ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL OF SCIENCE ENGINEERING AND TECHNOLOGY

A GRADUAL APPROACH IN PORTFOLIO SELECTION PROBLEM: OPTIMIZATION BY USING FUZZY APPROACH WITH SSD EFFICIENCY TEST

Ph.D. THESIS

Celal Barkan GÜRAN

Department of Management Engineering

Management Engineering Programme

Thesis Advisor: Prof Dr. Oktay TAŞ

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İSTANBUL TEKNİK ÜNİVERSİTESİ ★ FEN BİLİMLERİ ENSTİTÜSÜ

İKİNCİ DERECE STOKASTİK BASKINLIKTA VERİMLİLİK TESTİ VE BULANIK MANTIK YAKLAŞIMI İLE İKİ AŞAMALI BİR PORTFÖY OPTİMİZASYONU

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TEMMUZ 2015

Celal Barkan Güran, a **Ph.D.** student of ITU **Graduate School of Science Engineering and Technology** student ID 507092014, successfully defended the **thesis** entitled "**A GRADUAL APPROACH IN PORTFOLIO SELECTION PROBLEM: OPTIMIZATION BY USING FUZZY APPROACH WITH SSD EFFICIENCY TEST**", which he prepared after fulfilling the requirements specified in the associated legislations, before the jury whose signatures are below.

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Date of Submission : 07 June 2015
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To my spouse and son,

FOREWORD

This thesis is the end product of an intensive Ph.D. work of five years. In addition to the approval of the examining committee, the acceptances of the thesis related academic papers by the prestigious journals during the thesis progress has increased my motivation.

I would like to express my deep appreciation and thanks for my thesis advisor Prof. Dr. Oktay Taş who has helped me in all stages of this thesis process and also for my examining committee members Prof. Dr. Suat Teker and Prof. Dr. Cengiz Kahraman. Besides I want to thank to Institute of Science and Technology (TÜBİTAK) that has financially supported me during this long period.

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July 2015 Celal Barkan GÜRAN

(Mathematics Engineer)

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ABBREVIATIONS

CDF **:** Cumulative Distribution Function
FSD **:** First Order Stochastic Dominance
IPO **:** Initial Public Offering
MPT **:** Modern Portfolio Theory
MV **:** Mean-Variance
SR **:** Sharpe Ratio
SSD **:** Second Order Stochastic Dominance
SWF : Social Welfare Function
TR **:** Treynor Ratio

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A GRADUAL APPROACH IN PORTFOLIO SELECTION PROBLEM: OPTIMIZATION BY USING FUZZY APPROACH WITH SSD EFFICIENCY TEST

SUMMARY

Portfolio management is a trillion dollar business in today's financial world where every investor tries to increase the return of his portfolio while at the same time to decrease the risk of it. The classical and 60 years old Mean Variance (MV) portfolio optimization method has become old fashioned since it has some weaknesses which do not satisfy today's financial needs when working with real data. At the core, among other shortcomings, the requirement of normal distributed returns renders the MV optimized portfolios Second Order Stochastic Dominance (SSD) inefficient. In this thesis, a new two step gradual portfolio optimization method is introduced. In the first step of this method SSD inefficient stocks are eliminated after c(n,2) pairwise SSD comparisons of all stocks in the portfolio. At this point, a SSD inefficient stock means that it is second order stochastically dominated by at least one other stock. The second step of this gradual method is the application of the "fuzzy variance" minimization instead of MV. In this second step the future returns of the stocks are predicted with the help of the triangular fuzzy numbers where their centres are the average returns and their left and right deviations are relatively the worst and the best returns of the stocks in the observation period. As an empirical example, this suggested method is applied to the Turkish BIST-30 Index. Once the application is completed, the optimized portfolio of the suggested method is compared with both the MV optimized portfolio and the original BIST-30 portfolio according to most well known performance measurements, Sharpe Ratio (SR) and Treynor Ratio (TR). Detailed performance tests show that this new gradual method has overwhelming superiority over the classical method which requires normal distribution of stock returns that is nearly impossible in real data. In the near future, this novel gradual portfolio optimization method will be applied to other markets of the world to generalize its superiority over the MV.

İKİNCİ DERECE STOKASTİK BASKINLIKTA VERİMLİLİK TESTİ VE BULANIK MANTIK YAKLAŞIMI İLE İKİ AŞAMALI BİR PORTFÖY OPTİMİZASYONU

ÖZET

Yatırımcıların portföylerinin getirisini maksimize etmeye çalışırken aynı zamanda riskini de minimize etmeye çalıştıkları günümüzün finans dünyasında portföy yönetimi trilyon dolarlık bir pazar haline gelmiştir. 60 yıllık mazisi olan klasik varyans minimizasyonu yöntemi (MV) gerçek veri ile çalışırken günümüzün ihtiyaçlarını karşılamakta zayıf kalmaktadır. MV'nin diğer tüm zaaflarının yanında portföy varlıklarının getirileri üstüne getirdiği normal dağılım zorunluluğu bu yöntem ile optimize edilmiş portföylerin ikinci derece stokastik baskınlıkta (SSD) etkin olamaması sonucunu doğurmaktadır.

Bu tezde, ilk aşamasında SSD'de etkin olmayan hisse senetlerini eleyen ikinci aşamada da bulanık mantık yaklaşımı ile hesaplanan "bulanık varyans"ı minimize eden iki basamaklı yeni bir portföy optimizasyon metodu ortaya konmaktadır. Akabinde de bu metot BIST-30 endeksine uygulanmaktadır. Uygulama tamamlandıktan sonra, bu yeni iki aşamalı yöntemle oluşturulan portföy hem MV ile hesaplanan portföy hem de orijinal BIST-30 portföyü ile meşhur performans ölçütleri olan Sharpe Oranı (SR) ve Treynor Oranı (TR) kriterlerine gore kıyaslanmaktadır. Yapılan detaylı performans analizleri bu tezde ortaya konan yeni metodun gerçek veri ile yapılan çalışmalarda pratik olarak varolması neredeyse imkansız olan normal dağılım şartı yüzünden klasik metoda göre üstünlük sağladığını göstermektedir. Yakın gelecekte de bu yöntemin dünya çapındaki önemli endekslere uygulanarak performans sonuçlarının MV yöntemine karşı gösterdiği üstünlüğün ülkeden bağımsız olarak tüm finans piyasalarında genellenmesi hedeflenmektedir.

Portföy yönetimi ile ilgili genel açıklamalar içeren ve ilk paragrafta da belirtilmiş olan "Giriş" bölümünden sonra "Temel Kavramlar ve Teorik Altyapı" başlıklı ikinci bölümünde ortaya konan özgün modelin bu konular hakkında ileri düzeyde bilgi birikimine sahip olmayan bir okuyucu için bile anlaşılır olması için temel kavramlar ve bu kavramlara ait teorik bilgiler pekiştirici örnekler verilerek anlatılmıştır. Bu bölümde öncelikle Modern Portföy Teorisi (MPT) detaylandırılarak bu teori kapsamında Markowitz tarafından 1952'de ortaya konan Nobel ödüllü "Sabit getiride minimum Varyans" modeli (MV) tanıtılmıştır. Ayrıca MV'nin günümüzün finans dünyasında ortaya çıkan dezavantajları da tek tek listelenmiştir. İkinci kısımda, bulanık mantık teorisi ile ilgili tüm temel tanım ve kavramlar tanıtılmıştır. Bu esnada tezdeki modelde yoğun biçimde kullanılan üçgensel bulanık üyelik fonksiyonları da açıklayıcı örnekler ile anlatılmıştır. Üçüncü kısımda, Stokastik Baskınlık (SD) konusu hem birinci derece stokastik baskınlık (FSD) hem ikinci derece stokastik baskınlık (SSD) alt başlıkları altında teorik bilgilere ek açıklayıcı örneklerle anlatılmıştır. Sonrasında da tezde önemli bir yer tutan SSD etkinlik testinin mantığı ve uygulama biçimi anlatılmıştır. Dördüncü ve son kısımda ise portföylerin

performansının ölçülmesi safhasında çok önemli bir yeri olan Sharpe Oranı (SR) ve Treynor Oranı (TR) isimli performans kriterleri tanıtılmıştır.

“Literatür Taraması” isimli üçüncü bölümde, tezde ortaya konan modeldeki iki aşamanın temellerini oluşturan bulanık mantık ve stokastik baskınlık yaklaşımları ile ilgili bu güne kadar ekonomi ve finans dergilerinde yayınlanmış çalışmalar kronolojik sırada özetlenmiştir. Bu literatür özetleri okuyucuya bulanık mantık ve stokastik baskınlık konularının günümüzün finans dünyasındaki uygulamaları hakkında geniş bir perspektif kazandırmakla birlikte tezdeki modelin teorisinde kullanılacak kritik bazı tanımların ve denklemlerin nereden geldiğini de anlatmaktadır. Carlsson ve Fullër isimli akademisyenlerin 2001’de yayınladıkları makalelerinin özeti sırasında bulanık sayıların olasılıksal ortalama, varyans ve kovaryanslarının teorik olarak integraller yardımı ile nasıl tanımlandığı anlatılmıştır. Akabinde de bu tanımlar bir adım ileri götürülerek doğrusal olan üçgensel üyelik fonksiyonları ile çalışıldığında bu genel formüllerin kendilerini hangi sade formlara indirgediği de işlem detayları ile belirtilmiştir. Başka bir deyişle, doğrusal üyelik fonksiyonları ile çalışmanın hesaplama kolaylığı açısından ne kadar büyük avantajlar sağladığı bu literatür özetleri sırasında çok bariz bir biçimde ortaya konmuştur.

“Önerilen Modelin Teorisi” isimli dördüncü bölümde ise modelde ortaya konan iki basamaklı optimizasyonun teorik temelleri tüm detayları ile masaya yatırılmıştır. İlk etapta, SSD etkin olmayan hisse senetlerinin nasıl portföy dışında bırakıldığı anlatılmıştır. Bu bağlamda incelenen N hisse senedinde N’in ikili kombinasyonu adet ikili SSD kıyaslaması yapılarak her hisse senedinin bir diğerine göre SSD ilişkisi ortaya konmuştur. Akabinde de en az bir hisse senedi tarafından ikinci derece stokastik olarak domine edilen hisse senetleri “SSD verimsiz” olarak adlandırılıp kurulacak portföyün dışında bırakılmıştır. İkinci etapta ise, sadece “SSD verimli” hisse senetlerinin olduğu bir altkümede portföyün bulanık varyansının sabit bir getiri seviyesinde minimize edilmesi prensibine dayanan bir optimizasyon uygulanmıştır. Bu ikinci aşamadaki en önemli kısım, doğrusal olan üçgensel üyelik fonksiyonları ile oluşturulmuş bulanık ortalama, varyans ve kovaryans kavramlarının n adet “SSD verimli” hisse senedine sahip bir indirgenmiş portföye nasıl uygulandığı hususudur. Bu noktada, üçgensel üyelik fonksiyonları hisse senetlerinin gelecek dönemdeki getirilerini tahmin eden bir dağılımı temsil etmektedirler. Üçgenin tepe noktası ortalama getiriyi, sağ aşağı köşe noktası gelecek dönemde yapması olası en yüksek getiriyi, benzer şekilde sol aşağı köşe noktası da gelecek dönemde yapması olası en düşük getiriyi temsil etmektedir. Ortalama getirideki bu maksimum ve minimum sapmalar yardımı ile bir önceki bölümde çıkarılan indirgenmiş formüller n hisse senedinin tümüne uygulandığında artık portföyün tamamının bulanık varyansı ve bulanık ortalaması ortaya çıkmıştır. Son olarak, ortaya konan bu bulanık varyansın minimizasyonu aşamasına gelindiğinde ise bu formülün içinde ortaya çıkan ikinci dereceden terimler doğrusal bir modelde optimizasyon yapmaya engel olmuştur. Bu doğrusal olmama sorununu aşmak için de modelde bulanık varyans yerine bulanık standart sapmanın kullanılması önerilmiştir. Literatürdeki tanımındaki gibi bulanık varyansın karekökü alınarak hesaplanan bulanık standart sapmada bu ikinci dereceden terimler düşmüş ve geriye sadece doğrusal terimler kalmıştır. Sonuç olarak tezde ortaya konan modelin ikinci aşaması bulanık varyans yerine bulanık standart sapmanın belli bir bulanık getiri düzeyinde minimize edilerek optimize portföyler ortaya çıkarılması prensibine dayanmaktadır.

“Önerilen Modelin Uygulaması” isimli beşinci bölümde ise tezde önerilen iki aşamalı model Türkiye’deki BIST-30 hisse senetleri üstünde uygulanmıştır. Bu bölümde öncelikle uygulamanın yapılacağı veri seti detaylı olarak tanıtılmıştır. BIST-30'a dahil hisse senetlerinin piyasa değeri, halka arz zamanı ve portföy içindeki ağırlık değerleri gibi özellikleri tek tek incelenmiştir. Hisse senetlerinin bu detaylı analizlerinden sonra 2010 Aralık ayından 2013 Temmuz ayına kadarki kapanış değerleri haftalık olarak çekilmiştir ve sonrasında da bu değerler yardımı ile her hisse senedi için 135 adet haftalık getiri hesaplanmıştır. Bu veri kümesi üstünde 1952'de Markowitz tarafından ortaya konan klasik MV yönteminin herhangi bir yeni yaklaşım ortaya koymadan uygulaması sonucu etkin sınır üstünde ortaya çıkan portföylerden Sharpe oranını maksimize edeni A Portföyü olarak adlandırılmıştır. Kıyaslama yapabilmek için bir standartın yakalanması adına A Portföyündeki getiri düzeyi diğer portföylerde de aynı seviyede alınmıştır. Sadece birinci etaptaki yöntemin gücünün tek başına test edilmesi için BIST-30'daki 18 adet "SSD verimsiz" hisse senedi elenmiş ve geriye kalan 12 adet "SSD verimli" hisse senedine de gene Markowitz'in klasik MV optimizasyonunun uygulanıp akabinde de etkin sınır üstünde belirlenen getiri düzeyindeki portföye B portföyü denilmiştir. Son olarak da tezde ortaya konan model her iki aşamasıyla BIST-30'a uygulanmıştır. Bu bağlamda 18 adet "SSD verimsiz" elendikten sonra geriye kalan 12 tane "SSD verimli" hisse senedine tezde ortaya konan bulanık standart sapmanın minimizasyonu yöntemi ile optimizasyon yapılmıştır. Etkin sınır üstünde ortaya çıkan portföyler içinde gene belirlenen standart getiri düzeyi seçilerek bu portföye de Portföy C denilmiştir.

"Sonuç" başlıklı tezin son bölümünde ise öncelikle performans testleri ve bunların sonuçları değerlendirilmiştir. Değerlendirmeler yapılırken, ortaya konan A, B ve C portföylerinde verinin bittiği tarih olan Temmuz 2013'den ileriye doğru aynı hisse senetlerine ait bir yıllık veri çekilmiştir ve bu portföylerin bu gelecek verisi üstündeki performansları hem Sharpe hem Treynor oranına bakarak değerlendirilmiştir. Bu bağlamda 2013 Temmuz başlangıç olma üzere 10, 20, 30, 40 ve 50 haftalık farklı zaman dilimlerinde beş ayrı inceleme yapılmıştır. Ayrıca bu üç portföye ek olarak BIST-30'un orijinal portföyünde de aynı analizler yapılmıştır. Yapılan bu çok yönlü performans değerlendirmelerinin sonuçları ise ikinci kısımda detaylı olarak masaya yatırılmıştır. Grafiklerle de desteklenen bu analiz sonuçları tezdeki iki aşamalı model sonucunda ortaya çıkan C portföyünün diğer tüm portföylere tüm zaman dilimlerinde her iki performans kriterine göre de de üstünlük sağladığını göstermiştir. Ayrıca B portföyünün de A portföyü ve BIST-30'a göre üstünlük sağladığı sonucu çıkmıştır ki bu da modelin sadece birinci aşamasının bile tek başına önemli bir katma değer yarattığını göstermektedir. C portföyünün diğerlerine göre üstünlüğünü gözler önüne seren tablonun altında yatan nedenler de araştırılarak BIST-30 hisselerinin incelendiği 135 haftalık getirilerinin dağılımı incelenmiştir ve bu hisselerin neredeyse yarısının Shapiro-Wilk normallik testine istinaden normal dağılıma uymadığı görülmüştür. Bu sonuç da mazisi çok eskilere dayanan klasik MV metodunun tezde ortaya konan yönteme kıyasla neden başarısız sonuçlar ürettiği hakkında fikir üretmiştir. Tezin son kısmında da, bu modelin BIST-30 dışındaki başka piyasalara uygulanması ve akabinde de bu modeli uygulayan hazır bir bilgisayar programının hazırlanması gibi yazarın gelecekte planladığı çalışmalar anlatılmıştır.

1. INTRODUCTION

Portfolio management is a trillion dollar business. Nowadays investors are constantly faced with the dilemma of guessing the direction of market moves in order to meet the return target for assets. Because of the uncertainty inherent in financial markets, financial experts must be very cautious in expressing their market views. The information content in such circumstances can be best described as being “fuzzy”, in terms of both the direction and the size of market moves. Nevertheless, this is one of the best ways to structure portfolios so that the target return, which is assumed to be higher than the risk-free rate, is met. In general, achieving returns higher than the risk-free rate requires taking either market or credit risk.

In other words, generating returns higher than the risk-free rate requires investors to hold a portfolio of risky assets. Such portfolios may be structured around imprecise and potentially incorrect information of portfolio experts regarding the size and direction of market moves. In addition to that, investors may operate under strict constraints requiring a minimum rate of return for the assets being managed.

In mathematical terms, the target rate of return and the minimum rate of return for the portfolio will be a function of the investment horizon, the risk preference of the investor and the nature of assets that can be included in the portfolio. In the financial world, individuals and business firms make portfolio and investment decisions with the objective of maximising the expected income over a given time horizon. Such decisions are based on the subjective evaluation of income expectations and the risk preferences of the investors taking these decisions.

In this thesis a novel two-step gradual portfolio optimization method having lots of advantages compared to classical approaches is introduced.

The first step of this method deal with the Second-Order Stochastic Dominance (SSD) inefficiency problem so that the all SSD inefficient stocks are directly excluded from the portfolio and as a natural result a SSD efficient portfolio is constituted.

The second step is the variance minimization, but apart from the classical way which has serious shortcomings working with real data, the fuzzy variance is minimized by using triangular fuzzy numbers to forecast next movements of returns. This gradual method is then applied to Turkish BIST-30 Index and its performance is checked compared with other benchmark portfolios by using the most important Measurements Criteria.

Chapter 2 makes a deep summary about fundamental concepts and theoretical background. In the first part of this chapter the Modern Portfolio Theory (MPT) is summarized. After reminding the basic definitions the Markowitz's classical portfolio optimization model its shortcomings are also introduced. In the second part the basics of Fuzzy Logic is introduced. Firstly the fundamental differences between classical and fuzzy sets are determined, then basic definitions and operations of Fuzzy Sets are reported and lastly triangular and trapezoidal Fuzzy Numbers are introduced with instructive examples. In the third part the concept of Stochastic Dominance (SD) is explained deeply. After introducing first and second order Stochastic Dominance, FSD and SSD, the basics of the SSD efficiency analysis are introduced. In the fourth part Measurements Criteria of the Portfolio Performance, Sharper and Treynor Ratios, are defined.

Chapter 3 makes a detailed literature review. In its first part the studies including fuzzy approach in portfolio optimization; in its second part studies including Stochastic Dominance are summarized in a chronological order.

Chapter 4 presents the background theory of the proposed model in this thesis. This chapter fully explains the details of the first step which is "Elimination of SSD inefficient stock" and the second step which is "Minimizing the Fuzzy Variance at a given target return".

Chapter 5 is the application of the proposed model to the BIST-30 Stock Exchange. After a detailed examination of the companies in BIST-30, the portfolio of the proposed model and also its benchmark portfolios for comparison are constituted. Notice that all theoretical findings of the previous chapters are used in this application process.

Chapter 6, the conclusion part, begins with the presentation of the performance tests then these results are analysed and interpreted to focus the superiority of the proposed model. Lastly, the ideas of the future work are listed.

2. FUNDAMENTAL CONCEPTS AND THEORETICAL BACKGROUND

Before introducing the proposed portfolio optimization model of this thesis basic concepts and theoretical background is summarized.

2.1 Modern Portfolio Theory (MPT)

The fundamental key point behind MPT is that the assets in an investment portfolio should not be selected individually, each on their own merits. Rather, it is important to consider how each asset changes in price relative to how every other asset in the portfolio changes in price.

Investing is a tradeoff between risk and expected return. In general, assets with higher expected returns are riskier. For a given amount of risk, MPT describes how to select a portfolio with the highest possible expected return. Or, for a given expected return, MPT explains how to select a portfolio with the lowest possible risk. The targeted expected return cannot be more than the highest-returning available security, of course, unless negative holdings of assets are possible. (Elton and Gruber, 1997)

Therefore, MPT is a form of diversification. Under certain assumptions and for specific quantitative definitions of risk and return, MPT explains how to find the best possible diversification strategy. Markowitz (1952) introduced MPT in a article and later he wrote a book. (Markowitz, 1959)

MPT assumes that investors are risk averse, meaning that given two portfolios that offer the same expected return, investors will prefer the less risky one. Thus, an investor will take on increased risk only if compensated by higher expected returns.

Conversely, an investor who wants higher expected returns must accept more risk. The exact trade-off will be the same for all investors, but different investors will evaluate the trade-off differently based on individual risk aversion characteristics.

The implication is that a rational investor will not invest in a portfolio if a second portfolio exists with a more favorable risk-expected return profile – i.e., if for that level of risk an alternative portfolio exists which has better expected returns.

Note that the theory uses standard deviation of return as a proxy for risk, which is valid if asset returns are jointly normally distributed or otherwise elliptically distributed.

Under the model:

- Portfolio return is the proportion-weighted combination of the constituent assets' returns.
- Portfolio volatility is a function of the correlations ρ_{ij} of the component assets, for all asset pairs (i, j).

2.1.1 Mean-Variance (MV) optimization

$E(R_P)$: Expected return

$$E(\mathrm{R_P}) = \sum_i w_i \, E(R_i) \qquad \textbf{(2.1)}$$

where R_P is the return on the portfolio, R_i is the return on asset i (that is, the share of asset i in the portfolio) and w_i is the weighting of component asset.

$\sigma_p{}^2$: Portfolio return variance

$$\sigma_P^2 = \sum_i w_i^2 \, \sigma_i^2 + \sum_i \sum_{j \neq i} w_i w_j \sigma_i \sigma_j \rho_{ij} \qquad \textbf{(2.2)}$$

where ρ_{ij} is the correlation coefficient between the returns on assets i and j. Alternatively the expression can be written as

$$\sigma_P^2 = \sum_i \sum_j w_i w_j \sigma_i \sigma_j \rho_{ij} \qquad \textbf{(2.3)}$$

where $\rho_{ij}=1$ for i=j.

σ_p: Portfolio return volatility (standard deviation):

$$\sigma_p = \sqrt{\sigma_p^2} \qquad \textbf{(2.4)}$$

An investor can reduce portfolio risk simply by holding combinations of instruments which are not perfectly positively correlated (correlation coefficient $-1 \leq \rho_{ij} < 1$). In other words, investors can reduce their exposure to individual asset risk by holding a diversified portfolio of assets.

Diversification may allow for the same portfolio expected return with reduced risk. These ideas have been started with Markowitz and then reinforced by other economists and mathematicians who have expressed ideas in the limitation of variance through portfolio theory.

So according to Markowitz's findings, the model of an optimal portfolio with minimum variance, called classical Mean-Variance (MV) optimization, can be formulated as in (2.5).

$$\text{Min} \sum_i w_i^2 \, \sigma_i^2 + \sum_i \sum_{j \neq i} w_i w_j \sigma_i \sigma_j \rho_{ij}$$

$$\text{Subject to } \sum_i w_i \, E(R_i) \geq \mu \text{ and } \sum_i w_i = 1 \qquad \textbf{(2.5)}$$

If all the asset pairs have correlations of zero they are perfectly uncorrelated—the portfolio's return variance is the sum over all assets of the square of the fraction held in the asset times the asset's return variance (and the portfolio standard deviation is the square root of this sum).

As shown in the Figure 2.1, every possible combination of the risky assets, without including any holdings of the risk-free asset, can be plotted in risk-expected return space, and the collection of all such possible portfolios defines a region in this space. According to Merton (1972), the left boundary of this region is a hyperbola, and the upper edge of this region is the efficient frontier in the absence of a risk-free asset (sometimes called "the Markowitz bullet"). Combinations along this upper edge represent portfolios (including no holdings of the risk-free asset) for which there is lowest risk for a given level of expected return. Equivalently, a portfolio lying on the efficient frontier represents the combination offering the best possible expected return for given risk level.

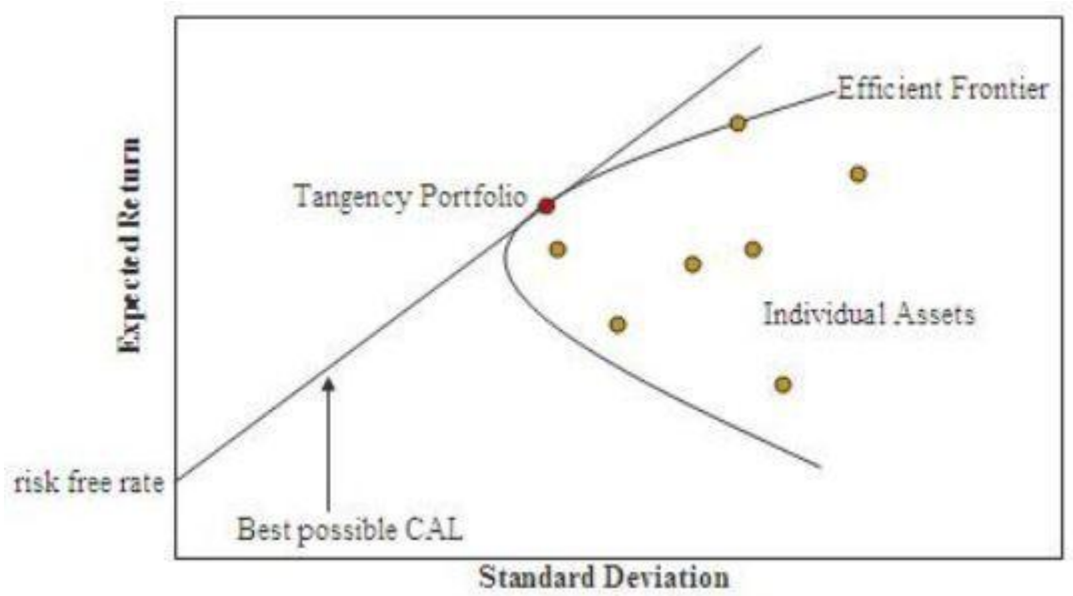


Figure 2.1 : Graphical display of the relationship between Standard Deviation vs Expected Return of a portfolio and its Efficient Frontier.

2.1.2 Shortcomings of MV optimization

MV is very common because of its simple algorithm which allows finding the optimal weights. But on the other side, in real life there are some very important shortcomings of MV.

Despite its theoretical importance, critics of MV Optimization question whether it is an ideal investing strategy, because its model of financial markets does not match the real world in many ways.

Efforts to translate the theoretical foundation into a viable portfolio construction algorithm have been plagued by technical difficulties stemming from the instability of the original optimization problem with respect to the available data. Brodie et al (2009) has shown recently that instabilities of this type disappear when a regularizing constraint or penalty term is incorporated in the optimization procedure.

Firstly, MV Optimization requires that the returns are normally distributed, but in the real data it is very rare to find normal distributed returns. Since this assumption brings some problems with it, the investor's problem is reduced to a one-period problem. Samuelson (1970) and Constandinides and Malliaris (1995) discuss this topic in detail and they work on the choice of MV optimal portfolios.

Secondly, MV Optimization is valid only for the quadratic utility functions but there are many other concave utility functions adopted by risk-averse investors.

Thirdly, MV Optimization deals only with two parameters - mean and variance - but there are two other significant parameters such as skewness and kurtosis. There is some research showing that risk averse investors prefer positive skewness and avoid kurtosis. For further details of MV Optimization's shortcomings the reader can examine the studies of Kraus and Litzenberger (1976), Athayde and Flores (1997), Fang and Lai (1997), Dittmar (2002), Post, Levy and Vliet (2008), Wong (2007).

Apart from them there are some other implicit assumptions and none of these assumptions are entirely true, and each of them compromises MV Optimization to some degree. These assumptions can be summarized as below:

- Correlations between assets are fixed and constant forever. Correlations depend on systemic relationships between the underlying assets, and change when these relationships change.
- All investors aim to maximize economic utility (in other words, to make as much money as possible, regardless of any other considerations).
- All investors are rational and risk-averse.
- All investors have access to the same information at the same time. In fact, real markets contain information asymmetry, insider trading, and those who are simply better informed than others.
- Investors have an accurate conception of possible returns, i.e., the probability beliefs of investors match the true distribution of returns. A different possibility is that investors' expectations are biased, causing market prices to be informationally inefficient. In fact, real markets contain information asymmetry, insider trading, and those who are simply better informed than others.
- There are no taxes or transaction costs. Real financial products are subject both to taxes and transaction costs (such as broker fees), and taking these into account will alter the composition of the optimum portfolio.
- All investors are price takers, i.e., their actions do not influence prices. In reality, sufficiently large sales or purchases of individual assets can shift market prices for that asset and others

- Any investor can lend and borrow an unlimited amount at the risk free rate of interest. In reality, every investor has a credit limit.
- All securities can be divided into parcels of any size. In reality, fractional shares usually cannot be bought or sold, and some assets have minimum orders sizes.
- Risk/Volatility of an asset is known in advance/is constant. In fact, markets often misprice risk (e.g. the US mortgage bubble or the european debt crisis) and volatility changes rapidly.

2.2 Basics of Fuzzy Logic

This chapter begins with a brief review of classical sets in order to facilitate the introduction of fuzzy sets. Next the concept of membership function is explained. It defines the degree to which an element under consideration belongs to a fuzzy set. Fuzzy numbers are described as a particular case of fuzzy sets. Fuzzy sets and fuzzy numbers will be used in fuzzy logic to model words such as profit, investment, cost, income, age, etc. Fuzzy relations together with some operations on fuzzy relations are introduced as a generalization of fuzzy sets and ordinary relations. They have application in database models. Fuzzy sets and fuzzy relations play an important role in fuzzy logic.

2.2.1 Classical sets: Relations and functions

This section reviews briefy the terminology, notations, and basic properties of classical sets, usually called sets. The concept of a set or collection of objects is common in our everyday experience. For instance, all persons listed in a certain telephone directory, all employees in a company, etc. There is a defining property that allows us to consider the objects as a whole. The objects in a set are called elements or members of the set. We will denote elements by small letters a,b,c,...,x,y,z and the sets by capital letters A,B,C,.....,X,Y,Z. Sets are also called ordinary or crisp in order to be distinguished from fuzzy sets.

The fundamental notion in set theory is that of belonging or membership. If an object x belongs to the set A we write $x \in A$. In other words for each object x there are only two possibilities: either x belongs to A or it does not.

A set containing finite number of members is called finite set; otherwise it is called infinite set. We present two methods of describing sets:

The set is described by listing its elements placed in braces; for example A={1,3,6,7,8},B={business, finance, managementg}.

The order in which elements are listed is of no importance. An element should be

listed only once.

The set is described by one or more properties to be satisfied only by objects in the set: A={x| x satifies some property or properties}

This reads: “A is the set of all x such that x satisfies some property or properties." For example R = {x| x is real number} reads: “R is the set of all x such that x is a real number"; R_+ ={x| x≥0; xϵR} reads R_+ is the set of all x which are nonnegative real numbers.

The set of all objects under consideration in a particular situation is called universal set or universe; it will be denoted by U. A set without elements is called empty; it is denoted by Φ. The set of all real numbers x such that $a_1<x<a_2$, where a_1 and a_2 are real numbers, form a closed interval $[a_1; a_2]$ = {x| $a_1<x<a_2$; xϵR} with boundaries a_1 and a_2. It is also called interval number.

If sets A and B are equal , it is denoted by A = B, they have the same elements. The set A is a subset of the set B (A is included in B), denoted by A c B, if every element of A is also an element of B. Every set is subset of itself, A c A. The empty set Φ is a subset of any set. It is assumed that each set we are dealing with is a subset of a universal set U.

A is a proper subset of B, denoted A c B, if A c B and there is at least one element in B which does not belong to A. For instance {a,b} C {a,b,c}. If A c B and B c C, then A c C.

The intersection of the sets A and B, denoted by A ∩ B, is defined by A ∩ B ={x| xϵA and xϵB} A ∩ B is a set whose elements are common to A and B. The union of A and B , denoted by A U B, is defined by A U B ={x| xϵA or xϵB} A U B is a set whose elements are in A or B, including any element that belongs to both A and B.

If the sets A and B have no elements in common, they are called disjoint. The complement of A c U, denoted by A, is the set A' = {xϵ U| x$\notin A$} The complement of a set consists of all elements in the universal set that are not in the given set.

Consider the universe U to be the set of real numbers R. A subset S of R is said to be convex if and only if, for all x_1, x_2 ϵ S and for every real number λ satisfying $0 \leq \lambda \leq 1$, we have $\lambda x_1 + (1 - \lambda) x_2 \in S$.

For example, any interval S = [a1; a2] is a convex set since the condition is satisfied; [0, 1] and [3, 4] are convex, but [0; 1]U[[3; 4] is not.

Sets are geometrically represented by circles inside a rectangle (the universal set U).

It was noted that the order of the elements of a set is not important. However there are cases when the order is important. To indicate that a set or pair of two elements a and b is ordered, we write (a,b), i.e. use parentheses instead of braces; a is called first element of the pair and b is called second element.

Cartesian product (or cross product) of the sets A and B denoted AxB is the set of ordered pairs AxB = {(a, b) | aϵA, bϵB}.

Given A = {1,2,3} and B={1,2}; then we find

AxB = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 1), (3, 2)}; geometrically it is presented as in Figure 2.2.

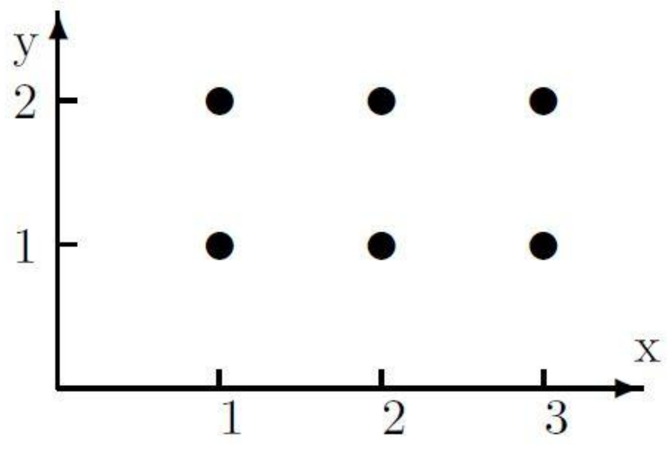


Figure 2.2 : Graphical Display of the Cartesian product with two finite sets.

If X, Y = R, the set of all real numbers, then XxY = {(x, y) | xϵX, yϵY}= RxR is the set of all ordered pairs which form the cartesian plane xy, geometrically it is presented as in Figure 2.3.

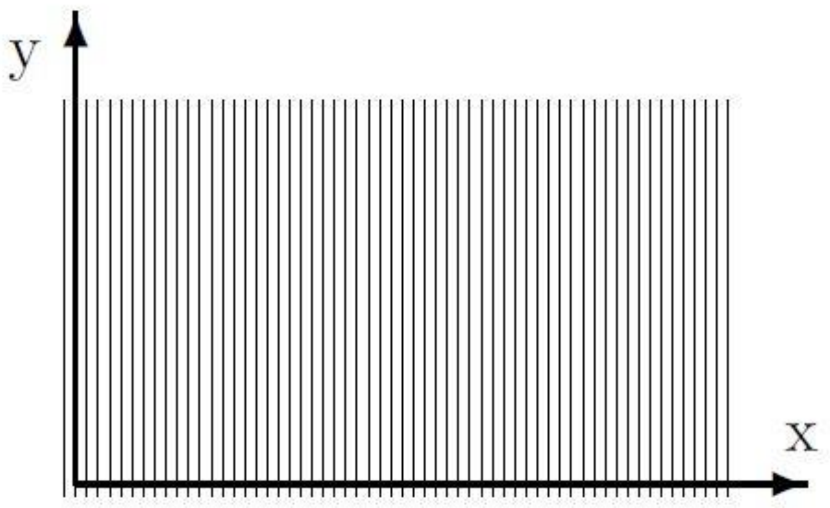


Figure 2.3 : Graphical Display of the Cartesian product with one finite and one infinite set.

The concept of relation is very general. It is based on the concepts of ordered pair (a,b), aϵA, bϵB, and cartesian product of the sets A and B.

A relation from A to B (or between A and B) is any subset R of the cartesian product AxB. We say that aϵA and bϵB are related by R; the elements a and b form the domain and range of the relation, correspondingly. Since a relation is a set, it may be described by either the listing method or the membership rule. The relation R is called binary relation since two sets, A and B, are related.

Let A = {x1; x2; x3} and B = {1; 2; 3; 4}.

We list some binary relations generated by A and B:

R1 = {(x1; 1); (x2; 1); (x3; 4)},

R2 = {(x1; 2); (x1; 3)};

R3 = {(x2; 2); (x3; 1)}

R4 = {(x1; 1); (x1; 2); (x1; 3); (x1; 4); (x2; 1); (x4; 1)}

are relations from A to B;

R5 = {(1; x2); (2; x3); (3; x1)},

R6 = {(1; x1); (2; x1)},

R7 = {(1; x1); (1; x2); (1; x4)},

R8 = {(2; x1); (3; x3)}

are relations from B to A; the empty set Φ is a relation; the cross product AxB is a relation from A to B and the cross product BxA is a relation from B to A.

A function f is a relation R such that for every element x in the domain of f there corresponds a unique element y in the range of f. For instance the relations in Figure 2.2 and Figure 2.3 are not functions.

We often say that f maps x onto y; y is the image of x under f. Then we can write

f: x →y. However, it is customary to use the notation y = f(x).

The notions of ordered pair, Cartesian product, relation, and function can be generalized for higher dimensions than two. For instance when n = 3 we have:

Ordered triple (a, b, c),

Cartesian product AxBxC = {(a,b,c) | a∈A, b∈B, c∈C};

Relation from AxBxC is any subset R of AxBxC.

Function z = f(x; y) is a relation such that for every pair (x; y) in the domain of f there corresponds a unique element z in its range.

The membership rule that characterizes the elements (members) of a set $A \subset U$ can be established by the concept of characteristic function (or membership function) $\mu_A(x)$ taking only two values, 1 and 0, indicating whether or not x∈U is a member of A:

$$\mu_A(x) = \begin{cases} 1 \; for \; x \in A \\ 0 \; for \; x \notin A \end{cases} \quad \textbf{(2.6)}$$

Hence $\mu_A(x) \in \{0,1\}$. Inversely, if a function $\mu_A(x)$ is defined as in (2.6), then it is the characteristic function for a set $A \subset U$ in the sense that A consists of the values of x∈U for which $\mu_A(x)$ is equal to 1. In other words every set is uniquely determined by its characteristic function.

The universal set U has for membership function $\mu_A(x)$ which is identically equal to 1, i.e. $\mu_U(x)= 1$. The empty set Φ has for membership function $\mu_\Phi(x)= 0$.

Consider the universe U = {x1, x2, x3, x4, x5, x6} and its subset A, A = {x2; x3; x5}

Only three of the six elements in U belong A. Using the embership notation gives

$\mu_A(x1)=0$, $\mu_A(x2)=1$, $\mu_A(x3)=1$, $\mu_A(x4)=0$, $\mu_A(x5)=1$, $\mu_A(x6)=0$

Hence the characteristic function of the set A is

$$\mu_A(x) = \begin{cases} 1 \; for \; x = x2, x3, x5 \\ 0 \; for \; x = x1, x4, x6 \end{cases}$$

The set A can be represented as

A = {(x1, 0), (x2, 1), (x3, 1), (x4, 0), (x5, 1), (x6, 0)}

Let us try to use crisp sets to describe tall men. Consider for instance a man as tall if his height is 180 cm or greater; otherwise the man is not tall. The characteristic function of the set A = {tall men} then is

$$\mu_A(x) = \begin{cases} 1 \; for \; 180 \leq x \\ 0 \; for \; 160 \leq x \leq 180 \end{cases}$$

It is shown in the in Figure 2.4, where the universe is U = {x|$160 \leq x \leq 200$}

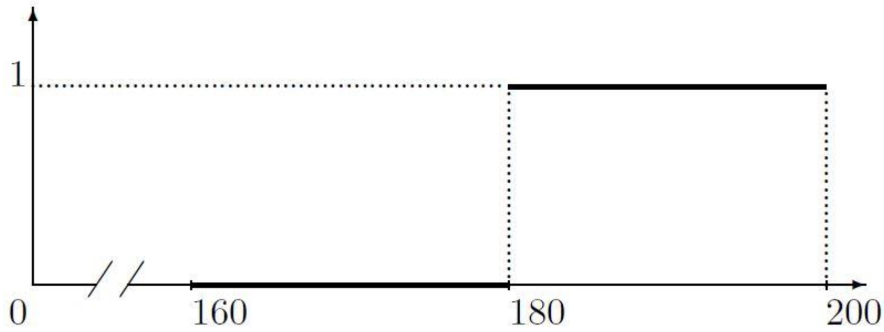


Figure 2.4 : Membership function of the set tall men.

Clearly this description of the set of tall men is not satisfactory since it does not allow gradation. The word tall is vague. For instance, a person whose height is 179 cm is not tall as well as a person whose height is 160 cm. Yet a person whose height is 180 is tall and so is a person with height 200 cm. Also this definition introduces a drastic difference between heights of 179 cm and 180 cm, thus fails to describe realistically borderline cases.

A paradox coming from ancient Greece has caused serious problems to logicians and mathematicians. Consider a heap of grains of sand. Take a grain and the heap is still there. Take another grain, and another grain, and continue the process. Eventually ten grains are left, then nine, and so on. When one grain is left, what happens with the heap. Is it still a heap? When the last grain is removed and there is nothing, does the heap cease to be a heap?

There are many paradoxes of similar nature called “sorites." This word comes from “soros" which is the Greek word for heap. For instance let us apply the above procedure to the cash (say, one million) of a rich person. He/she spends one dollar and is still rich; then another dollar and so on. When one hundred dollars are left, what happens to his/her richness? When does that person cease to be rich? In the crisp set theory such dilemmas are solved by sort of appropriate assumptions or by decree. In the case of the heap a certain natural number n is to be selected; if the number of sand grains is $\geq$ n, then the grains constitute a heap; n-1 sand grains does not form a heap anymore.

This defies common sense. Also how to select the number n? Is it 100, 1000, or 1,000,000, or larger? Common sense hints that the concept heap is a vague one. Hence a tool that can deal with vagueness is necessary. The concept of fuzzy set, a generalization of Cantor's sets, is such a tool. The following thoughts by Bertrand Russell (1923) are quoted very often: “All traditional logic habitually assumes that precise symbols are being employed. It is therefore not applicable to this terrestrial life, but only to an imagined celestial one. The law of excluded middle is true when precise symbols are employed but it is not true when symbols are vague, as, in fact, all symbols are." “All language is vague." “Vagueness, clearly, is a matter of degree."

An important step towards dealing with vagueness was made by the philosopher Max Black (1937) who introduced the concept of vague set.

2.2.2 Definition of fuzzy sets

We have seen that belonging or membership of an object to a set is a precise concept; the object is either a member to a set or it is not, hence the membership function can take only two values, 1 or 0. The example of the set tall men illustrates the need to increase the describing capabilities of classical sets while dealing with words.

To describe gradual transitions Zadeh (1965), the founder of fuzzy sets, introduced grades between 0 and 1 and the concept of graded membership.

Let us refer to previous example. Each of the six elements of the universal set U = {x1; x2; x3; x4; x5; x6} either belongs to or does not belong to the set A = {x2; x3; x5}. According to this, the characteristic function $\mu_A(x)$ takes only the values 1 or 0. Assume now that a characteristicfunction may take values in the interval [0, 1]. In this way the concept of membership is not any more crisp (either 1 or 0), but becomes fuzzy in the sense of representing partial belonging or degree of membership.

Consider a classical set A of the universe U. A fuzzy set A is defined by a set or ordered pairs, a binary relation,

$$A = \{(x, \mu_A(x)) \mid x \in A; \mu_A(x) \in [0; 1]\} \qquad \textbf{(2.7)}$$

where $\mu_A(x)$ is a function called membership function; $\mu_A(x)$ specifies the grade or degree to which any element x in A belongs to the fuzzy set A. Definition associates with each element x in A a real number $\mu_A(x)$ in the interval [0, 1] which is assigned to x. Larger values of $\mu_A(x)$ indicate higher degrees of membership.

Fuzzy, adv. fuzziness, in fuzzy logic is associated with the concept of graded membership which can be interpreted as degree of truth. The objects under study in fuzzy logic admit of degrees expressed by the membership functions of fuzzy sets Problems and events in reality involving components labeled as vague, ambiguous, uncertain, imprecise are considered in this thesis as fuzzy problems and events if graded membership is the tool for their description. In other words, when gradation is involved, vagueness, ambiguity, uncertainty, imprecision are included into the concept of fuzziness.

Beside the fundamental volume Fuzzy Sets and Applications: Selected Papers by L.A. Zadeh (1983), here we list several important books dealing with fuzzy sets and fuzzy logic used in this text: Kaufmann (1975), Dubois and Prade (1980), Zimmermann (1984), Kandel (1986), Klir and Folger (1988), Novak (1989), Terano, Asai, Sugeno (1992). Fascinating popular books on fuzzy logic are written by McNeill and Freiberger (1993) and Kosko (1993).

Let us express the meaning of this binary relationship A in a slightly modified way. The first elements x in the pair (x, $\mu_A(x)$) are given numbers or objects of the classical set A; they satisfy some property (P) under consideration partly (to various degrees). The second elements $\mu_A(x)$ belong to the interval (classical set) [0; 1]; they indicate to what extent (degree) the elements x satisfy the property P.

It is assumed here that the membership function $\mu_A(x)$ is either piecewise continuous or discrete.

The fuzzy set A according to the definition is formally equal to its membership function $\mu_A(x)$. We will identify any fuzzy set with its membership function and use these two concepts as interchangeable. Also we may look at a fuzzy set over a domain A as a function mapping A into [0, 1].

Fuzzy sets are denoted by letters A, B, C, ... and the corresponding membership functions by $\mu_A(x)$, $\mu_B(x)$, $\mu_C(x)$, ...

Elements with zero degree of membership in a fuzzy set are usually not listed. Classical sets can be considered as a special case of fuzzy sets with all membership grades equal to 1.

A fuzzy set is called normalized when at least one $x \epsilon A$ attains the maximum membership grade 1; otherwise the set is called nonnormalized. Assume the set A is nonnormalized; then max $\mu_A(x) < 1$. To normalize the set A means to normalize its membership function $\mu_A(x)$, i.e. to divide it by max $\mu_A(x)$, which gives $\frac{\mu_A(x)}{max\ \mu_A(x)}$.

A is called empty set labeled Φ if $\mu_A(x)=0$ for each $x \epsilon A$. The fuzzy set A = {(x1; $\mu_A(x_1)$}, where x_1 is the only value in $A \subset U$ and $\mu_A(x_1) \epsilon [0; 1]$ is called fuzzy singleton. While the set A is a subset of the universal set U which is crisp, the fuzzy set A is not.

Assume that x_i, i = 1,....,6 are integers, namely, x1 = 1, x2 = 2, x3 = 3, x4 = 4, x5 = 5, x6 = 6, they belong to the set A = {1; 2; 3; 4; 5; 6}, a subset of the universe U = N, the set of all integers. The fuzzy set A becomes

A = {(1, 0.1), (2, 0.5), (3, 0.3), (4, 0.8), (5, 1), (6, 0.2)},

its membership function $\mu_A(x)$ shown in Figure 2.5 by dots is a discrete one.

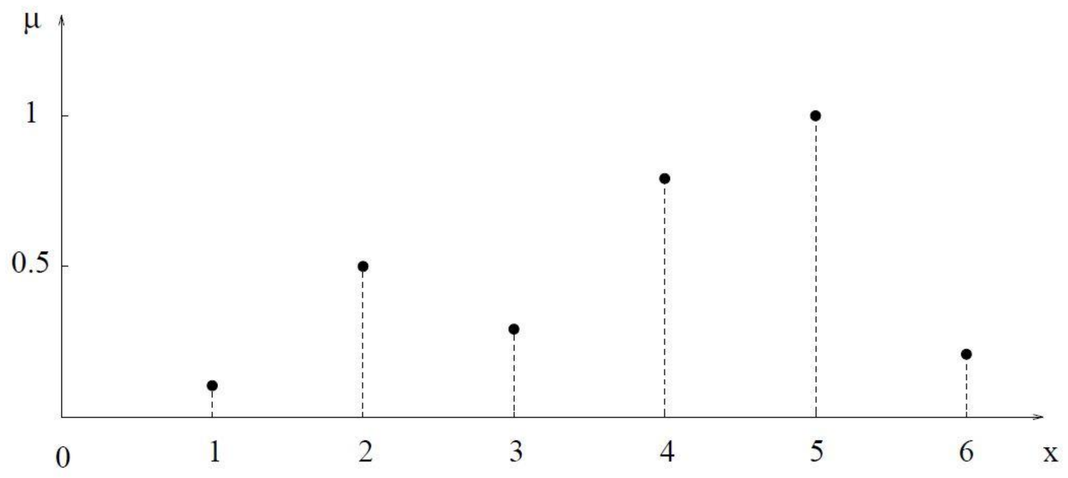


Figure 2.5 : Graphical Display of Fuzzy Set A.

Let us describe numbers close to 10. First consider the fuzzy set

$A_1 = \{(x, \mu_A(x)) \mid x \in [5; 15]; \mu_{A1}(x)=\frac{1}{1+(x-10)^2}\}$

where $\mu_{A1}(x)$ shown in Figure 2.6 is a continuous function.

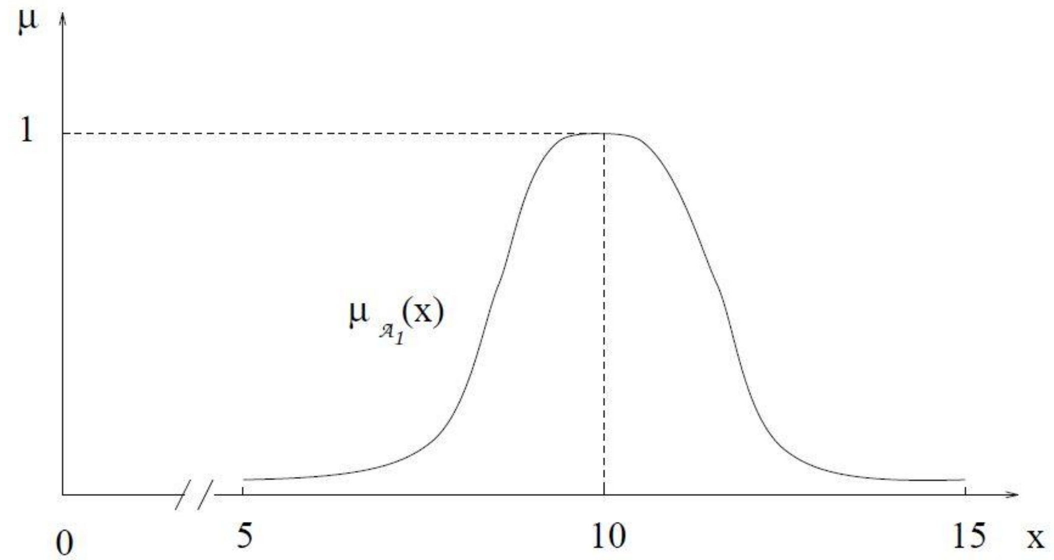


Figure 2.6 : The fuzzy set A_1 representing real numbers close to 10.

We have seen that the description of tall men by classical sets is not adequate. Now we employ for the same purpose the fuzzy set $T = \{(x; \mu_T(x)\}$, where x measured in cm belongs to the interval [160, 200] and $\mu_T(x)$ is defined by

$$\mu_T(x) = \begin{cases} \frac{1}{2(30)^2}(x-140)^2 \; for \; 160 \leq x \leq 170 \\ -\frac{1}{2(30)^2}(x-200)^2 + 1 \; for \; 170 \leq x \leq 200 \end{cases}$$

The membership function $\mu_T(x)$ is a continuous piecewise-quadratic function. The numbers on the horizontal axis x give height in cm and the vertical axis μ shows the degree to which a man can be labeled tall. According to the graph, if a person's height is 160 cm, the person is a little tall (degree 0.22), 180 cm stands for almost tall (degree 0.78), 200 cm for tall (degree 1). The segment [0.22, 1] of the vertical axis μ expresses the quantification of the degree of vagueness of the word tall. The graphical display is as in Figure 2.7.

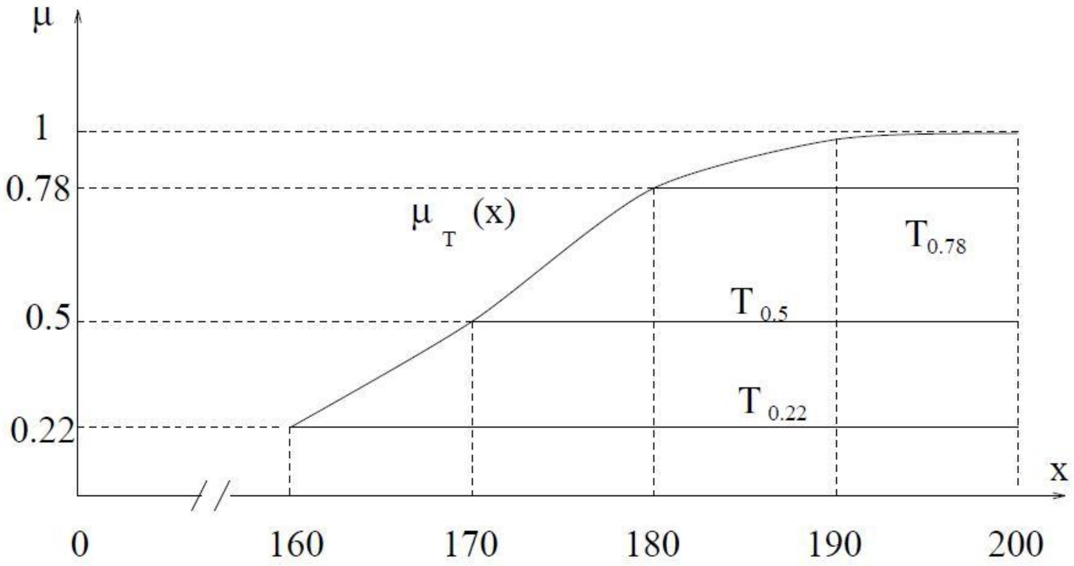


Figure 2.7 : Description of tall men by fuzzy set.

2.2.3 Basic operations on fuzzy sets

Consider the fuzzy sets A and B in the universe U,

$A = \{(x, \mu_A(x))\}$, $\mu_A(x) \in [0; 1]$,

$B = \{(x, \mu_B(x))\}$, $\mu_B(x) \in [0; 1]$,

The operations with A and B are introduced via operations on their membership functions $\mu_A(x)$ and $\mu_B(x)$.

The fuzzy sets A and B are equal denoted by A = B if and only if for every $x \in U$, $\mu_A(x) = \mu_B(x)$.

The fuzzy set A is included in the fuzzy set B denoted by $A \subseteq B$ if for every $x \in U$,

$\mu_A(x) \leq \mu_B(x)$. Then A is called a subset of B.

The fuzzy set A is called a proper subset of the fuzzy set B denoted $A \subset B$ when A is a subset of B and $A \neq B$, that is

$\mu_A(x) \leq \mu_B(x)$ for every $x \in U$,

$\mu_A(x) < \mu_B(x)$ for at least one $x \in U$.

The fuzzy sets A and A are complementary if

$\mu_{A'}(x) = 1 - \mu_A(x)$ or $\mu_{A'}(x) + \mu_A(x) = 1$

The membership function $\mu_{A'}(x)$ is symmetrical to $\mu_A(x)$ with respect to the line μ=0.5.

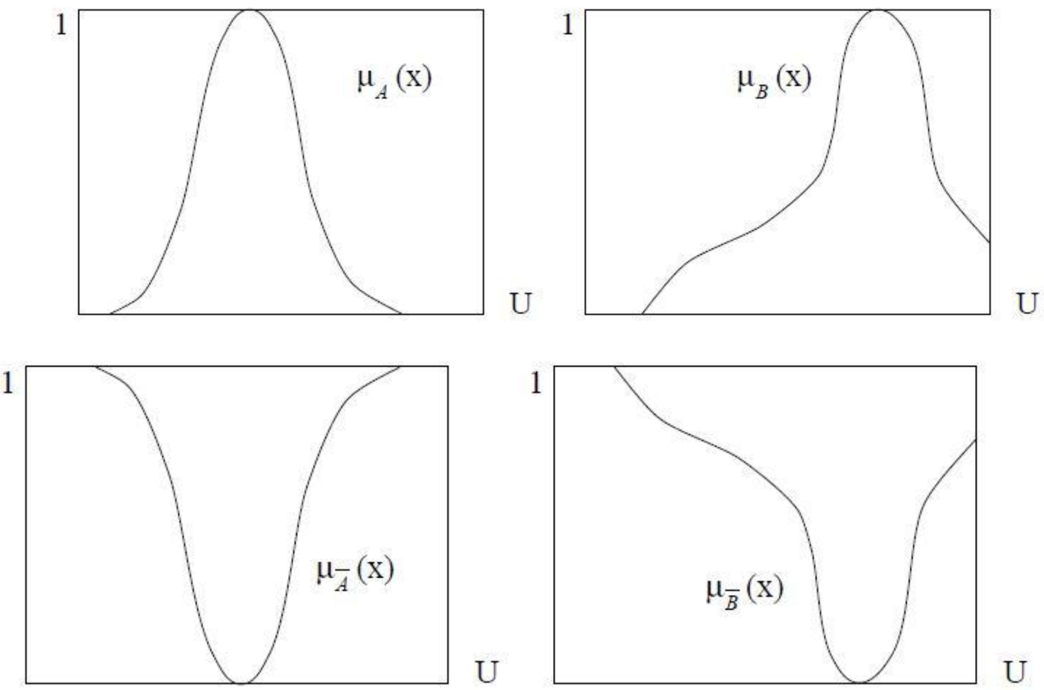
The operation intersection of A and B denoted as $A \cap B$ and the operation union of A and B denoted as $A \cup B$ are defined by

$$\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_A(x)), \quad \mu_{A \cup B}(x) = \max(\mu_A(x), \mu_A(x)), \; x \in U \quad \textbf{(2.8)}$$

If a1 < a2, min(a1,a2) = a1. For instance min(0.5, 0.7) = 0.5.

If a1 < a2, max(a1,a2) = a2. For instance min(0.5, 0.7) = 0.7.

Fuzzy sets are schematically represented by their membership functions (assumed continuous) inside of rectangles. In Figure 2.8 are shown $\mu_A(x)$ and $\mu_B(x)$ and their complementation intersection and union sets in graphical representation.



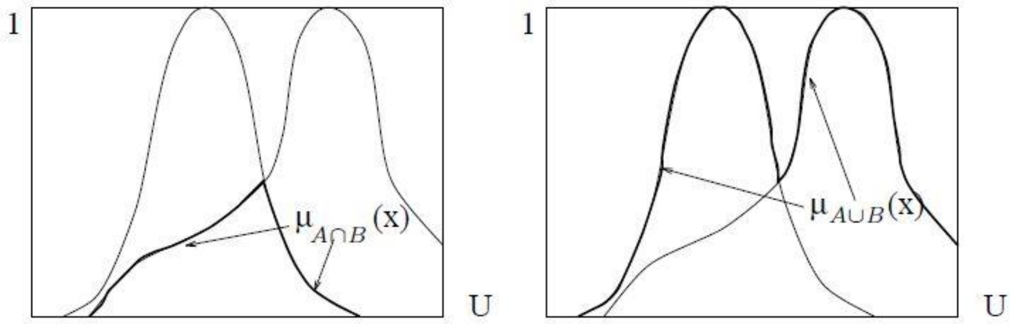


Figure 2.8 : μA(x) and μB(x) and their complementation intersection and union sets.

It is natural that the law of the excluded middle is not valid for fuzzy sets. In classical sets every object does or does not have a certain property, expressed by 1 or 0. Fuzzy sets were introduced to reflect the existence of objects in reality that have a property to a degree between 0 and 1. There are many shades of gray color between black and white.

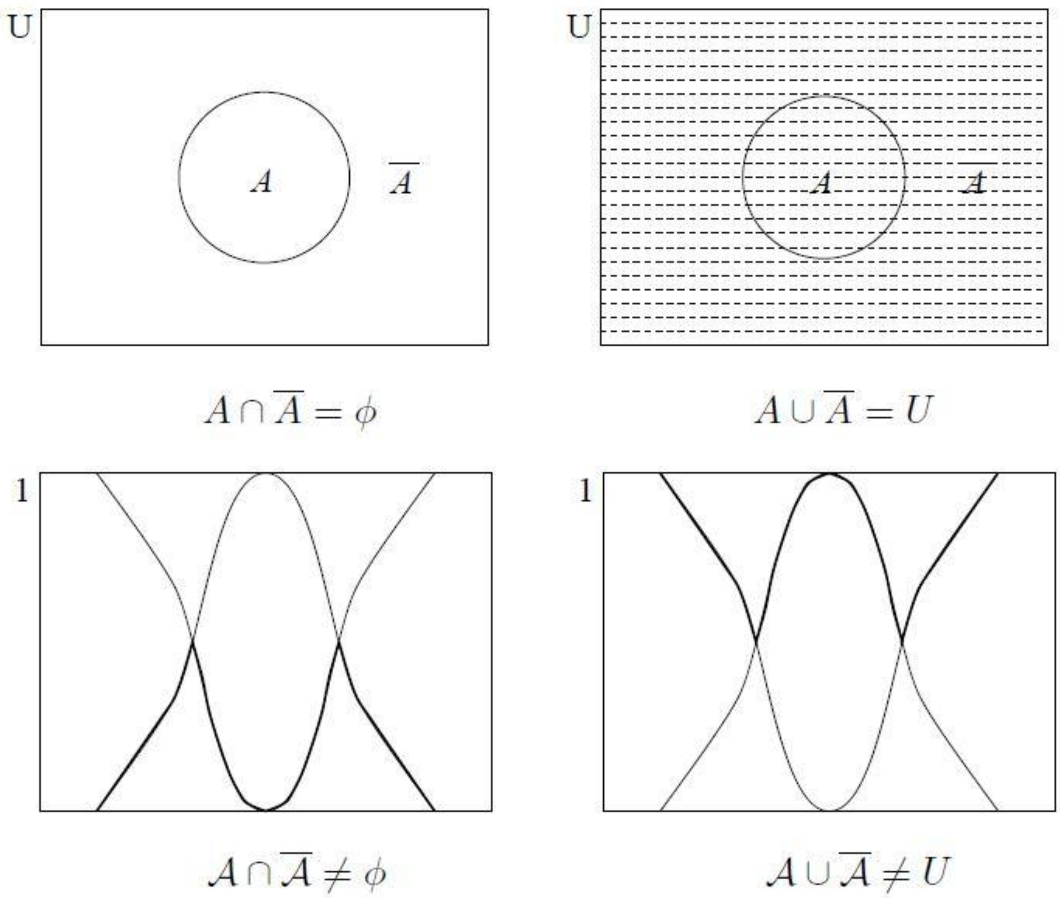


Figure 2.9 : The law of excluded middle both for classical and fuzzy sets.

The lack of the law of excluded middle in fuzzy set theory makes it less specific than that of classical set theory. However, at the same time, this lack makes fuzzy sets more general and flexible than classical sets and very suitable for describing vagueness and processes with incomplete and imprecise information.

2.2.4 Fuzzy numbers

A fuzzy number is defined on the universe R as a convex and normalized fuzzy set. The concept of fuzzy number was introduced after that of fuzzy set. Valuable contributions to fuzzy numbers were made by Nahmias (1977), Dubois and Prade (1978), and Kaufmann and Gupta (1985), see also G. Bojadziev and M. Bojadziev (1995).

The interval $[a_1, a_2]$ is called supporting interval for the fuzzy number. For $x = a_M$ the fuzzy number in Figure 2.10 (a) has a maximum. In Figure 2.10 (b) the flat segment has maximum height 1; actually it is the α-cut at the highest confidence level 1. Fuzzy numbers will be denoted by capital letters A,B,C, . . . and their membership functions by $\mu_A(x)$, $\mu_B(x)$, $\mu_C(x)$, ...

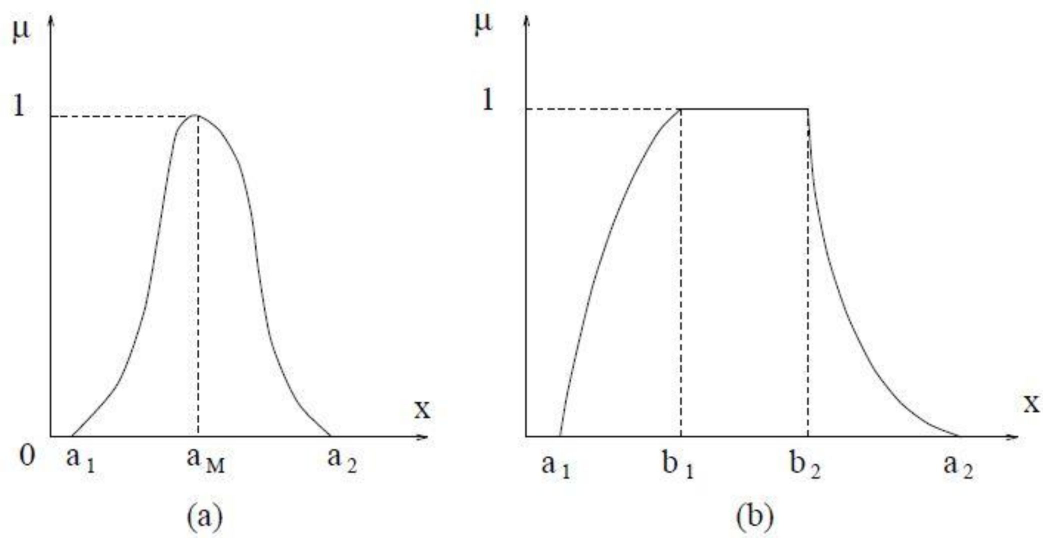


Figure 2.10 : Fuzzy numbers: (a) with a maximum; (b) with a flat.

The membership function $\mu_A(x)$ of a piecewise-quadratic fuzzy number shown in Figure 2.11 is bell-shaped, symmetric about the line x = p, has a supporting interval $A = [a_1, a_2]$, and is characterized by two parameters, $p=0.5(a_1+a_2)$ and $\beta \epsilon (0,a_2\text{-}p)$.

The peak-point (the maximum point) is (p,1); 2β called bandwidth is defined as the segment (α-cut) at level α=0.5 between the points (p-β,0.5) and (p+β,0.5), called crossover points.

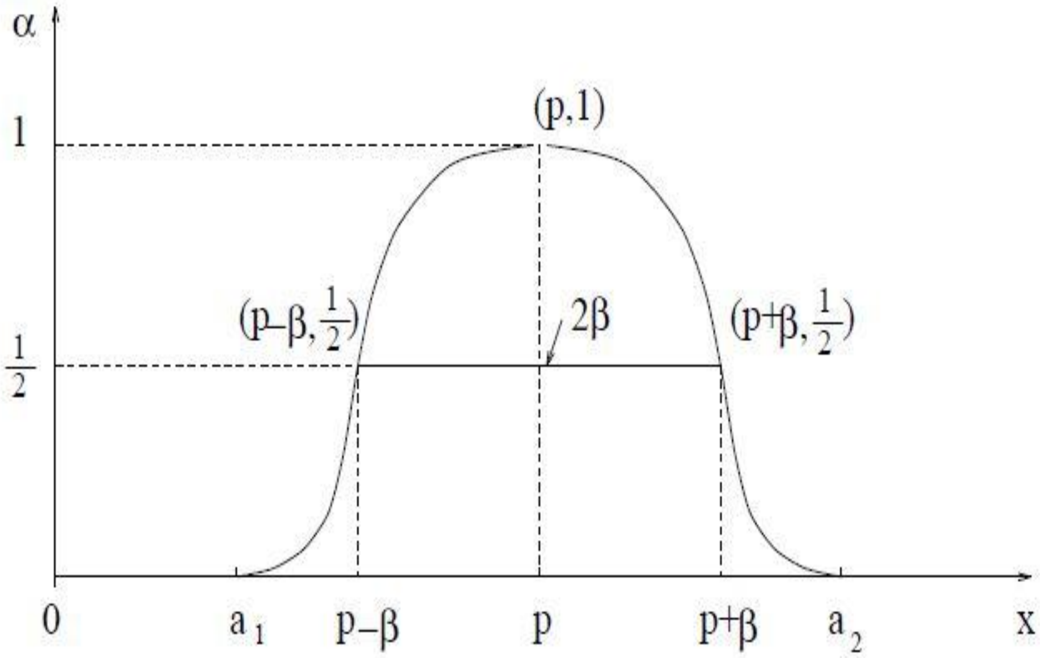


Figure 2.11 : The general graphical presentation of Piecewise-quadratic fuzzy number.

$$\mu_A(x) = \begin{cases} \frac{1}{2(p-\beta-a_1)^2}(x-a_1)^2 & for\ a_1 \le x \le p-\beta, \\ -\frac{1}{2\beta^2}(x-p)^2+1 & for\ p-\beta \le x \le p+\beta \\ \frac{1}{2(p+\beta-a_2)^2}(x-a_2)^2 & for\ p+\beta \le x \le a_2, \\ 0 & otherwise \end{cases} \quad \textbf{(2.9)}$$

The interpretation for the fuzzy number in (2.9) is real numbers close to the number p. Since the word close is vague and in that sense fuzzy, it cannot be defined uniquely. That depends on the selection of the supporting interval and the bandwidth which are supposed to reflect a particular situation. For instance the fuzzy set tall men is a particular case of this equation (left branch) on the interval [160, 200] with a_1=140, p=200 and β= 30.

2.2.5 Triangular fuzzy numbers

A triangular fuzzy number A or simply triangular number with membership function

$$\mu_A(x) = \begin{cases} \frac{x-a_1}{a_M-a_1} & for\ a_1 \le x \le a_M, \\ \frac{x-a_2}{a_M-a_2} & for\ a_M \le x \le a_2 \\ 0 & otherwise \end{cases} \quad \textbf{(2.10)}$$

where $[a_1, a_2]$ is the supporting interval and the point $(a_M,1)$ is the peak. The third line can be dropped.

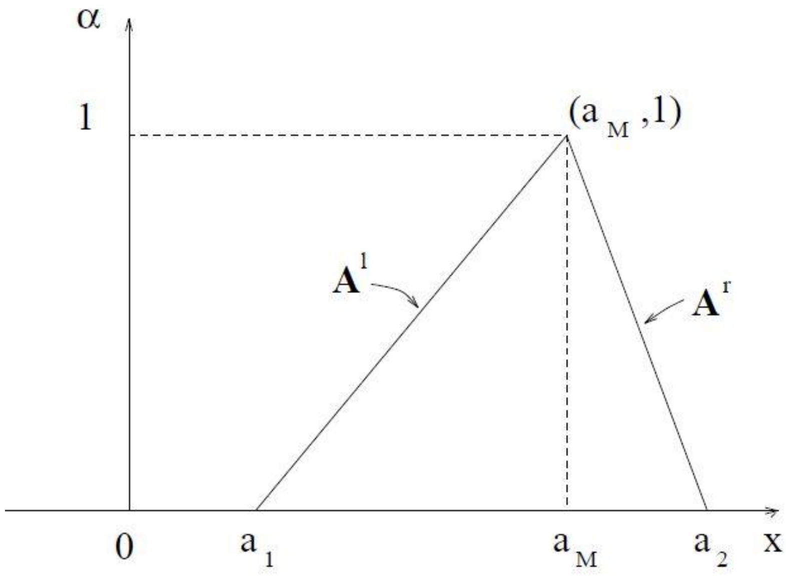


Figure 2.12 : Triangular fuzzy number with center a_M.

Often in applications the point $a_M \in (a_1,a_2)$ is located at the middle of the supporting interval, i.e. $a_M = 0.5\,(a_1+a_2)$. Then substituting this value in definition (2.10) gives

$$\mu_A(x) = \begin{cases} 2\,\frac{x-a_1}{a_2-a_1} & for\ a_1 \le x \le \frac{a_1+a_2}{2}, \\ 2\,\frac{x-a_2}{a_1-a_2} & for\ \frac{a_1+a_2}{2} \le x \le a_2 \\ 0 & otherwise \end{cases} \quad \textbf{(2.11)}$$

We say that the definition (2.11) represents a central triangular fuzzy number. Similarly to the piecewise-quadratic fuzzy number, it is very suitable to describe the word close (close to a_M). Triangular numbers are very often used in the applications (fuzzy controllers, managerial decision making, business and finance, social sciences, etc.).

They have a membership function consisting of two linear segments A^l (left) and A^r (right) joined at the peak $(a_M,1)$ which makes graphical representations and operations with triangular numbers very simple. Also it is important that they can be constructed easily on the basis of little information.

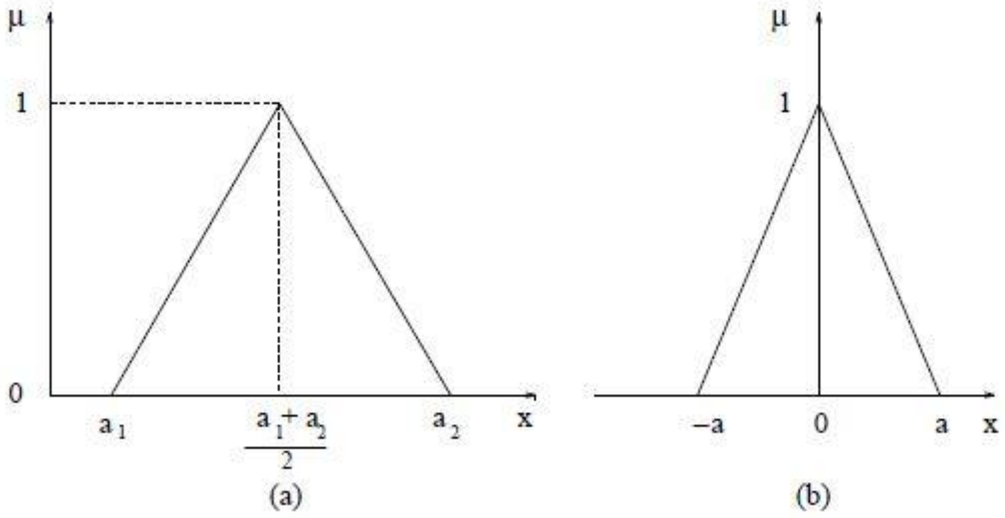


Figure 2.13 : (a) Central triangular number; (b) Central triangular number symmetrical about μ.

Assume while dealing with an uncertain value we are able to specify the smallest and largest possible values, i.e. the supporting interval $A = [a_1; a_2]$. If further we can indicate a value a_M in $[a_1; a_2]$ as most plausible to represent the uncertain value, then the peak will be the point $(a_M; 1)$. Hence with the three values a_1; a_2 and a_M, one can construct a triangular number and write down its membership function. That is why the triangular number is also denoted by $A = (a_1; a_M; a_2)$

A central triangular number is symmetrical with respect to the axis μ if a_1=-a; a_2=a, hence a_M=0. According to these substutions it is denoted by A = (-a, 0, a).

It is very suitable to express the word small. The right branch (segment) of A = (-a, 0, a), i.e. when $0 \leq x \leq a$, can be used to describe positive small (PS), for instance young age, small profit, small risk, etc. We can denote it by $A^r = (0, 0, a)$.

More generally, the left and right branches of the triangular number (1.14) can be denoted correspondingly by $A^l = (a_1, a_M, a_M)$ and $A^r = (a_M, a_M, a_2)$. They will be considered as triangular numbers and called correspondingly left and right triangular numbers.

The left triangular number A^l is suitable to represent positive large (PL) or words with similar meaning, for instance old age, big profit, high risk, etc. provided that a_M is large number.

2.2.6 Trapezoidal fuzzy numbers

A trapezoidal fuzzy number A or shortly trapezoidal number is defined on R by

$$A \cong \mu_A(x) = \begin{cases} \frac{x-a_1}{b_1-a_1} & for\ a_1 \leq x \leq b_1, \\ 1 & for\ b_1 \leq x \leq b_2, \\ \frac{x-a_2}{b_2-a_2} & for\ b_2 \leq x \leq a_2, \\ 0 & otherwise \end{cases} \qquad \textbf{(2.12)}$$

It is a particular case of a fuzzy number with a at.

The supporting interval is $A=[a_1,a_2]$ and the at segment on level $\alpha=1$ has projection $[b_1,b_2]$ on the x-axis. With the four values a_1, a_2, b_1 and b_2, we can construct the trapezoidal number in Figure 2.14. It can be denoted by $A = (a_1, b_1, b_2, a_2)$.

If $b_1 = b_2 = a_M$, the trapezoidal number reduces to a triangular fuzzy number and is denoted by (a_1, a_M, a_M, a_2). Hence a triangular number (a_1, a_M, a_2) can be written in the form of a trapezoidal number, i.e. $(a_1, a_M, a_2) = (a_1, a_M, a_M, a_2)$.

If $[a_1, b_1] = [b_2, a_2]$, the trapezoidal number is symmetrical with respect to the line $x=0.5(b_1+b_2)$. It is in central form and represents the interval $[b_1,b_2]$ and real number close to this interval.

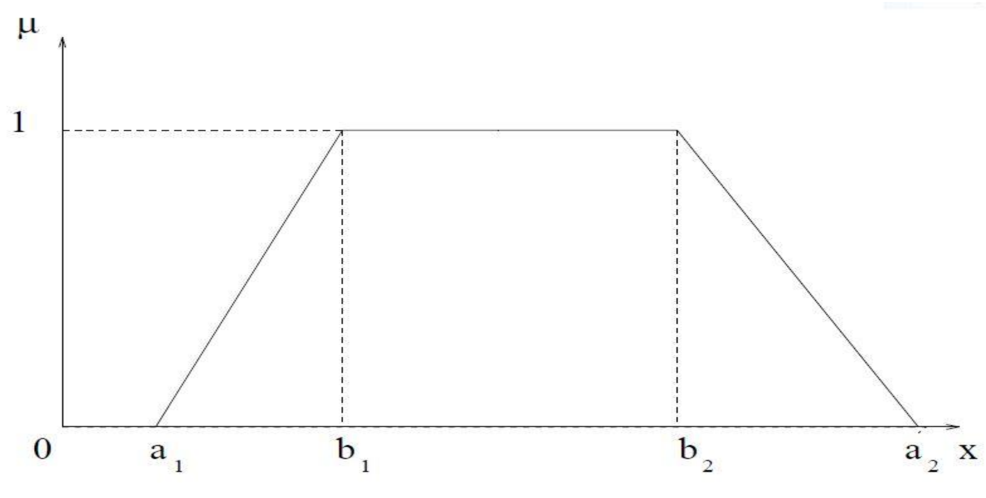


Figure 2.14 : Trapezoidal fuzzy number with a flat $[b_1,b_2]$.

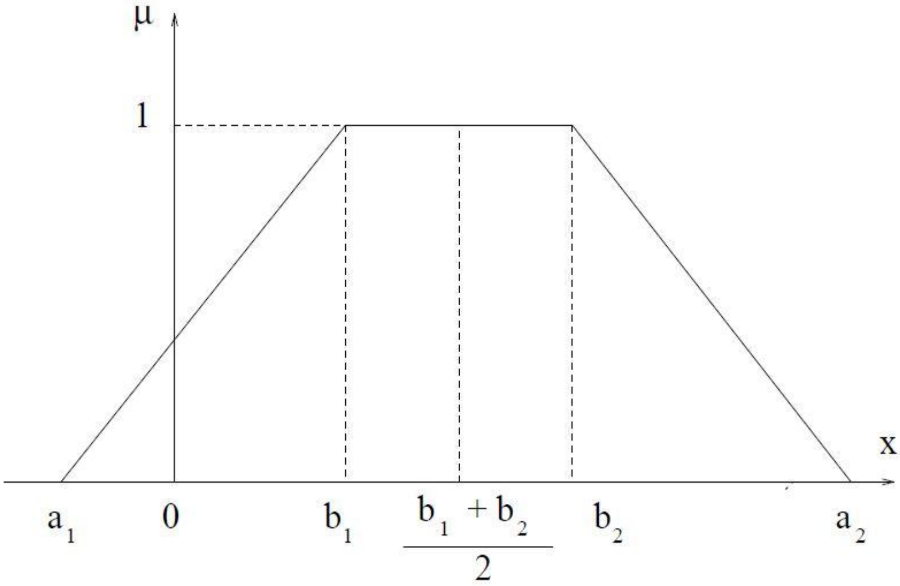


Figure 2.15 : Trapezoidal number in central form.

Similarly to right and left triangular numbers we can introduce right and left trapezoidal numbers as parts of a trapezoidal number.

The right trapezoidal number denoted $A^r = (b_1, b_1, b_2, a_2)$ has supporting interval $[b_1, a_2]$ and the left denoted $A^l = (a_1, b_1, b_2, b_2)$ has supporting interval $[a_1, b_2]$. Especially they are suitable to represent small$\cong A^r = (0, 0, b2, a2)$ and large$\cong A^l = (a_1, b_1, b_2, b_2)$ where b_1 is a large number.

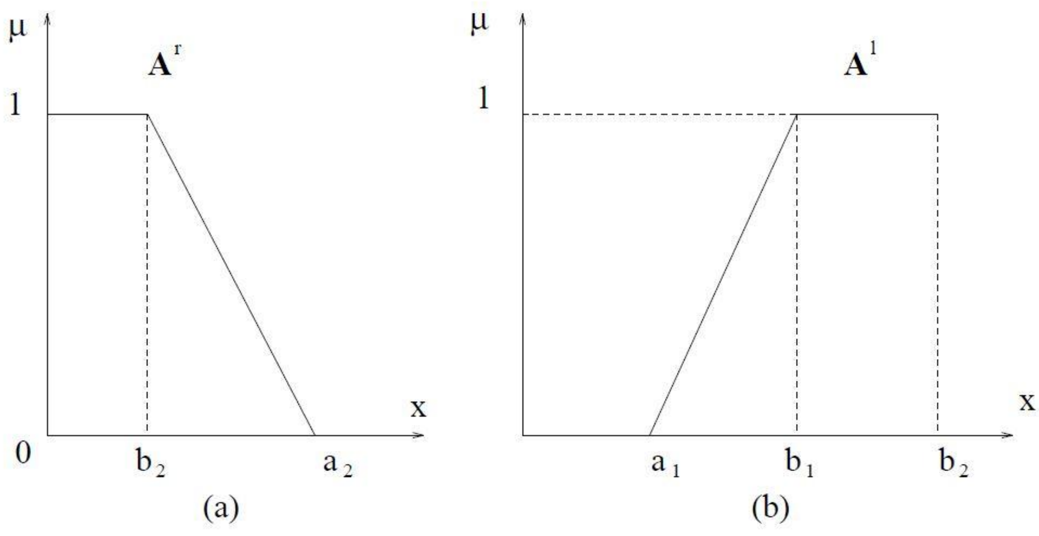


Figure 2.16 : (a) Right trapezoidal number Ar representing small;
(b) Left trapezoidal number Al representing large.

2.3 Stochastic Dominance (SD)

Stochastic Dominance (SD) is a fundamental concept in decision theory with uncertainty. It describes when a particular random prospect, such as a lottery or a stock, is better than another random prospect based on preferences regarding outcomes which may be expressed in terms of utility values.

The stochastic dominance is designed to capture the technical properties of statistical distributions for lotteries that enable broad rankings of those lotteries (with only limited information about the utility function of a particular consumer). Practically speaking, it is a way of comparing different lotteries or distributions of outcomes.

Let L1 be a lottery with cumulative distribution F(x) and L2 be a lottery with cumulative distribution G(x). One approach to comparing these lotteries (and thus examining stochastic dominance) is to ask the following two questions:

1- When can we say that everyone will prefer L1 to L2?

2- When can we say that anyone who is risk averse will prefer L1 to L2?

The answer to the first question is defined as the property of First-Order Stochastic Dominance (FSD), while the answer to the second question is the property of Second-Order Stochastic Dominance (SSD). A second approach to stochastic dominance asks two related questions:

1a) Can we write L1 = L2+ “something good”? If we can do so, then everyone should prefer L1 to L2 for the right definition of “something good.”

2a) Can we write L2 = L1+ “risk”? If we can do so, then every risk averse person should prefer L1 to L2 (and every risk loving person should prefer L2 to L1) for the right definition of risk.”

This section explains the definitions of “something good” and “risk, and then shows how the two approaches to stochastic dominance are equivalent for these definitions. There is also a separate set of technical conditions that can be used to check for FSD and SSD, but they are just simplified versions of the conditions for (1a) and (2a).

A final important general point is that FSD and SSD require only weak preference for L1 vs. L2, corresponding to weak conditions on utility functions (e.g. weak rather than strict concavity for risk aversion).

Two major types of SD, the FSD and SSD, with the latter being more common than the former in portfolio optimization since all investors are assumed to be risk-averse. The detailed explanations of FSD and SSD can be found in next parts.

2.3.1 First order stochastic dominance (FSD)

We want to find conditions where we can write L1 = L2+ “something good” and we want to find the appropriate definition (so that everyone will prefer L1 = L2) of “something good”. We will impose only the most minimal restriction on the utility function, specifying that u(x) is non-decreasing. This means that more wealth is at least as good as less wealth. For our definition, it must be that every person at least weakly prefers L1 to L2. No matter how strange the utility function, if it non-decreasing, it must be true that L1 $\geq$ L2.

In line with this restriction on u(x), if we can match up the outcomes in L1 and L2 so that the outcomes in L1 are at least as good as the outcomes in L2 (in pairwise fashion) and L1 is sometimes strictly better than L2, then everyone will prefer L1 to L2. If L1 and L2 are identical, then technically speaking, L1 FSDs L2, and L2 also FSDs L1, but this is not very interesting.

Convert a simple lottery into percentile terms as an example. There are four states with the results for L1 and L2 as shown in Figure 2.17.

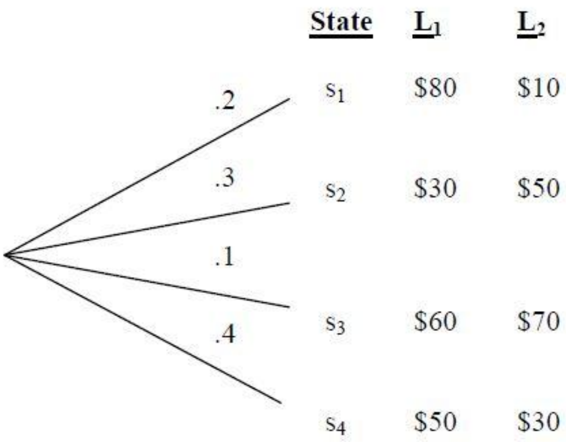


Figure 2.17 : An example distribution stating the output of rewards of two lotteries.

From these states the related percentiles can be calculated as in Table 2.1.

Table 2.1 : Percentiles of rewards of two lotteries.

Percentile	L1	L2
0%-10%	$30	$10
10%-20%	$30	$10
20%-30%	$30	$30
30%-40%	$50	$30
40%-50%	$50	$30
50%-60%	$50	$30
60%-70%	$50	$50
70%-80%	$60	$50
80%-90%	$80	$50
90%-100%	$80	$70

We can make two immediate observations: First, L2 is better in some states than L1. Second, L1 is at least as good as L2 at every percentile, and L1 is strictly better than L2 in 8 of the 10 percentiles. The second observation is important to the comparison of the lotteries in terms of expected utility because expected utility relies on a comparison of distributions of outcomes, not a comparison of outcomes in individual states.

We can compare the expected utility for each lottery

$$EU(L1) = 0.2\,u(80) + 0.3\,u(30) + 0.1\,u(60) + 0.4\,u(50)$$

$$EU(L2) = 0.2\,u(10) + 0.3\,u(50) + 0.1\,u(70) + 0.4\,u(30)$$

Now we can compare them directly by rewriting.

$$EU(L1) - EU(L2) = 0.2[u(30) - u(10)] + 0.3[u(50) - u(30)] + 0.1[u(60) - u(50)]$$
$$+0.1[u(80) - u(50)] + 0.1[u(80) - u(70)]$$

So we obtain

$$EU(L1) - EU(L2) \geq 0.$$

Note that each percentile contributes a term to the calculation of EU(L1) - EU(L2). In fact, it is not necessary to calculate EU(L1) and EU(L2) once we can rank order the terms from both lotteries and show that L1 is at least as good at every possible percentile.

With a finite number of outcomes in each lottery (a discrete distribution), we would have to find the least common denominator of probability outcomes in order to find the relevant percentiles that will enable comparison of L1 and L2. For example, if the L1 probabilities are in 1/5's, and the L2 probabilities are in 1/6's, then the relevant percentiles will be in 1/30's.

Formalizing this discussion, for a finite number of outcomes, if we can divide the distribution functions for L1 and L2 into probability ranges of size 1/M (where 1/M is the least common denominator) and L1 is at least as good as L2 for each range, then L1 $\geq$ L2.

With a continuous distribution of outcomes, where L1 is given by the cumulative density function (cdf) F(x); and L2 is given by the cdf G(x), the same condition would be that for each p between 0 and 1, for the values x_1 and x_2 such that $F(x_1) = G(x_2) = p$, then $x_1 \geq x_2$. Since the cdf F(x) is non-decreasing, this condition is equivalent to $F(x_2) \leq G(x_2)$ for x_2 such that $G(x_2) = p$. But there is nothing special about the particular value of p - this statement must hold for each and every p between 0 and 1, and therefore for each x.

Y is first-order stochastically dominant over X, if

$$F(t) \leq G(t) \qquad \textbf{(2.13)}$$

where F and G represents the cumulative probability distributions of Y and X respectively.

As an example let X is uniformly distributed on (0,1) and Y is uniformly distributed on (0,2). Since $F_X(x) \geq F_Y(x)$ for x value,Y first orderly dominates X.

$$F_X(x) = \begin{cases} x & ,0 \leq x \leq 1 \\ 1 & ,x \geq 1 \end{cases}$$

$$F_Y(x) = \begin{cases} x/2 & ,0 \leq x \leq 2 \\ 1 & ,x \geq 2 \end{cases}$$

Graphically

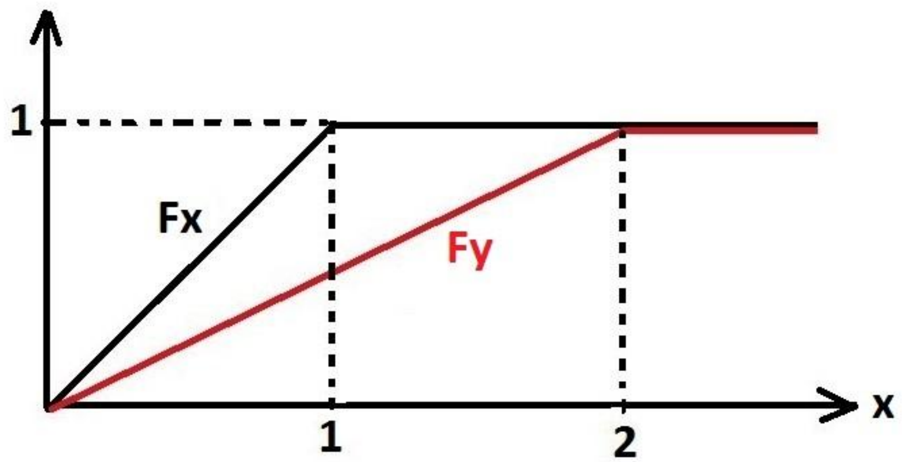


Figure 2.18 : Graphical representation of the cumulative probability distributions of F and G.

2.3.2 Second order stochastic dominance (SSD)

We want to write L2 = L1+ “risk”, and then find the appropriate definition for “risk” so that every risk averse person will prefer L1 to L2. Here we assume weak preference and weak concavity, so $u''(x) \leq 0$ is the condition for risk aversion. To isolate the effect of risk, we want the two distributions to have the same mean. This can be accomplished by specifying L2 = L1+L3, where L3 represents “risk”and has a conditional mean of 0 for each value of L1.

We will use this as our definition for “risk”: L2 = L1 + L3, where L3 is a mean-preserving spread for each possible value in L1. Here a mean-preserving spread is a lottery with mean 0 and some variation, meaning that it is not a degenerate lottery with 0 as the only possible outcome.

L3 is the 50%-50% lottery between +$25 and -$25 if L1 = $50, and L3 = 0 for certain if L1 = -$50. In simple lottery form, L2 can be written as: {+75, +25, -50; 1/4, 1/4, 1/2}. By Jensen’s inequality, if u(x) is concave: $u(50) \geq 0.5\,u(75)+ 0.5\,u(25)$. That is, every risk averse person prefers $50 for sure to $50 plus the mean-preserving spread of L3 (the lottery between an additional +$25 or -$25). The comparison between L1 and L2 depends only on the parts where they differ.

The result then is that L1 ≥ L2 if the consumer is risk averse. In equation form:

$EU(L1) = 0.5\,u(50) + 0.5\,u(-50) \geq 0.5\,[0.5\,u(75) + 0.5\,u(25)] + 0.5\,u(-50) = EU(L2)$

Again, the result is an application of Jensen’s inequality. Thus, L1 L2 for every risk averse person. If we can write L2 = L1+L3, where L3 is mean-preserving spread, or 0, for each value of L1 and there are always a finite number of values for each lottery, then repeated use of Jensen’s inequality, as in the example, will show that L1 ≥ L2 if u(x) is concave.

Although the derivation is slightly trickier if the lotteries have continuous distributions of values, the result is the same and the idea of the derivation is the same: repeated application of Jensen’s inequality to mean-preserving spreads shows that L1≥ L2.

The next step is to find a condition to check whether L2 = L1+L3, where L3 is mean-preserving spread or 0. It will not always be obvious how to create an L3 that transforms L1 into L2 even when L2 is clearly riskier than L1. The best way to proceed is to try some examples and see if we can discover, through experience, the appropriate conditions to check.

Here L1 and L2 each have the expected value $50, and it seems clear that L2 is riskier than L1 since its outcomes vary more from $50 than do the outcomes in L2. Suppose we try to addadditional mean-preserving spreads to L1 to create L2.

Step 1: Add a mean-preserving spread to +$20 to create outcomes $0 and $100. This will require a lottery with outcomes -$20 and +$80, so probabilities must be 4/5 and 1/5 to give the expected value of 0.In simplified form, this compound lottery reduces to: {$0, $100, $80; 2/5, 1/10, 1/2}. This step reduces expected utility because it adds risk to the certain outcome +$20.

Step 2: Now add a mean-preserving spread to +$80 to create outcomes $0 and $100. In simplified form, this compound lottery reduces to: {$0, $100; 1/2, 1/2}. Thus, we have recreated L2 by adding this pair of lotteries to L1. L2 = L1 + L3

Since L3 is a mean-preserving spread, L1 ≥ L2 for all concave u(x).

The preceding example suggests an algorithm for trying to transform L1 into L2 when there are a finite number of outcomes. Start with the lowest outcome in L1. Transform the lowest value in L1 into the two lowest values in L2. Then do the same for the second-lowest value in L1, and continue through all values in L1, subject to some checking.

So far, the two examples we've examined showed how to add mean-preserving spreads to L1 to recreate L2. Each addition of a mean-preserving spread makes L1 less attractive to a risk-averse consumer. This shows that if we can translate L1 into L2 by the addition of mean-preserving spreads, then every consumer who is risk averse will prefer L1 to L2. This preference is strict if the consumer is strictly risk averse (i.e. u''(x) < 0).

To derive a mathematical condition for SSD, suppose that L1 has a cdf F(z), and L2 has a cdf G(z), with associated pdf's f(z) and g(z) respectively. This analysis will assume continuous distributions of outcomes for L1 and L2, but the argument also holds for finite numbers of outcomes in each lottery.

Assume further that for some value x, that P(L1≤x)=P(L2≤x) and E(L1|L1≤x) < E(L2|L3≤x). Lastly, let the outcomes of both lotteries be distributed among values greater than or equal to zero. So, for some value of x, F(x) = G(x), and $\int_0^x z\,f(z)dz < \int_0^x z\,g(z)\,dz$ Use integration by parts for $\int_0^x z\,f(z)dz$, with u=z, du=dz; and v=F(z), dv=f(z)dz we have $\int_0^x z\,f(z)dz = zF(z)|_0^x - \int_0^x F(z)\,dz =$

$xF(x) - \int_0^x F(z)\,dz$. This lets us rewrite E(L1|L1≤x) < E(L2|L2≤x) as $xF(x) - \int_0^x F(z)\,dz < xG(x) - \int_0^x G(z)\,dz$

Since F(x) = G(x) by assumption, this is equivalent to: $\int_0^x F(z)\,dz > \int_0^x G(z)\,dz$

This condition turns out to be precisely the standard condition for SSD to fail at the value x (with the generalization that the outcomes for other lotteries might be negative). In other words, as long as $\int_{-\infty}^x F(z)\,dz > \int_{-\infty}^x G(z)\,dz$ for each and every x, we are guaranteed that the problems with the conversion algorithm that occurred in the preceding examples will not occur.

Comment: This discussion does not constitute a proof, and for this reason, MWG opted not to include it at all in the discussion in stochastic dominance. The important concepts to take away from this discussion are:

1) SSD cannot hold if F(x) = G(x) and E(L1|L1≤ x) < E(L2|L2≤ x) for any x;

2) Statement #1 is equivalent to saying that F(z) cannot SOSD G(z) if, for any x, $\int_0^x F(z)\,dz > \int_0^x G(z)\,dz$. Thus, to determine whether F(z) second-order stochastically dominates G(z), it is only necessary to check that:

$$\int_{-\infty}^x F(z)\,dz \le \int_{-\infty}^x G(z)\,dz \quad for\ all\ \boldsymbol{x} \qquad \textbf{(2.14)}$$

To understand the working principle of SSD in continuous functions let X is uniformly distributed on (0,2) and Y is uniformly distributed on (0.5,1.5). Then

$$F_X(x) = \begin{cases} \frac{x}{2} & ,0 \le x \le 2 \\ 1 & ,x \ge 2 \end{cases}$$

$$F_Y(x) = \begin{cases} 0 & ,0 \le x \le 0.5 \\ x - 0.5 & ,0.5 \le x \le 1.5 \\ 1 & ,x \ge 1.5 \end{cases}$$

A quick look at the graphs of F_X and F_Y shows that there is no FSD, e.g. $F_X(0.5)=0.25 > F_Y(0.5)=0$ but $F_X(1.5)=0.75 < F_Y(1.5)=1$. However

$$\int_{-\infty}^x F_X(y)\,dy = \begin{cases} \frac{x^2}{4} & ,0 \le x \le 2 \\ x - 1 & ,x \ge 2 \end{cases}$$

$$\int_{-\infty}^x F_Y(y)\,dy = \begin{cases} 0 & ,0 \le x \le 0.5 \\ \frac{x^2}{2} - \frac{x}{2} + \frac{1}{8} & ,0.5 \le x \le 1.5 \\ x - 1 & ,x \ge 1.5 \end{cases}$$

And so $\int_{-\infty}^{x} F_X(y)\,dy \geq \int_{-\infty}^{x} F_Y(y)\,dy$ with strict inequality for 0.5<x<2, and we have that Y is 2nd-order stochastically dominant over X. The easiest way to see this is from the graph in Figure 2.19.

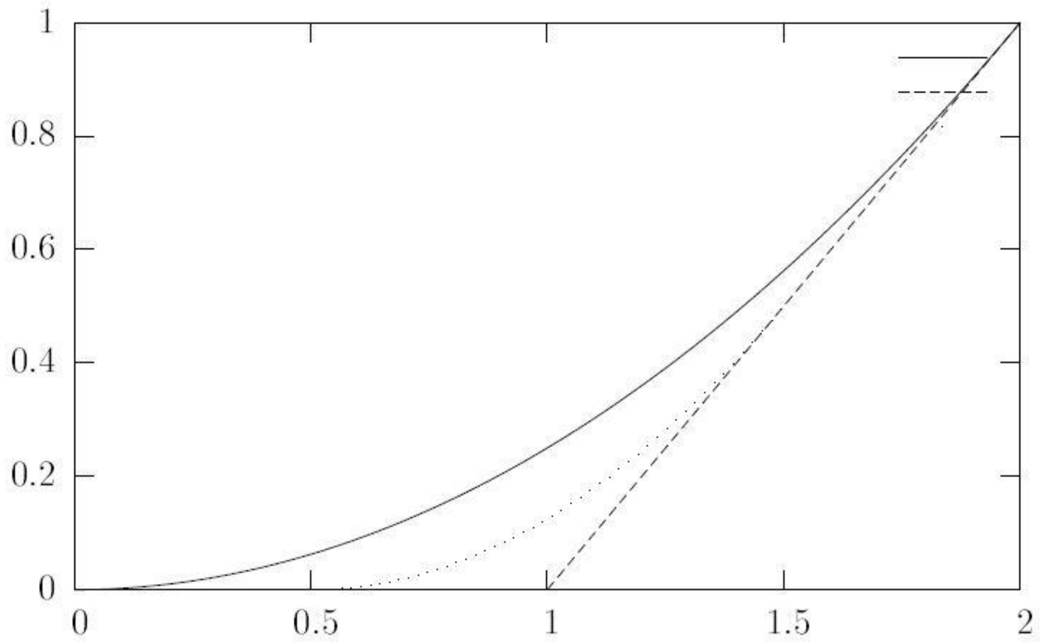


Figure 2.19 : Graphical representation of SSD where the solid curves shows $\int_{-\infty}^{x} \boldsymbol{F_X(y)\,dy}$, the dotted curve shows $\int_{-\infty}^{x} \boldsymbol{F_Y(y)\,dy}$ while the dashed straight line is x−1.

It can be directly realized that FSD is stronger than SSD. In terms of portfolio optimization, the results of FSD can be generalized for all investors while SSD is valid only for risk-averse investors, in mathematical terms, for all concave utility functions. Since all investors are assumed to be risk-averse, SSD must be preferred over FSD in an efficiency analysis of a portfolio.

2.3.3 Efficiency analysis by using SSD

To be efficient, a portfolio must not contain any SSD between any stock pairs. The set of all assets which are not dominated by other ones, according to SSD, is an SSD efficient set. As a further analysis of SSD efficiency, Yitzhaki and Mayshar (2001) provide some necessary and sufficient conditions which enable finding a direction for improving on an inefficient portfolio. More recently, Güran et al. (2013) applied SSD efficiency to the Turkish stock market.

SSD is used to determine the efficiency of the selected portfolio. Since SSD is a measure of a stock pair, it is obvious that SSD must be checked c(N,2)=N!/[2*(N-

2)!] times in a portfolio with N assets. To call the portfolio efficient, none of these c(N,2) pairwise comparisons must contain SSD.

2.4 Measurements Criteria of the Portfolio Performance

In this part two powerful performance criteria, Sharpe Ratio (SR) and Treynor Ratio (TR) are explained in detail that are very well known in financial world to measure the performance of portfolios.

2.4.1 Sharpe Ratio (SR)

Sharpe (1994) developed a ratio to measure risk-adjusted performance. The Sharpe Ratio is calculated by subtracting the risk-free rate from the rate of return for a portfolio and dividing the result by the standard deviation of the portfolio returns. The Sharpe Ratio (SR) formula is.

$$SR = \frac{R_p - R_f}{\sigma_p} \tag{2.15}$$

Where R_p is the portfolio return, R_f risk free return and σ_p portfolio standard deviation.

The Sharpe ratio tells us whether a portfolio's returns are due to smart investment decisions or a result of excess risk. This measurement is very useful because although one portfolio or fund can reap higher returns than its peers, it is only a good investment if those higher returns do not come with too much additional risk. The greater a portfolio's Sharpe ratio, the better its risk-adjusted performance has proven to be. A negative Sharpe ratio indicates that a risk-less asset would perform better than the security being analysed.

2.4.2 Treynor Ratio (TR)

The Treynor ratio (sometimes called the reward-to-volatility ratio or Treynor measure), named after Jack L. Treynor, is a measurement of the returns earned in excess of that which could have been earned on an investment that has no diversifiable risk (e.g., Treasury bills or a completely diversified portfolio), per each unit of market risk assumed. (Treynor, 1965) The Treynor Ratio (TR) formula is.

$$TR = \frac{R_p - R_f}{\beta_p} \tag{2.16}$$

Where R_p is the portfolio return, R_f risk free return and β_p beta of the portfolio.

The Treynor ratio relates excess return over the risk-free rate to the additional risk taken; however, systematic risk is used instead of total risk. The higher the Treynor ratio, the better the performance of the portfolio under analysis.

3. LITERATURE REVIEW

In this part the previous studies about economics, finance and portfolio optimization is summarized. In this literature review part analysed papers are grouped into two classes, fuzzy approach and stochastic dominance, in a chronological order.

3.1 Studies including Fuzzy Approach

Inuiguchi et al (1992) introduced the idea that the possibilistic programming can be applied to the portfolio selection problem. In the possibilistic programming approaches, the expected return rates are not handled as random variables but as possibilistic variables. According to them possibilistic programming has two main advantageous.

i. The financial experts can easily contribute their knowledge to the estimation of the return rates

ii. The reduced problem is more tractable than that of the stochastic programming approach of MPT

Inuiguchi et al (1992) introduced some possibilistic programming approaches Before introducing this minimax regret approach Inuiguchi and Tanino (2000) summarize three basic approaches based on possibilistic programming.

- Fractile Approach.

According to this method, the portfolio optimization problem is formulated as to maximize the Fractile z under a constraint that a necessity measure of the event that the objective function value is not less than z is greater than or equal to h^0, where h^0 is a given appropriate level lying on the interval of (0,1]. The model of this approach is as in (3.1) and (3.2).

$$\text{Maximize} \quad z,$$
$$\text{Subject to} \quad N_C(\{c|c^T x \geq z\}) \geq h^0,$$
$$e^T x = 1,\ x \geq 0. \tag{3.1}$$

where N_C shows a necessity degree represented as

$$N_C(\{c|c^T x \geq z\}) = \inf_c 1 - \pi_c(c)$$
$$C^T x < z \tag{3.2}$$

Under the possibilistic independence this model is reduced to a linear programming problem and in this case it can be solved easily.

- Modality Optimization Approach.

  According to this approach, the objective is to maximize the necessity measure in a given target value z^0, represented as in (3.3).

$$\text{Maximize} \quad N_C(\{c|c^T x \geq z^0\}),$$
$$\text{Subject to} \quad e^T x = 1,\ x \geq 0. \tag{3.3}$$

- Spread Minimization Approach.

  According to this approach the possibility distribution $\pi_{C^t x}$ on the objective function value can be minimized under the constraint $c^{\wedge T} x \geq z^0$. $h^0 \in (0,1]$ and z^0 are given and $\pi_{C^t x}$ is calculated by the extension principle as in (3.4).

$$\pi_{C^T X}(y) = \sup_c \pi_c(c),\ c^T x = y \tag{3.4}$$

  So the problem is formulated as

$$\text{Minimize} \quad w,$$
$$\text{Subject to} \quad \max\,(y^R - y^L) \leq w,$$
$$c^{\wedge T} x \geq z^0,$$
$$e^T x = 1,\ x \geq 0$$
$$\text{where } y^R, y^L \in [\,c^T x]_h{}^0 \tag{3.5}$$

Since there is a close relationship between the variance of a probability distribution and the spread of a possibility distribution, this model is a counterpart of the Markowitz model.

Inuiguchi and Tanino (2000) introduce a new possibilistic programming approach to the portfolio selection. This new approach is based on a regret which the decision maker may undertake.

It is shown that a distributive investment solution is obtained by this minimax regret approach to the possibilistic portfolio selection. At the end of the paper, they give numerical examples in order to compare the solutions obtained by the previous and proposed approaches.

This new possibilistic approach is based on the worst regret criterion. Suppose that an investor has invested his money in a bond according to a concentrated investment solution. If the return of another bond would be greater than that concentrated solution as a result, the investor feels regret. Since at the decision stage no one can determine the future returns, any concentrated investment solution may bring regret to the investor. In this logic, to minimize the worst regret of the investor a distributive investment solution must be preferable.

This Minimize Regret Model can be represented as in (3.6).

$$\begin{aligned} &\text{Minimize} \quad q, \\ &\text{Subject to} \;\; \max \;\; f(c_i)c^T x + g(c_i) \leq q, \\ &\qquad C \in cl(C)_{1-h0} \\ &\qquad i=1,2,...,n, \\ &\qquad e^T x=1,\; x \geq 0. \end{aligned} \tag{3.6}$$

An example of f(r) and g(r) functions can be given as in (3.7).

$$f(r) = -\frac{1}{1+r} \;\; and \;\; g(r) = \frac{r}{1+r} \tag{3.7}$$

At the end of the paper, Inuiguchi and Tanino (2000) compare all of these methods in different case scenarios. According to the results, the solution obtained from the minimax regret approach has a distributive investment solution, but not the other possibilistic methods such as Fractile, Modality and Spread Minimization approaches. And the solution of the minimax regret approach seems better than the classical Markowitz Method since it is following the return rate pattern.

Tanaka et al (2000) proposed a model based on fuzzy probabilities and possibility distributions which reflects experts' knowledge. As a main contribution of this model, the possibility grade h_i reflects a similarity degree between the future state of stock markets and the state of the *i*th sample offered by experts. These grades h_i (i=1,2,...,m) are used to determine the fuzzy average vector and covariance matrix for the analysed data where m shows the number of the observed periods.

Under the light of this possibility grade, the fuzzy average vector $\mathbf{a}=[\alpha_1,\ldots, \alpha_n]^t$ can be defined as follows where n donates the number of assets in the portfolio

$$\boldsymbol{a} = \sum_{i=1}^{m}(h_i r_i) / \sum_{i=1}^{m} h_i \qquad \textbf{(3.8)}$$

Similarly, the fuzzy weight covariance matrix $\sum=[\sigma_{ij}]$ can be defined by

$$\sigma_{ij} = \frac{\{\sum_{k=1}^{m}(r_{ki}-\alpha_i)(r_{kj}-\alpha_j)h_k\}}{\sum_{k=1}^{m} h_k} \quad (i, j = 1, \ldots, n) \qquad \textbf{(3.9)}$$

It can be noted that, with the same importance grade this fuzzy portfolio selection model is the same as the Markowitz's model.

Tanaka et al (2000) defined a performance function C_A where $z_i^t\, C_A\, z_i \leq -\ln(h_i)$, $i=1,\ldots,m$. By using this function with the transformations $y=r-a$ and $z=T^t y$, they designed the optimization problem as in (3.10).

$$\begin{gathered} \text{Max} \sum_{i=1}^{m} z_i^t C_A z_i \\ \text{s.t. } z_i^t\, C_A\, z_i \leq -\ln(h_i),\ i=1,\ldots,m \\ c_j \geq \varepsilon, \\ D_A=(T\, C_A\, T^t)^{-1} \end{gathered} \qquad \textbf{(3.10)}$$

Where T is the linear transformation matrix whose colums consist of the eigenvectors of the covariance matrix $\sum$ and D_A is the possibility distribution matrix.

In addition to these studies, Carlsson and Fullër (2001) defined possibilistic mean value, variance and covariance of fuzzy numbers by developing earlier works of Dubois and Prade (1987).

According to these works, A fuzzy number A is a fuzzy set of the real line R with a normal, fuzzy convex and continuous membership function of bounded support. The family of fuzzy numbers will be denoted by F. Let A be a fuzzy number with γ level set $[A]^\gamma = [a_1(\gamma), a_2(\gamma)] (\gamma > 0)$. So the possibilistic mean value of fuzzy numbers is as in (3.11).

$$\bar{M}(A) := \int_0^1 \gamma(a_1(\gamma) + a_2(\gamma))d\gamma = \frac{\displaystyle\int_0^1 \gamma \cdot \frac{a_1(\gamma)+a_2(\gamma)}{2}\, d\gamma}{\displaystyle\int_0^1 \gamma\, d\gamma}, \qquad \textbf{(3.11)}$$

It follows that M (A) is nothing else but the level-weighted average of the arithmetic means of all γ level sets, that is, the weight of the arithmetic mean of $a_1(\gamma)$ and $a_2(\gamma)$

is just γ. In a similar manner possibilistic variance of fuzzy numbers can be introduced as in (3.12).

$$\begin{aligned}\mathrm{Var}(A) &= \int_0^1 \mathrm{Pos}[A \le a_1(\gamma)] \left(\left[\frac{a_1(\gamma)+a_2(\gamma)}{2} - a_1(\gamma)\right]^2\right) d\gamma \\ &\quad + \int_0^1 \mathrm{Pos}[A \ge a_2(\gamma)] \left(\left[\frac{a_1(\gamma)+a_2(\gamma)}{2} - a_2(\gamma)\right]^2\right) d\gamma \\ &= \int_0^1 \gamma\left(\left[\frac{a_1(\gamma)+a_2(\gamma)}{2} - a_1(\gamma)\right]^2 \right. \\ &\quad \left. + \left[\frac{a_1(\gamma)+a_2(\gamma)}{2} - a_2(\gamma)\right]^2\right) d\gamma \\ &= \frac{1}{2}\int_0^1 \gamma\big(a_2(\gamma) - a_1(\gamma)\big)^2 d\gamma. \end{aligned} \quad \textbf{(3.12)}$$

Let apply these general definitions where $a_1(\gamma)$ and $a_2(\gamma)$ are linear functions in other words let them apply to a triangular fuzzy number (a,α,β) with center a, left with center a, leftwidth $\alpha > 0$ and right-width $\beta > 0$ as in Figure 3.1.

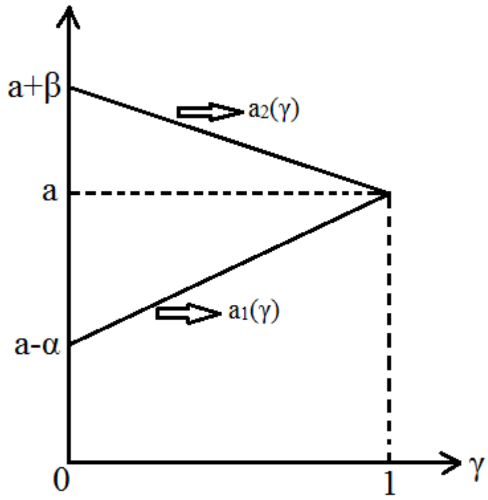


Figure 3.1 : Triangular fuzzy number (a,α,β) with center a and presentation of $a_1(\gamma)$ and $a_2(\gamma)$.

Because of their linear nature by using simple mathematic rules $a_i(\gamma)$, i=1,2 become

$$a_1(\gamma)=a-(1-\gamma)\alpha\ ,\ a_2(\gamma)=a+(1-\gamma)\beta\ ;\ \gamma\in[0,1] \quad \textbf{(3.13)}$$

When (3.13) is put on (3.11), possibilistic mean of the triangular fuzzy number $A=(a,\alpha,\beta)$ is calculated after a few integral calculations as in (3.14).

$$M(A) = a + \frac{\beta-\alpha}{6} \quad \textbf{(3.14)}$$

When (3.13) is put on (3.12), possibilistic variance $A=(a,\alpha,\beta)$ is calculated in a similar manner after a few more integral calculations as in (3.15).

$$Var(A) = \frac{(\alpha+\beta)^2}{24} \quad \textbf{(3.15)}$$

By considering two fuzzy numbers A and B such that $[A]_\gamma=[a_1(\gamma), a_2(\gamma)]$ and $[B]_\gamma=[b_1(\gamma), b_2(\gamma)]$ ($\gamma > 0$) the possibilistic covariance of A and B can be defined as in (3.16).

$$\mathrm{Cov}(A,B) = \frac{1}{2}\int_0^1 \gamma(a_2(\gamma) - a_1(\gamma))(b_2(\gamma) - b_1(\gamma))d\gamma \quad \textbf{(3.16)}$$

When $A=(a,\alpha,\beta)$ and $B=(b, \theta, \lambda)$ are triangular fuzzy numbers then the related functions become as in (3.17).

$$a_1(\gamma)=a-(1-\gamma)\alpha,\ a_2(\gamma)=a+(1-\gamma)\beta$$

$$b_1(\gamma)=b-(1-\gamma)\theta, b_2(\gamma)=b+(1-\gamma)\lambda \quad \textbf{(3.17)}$$

If (3.17) is put on (3.16) then Cov(A,B) can calculated as in (3.18).

$$Cov(A,B) = \frac{(\alpha+\beta)(\theta+\lambda)}{24} \quad \textbf{(3.18)}$$

It can be noticed when A=B then Cov(A,A) is equals to $(\alpha+\beta)^2/24$ which is Var(A) as expected according to statistics rules.

Apart from these definitions, let $\lambda, \mu \in R$ and let A and B be fuzzy numbers. Then the identity in (3.19) can be written.

$$\mathrm{Var}(\lambda A + \mu B) = \lambda^2\ \mathrm{Var}(A) + \mu^2\ \mathrm{Var}(B) + 2\ |\lambda\mu|\ \mathrm{Cov}(A,B) \quad \textbf{(3.19)}$$

where the addition and multiplication by a scalar of fuzzy numbers is defined by the sup-min extension principle. Carlsson and Fullër (2001) proved this theorem which forms the fundemantal of the portfolio variance in fuzzy environment.

If this theorem is generalized to n fuzzy numbers $A_1,...,A_n$ with constants $c_1,...,c_n \in R$, the generalized result in (3.20) can be obtained easily

$$\mathrm{Var}(c_1A_1+...+c_nA_n)=$$

$$c_1^2\mathrm{Var}(A_1)+...+c_n^2\mathrm{Var}(A_n) + 2|c_1c_2|\mathrm{Cov}(A_1,A_2) +...+ 2|c_{n-1}c_n|\mathrm{Cov}(A_{n-1},A_n) \quad \textbf{(3.20)}$$

3.2 Studies including Stochastic Dominance

When studying either income inequality or poverty, one is automatically in a normative context. Most modern studies make explicit or implicit use of a social welfare function (SWF). In a paper by Blackorby and Donaldson (1980), various ethically desirable criteria are developed and the sorts of SWF that respect these criteria are characterized.

One of these criteria is the anonymity of individuals. If we take all the worldly goods of a rich man and give them to a poor man, and then give the few worldly goods of the poor man to the rich, then social welfare should be unchanged. Formally, a SWF that respects this requirement is symmetric with respect to its arguments, which are the incomes of the members of society.

Another requirement is the Pareto principle. According to it, we should rank situation B better than situation A if at least one individual is better off in B than in A, and no one is worse off. In order for a SWF to respect the Pareto principle, it must be increasing in all its arguments.

As with welfare functions, this result can be extended. By progressively restricting the admissible class of poverty indices, in particular by imposing signs on the derivatives of $\pi(z) = \int_0^z \pi(z-y)dF(y)$, it can be seen that all poverty indices in these more restricted classes unanimously see more poverty in A than in B if there is a progressively higher order of stochastic dominance; see Davidson and Duclos (2000) for more details. An essential reference on poverty measurement is Atkinson (1987), in which the axiomatic approach is extended to poverty measurement. See also three papers by Foster and Shorrocks (1988a, b, and c).

If a richer person in distribution A transfers some income to a poorer person in such a way that the richer person stays richer after the transfer, the post-transfer distribution B stochastically dominates A at second order. The Pigou-Dalton principle of transfers says that “Robin-Hood” transfers of the sort described should improve welfare.

But it is easy to see that distribution B does not dominate A at first order, and indeed this is right and proper according to the Pareto principle, since the richer person is worse off after the transfer.

This example shows that, when we discuss inequality, we are not talking about the same thing as welfare. Any reasonable measure of inequality must declare that there is no inequality if everyone has the same income, even if everyone is in abject poverty.

The classical tool for studying inequality is the Lorenz curve. For any proportion p between zero and one, the ordinate of the corresponding point on the Lorenz curve for a given income distribution is the proportion of total income that accrues to the first 100p per cent of people when they are sorted in order of increasing income. By construction, the Lorenz curve fits into the unit square, lies below the 45-degree line that is the diagonal of that square, and is (weakly) convex. Figure 3.2 displays a typical Lorenz curve.

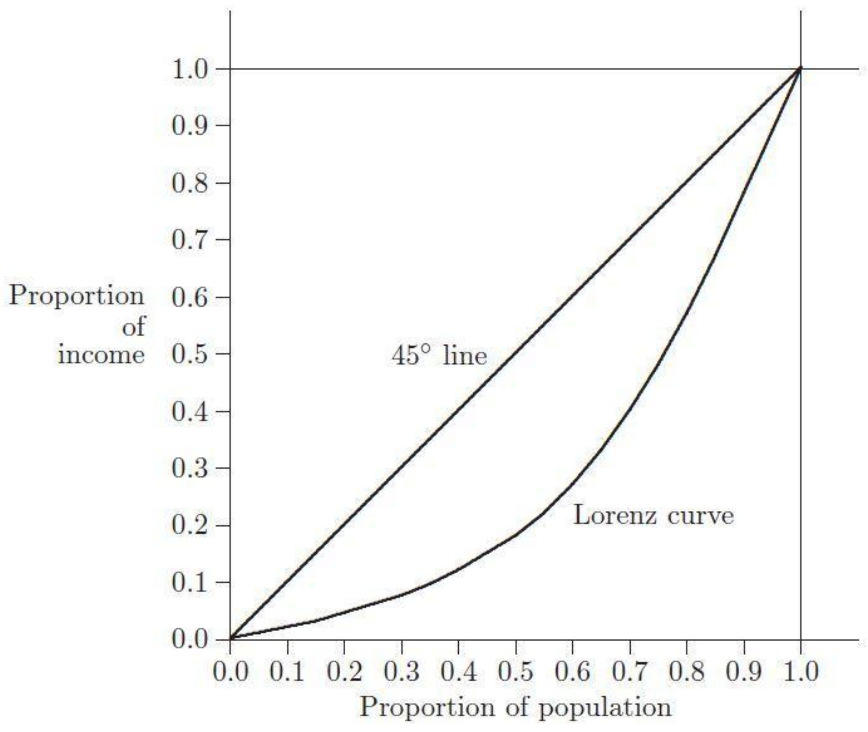


Figure 3.2 : Typical Lorenz Curve compared with the line y=x.

A distribution B is said to Lorenz dominate another distribution A if the Lorenz curve of B lies everywhere above that of A. We then say that there is less inequality in B than in A. But this comparison of A and B is not a welfare comparison, and, in particular, does not allow a comparison of poverty. This defect is remedied by the concept of generalized Lorenz dominance, based on the generalized Lorenz curve introduced by Shorrocks (1983). The ordinates of this curve are the Lorenz ordinates multiplied by the average income of the distribution. It turns out that generalized Lorenz dominance is the same thing as second-order stochastic dominance. Either one of these concepts implicitly mixes notions of welfare and inequality, as shown by the fact that the function u in a SWF that respects second-order dominance has a negative second derivative, which implies diminishing marginal (social) utility of income. The discussion of the previous section shows that higher-order dominance criteria put more and more weight on the welfare of the poorest members of society.

Consider the setup in Figure 3.3, where the CDFs of two distributions A and B are plotted.

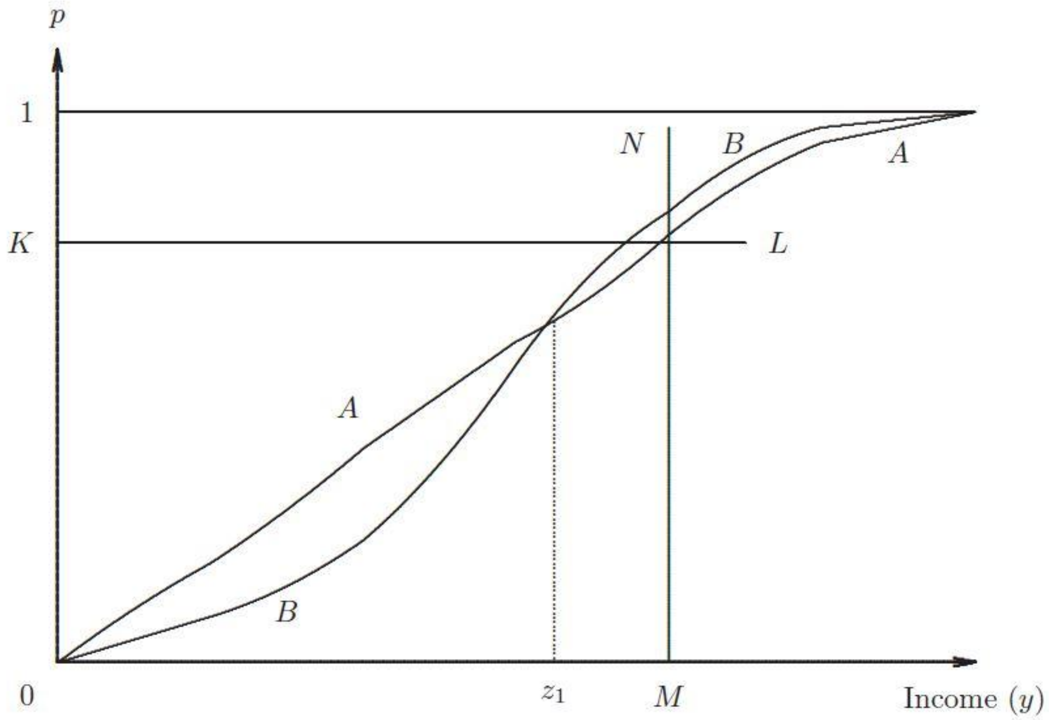


Figure 3.3 : Generalized Lorenz and Second Order Dominance.

The functions $D^2(z) = \int_0^z (z - y)dF(y)$ used for second-order dominance comparisons can be evaluated for a given argument, like z_1 in the figure, as the areas beneath the CDFs, by the usual geometric interpretation of the Riemann integral. We see that distribution B dominates A at second order because, although the CDFs cross, the areas between them are such that the condition for second-order dominance is always satisfied. Thus the vertical line MN marks off a large positive area between the graphs of the two CDFs up to the point at which they cross, and thereafter a small negative area bounded on the right by MN.

For generalized Lorenz dominance, it can be shown that what must be non-negative everywhere is the area between the two curves, bounded not by a vertical line like MN, but rather by a horizontal line like KL. This area is the difference between the areas under two quantile functions, a quantile function being by definition the inverse of the CDF. Although it is tedious to demonstrate it algebraically, it is intuitively clear that if the areas bounded on the right by vertical lines like MN are always positive, then so are the areas bounded above by horizontal lines like KL. This is why generalized Lorenz dominance and second-order stochastic dominance are equivalent conditions. The whole theory of stochastic dominance can be developed using quantiles rather than incomes; this is called a p-approach. Such approaches are used to advantage in Jenkins and Lambert (1997, 1998), Shorrocks (1998), and also Spencer and Fisher (1992).

Another thing that emerges clearly from Figure 3.3 is that the threshold income z_1 up to which first-order stochastic dominance holds is always smaller than the threshold z_2 up to which we have second-order dominance. In the Figure, we have second-order dominance everywhere, and so we can set z_2 equal to the highest income in either distribution. More generally, we can define a threshold z_s as the greatest income up to which we have dominance at order s. The z_s constitute an increasing sequence.

A result shown in Davidson and Duclos (2000) is that, if the distribution B dominates A at first-order over a range [0, z], with z > 0, then, no matter what happens for incomes above z, there is always some order s such that B dominates A at order s over the full range of the two distributions, provided only that that range is finite.

4. THEORY OF THE SUGGESTED MODEL

The gradual portfolio optimization method proposed in this thesis have two main steps. In the first step, the SSD inefficient stocks are excluded from the portfolio. In the second step, the optimized portfolio is found by minimizing the fuzzy variance for a given target return level.

4.1 First Step. Elimination of SSD inefficient stocks

To begin with this step, the SSD relationships between all stock pairs should be determined. To check an SSD between two assets is very easy if their cumulative distribution functions are known, but in this application there are n stocks to be examined and this equals c(n,2) pairwise SSD detections. It is obvious that this job cannot be done manually and because of that an algorithm, written in C++, is developed to check these SSDs among the stock pairs automatically. This code applies the same procedures for all stock pairs by taking the cumulative distributions and returns as inputs to detect the SSDs.

To explain the working principle of this algorithm, suppose that $f(r_i)$ and $g(r_i)$ are two cumulative distributions of a stock pair A and B respectively, where r_i represents the observed return levels of A and B. If (2.14) is applied to this A&B pair, taking into account that f and g are discrete functions, the inequality in (4.1) should be checked

$$\sum_{i=1}^{n}[f(r_i) - g(r_i)] \leq 0, \text{ for all i=1,..,n} \qquad \textbf{(4.1)}$$

According to the definition of SSD, if only non-positive terms are found such as in (4.1), it can be said that A dominates B in terms of SSD. On the other hand, only non-negative terms show that B dominates A. In a mixed situation of both positive and negative terms for i=1,..,n, there is no SSD between A and B.

4.2 Second Step. Minimizing the Fuzzy Variance at a given target return

In terms of fuzzy logic triangular membership functions are used to forecast future returns both because of its suitable nature to the portfolio selection problem and because of its linear structure which facilitates the optimization model. To predict the future return of a stock, it is assumed that the membership degree of the fuzzy average is always 1. But the membership degrees will change depending on the scenario whilst deviating vastly from the fuzzy average. This triangular membership function representing the future returns r_i of the i^{th} stock (i=1,...,n) as in (4.2).

$$\mu_i(u) = \begin{cases} \frac{1}{\propto_i} u + \frac{\propto_i - r_i}{\propto_i}, r_i - \propto_i \leq u \leq r_i \\ -\frac{1}{\beta_i} u + \frac{\beta_i + r_i}{\beta_i}, r_i \leq u \leq r_i + \beta_i \\ 0 \qquad\qquad , otherwise \end{cases} \tag{4.2}$$

In fuzzy terms this triangular membership function can be expressed by $(r_i-\alpha_i, r_i, r_i+\beta_i)$ where β_i and α_i represent maximum possible differences of future returns respectively in up and down directions and r_i is the expected centre future return with the highest membership value of 1.

After the calculation of r_i, α_i and β_i values of each stock, the fuzzy mean value of the whole portfolio return of n assets can be defined as in (4.3).

$$M(r) = \sum_{i=1}^{n} M(r_i) x_i \tag{4.3}$$

In this definition x_i (i=1,...,n) represents the weights of the stocks. These proportions satisfy the condition in (4.4) because in the proposed models short selling of any stock is not allowed.

$$\sum_{i=1}^{n} x_i = 1, \quad 0 \leq x_i \leq 1, i = 1, \dots, n \tag{4.4}$$

Since r_i is the midpoint of the triangular fuzzy number for stock i, by using (3.14) the expression in (4.5) can be obtained.

$$M(r_i) = r_i + \frac{\beta_i - \alpha_i}{6}, i = 1, \dots, n \tag{4.5}$$

If the equation in (4.5) is substituted into the equation in (4.3), the equation in (4.6) can be obtained for the fuzzy mean value of the whole portfolio return.

$$M(r) = \sum_{i=1}^{n} (r_i + \frac{\beta_i - \alpha_i}{6})\, x_i \tag{4.6}$$

According to portfolio theory, to solve the right portfolio selection problem, in other words to find the optimal weights, the fuzzy variance of the whole portfolio must be defined in addition to fuzzy portfolio return. For this purpose, the identity in (3.20) can be used to determine the portfolio variance as in (4.7).

$$Var(r) = Var\left(\sum_{i=1}^{n} r_i\, x_i\right) =$$

$$x_1^2\, Var(r_1)+...+x_n^2 Var(r_{10}) + 2|x_1x_2|Cov(r_1,r_2) +...+ 2|x_{n-1}x_n|Cov(r_{n-1},r_n) \qquad \textbf{(4.7)}$$

Because of the triangular feature of $r_1,...,r_n$ the identities in (3.15) and (3.18) can be applied in (4.7) and the formula in (4.8) can be concluded to determine the portfolio variance.

$$Var(r) = \sum_{i=1}^{n} \frac{(\alpha_i+\beta_i)^2}{24} x_i^2 + 2\sum_{i\neq j=1}^{n} \frac{(\alpha_i+\beta_i)^2(\alpha_j+\beta_j)^2}{24} x_i x_j = [\sum_{i=1}^{n} (\frac{\alpha_i+\beta_i}{2\sqrt{6}}) x_i]^2 \qquad \textbf{(4.8)}$$

According to MPT, the fuzzy mean-variance model can formulated as in (4.9).

$$Min \left\{ [\sum_{i=1}^{n} (\frac{\alpha_i+\beta_i}{2\sqrt{6}}) x_i]^2 \right\}$$

$$\text{Subject to } \sum_{i=1}^{n} (r_i + \frac{\beta_i - \alpha_i}{6})\, x_i \geq \mu,$$

$$\sum_{i=1}^{n} x_i = 1,$$

$$0 \leq x_i \leq 1, i = 1, \dots, n \qquad \textbf{(4.9)}$$

In this quadratic nonlinear optimization model, μ represents the desired minimum portfolio return level and this size of μ is left to the preference of the portfolio maker. It is obvious that the greater the risk, the greater the appetite of the investor and the bigger the size of the desired minimum portfolio return.

To facilitate the solution of this optimization model, standard deviation can be used instead of variance. When the square root of the objective function in (4.9) is taken, all nonlinear terms drop from the formula and the defuzzified standard deviation of the whole portfolio is as in (4.10).

$$\sqrt{Var(r)} = Std\ Dev(r) = \frac{\sqrt{6}}{12}\sum_{i=1}^{n}(\alpha_i + \beta_i)x_i \qquad \textbf{(4.10)}$$

When defuzzified standard deviation is minimized instead of fuzzy variance, the model in (4.9) is transformed to a linear optimization model as in (4.11).

$$Min\ \left\{\frac{\sqrt{6}}{12}\sum_{i=1}^{n}(\alpha_i + \beta_i)x_i\right\}$$

$$\text{Subject to } \sum_{i=1}^{n}(r_i + \frac{\beta_i - \alpha_i}{6})\ x_i \geq \mu,$$

$$\sum_{i=1}^{n} x_i = 1,$$

$$0 \leq x_i \leq 1, i = 1, \dots, n \qquad \textbf{(4.11)}$$

5. APPLICATION OF THE SUGGESTED MODEL

5.1 Main Structure

In this thesis three different model are applied in the BIST-30 Index.

- Model 1. MV Optimization

  In this model the classical MV optimization of Markowitz is applied without any modification. This model generates a benchmark portfolio which can be compared with other portfolios to test their performance.

- Model 2. MV Optimization after elimination of SSD inefficient stocks

  In this model the SSD inefficient stocks are eliminated in the first step and the MV optimization is applied in the second step. This model generates a portfolio which shows the effect of the only first step.

- Model 3. Fuzzy Variance Minimization after elimination of SSD inefficient stocks

  In this model the proposed two-step gradual method is completely applied. This model generates a portfolio which shows the effect of two steps together.

5.2 Data

BIST-30 Index consists of 30 stocks which belong to the largest companies in Turkey. An alphabetical list of these stocks including the company name, stock code, the weight of the stock in the index, initial public offering (IPO) date and market value in terms of USD can be found as in Table 5.1.

Table 5.1 : BIST-30 Index of Turkey.

	Name of the Company	Stock Code	Weight	İnitial Public Offering (IPO) Date	Market Value (USD)
1	AKBANK T.A.Ş.	AKBNK	11,33%	26.7.1990	10.966.789.668
2	ARÇELIK A.Ş.	ARCLK	1,97%	21.1.1986	3.702.791.086
3	ASELSAN ELEKTRONIK SANAYI VE TICARET A.Ş.	ASELS	0,72%	1.8.1990	2.767.527.675
4	ASYA KATILIM BANKASI	ASYAB	6,64%	5.5.2006	298.892.989
5	BİM BİRLEŞİK MAĞAZALAR A.Ş.	BIMAS	0,07%	8.7.2005	5.478.243.542
6	DOĞAN ŞIRKETLER GRUBU HOLDING A.Ş.	DOHOL	1,52%	14.6.1993	569.739.332
7	EMLAK KONUT GAYRİMENKUL YATIRIM ORTAKLIĞI A.Ş.	EKGYO	1,68%	24.11.2010	3.800.000.000
8	ENKA İNŞAAT VE SANAYİ A.Ş	ENKAI	1,93%	1.1.1970	7.380.073.801
9	EREGLI DEMIR VE ÇELIK FABRIKALARı T.A.Ş.	EREGL	14,28%	13.1.1986	5.269.372.694
10	T.GARANTI BANKASı A.Ş.	GARAN	8,46%	6.6.1990	12.630.996.310
11	TÜRKIYE HALK BANKASı A.Ş.	HALKB	0,34%	3.5.2007	5.581.180.812
12	İHLAS HOLDING A.Ş.	IHLAS	0,29%	7.12.1994	69.998.524
13	İPEK DOĞAL ENERJI KAYNAKLARı ARAŞTıRMA VE ÜRETIM A.Ş.	IPEKE	6,74%	27.6.2000	266.495.890
14	T.İŞ BANKASı A.Ş.	ISCTR	4,61%	16.11.1987	8.999.940.000
15	KOÇ HOLDING A.Ş.	KCHOL	0,48%	10.1.1986	11.743.734.512
16	KOZA ANADOLU METAL MADENCILIK ISLETMELERI A.Ş.	KOZAA	0,92%	5.2.2003	406.696.384
17	KOZA ALTIN İŞLETMELERİ A.Ş.	KOZAL	0,70%	5.2.2010	1.485.608.856
18	KARDEMIR KARABÜK DEMIR ÇELIK SANAYI VE TICARET A.Ş.	KRDMD	0,60%	1.6.1998	437.617.536
19	MIGROS TÜRK T.A.Ş.	MGROS	0,84%	27.2.1991	1.395.991.697
20	PETKIM PETROKIMYA HOLDING A.Ş.	PETKM	8,04%	9.7.1990	1.523.985.240
21	HACı ÖMER SABANCı HOLDING A.Ş.	SAHOL	1,05%	2.7.1997	7.393.640.813
22	T.ŞIŞE VE CAM FABRIKALARı A.Ş.	SISE	5,51%	13.1.1986	2.250.553.506
23	TAV HAVALIMANLARı HOLDING A.Ş.	TAVHL	4,62%	16.2.2007	2.949.146.679
24	TURKCELL İLETIŞIM HIZMETLERI A.Ş.	TCELL	1,37%	3.7.2000	10.228.782.288
25	TÜRK HAVA YOLLARı A.O.	THYAO	2,95%	20.12.1990	4.715.424.354
26	TOFAŞ TÜRK OTOMOBIL FABRIKASı A.Ş.	TOASO	4,84%	1.7.1991	3.477.859.779
27	TÜRK TELEKOMÜNIKASYON A.Ş.	TTKOM	2,52%	9.5.2008	8.898.523.985
28	TÜPRAŞ-TÜRKIYE PETROL RAFINERILERI A.Ş.	TUPRS	2,96%	30.5.1991	6.459.152.059
29	TÜRKİYE VAKIFLAR BANKASI T.A.O	VAKBN	0,56%	13.11.2005	3.791.512.915
30	YAPı VE KREDI BANKASı A.Ş.	YKBNK	1,50%	28.5.1987	6.031.333.147

Additional information about the companies such as general manager, chairman of the board and the official website are also listed in Table A.1.

To a better consideration of the BIST-30 indexes all of these 30 companies are sorted according the quantitave parameters which are stock code, the weight of the stock in the index, initial public offering (IPO) date and market value in terms of USD. The sorted tables can be found in Table 5.2, Table 5.3 and Table 5.4.

Table 5.2 : Bist-30 Companies sorted from largest to smallest according to their weight of the stock in the index.

Name of the Company	Stock Code	Weight
EREGLI DEMIR VE ÇELIK FABRIKALARı T.A.Ş.	EREGL	14,28%
AKBANK T.A.Ş.	AKBNK	11,33%
T.GARANTI BANKASı A.Ş.	GARAN	8,46%
PETKIM PETROKIMYA HOLDING A.Ş.	PETKM	8,04%
İPEK DOĞAL ENERJI KAYNAKLARı ARAŞTıRMA VE ÜRETIM A.Ş.	IPEKE	6,74%
ASYA KATILIM BANKASI	ASYAB	6,64%
T.ŞIŞE VE CAM FABRIKALARı A.Ş.	SISE	5,51%
TOFAŞ TÜRK OTOMOBIL FABRIKASı A.Ş.	TOASO	4,84%
TAV HAVALIMANLARı HOLDING A.Ş.	TAVHL	4,62%
T.İŞ BANKASı A.Ş.	ISCTR	4,61%
TÜPRAŞ-TÜRKIYE PETROL RAFINERILERI A.Ş.	TUPRS	2,96%
TÜRK HAVA YOLLARı A.O.	THYAO	2,95%
TÜRK TELEKOMÜNIKASYON A.Ş.	TTKOM	2,52%
ARÇELIK A.Ş.	ARCLK	1,97%
ENKA İNŞAAT VE SANAYİ A.Ş	ENKAI	1,93%
EMLAK KONUT GAYRİMENKUL YATIRIM ORTAKLIĞI A.Ş.	EKGYO	1,68%
DOĞAN ŞIRKETLER GRUBU HOLDING A.Ş.	DOHOL	1,52%
YAPı VE KREDI BANKASı A.Ş.	YKBNK	1,50%
TURKCELL İLETIŞIM HIZMETLERI A.Ş.	TCELL	1,37%
HACı ÖMER SABANCı HOLDING A.Ş.	SAHOL	1,05%
KOZA ANADOLU METAL MADENCILIK ISLETMELERI A.Ş.	KOZAA	0,92%
MIGROS TÜRK T.A.Ş.	MGROS	0,84%
ASELSAN ELEKTRONIK SANAYI VE TICARET A.Ş.	ASELS	0,72%
KOZA ALTIN İŞLETMELERİ A.Ş.	KOZAL	0,70%
KARDEMIR KARABÜK DEMIR ÇELIK SANAYI VE TICARET A.Ş.	KRDMD	0,60%
TÜRKİYE VAKIFLAR BANKASI T.A.O	VAKBN	0,56%
KOÇ HOLDING A.Ş.	KCHOL	0,48%
TÜRKIYE HALK BANKASı A.Ş.	HALKB	0,34%
İHLAS HOLDING A.Ş.	IHLAS	0,29%
BİM BİRLEŞİK MAĞAZALAR A.Ş.	BIMAS	0,07%

Table 5.3 : Bist-30 Companies sorted from largest to smallest according to their market value in USD.

Name of the Company	Stock Code	Market Value (USD)
T.GARANTI BANKASı A.Ş.	GARAN	12.630.996.310
KOÇ HOLDING A.Ş.	KCHOL	11.743.734.512
AKBANK T.A.Ş.	AKBNK	10.966.789.668
TURKCELL İLETIŞIM HIZMETLERI A.Ş.	TCELL	10.228.782.288
T.İŞ BANKASı A.Ş.	ISCTR	8.999.940.000
TÜRK TELEKOMÜNIKASYON A.Ş.	TTKOM	8.898.523.985
HACı ÖMER SABANCı HOLDING A.Ş.	SAHOL	7.393.640.813
ENKA İNŞAAT VE SANAYİ A.Ş	ENKAI	7.380.073.801
TÜPRAŞ-TÜRKIYE PETROL RAFINERILERI A.Ş.	TUPRS	6.459.152.059
YAPı VE KREDI BANKASı A.Ş.	YKBNK	6.031.333.147
TÜRKIYE HALK BANKASı A.Ş.	HALKB	5.581.180.812
BİM BİRLEŞİK MAĞAZALAR A.Ş.	BIMAS	5.478.243.542
EREGLI DEMIR VE ÇELIK FABRIKALARı T.A.Ş.	EREGL	5.269.372.694
TÜRK HAVA YOLLARı A.O.	THYAO	4.715.424.354
EMLAK KONUT GAYRİMENKUL YATIRIM ORTAKLIĞI A.Ş.	EKGYO	3.800.000.000
TÜRKİYE VAKIFLAR BANKASI T.A.O	VAKBN	3.791.512.915
ARÇELIK A.Ş.	ARCLK	3.702.791.086
TOFAŞ TÜRK OTOMOBIL FABRIKASı A.Ş.	TOASO	3.477.859.779
TAV HAVALIMANLARı HOLDING A.Ş.	TAVHL	2.949.146.679
ASELSAN ELEKTRONIK SANAYI VE TICARET A.Ş.	ASELS	2.767.527.675
T.ŞIŞE VE CAM FABRIKALARı A.Ş.	SISE	2.250.553.506
PETKIM PETROKIMYA HOLDING A.Ş.	PETKM	1.523.985.240
KOZA ALTIN İŞLETMELERİ A.Ş.	KOZAL	1.485.608.856
MIGROS TÜRK T.A.Ş.	MGROS	1.395.991.697
DOĞAN ŞIRKETLER GRUBU HOLDING A.Ş.	DOHOL	569.739.332
KARDEMIR KARABÜK DEMIR ÇELIK SANAYI VE TICARET A.Ş.	KRDMD	437.617.536
KOZA ANADOLU METAL MADENCILIK ISLETMELERI A.Ş.	KOZAA	406.696.384
ASYA KATILIM BANKASI	ASYAB	298.892.989
İPEK DOĞAL ENERJI KAYNAKLARı ARAŞTıRMA VE ÜRETIM A.Ş.	IPEKE	266.495.890
İHLAS HOLDING A.Ş.	IHLAS	69.998.524

Table 5.4 : Bist-30 Companies sorted from oldest to newest according to their IPO Date.

Name of the Company	Stock Code	İnitial Public Offering (IPO) Date
ENKA İNŞAAT VE SANAYİ A.Ş	ENKAI	1.1.1970
KOÇ HOLDING A.Ş.	KCHOL	10.1.1986
EREGLI DEMIR VE ÇELIK FABRIKALARı T.A.Ş.	EREGL	13.1.1986
T.ŞIŞE VE CAM FABRIKALARı A.Ş.	SISE	13.1.1986
ARÇELIK A.Ş.	ARCLK	21.1.1986
YAPı VE KREDI BANKASı A.Ş.	YKBNK	28.5.1987
T.İŞ BANKASı A.Ş.	ISCTR	16.11.1987
T.GARANTI BANKASı A.Ş.	GARAN	6.6.1990
PETKIM PETROKIMYA HOLDING A.Ş.	PETKM	9.7.1990
AKBANK T.A.Ş.	AKBNK	26.7.1990
ASELSAN ELEKTRONIK SANAYI VE TICARET A.Ş.	ASELS	1.8.1990
TÜRK HAVA YOLLARı A.O.	THYAO	20.12.1990
MIGROS TÜRK T.A.Ş.	MGROS	27.2.1991
TÜPRAŞ-TÜRKIYE PETROL RAFINERILERI A.Ş.	TUPRS	30.5.1991
TOFAŞ TÜRK OTOMOBIL FABRIKASı A.Ş.	TOASO	1.7.1991
DOĞAN ŞIRKETLER GRUBU HOLDING A.Ş.	DOHOL	14.6.1993
İHLAS HOLDING A.Ş.	IHLAS	7.12.1994
HACı ÖMER SABANCı HOLDING A.Ş.	SAHOL	2.7.1997
KARDEMIR KARABÜK DEMIR ÇELIK SANAYI VE TICARET A.Ş.	KRDMD	1.6.1998
İPEK DOĞAL ENERJI KAYNAKLARı ARAŞTıRMA VE ÜRETIM A.Ş.	IPEKE	27.6.2000
TURKCELL İLETIŞIM HIZMETLERI A.Ş.	TCELL	3.7.2000
KOZA ANADOLU METAL MADENCILIK ISLETMELERI A.Ş.	KOZAA	5.2.2003
BİM BİRLEŞİK MAĞAZALAR A.Ş.	BIMAS	8.7.2005
TÜRKİYE VAKIFLAR BANKASI T.A.O	VAKBN	13.11.2005
ASYA KATILIM BANKASI	ASYAB	5.5.2006
TAV HAVALIMANLARı HOLDING A.Ş.	TAVHL	16.2.2007
TÜRKIYE HALK BANKASı A.Ş.	HALKB	3.5.2007
TÜRK TELEKOMÜNIKASYON A.Ş.	TTKOM	9.5.2008
KOZA ALTIN İŞLETMELERİ A.Ş.	KOZAL	5.2.2010
EMLAK KONUT GAYRİMENKUL YATIRIM ORTAKLIĞI A.Ş.	EKGYO	24.11.2010

All data consisting of the closing values of these stocks from 03.12.2010 to 05.07.2013 are taken from the official web site of Istanbul Stock Exchange, borsaistanbul.com, on a weekly basis total of 135 observation periods that are displayed in Table A.2. The data before 03.12.2010 would be incomplete since some of these 30 companies have no returns before 03.12.2010.

5.3 Application of the Models

It is obvious that each model generates many portfolios lying on the efficient frontier according to their varying target returns. To prevent the confusion among hundreds of portfolios the target return level which maximizes the SR of Model 1 is taken as the standard level for all three portfolios. So under this condition just one portfolio is selected from each model.

Model 1 -> Portfolio-A

Model 2 -> Portfolio-B

Model 3 -> Portfolio-C

5.3.1 Model 1 -> Portfolio-A

The classical MV optimization will be applied on this data. As a first step of MV optimization variance-covariance matrix (displayed in the Table A.1) and average returns of the 30 stocks are determined. Next, MV optimization stated in (2.5) is applied with the help of MatLab Software (Version 7.9). The list of MV optimized portfolios according to 20 different return (μ) levels is in Table 5.5.

Table 5.5 : MV optimized portfolios of BIST-30 Stocks.

	Weights																																
Portfolio	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	Total	Return	Std Dev
1	0,00	0,00	0,04	0,00	0,03	0,00	0,00	0,01	0,06	0,00	0,00	0,07	0,00	0,00	0,00	0,00	0,03	0,00	0,00	0,11	0,00	0,00	0,18	0,23	0,00	0,00	0,23	0,00	0,00	0,00	1,00	0,0022	0,0239
2	0,00	0,01	0,05	0,00	0,02	0,00	0,01	0,00	0,03	0,00	0,00	0,06	0,00	0,00	0,00	0,00	0,03	0,02	0,00	0,11	0,00	0,00	0,19	0,23	0,00	0,00	0,22	0,00	0,00	0,00	1,00	0,0024	0,0240
3	0,00	0,03	0,05	0,00	0,01	0,00	0,01	0,00	0,01	0,00	0,00	0,06	0,00	0,00	0,00	0,00	0,04	0,03	0,00	0,11	0,00	0,00	0,20	0,23	0,00	0,00	0,22	0,00	0,00	0,00	1,00	0,0026	0,0241
4	0,00	0,04	0,06	0,00	0,01	0,00	0,01	0,00	0,00	0,00	0,00	0,05	0,01	0,00	0,00	0,00	0,04	0,05	0,00	0,10	0,00	0,00	0,21	0,22	0,00	0,00	0,21	0,00	0,00	0,00	1,00	0,0028	0,0242
5	0,00	0,06	0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,04	0,02	0,00	0,00	0,00	0,04	0,08	0,00	0,09	0,00	0,00	0,21	0,20	0,00	0,00	0,20	0,00	0,00	0,00	1,00	0,0031	0,0245
6	0,00	0,07	0,07	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,02	0,02	0,00	0,00	0,00	0,04	0,10	0,00	0,07	0,00	0,00	0,22	0,19	0,00	0,00	0,20	0,00	0,00	0,00	1,00	0,0033	0,0248
7	0,00	0,08	0,07	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,01	0,03	0,00	0,00	0,00	0,03	0,12	0,00	0,05	0,00	0,00	0,22	0,17	0,00	0,01	0,19	0,00	0,00	0,00	1,00	0,0035	0,0253
8	0,00	0,09	0,07	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,04	0,00	0,00	0,00	0,03	0,14	0,00	0,03	0,00	0,00	0,22	0,16	0,02	0,02	0,18	0,00	0,00	0,00	1,00	0,0037	0,0258
9	0,00	0,10	0,08	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,04	0,00	0,00	0,00	0,03	0,17	0,00	0,00	0,00	0,00	0,22	0,13	0,03	0,03	0,17	0,00	0,00	0,00	1,00	0,0040	0,0264
10	0,00	0,12	0,08	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,05	0,00	0,00	0,00	0,02	0,19	0,00	0,00	0,00	0,00	0,22	0,10	0,04	0,04	0,15	0,00	0,00	0,00	1,00	0,0042	0,0271
11	0,00	0,13	0,08	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,06	0,00	0,00	0,00	0,01	0,21	0,00	0,00	0,00	0,00	0,21	0,06	0,05	0,05	0,13	0,00	0,00	0,00	1,00	0,0044	0,0280
12	0,00	0,15	0,08	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,07	0,00	0,00	0,00	0,00	0,23	0,00	0,00	0,00	0,00	0,21	0,03	0,05	0,06	0,12	0,00	0,00	0,00	1,00	0,0046	0,0289
13	0,00	0,17	0,08	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,08	0,00	0,00	0,00	0,00	0,25	0,00	0,00	0,00	0,00	0,20	0,00	0,06	0,07	0,09	0,00	0,00	0,00	1,00	0,0049	0,0299
14	0,00	0,19	0,08	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,09	0,00	0,00	0,00	0,00	0,29	0,00	0,00	0,00	0,00	0,18	0,00	0,06	0,07	0,03	0,00	0,00	0,00	1,00	0,0051	0,0311
15	0,00	0,21	0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,11	0,00	0,00	0,00	0,00	0,33	0,00	0,00	0,00	0,00	0,14	0,00	0,07	0,08	0,00	0,00	0,00	0,00	1,00	0,0053	0,0324
16	0,00	0,24	0,02	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,13	0,00	0,00	0,00	0,00	0,38	0,00	0,00	0,00	0,00	0,07	0,00	0,08	0,10	0,00	0,00	0,00	0,00	1,00	0,0055	0,0343
17	0,00	0,27	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,16	0,00	0,00	0,00	0,00	0,44	0,00	0,00	0,00	0,00	0,00	0,00	0,03	0,09	0,00	0,00	0,00	0,00	1,00	0,0058	0,0366
18	0,00	0,20	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,38	0,00	0,00	0,00	0,00	0,42	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	0,0060	0,0429
19	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,68	0,00	0,00	0,00	0,00	0,32	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	0,0062	0,0577
20	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	0,0064	0,0779

It can be observed that in these portfolios the weights of some stocks are always zero. But the number of stocks, which do not enter the portfolio, is not constant; it is changeable from one portfolio to another. For that reason, the Portfolio A which maximizes the SR is chosen among these 20 portfolios.

To find the SR of each portfolio, risk free rates were collected from the Turkish central bank website (www.tcmb.gov.tr) and the weighted weekly risk free rate is calculated as 0.00110. Then the formula (2.15) is applied to these portfolios. The results are as in Table 5.6.

Table 5.6 : SR computation of the BIST-30 Portfolios.

Risk Free Rate	0,00110																			
Portfolio	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Return	0,0022	0,0024	0,0026	0,0028	0,0031	0,0033	0,0035	0,0037	0,0040	0,0042	0,0044	0,0046	0,0049	0,0051	0,0053	0,0055	0,0058	0,0060	0,0062	0,0064
Risk Free adjusted Return	0,0011	0,0013	0,0015	0,0017	0,0020	0,0022	0,0024	0,0026	0,0029	0,0031	0,0033	0,0035	0,0038	0,0040	0,0042	0,0044	0,0047	0,0049	0,0051	0,0053
Std Dev	0,0239	0,0240	0,0241	0,0242	0,0245	0,0248	0,0253	0,0258	0,0264	0,0271	0,0280	0,0289	0,0299	0,0311	0,0324	0,0343	0,0366	0,0429	0,0577	0,0779
Sharpe Ratio	0,0458	0,0540	0,0620	0,0700	0,0814	0,0885	0,0947	0,1006	0,1097	0,1142	0,1177	0,1209	0,1269	0,1285	**0,1295**	0,1281	0,1283	0,1141	0,0883	0,0680

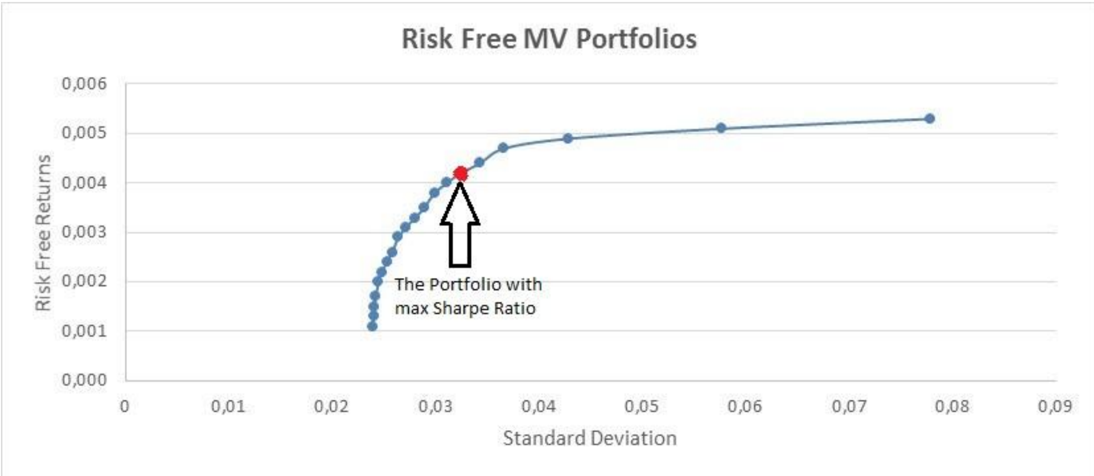


Figure 5.1 : Efficient Frontier of the MV Optimized BIST-30 Portfolios.

It is obvious from this table that the 15th portfolio has the maximum SR, so this portfolio can be selected as Portfolio-A. In this Portfolio-A only the stocks 2,3,13,18,23,25,26 have a positive weight and the other 23 stocks have a weight of zero, in other words, these stocks do not enter the portfolio.

5.3.2 Model 2 -> Portfolio-B

In this model the SSD inefficient stocks of BIST-30 are excluded in the first step. To identify these SSD inefficient stocks, c(30,2)=435 pairwise SSD comparisons are made with the help of C++ code. The comparison matrix of these stocks is as Table 5.7.

Table 5.7 : SSD output matrix showing all 435 SSD relationships among 30 assets.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	0	3	3	1	3	3	3	3	3	3	3	3	3	3	2	3	3	3	3	3	3	3	2	3	3	3	2	3	3	1
2	0	0	3	1	3	1	3	3	3	3	3	3	3	3	3	1	1	3	1	3	3	3	3	3	3	1	3	3	1	1
3	0	0	0	3	3	3	3	3	3	3	3	3	3	3	3	3	3	2	3	3	3	3	2	3	3	3	3	3	3	3
4	0	0	0	0	3	3	2	2	2	2	3	3	3	2	2	3	3	2	3	2	2	2	2	2	2	3	2	2	3	3
5	0	0	0	0	0	3	3	3	3	3	3	3	3	3	3	3	3	2	3	3	3	3	2	2	3	3	2	3	3	3
6	0	0	0	0	0	0	2	2	3	3	2	3	3	3	2	2	2	2	3	2	2	2	2	2	2	2	2	2	2	3
7	0	0	0	0	0	0	0	3	3	3	3	1	3	3	3	3	3	3	1	3	3	3	2	3	3	3	3	3	3	3
8	0	0	0	0	0	0	0	0	3	3	3	3	3	3	3	3	3	3	1	3	3	3	3	3	3	3	3	3	3	1
9	0	0	0	0	0	0	0	0	0	3	3	3	3	3	3	3	3	3	1	3	3	3	3	3	3	3	3	3	3	3
10	0	0	0	0	0	0	0	0	0	0	3	3	3	3	2	3	3	3	1	3	3	3	3	3	3	3	3	3	3	1
11	0	0	0	0	0	0	0	0	0	0	0	3	3	3	2	3	3	3	3	3	3	3	3	3	3	3	2	3	3	1
12	0	0	0	0	0	0	0	0	0	0	0	0	3	3	2	3	3	2	3	2	2	3	2	2	3	3	2	2	3	3
13	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3	3	1	3	3	3	3	3	3	3	2	3	3	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	3	1	3	3	1	3	3	3	3	3	3	1	1
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3	3	2	3	2	3	2	3	2	3	3	3
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3	3	3	2	3	2	3	3	3	3	3
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	3	3	3	3	3	3	3	3	3	3	3
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	2	2	2	2	2	3	3
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3	3	3	3	3	3	1	1
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	3	3	3	3	3	3	1	1
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3	3	2	3	3	3
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3	3	1	1	1
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3	3	3	3
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3	1	1
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3	3
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	1	1
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Suppose that a_{ij} is an element of this matrix, then "1" means that the stock i second orderly dominates the stock j. Similarly, "2" means that the stock j second orderly dominates the stock i. On the other side "3" represents no SSD between i and j.

As it is expected, this matrix has only 435 elements. Apart from the values "1,2,3" the zero values are attained in meaningless situations. All diagonal elements are zero since to check the SSD of two identical stocks is meaningless. Additionally because of the symmetric structure of SSD zeros are automatically allocated to the values below the diagonal elements. For example, if $a_{ij}=1$, i.e. i dominates j, then that also means $a_{ji}=2$. Therefore, all elements below the diagonal can be easily understood from the part above.

As it can be easily interpreted from this SSD matrix, the stocks can be grouped in four discrete clusters.

A. The stocks which dominate at least one stock and are not dominated by any other stock

B. The stocks which are dominated by at least one stock and do not dominate any other stock

C. The stocks which are dominated by at least one stock and dominate at least one other stock

D. The stocks which are not dominated by any other stock and do not dominate any other stock

According to the results of this matrix, the contents of these clusters are A={2, 8, 9, 15, 18, 20, 21, 23, 24, 25, 27}; B={3, 4, 5, 6, 12, 19, 30}; C={1, 7, 10, 11, 14, 16, 17, 22, 26, 28, 29}; D={13} which are displayed with the help of venn diagrams in Figure 5.2.

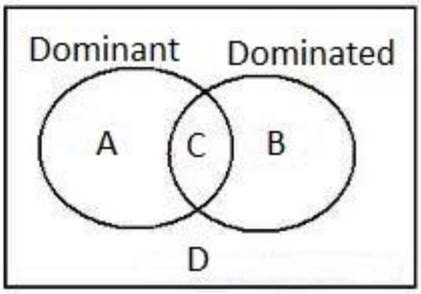


Figure 5.2 . The Venn diagram of the SSD clusters.

It is obvious that the stocks which are in the clusters B and C are dominated by at least one stock. Because of that these stocks must not be in a portfolio which is built by using these 30 stocks. In other words, the stocks in A and D should be taken in the portfolio optimization.

To apply Model 2, MV optimization stated in (2.15) is applied to this efficient set of A and D which consists of only 12 stocks with the help of MatLab Software (Version 7.9). These 12 stocks can be regarded as a SSD efficient subset of BIST-30 stocks. The list of MV optimized portfolios according to 20 different return (μ) levels is stated in Table 5.8.

Table 5.8 . MV optimized portfolios of 12 SSD Efficient Stocks.

	Weights													
Portfolio	**2**	**8**	**9**	**13**	**15**	**18**	**20**	**21**	**23**	**24**	**25**	**27**	**Return**	**Std Dev**
1	0,0000	0,0166	0,0846	0,0039	0,0000	0,0281	0,1186	0,0000	0,2206	0,2827	0,0000	0,2449	0,0023	0,0245
2	0,0022	0,0084	0,0584	0,0115	0,0000	0,0457	0,1199	0,0000	0,2355	0,2768	0,0000	0,2416	0,0025	0,0245
3	0,0185	0,0000	0,0343	0,0172	0,0000	0,0566	0,1221	0,0000	0,2450	0,2706	0,0000	0,2357	0,0028	0,0246
4	0,0337	0,0000	0,0079	0,0233	0,0000	0,0681	0,1222	0,0000	0,2525	0,2618	0,0000	0,2305	0,0030	0,0248
5	0,0518	0,0000	0,0000	0,0319	0,0000	0,0891	0,1077	0,0000	0,2571	0,2433	0,0000	0,2191	0,0032	0,0250
6	0,0685	0,0000	0,0000	0,0405	0,0000	0,1118	0,0844	0,0000	0,2582	0,2208	0,0092	0,2066	0,0034	0,0254
7	0,0829	0,0000	0,0000	0,0480	0,0000	0,1323	0,0587	0,0000	0,2572	0,1985	0,0266	0,1958	0,0036	0,0259
8	0,0973	0,0000	0,0000	0,0556	0,0000	0,1527	0,0331	0,0000	0,2561	0,1763	0,0440	0,1849	0,0038	0,0265
9	0,1118	0,0000	0,0000	0,0631	0,0000	0,1732	0,0075	0,0000	0,2551	0,1540	0,0613	0,1740	0,0040	0,0271
10	0,1291	0,0000	0,0000	0,0713	0,0000	0,1929	0,0000	0,0000	0,2529	0,1217	0,0746	0,1575	0,0043	0,0278
11	0,1477	0,0000	0,0000	0,0796	0,0000	0,2123	0,0000	0,0000	0,2501	0,0851	0,0866	0,1386	0,0045	0,0287
12	0,1663	0,0000	0,0000	0,0880	0,0000	0,2317	0,0000	0,0000	0,2474	0,0486	0,0983	0,1197	0,0047	0,0296
13	0,1849	0,0000	0,0000	0,0963	0,0000	0,2512	0,0000	0,0000	0,2447	0,0121	0,1100	0,1008	0,0049	0,0307
14	0,2089	0,0000	0,0000	0,1071	0,0000	0,2822	0,0000	0,0000	0,2294	0,0000	0,1168	0,0556	0,0051	0,0318
15	0,2362	0,0000	0,0000	0,1197	0,0000	0,3198	0,0000	0,0000	0,2035	0,0000	0,1208	0,0000	0,0053	0,0331
16	0,2749	0,0000	0,0000	0,1429	0,0000	0,3778	0,0000	0,0000	0,0857	0,0000	0,1187	0,0000	0,0055	0,0348
17	0,3122	0,0000	0,0000	0,1872	0,0000	0,4406	0,0000	0,0000	0,0000	0,0000	0,0600	0,0000	0,0058	0,0373
18	0,1827	0,0000	0,0000	0,4145	0,0000	0,4028	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0060	0,0441
19	0,0000	0,0000	0,0000	0,6942	0,0000	0,3058	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0062	0,0587
20	0,0000	0,0000	0,0000	1,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0064	0,0779

Similar to the previous parts risk free adjusted return, standard deviation and SR of each SSD efficient portfolio are computed. The results are shown in Table 5.9 and Figure 5.3.

Table 5.9 . Sharpe Ratio computation of the 12 SSD Efficient Stocks.

Risk Free Rate	0,00110																			
Portfolio	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Return	0,0023	0,0025	0,0028	0,0030	0,0032	0,0034	0,0036	0,0038	0,0040	0,0043	0,0045	0,0047	0,0049	0,0051	0,0053	0,0055	0,0058	0,0060	0,0062	0,0064
Risk Free adjusted Return	0,0012	0,0014	0,0017	0,0019	0,0021	0,0023	0,0025	0,0027	0,0029	0,0032	0,0034	0,0036	0,0038	0,0040	0,0042	0,0044	0,0047	0,0049	0,0051	0,0053
Std Dev	0,0245	0,0245	0,0246	0,0248	0,0250	0,0254	0,0259	0,0265	0,0271	0,0278	0,0287	0,0296	0,0307	0,0318	0,0331	0,0348	0,0373	0,0441	0,0587	0,0779
Sharpe Ratio	0,0488	0,0569	0,0689	0,0764	0,0838	0,0904	0,0963	0,1017	0,1068	0,1149	0,1183	0,1215	0,1236	0,1256	0,1267	0,1263	0,1259	0,1110	0,0868	0,0680

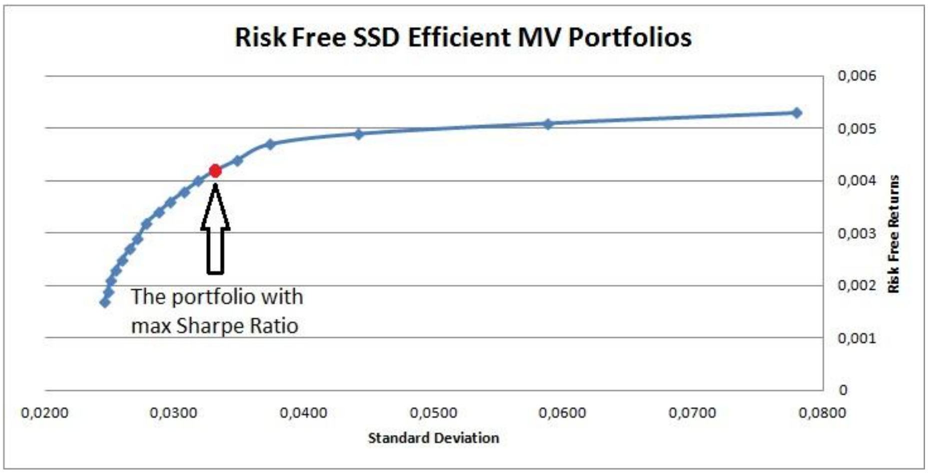


Figure 5.3 . Efficient Frontier of the MV Optimized SSD Efficient Portfolios.

As it is realized the 15th portfolio has the same risk free return of 0,0042 as the Portfolio-A, besides this portfolio has also the maximum SR among these SSD efficient portfolios. So this one can be selected as the Portfolio-B.

5.3.3 Model 3 -> Portfolio-C

In this Model 3, the suggested two-step gradual method of the thesis is applied completely. The first step of the Model 3, is the as Model 2. Then the fuzzy variance minimization is applied to the SSD efficient subset of 12 stocks of the clusters A and D which are explicitly stated in Figure 5.2.

To apply the fuzzy variance minimization, first of all, the membership function in (4.2) displaying the future return should be determined for each of these SSD efficient 12 stocks. To determine these functions, it is easy to compute the expected centre future return (r_i) since it can be accepted as the average return of the stock (i=1,...,12). But there is not just a certain rule to estimate the up direction (β_i) and the down direction (α_i) representing the maximum possible differences of future returns. Actually these deviations reflect the expert knowledge. That means this fuzzy model enables that up and down directions can be determined according to the next coming economic conditions. But in this application part of the thesis no subjective opinion is added to the model. The past observations of the stocks in the last 135 weeks are observed and under the lights of this information β_i and α_i are determined due to relatively the best and the worst returns in the past. The logic of β_i and α_i calculation can be explained by the "Arçelik" Stock which belongs to BIST-30 Portfolio. The related graph of this example is displayed in Figure 5.4.

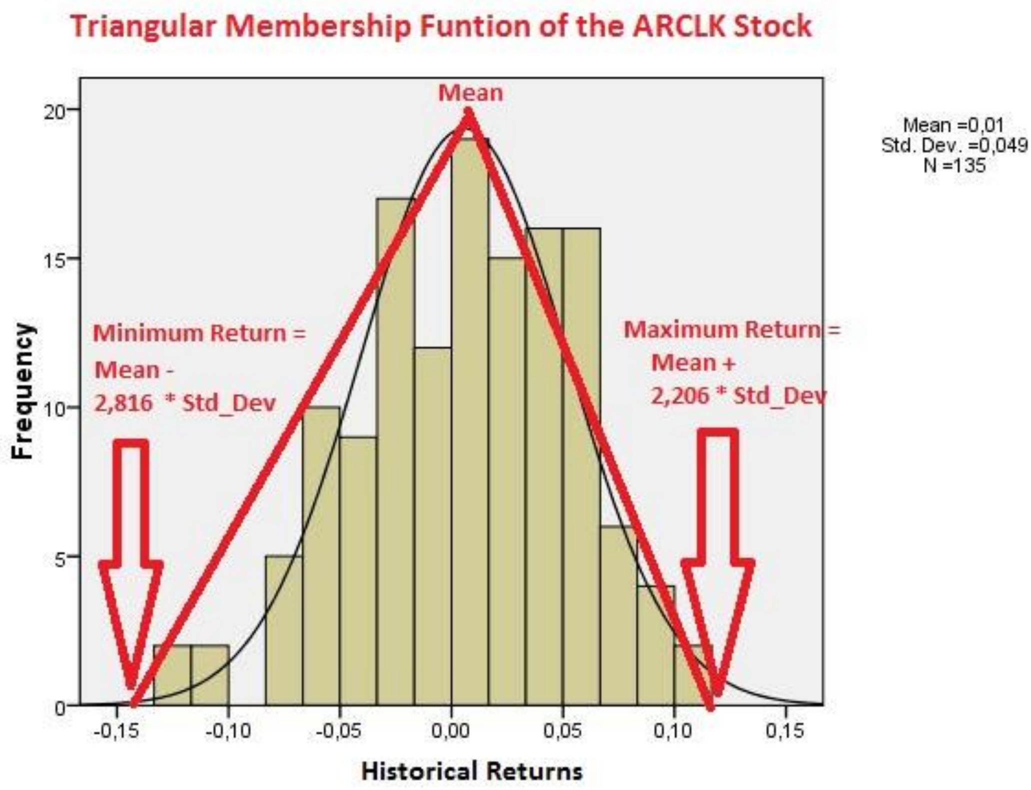


Figure 5.4: The calculation logic of up (β_i) and down (α_i) directions explained by the "Arçelik" Stock.

As it can be understood from Figure 5.4 the best and the worst returns of the stock constitute the end points of the triangular base side. To apply the optimization model in (4.11) the necessary calculations of the 12 SSD efficient stocks are made as in Table 5.10.

Table 5.10. The necessary calculations of the SSD efficient 12 stocks to minimize the fuzzy variance.

12 SSD Efficient Stocks	ARCLK	ENKAI	EREGL	IPEKE	KCHOL	KRDMD	PETKM	SAHOL	TAVHL	TCELL	THYAO	TTKOM
Average	0,006	0,002	-0,001	0,006	0,004	0,006	0,002	0,003	0,004	0,001	0,005	0,003
Risk Free Average	0,005	0,000	-0,002	0,005	0,002	0,005	0,001	0,002	0,003	0,000	0,004	0,002
Std Dev	0,049	0,046	0,042	0,078	0,045	0,045	0,039	0,046	0,039	0,036	0,049	0,038
Min Deviation= Average+ (x)* Std Dev	-2,816	-2,417	-2,582	-2,979	-2,524	-3,821	-3,400	-2,888	-3,293	-4,143	-2,757	-3,170
Max Deviation= Average+ (x)* Std Dev	2,206	3,233	2,413	6,334	3,620	2,386	3,586	2,939	2,478	2,833	3,139	3,070
β (deviation in up direction)	0,108	0,149	0,101	0,494	0,164	0,108	0,140	0,135	0,096	0,101	0,154	0,118
α (deviation in down direction)	0,138	0,112	0,108	0,232	0,114	0,172	0,133	0,133	0,128	0,148	0,135	0,122
[sqrt(6)/12]*(α+β)	0,050	0,053	0,043	0,148	0,057	0,057	0,056	0,055	0,046	0,051	0,059	0,049
Average+(β-α)/6	0,001	0,008	-0,002	0,050	0,012	-0,005	0,003	0,004	-0,001	-0,006	0,008	0,002
Risk Free Average + (β-α)/6	0,000	0,007	-0,003	0,049	0,011	-0,006	0,002	0,002	-0,002	-0,008	0,007	0,001

The linear optimization model is stated in Table 5.11.

Table 5.11 . The linear optimization model.

Min [0,050 x1 + 0,053 x2 + 0,043 x3 + 0,148 x4 + 0,057 x5 + 0,057 x6 + 0,056 x7 + 0,055 x8 + 0,046 x9 + 0,051 x10 + 0,059 x11 + 0,049 x12]
Subject to
-0,0005 x1 + 0,0068 x2 - 0,0031 x3 + 0,0489 x4 + 0,0107 x5 - 0,0062 x6 + 0,0022 x7 + 0,0025 x8 - 0,0024 x9 - 0,0075 x10 + 0,0070 x11 + 0,0010 x12 >= "Goal Return"
x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 = 1
x1, x2, x3, x4, x5, x6, x7, x8, x9, x10, x11, x12 >= 0

The results of the optimization are listed in Table 5.12.

Table 5.12. Fuzzy Varince minimization of the SSD efficient 12 stocks.

		Weights											
		ARCLK	ENKAI	EREGL	IPEKE	KCHOL	KRDMD	PETKM	SAHOL	TAVHL	TCELL	THYAO	TTKOM
Fuzzy Logic Std Dev	Risk Free Return	1	2	3	4	5	6	7	8	9	10	11	12
0,0487	0,0028	0	0	0,5711	0	0,4289	0	0	0	0	0	0	0
0,0491	0,0032	0	0	0,5420	0	0,4580	0	0	0	0	0	0	0
0,0495	0,0036	0	0	0,5130	0	0,4870	0	0	0	0	0	0	0
0,0499	0,004	0	0	0,4840	0	0,5160	0	0	0	0	0	0	0
0,0504	0,0045	0	0	0,4477	0	0,5523	0	0	0	0	0	0	0
0,0508	0,0049	0	0	0,4187	0	0,5813	0	0	0	0	0	0	0
0,0512	0,0053	0	0	0,3897	0	0,6103	0	0	0	0	0	0	0
0,0518	0,0058	0	0	0,3534	0	0,6466	0	0	0	0	0	0	0
0,0524	0,0064	0	0	0,3099	0	0,6901	0	0	0	0	0	0	0
0,0560	0,01	0	0	0,0487	0	0,9513	0	0	0	0	0	0	0
0,0671	0,015	0	0	0	0,1132	0,8868	0	0	0	0	0	0	0
0,0790	0,02	0	0	0	0,2440	0,7560	0	0	0	0	0	0	0
0,0910	0,025	0	0	0	0,3748	0,6252	0	0	0	0	0	0	0
0,1029	0,03	0	0	0	0,5056	0,4944	0	0	0	0	0	0	0
0,1149	0,035	0	0	0	0,6364	0,3636	0	0	0	0	0	0	0
0,1269	0,04	0	0	0	0,7672	0,2328	0	0	0	0	0	0	0
0,1388	0,045	0	0	0	0,8980	0,1020	0	0	0	0	0	0	0
0,1481	0,05	No Feasible Solution											
0,1481	0,06												
0,1481	0,07												
0,1481	0,08												

The portfolio with the fuzzy goal return of 0.053 is chosen as the Portfolio C among the optimized portfolios of Model 3 to ensure equal comparison conditions since the return of the SR max portfolios A and B is also 0.053 before the adjustment of risk free rate.

6. CONCLUSION

In this part the performance of the optimized portfolio created by the the new gradual method is tested in next coming future data and then the results are interpreted.

6.1 The results of the Performance Tests

In this part the portfolios A, B and C are tested in future data in various time horizons, such as next 10, 20, 30, 40 and 50 weeks from the end of the application period which is 12.07.2013.

As stated in the previous part, there are three models and in each method there are a lot of optimized portfolios. To be able to compare these methods with each other, the return level of SR maximizing Portfolio A is also accepted for Portfolio B and C to ensure equal comparison conditions. Apart from these three portfolios A, B and C, the BIST-30 Index, in other words the market portfolio, is also added into the comparison table.

To measure the performance of these portfolios the two most important criteria of the finance world, SR in (2.15) and TR in (2.16), are calculated.

Table 6.1. Performance results of the compared portfolios.

	10 Weeks after 12.07.2013		20 Weeks after 12.07.2013		30 Weeks after 12.07.2013		40 Weeks after 12.07.2013		50 Weeks after 12.07.2013	
	SR	**TR**	**SR**	**TR**	**SR**	**TR**	**SR**	**TR**	**SR**	**TR**
Market Potfolio	-0,0350		0,0429		-0,1428		-0,0053		0,0384	
Portfolio A	-0,0509	-0,0024	0,0956	0,0043	-0,0516	-0,0023	0,0368	0,0016	0,0873	0,0037
Portfolio B	0,0026	0,0001	0,1098	0,0050	-0,0292	-0,0013	0,0447	0,0020	0,0880	0,0038
Portfolio C	0,1473	0,0083	0,2195	0,0120	0,0156	0,0008	0,0963	0,0048	0,1784	0,0085

Notice that TR of the Market Portfolio is not calculated since beta of the market is always one. The bar charts for both SR and TR are displayed in Figure 6.1 for a visual perception.

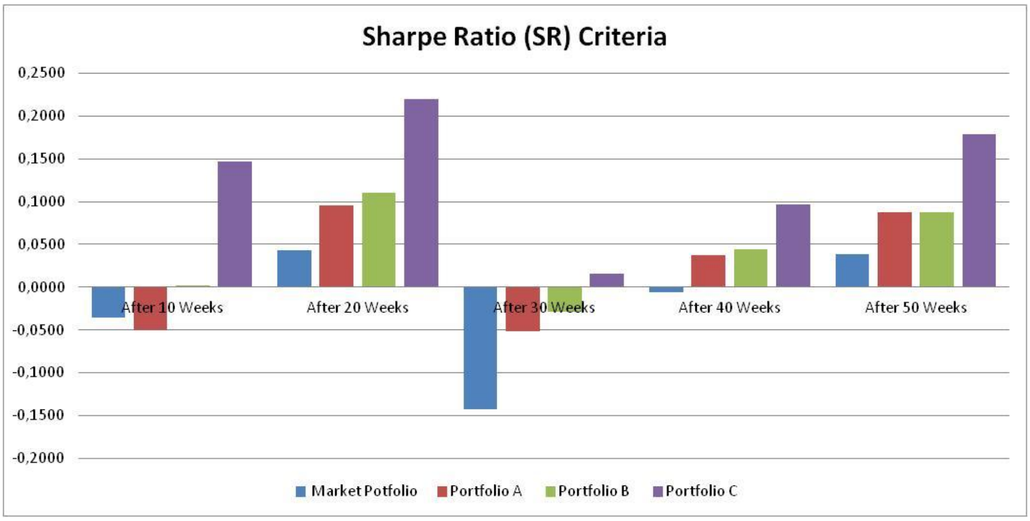


Figure 6.1. Sharpe Ratio (SR) results of the compared portfolios.

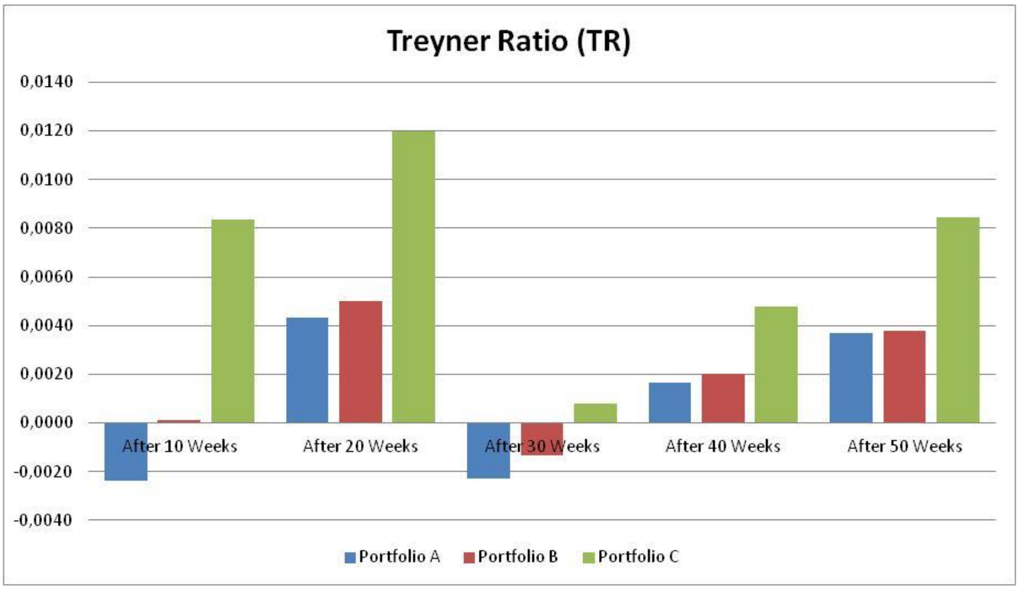


Figure 6.2. Treynor Ratio (TR) results of the compared portfolios.

6.2 Concluding Remarks

The Portfolio-B consists of the only first step of the suggested gradual method that is exclusion of the SSD inefficient stocks from the MV portfolio. According these results Portfolio-B performs better than the classical MV Portfolio (Portfolio-A) in all fields and this superiority indicates that the first step alone generates an improvement in the classical MV method.

But the Portfolio-C which also contains the second step, fuzzy variance minimization, in addition the first step performs far better than even Portfolio-B. That means in plain words, the both steps of the suggested gradual two-step method improves the classical MV method as plain as a pikestaff by generating a portfolio which has a higher performance in all time horizons and in both performance criteria SR and TR. Furthermore, SR of Portfolio-C is higher than the SR of the market portfolio again in all time intervals.

Finally, the technical reasons behind this superiority of the Portfolio-C among the others are explained. As it is stated in Part 2.1.2 MV has some shortcomings because of its assumptions which are practically not applicable. Among the others, the assumption requiring that the returns of all stocks in the portfolio must be distributed normally is probably the most impossible one in the real world. To check the validity of this normality assumption, Shapiro-Wilk Normality Test is applied to all stocks of BIST-30 data. The test outcomes are displayed in the Table A.2 and this table indicates that 14 of the 30 stocks are not normally distributed at the %5 confidence level.

In plain words, although MV requires that returns of the stocks in portfolio must be distributed normally, in the BIST-30 data nearly the half of them is not normally distributed. This striking result alone shows why the MV model generates such low performance portfolios. Since Fuzzy variance minimization does not require anything about the distribution of the stock returns apart from MV, the Portfolio-C has overwhelming superiority over the Portfolio-A and Portfolio-B. So it can be concluded from these findings that the suggested gradual optimization model works far better than classical MV model, especially in the real data, because of its simple algorithm which does not require any assumption over the return distributions.

6.3 Future Work

In this thesis a completely new gradual portfolio optimization method is suggested and this method is to the BIST-30 data applied. The performance tests show that this new method is more successful than the classical method.

The aim in the short run is to establish a series of academic papers which apply this new method to other markets of the world apart from Turkish Stock Exchange and generalize its results for the whole finance world.

The aim in the long run is creation of a user friendly software program which applies this method to any stock market and gives the optimized portfolio as the output.

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APPENDICES

APPENDIX A : Tables

Table A.1 : Additional Information about BIST-30 Companies.

	Stock Code	General Manager	Chairman of the Board	Web Site
1	AKBNK	S.HAKAN BİNBAŞGİL	SUZAN SABANCI DİNÇER	www.akbank.com
2	ARCLK	Levent Çakıroğlu	Mustafa Rahmi Koç	www.arcelik.com.tr
3	ASELS	FAİK EKEN	Mustafa Murat Şeker	www.aselsan.com.tr
4	ASYAB	Aydın Gündoğdu	MEHMET ALİ İSLAMOĞLU	www.bankasya.com.tr
5	BIMAS	Mustafa Latif Topbaş	Mustafa Latif TOPBAŞ	www.bim.com.tr
6	DOHOL	Yahya Üzdiyen	Yaşar Begümhan Doğan Faralyalı	www.doganholding.com.tr
7	EKGYO	MURAT KURUM	ERTAN YETİM	www.emlakkonut.com.tr
8	ENKAI	AGAH MEHMET TARA	M.Sinan Tara	www.enka.com
9	EREGL	Sedat ORHAN	Ali Aydın PANDIR	www.erdemirgrubu.com.tr
10	GARAN	SAİT ERGUN ÖZEN	FERİT FAİK ŞAHENK	www.garanti.com.tr
11	HALKB	Ali Fuat Taşkesenlioğlu	Hasan CEBECİ	www.halkbank.com.tr
12	IHLAS	Ahmet Mücahid Ören	Ahmet Mücahid Ören	www.ihlas.com.tr
13	IPEKE	C.Tekin İpek	Hamdi Akın İPEK	www.koza.com.tr
14	ISCTR	Adnan Bali	H. Ersin Özince	www.isbank.com.tr
15	KCHOL	Levent Çakıroğlu	Rahmi M. Koç	www.koc.com.tr
16	KOZAA	Şaban Yörüklü	Hamdi Akın İPEK	www.koza.com.tr
17	KOZAL	İSMET SİVRİOĞLU	HAMDİ AKIN İPEK	www.kozaaltin.com.tr
18	KRDMD	Uğur YILMAZ	MUTULLAH YOLBULAN	www.kardemir.com
19	MGROS	Ömer Özgür Tort	Fevzi Bülent Özaydınlı	www.migroskurumsal.com
20	PETKM	Sadettin Korkut	VAGIF ALIYEV	www.petkim.com.tr
21	SAHOL	ZAFER KURTUL	GÜLER SABANCI	www.sabanci.com.tr
22	SISE	Prof.Dr.Ahmet Kırman	Hakkı Ersin Özince	www.sisecam.com.tr
23	TAVHL	MUSTAFA SANİ ŞENER	Hamdi Akın	www.tav.com.tr
24	TCELL	Kaan Terzioğlu	Ahmet Akça	www.turkcell.com.tr
25	THYAO	Temel Kotil	M. İlker AYCI	www.thy.com
26	TOASO	Cengiz EROLDU	Mustafa Vehbi KOÇ	www.tofas.com.tr
27	TTKOM	Rami Aslan	Mohammed Hariri	www.turktelekom.com.tr
28	TUPRS	Yavuz Erkut	ÖMER MEHMET KOÇ	www.tupras.com.tr
29	VAKBN	Halil AYDOĞAN	Ramazan GÜNDÜZ	www.vakifbank.com.tr
30	YKBNK	Hüseyin Faik Açıkalın	Mustafa V. Koç	www.ykb.com.tr

Table A.2 : Closing values of 30 Stocks in BIST-30.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
	AKBNK	ARCLK	ASELS	ASYAB	BIMAS	DOHOL	EKGYO	ENKAI	EREGL	GARAN	HALKB	IHLAS	IPEKE	ISCTR	KCHOL	KOZAA	KOZAL	KRDMD	MGROS	PETKM	SAHOL	SISE	TAVHL	TCELL	THYAO	TOASO	TTKOM	TUPRS	VAKBN	YKBNK
3.12.2010	8,39	7,15	8,08	3,18	48,45	1,08	2,01	4,51	2,42	8,19	13,53	0,97	2,23	5,87	6,59	2,72	18,34	0,64	28,10	2,29	7,43	2,15	7,34	10,05	4,50	7,50	5,52	32,75	4,19	5,40
10.12.2010	8,20	6,72	8,26	3,04	49,89	1,08	1,96	4,28	2,33	7,81	12,34	1,02	2,19	5,49	6,38	2,78	19,46	0,66	27,50	2,23	6,90	2,14	7,32	10,00	4,17	7,05	5,37	32,92	3,95	4,87
17.12.2010	7,88	6,63	8,14	2,86	51,33	1,06	1,88	4,37	2,31	7,17	12,20	1,09	2,21	5,19	6,63	2,84	19,90	0,66	27,70	2,22	6,96	2,15	7,44	10,10	4,05	6,86	5,61	31,90	3,74	4,62
24.12.2010	8,43	6,87	8,36	2,92	51,09	1,10	1,96	4,49	2,44	7,63	12,43	1,35	2,26	5,49	6,74	2,90	20,04	0,69	28,70	2,29	7,10	2,18	7,66	10,60	4,37	7,11	5,69	33,86	3,98	4,95
31.12.2010	8,45	7,01	8,24	2,84	50,37	1,12	1,94	4,40	2,40	7,50	12,48	1,42	2,36	5,35	6,83	2,89	20,09	0,71	29,40	2,31	7,06	2,11	7,48	10,55	4,50	7,04	5,52	32,92	3,88	4,86
7.1.2011	8,39	7,59	8,54	3,07	52,77	1,16	2,25	4,62	2,48	7,84	13,24	1,43	2,38	5,52	6,94	3,11	20,39	0,75	31,10	2,46	7,12	2,32	7,68	10,85	4,52	8,03	5,81	35,91	4,16	5,22
14.1.2011	8,33	7,98	8,42	3,00	50,85	1,14	2,36	4,75	2,40	7,65	12,24	1,25	2,41	5,27	6,85	3,17	20,63	0,77	31,40	2,37	7,04	2,55	7,6	10,55	4,30	8,13	6,00	36,76	4,14	5,12
21.1.2011	7,94	7,87	8,28	2,80	51,57	1,12	2,51	4,77	2,39	7,25	11,77	1,25	2,25	5,04	6,56	2,96	20,19	0,84	32,70	2,35	6,98	2,60	7,6	10,05	4,15	7,64	5,88	36,59	3,89	4,75
28.1.2011	7,37	7,60	8,08	2,74	48,69	1,08	2,64	4,78	2,36	6,77	11,62	1,19	2,11	4,80	6,23	2,67	17,41	0,79	32,70	2,26	6,77	2,55	7,24	9,94	4,32	7,99	5,64	35,91	3,83	4,64
4.2.2011	7,80	7,33	7,86	2,86	48,21	1,23	2,64	4,78	2,35	7,27	12,63	1,20	2,17	5,31	6,50	2,70	18,44	0,80	35,50	2,25	6,77	2,56	7,08	10,00	4,27	7,83	5,64	35,65	4,02	4,92
11.2.2011	7,72	7,15	8,02	2,83	47,97	1,28	2,64	4,46	2,42	7,02	11,86	1,18	2,19	5,08	6,56	2,74	18,78	0,79	34,80	2,26	6,65	2,53	7,18	10,12	4,38	7,53	6,05	34,37	3,97	4,90
18.2.2011	7,92	6,87	8,1	2,86	50,61	1,27	2,77	4,46	2,45	7,42	12,24	1,17	2,21	5,23	6,45	2,86	19,02	0,78	35,60	2,30	6,81	2,43	7,28	10,55	4,22	7,21	5,98	34,29	4,10	4,94
25.2.2011	7,49	6,58	7,56	2,52	46,44	1,15	2,45	4,19	2,33	6,88	11,05	1,17	1,99	4,92	5,92	2,72	18,48	0,72	33,00	2,24	6,28	2,25	6,38	9,00	3,84	6,75	6,05	34,20	3,85	4,47
4.3.2011	6,99	6,26	7,32	2,64	49,17	1,18	2,57	4,14	2,28	6,83	10,77	1,35	2,08	4,71	5,87	2,70	19,17	0,73	32,60	2,22	6,94	2,21	6,62	8,94	3,69	7,37	5,83	33,86	3,71	4,35
11.3.2011	7,72	6,36	7,54	2,96	47,49	1,19	2,58	4,26	2,31	7,27	11,82	1,56	2,20	4,86	6,43	2,77	20,34	0,77	32,80	2,23	7,38	2,40	7,12	8,96	3,82	7,12	5,84	35,48	3,87	4,53
18.3.2011	7,47	6,40	7,82	2,88	44,04	1,21	2,60	4,23	2,54	7,23	11,72	1,50	2,25	4,88	6,43	2,87	21,80	0,77	33,00	2,24	7,30	2,44	7,24	9,06	3,82	7,18	5,96	36,67	3,92	4,62
25.3.2011	7,37	6,65	8,26	2,92	49,17	1,20	2,59	4,40	2,54	7,19	11,53	1,60	2,30	4,81	6,56	2,94	19,70	0,78	34,20	2,28	7,04	2,62	7,12	9,56	3,54	7,32	6,23	37,19	3,84	4,48
1.4.2011	7,63	6,90	8,52	3,12	51,57	1,22	2,69	4,69	2,60	7,04	11,67	1,67	2,27	4,80	6,76	2,96	19,70	0,82	34,90	2,32	7,32	2,73	7,42	9,50	3,63	7,74	6,74	39,32	3,91	4,53
8.4.2011	8,24	7,37	8,74	3,08	50,13	1,21	2,78	4,66	2,65	7,54	12,29	1,81	2,42	5,04	7,21	3,04	21,12	0,82	32,50	2,32	7,98	2,71	7,6	9,20	3,67	7,86	6,85	42,86	4,03	4,72
15.4.2011	8,24	7,37	8,78	3,04	50,61	1,21	2,86	4,80	2,65	7,67	12,72	1,65	2,52	5,19	7,40	3,12	21,12	0,83	26,60	2,35	7,81	2,80	7,62	8,96	3,69	8,26	6,83	43,14	4,05	4,80
22.4.2011	7,94	7,83	8,8	3,00	50,85	1,21	2,94	4,97	2,75	7,63	12,63	1,56	2,91	5,19	7,29	3,39	22,14	0,86	27,20	2,38	7,98	2,87	7,6	9,18	3,72	8,15	6,95	43,78	3,99	4,75
29.4.2011	7,78	8,17	8,98	3,01	50,85	1,21	2,94	5,09	2,87	7,69	12,53	1,45	2,93	5,23	7,65	3,39	23,26	0,86	26,30	2,56	7,98	2,90	7,68	8,98	3,74	8,18	6,73	45,05	4,01	4,79
6.5.2011	7,74	7,54	9,32	2,90	48,93	1,19	2,85	4,89	2,77	7,73	12,43	1,41	2,85	5,19	7,35	3,20	22,38	0,90	25,30	2,62	7,77	3,02	7,84	8,82	3,91	7,28	6,79	43,50	3,97	4,73
13.5.2011	7,33	7,41	8,5	2,57	50,13	1,14	2,86	4,62	2,65	7,02	11,53	1,26	2,59	5,00	7,03	3,17	21,84	0,82	23,40	2,61	7,16	3,08	7,3	8,86	3,67	6,97	6,73	39,85	3,79	4,23
20.5.2011	7,27	7,41	8,52	2,58	50,58	1,09	2,70	4,63	2,60	6,73	11,10	1,23	2,36	4,82	7,03	2,97	20,42	0,81	18,60	2,43	7,30	3,46	7,62	8,92	3,58	7,13	6,69	39,31	3,61	4,08
27.5.2011	7,03	7,47	8,64	2,56	51,07	1,09	2,73	4,36	2,60	6,75	11,01	1,17	2,37	4,77	6,56	2,93	21,20	0,81	19,95	2,39	7,04	3,69	7,9	8,96	3,47	7,15	6,76	38,12	3,57	3,83
3.6.2011	7,03	7,87	8,62	2,59	50,58	0,98	2,84	4,50	2,66	6,98	11,55	1,16	2,38	4,80	6,82	2,81	20,71	0,83	19,80	2,47	6,82	3,15	8,22	8,96	3,53	7,00	7,00	38,30	3,62	4,04
10.6.2011	7,49	7,51	8,52	2,65	50,09	0,94	2,96	4,54	2,69	7,22	11,60	1,21	2,50	4,96	6,71	2,92	21,94	0,86	19,10	2,46	6,94	3,10	8,48	8,68	3,52	6,76	7,46	38,85	3,65	4,05
17.6.2011	7,37	7,58	8,46	2,53	53,92	0,88	2,74	4,36	2,69	6,93	11,26	1,14	2,40	4,63	6,41	2,94	22,14	0,82	17,90	2,33	6,54	3,17	8,16	8,66	3,45	6,50	7,30	36,39	3,55	3,92
24.6.2011	7,43	7,45	8,94	2,50	50,34	0,87	2,62	4,30	2,61	7,16	11,41	1,16	2,39	4,71	6,30	2,92	20,76	0,83	17,75	2,31	6,74	2,99	8,22	9,00	3,57	7,02	7,46	35,75	3,51	3,96
1.7.2011	7,35	8,04	9	2,53	52,06	0,91	2,67	4,35	2,77	7,20	11,65	1,15	2,70	4,82	6,67	2,98	21,06	0,89	19,25	2,36	6,84	3,17	8,16	8,70	3,56	6,93	7,87	36,21	3,67	4,07
8.7.2011	7,33	7,88	8,96	2,53	52,79	0,88	2,65	4,33	2,74	7,36	11,55	1,14	2,61	4,84	6,65	2,97	22,28	0,91	19,35	2,41	6,84	3,24	8,1	8,80	3,45	6,89	7,85	37,48	3,69	4,09
15.7.2011	7,17	7,75	8,96	2,51	53,28	0,92	2,62	4,18	2,76	7,12	11,11	1,04	2,57	4,85	6,56	3,07	23,17	0,91	18,70	2,47	6,68	3,34	7,88	8,50	3,35	6,95	7,39	38,67	3,64	3,98
22.7.2011	6,88	7,17	8,68	2,32	54,76	0,86	2,42	3,96	2,68	6,83	10,77	1,08	2,44	4,56	6,36	3,01	22,92	0,89	17,60	2,35	6,40	3,15	7,88	8,46	3,10	5,97	7,24	35,29	3,36	3,80
29.7.2011	7,27	7,30	8,82	2,39	56,48	0,87	2,53	4,07	2,64	7,28	11,60	1,08	2,59	4,72	6,62	3,10	24,15	0,94	19,35	2,44	6,62	3,28	8,26	8,76	3,15	6,55	6,65	37,48	3,52	4,03
5.8.2011	6,44	6,33	8	2,15	54,27	0,62	2,14	3,82	2,37	6,54	10,38	1,04	2,42	4,37	5,96	2,95	23,17	0,87	17,00	2,12	6,08	3,02	7,58	8,38	2,74	5,66	6,35	31,01	3,37	3,60
12.8.2011	6,05	5,56	7,52	1,76	52,06	0,69	2,04	3,41	2,23	5,87	9,79	0,92	2,34	4,12	5,42	2,70	21,01	0,78	14,65	2,15	5,54	2,83	6,66	7,66	2,55	5,71	6,56	30,28	2,96	3,11
19.8.2011	6,13	5,86	7,36	1,88	51,32	0,65	2,02	3,30	2,22	5,97	9,69	0,94	2,32	4,26	5,79	2,75	23,76	0,81	14,15	2,27	5,52	2,82	6,66	7,62	2,41	5,34	6,98	32,47	3,04	3,35
26.8.2011	6,29	5,69	7,14	1,93	54,27	0,66	2,17	3,41	2,19	6,05	10,38	0,94	2,23	4,27	5,65	2,66	22,92	0,78	14,10	2,19	6,12	2,78	6,8	7,68	2,41	5,11	7,35	29,73	3,07	3,35
2.9.2011	6,34	5,88	7,38	1,93	55,00	0,66	2,23	3,52	2,28	6,34	10,77	0,95	2,29	4,36	6,06	2,73	24,44	0,80	14,45	2,23	6,24	2,89	7,28	7,72	2,45	5,81	7,50	30,55	3,22	3,53
9.9.2011	6,58	5,96	7,44	1,94	53,04	0,67	2,21	3,45	2,17	6,44	10,92	0,91	2,25	4,34	6,15	2,65	23,07	0,81	14,80	2,19	5,98	2,76	7,32	8,12	2,64	5,84	7,48	31,56	3,29	3,62
16.9.2011	6,76	6,20	7,48	1,96	54,27	0,68	2,27	3,44	2,17	6,79	11,55	0,91	2,29	4,42	6,43	2,57	22,53	0,81	15,50	2,26	6,32	2,83	7,16	8,48	2,81	6,16	7,48	32,56	3,51	3,80
23.9.2011	6,68	6,31	7,24	1,92	53,53	0,64	2,20	3,44	2,03	6,63	11,45	0,90	2,22	4,29	6,19	2,48	21,40	0,78	14,55	2,23	6,08	2,91	7	8,62	2,63	5,90	7,09	31,01	3,38	3,63
30.9.2011	7,23	6,92	7,34	1,94	50,58	0,64	2,35	3,96	2,15	7,06	13,12	0,88	2,24	4,68	6,47	2,49	21,79	0,79	15,10	2,33	6,50	3,16	7,36	8,56	2,72	6,22	7,37	35,11	3,71	4,13
7.10.2011	6,64	6,47	7,32	1,89	51,81	0,62	2,37	3,74	2,09	6,36	11,85	0,89	2,19	4,50	6,28	2,55	21,79	0,77	14,70	2,25	6,38	3,23	7,38	8,24	2,61	6,27	7,56	34,56	3,48	3,75
14.10.2011	7,05	6,58	7,68	1,94	53,28	0,64	2,51	3,96	2,22	6,77	12,34	0,89	2,21	4,63	6,52	2,59	23,12	0,78	15,10	2,30	6,48	3,15	7,82	8,86	2,63	6,50	7,08	34,93	3,54	4,02
21.10.2011	6,52	6,20	7,62	1,88	52,55	0,62	2,39	3,80	2,36	6,28	11,50	0,87	2,12	4,26	6,21	2,49	21,60	0,75	14,65	2,25	6,14	2,88	8	8,76	2,53	6,55	6,65	34,93	3,28	3,65
28.10.2011	6,52	6,52	7,66	1,91	54,51	0,60	2,42	4,00	2,56	6,20	11,06	0,86	2,18	4,12	6,11	2,60	23,17	0,77	16,20	2,29	6,20	2,99	8,4	8,92	2,63	6,67	6,89	37,48	3,10	3,41
4.11.2011	6,46	6,45	7,62	1,96	53,28	0,62	2,30	3,87	2,28	6,28	11,01	0,84	2,12	4,03	5,83	2,49	23,81	0,78	15,70	2,09	5,96	2,87	7,76	9,26	2,49	6,27	6,78	37,85	3,04	3,28
11.11.2011	6,50	6,26	7,58	1,94	52,06	0,61	2,27	3,84	2,24	6,38	11,06	0,83	2,23	3,95	5,83	2,49	23,85	0,79	15,65	2,05	5,92	2,84	7,72	9,54	2,44	6,14	6,67	37,85	3,06	3,35
18.11.2011	6,31	6,11	7,48	1,83	49,85	0,59	2,29	3,96	2,23	6,07	10,18	0,82	2,23	3,79	5,72	2,47	25,13	0,77	14,85	2,15	5,74	2,76	7,84	9,30	2,54	5,69	6,48	34,84	2,88	3,09
25.11.2011	5,87	5,46	7,44	1,63	48,13	0,51	2,02	3,64	2,06	5,77	9,63	0,80	2,04	3,47	5,14	2,28	24,20	0,71	13,90	2,02	5,36	2,50	7,76	9,00	2,30	5,43	6,37	33,74	2,54	2,80
2.12.2011	6,48	5,82	8,18	1,81	50,83	0,53	2,12	3,94	2,22	6,11	10,87	0,78	1,58	3,70	6,00	1,94	27,39	0,74	14,35	2,07	5,96	2,76	7,94	8,94	2,38	6,09	6,80	38,30	2,78	3,07
9.12.2011	6,29	5,62	7,88	1,76	51,57	0,56	2,04	3,79	2,30	6,09	10,38	0,79	1,58	3,45	5,78	1,99	28,47	0,75	13,85	2,01	5,96	2,67	7,68	8,82	2,30	5,90	6,80	37,39	2,67	2,91
16.12.2011	6,15	5,50	8	1,64	48,52	0,52	1,91	3,70	2,07	5,83	9,79	0,78	1,50	3,32	5,38	1,91	25,43	0,73	13,00	1,90	5,70	2,60	8,02	8,84	2,23	5,41	6,69	34,75	2,51	2,74
23.12.2011	5,99	5,63	8,26	1,63	51,07	0,53	1,96	3,94	2,23	5,83	9,79	0,78	1,42	3,27	5,46	1,83	26,21	0,72	12,80	1,88	5,60	2,57	7,98	9,06	2,21	5,51	6,50	35,75	2,46	2,71
30.12.2011	5,93	5,79	8,2	1,59	51,57	0,53	1,91	3,60	2,17	5,76	9,69	0,77	1,40	3,22	5,31	1,65	24,64	0,71	12,65	1,91	5,40	2,53	8,06	8,88	2,12	5,54	6,48	36,48	2,44	2,69
6.1.2012	5,67	6,28	8,18	1,56	51,81	0,52	1,84	3,74	2,21	5,48	9,28	0,76	1,25	3,21	5,33	1,52	23,27	0,68	11,80	1,80	5,36	2,47	7,96	8,84	2,06	5,67	6,45	34,84	2,30	2,61
13.1.2012	5,87	6,69	8,2	1,54	53,28	0,52	1,93	3,79	2,32	5,52	9,50	0,76	1,88	3,26	5,66	1,87	25,13	0,70	12,05	1,80	5,44	2,67	7,64	8,98	2,05	6,29	6,84	36,21	2,31	2,71
20.1.2012	6,64	6,73	8,86	1,64	54,76	0,63	1,99	4,27	2,31	6,15	10,52	0,78	2,13	3,48	5,87	2,00	28,47	0,73	13,00	1,95	6,04	2,85	7,8	9,00	2,19	6,69	6,78	36,12	2,59	3,05
27.1.2012	6,80	7,13	8,96	1,68	56,23	0,61	2,10	4,25	2,50	6,42	11,60	0,77	2,34	3,71	6,34	2,32	30,53	0,76	13,60	2,02	6,52	3,23	8	9,24	2,33	6,93	7,32	35,39	2,80	3,45
3.2.2012	7,13	7,43	9	1,77	57,70	0,66	2,21	4,50	2,69	6,71	12,19	0,79	2,37	3,83	6,77	2,52	30,24	0,81	14,30	2,22	7,08	3,08	8,8	9,46	2,70	7,30	7,24	39,03	3,08	3,46
10.2.2012	6,72	7,37	9,4	1,82	58,44	0,66	2,15	4,31	2,56	6,42	11,65	0,90	2,54	3,83	6,67	2,54	31,90	0,80	15,15	2,10	7,08	3,07	8,38	9,34	2,64	7,15	7,08	42,23	3,02	3,40
17.2.2012	6,99	7,28	9,66	1,90	59,91	0,78	2,23	4,43	2,64	6,61	12,14	0,97	2,44	4,17	6,80	2,56	33,08	0,81	15,65	2,17	7,40	3,07	8,66	9,76	2,76	7,30	6,87	41,13	3,11	3,53
24.2.2012	6,68	7,41	9,46	1,81	60,90	0,76	2,30	4,29	2,62	6,36	11,70	1,02	2,42	3,90	6,60	2,64	31,90	0,81	15,90	2,16	7,24	3,09	8,76	9,58	2,73	6,80	6,76	40,04	2,95	3,22
2.3.2012	6,96	7,32	9,38	1,83	60,90	0,73	2,30	4,50	2,70	6,59	11,90	1,07	2,55	4,05	6,82	2,96	33,77	0,83	16,55	2,17	7,46	3,12	8,94	9,52	2,62	7,15	6,89	39,12	3,11	3,38
9.3.2012	6,76	7,18	9,14	1,80	61,39	0,71	2,19	4,34	2,60	6,34	11,75	1,07	2,54	3,91	6,54	2,91	32,98	0,82	16,10	2,06	7,30	2,88	8,54	9,44	2,52	6,93	6,85	38,67	3,02	3,19
16.3.2012	7,27	7,56	9,86	1,96	64,33	0,76	2,27	4,45	2,63	6,73	12,58	1,16	2,50	4,20	6,73	3,02	31,41	0,84	16,30	2,08	8,00	3,06	8,44	9,70	2,65	6,98	7,13	40,31	3,43	3,62
23.3.2012	7,15	7,52	10,65	1,91	63,84	0,77	2,25	4,69	2,58	6,61	12,04	1,13	2,78	4,16	6,65	3,24	31,61	0,89	15,90	2,08	7,58	2,92	8,64	9,26	2,58	6,65	6,95	40,04	3,27	3,49
30.3.2012	6,90	7,52	10,6	1,96	66,30	0,84	2,27	4,99	2,32	6,89	12,48	1,08	2,75	4,27	6,77	3,16	33,18	0,88	16,25	2,15	7,66	3,01	8,88	9,12	2,60	7,13	7,15	41,50	3,36	3,60
6.4.2012	6,90	7,80	10,4	1,86	65,56	0,91	2,30	4,75	2,32	6,71	12,53	1,08	2,85	4,11	6,71	3,20	32,69	0,86	17,60	2,10	7,42	2,88	8,72	8,94	2,60	7,36	7,02	39,20	3,28	3,56
13.4.2012	6,82	7,80	10,4	1,82	67,03	0,72	2,30	5,01	2,29	6,77	12,34	1,07	2,92	4,13	6,51	3,17	32,69	0,84	16,95	2,05	7,42	2,77	9	8,92	2,85	7,22	6,84	38,50	3,25	3,46
20.4.2012	6,66	7,34	10,25	1,82	67,03	0,75	2,31	5,06	2,22	6,69	12,14	1,11	3,24	4,10	6,32	3,21	34,75	0,89	17,35	2,04	7,34	2,67	9,1	8,66	2,70	7,32	6,96	36,40	3,19	3,39
27.4.2012	6,70	7,62	10,5	1,83	70,72	0,87	2,30	4,82	2,29	6,48	12,14	1,13	3,45	4,08	6,30	3,39	36,71	0,88	17,30	2,04	7,42	2,67	9,12	8,82	2,69	7,76	7,26	37,00	3,17	3,37
4.5.2012	6,54	8,24	10,85	1,78	68,75	0,82	2,30	5,04	2,22	6,18	11,99	1,12	3,25	3,94	6,11	3,29	32,79	0,87	17,15	2,00	6,90	2,69	9,54	8,98	2,69	7,54	6,87	36,40	3,04	3,13
11.5.2012	6,38	8,28	10,9	1,78	69,74	0,79	2,34	4,78	2,10	6,13	12,09	1,09	3,46	4,05	6,06	3,29	32,69	0,89	17,50	1,96	6,88	2,68	9,44	8,52	2,78	7,36	6,74	36,60	3,18	3,23
18.5.2012	6,22	7,86	10,15	1,62	71,21	0,74	2,25	4,29	2,01	5,93	11,41	1,06	3,37	3,81	5,75	3,21	36,00	0,87	17,05	1,84	7,02	2,45	9,64	8,30	2,68	6,98	6,54	35,10	3,05	3,02
25.5.2012	5,44	7,62	9,62	1,60	74,75	0,68	2,08	4,02	1,98	5,72	11,00	1,04	3,27	3,75	5,56	2,95	32,90	0,82	15,50	1,78	6,80	2,32	8,92	8,40	2,53	6,24	6,47	34,10	2,98	3,00
1.6.2012	5,80	8,00	9,9	1,62	72,00	0,71	2,07	4,10	2,12	6,00	12,00	1,03	3,25	3,80	5,81	3,22	33,70	0,82	15,70	1,88	7,24	2,41	8,88	8,32	2,59	6,24	6,34	35,60	3,01	3,03
8.6.2012	5,98	7,96	10,45	1,68	67,25	0,76	2,22	4,32	2,15	6,46	12,85	1,03	3,41	4,00	6,04	3,34	34,00	0,88	16,05	1,95	7,08	2,46	9	8,20	2,73	6,56	6,34	35,90	3,20	3,22
15.6.2012	6,12	8,56	10,7	1,71	67,75	0,77	2,13	4,59	2,09	6,44	13,00	1,06	3,52	4,23	6,20	3,37	34,00	0,93	16,95	1,98	7,34	2,69	9,04	8,44	2,77	6,76	6,46	38,50	3,34	3,34
22.6.2012	6,34	8,78	10,9	1,76	69,00	0,79	2,19	4,50	2,14	6,76	14,15	1,12	3,69	4,50	6,70	3,54	34,80	0,97	17,35	2,01	7,64	2,65	9,58	8,68	3,03	7,48	6,86	37,00	3,49	3,52
29.6.2012	6,62	9,14	11,2	1,81	74,50	0,80	2,22	4,85	2,01	7,10	14,15	1,16	3,87	4,80	6,90	3,87	35,00	1,00	17,90	2,02	7,60	2,67	9,72	9,16	3,18	7,74	7,38	38,70	3,76	3,71
6.7.2012	6,72	8,60	12,2	1,82	74,25	0,87	2,20	4,70	2,05	6,98	14,40	1,11	3,97	4,89	6,86	3,67	37,10	0,98	18,85	2,01	7,78	2,70	9,5	9,34	3,19	7,96	7,04	38,80	3,72	3,75
13.7.2012	6,66	8,66	12,55	1,92	74,50	0,85	2,26	4,67	2,08	7,00	14,45	1,11	4,08	4,77	6,78	3,83	35,10	0,97	18,75	2,02	7,76	2,73	9,64	9,30	3,30	7,94	7,00	38,80	3,62	3,75
20.7.2012	6,52	8,24	12,35	1,85	78,75	0,79	2,25	4,77	1,93	6,88	13,60	1,09	4,07	4,73	6,52	3,87	34,90	0,97	18,35	2,00	7,44	2,51	9,66	9,34	3,18	7,40	7,08	39,30	3,56	3,54
27.7.2012	6,78	8,94	11,9	1,95	75,75	0,82	2,31	5,18	1,98	7,16	14,85	1,08	4,10	5,02	6,98	4,03	36,30	1,08	18,65	1,99	8,00	2,60	9,84	10,30	3,34	8,16	7,28	40,50	3,74	3,73
3.8.2012	6,82	8,78	12,25	1,95	79,75	0,82	2,39	5,10	1,97	7,24	16,25	1,11	4,21	5,32	7,06	4,21	35,70	1,10	19,90	2,00	7,94	2,59	9,72	10,25	3,43	8,30	7,12	40,00	3,90	3,86
10.8.2012	6,78	9,46	5,9	1,95	76,75	0,82	2,49	5,22	2,00	7,26	14,90	1,08	4,15	5,28	7,14	4,17	35,00	1,14	19,35	1,99	8,06	2,63	9,62	9,88	3,31	8,80	7,20	40,00	3,87	3,99
17.8.2012	7,10	9,84	5,88	1,93	73,75	0,81	2,58	4,90	2,02	7,56	15,10	1,09	4,12	5,38	7,10	4,10	35,00	1,18	18,75	2,02	7,92	2,61	9,58	10,12	3,52	8,42	7,04	40,20	4,01	4,05
24.8.2012	7,08	9,96	6,08	1,90	71,75	0,83	2,55	4,70	2,03	7,58	15,65	1,09	4,11	5,40	7,06	4,13	36,20	1,14	18,30	1,94	7,76	2,59	9,26	10,04	3,51	8,02	7,08	40,80	4,02	4,04
31.8.2012	7,18	10,06	6,6	1,92	74,75	0,84	2,66	5,04	2,03	7,80	16,40	1,15	4,11	5,54	7,40	4,12	35,80	1,16	19,05	2,00	7,82	2,67	9,46	10,80	3,60	8,80	7,20	41,60	4,12	4,24
7.9.2012	7,22	10,50	6,56	1,96	73,75	0,77	2,67	5,00	2,18	7,86	15,95	1,21	4,10	5,66	7,36	4,12	37,70	1,16	19,40	2,06	7,90	2,72	9,42	10,55	3,90	8,54	7,38	42,20	4,19	4,35
14.9.2012	7,22	10,80	6,34	2,05	75,00	0,77	2,60	4,69	2,26	7,76	15,05	1,30	4,14	5,66	7,30	4,30	40,10	1,16	19,55	2,08	7,96	2,74	9,1	11,05	3,83	9,28	7,24	43,80	4,20	4,42
21.9.2012	7,16	10,15	6,4	2,06	77,50	0,74	2,67	4,68	2,30	7,58	14,70	1,32	4,24	5,62	7,20	4,53	40,30	1,16	19,40	2,10	8,18	2,71	8,8	11,25	3,95	9,38	7,12	42,40	4,08	4,33
28.9.2012	7,10	9,56	6,5	1,98	75,00	0,72	2,51	4,50	2,21	7,50	14,00	1,35	4,10	5,64	7,18	4,35	38,60	1,17	18,70	2,01	7,88	2,49	8,9	10,95	3,76	9,08	7,16	41,00	3,88	4,28
5.10.2012	7,72	9,44	6,94	1,99	73,50	0,72	2,56	4,61	2,19	7,86	14,60	1,32	4,11	5,70	7,16	4,35	38,00	1,16	18,95	2,01	7,78	2,52	8,34	11,20	3,97	8,60	7,12	40,70	3,77	4,31
12.10.2012	8,16	10,04	6,86	1,99	75,25	0,75	2,59	4,72	2,22	8,30	15,25	1,31	4,17	5,88	7,50	4,37	38,40	1,23	19,35	2,06	8,28	2,55	8,64	10,95	4,04	9,00	7,06	41,70	4,16	4,53
19.10.2012	8,34	10,90	6,6	2,00	81,00	0,76	2,59	4,71	2,13	8,36	15,40	1,32	4,10	5,98	7,76	4,40	38,30	1,32	18,80	2,04	8,60	2,56	8,64	11,00	4,03	9,48	7,04	42,80	4,12	4,57
24.10.2012	8,32	11,10	6,66	1,99	81,50	0,77	2,59	4,72	2,11	8,22	15,15	1,30	4,16	6,00	7,88	4,53	37,50	1,31	18,85	2,04	8,92	2,54	8,58	10,90	4,06	9,36	6,82	43,00	4,14	4,52
2.11.2012	8,54	10,55	6,68	2,00	81,25	0,76	2,67	4,78	2,18	8,40	15,65	1,31	5,46	5,94	8,10	4,73	39,50	1,29	19,15	2,08	9,14	2,51	9,16	10,90	4,18	9,70	6,88	43,70	4,17	4,49
9.11.2012	8,56	11,25	6,8	1,96	80,00	0,80	2,78	4,67	2,18	8,46	15,90	1,31	4,80	5,86	8,30	4,74	42,80	1,26	19,20	2,08	9,44	2,67	8,94	10,80	4,20	10,14	6,74	43,00	4,22	4,63
16.11.2012	8,28	10,40	6,48	1,91	80,00	0,86	2,76	4,66	2,18	8,22	15,90	1,30	4,68	5,64	8,16	5,19	42,80	1,22	18,85	2,08	8,96	2,57	8,84	10,85	4,41	10,15	6,54	43,90	4,36	4,57
23.11.2012	8,04	10,80	6,72	1,91	79,00	0,87	2,81	4,80	2,15	8,28	16,00	1,30	5,62	5,68	8,40	5,30	42,40	1,23	18,90	2,13	9,18	2,59	8,62	10,60	4,81	10,70	6,36	44,10	4,35	4,59
30.11.2012	8,38	10,85	6,78	1,96	81,25	0,86	2,80	4,76	2,25	8,48	17,30	1,29	5,94	5,76	8,18	5,50	44,60	1,19	19,00	2,21	9,22	2,63	8,88	10,75	5,26	10,90	6,68	47,70	4,33	4,71
7.12.2012	8,72	10,90	7,06	2,03	81,75	0,84	2,97	5,10	2,38	8,90	17,50	1,35	6,00	6,08	8,74	5,80	44,50	1,25	21,75	2,40	9,88	2,74	8,76	11,60	5,84	11,00	6,90	49,50	4,73	5,13
14.12.2012	8,70	10,70	7,84	2,13	80,75	0,83	2,90	5,18	2,44	9,08	17,60	1,33	5,78	6,00	8,74	5,62	43,90	1,24	21,15	2,43	9,66	2,83	9,1	11,15	5,86	10,60	6,82	50,00	4,75	5,14
21.12.2012	8,68	11,05	7,9	2,17	84,50	0,89	3,04	5,22	2,43	9,06	17,20	1,29	5,60	6,08	9,04	5,38	40,00	1,27	21,25	2,69	9,80	2,95	9,24	11,25	6,30	10,05	6,78	49,50	4,46	5,14
28.12.2012	8,88	11,70	8,18	2,23	86,25	0,92	3,14	5,20	2,47	9,40	17,65	1,25	6,00	6,22	9,50	5,58	43,20	1,28	21,75	2,84	9,90	2,97	9,06	11,55	6,20	10,55	6,92	50,75	4,64	5,24
4.1.2013	8,98	12,10	8,42	2,25	87,50	0,91	2,95	5,46	2,54	9,46	17,95	1,24	5,98	6,24	9,54	5,64	43,50	1,28	22,05	2,81	9,86	3,15	9,22	11,75	6,32	10,35	7,04	52,25	4,88	5,36
11.1.2013	9,00	11,90	8,36	2,32	90,75	1,07	3,00	6,00	2,55	9,30	18,05	1,24	5,96	6,44	9,86	5,62	44,40	1,36	22,50	3,21	10,25	3,30	9,44	12,30	6,74	10,75	6,98	50,50	5,16	5,50
18.1.2013	9,86	12,60	8,28	2,51	92,50	1,02	3,10	5,88	2,64	9,86	19,80	1,21	5,76	6,96	10,45	5,58	44,60	1,46	22,65	3,16	10,80	3,24	10	12,25	6,78	10,95	7,46	52,25	5,72	5,84
25.1.2013	9,86	12,30	8,38	2,49	89,50	1,14	3,37	5,66	2,61	9,64	19,15	1,20	5,68	7,16	10,00	5,68	45,70	1,61	22,20	3,29	10,50	3,49	10,3	12,00	7,38	11,40	7,84	52,00	5,52	5,62
1.2.2013	9,10	12,20	8,36	2,25	87,25	1,09	3,20	5,36	2,49	9,08	18,15	1,18	5,10	6,66	9,46	5,14	44,30	1,59	20,80	3,11	10,10	3,22	10,9	10,80	6,54	10,60	7,44	49,50	5,38	5,28
8.2.2013	8,84	11,25	8,54	2,27	86,75	1,11	2,78	5,22	2,43	8,70	17,45	1,19	5,12	6,40	9,10	4,90	43,00	1,58	21,05	3,31	10,25	3,18	10,4	11,35	6,60	10,35	7,28	49,00	5,28	5,12
15.2.2013	8,60	10,65	9,08	2,23	84,00	1,10	2,77	5,36	2,35	8,64	16,80	1,15	5,24	6,54	9,04	5,06	42,20	1,56	20,95	3,19	10,25	2,94	10,6	11,20	7,08	10,60	7,14	48,80	5,42	5,14
22.2.2013	8,38	10,60	9,88	2,04	83,50	1,03	2,81	5,26	2,17	8,20	16,10	0,88	4,94	6,04	9,22	4,56	40,10	1,46	20,40	3,11	9,84	2,75	10,4	11,60	6,80	11,00	7,10	49,30	5,14	4,80
1.3.2013	8,78	11,15	10,55	2,22	85,50	1,08	2,91	5,48	2,31	8,86	17,70	0,90	5,24	6,44	9,66	4,80	42,10	1,55	21,60	3,11	9,88	2,94	11	11,90	7,34	11,20	7,40	50,50	5,62	5,14
8.3.2013	9,42	11,85	9,74	2,22	87,25	1,14	2,94	5,62	2,30	9,40	18,75	0,86	5,24	6,78	10,40	5,04	42,90	1,63	22,25	3,16	11,25	2,98	11,2	11,55	7,58	11,95	7,50	54,75	5,82	5,44
15.3.2013	9,30	12,00	10,3	2,24	88,75	1,08	2,93	5,36	2,22	9,48	19,25	0,79	5,18	6,74	9,66	5,01	41,30	1,62	22,05	3,19	10,50	3,00	11	12,15	7,30	12,10	7,74	54,00	5,72	5,40
22.3.2013	9,06	11,80	10,25	2,21	85,50	1,07	2,81	5,44	2,25	9,18	19,00	0,80	5,14	6,64	9,82	5,08	41,50	1,62	21,45	3,12	10,55	2,99	11,4	12,05	7,32	11,90	8,00	53,50	5,66	5,42
29.3.2013	9,46	13,00	10,6	2,38	88,50	1,08	2,92	5,66	2,34	9,66	19,55	0,82	5,32	6,96	10,50	5,18	42,00	1,67	22,65	3,13	10,85	3,13	12,5	12,15	7,48	13,05	8,06	55,00	5,84	5,68
5.4.2013	8,98	12,20	10,55	2,30	90,25	1,08	2,85	5,52	2,27	9,04	18,30	0,82	5,28	6,46	10,00	5,06	40,90	1,81	22,10	3,12	10,30	2,96	11,5	12,00	7,14	11,40	7,78	48,30	5,62	5,26
12.4.2013	9,48	12,65	10,6	2,35	92,25	1,13	2,87	5,68	2,06	9,56	18,65	0,81	5,60	6,72	10,70	5,10	38,80	1,84	22,80	3,11	10,90	3,07	12	11,95	6,90	11,60	7,92	49,80	5,92	5,54
19.4.2013	9,12	12,95	10,3	2,13	92,00	1,16	2,83	5,54	2,00	9,56	18,45	0,80	4,87	6,50	10,40	4,37	34,80	1,90	22,50	3,04	10,75	2,92	12,2	10,85	7,08	11,75	7,90	48,90	5,92	5,30
26.4.2013	9,36	13,10	10,6	2,13	92,25	1,21	3,01	5,46	2,05	9,70	19,35	0,81	4,94	6,80	10,40	4,45	35,20	2,05	22,80	3,07	11,00	2,95	12,55	10,80	7,62	12,30	8,22	49,30	6,20	5,46
3.5.2013	9,92	13,80	10,8	2,24	91,75	1,22	3,13	5,54	2,06	10,45	21,05	0,83	5,06	7,26	11,40	4,57	36,80	1,91	23,45	3,09	11,75	3,11	11,8	11,30	7,88	12,45	8,84	51,00	6,74	5,78
10.5.2013	9,98	13,50	11,1	2,24	93,25	1,22	3,06	5,52	2,07	10,45	20,85	0,86	4,95	7,42	11,30	4,21	32,00	1,97	24,05	3,13	12,20	3,20	10,9	11,05	8,04	12,90	8,66	52,25	6,90	6,00
17.5.2013	10,25	13,60	11,8	2,14	93,50	1,15	3,40	5,76	2,17	11,05	21,45	0,94	5,03	7,72	11,25	4,18	30,90	1,76	25,45	3,19	12,85	3,25	11,5	11,95	8,36	13,95	8,24	51,75	7,00	6,12
24.5.2013	9,95	14,15	11,3	2,12	90,75	1,12	3,25	5,90	2,16	10,95	21,05	1,12	4,77	7,54	11,00	4,16	31,70	1,75	25,70	3,14	12,40	3,26	12,2	11,80	8,84	14,45	8,24	50,25	6,84	6,08
31.5.2013	8,94	14,20	9,85	1,94	88,50	1,06	3,09	6,00	2,24	9,70	20,10	1,02	4,56	6,96	10,65	4,12	33,00	1,62	23,40	3,05	12,00	3,18	11,85	11,55	8,70	14,05	7,26	50,10	5,96	5,60
7.6.2013	8,28	12,70	9,54	1,85	82,00	0,93	3,09	5,34	2,14	8,80	17,90	0,93	4,07	6,12	9,90	3,58	28,60	1,54	22,00	2,89	10,44	2,93	11,25	10,60	8,08	12,85	6,64	44,60	5,40	5,04
14.6.2013	8,72	12,45	9,54	1,92	86,00	0,98	2,80	4,85	2,17	8,86	18,40	0,85	4,19	6,10	9,76	3,77	29,10	1,60	22,40	3,18	10,75	3,01	11,15	11,20	8,02	12,95	7,44	45,60	5,58	4,99
21.6.2013	7,62	11,45	8,96	1,76	38,60	0,95	2,68	4,94	2,02	8,00	16,20	0,83	3,41	5,46	8,68	3,26	25,30	1,39	19,10	2,98	9,88	2,73	11,05	10,90	7,64	12,55	7,00	43,50	4,79	4,28
28.6.2013	7,86	12,75	8,48	1,79	41,80	0,97	2,72	4,85	1,99	8,42	16,35	0,82	3,21	5,70	9,26	2,86	23,40	1,44	19,75	2,83	10,15	2,70	11,3	11,20	7,50	12,05	7,50	47,10	4,81	4,39
5.7.2013	7,34	13,00	8,94	1,72	40,50	0,96	2,69	4,85	1,92	7,74	15,15	0,82	3,65	5,28	9,24	3,11	22,90	1,20	19,70	2,73	9,90	2,57	11,35	11,10	7,48	12,60	7,20	43,50	4,47	4,19

Table A.3 : Variance-Covariance Matrix of Model 1

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0,0022	0,0011	0,0008	0,0015	0,0008	0,0013	0,0010	0,0010	0,0008	0,0018	0,0019	0,0007	0,0010	0,0016	0,0015
2	0,0011	0,0024	0,0001	0,0011	0,0007	0,0007	0,0010	0,0010	0,0009	0,0012	0,0013	0,0003	0,0007	0,0011	0,0014
3	0,0008	0,0001	0,0036	0,0008	0,0005	0,0007	0,0005	0,0006	0,0004	0,0008	0,0011	0,0004	0,0007	0,0007	0,0006
4	0,0015	0,0011	0,0008	0,0022	0,0009	0,0011	0,0010	0,0010	0,0009	0,0015	0,0016	0,0010	0,0009	0,0015	0,0014
5	0,0008	0,0007	0,0005	0,0009	0,0035	0,0006	0,0005	0,0002	0,0005	0,0007	0,0009	0,0004	0,0010	0,0008	0,0009
6	0,0013	0,0007	0,0007	0,0011	0,0006	0,0040	0,0011	0,0007	0,0007	0,0011	0,0014	0,0005	0,0009	0,0012	0,0011
7	0,0010	0,0010	0,0005	0,0010	0,0005	0,0011	0,0021	0,0010	0,0009	0,0011	0,0012	0,0003	0,0009	0,0011	0,0010
8	0,0010	0,0010	0,0006	0,0010	0,0002	0,0007	0,0010	0,0021	0,0007	0,0011	0,0012	0,0005	0,0007	0,0011	0,0011
9	0,0008	0,0009	0,0004	0,0009	0,0005	0,0007	0,0009	0,0007	0,0017	0,0008	0,0010	0,0005	0,0008	0,0007	0,0009
10	0,0018	0,0012	0,0008	0,0015	0,0007	0,0011	0,0011	0,0011	0,0008	0,0021	0,0020	0,0008	0,0011	0,0017	0,0015
11	0,0019	0,0013	0,0011	0,0016	0,0009	0,0014	0,0012	0,0012	0,0010	0,0027	0,0027	0,0006	0,0011	0,0019	0,0017
12	0,0007	0,0003	0,0004	0,0010	0,0004	0,0005	0,0003	0,0005	0,0005	0,0006	0,0031	0,0031	0,0007	0,0007	0,0006
13	0,0010	0,0007	0,0007	0,0009	0,0010	0,0009	0,0009	0,0007	0,0008	0,0011	0,0007	0,0060	0,0060	0,0010	0,0011
14	0,0016	0,0011	0,0007	0,0015	0,0008	0,0012	0,0011	0,0011	0,0007	0,0019	0,0007	0,0010	0,0019	0,0019	0,0014
15	0,0015	0,0014	0,0006	0,0014	0,0009	0,0011	0,0010	0,0011	0,0009	0,0017	0,0006	0,0011	0,0014	0,0020	0,0020
16	0,0010	0,0007	0,0004	0,0009	0,0009	0,0009	0,0008	0,0007	0,0009	0,0010	0,0007	0,0031	0,0010	0,0009	0,0031
17	0,0010	0,0006	0,0004	0,0010	0,0008	0,0010	0,0006	0,0005	0,0010	0,0010	0,0006	0,0016	0,0009	0,0011	0,0016
18	0,0011	0,0008	0,0002	0,0011	0,0009	0,0011	0,0008	0,0007	0,0005	0,0010	0,0006	0,0008	0,0011	0,0010	0,0007
19	0,0012	0,0010	0,0009	0,0013	0,0011	0,0013	0,0012	0,0009	0,0008	0,0014	0,0009	0,0015	0,0012	0,0011	0,0011
20	0,0009	0,0006	0,0005	0,0008	0,0005	0,0013	0,0008	0,0007	0,0008	0,0009	0,0002	0,0008	0,0009	0,0009	0,0008
21	0,0015	0,0011	0,0004	0,0014	0,0008	0,0012	0,0010	0,0010	0,0008	0,0016	0,0009	0,0009	0,0014	0,0015	0,0009
22	0,0011	0,0012	0,0006	0,0012	0,0008	0,0010	0,0010	0,0010	0,0008	0,0012	0,0003	0,0008	0,0011	0,0011	0,0010
23	0,0007	0,0007	0,0005	0,0007	0,0002	0,0003	0,0007	0,0008	0,0005	0,0008	0,0006	0,0004	0,0006	0,0007	0,0004
24	0,0007	0,0005	0,0006	0,0008	0,0006	0,0005	0,0005	0,0006	0,0004	0,0008	0,0004	0,0005	0,0006	0,0006	0,0006
25	0,0012	0,0009	0,0008	0,0011	0,0003	0,0011	0,0011	0,0011	0,0010	0,0014	0,0007	0,0009	0,0012	0,0011	0,0009
26	0,0010	0,0013	0,0005	0,0011	0,0005	0,0015	0,0013	0,0011	0,0009	0,0014	0,0004	0,0012	0,0012	0,0013	0,0010
27	0,0008	0,0006	0,0002	0,0007	0,0006	0,0005	0,0004	0,0002	0,0004	0,0008	0,0002	0,0006	0,0008	0,0007	0,0006
28	0,0012	0,0013	0,0006	0,0012	0,0005	0,0009	0,0008	0,0009	0,0008	0,0013	0,0004	0,0006	0,0012	0,0012	0,0006
29	0,0018	0,0013	0,0008	0,0017	0,0010	0,0012	0,0012	0,0012	0,0009	0,0021	0,0007	0,0011	0,0019	0,0017	0,0012
30	0,0020	0,0015	0,0008	0,0019	0,0009	0,0013	0,0013	0,0013	0,0011	0,0023	0,0009	0,0013	0,0021	0,0019	0,0012

	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	0,0010	0,0010	0,0011	0,0012	0,0009	0,0015	0,0011	0,0007	0,0007	0,0012	0,0010	0,0008	0,0012	0,0018	0,0020
2	0,0007	0,0006	0,0008	0,0010	0,0006	0,0011	0,0012	0,0007	0,0005	0,0009	0,0013	0,0006	0,0013	0,0013	0,0015
3	0,0004	0,0004	0,0002	0,0009	0,0005	0,0004	0,0006	0,0005	0,0006	0,0008	0,0005	0,0002	0,0006	0,0008	0,0008
4	0,0009	0,0010	0,0011	0,0013	0,0008	0,0014	0,0012	0,0007	0,0008	0,0011	0,0011	0,0007	0,0012	0,0017	0,0019
5	0,0009	0,0008	0,0009	0,0011	0,0005	0,0008	0,0008	0,0002	0,0006	0,0003	0,0005	0,0006	0,0005	0,0010	0,0009
6	0,0009	0,0010	0,0011	0,0013	0,0013	0,0012	0,0010	0,0003	0,0005	0,0011	0,0015	0,0005	0,0009	0,0012	0,0013
7	0,0008	0,0006	0,0008	0,0012	0,0008	0,0010	0,0010	0,0007	0,0005	0,0011	0,0013	0,0004	0,0008	0,0012	0,0013
8	0,0007	0,0005	0,0007	0,0009	0,0007	0,0010	0,0010	0,0008	0,0006	0,0011	0,0011	0,0002	0,0009	0,0012	0,0013
9	0,0009	0,0010	0,0005	0,0008	0,0008	0,0008	0,0008	0,0005	0,0004	0,0010	0,0009	0,0004	0,0008	0,0009	0,0011
10	0,0009	0,0010	0,0011	0,0013	0,0008	0,0014	0,0009	0,0008	0,0007	0,0012	0,0011	0,0007	0,0012	0,0020	0,0021
11	0,0010	0,0010	0,0010	0,0014	0,0009	0,0016	0,0012	0,0008	0,0008	0,0014	0,0014	0,0008	0,0013	0,0021	0,0023
12	0,0007	0,0006	0,0006	0,0009	0,0002	0,0009	0,0003	0,0006	0,0004	0,0007	0,0004	0,0002	0,0004	0,0007	0,0009
13	0,0031	0,0016	0,0008	0,0015	0,0008	0,0009	0,0008	0,0004	0,0005	0,0009	0,0012	0,0006	0,0006	0,0011	0,0013
14	0,0010	0,0009	0,0011	0,0012	0,0009	0,0014	0,0011	0,0006	0,0006	0,0012	0,0012	0,0008	0,0012	0,0019	0,0021
15	0,0009	0,0011	0,0010	0,0011	0,0009	0,0015	0,0011	0,0007	0,0006	0,0011	0,0013	0,0007	0,0012	0,0017	0,0019
16	0,0031	0,0016	0,0007	0,0011	0,0008	0,0009	0,0010	0,0004	0,0006	0,0009	0,0010	0,0006	0,0006	0,0012	0,0012
17	0,0029	0,0029	0,0007	0,0009	0,0006	0,0010	0,0009	0,0005	0,0003	0,0005	0,0008	0,0005	0,0009	0,0010	0,0012
18	0,0007	0,0020	0,0020	0,0010	0,0008	0,0009	0,0009	0,0007	0,0003	0,0008	0,0006	0,0006	0,0008	0,0012	0,0012
19	0,0009	0,0010	0,0026	0,0026	0,0009	0,0011	0,0009	0,0006	0,0008	0,0012	0,0012	0,0005	0,0010	0,0015	0,0016
20	0,0006	0,0008	0,0009	0,0015	0,0015	0,0008	0,0008	0,0004	0,0005	0,0011	0,0008	0,0004	0,0008	0,0010	0,0011
21	0,0010	0,0009	0,0011	0,0008	0,0021	0,0021	0,0011	0,0007	0,0006	0,0011	0,0013	0,0006	0,0011	0,0016	0,0018
22	0,0009	0,0009	0,0009	0,0008	0,0011	0,0024	0,0024	0,0004	0,0005	0,0008	0,0011	0,0007	0,0010	0,0012	0,0014
23	0,0005	0,0007	0,0006	0,0004	0,0007	0,0004	0,0015	0,0015	0,0003	0,0008	0,0007	0,0003	0,0005	0,0007	0,0008
24	0,0003	0,0003	0,0008	0,0005	0,0006	0,0005	0,0003	0,0013	0,0013	0,0006	0,0007	0,0003	0,0004	0,0007	0,0008
25	0,0005	0,0008	0,0012	0,0011	0,0011	0,0008	0,0008	0,0006	0,0024	0,0024	0,0011	0,0004	0,0009	0,0014	0,0015
26	0,0008	0,0006	0,0012	0,0008	0,0013	0,0011	0,0007	0,0007	0,0011	0,0029	0,0029	0,0005	0,0013	0,0013	0,0016
27	0,0005	0,0006	0,0005	0,0004	0,0006	0,0007	0,0003	0,0003	0,0004	0,0005	0,0015	0,0015	0,0007	0,0009	0,0009
28	0,0009	0,0008	0,0010	0,0008	0,0011	0,0010	0,0005	0,0004	0,0009	0,0013	0,0007	0,0021	0,0021	0,0013	0,0014
29	0,0010	0,0012	0,0015	0,0010	0,0016	0,0012	0,0007	0,0007	0,0014	0,0013	0,0009	0,0013	0,0026	0,0026	0,0025
30	0,0012	0,0012	0,0016	0,0011	0,0018	0,0014	0,0008	0,0008	0,0015	0,0016	0,0009	0,0014	0,0025	0,0025	0,0030

Table A.4 : Shapiro-Wilk Normality Test results of stock returns.

Stock No	Shapiro-Wilk Test Statistics	Significance
1	0.991	0.579
2	0.989	0.396
3	0.660	0.000*
4	0.971	0.005*
5	0.594	0.000*
6	0.926	0.000*
7	0.967	0.002*
8	0.992	0.694
9	0.991	0.558
10	0.983	0.097
11	0.991	0.523
12	0.897	0.000*
13	0.820	0.000*
14	0.993	0.712
15	0.987	0.257
16	0.957	0.000*
17	0.981	0.061
18	0.969	0.004*
19	0.947	0.000*
20	0.970	0.005*
21	0.988	0.322
22	0.992	0.629
23	0.976	0.019*
24	0.960	0.001*
25	0.995	0.918
26	0.992	0.653
27	0.988	0.293
28	0.973	0.009*
29	0.991	0.552
30	0.990	0.456

* Returns are not normally distributed at the 5% Confidence Level

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List of Publications:

- Taş, O., Tokmakçıoğlu, K. & **Güran, C. B.** (2010). Calculation of Credibility Scores of Major Sectors in Turkey. An Analytical Hierarchy Process Approach, *International Journal of Business and Social Science, 5 (1-2),* 75-88.

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