ISTANBUL TECHNICAL UNIVERSITY ★ INFORMATICS INSTITUTE

MODELING ELECTROMAGNETIC WAVE PROPAGATION THROUGH TIME - VARYING MEDIUM USING THE FINITE-DIFFERENCE TIME-DOMAIN (FDTD) METHOD

M.Sc. THESIS

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ISTANBUL TEKNİK ÜNİVERSİTESİ ★ BİLİŞİM ENSTİTÜSÜ

SONLU FARK ZAMAN ALANI (FDTD) YÖNTEMİ KULLANILARAK ZAMANLA DEĞİŞEN ORTAMDA ELEKTROMANYETİK DALGA YAYILMASININ MODELLENMESİ

YÜKSEK LİSANS TEZİ

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İletişim Sistemleri Anabilim Dalı

Uydu Haberleşmesi ve Uzaktan Algılama Programı

Tez Danışmanı: Dr. Öğr. Üyesi Sebahattin EKER

ŞUBAT 2026

İTÜ, Lisansüstü Eğitim Enstitüsü'nün .... numaralı Yüksek Lisans / Doktora Öğrencisi Adı SOYADI, ilgili yönetmeliklerin belirlediği gerekli tüm şartları yerine getirdikten sonra hazırladığı "TEZ BAŞLIĞI" başlıklı tezini aşağıda imzaları olan jüri önünde başarı ile sunmuştur.

Tez Danışmanı :	**Prof. Dr. Adı SOYADI** İstanbul Teknik Üniversitesi	....
Eş Danışman : **(Varsa)**	**Prof.Dr. Adı SOYADI** .... Üniversitesi	....
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(Varsa)	**Prof. Dr. Adı SOYADI** .... Üniversitesi	....

Teslim Tarihi : 17 Aralık 2014
Savunma Tarihi : 21 Ocak 2015

for my Mom,

FOREWORD

In this thesis, the aim is to model and simulate electromagnetic wave propagation in a time-varying medium. The finite difference time domain method is used for modeling and is the most suitable method.

I would like to express my gratitude to my thesis advisor, Sebahattin EKER, who patiently guided me at every stage of the preparation of this thesis and who supported me in its planning, writing, and creation. I am grateful to my family, who have always been with me throughout my life, and to my children Derin and Erk, who always bring tears to my eyes with their success and love.

January 2026

Deniz GÜNDOĞAN
(Electronics Engineer)

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ABBREVIATIONS

1D	: One-Dimensional
2D	: Two-Dimensional
3D	: Three-Dimensional
CFL	: Courant–Friedrichs–Lewy (stability condition)
EM	: Electromagnetic
FDTD	: Finite Difference Time Domain
FFT	: Fast Fourier Transform
GHz	: Gigahertz
MoM	: Method of Moments
FEM	: Finite Element Method
TLM	: Transmission Line Matrix
PML	: Perfectly Matched Layer
PIC	: Particle-in-Cell
RF	: Radio Frequency
TE	: Transverse Electric
TM	: Transverse Magnetic
Yee Cell	: Staggered Grid Cell Used in FDTD

SYMBOLS

$\mathbf{B}$	: Magnetic flux density
$\mathbf{D}$	: Electric flux density
$\mathbf{E}$	: Symbol of electric field intensity vector
$\mathbf{H}$	: Symbol of magnetic density vector
$\mathbf{J}$	: Current density
$\mathbf{c}$	: The speed of light
ε	: Electric permittivity
μ	: Magnetic permeability
c_0	: Speed of light in free space
μ_0	: Vacuum permeability
ε_0	: Vacuum permittivity
σ	: The conductivity
ρ	: Charge density
$\mathbf{t}$	: Time
ω	: Angular frequency
$\mathbf{f}$	: Frequency
Δx	: Spatial step
Δt	: Time step
$\mathbf{n}$	: A time step

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MODELING ELECTROMAGNETİC WAVE PROPAGATION THROUGH TIME-VARYING MEDIUM USING THE FINITE-DIFFERENCE TIME-DOMAIN (FDTD) METHOD

SUMMARY

Electromagnetic waves play a central role in modern communication, sensing, and space-based systems. Technologies such as satellite communication, radar, and remote sensing depend on the reliable transmission of electromagnetic signals over wide frequency ranges. The performance of these systems is strongly influenced by the characteristics of the propagation medium. Therefore, accurate numerical modeling of electromagnetic wave behavior is essential for reliable system design and analysis.

In many classical electromagnetic models, material properties such as electric permittivity (ε) and magnetic permeability (μ) are assumed to be constant in time. This assumption simplifies the mathematical formulation but limits the applicability of the models. In reality, many propagation environments exhibit time-dependent behavior. Such environments include plasma media, the upper atmosphere, and artificially engineered materials. Temporal variations in material parameters can significantly modify wave propagation by altering amplitude, phase, period, and frequency content.

The ionosphere is a well-known natural example of a time-varying electromagnetic medium. Extending from approximately 60 km to 1000 km above the Earth’s surface, the ionosphere is continuously influenced by solar radiation, geomagnetic activity, and seasonal effects. These processes cause time-dependent changes in electron density and, consequently, in the electromagnetic properties of the medium. As a result, electromagnetic waves propagating through the ionosphere may experience refraction, absorption, delay, and frequency shifts. Accurate modeling of such effects requires numerical methods that explicitly account for temporal variations.

This thesis investigates electromagnetic wave propagation in time-varying media using the Finite-Difference Time-Domain (FDTD) method. The FDTD technique directly solves Maxwell’s equations by discretizing both space and time. Electric and magnetic field components are updated sequentially using an explicit time-stepping scheme. This structure allows transient wave behavior to be observed naturally and enables the inclusion of material properties that vary with time. All numerical simulations are implemented in the MATLAB environment.

The primary application considered in this study is a plasma medium with time-dependent electromagnetic parameters. The standard FDTD update equations are modified so that the permittivity $\varepsilon(t)$ and permeability $\mu(t)$ can change at each time step. Simulations are carried out on a three-dimensional staggered grid based on the Yee lattice. The results demonstrate that temporal increases in permittivity lead to a measurable rise in wave frequency and a corresponding reduction in the wave period. These observations indicate strong interaction and energy exchange between the electromagnetic field and the evolving medium.

In addition to plasma modeling, the thesis examines electromagnetic wave propagation in complex media and nonlinear dielectric media. In these engineered materials, electromagnetic properties can be intentionally designed and externally modulated. By comparing the simulation results obtained from plasma and complex media, the study highlights the differences between naturally occurring and artificially controlled time-varying media. The comparison shows that while both environments produce frequency shifts and phase modulation, the underlying physical mechanisms differ.

The results of this work emphasize the importance of incorporating time-dependent material parameters into electromagnetic simulations. Models that neglect temporal variations may fail to predict critical propagation effects in dynamic environments. The numerical framework developed in this thesis provides a reliable tool for analyzing wave behavior in time-varying plasmas and engineered materials.

The findings are directly relevant to satellite communication systems, radar applications, remote sensing technologies, and the design of advanced electromagnetic materials. Overall, this study contributes to a deeper understanding of electromagnetic wave propagation in nonstationary media and offers a robust computational approach for future research in time-varying electromagnetic systems.

SONLU FARK ZAMAN ALANI (FDTD) YÖNTEMİ KULLANILARAK ZAMANLA DEĞİŞEN ORTAMDA ELEKTROMANYETİK DALGA YAYILMASININ MODELLENMESİ

ÖZET

Elektromanyetik dalgalar, çağdaş mühendislik ve fizik uygulamalarının temelini oluşturan en önemli fiziksel olgulardan biridir. Uydu haberleşme sistemleri, radar platformları, radyo frekans (RF) iletişim altyapıları, uzaktan algılama teknolojileri, küresel navigasyon sistemleri ve uzay tabanlı gözlem uygulamaları elektromanyetik dalgaların kontrollü, öngörülebilir ve düşük bozulmalı yayılımına dayanmaktadır. Bu sistemlerin doğruluğu, güvenilirliği ve performansı ise doğrudan dalganın yayıldığı ortamın elektromanyetik özellikleri ile ilişkilidir. Dolayısıyla elektromanyetik dalga yayılımının yalnızca ideal, homojen ve zamandan bağımsız ortamlar için değil; dinamik, zamana bağlı ve parametreleri değişken ortamlar için de ayrıntılı ve fiziksel olarak tutarlı biçimde modellenmesi büyük önem taşımaktadır.

Elektromanyetik alan davranışı Maxwell denklemleri ile tanımlanmaktadır. Bu denklemler, elektrik alan (E) ve manyetik alan (H) bileşenlerinin uzay ve zamandaki değişimini ortam parametreleri olan elektriksel geçirgenlik (ε), manyetik geçirgenlik (μ) ve iletkenlik (σ) aracılığıyla ilişkilendirir. Zamandan bağımsız, lineer ve izotropik ortamlarda Maxwell denklemlerinden türetilen klasik dalga denklemi, elektromanyetik dalganın sabit bir faz hızı ile yayıldığını ve frekansının uzayda korunduğunu göstermektedir. Bu yaklaşım, analitik çözüm kolaylığı sağlaması nedeniyle literatürde yaygın biçimde kullanılmaktadır. Ancak gerçek fiziksel ortamlarda, özellikle plazma ortamlarında ve mühendislik yoluyla tasarlanmış complex media yapılarda, elektromanyetik parametreler çoğu zaman zamanın fonksiyonu olarak değişmektedir. Bu durumda klasik dalga denklemi formu geçerliliğini yitirmekte ve Maxwell denklemlerinin zamana bağlı katsayılarla yeniden ele alınması gerekmektedir.

Zamana bağlı ortamlarda elektrik akı yoğunluğu $D = \varepsilon(t)E$ ve manyetik akı yoğunluğu $B = \mu(t)H$ bağıntıları dikkate alındığında, alan denklemlerinin zaman türevleri alınırken ε ve μ'nün zamansal değişimi ek terimler ortaya çıkarmaktadır. Bu ek terimler, Ampère–Maxwell yasasında ilave bir akım bileşeni gibi davranmakta ve ortam ile elektromanyetik alan arasında doğrudan enerji alışverişine imkân tanımaktadır. Sonuç olarak dalganın yalnızca genliği ve fazı değil, frekans içeriği de zamana bağlı olarak değişebilmektedir. Uzaysal süreksizliklerde frekans korunurken dalga sayısı değişmekte; buna karşılık zamansal süreksizliklerde dalga sayısı korunurken frekans değişebilmektedir. Bu temel fark, zamansal kırılma ve zamansal saçılma kavramlarının ortaya çıkmasına neden olmuş ve son yıllarda elektromanyetik literatürde artan bir ilgi görmüştür.

Zamana bağlı ortamların doğal örneklerinden en önemlisi iyonosferdir. İyonosfer, yaklaşık 60 km ile 1000 km yükseklikler arasında bulunan ve güneş radyasyonu ile sürekli etkileşim hâlinde olan iyonize bir plazma tabakasıdır. Elektron yoğunluğu;

güneş aktivitesi, jeomanyetik fırtınalar, mevsimsel değişimler ve gün içi varyasyonlar nedeniyle sürekli değişmektedir. Elektron yoğunluğundaki bu zamansal değişimler, ortamın etkin permittivitesini doğrudan etkilemekte ve elektromanyetik dalgaların faz hızını, grup hızını ve dispersiyon özelliklerini modifiye etmektedir. Bunun sonucunda gecikme, kırılma, sinyal zayıflaması, spektral genişleme ve frekans kayması gibi etkiler ortaya çıkmaktadır. Özellikle geniş bantlı uydu sistemlerinde ve hassas konumlama uygulamalarında bu etkilerin doğru modellenmemesi ciddi performans kayıplarına neden olabilmektedir.

Doğal plazma ortamlarının yanı sıra, elektromanyetik özellikleri mühendislik yoluyla tasarlanabilen complex media yapılar da zamana bağlı parametre değişimlerine olanak tanımaktadır. Bu tür ortamlarda permittivite ve/veya manyetik geçirgenlik kontrollü biçimde modüle edilerek dalga–ortam etkileşimi yönlendirilebilmektedir. Zamansal modülasyon sayesinde frekans dönüştürücü yapılar, faz manipülasyonu sağlayan sistemler ve enerji yönlendirme mekanizmaları tasarlanabilmektedir. Ancak bu tür sistemlerin doğru analiz edilebilmesi için zamana bağlı Maxwell denklemlerinin sayısal olarak kararlı ve fiziksel olarak tutarlı biçimde çözülmesi gerekmektedir.

Bu tez çalışmasında, zamana bağlı elektromanyetik parametrelere sahip ortamlarda elektromanyetik dalga yayılımı Sonlu Fark Zaman Alanı (Finite-Difference Time-Domain, FDTD) yöntemi kullanılarak incelenmiştir. FDTD yöntemi, Maxwell denklemlerinin diferansiyel formunun hem uzayda hem de zamanda sonlu fark yaklaşımları ile ayrıklaştırılmasına dayanmaktadır. Yee tarafından önerilen ayrıklaştırma şemasında elektrik ve manyetik alan bileşenleri uzaysal olarak yarım hücre kaydırmalı biçimde konumlandırılmakta ve zaman içerisinde sıçramalı (leapfrog) güncelleme algoritması ile ardışık olarak hesaplanmaktadır. Bu yapı, Maxwell denklemlerinin curl operatörlerini simetrik biçimde temsil etmekte ve alan bileşenleri arasındaki karşılıklı bağlaşımı doğal olarak korumaktadır. FDTD yöntemi özellikle geçici rejim analizlerinde, geniş bantlı uyarımlarda ve karmaşık geometriye sahip problemlerde güçlü bir sayısal araç olarak öne çıkmaktadır.

Çalışma kapsamında klasik FDTD güncelleme denklemleri, ortam parametrelerinin zamanın fonksiyonu olduğu durum için yeniden türetilmiştir. $\varepsilon = \varepsilon(t)$ ve $\mu = \mu(t)$ varsayımı altında, alan güncelleme ifadelerine parametrelerin zaman türevlerini içeren ek terimler dahil edilmiştir. Böylece her zaman adımında değişen elektromanyetik özellikler doğrudan sayısal şemaya entegre edilmiştir. Sayısal kararlılık Courant–Friedrichs–Lewy (CFL) koşulu gözetilerek sağlanmış; zaman adımı, uzaysal hücre boyutu ve ortamın maksimum faz hızı dikkate alınarak belirlenmiştir. Hesaplamalar MATLAB ortamında gerçekleştirilmiş; elde edilen sonuçlar hem zaman domeninde alan dağılımları hem de frekans domeninde spektral analizler aracılığıyla değerlendirilmiştir.

Tez kapsamında ilk olarak zamana bağlı bir plazma ortamı modellenmiştir. Plazma bölgesinde etkin permittivite zamanın belirli bir fonksiyonu olarak tanımlanmış ve elektromanyetik dalganın bu ortam içerisindeki uzay–zaman evrimi incelenmiştir. $E(z,t)$ ve $H(z,t)$ dağılımlarının analizi, permittivitedeki zamansal artışın dalga periyodunda azalmaya ve frekansın yukarı yönlü kaymasına neden olduğunu göstermiştir. Fourier dönüşümü sonuçları, spektral tepenin yer değiştirdiğini ve bazı durumlarda yeni frekans bileşenlerinin ortaya çıktığını ortaya koymuştur. Bu sonuçlar, plazma ortamının elektromanyetik alan ile aktif biçimde enerji alışverişi gerçekleştirdiğini ve frekans dönüşümünün zamansal modülasyondan kaynaklandığını doğrulamaktadır.

İkinci aşamada, elektromanyetik parametreleri mühendislik yoluyla kontrol edilebilen complex media ortamları incelenmiştir. Bu ortamlarda permittivite belirli bir zamansal modülasyon fonksiyonu ile değiştirilmiş ve dalganın genlik, faz ve spektral içeriği üzerindeki etkiler detaylı biçimde analiz edilmiştir. Simülasyon sonuçları, kontrollü zamansal modülasyonun yalnızca frekans kaymasına değil; aynı zamanda faz yapısında belirgin değişimlere ve enerji yoğunluğunda yeniden dağılıma yol açtığını göstermiştir. Zaman-frekans analizleri, frekans dönüşümünün ani bir olay olmadığını; ortam içinde ilerleyen ve parametre değişimi ile sürekli etkileşim hâlinde olan bir süreç olduğunu ortaya koymuştur.

Plazma ve complex media uygulamalarının karşılaştırmalı değerlendirmesi, her iki ortamda da frekans dönüşümünün gözlemlendiğini; ancak fiziksel kökenlerinin farklı olduğunu göstermiştir. Plazma ortamında değişim doğal elektron yoğunluğu dinamiklerinden kaynaklanırken, complex media ortamında frekans kayması tasarım parametreleri ile kontrol edilebilmektedir. Bu karşılaştırma, doğal ve mühendislik ürünü zamana bağlı ortamların ortak bir sayısal çerçevede analiz edilebileceğini göstermekte ve literatürde genellikle ayrı ayrı ele alınan iki yapıyı bütüncül bir perspektifte birleştirmektedir.

Bu tez çalışmasının literatüre temel katkılarından biri, zamana bağlı elektromanyetik parametrelerin FDTD çerçevesinde sistematik biçimde ele alınması ve hem doğal plazma ortamları hem de tasarlanmış complex media yapıları için ortak bir modelleme yaklaşımı sunulmasıdır. Çalışma, zamansal modülasyonun frekans dönüşümü üzerindeki etkisini nicel olarak ortaya koymakta ve enerji alışverişi mekanizmasını sayısal sonuçlarla desteklemektedir. Ayrıca geliştirilen model, gelecekte daha karmaşık geometriler, çok katmanlı yapılar ve üç boyutlu analizler için genişletilebilir bir altyapı sunmaktadır.

Sonuç olarak bu tez, zamana bağlı elektromanyetik ortamların dalga yayılımı üzerindeki etkilerini hem teorik hem de sayısal açıdan kapsamlı biçimde incelemekte ve dinamik elektromanyetik sistemlerin analizi için güçlü bir modelleme yaklaşımı ortaya koymaktadır. Elde edilen bulgular, uydu haberleşmesi, radar sistemleri, plazma etkileşimli elektromanyetik uygulamalar ve zamansal olarak modüle edilen complex media yapılarının tasarımı açısından önemli bilimsel ve mühendislik katkıları sunmaktadır. Çalışma, zamana bağlı ortamların ihmal edilmesinin yaratabileceği modelleme hatalarını açıkça ortaya koymakta ve dinamik elektromanyetik problemlerin daha doğru ve güvenilir biçimde analiz edilmesine yönelik sağlam bir teorik ve hesaplamalı temel oluşturmaktadır.

1. INTRODUCTION

Electromagnetic (EM) waves are fundamental to modern engineering systems. They enable communication, sensing, and detection technologies. Satellite communication systems rely on these waves. Radar systems also depend on electromagnetic propagation. Remote sensing applications use similar physical principles. In all these systems, signal reliability is essential. Accurate transmission is required. Small disturbances may cause large errors. Therefore, the propagation environment must be modeled correctly.

Electromagnetic wave behavior is governed by Maxwell's equations. These equations depend on material properties. Electric permittivity is one key parameter. Magnetic permeability is another. Together, they define the response of the medium. Classical electromagnetic models include simplifying assumptions. The medium is usually considered time-invariant. Material parameters are assumed constant. This approach reduces computational complexity. However, it limits physical realism.

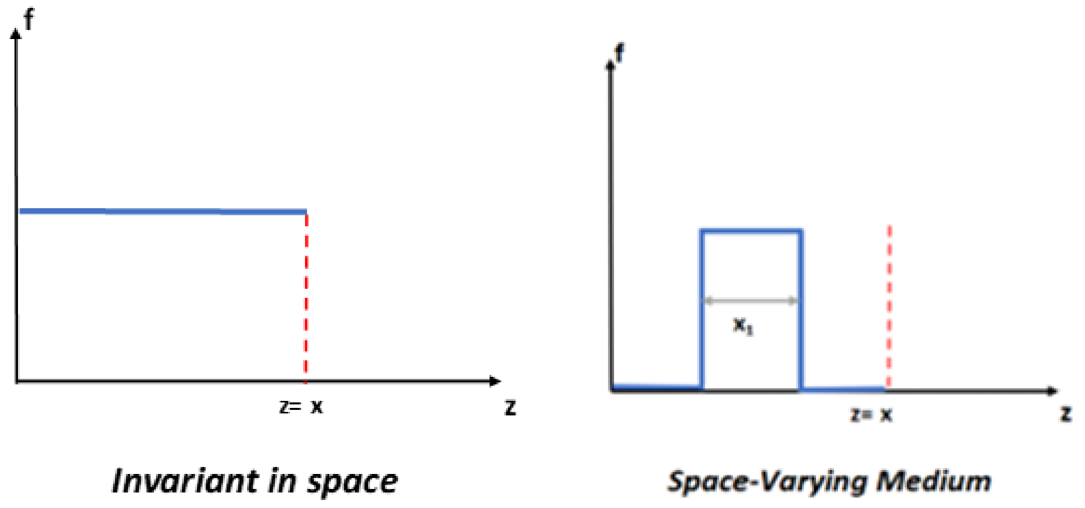

Figure 1.1: Variation of electromagnetic wave propagation with space

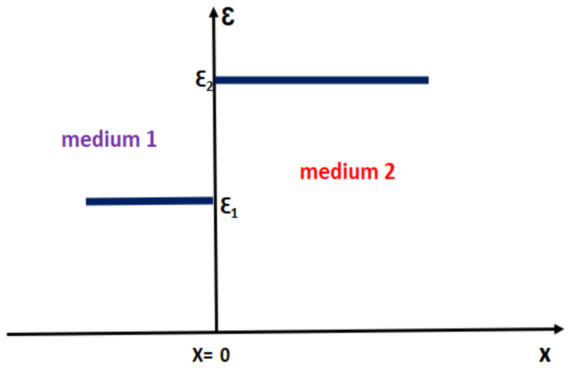

Figure 1.2: The effect of spatial discontinuity on dielectric constant (Ɛ)

Many real environments are not static. Material properties may change with time. They may also vary in space. In some cases, both variations occur simultaneously. Plasma environments exhibit strong temporal variation. The ionosphere is a typical example. It surrounds the Earth at high altitudes. Its properties change continuously. Solar radiation plays a major role.

Ionization levels vary with solar activity. Electron density is not constant. As a result, electromagnetic parameters change over time. Waves traveling through this region are affected. Signal delay may occur. Refraction is commonly observed. Frequency shifts can appear. Amplitude variations are also possible. Ignoring these effects leads to inaccurate predictions. Satellite communication links may degrade. Navigation systems may suffer errors. Radar performance may be reduced. For this reason, time-varying media must be considered. Advanced numerical methods are required. Analytical solutions are often impractical. Computational modeling becomes essential.

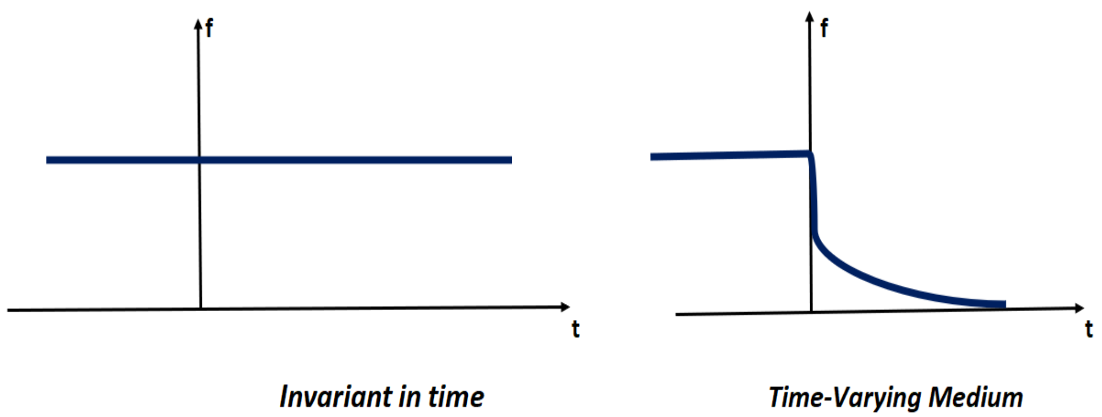

Figure 1.3: Variation of electromagnetic wave propagation with time

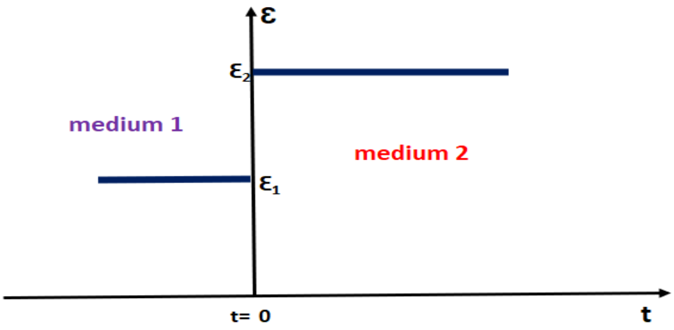

Figure 1.4: The effect of temporal discontinuity on dielectric constant (Ɛ)

In this thesis, electromagnetic wave propagation in time-varying media is investigated using the Finite-Difference Time-Domain (FDTD) method. FDTD is a fully explicit numerical technique that directly solves Maxwell's curl equations through discretization in both space and time. Originally introduced by Yee, the method employs a staggered grid arrangement in which electric and magnetic field components are spatially offset, ensuring numerical stability and accurate field coupling. One of the principal strengths of the FDTD method is its natural capability to model transient phenomena and broadband signals within a single simulation framework. Furthermore, time-dependent material properties can be incorporated directly into the field update equations, allowing dynamic wave–medium interactions to be captured explicitly.

Simulations are performed in MATLAB. Field evolution is monitored over time. Waveforms are analyzed in detail. Frequency behavior is evaluated. The results reveal important effects. Increasing permittivity leads to frequency upshift. Wave period decreases. Energy redistribution is observed.

The primary application focuses on plasma media with time-dependent permittivity and permeability. Such media are representative of ionospheric propagation conditions, where temporal variations arise due to changing electron density. Within the FDTD framework, material parameters are defined as explicit functions of time, and the electromagnetic field evolution is monitored as waves propagate through the dynamic plasma region. The simulations reveal that increasing permittivity over time leads to observable frequency upshift and a corresponding decrease in wave period, demonstrating clear energy exchange between the wave and the evolving medium.

The second application examines complex media and nonlinear dielectric media whose electromagnetic properties can be intentionally modulated in time through external control mechanisms. Unlike plasma environments, where parameter variations are governed by natural processes, complex media allow precise and programmable temporal modulation. Simulations performed for these engineered media exhibit distinct wave behaviors, including controlled frequency shifts, phase modulation, and amplitude variation, highlighting the flexibility of time-modulated artificial materials.
In this thesis, engineered time-varying metamaterials are treated within the broader framework of complex electromagnetic media. Therefore, the term complex media is used to represent both artificial metamaterial structures and other nonstationary electromagnetic environments.

A comparative analysis between plasma and complex media cases is conducted to evaluate similarities and differences in wave propagation behavior. While both media exhibit frequency shifting due to temporal parameter variation, the underlying mechanisms and resulting wave responses differ significantly. Plasma media demonstrate naturally induced, continuous temporal variation, whereas complex media responses reflect designed modulation profiles. This comparison provides valuable insight into how natural and engineered time-varying environments influence electromagnetic wave dynamics.

The results of these applications confirm the robustness and versatility of the proposed FDTD framework. By capturing frequency evolution, transient field behavior, and energy redistribution in both plasma and complex media, the study demonstrates the importance of including time-dependent material properties in electromagnetic simulations.

Overall, this thesis presents a comprehensive numerical investigation of electromagnetic wave propagation in time-varying media. The developed FDTD-based model provides a reliable and flexible tool for analyzing dynamic propagation environments. The findings are directly relevant to satellite communications, ionospheric modeling, radar systems, and emerging research on time-modulated complex media, contributing to a deeper understanding of wave–medium interaction under nonstationary conditions.

1.1 Objectives of Thesis

The primary objective of this thesis is to investigate electromagnetic wave propagation in media whose electromagnetic properties vary explicitly with time. The study aims to overcome the limitations of conventional electromagnetic models that assume time-invariant material parameters by developing a numerical framework capable of accurately capturing wave–medium interactions in nonstationary environments.

To achieve this goal, the Finite-Difference Time-Domain (FDTD) method is employed as the principal numerical technique. While classical FDTD formulations are well established for static or spatially varying media, they require modification when material parameters such as electric permittivity and magnetic permeability become time-dependent. This thesis extends the standard FDTD framework to incorporate temporally varying constitutive parameters, enabling realistic modeling of dynamic propagation environments.

A major objective of the study is to analyze the effects of temporal material variation on fundamental electromagnetic wave characteristics. In particular, the work focuses on how time-dependent permittivity and permeability influence wave frequency, phase evolution, amplitude, and energy distribution during propagation. Special attention is given to frequency shift phenomena arising from temporal modulation of the medium, as these effects are of critical importance in plasma physics, ionospheric studies, and time-modulated electromagnetic systems.

The developed numerical model is applied to two representative classes of time-varying media. The first application involves plasma environments with temporally varying electromagnetic properties, which serve as a realistic model for ionospheric propagation. The second application considers engineered complex media (including time-varying metamaterial structures) or nonlinear dielectric media, where temporal modulation of material parameters can be intentionally designed and controlled. Through these applications, the thesis aims to examine both naturally occurring and artificially engineered time-varying electromagnetic environments within a unified modeling framework.

Another key objective is to perform a comparative analysis between plasma and engineered complex media cases to identify similarities and differences in wave propagation behavior. By comparing frequency evolution, field distribution, and energy exchange mechanisms in these two media, the study seeks to clarify the physical origins of observed frequency shifts and to highlight the role of engineered temporal modulation versus naturally induced parameter variation.

Ultimately, the objective of this thesis is to provide a reliable and flexible numerical framework for analyzing electromagnetic wave propagation in dynamic environments. The results are intended to support the design and optimization of satellite communication systems, radar, and remote sensing applications, and advanced time-modulated engineered complex media devices. By enhancing the understanding of wave behavior in time-varying media, this work contributes to both theoretical and practical advancements in computational electromagnetics.

1.2 Literature Review

Electromagnetic wave propagation has been widely studied because of its importance in communication, radar, sensing, and plasma-related applications. Early research mainly focused on simple media. In these studies, material properties such as electric permittivity and magnetic permeability were assumed to be constant or dependent only on space. Under these conditions, Maxwell's equations can be solved analytically or with simplified numerical methods *(Balanis, 2012)*.

However, many real environments do not satisfy these assumptions. In natural media such as plasmas and the ionosphere, material properties change with time. These changes are caused by external factors such as solar radiation and geomagnetic activity *(Davies, 1990)*. As a result, electromagnetic waves propagating in such media experience effects that cannot be explained by time-invariant models. These effects include frequency shifts, phase changes, and variations in wave amplitude.

The study of time-varying electromagnetic media began with the analysis of temporal discontinuities. Early theoretical work showed that when material parameters change suddenly in time, the frequency of an electromagnetic wave can change even if there is no spatial boundary *(Morgenthaler, 1958)*. This behavior is different from spatial interfaces, where the frequency of the wave remains constant.

Later studies extended these ideas to continuously time-modulated media. In these systems, material parameters vary smoothly or periodically with time. Such modulation allows energy transfer between different frequency components. Time-varying media were therefore described as electromagnetic frequency transformers *(Kalluri, 2010)*. Kalluri provided a detailed theoretical explanation of how temporal modulation leads to frequency and polarization changes in electromagnetic waves.

Recent research has focused on engineered materials such as space–time modulated complex media materials. These materials are designed so that their electromagnetic properties vary in both space and time. Studies have shown that temporal modulation enables new wave phenomena, including frequency conversion and nonreciprocal propagation *(Caloz and Deck-Léger, 2019)*. These properties are useful for adaptive and reconfigurable electromagnetic devices.

Plasma media represent an important natural example of time-varying electromagnetic environments. The ionosphere is a partially ionized plasma layer located at high altitudes above the Earth. Its electron density changes continuously due to solar and geomagnetic effects *(Budden, 1985)*. These temporal variations modify the effective permittivity of the medium and influence wave propagation. As a result, electromagnetic waves traveling through the ionosphere may experience dispersion, absorption, delay, and frequency shifts.

To study electromagnetic wave propagation in such complex environments, numerical methods are widely used. Frequency-domain techniques such as the Finite Element Method and the Method of Moments are effective for steady-state problems but are less suitable for transient or broadband analysis *(Jin, 2014)*. Time-domain methods provide a more direct way to analyze temporal effects.

The Finite-Difference Time-Domain method is one of the most widely used time-domain techniques. It was originally introduced by Yee and later developed extensively by Taflove *(Yee, 1966; Taflove and Hagness, 2005)*. The method discretizes Maxwell’s equations in both space and time and updates the electric and magnetic fields iteratively. This structure makes FDTD well-suited for modeling transient phenomena and broadband signals.

Several studies have extended the FDTD method to include time-dependent material properties. These works demonstrated that temporal variations in permittivity can lead to frequency shifts and energy redistribution *(Taflove et al., 1998; Zhang et al., 2013; Feng and Qiu, 2017)*. Despite these advances, many studies remain limited to simplified models or lower-dimensional configurations.

In this thesis, a one-dimensional FDTD model is developed to study electromagnetic wave propagation in time-varying media. Both plasma environments and engineered complex media or nonlinear dielectric media are considered. By comparing these two cases, the study aims to clarify the effects of natural and engineered temporal modulation on wave behavior. The results contribute to a better understanding of frequency transformation and energy exchange in time-varying electromagnetic systems.

2. THEORETICAL BACKGROUND

2.1 Differential Forms of Maxwell Equations

A close analogy exists between electric and magnetic fields and the relationship between voltage and current in circuit theory. In electromagnetic theory, these interactions are governed by Maxwell's equations, which constitute the fundamental laws describing the generation, coupling, and propagation of electromagnetic fields (Griffiths, 2017; Jackson, 1999). The differential form of Maxwell's equations is expressed using the nabla (∇) operator, which represents spatial differentiation and enables the description of field variations at every point in space.

Gauss's law for electricity relates the electric flux passing through a closed surface to the total electric charge enclosed within that surface (Balanis, 2012):

$$\oint_{surf} D \cdot ds = \iiint_{vol} \rho \, dv \tag{2.1}$$

Applying the divergence theorem, the equivalent point (differential) form is obtained as:

$$\nabla \cdot D = \rho \tag{2.2}$$

Similarly, Gauss's law for magnetism states that the net magnetic flux through any closed surface is zero, implying the nonexistence of magnetic monopoles (Jackson, 1999). The integral and differential forms are given by:

$$\oint_{surf} B \cdot ds = 0 \tag{2.3}$$

$$\nabla \cdot B = 0 \tag{2.4}$$

Faraday's law of electromagnetic induction describes how a time-varying magnetic field induces an electric field (Griffiths, 2017):

$$\oint_{loop} E \cdot dl = -\frac{d}{dt}\int_{surf} B \cdot ds \tag{2.5}$$

Using Stokes' theorem, the differential form becomes:

$$\oint_{loop} E \cdot dl = -\frac{d}{dt}\int_{surf} (\nabla \times E) \cdot ds \tag{2.6}$$

$$\nabla \times \mathrm{E} = -\frac{\partial \mathrm{B}}{\partial t} \tag{2.7}$$

In Cartesian coordinates, the curl operator is written as (Sibley, 2021):

$$\nabla \times \mathrm{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} \tag{2.8}$$

2.2 Electromagnetic Wave Propagation in Free-Space

Electromagnetic wave propagation in free space refers to the transmission of time-varying electric and magnetic fields in a homogeneous, isotropic, lossless, and source-free medium (Balanis, 2012; Jackson, 1999). This idealized environment provides a reference framework for analyzing more complex propagation media.

In a source-free region, Maxwell's curl equations reduce to (Griffiths, 2017):

$$\nabla \times \mathrm{E} = -\mu_0 \frac{\partial \mathrm{H}}{\partial t} \tag{2.9}$$

$$\nabla \times \mathrm{H} = \varepsilon_0 \frac{\partial \mathrm{E}}{\partial t} \tag{2.10}$$

where $\varepsilon_0 = 8.854 \times \frac{10^{-12}\,\mathrm{F}}{\mathrm{m}}$ is the permittivity of free space and $\mu_0 = 4\pi \times \frac{10^{-7}\,\mathrm{H}}{\mathrm{m}}$ is the permeability of free space.

Taking the curl of Faraday's law and substituting Ampère–Maxwell's law yields the wave equation (Jackson, 1999):

$$\nabla^2 \mathrm{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \mathrm{E}}{\partial t^2} \tag{2.11}$$

Similarly, for the magnetic field:

$$\nabla^2 H = \mu_0 \varepsilon_0 \frac{\partial^2 \mathrm{H}}{\partial t^2} \tag{2.12}$$

2.3 Role of Maxwell's Equations in Electromagnetic Wave Modeling

H Maxwell's equations provide a unified mathematical framework for modeling electromagnetic wave propagation in both stationary and dynamic media (Taflove and Hagness, 2005; Griffiths, 2017).

$$\nabla \times \mathrm{H} = \frac{\partial \mathrm{D}}{\partial t} + \mathrm{J} \tag{2.13}$$

$$\nabla \times \mathrm{E} = -\frac{\partial \mathrm{B}}{\partial \mathrm{t}} \tag{2.14}$$

$$\nabla . \mathrm{D} = \rho \tag{2.15}$$

$$\nabla . \mathrm{B} = 0 \tag{2.16}$$

where

- E (V/m) is the electric field intensity,
- H (A/m) is the magnetic field intensity,
- D = εE (C/m^2) is the electric flux density,
- B = μH (Wb/m^2) is the magnetic flux density,
- ρ (C/m^3) is the charge density, and
- J (A/m^2) is the current density.

Together, these equations describe the coupling between the electric and magnetic fields and their evolution in a given medium characterized by its permittivity (ε), permeability (μ), and conductivity (σ).

Each of Maxwell's equations represents a distinct physical law:

Faraday's Law of Induction ($\nabla \times E = -\partial B/\partial t$)

Indicates that a time-varying magnetic field induces an electric field. This forms the basis for wave generation and electromagnetic induction.

Ampère–Maxwell Law ($\nabla \times H = J + \partial D/\partial t$)

Describes how magnetic fields are produced by electric currents and time-varying electric fields. The term $\partial D/\partial t$ introduces the concept of displacement current, allowing electromagnetic waves to exist in free space.

Gauss's Law for Electricity ($\nabla \cdot D = \rho$)

States that electric flux diverges from electric charges, defining how charge distributions generate electric fields.

Gauss's Law for Magnetism ($\nabla \cdot B = 0$)

Indicates that magnetic monopoles do not exist; magnetic field lines are continuous and always form closed loops.

Together, these four equations form a self-consistent system that links the temporal and spatial variations of E and H fields.

The constitutive relations are defined as (Balanis, 2012):

$$\mathrm{D} = \varepsilon \mathrm{E} \tag{2.17}$$

$$\mathrm{B} = \mu \mathrm{H} \tag{2.18}$$

$$\mathrm{J} = \sigma \mathrm{E} \tag{2.19}$$

where ε (F/m) represents the electric permittivity, μ (H/m) the magnetic permeability, and σ (S/m) the electrical conductivity.

In time-varying or inhomogeneous media, these parameters depend on time and space, significantly affecting wave dynamics (Kalluri, 2010).

In linear, isotropic, and time-invariant media, ε, μ, and σ are constants. However, in time-varying or inhomogeneous media, these parameters depend on both space and time, i.e., $\varepsilon = \varepsilon(r, t)$ and $\mu = \mu(r, t)$. Such dependence introduces significant complexity into Maxwell's equations, affecting both the amplitude and frequency of the propagating fields.

By combining Maxwell's curl equations and assuming a source-free region ($\rho = 0$, $J = 0$), the electromagnetic wave equation can be derived. Taking the curl of Faraday's law and substituting Ampère's law gives:

$$\nabla \times (\nabla \times \mathrm{E}) = -\, \mu \, \frac{\partial}{\partial t}\left(\frac{\partial D}{\partial t}\right) \tag{2.20}$$

$$\nabla^2 \mathrm{E} = \mu\varepsilon \frac{\partial^2 E}{\partial t^2} \tag{2.21}$$

This equation describes the propagation of electromagnetic waves through a uniform medium with speed:

$$\mathrm{v} = \frac{1}{\sqrt{\mu\varepsilon}} \tag{2.22}$$

In free space, $\mu = \mu_0$and $\varepsilon = \varepsilon_0$, giving $v = c = 3 \times 10^8$ m/s.
The same approach yields an equivalent wave equation for the magnetic field H.

Maxwell's equations serve as the starting point for any numerical or analytical model of electromagnetic wave propagation. They form the mathematical foundation for computational techniques such as the Finite-Difference Time-Domain (FDTD), Finite Element Method (FEM), and Method of Moments (MoM).

In the FDTD method, Maxwell's equations are discretized in both space and time, allowing the simulation of wave evolution in complex environments. The accuracy and stability of such simulations depend directly on how faithfully these equations are represented in their discrete form.

For time-varying media, Maxwell's equations are extended to include temporal derivatives of $\varepsilon(t)$ and $\mu(t)$. This extension captures the interaction between changing material properties and electromagnetic fields—an essential feature for accurately modeling plasma environments and nonstationary systems.

2.4 Time-Varying Media: Theoretical Background

Electromagnetic wave propagation is fundamentally influenced by the electromagnetic properties of the medium—namely, permittivity ε, permeability μ, and electrical conductivity σ. In most classical analyses, these parameters are assumed to be constant or dependent solely on spatial position. However, many natural and engineered media exhibit temporal variations in their constitutive parameters, leading to a class of problems collectively known as time-varying media. Such media introduce additional complexity into Maxwell’s equations and substantially modify wave dynamics, producing effects that are not observed in stationary environments.

Time-varying media arise in a wide range of physical systems, including the Earth's ionosphere, laser-induced plasmas, rapidly modulated engineered complex media, high-power microwave devices, and reconfigurable communication structures. In these environments, material parameters may change due to ionization, recombination, external electromagnetic excitation, or deliberate modulation. Understanding the behavior of electromagnetic waves under such conditions is essential for predicting frequency shifts, amplitude modulation, dispersion changes, and energy exchange mechanisms.

Time-varying media are characterized by constitutive parameters that explicitly depend on time (Kalluri, 2010; Caloz and Deck-Léger, 2019):

$$\varepsilon = \varepsilon(r, t), \mu = \mu(r, t), \sigma = \sigma(r, t) \tag{2.23}$$

Such media appear in ionospheric plasmas, laser-induced plasmas, and actively modulated complex media. (Kalluri, 2010).

Such temporal variations result in changes in the wave speed, characteristic impedance, and energy storage capacity of the medium. Because the electromagnetic fields interact dynamically with these changing parameters, the resulting waves may experience:

- frequency conversion or spectral broadening,
- amplitude modulation or gain,
- phase distortion,
- time-dependent dispersion,
- modified power flow.

These effects cannot be captured by conventional time-invariant models and therefore require a theoretical extension of Maxwell’s equations.

In standard media, the constitutive relation $D = \varepsilon E$ involves multiplication by a constant. However, when permittivity varies with time, this relation becomes:

$$D(r,t) = \varepsilon(r,t)E(r,t) \tag{2.24}$$

Taking the time derivative yields (Kalluri, 2010):

$$\frac{\partial D}{\partial t} = \varepsilon\frac{\partial E}{\partial t} + E\frac{\partial \varepsilon}{\partial t} \tag{2.25}$$

This additional term acts as an effective polarization current and leads to frequency conversion phenomena, often described as electromagnetic frequency transformation (Kalluri, 2010).

The second term,

$$E\frac{\partial \varepsilon}{\partial t},$$

is unique to time-varying media and represents an effective polarization current that contributes to the total current density:

$$J_{eff} = \sigma E + E\frac{\partial \varepsilon}{\partial t} \tag{2.26}$$

Substituting this into the Ampère–Maxwell law:

$$\nabla \times H = \frac{\partial D}{\partial t} + J \tag{2.27}$$

gives the modified form:

$$\nabla \times H = \varepsilon(r,t)\frac{\partial E}{\partial t} + E\frac{\partial \varepsilon}{\partial t} + \sigma(r,t)E \tag{2.28}$$

Equation (2.28) clearly shows that time-varying properties introduce an additional source term proportional to the rate of change of permittivity.

The same extension applies to magnetic fields if permeability is time-dependent:

$$\frac{\partial B}{\partial t} = \mu\frac{\partial H}{\partial t} + H\frac{\partial \mu}{\partial t} \tag{2.29}$$

These generalized Maxwell equations constitute the fundamental equations for modeling dynamic media.

Starting from the generalized Maxwell equations and assuming a source-free region ($\rho = 0,\ \mathrm{J} = 0$), the wave equation becomes significantly more complex than the classical form. By taking the curl of Faraday's law and using the modified Ampère–Maxwell law, the electric-field wave equation becomes:

$$\nabla^2 \mathrm{E} = \mu(\mathrm{r}, \mathrm{t})\varepsilon(\mathrm{r}, \mathrm{t}) \frac{\partial^2 \mathrm{E}}{\partial t^2} + 2\mu \frac{\partial \varepsilon}{\partial \mathrm{t}} \frac{\partial \mathrm{E}}{\partial \mathrm{t}} + \mu \mathrm{E} \frac{\partial^2 \varepsilon}{\partial t^2} \tag{2.30}$$

These equations form the theoretical foundation for numerical modeling of time-varying media using the FDTD method (Taflove and Hagness, 2005).

This expression contains additional first-order and zeroth-order time derivatives that do not appear in stationary media.

The coefficients:

- $2\mu \frac{\partial \varepsilon}{\partial \mathrm{t}}$
- $\mu \frac{\partial^2 \varepsilon}{\partial \mathrm{t}^2}$

introduce temporal modulation, responsible for frequency shifting and spectral mixing.

Similarly, the magnetic field satisfies:

$$\nabla^2 \mathrm{H} = \mu\varepsilon \frac{\partial^2 \mathrm{H}}{\partial t^2} + 2\varepsilon \frac{\partial \mu}{\partial \mathrm{t}} \frac{\partial \mathrm{H}}{\partial \mathrm{t}} + \varepsilon \mathrm{H} \frac{\partial^2 \mu}{\partial t^2} \tag{2.31}$$

Several physical systems exhibit inherent or externally induced temporal variations:

- Ionosphere: electron density varies with solar radiation.
- Plasma discharges: $\varepsilon(t)$ evolves with ionization and recombination.
- Engineered complex media: permittivity modulated by bias voltages or optical pumping.
- Reconfigurable RF devices: switching elements alter circuit permittivity/permeability.

These examples highlight the importance of accurately modeling time-dependent media for communication, radar, and remote-sensing applications.

Time-varying media require specialized numerical formulations because standard FDTD assumes constant material parameters. Incorporating $\varepsilon(t)$ or $\mu(t)$ necessitates:

- modifying update equations,
- adding temporal derivative terms,
- adjusting stability conditions (Courant criteria),
- carefully defining material interfaces.

3. FINITE-DIFFERENCE TIME-DOMAIN (FDTD) METHOD

3.1 Derivation of FDTD Equations

The Finite-Difference Time-Domain (FDTD) method is a numerical technique used to solve Maxwell's time-dependent curl equations by discretizing both spatial and temporal derivatives using finite-difference approximations. The method was originally introduced by Kane S. Yee (1966) and has since become one of the most widely used computational tools in electromagnetics due to its simplicity, explicit time-stepping structure, and direct correspondence with Maxwell's differential equations (Yee, 1966; Taflove & Hagness, 2005).

In a source-free, linear, isotropic medium, Maxwell's curl equations can be expressed as:

$$\nabla \times \mathrm{E} = -\mu \frac{\partial \mathrm{H}}{\partial t} \tag{3.1}$$

$$\nabla \times H = \varepsilon \frac{\partial \mathrm{E}}{\partial t} \tag{3.2}$$

These coupled equations describe the mutual interaction between electric and magnetic fields. The spatial variation of the electric field induces a temporal variation in the magnetic field and vice versa. The FDTD method numerically represents this behavior by replacing continuous derivatives with finite differences defined on a discrete grid.

Yee proposed a staggered grid arrangement, now known as the Yee lattice, in which electric and magnetic field components are offset in both space and time (Yee, 1966). This staggering allows central-difference approximations of curl operators and improves numerical stability and accuracy (Taflove & Hagness, 2005).

In a three-dimensional Cartesian coordinate system, the electric field components $E_x, E_y, E_z$are located along the edges of each grid cell, while the magnetic field components $H_x, H_y, H_z$are positioned on the faces of the cell. This configuration ensures that the discrete divergence conditions are automatically satisfied.

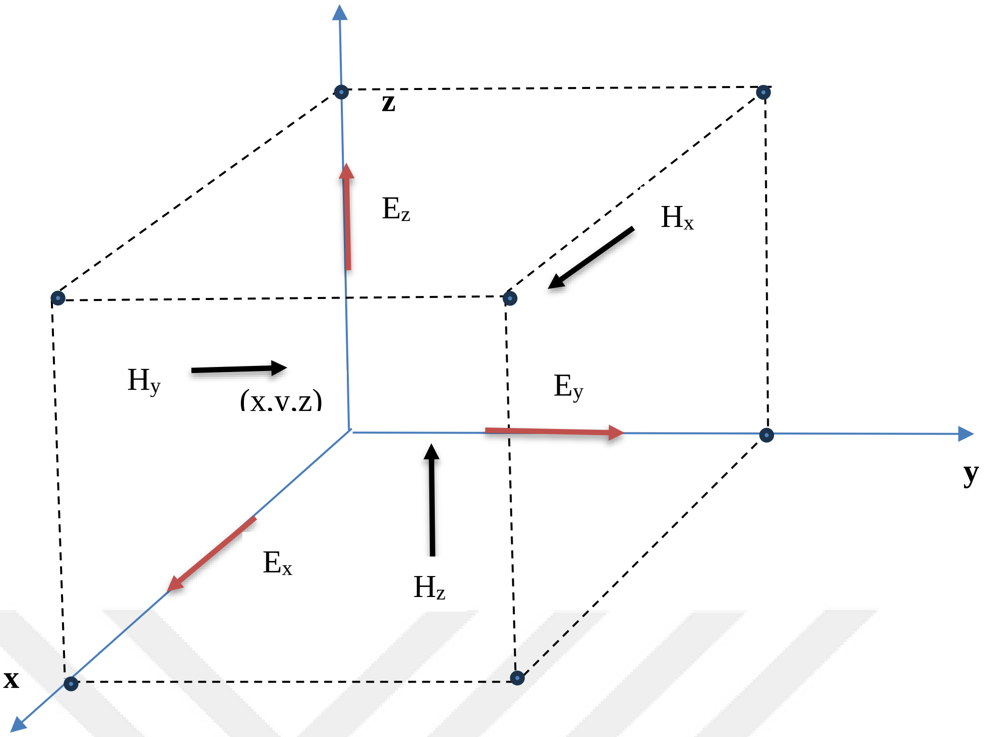

Figure 2.1: Yee Cell

In the Yee grid, the electric field components E_x, E_y, and E_z are positioned along the edges of each cubic cell, while the magnetic field components H_x, H_y, and H_z are located along the faces of the cell, offset by half a grid spacing. This staggered spatial arrangement allows the finite-difference approximations of Maxwell's curl equations to be implemented symmetrically and ensures that the discrete divergence conditions are automatically satisfied.

Furthermore, the time staggering of field updates—where the magnetic field is computed at half time steps and the electric field at full time steps—enables a leapfrog scheme, improving numerical stability and ensuring accurate coupling between E and H fields throughout the simulation domain.

The partial derivatives in Maxwell's equations are replaced by finite-difference approximations.

To illustrate the basic principles of the FDTD method, a one-dimensional (1D) formulation is first considered. The 1D case provides the foundation for extending the method to two- and three-dimensional problems (Taflove & Hagness, 2005).

For a plane electromagnetic wave propagating in the $+x$-direction in a source-free, lossless, and homogeneous medium, only two field components are nonzero: the electric field $E_z(x,t)$ and the magnetic field $H_y(x,t)$. This configuration corresponds to a transverse electromagnetic (TEM) mode (Balanis, 2012).

This configuration is known as a Transverse Electromagnetic (TEM) or E_z–H_y mode. The general Maxwell's curl equations in differential form are:

$$\nabla \times \mathrm{E} = -\mu \frac{\partial \mathrm{H}}{\partial t} \tag{3.3}$$

$$\nabla \times H = \varepsilon \frac{\partial \mathrm{E}}{\partial t} \tag{3.4}$$

For the 1D case, substituting $\mathrm{E} = \hat{z}E_z(x,t)$ and $\mathrm{H} = \hat{y}H_y(x,t)$, we obtain the following scalar equations:

$$\frac{\partial E_z(x,t)}{\partial \mathrm{x}} = -\mu \frac{\partial H_y(x,t)}{\partial t} \tag{3.5}$$

$$\frac{\partial H_y(x,t)}{\partial \mathrm{x}} = -\varepsilon \frac{\partial E_z(x,t)}{\partial t} \tag{3.6}$$

These two coupled partial differential equations describe the temporal and spatial evolution of the electric and magnetic fields as they propagate in the x-direction.

In the FDTD scheme, space and time are discretized into intervals Δx and Δt. The electric field is evaluated at integer grid points and integer time steps, while the magnetic field is evaluated at half-integer spatial and temporal positions. This leapfrog arrangement ensures accurate coupling between the field components (Yee, 1966).

Applying central-difference approximations to Eqs. (3.5) and (3.6) yield the update equations:

$$\left.\frac{\partial E_z}{\partial x}\right|_{i+\frac{1}{2}}^{n} \approx \frac{E_z(i+1,n) - E_z(i,n)}{\Delta x} \tag{3.7}$$

$$\left.\frac{\partial H_y}{\partial t}\right|_{i+\frac{1}{2}}^{n} \approx \frac{H_y\left(i+\frac{1}{2}, n+\frac{1}{2}\right) - H_y\left(i+\frac{1}{2}, n-\frac{1}{2}\right)}{\Delta t} \tag{3.8}$$

$$H_y^{n+\frac{1}{2}}\left(i+\frac{1}{2}\right) = H_y^{n-\frac{1}{2}}\left(i+\frac{1}{2}\right) - \frac{\Delta \mathrm{t}}{\mu \Delta \mathrm{x}}\left[E_z^n(i+1) - E_z^n(i)\right] \tag{3.9}$$

$$\left.\frac{\partial H_y}{\partial x}\right|_{i}^{n+\frac{1}{2}} \approx \frac{H_y\left(i+\frac{1}{2}, n+\frac{1}{2}\right) - H_y\left(i-\frac{1}{2}, n+\frac{1}{2}\right)}{\Delta x} \tag{3.10}$$

$$\left.\frac{\partial E_z}{\partial t}\right|_{i}^{n} \approx \frac{E_z^{n+1}(i) - E_z^n(i)}{\Delta t} \tag{3.11}$$

Substituting into Eq. (3.11) yields the update equation for the electric field:

$$E_z^{n+1}(i) = E_z^n(i) - \frac{\Delta t}{\varepsilon \Delta \mathrm{x}}\left[H_y^{n+\frac{1}{2}}\left(i+\frac{1}{2}\right) - H_y^{n+\frac{1}{2}}\left(i-\frac{1}{2}\right)\right] \tag{3.12}$$

For the numerical scheme to remain stable, the Courant–Friedrichs–Lewy (CFL) condition must be satisfied:

$$\Delta \mathrm{t} \leq \frac{\Delta \mathrm{x}}{\mathrm{c}} \tag{3.13}$$

where $\boldsymbol{c} = \frac{\mathbf{1}}{\sqrt{\boldsymbol{\mu\varepsilon}}}$ is the wave propagation speed in the medium. This condition ensures that the numerical domain of dependence encompasses the physical domain of dependence of the wave.

The 1-D FDTD formulation provides the conceptual foundation for electromagnetic wave simulation; however, most practical problems require two- or three-dimensional modeling. Extending the FDTD method to higher dimensions involves incorporating additional field components and applying the Yee staggered grid in multiple spatial directions.

In two-dimensional simulations, electromagnetic waves propagate in a plane (typically the x-y plane), and either transverse electric (TE) or transverse magnetic (TM) polarization is considered (Balanis, 2012). In three-dimensional simulations, all six field components are nonzero and are defined on a three-dimensional Yee lattice.

The general update equations for electric and magnetic field components are obtained by discretizing Maxwell's curl equations in all spatial directions. The staggered spatial and temporal arrangement preserves numerical stability and ensures accurate curl approximations (Taflove & Hagness, 2005).

1. Transverse Electric (TE) Mode:
    - Non-zero components: E_x, E_y, H_z
    - Magnetic field $H_z$is perpendicular to the x–y plane.
2. Transverse Magnetic (TM) Mode:
    - Non-zero components: H_x, H_y, E_z
    - Electric field $E_z$is perpendicular to the x–y plane.

The update equations for each non-zero component are derived from Maxwell's curl equations, discretized using central differences. The Yee grid is extended such that field components are staggered in both x and y directions, maintaining the leapfrog time-stepping scheme.

In 3-D simulations, all six field components ($E_x, E_y, E_z, H_x, H_y, H_z$) are non-zero and are defined on a 3-D Yee lattice. Each cubic cell contains:

- Electric field components along edges,
- Magnetic field components on faces, offset by half a grid spacing.

The update equations for each component are derived by discretizing the corresponding Maxwell curl equation in all three dimensions. The staggered spatial and temporal arrangement ensures numerical stability, accurate curl approximations, and divergence preservation.

For a field component $f(x, y, z, t)$The first-order central-difference approximations are:

$$\frac{\partial \mathrm{f}}{\partial \mathrm{x}} \approx \frac{f\left(i+\frac{1}{2}, j, k\right)-f\left(i-\frac{1}{2}, j, k\right)}{\nabla \mathrm{x}} \tag{3.14}$$

$$\frac{\partial \mathrm{f}}{\partial \mathrm{t}} \approx \frac{f^{n+\frac{1}{2}} - f^{n-\frac{1}{2}}}{\nabla \mathrm{t}} \tag{3.15}$$

where

- $\Delta x, \Delta y, \Delta z$ are the spatial step sizes,
- Δt is the time step, and
- The superscript n denotes the discrete time level.

By applying these finite-difference approximations to Maxwell's curl equations, the FDTD updating equations are derived. For example, the x-component of the electric field is updated as:

$$E_x^{n+1}(i,j,k) = E_x^n(i,j,k) + \frac{\Delta \mathrm{t}}{\varepsilon(\mathrm{i,j,k})}\left[\frac{H_z^{n+\frac{1}{2}}(i,j,k) - H_z^{n+\frac{1}{2}}(i,j-1,k)}{\Delta y} - \frac{H_y^{n+\frac{1}{2}}(i,j,k) - H_y^{n+\frac{1}{2}}(i,j,k-1)}{\Delta z}\right] \tag{3.16}$$

Similarly, the y- and z-components of the electric field can be written as:

$$E_y^{n+1}(i,j,k) = E_y^n(i,j,k) + \frac{\Delta \mathrm{t}}{\varepsilon(\mathrm{i,j,k})}\left[\frac{H_x^{n+\frac{1}{2}}(i,j,k) - H_x^{n+\frac{1}{2}}(i,j,k-1)}{\Delta \mathrm{z}} - \frac{H_z^{n+\frac{1}{2}}(i,j,k) - H_z^{n+\frac{1}{2}}(i-1,j,k)}{\Delta x}\right] \tag{3.17}$$

$$E_z^{n+1}(i,j,k) = E_z^n(i,j,k) + \frac{\Delta \mathrm{t}}{\varepsilon(\mathrm{i,j,k})}\left[\frac{H_y^{n+\frac{1}{2}}(i,j,k) - H_y^{n+\frac{1}{2}}(i-1,j,k)}{\Delta \mathrm{x}} - \frac{H_x^{n+\frac{1}{2}}(i,j,k) - H_x^{n+\frac{1}{2}}(i,j-1,k)}{\Delta \mathrm{y}}\right] \tag{3.18}$$

The magnetic field components are updated similarly but staggered by half a time step, as follows:

$$H_x^{n+\frac{1}{2}}(i,j,k) = H_x^{n-\frac{1}{2}}(i,j,k) + \frac{\Delta \mathrm{t}}{\mu(\mathrm{i,j,k})}\left[\frac{E_z^n(i,j+1,k) - E_z^n(i,j,k)}{\Delta \mathrm{y}} - \frac{E_y^n(i,j,k+1) - E_y^n(i,j,k)}{\Delta \mathrm{z}}\right] \tag{3.19}$$

and analogously for $H_y$and H_z.

These update equations form the core of the FDTD algorithm, allowing the electric and magnetic fields to be computed iteratively in a leapfrog fashion: magnetic fields are updated at half time steps, and electric fields at full time steps.

The numerical stability of the FDTD method is governed by the Courant–Friedrichs–Lewy (CFL) condition, which determines the maximum allowable time step:

$$\Delta t \leq \frac{1}{c\sqrt{\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}\right)}} \tag{3.20}$$

where c is the speed of light in the medium.

Violating this condition leads to numerical divergence and nonphysical results. Therefore, the time step must always satisfy the Courant limit for stable simulations.

For inhomogeneous media, spatial variations in ε, μ, and $\sigma$are incorporated directly into the update equations.

When the medium parameters are time-dependent, i.e., $\varepsilon = \varepsilon(t)$and $\mu = \mu(t)$, additional terms involving their time derivatives appear. The electric field update, for instance, becomes:

$$\frac{\partial E}{\partial t} = \frac{1}{\varepsilon(t)}\left[(\nabla \times H) - \frac{d\varepsilon(t)}{dt}E\right] \tag{3.21}$$

Such formulations enable accurate modeling of time-varying plasmas and dynamic engineered complex media materials, where the electromagnetic properties change as a function of external stimuli or plasma density fluctuations.

In summary, the FDTD method transforms Maxwell’s continuous curl equations into discrete time-marching relations that describe how electric and magnetic fields evolve within a computational grid.

By using Yee's staggered lattice and central-difference discretization, the method provides a robust and explicit scheme for simulating electromagnetic wave propagation in three-dimensional, inhomogeneous, and time-varying environments.

3.2 FDTD Formulation For Time-Varying Media

In many practical applications, such as plasma environments, ionospheric propagation, and dynamically engineered complex media, material parameters such as permittivity and permeability vary with time. In such cases, Maxwell's equations must be modified to include temporal derivatives of material parameters (Kalluri, 2010).

For a medium with time-dependent permittivity and permeability, the Maxwell curl equations become:

$$\nabla \times H = \sigma E + \varepsilon(t)\frac{\partial E}{\partial t} + E\frac{\partial \varepsilon(t)}{\partial t} \tag{3.22}$$

$$\nabla \times E = -\mu(t)\frac{\partial E}{\partial t} - H\frac{\partial \mu(t)}{\partial t} \tag{3.23}$$

The additional terms represent temporal modulation and are responsible for frequency shifting and energy exchange between the electromagnetic field and the medium (Kalluri, 2010; Caloz & Deck-Léger, 2019).

The FDTD update equations are modified accordingly to include these temporal derivatives, enabling accurate modeling of plasmas and time-modulated complex media materials.

Electric field update:

$$E^{n+1}(i,j,k) = E^{n}(i,j,k) - \frac{\Delta t}{\varepsilon(i,j,k,t)}[\nabla \times H] + \frac{\Delta t}{\varepsilon(i,j,k,t)}\frac{\partial \varepsilon(i,j,k,t)}{\partial t}E^{n}(i,j,k) \tag{3.24}$$

Magnetic field update:

$$H^{n+\frac{1}{2}}(i,j,k) = H^{n-\frac{1}{2}}(i,j,k) - \frac{\Delta t}{\mu(i,j,k,t)}[\nabla \times E] - \frac{\Delta t}{\mu(i,j,k,t)}\frac{\partial \mu(i,j,k,t)}{\partial t}H^{n-\frac{1}{2}}(i,j,k) \tag{3.25}$$

Here:

- Δt is the time step,
- $\nabla \times \mathrm{E}$ and $\nabla \times \mathrm{H}$ are approximated using central differences on the staggered Yee grid,
- $\varepsilon(i,j,k,t)$ and $\mu(i,j,k,t)$ are updated at each time step based on the medium model (e.g., plasma density variation).

And here are the implementation considerations:

1. Stability:
   The CFL condition must account for the maximum wave speed over all time steps:

$$\Delta t \leq \frac{1}{c_{\max}\sqrt{\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}}} \tag{3.26}$$

where $c_{\max} = 1/\sqrt{\varepsilon_{\min}\mu_{\min}}$.

2. Temporal Derivatives of Material Parameters:
   - In many practical cases, $\partial\varepsilon/\partial t$ and $\partial\mu/\partial t$ can be computed analytically from the plasma or material model.
   - If data are discrete, finite-difference approximations may be used.
3. Boundary Conditions:
   - Time-varying media may require modified absorbing boundary conditions (ABC) or perfectly matched layers (PML) to handle reflections caused by changing material properties.
4. Computational Load:
   - Each time step now requires evaluating additional terms, increasing the computational complexity, particularly for 3-D simulations.

The FDTD method for time-varying media enables simulation of:

- Ionospheric wave propagation: modeling changes in electron density over time due to solar activity, day-night cycles, or geomagnetic storms.

- Plasma environments: examining frequency shifts and dispersion caused by temporal variations in plasma density.
- Dynamic engineered complex media: simulating engineered media with programmable permittivity or permeability.

3.3 The Advantages of the FDTD

The Finite-Difference Time-Domain (FDTD) method is one of the most widely used numerical techniques in computational electromagnetics. Its popularity arises from its simplicity, flexibility, and ability to model complex electromagnetic phenomena. Compared to other numerical approaches, such as the Finite Element Method (FEM), Method of Moments (MoM), and Transmission Line Matrix (TLM) method, FDTD offers several important advantages.

The FDTD method solves Maxwell's curl equations directly in the time domain. No matrix inversion or frequency-domain transformation is required. Electric and magnetic field components are updated explicitly at each time step. This structure allows the method to naturally model transient and pulsed electromagnetic signals. Broadband wave behavior can be captured within a single simulation. In addition, nonlinear and dispersive material properties can be incorporated directly into the update equations. As a result, the full temporal evolution of electromagnetic fields is obtained straightforwardly.

A major advantage of FDTD is its broadband capability. A single time-domain simulation contains information over a wide frequency range. By applying Fourier transform techniques to the recorded field data, frequency-domain characteristics can be extracted. These include transmission and reflection coefficients, scattering parameters, and frequency-dependent absorption behavior. Unlike frequency-domain methods, separate simulations for each frequency are not required. This feature makes FDTD especially suitable for wideband antennas, plasma environments, engineered complex media, and ionospheric propagation studies.

FDTD employs the Yee lattice, which is based on a structured Cartesian grid. Electric and magnetic field components are placed at staggered positions in space and time. This arrangement leads to simple finite-difference expressions and a clear numerical algorithm. The grid structure is easy to implement and efficient for programming in environments such as MATLAB, C/C++, and Python. For standard central-difference formulations, the method achieves second-order accuracy in both space and time.

The FDTD method is highly effective for modeling complex material properties. Lossy, dispersive, anisotropic, and nonlinear materials can be included without major modifications to the algorithm. Material parameters may vary from one grid cell to another and can also change with time. This makes FDTD particularly suitable for simulating time-varying media such as plasmas, the ionosphere, and actively controlled engineered complex media. Temporal changes in permittivity and permeability can be incorporated directly into the update equations at each time step.

FDTD provides direct access to the spatial and temporal evolution of electromagnetic fields. Field distributions can be visualized at any time step within the computational domain. This allows observation of wave propagation, reflection, transmission, and diffraction effects. Energy flow, standing waves, and resonant behavior can also be identified clearly. Such visual insight is difficult to achieve with purely analytical methods and greatly enhances the physical interpretation of simulation results.

The update equations in FDTD are explicit. This means that no system of linear equations needs to be solved. The computational process is predictable and uniform at each time step. Because of this structure, FDTD is well-suited for parallel computing. Modern implementations can efficiently use multi-core processors, graphics processing units (GPUs), and distributed computing platforms. Large three-dimensional problems can therefore be simulated with reasonable computational cost.

FDTD supports several absorbing boundary techniques for modeling open-region problems. These include Mur and Liao boundary conditions, as well as the Perfectly Matched Layer (PML) method. PML is especially effective in minimizing artificial reflections at the boundaries of the computational domain. This allows accurate simulation of radiation, scattering, and wave propagation in unbounded space.

Another important advantage of FDTD is its compatibility with hybrid simulation approaches. The method can be combined with FEM for complex geometries, MoM for large-scale scattering problems, and Particle-in-Cell (PIC) methods for plasma simulations. Ray tracing techniques can also be integrated for channel modeling applications. This flexibility extends the applicability of FDTD to a wide range of electromagnetic problems.

3.4 Limitations of the FDTD Method

Despite its many advantages, the Finite-Difference Time-Domain (FDTD) method also has several limitations. These limitations should be considered carefully when selecting FDTD for large-scale or highly complex electromagnetic problems.

One of the main limitations of FDTD is its computational cost. The method requires the discretization of the entire computational domain. As the size of the domain increases, the number of grid cells grows rapidly. This leads to increased memory usage and longer simulation times. Three-dimensional simulations are particularly demanding. High spatial resolution further increases the computational burden. As a result, FDTD may become impractical for electrically large structures or very fine geometries without access to high-performance computing resources.

The time step in FDTD is limited by the Courant–Friedrichs–Lewy (CFL) stability condition. The maximum allowable time step depends on the smallest spatial cell size and the wave propagation speed. When very small grid cells are used, the time step must be reduced accordingly. This increases the total number of time steps required for a simulation. Consequently, simulations with fine spatial resolution may become time-consuming even for moderate domain sizes.

FDTD typically uses a Cartesian grid. Curved or oblique material boundaries must be approximated using stair-step representations. This approximation introduces geometric errors known as staircasing effects. These errors can lead to inaccurate field distributions, especially near curved surfaces. While advanced techniques such as subcell modeling and conformal FDTD exist, they increase algorithmic complexity and computational cost.

Numerical dispersion is an inherent limitation of the FDTD method. The numerical phase velocity differs from the true physical phase velocity, particularly when the grid resolution is insufficient. This effect causes waveform distortion and phase errors during propagation. Numerical dispersion becomes more pronounced for high-frequency components and long propagation distances. Careful selection of grid spacing relative to the wavelength is required to minimize this error.

Low-frequency simulations require long time durations to reach steady-state behavior. Since FDTD operates in the time domain, many time steps may be needed to capture slow field variations. This makes FDTD less efficient for narrowband or low-frequency steady-state problems compared to frequency-domain methods.

Although FDTD can model dispersive materials, such implementations often require additional auxiliary equations. These increase memory requirements and computational complexity. Improper modeling of dispersion can lead to numerical instability or inaccurate results. Careful parameter selection and validation are therefore necessary when simulating strongly dispersive or resonant materials.

Absorbing boundary conditions play a critical role in FDTD accuracy. Although Perfectly Matched Layer (PML) boundaries are highly effective, their performance depends on proper parameter selection. Incorrect PML thickness or conductivity profiles may cause unwanted reflections. This can contaminate simulation results, especially in long-duration or broadband analyses.

FDTD simulations often generate large amounts of data. Storing field values over time and space requires significant memory. Post-processing tasks such as Fourier analysis, spectrogram generation, and field visualization may further increase computational load. Efficient data management strategies are therefore necessary, particularly for three-dimensional simulations.

4. NUMERICAL SIMULATIONS AND RESULTS

This chapter presents numerical results obtained using a one-dimensional (1D) Finite-Difference Time-Domain (FDTD) model. The objective is to analyze electromagnetic wave propagation in time-varying media using a simplified but physically meaningful computational framework. A 1D grid is selected to isolate temporal material effects and to clearly observe wave–medium interactions.

4.1 Wave Propagation in a Plasma Medium

This section presents numerical results obtained from one-dimensional FDTD simulations of electromagnetic wave propagation in a time-varying plasma medium. The plasma region is modeled with temporally modulated permittivity, while the surrounding space is assumed to be free space. The objective is to observe how temporal changes in the plasma parameters influence the spatial and spectral characteristics of the propagating wave.

4.1.1 Incident wave characteristics

Before interacting with the plasma region, the electromagnetic wave propagates in free space without distortion.

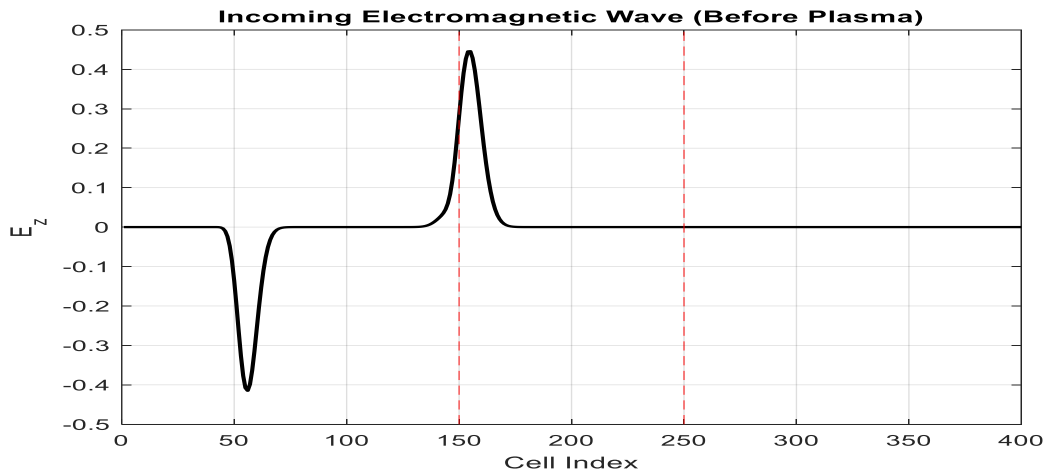

Figure 4.1: Incident electromagnetic wave propagating in free space before entering the time-varying plasma region.

Figure 4.1 illustrates the incident electric field distribution measured before the plasma interface. The wave maintains a stable amplitude and a well-defined wavelength, indicating undisturbed propagation. At this stage, no frequency modulation or amplitude variation is observed, as the medium parameters remain constant in time.

4.1.2 Field distribution inside and beyond the plasma

In Figure 4.2, the snapshot corresponds to the first stored field profile and illustrates the initial modification of the incident wave as it begins to interact with the time-dependent permittivity of the plasma.

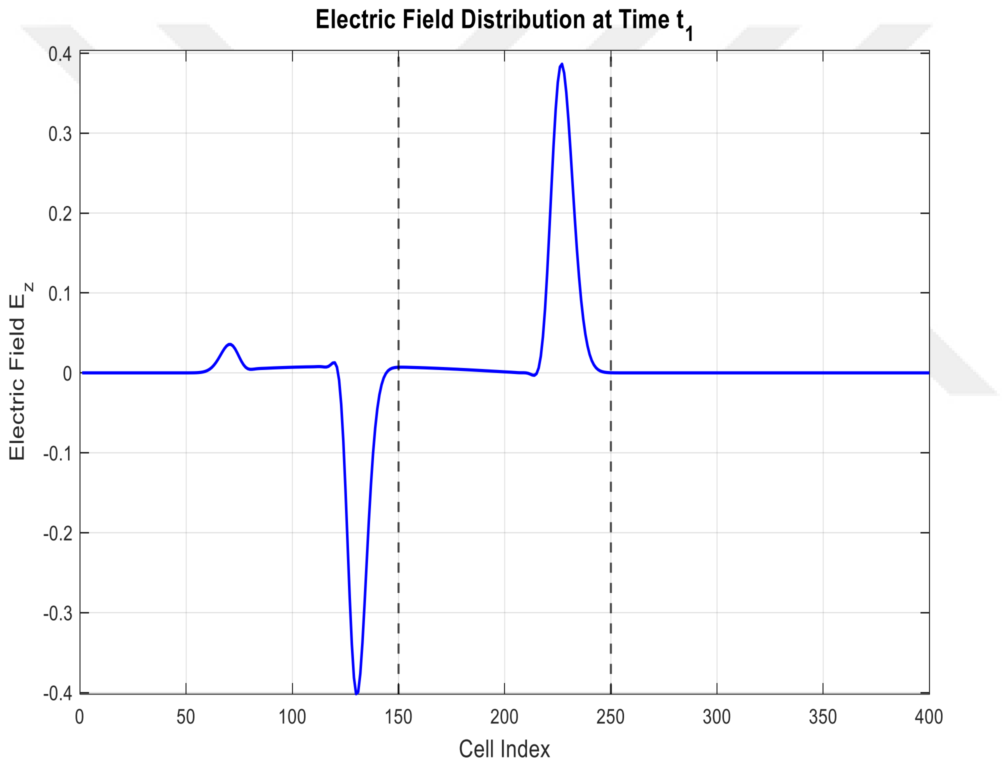

Figure 4.2: Electric field distribution during propagation inside and beyond the time-varying plasma region.

Figure 4.3 shows electric field snapshots at time instants t_1, t_2, and t_3. The three curves represent the temporal evolution of the electric field during propagation through the plasma, clearly showing waveform distortion and phase variation induced by the time-varying medium.

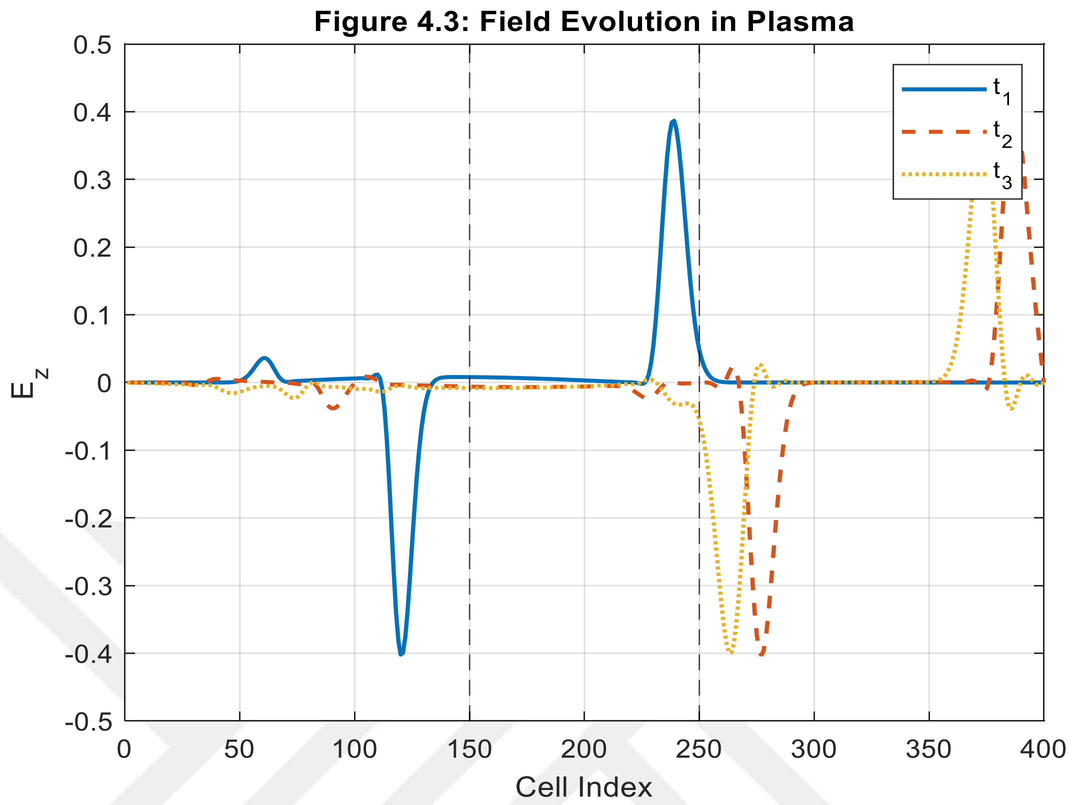

Figure 4.3: Snapshots of the electric field at three different time instants during propagation through the plasma medium.

As the wave enters the plasma region, changes in the field amplitude and phase become noticeable. This behavior results from the time-dependent permittivity, which alters the local wave velocity. Inside the plasma, the wavelength is modified, and slight amplitude redistribution is observed. After leaving the plasma region, the wave continues to propagate in free space, but its characteristics differ from those of the original incident wave.

The comparison of these snapshots clearly demonstrates that temporal modulation of the plasma parameters affects the wave continuously, rather than only at spatial interfaces.

4.1.3 Frequency shift analysis

To further analyze the effect of the time-varying plasma, frequency-domain representations of the electric field are examined. Figure 4.4 presents the FFT spectra of the field.

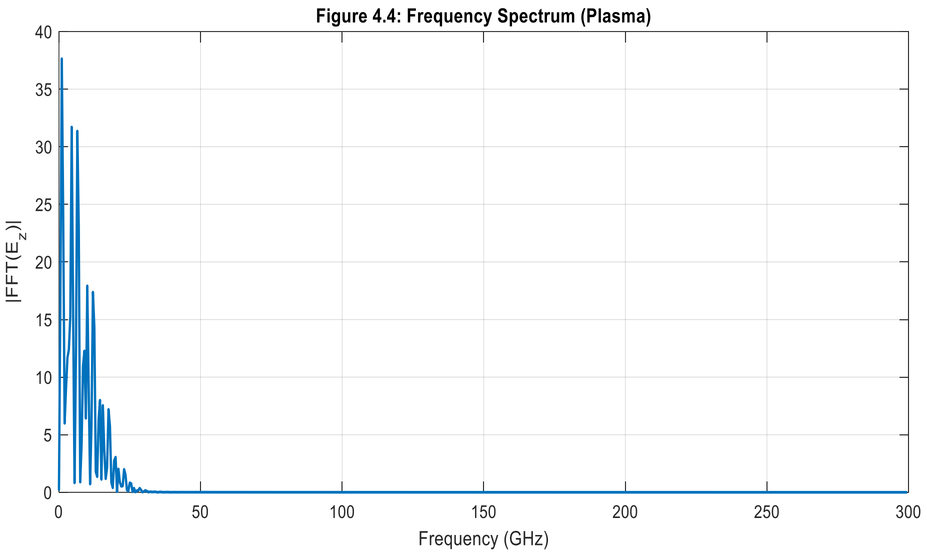

Figure 4.4: Frequency spectrum of the incident and transmitted electric field obtained using Fourier analysis.

The spectrum measured after the plasma shows a distinct shift toward higher frequencies compared to the incident wave. This frequency upshift indicates an exchange of energy between the electromagnetic field and the time-varying medium. Unlike conventional dispersive effects, this phenomenon originates from temporal modulation rather than spatial inhomogeneity.

The FFT results confirm that the plasma acts as a temporal modulator, producing frequency conversion effects that are not present in time-invariant media.

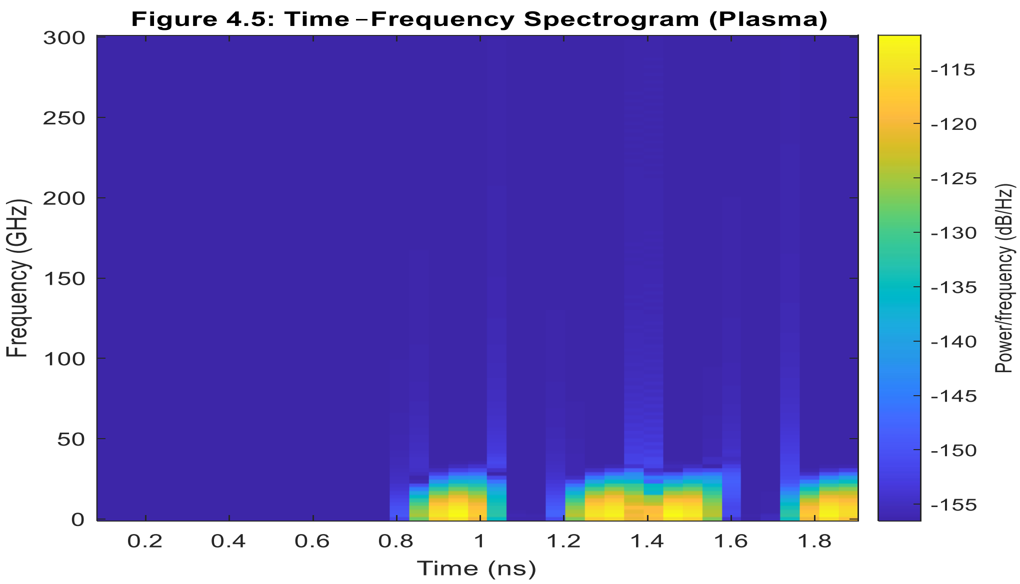

Figure 4.5: Time–frequency spectrogram of the transmitted electromagnetic wave in the plasma medium.

Figure 4.5 illustrates the time–frequency representation of the electric field recorded after the plasma region. The spectrogram reveals a gradual upward shift in frequency as the wave propagates through the time-varying plasma. This behavior confirms the occurrence of temporal refraction and energy exchange between the electromagnetic wave and the dynamic medium.

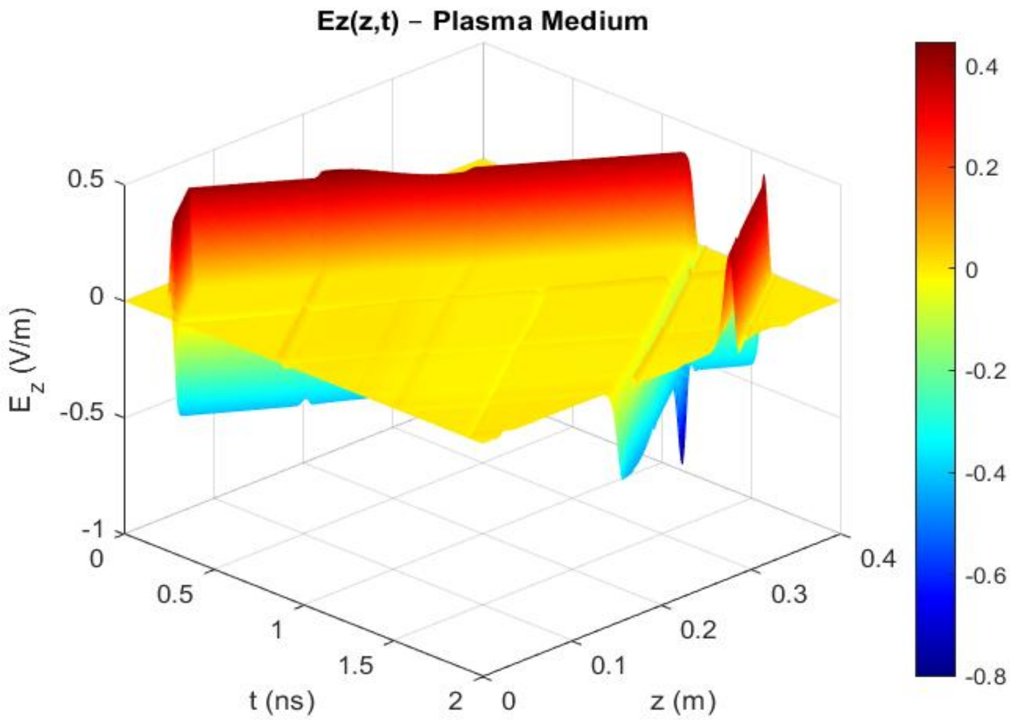

Figure 4.6: Three-dimensional distribution of the electric field component $E_z(z,t)$during wave propagation in the time-varying plasma medium.

Figure 4.6 shows the three-dimensional distribution of the electric field component, $E_x(z,t)$in the time-varying plasma medium. The figure presents how the electric field changes along the propagation direction over time. As the wave enters the plasma region, the spacing between field oscillations in time becomes smaller. This indicates a decrease in the wave period and a corresponding increase in frequency.

The frequency change does not occur at a single point. Instead, it develops gradually while the wave propagates inside the plasma. This behavior is caused by the time-dependent permittivity of the medium. Unlike spatial interfaces, the temporal variation modifies the wave frequency without changing its propagation direction. The Ex(z,t) distribution clearly shows this continuous frequency transformation. These results confirm that the plasma medium actively alters the spectral content of the electromagnetic wave through temporal interaction.

4.1.4 Discussion

The results show that time-dependent plasma parameters strongly affect the magnetic field behavior. The Hy(z,t) distribution clearly reflects the dynamic interaction between the electromagnetic wave and the plasma medium. Temporal changes in permittivity modify the magnetic field amplitude and phase as the wave propagates through the plasma region.

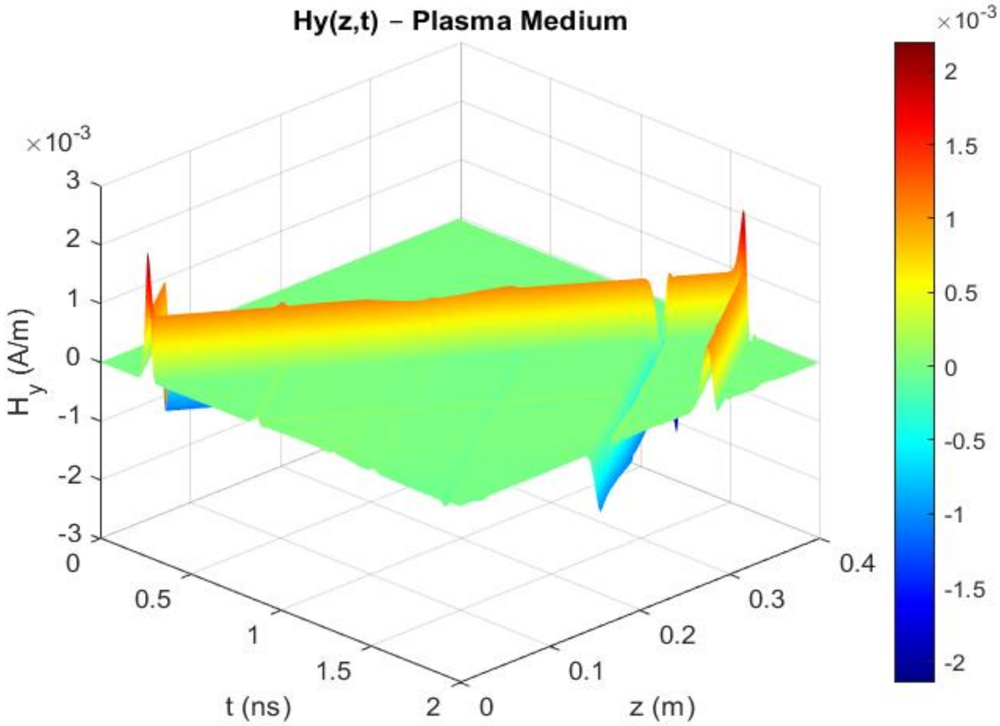

Figure 4.7 Three-dimensional distribution of the magnetic field component $H_y(z,t)$in the time-varying plasma medium.

The magnetic field also exhibits frequency upshift behavior that is consistent with the electric field response. This confirms that temporal modulation influences both field components simultaneously. The observed variations indicate continuous energy exchange between the wave and the time-varying plasma.

These effects are especially important for ionospheric propagation and plasma-based communication systems, where magnetic field behavior directly impacts wave impedance and power flow. The results demonstrate that neglecting time-varying plasma properties may lead to inaccurate predictions of wave behavior.

Overall, the Hy(z,t) results confirm that the one-dimensional FDTD model successfully captures dynamic wave–plasma interactions. They also emphasize the necessity of including time-dependent material parameters for realistic electromagnetic simulations in plasma environments.

4.2 Wave Propagation in Complex Media

Engineered complex media and nonlinear dielectric media are artificial structures whose electromagnetic behavior can be tailored through design. Their effective permittivity and permeability can be adjusted to achieve responses that do not occur in natural materials. This capability enables advanced control over electromagnetic wave propagation.

Unlike conventional dielectrics, the properties of engineered complex media materials are often not constant. External modulation, active components, or nonlinear effects may cause their electromagnetic parameters to change with time. When this occurs, wave propagation becomes a dynamic process. Classical time-invariant models are not sufficient to describe such behavior.

In this work, electromagnetic wave propagation in an engineered complex media material or nonlinear dielectric medium is investigated using a one-dimensional Finite-Difference Time-Domain (FDTD) approach. The numerical scheme is modified to include time-dependent material properties. Only the electric permittivity is allowed to vary with time, while the magnetic permeability is assumed constant. This assumption simplifies the analysis and isolates the impact of temporal permittivity variation.

The complex media region is defined over a limited portion of the computational domain. Outside this region, free-space conditions are assumed. Inside the complex media section, permittivity follows a prescribed time-dependent function. This function represents either an externally controlled modulation or an effective nonlinear response of the medium. The transition between regions is naturally handled by the FDTD update equations.

A Gaussian-modulated sinusoidal source is used to generate the incident wave. The wave propagates toward the complex media region and interacts with the time-varying permittivity. During this interaction, the temporal modulation modifies the wave phase and redistributes its spectral content. Unlike spatial interfaces, temporal variation leads to frequency conversion without altering the propagation direction.

Numerical results show clear differences between the incident and transmitted fields. Changes in amplitude and phase are observed after the wave exits the complex media region. Frequency-domain analysis reveals additional spectral components and frequency shifts. These features indicate temporal scattering caused by the time-dependent dielectric response.

A comparison with the plasma case highlights fundamental differences between natural and engineered media. In plasma, frequency variation is linked to electron dynamics and dispersion. In engineered complex media, frequency changes arise directly from designed temporal modulation. This distinction emphasizes the flexibility of artificial media in controlling wave behavior.

Time–frequency analysis confirms that electromagnetic energy is redistributed among multiple frequency components. The dominant frequency evolves as the wave propagates through the modulated region. This behavior demonstrates controlled energy exchange between the field and the medium.

The results confirm that the FDTD method is well-suited for studying engineered complex media and nonlinear dielectric environments. Its time-domain formulation captures transient effects and broadband behavior accurately. The proposed model provides a reliable framework for analyzing time-varying artificial media and supports the development of active and reconfigurable electromagnetic devices.

4.2.1 Simulation setup

In this section, electromagnetic wave propagation in a complex medium or a nonlinear dielectric medium is investigated using a one-dimensional Finite-Difference Time-Domain (FDTD) framework. The computational domain is discretized into uniform spatial cells, and the electric and magnetic field components are updated in time using Maxwell's curl equations.

The medium consists of two regions. The first region represents free space, while the second corresponds to an engineered complex media or nonlinear dielectric layer. Within the complex media region, the electric permittivity is defined as a time-dependent function. Outside this region, the permittivity remains constant and equal to the free-space value. The magnetic permeability is assumed constant throughout the domain to isolate the effects caused by temporal permittivity variation.

A Gaussian-modulated sinusoidal source is used to excite the electromagnetic wave. The source generates a broadband pulse centered at the desired carrier frequency. The wave propagates toward the complex media region, interacts with the time-varying dielectric properties, and then exits the medium. Absorbing boundary conditions are applied at the domain edges to prevent artificial reflections.

4.2.2 Incident wave characteristics

Before interacting with the complex media region, the incident electromagnetic wave propagates through free space without distortion. The field distribution in this region exhibits a stable waveform, with constant frequency and amplitude. This behavior confirms that the numerical grid and source implementation introduce no unintended modulation effects.

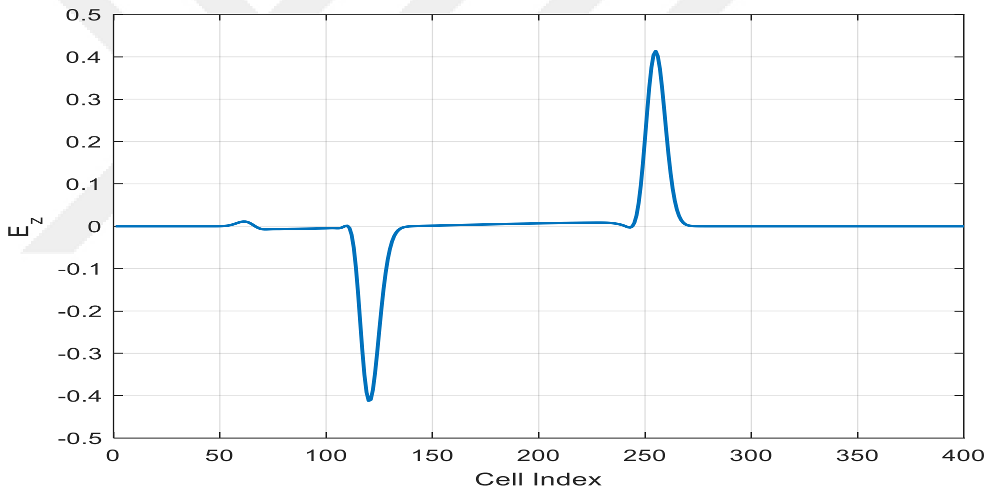

Figure 4.8: Incident electric field distribution Ez propagating toward the complex media region.

The incident wave serves as a reference for evaluating the influence of the complex media. By observing the wave before entering the nonlinear region, it becomes possible to distinguish changes caused solely by the temporal variation of the permittivity. The incoming field maintains its original spectral content and phase structure, providing a baseline for subsequent comparisons.

4.2.3 Field evolution inside the engineered complex media

As the electromagnetic wave enters the complex media region, it begins to interact with the time-dependent permittivity. The electric field distribution evolves differently from the free-space case. Temporal modulation of the material properties introduces phase variations and modifies the local wave impedance.

Snapshots of the electric field taken at different time instants reveal progressive changes in waveform structure as the wave propagates through the complex media. The field amplitude varies smoothly, and localized distortions develop inside the nonlinear region. These changes are not associated with spatial discontinuities, but rather with the temporal evolution of the material parameters.

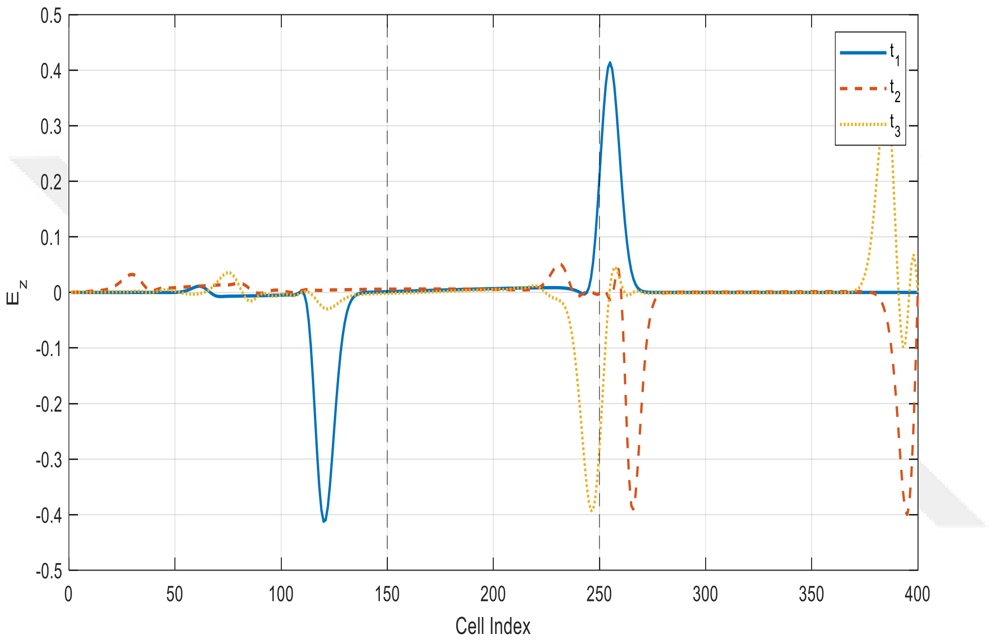

Figure 4.9: Electric field evolution Ez_snapshots inside the complex media region at different time instants.

As time advances, the wave exits the complex media region with a modified temporal structure. The transmitted field differs from the incident field in both phase and amplitude. This observation indicates that energy exchange occurs between the electromagnetic wave and the time-varying medium.

4.2.4 Frequency shift analysis

To quantify the effect of temporal modulation, a spectral analysis of the transmitted wave is performed. The electric field is recorded at a probe location beyond the complex media region. The recorded time-domain signal is transformed into the frequency domain using the Fast Fourier Transform (FFT).

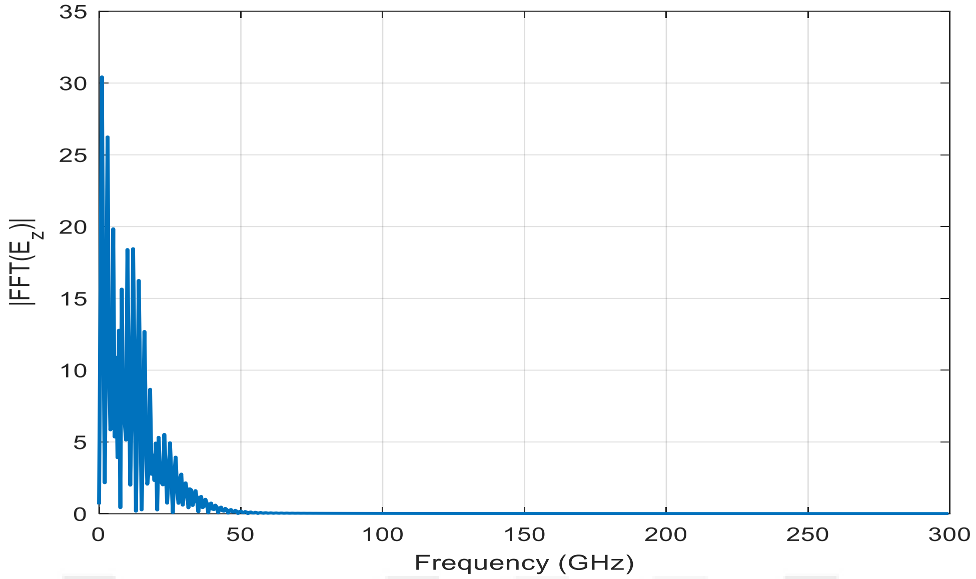

Figure 4.10: Frequency spectrum of the transmitted electric field in the engineered complex mediumderived from FFT analysis.

In addition to the frequency spectrum, the spatiotemporal evolution of the electric field is analyzed using the Ex(z,t) distribution. This three-dimensional field representation illustrates how the electric field evolves simultaneously in space and time while propagating through the time-varying complex media. The Ex(z,t) figure clearly shows that the wavefront undergoes temporal distortion within the modulated region.

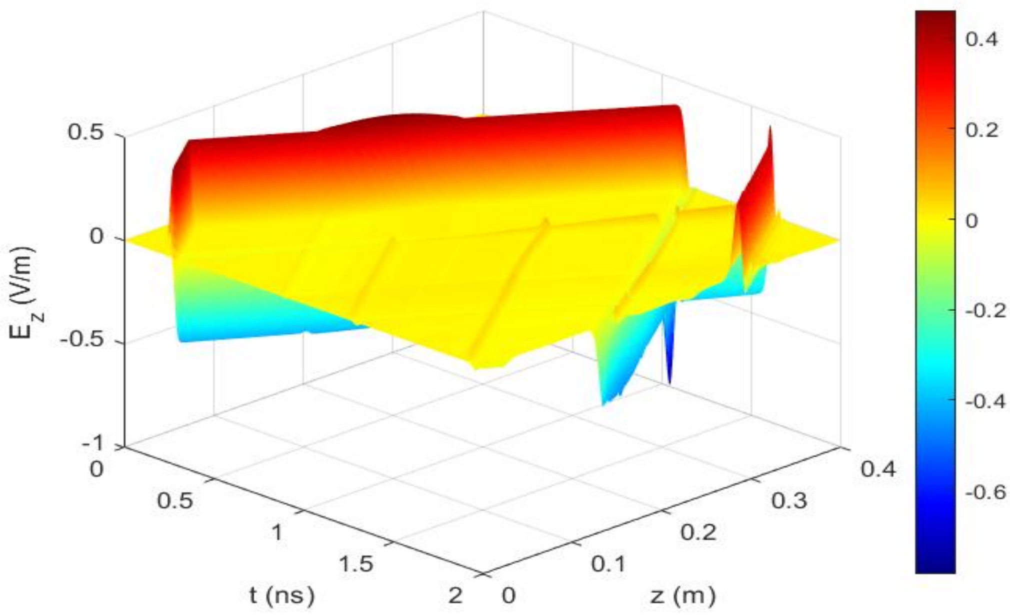

Figure 4.11: Three-dimensional evolution of the electric field component $E_z(z,t)$in the time-varying engineered complex medium.

The frequency spectrum of the transmitted signal shows a clear deviation from the source frequency. New frequency components appear, and the dominant spectral peak shifts relative to the incident wave. This behavior confirms the presence of frequency upshift induced by the temporal modulation of the permittivity.

Unlike spatial interfaces, where the frequency of the wave is conserved, temporal modulation enables direct frequency conversion without changing the propagation direction. The observed frequency shift is a signature of temporal scattering. The Ex(z,t) distribution visually supports this phenomenon by revealing gradual changes in the wave's temporal structure as it traverses the complex media region.

These results demonstrate controlled energy exchange between the electromagnetic wave and the time-varying medium. The combined frequency-domain and spatiotemporal analyses confirm that temporal modulation plays a dominant role in shaping wave behavior in an engineered complex electromagnetic structure.

4.2.5 Time–frequency behavior

To further investigate the dynamic nature of the interaction, a time–frequency spectrogram of the transmitted wave is computed. This analysis reveals how the frequency content of the wave evolves during propagation.

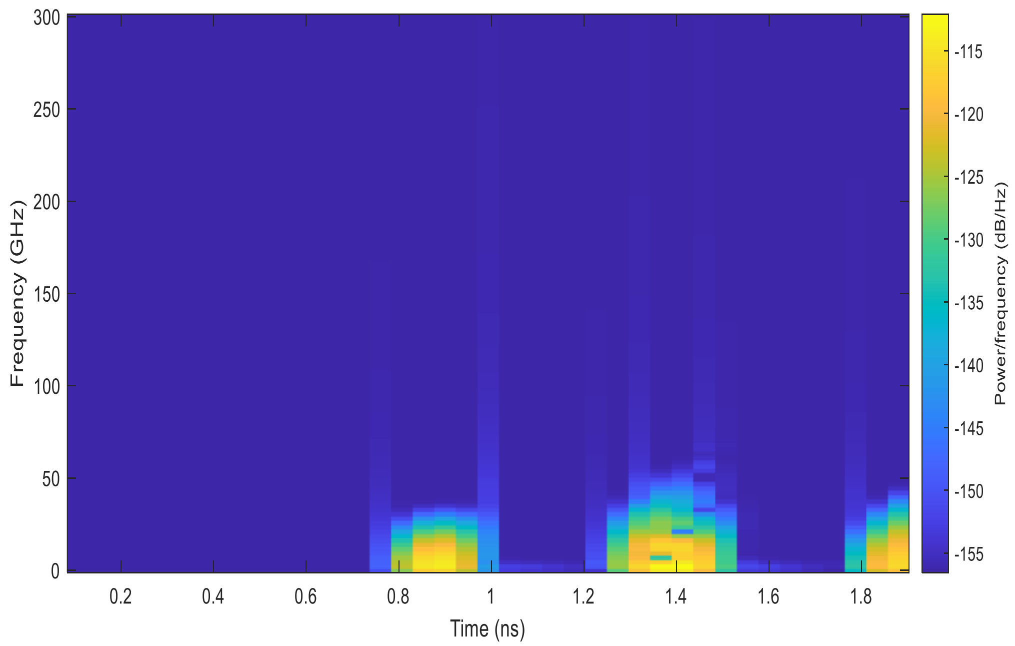

Figure 4.12: Time–frequency spectrogram of the transmitted wave in the engineered complex medium.

The spectrogram shows a gradual migration of energy from the original carrier frequency toward higher frequencies as the wave passes through the complex media region. This continuous transition confirms that the frequency shift is not instantaneous but develops over time due to sustained interaction with the temporally modulated medium.

The redistribution of spectral energy highlights the nonstationary nature of the propagation process. Such behavior cannot be captured using time-invariant material models and emphasizes the importance of time-domain numerical techniques.

4.2.6 Discussion

The Hy(z,t) distribution provides complementary insight into the magnetic field response in the time-varying medium. Unlike the electric field, the magnetic field is updated through the temporal variation of the electric field, making it an indirect indicator of time-dependent material interaction.

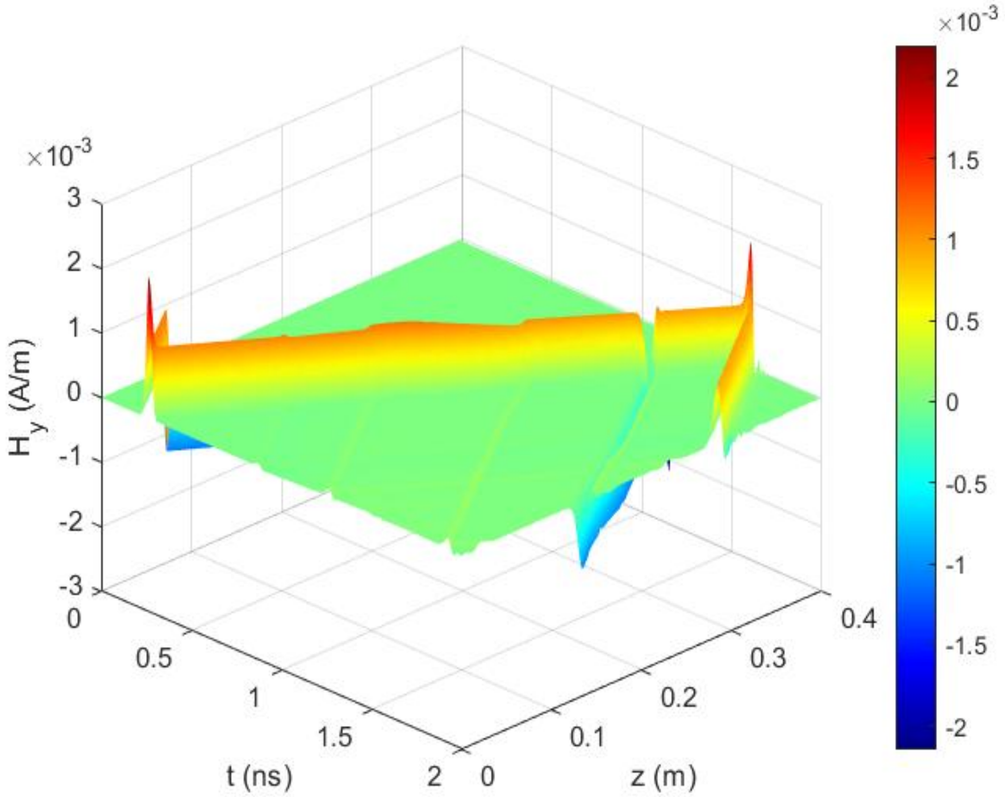

Figure 4.13: Three-dimensional distribution of the magnetic field component $H_y(z,t)$in the time-varying engineered complex medium.

The Hy(z,t) figure shows that temporal modulation of permittivity affects not only the electric field but also the magnetic field evolution. Variations in field amplitude and temporal spacing are observed as the wave propagates through the modulated region. These changes indicate that the electromagnetic energy exchange influences both field components.

The gradual distortion of the magnetic field waveform confirms that the frequency upshift observed in the electric field is not an isolated effect. Instead, it is a coupled electromagnetic phenomenon governed by Maxwell’s equations. The Hy(z,t) distribution demonstrates that temporal modulation alters the local phase velocity, leading to synchronized changes in both electric and magnetic fields.

From a physical perspective, the observed behavior reflects the conservation of electromagnetic coupling in a nonstationary medium. As the electric field undergoes temporal scattering, the magnetic field responds accordingly to maintain field continuity. This confirms the consistency and stability of the numerical model.

Overall, the Hy(z,t) results support the conclusion that time-varying material parameters modify the complete electromagnetic field structure. The combined analysis of Ex(z,t) and Hy(z,t) provides a comprehensive understanding of wave propagation in dynamic plasma and complex media environments.

4.3 Comparative Analysis and Discussion

This section presents a comparative analysis of electromagnetic wave propagation in time-varying plasma and complex (nonlinear dielectric) media. The objective is to identify the similarities and fundamental differences in wave behavior arising from natural and engineered temporal modulation mechanisms and to interpret the numerical results within the framework of time-varying media acting as electromagnetic frequency transformers (Kalluri, 2010).

In the plasma case, the temporal variation of permittivity is governed by changes in the effective electron density, which results in a smooth and continuous modulation of the medium (Ginzburg, 1970; Budden, 1985). As shown in Figures 4.3–4.5, the propagating electromagnetic wave experiences a clear frequency upshift accompanied by gradual phase variation and amplitude modulation. The time–frequency spectrogram reveals a continuous migration of spectral energy toward higher frequencies, indicating a progressive frequency transformation process distributed over time. This behavior confirms that frequency conversion in plasma media can occur without spatial discontinuities and is consistent with classical theoretical models of time-varying plasmas (Budden, 1985).

In contrast, the engineered complex medium exhibits frequency transformation through deliberately engineered temporal modulation of its constitutive parameters. The numerical results presented in Figures 4.7–4.10 demonstrate that the transmitted wave contains multiple frequency components, including distinct sidebands around the original carrier frequency. The FFT results show a broader spectral distribution compared to the plasma case, while the time–frequency spectrogram indicates abrupt temporal scattering events and dynamic redistribution of energy among frequency components. These characteristics are typical of temporally modulated complex media materials and reflect higher-order temporal harmonics generated by controlled modulation schemes (Caloz & Deck-Léger, 2019).

A direct comparison of the frequency spectra in Figure 4.11 highlights a fundamental difference between the two media. Plasma-induced frequency shifts result in a dominant spectral peak that gradually shifts toward higher frequencies, whereas engineered complex media modulation produces multiple spectral peaks due to stronger and intentionally designed temporal variations. This distinction reflects the difference between naturally occurring temporal modulation and engineered modulation, where the latter allows precise control over modulation depth, frequency, and waveform shape (Kalluri, 2010).

The three-dimensional field distributions of $E_x(z,t)$and $H_y(z,t)$provide further physical insight into the frequency transformation mechanisms. In the plasma medium, the field evolution shows smooth temporal compression of wavefronts, which is consistent with a gradual frequency upshift caused by continuous permittivity variation. In the complex media case, the field distributions exhibit localized temporal discontinuities and stronger field distortion, confirming the presence of temporal scattering and enhanced energy exchange between the electromagnetic wave and the medium. These observations align with Kalluri's interpretation of time-varying media as electromagnetic frequency transformers, where temporal modulation directly converts wave energy from one frequency to another without altering the propagation direction (Kalluri, 2010).

From an application perspective, the plasma results are particularly relevant to ionospheric propagation, satellite communication links, and radar systems operating in naturally time-varying environments (Ginzburg, 1970). The engineered complex media results, on the other hand, demonstrate the potential for engineered frequency conversion, adaptive signal processing, and reconfigurable electromagnetic devices enabled by programmable temporal modulation (Caloz & Deck-Léger, 2019).

Overall, this comparative study demonstrates that both plasma and engineered complex media can function as effective electromagnetic frequency transformers, although through fundamentally different mechanisms. Plasma media produce smooth and passive frequency conversion driven by natural temporal variations, while engineered complex media enable active and controllable frequency transformation through engineered modulation. The combined use of one-dimensional FDTD simulations, FFT-based spectral analysis, time–frequency spectrograms, and three-dimensional field visualizations provides a robust numerical framework for analyzing electromagnetic wave propagation in time-varying media (Taflove & Hagness, 2005; Archambeault, 2017).

Table 4.1: Comparative Summary of Electromagnetic Wave Propagation in Time-Varying Plasma and Engineered Complex Media

Feature/ Metric	Plasma Medium	Complex Media / Nonlineer Dielectric Medium
Nature of Medium	Naturally occurring, time-varying ionized medium	Artificially engineered medium with designed temporal modulation
Source of Temporal Variation	Natural variation of electron density due to ionization processes	Deliberate temporal modulation of permittivity
Permittivity Behavior, ε(t)	Smooth, continuous sinusoidal variation	Engineered modulation with controllable depth and frequency
Permeability, μ	Constant	Constant (assumed for numerical stability)
Primary Frequency Effect	Gradual frequency upshift	Strong frequency conversion with multiple sidebands
Spectral Characteristics (FFT)	Single dominant peak shifted to higher frequency	Multiple spectral peaks and broadened frequency content
Time–Frequency Behavior	Smooth and continuous frequency evolution	Dynamic temporal scattering and sideband generation
Field Evolution (Ex(z,t), Hy(z,t))	Gradual temporal compression of wavefronts	Localized temporal distortion and enhanced field modulation
Energy Exchange Mechanism	Passive energy redistribution driven by natural temporal variation	Active and controllable energy exchange enabled by temporal modulation
Frequency Transformer Interpretation	Acts as a passive electromagnetic frequency transformer	Acts as an engineered electromagnetic frequency transformer
Predictability and Control	Limited control due to natural variability	High controllability through design parameters
Numerical Observations (FDTD)	Stable propagation with smooth spectral shift	Strong spectral complexity and enhanced modulation effects
Primary Applications	Ionospheric propagation, satellite communication, and radar systems	Frequency conversion, adaptive signal processing, reconfigurable EM devices

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5. CONCLUSIONS AND FUTURE WORKS

This thesis investigated electromagnetic wave propagation in time-varying media using the Finite-Difference Time-Domain (FDTD) method. By extending the conventional FDTD formulation to include temporally varying permittivity, wave–medium interactions in both plasma and engineered complex media were successfully modeled. The numerical results demonstrate that temporal modulation leads to frequency upshift, phase variation, and energy redistribution, which are clearly observed through time-domain field evolution, FFT-based spectral analysis, and time–frequency spectrograms. The findings confirm that frequency is not conserved in time-varying media and that electromagnetic waves can exchange energy with the medium during propagation.

The results further support the interpretation of time-varying media as electromagnetic frequency transformers**,** as described in Kalluri's theoretical framework. Plasma media act as naturally varying frequency transformers, while complex media enable engineered and controllable frequency conversion. These insights are directly relevant to applications in ionospheric propagation, satellite communications, radar systems, and reconfigurable electromagnetic devices. Future work may focus on incorporating fully dispersive plasma models, time-varying permeability, higher-dimensional simulations, and experimental validation to further explore and exploit temporal modulation for advanced electromagnetic system design.

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