture

Force and moment effects of hygrothermal situations are related with ABD matrices.

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} + \begin{bmatrix} N_x^C \\ N_y^C \\ N_{xy}^C \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \quad (2.30)$$

$$\begin{bmatrix} M_x^T \\ M_y^T \\ M_{xy}^T \end{bmatrix} + \begin{bmatrix} M_x^C \\ M_y^C \\ M_{xy}^C \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

Composite materials have components of the coefficient of thermal expansion and coefficient of moisture expansion as material properties.

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{bmatrix} : \text{coefficient of thermal expansion} \quad (2.31)$$

$$\begin{bmatrix} \beta_1 \\ \beta_2 \\ 0 \end{bmatrix} : \text{coefficient of moisture expansion}$$

Unidirectional laminas can be utilized directly from equation (2.31). However, a transformation matrix needs to be used in the calculation of oriented laminas as in equation (2.32).

$$\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy}/2 \end{bmatrix} = [T]^{-1} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \beta_x \\ \beta_y \\ \beta_{xy}/2 \end{bmatrix} = [T]^{-1} \begin{bmatrix} \beta_1 \\ \beta_2 \\ 0 \end{bmatrix} \quad (2.32)$$

A clearer and more understandable version of equation (2.30) with the relation of equation (2.32) can be seen in equation (2.33).

$$[N^T] = \begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \Delta T \sum_{k=1}^n \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_k (h_k - h_{k-1})$$

$$[M^T] = \begin{bmatrix} M_x^T \\ M_y^T \\ M_{xy}^T \end{bmatrix} = \frac{1}{2} \Delta T \sum_{k=1}^n \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_k (h_k^2 - h_{k-1}^2)$$

$$[N^C] = \begin{bmatrix} N_x^C \\ N_y^C \\ N_{xy}^C \end{bmatrix} = \Delta C \sum_{k=1}^n \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{bmatrix}_k (h_k - h_{k-1})$$

$$[M^C] = \begin{bmatrix} M_x^C \\ M_y^C \\ M_{xy}^C \end{bmatrix} = \frac{1}{2} \Delta C \sum_{k=1}^n \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{bmatrix}_k (h_k^2 - h_{k-1}^2) \quad (2.33)$$

2.3 Variable Angle Tows-Composites

In this section, the fiber path difference of variable angle tow will be emphasized and as a result of that suitable modeling techniques will be applied.

In the 2.2 Classical Lamination Theory, it has been seen that stiffness values of ABD matrices have functions of material mechanical property, thickness, and laminate orientation stackings. There was no relation between lateral or longitudinal location and ABD matrices, since the fiber path is constant for each lamina. In variable angle tows, the laminate will consist of a single lamina in the direction of thickness. However, the fiber path will have different orientations for lateral or longitudinal directions.

In order to simplify modeling, the fiber path generally has a function of x direction (lateral). The fiber path is related with x locations by using edge and mid orientations as in Figure 2.6.

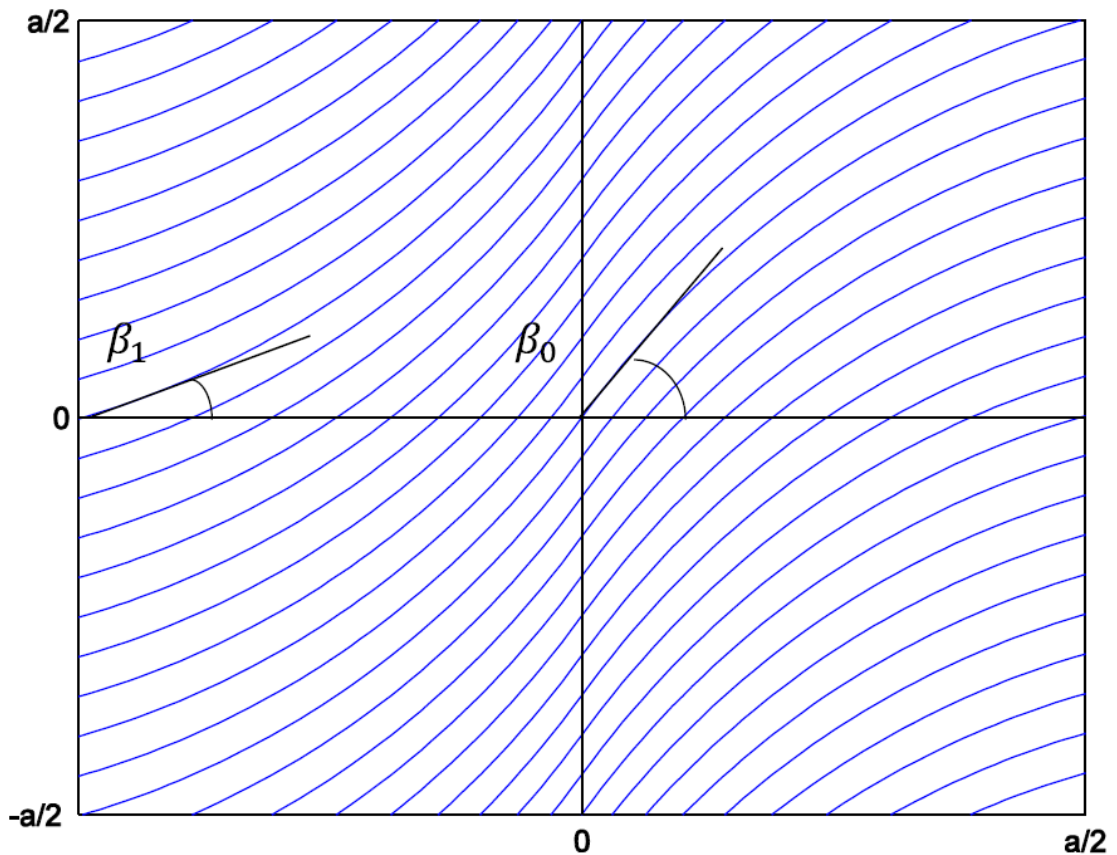


Figure 2.6 Fiber path for VAT [13].

As a summary, the modeling can be formulated as in equation (2.34), the illustration of the equation can be examined in Figure 2.6.

$$\theta(x) = \begin{cases} \frac{2(\beta_1 - \beta_0)}{a}x + \beta_0, & 0 \leq x < \frac{a}{2} \\ \frac{2(\beta_0 - \beta_1)}{a}x + \beta_0, & \frac{-a}{2} \leq x < 0 \end{cases}$$

β_0, β_1 : Orientation of geometric center and plate edges, respectively (2.34)

a: plate width

x: lateral direction for tows



3. PLATE THEORY

Plates can be subjected to large in-plane loads. In this case, since the stabilized state of the plate will be interrupted, it is necessary to perform a stability analysis.

3.1 Elasticity Relations

The Classical Plate Theory (CPT) is obtained for isotropic materials by the Kirchhoff hypothesis. Kirchhoff assumes that if a plate deforms, midplane normals of the plate will remain straight and normal [30]. While the CPT is valid for thin plates, another plate theory, first-order shear deformation theory (FSDT) also known as Mindlin plate theory, is suitable for thick plates, it calculates the deformations through rotations in the x-y axis and the cross-sectional area does not have to remain normal for midplanes [31]. If the x-y axis is located at the midplane and z-axes refer to thickness axes, the kinematical behaviour of the plate is explained by Kirchhoff as in equation (3.1).

$$u = u_0 - z \frac{\partial w}{\partial x} \quad v = v_0 - z \frac{\partial w}{\partial y}$$

u, v, w : displacements of a typical point x,y,z respectively (3.1)

u_0, v_0 : displacements of a inplane point x,y respectively

The classical plane elasticity theory (CPET) states that there is a derivational relation between strain and displacement as seen in equation (3.2).

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \quad \gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

$\varepsilon_x, \varepsilon_y$: in-plane normal strains (3.2)

γ_{xy} : engineering shear strain

If equation (3.2) is rewritten by relations between thickness and strains, equation (3.3) can be obtained.

$$\begin{aligned} \varepsilon_x &= \varepsilon_x^0 - z\kappa_x \\ \varepsilon_y &= \varepsilon_y^0 - z\kappa_y \end{aligned} \quad (3.3)$$

$$\gamma_{xy} = \gamma_{xy}^0 - z\kappa_{xy}$$

$$\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0 : \text{Midplane strains}$$

$$\kappa_x, \kappa_y, \kappa_{xy} : \text{Midplane curvature changes}$$

The equation (3.3) ingredients are simply explained in equation (3.4).

$$\begin{aligned} \varepsilon_x^0 &= \frac{\partial u_0}{\partial x} , & \varepsilon_y^0 &= \frac{\partial v_0}{\partial y} , & \gamma_{xy} &= \frac{\partial v_0}{\partial x} + \frac{\partial u_0}{\partial y} \\ \kappa_x &= \frac{\partial^2 w_0}{\partial x^2} , & \kappa_y &= \frac{\partial^2 w_0}{\partial y^2} , & \kappa_{xy} &= 2 \frac{\partial^2 w_0}{\partial x \partial y} \end{aligned} \quad (3.4)$$

By expanding and developing the assumptions on the CPT, Classical Laminated Plate Theory (CLPT) is achieved.

The CLPT can be applied if a composite structure has a single layer, or a structure has symmetrical multiple layers [30]. Under these conditions, buckling for simply supported composite plates can be governed by Von Karman's differential solutions as in equation (3.5). If there will be coupling between bending and twist because of the unsymmetrical stacking, there will be additional D_{16} , D_{26} parts in the equation (3.5).

$$\begin{aligned} D_{11} \frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 w}{\partial y^4} \\ = h \left(\sigma_x \frac{\partial^2 w}{\partial x^2} + 2\tau_{xy} \frac{\partial^2 w}{\partial x \partial y} + \sigma_y \frac{\partial^2 w}{\partial y^2} \right) \end{aligned} \quad (3.5)$$

$$D_{ij} : \text{Bending stiffness matrix}$$

$$h : \text{thickness}$$

3.2 Governing Equations

The summation of the directional forces (x-y-z) is generalized in equation (3.6) sourced from pressure and in-plane stress resultants.

$$\begin{aligned}
\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} - Q_x \frac{\partial w}{\partial x} + p_x &= 0 \\
\frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} - Q_y \frac{\partial w}{\partial y} + p_y &= 0 \\
\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + q &= 0
\end{aligned} \tag{3.6}$$

p_x, p_y, q : Pressure about x-y-z respectively

N_x, N_y, N_{xy} : In-plane force resultants

Q_x, Q_y : Transverse shear forces

Similarly, the summation of the moments about x-y is generalized in the equation (3.7).

$$\begin{aligned}
Q_x &= \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} \\
Q_y &= \frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x}
\end{aligned} \tag{3.7}$$

M_x, M_y, M_{xy} : In-plane moment resultants

The first two parts of equation (3.6) includes Q_x, Q_y which are relatively small and can be neglected. Therefore, rewritten version of equation (3.6) is equation (3.8) by using equation (3.7).

$$\begin{aligned}
\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} + p_x &= 0 \\
\frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} + p_y &= 0 \\
\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_y}{\partial y^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + q &= 0
\end{aligned} \tag{3.8}$$

Consider the N_i and M_i in the equation (3.8) have two parts which are before the buckling and due to buckling. Therefore, equation (3.8) can be rewritten by separating the parts.

$$\begin{aligned}
N_x &= N_x^i + N_x^b, \quad \text{etc.} \\
M_x &= M_x^i + M_x^b, \quad \text{etc.}
\end{aligned}
\tag{3.9}$$

i: before the buckling

b: due to buckling

The assumption of the separation process is, the plate remains the same (flat) and there will be no additional body forces (p,q) along the buckling. Before the buckling, equation (3.8) will be equation (3.10).

$$\begin{aligned}
\frac{\partial N_x^1}{\partial x} + \frac{\partial N_{xy}^1}{\partial y} + p_x &= 0 \\
\frac{\partial N_y^1}{\partial y} + \frac{\partial N_{xy}^1}{\partial x} + p_y &= 0 \\
\frac{\partial^2 M_x^1}{\partial x^2} + \frac{\partial^2 M_y^1}{\partial y^2} + 2 \frac{\partial^2 M_{xy}^1}{\partial x \partial y} + q &= 0
\end{aligned}
\tag{3.10}$$

If we subtract equation (3.10) from equation (3.8), as in the equation (3.9), equations due to the buckling will be achieved.

$$\begin{aligned}
\frac{\partial N_x^b}{\partial x} + \frac{\partial N_{xy}^b}{\partial y} + p_x &= 0 \\
\frac{\partial N_y^b}{\partial y} + \frac{\partial N_{xy}^b}{\partial x} + p_y &= 0 \\
\frac{\partial^2 M_x^b}{\partial x^2} + \frac{\partial^2 M_y^b}{\partial y^2} + 2 \frac{\partial^2 M_{xy}^b}{\partial x \partial y} + N_x^1 \frac{\partial^2 w}{\partial x^2} + N_y^1 \frac{\partial^2 w}{\partial y^2} + 2 N_{xy}^1 \frac{\partial^2 w}{\partial x \partial y} &= 0
\end{aligned}
\tag{3.11}$$

The last three term of equation (3.11) is the same before buckling and due to buckling situations, and since due to buckling situation these terms are functions of w and they are nonlinear, they need to be removed to linearize the formula.

3.2.1 Boundary condition applications

In many books and articles, the right side of equation (3.5) is also shown as force resultants instead of stresses as in equation (3.12).

$$\begin{aligned}
D_{11} \frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 w}{\partial y^4} \\
= N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2}
\end{aligned} \tag{3.12}$$

D_{ij} : Bending stiffness matrix

This is the generalized formula for simply supported composite plate buckling calculations.

If the loading type is biaxial it means there will be no shear loadings. The general application for simply support plates can be seen in equation (3.13). The geometrical sketch for this equation can be examined in Figure 3.1.

$$\begin{aligned}
w = M_x = -D_{11} \frac{\partial^2 w}{\partial x^2} - D_{12} \frac{\partial^2 w}{\partial y^2} = 0 \quad \text{at } x = 0 \text{ and } x = a \\
w = M_y = -D_{12} \frac{\partial^2 w}{\partial x^2} - D_{22} \frac{\partial^2 w}{\partial y^2} = 0 \quad \text{at } y = 0 \text{ and } y = b
\end{aligned} \tag{3.13}$$

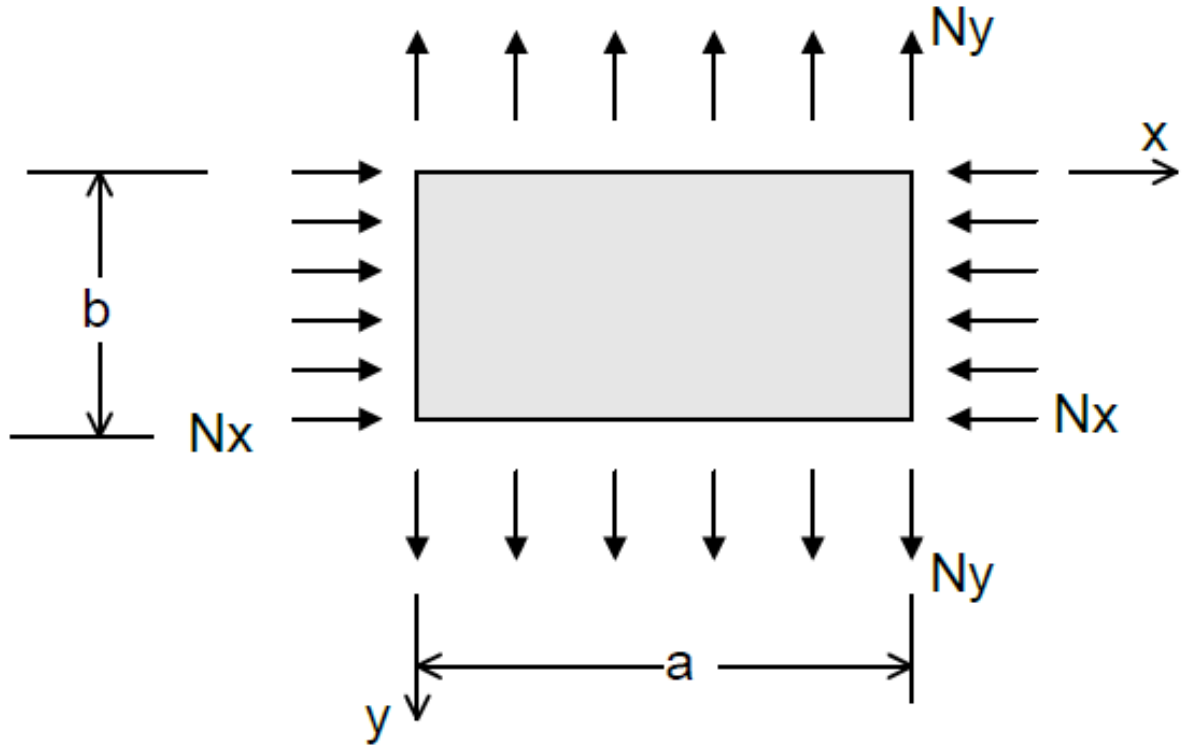


Figure 3.1 Composite plate under biaxial loading [27].

The out of plane displacement 'w' can be given as in equation (3.14).

$$w = \sum \sum A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

m : number of half waves in the x direction (3.14)

n : number of half waves in the y direction

If necessary arrangements are realized (taking derivation of w etc.) and equation (3.14) is substituted into equation (3.12), the new general composite plate simply support the plate buckling formula will be achieved as in equation (3.15).

$$\begin{aligned} \pi^2 A_{mn} [D_{11}m^4 + 2(D_{12} + 2D_{66})m^2n^2(AR)^2 + D_{22}n^4(AR)^4] \\ = -A_{mn}a^2 [N_xm^2 + N_yn^2(AR)^2] \end{aligned} \quad (3.15)$$

AR= aspect ratio = a/b

If buckling occurs, the out-of-plane displacement w will not be zero. Therefore, A_{mn} can be removed from equation (3.15). Finally, the buckling load can be calculated with a help of simplification as $k = N_y/N_x$.

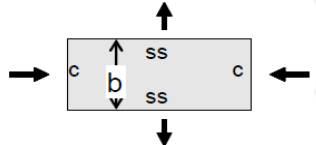
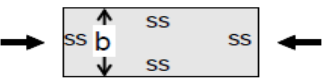
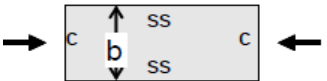
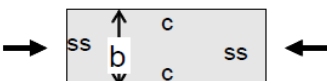
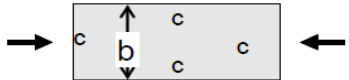
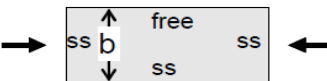
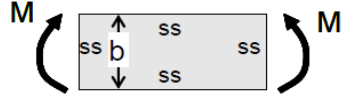
$$N_0 = \frac{\pi^2 [D_{11}m^4 + 2(D_{12} + 2D_{66})m^2n^2(AR)^2 + D_{22}n^4(AR)^4]}{a^2 [m^2 + kn^2(AR)^2]} \quad (3.16)$$

$$k = N_y/N_x$$

By using equation (3.15), equation (3.16), and equation (3.17) all loading types can be summarized as next page at Table 3.1.

$$\lambda = \frac{a}{b} \left(\frac{D_{22}}{D_{11}} \right)^{1/4} \quad (3.17)$$

Table 3.1 Buckling loads for different boundary conditions [27].

Boundary Condition	Buckling Formula
	$N_0 = \frac{\pi^2 [D_{11}m^4 + 2(D_{12} + 2D_{66})m^2n^2(AR)^2 + D_{22}n^4(AR)^4]}{a^2[m^2 + kn^2(AR)^2]}$
	$N_0 = \frac{\pi^2 [D_{11}m^4 + 2(D_{12} + 2D_{66})m^2n^2(AR)^2 + D_{22}n^4(AR)^4]}{a^2m^2}$
	$N_0 = \frac{\pi^2}{b^2} \sqrt{D_{11}D_{22}}(K) \quad K = \begin{cases} \frac{4}{\lambda^2} + \frac{2(D_{12}+2D_{66})}{\sqrt{D_{11}D_{22}}} + \frac{3}{4}\lambda^2, & 0 < \lambda < 1.662 \\ \frac{m^4+8m^2+1}{\lambda^2(m+1)} + \frac{2(D_{12}+2D_{66})}{\sqrt{D_{11}D_{22}}} + \frac{\lambda^2}{m^2+1}, & \lambda > 1.662 \end{cases}$
	$N_0 = \frac{\pi^2}{b^2} \sqrt{D_{11}D_{22}}(K) \quad K = \frac{m^2}{\lambda^2} + \frac{2(D_{12}+2D_{66})}{\sqrt{D_{11}D_{22}}} + \frac{16}{3} \frac{\lambda^2}{m^2}$
	$N_0 = \frac{\pi^2}{b^2} \sqrt{D_{11}D_{22}}(K) \quad K = \begin{cases} \frac{4}{\lambda^2} + \frac{8(D_{12}+2D_{66})}{3\sqrt{D_{11}D_{22}}} + 4\lambda^2, & 0 < \lambda < 1.094 \\ \frac{m^4+8m^2+1}{\lambda^2(m+1)} + \frac{2(D_{12}+2D_{66})}{\sqrt{D_{11}D_{22}}} + \frac{\lambda^2}{m^2+1}, & \lambda > 1.094 \end{cases}$
	$N_0 = \frac{\pi^2}{b^2} \sqrt{D_{11}D_{22}}(K) \quad K = \frac{12}{\pi^2} \frac{D_{66}}{\sqrt{D_{11}D_{22}}} + \frac{1}{\lambda^2}$
	$M_0 = \frac{\pi^2}{b^2} \sqrt{D_{11}D_{22}}(K) \quad K = 0.047\pi^2 b^2 \sqrt{\left(\frac{m^2}{\lambda^2} + \frac{2(D_{12}+2D_{66})}{\sqrt{D_{11}D_{22}}} + \frac{\lambda^2}{m^2}\right) \left(\frac{m^2}{\lambda^2} + \frac{8(D_{12}+2D_{66})}{\sqrt{D_{11}D_{22}}} + 16 \frac{\lambda^2}{m^2}\right)}$

ss: Simply supported c: Clamped

In addition to equation (3.16), the critical value of temperature change is found by Mossavarali under no x-y displacement as all edges simply supported for plates as in equation (3.18) using equation (3.16) [37].

$$\Delta T = \frac{\delta + D_{11}(m\pi/a)^4 + 2(D_{12} + 2D_{66})(m\pi/a)^2(n\pi/b)^2 + D_{22}(n\pi/b)^4}{(A_{11}\alpha_x + A_{12}\alpha_y)(m\pi/a)^2 + (A_{12}\alpha_x + A_{22}\alpha_y)(n\pi/b)^2} \quad (3.18)$$

$\delta = 0$ for symmetric orthotropic materials

For all other cases, especially free and clamped cases, analytical formulas can be derived with the help of Table 3.1.



4. FINITE ELEMENT THEORY for PLATES

4.1 Definiton of Finite Element Method

The finite element method (FEM) is a numerical analysis method that provides approximate solutions to many engineering problems such as stress analysis, heat transfer, fluid flow, etc. The beginning of the method is the usage of the bar and beam elements by Hrennikoff in 1941 and McHenry in 1943 to obtain stress solutions of solids. In the 1950s, Turner succeed to derive stiffness matrices for one-dimensional elements and two-dimensional triangular-rectangular elements. The direct stiffness method which helps to find the total stiffness of the structure is started to use at these years. In the early 1960s, Clough named the process as the finite element and detailed usage of in-plane stress elements was started [32].

In the FEM, the structure is created one by one. Some points serve to connect the parts. These points are named nodes and they are pedestals of FEM. A node group creates elements that give the shape of the structure. These elements are named after their shape and the node numbers they have. There are mainly 3 types of elements as in Table 4.1:.

Table 4.1: Element Types [32].

Dimension	Elements
1D	Rod
	Bar
	Beam
2D	Triangular
	Quadrilaterals
3D	Tetrahedrals
	Hexahedral

If nodes are located just at the corners of elements, they are first-order elements. However, if there are some mid-nodes between the corner nodes, they are named higher-order elements. Different degree elements can be preferred according to the purpose of use. In Figure 4.1, first-order and higher-order elements can be discriminated.

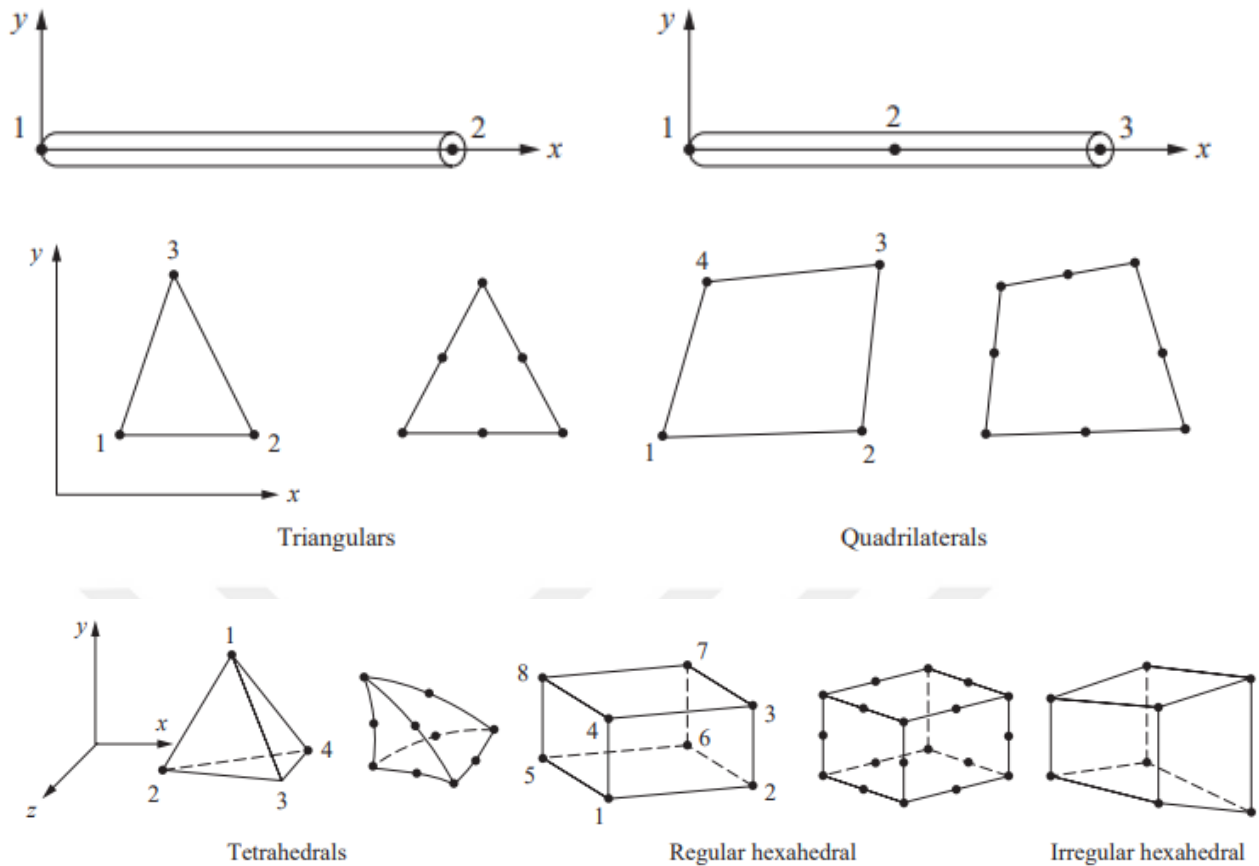


Figure 4.1 1D-2D-3D Element Types [32].

The left elements are first-degree, right elements are second-degree elements.

4.2 Process of Finite Element Method

4.2.1 Selection of element type

There are many element selection criteria such as; dividing structure into small enough pieces, resembling, dimension of element, working principles (carried load types), order level, and analysis type (stress, heat transfer, fluid flow etc).

Surrounding the structure is very important for the convergence of the result and for getting a logical result. Therefore, the structure should be knitted with enough number of elements. High-level elements can be used depending on the situation.

The similarity between structure and element is important to correct identification of structure. For instance, if we use a surface element to model a beam element or truss element, it can be quite difficult to simulate geometries. In addition to that, usage of

quadrilateral elements can be hard, therefore, changing element type to triangular elements may help to cover geometry better.

Coarsely constructed models do not have enough nodes on them. Therefore, value distributions do not make sense when taking solutions and unrealistic results can be seen. In order to avoid this, a sufficiently large number of elements should be used to see the convergence of the result. However, the elements that are used more than necessary are of no use other than prolonging the solution time.

Table 4.2 includes 3 different 1D element types; rods, beams, and bars. They all same number of nodes in the same element order and their dimensions are also the same. However, their working principles are not the same and that is the distinguishing feature of them.

Table 4.2 1D Elements working principles [4].

Element Type	Principle
Rod	Carries tension-compression only
Bar	Carries tension-compression, bending Symmetric and constant cross section Coincided shear center and neutral axis
Beam	Carries tension-compression, bending Complex and open section cross section Noncoincided shear center and neutral axis

4.2.2 Selection of displacement function

Some of the important cases for FEM analyzes are related to solvers. The FEM model is need to be defined as achieving results with high accuracy and high resolution with little effort. Higher-order elements can provide these by reducing mesh density [9]. Especially, the volume bodies need to mesh by 3D elements and higher-order elements. In addition to that, usage areas can be key point for element selection. For instance, CFD (computational fluid dynamics) analyzes need volume meshing. Therefore, higher-order meshes can be used in these type areas. To sum up, second-degree (quadratic) or fourth-degree (quartic) element usage can be more suitable instead of using first-order elements (linear).

4.2.3 Definiton of strain-displacement stress-strain relations

In structural analysis, two main objectives are achieving the displacement and stress results. Since the strain is the intersection of the relations definition of that is important. The strain has a derivational relation with displacement. After the definition of strain, the Hooke Law has to be applied to obtain stress. Therefore, the goals can be achieved through these relations.

4.2.4 Element stiffness matrix

All the elements inside a model will have their stiffness sides. In a structural system, a node will have motion directions which are called degrees of freedom (DOF). These directions will specify the stiffness directions of a node, and herewith they will specify for elements also. The stiffness values will limit the response that elements can give to the faced loads.

4.2.5 Global stiffness matrix and boundary conditions

The element stiffness matrix gives the individual stiffness values that an element have. However, structures generally have more than one element. Therefore, a structure that has more than one element x cannot consist of just an element's stiffness matrix. The assembly method is up to node connections. The connected nodes in the elements will give more than an individual stiffness value. Therefore, they will affect all elements that they are included.

1D elements are generally assembled by direct equilibrium or stiffness method because of the convenience of the method. 2D, and 3D elements benefit from work or energy methods. Galerkin Method is also applicable to the systems [32]. At the end of the assembly of the stiffness matrix, the system is ready to apply the boundary conditions.

Boundary conditions include the effects of system connections and loads. If the used connection system limits displacement at any side, it has to be imported into the displacement matrix. Similarly, applied loads are also imported into related indices of force matrix. The final version of the matrix system will comprise the equation system (4.1). The n is the total DOF number which is the multiplication of the node number and DOF number for a single node. $\{f\}$, $[k]$, $\{d\}$ are force, stiffness, and displacement matrix respectively.

$$\{f\} = [k]\{d\}$$

$$\begin{Bmatrix} F_1 \\ \vdots \\ F_n \end{Bmatrix} = \begin{bmatrix} K_{11} & \cdots & K_{1n} \\ \vdots & \cdots & \vdots \\ K_{n1} & \cdots & K_{nn} \end{bmatrix} \begin{Bmatrix} D_1 \\ \vdots \\ D_n \end{Bmatrix} \quad (4.1)$$

4.2.6 Solution of global matrices

After the assembly of the equation system (4.1), applying the boundary conditions will help the solving of the system. Some solving methods must be used such as Gauss Elimination Method and Gauss-Seidel Method [32]. These methods help the elimination of unnecessary indices of the matrix and the solution of the matrix can be achieved. Since equation (4.1) gives a displacement matrix as a result, strain and stress can be calculated by elasticity and strength laws as a postprocess. The remaining part is the interpretation of the results. This part is the insurance of all steps. The consistency of the results must be evaluated in this section. Finally, an engineer can decide whether or not the structure is safe enough with the material mechanical values.

4.2.7 Displacement field and fem

All nodes inside an element have their displacement. In order to define displacement vectors of elements, nodal displacements have to be known. However, the nodal location in an element is also important for displacement vectors. The element shape functions provide the nodal locations' importance [15].

$$u = \sum_{i=1}^n N_i^e u_i \quad v = \sum_{i=1}^n N_i^e v_i$$

$$u, v: \text{nodal displacement directions} \quad (4.2)$$

$$n: \text{node number}$$

$$N: \text{shape function}$$

Shape functions helps the interpolation of displacements with the equation (4.2) [11]. A rectangular plane element's (Q4) node numbering and nodal displacement vectors can be seen in Figure 4.2.

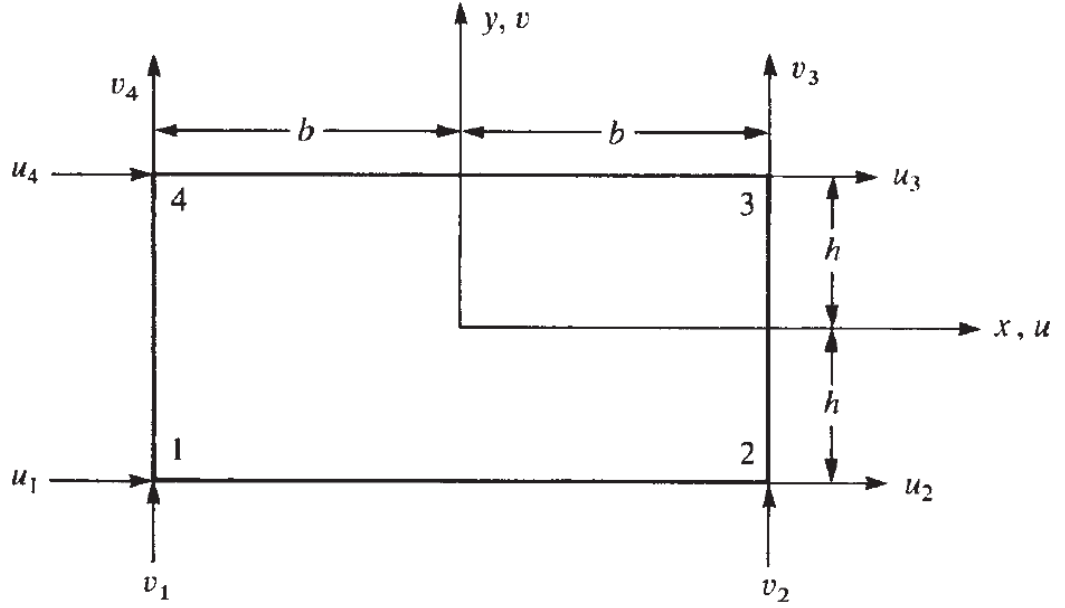


Figure 4.2 Q4 nodal displacement [32].

Shape function of a Q4 element can be formulated as in equation (4.3).

$$\begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 \end{bmatrix} \quad (4.3)$$

Final expression for equation (4.2) is equation (4.4).

$$\{u\} = [N]\{d\} \quad (4.4)$$

There was a derivational relation between strain and displacement for 1D elements.

The relation is valid again, instead of an equation there is a matrix that relates strain and displacement as in equation (4.5).

$$[B] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & \frac{\partial N_2}{\partial x} & 0 & \dots & \frac{\partial N_n}{\partial x} & 0 \\ 0 & \frac{\partial N_1}{\partial y} & 0 & \frac{\partial N_2}{\partial y} & 0 & \dots & \frac{\partial N_n}{\partial y} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial y} & \frac{\partial N_2}{\partial x} & \dots & \frac{\partial N_n}{\partial y} & \frac{\partial N_n}{\partial x} \end{bmatrix} \quad (4.5)$$

B: Strain-displacement matrix

Finally, the element stiffness matrix can be formed with the relations as in equation (4.6).

$$[k] = \int [B]^T [E] [B] dV$$

B: Strain-displacement matrix (4.6)

E: Material Property matrix

As an example, the simple version of equation (4.6) for constant strain triangle (CST) element, also known as TRIA, can be evaluated similar to equation (4.7) [11].

$$[k] = \iint [B]_{6 \times 3}^T [E]_{3 \times 3} [B]_{3 \times 6} t dx dy = At [B]^T [E] [B]$$

t: thickness

(4.7)

A: area

4.2.8 Mapping of nodes

4.2.8.1 Coordinate systems in mapping

Every structure has a global coordinate system to define its geometry. However, it makes the calculations harder and more complicate if there are too many elements and a complex body. The engineers and scientists handle this problem by using a new coordinate system which is named natural coordinates [15]. This coordinate system is set in the middle of every element similarly in Figure 4.3.

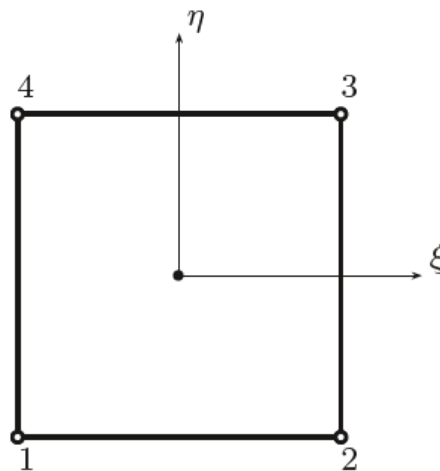


Figure 4.3 Q4 natural coordinate system [15].

The natural coordinates (ξ, η) have formulations for shape function. As stated in equation (4.2), shape function has components as the number of nodes in the element. These Lagrangian functions can be showed in (4.8). The nodal ordering can be examined from Figure 4.3.

$$\begin{aligned}
 N_1(\xi, \eta) &= l_1(\xi)l_1(\eta) = \frac{1}{4}(1 - \xi)(1 - \eta) \\
 N_2(\xi, \eta) &= l_2(\xi)l_1(\eta) = \frac{1}{4}(1 + \xi)(1 - \eta) \\
 N_3(\xi, \eta) &= l_2(\xi)l_2(\eta) = \frac{1}{4}(1 + \xi)(1 + \eta) \\
 N_4(\xi, \eta) &= l_1(\xi)l_2(\eta) = \frac{1}{4}(1 - \xi)(1 + \eta)
 \end{aligned} \tag{4.8}$$

In order to relate global and natural coordinates, a derivation operation must be applied as in equation (4.9).

$$\begin{bmatrix} \frac{\partial}{\partial \xi} \\ \frac{\partial}{\partial \eta} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{bmatrix} \tag{4.9}$$

The relational operator has a special name Jacobian, and is expressed with J. Inverse of the Jacobian reverses the equation (4.9).

$$\begin{bmatrix} \frac{\partial}{\partial \xi} \\ \frac{\partial}{\partial \eta} \end{bmatrix} = J \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{bmatrix} = J^{-1} \begin{bmatrix} \frac{\partial}{\partial \xi} \\ \frac{\partial}{\partial \eta} \end{bmatrix} \tag{4.10}$$

In order to exemplify, elemental volume in the equation (4.6) can be summarized as equation (4.11) where det means determinant, in the case of strain displacement matrix B depended on natural coordinates [32].

$$dV = t \det J d\xi d\eta \tag{4.11}$$

4.2.8.2 Gaussian quadrature

In order to calculate the integral (4.12), the midpoint can be selected as a sample and multiplied by the path length.

$$I = \int_{-1}^1 y(x) dx \quad (4.12)$$

This process is called Gaussian Quadrature. The rule can be extended by a selection of more sample points. The general form of the process can be seen in equation (4.13). In the formula, n states the sampling point number, and W states the weight. The sample points' locations are symmetrical. In this way, the integral is tried to be evaluated precisely.

$$I = \int_{-1}^1 y(x) dx = \sum_{i=1}^n W_i y_i \quad (4.13)$$

The sampling number can be used as 2 to optimize processing time and precise of result. In the case of n=2, the coordinates of sample points are as in Table 4.3.

Table 4.3 2 Points Gaussian locations [32].

Gauss Point	Formula	Gauss Location
1	$(-\xi, -\eta)$	$(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}})$
2	$(\xi, -\eta)$	$(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}})$
3	(ξ, η)	$(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$
4	$(-\xi, \eta)$	$(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$

The locations in Table 4.3 can be seen at Figure 4.4.

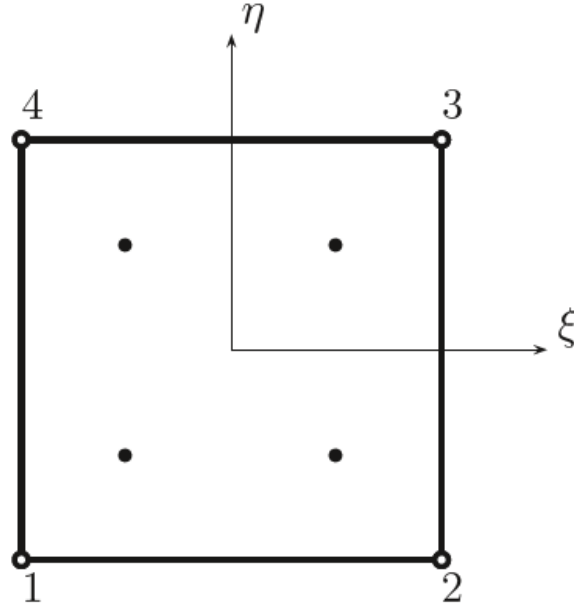


Figure 4.4 2 Points Gaussian locations [15].

Under these relations, equation (4.7) can be rewritten as equation (4.14) [15].

$$\begin{aligned}
 [k] &= \int [B]^T [E] [B] dV = \int_{-1}^1 \int_{-1}^1 t [B]^T [E] [B] \det J d\xi d\eta \\
 &= t \sum_{i=1}^2 \sum_{j=1}^2 [B]^T [E] [B] \det J w_i w_j
 \end{aligned} \tag{4.14}$$

Finally, the global coordinates are mapped into the natural coordinates, and stiffness matrices can be evaluated easily.

4.2.9 Forces and boundary conditions

In this section, applied forces and boundary conditions will be explained in terms of FEM rules.

Since thermal loads are preferred as the main loading type in this study, analyses will be carried out under these loads. As explained in equation (2.33), temperature changes cause thermal force and moment resultants. Therefore, these loads will be applied in the FEM code. The loading can be changed due to the environmental conditions and composite material stacking sequence. In this study, compressive loads of thermal application will be concerned since the loads are the main reason for buckling. The

loads will be imported into the code by taking into consideration DOF relation for nodes.

Boundary conditions include support effects in the displacement matrix. Consequently, different support types will give different displacement matrices. There are 3 main connection types for plates which are simply supporting (S or SS), clamped (C) and free (F). Since there are 4 edges in a plate, the contraction for boundary condition is done by 4 letters e.g. SSSS, CCCC. Chapter 3.1 Elasticity Relations includes deformation relation for Kirchhoff Plate Theory which is used in this study. According to this theory, there are 3 DOFs in each node as in equation (4.15). In order to define the motion of a plate, these 3 DOFs will be used in elastic relations.

w : out of plane displacement

$$\theta_x = \frac{\partial w}{\partial x} \text{ rotation about x axis} \quad (4.15)$$

$$\theta_y = \frac{\partial w}{\partial y} \text{ rotation about y axis}$$

Simply support edges will have boundary condition: at x edges $w=0$ and $\frac{\partial^2 w}{\partial x^2} = 0$, at y edges $w=0$ and $\frac{\partial^2 w}{\partial y^2} = 0$ [49].

Clamped edges will have boundary condition: at x edges $w=0$ and $\frac{\partial w}{\partial x} = 0$, at y edges $w=0$ and $\frac{\partial w}{\partial y} = 0$ [7].

4.2.10 Buckling analysis by fem

Buckling analysis includes several calculation steps. Since buckling is a stability problem, the geometry of a structure has crucial importance. For instance, the analysis of columns and plates has huge differences since the stabilities of the structures are different because of the geometry. However, equation (4.16) can be used to generalize the buckling analysis [13].

$$|K_m + \lambda K_g| = 0$$

$$K_m, K_g: \text{Material and Geometric Stiffness Matrices} \quad (4.16)$$

λ : Buckling Load Factor

In the equation (4.16), K_m is the material matrix which can be formulated as equation (4.14) shows how much powerful the structure is. The K_m is commonly used on static analysis and provides to calculate displacement and stresses on the body.

Another stiffness matrix K_g is named as geometric stiffness matrix or initial stress matrix. This matrix has no part related to the mechanical properties of the material, however it is dependent on the geometrical values and stresses on the body [53]. This matrix is a special matrix for buckling analysis, and the usage is not common unsimilar to K_m .

The calculation of the geometric stiffness matrix has the same procedures with equation (4.14). Instead of strain-displacement matrix B and material matrix E, geometric matrix G and initial stress matrix σ^0 is used respectively. G matrix uses shape functions for contributions. σ^0 matrix shows the effects of stress-state on the body. Generally, it is utilized to see permanent load effects [41].

$$[k] = \int [G]^T [\sigma^0] [G] dV \quad (4.17)$$

Last part of the equation (4.16) is, buckling load factor, shows how safe the structure is. In general structural analysis, buckling load factor has same sense with reserve factor (RF) which is a criteria for failure. Therefore, the critical buckling load factor, λ_{min} , has to be not lower than 1 for a safe structure as stated in equation (4.18).

$$RF = \frac{\text{Allowable}}{\text{Applied}} \quad \begin{cases} RF \geq 1, & \text{structure is safe} \\ RF < 1, & \text{structure is not safe} \end{cases} \quad (4.18)$$

Therefore, the general steps for this study are; to determine the path of the fiber using edge and middle orientations for VAT modeling, to generate the ABD matrices using CLT for each determined station of the plate and thermal stress resultants using the material properties under the specified temperature change, to generate the material stiffness matrix with the help of the orientation dependent ABD matrices at each station, creating the geometric stiffness matrix that depends on the geometry of the structure, and finally to estimate the critical temperature of the system before buckling by calculating the buckling load factor under the calculations in equation (4.16).

$$T_{critical} = \lambda T \quad (4.19)$$

5. RESULT & DISCUSSION

In this study, the code is verified by finite element analysis. In the modeling of the composite plate MSC Patran software is used, and the solutions are obtained by MSC Nastran while the code is written on the MATLAB software. In the verification process, many cases are examined which are basically isotropic materials, cross-ply layup, and angle-ply layup.

After the verification, a VAT will be analyzed to show comparison of buckling result for common layups and VAT.

5.1 Isotropic Plate Verification

In this section; the result of the code, the analytical method, and the FEM result will be compared. Shanmugam and Narayanan stated the buckling factor for the isotropic materials as the [43].

$$\sigma_{cr} = k \frac{\pi^2 E}{12(1 - \nu^2)} \left(\frac{t}{b}\right)^2 \quad (5.1)$$

k: coefficient of BC

Coefficient k can be taken from Figure 5.1. An isotropic material with the below properties is used for verification. The analysis will be done for a simply supported plate.

Table 5.1 Material properties for isotropic verification.

Property	Value
E	70 GPa
ν	0.30
a=b	1m
t	0.001 m
k	4.0 (SS)
$\sigma_{applied}$	1 MPa

In Table 5.1, a and b are plate length and width, k is coefficient for simply supported plates, and σ is the compression stress which causes buckling.

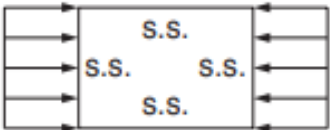
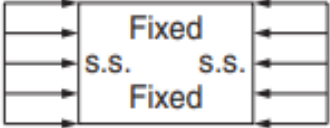
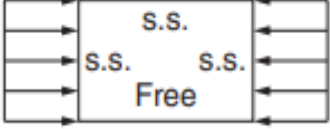
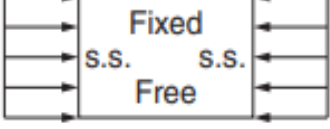
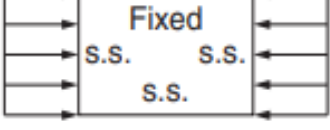
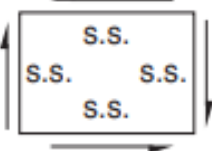
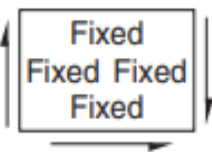
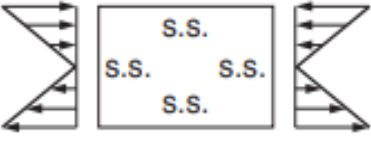
Case	Boundary and loading condition	Type of stress	Value of k for long plate
(a)		Compression	4.0
(b)		Compression	6.97
(c)		Compression	0.425
(d)		Compression	1.277
(e)		Compression	5.42
(f)		Shear	5.34
(g)		Shear	8.98
(h)		Bending	23.9

Figure 5.1 Coefficient for different BCs and loadings [43].

Under this geometry and material properties; the most critical eigen value can be seen below table. Error is based on the analytical formula.

Table 5.2 Isotropic mat. verification

Analytical Formula	The Code	FEM	Error
0.2530	0.2413	0.2587	2.24 %

In the creation process of the finite element model on the MSC Patran software, the boundary conditions and distributed load sourced from stress are applied as the Figure 5.2.

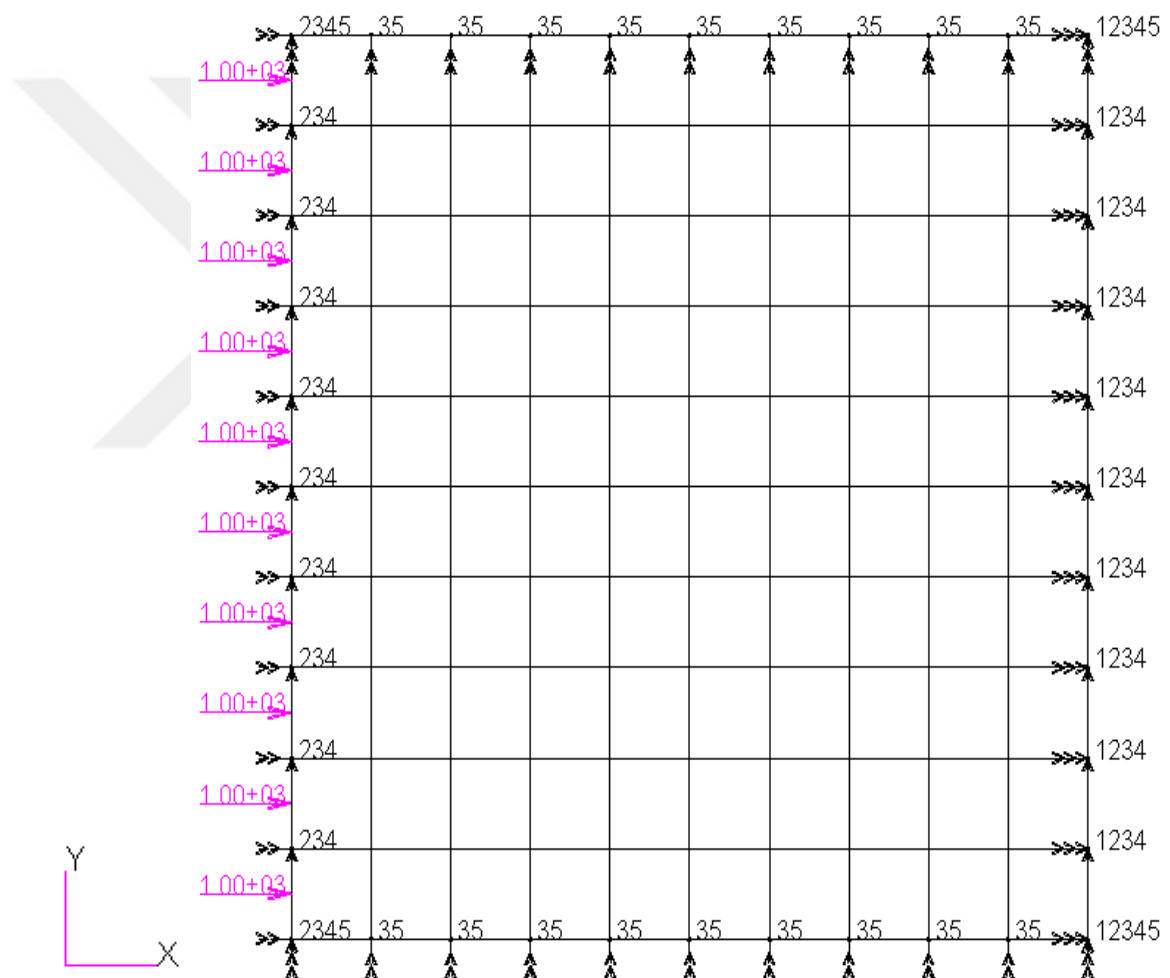


Figure 5.2 FEM for isotropic model

5.2 Composite Plate Verification

In this section, composite plates will be tried to be veriflicated. In order to do that, classical lamination theory will be utilized. In-house code and FEM on Patran will be

developed under a unit temperature change assumption as loading. The unit temperature loading is entered on uniform elements, since the loading is tried to be entered as unit pressure not a nodal force. Application on MSC Patran can be seen in Figure 5.3.

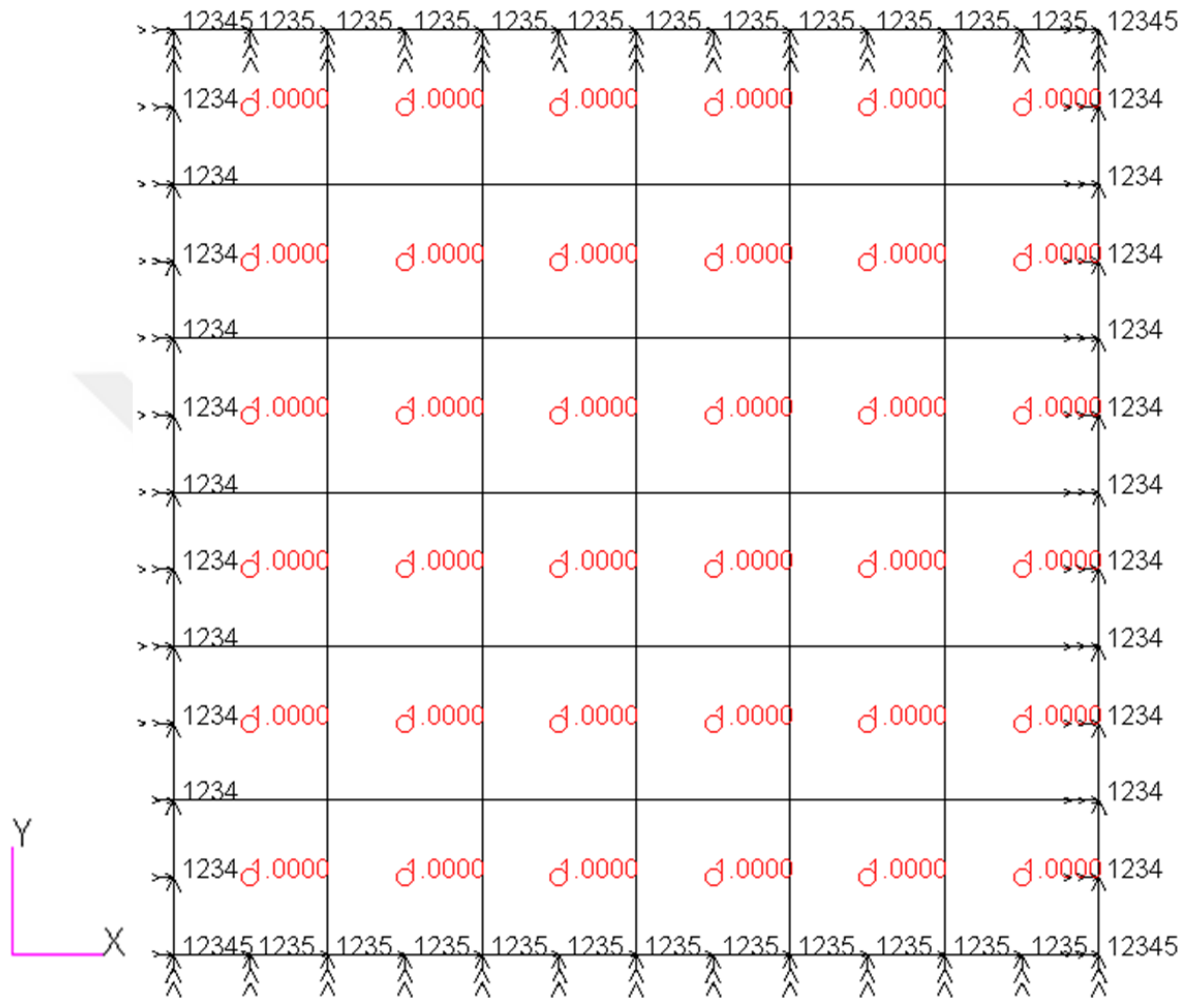


Figure 5.3 FEM for composite plates.

5.2.1 Cross-Ply layup

As explained before, cross-ply refer to laminates that have $0^\circ, 90^\circ$ orientation symmetrically. Different ply numbers will be examined for cross-ply layup. The study of Babu and Kant is chosen to verification the code [26].

5.2.1.1 Laminate with $a/h=100$

The material and geometry properties that are retrieved from the study of Babu can be seen as Table 5.3.

Table 5.3 Properties of cross ply (a/h=100).

Property	Value
Elasticity M.	$E_1/E_2 = 15$
Shear M.	$G_{12}/E_2 = 0.5$
ν_{12}	0.30
Thermal Exp. Coef.	$\alpha_1/\alpha_0 = 0.015$
Orientation	$[0/90]_s$
a/h	100
a	0.15 m
Ply thickness	$3.75 * 10^{-4}$ m

Under this geometry and material properties; the thermal force is calculated by equation (2.33) for composite laminates. After that, the model with 6x6 elements is loaded by the calculated biaxial compression forces due to unit temperature change. The results for this case can be seen below table Table 5.4. The study of Babu and Kant is based on error calculation.

Table 5.4 Cross-ply verification (a/h=100).

Babu	The Code	FEM	Error
0.9960	0.9928	1.0000	0.73 %

* The correction parameter 10^3 is used to simplification of results.

As it can be seen, the values of the code and Babu are nearly same.

In order to see the continuance of eigen values, the critical 10 eigen values are elaborated. The results and errors for these cases can be seen in Table 5.5.

Table 5.5 The most critical 10 eigen values.

Eigen Value (#)	The Code	FEM	Error
1	0.9928	1.0000	0.73
2	1.4499	1.4620	0.83
3	2.8126	2.8256	0.46
4	3.9976	4.1034	2.58
5	4.5259	4.7304	4.32
6	4.8030	4.8558	1.09
7	4.9826	4.9226	1.22
8	6.4601	6.7105	3.73
9	8.1239	7.7119	5.34
10	9.6869	10.012	3.25

The code and FEM have averagely 2% difference except 9th case. Therefore, it can be accepted that the results are converged with FEM software.

5.2.1.2 Laminate with $a/h=10$

The same material is used for this analyze, the only alteration is the a/h ratio with the increment of thickness for 4 layers.

Table 5.6 Properties of cross ply ($a/h=10$).

Property	Value
Elasticity M.	$E_1/E_2 = 15$
Shear M.	$G_{12}/E_2 = 0.5$
ν_{12}	0.30
Thermal Exp. Coef.	$\alpha_1/\alpha_0 = 0.015$
Orientation	$[0/90]_s$
a/h	10
a	0.15 m
Ply thickness	$3.75 * 10^{-3}$ m

Under this geometry and material properties; the thermal force is calculated as previously by equation (2.33) for composite laminates. After that, the model with 6x6 elements is loaded by the calculated biaxial compression forces due to unit temperature change. The results for this case can be seen Table 5.7. The study of Babu and Kant is based on error calculation.

Table 5.7 Cross-ply verification ($a/h=10$).

Babu	The Code	FEM	Error
0.0757	0.0782	0.0996	3.35 %

As it can be seen, error value of code is around 3% which is highly confidential.

5.2.2 Angle-ply layup

In this section, one of the very common orientations, ($45^\circ, -45^\circ \dots$) will be focused. These analyzes will be repeated with different layer numbers and the difference between the code and the article will be examined.

5.2.2.1 Laminate with $a/h=100$

As stated, the changed orientation which is angle ply will be examined, with $a/h=100$ ratio. Material properties can be seen at Table 5.8.

Table 5.8 Properties of angle ply (a/h=100).

Property	Value
Elasticity M.	$E_1/E_2 = 15$
Shear M.	$G_{12}/E_2 = 0.5$
ν_{12}	0.30
Thermal Exp. Coef.	$\alpha_1/\alpha_0 = 0.015$
Orientation	[45/-45/45/-45]
a/h	100
a	0.15 m
Ply thickness	$3.75 * 10^{-4}$ m

The model with 6x6 elements, in accordance with the study of Babu and Kant, is loaded by the calculated biaxial compression forces due to unit temperature change. The results for this case can be seen in Table 5.9. The study of Babu and Kant is based on error calculation.

Table 5.9 Angle-ply verification (a/h=100).

Babu	The Code	FEM	Error
1.4680	1.6555	1.5560	12.77 %

* The correction parameter 10^3 is used to simplification of results.

As it can be seen, the result of code is not good as cross-ply layup. However, the difference between the article and the code is still under 15%.

5.2.2.2 Laminate with a/h=10

The same material is used for this analyze, the only alteration is the a/h ratio with the increment of thickness for 4 layers. Material properties can be seen at Table 5.10.

Table 5.10 Properties of angle ply (a/h=10).

Property	Value
Elasticity M.	$E_1/E_2 = 15$
Shear M.	$G_{12}/E_2 = 0.5$
ν_{12}	0.30
Thermal Exp. Coef.	$\alpha_1/\alpha_0 = 0.015$
Orientation	[45/-45/45/-45]
a/h	100
a	0.15 m
Ply thickness	$3.75 * 10^{-3}$ m

Under this geometry and material properties; the thermal force is calculated as previously by equation (2.33) for composite laminates. After that the model with 6x6 elements is loaded by the calculated biaxial compression forces due to unit temperature change. The results for this case can be seen in Table 5.11. The study of Babu and Kant is based on error calculation.

Table 5.11 Angle-ply verification (a/h=10).

Babu	The Code	FEM	Error
0.1061	0.1029	0.1553	3.03 %

As it can be seen, error value of code is around 3% which is highly confidential.

5.2.3 Complex layup

In this part, the analysis will be one step further complicated and a similar orientation with Duran's study will be used [13]. Graphite-Epoxy is used to ensure compatibility with the other work. The material properties and geometry can be seen in Table 5.12.

Table 5.12 Properties of complex layup.

Property	Value
E_1	155 GPa
E_2	8.07 GPa
G_{12}	4.55 GPa
ν_{12}	0.22
α_1	$-0.07 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$
α_2	$30.1 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$
Orientation	$[45/-45/0/90]_s$
a	0.15 m
Ply thickness	$3.75 * 10^{-4} \text{ m}$

By utilizing the data at Table 5.12, the analysis process is done. The result of the code is compared with Rayleigh-Ritz formulation result [35], and the study of Duran [13].

Table 5.13 Complex layup verification.

Rayleigh-Ritz	Duran	Code	FEM	Error (Rayleigh-Ritz)	Error (Duran)
38.60	39.40	38.51	38.72	0.25%	2.27%

As can be seen from table Table 5.13, the error ratios are small and negligible. In this verification, both orientation and layer number complexity are tested for code.

To summarize, the error of the code for different types of composite layup and aspect ratios are tested. Error rates are acceptable, even below 1% in some cases. Therefore,

the code can be used trustingly for common straight layups. Now, it is necessary to verify the variable angle capabilities of the code.

5.2.4 Variable angle layups

In this section, the variable angled tows will be tried to be verified by using the study of Duran and FEM model. In the process of FEM model, a code is created so that each element in the model has different properties. By this way, each element can have different orientations. Here, the x-coordinates of elemental centers are used to obtain average orientations for elements, $\hat{\theta}$. In the modelling of laminate for VATs, $[\widehat{+\theta-\theta}]_s$ stacking is used in the FEM modelling. There are three different materials all have different edge and center orientations for plates. In Table 5.14, the mechanical properties of materials can be seen.

Table 5.14 Mechanical properties for VATs.

Material	E_1 [GPa]	E_2 [GPa]	G_{12} [GPa]	ν_{12}
E-Glass/Epoxy	41	10.04	4.3	0.28
S-Glass/Epoxy	45	11.0	4.5	0.29
Carbon/Polyimide	216	5.0	4.5	0.25

In Table 5.15, thermal properties and orientations of materials can be seen.

Table 5.15 Thermal properties and orientations for VATs.

Material	α_1 [$^{\circ}\text{C}^{-1} \times 10^{-6}$]	α_2 [$^{\circ}\text{C}^{-1} \times 10^{-6}$]	Center Orientation	Edge Orientation
E-Glass/Epoxy	7.0	26	6.71 $^{\circ}$	58.04 $^{\circ}$
S-Glass/Epoxy	7.1	30	16.12 $^{\circ}$	54.74 $^{\circ}$
Carbon/Polyimide	0.0	25	56.30 $^{\circ}$	36.68 $^{\circ}$

All these cases will be considered as a verification process of VAT. All cases will cover the same geometry properties stated below. According to the result of this verification part, the usability of the code can be decided.

5.2.4.1 E-glass/epoxy

Firstly, the start will be with what's called E-Glass/Epoxy or Borosilicate Glass Reinforced epoxy. This material has strong mechanical properties as can be seen in the table above. Although it was initially used in electrical cables, it is now also used in structural areas [17].

In the verification process, unlike straight layups, two orientation angles are required as input here. The preferred edge and center orientation for conformality with Duran's study can be seen in Table 5.15.

By using these orientations and the equation (2.34) all orientations on the plate are obtained. The result with these orientations and geometrical values can be seen in Table 5.16.

Table 5.16 E-Glass/Epoxy verification.

Duran	The Code	FEM	Error
5.58	5.47	5.10	1.87 %

The code gave good result with a low error level. Therefore, the code can be accepted as usable for this material.

5.2.4.2 S-glass/epoxy

S-Glass/Epoxy material has greater stiffness values compared to E-Glass/Epoxy. Due to its temperature resistance, it is used in the cargo part of aircraft and connections such as gaskets [54].

The optimum thermal buckling orientation for this plate is indicated in Table 5.15. Therefore, these values will be used for the verification. The analysis process of the code is ended in the Table 5.17.

Table 5.17 S-Glass/Epoxy verification.

Duran	The Code	FEM	Error
5.04	4.97	4.75	1.26 %

When the results of Duran and the code are compared, it can be seen that they are very similar. The difference is around 1% and this means this part has high accuracy.

5.2.4.3 Carbon/polyimide

Carbon polyimide can be classified as a reinforced polymer composite. This class of composites is extensively preferred in various areas such as aerospace, and civil due to their outstanding mechanical properties [48]. In order to analyze this material, the orientation for the edges and center can be seen in Table 5.15.

Since Carbon/Polyimide has greater mechanical stiffness than the other material listed in Table 5.14, the eigen value or critical temperature value will have to be greater than

others. The result can be explained by that idea. When it is turned to the verification part, the comparison can be done with Table 5.18.

Table 5.18 Carbon/Polyimide verification.

Duran	The Code	FEM	Error
78.28	77.31	71.45	1.24 %

As expected, the results have high critical temperature values. The results are corroborating each other with a low difference of around 1%. Hence, the code can be seen as safe for this material also.

In the validation part of VATs, the error level occurs around 2% except in one case. This dispensation has 6% error which is under 10% and acceptable. Therefore, the code can be used for VATs with the support of these validations.

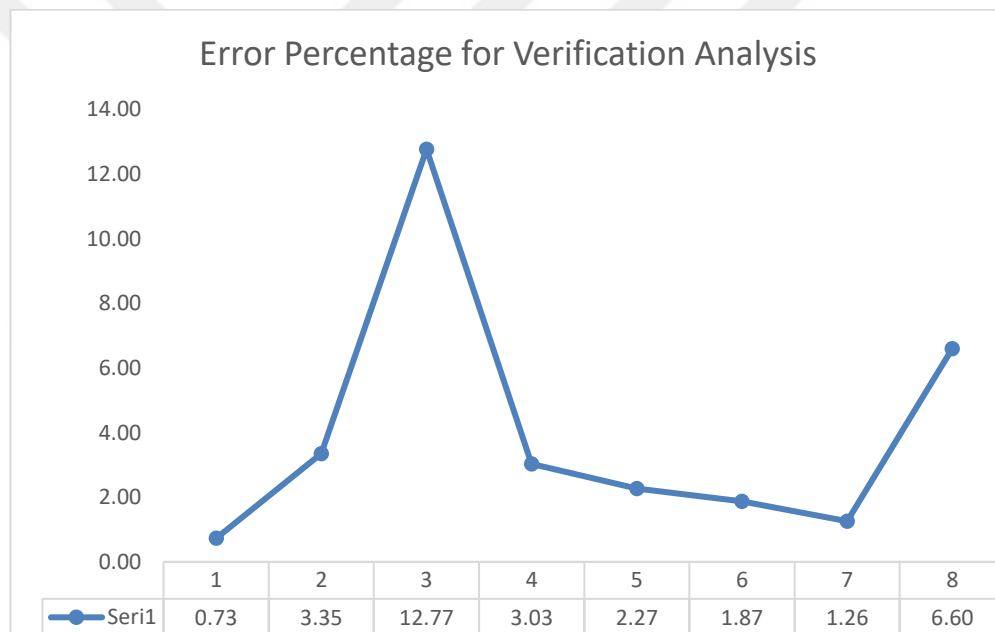


Figure 5.4 Error percentages for verification analysis.

Figure 5.4 summarizes the 8 validation analysis results. As it can be observed, the pick of error is 12% and the average is around 3%. Thus, the code can be confidently used for calculating the thermal buckling factor of variable angled composites.

5.3 Result for a VAT

In this section, a sample composite material will be selected and modeled as a variable angle composite plate. Later, this model will be compared with the straight angle

composite plate, with the same geometrical and material properties, and the effectiveness of the variable-angle composites will be investigated.

One of the very common material in the aviation industry, boron epoxy, is examined in order to show the effective buckling resistance of VATs. The necessary material properties for the analysis can be taken from Table 5.14 and Table 5.15.

The preferred geometry is generated with a ratio of $a/h=100$ as in Table 5.19.

Table 5.19 Geometry for the analysis.

Property	Value
Length & Width	0.1016 m
Laminate Thickness	1.016 mm

Each of the configurations is under a unit thermal load with the same geometrical and material property. The only difference between the configurations is the orientation of laminates.

Table 5.20 Buckling factor improvement.

	Common Layups		VAT	Increment
Orientations	$[0/90]_s$	$[45/-45]_s$	(5,-60)	
Eigen Values	10.2286	14.4204	15.6604	8.60 %

In the comparison analysis, two common straight angle layups which are $[0/90]_s$, $[45/-45]_s$ are investigated. Under the given geometry, and material conditions with the given orientations, it is found that in the Table 5.20, the VAT is 8.60% more effective than the $[45/-45]_s$ layup (the most buckling resistant straight layup) and 53.10% more effective than $[0/90]_s$ layup. Even the VAT orientation is not an optimized orientation in this case, buckling resistance is increased approximately 10% compared to the most effective straight layup. This value can be increased by optimizing the VAT orientation.

In the study of Duran, it is emphasized that the buckling resistance of Carbon/Epoxy $[45/-45]_s$ can be improved by 37% with an optimized orientation version of VAT [13]. At this level of optimization for thermal buckling enables decreased material usage. Since the plate would have more stiffness, the thickness can be decreased. Production costs that may be occurred can be covered by the material effectiveness of VAT.

6. CONCLUSION

Since many vehicles, especially aircraft, operate in environments where temperature differences occur, they are faced with the breaking criterion called thermal buckling, which makes the structure unstable. While this temperature difference is sometimes related to the altitude of the vehicles, sometimes it can be caused by the speed of the vehicle and the atmospheric aerodynamic effects caused by this velocity. Therefore, this breaking criterion is very important, especially for vehicles operating at high velocity levels. Since the parts that give the outer shape of the vehicles are plate parts due to aerodynamic reasons, the thermal buckling of the plates is examined as a sub-branch of this criterion.

Although metal parts came to the forefront in terms of production, ease of processing, and price in the first years of aviation, composite materials began to be preferred in many aircraft and spacecraft parts with the developing production technologies and analysis capacities. As production technologies progress, the preference for variable angle composites has increased instead of composite plates consisting of straight angles. In particular, the fact that the robotic arm of the automatic fiber placement machine allows production in very different orientations has made this area the focus of attention. Scientists and engineers started to work in this area considering the possibility of improving the thermal buckling criterion of plates with variable-angle composites. In the course of these investigations, the edges and centers of the plates do not require the same angle orientation, as expected. However, flat composite plates have the same angle throughout the layer. Depending on this angle preference, although some of the plates can be well supported, some of them do not have sufficient stiffness and this does not cause unnecessary material use. This causes a situation against the principle of lightness, which is perhaps the most important criterion of aviation. In order to prevent this, scientists and engineers have aimed to have similar stiffness by working on composite plates with variable angles that can work more effectively, by consuming less material.

In this study, a code was created to reveal whether the variable angle composite plates will be subjected to buckling under thermal load, and as a result of this code, the load factor or critical temperature value was tried to be found. In the study, simple support

was preferred as the boundary condition of the plates, and analyzes were carried out accordingly. As a geometric feature, square shaped plates were preferred.

While modeling the plates, the classical lamination theory, which is the basis of composite materials, was used. With the help of this theory, the stiffness matrices (A, B, D) of the composites and the thermal loads they are exposed to under unit temperature were calculated. In the process of making this layer suitable for the finite element method, the necessary strain-displacement matrices were created using Kirchhoff plate theory. In order to create the plate shape properly, it is possible to create a sufficient number of elements with nodal points by using the necessary shape functions. The nodes and elements created were helped calculate the required loads.

In order for the user to decide how the plates will be modeled structurally, besides the geometric data, the number of elements and material preference are left to the discretion of the user. After the creation of the element and nodes, material and geometry stiffness matrices were created with the help of the composite's stiffness matrices (A, B, D) and strain displacement matrices. Matrix operations are built into the code to provide the necessary boundary conditions (simple support). In this way, it is planned that the code will understand which nodes are the boundary points and define the relevant boundary conditions for these points without the need for the user to specify. In addition, a path is drawn so that the calculated loads are placed on the respective degrees of freedom of the nodal points. Using the generated stiffness matrices and equation 4.16, the thermal buckling factor of the variable angle plates was calculated. The user's geometry, material, element-node number, plate edge-center orientation inputs, and thermal buckling factor output of the plates are provided.

Many analyzes were carried out during the verification phase of the code created in this study. Basically, the studies of Duran and Babu are used and comparisons are made with some analytical formulas. In the verification phase, the following steps were carried out, respectively, in order to reach from simple to complex.

The buckling factor of an isotropic plate under structural static loading; the comparison was made with the analytical formula and the FEM model created in the Patran software. Here, with the assumption that the error obtained around 2% is acceptable, the next step is taken.

At this stage, it is aimed to verify the code with different a/h ratios of cross-layered composite plates. In the comparison made based on Babu's study and the model created in Patran, an error rate of less than 1% was observed for the $a/h=100$ condition. A comparison was made between the results of the Patran model and the code over the 10 most critical load factors, and it was shown that all values converged to a certain extent. When the analysis was performed with the same material and layer distribution for the other case, $a/h=10$, it was observed that the error rate was around 3%. It has emerged as a result of these two analyzes that the load factor is closely related to the a/h ratio.

In the condition of verifying the angled layup, which is the next step, the same geometry, and material properties as the previous analysis type was used, and comparisons were provided by Babu's study. Only the orientations of the layers were changed. The error rate for the distribution of $a/h=100$ and $[45/-45/45/-45]$, which is the first case, is around 10%. Although this rate is high compared to other situations, it is considered an exception and still an acceptable error rate for the industry. In the second case, again $a/h=10$ was preferred and an error of around 3% was encountered. It has been determined that this layer type is more effective for buckling loading than the cross-layer type. The code was also verified according to this layer type and the next stage was passed.

Here, complex layers with different layer angles were preferred and Duran's work, Rayleigh-Ritz analytical formula, and Patran models were used during verification. When the analysis was performed for 3mm laminate thickness and $[45/-45/0/90]_s$ layer distribution, the deviation rates were found to be 0.25% (Rayleigh-Ritz) and 2.27% (Duran). The code has gone through this phase as well, completing the validation for straight angled composites, as the comparison to Rayleigh-Ritz is fairly consistent.

In the last and most important verification stage, variable angle composites were tried to be verified with 3 different materials and orientations based on Duran's work. E-Glass/Epoxy, S-Glass/Epoxy, and Carbon/Polyimide materials were modeled as variable angle composite laminates and analyzed with $h=1.016\text{mm}$ and $a=0.15\text{m}$, respectively. In comparisons as a result of the analysis, an average of 2% error were observed. Thus, it was concluded that the code can be used reliably.

After the completion of the validation part of the study, the cross-layup, angle-layup, and variable-angle (non-optimized) states of the plate were compared under the same geometric conditions using Boron/Epoxy material. Even in the unoptimized state, the variable angled configuration was found to be around 9% more effective against angled layup, which is the most resistant of the straight-angle layered states. This ratio can be improved by optimization analysis. In Duran's study, it was stated that about 37% improvement could be made for Carbon/Epoxy.

As a result of the study, it was seen that the buckling analysis depends on the a/h ratio, the orientation of the layers, and the elastic modulus of the material. It has been observed that variable angled composites can easily outperform straight angled composites in thermal buckling factor.

In the next studies; different boundary conditions options, the possibility of compatibility with different geometric surfaces (rectangular plate), and analysis of plates with stiffener elements can be added to the code.

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