İSTANBUL TECHNICAL UNIVERSITY ★ INSTITUTE OF SCIENCE AND TECHNOLOGY

DECIDING ON RFID TAGGING LEVEL OF INVENTORIES

M.Sc. Thesis by
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Department: Industrial Engineering

Programme: Industrial Engineering

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ENVANTERLERİN ÜZERİNDE RFID ETİKETLERİNİN DÜZEYİNİN BELİRLENMESİ

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December, 2007 Evşen Korkmaz

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ABBREVIATIONS

RFID	**:** Radio Frequency Identification
SKU	**:** Stock Keeping Unit
CSL	**:** Customer Service Level
IRI	**:** Inventory Record Inaccuracy
ROI	**:** Return on Investment
NSRG	**:** National Supermarket Research Group
ITU	**:** İstanbul Teknik Üniversitesi
TU/e	**:**Technische Universiteit Eindhoven

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DECIDING ON RFID TAGGING LEVEL OF INVENTORIES

SUMMARY

Radio Frequency Identification (RFID) technology is the novel automatic identification technology that uses radio waves to automatically identify, track and categorize individual items. It is estimated that this new auto-ID technology will supersede barcode technology in the near future. However, there are some obstacles that stands front of the proliferation of item-level RFID technology. The main obstacle is the high cost rates of RFID systems. In this thesis, we have presented an analytical model on optimum RFID tagging level decision in the retail supply chain that is subject to inventory record errors. Our model captures the most important benefits of RFID tagging such as decreasing the rate of inventory discrepancy and decreasing the labor cost, which is the biggest cost account at the retailer inventory system, and attempts to reflect the real-world cost considerations in the deployment of RFID technology. To model this problem we revised the classical Newsvendor model. Classic Newsvendor model does not take into account the errors occurred on the information flows during the physical flow. In other words it does not take into account the misalignment between physical and information flows. But in real world, the errors may be occurred in many stages of the chain. When it is focused on the flow of goods through on the supply chain stages, it can be observed that there may occur some misalignments. According to the extended literature review part of this study, we choose two main error factors to take into consideration in our model which are called *misplacement* and *shrinkage* since their impact on inventory inaccuracy can be easily measured. After explaining the need for this study, the five steps that are followed has been showed.

In the first step, the profit functions of four different application forms of RFID tagging are defined. This application forms are item-level tagging, case-level tagging, pallet-level tagging and no tagging.

In the second step, expectations of the previously defined profit functions are taken and the equations that give the optimum order quantity and maximum expected profit values are derived for four cases.

In the third step, through defining a continuous tagging level decision variable, we obtain a more flexible model that gives us the possibility of analyzing different tagging levels, i.e. different capacity of cases and pallets. So, the equations of four different cases are revised by the functions of the newly defined tagging level variable.

In the fourth step, under the assumptions of normally and uniform distributed demand, the equation of maximum expected profit is re-derived.

In the fifth step, a numerical study is conducted to test the outcomes of the model.

As a result, the parameters that affect RFID tagging level decision are found out and their impacts on the results are discussed.

ENVANTERLER ÜZERİNDE RFID ETİKETLEME DÜZEYİNİN BELİRLENMESİ

ÖZET

Radyo Frekanslı Tanıma (RFID) teknolojisi ürünleri radyo dalgaları kullanarak tanımlayan, takip eden ve sınıflandıran yeni bir otomatik tanımlama teknolojisidir. Bu yeni teknolojinin yakın zamanda barkod teknolojisinin yerini alacağı tahmin edilmektedir. Ancak birim-düzeyli RFID etiketleme teknolojisinin yayılması önünde bazı engeller bulunmaktadır. RFID sistemlerinin yüksek maliyet oranları temel engeli teşkil etmektedir. Bu çalışmada, çeşitli envanter kayıt hatalarına maruz kalan perakende tedarik zincirinde, en uygun RFID etiketleme düzeyi kararı üzerine kurulmuş bir analitik model sunuyoruz. Modelimiz, RFID etiketlemesinin envanter kayıt hata oranlarını azaltması ve prakendecilik için en büyük maliyet kalemi olan işçilik maliyetlerini azaltması gibi faydalarını içermekle birlikte, RFID teknolojisinin kurulması ile ilgili gerçek hayat maliyetlerini yansıtmayı da hedeflemektedir. Bu problemi modellemek için klasik Gazeteci Çocuk problemini revize ettik. Klasik Gazeteci Çocuk problemi, fiziksel akış sırasında gerçekleşen bilgi akışında oluşabilecek hataları gözönüne almaz. Diğer bir kelimeyle, fiziksel ve bilgi akışı arasındaki yanlış kayıdı gözönüne almaz. Fakat, gerçek hayatta zincirin halkaları arasında pek çok noktada bu tarz hatalar oluşabilmektedir. Kapsamlı bir literatür araştırması üzerine, modelimize *yanlış yerleştirme* ile *kayıp* hataları olarak isimlendiren iki hata faktörünü dahil etmeye karar verdik. Bu çalışmaya neden gerek olduğunu açıkladıktan sonra, takip edilen beş basamak açıklanmıştır:

Birinci aşama olarak, dört farklı RFID etiketleme düzeyi için kar fonksiyonlarımız tanımlanmıştır. Bu dört farklı etiketleme düzeyi; birim-düzeyli etiketleme, kutu-düzeyli etiketleme, palet-düzeyli etiketleme ve hiç etiketlememe düzeyleridir.

İkinci aşamada, tanımlanan kar fonksiyonlarının beklenen değerleri alınarak, dört farklı durum için en iyi sipariş miktarı ile beklenen en büyük kar değerleri elde edilmiştir.

Üçüncü aşamada, sürekli bir etiketleme düzeyi karar değişkeni tanımlanarak, bu yeni değişkenin fonksiyonları ile dört durum için tanımlanmış eşitlikler revize edilmiştir.

Dördüncü aşamada, normal ve uniform dağılmış talep varsayımı altında elde edilmiş eşitlikler tekrardan düzenlenmişlerdir.

Beşinci aşamada ise, sayısal bir çalışma yapılarak modelin çıktıları test edilmiştir.

Sonuç olarak, RFID etiketleme düzeyi karar problemini etkileyen parametreler bulunmuş ve bunların sonuç üzerindeki etkisi tartışılmıştır.

1. INTRODUCTION

1.1 Purpose of the Thesis

The main objective of this study is to decide on the level of RFID tagging for each Stock Keeping Unit (SKU) in the inventory: item, case, pallet, or none. Revising the classical newsvendor model within the consideration of misplacement and shrinkage rates and labor cost, is the methodology to determine the optimal tagging level of items.

1.2 Background

For some industries operating inventory-carrying facilities, the key success factor has always been to minimize the inventory carrying cost while providing a high customer service level. This situation is also valid for retail industry. To survive in the competitive, rapidly and ever-changing retail industry, because of the cardinal importance of purchasing power, merging strategy is one of the main trends of this market. Due to the coercion of merging, fierce of competition, low profit margins and lack of customer loyalty, retail firms have been making big investments to automation technologies since the early 1980’s (Lee and Özer, 2005). Technology is needed to manage huge systems. Even a medium-sized retail chain carries thousands of Stock Keeping Units (SKUs). Tracking the inventory record of every SKU and managing the replenishment policy for the whole system manually is a very time-consuming process. In this context, it can be easily said that keeping track of the location of items and making sure that the inventory record is equal to the actual stock quantity is a task of primary importance.

The improvements on automatic tracking technology and inventory management literature help the retail industry with the investments of automation. As its name implies, Radio Frequency Identification (RFID) uses radio signals to automatically identify the individual items. Using radio-frequency waves, data and energy are transferred between a reader and a tag to identify, categorize and track objects. With the properties of this novel Auto-ID technology, it is possible to improve the performance of a supply chain process in terms of operational efficiency, accuracy and security. There are three main components of RFID systems. These are tag, reader and back-end database (Weis, 2003). The communication between these components is provided by radio waves like the other wireless technologies as Bluetooth. RFID represents several distinctions over barcodes in terms of (1) non optical proximity communication, (2) information density, (3) two way communication ability and (4) multiple simultaneous reading (read more than one item at once) (Korkmaz et al, 2006). There are many industry reports and scientific papers available claims that by the development of a new auto-ID technology, RFID, automated inventory control systems are going to be adapted with this new technology (see "current views on the value of RFID" section in Lee and Özer, 2005). AMR Report stated on 2002 that the total supply chain cost can decrease by 3-5%, while revenues can increase by 2-7% at early adapters (Abell and Quirk, 2002). A more recent report presented by GMA gives a very comprehensive discussion of the benefits of RFID (Lee and Özer, 2005).

On the other hand the fixed cost (including first investment and maintenance costs) and variable costs (tag cost) of RFID systems are quite high. Consequently the first question in minds was "Is it really necessary to deploy this novel, expensive auto-ID technology to improve supply chain performance". With the motivation of this question there are some papers published in literature that focus on the assessment of RFID systems (Kök et al. 2004; Şahin et al. 2005; Gel et al. 2006).

Nevermore, the occurrence of out-of-stock is still a significant issue in the retail supply chain. Several factors that cause out-of-stock are identified in inventory inaccuracy literature. The main reasons that form a substantial portion of out-of-stock cost are as following:

- *Retail store ordering and forecasting problems;* the ordered quantity may be insufficient to meet the actual consumer demand or some problems may exist during the ordering process (problems on the automatic ordering system).
- *Misplacement of items on shelf or in the backroom;* errors may be occurred during store shelving and replenishment practices (although store has items, some of them may be unavailable because of not being in the correct place).

- *Shrinkage;* Vendor fraud (also called random yield problem in literature), inner and outer theft and undetected spoilage form this group of errors.
- *Transaction errors;* may occur because of scanning errors during sales and receipt of shipments.

There is a very limited amount of detailed and model based analysis that guaranties the promises of RFID deployment. On the other hand industry reports and whitepapers support the deployment of RFID systems which are rapidly rising. In a recent survey of retail chief information officers, it is reported that 55% of respondents plan to implement RFID capabilities in a near future (MIT Auto-ID Center, Industry Reports – IBM, 2004).

Accordingly, the main question in minds should be changed by "As the RFID technology spreads, is it suitable to justify tagging on item-level for all SKUs, if not (because it might be still very expensive to justify tagging on item-level for all SKUs), on which level of tagging will be justified for each SKUs in the inventory; item, case, pallet, or none?"

1.3 Hypothesis

This study provided a unique opportunity to discuss the implication of RFID systems in a more realistic manner by considering the problem of tagging level for items. To our knowledge this study is the first study that focuses on the problem of RFID tagging level for each SKU.

2. LITERATURE REVIEW

Literature on inventory management has been growing up since 1950s' and 1960s' by developing some of the earliest qualitative automated approaches. Although a vast area related with inventory management issues can be accessed in literature, one of the sub-issues of inventory management literature that is focused on *inventory management with inaccurate inventory record* is not so profound. The reason of this mainly comes from the general assumption of "*perfect knowledge of the inventory is available*". Although the huge difference obtained when the comparison of the accumulated literature on inventory management and inventory management with inaccurate inventory record made, nevermore there are some very important papers published in literature related with this sub-area. Essentially it is highlighted that this part of literature needs more detailed and model-based studies. As detailed in the study of Kang and Gershwin (2004), there have been some papers written about different aspects of discrepancy between real inventory that physically exist in inventory carrying facilities and recorded inventory that is obtained by IT system. In this thesis, the related literature focused on the error between real and recorded inventory is separated into two parts, *pre-RFID* and *post-RFID*.

As forementioned, literature related with inventory record inaccuracy before RFID adoption is reviewed by Kang and Gershwin (2004). To sum up, the studies started by considering the uncertainty about inventory record error to the determination of reorder point process (Iglehart et al., 1972). The authors added one more factor (error of inventory record) that affects customer service level (CSL). They focused to find an optimal combination of safety stock and frequency of cycle counts. Morey, 1986, also focused on the frequency, types of inventory counts - i.e. perfect and imperfect audits that leave errors in record. Effective timing of cycle counts in multiple SKU environments was discussed by ABC analysis and it was found that different methods for different kinds of SKUs such as high activity SKUs (Cantwell, 1985, Edelman, 1984, Reddock, 1984 and Neely, 1987). Buck & Sadowski (1983), Dalenius & Hodges (1959), Cochran (1977), Arens & Loebecke (1981), and Martin

& Goodrich (1987) focused on sampling techniques under the conditions where there are many SKUs and counting costs are very high. They try to help the managers for choosing and counting only a portion of the whole inventory, not all. While some authors are discussing the managerial steps to make the cycle counts more effective (Bernard, 1985 and Graff, 1987), some of them address inventory inaccuracy in MRP systems (Frech, 1980 and Krajewski, 1987).

Concerning this accumulation in literature about inventory cycle counts, it can be generalized that the literature of inventory record inaccuracy is built on determination of frequency and sampling of inventory cycle counts before the development of RFID technology. After the development of this new auto-ID sub-component, the only way of reducing the difference between physical and digital world is not only costly audits anymore.

In spite of developed automatic tracking technologies and advanced inventory replenishment models, it is observed for many retail firms that targeted customer service level and inventory holding cost may not be achieved. These kinds of observations are made by many audit firms and scientific papers. Raman et al. reports that more than 65% of the inventory records did not match with the physical inventory at the store SKU level (2004). Kang and Gershwin also point out that retailers are not very good at knowing the exact quantities that exist in their warehouses and stores (2004). The recorded quantities in their inventory management systems are only rough estimates of the real inventory quantities. In their study, it is explicitly shown that the inventory accuracy of one of the well-known retail firm's store is only 51% on average. One more measurement of inventory inaccuracy is used in this study. According to this measurement, the inventory record of a SKU is considered accurate if it agrees with the actual stock within the interval of 5 units deviation. Under this measurement the average accuracy of the same firm is 76%. In another study, it is found that median of 3,4% of SKUs were not found on the sales floor although inventory was available in the store (Tellkamp and Fleisch, 2004). According to a survey data, internal and external theft, administrative errors and vendor fraud accounted for an estimated 1,8% of sales in US retail industry in 2001 (Hollinger and Davis, 2001). National Supermarket Research Group (NSRG) survey estimates that internal and external theft, receiving errors, damage, accounting errors and retail pricing errors amount to

2,3% of sales that reflects a much higher percentage of profit (Tellkamp and Fleisch, 2004). One of the important reports related with US retail industry is published by University of Florida. In this industry-wide empirical research, it is presented that an average stock loss amounts to 1,75% of sales occurs annually for 118 retailers from 22 different retail markets. The same situation is also valid for Europe. According to an extensive report prepared by ECREurope in 2001, on a sampling of 200 companies with dominant shared of consumer goods industry in Europe, 1,75% of sales are lost annually for the retailers (Kang and Gershwin, 2004). A recent report published by ECREurope also shows that shrinkage rates were 0,56% for manufacturers while 1,75% for retailers (Atalı et al, 2005). In the same study it is also presented that US retailers suffered a $31.3 billion loss due to shrinkage in 2002. In one of the whitepapers prepared by IBM, the same amounts are mentioned as approximately $30 billion annually loss in US retail industry (Retail on Demand Solutions, IBM). Andersen Consulting (1996) estimates that sales lost due to products that are present in storage areas but not on the selling floor amount to $560-$960 million per year in the US supermarket industry. Further more, several surveys show that a significant number of customers leave retail stores because they cannot find the products for which they are looking (Emmelhainz et al., 1991; Andersen Consulting, 1996; Gruen et al., 2002; Kurt Salmon Associates, 2002). Ton (2002), point out that even though the application of algorithms select the appropriate stocking quantity and appropriate store assortment, the right product may be still unavailable to retail customers. For example, audition of 50 products at ten different stores yielded that only 16% of the stockouts could be attributed to statistical stockouts (Ton, 2002). Instead, 24% of the stockouts were due to inventory record inaccuracy, discrepancies between the recorded and actual on-hand inventory quantity, and 60% were due to misplaced products, products that were physically present at the store but in locations where customers could not find them. For the sake of visualization, two histograms shown on Figure 2.1 and Figure 2.2 demonstrate the absolute value difference between system and actual inventory measured in units and the fraction of products that are not available on the sales floor (DeHoratius and Ton, 2006). These empirical studies motivated some authors to study on the effects of inventory inaccuracies.

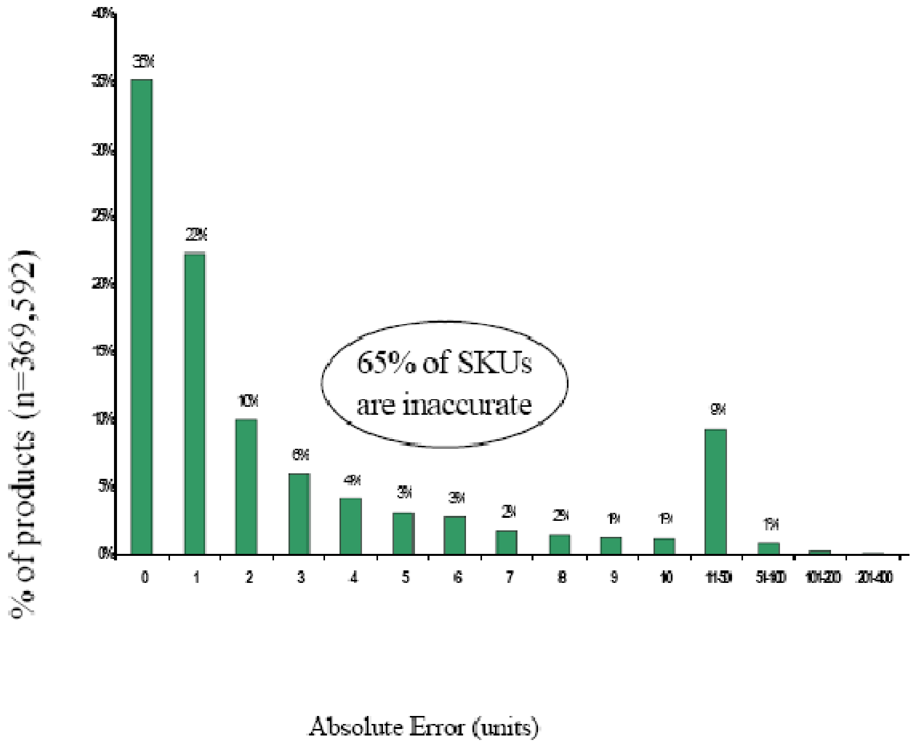

Figure 2.1 : Histogram of the absolute value difference between system and actual inventory measured in units

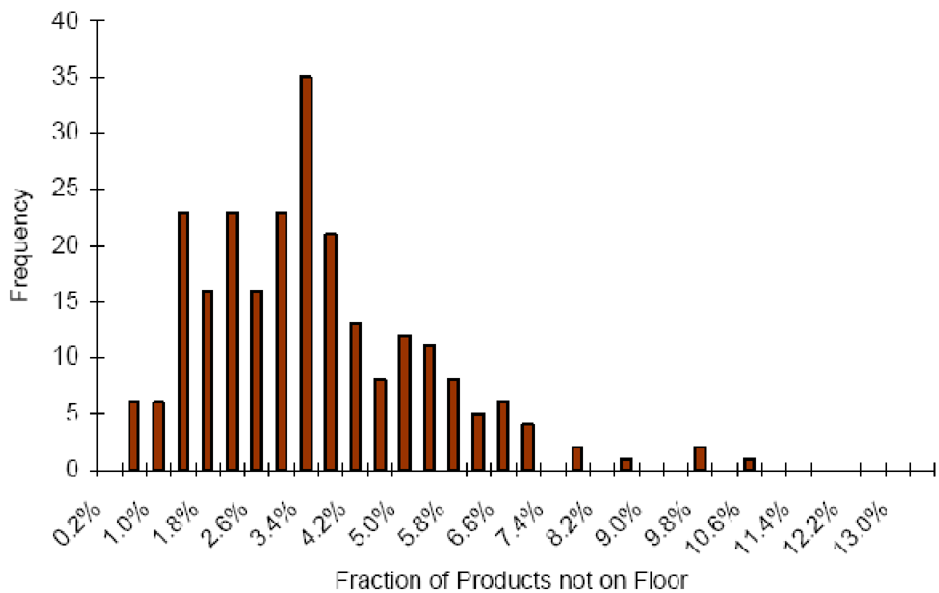

Figure 2.2 : Histogram for the fraction of products that are not available on the sales floor.

Before the detailed analysis of related articles in post-RFID literature, it would be more effective to sum up some fixed definitions related with inventory inaccuracy literature. First of all, the causes of inventory discrepancy between the actual stock level that physically exist in warehouse and inventory records in information system are discussed variously. Atalı et al. (2005), classifies the main sources of discrepancy in three groups: shrinkage, misplacement and transaction errors. This classification is one of the most accepted one in inventory inaccuracy literature. According to this categorization, (1) shrinkage that contains theft and undetected spoilage, (2) transaction errors that contain scanning errors during sales and receipt of shipments and (3) misplacement which causes the distinction between physical and sales-available inventory are the main sources of inventory discrepancy. All of the causes have different properties and effects on system that should be considered during the modeling process. Misplacement is one of the most challenging factors in respect of the modeling studies because of reducing the sales-available inventory during periods between inventory counts, and then increasing the level after an inventory count. Misplacement reduces the level of sales-available inventory, but at the same time leaves physical inventory unchanged. DeHoratius and Ton classifies the whole discrepancy factors such as misplacement and inventory record inaccuracy (2006). Despite causing the same effect on the inventory record (cause to make the inventory record more than available one) with shrinkage, it actually differs by making the inventory manager to take the holding cost for misplaced item into account. As mentioned before, shrinkage also affects the physical and sales-available on-hand inventory but leaves the inventory record unchanged. In opposition to misplacement, the loss due to shrinkage can not return back to inventory after an audit. Instead of holding cost, retailer incurs the direct cost of shrunk items. Transaction errors is the one that only affects the inventory record but leave the physical inventory unchanged. Transaction errors also affect the recorded inventory by two ways contrary to misplacement and shrinkage. The inventory record are always higher than the real quantity in store when shrinkage and misplacement errors occur, but when transaction errors occur, the recorded level may be more or less than the physical level. This feature also causes some difficulties during the modeling phase. However, as it will be seen in the next part of the detailed review, most of the studies don’t consider these causes all together. The authors mainly chose one of these subjects and focused only on it. Atalı et al. and Tellkamp & Fleisch are one of the authors who take into consideration of these three causes together in their research. Tellkamp and Fleisch (2005), classify the causes in a different way. According to their simulation model, the main causes of inventory discrepancy are: incorrect deliveries, misplaced items, theft and unsaleables. Incorrect deliveries refer to the undetected deliveries with fewer quantities that make the inventory record level seem more than the actual level (this error is analyzed under the title of “random yield problem” in inventory literature). Misplacement and theft discrepancy factors have the same meaning with the classification of Atali et al. (misplaced and shrinkage). Unsaleables are items that have been damaged during the handling process or have exceeded their shelf life (undetected spoilage). Kang and Gershwin (2004) discussed the main reasons of inventory errors in the IT system under the following titles; stock loss, transaction error, inaccessible inventory and incorrect product identification. Stock loss refers to shrinkage. Transaction error is the same with the previous mentioned. Inaccessible inventory term defined as the inventory that is not available because they cannot be found (such as misplacement). Incorrect product identification is the cause that takes root from wrong labeling.

The reasons that make inventory records different from actual inventory finally affect the replenishment process causing wrong order quantities or untimely triggering. The breakdown occured in the replenishment process causes decrease in revenue at the final stage. Figure 2.3 shows the error chain of discrepancy factors. The results obtained from searches show that, the chain reaction (i.e. wrong replenishment time and quantity, and out-of-stock) caused by inventory discrepancy is higher than the direct loss incurred by retailer (Kang and Gershwin, 2004).

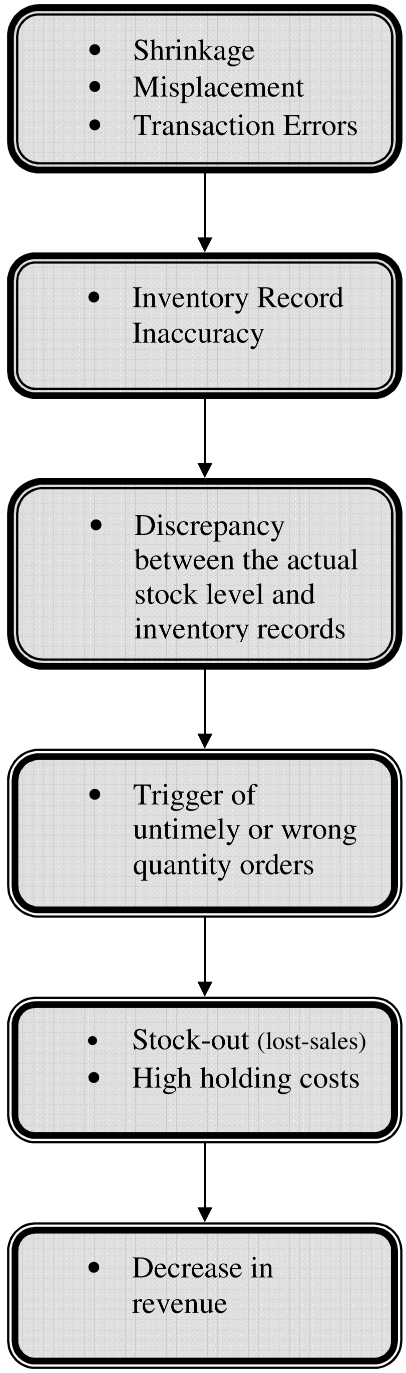

Figure 2.3 : The error chain of inventory discrepancy

Hereafter, we try to classify and analyze the studies which are focused on inventory inaccuracy issue after the development of RFID technology in detail. There are different classification criteria of these related works. One classification was made by Lee and Özer (2005) in terms of the implication of inaccuracy causes to the proposed model. According to this classification, there are three groups. In the first group, only transaction errors are considered as the reason of inventory inaccuracy. For the second group, only shrinkage errors are included by the proposed models. And finally for the third group, misplacement, shrinkage and transaction errors all considered together for modeling phase. We can enhance this classification by adding one more group of proposals that focus only on misplacement error factor. For instance, the paper titled "Analysis of the impact of the RFID technology on reducing product misplacement errors at retail stores" is one of the studies that just focused on misplacement error factor as a discrepancy factor (Rekik et al., 2006a).

The second classification criteria was first presented by DeHoratius et al. (2005), and then developed by Gel et al. in 2006. They grouped the actions of companies (and also the aims of proposals of scientific papers) into three categories; according to their response to inventory inaccuracy problem: (1) prevention, (2) correction, and (3) integration. In our study, this classification is amplified by updating with the recently published papers. Studies that focus on prevention strategies aim to reduce or eliminate the root causes of inaccuracies. This can be achieved by different kinds of methods such as education of workforce, redesign of warehousing processes or by RFID deployment for automatic inventory management systems. As one might guess, the methods of this first category are more costly than the others. Correction category mainly consists of auditing policies which is well-investigated by the early inventory inaccuracy literature. Finally the third category, integration, covers the use of inventory management strategies which takes into consideration the inventory record error factor and also incorporate this discrepancy into the decision making process by stochastic modeling tools.

Regarding these two classification types, the studies related with inventory inaccuracy issue that are written after the development of RFID, are examined in detail on the following part.

One of the studies in this related literature that focus on the evaluation of "economic impact of inventory record inaccuracy" was written by Gel et al. (2006). To reach the objective (i.e. the evaluation of the economic impact), they used a simulation model. Gel et al. consider a continuous-review inventory system with execution errors, controlled by (Q, r) inventory control policy. Costs are considered over a finite-horizon. There are two main contributions of the paper to the literature. Firstly, they paid attention to the modeling of "transaction errors" when determining the economic impact of inventory inaccuracy. As opposed to other studies which mainly focus on shrinkage factor (Gershwin & Kang, 2004 and de Kok et al., 2006), they isolated transaction errors and focused on it. As a result of focusing on this discrepancy cause, their approach allows positive and negative record errors because of the characteristics of transaction errors. Transaction errors were defined previously as incorrect accounting of inbound shipments because of the mistakes in product identification or labeling to errors occurred at cash registers. Consideration of execution errors results both over-recording and under-recording of demand. Second main contribution is related with the consideration of correction epochs. They modeled correction epochs which are the actions giving the chance of fully or partially correcting records. They define two events that signal the existence of errors and refer them as "correction opportunities". The closest study in literature in respect of correction opportunities is Mosconi et al. (2004). Due to the over-registering (actual inventory higher than the recorded one) and under-registering (actual inventory higher than the recorded one), the inventory system does not behave as anticipated (i.e., a customer demand may be lost when IT system shows positive inventory or vice versa).

In the paper, the correction opportunities are grouped as two;

- CO-1: The store experiences a stock-out while the inventory record shows positive inventory (over-registering situation). This situation is an opportunity because of the customer feedbacks. Stock-out can be realized and the system can be corrected by this feedbacks.
- CO-2: A customer finds an item and brings it to the cash register when the inventory record for that item is zero. Contrary to CO-1, CO-2 provides only a partial correction, not completely.

One reason of considering these two correction opportunities is that the model reflects the behavior and also the performance of the inventory system correctly such as in a real system. Second reason for the existence of correction actions is having an important effect on reduction of errors. One of the most important features of these correction opportunities is that they do not have cost such as the other correction actions like costly audits.

If we sum up the acquirements of the paper:

- It is explicitly shown that the economic impact of inventory record inaccuracies caused by transaction errors can be more significant than estimated. This result is obtained by considering a simple model with deterministic demand. Even for the simple deterministic case considered, determination of optimal ordering quantity becomes more complex according to the situation that there are any incorrect data. It makes sense that whenever the execution errors get higher, expected cost of the system also increases. From this respect, the analytical results of experimentation support the empirical research of Raman et al. (2001) that says 10% of profit is lost due to inaccuracies. Related with all this findings, one more acquisition is that the economic impact of inventory record inaccuracy increases for some systems where small order sizes and low reorder levels exist.

- After observing the great impact of inventory record inaccuracies on the behaviour and performance of the system, the second insight obtained from the paper is; as the level of errors increased, the variability of inventory related costs also increases. This also means that the risk of the system increases.

- When a customer demands multiple units of products (called batch demand), the economic effects of execution errors become more prominent.

- If it is aimed to make the performance of the inventory system higher, it is needed to increase the number of audits more and more (i.e., low frequencies of inventory audits has only small effects in controlling the impact of record inaccuracies. However, it is also known that these frequent audits are costly to implement, so not always feasible.

- It is also highlighted that prevention and correction actions such as training of workers, better labeling of products and improved shelving schemes, reduction on the number of SKUs minimize errors.

The presented paper does not focus on the valuation of the new auto-id technology, RFID. In this context, the paper can be classified in the group of correction strategies.

Kang and Gershwin (2004) investigated the information inaccuracy problem in inventory systems and tried to find answers to the following questions; what the inaccuracy is, what its causes are, what kind of impacts it has on the performance of the inventory system. Furthermore, various methods developed to cope with the inventory inaccuracy problem are presented and evaluated. The paper can be analyzed by two main parts. For the first part, the impact of inventory inaccuracy on the performance of inventory management system is investigated under the continuous (Q, R) policy. The second part addresses the question of what can be done to deal with the inventory record error and also how to improve the performance of the system. Hence, some well-known compensation methods are analyzed.

In the first part, the inventory system is analyzed under two situations. For the first situation (stochastic simulation model), single item inventory model is analyzed under the assumptions of demand and stock loss assumed stochastic. It is assumed that demand for purchase is distributed according to normal distribution and stock loss assumed to be distributed according to Poisson distribution. For the second situation it is assumed that demand and also the stock-loss are constant. The main reason of the inventory record error is shrinkage for both of the situations. For both of the situations, after conducting the simulation runs, the same result gained with Gel et al. (2006) which is inventory inaccuracy impacts the replenishment process more than anticipated. One of the most important contributions of the paper is to show explicitly by analytical results that lost of items directly charged to retailer is only a small portion of the total impact. The cost of lost sales has larger proportion of the loss. In the second part firstly compensation methods are presented and then evaluated according to simulation results and empirical evidences. The presented methods are safety stock, manual inventory verification, manual reset of inventory record, constant decrement of the inventory record and new auto-ID technology, RFID. According to performance comparisons of each method, it can be said that decrementing the inventory record performs remarkably well. Although the deployment of RFID system with the inventory tracking system is able to attain the lowest inventory for any given stock-out, when it is also considered the cost of implementing RFID system, it would be a challenging decision. One of the main assumptions of the paper is to reset the all errors after RFID implementation. But Lee and Özer (2005) report that between 10% and 66% of the original shrinkage observed is reduced after implementing RFID technologies. Kok et al. (2006) took this information into consideration. On the contrary, Kang & Gershwin and Rekik et al. (2006a, 2006b) assumes that after RFID adoption, there would be no error occurred in the system.

The aim of the paper that is presented by de Kok et al. (2006) is to determine the additional cost per time unit caused by shrinkage and with this additional cost determination; they try to adopt the inventory policy by including both the shrinkage fraction and impact of RFID technology.

One of the main contributions of the paper to the related literature is; they derive an analytical expression about RFID implementation cost by determining break-even tag prices. The authors obtained these break-even tag prices by comparing the situation with RFID and the one without RFID in terms of cost. Through the determination of these tag prices, a manager can determine the maximum amount of money for RFID investment. In spite of being one of the important studies in this scarce area, the paper presents insufficient cost values, because of not taking into account the hardware, software and also implementation costs. To our knowledge, there is no comprehensive study in literature that considers all earnings and costs of RFID investments.

De Kok et al. (2006) also show that break-even prices are highly related with:

- The value of items that are lost
- The shrinkage fraction
- The remaining shrinkage after implementing RFID.

These factors are important due to the lack of analytical RFID implementation studies.

Assumptions made by the authors for the modeling phase are; inventory control system is a periodic-review base stock policy (R, S), demand not remedied from shelf is backordered. They only considered the shrinkage factor determined by Atalı et al. (2005). A fraction of demand per period (π) disappears due to shrinkage and the disappearance process is a pure Poisson process. One of the main assumptions of the paper that makes it distinct in this sub-literature is the assumption that RFID systems do not vanish all of the shrinkage errors in the system (in contrary to Gershwin at al. and Rekik et al.). They make their assumption depending on the study of Lee and Özer (2005). They consider this insight in their study by determining α value representing the fraction of disappearance (α) that cannot be prevented from disappearing. In their experimental design, four different fractions of α is taken into account.

With the help of some parameters such as price of product, annual interest rate, auditing cost etc., they evaluated the total relevant annual cost of both of two situations, with RFID and without RFID. By comparing these two total cost functions (in the case of inspection cycles are equal), they obtained an equation that contains parameter values. To analyze the effects of different parameters on the expected break-even prices, they performed a closed experimental design. As mentioned before, after executing the design for 62.208 observations, it is founded that the value of item (ν), theft fraction (π) and the remaining theft fraction (α) have important effects on break-even prices.

Besides quantifying the potential gains of using RFID technology and also presenting a detailed cost-benefit analysis that helps managers in respect of RFID investments, it is also presented that the inspection cycle has an important effect on the break-even prices. So, one might guess that they both take into consideration the prevention (RFID technology implication) and correction (determining the optimum frequency of audits) strategies together.

Kök and Shang (2004) aimed to minimize total inspection and inventory costs. To reach this aim, they developed an effective policy, the inspection adjusted base-stock (IABS) policy. According to this newly-developed policy; an inventory manager decides to make an inspection whether the recorded inventory level is less than a threshold level and gives order up to a base-stock level that depends on the inspection decision. They consider a single-product, single stage inventory system in which inventory records are inaccurate. According to their replenishment system, the manager decides the inventory inspection and replenishments at the beginning of each period. According to their assumptions, in each period random transaction errors occur that cause discrepancy between the recorded and actual inventory. The errors are accumulated until an inspection is conducted. The total error is the accumulated error since the last audit. Customer demand is also assumed stochastic for each period.

There are two main decision problems for the inventory manager; (1) decisions of conducting or not conducting an inspection at the beginning of the period, (2) decision of order quantity. Their main objective is to find an effective method for these two decision problems together. Both of them are important decisions due to the cost of audits and also costs of ordering wrong quantities (i.e. lost sales or carrying extensive quantities). As mentioned before, according to the IABS policy for single period decisions; there exists a threshold level that helps to determine whether conduct or not an inspection. If the initial inventory record is less than or equal to this threshold level, the manager should perform the inspection. And the optimal base-stock level is adjusted to the decision of conducting an inspection (i.e. the optimal order quantity depends the existence of inspection; with inspection, it is smaller than without inspection).

As the optimal policy is only valid for single-period decisions; the authors extend the optimal solution to multi-period problems by constructing two heuristics. These heuristics are lower-bound (LB) and myopic (MP) heuristics. According to their numerical experimentations that aim to find the quality of these heuristics in respect of optimal cost and to quantify the effectiveness and total cost due to inventory record inaccuracy; it is proved that the lower-bound heuristics gives the optimal policy within the interval of 0,4% deviation on average (near-optimal). But myopic (MP) heuristics is not effective as well as the other -LB- heuristics.

Other than finding optimal policy for the decisions of both inspection time and order quantity for single period and near optimal policy for multi-period, they also compared the potential benefits of implementing advanced inventory tracking technologies such as RFID, and the LB heuristics. Comparison is made with the RFID system, LB heuristics implemented system and the base case in which any corrective and preventive activities engaged. The results showed that RFID systems can reduce cost by an average of 11,5% and LB heuristics can reduce the costs by an average of 5,5%. This also means that total potential benefit of using advanced systems or preventive strategies is 11,5% of total cost, for corrective strategies such as LB heuristics is 5,5%.

However obtained values do not contain the investment costs of advanced inventory tracking systems. Therefore the authors claimed that with lower implementation costs and significant progress rates on the system, corrective strategies are feasible for implementation. Especially for companies not able to implement costly preventive approaches such as RFID systems, corrective strategies present effective solution for inventory inaccuracies.

The most commonly-used automatic-identification technology in the world is doubtlessly barcode technology. But it can not be said that it works without errors. The paper presented by Şahin et al. (2005) considers that barcode system used in inventory control system may sometimes be erroneous. By this assumption they developed a single-period model. The chain that they consider contains three stages which are supplier, wholesaler and retail stores. They take into consideration a single seasonal product with a short product life-cycle and a short selling season. The main decision maker in the model is the wholesaler. Wholesaler acts as a newsvendor. Classic Newsvendor model does not take into account the errors occurred on the information flows during the physical flow. In other words it does not take into account the misalignment between physical and information flows. But in real world, the errors can be occurred in many stages of the chain. When it is focused on the flow of goods throughout the supply chain stages, it can be seen that there can be some misalignments. These places where it is possible to happen a misalignment are;

- When the wholesaler orders the amount of Q to meet the demand of retailer, supplier delivers the quantity that is ordered. But at this stage there may be some errors due to unreliable supplier policies (vendor fraud), thefts that happened during transportation phase or even in the inbound part of warehouse of wholesaler. So, the ordered quantity Q may not be equal to the physical inventory in the warehouse. This can be denoted as $Q \neq Q_{ph}$.

- The second place that some discrepancies may be occurred is registering the amount that exists physically in warehouse to the information system. The available information of system quantity may differ from Q_{ph} due to some errors occurred during the data collection process. So, physical quantity (Q_{ph}) may not be equal to the available information system quantity (Q_{is}), $Q_{is} \neq Q_{ph}$. Hence, by these two stages where errors may be occured, it can be said that the ordered quantity and the quantity on the information system may be differed. During these differentiation stage, there are two random variables determined.

$Q_{ph} = Q_a$ (random variable function of Q)

$Q_{is} = Q_b$ (random variable function of Q_{ph})

With these two random variables representing errors, the model contains three random variables (with the uncertainity of demand) that make the analysis process of the system more complex.

As an analysis of a three random variable model is very tedious, the authors decided to decrease the number of random variables in the model by assuming that no errors occurred during the physical flow phase ($Q=Q_{ph}$). Thus, the only place that an error occurres is the information flow stage ($Q=Q_{ph} \neq Q_{is}$). The second reason of the elimination of the random variable Q_a is that the issue is a well-investigated one which is known as "random yield problem". The authors preferred to focus on the other situation which has not been developed well yet.

In the paper two different situations are analyzed in detail. For the first situation, wholesaler is unaware of the errors occurred in data collection process. So, all the decisions are independent of errors. The trust level to barcode system is 100%. This situation is also called the "ignorance strategy" (Lee & Özer, 2005). For the second situation the wholesaler is aware of the errors occured during the registration of products to the information system. In this situation they tried to determine the behavior of error by using mean error rate and variance.

The paper only analyzed these two situations, not a third choice such as the choice of a more effective auto-id system such as RFID system. The first situation is a classical newsvendor problem. For the second one, wholesaler tries to optimize its system by taking into consideration the errors.

The analysis of the second situation is conducted in detail (determination of the expected cost function and optimal order quantities) and the penalty cost of managing error including systems is evaluated by comparing the systems, not considering the errors.

The cost function defined in this study contains three parts;

- Cost of products unsold at the end of the season
- Shortage cost of demands that are rejected by the wholesaler

- Cost of accepted demand by wholesaler that could not be satisfied effectively

Normally a classical newsvendor problem contains the first two one. The third cost account is specific for this study. Denoted by u_2, unit cost means the time consumption of the employees that is spent to find a good on shelf recorded in the information system but actually does not exist on the shelf and also the loss of reliability of the retailer to wholesaler. So this unit cost is also important for real life applications.

It is assumed that demand and record errors are uniformly distributed.

According to the unit cost multipliers h, u_1 and u_2, unit shortage type 1 ($k=u_1/h$) and m unit shortage type 2 ($m=u_2/u_1$) are defined. The values of these multipliers can vary according to industry and product category. But in the paper it is assumed that $0.5<k<5$ and $1<m<5$.

The analysis of the optimal policy is achieved for three cases which are, mean of error is smaller than zero, equal to zero and greater than zero.

The analysis of the three situations is done with the three principles which are defined before. These principles are;

- Different configurations correspond to a specific position between the distribution of Q_{is} and D are taken into account. For each configuration there are different intervals of realization of ε, due to randomness of errors made.
- For each configuration, expected cost function are obtained.
- By differentiating the cost functions according to Q leads some analytical solutions for the optimal policy.

For each case (mean of error is smaller than zero, equal to zero and greater than zero), analysis of the cases are conducted firstly by determining the configurations then according to each configuration optimal order quantity and optimal cost functions are defined with respect to k, m and standard variation of demand and error.

After getting the optimal order quantity and cost functions, the sensitivity analysis of Q* varying values of standard variation of error and m values are done. Finally a ratio (R) is defined to evaluate the benefit of eliminating inaccuracies and analysis of the variations with error parameters and u_2 are done.

In conclusion, the analysis of penalty of costs in each situation is conducted.

The research proposed by Şahin et al. (2005) was developed by Rekik et al. in 2006a, by adding the evaluation of a third situation in which the retailer deploys an advanced automatic identification technology. The main issue in the paper is the inventory inaccuracies caused by misplacement errors. A newsvendor model is developed in a simple way. For a given quantity of products ordered from the supplier, only a random fraction is available for sales. As mentioned before three situations are analyzed.

They try to provide an analytical expression of the tag of RFID tag costs (not the other hardware, software and implementation costs). But one of the main assumption made in this paper is, after implementing RFID system, the whole system reaches an error purified situation on the contrary to the proposal of Lee and Özer (2005).

In the previously analyzed study, the main decision maker is the wholesaler whose customer is retailer. In this study, the main decision maker is the retailer that is prone to be the end customer.

One of the underlying assumptions in the formulation of the classical newsvendor problem is that there is no misalignment between the physical and information flows, meaning that the retailer operates without execution errors.

In this paper, a θ value is defined as a ratio between the quantity on shelf which is available for sales and the total physical quantity available in the store. In other words θ is the random variable which reflects the effect of misplacement errors on the real quantity which is available on shelf for consumers.

If the order quantity is Q, the available inventory in the store is θ*Q. This means (1-θ)*Q is the missing part of the order. The authors assume that when RFID is implemented to their warehouse management system, θ is equal to 1 (no remaining misplacement nor shrinkage).

Additionally, there is one more assumption about missed goods. At the end of the selling season all lost items are found and if there are unsold goods also, they are together sold by a salvage value s.

The sequence of events is basically as follows: The retailer receives its order quantity (Q) – there is no lost in these stage. Due to the internal errors occurring in the store, the available quantity of goods for the customer is q*Q.

The actual demand is satisfied from the available inventory on shelf. At the end of the season, remaining items on shelf and also the misplaced items are sold.

After the profit and expected profit function is calculated, differentiating the expected profit function with respect to Q leads to find the optimal Q*.

Previously discussed study is extended to a two-staged supply chain by Rekik et al. (2006b). Two supply chain actors, manufacturer and retailer considered for this model. There are two situation compared with each other. In the first situation it is assumed that two supply chain actors are aware of errors and optimize their operation with this awareness. In the second situation RFID technology is deployed by both of the actors. For both of these situations, there are three scenarios examined. These scenarios are: centralized scenario in which it is assumed that there is one decision maker that aims to maximize the profit of entire chain, decentralized uncoordinated scenario where it is considered two different decision-makers and each of them aims to optimize his own profit function, and finally decentralized coordinated scenario that only differs from the second one by the cooperation between entities. Because of being a multi-stage research, game theory and pay-back contracting issues are also discussed. One of the main assumptions which do not take place for the other studies is related with these multi-stage structures. According to this assumption, manufacturer shares the loss of retailer caused by uncertainty. However, it does not take responsibility for the misplaced items in retail stores. It is shown explicitly that coordinating the channel can lead to important savings may not necessitate the deployment of any advanced inventory tracking system with high investment costs.

One of the studies that took place in inventory inaccuracy literature is presented by Fleisch and Tellkamp (2004). Similar to the study of Lee and Özer (2005), the authors take into consideration more than one cause of inventory discrepancy. They constructed a simulation model in a three-echelon supply chain environment and with one product. They compare the base case and modified case that eliminates inventory inaccuracy by the alignment of physical and information system inventory in each period time. The main result that gained from simulation is on the same way with the other studies (Kök and Shang (2004); Gel et al. (2006) and de Kok et al.(2006)).

Atalı, Lee and Özer (2004) characterized three different kinds of factors that result in inventory discrepancy. They consider all of these discrepancy causes in their model. The model has been constructed on the basic assumptions of a finite horizon, single-item, periodic-review inventory problem. They showed how an optimal inventory control can be designed in the presence of record errors, using only statistical estimates, such as their distributions, of the demand and error factors. The model is also used to assess the value of having visibility of inventory and the elimination or reduction of some of the causes of inventory discrepancy. By courtesy of the comparison made between lower and upper bound models, the value of RFID is explicitly presented under two factors; visibility and prevention.

DeHoratius et al. (2005) contributes this area of study by considering an intelligent inventory management tool that accounts for record inaccuracy using a Bayesian belief of the physical inventory level. They show that a probability distribution on physical inventory levels is a sufficient summary of the past sales and replenishment observations, and that this probability distribution can be efficiently updated in a Bayesian fashion as observations are accumulated. They conclude by a Bayesian inventory management record that accounts for record inaccuracy is a viable alternative to the traditional point estimate inventory record frequently used in inventory management models and in practice.

Gaukler et al. (2006) also focused on calculating the tag prices that make item-level tagging of RFID economically feasible, such as Kok et al. The other focus of their study is to analyze the impact of item-level RFID tagging on the decentralized supply chain. By considering a supply chain with one manufacturer and one retailer, through an analytical model, they describe the dilemma of a supply chain in which costs and benefits of a collaborative technology are distributed in a asymmetric fashion. They investigate the relative market power of the retailer vs. the manufacturer affect the conclusion of sharing of tag cost amaong the supply chain partners.

In another study of Gaukler (2007), he characterizes some of the operational benefits of item-level RFID tagging in a retail environment under the assumption of multiple replenishment and sales periods. The retailer's goal is to reduce out-of-stock situations at the shelf to avoid lost sales. The role of item-level RFID is to alert the retailer to impending stockouts at the shelf so that product can be replenished in time from the backroom to the shelf. This approach points out that the "future store" project of Metro will become a practical application soon (Roberti, 2003).

In the research done by DeHoratius and Raman (2004), drivers of inventory record analysis is examined through the data obtained from physical audits of Gamma Corporation's 37 stores. This analysis is done with the dependent variable IRI (Inventory record inaccuracy) and independent variables of the cost of the item, unit sales per store and shipment route. Due to the multi-level structure of this data, a series of hierarchical linear models have been fit.

In conclusion, the common point of the studies proposed in the inventory inaccuracy area is that inventory discrepancy causes huge amount of losses than anticipated. There are many compensation methods developed to reduce this loss. The two main methods are the awareness strategy that leads to redesign of the inventory management systems with stochastic modeling techniques (also called "decrement of inventory record" by Kang and Gershwin) and deployment of an advanced inventory tracking system, RFID. Although the results of providing more accurate tracking capability with RFID systems are reached, there is still a gap in the literature in respect of extensive valuation models that takes into account all cost and benefit factors together. For most of the studies, redesign of the inventory management system with stochastic modeling techniques is one of the most recommended solutions for inventory record errors (Gel et al. (2006); Kök & Shang (2004); Şahin et al. (2007); Kang & Gershwin (2004)). However, for huge inventory keeping facilities, this method can also be hardly-implemented. In spite of stressing the risk of RFID investments in literature, it can easily be observed that the deployment of RFID technology is expanding rapidly. According to a recent survey of retail chief information officers, 55% of respondents reported they already had plans to implement RFID capabilities within three to five years (MIT Auto-ID Center, Industry Reports – IBM, 2004). In this context, it is predicted that the next question that will preoccupy the retailers' mind will be the decision problem of item, case and pallet-level tagging.

Table 2.1 : Classification of research according to three criteria

Author & Year	Title	Inventory Error Factor	Kind of Actions	Methodology
Gel, E., Erkip, N. & Thulaseedas, A. 2006	*Analysis of Simple Inventory Control with Execution Errors: Economic Impact under Correction Opportunities*	Transaction Errors	Correction	Simulation model on a continuous review (Q, r) inventory control policy over a finite horizon
Kang, Y. & Gershwin, S. B. 2004	*Information Inaccuracy in Inventory Systems – Stock Loss and Stockout*	Shrinkage	Prevention	Stochastic simulation model over a continuous (Q, R) policy of a single item inventory model
Kok, A. G., Donselaar, K. H. & Woensel, T. 2006	*A Break-even Analysis of RID Technology for Inventory Sensitive to Shrinkage.*	Shrinkage	Prevention and Correction	Periodic system base stock policy (R, S) and closed experimental design
Kök, A. G. & Shang, K. H. 2004.	*Replenishment and Inspection Policies for Systems with Inventory Record Inaccuracy*	Transaction Errors	Prevention and Correction	Inspection adjusted base-stock (IABS) policy
Sahin, E., Buzacott, J. & Dallery, Y. 2005.	*Analysis of a Newsvendor which is Subject to Errors in Inventory Records.*	Transaction Errors	Correction	Revised single-period model with *unit shortage type 2* cost factor
Rekik, Y., Sahin, E. & Dallery, Y. 2006.	*Anaysis of the Impact of the RFID Technology on Reducing Misplacement Errors at Retail Stores.*	Misplacement	Prevention and Integration	Single period model revised with misplacement error factor
Rekik, Y., Jemai, Z., Sahin, E. & Dallery, Y. 2006	*Improving the Performance of Retail Stores Subject to Execution Errors: Coordination Versus RFID Technology*	Misplacement	Prevention and Integration	Single period model revised with misplacement error factor under a multi stage problem setting
Flesich, E. & Tellkamp, C. 2004	Inventory Inaccuracy and Supply Chain Performance: A Simulation Study of a Retail Supply Chain	Transaction Errors, Misplacement & Shrinkage	Prevention and Correction	Simulation model under a three-echelon supply chain environment
Atali, A., Lee, L. & Ozer, O. 2005	*If the Inventory Manager Knew: Value of RFID under Imperfect Inventory Information*	Transaction Errors, Misplacement & Shrinkage	Prevention	Finite horizon single item periodic review inventory problem utilized by dynamic programming
DeHoratius, N. Mersereau, A. J. & Scharage, L. 2005.	*Retail Inventory Management When Records are Inaccurate*	Transaction Errors	Integration	Bayesian Inventory Control Management
Gaukler, G., 2007.	*The Impact of Item-level RFID on the Retail Supply Chain:Product Availability and Demand Forecasting*	--	Prevention	Multi Period Model with lost sales under a multi stage environment

3. MATHEMATICAL MODEL

In this chapter of the study, some mathematical modeling techniques are used to decide on the optimum level of RFID tagging on inventories. As mentioned before, this model is a newsvendor model that utilizes single period settings. A retail store that makes its ordering decisions within this one-period newsvendor framework is considered. Under the single period framework, we take into consideration error factors that cause inventory inaccuracy.

Since the main aim of this study is to determine the level of RFID tagging on inventories considering the labor cost, overage-underage costs and the costs caused by two error factors called misplacement and shrinkage, the constructed model contains all of these factors.

3.1 Notations

The notations used throughout this study are presented as the following:

r = the unit product selling price (retail price)

w = the unit product purchase cost (wholesale price)

s = the unit product salvage price

t = the unit RFID tag cost

l_i = labor cost parameter on the ith level of tagging (i = 1: no tagging; i = 2: pallet level tagging; i = 3: case level tagging; i = 4: item level tagging)

θ = the random variable representing the rate of remaining units of item after misplacement occurs

β = the random variable representing the rate of remaining units of item after shrinkage occurs

x = the random variable representing demand

f = pdf characterizing x

g = pdf characterizing θ

j = pdf characterizing β

F = cdf characterizing x

μ_x = the expected value of x

σ_x = the standard deviation of x

μ_θ = the expected value of θ

μ_β = the expected value of β

Q_i = the order quantity in case i (i= 1, 2, 3, 4)

Q_i^* = the optimal value of Q_i

π_i = the expected profit function in case i (i= 1, 2, 3, 4)

π_i^* = the maximum value of π_i

p = the number of items in a pallet (pallet capacity)

c = the number of items in a case (case capacity)

y = variable representing level of tagging, $y \in [0,1]$

$L(y)$ = function characterizing labor cost with respect to y

$\beta(y)$ = function characterizing the rate of remaining items after shrinkage with respect to y

$\theta(y)$ = function characterizing the rate of remaining items after misplacement errors with respect to y

$C(y)$ = function characterizing tagging cost with respect to y

3.2 Assumptions

The first assumption of the model: It is assumed that there are two main error factors affecting the inventory level. These are shrinkage and misplacement. In order to model these two error factors; we define two inventory discrepancy factors through two random variables that build a one to one correspondence between the accuracy of the inventory level and rates. These are;

$$\theta = \frac{effective\ \ quantity\ \ that\ \ is\ \ on\ \ the\ \ right\ \ shelf}{ordered\ \ quantity\ \ at\ the\ beginning\ \ of\ \ the\ season}$$

$$\beta = \frac{effective\ \ remaining\ \ quantity\ \ after\ \ the\ occurrence\ \ of\ \ shrinkage}{ordered\ \ quantity\ \ at\ \ the\ \ beginning\ \ of\ \ the\ \ season}$$

Second assumption of the model: It is assumed that misplacement and shrinkage occur just after the retailer receives its order. So the available quantity for customers can be calculated at the beginning of the season through θ and β random variables.

Third assumption of the model: It is assumed that θ and β random variables corresponding misplacement and shrinkage errors on inventory are *independent* that also means an item which is misplaced to a shelf may be shrunk as well. In other words the intersection set of two events which are misplacement and shrinkage is not empty, i.e. not mutually exclusive sets.

To calculate the available quantity for customers under these error factors, the number of elements in the set of misplaced or shrunk items are computed, as follows:

Note that $P(A \cup B) = P(A) + P(B) - P(AB)$ for any events A and B, $P(A \cap B) = P(A).P(B)$ for any independent events.

The expected number of the units of any item in the union set of lost ones caused by misplacement or shrinkage $= Q\big((1-\theta)+(1-\beta)-(1-\theta)(1-\beta)\big)$

$= Q - \theta\beta Q$

Then the expected value of available quantity is equal to;

$$= Q - (Q - \theta\beta Q)$$

$$= \theta\beta Q$$

Fourth assumption of the model: It is assumed that misplaced items can be found at the end of the season with the possibility of selling with a salvage value s ($s < w$). Retailer incurs a holding cost for this "misplaced but not shrunk" quantity which is equal to $(1-\theta)\beta Q$.

Fifth assumption of the model: It is assumed that underage cost, in other words opportunity cost, consists of only the difference between selling price and purchasing cost ($u = r - w$), not the *loss of goodwill*.

Under four different cases (no-tagging, pallet-level tagging, case-level tagging, item-level tagging), a profit function is defined as the first step. The labor cost, the cost caused by errors, underage and overage costs that all varies according to the level of tagging are taken into consideration. As tagging level increases; labor cost (per item) and the cost caused by error factors decreases whilst the tagging cost increases (Figure 3.1).

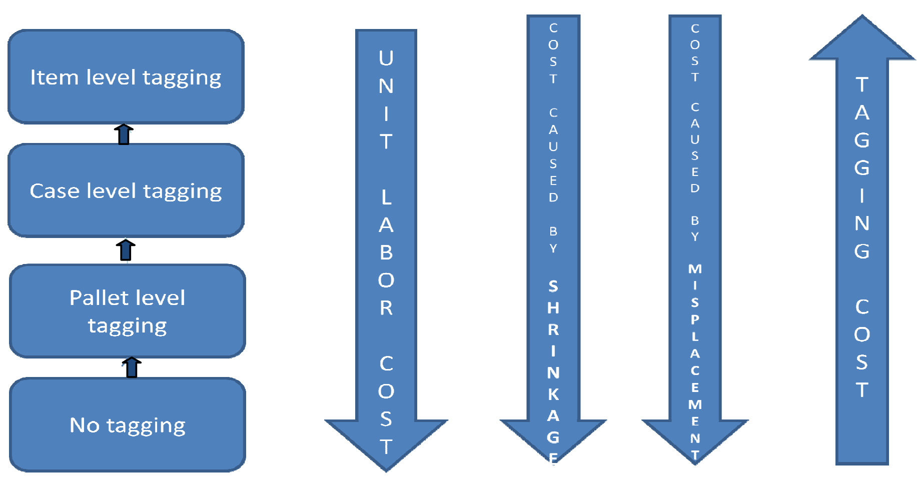

Figure 3.1 : Behaviours of cost factors according to four tagging levels transitions

3.3 Four Different Cases for Tagging Level Decision

3.3.1 Case I (No Tagging Case)

Profit function for no-tagging case contains the direct income of selling units of items, the direct outcome of purchasing the units and the indirect outcomes that are the underage cost caused by the excess demand, the overage costs caused by the higher order quantity which causes difference between available quantity of that single item and the demand of customer for this specific product, overage cost caused by misplaced but not-shrunk quantity found at the end of season and labor cost. The main distinction of the profit function of no-tagging case from other cases is naturally not considerating the tagging costs.

$$profit_1 = rx - wQ_1 - u(x - \theta\beta Q_1)^+ - h(\theta\beta Q_1 - x)^+ - h(1-\theta)\beta Q_1 - l_1 Q_1 \quad \textbf{(3.1)}$$

$$\text{expected profit function}_1 = \pi(Q_1) = r\mu_x - wQ_1 - u\int_{L_\beta}^{U_\beta}\int_{L_\theta}^{U_\theta}\int_{x=\theta\beta Q_1}^{\infty}(x-\theta\beta Q_1)f(x)g(\theta)j(\beta)dxd\theta d\beta$$
$$-h\int_{L_\beta}^{U_\beta}\int_{L_\theta}^{U_\theta}\int_{x=-\infty}^{\theta\beta Q_1}(\theta\beta Q_1 - x)f(x)g(\theta)j(\beta)dxd\theta d\beta - h\int_{L_\beta}^{U_\beta}\int_{L_\theta}^{U_\theta}(1-\theta)\beta Q_1 g(\theta)j(\beta)d\theta d\beta - l_1 Q_1 \quad \textbf{(3.2)}$$

$$= r\mu_x - wQ_1 - u\int_{\beta=L_\beta}^{U_\beta} j(\beta)\int_{\theta=L_\theta}^{U_\theta} g(\theta)\left[\int_{x=\theta\beta Q_1}^{\infty} xf(x)dx - \theta\beta Q_1(1-F(\theta\beta Q_1))\right]d\theta d\beta$$
$$+h\int_{\beta=L_\beta}^{U_\beta} j(\beta)\int_{\theta=L_\theta}^{U_\theta} g(\theta)\left[\int_{x=-\infty}^{\theta\beta Q_1} xf(x)dx - \theta\beta Q_1 F(\theta\beta Q_1)\right]d\theta d\beta - h(1-\mu_\theta)\mu_\beta Q_1 - l_1 Q_1 \quad \textbf{(3.2b)}$$

In order to find the maximum profit value through first order conditions, one should check whether this function is concave or not.

$$\frac{d^2(\pi(Q_1))}{(dQ_1)^2} = -(u+h)\int_{L_\beta}^{U_\beta} j(\beta)\int_{L_\theta}^{U_\theta}(\theta\beta)^2 g(\theta)f(\theta\beta Q_1)d\theta d\beta \le 0 \quad \textbf{(3.3)}$$

Hence $\pi(Q_1)$ is concave in Q_1, when the first derivative of expected profit function is equal to zero ($\frac{d(\pi(Q_1))}{dQ_1} = 0$), the optimum order quantity (Q_1^*) that maximizes expected profit function can be obtained.

Calculations for 1st and 2nd derivative of expected profit function with respect to Q_1 are shown below:

$$\frac{d\pi(Q_1)}{dQ_1} = -w - \frac{d\left(u\int_{L_\beta}^{U\beta} j(\beta)\int_{L_\theta}^{U_\theta} g(\theta)\left[\int_{x=\theta\beta Q_1}^{\infty} xf(x)dx - \theta\beta Q_1(1-F(\theta\beta Q_1))\right]d\theta d\beta\right)}{dQ_1}$$

$$+\frac{d\left(h\int_{L_\beta}^{U\beta} j(\beta)\int_{L_\theta}^{U_\theta} g(\theta)\left[\int_{x=0}^{\theta\beta Q_1} xf(x)dx - \theta\beta Q_1 F(\theta\beta Q_1)\right]d\theta\right)}{dQ_1} - h(1-\mu_\theta)\mu_\beta - l_1 \tag{3.4a}$$

$$= -w - u\int_{L_\beta}^{U\beta} j(\beta)\int_{L_\theta}^{U_\theta} g(\theta)\left[\frac{d\left(\int_{x=\theta\beta Q_1}^{\infty} xf(x)dx\right)}{dQ_1} + \frac{d\left(-\theta\beta Q + \theta\beta Q F(\theta\beta Q)\right)}{dQ_1}\right]d\theta d\beta$$

$$+h\int_{L_\beta}^{U_\beta} j(\beta)\int_{L_\theta}^{U_\theta} g(\theta)\left[\frac{d\left(\int_{x=-\infty}^{\theta\beta Q} xf(x)dx\right)}{dQ_1} - \frac{d\left(\theta\beta Q F(\theta\beta Q)\right)}{dQ_1}\right]d\theta d\beta - h(1-\mu_\theta)\mu_\beta - l_1 \tag{3.4b}$$

$$= -w - u\int_{L_\beta}^{U_\beta} j(\beta)\int_{L_\theta}^{U_\theta} g(\theta)\left[\frac{d\left(\int_{x=\theta\beta Q_1}^{\infty} xf(x)dx\right)}{dQ_1} - \theta\beta + \theta\beta F(\theta\beta Q_1) + (\theta\beta)^2 Q_1 f(\theta\beta Q_1)\right]d\theta d\beta$$

$$+h\int_{L_\beta}^{U_\beta} j(\beta)\int_{L_\theta}^{U_\theta} g(\theta)\left[\frac{d\left(\int_{x=-\infty}^{\theta\beta Q_1} xf(x)dx\right)}{dQ_1} - \theta\beta F(\theta\beta Q_1) - (\theta\beta)^2 Q_1 f(\theta\beta Q_1)\right]d\theta d\beta - h(1-\mu_\theta)\mu_\beta - l_1 \tag{3.4c}$$

$$= u\int_{L_\beta}^{U_\beta} j(\beta)\int_{L_\theta}^{U_\theta} \theta\beta g(\theta)\left[1-F(\theta\beta Q_1)\right]d\theta d\beta + h\int_{L_\beta}^{U_\beta} j(\beta)\int_{L_\theta}^{U_\theta}(-g(\theta)\theta\beta F(\theta\beta Q_1) + \theta\beta g(\theta)$$

$$-\theta\beta g(\theta))d\theta d\beta - h(1-\mu_\theta)\mu_\beta - l_1 - w \tag{3.4d}$$

$$= (u+h)\int_{L_\beta}^{U_\beta} j(\beta)\int_{L_\theta}^{U_\theta} \theta\beta g(\theta)\left[1-F(\theta\beta Q_1)\right]d\theta d\beta - h\int_{L_\beta}^{U_\beta} j(\beta)\int_{L_\theta}^{U_\theta} \theta\beta g(\theta) d\theta d\beta - h(1-\mu_\theta)\mu_\beta - l_1 - w \tag{3.4e}$$

$$= (u+h)\int_{L_\beta}^{U_\beta} j(\beta)\int_{L_\theta}^{U_\theta} \theta\beta g(\theta)\left[1-F(\theta\beta Q_1)\right]d\theta d\beta - h\mu_\theta\mu_\beta - h\mu_\beta + h\mu_\theta\mu_\beta - l_1 - w \tag{3.4f}$$

$$\Rightarrow \frac{d\pi(Q_1)}{dQ_1} = (u+h)\int_{L_\beta}^{U_\beta} j(\beta)\int_{L_\theta}^{U_\theta} \theta\beta g(\theta)\left[1-F(\theta\beta Q_1)\right]d\theta d\beta - h\mu_\beta - l_1 - w \tag{3.4g}$$

$$\frac{d\pi(Q_1)}{dQ_1} = 0$$

$$\Rightarrow (u+h)\int_{L_\beta}^{U_\beta} j(\beta)\int_{L_\theta}^{U_\theta} \theta\beta g(\theta)\left[1-F(\theta\beta Q_1^{*})\right]d\theta d\beta - h\mu_\beta - l_1 - w = 0 \tag{3.4h}$$

The equation that gives the optimum order quantity for no-tagging approach maximizing profit value;

$$\int_{L_\beta}^{U_\beta} j(\beta)\int_{L_\theta}^{U_\theta} \theta\beta g(\theta)\left[1-F(\theta\beta Q_1^{*})\right]d\theta d\beta = \frac{h\mu_\beta + l_1 + w}{(u+h)} \tag{3.4i}$$

which is concave, due to:

$$\Rightarrow \frac{d^2\pi(Q_1)}{dQ_1^{\,2}} = (u+h)\int_{L_\beta}^{U_\beta} j(\beta)\int_{L_\theta}^{U_\theta} \theta\beta g(\theta)\frac{d\left[1-F(\theta\beta Q_1)\right]}{dQ_1}d\theta d\beta$$

$$= -(u+h)\int_{L_\beta}^{U_\beta} j(\beta)\int_{L_\theta}^{U_\theta} (\theta\beta)^2 g(\theta) f(\theta\beta Q_1) d\theta d\beta \le 0 \tag{3.4j}$$

Under deterministic errors setting, the equation that gives optimum order quantity is given as;

$$\mu_\theta \mu_\beta \left[1 - F(\theta \beta Q_1^*)\right] = \frac{h\mu_\beta + l_1 + w}{u+h} \tag{3.5}$$

$$F(\mu_\theta \mu_\beta Q_1^*) = 1 - \frac{h\mu_\beta + l_1 + w}{(u+h)\mu_\theta \mu_\beta} \quad for \quad \frac{h\mu_\beta + l_1 + w}{\mu_\theta \mu_\beta} \leq u+h \tag{3.6}$$

$$Q_1^* = 0 \qquad\qquad otherwise$$

Under deterministic error setting, expected profit function for Case I is given as;

$$\pi(Q_1) = r\mu_x - wQ_1 - u\left[\int_{x=\mu_\theta\mu_\beta Q_1}^{\infty} xf(x)dx - \mu_\theta\mu_\beta Q_1 + \mu_\theta\mu_\beta Q_1 F(\mu_\theta\mu_\beta Q_1)\right]$$
$$+ h\left[\int_{x=-\infty}^{\mu_\theta\mu_\beta Q_1} xf(x)dx - \mu_\theta\mu_\beta Q_1 F(\mu_\theta\mu_\beta Q_1)\right] - h(1-\mu_\theta)\mu_\beta Q_1 - l_1 Q_1 \tag{3.7}$$

Noting that $F(\mu_\theta \mu_\beta Q_1^*) = 1 - \frac{h\mu_\beta + l_1 + w}{(u+h)\mu_\theta \mu_\beta} \quad for \quad \frac{h\mu_\beta + l_1 + w}{\mu_\theta \mu_\beta} \leq u+h$, we replace $F(\mu_\theta\mu_\beta Q_1) \quad with \quad F(\mu_\theta \mu_\beta Q_1^*) = 1 - \frac{h\mu_\beta + l_1 + w}{(u+h)\mu_\theta \mu_\beta}$.

With Q_1^* optimum order quantity, maximum expected profit value is given as;

$$\pi(Q_1^*) = r\mu_x - wQ_1^* - u\left[\int_{x=\mu_\theta\mu_\beta Q_1^*}^{\infty} xf(x)dx - \mu_\theta\mu_\beta Q_1^* + \mu_\theta\mu_\beta Q_1^* \left(1 - \frac{h\mu_\beta + l_1 + w}{(u+h)\mu_\theta\mu_\beta}\right)\right]$$
$$+ h\left[\int_{x=-\infty}^{\mu_\theta\mu_\beta Q_1} xf(x)dx - \mu_\theta\mu_\beta Q_1^* \left(1 - \frac{h\mu_\beta + l_1 + w}{(u+h)\mu_\theta\mu_\beta}\right)\right] - h(1-\mu_\theta)\mu_\beta Q_1^* - l_1 Q_1^* \tag{3.8a}$$

$$= r\mu_x - wQ_1^* - u\int_{x=\mu_\theta\mu_\beta Q_1^*}^{\infty} xf(x)dx + uQ_1^* \frac{h\mu_\beta + l_1 + w}{u+h}$$

$$+ h\int_{x=-\infty}^{\mu_\theta\mu_\beta Q} xf(x)dx - h\mu_\theta\mu_\beta Q_1^* + hQ_1^* \frac{h\mu_\beta + l_1 + w}{u+h} - h\mu_\beta Q_1^* + h\mu_\theta\mu_\beta Q_1^* - l_1 Q_1^* \quad \textbf{(3.8b)}$$

$$= r\mu_x - wQ_1^* - u\int_{x=\mu_\theta\mu_\beta Q_1^*}^{\infty} xf(x)dx + +h\int_{x=-\infty}^{\mu_\theta\mu_\beta Q} xf(x)dx + (u+h)Q_1^* \frac{h\mu_\beta + l_1 + w}{u+h} - h\mu_\beta Q_1^* - l_1 Q_1^* \quad \textbf{(3.8c)}$$

$$= u\mu_x + w\mu_x - wQ_1^* - u\int_{x=\mu_\theta\mu_\beta Q_1^*}^{\infty} xf(x)dx + +h\int_{x=-\infty}^{\mu_\theta\mu_\beta Q_1^*} xf(x)dx + Q_1^* h\mu_\beta + l_1 Q_1^* + wQ_1^* - h\mu_\beta Q_1^* - l_1 Q_1^* \quad \textbf{(3.8d)}$$

$$= u\int_{-\infty}^{\infty} xf(x)dx - u\int_{x=\mu_\theta\mu_\beta Q_1^*}^{\infty} xf(x)dx + +h\int_{x=-\infty}^{\mu_\theta\mu_\beta Q_1^*} xf(x)dx + w\mu_x \quad \textbf{(3.8e)}$$

Optimum expected profit value for no-tagging case :

$$\pi(Q_1^*) = (u+h)\int_{x=-\infty}^{\mu_\theta\mu_\beta Q_1^*} xf(x)dx + w\mu_x \qquad for \quad \frac{h\mu_\beta + l_1 + w}{\mu_\theta\mu_\beta} \le u+h \quad \textbf{(3.9)}$$

$$\pi(Q_1^*) = 0 \qquad\qquad otherwise$$

For different tagging level cases, it is necessary to explain how we have included the tagging cost for different cases into the model. The main logic of modeling tagging cost is to charge the tag cost on a single unit of item. Figure 3.2 demonstrates this logic and shows the revised equations of underage and overage costs for different cases.

$w \Rightarrow w + t / p$	$w \Rightarrow w + t / c$	$w \Rightarrow w + t$
$u \Rightarrow u - \rho$	$u \Rightarrow u - v$	$u \Rightarrow u - t$
$h \Rightarrow h + \rho$	$h \Rightarrow h + v$	$h \Rightarrow h + t$

Figure 3.2 : Revised Underage and Overage Costs for Pallet, Case and Item Level Tagging

3.3.2 Case II (Pallet-Level Tagging)

As mentioned before, tagging price is charged on purchasing cost of a unit of an item. Since the price of a tag is denoted by t, we calculate the purchasing cost of an item for pallet-level tagging by;

$\Rightarrow$. *p* is the capacity of a pallet and *t/p* is the tagging cost per item, we denote *t/p* as ρ.

Since $u = r - w \Rightarrow$ Underage and overage costs for pallet level tagging are equal to $u = u - \rho$ and $h = h + \rho$

As it was mentioned before, while the level of tagging increases, tagging cost per units of item and the values that θ and β random variables get are getting higher. Meanwhile labor cost parameter per item is getting lower.

$$profit_2 = rx - (w+\rho)Q_2 - (u-\rho)(x-\theta\beta Q_2)^+ - (h+\rho)(\theta\beta Q_2 - x)^+ - (h+\rho)(1-\theta)\beta Q_2 - l_2 Q_2 \qquad \textbf{(3.10)}$$

$$\text{exp}\,ected\ profit\ function_2 = \pi(Q_2) = r\mu_x - (w+\rho)Q_2 - (u-\rho)\int_{L_\beta}^{U_\beta}\int_{L_\theta}^{U_\theta}\int_{x=\theta\beta Q_2}^{\infty}(x-\theta\beta Q_2)f(x)g(\theta)j(\beta)\,dx\,d\theta\,d\beta$$

$$-(h+\rho)\int_{L_\beta}^{U_\beta}\int_{L_\theta}^{U_\theta}\int_{x=-\infty}^{\theta\beta Q_2}(\theta\beta Q_2 - x)f(x)g(\theta)j(\beta)\,dx\,d\theta\,d\beta - (h+\rho)\int_{L_\beta}^{U_\beta}\int_{L_\theta}^{U_\theta}(1-\theta)\beta Q_2\, g(\theta)j(\beta)\,d\theta\,d\beta - l_2 Q_2$$

$$= r\mu_x - (w+\rho)Q_2 - (u-\rho)\int_{\beta=L_\beta}^{U_\beta} j(\beta)\int_{\theta=L_\theta}^{U_\theta} g(\theta)\left[\int_{x=\theta\beta Q_2}^{\infty} xf(x)dx - \theta\beta Q_2(1-F(\theta\beta Q_2))\right]d\theta d\beta \tag{3.11}$$

$$+ (h+\rho)\int_{\beta=L_\beta}^{U_\beta} j(\beta)\int_{\theta=L_\theta}^{U_\theta} g(\theta)\left[\int_{x=-\infty}^{\theta\beta Q_2} xf(x)dx - \theta\beta Q_2 F(\theta\beta Q_2)\right]d\theta d\beta - (h+\rho)(1-\mu_\theta)\mu_\beta Q_2 - l_2 Q_2$$

$$\frac{d^2(\pi(Q_2))}{(dQ_2)^2} = -(u+h)\int_{L_\beta}^{U_\beta} j(\beta)\int_{L_\theta}^{U_\theta} (\theta\beta)^2 g(\theta) f(\theta\beta Q_2) d\theta d\beta \le 0 \tag{3.12}$$

Hence $\pi(Q_2)$ is concave in Q_2, we can obtain the optimum order quantity (Q_2^*) that maximizes expected profit function by making the first derivative of expected profit function equal to zero ($\frac{d\pi(Q_2)}{dQ_2} = 0$).

$$\frac{d\pi(Q_2)}{dQ_2} = (u+h)\int_{L_\beta}^{U_\beta} j(\beta)\int_{L_\theta}^{U_\theta} \theta\beta g(\theta)\left[1 - F(\theta\beta Q_2)\right]d\theta d\beta - (h+\rho)\mu_\beta - l_2 - (w+\rho) \tag{3.13}$$

$$\frac{d\pi(Q_2)}{dQ_2} = 0$$

$$\Rightarrow (u+h)\int_{L_\beta}^{U_\beta} j(\beta)\int_{L_\theta}^{U_\theta} \theta\beta g(\theta)\left[1 - F(\theta\beta Q_2^*)\right]d\theta d\beta - (h+\rho)\mu_\beta - l_2 - (w+\rho) = 0 \tag{3.14}$$

The equation that gives the optimum order quantity for pallet-level tagging case maximizing profit value:

$$\int_{L_\beta}^{U_\beta} j(\beta)\int_{L_\theta}^{U_\theta} \theta\beta g(\theta)\left[1 - F(\theta\beta Q_2^*)\right]d\theta d\beta = \frac{(h+\rho)\mu_\beta + l_2 + (w+\rho)}{(u+h)} \tag{3.15}$$

Under deterministic errors setting; the equation that gives optimum order quantity is as follows;

$$\mu_\theta \mu_\beta \left[1 - F(\theta\beta Q_2^{*})\right] = \frac{(h+\rho)\mu_\beta + l_2 + (w+\rho)}{u+h} \qquad \textbf{(3.16)}$$

$$F(\mu_\theta \mu_\beta Q_2^{*}) = 1 - \frac{(h+\rho)\mu_\beta + l_2 + (w+\rho)}{(u+h)\mu_\theta \mu_\beta} \quad for \quad \frac{(h+\rho)\mu_\beta + l_2 + (w+\rho)}{\mu_\theta \mu_\beta} \le u+h$$

$$\textbf{(3.17)}$$

$$Q_2^{*} = 0 \qquad\qquad otherwise$$

Under deterministic error setting, expected profit function for case II is as following;

$$\pi(Q_2) = r\mu_x - (w+\rho)Q_2 - (u-\rho)\left[\int_{x=\mu_\theta\mu_\beta Q_2}^{\infty} xf(x)dx - \mu_\theta\mu_\beta Q_2 + \mu_\theta\mu_\beta Q_2 F(\mu_\theta\mu_\beta Q_2)\right]$$
$$+ (h+\rho)\left[\int_{x=-\infty}^{\mu_\theta\mu_\beta Q_2} xf(x)dx - \mu_\theta\mu_\beta Q_2 F(\mu_\theta\mu_\beta Q_2)\right] - (h+\rho)(1-\mu_\theta)\mu_\beta Q_2 - l_2 Q_2 \qquad \textbf{(3.18)}$$

Noting that for $\frac{(h+\rho)\mu_\beta + l_2 + (w+\rho)}{\mu_\theta \mu_\beta} \le u+h$, we replace $F(\mu_\theta\mu_\beta Q_2)$ with $F(\mu_\theta \mu_\beta Q_2^{*}) = 1 - \frac{(h+\rho)\mu_\beta + l_2 + (w+\rho)}{(u+h)\mu_\theta \mu_\beta}$.

With Q_2^* optimum order quantity, maximum expected profit value for pallet-level tagging case:

$$\pi(Q_2^{*}) = (u+h)\int_{x=-\infty}^{\mu_\theta\mu_\beta Q_2^*} xf(x)dx + (w+\rho)\mu_x \quad for \quad \frac{(h+\rho)\mu_\beta + l_2 + (w+\rho)}{\mu_\theta \mu_\beta} \le u+h$$

$$\textbf{(3.19)}$$

$$\pi(Q_2^{*}) = 0 \qquad\qquad otherwise$$

3.3.3 Case III (Case-Level Tagging)

Similarly, the tagging price of cases can be charged on purchasing cost of a unit of an item. Since the price of a tag is denoted by t, we calculate the purchasing cost of an item for case-level tagging by;

$w \Rightarrow q + t/c$. *c* is the capacity of a case and *t/c* is the tagging cost per item, we denote *t/c* as ν.

$u = r - w \ \Rightarrow \ u = u - \nu \qquad h = h + \nu$

$$profit_3 = rx - (w+\nu)Q_3 - (u-\nu)(x - \theta\beta Q_3)^+ - (h+\nu)(\theta\beta Q_3 - x)^+ - (h+\nu)(1-\theta)\beta Q_3 - l_3 Q_3 \quad \textbf{(3.20)}$$

$$\text{expected profit function} = \pi(Q_3) = r\mu_x - (w+\nu)Q_3 - (u-\nu)\int_{\beta=L_\beta}^{U_\beta} j(\beta)\int_{\theta=L_\theta}^{U_\theta} g(\theta)\left[\int_{x=\theta\beta Q_3}^{\infty} xf(x)dx - \theta\beta Q_3(1 - F(\theta\beta Q_3))\right]d\theta d\beta$$
$$+(h+\nu)\int_{\beta=L_\beta}^{U_\beta} j(\beta)\int_{\theta=L_\theta}^{U_\theta} g(\theta)\left[\int_{x=-\infty}^{\theta\beta Q_3} xf(x)dx - \theta\beta Q_3 F(\theta\beta Q_3)\right]d\theta d\beta - (h+\nu)(1-\mu_\theta)\mu_\beta Q_3 - l_3 Q_3 \quad \textbf{(3.21)}$$

$$\frac{d\pi(Q_3)}{dQ_3} = 0$$

$$\Rightarrow \int_{L_\beta}^{U_\beta} j(\beta)\int_{L_\theta}^{U_\theta} \theta\beta g(\theta)\left[1 - F(\theta\beta Q_3^{*})\right]d\theta d\beta = \frac{(h+\nu)\mu_\beta + l_3 + (w+\nu)}{u+h} \quad \textbf{(3.22)}$$

Under deterministic errors setting, the equations that give optimum order quantity and maximum expected profit value are calculated as below:

$$\mu_\theta\mu_\beta\left[1 - F(\theta\beta Q_3^{*})\right] = \frac{(h+\nu)\mu_\beta + l_3 + (w+\nu)}{u+h} \quad \textbf{(3.23)}$$

$$F(\mu_\theta\mu_\beta Q_3^{*}) = 1 - \frac{(h+\nu)\mu_\beta + l_3 + (w+\nu)}{(u+h)\mu_\theta\mu_\beta} \quad for \quad \frac{(h+\nu)\mu_\beta + l_3 + (w+\nu)}{\mu_\theta\mu_\beta} \le u+h$$
$$Q_3^{*} = 0 \qquad otherwise \quad \textbf{(3.24)}$$

With Q_3^* optimum order quantity, maximum expected profit value for case-level tagging case;

$$\pi(Q_3^*) = (u+h)\int_{x=-\infty}^{\mu_\theta \mu_\beta Q_3^*} xf(x)dx + (w+v)\mu_x \quad for \quad \frac{(h+v)\mu_\beta + l_3 + (w+v)}{\mu_\theta \mu_\beta} \le u+h$$

$$\pi(Q_3^*) = 0 \qquad\qquad otherwise \tag{3.25}$$

3.3.4 Case IV (Item-Level Tagging)

At this level of tagging, we charge the entire tag price to a single item. So the purchasing cost of the item increases to $w+t$, that causes similar changes on underage and overage costs;

$u \Rightarrow u-t \qquad h \Rightarrow h+t$

$$profit_4 = rx - (w+t)Q_4 - (u-t)(x-\theta\beta Q_4)^+ - (h+t)(\theta\beta Q_4 - x)^+ - (h+t)(1-\theta)\beta Q_4 - l_4 Q_4 \tag{3.26}$$

$$\text{expected } profit function_4 = \pi(Q_4) = r\mu_x - (w+t)Q_4 - (u-t)\int_{\beta=L_\beta}^{U_\beta} j(\beta)\int_{\theta=L_\theta}^{U_\theta} g(\theta)\left[\int_{x=\theta\beta Q_4}^{\infty} xf(x)dx - \theta\beta Q_4(1-F(\theta\beta Q_4))\right]d\theta d\beta$$
$$+(h+t)\int_{\beta=L_\beta}^{U_\beta} j(\beta)\int_{\theta=L_\theta}^{U_\theta} g(\theta)\left[\int_{x=-\infty}^{\theta\beta Q_4} xf(x)dx - \theta\beta Q_4 F(\theta\beta Q_4)\right]d\theta d\beta - (h+t)(1-\mu_\theta)\mu_\beta Q_4 - l_4 Q_4 \tag{3.27}$$

$$\frac{d\pi(Q_4)}{dQ_4} = 0$$

$$=> \int_{L_\beta}^{U_\beta} j(\beta)\int_{L_\theta}^{U_\theta} \theta\beta g(\theta)\left[1 - F(\theta\beta Q_4^*)\right]d\theta d\beta = \frac{(h+t)\mu_\beta + l_4 + (w+t)}{u+h} \tag{3.28}$$

Under deterministic errors setting, the equations that give optimum order quantity and maximum expected profit value are calculated as below:

$$\mu_\theta \mu_\beta \left[1 - F(\theta\beta Q_4^{\ *})\right] = \frac{(h+t)\mu_\beta + l_4 + (w+t)}{u+h} \qquad \textbf{(3.30)}$$

$$F(\mu_\theta \mu_\beta Q_4^{\ *}) = 1 - \frac{(h+t)\mu_\beta + l_4 + (w+t)}{(u+h)\mu_\theta \mu_\beta} \quad for \quad \frac{(h+t)\mu_\beta + l_4 + (w+t)}{\mu_\theta \mu_\beta} \le u+h \qquad \textbf{(3.31)}$$

$$Q_4^{\ *} = 0 \qquad otherwise$$

With Q_4^* optimum order quantity, maximum expected profit value for item-level tagging case;

$$\pi(Q_4^{\ *}) = (u+h) \int_{x=-\infty}^{\mu_\theta \mu_\beta Q_4^*} x f(x) dx + (w+t)\mu_x \quad for \quad \frac{(h+t)\mu_\beta + l_4 + (w+t)}{\mu_\theta \mu_\beta} \le u+h$$

$$\pi(Q_4^{\ *}) = 0 \qquad otherwise \qquad \textbf{(3.32)}$$

In order to compare the expected maximum profit values, all the results are written together in the following Table 3.1 under deterministic θ and β.

Table 3.1 : Optimum Order Quantity and Expected Maximum Profit Values for Each Tagging Cases

No-tagging level	Pallet-level tagging
$F(\mu_\theta \mu_\beta Q_1^*) = 1 - \frac{h\mu_\beta + l_1 + w}{(u+h)\mu_\theta \mu_\beta}$ for $\frac{h\mu_\beta + l_1 + w}{\mu_\theta \mu_\beta} \le u+h$ $Q_1^* = 0$ otherwise $\pi(Q_1^*) = (u+h)\int_{x=-\infty}^{\mu_\theta \mu_\beta Q_1^*} xf(x)dx + w\mu_x$ for $\frac{h\mu_\beta + l_1 + w}{\mu_\theta \mu_\beta} \le u+h$ $\pi(Q_1^*) = 0$ otherwise	$F(\mu_\theta \mu_\beta Q_2^*) = 1 - \frac{(h+\rho)\mu_\beta + l_2 + (w+\rho)}{(u+h)\mu_\theta \mu_\beta}$ for $\frac{(h+\rho)\mu_\beta + l_2 + (w+\rho)}{\mu_\theta \mu_\beta} \le u+h$ $Q_2^* = 0$ otherwise $\pi(Q_2^*) = (u+h)\int_{x=-\infty}^{\mu_\theta \mu_\beta Q_2^*} xf(x)dx + (w+\rho)\mu_x$ for $\frac{(h+\rho)\mu_\beta + l_2 + (w+\rho)}{\mu_\theta \mu_\beta} \le u+h$ $\pi(Q_2^*) = 0$ otherwise
Case-level tagging	Item-level tagging
$F(\mu_\theta \mu_\beta Q_3^*) = 1 - \frac{(h+v)\mu_\beta + l_3 + (w+v)}{(u+h)\mu_\theta \mu_\beta}$ for $\frac{(h+v)\mu_\beta + l_3 + (w+v)}{\mu_\theta \mu_\beta} \le u+h$ $Q_3^* = 0$ otherwise $\pi(Q_3^*) = (u+h)\int_{x=-\infty}^{\mu_\theta \mu_\beta Q_3^*} xf(x)dx + (w+v)\mu_x$ for $\frac{(h+v)\mu_\beta + l_3 + (w+v)}{\mu_\theta \mu_\beta} \le u+h$ $\pi(Q_3^*) = 0$ otherwise	$F(\mu_\theta \mu_\beta Q_4^*) = 1 - \frac{(h+t)\mu_\beta + l_4 + (w+t)}{(u+h)\mu_\theta \mu_\beta}$ for $\frac{(h+t)\mu_\beta + l_4 + (w+t)}{\mu_\theta \mu_\beta} \le u+h$ $Q_4^* = 0$ otherwise $\pi(Q_4^*) = (u+h)\int_{x=-\infty}^{\mu_\theta \mu_\beta Q_4^*} xf(x)dx + (w+t)\mu_x$ for $\frac{(h+t)\mu_\beta + l_4 + (w+t)}{\mu_\theta \mu_\beta} \le u+h$ $\pi(Q_4^*) = 0$ otherwise

3.4 Continuous Tagging Level Approach

We have noticed the similarity between the optimum order quantity and maximum expected profit values and decided to define a continuous level of tagging variable, $y \in [0,1]$ in order to analyze tagging level problem in a profound mathematical way. Through y tagging level variable, we are able to determine the optimal tagging level in a continuous fashion, instead of analyzing only four different alternatives and choosing the best among them.

In fact, y tagging level variable represents a discrete tagging level in real life applications, since $y = \frac{1}{number of units of a single item that are packed and tagged together}$.

Here for the sake of simplicity, we are assuming y is a continuous variable in our mathematical analyses.

We define some kind of functions of y characterizing the tagging cost, the rate of remaining items after misplacement and shrinkage, and labor cost.

$L_{(y)}$ = function characterizing the variation on labor cost with respect to y

$\beta_{(y)}$ = function characterizing the rate of remaining items after shrinkage with respect to y

$\theta_{(y)}$ = function characterizing the rate of remaining items after misplacement errors with respect to y

$C_{(y)}$ = function characterizing tagging cost with respect to y

We have derived the following (3.33) and (3.34) identities that give the optimum order quantity and maximum profit values.

$$F(\theta_y \beta_y Q_y^*) = 1 - \frac{(h+C_y)\beta_y + L_y + C_y + w}{(u+h)\theta_y \beta_y} \quad for \quad \frac{(h+C_y)\beta_y + L_y + (w+C_y)}{\theta_y \beta_y} \le u+h \qquad \textbf{(3.33)}$$

$$\pi(Q_y^*) = (u+h)\int_{-\infty}^{\theta_y \beta_y Q_y^*} xf(x)dx + (w+C_y).\mu_x \quad for \quad \frac{(h+C_y)\beta_y + L_y + (w+C_y)}{\theta_y \beta_y} \le u+h \qquad \textbf{(3.34)}$$

Up to here for four different cases of tagging level, optimum order quantity and expected maximum profit values are calculated and then, this equations are dealt with the y tagging level decision variable and the functions of y that gives us the chance for also determining the packaging size of units. In this part of the study under two different assumptions of the distribution of demand, the explicit equations are calculated for expected maximum profit value. These demand distributions are normal and uniform distribution.

3.5 Expected Maximum Profit Values Under Normally and Uniform Distributed Demand

3.5.1 Expected Maximum Profit Value Under Normally Distributed Demand Assumption

Let's assume, demand is normally distributed with mean of μ_x and standard deviation of σ_x.

$$F(\theta_y \beta_y Q_y^*) = P(X \le \theta_y \beta_y Q_y^*) \tag{3.35}$$

$$P\left(\frac{X-\mu_x}{\sigma_x} \le \frac{\theta_y \beta_y Q_y^* - \mu_x}{\sigma_x}\right) = \Phi\left(\frac{\theta_y \beta_y Q_y^* - \mu_x}{\sigma_x}\right) \tag{3.36}$$

$$F(\theta_y \beta_y Q_y^*) = \Phi\left(\frac{\theta_y \beta_y Q_y^* - \mu_x}{\sigma_x}\right) \tag{3.37}$$

$$\Phi\left(\frac{\theta_y \beta_y Q_y^* - \mu_x}{\sigma_x}\right) = 1 - \frac{(h+C_y)\beta_y + L_y + C_y + w}{(u+h)\theta_y \beta_y} \quad \text{for} \quad \frac{(h+C_y)\beta_y + L_y + C_y + w}{\theta_y \beta_y} \le u+h \tag{3.38}$$

$$\theta_y \beta_y Q_y^* = \mu_x + \sigma_x \Phi^{-1}\left(1 - \frac{(h+C_y)\beta_y + L_y + C_y + w}{(u+h)\theta_y \beta_y}\right) \quad \text{for} \quad \frac{(h+C_y)\mu_\beta + L_y + (w+C_y)}{\theta_y \beta_y} \le u+h \tag{3.39}$$

$$\pi(Q_y^*) = (u+h) \int_{-\infty}^{\theta_y \beta_y Q_y^*} x f(x) dx + (w + C_y).\mu_x \tag{3.40}$$

$$\pi(Q_y^*) = (u+h) \int_{-\infty}^{\mu_x + \sigma_x.\Phi^{-1}\left(1 - \frac{(h+C_y).\beta_y + L_y + w + C_y}{(u+h).(\theta_y + \beta_y - 1)}\right)} x.\frac{1}{\sqrt{2\pi}\sigma_x}.\exp(-(x-\mu_x)^2 / 2\sigma_x^2) dx + (w+C_y).\mu_x \tag{3.41}$$

Let $t = \frac{x - \mu_x}{\sigma_x}$. Then $dt = \frac{dx}{\sigma_x}$ and $x = \sigma_x \cdot t + \mu_x$.

$$\pi(Q_y^*) = (u+h)\int_{-\infty}^{k} (\sigma_x.t + \mu_x).\frac{1}{\sqrt{2\pi}}.\exp(-t^2/2)dt + (w + C_y).\mu_x \tag{3.42}$$

Where k is the modified upper bound that is equal to $\Phi^{-1}(1 - \frac{(h+C_y).\beta_y + L_y + w + C_y}{(u+h)\theta_y\beta_y})$.

$$\pi(Q_y^*) = (u+h)\frac{1}{\sqrt{2\pi}}\sigma_x \int_{-\infty}^{k} (t + \frac{\mu_x}{\sigma_x}).\exp(-t^2/2)dt + (w + C_y).\mu_x \tag{3.43}$$

$$\pi(Q_y^*) = (u+h)\frac{1}{\sqrt{2\pi}}\sigma_x \int_{-\infty}^{k} (t + \frac{\mu_x}{\sigma_x} + k - k).\exp(-t^2/2)dt + (w + C_y).\mu_x \tag{3.44}$$

$$\pi(Q_y^*) = (u+h)\sigma_x \frac{1}{\sqrt{2\pi}}\left[\int_{-\infty}^{k} (t-k)\exp(-t^2/2)dt + \left(\frac{\mu_x}{\sigma_x} + k\right)\int_{-\infty}^{k} \exp(-t^2/2)dt\right] + (w + C_y).\mu_x \tag{3.45}$$

Let say $\phi_y = \frac{1}{\sqrt{2\pi}}\exp(-y^2/2)dy$. Note that Y is a standard normal random variable, we can say that;

$$E[Y] = \int_{-\infty}^{\infty} y\phi_y dy = 0 \tag{3.46}$$

$$\int_{-\infty}^{\infty} y\phi_y dy = \int_{-\infty}^{k} y\phi_y dy + \int_{k}^{\infty} y\phi_y dy = 0 \tag{3.47}$$

$$\int_{-\infty}^{k} y\phi_y dy = -\int_{k}^{\infty} y\phi_y dy \tag{3.48}$$

Note that $\int_{-\infty}^{k} y\phi_y dy = -\int_{k}^{\infty} y\phi_y dy$, $\Phi(z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} \exp[-{y^2}/{2}]dy$ and

$$G_u(k) = \frac{1}{\sqrt{2\pi}}\int_{k}^{\infty}(y-k)\exp(-y^2/2)dy$$

$$\pi(Q_y^*) = -(u+h)\sigma_x\left[\frac{1}{\sqrt{2\pi}}\int_{k}^{\infty}(t-k)\exp(-t^2/2)dt + \left(\frac{\mu_x}{\sigma_x}+k\right)\frac{1}{\sqrt{2\pi}}\int_{k}^{\infty}\exp(-t^2/2)dt\right] + (w+C_y).\mu_x \tag{3.49}$$

$$\pi(Q_y^*) = -(u+h)\sigma_x\left[G_u(k) + \left(\frac{\mu_x}{\sigma_x}+k\right)[1-\Phi(k)]\right] + (w+C_y).\mu_x \tag{3.50}$$

$$\pi(Q_y^*) = -(u+h)\sigma_x\left[G_u\left(\Phi^{-1}\left(1-\frac{(h+C_y)\beta_y+L_y+C_y+w}{u+h}.\frac{1}{\theta_y+\beta_y-1}\right)\right) + \left(\frac{\mu_x}{\sigma_x}+\Phi^{-1}\left(1-\frac{(h+C_y)\beta_y+L_y+C_y+w}{u+h}.\frac{1}{\theta_y+\beta_y-1}\right)\right)\left(\frac{(h+C_y)\beta_y+L_y+C_y+w}{u+h}.\frac{1}{\theta_y+\beta_y-1}\right)\right] + (w+C_y)\mu_x \tag{3.51}$$

An alternative calculation could be given as the following:

$$\pi(Q_y^*) = (u+h)\int_{-\infty}^{\Phi^{-1}(1-\frac{(h+C_y).\beta_y+L_y+w+C_y}{(u+h)\theta_y\beta_y})}(\sigma_x.t+\mu_x).\frac{1}{\sqrt{2\pi}}.\exp(-t^2/2)dt + (w+C_y).\mu_x \tag{3.52}$$

$$\pi(Q_y^*) = (u+h)\left[\sigma_x\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\Phi^{-1}(1-\frac{(h+C_y).\beta_y+L_y+w+C_y}{(u+h)\theta_y\beta_y})} t\exp(-t^2/2)dt + \mu_x\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\Phi^{-1}(1-\frac{(h+C_y).\beta_y+L_y+w+C_y}{(u+h)\theta_y\beta_y})}\exp(-t^2/2)dt\right] + (w+C_y).\mu_x \tag{3.53}$$

$$\pi(Q_y^*) = (u+h)\left[\sigma_x\frac{1}{\sqrt{2\pi}}\exp(-t^2/2)\Big|_{-\infty}^{\Phi^{-1}(1-\frac{(h+C_y).\beta_y+L_y+w+C_y}{(u+h)\theta_y\beta_y})} + \mu_x\Phi\left(\Phi^{-1}(1-\frac{(h+C_y).\beta_y+L_y+w+C_y}{(u+h)\theta_y\beta_y})\right)\right] + (w+C_y).\mu_x \tag{3.54}$$

$$\pi(Q_y^*) = (u+h)\left[\sigma_x \frac{1}{\sqrt{2\pi}}\exp(-t^2/2)\Big|_{-\infty}^{\Phi^{-1}(1-\frac{(h+C_y).\beta_y+L_y+w+C_y}{(u+h)\theta_y\beta_y})} + \mu_x\left((1-\frac{(h+C_y).\beta_y+L_y+w+C_y}{(u+h)\theta_y\beta_y})\right)\right] + (w+C_y).\mu_x \quad \textbf{(3.55)}$$

$$\pi(Q_y^*) = (u+h)\left[\sigma_x \frac{1}{\sqrt{2\pi}}\exp\left(-\left(\Phi^{-1}(1-\frac{(h+C_y)\beta_y+L_y+w+C_y}{(u+h)\theta_y\beta_y}\right)^2/2\right) + \mu_x\left((1-\frac{(h+C_y)\beta_y+L_y+w+C_y}{(u+h)\theta_y\beta_y})\right)\right] + (w+C_y).\mu_x \quad \textbf{(3.56)}$$

3.5.2 Expected Maximum Profit Value Under Uniformly Distributed Demand Assumption

Let's assume, demand is uniformly distributed with upper limit U_x and lower limit L_x ($U_x = \mu_x + \sqrt{3}\sigma_x$ and $L_x = \mu_x - \sqrt{3}\sigma_x$).

$$F(\theta_y\beta_y Q_y^*) = 1 - \frac{(h+C_y)\beta_y + L_y + C_y + w}{(u+h)\theta_y\beta_y} \text{ for } \frac{(h+C_y)\beta_y + L_y + (w+C_y)}{\theta_y\beta_y} \le u+h \quad \textbf{(3.57)}$$

$$F(\theta_y\beta_y Q_y^*) = \frac{\theta_y\beta_y Q_y^* - L_x}{U_x - L_x} = 1 - \frac{(h+C_y)\beta_y + L_y + C_y + w}{(u+h)\theta_y\beta_y} \quad \textbf{(3.58)}$$

$$\theta_y\beta_y Q_y^* = L_x + (U_x - L_x)\left(1 - \frac{(h+C_y)\beta_y + L_y + C_y + w}{(u+h)\theta_y\beta_y}\right) \quad \textbf{(3.59)}$$

$$\pi(Q_y^*) = (u+h)\int_{-\infty}^{\theta_y\beta_y Q_y^*} xf(x)dx + (w+C_y).\mu_x \quad \textbf{(3.60a)}$$

$$= (u+h)\int_{-\infty}^{L_x+(U_x-L_x)\left(1-\frac{(h+C_y)\beta_y+L_y+C_y+w}{(u+h)\theta_y\beta_y}\right)} x\frac{1}{U_x-L_x}dx + (w+C_y).\mu_x \quad \textbf{(3.60b)}$$

$$
=\frac{u+h}{2(U_x-L_x)}\left[L_x^{\,2}+2L_x(U_x-L_x)F(\theta_y\beta_yQ_y^{\,*})+(U_x-L_x)^2F^2(\theta_y\beta_yQ_y^{\,*})-L_x^{\,2}\right](w+C_y).\mu_x \qquad \textbf{(3.60c)}
$$

$$
=\frac{u+h}{U_x-L_x}\frac{x^2}{2}\Bigg|_{L_x}^{L_x+(U_x-L_x)\left(1-\frac{(h+C_y)\beta_y+L_y+C_y+w}{(u+h)\theta_y\beta_y}\right)}(w+C_y).\mu_x \qquad \textbf{(3.60d)}
$$

$$
\pi(Q_y^{\,*})=L_x(u+h)F(\theta_y\beta_yQ_y^{\,*})+(U_x-L_x)\frac{u+h}{2}F^2(\theta_y\beta_yQ_y^{\,*})+(w+C_y)\frac{U_x-L_x}{2} \qquad \textbf{(3.61)}
$$

Up to here; we have analyzed a newsvendor model under the assumption that there exist two different error factors. We assume that these two error factors are independent and have a deterministic manner. Through normally and uniform distributed demand assumption maximum profit function with respect to tagging variable y should be as the following;

Normally distributed demand;

$$
\pi(Q_y^{*})=-(u+h)\sigma_x\left[G_u\left(\Phi^{-1}\left(1-\frac{(h+C_y)\beta_y+L_y+C_y+w}{u+h}.\frac{1}{\theta_y+\beta_y-1}\right)\right)+\left(\frac{\mu_x}{\sigma_x}+\Phi^{-1}\left(1-\frac{(h+C_y)\beta_y+L_y+C_y+w}{u+h}.\frac{1}{\theta_y+\beta_y-1}\right)\right)\left(\frac{(h+C_y)\beta_y+L_y+C_y+w}{u+h}.\frac{1}{\theta_y+\beta_y-1}\right)\right]+(w+C_y)\mu_x \qquad \textbf{(3.62)}
$$

$$
\pi(Q_y^{*})=(u+h)\left[\sigma_x\frac{1}{\sqrt{2\pi}}\exp\left(-\left(\Phi^{-1}(1-\frac{(h+C_y)\beta_y+L_y+w+C_y}{(u+h)\theta_y\beta_y}\right)^2/2\right)+\mu_x\left((1-\frac{(h+C_y)\beta_y+L_y+w+C_y}{(u+h)\theta_y\beta_y})\right)\right]+(w+C_y)\mu_x \qquad \textbf{(3.63)}
$$

Uniform distributed demand;

$$
\pi(Q_y^{\,*})=L_x(u+h)F(\theta_y\beta_yQ_y^{\,*})+(U_x-L_x)\frac{u+h}{2}F^2(\theta_y\beta_yQ_y^{\,*})+(w+C_y)\frac{U_x-L_x}{2} \qquad \textbf{(3.64)}
$$

In the following parts of this study, through different functions and parameter values, we conduct a numerical study by the help of MATLAB that tests our model.

4. NUMERICAL STUDY & RESULTS

4.1 Numerical Study & Results for Uniform Distributed Demand

To analyze the effects of different parameters on expected maximum profit value, we conduct a numerical study with the parameter set of 6 different parameters (t, y, σ_x, l_{min}, $\mu_{\theta max}$, $\mu_{\beta max}$) under uniform distributed demand assumption. All the calculations and graphs that are required to present the inferences are done by the help of MATLAB 7.3.0.R2006b. Note that throughout the paper in our numerical examples, we set the function of tagging cost $C_{(y)}$ with respect to tagging level y linear ($C_{(y)} = t.y$), and the other functions of y linear, convex and concave such as ($\theta(y)$ and $\beta(y)$ has the same functions);

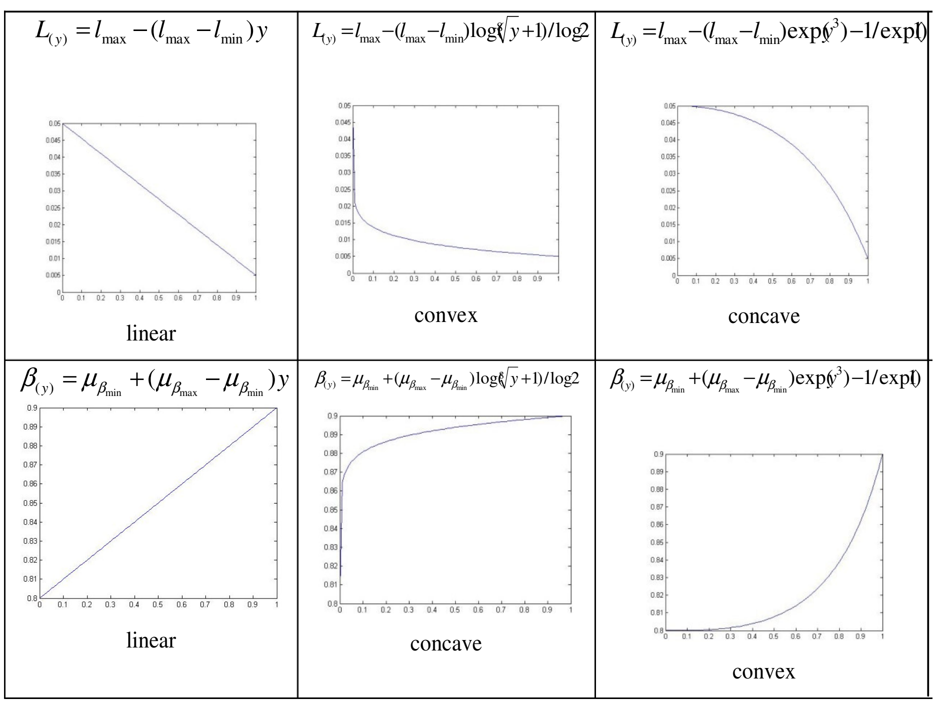

Figure 4.1 : Linear, Convex and Concave Functions of Labor, Misplacement and Shrinkage Costs of Tagging Level Variable y

The main reason of assuming three different functions of y for misplacement, shrinkage and labor costs ($\theta(y)$, $\beta(y)$ and $L(y)$) is to provide an overall model for different inventory environments. We can not know the exact behavior, response of the system when pallet level tagging decision holds. Some inventory systems may response with high decrease rate on error factors and some may not.

While fixing the parameter values, we considered some related industry reports. By the deployment of RFID systems, IBM reported 47% of reduction, Metro Group reported 11-18% of reduction, based on a survey of 500 respondents and Clark reported 12,3% of inventory inaccuracy reduction and finally 40-50% of reduction on inventory inaccuracy is reported by SAP on shrinkage and misplacement (Lee and Özer, 2005). Throughout the numerical examples, we set the values for misplacement and shrinkage with high reduction rates (equal and more than 50%) from no-tagging case to item level tagging case ($y = 0 \Rightarrow y = 1$).

One should pay attention to the point that tag prices are relative values according to retailing price. So we fixed retail price at 1 unit of money and change the tag price with respect to this value. The table representing all possible values of parameters is shown below:

Table 4.1 : Parameter Set

Parameters					
r	1				
w	0.4				
t	0.002	0.005	0.01	0.1	0.5
y	0:0.001:1				
$\mu_{\beta\ min}$	0.8				
$\mu_{\beta\ max}$	0.9	0.95	0.98		
$\mu_{\theta\ min}$	0.7				
$\mu_{\theta\ max}$	0.85	0.90	0.95		
l_{min}	0.003	0.001	0.0005		
l_{max}	0.01				
μ_x	100				
σ_x	10	20	30		

4.1.1 Results on The Effect of Different Tag Prices on Tagging Level Decision

To analyze the effect of tag price on optimum tagging level decision, we graph the expected maximum profit function with respect to tagging level y. Figure 4.3 shows the behavior of expected maximum profit function under the parameter set that is shown in Table 4.2. The functions of $\theta(y)$, $\beta(y)$ and $L(y)$ that are used for this calculation are on Figure 4.2.

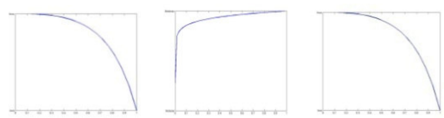

Figure 4.2 : Different Shapes of Functions *L(y), θ(y)* and *β(y)*

Table 4.2 : Parameters used for Analyzing the Effect of Tag Price on Optimum Tagging Level

Parameters	**Values**
$\mu_{\beta\ min}$	0.8
$\mu_{\beta\ max}$	0.9
$\mu_{\theta\ min}$	0.7
$\mu_{\theta\ max}$	0.85
l_{min}	0.003
l_{max}	0.01
μ_x	100
σ_x	10

As can be seen from Figure 4.3, it is obvious that while tag price is increasing, y value is getting smaller, which also means there is a tendency of packing and tagging more units of items together, i.e. from item level tagging to pallet level tagging. Table 4.3 shows us optimum y values and also the optimum packing sizes for different tag prices. For example for relatively cheap tag prices according to retail price of the item (t = 0,01, 0,005 and 0,002), item level tagging is the best option, for t = 0,1 (that means tag price is equal to 1/10 of retail price) the best option is to tag cases of 12 items capacity, for t = 0,5 (that is a quite high tag price for that specific item- half of the retail price) the best option is to tag pallets or cases of 67 items capacity.

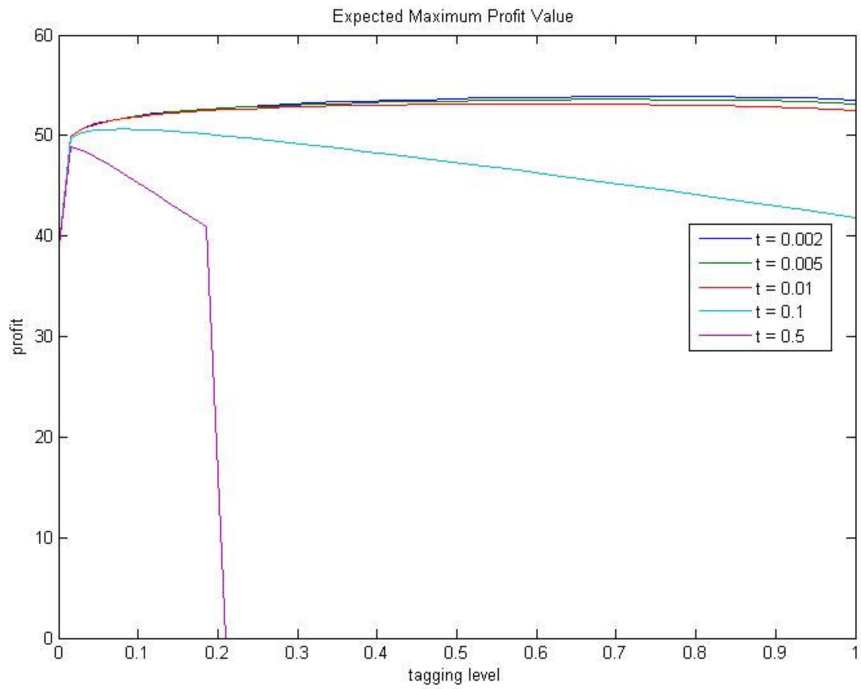

Figure 4.3 : Expected Maximum Profit Function with Respect to Tagging Level Under 5 Different Tag Prices

Table 4.3 : Optimum Tagging Level and Packing Size Values

Tag Price (t)	Optimum Tagging Level Value	Optimum Packing Size Value
<0.01	1	1
0.1	0.08	12
0.5	0.015	67

4.1.2 Results on The Effect of Different Tag Prices on Tagging Level Decision Under High Performance of RFID Systems (Improved Inventory Accuracy Levels)

To analyze the effect of inventory accuracy on optimum tagging level decision while the accuracy rate with RFID is higher, we do the same calculations with higher $\mu_{\theta max}$, $\mu_{\beta max}$ and lower l_{min} values. The results are quite intuitive because for higher gains of RFID technology, optimum tagging level value skews to the right, i.e. to item level tagging. Figure 4.4 shows the behavior of expected maximum profit function under the parameter set that is shown on Table 4.4.

Table 4.4 : Parameters used for Analyzing the Effect of Increased Inventory Accuracy on Optimum Tagging Level

Parameters	**Values**
$\mu_{\beta\ min}$	0.8
$\mu_{\beta\ max}$	0.98
$\mu_{\theta\ min}$	0.7
$\mu_{\theta\ max}$	0.95
l_{min}	0.0005
l_{max}	0.01
μ_x	100
σ_x	10

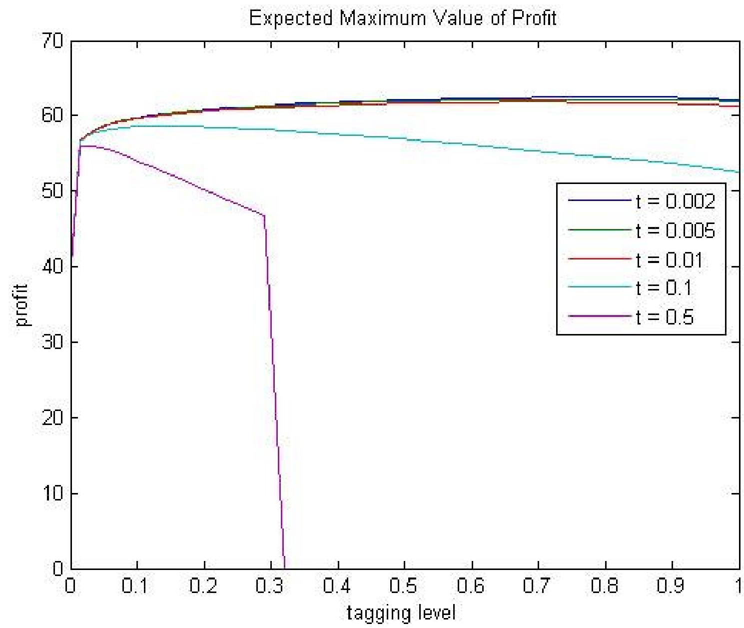

Figure 4.4 : Expected Maximum Profit Values with Respect to Tagging Level under Improved Inventory Accuracy Levels

Table 4.5 shows us optimum y values and also the optimum packing sizes for higher benefit rates of RFID tagging.

Table 4.5 : Optimum Tagging Level and Packing Size Values

Tag Price (t)	**Optimum Tagging Level Value**	**Optimum Packing Size Value**
<0.01	1	1
0.1	0.14	7
0.5	0.03	33

4.1.3 Results on the Effect of the Variance of Demand on Tagging Level Decision

To analyze the effect of variance of demand on optimum tagging level decision, we graph the expected maximum profit function with respect to tagging level y. Figure 4.6 shows the behavior of expected maximum profit function under the parameter set that is shown on Table 4.6. The functions that are used for this calculation are as following.

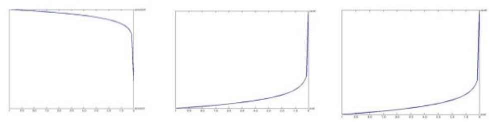

Figure 4.5 : Different Shapes of Functions *L(y), θ(y)* and *β(y)*

Table 4.6 : Parameters used for Analyzing the Effect of Increased Deviation of Demand on Tagging Level Decision

Parameters	**Values**
$\mu_{\beta\ min}$	0.8
$\mu_{\beta\ max}$	0.98
$\mu_{\theta\ min}$	0.7
$\mu_{\theta\ max}$	0.85
l_{min}	0.003
l_{max}	0.01
μ_x	100
t	0,1

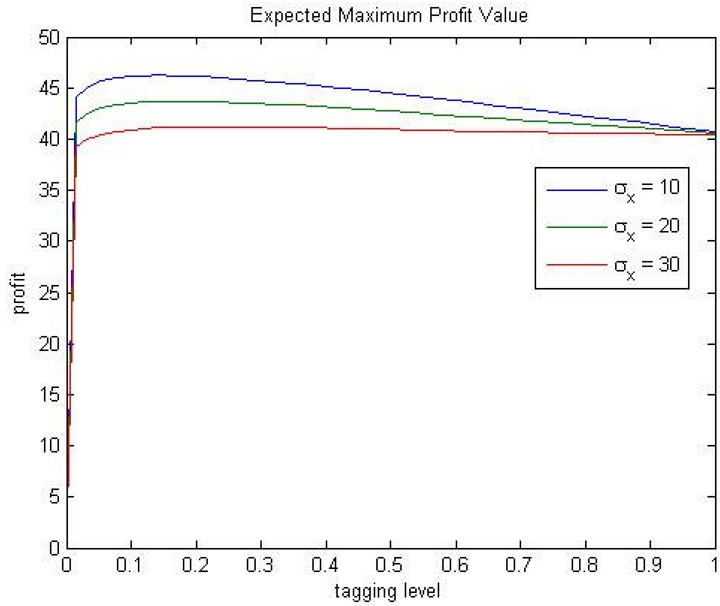

Figure 4.6 : Expected Maximum Profit Values with Respect to Tagging Level under Different Standard Deviations of Demand

As can be seen from Figure 4.6, while the variance of demand is increasing, y^* value is getting higher, which also means there is a tendency of packing and tagging less units of items together, i.e. from pallet level tagging to item level tagging. Table 4.7 shows us optimum y values and also the optimum packing sizes under different standard deviation levels of demand. For example for higher variance of demand (σ_x = 30), packing and tagging four units of the item together is the best option; while relatively lower deviation of demand (σ_x = 10), the best option is packing and tagging seven units of the item together.

Table 4.7 : Optimum Tagging Level and Packing Size Values

Variance of Demand (σ)	Optimum Tagging Level Value	Optimum Packing Size Value
σ = 10	0.14	7
σ = 20	0.16	6
σ = 30	0.24	4

4.1.4 Results on the Effect of Different Functions of *L(y)*, *θ(y)* and *β(y)* on Tagging Level Decision

To analyze the effect of different functions $L_{(y)}$, $\beta_{(y)}$ and $\theta_{(y)}$ on optimum tagging level decision, under the same parameter set, we draw the graphs that show us how the system response to tagging level decision problem under different behaviors of system according to different functions of $L_{(y)}$, $\beta_{(y)}$ and $\theta_{(y)}$. So we fixed all parameter values including t, y, σ_x, l_{min}, $\mu_{\theta max}$ and $\mu_{\beta max}$ and set the functions $\beta_{(y)}$ and $\theta_{(y)}$ function linear then change the type of $L_{(y)}$ function, concave, convex and linear to find the effect of the changes on $L_{(y)}$. Then we repeat this by fixing $L_{(y)}$, $\beta_{(y)}$ linear and changing the types of $\theta_{(y)}$, similarly fixing $L_{(y)}$, $\theta_{(y)}$ linear and changing the types of $\beta_{(y)}$. If the system has already had a big response on even pallet level tagging (see the concave function of $\beta_{(y)}$ and $\theta_{(y)}$), in other words even applying pallet level tagging; y value skews to left, i.e. to pallet level tagging. The same logic is also valid for convex function of $L_{(y)}$.

Concave function of $\theta_{(y)}$ has the meaning of that the inventory system responses with high improvement on inventory error factors even for pallet level tagging. On the other hand convex function of $\theta_{(y)}$ means that the system response with a meaningful improvement rate on misplacement error factor only for item level tagging.

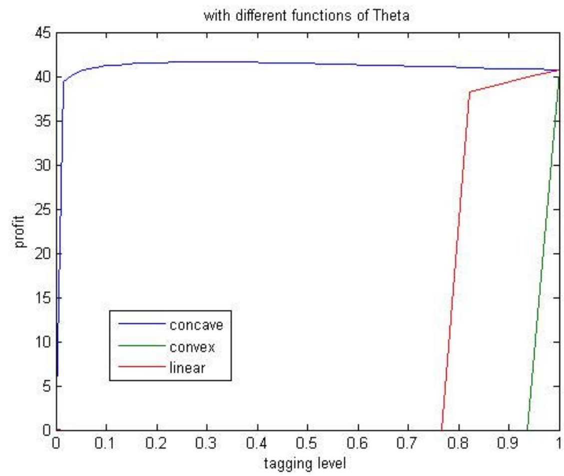

Figure 4.7 : Expected Maximum Profit Values with Respect to Tagging Level with Different Functions of Theta

Concave function of $\beta_{(y)}$ has the meaning of the inventory system responses with high improvement on inventory error factors even for pallet level tagging. On the other hand convex function of $\beta_{(y)}$ means that the system response with a meaningful improvement rate on shrinkage error factor only for item level tagging.

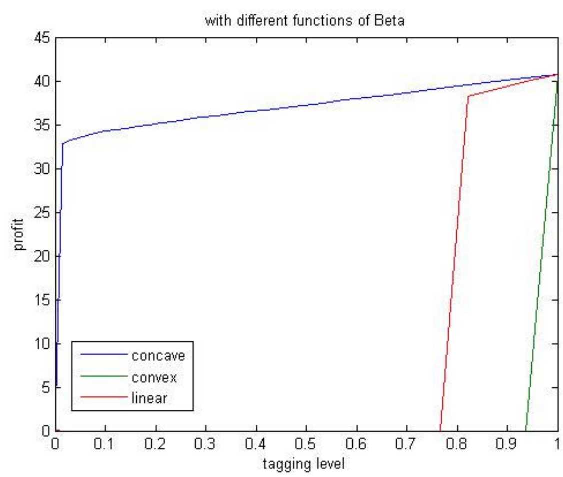

Figure 4.8 : Expected Maximum Profit Values with Respect to Tagging Level with Different Functions of Beta

The effects of different functions of $L_{(y)}$ on inventory system has different characteristics according to $\beta_{(y)}$ and $\theta_{(y)}$. The system is not sensitive to the different behaviors of labor cost function of tagging level variable ($L_{(y)}$) as much as the others ($\beta_{(y)}$ and $\theta_{(y)}$). But in any case there is a distinction between convex, linear and concave functions.

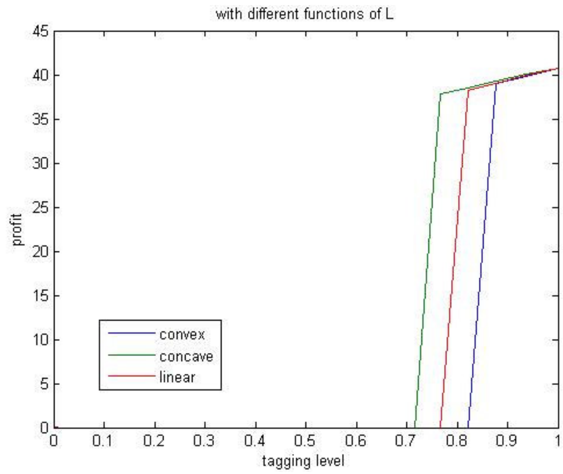

Figure 4.9 : Expected Maximum Profit Values with Respect to Tagging Level with Different Functions of Labor Cost

In this part of the study, under uniform distributed demand assumption, we have reached some inferences. To sum up:

- While the relative tag price according to retail price is getting higher, optimum tagging level moves to the pallet level tagging decision.
- While the benefit provided by RFID systems increases, optimum tagging level moves to item level tagging side.
- While the deviation of demand increases, optimum tagging level moves to item level tagging side.
- $\theta_{(y)}$ has bigger effect on systems according to $\beta_{(y)}$, and $\beta_{(y)}$ has bigger effect on system according to $L_{(y)}$.

4.2 Numerical Study & Results for Normally Distributed Demand

To analyze the effects of different parameters on expected maximum profit value under normally distributed demand assumption, we preserve the parameter set that we used for numerical study of uniform distributed demand assumption. All the calculations and graphs that are required to present the results are done by the help of MATLAB 7.3.0.R2006b. Note that throughout the paper in our numerical examples, we set the function of tagging cost $C_{(y)}$ with respect to tagging level linear (

$C_{(y)} = t.y$), and the other functions of y ($\theta_{(y)}$ and $\beta_{(y)}$) linear, convex and concave such as the previous part.

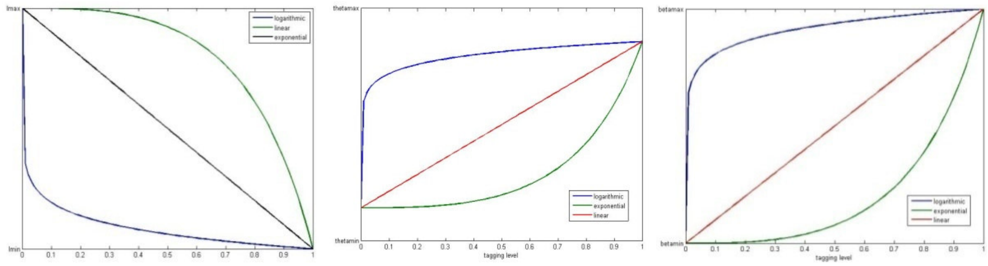

Figure 4.10 : Linear, Convex and Concave Functions of Labor, Misplacement and Shrinkage Costs of Tagging Level Variable *y*

4.2.1 Results on The Effect of Different Tag Prices on Tagging Level Decision

To analyze the effect of tag price on optimum tagging level decision under normally distributed demand, we graph the expected maximum profit function with respect to tagging level y. Figure 4.12 shows the behavior of expected maximum profit function under the parameter set that is shown on Table 4.8. The functions that are used for this calculation are as following.

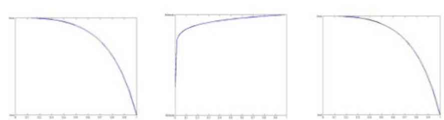

Figure 4.11 : Different Shapes of Functions *L(y), θ(y)* and *β(y)*

Table 4.8 : Parameters used for Analyzing the Effect of Tag Price on Optimum Tagging Level

Parameters	**Values**
$\mu_{\beta\ min}$	0.8
$\mu_{\beta\ max}$	0.9
$\mu_{\theta\ min}$	0.7
$\mu_{\theta\ max}$	0.85
l_{min}	0.003
l_{max}	0.01
μ_x	100
σ_x	10

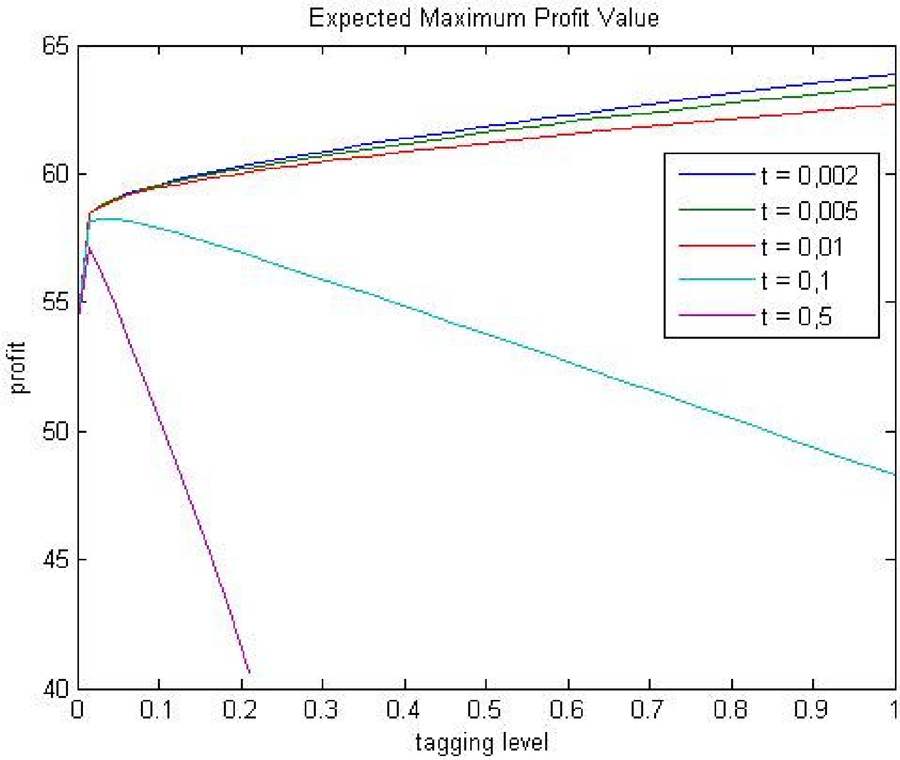

Figure 4.12 : Expected Maximum Profit Function with Respect to Tagging Level Under 5 Different Tag Prices

As can be seen from Figure 4.12, it is obvious that while tag price is increasing y^* value is getting smaller, which also means there is a tendency of packing and tagging more units of items together, i.e. from item level tagging to pallet level tagging. The following table shows us optimum y^* values and also the optimum packing sizes. For example for relatively cheap tag prices according to retail price of the item (t = 0,01, 0,005 and 0,002), item level tagging is the best option; for t = 0,1, the best option is to tag cases of 12 units of that item capacity; for t = 0,5, the best option is to tag pallets or cases of 67 units capacity.

Table 4.9 : Optimum Tagging Level and Packing Size Values

Tag Price (t)	Optimum Tagging Level Value	Optimum Packing Size Value
<0.01	1	1
0.1	0.03	33
0.5	0.15	67

4.2.2 Results on The Effect of Different Tag Prices on Tagging Level Decision Under High Performance of RFID Systems (Improved Inventory Accuracy Levels)

Under normally distributed demand, analyzing the effect of improved inventory accuracy rates on optimum tagging level decision, we do the same calculation with higher $\mu_{\theta max}$, $\mu_{\beta max}$ and lower l_{min} values. The results are quite intuitive because for higher gains of RFID technology optimum tagging level value skews to the right, i.e. to item level tagging. Figure 4.13 shows the behavior of expected maximum profit function under the parameter set that is shown on Table 4.10.

Table 4.10 : Parameters used for Analyzing the Effect of Increased Inventory Accuracy on Optimum Tagging Level

Parameters	Values
$\mu_{\beta\ min}$	0.8
$\mu_{\beta\ max}$	0.98
$\mu_{\theta\ min}$	0.7
$\mu_{\theta\ max}$	0.95
l_{min}	0.0005
l_{max}	0.01
μ_x	100
σ_x	10

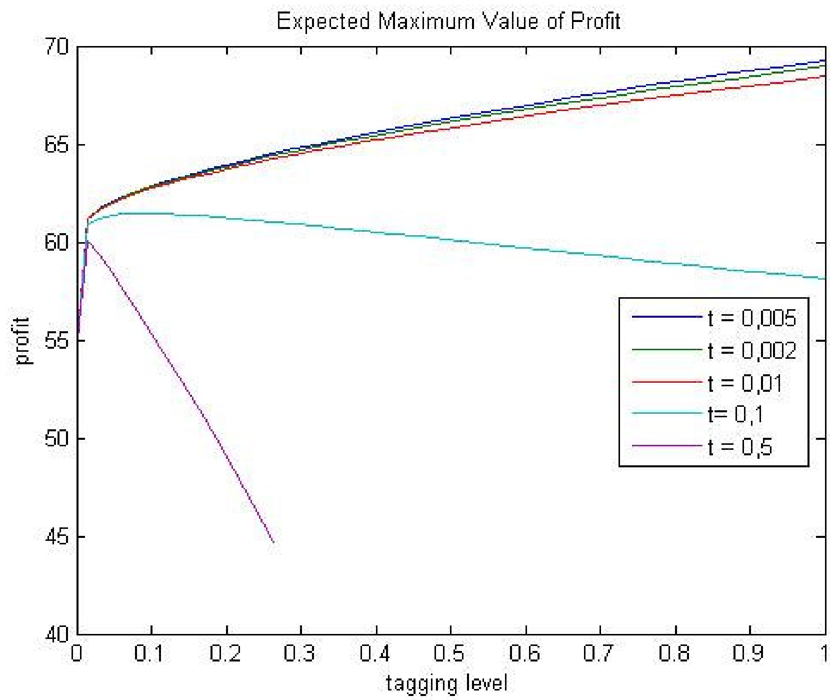

Figure 4.13 Expected Maximum Profit Values with Respect to Tagging Level under Improved Inventory Accuracy Levels

Table 4.11 shows us optimum y values and also the optimum packing sizes for higher benefit rates of RFID tagging.

Table 4.11 : Optimum Tagging Level and Packing Size Values

Tag Price (t)	Optimum Tagging Level Value	Optimum Packing Size Value
<0.01	1	1
0.1	0.08	12
0.5	0.15	67

4.2.3 Results on the Effect of the Variance of Demand on Tagging Level Decision

Under normally distributed demand, analyzing the effect of variance of demand on optimum tagging level decision, we graph the expected maximum profit function with respect to tagging level y. Figure 4.15 (maximum benefit of RFID), 4.16 and 4.17 (minimum benefit of RFID) show the behavior of expected maximum profit function under the parameter sets that is shown on Table 4.12. The functions that are used for this calculation are as following.

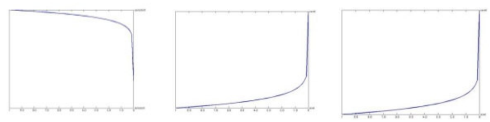

Figure 4.14 : Different Shapes of Functions *L(y), θ(y)* and *β(y)*

Table 4.12 : Parameters used for Analyzing the Effect of Increased Deviation of Demand on Tagging Level Decision

Parameters	**Values**
$\mu_{\beta\ min}$	0.8
$\mu_{\beta\ max}$	0.98 - 0.95 - 0.90
$\mu_{\theta\ min}$	0.7
$\mu_{\theta\ max}$	0.95 – 0.90 - 0.85
l_{min}	0.0005 - 0.001 - 0.003
l_{max}	0.01
μ_x	100
t	0,1

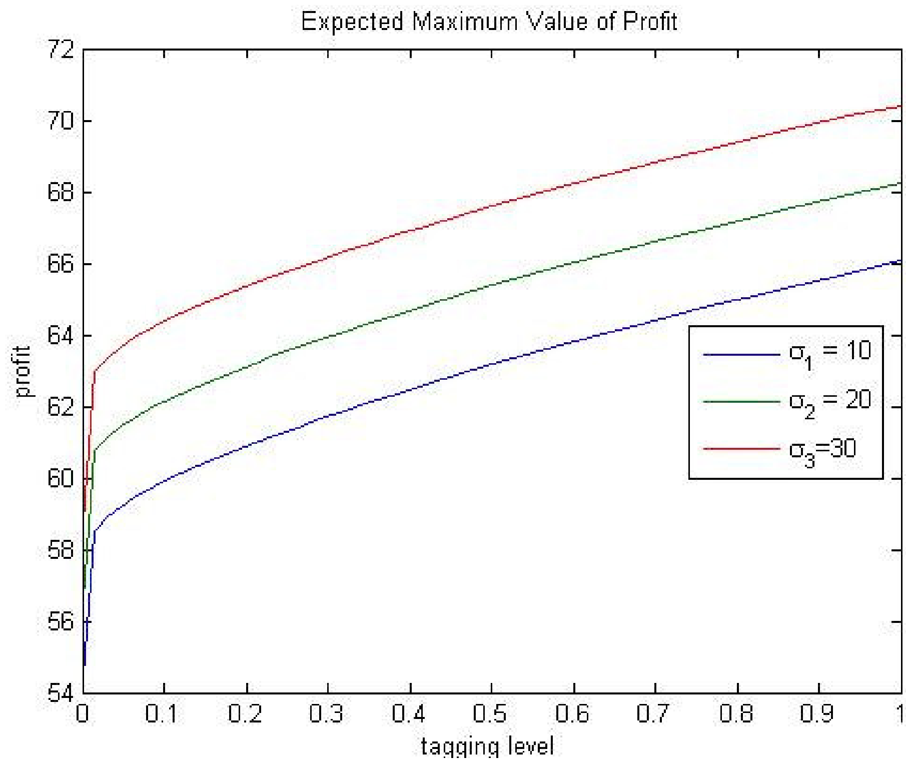

Figure 4.15 : Expected Maximum Profit Values with Respect to Tagging Level under Different Deviations of Demand (maximum benefit of RFID)

As can be seen from Figure 4.15, under the case of gaining maximum profit through RFID technology, while the variance of demand is increasing, y^* value is not changing contrary to the uniform distributed demand case. As can be seen on Table 4.13 the optimum y values are all equal to 1 that also means item level tagging level is the most appropriate case.

Table 4.13 : Optimum Tagging Level and Packing Size Values

Variance of Demand (σ)	**Optimum Tagging Level Value**	**Optimum Packing Size Value**
σ = 10	1	1
σ = 20	1	1
σ = 30	1	1

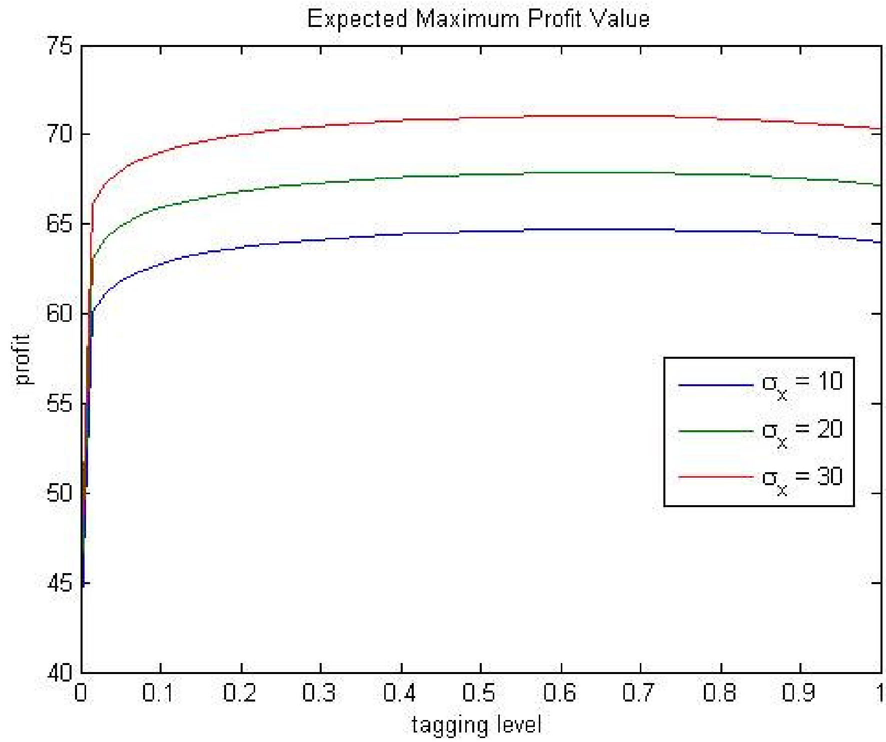

Figure 4.16 : Expected Maximum Profit Values with Respect to Tagging Level under Different Deviations of Demand (average benefit of RFID)

As can be seen from Figure 4.16, while the variance of demand is increasing, y^* value is not changing contrary to the uniform distributed demand case, under the parameter values of $\mu_{\theta max\ =\ 0,90}$, $\mu_{\beta max\ =\ 0,95}$ and $l_{min\ =\ 0,001}$. As can be seen on Table 4.14 the optimum y values are all equal to 0,67.

Table 4.14 : Optimum Tagging Level and Packing Size Values

Variance of Demand (σ)	Optimum Tagging Level Value
σ = 10	0.67
σ = 20	0.67
σ = 30	0.67

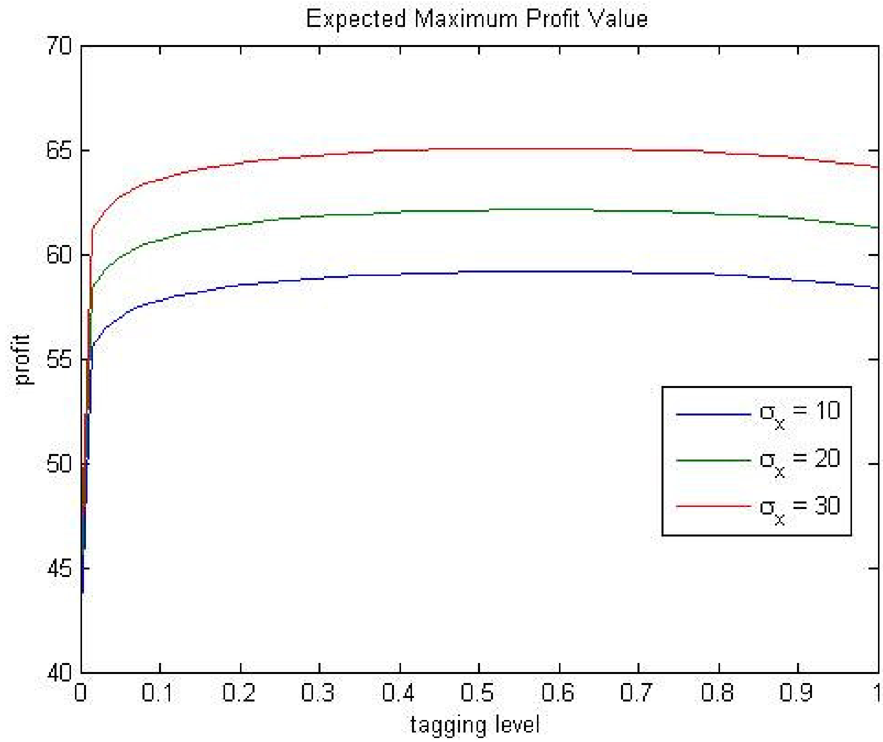

Figure 4.17 : Expected Maximum Profit Values with Respect to Tagging Level under Different Deviations of Demand (minimum benefit of RFID)

As can be seen from Figure 4.17, while the variance of demand is increasing, y^* value is not changing contrary to the uniform distributed demand case. As can be seen on Table 4.15 the optimum y values are all equal to 0,58.

Table 4.15 : Optimum Tagging Level and Packing Size Values

Variance of Demand (σ)	**Tagging Level**
σ = 10	0.58
σ = 20	0.58
σ = 30	0.58

4.2.4 Results on the Effect of Different Functions of *L(y)*, *θ(y)* and *β(y)* on Tagging Level Decision

To analyze the effect of different functions $L_{(y)}$, $\beta_{(y)}$ and $\theta_{(y)}$ on optimum tagging level decision, under the same parameter set and normally distributed demand assumption, we draw the graphs that show us how the system response to tagging level decision problem under different behaviors of system according to different functions of $L_{(y)}$, $\beta_{(y)}$ and $\theta_{(y)}$. So we fixed all parameter values including t, y, σ_x, l_{min}, $\mu_{\theta max}$ and $\mu_{\beta max}$ and set the functions $\beta_{(y)}$ and $\theta_{(y)}$ functions linear, then change the type of $L_{(y)}$ function; logarithmic, exponential and linear to find the effect of the changes on $L_{(y)}$. Then we repeat this by fixing $L_{(y)}$, $\beta_{(y)}$ linear and changing the types of $\theta_{(y)}$, similarly fixing $L_{(y)}$, $\theta_{(y)}$ linear and changing the types of $\beta_{(y)}$. As can be seen from the following graphs, results are quite intuitive. If the system has already had a big response on even pallet level tagging (see the concave function of $\beta_{(y)}$ and $\theta_{(y)}$), in other words even applying tagging on pallet level, the improvement on the rates of misplacement and shrinkage is high; y value skews to left, i.e. to pallet level tagging. The same logic is also valid for convex function of $L_{(y)}$.

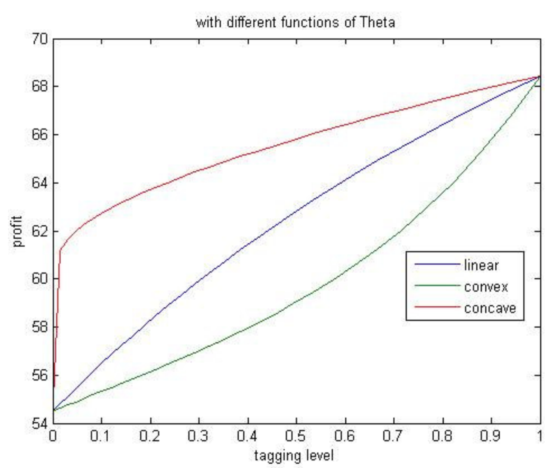

Figure 4.18 : Expected Maximum Profit Values with Respect to Tagging Level with Different Functions of Theta

Concavity of the function $\theta_{(y)}$ means that the inventory system is sensitive to high improvement on inventory error factors even for pallet level tagging. On the other hand convex function of $\theta_{(y)}$ means that the system response with a meaningful improvement rate on misplacement error factor only for item level tagging.

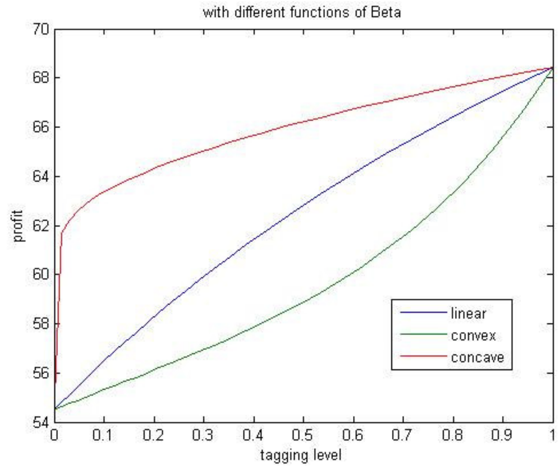

Figure 4.19 : Expected Maximum Profit Values with Respect to Tagging Level with Different Functions of Beta

Concaveness of the function $\beta_{(y)}$ means that the inventory system response with high improvement on inventory error factors even for pallet level tagging. On the other hand convex function of $\beta_{(y)}$ means that the system response with a meaningful improvement rate on error factors only for item level tagging.

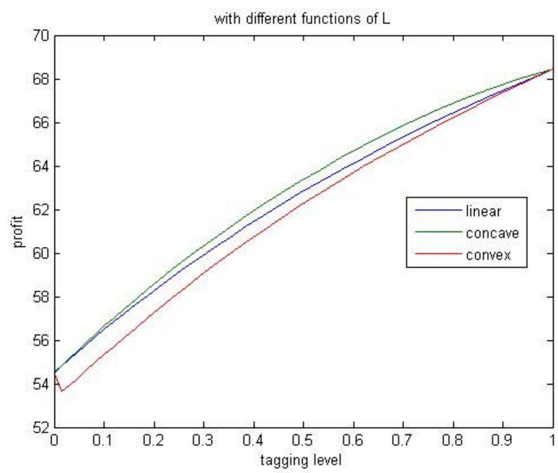

Figure 4.20 : Expected Maximum Profit Values with Respect to Tagging Level with Different Functions of Labor Cost

The effects of different functions of $L_{(y)}$ on system has different characteristics according to $\beta_{(y)}$ and $\theta_{(y)}$. The system is not sensitive to the different behaviors of labor cost function of tagging level variable as much as the others. But in any case there is a distinction between convex, linear and concave functions of *L*.

In this part of the study, under normally distributed demand assumption, we have reached some results. To sum up:

- While the relative tag price according to retail price is getting higher, optimum tagging level moves to lower level of tagging decision.
- While the benefit provided by RFID systems increases, optimum tagging level has the tendency of moving to item level tagging side.
- Optimum tagging level decision is not affected by the different deviation levels of demand under normally distributed demand assumption as much as under uniform distributed demand assumption.

5. CONCLUSION

The major purpose of this research was to achieve the hypothesis mentioned in the first section. Since we aimed to model a real life problem, our model attempts to reflect the real-world cost considerations and expected gains in the deployment of RFID tagging. To our knowledge this thesis is the first study that focuses on the problem of RFID tagging level for each SKU. This study can be extended in such a way that multi priod models can be analyzed.

5.1 Application of the work

In this thesis, the necessary steps for constructing an analytical model to decide on the optimum tagging decision were discussed. We firstly considered four different cases of tagging levels which are item-level tagging, case-level tagging, pallet-level tagging and no tagging. After defining the profit functions and taking the expectations of profit functions, we derived the equations that give us the optimum order quantity and maximum expected profit values for each cases. To analyse the problem more deeply we defined a continuous tagging level decision variable. After representing the parameters, which we consider as the inputs of our model, as the function of the lately defined tagging level decision variable, we are able to decide on the packing size of the units as well. We checked the outcomes of our model with numerical studies and the results are as follows: tagging level decision is affected by the relative price of RFID tags, the existent rates of misplacement and shrinkage errors, the improved rates of misplacement and shrinkage errors through RFID deployment and the variance of demand. The optimum tagging level moves to lower level of tagging while the concavity degree of $\theta(y)$ and $\beta(y)$ get higher. On the other hand if the concavity degree of $L(y)$ increases, optimum tagging level moves to high level of tagging.

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APPENDICES

APPENDIX A.1 : Different kind of RFID tags, adapted from Url-1 and Url-2

APPENDIX A.1

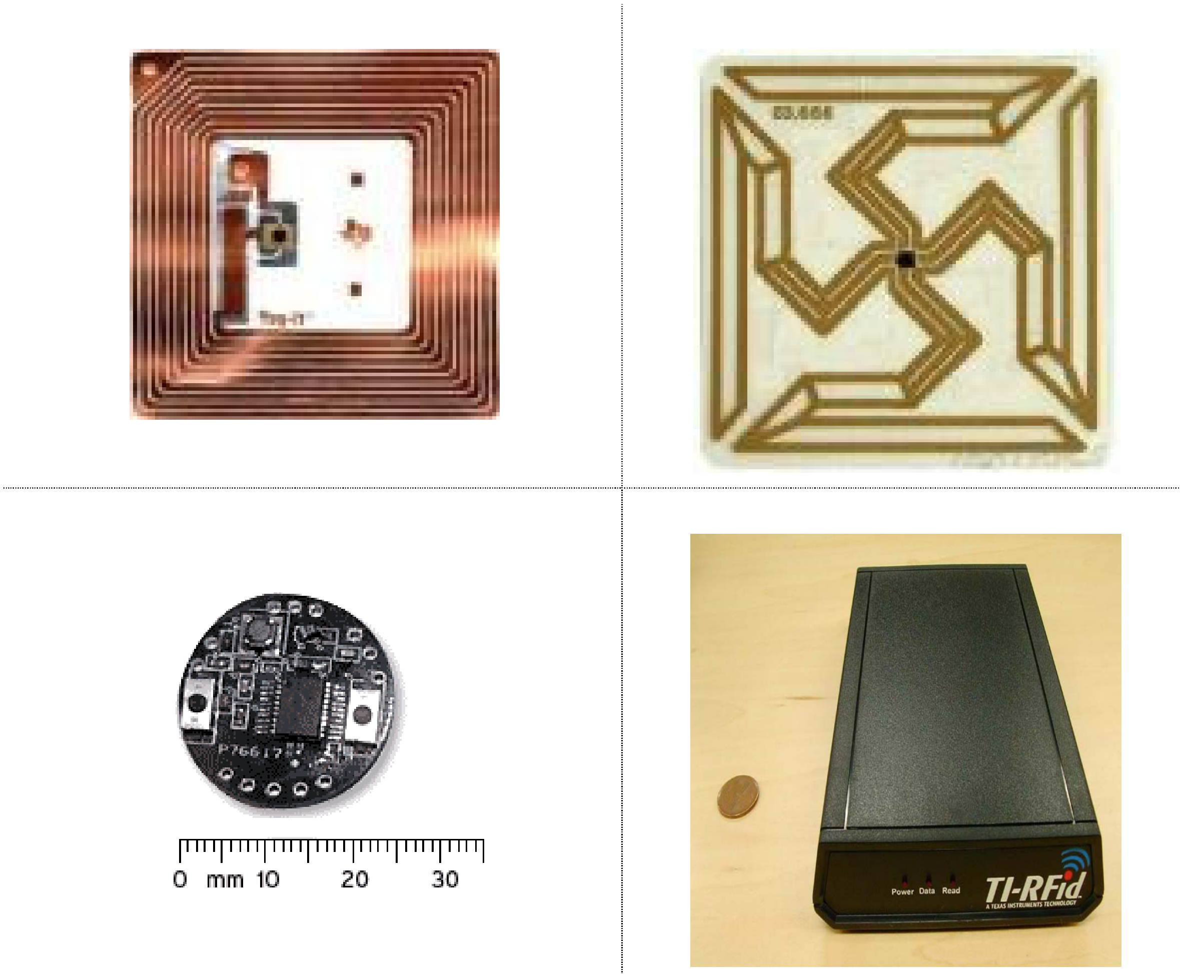

Figure A. 1 : Different Kinds of RFID Tags

Figure A. 2 : Different RFID Reader Systems

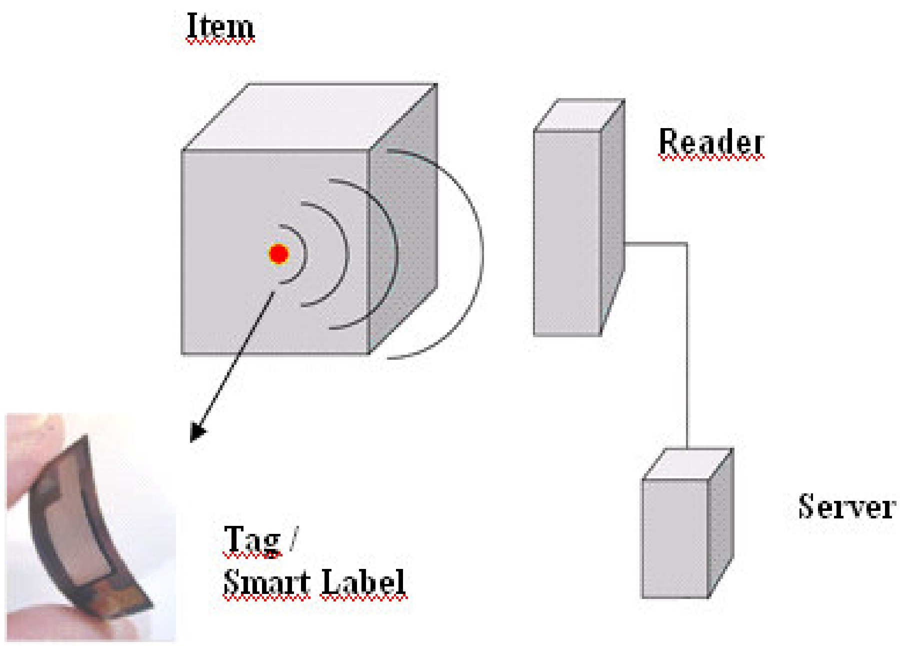

Figure A. 3 : Basic Components of an RFID System

VITA

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Publications:

- **Korkmaz E**. and Tan T., 2007: Deciding on the Level of RFID Tagging on Inventories. *INFORMS Annual Meeting 2007 (The Institute for Operations Research and the Management Sciences)*, 4-7 November, Seattle, USA.

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