**İSTANBUL TECHNICAL UNIVERSITY ★ INSTITUTE OF SCIENCE AND TECHNOLOGY**

# EFFECTS OF VISCOELASTIC DAMPERS ON TWO DIMENSIONAL FRAMES

**M.Sc. Thesis by**

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**Department : Civil Engineering**

**Programme: Structural Engineering**

**MAY 2002**

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**MAY 2002**

**<u>İSTANBUL TEKNİK ÜNİVERSİTESİ ★ FEN BİLİMLERİ ENSTİTÜSÜ</u>**

# VİSKOELASTİK SÖNÜMLEYİCİLERİN İKİ BOYUTLU ÇERÇEVELER ÜZERİNDEKİ ETKİLERİ

**YÜKSEK LİSANS TEZİ**

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## PREFACE

In this study, the effects of viscoelastic dampers on two dimensional frames are dealt by using Wilson θ step by step integration method and a comparison of the response is made between traditional structures and damper added structures.

I would sincerely like to thank Prof. Dr. Hasan Boduroğlu for his guidance during my study.

13.05.2002 M.Ümit ÖZKAN

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## ABBREVIATIONS

| | |
|---|---|
| **VE** | : Viscoelastic |
| **SDOF** | : Single Degree of Freedom |
| **MDOF** | : Multi Degree of Freedom |
| **Acc**. | : Acceleration |
| **Vel.** | : Velocity |
| **Disp.** | : Displacement |

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| | |
|---|---|
| **G’** | : Elastic shear stiffness |
| **G’’** | : Viscous stiffness |
| $\beta$ | : Damping ratio |
| $\gamma$ | : Shear strain rate |
| $\omega$ | : Frequency |
| $k_d$ | : Damper stiffness |
| $k_s$ | : System stiffness |
| **A** | : Area |
| **h** | : Height |
| $c_d$ | : Damper damping coefficient |
| $c_s$ | : System damping coefficient |
| $\eta$ | : Loss factor |
| **K** | : Stiffness matrix |
| **C** | : Damping matrix |
| **M** | : Mass matrix |
| $\alpha$ | : Attachment coefficient |
| **p** | : Force |
| **u** | : Displacement |
| **u** | : Velocity |
| **ü** | : Acceleration |
| $\Delta t$ | : Time step |
| $\delta$ | : Increase in time |
| **T** | : Period |
| $\theta$ | : Wilson’s parameter |
| **R** | : Load matrix |
| **A** | : Wilson’s integration constants |

# ÖZET

Günümüzde, deprem yönetmeliklerindeki minimum yatay kuvvet şartlarına uygun olarak tasarlanan yapıların beklendiği şekilde davranmadığı görülmüştür. Yakın geçmişteki depremler göstermiştir ki, en yeni yönetmeliklerle tasarlanan ve inşaa edilen yapılar da bile hasarların maddi büyüklüğü ve bu hasarların onarımı için gereken süre tahmin edilenden fazladır. Birçok araştırmaya göre yalnızca yatay tasarım kuvvetlerini arttırmak bir çözüm olmamaktadır. Yapıların sismik kuvvetlere karşı dayanımını arttırmak için birçok yeni teknikler uygulanmaktadır. Bu çalışma ilave enerji yutucu sistemlerden olan katı viskoelastik sönümleyiciler üzerine odaklanmaktadır.

Enerjinin yutulması, dinamik yükler altında mekanik ve yapısal sistemlerin aşırı vibrasyonunun etkili şekilde kontrol edilmesini sağlayan bir sistem olarak tanınmıştır. Deprem yönetmelikleri sabit bir yatay yük katsayısı kabulü üstüne kurulmuş olup bugün ise spektral yaklaşımlar kullanılmaktadır. Bu tasarım spektrumu yaklaşımında yapının belli bir tahmini yüzdede içsel sönüme sahip olduğu varsayılmaktadır. Yapının tasarım depreminde inelastik davranış göstereceği varsayılır ve yatay deprem kuvvetleri azaltılır. Elastik davranış kabulü ve yüklerin azaltılması ile birlikte yapılar, yapısal elemanlarının kapasiteleri kadar sönüme sahip olurlar. Bu durumda elemanlar büyük deplasmanları yapabilecek şekilde boyutlandırılmalıdır. Bu şekilde oluşan hasar genellikle tamir edilebilir düzeyde olmalıdır. Hasarın boyutu, tamirat için gereken finans ve zamanı etkiler.[1]

Bu çalışma deprem yüklerine karşı tasarlanan yeni yapıların ve mevcut yapıların sismik performanslarının enerji yutucu cihazlarla iyileştirilmesini incelemektedir. Görülmüştür ki yapılar ilave edilen sönümleyiciler sayesinde sönüm yüzdeleri arttığı için daha az yanal deplasman yapmakta böylece hasara yol açan yerdeğiştirmeler sınırlandırılmış olmaktadır.

# SUMMARY

Buildings designed in accordance with minimum code lateral force requirements do not necessarily produce buildings that behave as expected. Recent earthquakes have shown that buildings designed and constructed in accordance with the newest codes post of damage repair and the time needed to implement these repairs are greater than anticipated. Many researches have proved that increasing the design force levels alone doesn't improve aspects of the performance. New techniques have been proposed for use individually or in combination to improve earthquake building performace and are at various stages of development and acceptance. This thesis is focused on supplemental energy dissipation system approach of viscoelastic solid dampers.

Energy dissipation has long been recognised as an effective means for controlling excessive vibration of mechanical and structural systems under dynamic loads. Earthquake resistant design requirements in building codes have evolved from a constant lateral force coefficient to current code requirements that are based on design spectral approaches. These design spectra assume that the elastic structural system's inherent damping is a guessed acceptable percentage of critical damping. Assuming that the building will go into inelastic response during the design level earthquake allows further reduction of the seismic design lateral forces. The reduction from elastic responce forces is attributed to the limits of elastic member capacities and to the increase in effective damping caused by the nonlinear hysteretic energy dissipation. In this case, components and systems must be constructed so they can sustain their load capacities while undergoing large deformations. This action results in damage that usually must be repaired. The extent of the damage can influence the time and cost required to make the repairs. [1]

The study focuses on the information of dissipation devices in designing new earthquake resistant buildings and also upgrading the seismic performance of existing buildings. It is seen that buildings denote less lateral drift when their damping ratios are increased by adding dampers to the structure which limits the damage dependent to the displacements.

# SUMMARY

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# 1 INTRODUCTION

A number of imaginative approaches for improving eartquake responce performance and damage control of buildings, bridges and other structures have been developed. One of these approaches is passive systems such as supplemental energy dissipation devices. Viscoelastic energy dissipation devices will be focused on and basic working principles, application of these devices will be dealt. Some applications will be shown and the difference between the systems without dampers case will be compared with systems with dissipated energy cases. Approaches will be made by using Wilson-θ method of step by step integration.

## 1.1. Energy Dissipation Devices: Viscoelastic Devices.

VE devices are solid or fluid devices, which dissipate energy through deformation of VE polymers , deformation of viscous fluids, or fluid orificing. Their energy dissipation depends on both relative displacements and relative velocities within the device. In this study only the VE solid devices will be focused.

The use of VE solid devices in civil engineering structures appears to have begun in 1969, when 10,000 VE dampers were installed in each of the twin towers of New York's World Trade Center to help resist wind loads. More recently, further analytical and experimental studies on the dynamic response of VE energy dissipators and seismic response of viscoelastically damped structures have been carried out. VE materials used in civil engineering structures are typically copolymers or glassy substances. A typical VE solid device, consists of VE layers bonded with steel plates is shown in figure 1.1. VE solid devices dissipate energy through shear deformation of the VE layers, which also depends on the vibrational frequency, strain, and ambient temperature.[1]

When incorporated to a structure, dampers dissipate earthquake induced energy and may add stiffness and strength to the structure.

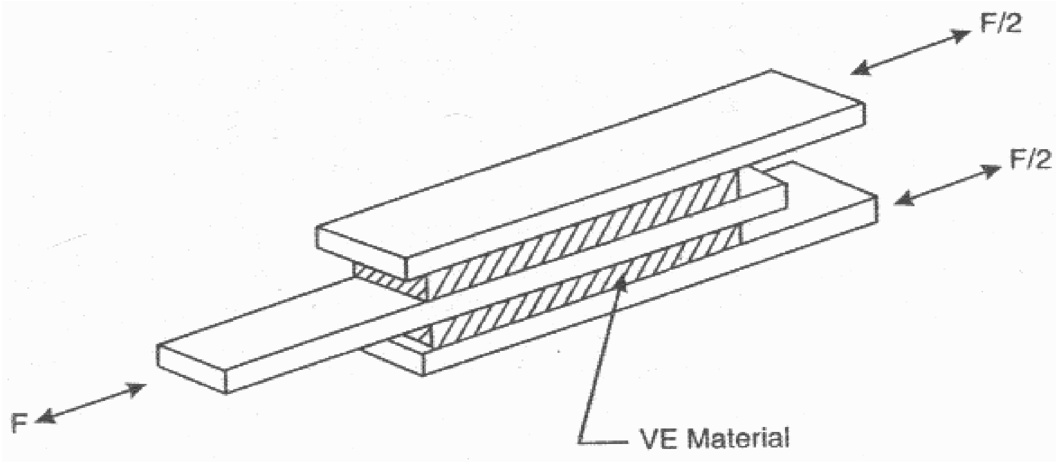


Figure 1.1: Viscoelastic Solid Damper.

## 1.2. Damping Changes Under Effect of Ground Motions.

The presence of some damping in conventional buildings has been recognized and accepted. Although the nature of energy dissipation inherent in buildings has not been explictly identified, inherent equivalent viscous damping of about 2-5% of critical damping has become accepted in practice for linear response analyses of typical buildings. In fact, most of the design spectra developed assume 5% of critical viscous damping in the system. However the addition of dampers can substantially increase damping.

As an example a single degree of freedom system that has less than critical viscous damping initially when the system is at rest is considered. It’s subjected to a ground motion impulse that creates an initial velocity, much like an earthquake ground acceleration pulse. The resulting free vibration of the structure is shown in Figure 2 for 2%, 5%, 10% and 50% of critical viscous damping. Two primary effects of increasing the damping are seen. The initial amplitude of the structural response decreases with increases in damping. The number of cycles to reduce the initial amplitude to half that amplitude are also decreased.

When the earthquake ground motion is thought of as a series of individual pulses, the response of a building to the series of pulses would be just a summation of the motions from each pulse, as shown in figure 1.2, shifted in time to the time when the pulse occured. It's easily concluded that the response of more highly damped systems will be smaller that of the lightly damped systems, because the initial amplitudes are smaller, and the responses decay more quickly.

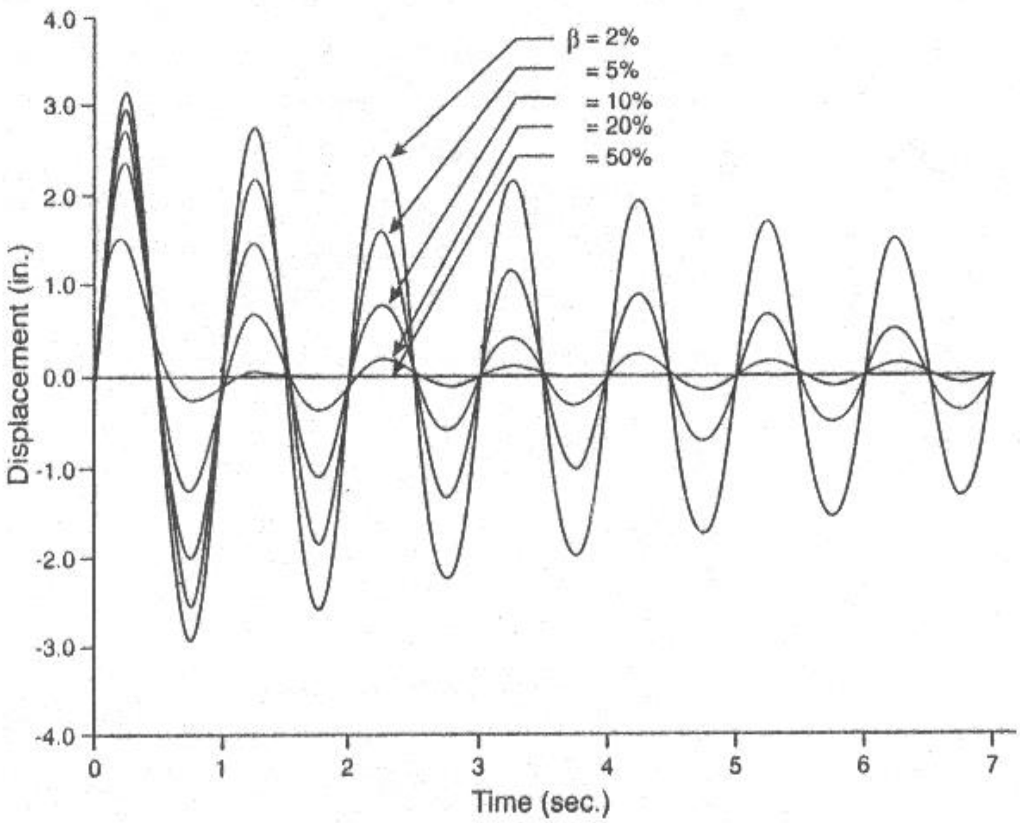


Figure 1.2: Damping Effects on SDOF system.

## 1.3. Significant Increases in Damping.

Damper added structures exhibit significantly higher modal damping ratios than those associated with traditional structures. This is particularly true in higher modes, where damping ratios can reach values close to or even exceeding their critical values. Hence, the damping term in the equation of motion of a damper added structure becomes important in determining the structure's modal properties.

Moreover, the effect of adding dampers to a structure is not only a significant increase in but also a redistribution of modal dampings. Some modal response components that have minor contributions to the total response of the structure may become important after dampers are added.

### 1.4. Nonproportional Damping.

For analytical convenience, proportional damping is usually assumed in the analysis of a traditional structure. This simplifies the structural analysis by using modal superposition. The consequence of adding dampers to a traditional structure depends on the locations and characteristics of the selected devices. If the added damping is proportional-that is, if the undamped mode shapes of the structure with added stiffnesses that are due to the damping devices diagonalize the structure's damping matrix- then the structure has proportional damping. In this case, the traditional modal analysis approaches work well. That is, the normal modes of vibration of the damped system are identical to those for the undamped structure, making the calculation of modal properties a routine procedure.

The proportional damping assumption, however, is generally not valid for damper added structures, because it may not be practical to try to match added damper characteristics to the variations in the structural stiffness and mass of the building. In fact in some cases it may be desirable to add dampers only at spesific floors in the building. Thus, the distribution of the damping properties within the structure will probably not be proportional. In this situation, modifications of the traditional model analysis must be considered.[1]

# 2 CHARACTERISATION AND APPLICATION OF VISCOELASTIC SOLID DAMPERS

Mechanical properties and mathematical modelling of dampers and some general discussion on their applicability is considered. The concept of replacing complicated and often nonlinear behaviour of dampers by equivalent linear stiffness and viscous characteristics has enormous benefits for the preliminary analysis and design of damper added structures. This linear approximation is emphasized.

## 2.1 Viscoelastic Solid Devices.

VE materials used in structural applications are typically copolymers or elastomeric substances that dissipate energy when subjected to shear deformation. A typical VE device consists of one or more layers of VE material bonded to steel plates. The device is mounted in the structure so that relative floor displacement causes shear deformation of the device. The mechanical properties of VE materials depends on temperature and frequency. The expected frequencies of the device motion can be approximated for sufficient accurancy to establish the proper frequency property for a spesific application. The device temperature will increase from the initial ambient temperature of the device as the dissipated energy is converted to heat. This range of expected temperature for which the device operates must be included in the design of the device for a spesific application.

The main VE material properties used in designing VE devices are the shear storage modulus, G' , which provides the "elastic" shear stiffness of the material, and the shear loss modulus, G'' , which represents the velocity-dependent or viscous stiffness of the material. The material stress-strain relationship can be expressed as

$$\tau(t) = G'\gamma(t) \pm G''\,\gamma'(t) / \omega \qquad (2.1)$$

where $\tau(t)$ is the shear stress as a function of time, t; $\gamma'(t)$ is the shear strain as a function of time; $\gamma'(t)$ is the shear strain rate of change (shear velocity) as a function of time; and $\omega$ is the circular cyclic frequency in radians per seconds. This relationship is illustrated in figure 2.1. It's seen that the stress-strain relationship is an ellipse with a nonzero slope. The slope is associated with the G' term, and the area of the ellipse is related to the G''term. Thus a simple relationship between the energy dissipated by VE materials and viscous dampers can be established. Figure 2.1 shows its dependence on cyclic amplitude and frequency, or rate dependence.

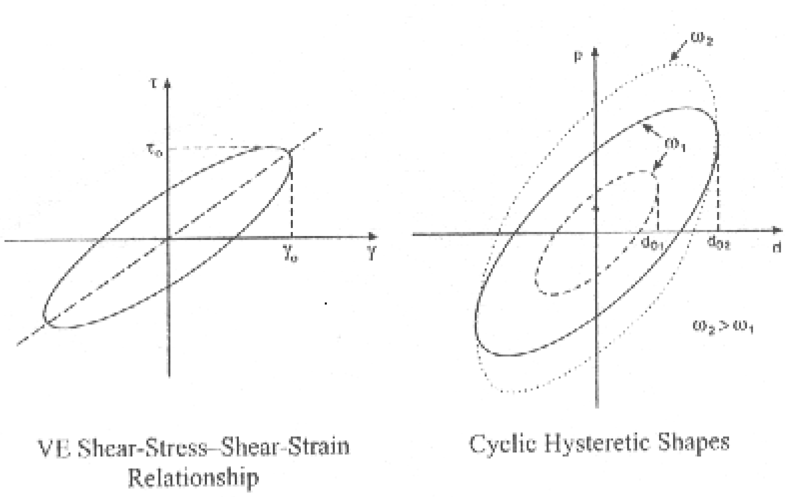


Figure 2.1: VE shear stress-strain relationship.

Consider a sheet of viscoelastic material bonded between two plates with area A and thickness h. The effective or equivalent stiffness of the device is

$$k_d = AG'(\omega) / h \tag{2.2}$$

and the effective or equivalent viscous damping coefficient is

$$c_d = AG''(\omega) / \omega h \tag{2.3}$$

It's convenient to define a loss factor, $\eta$, as the ratio of the shear loss modulus to the shear storage modulus.

That is ,

$$\eta = G''(\omega) / G'(\omega) \tag{2.4}$$

Experimental data have shown that, although $G'(\omega)$ and $G''(\omega)$ are functions of excitation frequency, the loss factor, $\eta$, is generally not sensitive to $\omega$**.** When the loss factor is used, the effective viscous damping can be expressed as

$$c_d = k_d \eta / \omega \tag{2.5}$$

At a given frequency $\omega$, it is seen that the damping coefficient is proportional to the stiffness. [1],[5],[6]

**2.1.1. Frequency, Temperature and Strain Amplitude Effects.**

It has been observed that the variations in G' and G'' fall into straight lines on a log-log plot with respect to cyclic frequency. Thus, at a given temperature, only two tests at different frequencies are needed to define this log-log straightline relationship.

The properties of VE materials change from galssy behaviour at low temperatutes to rubby behaviour at high temperatures, as illustrated in figure 2.2. The highest loss factor materials usually have the smallest temperature transition range, which is the change in temperature from glassy behaviour to rubbery behaviou, and the highest loss factor occur near the midpoint of this range. Materials science research of VE materials has developed numerous approaches for defining the effects of ambient temperature anf changes in material temperatures during operation. In general, the effects of temperature can be combined with the efects of frequency in simple log-log plots that characterise both efects. This variation with frequency and material temperature is shown in figure 2.2. This characterization provides a simple means for establishing the frequency dependence of the VE material properties.

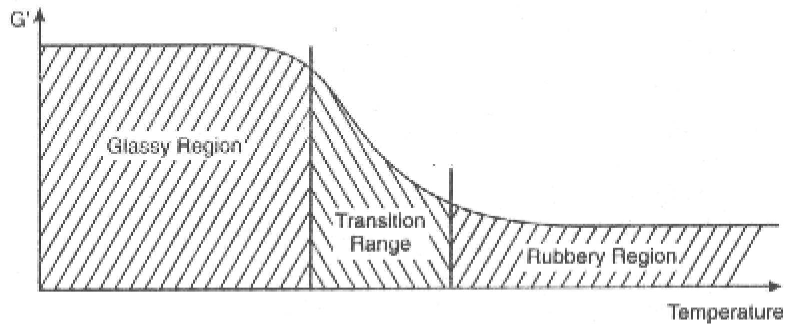


Figure 2.2: Temperature effect on VE materials.

A concern has been that VE materials exhibit nonlinear behaviour when subjected to large strains. It has been found that single-amplitude cyclic strains as large as 125% are essentially linear, as long as the temperature rise is taken into account, linear properties can be used to predict large strain behaviour. The material properties return to their initial properties when the device material has turned to its initial temperature.[5]

## 2.2. Applicability.

A direct consequence of adding dampers to a structure is to increase its energy dissipation capacity. More importantly, dampers dissipate the energy that must otherwise be dissipated by the structural lateral system, thus eliminating or reducing potential damage to structural elements or connections. Different devices provide different means by which energy is dissipated. Viscoelastic devices also add structural strength and stiffness. Though energy dissipation, the addition of dampers to a structure results in a reduction in drift and hence reduction of damage. An increase in strength or stiffness reduces the effective structural period, thus further reducing the maximum displacement; on the other hand, such an increase may also increase the total lateral force exerted on the structure.

On the basis of the dynamic characteristics and practical limitations of dampers, some general observations can be made about how their applicability to structures compares with more conventional strengthening schemes, such as base isolation or installation of braced braced frames or shear walls.[1]

Building types which dampers are added, applying dampers to existing structures and new constructed ones, effects of dampers on architecture, envoirmental effects, aging, cost and selection of damper types are things to consider more over VE dampers.

# 3 ANALYSIS OF DAMPER ADDED STRUCTURES

The objective is to provide analysis over structures with added damping and to make a relation with the analysis of traditional structures. Since nonproportional damping occurs in this systems usage of step by step integration to solve the system as with proportional damping is required. A method of this special case , Wilson-$\theta$ method for analysing will be used in further sections.

## 3.1 Nonlinear Structure and Linearized Damper Procedure.

The equivalent linear damping and sriffness characterisrics of the dampers are first integrated with those of the structural elements. This step allows nonlinear analysis of damper added structures to follow the procedures used with traditional structures with modified damping and stiffness elements.

Before the development of this integration scheme, it should be noted that dampers are usually incorporated into structures through bracing. Typical damper bracing forms are shown in figure 3.1. Figure 3.2 shows the more recent used toggle or scissor-jack configurations.

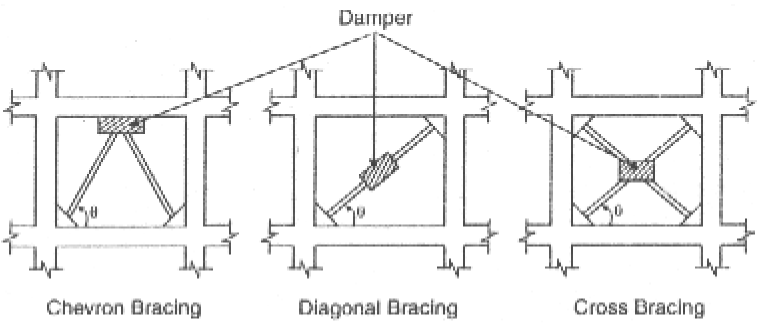


Figure 3.1: Typical damper bracings.

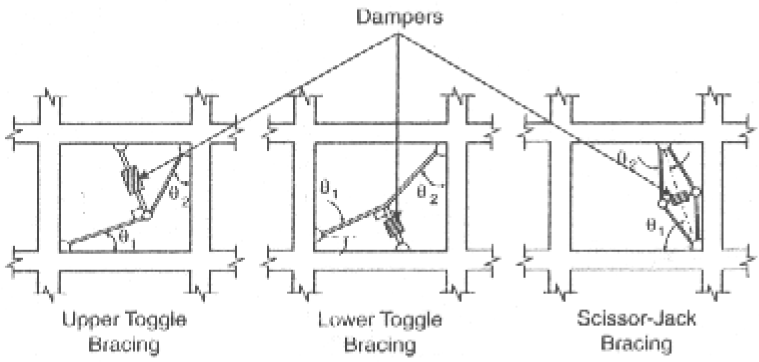


Figure 3.2: More recent used bracings.

In each case, bracing has finite stiffness so that the behaviour of the damper bracing assembly is such that one or more spring elements, representing bracing stiffness, are in series with the damper. Conventional analytical models of the traditional structure assume a discrete spring dashpot mass system for each storey as shown in figure 3.3. For damper added structures, when linear characterization ofthe damper is used, there is a more complicated spring dashpot arrengement, as shown in figure 3.3. In this figure, $k_s$ and $c_s$ are, respectively, lateral structural storey stiffness and the damping coefficient; $k_d$ and $c_d$ are respetively, axial damper stiffness and the damping coefficient; $k_b$ is the axial bracing stiffness; and $\alpha_b$ and $\alpha_d$ are, respectively, attachment coefficients that take into acount the effect of brace-damper configuration as depicted in figures 3.1 and 3.2 and the transformation from axial coordinates to the horizontal coordinates of the brace/damper force-displacement relationship. Their expressions are given in table 3.1. The damper assemblies cited in this table are shown in figures 3.1 and 3.2. Depending upon the values of $\theta_1$ and $\theta_2$, attachment coefficients $\gamma_1$, $\gamma_2$, and $\gamma_3$ can be significantly greater than one.

In table 3.1, $\gamma_1$, $\gamma_2$ and $\gamma_3$ are derived to be

$$\gamma_1 = [\sin\theta_2 / \cos(\theta_1+\theta_2)] + \sin\theta_1 \qquad (3.1)$$

$$\gamma_2 = \sin\theta_2 / \cos(\theta_1+\theta_2) \qquad (3.2)$$

$$\gamma_3 = \cos\theta_1 / \tan\theta_2)] \qquad (3.3)$$

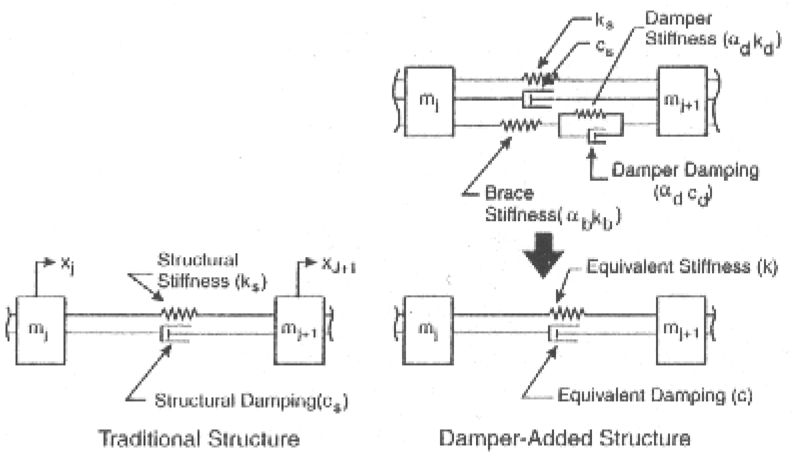


Figure 3.3: Spring dashpot arrangements.

Table 3.1: Attachment coefficients.

| *Damper Assembly* | $\alpha_b$ | $\alpha_d$ |
|---|---|---|
| Diagonal bracing | $\cos^2\theta$ | $\cos^2\theta$ |
| Chevron or cross bracing | $2\cos^2\theta$ | 1 |
| Upper toggle bracing | $\gamma_1^2$ | $\gamma_1^2$ |
| Lower toggle bracing | $\gamma_2^2$ | $\gamma_2^2$ |
| Scissor-jack bracing | $\gamma_3^2$ | $\gamma_3^2$ |

Considering the equivalent stiffness and damping coefficients of a damper added structure, taking into account a spesific brace damper configuration. Under harmonic oscillations with frequency ω, equivalent storey stiffness and damping of the damper added structure, k and c can be derived to be

$$k = k_s + \frac{\alpha_b\alpha_d k_b k_d(\alpha_b k_b+\alpha_d k_d)+\omega^2\alpha_b\alpha_d^2 k_b c_d^2}{(\alpha_b k_b+\alpha_d k_d)^2+\omega^2\alpha_d^2 c_d^2} \tag{3.4}$$

$$c = c_s + \frac{\alpha_d \alpha_b^2 c_d k_b^2}{(\alpha_b k_b + \alpha_d k_d)^2 + \omega^2 \alpha_d^2 c_d^2} \tag{3.5}$$

If $k_b \approx k_d$ then k and c are reduced to

$$k = k_s + \frac{\alpha_b \alpha_d k_b (\alpha_b k_b k_d + \omega^2 \alpha_d c_d^2)}{\alpha_b^2 k_b^2 + \omega^2 \alpha_d^2 c_d^2} \tag{3.6}$$

$$c = c_s + \frac{\alpha_b^2 k_b^2 c_d}{\alpha_b (k_d^2 + \omega^2 c_d^2)} \tag{3.7}$$

As $k_b \rightarrow \infty$, as with rigid bracing, k and c becomes, as expected,

$$k = k_s + \alpha_d k_d \tag{3.8}$$

$$c = c_s + \alpha_d c_d \tag{3.9}$$

If $k_s$ and $c_s$ are replaced in the traditional structure at each floor by k and c as derived above, then the analysis of a damper added structure, either linear or nonlinear, can be carried out by conventional procedures within the following important exceptions:

A damping increase can be significiant in the damper added structure. Hence the usual assumption that the modal periods and mode shapes of the damped structure are identical to those of the undamped structure may no longer be valid. Sample calculations have shown that, for proportional damping of up to about 50% of critical damping, the undamped mode shapes can be retained, and the damped natural periods can be calculated.

Adding dampers to a traditional structure generally results in nonproportional damping. However the effects of this nonproportionality usually can be ignored when the added dampers are distributed throughout the building height. When there is any doubt, the potential significance of nonproportional damping can be determined by premultiplying and postmultiplying the added damping matrix by the undamped elastic mode shapes. If the off diagonal terms of this uncoupled damping matrix calculation are not sufficiently smaller than the diagonal terms, then dynamic coupling will occur and the nonproportionality effects can be significant. If proportional damping is assumed in the design steps and the nonproportionality effects remain in doubt, then a response history analsis of the completed design should be made to verify the expected dynamic response. Response history by direct integration ofthe equations of motion can be used, but combining modal integration solutions cannot be used, because it will not correctly represent the effects of damping nonproportionality.[1]

# 4 COMPARISON OF TRADITIONAL STRUCTURES AND DAMPER ADDED STRUCTURES

The behaviour of damper added structures forces the designer to use direct integration methods in solving the problems of nonproportional damping. To do so numerical evaluation of dynamic responses and time stepping methods are used. As mentioned before in direct integration solutions, Wilson-$\theta$ method is used.

## 4.1. Time stepping Methods.

The objective is to solve numerically the system of different equations governing the response of multi degree of freedom systems.

$$m\ddot{u} + cu + ku = p(t) \tag{4.1}$$

with the initial conditions

$$u=u(0) \qquad \text{and} \qquad u=u(0) \tag{4.2}$$

at $t = 0$. The solution will provide the displacement vector $u(t)$ as a function of time.

The time scale is divided into a series of time steps, usually of constant duration $\Delta t$. The excitation is defined at discrete time instants $t_i = i\Delta t$; at this time, denoted as time i, the excitation vector is $p_i \equiv p(t_i)$. The response will be determined at the same time instants and is denoted by $\ddot{u}_i \equiv \ddot{u}_i(t)$, $u_i \equiv u(t_i)$, and $u_i \equiv u(t_i)$.

The known response of the system at time i ;

$$m\ddot{u}_i + cu_i + ku_i = P_i \tag{4.3}$$

time stepping methods enables to step ahead to determine the response $\ddot{u}_{i+1}$, $u_{i+1}$, and $u_{i+1}$ of the system at time i+1 ;

$$m\ddot{u}_{i+1} + cu_{i+1} + ku_{i+1} = P_{i+1} \tag{4.4}$$

When applied succesively with i= 0,1,2,3,.......... the time stepping procedure gives the desired response at all time instants i=1,2,3,......... The known initial conditions at time i=0 provide the information necessary to start the procedure.

The numerical procedure requires three matrix equations to determine the three unknown vectors $\ddot{u}_{i+1}$, $u_{i+1}$, and $u_{i+1}$. Two of these equations are derived from either finite difference equations for the velocity and acceleration vectors or from an assumption on how the response varies during a time step. The third is equation (4.1) at a selected time instant. If it is the current time i, the method of integration is said to be an explicit method. If the time i+1 at the end of the time step is used, the method is known as an implicit method.

For a numerical procedure to be useful, it should converge to the exact solution as $\Delta t$ decreases, be stable in the presence of numerical round-off errors and be accurate. The stability criteria were shown not to be restrictive in the response analysis of SDF systems because $\Delta t$ must be considerably smaller than the stability limit to ensure adequate accurancy in the numerical results. Stability of the numerical method is a critical consideration, in the analysis of MDF systems. In particular, conditionally stable procedures can be used effectively for analysis of linear response of large MDF systems, but unconditionally stable procedures are generally necessary for nonlinear response analysis of such systems. [2]

### 4.1.1 Analysis of Nonlinear Systems.

Numerical evaluation of the dynamic response of systems responding beyond their linearly elastic range is computationally demanding for systems with a large number of degrees of freedom. The N equations for an N-DOF system are usually solved in their original form, because classical modal analysis is not applicable to nonlinear systems. However, even the displacements of nonlinear system can be expressed as a combination of the natural modes of the undamped system vibrating within the range of its linear behaviour:

$$u(t) = \sum_{n=1}^{N} \phi_n q_n(t) \tag{4.5}$$

This transformation will serve to uncouple the equations of motion of a classicaly damped system only as long as the structure remains linear. After yielding, the modal equations would become coupled, precluding classical modal analysis. Despite this complication, it may seem attractive to truncate the modal transformation of eq.(4.5) to include only th first J (typically J « N) modes that contribute significantly to the response, and then solve the J coupled equations in modal coordinates instead of the N equations in phsical coordinates. However, this approach is usually not effective for general nonlinear systems but can be used with advantage for structures composed of linear subsystems connected through nonlinear systems . Although the equations being solved are not uncoupled equations, it is convenient for the discussion to follow to think of the response in terms of its modal decomposition.

Direct solution of eq.(4.1) is equivalent to including all the N modes in the analysis, although only the first J terms may be sufficient tı represent accurately the structural response. It would seem that the choice of $\Delta t$ should be based on the accurancy requirements for the Jth mode, $\Delta t = T_j/10$ where $T_j$ is the period of the Jth mode of undamped linear vibration. This choice of $\Delta t$ implies that the higher mode ( J+1 to N) terms in eq.(4.5) would be inaccurate, but this should not be of concern because its concluded that these higher mode contributions to the response were negligible. Although this choise of $\Delta t$ would seem to provide accurate results, it may not be sufficiently small to ensure stability of the numerical procedure. Accurancy is required only for the first J modes, but stability must be ensured for all modes because even if the response in the higher modes is insignificant, it will “blow up” if the stability equirements are not satisfied relative to these modes.

This problem is illustrated figure 4.1

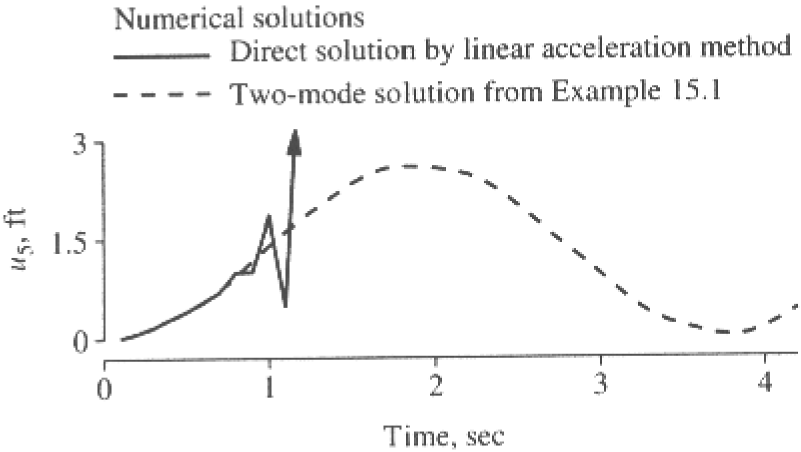


Figure 4.1: Blow up.

The curve shows a solution of an example using linear acceleration method with $\Delta t$=0.1sec. When the original equations are solved by the same method using the same time step, this direct solution “blows up” around t=1 sec.

Requiring stability for all modes imposes very severe restrictions on $\Delta t$. Consider a system in which the highest mode with significant response contribution has a period $T_j$ =0,10 sec., whereas the period of the highest mode is $T_N$ =0,001 sec. If the linear acceleration method is used, the numerical solution would be reasonably accurate if $\Delta t$ is chosen as $T_j/10$. To ensure stability of the procedure, however, $\Delta t$ should be less than $0{,}551T_N$. This choice of $\Delta t$ implies that about 2000 time steps are necessary to compute the response of the system for 1 sec. of the excitation. It is obvious that the numerical procedure used should be unconditionally stable. Then a time step of 0,01 sec should be used in this example without the solution blowing up. The same conclusion applies to linear systems if their equations of motion are not transformed to a truncated set of J modal coordinates.

Only Wilson-$\theta$ method will be presented in this section. It is intended for the solution of eq.(4.1) for linear or nonlinear systems. [2]

### 4.1.2. Wilson-θ Method.

A method developed by E.L. Wilson is a modification of the conditionally stable linear acceleration method that makes it unconditionally stable. This modification is based on the assumption that the acceleration varies linearly over an extended time step $\delta t = \theta \Delta t$, as shown in figure 4.2. The accurancy and stability properties of the method depend on the value of the parameter θ, which is always greater than 1.

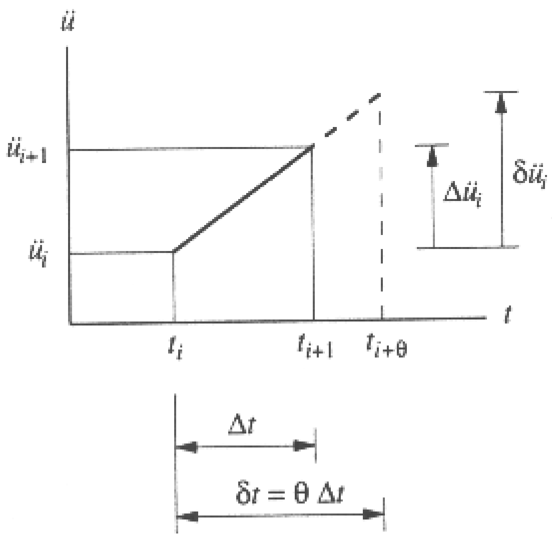


Figure 4.2: Linear variation assumption of Wilson-θ method.

Let δ denote the increase in time, where 0 [ δ [ θ Δt ; then for the time interval t to t+θ Δt, it is assumed that

$$\ddot{U}^{t+\delta} = \ddot{U}^{t} + \frac{\delta}{\theta\Delta t}(\ddot{U}^{t+\theta\Delta t} - \ddot{U}^{t}) \tag{4.6}$$

Integrating eq.(4.6)

$$\ddot{U}^{t+\delta} = \dot{U}^{t} + \ddot{U}^{t}\delta + \frac{\delta^2}{2\theta\Delta t}(\ddot{U}^{t+\theta\Delta t} - \ddot{U}^{t}) \tag{4.7}$$

$$U^{t+\delta} = U^{t} + \ddot{U}^{t}\delta + \frac{1}{2}\ddot{U}\delta^{2} + \frac{1}{6\theta\Delta t}\delta^{3}(\ddot{U}^{t+\theta\Delta t} - \ddot{U}^{t}) \tag{4.8}$$

Using (4.7) and (4.8) we have at time t + θΔt,

$$\dot{U}^{t+\theta\Delta t} = \dot{U}^{t} + \frac{\theta\Delta t}{2}(\ddot{U}^{t+\theta\Delta t} + \ddot{U}^{t}) \tag{4.9}$$

$$U^{t+\theta\Delta t} = U^{t} + \theta\Delta t\,\dot{U}^{t} + \frac{\theta^{2}\Delta t^{2}}{6}(\ddot{U}^{t+\theta\Delta t} + 2\ddot{U}^{t}) \tag{4.10}$$

from which we can solve for ${}^{t+\theta\Delta t}\ddot{U}$ and ${}^{t+\theta\Delta t}U$ in terms of ${}^{t+\theta\Delta t}U$ :

$$\ddot{U}^{t+\theta\Delta t} = \frac{6}{\theta^{2}\Delta t^{2}}(U^{t+\theta\Delta t} - U^{t}) - \frac{6}{\theta\Delta t}\dot{U}^{t} - 2\ddot{U}^{t} \tag{4.11}$$

$$\dot{U}^{t+\theta\Delta t} = \frac{3}{\theta\Delta t}(U^{t+\theta\Delta t} - U^{t}) - 2\dot{U}^{t} - \frac{\theta\Delta t}{2}\ddot{U}^{t} \tag{4.12}$$

To obtain the solution for the displacements, velocities and accelerations at time t + Δt, the equilibrium equations are considered at time t + θΔt. However, because the accelerations are assumed to vary linearly, a linearly projected load vector is used . The equation employed is

$$M\,{}^{t+\theta\Delta t}\ddot{U} + C\,{}^{t+\theta\Delta t}U + K\,{}^{t+\theta\Delta t}U = {}^{t+\theta\Delta t}R \tag{4.13}$$

where $${}^{t+\theta\Delta t}R = {}^{t}R + \theta\,({}^{t+\Delta t}R - {}^{t}R) \tag{4.14}$$

Substituting eq(4.11). and (4.12) into (4.13), an equation is obtained from which ${}^{t+\theta\Delta t}U$ can be solved. Then substituting ${}^{t+\theta\Delta t}U$ into(4.11) we obtain ${}^{t+\theta\Delta t}\ddot{U}$, which is used in (4.12), (4.13) and (4.14), all evaluated at $\delta = \Delta t$ to calculate $^{t+\Delta t}\ddot{U}$, $^{t+\Delta t}U$ and $^{t+\Delta t}U$. The complete algorithm used in the integration is given in table 4.1.

The Wilson θ method is also an implict integration method, because the stiffness matrix K is a coefficient matrix to the unknown displacement vector. It may also be noted that no special starting procedures are needed, since the displacements, velocities and accelerations at time t+Δt are expressed in terms of the same quantities at time t only.[2],[4]

Table 4.1: Calculation steps.

A: Initial Calculations:

---

1. Form stiffness matrix K, mass matrix M and damping matrix C.
2. Initialize $U^0$, $U^0$ and $\ddot{U}^0$.
3. Select time step Δt and calculate integration constants, θ =1.4 (usually):

   $a_0 = 6 / (\theta\Delta t^2)$ ; $a_1 = 3 / \theta\Delta t$ ; $a_2 = 2a_1$ ; $a_3 = \theta\Delta t / 2$ ; $a_4 = a_0 / \theta$ ;

   $a_5 = -a_2 / \theta$ ; $a_6 = 1 - (3 / \theta)$ ; $a_7 = \Delta t / 2$ ; $a_8 = \Delta t^2 / 6$
4. Form effective stiffness matrix $\cancel{K}$ : $\cancel{K} = K + a_0 M + a_1 C$.
5. Triangularize $\cancel{K}$ : $\cancel{K} = LDL^T$.

---

B: For each time step :

---

1. Calculate effective loads ar time t + Δt:

   $$^{t+\theta\Delta t}R = {}^{t}R + \theta\,({}^{t+\Delta t}R - {}^{t}R) + M\,(a_0\,{}^{t}U + a_2\,{}^{t}U + 2\,{}^{t}\ddot{U}) + C\,(a_1\,{}^{t}U + 2\,{}^{t}U + a_3\,{}^{t}\ddot{U})$$
2. Solve for displacements at time t + θΔt:

   $$LDL^T\,{}^{t+\theta\Delta t}U = {}^{t+\theta\Delta t}R$$
3. Calculate displacements, velocities and accelerations at time t + Δt:

   $$^{t+\Delta t}\ddot{U} = a_4\,({}^{t+\theta\Delta t}U - {}^{t}U) + a_5\,{}^{t}U + a_6\,{}^{t}\ddot{U}$$

   $$^{t+\Delta t}U = a_7\,({}^{t+\Delta t}U + {}^{t}\ddot{U}) + {}^{t}U$$

   $$^{t+\Delta t}U = a_8\,({}^{t+\Delta t}\ddot{U} + 2\,{}^{t}\ddot{U}) + {}^{t}U + \Delta t\,{}^{t}U$$

---

# 5 EXAMPLES

## 5.1. Example 1: Two Storey Shear Frame.

A system given is solved for its given values and $\omega_1$=6,279 rad/sec. and $\omega_2$=13,364 rad/sec. for the system in undamped case is found. Then the system is loaded with R=44.4 kNs. The load is applied to the system from the second storey height. A solution with Wilson-$\theta$ method is done. After that, two viscoelastic dampers, one damper per each storey, are installed to the system. It is assumed that the VE dampers are rigidly braced. The same load, R=44.4 kN is applied and the system is solved. In the first case only the systems inherent damping is effective but in the second case the VE dampers are effecting the systems stiffness and damping matrices. It can be easily seen that the displacements are reduced nearly 60% when VE dampers are added to the system. In order to find out the optimum solution for the system, three different cases of damper installations are used and a comparison is made .

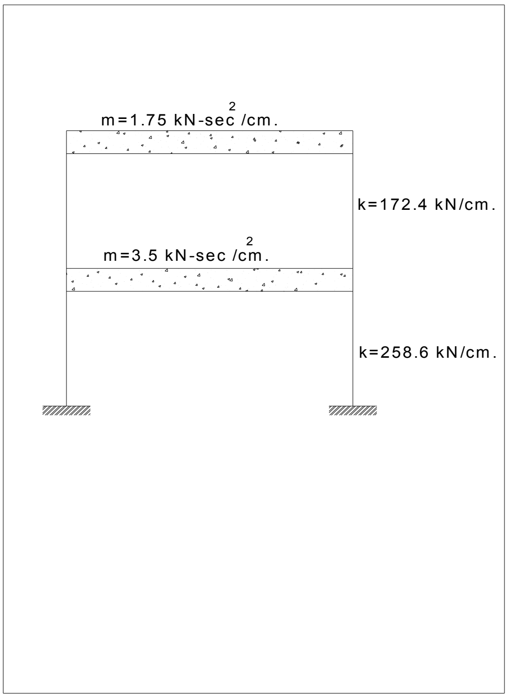


Figure 5.1: Two storey shear frame.

The system is shown in figure 5.1. A constant load R=44,4 kN is applied.
$\theta$ = 1,4 and $\Delta t$ = 0,25 sec. The stiffness, mass and the damping matrices are ;

$$K = (kN/cm) \begin{vmatrix} 431 & -172{,}4 \\ -172{,}4 & 172{,}4 \end{vmatrix} \quad M = (kN\text{-}s^2/cm) \begin{vmatrix} 3{,}5 & 0 \\ 0 & 1{,}75 \end{vmatrix} \quad C = (kN\text{-}s/cm) \begin{vmatrix} 1{,}64 & -0{,}87 \\ -0{,}87 & 3{,}74 \end{vmatrix}$$

Initial conditions are found to be ;

$$U^0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad U^0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \ddot{U}^0 = \begin{bmatrix} 0 \\ 50 \end{bmatrix}$$

Constant values for the solution with the current $\theta$ and $\Delta t$ values are;
$a_0$= 306,12 , $a_1$= 21,42 , $a_2$= 42,84 , $a_3$= 0,07 , $a_4$= 218,65 , $a_5$= -15,3 ,
$a_6$= -1,14 , $a_7$= 0,05 , $a_8$= 0,00166
respectively.

The selection of VE damper stiffness $k_d$ and loss factor $\eta$ is a trial-and-error procedure. These values can also be determined on the basis of the principle that the added stiffness that is due to VE dampers should be proportional to the storey stiffness of the primary structure. This is obtained from modifiying the modal strain energy method for each storey as

$$\alpha_{di}k_{di} = k_{si}\,(2\beta/\eta\text{-}2\beta) \qquad (5.1)$$

where $\beta$ is the targeted added damping ratio; $\alpha_{di}$ is the attachement coefficient from table 1 and $k_{di}$ and $k_{si}$ are the contributions of damper added stiffness and the structural storey stiffness without added dampers at the ith story, respectively. For this example the thickness of the VE dampers are 0,76 inches, G'=17.6 kg/cm$^2$ and constant temperature is 25$^{o}$C. An area of 45 cm$^2$ is chosen. Loss factor $\eta$=1,1

$C_d$=2*45*1,1*17,6/6,279*0,76=3,65 kN-sec/cm.

$\alpha_d=\cos^2\theta$=0,8

$\alpha_d c_d$=2,92 kN-sec./cm.

$k_d$=2*17,6*45/6,279*0,76=3,32 kN/cm.

$\alpha_d k_d$=2,655 kN/cm.

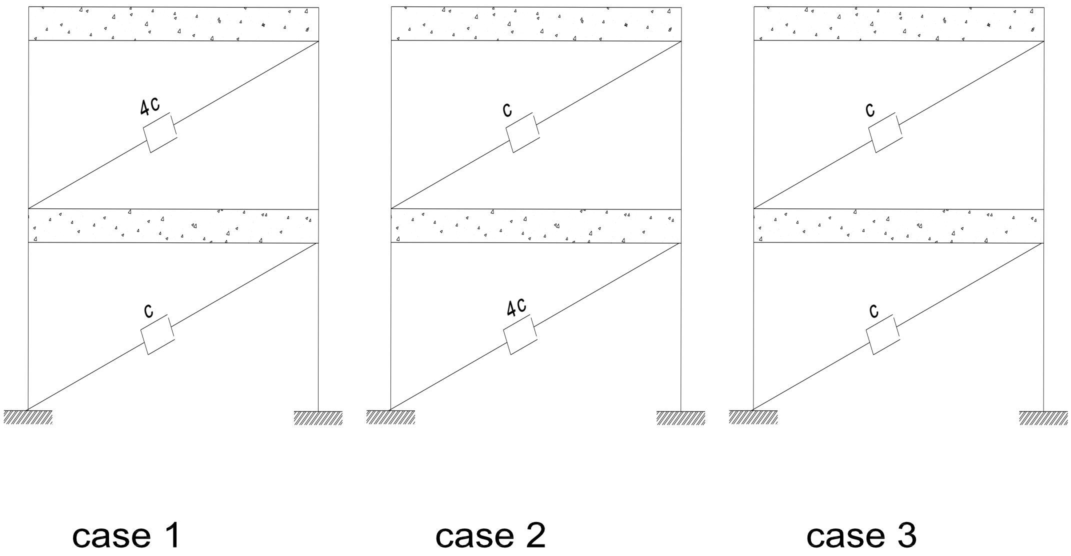


Figure 5.2: Damper added frames.

Added damper stiffnesses are shown in figure 5.2. After including dampers values K and C are calculated again. After that Wilson θ method is used. Accelerations, velocities and displacements are found for both two cases and plots are made to show the results. Table 5.1 shows all the results. Figure 5.3 and figure 5.4 shows the displacement comparisons. Figures 5.5 and 5.6 compares the velocities and figures 5.7 and 5.8 compares the accelerations of the two storey shear frames. The added stiffness and damping matrices are given below for each case.

| | | | | | | |
|---|---|---|---|---|---|---|
| | For case 1; | | | | | |
| | | | | | | |
| K= | 444,27 | -183,02 | | C= | 16,24 | -12,55 |
| | -183,02 | 183,02 | | | -12,55 | 15,42 |
| | | | | | | |
| | For case 2; | | | | | |
| | | | | | | |
| K= | 444,29 | -175,05 | | C= | 16,24 | -3,79 |
| | -175,05 | 175,05 | | | -3,79 | 6,66 |
| | | | | | | |
| | For case 3; | | | | | |
| | | | | | | |
| K= | 436,31 | -175,06 | | C= | 7,48 | -3,79 |
| | -175,06 | 175,055 | | | -3,79 | 6,66 |

### 5.2. Example 2: Five Storey Frame.

A five story frame system is given as in figure 5.3. The period of the system is 0.3 seconds. The stiffness and the mass matrices are given as follows.

$$\mathbf{K} = \begin{vmatrix} 84{,}372 & -26{,}23 & 0 & 0 & 0 \\ -26{,}23 & 52{,}44 & -26{,}23 & 0 & 0 \\ 0 & -26{,}23 & 52{,}44 & -26{,}23 & 0 \\ 0 & 0 & -26{,}23 & 52{,}44 & -26{,}23 \\ 0 & 0 & 0 & -26{,}23 & 26{,}23 \end{vmatrix} (\mathrm{kN/cm})$$

$$\mathbf{M} = \begin{vmatrix} 5{,}83 & 0 & 0 & 0 & 0 \\ 0 & 5{,}65 & 0 & 0 & 0 \\ 0 & 0 & 5{,}65 & 0 & 0 \\ 0 & 0 & 0 & 5{,}65 & 0 \\ 0 & 0 & 0 & 0 & 4{,}982 \end{vmatrix} (\mathrm{kN})$$

The traditional structure is loaded by using El Centro May 1940 NS record. The accelerations, velocities and displacements are found by using Wilson-$\theta$ method. Then, the same dampers with the same damping coefficient are installed at each storey and the system is solved under the same dynamic loads. The system with added dampers are shown in figure 5.4. The results are ploted and the comparison is made. It is seen that using VE solid dampers effects on the drift of the storeys and reduces the displacements.

The dampers used in the systems has a stiffness of 13,04 kN/cm. at the first storey and 10,62 kN/cm. at typical storeys. The damping at the first storey is 0,926 kN-sec/cm. and 0,755 kN-sec/cm. at the typical storeys. These are the $\alpha_d c_d$ and $\alpha_d k_d$ values which are directly inserted into the relevant matrices. The final added stiffness and damping matrices are as follows.

$$\mathbf{K} = \begin{vmatrix} 103{,}3 & -34{,}73 & 0 & 0 & 0 \\ -34{,}73 & 69{,}44 & -34{,}73 & 0 & 0 \\ 0 & -34{,}73 & 69{,}44 & -34{,}73 & 0 \\ 0 & 0 & -26{,}23 & 69{,}44 & -34{,}73 \\ 0 & 0 & 0 & -34{,}73 & 34{,}73 \end{vmatrix} (\mathrm{kN/cm})$$

$$\mathbf{C} = \begin{vmatrix} 1{,}34 & -0{,}6 & 0 & 0 & 0 \\ -0{,}6 & 1{,}2 & -0{,}6 & 0 & 0 \\ 0 & -0{,}6 & 1{,}2 & -0{,}6 & 0 \\ 0 & 0 & -0{,}6 & 1{,}2 & -0{,}6 \\ 0 & 0 & 0 & -0{,}6 & 0{,}6 \end{vmatrix} (\mathrm{kN/cm})$$

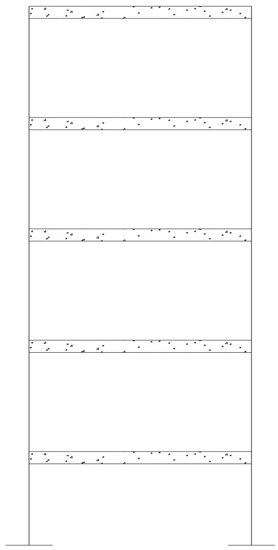

Figure 5.3a: Five storey shear frame.

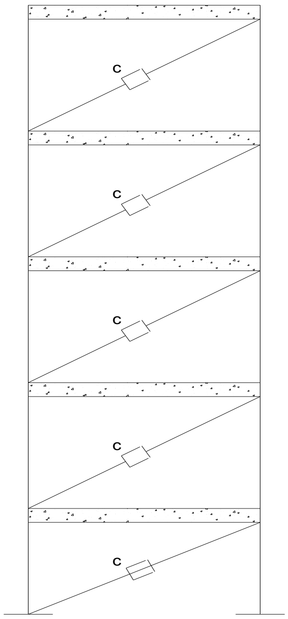


Figure 5.3b: Damper added frame.

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### 5.3. Example 3: Five Storey RC Frame.

A five story frame system is given as in figure 5.24. The system is assumed to be an existing structure with low stiffnesses and material quality. The concrete used in the structure is C16 and the cross sections are 30*30 cm. in the first three storeys and 25*25 in the top two storeys. The period of the system is calculated as 0,52 seconds by using earthquake resistant code of Turkey 1975. The stiffness and the mass matrices are given as follows.

$$\mathbf{K} = \begin{vmatrix} 162 & -81 & 0 & 0 & 0 \\ -81 & 162 & -81 & 0 & 0 \\ 0 & -81 & 120{,}1 & -39{,}1 & 0 \\ 0 & 0 & -39{,}1 & 78{,}2 & -39{,}1 \\ 0 & 0 & 0 & -39{,}1 & 39{,}1 \end{vmatrix} (kN/cm)$$

$$\mathbf{M} = \begin{vmatrix} 300 & 0 & 0 & 0 & 0 \\ 0 & 250 & 0 & 0 & 0 \\ 0 & 0 & 250 & 0 & 0 \\ 0 & 0 & 0 & 250 & 0 \\ 0 & 0 & 0 & 0 & 200 \end{vmatrix} (kN)$$

The traditional structure is loaded by using El Centro May 1940 NS record. The accelerations, velocities and displacements are found by using Wilson-θ method. Then, the same dampers with the same damping coefficient are installed at each storey and the system is solved under the same dynamic loads. The system with added dampers are shown in figure 5.25. The results are ploted and the comparison is made. It is seen that using VE solid dampers effects on the drift of the storeys and reduces the displacements. Another solution by using dampers with a damping ratio that is five times greater than the first case is made to compare the effects of the damping. The results are shown in table 5.5-13 and figure 5.26-40. The base shear forces are also compared in table 5.14 and shown in figure 5.41.

The dampers used in the systems has a stiffness of 10,62 kN/cm. at all storeys. The damping is 0,926 kN-sec/cm. at all storeys. For the second case the values are five times greater than the first one. These are the $\alpha_d c_d$ and $\alpha_d k_d$ values which are directly inserted into the relevant matrices. The added stiffness and damping matrices are as follows.

For the first case;

$$\mathbf{K}=\begin{vmatrix} 179 & -89,5 & 0 & 0 & 0 \\ -89,5 & 179 & -89,5 & 0 & 0 \\ 0 & -89,5 & 137,1 & -47,6 & 0 \\ 0 & 0 & -47,6 & 95,2 & -47,6 \\ 0 & 0 & 0 & -47,6 & 47,6 \end{vmatrix}(kN/cm)$$

$$\mathbf{C}=\begin{vmatrix} 1,48 & -0,74 & 0 & 0 & 0 \\ -0,74 & 1,48 & -0,74 & 0 & 0 \\ 0 & -0,74 & 1,48 & 74 & 0 \\ 0 & 0 & -0,74 & 1,48 & -0,74 \\ 0 & 0 & 0 & -0,74 & 0,74 \end{vmatrix}(kN/cm)$$

For the second case;

$$\mathbf{K}=\begin{vmatrix} 247 & -123,5 & 0 & 0 & 0 \\ -123,5 & 247 & -123,5 & 0 & 0 \\ 0 & -123,5 & 205,1 & -81,6 & 0 \\ 0 & 0 & -81,6 & 163,2 & -81,6 \\ 0 & 0 & 0 & -81,6 & 81,6 \end{vmatrix}(kN/cm)$$

$$\mathbf{C}=\begin{vmatrix} 7,4 & -3,7 & 0 & 0 & 0 \\ -3,7 & 7,4 & -3,7 & 0 & 0 \\ 0 & -3,7 & 7,4 & -3,7 & 0 \\ 0 & 0 & -3,7 & 7,4 & -3,7 \\ 0 & 0 & 0 & -3,7 & 3,7 \end{vmatrix}(kN/cm)$$

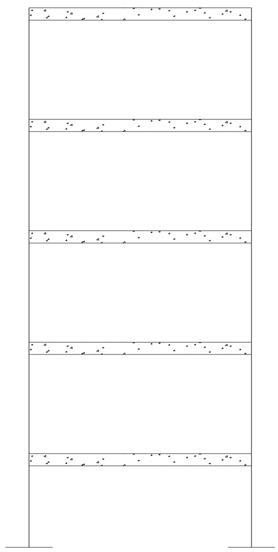

Figure 5.24: Five storey RC shear frame.

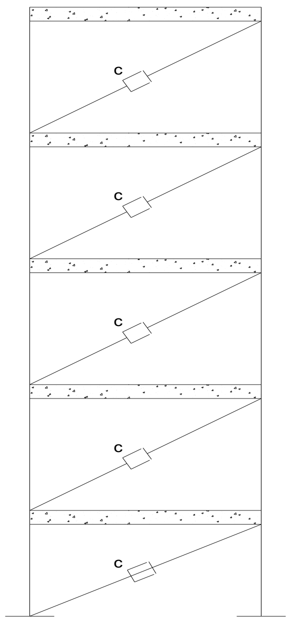


Figure 5.25: Damper added RC frame.

# 6 CONCLUSION

This study is presenting the effects of viscoelastic solid dampers on two dimensional frames by solving some examples, using a step by step integration method to make a time history analysis. The mathematical method is chosen to be Wilson-$\theta$ method since the damping mentioned is to be nonproportional and it is one of the best ways to produce appropriate results.

The behaviour of the traditional structures are kept and adding dampers have increased stiffness and damping in the systems. The strong ground motion record El Centro was used. When the damper added structures are compared with their traditional forms after loading , it was clearly seen that the reduction in the displacements were reaching 60% and the velocities and accelerations were also decreasing. The VE dampers dissipate the earthquake energy and turn it into heat by doing some shear deformation. The rate of the deformation depends on the materials used in the damper layers and the temperature is also an effect in the dampers performance. The bracing type and members are directly effecting the case, because of their inherent stiffnesses and damping constants. It can be seen that all this facts are directly effecting the systems behaviour.

The results show that the usage of energy dissipation devices on structures are increasing the systems damping and stiffness. The storey drifts are getting smaller than they were before and it can be expected that damages connected to large displacements can be minimized by using this devices. Even if damage occurs, the time needed to implement the repairs and the cost will be lowered.

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**Appendix A:**

The solutions of the examples in section 5 are made by using a simple programme in MS Excel. In order to prove that the programme is running correctly, an example with a known solution is done by using it. The known answers of the example are given in table A.1 and the displacement graph is drawn in figure A.1. The results of the example found with MS Excel is presented in table A.2 and plotted in figure A.2. A final plot which compares the two solutions is made as figure A.3.

$$K = \begin{matrix} 6 & -2 \\ -2 & 4 \end{matrix} \qquad M = \begin{matrix} 2 & 0 \\ 0 & 1 \end{matrix}$$

As the parameter of Wilson $\theta$ =1,4 and $\Delta$t=0,28 . R=10 kips constant force. [4]

## AUTOBIOGRAPHY

Born in İstanbul in 1977. Graduated from Kadıköy Anatolian High School in 1995 and started undergraduate education in Yıldız Technical University Civil Engineering Faculty Civil Engineering Department. Recieved Bachelor of Science degree in Civil Engineering in 2000. Still working in a private company as project controller engineer.