

İSTANBUL TEKNİK ÜNİVERSİTESİ ★ FEN BİLİMLERİ ENSTİTÜSÜ

**COMPARISON OF SINGLE STATE DYNAMIC
FRICTION MODELS AND FRICTION OBSERVERS**

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Anabilim Dalı : KONTROL VE BILGISAYAR MÜHENDİSLİĞİ

Programı : KONTROL VE BILGISAYAR MÜHENDİSLİĞİ

ARALIK 2002

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**Tezin Enstitüye Verildiği Tarih: 24 Aralık 2002
Tezin Savunulduğu Tarih: 15 Ocak 2003**

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ARALIK 2002

ÖNSÖZ

Bu tez çalışmasının hazırlanmasında bana her konuda destek veren sevgili tez danışmanım Doç.Dr.Hakan Temeltaş'a, tez çalışmasındaki deneyimlerini bana aktaran Y.Müh. Murat Aksakal'a ve manevi destekleriyle her zaman yanımda olan aileme teşekkür ederim.

Gürcan AKTAŞ

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Symbol List

x_b	: break point position
k_t	: hardness coefficient
F_b	: called break away force
$\dot{x}$	: Velocity.
σ_2	: Viscous friction coefficient
σ_0	: Surface hardness coefficient in static region
σ_1	: Damping factor
J	: Inertia
F_C	: Coulomb friction
F_G	: Friction caused by gravity
F_v	: Viscous friction
F_e	: Applied force
F_S	: Stiction force
$F(x(t))$	: Instantaneous friction force
$F_{s,a}$	: Stribeck value of the last slip movement
$F_{s,\infty}$	: Steady state Stribeck value
V_s	: The characteristic velocity of Stribeck friction
τ_l	: Time constant of the friction lag

γ	: Transient condition parameter of the rising friction
t_d	: Spent time in the stick region
σ	: Stiffness coefficient
x	: Position
z	: The average bristle deviation (elastic displacement)
ω	: Plastic displacement
J_d	: Motor and incremental encoder inertia
F_l	: External forces
M_d	: Motor torque
φ_d	: Drive side angular displacements
φ_l	: Load side angular displacements
Ω_d	: Drive side angular velocity
Ω_l	: Load side angular velocity
J_l	: Screw, encoder and carriage inertia
c_{dl}	: Spring stiffens
b_{dl}	: Damping constant
b_d	: Drive side damping constant
b_l	: Load side damping constant
M_{fl}	: Internal load side friction torque

ÖZET

Bu tez çalışması iki bölümden oluşmaktadır. Birinci bölümde sürtünmenin ne olduğu, nasıl modellendiği hakkında bilgi verilmiştir. Coulomb ve vizkoz sürtünme modelleri gibi en temel statik sürtünme kuvvetinden başlanıp Karnopp, Dahl, LuGre ve Elasto-Plastik sürtünme modellerine kadar geniş çapta birçok statik ve dinamik sürtünme modeli incelenmiştir. Ayrıca yapışma, Stribeck etkisi, sürtünme gecikmesi, kayma öncesi hareket, hız değişim noktalarında sürtünme davranışları incelenmiştir. Birinci bölümün son kısmında da dinamik modellerden Lugre ve Elasto-Plastik sürtünme modelleri daha ayrıntılı olarak incelenmiş ve çalışma simulasyon ve gerçek elektro-mekanik sistem üzerindeki deneylerin sonuçlarının karşılaştırılması ile sonlandırılmıştır. Bu karşılaştırmada Elasto-Plastik sürtünme modelinin gerçeğe daha yakın sonuç verdiği ortaya konulmuştur. Elektro-Mekanik sistemde gerçekleştirilen deneylerin doğruluk seviyesini arttırmak için sürekli durum testi, PRBS testi ve Dahl eğrisi testi gerçekleştirilerek sistem parametrelerini tanımlama çalışmaları yapılmıştır.

İkinci bölümde Elasto-Plastik sürtünme modelinin daha doğru sonuçlar verdiği kabul edilerek sürtünme gözleyicileri karşılaştırılmıştır. Bu çalışma için Luenberger tipi gözleyici ile Ayırık tip gözleyici ele alınmıştır. Her iki gözleyici de öncelikle Matlab/Simulink ortamında gerçekleştirilip daha sonra da elektro-mekanik sistem üzerine uygulanmıştır. Simulasyon ve gerçek sistem test sonuçları karşılaştırmalı olarak verilmiştir.

Bu çalışmada simulasyon ve modelleme ortamı olarak Matlab/ Simulink yazılımı, gerçek sistem uygulamaları için dSPACE DS1104 DSP kartı, fırçasız doğru akım motoru ve sürücü donanımı ile ControlDesk isimli gerçek zaman veri yakalama yazılımı kullanılmıştır. Simulasyon ve gerçek sistem sonuçları karşılaştırıldığında, Ayırık tip gözleyici kullanıldığında elastik yer değiştirme parametresi z 'nin gözlenen değerlerinin simulasyon değerlerine daha yakın olduğu görülmüştür. Fakat Ayırık tip gözleyicide hızın gözlenmesi mümkün olmadığından hız ölçülen pozisyon değerinin türevi alınarak elde edilmiştir, bu da gerçek sistemde hızın olduğundan daha yüksek bir şekilde elde edilmesine sebep olmuştur. Luenberger tipi gözleyicide de elastik yer değiştirme için benzer sonuçlar elde edilmesine rağmen gözlenen z değerlerinde parazitik etki gösteren gürültüler görülmüştür.

SUMMARY

This thesis study contains two main chapters. In the first chapter, detailed information was given about friction. Beginning with basic static friction models like Coulomb and viscous friction models, variety of different static and dynamic friction models were studied. Moreover, different friction behaviors like Stiction, Stribeck effect, Friction lag, Presliding movement, friction at velocity reversal point were examined. At the end of chapter one, LuGre and Elasto-Plastic dynamic friction models were examined in detail, and comparing the results of simulations and experiments on the real electro-mechanical system, the study was ended. In that comparison, it was stated that Elasto-Plastic friction model gives closer results to the real system results. In order to increase the accuracy of electro-mechanical system experiments, applying Steady State test, PRBS test and Dahl curve test, a system parameter identification study was held.

In the second chapter, assuming Elasto-Plastic friction model gives better results, the friction observers were compared. For this study, Luenberger-like observer and Discrete observer were used. After realizing both of the observers in Matlab/Simulink simulation environment, they were applied to the real electro-mechanical positioning system. Simulation and real system results were given with comparison.

In this study, Matlab/Simulink software was used as simulation and modelling environment, dSPACE DS1104 DSP card, brushless D.C. motor hardware and ControlDesk tracing and capturing software were used as real-time system applications. When the simulation and real system results were compared, it was seen that observed z elastic displacement values in real system were closer to observed values in simulation if we use Discrete observer. However, since it is not possible to observe velocity in Discrete observer, the velocity was derived from the measured position value, and this caused the velocity to be estimated bigger than its real value. Although the similar results were obtained for z elastic displacement using Luenberger-like observer, it was seen that there are noises on the observed z values.

1. Introduction

The friction force, which occurs inevitably between machine parts in the mechanical systems, limits the performance considerably in precise motion control. It is obvious that friction effect introduces non-linear dynamics, while the system operates in low velocities or includes sing changes of velocity also called velocity reversal operations. It basically causes tracking errors, Stribeck effect and stick-slip movement in motion control systems [1-2]. When the system operates in high velocities, the friction effect becomes simple pure damping and it can be represented by a coefficient called viscous damping.

One of the ways to minimise the friction effect is to reduce actual friction by choosing proper lubricants or to use better materials in mechanical bearings in the system. Although those methods reduce the friction effect, they do not remove it completely. Thus, an efficient approach to solve the problem can be found in control engineering domain by introducing feedback control systems compensating undesired effect of the friction. It is possible to find, by far, some certain amount of studies carried out on modelling, observing and compensating of the friction

2: Friction physics and modeling

2.1- Fundamentals of friction

Friction is a force that occurs between two surfaces in contact. Those surfaces, either moving or motionless, are always affected different kinds of friction. Surfaces of all the materials are formed of microscobical protrusions called asperit. See figure 1.

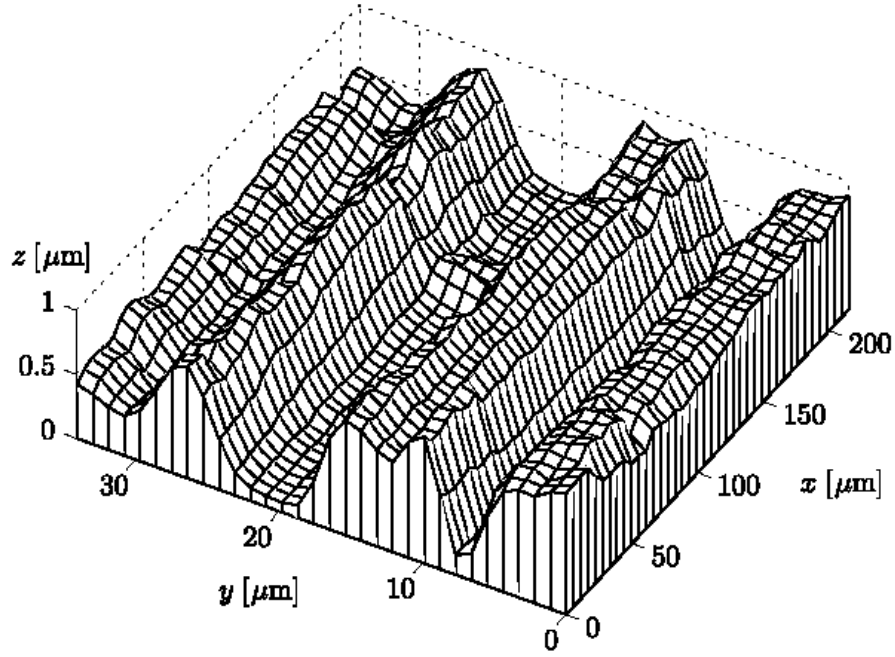


Figure 1: Three dimension surface model

When the surfaces are motionless, those asperits on the surfaces penetrate into each other. Friction is, simply, asperits' grabbing and stiction force. At the beginning of the movement, firstly, asperits bend and allow the movement, but their grabbing force still exists. Meanwhile, the more asperits bend during the beginning of the movement, the more energy is stored in asperits, similarly to spring characteristic. That kind of movement continues until asperits elasticity limit (break point) is exceeded. See figure 2. This force (F_b) is called break away force. Thus, if we call x_b as break point position, where k_t is hardness coefficient.

$$k_t = \frac{F_b}{x_b} \quad (2.1)$$

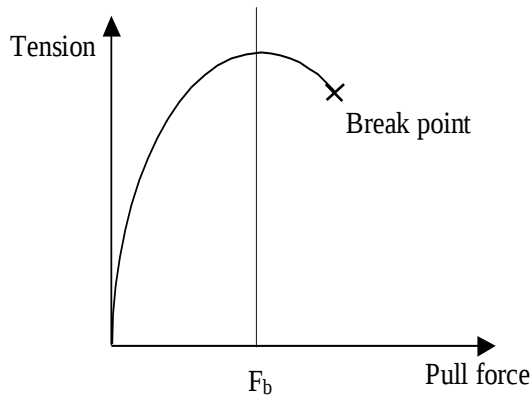


Figure 2: Asperits' pull force-tension characteristic curve

The situation of the friction that happens before the break away occurrence is called “stiction friction” or “dynamic friction”. After the break away occurs the situation of the friction is called “static friction”.

Lubricated contacts friction force

The lubrication between the surfaces affects the friction force characteristic. The lubricant takes place between the surfaces as a thin surface film. The behavior of the friction force was examined by Stribeck. He proved that the friction force at very low speeds decreases exponentially until a certain velocity is reached and from this point, the friction force increases proportionally to velocity.

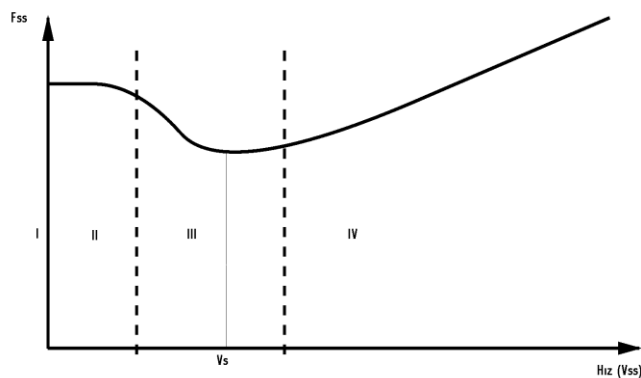


Figure 3: Friction- velocity curve of the lubricated surfaces

As it can be seen in figure 3, the friction force between lubricated surfaces should be 4 different type of behaviour

I- Boun Lubrication

As there isn't a full lubricant layer at low speeds, there is contacts between the asperits partially.

II- Partial Fluid Lubrication

In this region, There is a lubricant surface film, but it isn't thick enough to remove the asperits contacts. Therefore, the fluid lubricant film and the asperits elastic anda plastic deformation tensions shares the load force. The more the velocity is high, the thicker the lubricant film is. That situation causes the friction force to decrease, because the friction force between lubricant film and asperits is lower than the one between the asperits each other. This situation is called "Stribeck effect".

III- Full Lubrication

In this region the thickness of the fluid lubricant layer is enough to remove asperits contacts, therefore only viscous friction force can be mentioned. That viscous friction force increases proportionally to velocity.

Mathematical Model of Stribeck Effect

Stribeck effect can be mathematically modelled in order to analyse the system tracking performance and stability. The equation between velocity and friction force in steady state characteristic can be given as:

$$F(\dot{x}) = \sigma_2 \dot{x} + \sigma_0 g(\dot{x}) \operatorname{sgn} \dot{x} \quad (2.2)$$

that $\dot{x}$, σ_2 , $\sigma_0 g(\dot{x})$ stand for, velocity, viscous friction coefficient and non-linear part of the velocity-friction force characteristic, respectively. See figure 4.

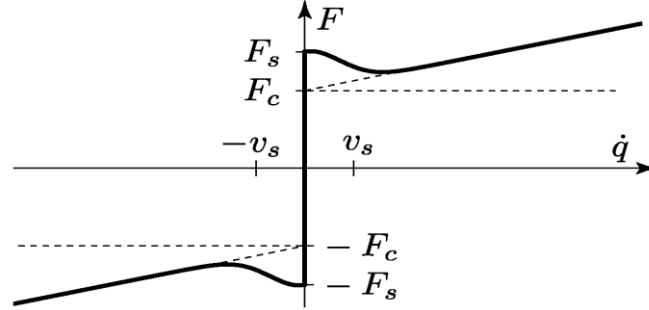


Figure 4: Friction-Velocity relationship

Some mathematical formulation can be seen related to non-linear part of the velocity-friction characteristic $\sigma_0 g(\dot{x})$ can be given as below:

Type of the formulation	non-linear $\sigma_0 g(\dot{x})$
Linear	F_c
Piecewise Linear	$F_S - \text{sat}(R \dot{x} , F_S - F_C)$
Exponential	$F_C + (F_S - F_C)e^{- \dot{x} /v_s}$
Gaussian	$F_C + (F_S - F_C)e^{-\left(\frac{\dot{x}}{v_s}\right)^2}$
Generalised Exponential	$F_C + (F_S - F_C)e^{-\alpha \dot{x} ^\delta}$
Laurentzian	$F_C + (F_S - F_C)\frac{1}{1 + (\dot{x}/v_s)^2}$

Frictional Lag

It is also called friction memory. When the velocity or the load in the normal direction is changed, friction is changed, too. However, the friction changes with delay. This means that the friction force doesn't depends on only the instantaneous velocity and load values, but also it depends on the former values of them.

Pre-Sliding Displacement

Pre-Sliding displacement occurs just before the sliding begins. It is caused by the elastic or plastic deformations of the asperities.

2-2 Friction Models

All the modelling studies of the friction can be classified in two groups: static models and dynamic models. Static models are the static functions of the friction force and the stable velocity. As those models can't model the observed friction effects, dynamic models are needed and studied. Furthermore a friction model should have some features as below:

1. Since friction dissipates energy the model should be dissipative.
2. It must be easy to perform identification of the model parameters.
3. The friction model together with the equation of motion (including control feedback) must constitute a well posed set of equations.
4. The model shall include the Coulomb friction as a special case.
5. The model must be simple enough in order to be used in real time algorithms.

2.3 Static Friction Models

2.3.1 Classical Friction Models

The classical friction models include different kind of friction components. All those friction components model different features of the friction. Basically, friction force is known as an opposite force to movement the direction. . However, it is on the movement direction when the moving surfaces begin the movement which is called the presliding movement. The friction generally absorbs the energy, but it also can store the energy by means of the bend of the asperities in the presliding movement region. This feature of the friction can not be modelled by the static friction modelling approaches, but the dynamic friction approaches.

$F = F_C \text{sgn}(v)$ The most primitive static friction model is Coulomb Friction Model (see Figure 5-a) that models the friction as the friction force F_C is proportional to the load force ($F_C = \mu F_N$).

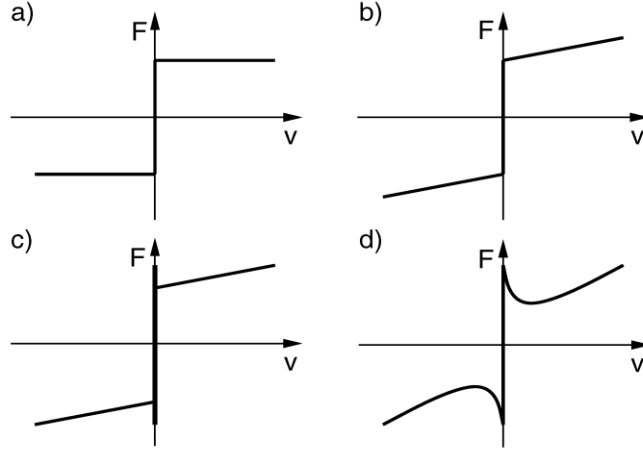


Figure 5: Basic friction models.

This model doesn't have a definition of the friction when the velocity is zero. The friction force changes its direction according to opposite of the movement direction. The friction force that is between the lubricated surfaces can be

$$F = F_v v + F_C \quad (2.3)$$

modelled by Coulomb Friction added by viscous friction (see, Figure 5-b). Stiction is one of the static components of the friction and defined in zero velocity condition. Furthermore, value of the stiction friction is higher than the value of Coulomb Friction. Stiction absorbs the energy and doesn't allow the subject to move unless maximum stiction value is exceeded.(see Figure 5-c). This model doesn't have a definition of the friction when the velocity is zero, either. Stiction friction can be formulated as follow:

$$F = \begin{cases} F_e & v = 0 \quad \text{and} \quad |F_e| < F_s \\ F_s \text{sgn}(F_e) & v = 0 \quad \text{and} \quad |F_e| \geq F_s \end{cases} \quad (2.4)$$

In this formulation, F_e , F_s , F stand for applied force, stiction force and friction, respectively. It is observed that the friction force doesn't drop down from the stiction to Coulomb friction level, therefore a new non-linear velocity dependent model was introduced by Stribeck (see Figure 5-d). A general friction model which consists of the several classical friction models can be given as:

$$F = \begin{cases} F(v) & v \neq 0 \\ F_e & v = 0 \text{ and } |F_e| < F_s \\ F_s \operatorname{sgn}(F_e) & v = 0 \text{ and } |F_e| \geq F_s \end{cases} \quad (2.5)$$

These kinds of the models have been used for modelling the friction force, however they are incapable of modelling the dynamic features of the friction.

2.3.2 Karnopp Friction Model

One of the most important reasons which prove that the classical friction models are incapable is their insufficiency to model the friction when zero velocity. A model which includes zero velocity region has been introduced by Karnopp.

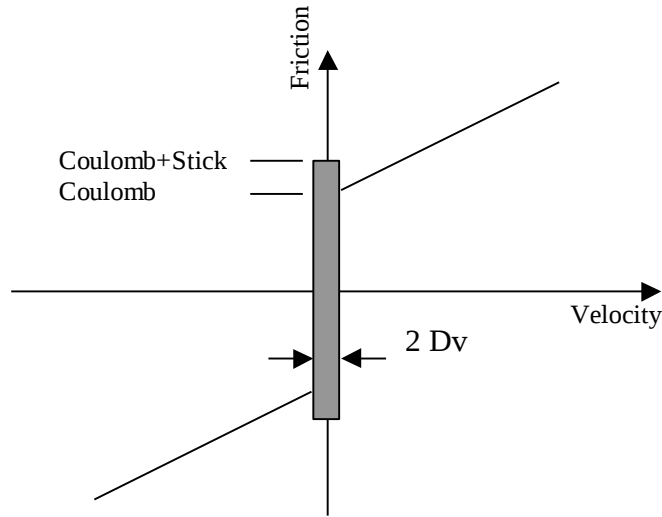


Figure 6: The velocity-friction curve of Karnopp friction model

Karnopp model can be explained mathematically as below:

$$F_m(v(t), F_a(t)) = \begin{cases} -\operatorname{sgn}[F_a(t)] \max[|F_a(t)|, (F_c + F_s)] & |v(t)| < Dv \\ -\operatorname{sgn}[v(t)] F_c + \alpha v(t) & |v(t)| \geq Dv \end{cases} \quad (2.6)$$

$F_a(t)$, $F_m(t)$ and α stand for applied force, modelled friction force and viscous friction coefficient. The zero velocity condition (or close to zero) is defined in Dv interval. Outside of that interval, friction is a function of the velocity and can be defined by one of the other static models. The friction force inside the Dv interval is defined as reverse direction of the applied force.

2.3.3. Armstrong Friction Model

Armstrong friction contains Coulomb and viscous frictions included some of the dynamic features of the friction. The model is defined separately in stick and slip regions.

$F(x) = \sigma_0 x$; Stick region

$$F(v, t) = \left(F_c + F_s(\gamma, t_d) \frac{1}{1 + \left(\frac{v(t - \tau_l)}{v_s} \right)^2} \right) \text{sgn}(v) + F_v v \quad ; \text{ Slip region} \quad (2.7)$$

The model includes Coulomb, viscous, Stribeck effect and friction lag in slip region. The rising static friction force is defined in Armstrong model as:

$$F_s(\gamma, t_d) = F_{s,a} + \left(F_{s,\infty} - F_{s,a} \frac{t_d}{t_d + \gamma} \right). \quad (2.8)$$

All the parameters are described as below:

$F(x(t))$: Instantaneous friction force	F_c : Coulomb friction force
F_v : Viscous friction force	F_s : Stiction friction
$F_{s,a}$: Stribeck value of the last slip movement	$F_{s,\infty}$: Steady state Stribeck value
σ_0 : Surface hardness coefficient in static region	
V_s : The characteristic velocity of Stribeck friction	
τ_l : Time constant of the friction lag	
γ : Transient condition parameter of the rising friction	
t_d : Spent time in the stick region	

2.4 Dynamic Friction Models

One of the first dynamic friction models was introduced by Dahl in 1976. This model is based on the stress-strain curve, which occurs dry surface contacts. That stress-strain curve is modelled by differential equations given as below:

$$\frac{dF}{dx} = \sigma \left(1 - \frac{F}{F_c} \text{sgn } v \right)^\alpha \quad (2.9)$$

x , F , σ , α stand for position, friction, stiffness coefficient, stress-strain curve parameter respectively. Since the friction force can never exceed the F_c coulomb friction force in this model, initial value of F is lower than F_c $|F(0)| < F_c$.

The friction force depends on the position and the direction of the velocity. Dahl model can be defined in time domain as below:

$$\frac{dF}{dt} = \frac{dF}{dx} \frac{dx}{dt} = \frac{dF}{dx} v = \sigma \left(1 - \frac{F}{F_c} \text{sgn } v \right)^\alpha v \quad (2.10)$$

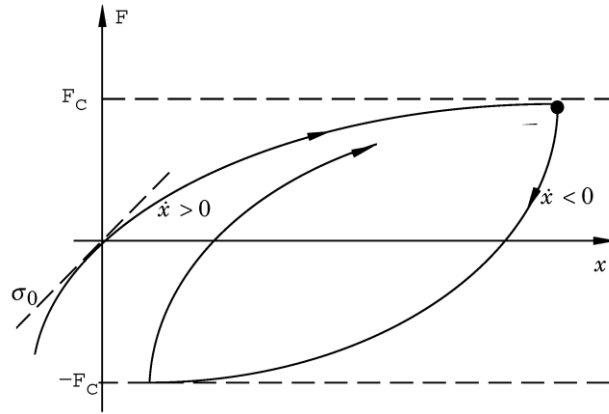


Figure 7: Dahl friction model

This mathematical equation can give coulomb friction dynamically, however Stribeck effect, dependently stiction friction and (because of the independence from the velocity) the friction lag can not be modelled by Dahl model. Therefore, improved dynamic models based on Dahl model have been introduced.

Another dynamic friction model, which is based on bristle method was introduced by Haessig and Friedland in 1991. In this modelling, in order to simulate the irregularity of the contacting surfaces, number of the contacting points and their location are distributed randomly. When the contacting surfaces make a movement in opposite direction to each other, the stress on the contacting points increases and causes the bristles to behave in spring characteristic, and results in rising friction force. This force can be given as below:

$$F = \sum_{i=1}^N \sigma_0 (x_i - b_i) \quad (2.11)$$

N , σ_0 , x_i , b_i stand for number of bristles, hardness factor of the bristles, position of the bristles, locations of the contacting points, respectively. $|x_i - b_i| = \delta_s$. The complexity of the model depends on the number of the bristles on the surface. However, this model is not appropriate for the simulations because of its complexity. Since there is no damping factor in modelling the bristles, the movement in the stick region oscillates.

Moreover, Bliman-Sorine dynamic friction modelling method is one of the recent models, which is independent from the velocity. In this friction model, the magnitude of the friction force depends on the direction of the velocity and the state variable that is called “s”.

$$s = \int_0^t |v(\tau)| d\tau \quad (2.12)$$

The friction force depends on only the position, not the velocity. The model can be shown as linear system equations as below:

$$\begin{aligned} \frac{dx_s}{ds} &= Ax_s + Bv_s \\ F &= Cx_s \end{aligned} \quad (2.13)$$

Since v_s variable is $\text{sgn}(v)$, it is enough to determine the direction (sign) of the velocity for input signal. Bliman-Sorine model can be defined in different complexity. First degree model can be given as below:

$$A = -\frac{1}{\varepsilon_f} \quad B = \frac{f_1}{\varepsilon_f} \quad \text{ve} \quad C = 1$$

Organising the model:

$$\frac{dF}{dt} = \frac{dF}{ds} \frac{ds}{dt} = |v| \frac{dF}{ds} = \frac{f_1}{\varepsilon_f} \left(v - |v| \frac{F}{f_1} \right) \quad (2.14)$$

As it is seen, first degree Bliman-Sorine model is similar to Dahl model.

$$F_c = f_1, \quad \sigma = \frac{f_1}{\varepsilon_f} \quad \text{ve} \quad \alpha = 1$$

However, the first degree Bliman-Sorine model isn't capable of modelling the stiction friction and break-away friction, therefore the second order model can be applied.

$$A = \begin{pmatrix} -\frac{1}{\eta\varepsilon_f} & 0 \\ 0 & -\frac{1}{\varepsilon_f} \end{pmatrix} \quad B = \begin{pmatrix} \frac{f_1}{\eta\varepsilon_f} \\ -\frac{f_2}{\varepsilon_f} \end{pmatrix} \quad (2.15)$$

$$C = (1 \quad 1)$$

This model is obtained by adding a first order slow Dahl model to a first order fast Dahl model. The fast model forms the stiction force and the friction force of the slow one is derived from the fast model. The both of the models are dissipative.

2.4.1 LuGre Friction Model

LuGre friction model has been derivated from the bristle approach. The friction force is derived from the average elastical deviation of the bristels. When a tangential force is applied to the object, the bristels on the contacting surface bend like a spring. If the break point is exceeded, the bristles begin to slip. In steady state condition, the average bristle deviation can be determined by the velocity; the more velocity is high the more the average bristle deviation gets bigger. LuGre friction model also models the Stribeck effect including breakaway and frictional lag phenomenas and can be defined as below:

$$\frac{dz}{dt} = v - \sigma_0 \frac{|v|}{g(v)} z \quad (2.16)$$

$$F = \sigma_0 z + \sigma_1(v) \frac{dz}{dt} + f(v) \quad (2.17)$$

Here, “z” is the average bristle deviation, when the model is linearised:

$$\frac{d(\delta z)}{dt} = \delta v, \quad (2.18)$$

$$\delta F = \sigma_0 \delta z + (\sigma_1(0) + f(0)) \delta v$$

σ_0 , $\sigma_1(v)$ stand for the hardness factor of the bristles and damping factor, respectively. The steady state friction force is

$$F = g(v) \operatorname{sgn}(v) + f(v) \quad (2.19)$$

Here, $g(v)$ models the Stribeck effect and the $f(v)$ models the viscous friction force. It is important to choose the appropriate $g(v)$ function to make the modelled Stribeck curve more accurate.

$$g(v) = \alpha_0 + \alpha_1 e^{-\left(\frac{v}{v_0}\right)^2} \quad (2.20)$$

$\alpha_0 + \alpha_1$ and α_0 denote the stiction friction, coulomb friction respectively and v_0 parameter defines the variation of the $g(v)$ function in $\alpha_0 < g(v) \leq \alpha_0 + \alpha_1$ interval. Usually, $f(v)$ viscous friction model function is chosen as $f(v) = \alpha_2 v$. LuGre model can be defined with the constant σ_1 as below:

$$\frac{dz}{dt} = v - \frac{|v|}{g(v)} z$$

$$g(v) = \frac{1}{\sigma_0} \left(F_c + (F_s - F_c) e^{-\left(\frac{v}{v_s}\right)^2} \right) \quad (2.21)$$

$$F = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 v$$

Moreover, The better result can be achieved when the appropriate damping factor, which gets lower when the velocity gets higher, is used. That damping parameter can be defined as below:

$$\sigma_1(v) = \sigma_1 e^{-\left(\frac{v}{v_d}\right)^2} \quad (2.22)$$

2.4.2 Elasto-plastic Friction Model

Also, Elasto-plastic friction model is a modelling approach which uses first order differential equations for modelling friction. One of the important features of this model is modelling the presliding displacement in three temporary region. If we make a definition for presliding movement as

$$x = z + \omega$$

Where x is rigid body displacement, z is elastic component of the displacement and ω is plastic component of the displacement. Presliding displacement can be formulised as below:

$$\left. \begin{array}{l} \dot{x} = \dot{z} \\ \dot{\omega} = 0 \end{array} \right\} \quad \text{elastic displacement}$$

$$\dot{x} = \dot{z} + \dot{\omega} \quad \text{mixed elastic and plastic displacement}$$

$$\left. \begin{array}{l} \dot{x} = \dot{\omega} \\ \dot{z} = 0 \end{array} \right\} \quad \text{plastic displacement}$$

Using these equations, elasto-plastic presliding can be said that to begin with elastic displacement, continue with mixed, and then end with plastic displacement. Before giving the definition of the elasto-plastic friction model, we had better describe the elasto-plastic presliding in three temporary regime condition, since they are bound of the $\alpha(z, x')$ auxiliary elasto-plastic modelling function.

Temp. Regime 1: Transition from sticking to sliding

Here we have x', z', ω' in same direction.

$$\text{sgn}(\dot{x}) = \text{sgn}(\dot{\omega}) = \text{sgn}(\dot{z})$$

During this transition, the contact of the asperits in other words penetrated structure deforms. That beginning of the movement contains both of the elastic and plastic displacement.

Temp Regime 2: Elastic relaxation caused by Stribeck effect

As it can be seen from Stribeck curve, when the velocity increases, although plastic deformation increases, on the other hand, elastic deformation must decrease, resulting in decrease of the steady-state friction force. Those means that

$$\text{sgn}(\dot{x}) \neq \text{sgn}(\dot{z})$$

Temp Regime 3: Elastic super relaxation following motion reversal

Immediate change of direction of the velocity causes the elastic deformation to decrease, the elastic tension of the asperits must decrease, before the opposite direction movement causes reverse elastic deformation. This phenomena is called “super relaxation”.

The main purpose of the introduction of the elasto-plastic friction modelling idea is to simulate all the friction phenomenal like presliding displacement, stiction, super relaxation, Stribeck effect and viscous friction.

If we give the dynamic friction state variable “z” as

$$\dot{z} = \dot{x} \left(1 - \alpha(z, \dot{x}) \frac{\sigma_0}{g(v)} \text{sgn}(v) z \right)^i \quad (2.23)$$

In order to include all these the piecewise continuous function $\alpha(z, \dot{x})$ is defined as

When $\text{sgn}(\dot{x}) = \text{sgn}(z)$

$$\alpha(z, \dot{x}) = \begin{cases} 0 & |z| \leq z_{ba} \\ 0 < \alpha_m < 1 & z_{ba} < |z| < z_{\max}(\dot{x}) \\ 1 & |z| \geq z_{\max}(\dot{x}) \end{cases}$$

$$0 < z_{ba} < z_{\max}(\dot{x}) = \frac{g(v)}{\sigma_0}$$

$$\alpha_m(z, z_{BA}, z_{SS}) = \frac{1}{2} \sin\left(\pi \frac{z - \left(\frac{z_{SS} + z_{BA}}{2}\right)}{z_{SS} - z_{BA}}\right) + \frac{1}{2} \quad (2.24)$$

When $\text{sgn}(\dot{x}) \neq \text{sgn}(z)$

$$\alpha(z, \dot{x}) = 0$$

2.5 Model Parameter Identification

In this study the electromechanical system that is seen below will be used for model parameter identification studies.

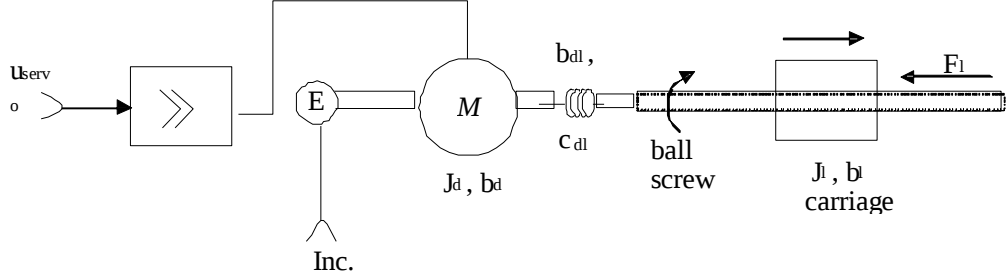


Figure 8: The Electro Mechanical System

The EMPS plant consists of a DC motor with a current controlled servo amplifier and a linear-positioning unit. Backlash-free ball screw drive converts the rotary motion of the motor to the linear carriage displacement. Incremental encoders provide motor position and carriage position. Mathematical model for control design is developed

$$\begin{aligned}
 J_d \dot{\Omega}_d &= M_d - b_d \Omega_d - b_{dl} (\Omega_d - \Omega_l) - c_{dl} (\varphi_d - \varphi_l) \\
 J_l \dot{\Omega}_l &= b_{dl} (\Omega_d - \Omega_l) + c_{dl} (\varphi_d - \varphi_l) - b_l \Omega_l - (M_l + M_{frl})
 \end{aligned} \tag{2.25}$$

$$\begin{bmatrix} \dot{\varphi}_d \\ \dot{\Omega}_d \\ \dot{\varphi}_l \\ \dot{\Omega}_l \\ \dot{M}_d \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\frac{c_{dl}}{J_d} & -\frac{b_d + b_{dl}}{J_d} & \frac{c_{dl}}{J_d} & \frac{b_{dl}}{J_d} & \frac{1}{J_d} \\ 0 & 0 & 0 & 1 & 0 \\ -\frac{c_{dl}}{J_l} & -\frac{b_{dl}}{J_l} & -\frac{c_{dl}}{J_l} & -\frac{b_{dl}}{J_l} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{T_{servo}} \end{bmatrix} \begin{bmatrix} \varphi_d \\ \Omega_d \\ \varphi_l \\ \Omega_l \\ M_d \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{k_{servo}}{T_{servo}} \end{bmatrix} u_{servo} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{1}{J_l} \\ 0 \end{bmatrix} (M_l + M_{frl})$$

$$\begin{aligned}
\begin{bmatrix} inc. \\ inc \end{bmatrix} \dot{} &= \begin{bmatrix} k_\varphi & 0 & 0 & 0 & 0 \\ 0 & 0 & k_\varphi & 0 & 0 \end{bmatrix} \begin{bmatrix} \varphi_d \\ \Omega_d \\ \varphi_l \\ \Omega_l \\ M_d \end{bmatrix} \quad \begin{aligned} \dot{\mathbf{X}}_p &= A_p \mathbf{X}_p + B_{pc} u_{pc} + B u_{pd} \\ Y &= C_{pm} \mathbf{X}_p \end{aligned}
\end{aligned} \tag{2.26}$$

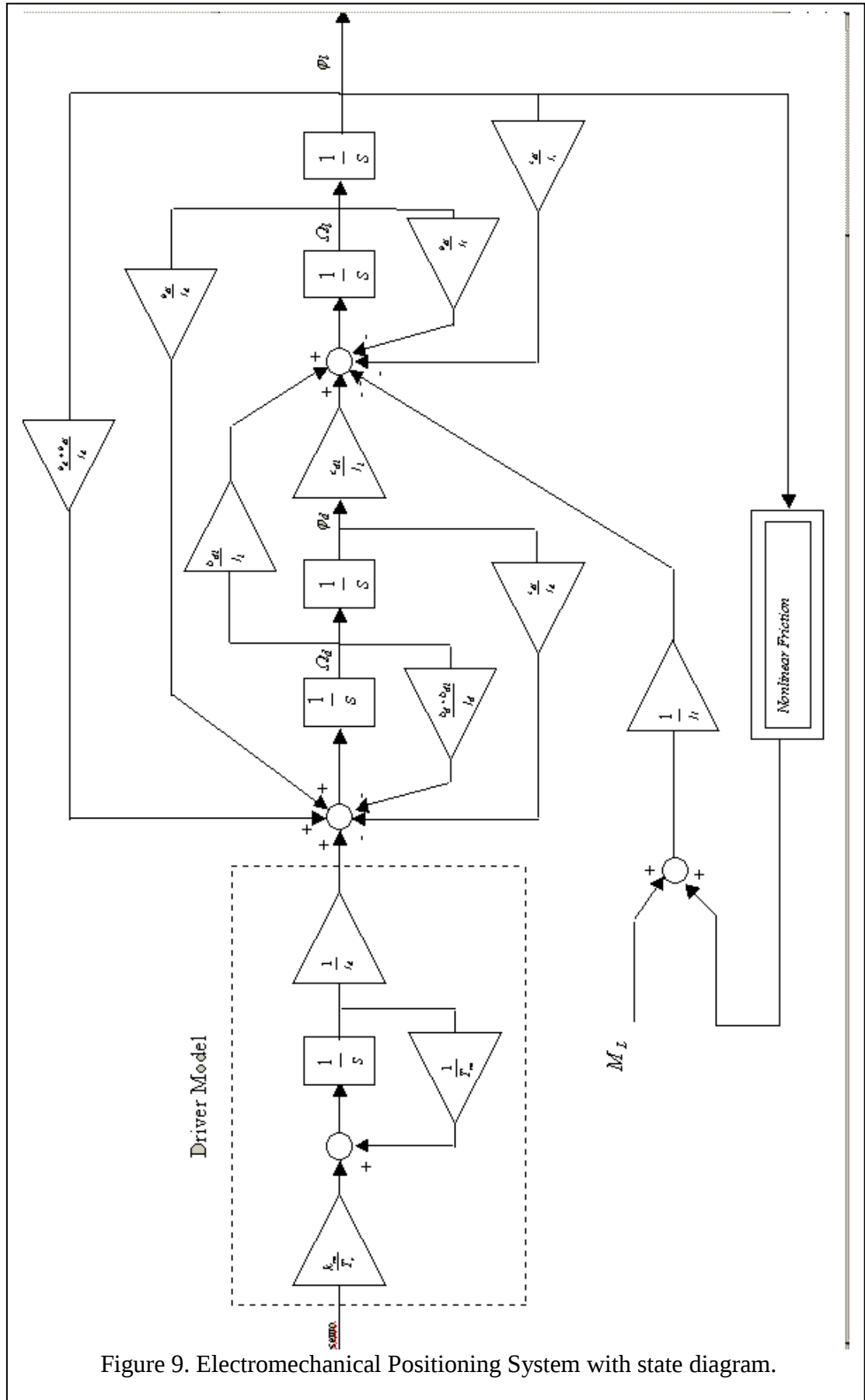


Figure 9. Electromechanical Positioning System with state diagram.

J_d :	Motor and Incremental Encoder inertia	J_l :	Screw, encoder and carriage Inertia
F_l :	External forces,	c_{dl} :	Spring stiffens
M_d :	Motor torque,	b_{dl} :	Damping constant
φ_d :	Drive side angular displacements	b_d :	Drive side damping constant
φ_l :	Load side angular displacements	b_l :	Load side damping constant
Ω_d :	Drive side angular velocity	M_{frl} :	Internal load side friction torque
Ω_l :	Load side angular velocity		

The angular velocities, positions and motor torque are chosen as state variables, the state space equations of the linear plant model with the state differential equation and measurement output equation become.

Although mathematical model of the EMPS shows linear characteristics as it can be sensed from its state space equations, the friction effect stated by M_{fr} represents non-linear behaviour. Therefore input-output relations of the EMPS to have linear structure as it can be seen from its state state equations, the friction effect stated by M_{frl} in the equations, has represents a non-linear characteristic. Therefore the input-output relation of the EMPS is non-linear, if dynamics model of the friction effect is taken in to the account.

One of the common techniques in control of non-linear systems is to make linearisation of the input-output behaviour of the system. An EMPS is a typical example of such a non-linear system because of dynamic friction in contacting parts. The dynamics friction can be either modelled by Lugre or Elasto Plastic dynamical friction models. One of the common feature of those model is to include pre-sliding displacement variable represented by z . Friction compensation techniques, the most generally rely on a friction model which are parts the general model of motion control systems. However they require model validation and they should be compared with a real friction effect on especially electromechanical motion control systems.

The major source of dry friction is the ball screw drive. A preloading of the ball screw system, which is applied to avoid backlash, causes this friction. Both interpretation of experimental studies and realization of simulations required for the

friction model analysis rely on the correct values of the model parameters. Since dynamical friction models have nonlinear nature there is no unique identification technique covering all parameters in the friction model. Therefore identification process is divided into several steps.

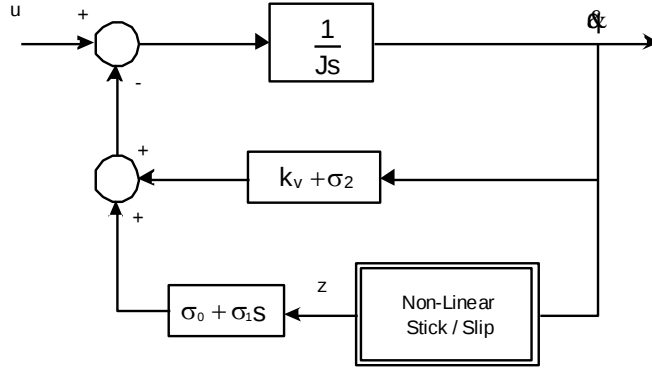


Figure 10: Block Diagram of an EMPS with nonlinear friction

In our effort we applied three steps for different kind of parameters sets. These are : steady state case in which applied velocity is kept constant in order to generate torque versus velocity characteristics. Presliding case required for parameter set in presliding regime and Dahl's curve step realized in very low velocities.

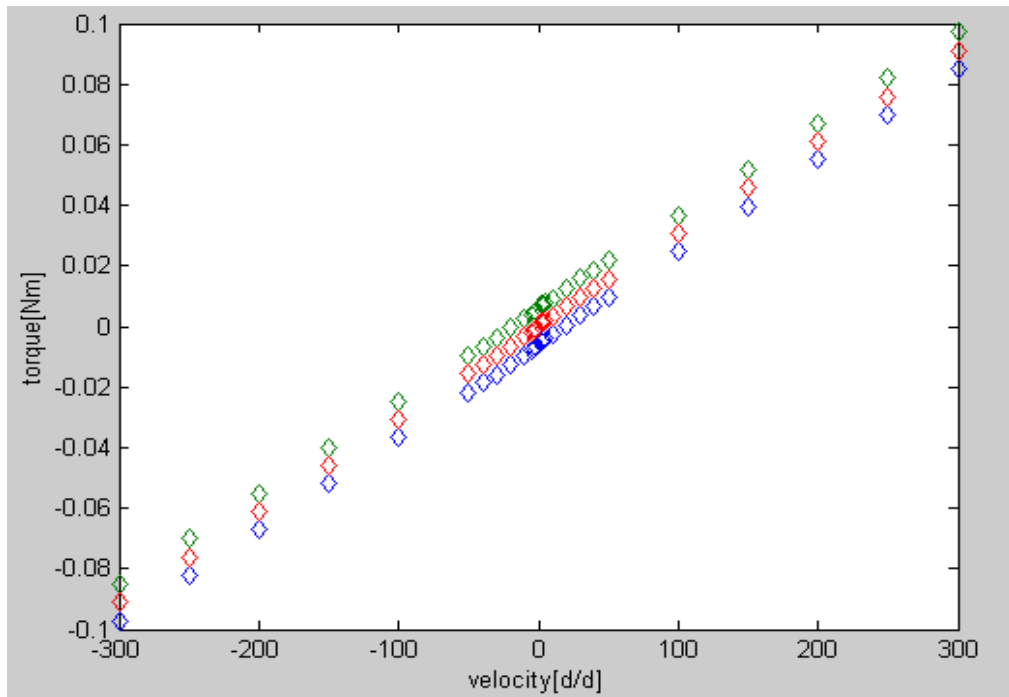


Figure 11: Steady state torque-velocity relation

In the first test step, steady state, applying different speed levels from high speed to very low speeds the torque-velocity relationship is achieved together with standard deviations, see Figure 11. When this graph is zoomed, it can be seen that the relationship is not continuous at very low speeds close to zero, see Figure 12.

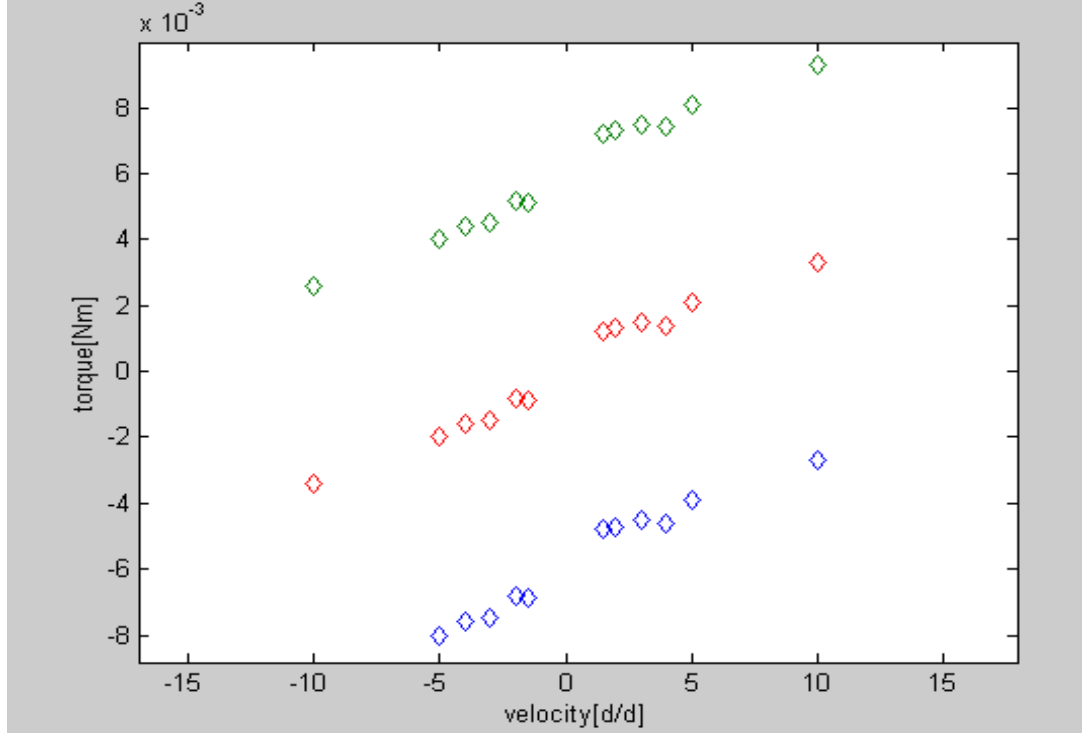


Figure 12: Close-up of the torque-velocity relationship at very low speeds.

This is the stiction region in which the presliding movements are observed. In the steady state case, we can define $\dot{z} = 0$ and $\ddot{q} = 0$. Replacing the values of z and q in the general mechanical equation, we obtain:

$$u = F_G + \sigma_2 \dot{q} + (F_C + (F_S - F_C)e^{-\left(\frac{\dot{q}}{V_s}\right)^2}) \text{sign} \dot{q} \quad (2.27)$$

Since this equation is non-linear, a classical least-squares algorithm does not apply. Therefore, using nonlinear optimization techniques, a prediction error vector should be minimised. The curve fitting method can be applied to this problem. By using Matlab-Optimization functions like “fminsearch”, the parameters can be fitted to given equation, minimizing the prediction cost function. This test gives us F_G , σ_2 , F_C , F_S and V_s estimations. Matlab studies can be seen in appendix.

In the other identification level (presliding displacement regime), presuming that $|z| \ll g(\dot{q})$, the system dynamics are described by [17]:

$$J\ddot{q} + (\sigma_1 + \sigma_2)\dot{q} + \sigma_0(q + q_0) = u - F_g \quad (2.28)$$

In order to keep the system in the presliding regime, the torque signals of amplitude much lower than the break-away torque should be applied. Therefore, with the proposed excitation signal $u(t) = u_0 + u_\delta(t)$, where $|u_\delta(t)| < 0.1 \min\{F_c; F_s\}$, open-loop experiments become possible. Obviously, moving into and keeping the system within the region around $z \approx 0$ is a very critical point. In the presliding experiments, only a little vibration can be observed by human eye. If we replace $(q + q_0)$ and $(u - F_g)$ by $\tilde{q}$ and $\tilde{u}$, we can obtain a transfer function like:

$$\frac{\tilde{Q}(s)}{\tilde{U}(s)} = \frac{1/\sigma_0}{\frac{J}{\sigma_0}s^2 + \frac{\sigma_1 + \sigma_2}{\sigma_0}s + 1} \quad (2.29)$$

In the experiments, a PRBS (Pseudo random binary signal) with very low level amplitude, can be applied to the system, as below in Figure 13. Applying that low level torque input, the positional displacements can be observed in vibration movements, see Figure 14. In order to increase the identification quality, it is better to achieve at least 5 data sets of experiments. Also, realization of the presliding experiment in different positions of the mechanical system maybe necessary.

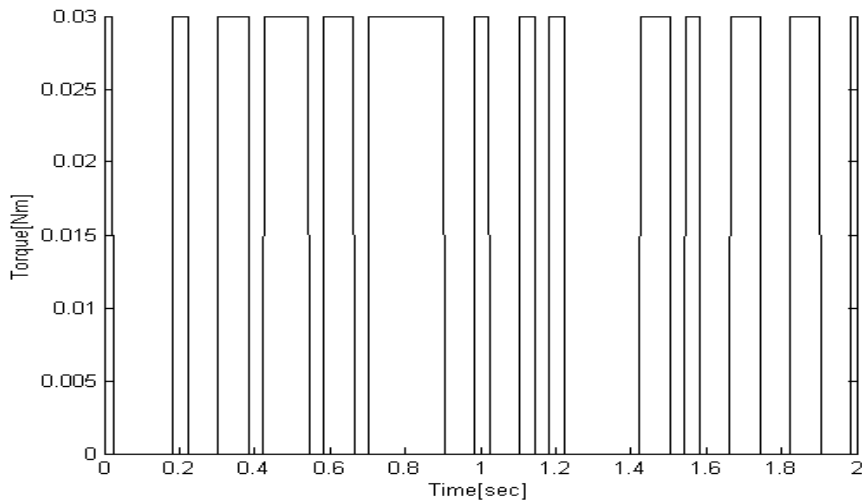


Figure 13: Example PRBS torque input

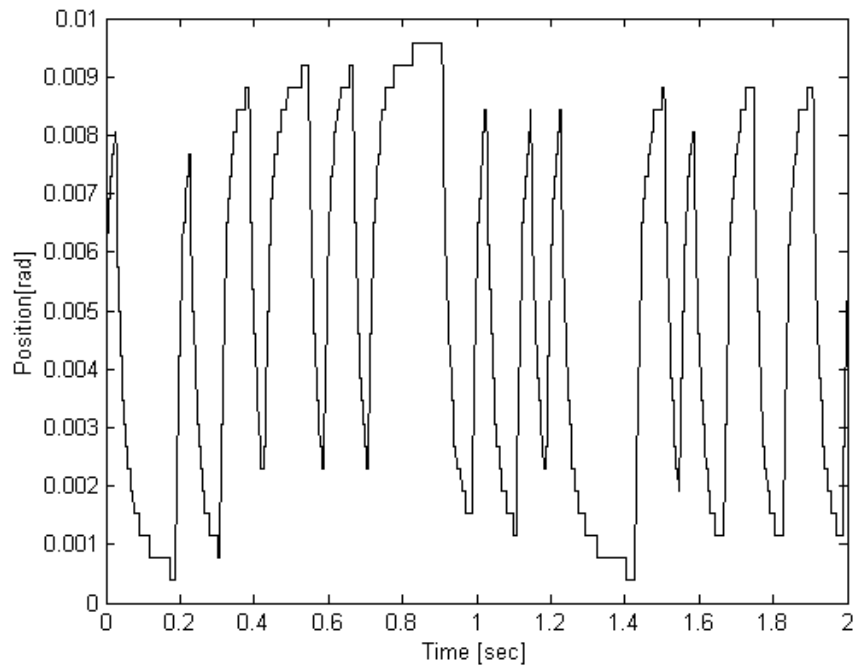


Figure 14: Example PRBS position output

Since the discrete model has the form of an auto-regressive system with linear dependence on parameters, the least square, instrumental variables, or ARX method can be applied. **ARX** method uses “ $A(q) y(t) = B(q) u(t-nk) + e(t)$ ” approach to find the system parameters. In our case $y(t)$ means position and $u(t)$ torque input. Applying ARX model, we obtain a differential equation in inverse z-domain format. All the calculations are given in Appendix.

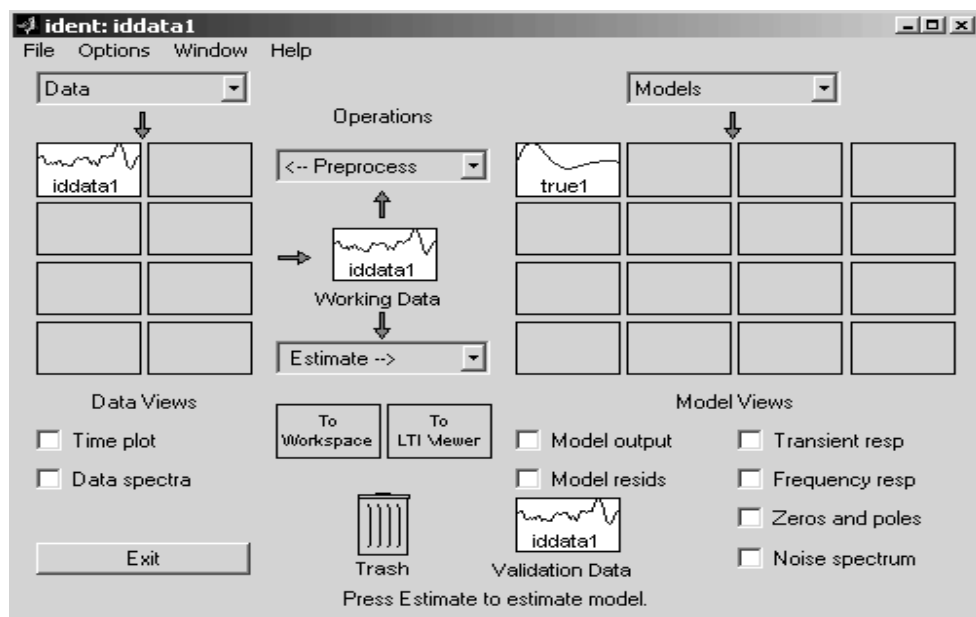


Figure 15: Matlab, System Identification Toolbox

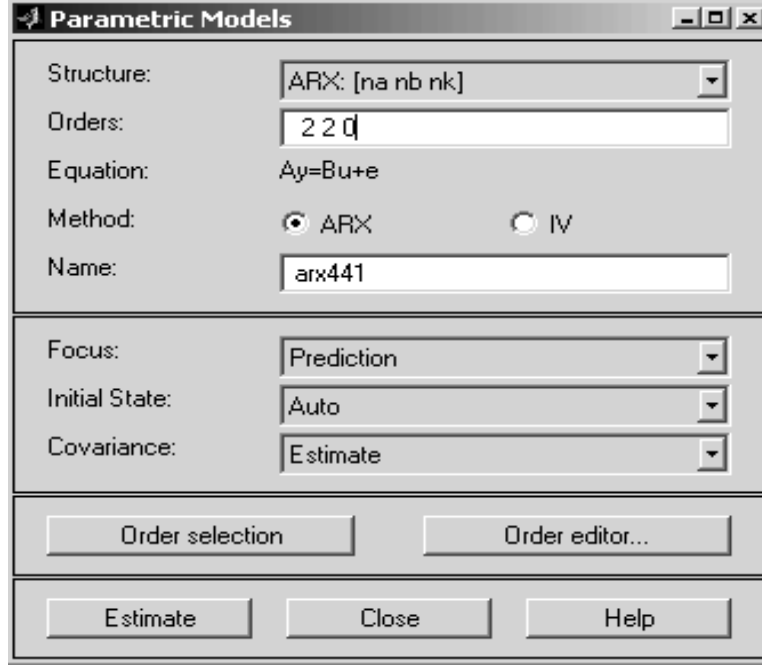


Figure 16: Matlab, System Identification Toolbox, Model selection window

The Last Identification level is Dahl's Curve. Dahl's curve is a position-force plot acquired for very low velocities. It illustrates the fact that friction acts like a filter, where time scale is replaced by space. Dahl's curve can be used for parameter identification by means of linear methods. Under the assumption that velocity is constant and small, resulting in $\ddot{q} \approx 0$ and $g(\dot{q}) = F_s / \sigma_0$, and then, we obtain:

$$\begin{aligned} \dot{z} &= \dot{q} - \sigma_0 \frac{|\dot{q}|}{F_s} z \\ u &= \sigma_0 z + F_G \end{aligned} \quad (2.30)$$

Introducing the discrete derivation approximation:

$$\Delta u(kh) = \sigma \Delta q(kh) - \frac{\sigma_0}{F_s} |\Delta q(kh)| u(kh) + \frac{\sigma_0 F_G}{F_s} |\Delta q(kh)| \quad (2.31)$$

3. Comparison of the Friction Models

In the first comparison test, the EMPS system, that is set up in open loop, is applied by a torque reference signal which provides a pulse in order to make the system exceed the break-away point, and provides a fluctuating like sinus wave in order to test the stiction performances of the friction models, see Figure 17.

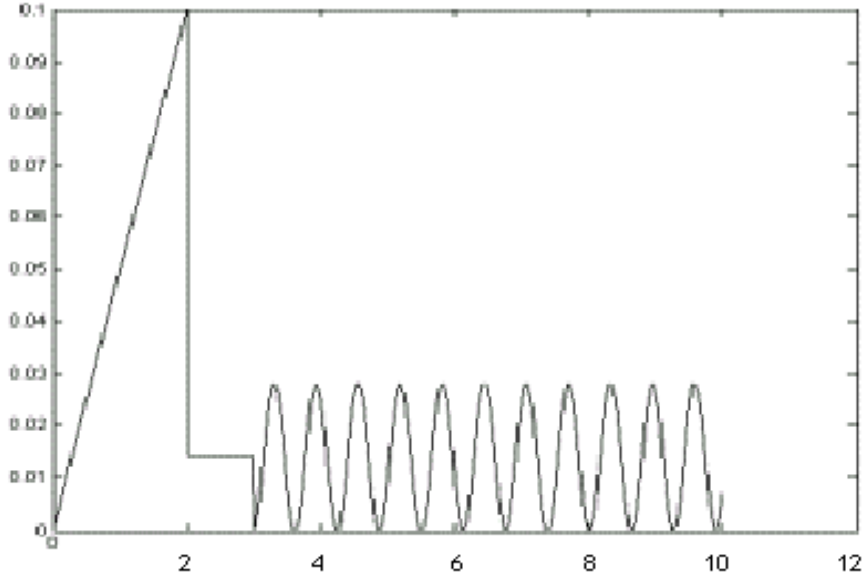


Figure17: Applied reference torque signal

When you apply such a torque reference, the electro-mechanical system makes a movement according to initial state of reference signal, because the reference signal value is bigger than the break-away. In the second state of the reference signal, since the applied reference is too small, the electro-mechanical system doesn't make any movement. Actually you can only feel a vibration when you touch the system.

The LuGre model can present Coulomb+viscous friction, Stribeck friction curve, frictional memory and rising static friction. However, it cannot render stiction although it can render the presliding displacement. On the other hand, The Elasto-plastic model can render both presliding and stiction and gives closer result to real system response.

The responses of the models can be obtained as in Figure 18, 19 and the EMPS response, Figure 20.

As it can be seen in Figure 18, there is also a steady drift in position in LuGre model response.

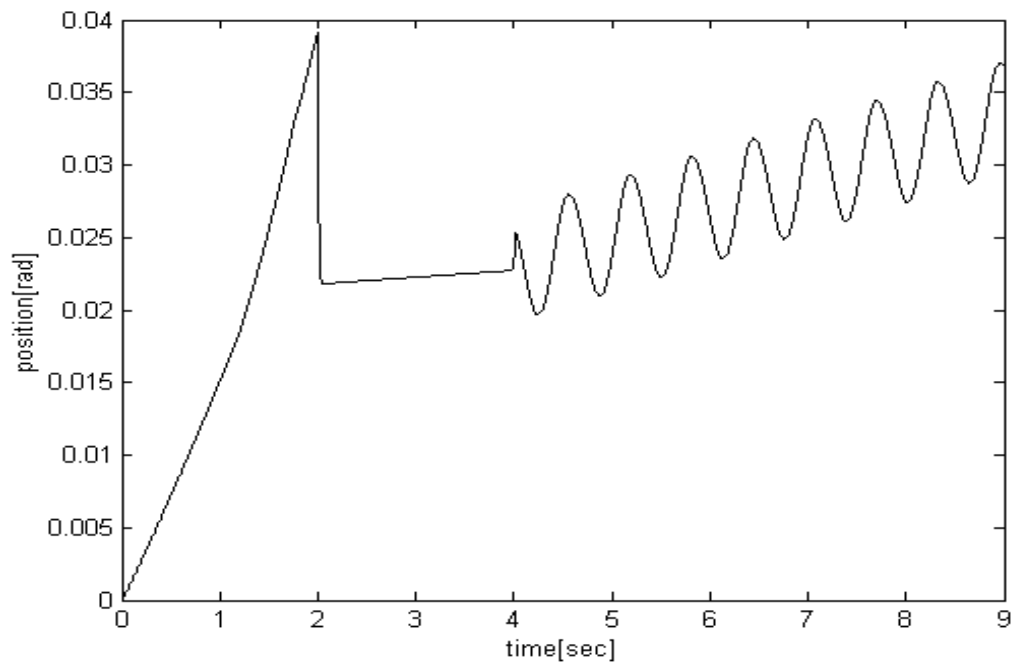


Figure 18: The response of the LuGre friction model

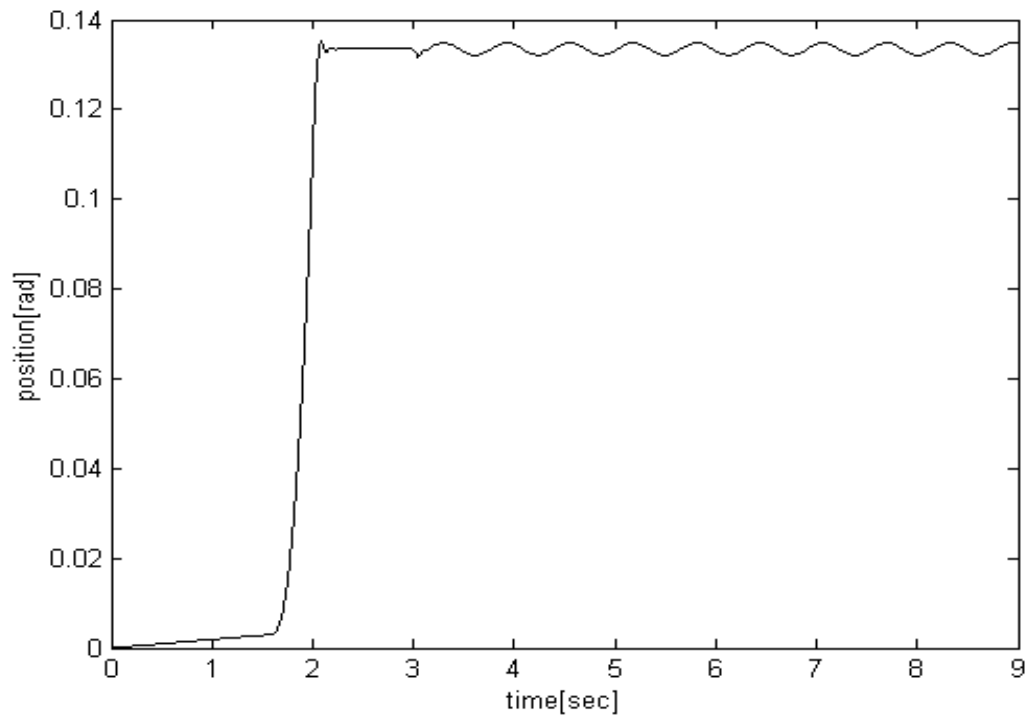


Figure19: The response of the Elasto-plastic model

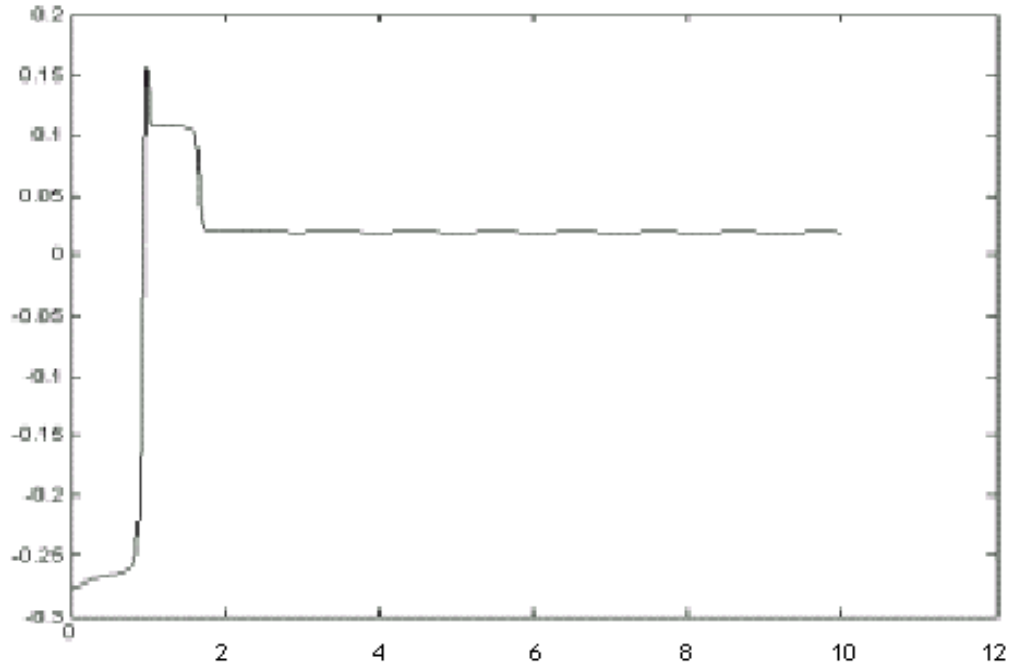


Figure 20: The response of the real EMPS

One of the other important point, that a friction model is expected to simulate, is velocity reversal point. Both of the models can render the velocity reversal point, however, Elasto-plastic model gives better results which are much closer to real EMPS response than LuGre model's results. Applying the same position reference (see Figure 21), to all friction models and EMPS in closed loop, all of the responses can be obtained as below, see Figure 22, 23 and 24.

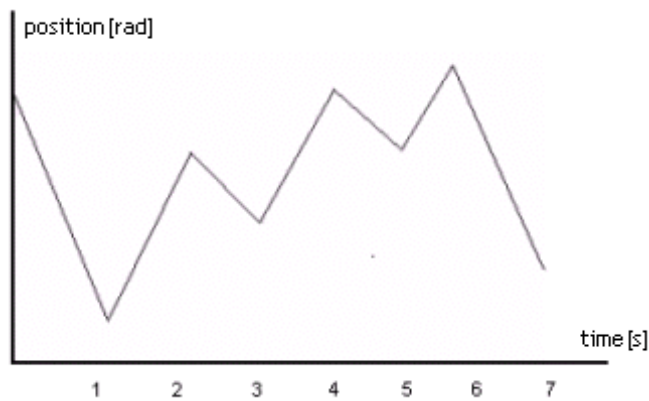


Figure 21: Applied position reference

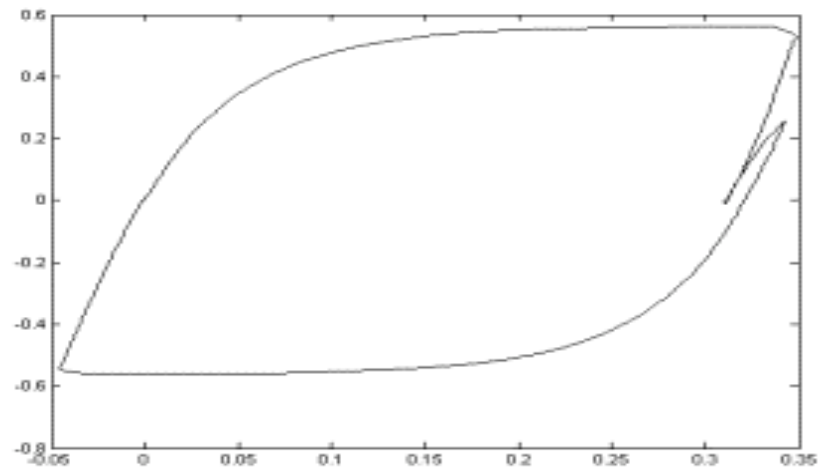


Figure 22: The response of the LuGre friction model

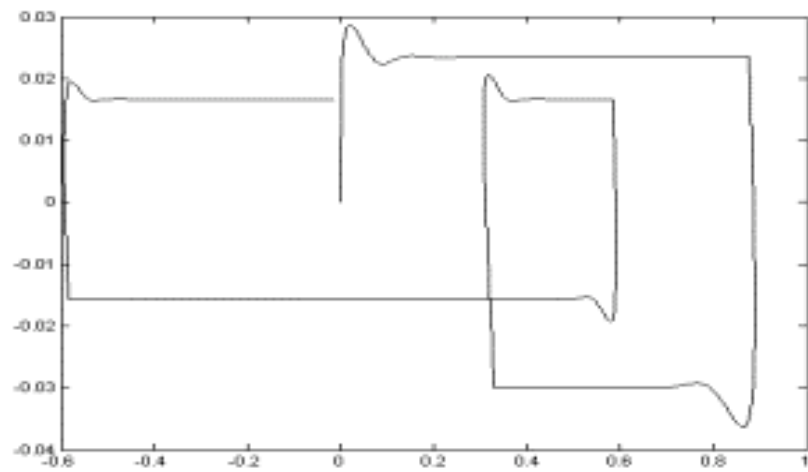


Figure 23: The response of the Elasto-plastic model

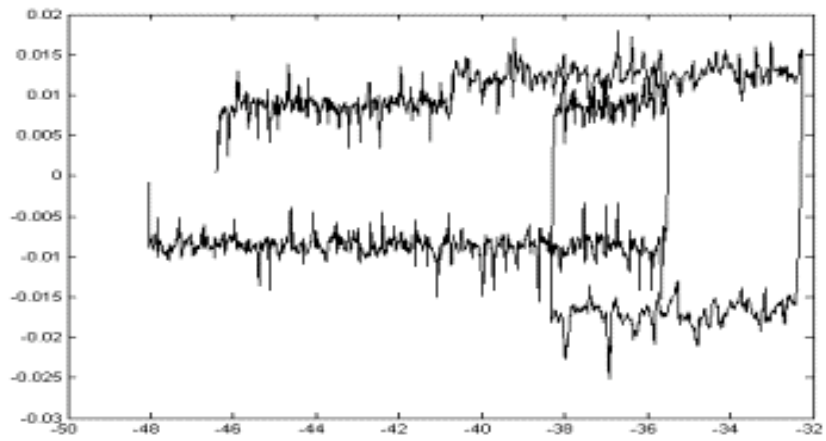


Figure 24: The response of the real EMPS

4. Comparison of the Friction Observers

In general, the cost of industrial installations is reduced by measuring a limited number of signals only, i.e. the full state vector is not available to the controller. Furthermore, in the particular case of drives with LuGre or Elasto-Plastic models friction, it is not possible to measure the friction state z since this signal is not a physical quantity. Therefore, it is necessary to present an approach for efficient state estimation because enhanced feedback compensation methods.

4.1 Introduction

This comparison study consists of two sections: simulation studies and experimental studies on a real system. As we can see that elasto-plastic friction modeling method gives better results, observation studies are based on only elasto-plastic friction model. Two types of observation methods were used. One of them is classical Luenberger-like observer that is used for non-linear systems. Other one which is easy to implement, better suited to DSP type digital systems, discrete observer.

4.2 Luenberger-like Reduced Order State Observer

When some of the state variables can be accurately measured, it is not necessary to observe such state variables. Moreover this measurement can be used to increase observer efficiency. We can consider system

$$[x] = [A]x + [B]u \quad (4.1a)$$

$$[y] = Cx \quad (4.1b)$$

and, let's separate the x vector into two parts as:

$$x = \begin{bmatrix} y \\ \omega \end{bmatrix}$$

In this case, we can rewrite the state equation:

$$\dot{x} = \begin{bmatrix} \dot{y}(t) \\ \dot{\omega}(t) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} y(t) \\ \omega(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u(t) \quad (4.2)$$

and accordingly write the system in the form:

$$\dot{y}(t) = A_{11}y(t) + A_{12}\omega(t) + B_1u(t) \quad (4.3a)$$

$$\dot{\omega}(t) = A_{21}y(t) + A_{22}\omega(t) + B_2u(t) \quad (4.3b)$$

The vector $y(t)$ is available for measurement, and if we differentiate it, we can obtain $\dot{y}(t)$. Since $u(t)$ is also measurable (4.3a) provides the measurement $A_{12}w(t)$ for the equation (4.3b) that has state vector $w(t)$ and input $A_{21}w(t) + B_2u(t)$, so a reduced order observer can be designed. To construct the observer, we can give a initial definition in the form:

$$\dot{\hat{w}}(t) = (A_{22} - LA_{12})\hat{w}(t) + A_{21}y(t) + B_2u(t) + L(\dot{y}(t) - A_{11}y(t)) - LB_1u(t) \quad (4.4)$$

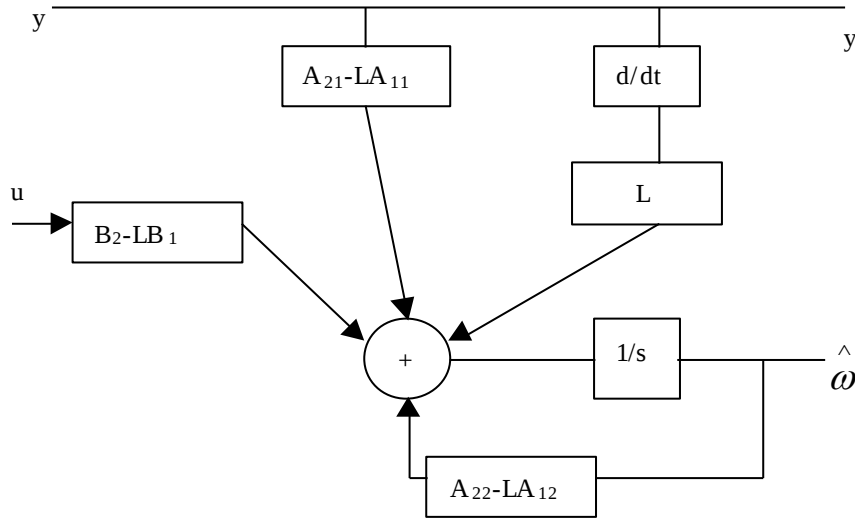


Figure 25: Reduced order observer using derivative

L can be selected so that $A_{22} - LA_{12}$ has arbitrary eigenvalues, see figure 25. The required differentiation of y can be dismissed by modifying the block diagram of

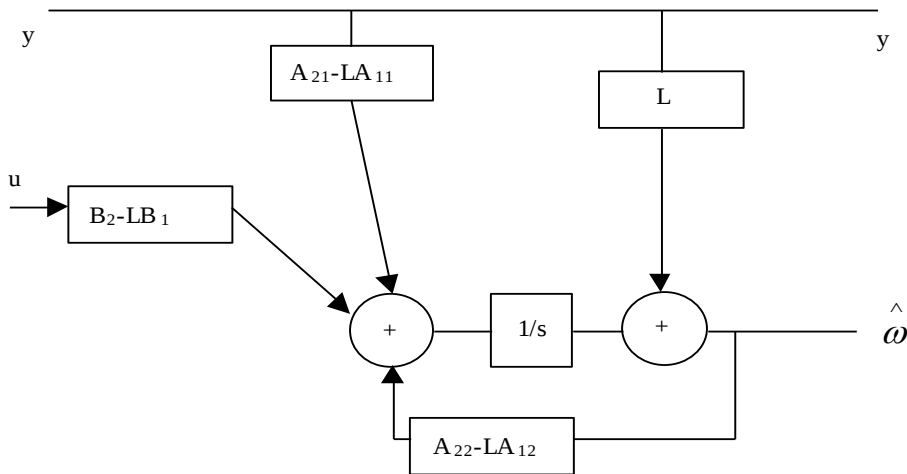


Figure 26: Reduced order observer

figure 23 to block diagram of figure 26. Modifying the state variable as

$$z(t) = \hat{\omega}(t) - Ly(t) \quad (4.5)$$

We can obtain the demanded form of the observer:

$$\dot{z}(t) = (A_{22} - LA_{12})z(t) + (A_{22} - LA_{12})Ly(t) + (A_{21} - LA_{11})y(t) + (B_2 - LB_1)u(t) \quad (4.6)$$

4.3 Luenberger-like Reduced Order Friction State Observer

In certain applications and for economic reasons only position measurement is available. Thus, not only the friction state z should be observed, but also the velocity q' . In addition, it is known that the order of the state observer should be as small as possible in order to simplify the problem of stabilization of the closed-loop system. The method introduced for linear systems [15] allows reduction of the order of the state observer to the system's order minus the number of outputs. A friction state observer that applies this concept and requires only position measurement has been proposed for a simple drive with friction [2]. The system (3.5), (3.8), and (3.10) is written in a state space form, with position output $y = q$ appearing explicitly, i.e.

$$[x] = [A]x + [B]u \quad (4.7)$$

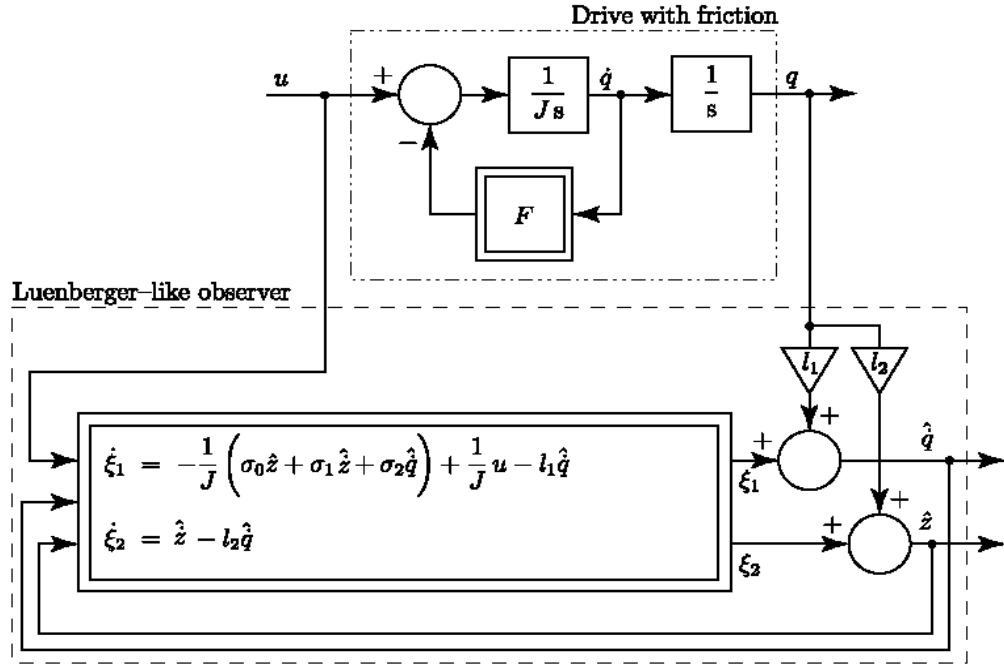


Figure 27: Luenberger-like friction observer system

$$\begin{aligned} y &= q \\ \dot{y} &= \begin{bmatrix} 1 & 0 \end{bmatrix} \omega \end{aligned} \quad \dot{\omega} = \begin{bmatrix} \ddot{q} \\ \dot{q} \\ \dot{z} \end{bmatrix} = f(\omega) + \begin{bmatrix} \frac{1}{J} \\ 0 \\ 0 \end{bmatrix} u \quad (4.8)$$

Where ω denotes the part of the state vector that is observed and

$$\omega = \begin{bmatrix} \dot{q} \\ q \\ z \end{bmatrix} \quad f(\omega) = \begin{bmatrix} -\frac{1}{J} \left(\sigma_0 z + \sigma_1 \dot{z} + \sigma_2 \dot{q} \right) \\ \dot{q} \left(1 - \alpha(z, \dot{q}) \frac{\sigma_0}{f_{ss}(\dot{q})} \text{sgn}(\dot{q}) z \right) \\ f_{ss}(\dot{q}) \end{bmatrix} \quad (4.9)$$

for elasto-plastic friction model approach. Moreover, we can introduce observer gain vector L and extended observer state variable θ as below:

$$L = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} \quad \theta = \hat{\omega} - Lq \quad (4.10)$$

In that case, Luenberger-like observer for the system with friction disturbance can be stated as below, where “ u ” is applied torque to the system and measured position “ q ”. Also see figure...

$$\dot{\theta} = f(\theta + Lq) + \begin{bmatrix} \frac{1}{J} \\ 0 \end{bmatrix} u - \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} (\theta + Lq) \quad (4.11)$$

The observation error $e_{obs} = \omega - \hat{\omega}$. Because our aim is to decrease the observation error as much as possible, the definition of the L matrix is very important for the system stability.

$$e_{obs} = \omega - \hat{\omega} \quad (4.12)$$

$$\dot{e}_{obs} = \dot{\omega} - \dot{\hat{\omega}} = f(\omega) + \begin{bmatrix} \frac{1}{J} \\ 0 \end{bmatrix} u - \left(f(\hat{\omega}) + \begin{bmatrix} \frac{1}{J} \\ 0 \end{bmatrix} u - \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} (\theta + Lq) + L\dot{q} \right) \quad (4.13)$$

$$\dot{e}_{obs} = f(\omega) - f(\hat{\omega}) - \begin{bmatrix} l_1 & 0 \\ l_2 & 0 \end{bmatrix} (\omega - \hat{\omega}) \quad (4.14)$$

The $f(\omega) - f(\hat{\omega})$ term can be approximated after first order Taylor expansion of function around $\hat{\omega}$ as below:

$$f(\omega) = f(\hat{\omega}) + \frac{\partial f}{\partial \omega} \bigg|_{\omega=\hat{\omega}} (\omega - \hat{\omega}) \quad (4.15)$$

Then,

$$\dot{e}_{obs} = \omega - \hat{\omega} \approx \frac{\partial f}{\partial \omega} \bigg|_{\omega=\hat{\omega}} \left(\omega - \hat{\omega} \right) - \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} \begin{pmatrix} \dot{q} - \hat{\dot{q}} \\ \dot{q} - \hat{\dot{q}} \end{pmatrix} = A e_{obs} \quad (4.16)$$

The last equation expresses that the system stability depends on the eigenvalues of A given by

$$A = \frac{\partial f}{\partial \omega} \bigg|_{\omega=\hat{\omega}} - \begin{bmatrix} l_1 & 0 \\ l_2 & 0 \end{bmatrix} \quad (4.17)$$

Finally, if we use elasto-plastic model equations in the definition of matrix A, we obtain:

$$A = \begin{bmatrix} \frac{-1}{J} \left(\sigma_2 + \sigma_1 \left(1 - \alpha(\hat{z}) \hat{z} \left(\frac{g(\hat{q}) \operatorname{sgn} \hat{\dot{q}} - |\hat{\dot{q}}| g'(\hat{q})}{g^2(\hat{q})} \right) \right) \right) - l_1 & \frac{-1}{J} \left(\sigma_0 - \sigma_1 \frac{|\hat{\dot{q}}|}{g(\hat{q})} \left(\alpha'(\hat{z}) \hat{z} + \alpha(\hat{z}) \right) \right) \\ 1 - \alpha(\hat{z}) \hat{z} \left(\frac{g(\hat{q}) \operatorname{sgn} \hat{\dot{q}} - |\hat{\dot{q}}| g'(\hat{q})}{g^2(\hat{q})} \right) - l_2 & \frac{|\hat{\dot{q}}|}{g(\hat{q})} \left(\alpha'(\hat{z}) \hat{z} + \alpha(\hat{z}) \right) \end{bmatrix} \quad (4.18)$$

Obtaining matrix A, we have to discuss three states of the function $\alpha(z, \dot{q})$, see 2.24. Actually we should obtain three different A matrices according to these $\alpha(z, \dot{q})$ variations. Therefore the selection of L gain matrix depends on these three states. As we remember, these state are given according to z, z_{ba}, z_{ss} relationship. Let's examine the variations of matrix A:

- $z < z_{ba}, \alpha(z, \dot{q}) = 0$

$$A = \begin{bmatrix} \frac{-1}{J} (\sigma_1 + \sigma_2) - l_1 & \frac{-1}{J} \sigma_0 \\ 1 - l_2 & 0 \end{bmatrix} \quad (4.19)$$

- $z \geq z_{ss}, \alpha(z, \dot{q}) = 1$

$$A = \begin{bmatrix} \frac{-1}{J} \left(\sigma_2 + \sigma_1 \left(1 - \hat{z} \left(\frac{g(\dot{q}) \operatorname{sgn} \dot{q} - |\dot{q}| g'(\dot{q})}{g^2(\dot{q})} \right) \right) \right) - l_1 & \frac{-1}{J} \left(\sigma_0 - \sigma_1 \frac{|\dot{q}|}{g(\dot{q})} \right) \\ 1 - \hat{z} \left(\frac{g(\dot{q}) \operatorname{sgn} \dot{q} - |\dot{q}| g'(\dot{q})}{g^2(\dot{q})} \right) - l_2 & \frac{|\dot{q}|}{g(\dot{q})} \end{bmatrix} \quad (4.20)$$

- $z_{ba} < |z| < z_{ss}, \alpha(z, \dot{q}) = \alpha_m$

$$A = \begin{bmatrix} \frac{-1}{J} \left(\sigma_2 + \sigma_1 \left(1 - \alpha_m \hat{z} \left(\frac{g(\dot{q}) \operatorname{sgn} \dot{q} - |\dot{q}| g'(\dot{q})}{g^2(\dot{q})} \right) \right) \right) - l_1 & \frac{-1}{J} \left(\sigma_0 - \sigma_1 \frac{|\dot{q}|}{g(\dot{q})} \left(\alpha'_m \hat{z} + \alpha_m \right) \right) \\ 1 - \alpha_m \hat{z} \left(\frac{g(\dot{q}) \operatorname{sgn} \dot{q} - |\dot{q}| g'(\dot{q})}{g^2(\dot{q})} \right) - l_2 & \frac{|\dot{q}|}{g(\dot{q})} \left(\alpha'_m \hat{z} + \alpha_m \right) \end{bmatrix} \quad (4.21)$$

The structure of (5.26) implies that it is not possible to assign any observer error dynamics with an appropriate choice of the observer gain L . Extended simulation of the continuous-time closed-loop system has shown that l_2 does not improve performance. However, a large l_1 leads to a fast response of the velocity estimation error and therefore to a good perturbation rejection: it results directly from (4.19), (4.20) and (4.21) that, to have a considerable effect on damping, gain l_1 has to be chosen such that $l_1 > \frac{\sigma_1 + \sigma_2}{J}$

In our simulations and experiments, we applied a sinus form input-reference torque. Related Simulink models can be seen in “Chapter 5, Experimental Studies”.

4.4 Discrete Time-free Elasto-plastic Friction Observer

It is possible to express the equations describing the dynamics of friction in a time-free style, relating differential quantities of displacement rather than of time. We can express equation (2.23) in time-free form as below[3]:

$$\frac{dz}{dq} = 1 - \alpha(z, \dot{q}) \frac{\sigma_0}{f_{ss}(\dot{q})} \text{sgn}(\dot{q}) \quad (4.22)$$

If the measurements of position and the velocity are available, integrating over space, instead of time, friction force can be obtained. Let's integrate equation (4.22) with respect to q over an interval h of any length between two sample points:

$$z = q - \int_h \frac{\sigma_0}{f_{ss}(\dot{q})} \alpha(z, \dot{q}) \text{sgn}(\dot{q}) dq \quad (4.23)$$

$$z = q - \omega \quad (4.24)$$

An online solution involves the calculation of the successive values of the inelastic displacement ω_k from measurements $\bar{q}_k$ of the rigid body displacements and velocity estimates $\bar{v}_k$. Let's say sampled versions of $z(t)$ and $\omega(t)$ are z_k and ω_k . Therefore, we can write:

$$z_k = \bar{q}_k - \omega_k \quad (4.25)$$

The equation $\omega_{k+1} = \omega_k + \Delta\omega_k$ gives the progress of ω_k at each update, where $\Delta\omega_k$ is the discrete counterpart of the integrand of equation (4.23). If we make a redefinition for convenience as

$$\alpha'(z, \dot{q}) = \frac{\sigma_0 \alpha(z, \dot{q})}{f_{ss}(\dot{q})} \quad (4.26)$$

We obtain that:

$$\Delta\omega_k = \alpha'(z_k, \bar{v}_k) z_k |\Delta \bar{q}_k| \quad (4.27)$$

The sequence $\{\omega_k\}$ can be obtained from explicit Euler integration giving:

$$\omega_{k+1} = \omega_k + \alpha'(z_k, \bar{v}_k) z_k |\Delta \bar{q}_k| \quad (4.28)$$

The observer should provide that presliding displacement is bounded. From equations (4.25) and (4.26) :

$$\Delta z_k = \Delta \bar{q}_k - \alpha'(z_k, \bar{v}_k) z_k |\Delta \bar{q}_k| \quad (4.29)$$

And so

$$\frac{\Delta z_k}{\Delta q_k} = 1 - \alpha'(z_k, \bar{v}_k) z_k \operatorname{sgn}(\Delta \bar{q}_k) \quad (4.30)$$

which is always defined.

Because $\bar{q}_k$ are the measurements, their magnitudes can not be known. For a large enough $\bar{q}_k$, z_{k+1} can exceed $z_{\max}$ and the right hand side of equation (4.30) can become negative or exceed one. The process would diverge for all input values. A sufficient condition to guarantee the convergence is to provide:

$$0 \leq 1 - \alpha'(z_k, \bar{v}_k) z_k \operatorname{sgn}(\Delta \bar{q}_k) \leq 1 \quad (4.31)$$

That situation forces us to give a modified observer law like:

$$\omega_k = \begin{cases} \omega_k + |\Delta \bar{q}_k| \alpha'(z_k, \bar{v}_k) z_k & \text{If } \alpha'(z_k, \bar{v}_k) |z_k| < 1, \\ q_{k+1} - \operatorname{sgn}(z_k) z_{\max}(\bar{v}_k) & \text{Otherwise.} \end{cases} \quad (4.32)$$

$$z_{k+1} = q_{k+1} - \omega_{k+1} \quad (4.33)$$

During steady sliding, the discrete observer given by equation (4.32) converges to a steady state value equivalent to value that is given continuous, time-free model, equation (4.22). When $dz/dx = 0$, the fixed point time-free model is given as:

$$z_{ss} = \frac{f_{ss}(\bar{q})}{\alpha(z, \bar{q}) \sigma_0 \operatorname{sgn}(dq)} \quad (4.34)$$

Likewise, when

$\Delta z_k / \Delta \bar{q}_k = 0$, from equation (4.30), the fixed-point of the discrete observer is:

$$z_{ss,k} = \frac{1}{\alpha'(z_k, \bar{v}_k) \operatorname{sgn}(\Delta \bar{q}_k)} = \frac{f_{ss}(\bar{v}_k)}{\alpha(z_k, \bar{v}_k) \sigma_0 \operatorname{sgn}(\Delta \bar{q}_k)} \quad (4.35)$$

The stability of the observer is seen in the boundness of $\Delta z_k / \Delta \bar{q}_k$ between 0 and 1.

4.5 Simulations

In simulation studies, sinus type input torque reference was applied to Simulink model. The sinus form torque reference shows us negative and positive side movement, velocity reversal. Thus, it let's us see the observer behaviors better. Related Simulink models can be seen in Appendix.

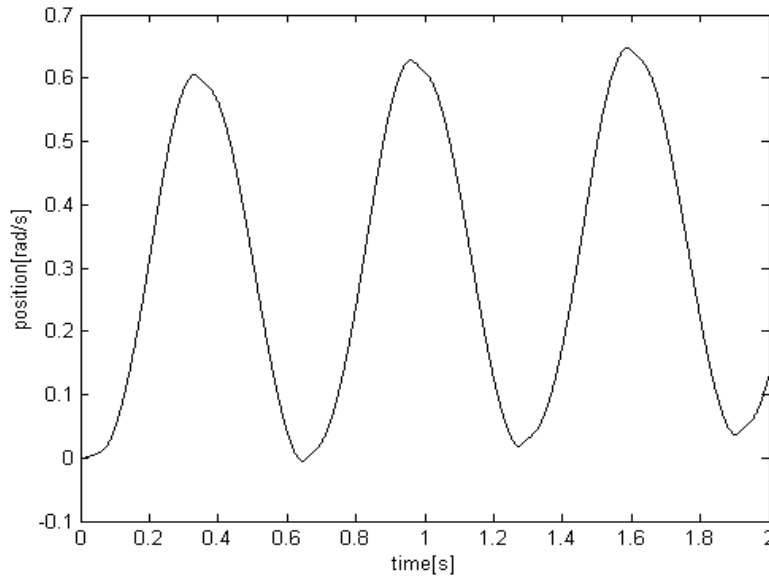


Figure 28: Position output in the simulations.

When we look at the simulations both Luenberger-like observer and discrete observer system, the position output is as below, figure 28. The estimated velocities

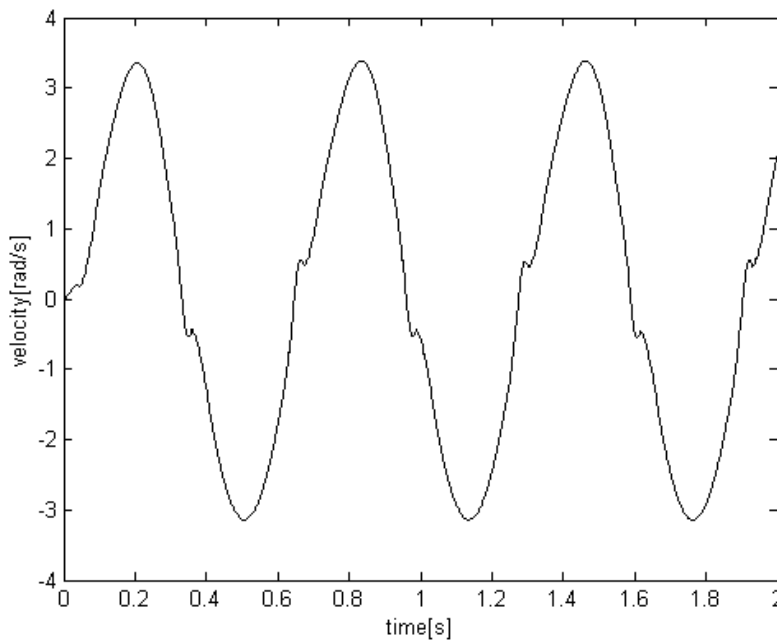


Figure 29: Luenberger-like observer, velocity estimation in the simulation.

are given in figure 29 and 30. However, since the discrete observer structure doesn't include velocity estimation, the velocity values are derived from $\Delta q / \Delta t$ (successive discrete position value difference and time difference).

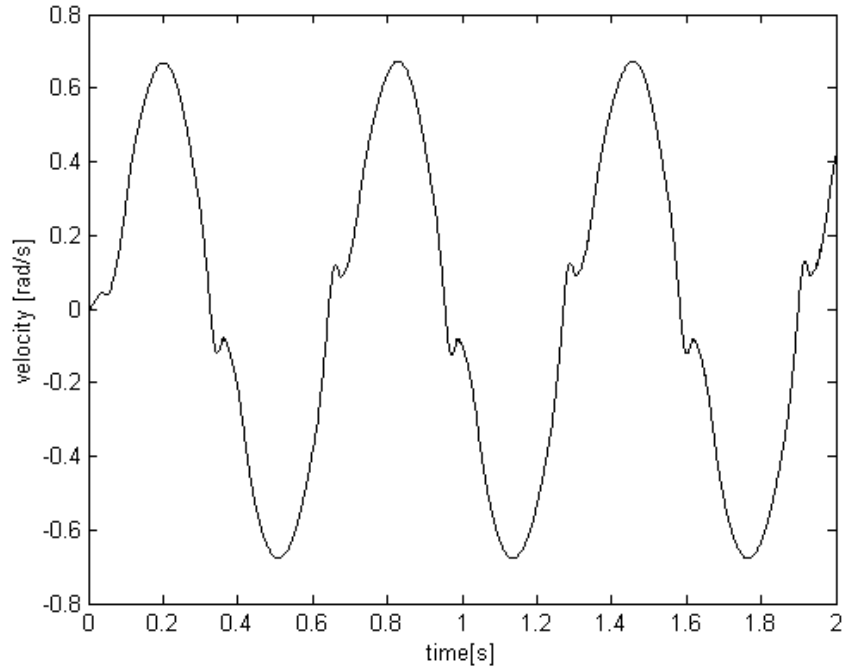


Figure 30: Discrete observer, velocity estimation in the simulation.

The estimated z elastic displacement values for each observer simulations can be seen in figure 31 and 32.

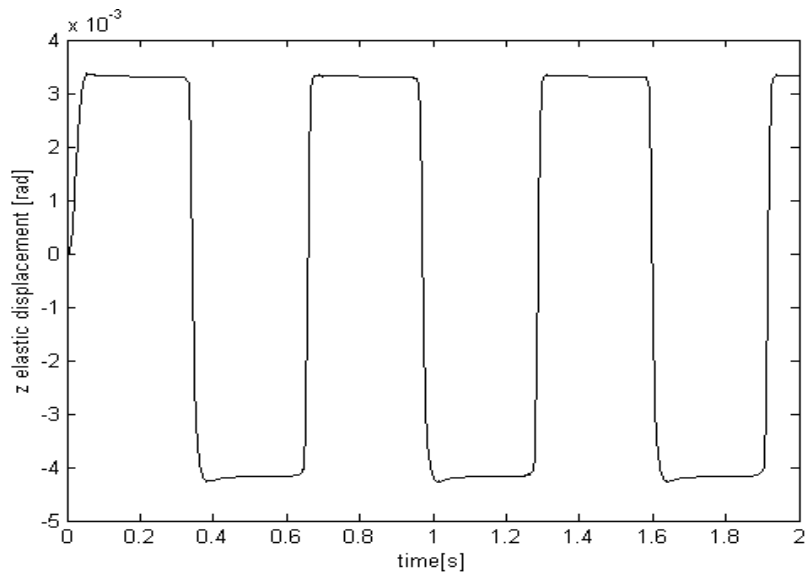


Figure 31: Luenberger-like observer, z elastic displacement estimation in the simulation.

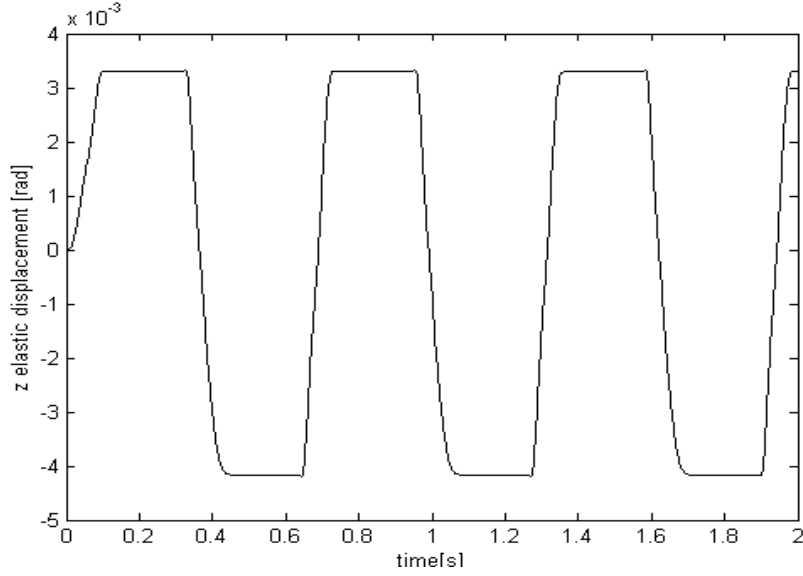


Figure 32: Discrete observer, z elastic displacement estimation in the simulation.

Simulink models which were used for simulation can be seen in Appendix. In this simulation study the system parameters, which were identified in Chapter 2.5, were used. L_1 parameter was chosen as 1700 in order to obtain fast observer response. In Simulink simulations, L_1 value was very critical. It frequently caused to simulation failures for other values. For example, although 500 is big enough to make the observer stable, it was seen that estimated velocity and z elastic displacement values went to inf. On the other hand, as it is expected, selection of sampling period T is very important for Discrete observer. T was chosen 0.1 ms for both simulations and real system experiments. Both of the observer simulations resulted in basically same velocity and z elastic displacement observation.

5. Experimental Studies

In order to realise model validation an experimental set-up is used so that validation of the friction models can be shown experimentally. The experiment setup includes a PC, DSpace 1104 DSP card which is able to download MATLAB/SIMULINK codes into the real-time, an AC motor and driver, a coupling, a 4x4906 step incremental encoder and a ball-screw table system. Model parameters, particularly required for components on the experiment are assumed that they exist, actually they obtained by the previous study. A reference input signals both applied simulated models and experimental set in order to notice error between the models. Friction effect on the EMPS are tried to be exaggerated by slowing down to velocity and adding extra friction force on the experiment.

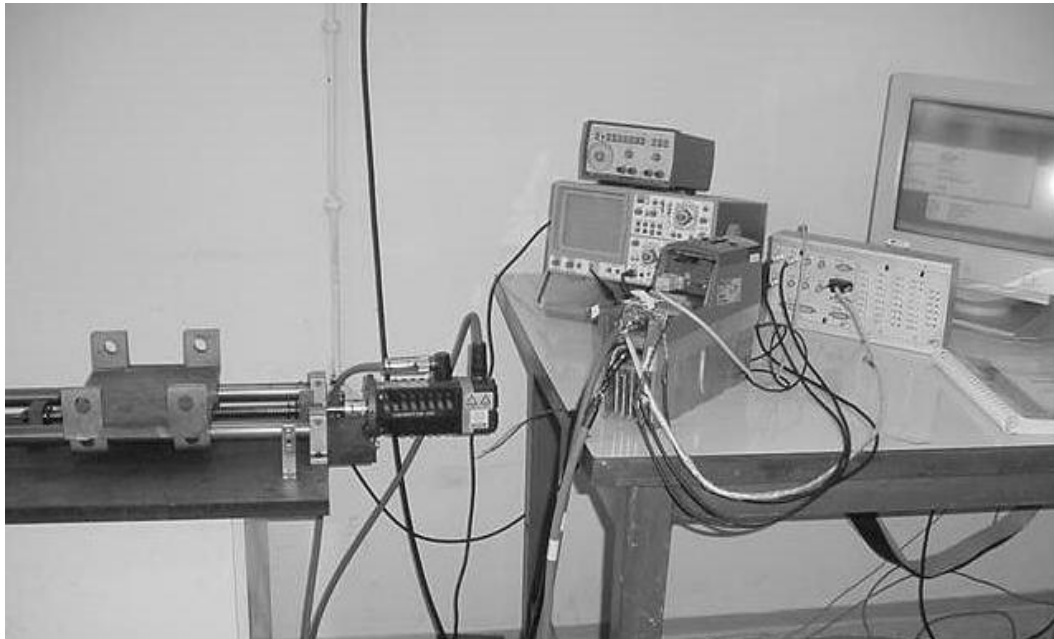


Figure 33: Experiment environment.

A photograph of the EPMS can be seen in figure 33 and schematic diagram in figure 34.

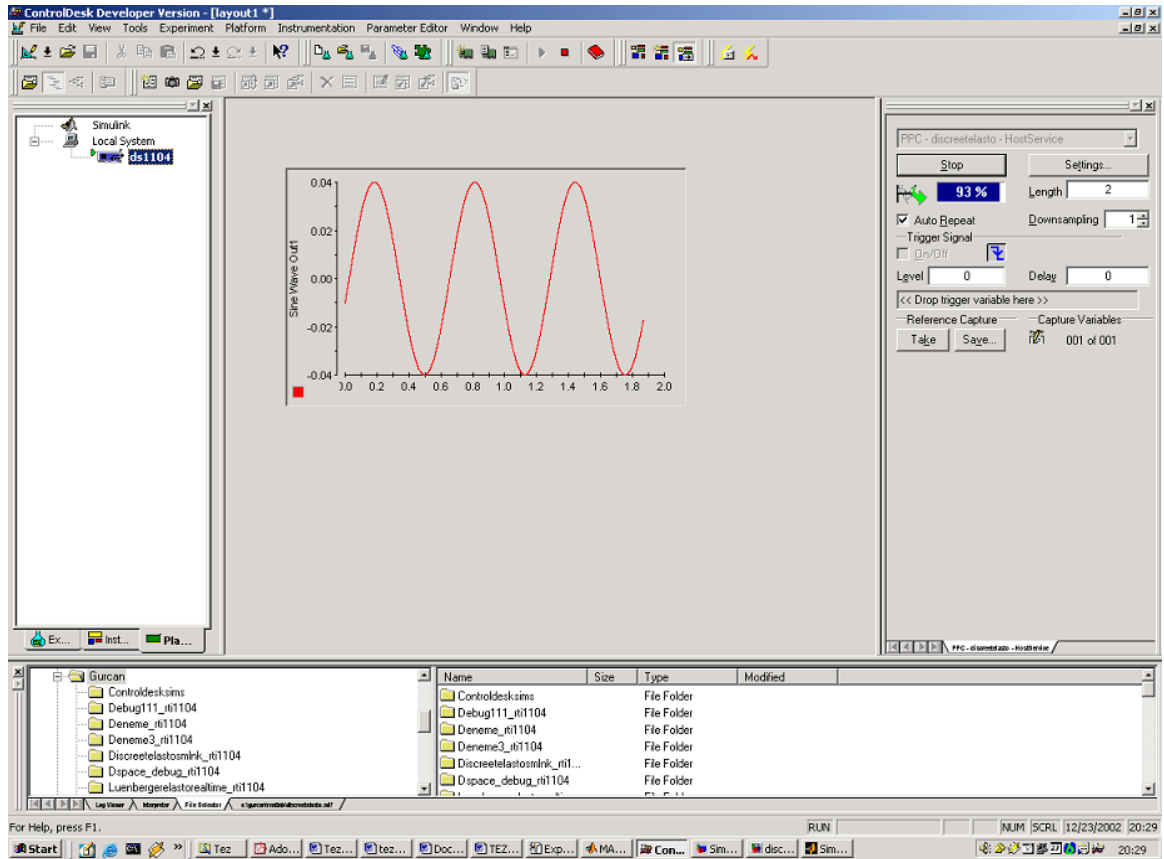


Figure 35: ControlDesk software.

torque reference was used and in steady state test, position, in other words, speed reference mode was used. In order to set the operation type of the driver an interface software, which is called DPL Toolkit, is used. This software has a user interface and a kind of compiler, in order to generate an executable output for driver. The operation mode setting codes are given in Appendix.

The motor is 3000 r.p.m. and 0.98kW AC servo motor. Encoder that is coupled to motor has 4 x 4096 steps. On the other hand DS 1104 DSP kit's encoder structure has 2^{11} steps and also one bit for position direction. The conversion between those encoder structures can be seen in Simulink models in Appendix.

In the same way with simulations, a sinus formed reference torque was applied to the system with a peak value of 0.035N.m. and 1s period.

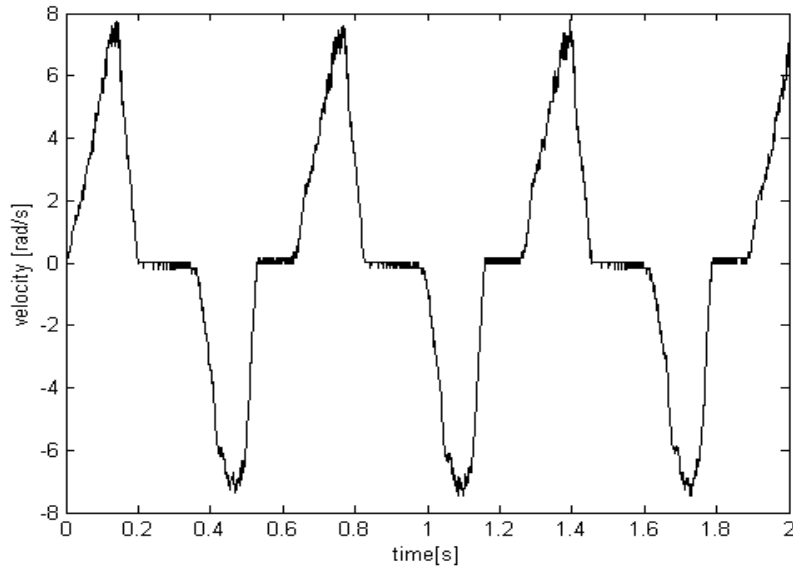


Figure 36: Luenberger-like observer velocity estimation in EMPS.

The estimated velocity values in real-time experiment system are shown in figure 36 and 37. However, since the discrete observer structure doesn't include velocity estimation, the velocity values are derived from $\Delta q / \Delta t$ (successive discrete position value difference and time difference).

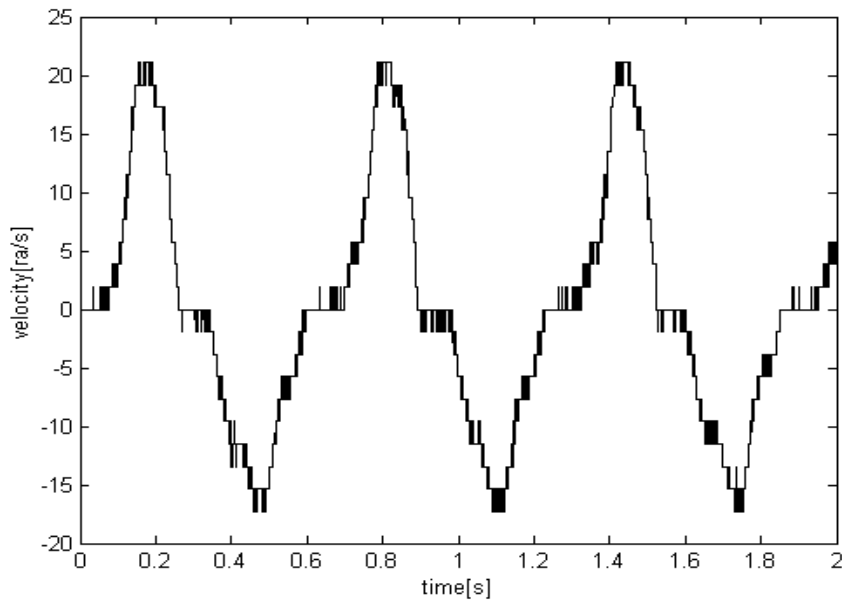


Figure 37: Discrete observer velocity estimation in EMPS

Estimated z elastic displacement values are shown in figure 38 and 39. Both of the observer types give similar estimations for velocity and z elastic displacement. However, z estimation graphs shows us that Luenberger-like observer estimation is

effected by noise. In Luenberger-like observer structure, we estimated both velocity and z parameter. Since both of those estimations affect each other's estimation, it is

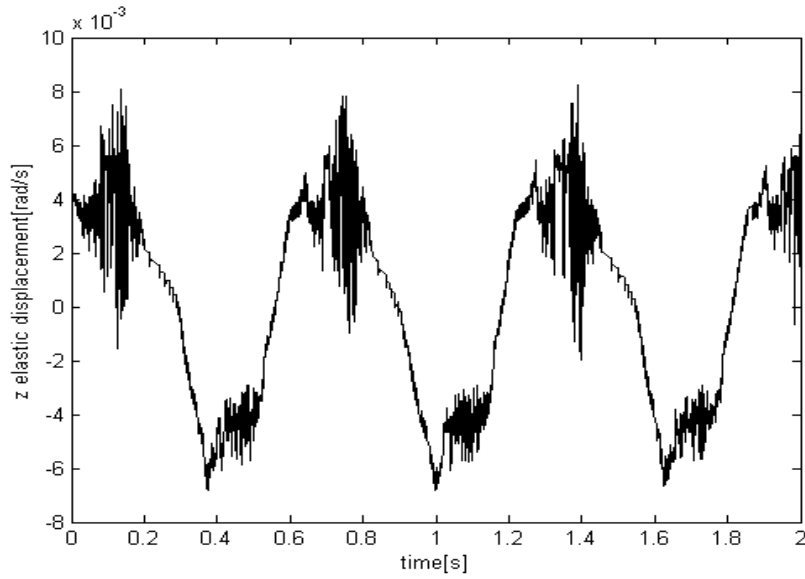


Figure 38: Luenberger-like observer z elastic displacement estimation in EMPS

natural for Luenberger-like observer to have noises in estimations. On the other hand, as it has been discussed before, Discrete observer doesn't really estimates the velocity, so it simply means that, in Discrete observer application, z estimation is not affected from velocity values. Therefore Discrete observer is more appropriate for the system where the velocity is somehow obtained by measurement.

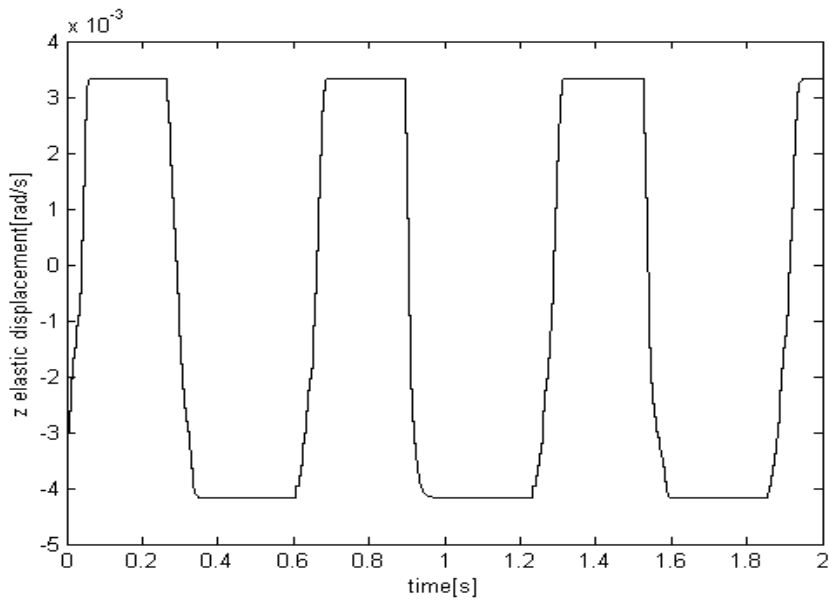


Figure 39: Discrete observer z elastic displacement estimation in EMPS

5.2 Tips For Real-time Application on Matlab,dSPACE,ControlDesk System

1. Run Matlab, ControlDesk and DPL Toolkit.
2. Run Simulink in Matlab window.
3. After running the model, click “Simulation parameter” on “Simulation” menu in the model window in order to build the model for real-time experiment.
4. In the “Solver” tab, start time must be zero and stop time must be inf.
5. Open “Real Time Workshop” tab, Click “Browse” button in order to specify compiler type, only one time is enough for whole experiment. For RT1104 set-up, choose “rti1104.tlc dSPACE DS1104 PPC Real-Time System” line in the compiler type selection window.
6. Press “Build” in order to compile the model.
7. After compilation and linking (totally building) the output file is loaded to dSPACE card, automatically.
8. “Tool window” and “Navigator window” should be open in ControlDesk
9. “Simulink” and “ds1104” are displayed under “Local Systems” in “Navigator window”. You can activate each of them by right clicking and choosing “Make working board” line. The activated one is displayed bold.
10. Open “File Selector” in “Tool window”. On the right hand side of the “Tool window”, there is a “File Explorer”. Drag the file ,that is generated after building the model, and drop on to activated working board (i.e. ds1104)
11. There are “Play” and “Pause” buttons on the upper menu. This buttons activate and deactivate the real-time ds1104 DSP card.
12. For beginning “Trace” and “Capture” operations, a new layout must be opened from the shortcut on the left-upper corner.
13. Several virtual instruments can be deployed in this layout. Click the menu “view->Controlbars->Instrument Selector”.
14. After deploying proper instruments on the layout, click the tab called *.sdf in the “Tool window”.
15. See your model.sdf file’s name in left hand side of the “Tool window”. You will see all the components of your model under “Model Root” and variables of those components on the right hand side.
16. You can drag several of those component variables and drop in to instruments in the layout in order to trace the real-time values of them.

17. In order to obtain those variables' real-time changing and values, you must click the "Animation Mode" in the "Instrumentation" menu. You are not allowed to make a modification on the instruments in the layout and not allowed to stop the real-time system, unless you click "Edit Mode" in the "Instrumentation" menu.
18. To capture and save the values of the variables, you must open "Capture Setting Window" from "View->ControlBars" menu.

Note:

When you are not sure about dSPACE card's working properly, it is suggested to build the Simulink model in Matlab completely, and to reload it. After parametrical modifications in the Simulink model, Matlab doesn't need to recompile it. It progresses only linking. Be sure the building operation also involves compiling.

6. Results

In the first part variety of friction models from static to dynamic was examined. System parameter identification study for Electro-mechanical positioning system was held. Applying to the simulation and Electro-mechanical positioning system, two of the latest dynamic friction models, LuGre friction model and Elasto-Plastic friction model, were compared in detail. It was stated that Elasto-Plastic friction model gives closer results to the real system behaviors. In the second part Luenberger-like friction observer and Discrete friction observer were implemented based on Elasto-Plastic friction model. Simulation and experimental studies were held in order to make a detailed comparison. In those simulations and experiments, velocity and z elastic displacement values were observed. Since it is not possible to measure z elastic displacement in a real system, it is aimed to make a compensation study for z elastic displacement, using those observer structures, in future studies.

7. Appendixes

Appendix 1: Parameter Identification Studies, Steady State Experiment

DPL code:

```
$TITLE Fric_id0
$AUTHOR gurcan
$COMPANY ITU
$VERSION 1
$DRIVE Unidrive
INITIAL {
; Bu ayarlamalar hiz giris ve moment(akim) cikisinin
karsilastirilip skalalandirilmesi icin yapildi.
; Tarih : 22.04.2002
; *****Onemli*****
; 7.30 parametresi 1 yapildi, bu durumda ornekleme periyodu 350uS'
ye iniyor, ve daha dogru sonuclar elde ediliyor.
; Ayni zamanda motorun max speed degeri 3000rpm'ye karsilik gelen
port cikis/ ADC giris gerilimi 5.5 V
; *****

#00.00 = 1000
#06.04 = 4
#01.14 = 2
#01.10 = 1
#02.02 = 0
#03.22 = 0
#03.10 = 200 ; speed proportional
#03.11 = 150 ; speed integral
#03.12 = 25 ; speed derivative
#4.11 = 0 ; speed mode selected

; SETUP Analog Input Parameters:

; Analog Outputs:
#07.30 =1
#07.20=1 ;Output scaling
#07.21 = 0 ;Voltage Mode selected for Analog Output 1
#07.19 =4.02 ;Motor Active Current connected to Analog
;Output 1 (Terminal 9 -- Vinput1)

#16.06 = 1 ;Encoder1 selected

; Analog Input;
#7.11=0 ;Voltage Input mode selected
#7.14=1.37 ;Speed reference connected to Analog
;Input 2 (terminal 7)
}

Background {
top:
goto top: // main background loop
}
```

Simulink Model:

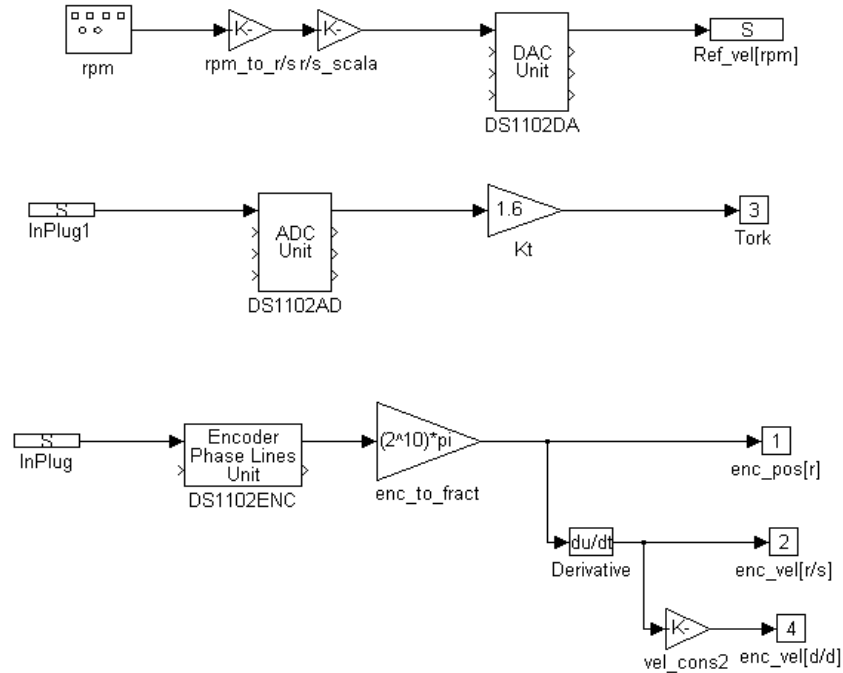


Figure 40: Parameter identification, steady state experiments, EMPS-Simulink model. Speed reference is given.

Matlab Studies:

FindPar function:

```
function err = FindPar(C)
```

```
%U = Tork- (Fg + To2*Speed +(Fc+(Fs-Fc)*exp(-((Speed/Vs)^2)))) ;
```

```
global StdStt1 Plothandle
```

```
SS = [-100.0000 -75.0000 -50.0000 -40.0000 -30.0000 -20.0000 -10.0000 -  
5.0000 -4.0000 -3.0000 -2.0000 -1.5000 1.5000 2.0000 3.0000 4.0000  
5.0000 10.0000 20.0000 30.0000 40.0000 50.0000 75.0000 100.0000 ; -  
0.0307 -0.0233 -0.0159 -0.0127 -0.0097 -0.0064 -0.0034 -0.0020 -0.0014 -  
0.0014 -0.000915 -0.000929 0.0011 0.0013 0.0013 0.0016 0.0021  
0.0034 0.0066 0.0096 0.0124 0.0158 0.0233 0.0308 ];
```

```
SS1= SS(:,12:24);
```

```
t = SS1(1,:);
```

```
y = SS1(2,:);
```

```

tRad=t.*(3.14/30);
z = C(1)+C(2)*tRad+(C(3)+C(4)*exp(-(tRad/C(5)).^2));
err = norm(z-y);

```

Matlab Logs:

```
>> C=[0.1 0.1 0.1 0.1 0.1] (initial conditions)
```

```
C =
```

```
    0.1000    0.1000    0.1000    0.1000    0.1000
```

```
>> FindPar(C)
```

```
ans =
```

```
    5.3989
```

```
>> options=optimset('Display','off','TolX',0.01)
```

```
options =
```

```
    ActiveConstrTol      : []
```

```
    DerivativeCheck      : []
```

```
        Diagnostics      : []
```

```
    DiffMaxChange        : []
```

```
    DiffMinChange        : []
```

```
        Display: 'off'
```

```
    GoalsExactAchieve    : []
```

```
    GradConstr            : []
```

```
    GradObj               : []
```

```
    Hessian               : []
```

```
    HessMult              : []
```

```
    HessPattern           : []
```

```
    HessUpdate            : []
```

```
    Jacobian              : []
```

```
    JacobMult             : []
```

```
    JacobPattern          : []
```

```

        LargeScale      : []
LevenbergMarquardt    : []
        LineSearchType  : []
        MaxFunEvals     : []
        MaxIter         : []
        MaxPCGIter      : []
        MaxSQPIter      : []
        MeritFunction   : []
        MinAbsMax       : []
        Preconditioner   : []
        PrecondBandWidth : []
        ShowStatusWindow : []
        TolCon          : []
        TolFun          : []
        TolPCG          : []
        TolX            : 0.0100
        TypicalX        : []

```

```
>> lambda=fminsearch('FindPar',C,options)
```

```
lambda =
```

```
-0.0528  0.0029  0.0533  0.4170 -0.0171
```

$F_G = -0.0528$ $\sigma_2 = 0.0029$ $F_C = 0.0533$ $F_s = 0.04703$ $V_s = 0.0171$

```
f=inline(' -0.0614 + 0.0029*tRad +(0.0621 + -0.0005*exp(-(tRad/0.377)).^2)')
```

```
f =
```

```
Inline function:
```

```
f(tRad) = -0.0614 + 0.0029*tRad +(0.0621 + -0.0005*exp(-(tRad/0.377)).^2)
```

```
>> fplot(f,[0 10])
```

```
>> fplot(f,[0 1])
```

Appendix 2: Parameter Identification Studies, PRBS Experiment

DPL code:

```
$TITLE Fric_ID1
$AUTHOR Gurcan
$COMPANY ITU
$VERSION 1
$DRIVE Unidrive
INITIAL {
; Bu ayarlamalar hiz giris ve moment(akim) cikisinin
karsilastirilip skalalandirilmesi icin yapildi.
; Tarih : 22.04.2002
; *****Onemli*****
; 7.30 parametresi 1 yapildi, bu durumda ornekleme periyodu 350uS'
ye iniyor, ve daha dogru sonuclar elde ediliyor.
; Ayni zamanda motorun max speed degeri 3000rpm'ye karsilik gelen
port cikis/ ADC giris gerilimi 5.5 V
; *****

#00.00 = 1000
#01.10 = 1
#4.11 = 1 ; tork mode selected

; SETUP Analog Input Parameters:

; Analog Outputs:
#07.30 =1
#07.20=1 ;Output scaling
#07.21 = 0 ;Voltage Mode selected for Analog
Output 1
#07.19 =4.02 ;Motor Active Current connected to
;Analog Output 1 (Terminal 9 --
Vinput1)

#16.06 = 1 ;Encoder1 selected

; Analog Input;
#7.11=0 ;Voltage Input mode selected
#7.14=4.08 ;Torque Reference connected connected
;to Analog Input 2 (terminal 7)
}

Background {
top:
goto top: // main background loop
}
```

Simulink Model:

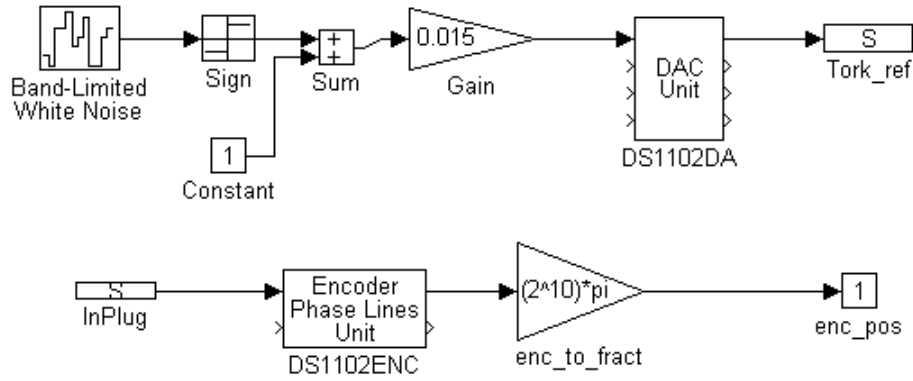


Figure 41: Parameter Identification, PRBS Experiment, torque reference is given.

Matlab Logs:

System Identification Toolbox Studies:

$$A(q) = 1 - 0.9201q^{-1} - 0.04387q^{-2}$$

$$B(q) = -0.000821 + 0.01257q^{-1}$$

$$A(q)y(t) = B(q)u(t) + e(t)$$

$$y(t) = \frac{B(q)}{A(q)}u(t) + \frac{e(t)}{A(q)}$$

$$y(t) = \frac{-0.000821 + 0.01257q^{-1}}{1 - 0.92q^{-1} - 0.0438q^{-2}}u(t) + \frac{e(t)}{A(q)} \implies \frac{y(z)}{u(z)} = \frac{-0.000821z + 0.01257}{z^2 - 0.92z + 0.0438}$$

$$\frac{y(s)}{u(s)} = \frac{-32.6s + 3.96 \cdot 10^4}{s^2 + 3127s + 4.171 \cdot 10^5}$$

$$\sigma_0 = 10.53 \quad J = 2.5246 \cdot 10^{-5} \quad \sigma_1 = 0.076$$

Appendix 3: Parameter Identification Studies, Dahl Curve Experiment

Simulink Model:

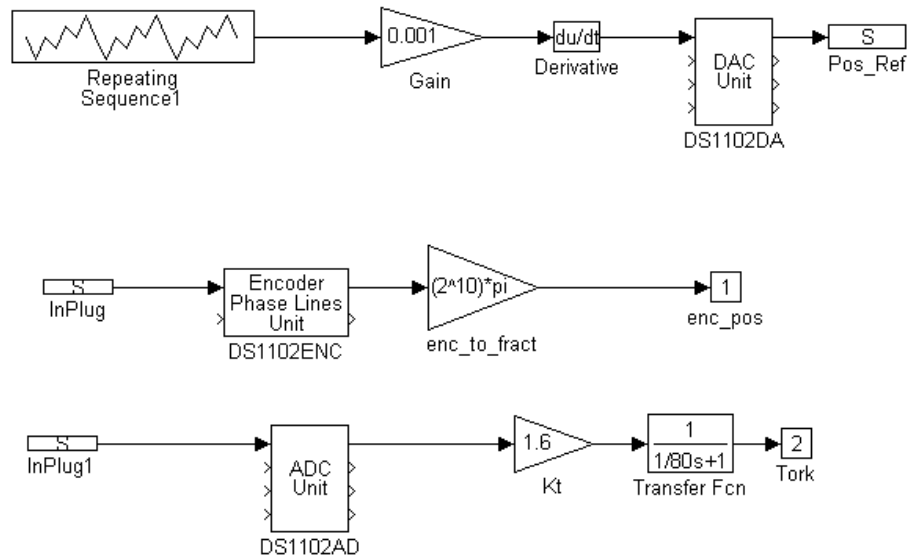


Figure 42: Parameter Identification, Dahl Curve Experiment

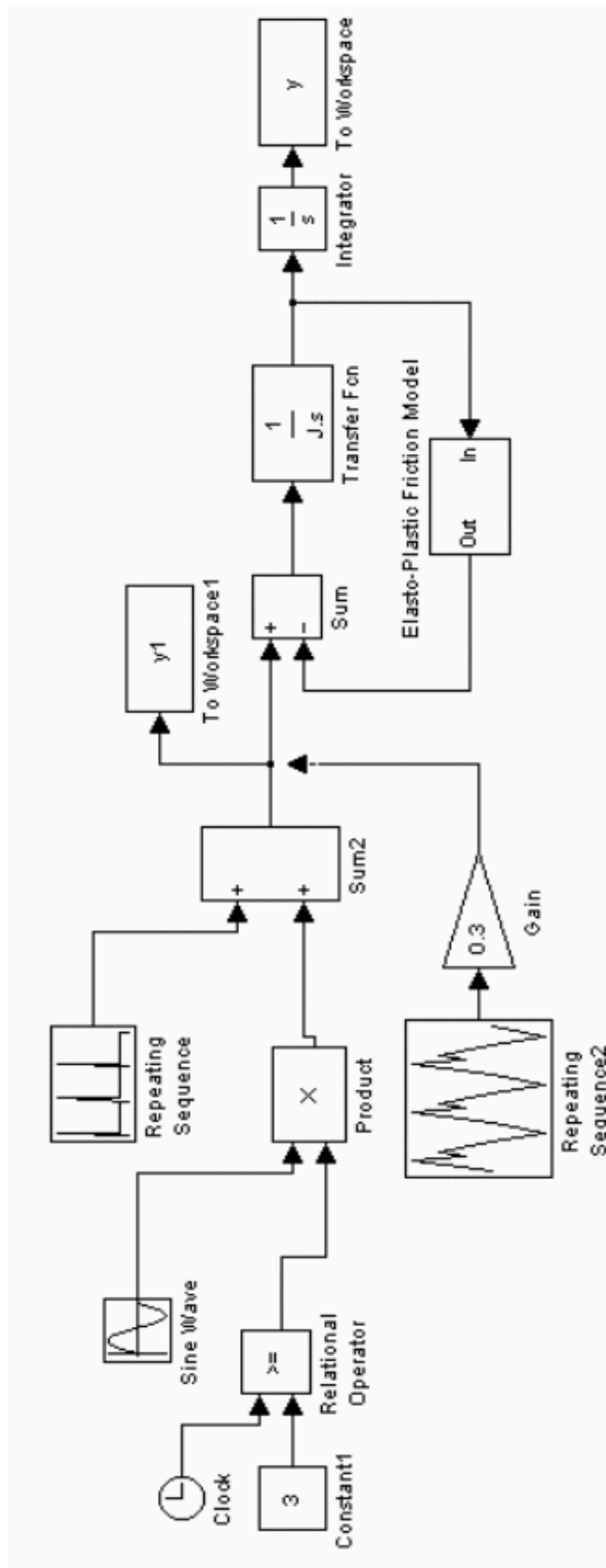


Figure 43: Dahl Curve Experiment, Simulink model of the EMPS with Elasto-Plastic friction model.

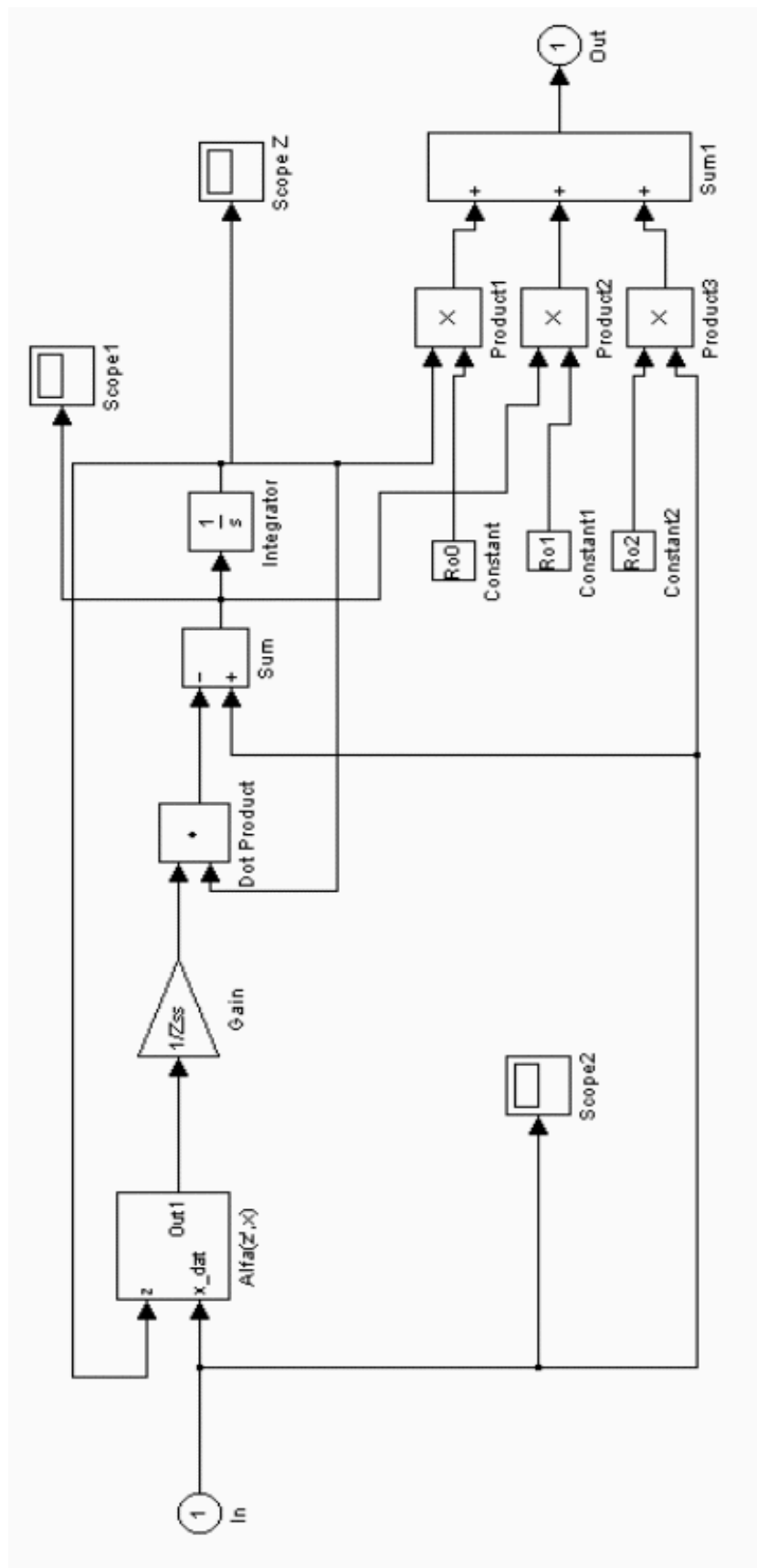


Figure 44: “Elasto-Plastic Friction Model” Subsystem.

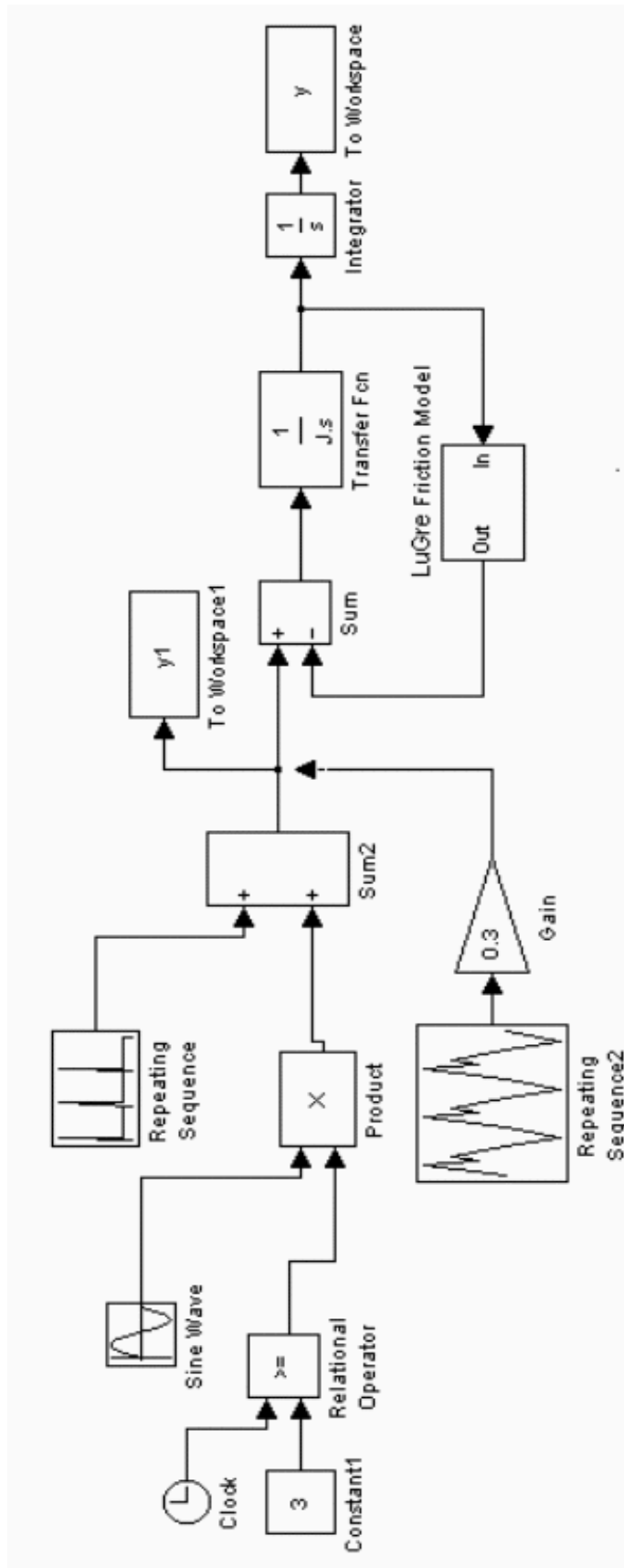


Figure 46: Dahl Curve Experiment, Simulink model of the EMPS with LuGre friction model.

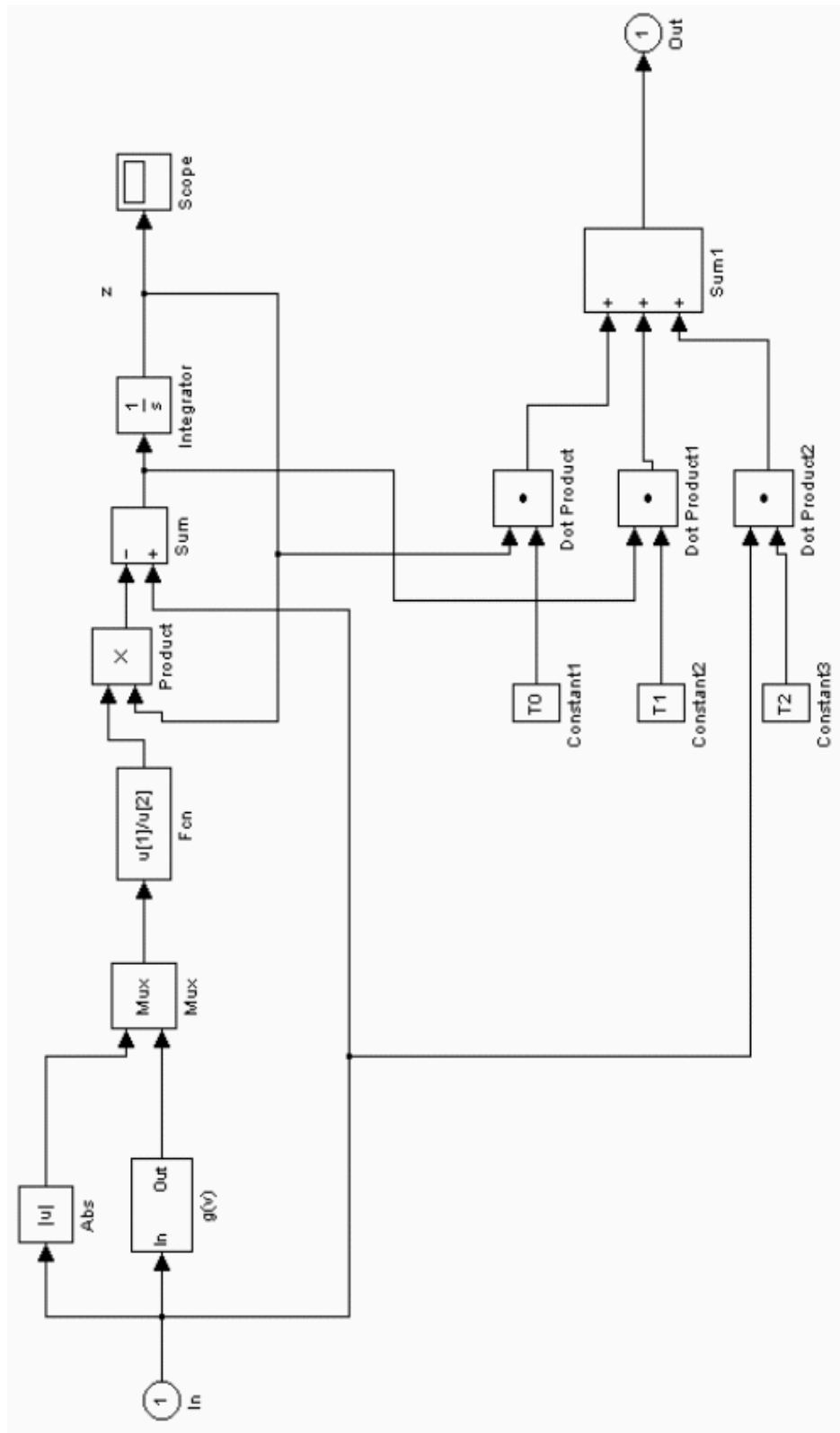


Figure 47: “LuGre Friction Model” Subsystem.

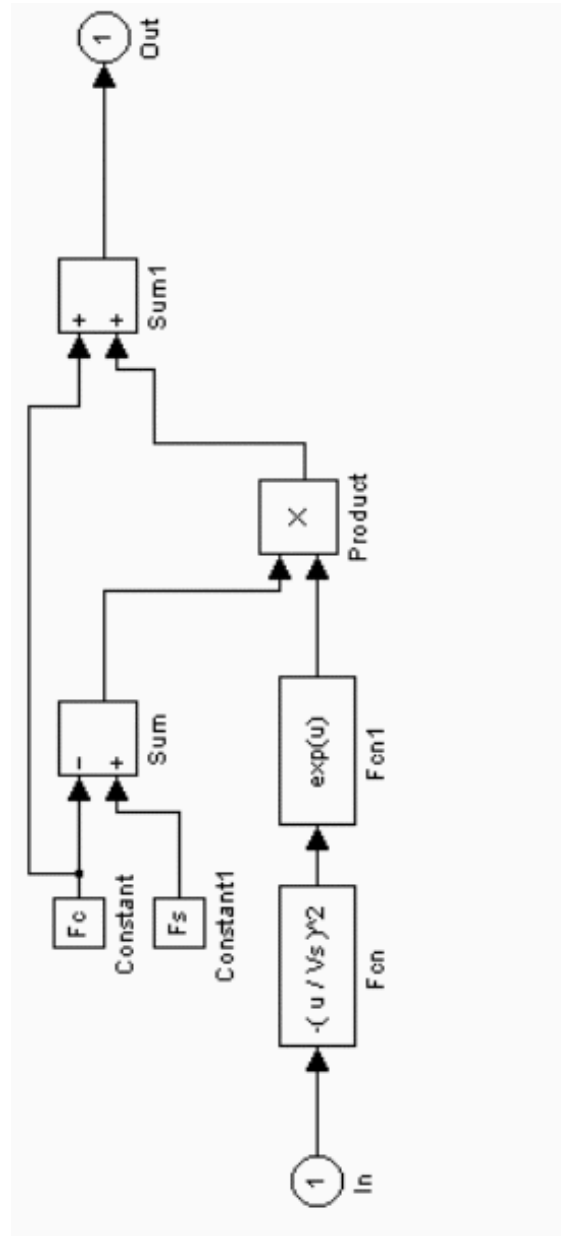


Figure 48: $g(v)$ function.

Appendix 4: Comparison of The Friction Models Experiment

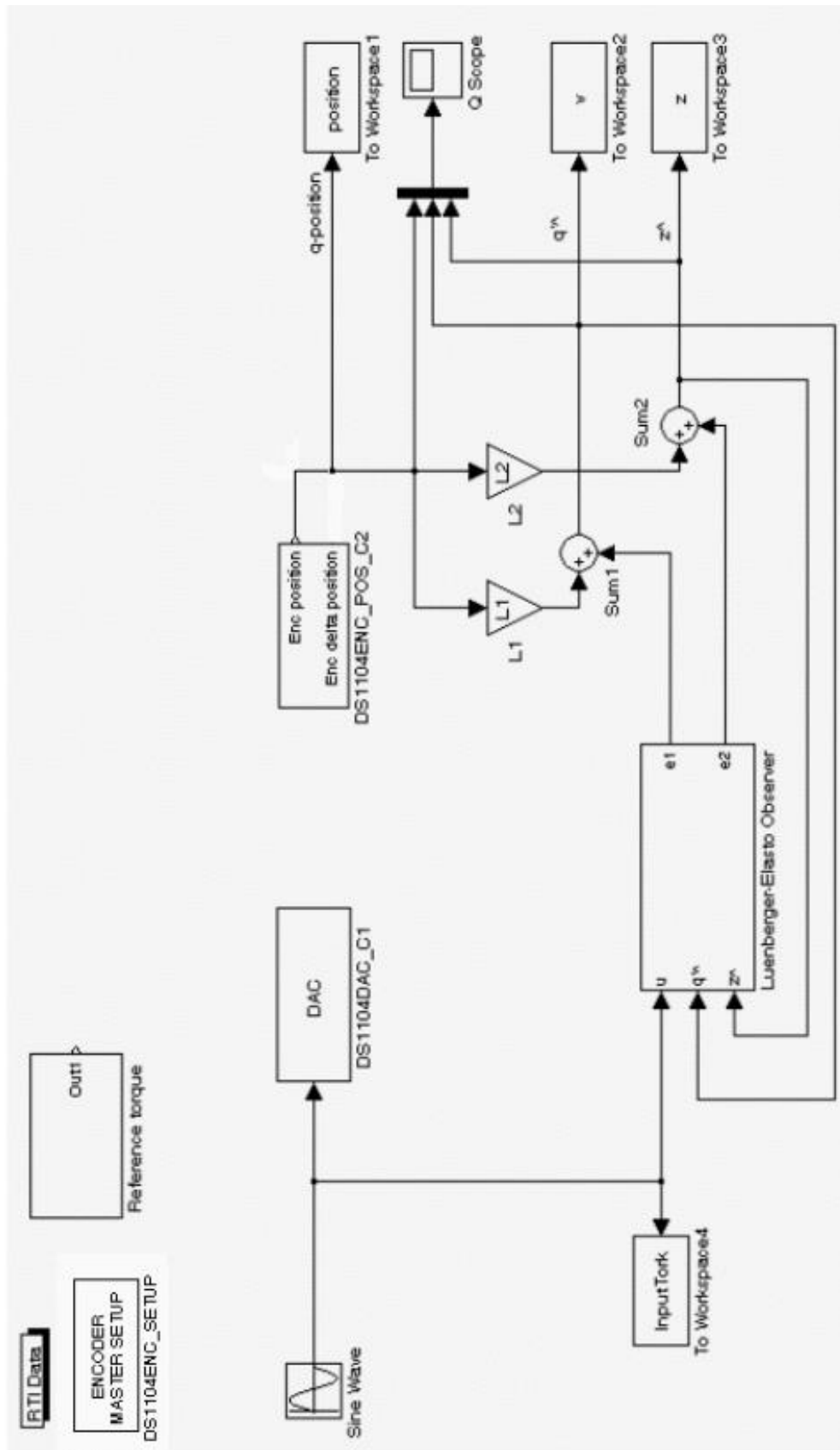


Figure 49: Real-time system with Luenberger-like Elasto-Plastic Friction Observer



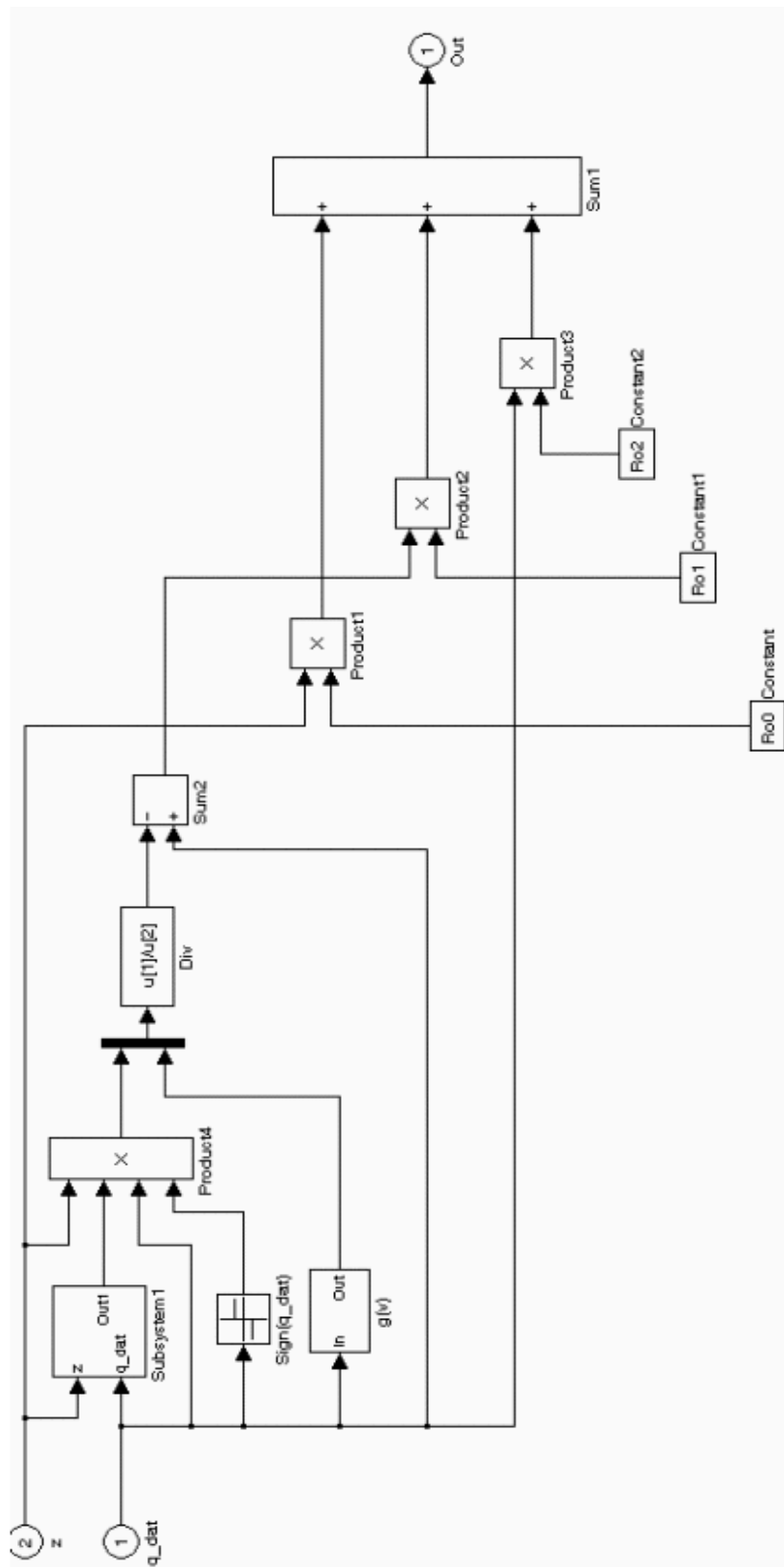


Figure 51: “Elasto Friction Force” Subsystem.

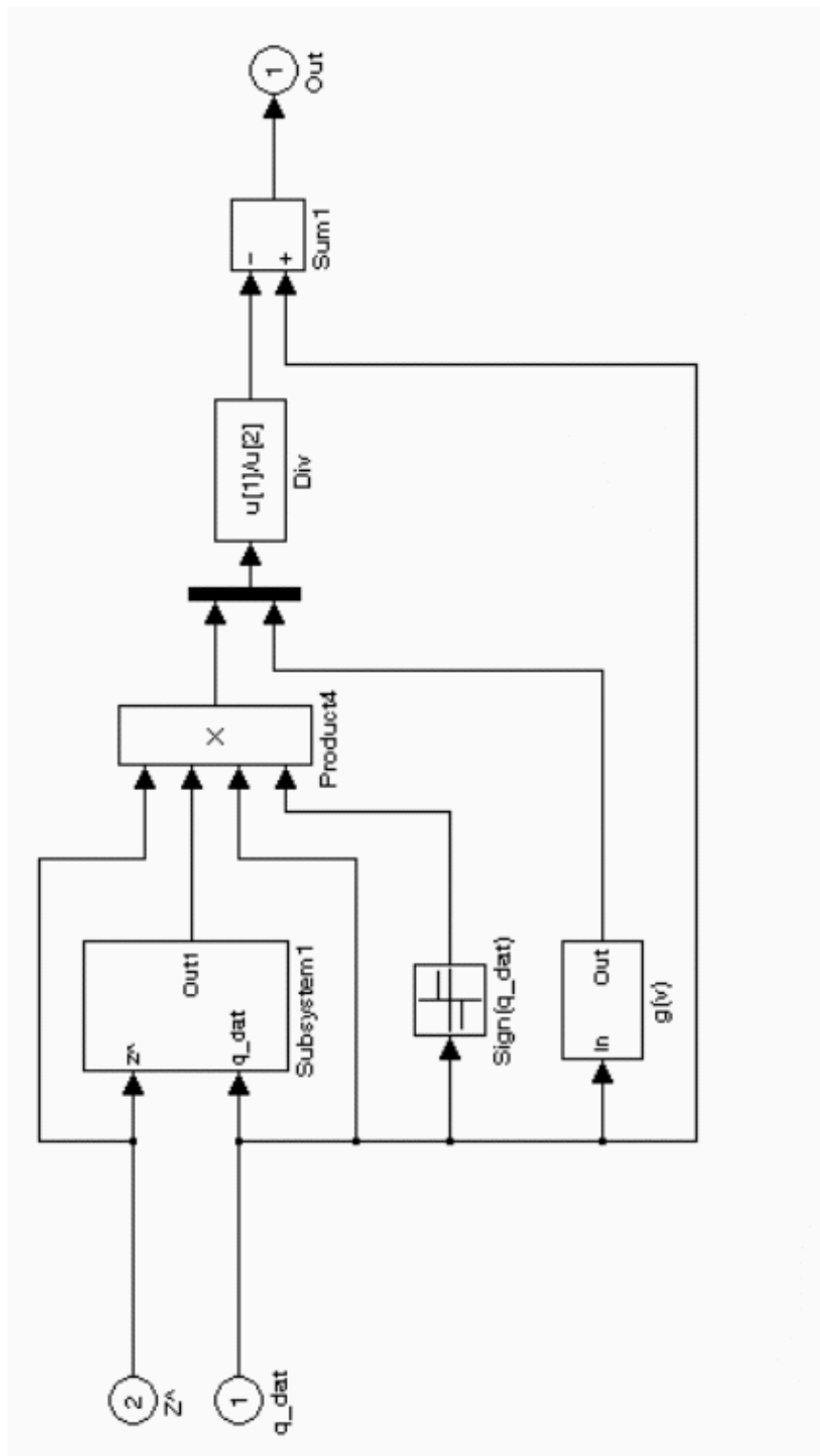


Figure 52: “ElastoZ” Subsystem.

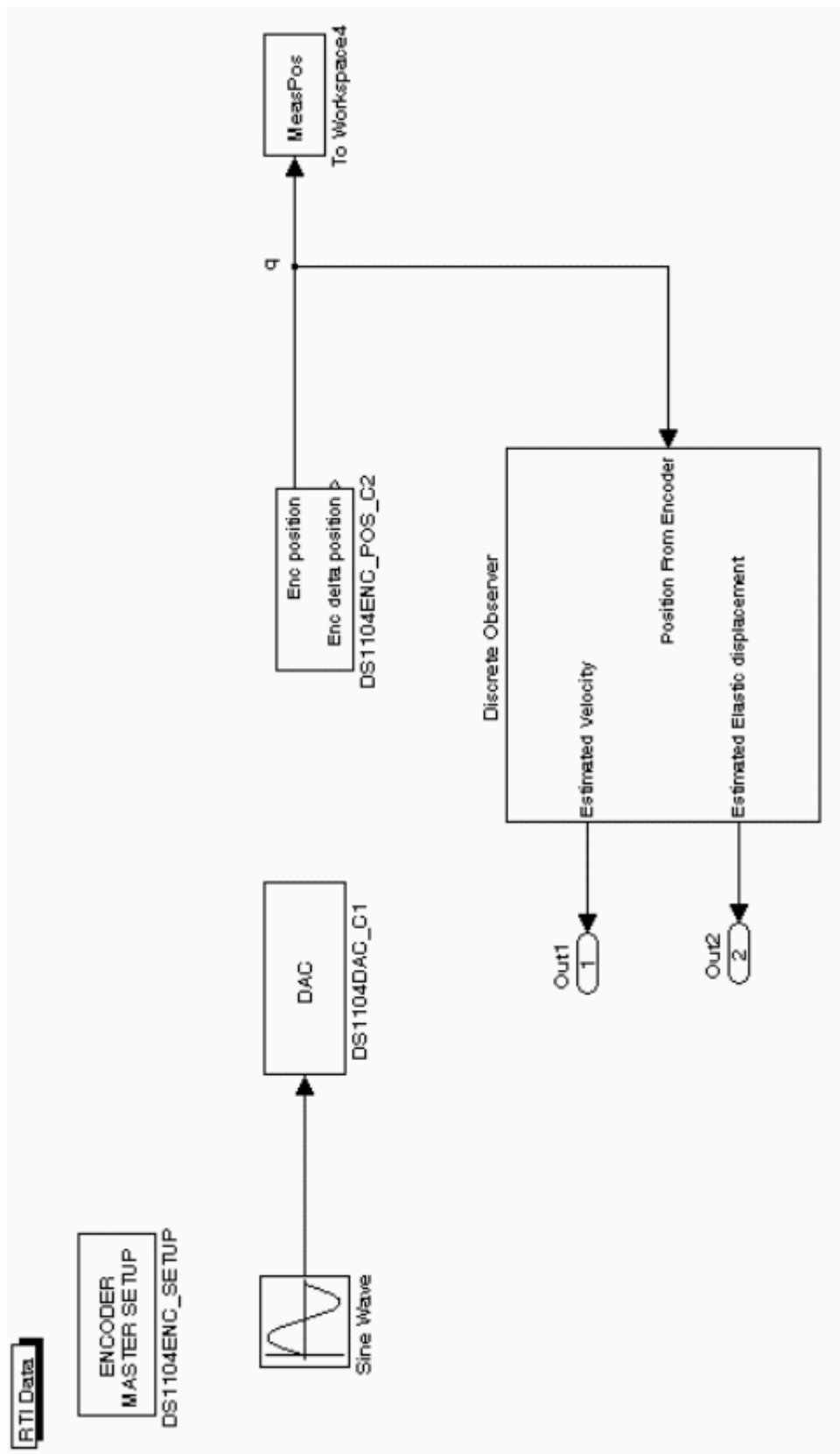


Figure 53: Discrete Elasto-Plastic Model Observer.

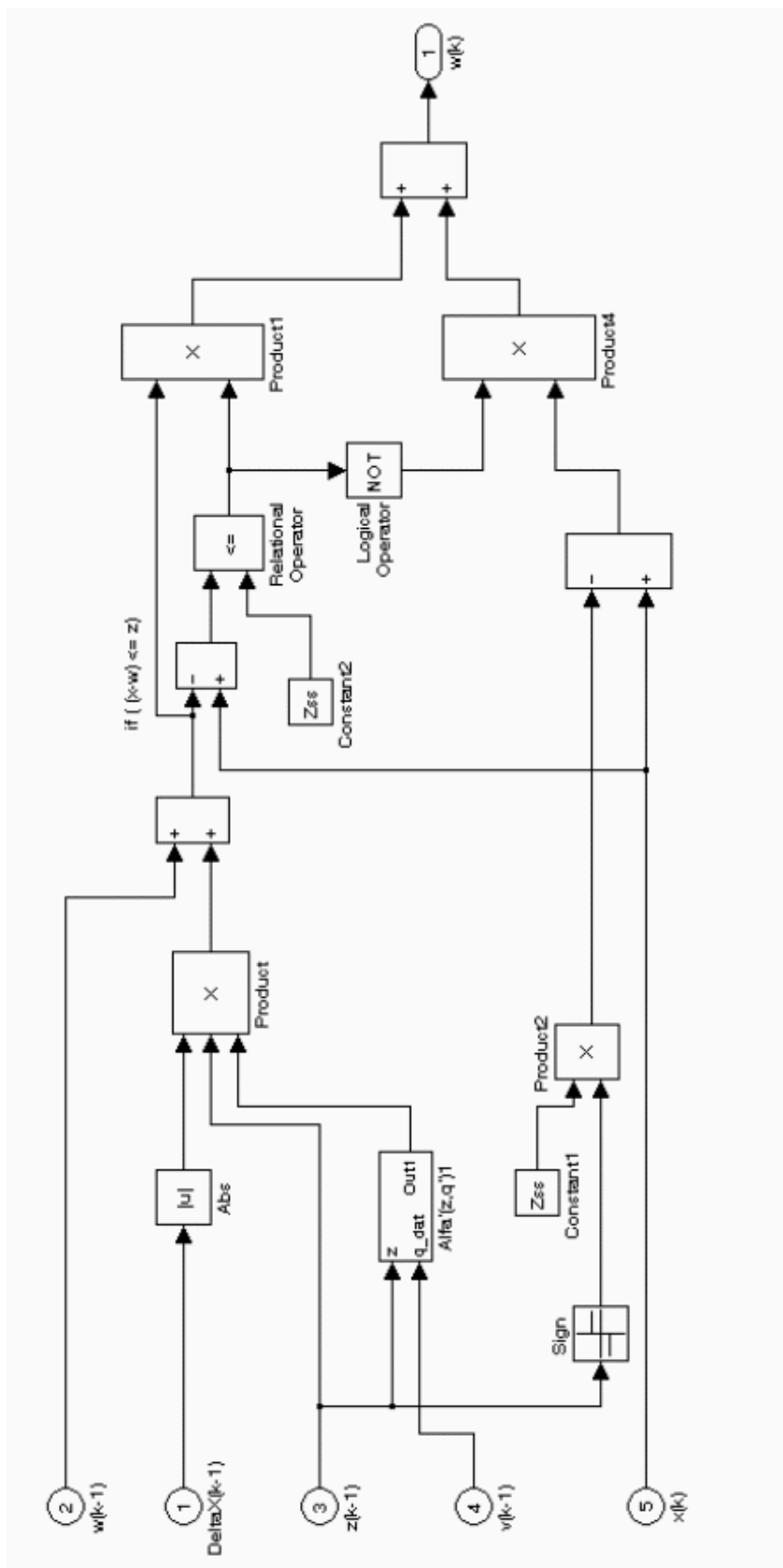


Figure 55: “w (k) function” Subsystem.

References:

- [1]. C.Canudas De Wit, H. B. Armstrong, P. Dupont “Survey of models, analysis tools and compensation methods for the control of machines with friction”,Automatica v 30 n 7 July 1994 p 1083-1138.
- [2]. C.Canudas De Wit, H. Olsson, K. J. Aström and P.Lischinsky, “ A New Model for Control of Systems with Friction ” IEEE Trans. Of Automatic Control, Vol. 40, No. 3, p.p. 419 –425, 1995.
- [3]. Pierre Dupont, Vincent Hayward, and Brian Armstrong. *Single state elasto-plastic models for friction compensation*. IEEE Trans. on Automatic Control, 2000. Under Review.
- [4]. Iwasaki, Makoto; Shibata, Tomohiro; Matsui, Nobuyuki “Disturbance-Observer Based Non-linear Friction Compensation in Table Drive System”,IEEE\ASME Transactions on Mechatronics Vol 4 No 1 March 1999.
- [5]. Misovec, Kathleen M.; Annaswamy, Anuradha M. “Friction Compensation using adaptive non-linear control with persistent excitation”,Int.J.Control 1999 vol 72 No 5 457-479
- [6]. Nuninger, B. Balaud and F. Kratz “Disturbance Rejection using output and input estimation. Application to the Friction Compensation of a DC Motor” Contr. Eng. Practice vol 5 no 4 pp 477-483, 1997.
- [7]. C. Canudas De Wit, G. Morel, P. Lischinsky, “Friction Compensation for an Industrial Hydraulic Robot “, IEEE Control Systems 1999.
- [8]. H.olsson, K. J. Astrom, C. Canudas De Wit, P. Lischinsky, “ Dynamic friction models and control design “, American Cont. Conf. 1993 p.p. 1920-1926.
- [9]. S. Meshkat, “Observer algorithm compensates for machine tool friction torque”, The Fusion of Power & Motion Technology & Applications v 21 n 4 Apr 1995 p 50-53.
- [10]. J. Amin, B. Friedland, A. Harnoy, “Implementation of a friction estimation and compensation technique”, IEEE Conf. On Control Applications-Proc. 1996 p 804-808.

- [11]. H.Olsson, K. J. Aström, C. Canudas De Wit, M. Gafvert, P. Lischinsky, “Friction Models and Friction Compensation” *European Journal of Control* v 4 n 3 1998. p 176-195,
- [12]. F. Altpeter “Friction Modeling, Identification and Compensation” PhD thesis Lausanne 1999.
- [13]. S. Neculescu, J. DE Carefel, C. Canudas De Wit, “Investigation on the efficiency of acceleration feedback in servomechanism with friction”, *Dynamics and Control* v 7 n 4 Oct. 1997 p 377-397.
- [14]. M. Gafvert, “Comparison of two Frictions Models “, M. Thesis, June 1997.
- [15]. P. Dahl, “A Solid friction model” Technical report TOR –0158 (3107-18)-1 The Aerospace Corporation El Segundo, CA, 1968

Curriculum Vitae

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