ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL OF SCIENCE ENGINEERING AND TECHNOLOGY

HEAT TRANSFER ENHANCEMENT WITH NANOFLUIDS IN LAMINAR FLOW

M.Sc. THESIS

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İSTANBUL TEKNİK ÜNİVERSİTESİ ★ FEN BİLİMLERİ ENSTİTÜSÜ

NANOAKIŞKAN KULLANARAK LAMİNAR AKIŞTA ISI TRANSFERİNİN ARTIRILMASI

YÜKSEK LİSANS TEZİ

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Mehrdad-KARIMZADEHKHOUEI, a **M.Sc.** student of ITU **Graduate School of Scince Engineering and Technology** student ID **503091149**, successfully defended the **thesis** entitled "**HEAT TRANSFER ENHANCEMENT WITH NANOFLUIDS IN LAMINAR FLOW**", which he prepared after fulfilling the requirements specified in the associated legislations, before the jury whose signatures are below.

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To my Mother and Brothers,

FOREWORD

I am heartily thankful to my supervisor, Prof. Dr. Lütfullah KUDDUSİ, whose encouragement, guidance and support from the initial to the final level enabled me to develop an understanding of the subject. I have experienced an excellent academic study under his supervision and his invaluable recommendations and support helped me to plan my further academic studies in the best way.

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Lastly, I offer my regards and blessings to all of those who supported me in any respect during the completion of the project.

May 2012

Mehrdad KARIMZADEHKHOUEI
(Mechanical Engineer)

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ABBREVIATIONS

CE	: Continuity Equation
DWCNT	: Double Walled Carbon Nano Tube
EE	: Energy Equation
ME	: Momentum Equation
MWCNT	: Multi Walled Carbon Nano Tube
SWCNT	: Single Walled Carbon Nano Tube

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LIST OF SYMBOLS

c_p : Specific heat capacity
c_{pm} : Mean specific heat capacity
C_1, C_2 : Constant coefficients
d : Molecular diameter
D : Tube diameter
f_r : Radial body force
f_θ : Tangential body force
f_z : Axial body force
g : Earth's gravity
Gr : Grashof number
h_x : Local convective heat transfer coefficient
k : Thermal conductivity
$\bar{k}$: Apparent thermal conductivity
$k_{\bar{D}}$: Effective apparent thermal diffusivity
$k_{D,r}$: Radial thermal dispersion coefficient
$k_{D,x}$: Axial thermal dispersion coefficient
m : Mass
$\dot{m}$: Mass flow rate
n : Shape factor
Nu : Nusselt number
p : Pressure
Pe : Peclet number
Pr : Prandtl number
$q_w^{''}$: Wall heat flux
Q : Heat flow
$\dot{Q}$: Total volume flow rate
r_0 : Tube radius
Ra : Rayleigh number
Re : Reynolds number
T : temperature
T_c : tube center temperature
T_m : Mean temperature
u : Axial flow velocity
u_m : Mean velocity
U_m : Mean flow velocity
U_c : Maximum velocity
V : Volume
v_r : Radial flow velocity
v_θ : Tangential flow velocity

GREEK LETTERS

α	: Thermal diffusivity
α_{nf}^{*}	: Effective apparent thermal diffusivity
β	: Volumetric thermal expansion coefficient
Θ	: Dimensionless temperature
κ	: Boltzman constant, 1.381 x 10^{-23} J/K
μ	: Dynamic viscosity
ν	: Kinematic viscosity
ρ	: Density
τ	: Shear stress
ϕ	: Particle volume fraction
ψ	: Empirical shape factor

SUBCRIPTS

c	: Center
f	: Base fluid
nf	: Nanofluid
p	: Nanoparticle
w	: Wall

HEAT TRANSFER ENHANCEMENT WITH NANOFLUIDSIN LAMINAR FLOW

SUMMARY

Heat transfer is one of the most important processes in many industrial and consumer products. Poor thermal conductivity of conventional fluids is one of the most significant limitations in heat transfer process. For more than a century scientists have made great efforts to solve this fundamental limit by using milli- and micro-sized particles in base fluids. In addition, in many industrial technologies cooling is one of the most vital needs. In providing of this vital need there is a main limitation which that is low thermal conductivity.

Enhancement of heat transfer can be done by using various different techniques and methods, such as increasing the heat transfer surface or the convective heat transfer coefficient. However, there is another state-of-the-art method, which that is adding milli-, micro- and even nano-sized of metallic or nonmetallic particles to the base fluids for increasing thermal conductivity of resultant fluid.

The nanoparticles are ultrafine particles in the size of nanometer order. Nano is a prefix denoting the minus 9^{th} power of ten. The definition of nanoparticles differs depending upon materials, fields and applications concerned. In the narrower sense particles smaller than 10-20nm is nanoparticles, but in physical properties and in most of the cases particles in the range of 1-100nm are nanoparticles.

There are many types of materials, which have been used as nanoparticles such as oxide ceramics, nitride ceramics, carbide ceramics, metals, semiconductors, carbon nanotubes, composite materials such as alloyed nanoparticles, nanoparticle core-polymer shell composites. Nowadays in addition to nonmetallic, metallic and other materials for nanoparticles new materials and structures, such as materials doped with molecules in their solid-liquid interface structure, are used in technology. Also there are many types of liquids which have been used as base fluids such as water, ethylene glycol and oil.

Nanomaterials have unique mechanical, optical, electrical, magnetic and thermal properties. This area of research is active and attractive because of its usage in many applications, such as using nanofluids as coolants in the automobile and electronics industries. Of course using this method has its own problems like not having the stable solution, causing erosion and clogging in the channels. Using nanofluids, which are a colloidal mixture of nanoparticle and a base liquid, in last decades provide impressive improvements in thermal properties of base fluids.

In this thesis, firstly continuity, momentum and energy equations are solved for pure water in circular tube. Secondly, the solutions are repeated for mixture of nanoparticles in base fluid by supposing the mixture as a single-phase fluid. Finally, by supposing the mixture as two-phase the calculations of the fundamental heat transfer equations are repeated for dispersed model. The third part is repeated for a nanofluid in between two parallel plates. All of calculations are calculated for both

plug and parabolic velocity profile for different kinds of thermal boundary conditions, such as constant wall temperature and constant heat flux boundary conditions. Nanoparticle volume fractions differ from one to 5 percent in nanofluids with different nanoparticle materials such as alumina etc.

NANOAKIŞKAN KULLANARAK LAMİNAR AKIŞTA ISI TRANSFERİNİN ARTIRILMASI

ÖZET

Isı transferi, pek çok endüstriyel ve tüketici ürünlerin en önemli aşamaların biridir. Düşük ısıl iletkenlik, ısı transferi sürecinde en önemli sınırlamalardan biridir. Yüzyılı aşkın bir süredir bilim adamları, bir temel akışkanın içinde mili-ölçekli veya mikro-ölçekli parçacıkları kullanarak bu limitin üstesinden gelmek için çaba harcamaktadırlar. Temel akışın içine bu parçacıları kattıktan sonra bu akışa "nanofluid" denilmektedir. Ayrıca, soğutma pek çok endüstriyel teknolojilerde en önemli ihtiyaçlardan biridir. Bu hayati ihtiyacı sağlanması yolunda düşük ısıl iletkenlik en önemli sınırlardan biridir.

Isı transferinin artırılması için ısı transfer yüzeyini arttırmak veya ısı taşınım katsayısının arttırılması gibi çeşitli teknikler ve yöntemler kullanılarak yapılabilir. Ancak, ısı taşınım katsayısını arttırmak için mili-, mikro- hatta nano-ölçekli metal veya metal olmayan parçacıkları akışkana ilave etmek başka bir modern yöntemdir. Son yüzyılda nano-ölçekli partiküller daha da fazla araştırlmakta ve deneysel çalışmalar yapılmaktadır. Nanoakışları kullanıldığı zaman en buyuk hedeflerden birisi teorik ve amperik formülleri iyi bir şekilde ve en az hatayla doğrulamak ve bir sonraki aşamalarda uygulamaktır.

Sınırlı enerji kaynaklarının endüstrinin tüm bölümlerinde verimli ve etkili kullanılması için bilim insanları ısı transferinin iyileştirmesi için yeni metodları araştırmaktalardır. Isı bilimi ve mühendisleri çaba harcamakalrının, ısı transferini artırmak için, temel nedeni enerji kaynaklarının daha verimli bir şekilde kullanılmasıdır. Son yüzyılda bilim adamları ısı transferi konusunda modern yöntemler üretmektedir. Modern metodların en iyi örneği akışkanların içerisine ısıl transferini iyileştirmek amacıyla nano boyutunda metal parçacıkalrı eklemekdir. Bu iyileştirmek, metal parçacıkların ısı transfer katsayısının temel akışkanın ısı transfer katsayısından daha fazla olduğu sebebiyle meydana gelmektedir.

Nanopartiküller, nanometre boyutunda ultra ince parçacıklardır. Nano eki on üzeri eksi 9 demektir. Bazi alanlarda 10-20nm'den daha küçük olan parçacıklar, nanopartikül olarak tanımlanmakta, ancak fiziksel özelliklerde ve bir çok alanlarda 1 ve 100nm arlıklarında olan parçacıklara nanopartiküller denilmektedir. Bu nanopartiküller ile hazırlanan nanoakışkanın tam, kararlı ve uzun ömürlü bir süspansiyon olması için akışkanın kimyasal özelliklerinin değişmemesi gerekmektedir.

Oksit seramik, nitrür seramik, karbür seramik, metal, yarı iletkenler ve karbon nanotüpler gibi veya alaşımlı nanopartiküller ve nanoparçacık çekirdekli-polimer kabuklu kompozitler gibi bir çok değişik nanopartikül türleri kullanılmaktadır. Günümüzde metal, metal olmayan ve diğer yeni malzemeler ve yapılar nanopartikül olarak kullanılmaktadır. Ayrıca su, etilen glikol ve yağ gibi sıvılar temel akışkan olarak kullanılmaktadır. Bu geleneksel akışkanların ısı transfer katsayıları düşük

olması nedeniyle ısı transferinin verimini daha az iyileştirmektedir. Ama nanoparçacıkalrı kattıktan sonra artık bir iki fazlı akışkan vardır ki ısı transferi katsayısı bahs ettiğimiz geleneksel sıvılardan daha yüksektir. Bir nanoakışkanda sıvı-katı karışımı sonraki bölümlerde bahs edilecek gibi bir basit karışım anlamına gelememektedir. Hazırlanan nanoakışkanın ısı transfer katsayısı temel akışkan içerisinde olan nanopartiküllerin hacimsel oran ile ilişkilidir. Ayrıca, nanoparçacıkların şekline ve boyutuna da bağlıdır. Son yıllara ait olan çalışmalara göre başka faktürler de, partiküllerin tipi, çapı, temel akışkanın laminar veya türbülanslı olamsı ve tabii ki kanalın kesiti, ısı transfer katsayısında rol oynamaktadır.

Nanomalzemeler benzersiz mekanik, optik, elektrik, manyetik ve ısıl özelikleri olmaktadır. Bu araştırma alanı nanoparçacıkalrı otomotiv ve elektronik sanayilerinde soğutucu olarak kullanmak gibi bir çok alanlarda kullanım nedeniyle aktif ve çekicidir. Tabii ki bu yöntemin eroziyona neden olamsı ve kanalların tıkanması gibi kendine ait sorunları vardır. Son yıllarda, nanoakışkanların kullanımı akışkanların ısıl özelikleri üzerine etkileyici gelişmeler sağlamaktadır.

Bu tezde, öncelikle süreklilik, momentum ve enerji denklemleri, dairesel tüp içinde saf su için çözülmüştür. Nanopartikülleri suya ilave etmeden saf suyun ısıl iletkenliğini bulmak için deneysel çalışmalar yapılmış ve bu deneysel çalışmalar literatürde yayınlanmıştır. Bu tezde literatürü ve farklı kaynakları kullanarak saf su için ısıl iletkenlik katsayıları elde edilmiştir. Bir boru içinde momentum ve enerji denklemlerini çözerken giriş hızı iki farklı profil olarak göz önüne alınmıştır. Birinci tür hız profili sabit hız profilidir. Bu hız profiline literatürde “Plug Flow” denilmektedir. İkinci tür hız profili ise parabolik hız profilidir.Literatürde farklı ısıl sınır koşulları kullanılmaktadır ve en önemli ve fazla kullanılan ısıl sınır koşulları H1, H2 ve T den ibarettir. T ısıl sınır koşulunda duvar sıcaklığı kanal boyunca eksenel ve çevresel olarak sabittir. H1 sınır koşulunda ise kanal boyunca eksenel yönde duvara sabit ısı transferi verilmektedir ve herhangi bir kesitte sabit duvar sıcaklığı olmaktadır. H2 sınır koşulunda eksenel ve çevresel yönde kanal boyunca duvara sabit ısı akısı söz konusudur. Bu tezde 3 ısıl sınır koşulları arasında H1 ve H2 sınır koşulları seçilmektedir.

Sonraki aşamada, çözümler nanoakışkanın tek fazlı varsayımını kabul ederek tekrarlanmıştır. Nanoakışkanın ısı transfer katsayısını bulmak için farklı yöntemler kullanılmaktadır. En önemli yöntem deneysel çalışmalardır ve bu deneysel çalışmaları kullanarak teorik çözümler üretilmektir. Deneysel çalışmalarda çok sayıda faktör ısı transfer katsayısını etkilemektedir. En önemli faktörlerden kaç tanesi aşağıda verilmektedir: nanoparçacıkların oranı, parçacıkların üretilmesi için kullanılan malzeme, kullanılan temel sıvı, parçacıkların çapı ve şekli. Bu konuda zorlanmış ve doğal taşınımla nanoakışkalnların ısı transfer araştırmaları yapılmaktadır.

Kullanılan denklemlerde nanoakışın için Einstein’in teorik formülü kullanılmaktadır. Bu formül nanoparçacıkların oranı %5’den daha az olduğu takdirde geçerlidir. Bir sonraki aşamalarda ısı transfer katsayısı 3 farklı formül ile elde edilmektedir. Bu 3 farklı formülün adı şöyledir: Maxwell, Hamilton-Crosser ve Bruggeman. Bu formüllerden elde edilen ısı transfer katsayılarını daha kolay anlaşılmak amacıyla her bir formül için birer çizelge ile gösterilmektedir. Bazi çizelgelerde bir formülden, 3 farkli akış için, elde edilmiş katsayılar karşılaştırılmıştır. Diğer çizelgelerde ısı

transfer katsayıları aynı 3 nanoakışkan için iki farklı formüllerden elede edilip karşılaştırılmıştır.

Son olarak, ısı transferinin temel denklemleri nanoakışkanı iki fazlı bir karışım kabul ederek dağınık model için çözülmüştür. Üçüncü kısımın hesapları, birbirine paralel iki plaka arasındaki bir nanoakışkan için yapılmıştır. Tüm hesaplamalar sabit ve parabolik hız profili için sabit duvar sıcaklığı ve sabit ısı akısı gibi farklı ısıl sınır şartlara yapılmıştır. Alumina ve diğer malzemeler gibi %1 ve %5 aralıklarda farklı hacimsel parçacık oranlarına hesaplar yapılmıştır. Farklı bölümlerde farklı metodlar kullanarak bazi ortak sonuçlar elde edilmektedir ki bu sonuçların değerleri metod değişince değişmektedir. Son bölümde bazi önemli faktürleri göz önüne alarak bir kaç karşılaştırma yapılmakta ve daha anlaşılacak bir şekilde onlara ait tablolar ve çizelgeler çizilmektedir.

1. INTRODUCTION

Heat transfer in heating and cooling processes play an important role in our world. Then trying to find new ways to improve thermal efficiency is scientists' big responsibility. There are a few ways to do that especially in conductive heat transfer, for example extending surface of fins, increasing temperature gradient or increasing thermal conductivity. For increasing thermal properties, especially thermal conductivity using fluids such as water and ethylene glycol is not suitable. On the other hand, adding some solid particles to these base fluids with size on the order of millimeter or micrometer will be helpful. Even solid particles with size on the order of nanometer will be more effective than the other ones [1].

Choi [2] was the first person who used the word nanofluid to refer to the fluid with suspended nanoparticles.

In Table 1.1 the numbers of papers written on nanofluids, heat transfer with using nanofluids and properties of nanofluids is summarized from the beginning year of publications up to the year 2010 [3].

Table 1.1 : Number of papers on "nanofluids", "nanofluids and heat transfer" and "nanofluids and properties" by SCOPUS database.

Year	Nanofluids	Nanofluids and heat transfer	Nanofluids and properties
1993	1	0	0
1995	1	1	0
1996	2	2	0
1997	2	1	1
1999	2	2	1
2000	4	3	3
2001	5	2	2
2002	5	2	2
2003	19	9	6
2004	35	23	8
2005	90	50	34
2006	124	62	32
2007	175	89	50
2008	225	107	91
2009	222	109	96
2010	95	54	25
total	1007	516	351

Maxwell [4, 5] showed the more increasing of thermal conductivity of a mixture, byusing more volume fractions. By that it means, the more volume fraction of nanoparticle, the more thermal conductivity of mixture.

Nanofluid is a mixture of nanoparticles and base fluid, with dimension of nanoparticles varying from 1 to 100 nanometers [6].

There is a good example of nanometer in comparison with millimeter and micrometer in Serrano el al [7] in Fig. 1.1.

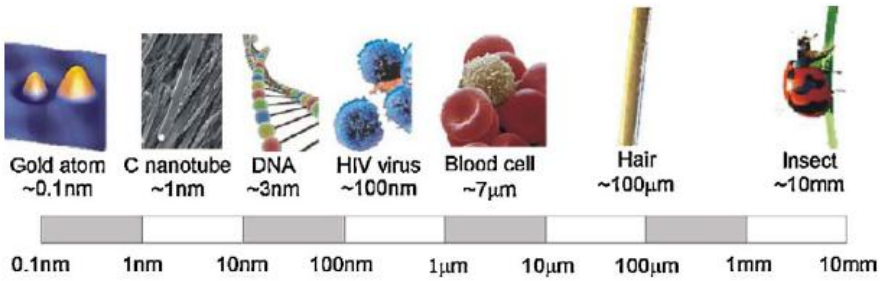

Figure 1.1 : Comparing nanometer with micrometer and millimeter.

There are different nanoparticles that researchers are using them in their researches, for example metal oxides like Al_2O_3 (Alumina), CuO and TiO_2, metals like Al and

Cu and fluids like water, Ethylene Glycol and Engine Oil. In Fig 1.2 thermal conductivity of a few metal oxides, metals and fluids has been shown [8].

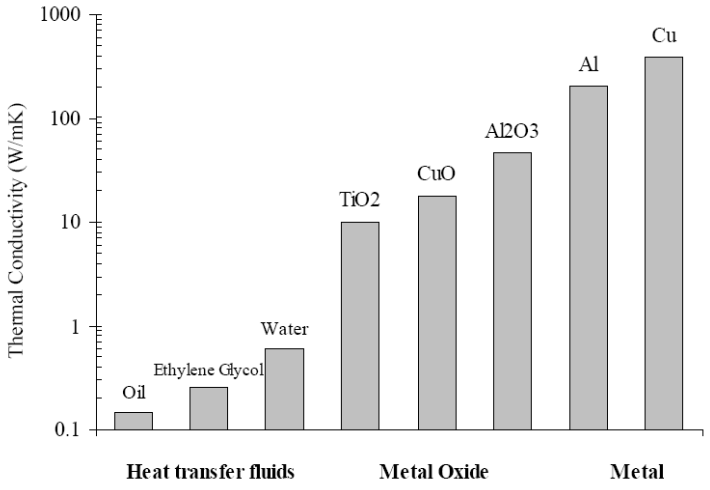

Figure 1.2 : Thermal conductivity of a few metal oxides, metals and fluids.

Another most used nanoparticle is Carbon Nanotube (CNT) because of its high thermal conductivity [6].Researchers prefer to use single-, double-, or multi-walled carbon nanotubes (SWCNT, DWCNT, MWCNT) [9, 10] for their experiments.

Because of existence of nanoparticles, nanofluids have high thermal conductivities at very low concentrations and make a considerable enhancement of convection [11]. These variations of thermal conductivity not only are relevant to heat transfer performance of heat exchangers, but also useful in cooling devices in many industries [12]. A simple mixture of solid particles and base fluid does not call nanofluid, proper mixture of them is required. Three effective methods used to attain stability of suspension against sedimentation are: controlling the ph value, adding surface activators or surfactants and using ultrasonic vibration. There are some advantages of nanofluids that are listed below [6]:

- ✓ More heat transfer surface between particles because of high specific surface area
- ✓ Less pumping power for nanofluids in comparison with pure liquid for reaching the same heat transfer intensification
- ✓ Reduced particle clogging in comparison with common suspensions of insolvable particles in liquid thus promoting to miniaturization of system

- ✓ Adaptable thermal conductivity by changing nanoparticle concentration for using in different applications

Choi et al. [13] showed that addition of small amount of nanoparticles (even less than 1%) increases conductivity of the fluid up to nearly two times.

In some researches, it was shown that the heat transfer coefficient increased more than 20% by using nanoparticles (1-5 Vol. %) [14-17].

Nowadays there are many research activities in this heat transfer area because of fast growing of usage of these devices in different branch of applications [18-22].

These are some of the most important and specific applications in different branches like industrial one: engine cooling, engine transmission oil, in diesel electric generator as jacket water coolant, boiler exhaust flue gas recovery, heating and cooling of buildings, cooling of electronics, cooling of welding, nanofluids in transformer cooling oil, nuclear systems cooling, solar water heating, nanofluids in drilling, refrigeration (domestic refrigerator, chillers), defense, space, high power laser, microwave tubes, biomedical applications, drilling, lubrications, thermal storage and drag reductions [23-25].

Preparation of nanoparticles generally divides to two main techniques, physical or chemical synthesis techniques. Physical synthesis itself includes mechanical grinding and the inert-gas-condensation techniques. Chemical synthesis includes chemical precipitation, chemical vapor deposition, micro-emulsions, spray pyrolysis, thermal spraying and recently developed sonochemical method to make suspensions of iron nanoparticles. Preparation of nanofluids also divides to two main methods, two-step and one-step method [10]. Two-step method has an advantage of producing economically tonnage quantities of nanoparticles and one-step method is preferable for nanofluids containing high-conductivity metals such as copper to the two-step method to prevent oxidation of the particles [26].

2. HEAT TRANSFER IN CIRCULAR PIPE FOR PURE WATER

There are three set of equations for analyzing flow in the pipe namely continuity, momentum and energy equations [27].

2.1 Conservation of Mass (C.E.)

Continuity equation is application of mathematical expression of conservation of mass to a control volume in the fluid in motion and can be written as,

$$div(\rho\vec{V}) = 0 \tag{2.1}$$

By using cylindrical coordinate as default coordinate system for a flow in a pipe, which flow is in x-direction, we can write general equation as below,

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial u}{\partial x} = 0 \tag{2.2}$$

Velocity in radial and θ directions are zero then variations of them are zero, too,

$$v_r = 0 \quad v_\theta = 0 \quad \frac{\partial v_r}{\partial r} = 0 \quad \frac{\partial v_\theta}{\partial \theta} = 0 \tag{2.3}$$

So,

$$\frac{\partial u}{\partial x} = 0 \tag{2.4}$$

Then it can be resulted that the variation of velocity along the pipe is zero.

2.2 Conservation of Momentum (M.E.)

Set of equations govern dynamic behavior of fluid are called momentum equations which obtained by applying law of conservation of momentum to a control volume in the fluid.

$$div(\rho\vec{V}\vec{V}) = -gradP + \mu\nabla^2\vec{V} \tag{2.5}$$

The above-mentioned general equation can be written in cylindrical coordinates.

The r-component of equation is,

$$\begin{aligned}\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{{v_\theta}^2}{r} + u\frac{\partial v_r}{\partial x} \\ = f_r - \frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\{\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right] \\ + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial x^2}\}\end{aligned} \tag{2.6}$$

By using these assumptions,

$$\begin{aligned}&\frac{\partial v_r}{\partial t} = 0 \qquad v_r = 0 \qquad v_\theta = 0 \qquad \frac{\partial v_r}{\partial x} = 0 \\ &f_r = 0 \qquad \frac{\partial^2 v_r}{\partial \theta^2} = 0 \qquad \frac{\partial v_\theta}{\partial \theta} = 0 \qquad \frac{\partial^2 v_r}{\partial x^2} = 0\end{aligned} \tag{2.7}$$

So equation can be simplified to,

$$\frac{\partial p}{\partial r} = 0 \tag{2.8}$$

The θ-component of equation is,

$$\begin{aligned}\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\theta v_r}{r} + u\frac{\partial v_\theta}{\partial x} \\ = f_\theta - \frac{1}{\rho r}\frac{\partial p}{\partial \theta} + \nu\{\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right] \\ + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial x^2}\}\end{aligned} \tag{2.9}$$

By using these assumptions,

$$\begin{aligned}&\frac{\partial v_\theta}{\partial t} = 0 \qquad v_r = 0 \qquad v_\theta = 0 \qquad \frac{\partial v_\theta}{\partial x} = 0 \\ &f_\theta = 0\,\frac{\partial^2 v_\theta}{\partial \theta^2} = 0 \qquad \frac{\partial v_r}{\partial \theta} = 0 \qquad \frac{\partial^2 v_\theta}{\partial x^2} = 0\end{aligned} \tag{2.10}$$

So equation can be simplified as,

$$\frac{\partial p}{\partial \theta} = 0 \tag{2.11}$$

Therefore, we can result that pressure only is a function of length. Then we can say that,

$$p = p(x) \tag{2.12}$$

The x-component of equation is,

$$\begin{aligned} &\frac{\partial u}{\partial t} + v_r \frac{\partial u}{\partial r} + \frac{v_\theta}{r}\frac{\partial u}{\partial \theta} + u\frac{\partial u}{\partial x} \\ &= f_z - \frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\{\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)\right] + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial x^2}\} \end{aligned} \tag{2.13}$$

By using these assumptions,

$$\begin{aligned} &\frac{\partial u}{\partial t} = 0 && v_r = 0 && v_\theta = 0 \\ &f_x = 0 && \frac{\partial^2 u}{\partial \theta^2} = 0 && \frac{\partial^2 u}{\partial x^2} = 0 \end{aligned} \tag{2.14}$$

From continuity equation we have,

$$\frac{\partial u}{\partial x} = 0 \tag{2.15}$$

So equation can be simplified as,

$$0 = -\frac{1}{\rho}\frac{dp}{dx} + \nu\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)\right] \tag{2.16}$$

The pressure at any cross section is uniform and decreases only in the flow direction, in a circular pipe, far enough from entrance region,

$$\Delta p \pi r^2 = 2\pi r \Delta x \tau \tag{2.17}$$

Here pressure difference is,

$$\Delta p = p_1 - p_2 \tag{2.18}$$

Therefore, shear stress is,

$$\tau = \frac{r}{2}\frac{\Delta p}{\Delta x} \tag{2.19}$$

According to Newton's law of viscosity,

$$\tau = -\frac{du}{dr} \tag{2.20}$$

By substituting shear stresses in momentum equation and integrating with respect to r:

$$u = \frac{1}{4\mu}\frac{\Delta p}{\Delta x}(r_0^2 - r^2) \tag{2.21}$$

Or in the other form parabolic profile of velocity in terms of maximum velocity can be written as,

$$u = U_c\left[1 - \left(\frac{r}{r_0}\right)^2\right] \tag{2.22}$$

Where maximum velocity is,

$$U_c = \frac{r_0^2}{4\mu}\frac{\Delta p}{\Delta x} \tag{2.23}$$

The total volume flow rate is,

$$\dot{Q} = \int_0^{r_0} u(2\pi r)dr = \frac{\pi r_0^4}{8\mu}\frac{\Delta p}{\Delta x} \tag{2.24}$$

Then the mean flow velocity can be obtained as,

$$U_m = \frac{\dot{Q}}{\pi r_0^2} = \frac{r_0^2}{8\mu}\frac{\Delta p}{\Delta x} = \frac{1}{2}U_c \tag{2.25}$$

Then parabolic velocity profile in terms of mean velocity can be written as,

$$\frac{u}{U_m} = 2\left[1 - \left(\frac{r}{r_0}\right)^2\right] \tag{2.26}$$

2.3 Conservation of Energy (E.E.)

Energy equation can be obtained by applying the first law of thermodynamics to a control volume in the fluid.

$$div(\rho \vec{V} c_p T) = div(k grad T) \tag{2.27}$$

$$\rho c_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} + u\frac{\partial T}{\partial x}\right) = \frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k\frac{\partial T}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial x}\right) \tag{2.28}$$

By using these assumptions,

$$\frac{\partial T}{\partial t} = 0 \qquad v_r = 0 \qquad v_\theta = 0 \qquad \frac{\partial T}{\partial \theta} = 0 \tag{2.29}$$

So equation can be simplified as,

$$\rho c_p \left(u\frac{\partial T}{\partial x}\right) = \frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) \tag{2.30}$$

By neglecting axial conduction equation above reduces to,

$$\frac{u}{\alpha}\frac{\partial T}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{2.31}$$

Now for defining a heat transfer coefficient three variables are needed, the heat flux across the surface, wall temperature and fluid temperature. Because of varying the temperature of the fluid at any axial location across the pipe cross-section, it is better to define an appropriate mean temperature. By using the definition of rate of thermal energy,

$$T_m c_{pm} \dot{m} = \int_A T c_p \rho u dA \tag{2.32}$$

The mean temperature of the fluid can be calculated as below,

$$T_m = \frac{\int_A T c_p \rho u dA}{c_{pm}\dot{m}} = \frac{\int_A T c_p \rho u dA}{c_{pm}\int_A \rho u dA} \tag{2.33}$$

For an incompressible fluid with constant specific heat mean temperature can be written as,

$$T_m = \frac{\int_A TudA}{\int_A udA} \tag{2.34}$$

This means temperature can be used not only for pipes but also for ducts as well. The mean temperature is referred to the bulk temperature or mixing-cup temperature, too.

General form of nondimensional temperature defined as below,

$$\Theta = \frac{T - T_w}{T_m - T_w} \tag{2.35}$$

It is invariant with the axial coordinates along the pipe,

$$\frac{\partial \Theta}{\partial x} = \frac{\partial}{\partial x}\left(\frac{T - T_w}{T_m - T_w}\right) = 0 \tag{2.36}$$

$$\frac{\partial \mathrm{T}}{\partial x} = \frac{\partial T_w}{\partial x} - \left(\frac{T - T_w}{T_m - T_w}\right)\frac{\partial T_w}{\partial x} + \left(\frac{T - T_w}{T_m - T_w}\right)\frac{\partial T_m}{\partial x} \tag{2.37}$$

On the other hand,by using the definition of local heat transfer coefficient we may write,

$$h_x = \frac{q''}{T_w - T_m} \tag{2.38}$$

In the case of circular pipe heat transfer coefficient can be written as,

$$h_x = \frac{k\left(\frac{\partial T}{\partial r}\right)_{r=r_0}}{T_w - T_m} = -k\frac{\partial}{\partial r}\left[\frac{T - T_w}{T_m - T_w}\right]_{r=r_0} = -k\left(\frac{\partial \Theta}{\partial r}\right)_{r=r_0} \tag{2.39}$$

In fully developed flow $\left(\frac{\partial \Theta}{\partial r}\right)_{r=r_0} = constant$ is constant, therefore heat transfer coefficient is constant along the pipe.

2.4 Thermal Boundary Conditions

There are different types of thermal boundary conditions according to Shah and London [28]. Here are three important boundary conditions namely the T, H1 and H2. In the T boundary condition wall temperature is constant axially and peripherally throughout the channel or passage length. The H1 boundary condition refers to constant wall heat transfer rate along the axial direction and having constant wall temperature in the peripheral direction at any cross-section and in the H2 boundary condition wall heat transfer rate along the axial and peripheral directions is constant as can be seen in Fig. 2.1.

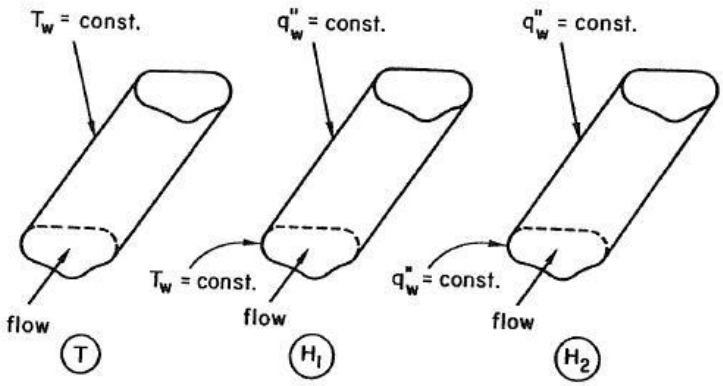

Figure 2.1 : Different thermal boundary conditions.

Two main boundary conditions are interested in this work; uniform heat flux and constant wall temperature boundary conditions.

2.5 Energy Equation for Uniform Heat Flux Boundary Condition

The last form of energy equation solved here for uniform heat flux boundary condition for two types of flow, namely plug flow and flow with parabolic velocity profile.

2.5.1 Plug flow (constant velocity)

In fluids mechanics plug flow is the simplest velocity profile in the pipe that the velocity profile is assumed constant across any cross-section of the pipe perpendicular to the axis of the pipe. In this boundary condition heat flux along the wall is constant, and also flow in the pipe is constant (plug flow) [27].

$$q_w'' = h(T_w - T_m) = const. \tag{2.40}$$

$$u = const. \tag{2.41}$$

From the previous section, we know that heat transfer coefficient is constant then,

$$\frac{\partial T_w}{\partial x} = \frac{\partial T_m}{\partial x} \tag{2.42}$$

We may apply energy balance to a control volume as shown in Fig. 2.2.,

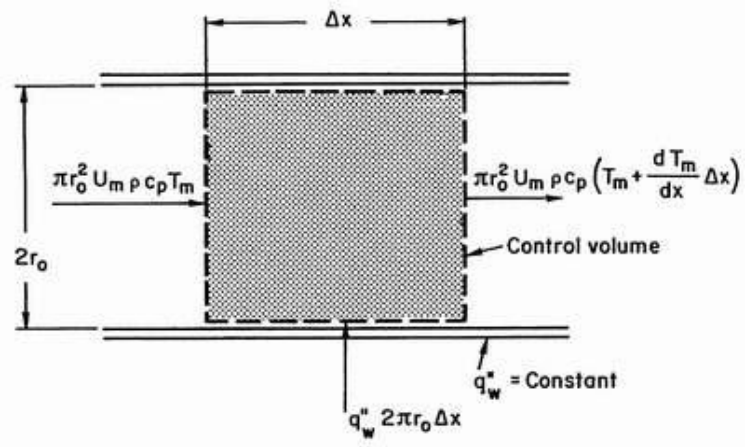

Figure 2.2 : Energy balance of a control volume.

$$q_w'' = \frac{r_0 U_m \rho c_p}{2} \frac{\partial T_m}{\partial x} \tag{2.43}$$

By using these equations it can be resulted that,

$$\frac{\partial T_w}{\partial x} = \frac{\partial T_m}{\partial x} = const. \tag{2.44}$$

Also,

$$\frac{\partial \mathrm{T}}{\partial x} = \frac{\partial T_w}{\partial x} = \frac{\partial T_m}{\partial x} = const. \tag{2.45}$$

Other form of this boundary condition can be written as,

$$\frac{\partial \mathrm{T}(\mathrm{x}, 0)}{\partial r} = 0 \tag{2.46}$$

Using equation below as second condition,

$$k\frac{\partial \mathrm{T}(\mathrm{x},\mathrm{r}_0)}{\partial r} = q_w^{''} = const. \tag{2.47}$$

Integrating energy equation twice with respect to r,

$$T(x,r) = \frac{ur^2}{4\alpha}\frac{\partial T}{\partial x} + C_1 lnr + C_2 \tag{2.48}$$

Using first boundary condition at $r = 0$ we obtain $C_1 = 0$,

$$T(x,0) = \frac{ur^2}{4\alpha}\frac{\partial T}{\partial x} + C_2 = T_c \tag{2.49}$$

Then,

$$C_2 = T_c \tag{2.50}$$

By substituting constants in equation we have,

$$T = \frac{ur^2}{4\alpha}\frac{\partial T}{\partial x} + T_c \tag{2.51}$$

Bulk temperature can be calculated as,

$$T_m = \frac{\int_0^{r_0}\left(\frac{ur^2}{4\alpha}\frac{\partial T}{\partial x} + T_c\right)rdr}{\int_0^{r_0} rdr} \tag{2.52}$$

$$T_m = \frac{ur_0^2}{8\alpha}\frac{\partial T}{\partial x} + T_c \tag{2.53}$$

Temperature on the wall is,

$$T_w = \frac{ur_0^2}{4\alpha}\frac{\partial T}{\partial x} + T_c \tag{2.54}$$

Then,

$$T_w - T_m = \frac{ur_0^2}{8\alpha}\frac{\partial T}{\partial x} \tag{2.55}$$

In addition, heat transfer coefficient is,

$$h_x = \frac{k\left(\frac{\partial T}{\partial r}\right)_{r=r_0}}{T_w - T_m} = \frac{4k}{r_0} \tag{2.56}$$

Finally, Nusselt number can be calculated as,

$$Nu = \frac{h_x d}{k} = \frac{2h_x r_0}{k} = \frac{2r_0}{k}\frac{4k}{r_0} = 8 \tag{2.57}$$

Here we can see that Nusselt number which is constant along the pipe but heat transfer coefficient varies in the pipe because of variation of thermal conductivity and radius of the pipe.

2.5.2 Parabolic velocity profile

In this boundary condition heat flux along the wall is constant, but velocity profile in the pipe is parabolic.

Integrating last form of energy equation twice with respect to r,

$$T(x,r) = \frac{1}{\alpha}\frac{\partial \mathrm{T}}{\partial x} U_c \left(\frac{r^2}{4} - \frac{r^4}{16r_0^2}\right) + C_1 lnr + C_2 \tag{2.58}$$

By using first boundary condition at $r = 0$ we obtain $C_1 = 0$,

$$T(x,0) = \frac{1}{\alpha}\frac{\partial \mathrm{T}}{\partial x} U_c \left(\frac{r^2}{4} - \frac{r^4}{16r_0^2}\right) + C_2 = T_c \tag{2.59}$$

Then,

$$C_2 = T_c \tag{2.60}$$

By substituting constants in equation we have,

$$T = \frac{1}{\alpha}\frac{\partial \mathrm{T}}{\partial x} U_c \left(\frac{r^2}{4} - \frac{r^4}{16r_0^2}\right) + T_c \tag{2.61}$$

$$T - T_c = \frac{1}{\alpha}\frac{\partial \mathrm{T}}{\partial x}\frac{U_c r_0^2}{4}\left[\left(\frac{r}{r_0}\right)^2 - \frac{1}{4}\left(\frac{r}{r_0}\right)^4\right] \tag{2.62}$$

By calculating bulk temperature,

$$T_m = T_c + \frac{7}{96}\frac{U_c r_0^2}{\alpha}\frac{\partial \mathrm{T}}{\partial x} \tag{2.63}$$

And using it for calculating wall temperature, we have,

$$T_w = T_c + \frac{3}{16}\frac{U_c r_0^2}{\alpha}\frac{\partial \mathrm{T}}{\partial x} \tag{2.64}$$

Gradient of the fluid temperature at the wall is,

$$\left(\frac{\partial \mathrm{T}}{\partial x}\right)_{r=r_0} = \frac{U_c r_0}{4\alpha}\frac{\partial \mathrm{T}}{\partial x} \tag{2.65}$$

Then,

$$T_w - T_m = \frac{11}{96}\frac{U_c r_0^2}{\alpha}\frac{\partial \mathrm{T}}{\partial x} \tag{2.66}$$

Then heat transfer coefficient in circular pipe for constant heat flux for parabolic velocity profile is,

$$h = \frac{24}{11}\frac{k}{r_0} = \frac{48}{11}\frac{k}{d} \tag{2.67}$$

And Nusselt number can be obtained as,

$$Nu = \frac{hd}{k} = 4.364 \tag{2.68}$$

It can be seen that the Nusselt numbers for different velocity profiles are constant but different from each other.

2.6 Constant Wall Temperature Boundary Condition

Energy equation is solved for constant wall temperature boundary condition for two types of flows; plug flow and flow with parabolic velocity profile.

2.6.1 Plug flow

In this boundary condition wall temperature is constant and flow in the pipe is plug flow (constant velocity).

$$\frac{dT_w}{dx} = 0 \tag{2.69}$$

We can use above-mentioned equation. Then,

$$\frac{\partial \mathrm{T}}{\partial x} = \frac{\partial T_w}{\partial x} - \left(\frac{T - T_w}{T_m - T_w}\right)\frac{\partial T_w}{\partial x} + \left(\frac{T - T_w}{T_m - T_w}\right)\frac{\partial T_m}{\partial x} \tag{2.70}$$

$$\frac{\partial \mathrm{T}}{\partial x} = \left(\frac{T - T_w}{T_m - T_w}\right)\frac{\partial T_m}{\partial x} \tag{2.71}$$

Substituting in energy equation,

$$\frac{u}{\alpha}\left(\frac{T - T_w}{T_m - T_w}\right)\frac{\partial T_m}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{2.72}$$

Using conditions below,

$$T(x, r_0) = T_w = const. \tag{2.73}$$

$$\frac{\partial T(x, 0)}{\partial r} = 0 \tag{2.74}$$

And using constant velocity profile for solving energy equation, we have,

$$\frac{u}{\alpha}\left(\frac{T - T_w}{T_m - T_w}\right)\frac{\partial T_m}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{2.75}$$

For solving this equation we can use temperature distribution, wall and mean temperatures obtained from constant heat flux by substituting in the left hand side of equation,

$$\frac{u}{\alpha}\frac{\left(\frac{1}{\alpha}\frac{\partial \mathrm{T}}{\partial x}\frac{U_c r_0^2}{4}\left[\left(\frac{r}{r_0}\right)^2 - \frac{1}{4}\left(\frac{r}{r_0}\right)^4\right] + T_c\right) - \left(T_c + \frac{3}{16}\frac{U_c r_0^2}{\alpha}\frac{\partial \mathrm{T}}{\partial x}\right)}{-\frac{11}{96}\frac{U_c r_0^2}{\alpha}\frac{\partial \mathrm{T}}{\partial x}}\frac{\partial T_m}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{2.76}$$

Then by integrating twice with respect to r,

$$\frac{2u}{\alpha}\left(1-\frac{r^2}{r_0^2}\right)\frac{\partial T_m}{\partial x}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{2.77}$$

$$\frac{2u}{\alpha}\left(r-\frac{r^3}{r_0^2}\right)\frac{\partial T_m}{\partial x}=\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\left(\frac{T-T_w}{T_m-T_w}\right)\frac{\partial T_m}{\partial x}$$
$$=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{2.78}$$

$$\frac{2u}{\alpha}\left(\frac{r^2}{2}-\frac{r^4}{4r_0^2}\right)\frac{\partial T_m}{\partial x}+C_1=r\frac{\partial T}{\partial r} \tag{2.79}$$

$$\frac{2u}{\alpha}\left(\frac{r}{2}-\frac{r^3}{4r_0^2}\right)\frac{\partial T_m}{\partial x}+\frac{C_1}{r}=\frac{\partial T}{\partial r} \tag{2.80}$$

The new temperature will be obtained as,

$$T=\frac{2u}{\alpha}\left(\frac{r^2}{4}-\frac{r^4}{16r_0^2}\right)\frac{\partial T_m}{\partial x}+C_1 lnr+C_2 \tag{2.81}$$

By using second boundary condition $C_1=0$ and from the first boundary condition we have,

$$T_c=C_2 \tag{2.82}$$

After substituting constants temperature distribution can be written as,

$$T=\frac{2u}{\alpha}\left(\frac{r^2}{4}-\frac{r^4}{16r_0^2}\right)\frac{\partial T_m}{\partial x}+T_c \tag{2.83}$$

This new temperature enables one to obtain mean and wall temperatures. By substituting this temperature profile, the newer temperature can be obtained. This process should be repeated until the Nusselt number converges to a limit. In this work Mathematica 7.0 software was used for doing iterations.

From the last iteration heat transfer coefficient can be written as,

$$h=\frac{1368}{473}\frac{k}{r_0} \tag{2.84}$$

In this case Nusselt number is,

$$Nu = 5.783 \tag{2.85}$$

2.6.2 Parabolic velocity profile

In this boundary condition wall temperature is constant and velocity profile in the pipe is parabolic.

$$\frac{dT_w}{dx} = 0 \tag{2.86}$$

By using this equation we can write,

$$\frac{\partial \mathrm{T}}{\partial x} = \frac{\partial T_w}{\partial x} - \left(\frac{T - T_w}{T_m - T_w}\right)\frac{\partial T_w}{\partial x} + \left(\frac{T - T_w}{T_m - T_w}\right)\frac{\partial T_m}{\partial x} \tag{2.87}$$

$$\frac{\partial \mathrm{T}}{\partial x} = \left(\frac{T - T_w}{T_m - T_w}\right)\frac{\partial T_m}{\partial x} \tag{2.88}$$

Substituting in energy equation,

$$\frac{u}{\alpha}\left(\frac{T - T_w}{T_m - T_w}\right)\frac{\partial T_m}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{2.89}$$

Using conditions below,

$$T(x, r_0) = T_w = const. \tag{2.90}$$

$$\frac{\partial T(x, 0)}{\partial r} = 0 \tag{2.91}$$

And using parabolic velocity profile for solving energy equation, we have,

$$\frac{2U_m}{\alpha}\left(1 - \frac{r^2}{r_0^2}\right)\left(\frac{T - T_w}{T_m - T_w}\right)\frac{\partial T_m}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{2.92}$$

For solving this equation, a few iterations should be done in Mathematica 7.0. Here is the first iteration,

$$T = \frac{U_m}{11\alpha}\left(9r^2 - \frac{3r^8}{16r_0^6} + \frac{10r^6}{6r_0^4} - \frac{21r^4}{4r_0^2}\right)\frac{\partial T_m}{\partial x} + C_1 lnr + C_2 \tag{2.93}$$

By using second boundary condition $C_1 = 0$ and from the first boundary condition we have,

$$T_c = C_2 \tag{2.94}$$

Then temperature distribution can be written as,

$$T = \frac{U_m}{11\alpha}\left(9r^2 - \frac{3r^8}{16r_0^6} + \frac{10r^6}{6r_0^4} - \frac{21r^4}{4r_0^2}\right)\frac{\partial T_m}{\partial x} + T_c \tag{2.95}$$

From here new temperature distribution, mean and wall temperatures can be obtain. By substituting this temperature profile, the newer temperature can be obtained. This process should be repeated until the Nusselt number converges to a limit.

From the last iteration heat transfer coefficient can be obtained as,

$$h = \frac{778422}{425549}\frac{k}{r_0} \tag{2.96}$$

In this case Nusselt number is,

$$Nu = 3.658 \tag{2.97}$$

3. THERMAL PROPERTIES OF NANOFLUIDS AND NONDIMENSIONAL NUMBERS

There are two ways for making use of thermal properties of nanofluids, experimental and theoretical investigations. In experimental investigations section, some experiments will be compared from different points of view. In theoretical investigations section, empirical formulations for calculating thermal properties, especially thermal conductivity of nanofluids, will be given.

3.1 Experimental Investigations

There are some methods for thermal conductivity measurements like transient hot-wire, modified transient hot-wire, steady state parallel-plate, temperature oscillation, microhot strip, and optical beam deflection techniques. Among theabove-mentionedmethods, the most commonly used method is the transient hot-wire technique [29].

Many parameters affect thermal conductivity of nanofluids like: size, shape, material and volume fraction of particles, material and temperature of base fluid etc. These are effects of some of the parameters.

- Volume fraction of nanoparticles

One of the most important parameter is volume fraction of nanoparticles. According to some researches, there is a linear relationship between increasing volume fraction of nanoparticles and increasing thermal conductivity of nanofluid. In most of the researches, volume fraction of nanoparticles was varied between 0.5 and 5%. For example [30-32]. However, On the other hand, in some other researches this relationship between them is nonlinear.For example [13, 33].

- Material of nanoparticles

Another important parameter is material of nanoparticles, which have been used in nanofluids. According to table of thermal properties, different materials have

different thermal conductivities but it does not mean that materials with high thermal conductivity have better enhancement of heat transfer because of their higher thermal conductivities. Another parameter may affect heat transfer enhancement. For example, Al_2O_3 has higher thermal conductivity than CuO, but according to Lee et al. [31] heat transfer enhancement of CuO is better than Alumina because clusters that form Alumina are larger than ones in CuO.

There are different materials that can be used in experimental investigations like diamond, silver, copper, silicon carbide, titanium carbide, gold, aluminum nitride, aluminum, silicon, graphite, sodium, Alumina, copper oxide, titanium dioxide and zirconia. Alumina (Al_2O_3) and CuO are the most common nanoparticles that have been used in experiments, in different diameters with different base fluids [34]. These are some experimental researches on thermal conductivity of nanofluids.

- Base fluid

Effects of base fluid are a bit more complicated than nanoparticles itself because viscosity of base fluid affects the Brownian motion in nanofluids and that is why affects thermal conductivity of nanofluids. There are different fluids as base fluid like water, ethylene glycol, engine oil and mixture of glycol-water in different proportions. In study of Wang et al. [31] two kinds of nanoparticles were used, Al_2O_3 and CuO, in different base fluids, water, ethylene glycol, vacuum pump fluid and engine oil. In Al_2O_3 case, the highest thermal conductivity was observed when ethylene glycol used as base fluid. In CuO case the same thermal conductivity ratio, thermal conductivity of nanofluid divided by thermal conductivity of base fluid, was observed when ethylene glycol was used.

- Particle size

Size of the particle or mean diameter of nanoparticles is another important parameter that affects thermal conductivity of nanofluid. According to researches the smaller the nanoparticles, the higher the thermal conductivity of nanofluids. Teng et al. [35] used Al_2O_3 nanoparticles in water base fluid with different volume fraction, from 0.5 to 2 percent, and different particle sizes, 20, 50 and 100 nm, were shown that in the same volume fraction and different particles sizes nanofluid with smaller diameter of particles had higher thermal conductivity ratio. For example, Fig. 3.1 compares ratio

of effective thermal conductivity of nanofluid for Alumina in water and base fluid for different particle sizes, 5-500 nm, according to experiments [36].

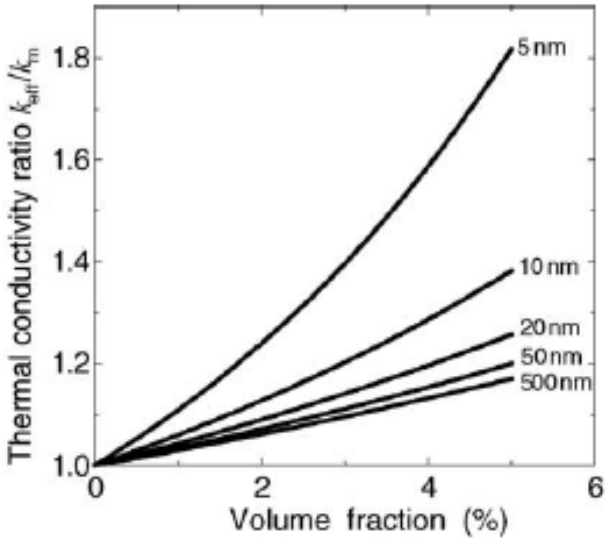

Figure 3.1 : Comparison of ratio of thermal conductivities for different particle sizes for Alumina-water.

- Particle shape

According to experimental investigations between three main particle shapes of nanoparticles, spherical, disk-shape and cylindrical, cylindrical shape has better heat transfer enhancement than the other type. Murshed et al. [33] showed that thermal enhancement of rod-shaped type of TiO_2, with 10 nm in diameter and 40 nm in length, in deionized water is higher than spherical type, with 15 nm diameter.

3.2 Theoretical Investigations

Density of nanofluids by using physical rules of mixture can be written as,

$$\rho_{nf} = \left(\frac{m}{V}\right)_{nf} = \frac{m_f + m_p}{V_f + V_p} = \frac{\rho_f V_f + \rho_p V_p}{V_f + V_p} = (1-\phi)\rho_f + \phi\rho_p \qquad \textbf{(3.1)}$$

Where volume fraction is defined as,

$$\phi = \frac{V_p}{V_f + V_p} \qquad \textbf{(3.2)}$$

Another form of combination of density and thermal capacity of nanofluids is,

$$(\rho c_p)_{nf} = \rho_{nf} \left(\frac{Q}{m\Delta T}\right)_{nf} \quad \textbf{(3.3)}$$

$$(\rho c_p)_{nf} = \rho_{nf} \frac{Q_f + Q_p}{(m_f + m_p)\Delta T} \quad \textbf{(3.4)}$$

$$(\rho c_p)_{nf} = \rho_{nf} \frac{(mc_p)_f \Delta T + (mc_p)_p \Delta T}{(m_f + m_p)\Delta T} \quad \textbf{(3.5)}$$

$$(\rho c_p)_{nf} = (1 - \phi)(\rho c_p)_f + \phi(\rho c_p)_p \quad \textbf{(3.6)}$$

In Fig. 3.2 clear increase in density of nanofluid by increasing volume fraction of nanoparticles is shown for three different nanoparticles, Al_2O_3 (Alumina), CuO (Cupper II oxide) and TiO_2 (Titanium dioxide),

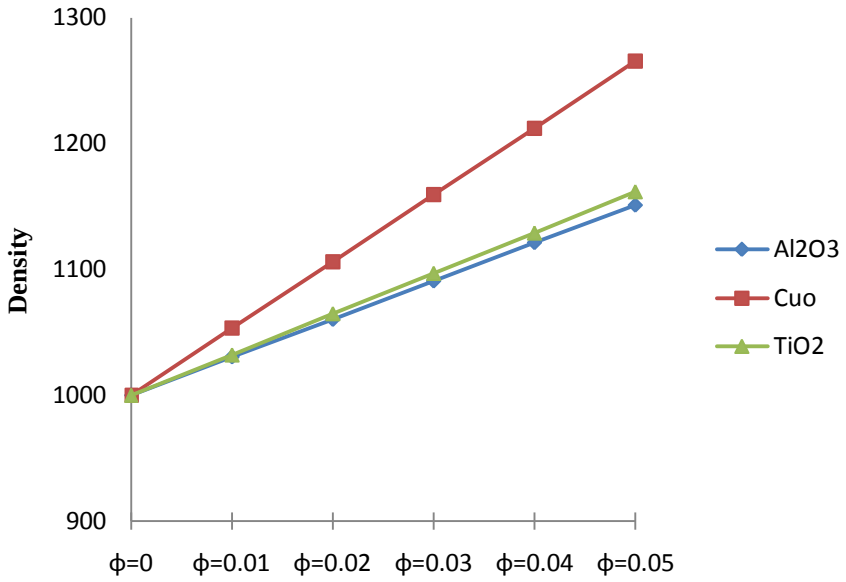

Figure 3.2 : Increasing of density of nanofluid by increasing volume fraction of nanoparticles.

By simplifying combination formula of density and capacity of nanofluid the general formula can be calculated as below,

$$c_{p,nf} = \frac{(1-\phi)\rho_f c_{p,f} + \phi \rho_p c_{p,p}}{\rho_{nf}} \quad \textbf{(3.7)}$$

And a simpler expression for thermal capacity of nanofluids, which most of the authors prefer to use it, can be defined by equation below,

$$c_{p,nf} = (1-\phi)c_{p,f} + \phi c_{p,p} \tag{3.8}$$

In Fig. 3.3 decreasing of thermal capacity of the same three different nanofluids by increasing volume fraction of nanoparticles is shown,

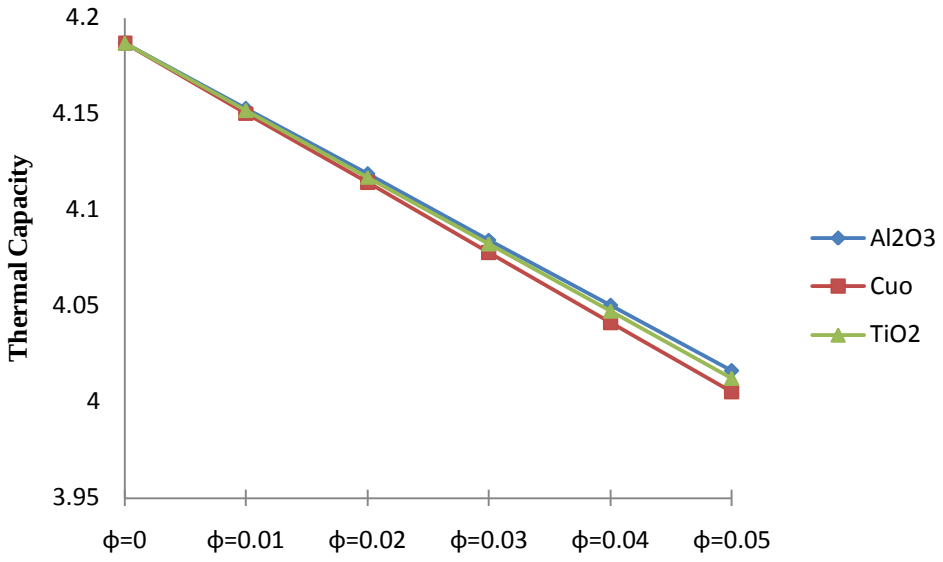

Figure 3.3 : Decreasing of thermal capacity of different nanofluids by increasing volume fraction of nanoparticles.

Einstein's formula for evaluating the viscosity of nanofluid is,

$$\mu_{nf} = \mu_f(1 + 2.5\phi) \qquad \phi < 0.05 \tag{3.9}$$

In Fig. 3.4 a sample increase of viscosity of nanofluid, Alumina, with water as base fluid by increasing volume fraction at 25^{o}C is shown,

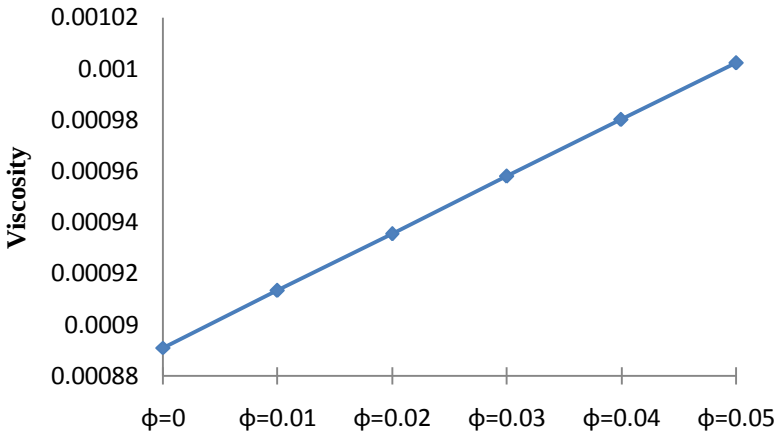

Figure 3.4 : Increasing viscosity by increasing volume fraction.

In theoretical domain, there are different theories for calculating thermal conductivity of nanofluids. Here a few oldest and the most important ones will be mentioned,

- Maxwell

Thermal conductivity is given by Maxwell [4] as below,

$$k_{nf,Maxwell} = \frac{2k_p + k_f + \phi(k_p - k_f)}{2k_p + k_f - 2\phi(k_p - k_f)} k_f \qquad \textbf{(3.10)}$$

Fig. 3.5 is a graph for thermal conductivity of three sample nanoparticles,

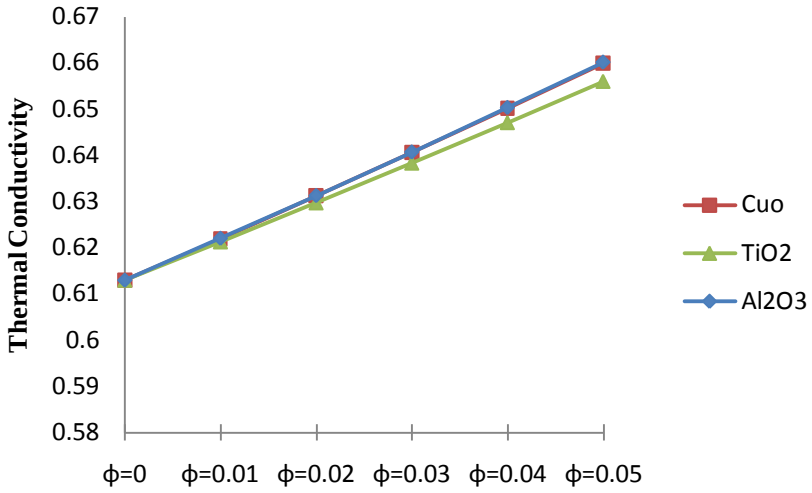

Figure 3.5 : Thermal conductivity of three diferent nanoparticles in Maxwell model.

In Fig. 3.6 ratio of thermal conductivity of nanofluid and base fluid, water, was compared for the three different nanoparticles,

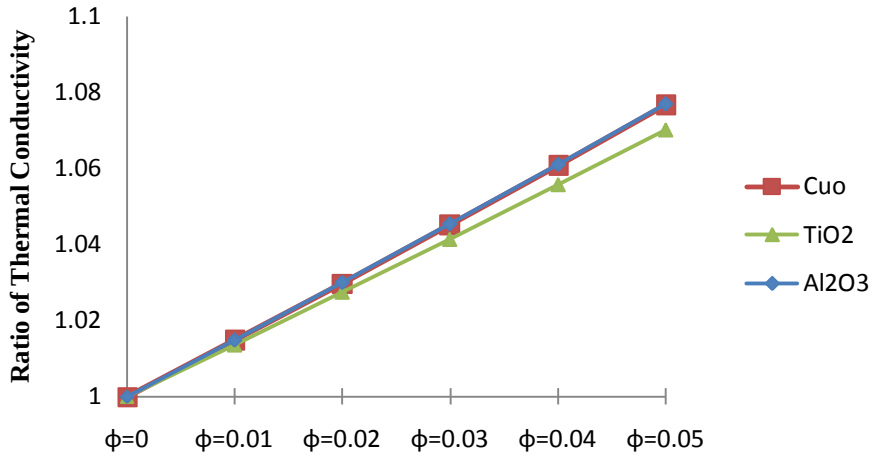

Figure 3.6 : Comparison of ratio of thermal conductivity of nanofluid and water between three different nanoparticles.

- Hamilton-Crosser

There is a comprehensive model in addition to Maxwell model which is used in none spherical nanoparticles. Hamilton and Crosser [37] extended formula below and introduced n, as shape factor for different geometries, which defined as empirical shape factor.

$$k_{nf,Hamilton\ -Crosser} = k_f \left[\frac{k_p+(n-1)k_f-(n-1)\phi(k_f-k_p)}{k_p+(n-1)k_f+\phi(k_f-k_p)}\right] \quad \textbf{(3.11)}$$

Where empirical shape factor is defined by $n = 3/\psi$ and ψ is the sphericity defined as the ratio of the surface areas of a sphere with the volume equal to that of the particle. Maxwell is a reduction of Hamilton-Crosser formula when $\psi = 1$,

$$k_{nf,Hamilton\ -Crosser} = k_f \left[\frac{k_p+2k_f-2\phi(k_f-k_p)}{k_p+2k_f+\phi(k_f-k_p)}\right] \quad \textbf{(3.12)}$$

In Fig. 3.7 calculated thermal conductivity of three nanoparticles by Hamilton-Crosser model is shown,

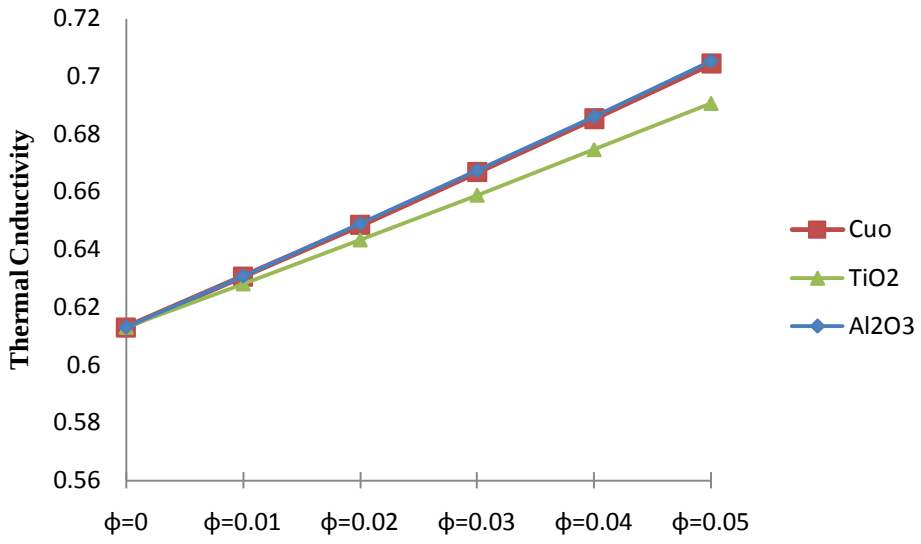

Figure 3.7 : Thermal conductivity of three sample nanoparticles by Hamilton-Crosser.

In Fig. 3.8 thermal conductivity of three different nanofluids for both models, i.e. Maxwell and Hamilton-Crosser, is shown,

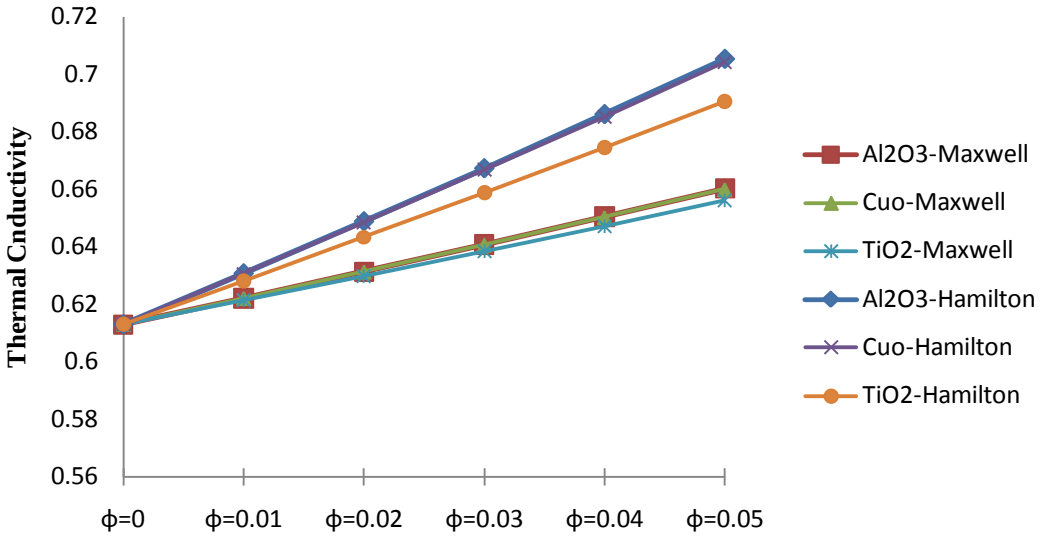

Figure 3.8 : Comparison of thermal conductivities of Maxwell and Hamilton-Crosser models for mentioned nanofluids.

In Fig. 3.9 ratio of thermal conductivity of three different nanofluids in water base fluid, which was calculated by Hamilton-Crosser model, is shown,

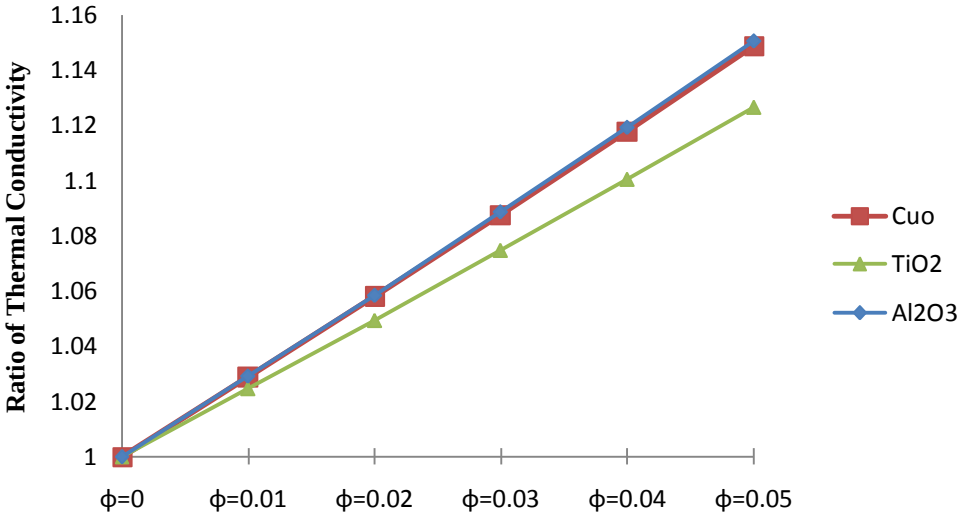

Figure 3.9 : Comparison of ratio of thermal conductivity of three nanoparticles in water in Hamilton-Crosser model.

In Fig. 3.10 ratio of thermal conductivities of nanofluid to base fluid for both models, i.e. Maxwell and Hamilton-Crosser models, for three nanoparticles is shown,

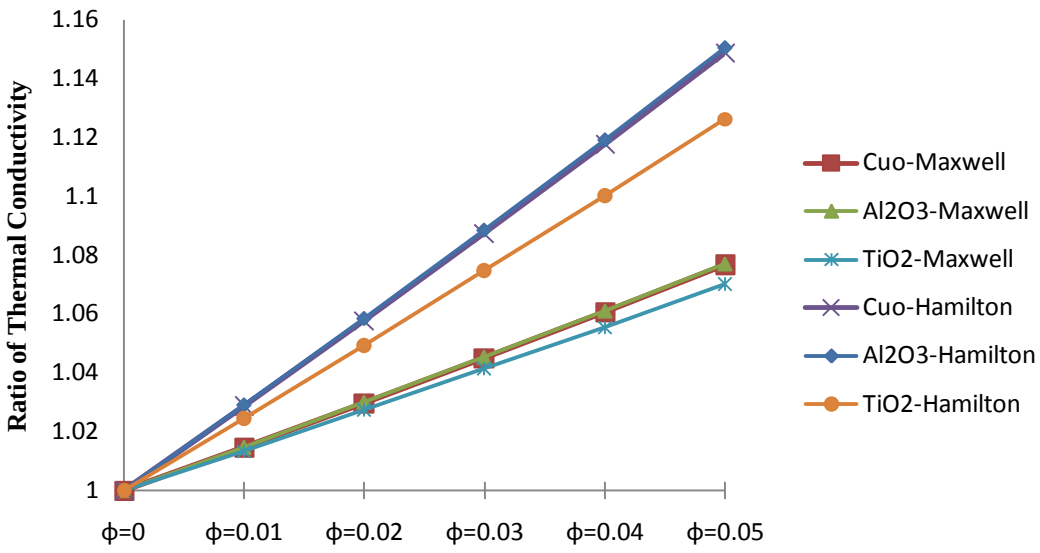

Figure 3.10 : Comparison of ratio of thermal conductivities of nanofluid to base fluid for Maxwell and Hamilton-Crosser models for mentioned nanoparticles.

There is little difference between above-mentioned models namely Maxwell and Hamilton-Crosser in calculation of thermal conductivity. In Fig. 3.11 ratio of Alumina in water that calculated with both of the models is compared.

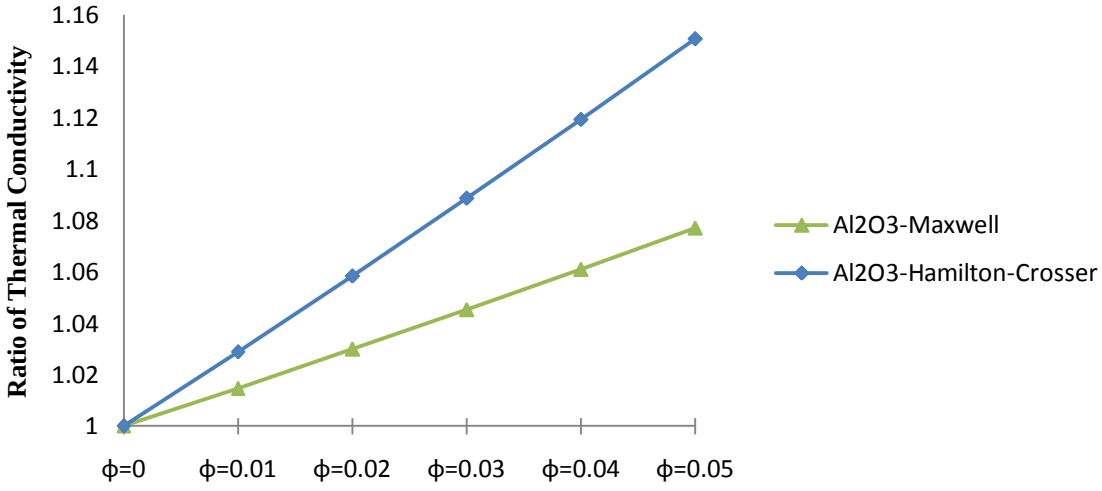

Figure 3.11 : Comparison of thermal conductivities of Alumina in water calculated by two different models, Maxwell and Hamilton-Crosser.

- Bruggeman

There is another theory for calculating thermal conductivity, Bruggeman [38] model, which has no limitation on the contraction of inclusions, and can be used for particle percolation in suspensions.

$$\phi\left(\frac{k_p - k_{nf}}{k_p + 2k_{nf}}\right) + (1-\phi)\left(\frac{k_f - k_{nf}}{k_f + 2k_{nf}}\right) = 0 \quad \textbf{(3.13)}$$

And solution of this quadratic equation is as below,

$$k_{nf,Bruggeman} = (3\phi - 1)k_p + [3(1-\phi) - 1]k_f + \sqrt{\Delta} \quad \textbf{(3.14)}$$

$$\Delta = (3\phi - 1)^2 k_p^2 + [3(1-\phi) - 1]^2 k_f^2 + 2[2 + 9\phi(1-\phi)]k_p k_f \quad \textbf{(3.15)}$$

From the above-mentioned formulas, it can be concluded that thermal conductivity of nanofluids dependent on a few factors. Another important factor that affects thermal conductivity is diameter of nanoparticle. According to some articles, there is no exact formula for showing this effect. Based on Buckingham-Pi theorem and experimental resources there is an empirical correlation between some of the thermal properties and nondimensional numbers as below [39],

$$\frac{k_{nf}}{k_f} = 1 + 64.7\phi^{0.7460}\left(\frac{d_f}{d_p}\right)^{0.3690}\left(\frac{k_p}{k_f}\right)^{0.7476} Pr^{0.9955} Re^{1.2321} \quad \textbf{(3.16)}$$

Here nondimensional Reynolds number and Prandtl number are,

$$Re = \frac{\rho_{bf}\kappa T}{3\pi\mu_{bf}^2 l_{bf}} \quad \textbf{(3.17)}$$

$$Pr = \frac{c_{pbf}\,\mu_{bf}}{k_{bf}} \quad \textbf{(3.18)}$$

In this formula l_{bf} is the mean-free path for the base fluid. A constant value of 0.17 nm for the mean-free path l_{bf} was used in their paper for the water for the entire tested temperature range.

The effective diffusivity of nanofluid can be written as,

$$\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}} = \frac{k_{nf}}{(1-\phi)(\rho c_p)_f + \phi(\rho c_p)_p} \quad \textbf{(3.19)}$$

In Fig. 3.12 comparison between their effective diffusivities is shown,

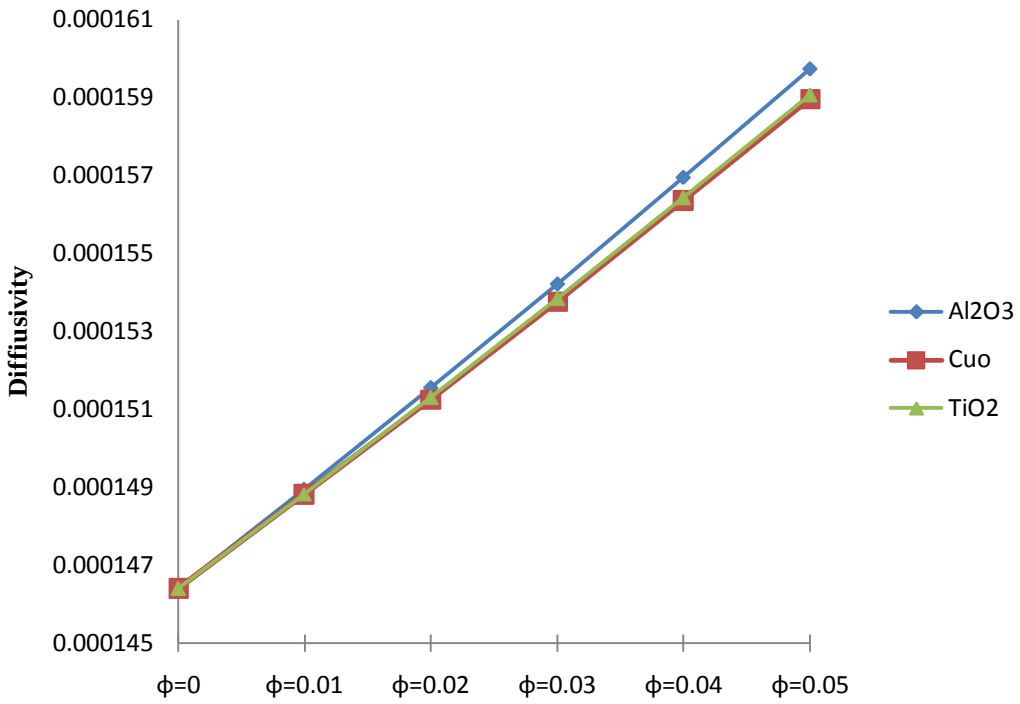

Figure 3.12 : Comparison of effective diffusivity between three different nanoparticles.

Thermal properties for Alumina (Al_2O_3), water and some other materials are listed in the Table 3.1,

Table 3.1 : Specific heat capacity and thermal conductivity of some nanoparticles.

Materials	Specific heat (kJ/kgK)	Thermal conductivity (W/mK)
Diamond	0.509	3300
Carbon nanotube	–	3000
Silver (Ag)	0.235	429
Copper (Cu)	0.385	401
Silicon carbide (SiC)	1.340	350
Titanium carbide (TiC)	0.711	330
Gold (Au)	0.129	317
Aluminum nitride (AlN)	0.740	285
Aluminum (Al)	0.904	237
Silicon (Si)	0.714	148
Graphite	0.701	120
Sodium (Na)	1.230	72.3
Alumina (Al_2O_3)	0.773	40
Copper oxide (CuO)	0.551	32.9
Titanium dioxide (TiO_2)	0.692	8.4
Zirconia	0.418	
Water (base fluid)	4.187	0.613

A few thermal properties of Alumina as a nanoparticle and water as base fluid are listed in Table 3.2:

Table 3.2 : Physical properties of Alumina and water.

Physical properties	Base fluid (water)	Nanoparticles (Al_2O_3)
C_p (J/kg K)	4179	765
ρ (kg/m^3)	997.1	3970
k (W/m K)	0.613	25
β (K^{-1})	21×10^{-5}	0.85×10^{-5}
μ (kg/m s)	10.03×10^{-4}	–

3.3 Nondimensional Numbers

For the correlations these nondimensional numbers, the Reynolds, Prandtl, Grashof, Rayleigh and Peclet numbers, are introduced:

Reynolds number, which will be helpful for defining flow as laminar or turbulent,

$$Re = \frac{\rho u_m D}{\mu} \tag{3.20}$$

Prandtl number,

$$Pr = \frac{c_p \mu}{k} \tag{3.21}$$

Grashof number,

$$Gr = \frac{\rho^2 g \beta q^{"} D^4}{k \mu^2} \tag{3.22}$$

Rayleigh number,

$$Ra = \frac{\rho g \beta q^{"} D^4}{\alpha \mu k} \tag{3.23}$$

Peclet number,

$$Pe_d = \frac{u_m d_p}{\alpha} \tag{3.24}$$

4. HEAT TRANSFER IN CIRCULAR PIPE FOR NANOFLUID BY USING SINGLE-PHASE MODEL

In this model, mixture of nanoparticle in fluid assumes as a single phase. The only difference between this model and pure water is in thermal properties. By substituting thermal properties of nanofluid in equations related to pure water Nusselt number for different boundary conditions can be obtained.

4.1 Continuity Equation

In this equation, we should substitute density of pure water with effective density of nanofluid,

$$div(\rho_{nf}\vec{V}) = 0 \tag{4.1}$$

So by using the same assumptions which are used in chapter 2 it can be written,

$$\frac{\partial u}{\partial x} = 0 \tag{4.2}$$

4.2 Momentum Equation

In this part momentum equation which has been used for pure water can be used but thermal properties of pure water should be substituted with thermal properties of nanofluid.

$$div\left(\rho_{nf}\vec{V}\vec{V}\right) = -gradP + \mu_{nf}\nabla^2\vec{V} \tag{4.3}$$

So by considering the difference between pure water and single-phase same assumptions can be used in r-component of momentum equation,

$$\frac{\partial p}{\partial r} = 0 \tag{4.4}$$

And θ-component of momentum equation can be resulted as,

$$\frac{\partial p}{\partial \theta} = 0 \tag{4.5}$$

Therefore, we can result that pressure only is a function of z.

Then we can say that$p = p(z)$.

Last form of z-component of momentum equation by using results of continuity equation can be written as,

$$0 = -\frac{1}{\rho_{nf}}\frac{dp}{dx} + \nu_{nf}\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)\right] \tag{4.6}$$

4.3 Energy Equation

In single-phase method for solving energy equation using the same equations, which have been used for pure water by substituting thermal properties of pure water with thermal properties of nanofluid, will be enough.

$$div(\rho_{nf}\vec{V}c_{p,nf}T) = div(k_{nf}\,gradT) \tag{4.7}$$

By using the same assuptions last form of energy equation can be written as,

$$\rho_{nf}\,c_{p,nf}\left(u\frac{\partial T}{\partial x}\right) = \frac{1}{r}\frac{\partial}{\partial r}\left(rk_{nf}\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial x}\left(k_{nf}\frac{\partial T}{\partial x}\right) \tag{4.8}$$

By neglecting axial conduction equation above reduces to,

$$\frac{u}{\alpha_{nf}}\frac{\partial T}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{4.9}$$

By using definitions of mean temperatureheat transfer coefficient can be written as,

$$h_{x,nf} = \frac{q''}{T_w - T_m} \tag{4.10}$$

In the case of circular pipe heat transfer coefficient can be written as,

$$h_{x,nf} = \frac{k_{nf}\left(\frac{\partial T}{\partial r}\right)_{r=r_0}}{T_w - T_m} = -k_{nf}\frac{\partial}{\partial r}\left[\frac{T-T_w}{T_m - T_w}\right]_{r=r_0} = -k_{nf}\left(\frac{\partial \Theta}{\partial r}\right)_{r=r_0} \tag{4.11}$$

In fully developed flow $\left(\frac{\partial \Theta}{\partial r}\right)_{r=r_0}$ is constant, therefore heat transfer coefficent is constant along the pipe.

For flow with nanoparticles in it (nanofluids) equations solved for two thermal boundary conditions; uniform heat flux and constant wall temperature boundary conditions.

4.3.1 Uniform heat flux boundary condition

In this type of boundary condition energy equation solved for both plug flow and flow with parabolic velocity profile.

4.3.1.1 Plug flow

In this boundary condition heat flux along the wall and is constant, and flow in the pipe is constant (plug flow). By solving energy equation and substituting thermal properties of pure water by properties of nanofluid it can been resulted that heat transfer coefficient for nanofluid in this kind of boundary condition if different from its value for pure water but Nusselt numbers for these different substances are the same.

$$q_w^{''} = h(T_w - T_m) = const. \tag{4.12}$$

$$u = const. \tag{4.13}$$

From the previous section, we know that heat transfer coefficient is constant then,

$$\frac{\partial T_w}{\partial x} = \frac{\partial T_m}{\partial x} \tag{4.14}$$

We may apply energy balance to a control volume.After that by using assumptions that mentioned in chapter 2 heat transfer coefficient can be written as,

$$h_x = \frac{k_{nf}\left(\frac{\partial T}{\partial r}\right)_{r=r_0}}{T_w - T_m} = \frac{4k_{nf}}{r_0} \tag{4.15}$$

Finally, Nusselt number can be calculated,

$$Nu = \frac{h_x d}{k_{nf}} = \frac{2h_x r_0}{k_{nf}} = \frac{2r_0}{k_{nf}}\frac{4k_{nf}}{r_0} = 8 \tag{4.16}$$

4.3.1.2 Parabolic velocity profile

In this boundary condition heat flux along the wall is constant and velocity profile in the pipe is parabolic. By substituting thermal properties of pure water with thermal properties of nanofluid, it can be resulted that there will be no difference between Nusselt number in pure water and nanofluid cases. Heat transfer coefficient is different from the case of pure water and Nusselt number is constant along the pipe.

$$h_{nf} = \frac{24}{11}\frac{k_{nf}}{r_0} = \frac{48}{11}\frac{k_{nf}}{d} \quad \textbf{(4.17)}$$

And Nusselt number can be obtained as,

$$Nu = \frac{h_{nf}\, d}{k_{nf}} = 4.364 \quad \textbf{(4.18)}$$

4.3.2 Constant wall temperature boundary condition

In constant wall boundary condition both types of flows, plug and parabolic velocity profile, discussed.

4.3.2.1 Plug flow

In this case, wall temperature during the pipe and velocity profile in the pipe is constant. Again, here just like the same case in pure water section, Nusselt numbers are the same but heat transfer coefficients are different. By using the same code of Mathematica 7.0 the last heat transfer coefficient after a few iterations is,

$$h = \frac{1368}{473}\frac{k_{nf}}{r_0} \quad \textbf{(4.19)}$$

And Nusselt number is,

$$Nu = 5.784 \quad \textbf{(4.20)}$$

4.3.2.2 Parabolic velocity profile

In this boundary condition wall temperature is constant and velocity profile in the pipe is parabolic. In this case, just like uniform heat flux boundary condition there is no difference between Nusselt numbers by substituting thermal properties of pure

water with thermal properties of nanofluid and using the same method for obtaining Nusselt number.

Here the last heat transfer coefficient after a few iterations is,

$$h = \frac{778422}{425549} \frac{k_{nf}}{r_0} \tag{4.21}$$

And Nusselt number is,

$$Nu = 3.658 \tag{4.22}$$

All of the above equations which used for macroscale tubes are applicable for microscale ones, too, just because of supposed fluid with nanoparticles as water or other liquids. On the other hand, these equations are not useful for mix of air and nanoparticles.

5. SOLVING ENERGY EQUATION BY USING DISPERSED MODEL

5.1 Constant Wall Temperature

5.1.1 Plug flow

In this model, random movement of nanoparticles is taken into account. The energy equation in dispersed model can be expressed as:

$$u\frac{\partial T}{\partial x} = \left(\alpha_{nf} + \frac{k_{D,x}}{(\rho c_p)_{nf}}\right)\frac{\partial^2 T}{\partial x^2} + \frac{1}{r}\frac{\partial}{\partial r}\left[\left(\alpha_{nf} + \frac{k_{D,r}}{(\rho c_p)_{nf}}\right) r\frac{\partial T}{\partial r}\right] \quad \textbf{(5.1)}$$

Where $k_{D,x}$ and $k_{D,r}$ is thermal dispersion coefficient in axial and radial directions, respectively.

Formula above by neglecting axial conduction can be written as,

$$u\frac{\partial T}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[\left(\alpha_{nf} + \frac{k_{D,r}}{(\rho c_p)_{nf}}\right) r\frac{\partial T}{\partial r}\right] \quad \textbf{(5.2)}$$

If we assume the effective apparent thermal diffusivity as,

$$k_{\bar{D}} = \alpha^*_{nf} = \alpha_{nf} + \frac{k_{D,r}}{(\rho c_p)_{nf}} \quad \textbf{(5.3)}$$

Apparent thermal conductivity is,

$$\bar{k} = k_{nf} + k \quad \textbf{(5.4)}$$

Then the last form of energy equation can be written as,

$$u\frac{\partial T}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[k_{\bar{D}} r\frac{\partial T}{\partial r}\right] \quad \textbf{(5.5)}$$

By using classical Graetz problem assumptions for constant velocity profile the equation to be solved for temperature distribution is [27],

$$u\frac{\partial\Theta}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[k_{\bar{D}} r \frac{\partial\Theta}{\partial r}\right] \quad \textbf{(5.6)}$$

Where,

$$\Theta = \frac{T - T_w}{T_0 - T_w} \quad \textbf{(5.7)}$$

Here,

$$T|_{r=R} = T_w \text{ and } T|_{x=0} = T_0 \quad \textbf{(5.8)}$$

Then the boundary conditions are:

$$\Theta(0, r) = 1 \quad \textbf{(5.9)}$$

And,

$$\Theta(x, r_0) = 0 \quad \textbf{(5.10)}$$

$$\frac{\partial\Theta(x,0)}{\partial r} = 0 \quad \textbf{(5.11)}$$

One of methods for solving this equation is method of separating variables and making use of Bessel functions.

Separating of variables of this equation,

$$\Theta(x, r) = X(x) \,.\, R(r) \quad \textbf{(5.12)}$$

First derivative of function with respect to x is,

$$\frac{\partial\Theta}{\partial x} = X'(x).\, R(r) \quad \textbf{(5.13)}$$

First and second derivatives of function with respect to r is,

$$\frac{\partial\Theta}{\partial r} = X(x).\, \dot{R}(r) \quad \textbf{(5.14)}$$

$$\frac{\partial^2\Theta}{\partial r^2} = X(x).\, \ddot{R}(r) \quad \textbf{(5.15)}$$

By solving energy equation for plug flow we have,

$$\frac{u}{k_{\bar{D}}}\frac{X'}{X} = \frac{1}{r}\frac{\dot{R}}{R} + \frac{\ddot{R}}{R} = -\lambda^2 \tag{5.16}$$

Solving the left hand side of equation,

$$\frac{u}{k_{\bar{D}}}\frac{X'}{X} = -\lambda^2 \tag{5.17}$$

$$X(x) = Ae^{-\frac{\lambda^2 k_{\bar{D}}}{u}x} \tag{5.18}$$

Solving the right hand side of equation,

$$r^2\ddot{R} + r\dot{R} + \lambda^2 r^2 R = 0 \tag{5.19}$$

Making use of Bessel function,

$$R(r) = C_1 J_0(\lambda r) + C_2 Y_0(\lambda r) \tag{5.20}$$

We have,

$$\Theta(x,r) = Ae^{-\frac{\lambda^2 k_{\bar{D}}}{u}x}\left(C_1 J_0(\lambda r) + C_2 Y_0(\lambda r)\right) \tag{5.21}$$

From the third boundary condition we have,

$$C_2 = 0 \tag{5.22}$$

And from the second boundary condition,

$$J_0(\lambda r_0) = 0 \tag{5.23}$$

There are many values for λ then we have,

$$\Theta_{\mathrm{n}}(x,r) = A_n e^{-\frac{\lambda_n^2 k_{\bar{D}}}{u}x} J_0(\lambda_n r) \tag{5.24}$$

$$\Theta(x,r) = \sum_{n=1}^{\infty} A_n e^{-\frac{\lambda_n^2 k_{\bar{D}}}{u}x} J_0(\lambda_n r) \tag{5.25}$$

From the first boundary condition we have,

$$1 = \sum_{n=1}^{\infty} A_n J_0(\lambda_n r) \tag{5.26}$$

By using Fourier expansion of 1 in the left hand side and multiplying both sides with$rJ_0(\lambda_n r)$,

$$\int_0^{r_0} rJ_0(\lambda_n r)dr = A_n \int_0^{r_0} rJ_0^2(\lambda_n r)dr \tag{5.27}$$

By using orthogonality relation of the Bessel functions of the first kind of zero and integrating equation above,

$$\int_0^{r_0} rJ_0(\lambda_n r)J_0(\lambda_m r)dr = 0, if m \neq n \tag{5.28}$$

$$A_n = \frac{2}{\lambda_n r_0} \frac{1}{J_1(\lambda_n r_0)} \tag{5.29}$$

Then the temperature distribution can be written as,

$$\Theta(x,r) = \sum_{n=1}^{\infty} \frac{2}{\lambda_n r_0} \frac{1}{J_1(\lambda_n r_0)} e^{-\frac{\lambda_n^2 k_{\bar{D}}}{u}x} J_0(\lambda_n r) \tag{5.30}$$

$$\mathrm{T} = 2(\mathrm{T_0} - \mathrm{T_w}) \sum_{n=1}^{\infty} \frac{1}{\lambda_n r_0} \frac{J_0(\lambda_n r)}{J_1(\lambda_n r_0)} e^{-\frac{\lambda_n^2 k_{\bar{D}}}{u}x} + \mathrm{T_w} \tag{5.31}$$

$$\frac{\partial T}{\partial r} = 2(\mathrm{T_0} - \mathrm{T_w}) \sum_{n=1}^{\infty} \frac{1}{\lambda_n r_0} \frac{\frac{\partial}{\partial r}(J_0(\lambda_n r))}{J_1(\lambda_n r_0)} e^{-\frac{\lambda_n^2 k_{\bar{D}}}{u}x} \tag{5.32}$$

$$\left(\frac{\partial T}{\partial r}\right)_{r=r_0} = 2(\mathrm{T_w} - \mathrm{T_0}) \sum_{n=1}^{\infty} \frac{1}{r_0} e^{-\frac{\lambda_n^2 k_{\bar{D}}}{u}x} \tag{5.33}$$

We can determine bulk or mean temperature at any distance,

$$\Theta_{\mathrm{m}} = \frac{\mathrm{T_m} - \mathrm{T_w}}{\mathrm{T_0} - \mathrm{T_w}} = 4 \sum_{n=1}^{\infty} \frac{1}{(\lambda_n r_0)^2} e^{-\frac{\lambda_n^2 k_{\bar{D}}}{u}x} \tag{5.34}$$

$$\mathrm{T_w} - \mathrm{T_m} = 4(\mathrm{T_w} - \mathrm{T_0}) \sum_{n=1}^{\infty} \frac{1}{(\lambda_n r_0)^2} e^{-\frac{\lambda_n^2 k_{\bar{D}}}{u}x} \tag{5.35}$$

Heat transfer coefficient can be written as,

$$h_x = \frac{\bar{k}\left(\frac{\partial T}{\partial r}\right)_{r=r_0}}{T_w - T_m} = \frac{\bar{k}\sum_{n=1}^{\infty}\frac{1}{r_0}e^{-\frac{\lambda_n^2 k_{\bar{D}}}{u}x}}{2\sum_{n=1}^{\infty}\frac{1}{(\lambda_n r_0)^2}e^{-\frac{\lambda_n^2 k_{\bar{D}}}{u}x}} \quad \textbf{(5.36)}$$

Nusselt number cab be calculated as,

$$Nu = \frac{h_x d}{\bar{k}} = \frac{\sum_{n=1}^{\infty} e^{-\frac{\lambda_n^2 k_{\bar{D}}}{u}x}}{\sum_{n=1}^{\infty}\frac{1}{(\lambda_n r_0)^2}e^{-\frac{\lambda_n^2 k_{\bar{D}}}{u}x}} \quad \textbf{(5.37)}$$

5.1.2 Parabolic velocity profile

By assuming velocity profile as parabolic, we have [27],

$$u = 2U_m\left[1 - \left(\frac{r}{r_0}\right)^2\right] \quad \textbf{(5.38)}$$

Just like whatsoever shown in Fig. 5.1,

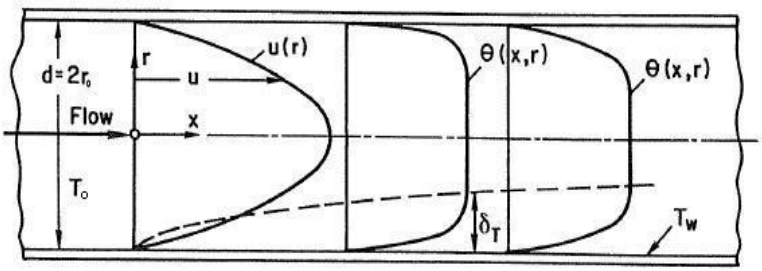

Figure 5.1 : Parabolic profile of velocity.

By using these dimensionless variables,

$$\Theta = \frac{T - T_w}{T_0 - T_w} \quad \textbf{(5.39)}$$

$$\eta = \frac{r}{r_0} \quad \textbf{(5.40)}$$

$$\xi = \frac{x/r_0}{RePr} = \frac{x/r_0}{Pe_{nf}} \quad \textbf{(5.41)}$$

Here,

$$\mathrm{Pe_{nf}} = \frac{\mathrm{U_m\, d}}{k_{\bar{D}}} \tag{5.42}$$

Then parabolic velocity can be written as,

$$u = 2U_m(1-\eta^2) \tag{5.43}$$

The final form of energy equation can be written as,

$$(1-\eta^2)\frac{\partial\Theta}{\partial\xi} = \frac{1}{\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial\Theta}{\partial\eta}\right) \tag{5.44}$$

Where the boundary conditions are,

$$\Theta(0,\eta) = 1 \tag{5.45}$$

$$\frac{\partial\Theta(\xi,0)}{\partial\eta} = 0 \tag{5.46}$$

$$\Theta(\xi,1) = 0 \tag{5.47}$$

Again, by using method of separating variables we have,

$$\Theta(\xi,\eta) = X(\xi)\,.R(\eta) \tag{5.48}$$

$$\frac{\partial\Theta}{\partial\xi} = \frac{\partial X(\xi)}{\partial\xi}.R(\eta) \tag{5.49}$$

$$\frac{\partial\Theta}{\partial\eta} = X(\xi).\frac{\partial R(\eta)}{\partial\eta} \tag{5.50}$$

$$\frac{\partial^2\Theta}{\partial\eta^2} = X(\xi).\frac{\partial^2 R(\eta)}{\partial\eta^2} \tag{5.51}$$

$$\frac{1}{X}\frac{dX}{d\xi} = \frac{1}{R(1-\eta^2)}\frac{1}{\eta}\frac{d}{d\eta}\left(\eta\frac{dR}{d\eta}\right) = -\lambda^2 \tag{5.52}$$

Solving the left hand side of general equation,

$$\frac{1}{X}\frac{dX}{d\xi} = -\lambda^2 \tag{5.53}$$

$$\frac{dX}{d\xi} + \lambda^2 X = 0 \tag{5.54}$$

$$X(\xi) = Ae^{-\lambda^2 \xi} \tag{5.55}$$

Solving the right hand side of equation,

$$\frac{d^2R}{d\eta^2} + \frac{1}{\eta}\frac{dR}{d\eta} + \lambda^2(1-\eta^2)R = 0 \tag{5.56}$$

Second and third conditions can be rewritten as,

$$\frac{dR(0)}{d\eta} = 0 \tag{5.57}$$

$$R(1) = 0 \tag{5.58}$$

According to orthogonality relation we have,

$$\int_0^1 \eta(1-\eta^2)R_m(\eta)R_n(\eta)d\eta = 0, if m \neq n \tag{5.59}$$

$$\int_0^1 \eta(1-\eta^2)R_n^2(\eta)d\eta = \frac{1}{2\lambda_n}\left(\frac{\partial R_n}{\partial \lambda_n}\frac{dR_n}{d\eta}\right)_{\eta=1} \tag{5.60}$$

By using series method of solution,

$$\Theta(\xi,\eta) = \sum_{n=0}^{\infty} C_n R_n(\eta) e^{-\lambda_n^2 \xi} \tag{5.61}$$

By using first boundary condition we have,

$$1 = \sum_{n=0}^{\infty} C_n R_n(\eta) \tag{5.62}$$

By using orthogonality relation and below equation we have,

$$\int_0^1 \eta(1-\eta^2)R_n(\eta)d\eta = -\frac{1}{\lambda_n^2}\left(\frac{\partial R_n}{\partial \lambda_n}\right)_{\eta=1} \tag{5.63}$$

$$C_n = \frac{\int_0^1 \eta(1-\eta^2)R_n(\eta)d\eta}{\int_0^1 \eta(1-\eta^2)R_n^2(\eta)d\eta} = -\frac{2}{\lambda_n\left(\frac{\partial R_n}{\partial \lambda_n}\right)_{\eta=1}} \tag{5.64}$$

By substituting above equation we have,

$$\Theta(\xi,\eta) = -2\sum_{n=0}^{\infty} \frac{R_n(\eta)e^{-\lambda_n^2\xi}}{\lambda_n\left(\frac{\partial R_n}{\partial \lambda_n}\right)_{\eta=1}} \quad \textbf{(5.65)}$$

We know that local heat transfer coefficient is,

$$h_x = \frac{\bar{k}\left(\frac{\partial T}{\partial r}\right)_{r=r_0}}{T_w - T_m} \quad \textbf{(5.66)}$$

In addition, in the form of dimensionless variables can be written as,

$$h_x = -\frac{\bar{k}}{r_0}\frac{1}{\Theta_m}\left(\frac{\partial \Theta}{\partial \eta}\right)_{\eta=1} \quad \textbf{(5.67)}$$

The formulation for Nusselt number in the form of dimensionless variables can be written as,

$$Nu_x = \frac{2r_0h_x}{\bar{k}} = -\frac{2}{\Theta_m}\left(\frac{\partial \Theta}{\partial \eta}\right)_{\eta=1} \quad \textbf{(5.68)}$$

The bulk temperature in any distance can be written as,

$$\Theta_m = 8\sum_{n=0}^{\infty} \frac{\left(\frac{dR_n}{d\eta}\right)_{\eta=1}}{\lambda_n^3\left(\frac{\partial R_n}{\partial \lambda_n}\right)_{\eta=1}} e^{-\lambda_n^2\xi} \quad \textbf{(5.69)}$$

And from the temperature distribution we have,

$$\left(\frac{\partial \Theta}{\partial \eta}\right)_{\eta=1} = -2\sum_{n=0}^{\infty} \frac{\left(\frac{dR_n}{d\eta}\right)_{\eta=1}}{\lambda_n\left(\frac{\partial R_n}{\partial \lambda_n}\right)_{\eta=1}} e^{-\lambda_n^2\xi} \quad \textbf{(5.70)}$$

Thus the Nusselt number become,

$$Nu_x = \frac{\sum_{n=0}^{\infty} A_n e^{-\lambda_n^2\xi}}{2\sum_{n=0}^{\infty} \frac{A_n}{\lambda_n^2} e^{-\lambda_n^2\xi}} \quad \textbf{(5.71)}$$

Equation above gives the variation of Nusselt number in entrance region of the pipe for constant wall temperature. Where,

$$A_n = \frac{\left(\frac{dR_n}{d\eta}\right)_{\eta=1}}{\lambda_n \left(\frac{\partial R_n}{\partial \lambda_n}\right)_{\eta=1}} \tag{5.72}$$

5.2 Uniform Heat Flux Boundary Condition

5.2.1 Parabolic velocity profile

Solution for extended classical Graetz problem for constant heat flux boundary condition has been extended by Sellars et al. [27],

$$u\frac{\partial T}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[k_{\bar{D}} r \frac{\partial T}{\partial r}\right] \tag{5.73}$$

$$(1-\eta^2)\frac{\partial T}{\partial \xi} = \frac{1}{\eta}\frac{\partial}{\partial \eta}\left(\eta \frac{\partial T}{\partial \eta}\right) \tag{5.74}$$

Where nondimensional variables are,

$$\eta = \frac{r}{r_0} \tag{5.75}$$

$$\xi = \frac{x/r_0}{RePr} = \frac{x/r_0}{Pe_{nf}} \tag{5.76}$$

For this case the inlet and boundary conditions are,

$$T(0,\eta) = T_0 \tag{5.77}$$

$$\frac{\partial T(\xi,1)}{\partial \eta} = \frac{q_w'' r_0}{k} = const. \tag{5.78}$$

$$\frac{\partial T(\xi,0)}{\partial \eta} = 0 \tag{5.79}$$

According to Sellars et al. [27] the local Nusselt number for the thermal entrance region is given by,

$$Nu_x = \frac{1}{\frac{11}{48} + \frac{1}{2}\sum_{n=1}^{\infty} \frac{e^{-\beta_n^2 \xi}}{\beta_n^4 R'(-\beta_n^2)}} \tag{5.80}$$

Where the first three values are given in Table 5.1,

Table 5.1 : First three values for coefficients mentioned in local Nusselt number.

n	β_n^2	$-R'(-\beta_n^2)$
1	25.639	8.854×10^{-3}
2	84.624	2.062×10^{-3}
3	176.40	9.434×10^{-4}

6. EULERIAN-EULERIAN TWO-PHASE NUMERICAL SIMULATION OF NANOFLUID

6.1 Problem Statement and Assumptions

Here is a problem that consists of two parallel plates with below mentioned dimensions:

Height of microchannel is 200 micrometer and length of microchannel is 100 times larger than its height. The origin of Cartesian coordinates considered at the plate symmetry axis and only the top of the channel used for numerical solutions. Used nanofluid is mixture of copper nanoparticles and water. The entrance properties of nanofluid are: Nanofluid enters the microchannel with uniform velocity and temperature and walls of microchannel is isothermal.

Continuity, momentum and energy equations were written for laminar, steady-state and two-dimensional flow in microchannel for fluid and particle phase, in Eulerian-Eulerian two-phase flow. Energy equations were solved for constant wall temperature boundary condition.

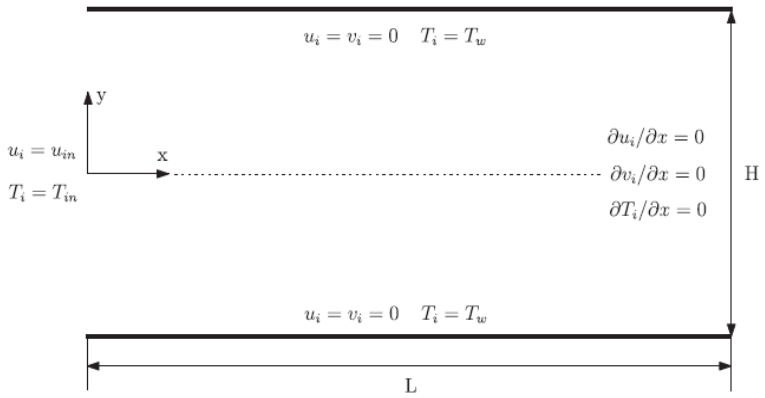

Figure 6.1 : Geometry of microchannel.

According to Fig. 6.1, these are some assumptions in fully developed region in microchannel,

$$\frac{\partial u_l}{\partial x} = 0, \qquad \frac{\partial v_l}{\partial x} = 0, \qquad \frac{\partial T_l}{\partial x} = 0 \tag{6.1}$$

$$\frac{\partial u_p}{\partial x} = 0, \qquad \frac{\partial v_p}{\partial x} = 0, \qquad \frac{\partial T_p}{\partial x} = 0 \tag{6.2}$$

Kalteh et al. [40] and Hao et al. [41] wrote governing equations, such as continuity, momentum and energy equations, for liquid and particle phases separately.

6.2 Continuity Equations

General form of continuity equation for liquid phase can be written as,

$$\frac{\partial(\varphi_l \rho_l u_l)}{\partial x} + \frac{\partial(\varphi_l \rho_l v_l)}{\partial y} = 0 \tag{6.3}$$

By knowing this fact that liquid fraction ratio and density are constant with x and y, general form of continuity equation can be simplified as,

$$\varphi_l \rho_l \left[\frac{\partial u_l}{\partial x} + \frac{\partial v_l}{\partial y}\right] = 0 \tag{6.4}$$

$$\frac{\partial u_l}{\partial x} + \frac{\partial v_l}{\partial y} = 0 \tag{6.5}$$

In the fully developed region by using assumptions mentioned in first part we have,

$$\frac{\partial v_l}{\partial y} = 0 \tag{6.6}$$

According to assumptions and by knowing this fact that derivation of v_l by x and y is zero we have,

$$v_l = constant \tag{6.7}$$

$$u_l = u_l(y) \tag{6.8}$$

By using these equations velocity vector for liquid can be written as,

$$\vec{V_l} = u_l \vec{i} + v_l \vec{j} \tag{6.9}$$

And,

$$\vec{V_l} = u_l(y)\vec{i} + v_l\vec{j} \quad \textbf{(6.10)}$$

On the other hand,general form of continuity equation for particle phase can be written as,

$$\frac{\partial(\varphi_p \rho_p u_p)}{\partial x} + \frac{\partial(\varphi_p \rho_p v_p)}{\partial y} = 0 \quad \textbf{(6.11)}$$

Fraction ratio and density of particle also are constant,

$$\varphi_p \rho_p \left[\frac{\partial(u_p)}{\partial x} + \frac{\partial(v_p)}{\partial y}\right] = 0 \quad \textbf{(6.12)}$$

$$\frac{\partial u_p}{\partial x} + \frac{\partial v_p}{\partial y} = 0 \quad \textbf{(6.13)}$$

And by using assumptions in the fully developed region we have,

$$\frac{\partial v_p}{\partial y} = 0 \quad \textbf{(6.14)}$$

By the same method it can be written for velocity of particle,

$$v_p = constant \quad \textbf{(6.15)}$$

$$u_p = u_p(y) \quad \textbf{(6.16)}$$

By using above-mentioned equations velocity vector for particle can be written as,

$$\vec{V_p} = u_p\vec{i} + v_p\vec{j} \quad \textbf{(6.17)}$$

$$\vec{V_p} = u_p(y)\vec{i} + v_p\vec{j} \quad \textbf{(6.18)}$$

There is an obvious rule about volume concentrations of liquid and particle,

$$\varphi_l + \varphi_p = 1 \quad \textbf{(6.19)}$$

6.3 Momentum Equations

General form of momentum equation for liquid phase in x-direction can be written as,

$$\frac{\partial(\varphi_l\rho_l u_l u_l)}{\partial x}+\frac{\partial(\varphi_l\rho_l v_l u_l)}{\partial y}=-\varphi_l\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left(\varphi_l\mu_l\frac{\partial u_l}{\partial x}\right)+\frac{\partial}{\partial y}\left(\varphi_l\mu_l\frac{\partial u_l}{\partial y}\right)+(F_d)_x+(F_{vm})_x \quad \textbf{(6.20)}$$

By doing some simplifications,

$$\varphi_l\rho_l\left(\frac{\partial(u_l u_l)}{\partial x}+\frac{\partial(v_l u_l)}{\partial y}\right)=-\varphi_l\frac{\partial p}{\partial x}+\varphi_l\mu_l\left(\frac{\partial}{\partial x}\left(\frac{\partial u_l}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\partial u_l}{\partial y}\right)\right)+(F_d)_x+(F_{vm})_x \quad \textbf{(6.21)}$$

WhereF_d is the drag force between the phases and F_{vm} is the virtual mass force, which is proportional to the relative acceleration of the two phases and can be calculated as,

$$F_{vm}=0.5\varphi_p\rho_l\frac{D}{Dt}\left(\vec{V}_l-\vec{V}_p\right) \quad \textbf{(6.22)}$$

By doing these simplification in fully developed region,

$$\frac{\partial(u_l u_l)}{\partial x}=2u_l\frac{\partial(u_l)}{\partial x}=0 \quad \textbf{(6.23)}$$

And,

$$\frac{\partial}{\partial x}\left(\frac{\partial u_l}{\partial x}\right)=0 \quad \textbf{(6.24)}$$

And,

$$\frac{\partial(v_l u_l)}{\partial y}=\frac{\partial(v_l)}{\partial y}u_l+v_l\frac{\partial(u_l)}{\partial y}=v_l\frac{\partial(u_l)}{\partial y} \quad \textbf{(6.25)}$$

Simplified momentum equation of liquid phase in x-direction can be written as,

$$\frac{\partial^2 u_l}{\partial y^2}-\frac{\rho_l v_l}{\mu_l}\frac{\partial u_l}{\partial y}=\frac{1}{\mu_l}\frac{\partial p}{\partial x}-\frac{(F_d)_x}{\varphi_l\mu_l}-\frac{(F_{vm})_x}{\varphi_l\mu_l} \quad \textbf{(6.26)}$$

Because of small size of the nanoparticles, it can be assumed that the lift force between phases is negligible in x-direction of momentum equation.

On the other hand, general form of momentum equation of particle phase in x-direction can be written as,

$$\frac{\partial(\varphi_p \rho_p u_p u_p)}{\partial x} + \frac{\partial(\varphi_p \rho_p v_p u_p)}{\partial y} = -\varphi_p \frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(\varphi_p \mu_p \frac{\partial u_p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\varphi_p \mu_p \frac{\partial u_p}{\partial y}\right) + (F_{col})_x - (F_d)_x - (F_{vm})_x \quad \textbf{(6.27)}$$

WhereF_{col} is the particle-particle interaction force and can be calculated by this formula,

$$F_{col} = G(\varphi_l)\vec{\nabla}\varphi_l \quad \textbf{(6.28)}$$

Where particle-particle interaction module is,

$$G(\varphi_l) = 1.0 \exp(-600[\varphi_l - 0.376]) \quad \textbf{(6.29)}$$

By doing some simplifications on momentum equation,

$$\varphi_p \rho_p \left(\frac{\partial(u_p u_p)}{\partial x} + \frac{\partial(v_p u_p)}{\partial y}\right) = -\varphi_p \frac{\partial p}{\partial x} + \varphi_p \mu_p \left(\frac{\partial}{\partial x}\left(\frac{\partial u_p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial u_p}{\partial y}\right)\right) + (F_{col})_x - (F_d)_x - (F_{vm})_x \quad \textbf{(6.30)}$$

By using these assumptions of fully developed region,

$$\frac{\partial(u_p u_p)}{\partial x} = 2u_p \frac{\partial(u_p)}{\partial x} = 0 \quad \textbf{(6.31)}$$

And,

$$\frac{\partial}{\partial x}\left(\frac{\partial u_p}{\partial x}\right) = 0 \quad \textbf{(6.32)}$$

And,

$$\frac{\partial(v_p u_p)}{\partial y} = \frac{\partial(v_p)}{\partial y} u_p + v_p \frac{\partial(u_p)}{\partial y} = v_p \frac{\partial(u_p)}{\partial y} \quad \textbf{(6.33)}$$

Then simplified form of momentum equation for particle phase is,

$$\frac{\partial^2 u_p}{\partial y^2} - \frac{\rho_p v_p}{\mu_p}\frac{\partial u_p}{\partial y} = \frac{1}{\mu_p}\frac{\partial p}{\partial x} - \frac{(F_{col})_x}{\varphi_p \mu_p} + \frac{(F_d)_x}{\varphi_p \mu_p} + \frac{(F_{vm})_x}{\varphi_p \mu_p} \quad \textbf{(6.34)}$$

General form of momentum equation for liquid phase in y-direction is,

$$\frac{\partial(\varphi_l \rho_l u_l v_l)}{\partial x} + \frac{\partial(\varphi_l \rho_l v_l v_l)}{\partial y}$$
$$= -\varphi_l \frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left(\varphi_l \mu_l \frac{\partial v_l}{\partial x}\right) + \frac{\partial}{\partial y}\left(\varphi_l \mu_l \frac{\partial v_l}{\partial y}\right) + (F_d)_y + (F_{vm})_y \quad \textbf{(6.35)}$$

By doing some simplifications according to constant quantity of liquid fraction ratio and density,

$$\varphi_l \rho_l \left(\frac{\partial(u_l v_l)}{\partial x} + \frac{\partial(v_l v_l)}{\partial y}\right) = -\varphi_l \frac{\partial p}{\partial y} + \varphi_l \mu_l \left(\frac{\partial}{\partial x}\left(\frac{\partial v_l}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial v_l}{\partial y}\right)\right) + (F_d)_y + (F_{vm})_y \quad \textbf{(6.36)}$$

By knowing these facts,

$$\frac{\partial(u_l v_l)}{\partial x} = \frac{\partial(u_l)}{\partial x} v_l + u_l \frac{\partial(v_l)}{\partial x} = 0 \quad \textbf{(6.37)}$$

And,

$$\frac{\partial(v_l v_l)}{\partial y} = 2 v_l \frac{\partial(v_l)}{\partial y} = 0 \quad \textbf{(6.38)}$$

And,

$$\frac{\partial}{\partial y}\left(\frac{\partial v_l}{\partial y}\right) = 0 \quad \textbf{(6.39)}$$

Then the last form of momentum equation in y-direction for liquid phase is,

$$0 = -\varphi_l \frac{\partial p}{\partial y} + (F_d)_y + (F_{vm})_y \quad \textbf{(6.40)}$$

On the other hand, general form of momentum equation for particle phase in y-direction is,

$$\frac{\partial(\varphi_p \rho_p u_p v_p)}{\partial x} + \frac{\partial(\varphi_p \rho_p v_p v_p)}{\partial y}$$
$$= -\varphi_p \frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left(\varphi_p \mu_p \frac{\partial v_p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\varphi_p \mu_p \frac{\partial v_p}{\partial y}\right) \quad \textbf{(6.41)}$$
$$+ (F_{col})_y - (F_d)_y - (F_{vm})_y$$

Because of the small size of the channel, gravitational force can be negligible assumed in y-direction of momentum equation for particle phase.

By doing some simplifications,

$$\varphi_p \rho_p \left(\frac{\partial(u_p v_p)}{\partial x} + \frac{\partial(v_p v_p)}{\partial y}\right) = -\varphi_p \frac{\partial p}{\partial y} + \varphi_p \mu_p \left(\frac{\partial}{\partial x}\left(\frac{\partial v_p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial v_p}{\partial y}\right)\right) + (F_{col})_y - (F_d)_y - (F_{vm})_y \quad \textbf{(6.42)}$$

Using these assumptions can be useful for simplifying above-mentioned equation,

$$\frac{\partial(u_p v_p)}{\partial x} = \frac{\partial(u_p)}{\partial x} v_p + u_p \frac{\partial(v_p)}{\partial x} = 0 \quad \textbf{(6.43)}$$

And,

$$\frac{\partial(v_p v_p)}{\partial y} = 2 v_p \frac{\partial(v_p)}{\partial y} = 0 \quad \textbf{(6.44)}$$

And,

$$\frac{\partial}{\partial y}\left(\frac{\partial v_p}{\partial y}\right) = 0 \quad \textbf{(6.45)}$$

The last form of momentum equation in y-direction for particle phase is,

$$0 = -\varphi_p \frac{\partial p}{\partial y} + (F_{col})_y - (F_d)_y - (F_{vm})_y \quad \textbf{(6.46)}$$

Where drag force can be calculated as,

$$F_d = -\beta(\vec{V}_l - \vec{V}_p) \quad \textbf{(6.47)}$$

By substituting velocity vectors of liquid and particle drag force is,

$$F_d = -\beta\left(u_l(y)\vec{i} + v_l\vec{j} - \left(u_p(y)\vec{i} + v_p\vec{j}\right)\right) \quad \textbf{(6.48)}$$

$$F_d = -\beta\left(\left(u_l(y) - u_p(y)\right)\vec{i} + \left(v_l - v_p\right)\vec{j}\right) \quad \textbf{(6.49)}$$

Then x and y components of drag force are,

$$(F_d)_x = -\beta\left(u_l(y) - u_p(y)\right) \quad \textbf{(6.50)}$$

$$(F_d)_y = -\beta\left(v_l - v_p\right) \quad \textbf{(6.51)}$$

In this equation β is friction coefficient and can be calculated for liquid fraction ratio more than 0.8 as,

$$\beta = \frac{3}{4} C_d \frac{\varphi_l(1-\varphi_l)}{d_p} \left|\vec{V}_l - \vec{V}_p\right| \varphi_l^{-2.65} \quad \textbf{(6.52)}$$

By substituting velocity vectors in above equation useful formula for calculating of friction coefficient is,

$$\beta = \frac{3}{4} C_d \frac{\varphi_l(1-\varphi_l)}{d_p} \left|\left(u_l(y) - u_p(y)\right)\vec{i} + \left(v_l - v_p\right)\vec{j}\right| \varphi_l^{-2.65} \quad \textbf{(6.53)}$$

In friction coefficient formula C_d is drag coefficient which is different in different Reynolds numbers and can be calculated as,

$$C_d = \begin{cases} \frac{24}{Re_p}\left(1 + 0.15 Re_p^{0.697}\right), & Re_p < 1000 \\ 0.44 & , \; Re_p \geq 1000 \end{cases} \quad \textbf{(6.54)}$$

Reynolds number in this formula is,

$$Re_p = \frac{\varphi_l \rho_l \left|\vec{V}_l - \vec{V}_p\right| d_p}{\mu_l} \quad \textbf{(6.55)}$$

By substituting velocity vectors of liquid and particle it can changed to,

$$Re_p = \frac{\varphi_l \rho_l \left|\left(u_l(y) - u_p(y)\right)\vec{i} + \left(v_l - v_p\right)\vec{j}\right| d_p}{\mu_l} \quad \textbf{(6.56)}$$

Equations of drag force, friction coefficient, drag coefficient, virtual mass force, particle-particle interaction force and particle-particle interaction modules are not for nano-sized particles but because there are no any relations for nano-sized, these equations are used.

6.4 Energy Equations

One of the assumptions in solving energy equation is neglecting viscous dissipation. Another term in energy equation that can be neglected is radiation. As written momentum equations for liquid and particle phases, in energy equation also these phases will be analyzed by using separate equations but coupled.

General form of energy equation for liquid phase is,

$$\frac{\partial}{\partial x}\left(\varphi_l \rho_l u_l {c_p}_l T_l\right) + \frac{\partial}{\partial y}\left(\varphi_l \rho_l v_l {c_p}_l T_l\right) = \frac{\partial}{\partial x}\left(\varphi_l k_{eff,l} \frac{\partial T_l}{\partial x}\right) + \frac{\partial}{\partial y}\left(\varphi_l k_{eff,l} \frac{\partial T_l}{\partial y}\right) - h_v\left(T_l - T_p\right) \quad \textbf{(6.57)}$$

In this equation in addition to constant quantity of fraction ratio and density, heat capacity also is constant with x and y for liquid and particle,

$$\varphi_l \rho_l {c_p}_l \left(\frac{\partial}{\partial x}(u_l T_l) + \frac{\partial}{\partial y}(v_l T_l)\right) = \varphi_l k_{eff,l} \left(\frac{\partial}{\partial x}\left(\frac{\partial T_l}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial T_l}{\partial y}\right)\right) - h_v\left(T_l - T_p\right) \quad \textbf{(6.58)}$$

By using these simplifications,

$$\frac{\partial}{\partial x}(u_l T_l) = T_l \frac{\partial}{\partial x}(u_l) + u_l \frac{\partial}{\partial x}(T_l) = u_l \frac{\partial T_l}{\partial x} \quad \textbf{(6.59)}$$

And in fully-developed region we can neglect the y-component of velocity of liquid, by that means,

$$v_l = 0 \quad \textbf{(6.60)}$$

So,

$$\frac{\partial}{\partial y}(v_l T_l) = 0 \quad \textbf{(6.61)}$$

By substituting above-mentioned equations in energy equation the last form of energy equation for liquid can be written as,

$$\varphi_l \rho_l c_{p_l} \left(u_l \frac{\partial T_l}{\partial x}\right) = \varphi_l k_{eff,l} \left(\frac{\partial^2 T_l}{\partial x^2} + \frac{\partial^2 T_l}{\partial y^2}\right) - h_v (T_l - T_p) \quad \textbf{(6.62)}$$

$$\frac{\partial^2 T_l}{\partial y^2} + \frac{\partial^2 T_l}{\partial x^2} - \frac{\rho_l c_{p_l}}{k_{eff,l}} u_l \frac{\partial T_l}{\partial x} = \frac{h_v}{\varphi_l k_{eff,l}} (T_l - T_p) \quad \textbf{(6.63)}$$

On the other hand, general form of energy equation for particle phase is,

$$\frac{\partial}{\partial x}\left(\varphi_p \rho_p u_p c_{p_p} T_p\right) + \frac{\partial}{\partial y}\left(\varphi_p \rho_p v_p c_{p_p} T_p\right) = \frac{\partial}{\partial x}\left(\varphi_p k_{eff,p} \frac{\partial T_p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\varphi_p k_{eff,p} \frac{\partial T_p}{\partial y}\right) + h_v (T_l - T_p) \quad \textbf{(6.64)}$$

By using this equation,

$$\frac{\partial}{\partial x}(u_p T_p) = T_p \frac{\partial}{\partial x}(u_p) + u_p \frac{\partial}{\partial x}(T_p) = u_p \frac{\partial T_p}{\partial x} \quad \textbf{(6.65)}$$

In fully developedregion, we can neglect the y-comp of velocity of liquid, by that means,

$$v_l = 0 \quad \textbf{(6.66)}$$

So,

$$\frac{\partial}{\partial y}(v_p T_p) = 0 \quad \textbf{(6.67)}$$

By substituting in general form of energy equation,

$$\frac{\partial^2 T_p}{\partial y^2} + \frac{\partial^2 T_p}{\partial x^2} - \frac{\rho_p c_{p_p}}{k_{eff,p}} u_p \frac{\partial T_p}{\partial x} = \frac{-h_v}{\varphi_p k_{eff,p}} (T_l - T_p) \quad \textbf{(6.68)}$$

Where volumetric interphase heat transfer coefficient can be calculated as,

$$h_v = \frac{6(1-\varphi_l)}{d_p} h_p \quad \textbf{(6.69)}$$

Whereh_p is fluid-particle heat transfer coefficient. For calculating this coefficient formula of Nusselt number should be used as below,

$$Nu_p = \frac{h_p d_p}{k_l} = 2 + 1.1 Re_p^{0.6} Pr^{\frac{1}{3}} \quad \textbf{(6.70)}$$

So,

$$h_p = \frac{k_l\left(2+1.1 Re_p^{0.6} Pr^{\frac{1}{3}}\right)}{d_p} \quad \textbf{(6.71)}$$

In this case effective conduction coefficients of liquid and particle are,

$$k_{eff,l} = \frac{k_{b,l}}{\varphi_l} \quad \textbf{(6.72)}$$

$$k_{eff,p} = \frac{k_{b,p}}{\varphi_p} \quad \textbf{(6.73)}$$

Where,

$$k_{b,l} = \left(1 - \sqrt{(1-\varphi_l)}\right) k_l \quad \textbf{(6.74)}$$

$$k_{b,p} = \sqrt{(1-\varphi_l)}(\omega A + [1-\omega]\Gamma) k_l \quad \textbf{(6.75)}$$

$$\Gamma = \frac{2}{\left(1-\frac{\mathrm{B}}{\mathrm{A}}\right)}\left\{\frac{\mathrm{B}(\mathrm{A}-1)}{\mathrm{A}\left(1-\frac{\mathrm{B}}{\mathrm{A}}\right)^2}\ln\left(\frac{\mathrm{A}}{\mathrm{B}}\right) - \frac{(B-1)}{\left(1-\frac{\mathrm{B}}{\mathrm{A}}\right)} - \frac{B+1}{2}\right\} \quad \textbf{(6.76)}$$

$$\mathrm{B} = 1.25\left(\frac{[1-\varphi_l]}{\varphi_l}\right)^{\frac{10}{9}} \quad \textbf{(6.77)}$$

$$\mathrm{A} = \frac{k_p}{k_l} \text{ and } \omega = 7.26 \times 10^{-3} \quad \textbf{(6.78)}$$

After solving energy equations local Nusselt number can be calculated as,

$$Nu = \frac{(hD_h)}{k_l} = q^{"} D_h / k_l (T_w - T_m) \quad \textbf{(6.79)}$$

Where mean temperature is,

$$T_m = \frac{\sum_{i=1}^{p}\left(\iint \rho_i u_i c_{pi} T_i dA\right)}{\sum_{i=1}^{p}\left(\iint \rho_i u_i c_{pi} dA\right)} \quad \textbf{(6.80)}$$

And wall convective heat transfer flux is,

$$q^{"} = \varphi_l k_{eff,l} \frac{\partial T_l}{\partial y}\Big|_w + \varphi_p k_{eff,p} \frac{\partial T_p}{\partial y}\Big|_w \quad \textbf{(6.81)}$$

The average Nusselt number can be calculated as,

$$\overline{Nu} = \frac{1}{L}\int_0^L Nu dx \quad \textbf{(6.82)}$$

6.5 Non-dimensionalization

For solving momentum equations for calculating velocity profiles of liquid and particle and then using them in energy equations these non-dimensional equations should be used,

$$X = \frac{x}{D_h}, \qquad Y = \frac{y}{D_h}, \qquad U_i = \frac{u_i}{u_{in}}$$
$$V_i = \frac{v_i}{u_{in}}, \qquad P = \frac{p - p_{in}}{\rho_l u_{in}^2}, \qquad \theta_i = \frac{T_i - T_{in}}{T_w - T_{in}} \quad \textbf{(6.83)}$$

Where i represents liquid and particle phases,

Non-dimensional forms can be written seperately as,

$$x = XD_h, \qquad y = YD_h$$
$$u_l = U_l u_{in}, \; v_l = V_l u_{in}, \quad u_p = U_p u_{in}, \; v_p = V_p u_{in}$$
$$p = P\rho_l u_{in}^2 + p_{in} \quad \textbf{(6.84)}$$
$$T_l = \theta_l (T_w - T_{in}) + T_{in}, \qquad T_p = \theta_p (T_w - T_{in}) + T_{in}$$

By using these variables for nondimensionalization, continuity equations for both phases can be written as,

$$\frac{\partial (V_l u_{in})}{\partial (YD_h)} = 0 \quad \textbf{(6.85)}$$

$$\frac{\partial V_l}{\partial Y} = 0 \quad \textbf{(6.86)}$$

So,

$$V_l = constant \quad \textbf{(6.87)}$$

And,

$$\frac{\partial\left(V_p u_{in}\right)}{\partial\left(YD_h\right)} = 0 \tag{6.88}$$

$$\frac{\partial V_p}{\partial Y} = 0 \tag{6.89}$$

So,

$$V_p = constant \tag{6.90}$$

According to experimental investigation of Kalteh et al. [40] virtual mass and particle-particle interaction forces are ignorable.

$$(F_{vm})_x = 0 \tag{6.91}$$

Then this term can be neglected in momentum equation of liquid phase in x-direction,

$$\frac{\partial^2 u_l}{\partial y^2} - \frac{\rho_l v_l}{\mu_l}\frac{\partial u_l}{\partial y} = \frac{1}{\mu_l}\frac{\partial p}{\partial x} - \frac{(F_d)_x}{\varphi_l \mu_l} \tag{6.92}$$

$$\frac{\partial^2 u_l}{\partial y^2} - \frac{\rho_l v_l}{\mu_l}\frac{\partial u_l}{\partial y} = \frac{1}{\mu_l}\frac{\partial p}{\partial x} + \frac{\beta\left(u_l(y) - u_p(y)\right)}{\varphi_l \mu_l} \tag{6.93}$$

By using the same non-dimensional variables momentum equations can be written as,

$$\frac{\partial^2 U_l}{\partial Y^2} - \frac{\rho_l u_{in} D_h}{\mu_l} V_l \frac{\partial U_l}{\partial Y} - \frac{\beta D_h^2}{\varphi_l \mu_l}\left(U_l - U_p\right) = \frac{\rho_l u_{in} D_h}{\mu_l}\frac{\partial P}{\partial X} \tag{6.94}$$

In fully developedregion, we can neglect the y-comp of velocity of liquid, by that means,

$$V_l = 0 \tag{6.95}$$

Last form of non-dimensional momentum equation for liquid phase is,

$$\frac{\partial^2 U_l}{\partial Y^2} - \frac{\beta D_h^2}{\varphi_l \mu_l}\left(U_l - U_p\right) = \frac{\rho_l u_{in} D_h}{\mu_l}\frac{\partial P}{\partial X} \tag{6.96}$$

By replacing group of constants with these constants,

$$a_1 = -\frac{\beta D_h^2}{\varphi_l \mu_l} \quad \textbf{(6.97)}$$

$$b_1 = \frac{\rho_l u_{in} D_h}{\mu_l} \frac{\partial P}{\partial X} \quad \textbf{(6.98)}$$

Then,

$$\frac{\partial^2 U_l}{\partial Y^2} + a_1 (U_l - U_p) = b_1 \quad \textbf{(6.99)}$$

For solving this second order equation we should integrate twice with respect to Y,

$$\frac{\partial^2 U_l}{\partial Y^2} + a_1 (U_l - U_p) = b_1 \quad \textbf{(6.100)}$$

$$\frac{\partial U_l}{\partial Y} + a_1 \int_0^Y (U_l - U_p) dY = b_1 Y + c_1 \quad \textbf{(6.101)}$$

$$U_l + a_1 \int_0^Y \int_0^Y (U_l - U_p) dY \, dY = \frac{b_1}{2} Y^2 + c_1 Y + d_1 \quad \textbf{(6.102)}$$

After integrating with respect to Y we have 2 integral constants. Two boundary conditions are needed for calculating these integral constants, c_1 and d_1,

For the first boundary condition, it can be assumed that at the center of pipe derivative of velocity of liquid is zero.

$$aty = 0, \qquad \frac{\partial u_l}{\partial y} = 0 \quad \textbf{(6.103)}$$

Nondimensional form of first boundary condition can be written as,

$$atY = 0, \qquad \frac{\partial U_l}{\partial Y} = 0 \quad \textbf{(6.104)}$$

By substituting this boundary condition it can resulted that,

$$c_1 = 0 \quad \textbf{(6.105)}$$

For the second boundary condition properties of nonslip flow can be used,

$$aty = \frac{d}{2} = r, \qquad u_l = 0, \qquad v_l = 0 \quad \textbf{(6.106)}$$

Nondimensional form of boundary condition is,

$$atY = \frac{H}{2D_h}, \qquad U_l = 0, \qquad V_l = 0 \tag{6.107}$$

By substituting this boundary condition,

$$0 + a_1 \int_0^{\frac{H}{2D_h}} \int_0^{\frac{H}{2D_h}} (0-0)\,dY\,dY = \frac{b_1}{2}\left(\frac{H}{2D_h}\right)^2 + (0)\left(\frac{H}{2D_h}\right) + d_1 \tag{6.108}$$

$$d_1 = -\frac{b_1 H^2}{8D_h^2} \tag{6.109}$$

On the other hand, in momentum equation for particle phase also virtual mass and particle-particle forces are ignorable.

$$(F_{col})_x = 0 \tag{6.110}$$

Then momentum equation of particle phase in y-direction is,

$$\frac{\partial^2 u_p}{\partial y^2} - \frac{\rho_p v_p}{\mu_p}\frac{\partial u_p}{\partial y} = \frac{1}{\mu_p}\frac{\partial p}{\partial x} + \frac{(F_d)_x}{\varphi_p \mu_p} \tag{6.111}$$

$$\frac{\partial^2 u_p}{\partial y^2} - \frac{\rho_p v_p}{\mu_p}\frac{\partial u_p}{\partial y} = \frac{1}{\mu_p}\frac{\partial p}{\partial x} - \frac{\beta\left(u_l(y) - u_p(y)\right)}{\varphi_p \mu_p} \tag{6.112}$$

By using nondimensionalized variables,

$$\frac{\partial^2 (U_p u_{in})}{\partial (YD_h)^2} - \frac{\rho_p (V_p u_{in})}{\mu_p}\frac{\partial (U_p u_{in})}{\partial (YD_h)} = \frac{1}{\mu_p}\frac{\partial \left(P\rho_l u_{in}^2 + p_{in}\right)}{\partial (XD_h)} - \frac{\beta (U_l u_{in} - U_p u_{in})}{\varphi_p \mu_p} \tag{6.113}$$

$$\frac{\partial^2 U_p}{\partial Y^2} - \frac{\rho_p u_{in} D_h}{\mu_p} V_p \frac{\partial U_p}{\partial Y} + \frac{\beta D_h^2}{\varphi_p \mu_p}\left(U_l - U_p\right) = \frac{\rho_l u_{in} D_h}{\mu_p}\frac{\partial P}{\partial X} \tag{6.114}$$

In fully developedregion, we can neglect the y-comp of velocity of particle, by that means,

$$V_p = 0 \tag{6.115}$$

Then,

$$\frac{\partial^2 U_p}{\partial Y^2} + \frac{\beta D_h^2}{\varphi_p \mu_p}\left(U_l - U_p\right) = \frac{\rho_l u_{in} D_h}{\mu_p}\frac{\partial P}{\partial X} \quad \textbf{(6.116)}$$

By replacing group of constants with these constant we have,

$$a_2 = \frac{\beta D_h^2}{\varphi_p \mu_p} \quad \textbf{(6.117)}$$

$$b_2 = \frac{\rho_l u_{in} D_h}{\mu_p}\frac{\partial P}{\partial X} \quad \textbf{(6.118)}$$

Then,

$$\frac{\partial^2 U_p}{\partial Y^2} + a_2\left(U_l - U_p\right) = b_2 \quad \textbf{(6.119)}$$

For solving this second order equation we should twice integrate with respect to Y,

$$\frac{\partial U_p}{\partial Y} + a_2 \int_0^Y \left(U_l - U_p\right) dY = b_2 Y + c_2 \quad \textbf{(6.120)}$$

$$U_p + a_2 \int_0^Y \int_0^Y \left(U_l - U_p\right) dY\, dY = \frac{b_2}{2} Y^2 + c_2 Y + d_2 \quad \textbf{(6.121)}$$

In this equation, also two boundary conditions are needed.

For the first boundary condition, it can be assumed that at the center of pipe derivative of velocity of particle is zero.

$$at\ y = 0, \qquad \frac{\partial u_p}{\partial y} = 0 \quad \textbf{(6.122)}$$

Then,

$$at\ Y = 0, \qquad \frac{\partial U_p}{\partial Y} = 0 \quad \textbf{(6.123)}$$

By substituting this boundary condition it can resulted that,

$$c_2 = 0 \quad \textbf{(6.124)}$$

For the second boundary condition properties of nonslip flow can be used,

$$at\ y = \frac{d}{2} = r, \qquad u_p = 0, \qquad v_p = 0 \quad \textbf{(6.125)}$$

Non-dimensional form of boundary condition can be written as,

$$at\ Y = \frac{H}{2D_h}, \qquad U_p = 0, \qquad V_p = 0 \tag{6.126}$$

By substituting this boundary condition,

$$0 + a_2 \int_0^{\frac{H}{2D_h}} \int_0^{\frac{H}{2D_h}} (0-0)\,dY\,dY = \frac{b_2}{2}\left(\frac{H}{2D_h}\right)^2 + (0)\frac{H}{2D_h} + d_2 \tag{6.127}$$

$$d_2 = -\frac{b_2 H^2}{8D_h^2} \tag{6.128}$$

Momentum equation for liquid phase in y-direction by neglecting virtual mass particle-particle interaction forces is,

$$0 = -\varphi_l \frac{\partial p}{\partial y} + (F_d)_y \tag{6.129}$$

$$0 = -\varphi_l \frac{\partial p}{\partial y} + \left(-\beta\left(v_l - v_p\right)\right) \tag{6.130}$$

By using nondimensionalized variables,

$$0 = -\varphi_l \frac{\partial\left(P\rho_l u_{in}^2 + p_{in}\right)}{\partial(YD_h)} + \left(-\beta\left(V_l u_{in} - V_p u_{in}\right)\right) \tag{6.131}$$

$$0 = -\varphi_l \frac{\rho_l u_{in}^2}{D_h}\frac{\partial P}{\partial Y} - \beta u_{in}\left(V_l - V_p\right) \tag{6.132}$$

On the other hand, momentum equation for particle phase in y-direction by neglecting virtual mass and particle-particle interaction forces is,

$$0 = -\varphi_p \frac{\partial p}{\partial y} - (F_d)_y \tag{6.133}$$

$$0 = -\varphi_p \frac{\partial p}{\partial y} - \left(-\beta\left(v_l - v_p\right)\right) \tag{6.134}$$

By using nondimensionalized variables,

$$0 = -\varphi_p \frac{\partial\left(P\rho_l u_{in}^2 + p_{in}\right)}{\partial(YD_h)} - \left(-\beta\left(V_l u_{in} - V_p u_{in}\right)\right) \tag{6.135}$$

$$0 = -\varphi_p \frac{\rho_l u_{in}^2}{D_h} \frac{\partial P}{\partial Y} + \beta u_{in}\left(V_l - V_p\right) \tag{6.136}$$

By summing up these momentum equations in y-direction,

$$0 = -\varphi_l \frac{\rho_l u_{in}^2}{D_h} \frac{\partial P}{\partial Y} - \beta u_{in}\left(V_l - V_p\right) - \varphi_p \frac{\rho_l u_{in}^2}{D_h} \frac{\partial P}{\partial Y} + \beta u_{in}\left(V_l - V_p\right) \tag{6.137}$$

$$\frac{\partial P}{\partial Y} = 0 \tag{6.138}$$

It can be resulted that pressure is constant in y-direction,

$$P = constant \tag{6.139}$$

By using the same non-dimensional variables energy equation for liquid phase is,

$$\frac{\partial^2(\theta_l(T_w - T_{in}) + T_{in})}{\partial(YD_h)^2} + \frac{\partial^2(\theta_l(T_w - T_{in}) + T_{in})}{\partial(XD_h)^2} - \frac{\rho_l c_{p_l}}{k_{eff,l}}(U_l u_{in}) \frac{\partial(\theta_l(T_w - T_{in}) + T_{in})}{\partial(XD_h)} = \frac{h_v}{\varphi_l k_{eff,l}}\Big((\theta_l(T_w - T_{in}) + T_{in}) - \left(\theta_p(T_w - T_{in}) + T_{in}\right)\Big) \tag{6.140}$$

$$\frac{\partial^2\theta_l}{\partial Y^2} + \frac{\partial^2\theta_l}{\partial X^2} - \frac{D_h \rho_l c_{p_l} u_{in}}{k_{eff,l}} U_l \frac{\partial\theta_l}{\partial X} - \frac{D_h^2 h_v}{\varphi_l k_{eff,l}}\left(\theta_l - \theta_p\right) = 0 \tag{6.141}$$

By replacing these constants instead of group of constants,

$$s_1 = -\frac{D_h \rho_l c_{p_l} u_{in}}{k_{eff,l}} \tag{6.142}$$

$$t_1 = -\frac{D_h^2 h_v}{\varphi_l k_{eff,l}} \tag{6.143}$$

Then,

$$\frac{\partial^2\theta_l}{\partial Y^2} + \frac{\partial^2\theta_l}{\partial X^2} + s_1 U_l \frac{\partial\theta_l}{\partial X} + t_1\left(\theta_l - \theta_p\right) = 0 \tag{6.144}$$

On the other hand, energy equation for particle phase is,

$$\frac{\partial^2\theta_p}{\partial Y^2} + \frac{\partial^2\theta_p}{\partial X^2} - \frac{D_h \rho_p c_{p_p} u_{in}}{k_{eff,p}} U_p \frac{\partial\theta_p}{\partial X} + \frac{D_h^2 h_v}{\varphi_p k_{eff,p}}\left(\theta_l - \theta_p\right) = 0 \tag{6.145}$$

By replacing these constant instead of group of constants,

$$s_2 = -\frac{D_h \rho_p {c_p}_p u_{in}}{k_{eff,p}} \tag{6.146}$$

$$t_2 = \frac{D_h^2 h_v}{\varphi_p k_{eff,p}} \tag{6.147}$$

Then,

$$\frac{\partial^2 \theta_p}{\partial Y^2} + \frac{\partial^2 \theta_p}{\partial X^2} + s_2 U_p \frac{\partial \theta_p}{\partial X} + t_2(\theta_l - \theta_p) = 0 \tag{6.148}$$

According to two energy equations, liquid and particle phases, two second order coupled equations should be solved. In this thesis, these coupled equations solved numerically in Mathematica 7.0 program.

6.6 Energy Equations for Long Parallel Plates

By using the same non-dimensional equations, energy equation for liquid phase for long parallel plates can be written as,

$$\frac{\partial}{\partial x}\left(\varphi_l \rho_l u_l {c_p}_l T_l\right) + \frac{\partial}{\partial y}\left(\varphi_l \rho_l v_l {c_p}_l T_l\right) = \frac{\partial}{\partial x}\left(\varphi_l k_{eff,l} \frac{\partial T_l}{\partial x}\right) + \frac{\partial}{\partial y}\left(\varphi_l k_{eff,l} \frac{\partial T_l}{\partial y}\right) - h_v(T_l - T_p) \tag{6.149}$$

In this equation in addition to constant quantity of fraction ratio and density, heat capacity also is constant with x and y for liquid and particle, then,

$$\varphi_l \rho_l {c_p}_l \left(\frac{\partial}{\partial x}(u_l T_l) + \frac{\partial}{\partial y}(v_l T_l)\right) = \varphi_l k_{eff,l} \left(\frac{\partial}{\partial x}\left(\frac{\partial T_l}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial T_l}{\partial y}\right)\right) - h_v(T_l - T_p) \tag{6.150}$$

In this case, because of long length of plates fluid is fully developed thermally and hydraulically.

By using these simplifications,

$$\frac{\partial}{\partial x}(u_l T_l) = T_l \frac{\partial}{\partial x}(u_l) + u_l \frac{\partial}{\partial x}(T_l) = 0 \tag{6.151}$$

And in fully-developed region we can neglect the y-comp of velocity of liquid, by that means,

$$v_l = 0 \tag{6.152}$$

Then,

$$\frac{\partial}{\partial y}(v_l T_l) = 0 \tag{6.153}$$

By substituting above-mentioned equations in energy equation the last form of energy equation for liquid can be written as,

$$\frac{\partial^2 T_l}{\partial y^2} = \frac{h_v(T_l - T_p)}{\varphi_l k_{eff,l}} \tag{6.154}$$

By using nondimensionalized variables,

$$\frac{\partial^2(\theta_l(T_w - T_{in}) + T_{in})}{\partial(YD_h)^2} = \frac{h_v\left((\theta_l(T_w - T_{in}) + T_{in}) - (\theta_p(T_w - T_{in}) + T_{in})\right)}{\varphi_l k_{eff,l}} \tag{6.155}$$

$$\frac{\partial^2 \theta_l}{\partial Y^2} = \frac{h_v D_h^2(\theta_l - \theta_p)}{\varphi_l k_{eff,l}} \tag{6.156}$$

For solving this equation,

$$\frac{\partial^2 \theta_l}{\partial Y^2} = \frac{h_v D_h^2(\theta_l - \theta_p)}{\varphi_l k_{eff,l}} \tag{6.157}$$

$$\frac{\partial \theta_l}{\partial Y} = \frac{h_v D_h^2}{\varphi_l k_{eff,l}} \int_0^Y (\theta_l - \theta_p) dY + f_1 \tag{6.158}$$

$$\theta_l = \frac{h_v D_h^2}{\varphi_l k_{eff,l}} \int_0^Y \int_0^Y (\theta_l - \theta_p) dY\, dY + f_1 Y + g_1 \tag{6.159}$$

$$\theta_l - \frac{h_v D_h^2}{\varphi_l k_{eff,l}} \int_0^Y \int_0^Y (\theta_l - \theta_p) dY\, dY = f_1 Y + g_1 \tag{6.160}$$

By replacing this constant instead of group of constants,

$$s_1 = -\frac{h_v D_h^2}{\varphi_l k_{eff,l}} \tag{6.161}$$

Then,

$$\theta_l + s_1 \int_0^Y \int_0^Y (\theta_l - \theta_p) dY\, dY = f_1 Y + g_1 \quad \textbf{(6.162)}$$

For finding integral constants,two boundary conditions are needed.

At the center of pipe derivative of temperature profile is zero,

$$at\ y = 0, \qquad \frac{\partial T_l}{\partial y} = 0 \quad \textbf{(6.163)}$$

Then,

$$at\ Y = 0, \qquad \frac{\partial \theta_l}{\partial Y} = 0 \quad \textbf{(6.164)}$$

By using this boundary condition,

$$0 = -s_1 \int_0^0 (\theta_l - \theta_p) dY + f_1 \quad \textbf{(6.165)}$$

$$f_1 = 0 \quad \textbf{(6.166)}$$

In this thesis equations were solved for one of thermal boundary conditions, constant temperature on the wall, by that I means,

$$at\ y = \frac{d}{2} = r, \qquad T_l = T_w \quad \textbf{(6.167)}$$

This boundary condition in nondimensional form can be written as,

$$at\ Y = \frac{H}{2D_h}, \qquad \theta_l = 1 \quad \textbf{(6.168)}$$

And by using this boundary condition,

$$1 + s_1 \int_0^{\frac{H}{2D_h}} \int_0^{\frac{H}{2D_h}} (1 - 1) dY\, dY = (0) \frac{H}{2D_h} + g_1 \quad \textbf{(6.169)}$$

$$g_1 = 1 \quad \textbf{(6.170)}$$

On the other hand, energy equation for particle phase for long parallel plates is,

$$\frac{\partial}{\partial x}\left(\varphi_p \rho_p u_p c_{p_p} T_p\right) + \frac{\partial}{\partial y}\left(\varphi_p \rho_p v_p c_{p_p} T_p\right) = \frac{\partial}{\partial x}\left(\varphi_p k_{eff,p} \frac{\partial T_p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\varphi_p k_{eff,p} \frac{\partial T_p}{\partial y}\right) + h_v\left(T_l - T_p\right) \quad \textbf{(6.171)}$$

In this case, because of long length of plates fluid is fully developed thermally and hydraulically.

By using these simplifications,

$$\frac{\partial}{\partial x}\left(u_p T_p\right) = T_p \frac{\partial}{\partial x}\left(u_p\right) + u_p \frac{\partial}{\partial x}\left(T_p\right) = 0 \quad \textbf{(6.172)}$$

And in fully-developed region we can neglect the y-component of velocity of liquid, by that means,

$$v_p = 0 \quad \textbf{(6.173)}$$

So,

$$\frac{\partial}{\partial y}\left(v_p T_p\right) = 0 \quad \textbf{(6.174)}$$

By substituting above-mentioned equations in energy equation the last form of energy equation for liquid can be written as,

$$\frac{\partial^2 T_p}{\partial y^2} = \frac{-h_v\left(T_l - T_p\right)}{\varphi_p k_{eff,p}} \quad \textbf{(6.175)}$$

By using nondimensionalized variables,

$$\frac{\partial^2\left(\theta_p\left(T_w - T_{in}\right) + T_{in}\right)}{\partial\left(Y D_h\right)^2} = \frac{-h_v\left(\left(\theta_l\left(T_w - T_{in}\right) + T_{in}\right) - \left(\theta_p\left(T_w - T_{in}\right) + T_{in}\right)\right)}{\varphi_p k_{eff,p}} \quad \textbf{(6.176)}$$

$$\frac{1}{D_h^2}\frac{\partial^2 \theta_p}{\partial Y^2} = \frac{-h_v\left(\theta_l - \theta_p\right)}{\varphi_p k_{eff,p}} \quad \textbf{(6.177)}$$

For solving this equation, twice integrating with respect to Y is needed,

$$\frac{\partial^2 \theta_p}{\partial Y^2} = \frac{-h_v D_h^2 (\theta_l - \theta_p)}{\varphi_p k_{eff,p}} \quad \textbf{(6.178)}$$

$$\frac{\partial \theta_p}{\partial Y} = \frac{-h_v D_h^2}{\varphi_p k_{eff,p}} \int_0^Y (\theta_l - \theta_p) dY + f_2 \quad \textbf{(6.179)}$$

$$\theta_p = \frac{-h_v D_h^2}{\varphi_p k_{eff,p}} \int_0^Y \int_0^Y (\theta_l - \theta_p) dY\, dY + f_2 Y + g_2 \quad \textbf{(6.180)}$$

$$\theta_p + \frac{h_v D_h^2}{\varphi_p k_{eff,p}} \int_0^Y \int_0^Y (\theta_l - \theta_p) dY\, dY = f_2 Y + g_2 \quad \textbf{(6.181)}$$

By replacing this constant instead of group of constants,

$$s_2 = \frac{h_v D_h^2}{\varphi_p k_{eff,p}} \quad \textbf{(6.182)}$$

We have,

$$\theta_p + s_2 \int_0^Y \int_0^Y (\theta_l - \theta_p) dY\, dY = f_2 Y + g_2 \quad \textbf{(6.183)}$$

For finding integral constants,two boundary conditions are needed.

At the center of pipe derivative of temperature profile is zero,

$$at\ y = 0, \qquad \frac{\partial T_p}{\partial y} = 0 \quad \textbf{(6.184)}$$

By writing in nondimensional form we have,

$$at\ Y = 0, \qquad \frac{\partial \theta_p}{\partial Y} = 0 \quad \textbf{(6.185)}$$

By using this boundary condition,

$$0 = -s_2 \int_0^0 (\theta_l - \theta_p) dY + f_2 \quad \textbf{(6.186)}$$

$$f_2 = 0 \quad \textbf{(6.187)}$$

In this thesis this equation was solved for constant wall temperature boundary condition only,

$$at\ y = \frac{d}{2} = r, \qquad T_p = T_w \tag{6.188}$$

Nondimensional form of boundary condition can be written as,

$$at\ Y = \frac{H}{2D_h}, \qquad \theta_p = 1 \tag{6.189}$$

For finding integral constant,above-mentioned boundary condition should be used,

$$1 + s_2 \int_0^{\frac{H}{2D_h}} \int_0^{\frac{H}{2D_h}} (1-1)dY\,dY = (0)\frac{H}{2D_h} + g_2 \tag{6.190}$$

$$g_2 = 1 \tag{6.191}$$

Then energy equations for liquid and particle phase for long parallel plates are,

$$\theta_l + s_1 \int_0^Y \int_0^Y (\theta_l - \theta_p)dY\,dY = 1 \tag{6.192}$$

$$\theta_p + s_2 \int_0^Y \int_0^Y (\theta_l - \theta_p)dY\,dY = 1 \tag{6.193}$$

These energy equations solved together numerically in Mathematica 7.0 program for constant wall temperature boundary condition. By looking on the results of these coupled equations it can be resulted that for a nanofluid between long parallel plates, temperature in long distance from entrance, where flow is fully developed thermally and hydraulically, is equal to temperature of wall.

7. CONCLUSIONS AND RECOMMENDATIONS

7.1 Pure Water and Single-phase Models

From the previous sections, it can be concluded that, in constant wall heat flux boundary condition, Nusselt number is constant along the pipe for pure water flow in circular pipe in both velocity profiles, i.e. plug flow and parabolic velocity profile. Nusselt number for plug flow is higher than parabolic velocity profile, by that it means heat transfer for plug flow is better than parabolic velocity profile. The same general results can be concluded in constant wall temperature boundary condition. Nusselt numbers of pure water flow in circular pipe in constant wall tepmerature boundary condition are constant along the pipe but are different from each other in different types of velocity profiles.

Thermal properties of fluid will be changed by adding nanoparticles to base fluid and that is why heat transfer coefficient of nanofluid should be used for calculating Nusselt number instead of heat transfer coefficient of pure water. In general, heat transfer coefficient of nanofluid will be increased but amount of increase depend on methods that are used.

In single-phase model according to analytical calculations, there is no difference between Nusselt numbers of this model and values of Nusselt numbers of pure water. It can be resulted that using single-phase model for mixture of base fluid and nanoparticles is not an accurate model in calculating Nusselt numbers. However, heat transfer coefficients of these two models are different from each other because of different values of thermal conductivities of nanofluid and pure water.

Here are four diagrams for comparing heat transfer coefficients of mixture of Alumina in water for different thermal boundary conditions, i.e. constant heat flux and constant wall temperature thermal boundary conditions, and different velocity profiles, i.e. coantant and parabolic velocity profiles, for flow in circular pipe with different radiuses.

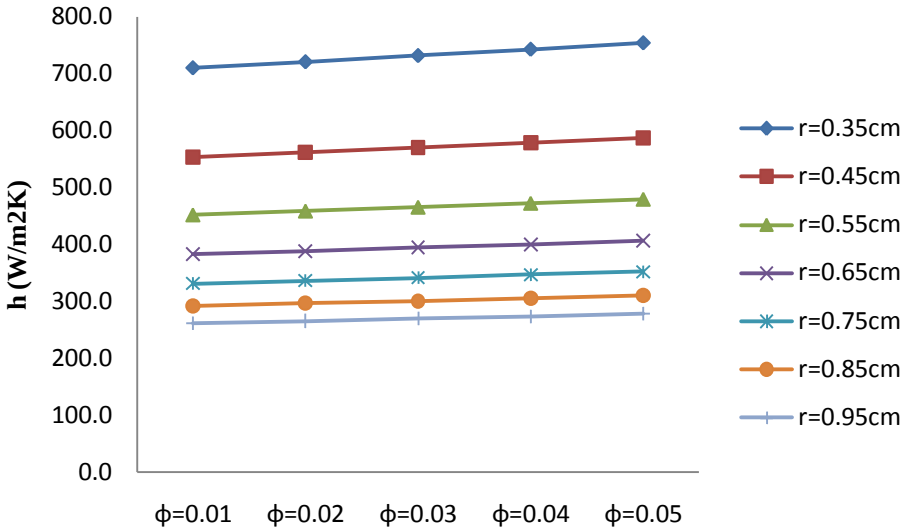

Figure 7.1 : Comparing heat transfer coefficients of Alumina in constant heat flux thermal boundary condition for plug flow.

In Fig. 7.1 it can be seen that in constant heat flux thermal boundary condition for plug flowby increasing nanoparticle volume concentration, heat transfer coefficient is increasing slightly in all of the pipes radiuses. On the other hand, by decreasing radius of pipe, heat transfer coefficientshave obvious increase in all of the nanoparticle volume concentrations.

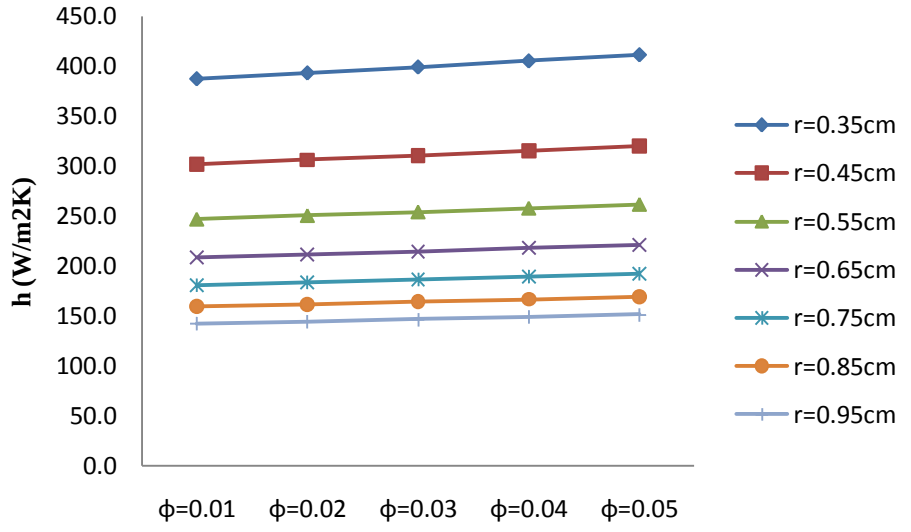

Figure 7.2 : Comparing heat transfer coefficients of Alumina in constant heat flux thermal boundary condition for parabolic velocity profile.

The same style of increasing in heat transfer coefficients by increasing nanoparticle concentrations exist in Fig. 7.2 for constant heat flux thermal boundary condition for

parabolic velocity profile as well. In an assumed nanoparticle volume concentration, there is a sharp increase of heat transfer coefficient by decreasing radius of pipe.

In constant wall temperature thermal boundary condition, as can be seen in Fig. 7.3**,** in plug flow there is a slight increase of heat transfer coefficient in constant radius by increasing nanoparticle volume concentration.According to figure, sharp increase of heat transfer by decreasing radius is obvious.

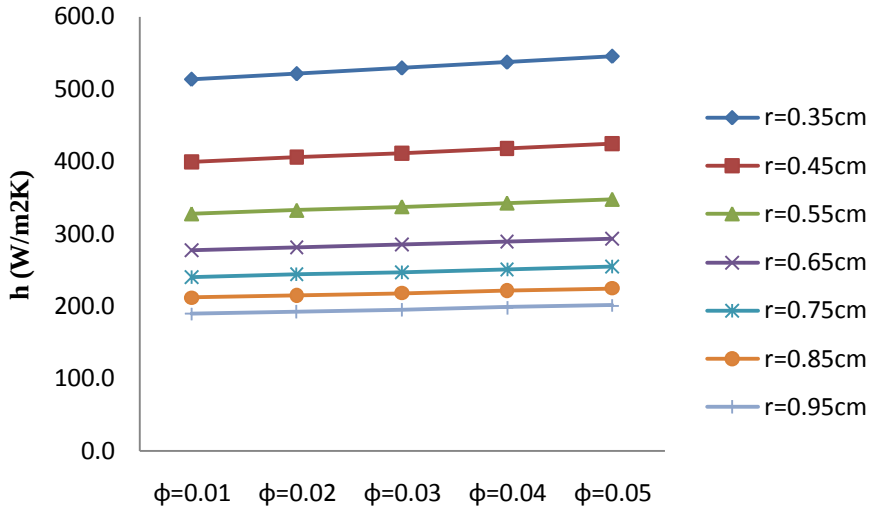

Figure 7.3 : Comparing heat transfer coefficients of Alumina in constant wall temperature thermal boundary condition for plug flow.

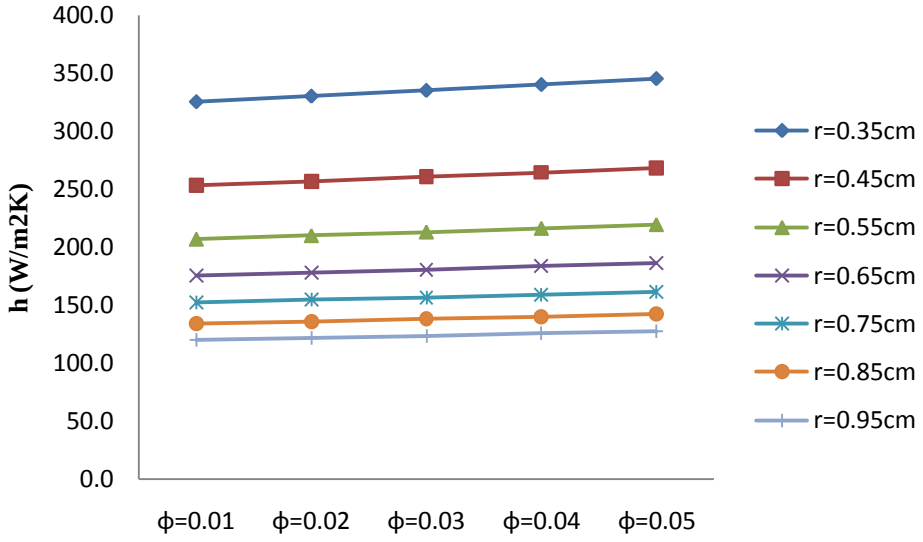

Figure 7.4 : Comparing heat transfer coefficients of Alumina in constant wall temperature thermal boundary condition for parabolic velocity profile.

For the last boundary condition, it means constant wall temperature thermal boundary condition and parabolic velocity profile, just like previous one heat transfer is increasing by increasing of nanoparticle volume concentration and decreasing of radius, as can be seen in Fig. 7.4**.**

It can be concluded that in the entire mentioned boundary conditions heat transfer coefficient is increasing approximately 6% by increasing nanoparticle volume concentration in constant radius.On the other hand, by decreasing radius of the pipe from 0.95cm to 0.35cm heat transfer coefficient is increasing three times in constant nanoparticle volume concentration.

7.2 Two-phase Model

In Eulerian-Eulerian two-phase model according to experimental and theoretical investigations, a few important factors affect the heat transfer coefficient, Nusselt number and heat transfer enhancement by using nanoparticles mixtures. In this section, a few of them is discussed.

7.2.1 Drag, particle-particle and virtual mass forces

These forces, drag, particle-particle and virtual mass forces, exist in general form of momentum equations but according to experiments of Kalteh et al. [40] two of them, i.e. particle-particle and virtual mass forces, can be neglected. By increasing volume concentration of nanoparticles, drag force affects the Nusselt number by increasing its value and that is why drag force cannot be neglected in momentum equations as mentioned in Table 7.1 which is for different volume concentrations.

Table 7.1 : Comparison of Nusselt numbers for different fraction ratios when drag, particle-particle and virtual mass forces neglected.

Volume concentration (%)	Considering all terms	Neglecting the drag term	Neglecting the particle–particle interaction term	Neglecting the virtual mass term
1	12.343	12.385	12.343	12.343
2	14.068	14.151	14.068	14.068
3	15.507	15.637	15.507	15.507
4	16.818	16.998	16.818	16.818
5	18.051	18.292	18.051	18.051

7.2.2 Two-phase model comparing with single-phase homogenous model

It can be seen in Fig. 7.5 that percentage of average Nusselt number enhancement of two-phase model [40] is higher than single-phase homogenous model [42]. It can be seen that for Re=100 and nanoparticle diameter of 100nm, enhancement of Nusselt

number of homogenous model increases up to 15% when nanoparticle volume concentration is 4%. On the other hand, this amount for two-phase model is nearly twice as homogenous model for the same properties for fluid and nanoparticles. Enhancement of Nusselt number in homogenous model is linear from 1 to 5% but changing of enhancement is not linear for two-phase model.

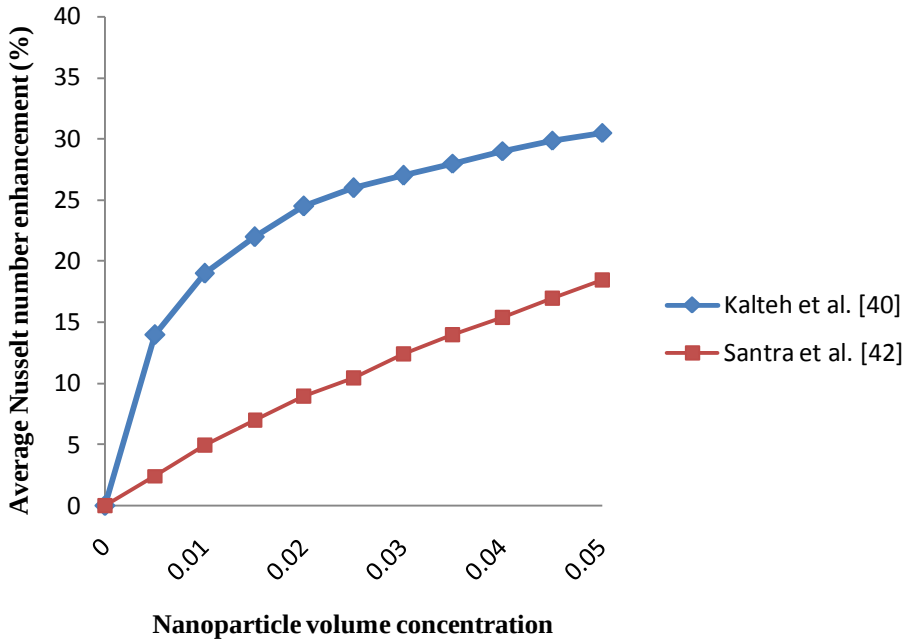

Figure 7.5 : Comparing average Nusselt number enhancement in two-phase and homogenous models.

In Table 7.2 percentage of enhancement of Nusselt number of two-phase and homogenous single-phase with respect to pure water are given and it can be seen that in different Reynolds numbers enhancement of heat transfer for two-phase model is morethan homogenous one. It can be seen that in the entire Reynolds numbers enhancement of Nusselt number of two-phase model is at least twice as homogenous single-phase model. In lower nanoparticle volume concentrations enhancement increases even up to approximately seven-folded.

Table 7.2 : Comparing enhancement of Nusselt number for two-phase and homogenous models in different Reynolds numbers and volume concentrations.

φ_p (%)	Re_H = 500		Re_H = 1000		Re_H = 1500	
	Kalteh et al. [40]	Santra et al. [42]	Kalteh et al. [40]	Santra et al. [42]	Kalteh et al. [40]	Santra et al. [42]
0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.5	20.75917	3.21628	20.90864	3.26732	20.94575	3.30307
1.0	29.31589	6.40564	29.5805	6.51014	29.62227	6.5825
1.5	35.96397	9.56952	36.36213	9.72955	36.42431	9.83894
2.0	41.64701	12.70968	42.21522	12.9274	42.30332	13.07415
2.5	46.75102	15.82767	47.52114	16.10541	47.62851	16.28976
3.0	51.49046	18.92487	52.42912	19.26517	52.58449	19.48727
3.5	55.84388	22.00248	57.05521	22.40813	57.25646	22.66808
4.0	60.02573	25.06157	61.49892	25.53565	61.70122	25.83347
4.5	64.07892	28.10305	65.7105	28.64899	66.01818	28.98467
5.0	67.76753	31.12774	69.88891	31.74931	70.19313	32.12279

7.2.3 Particle volume concentration

In Fig. 7.6 obvious increase of average Nusselt number with increasing nanoparticle volume concentration and increasing Reynolds number is shown. The amount of increase of Nusselt number with increasing of Reynolds number is the same for all volume concentrations. On the other hand, there is a big difference between average Nusselt number of pure water and other mixed nanofluids.

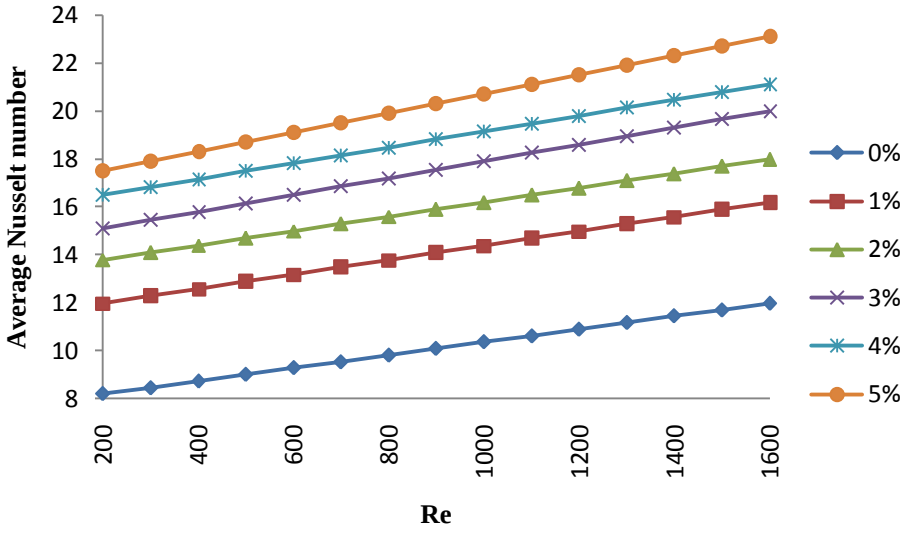

Figure 7.6 : Comparing average Nusselt numbers for different nanoparticle volume concentrations in different Reynolds numbers.

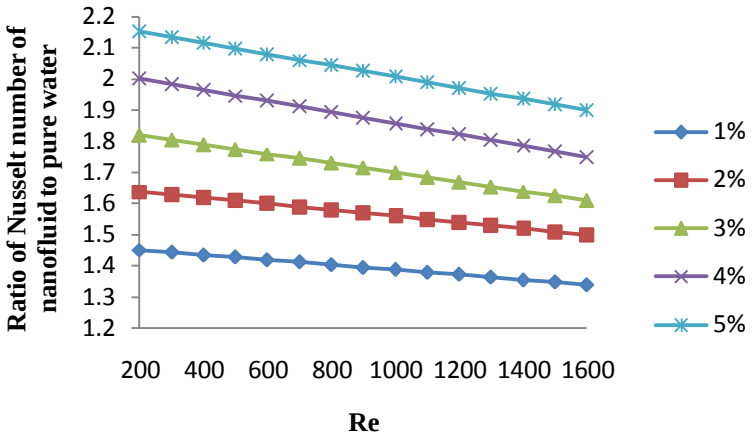

Figure 7.7 : Ratio of average Nusselt number of nanofluid to pure water for different Reynolds numbers and nanoparticle volume concentrations.

By using the same data, Fig. 7.7 shows the ratio of average Nusselt number of nanofluid to pure water for different volume concentrations and Reynolds numbers. It can be seen that the ratio of heat transfer coefficient of nanofluid to base fluid is increasing by increasing of nanoparticle volume concentration.

7.2.4 Nanoparticle diameter

In Fig. 7.8 effect of nanoparticle diameter on average Nusselt number enhancement is shown. It can be seen that by decreasing diameter of nanoparticles, average Nusselt number is increasing. Increasing in high nanoparticle volume concentrations, for example 5% even is more than lower nanoparticle volume concentrations.

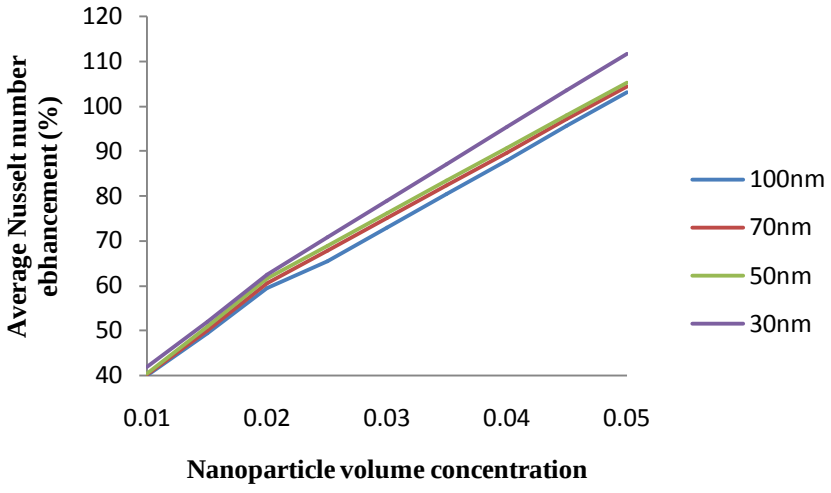

Figure 7.8 : Effect of nanoparticles diameter on average Nusselt number enhancement of nanofluid.

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