ISTANBUL TECHNICAL UNIVERSITY ★ INSTITUTE OF SCIENCE AND TECHNOLOGY

DEVELOPING DRILLING OPTIMIZATION PROGRAM FOR GALLE AND WOODS METHOD

M.Sc. Thesis by
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Programme : **Petroleum and Natural Gas**

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SONDAJ OPTİMİZASYONU PROGRAMININ GALLE VE WOODS METODU İÇİN GELİŞTİRİLMESİ

YÜKSEK LİSANS TEZİ

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FOREWORD

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TABLE OF CONTENTS

Page

ABBREVIATIONS

$\boldsymbol{A}$	: $\frac{\text{bit cost}}{\text{rig cost per hr}} + \text{trip time, hours}$
$\boldsymbol{A_f}$	: formation abrasiveness parameter
$\boldsymbol{A_n}$	: A/A_f
$\boldsymbol{B}$	: bit bearing life, hours
$\boldsymbol{B_x}$	: bearing life expended, fraction
$\boldsymbol{B_{xf}}$	: bearing life expended, final fraction
$\boldsymbol{C}$	: drilling cost, rig hours per foot
$\boldsymbol{C_b}$	: bit cost, $
$\boldsymbol{C_r}$	: rig cost, $/hr
$\boldsymbol{C_f}$	: formation drillability parameter in Galle and Woods study
$\boldsymbol{C_f}$	: drilling cost, $/ft in Bourgoyne study
$\boldsymbol{C_2}$	: constant
$\boldsymbol{D}$	: bit tooth dullness, fraction of original tooth height worn away
$\boldsymbol{D_f}$	: bit tooth dullness, final fraction
$\boldsymbol{F}$	: distance drilled by bit, ft
$\boldsymbol{F_f}$	: final distance drilled by bit, ft
$\boldsymbol{F_t}$	: footage drilled, ft
$\boldsymbol{H}$	: hole or bit diameter, in
$\boldsymbol{H_t}$	: normalized tooth height
$\boldsymbol{H_1 - H_2}$	: tooth geometry constants
$\boldsymbol{J_1 - J_2}$	: composite functions of bit weight and rotary speed
$\boldsymbol{K}$	: $C * C_f$
$\boldsymbol{L}$	: tabulated function of $\overline{W}$

M	:	bit weight extrapolated to zero drilling rate, lbf
N	:	rotary speed, rpm
$\mathbf{P_1}$	:	$\int_0^{D_f} f_1\, dD$
$\mathbf{P_2}$	:	$\int_0^{D_f} f_2\, dD$
$\mathbf{P_3}$	:	$\int_0^{D_f} f_3\, dD$
R_t	:	rotating time, hr
TT	:	trip time, hr
S	:	drilling fluid parameter
S_n	:	S/A_f
$\mathbf{S_n}^*$	:	$\int_0^{D_f} \frac{Nma}{Li}\, dD$
T	:	rotating time, hours
T_f	:	final rotating time, hours
$\mathbf{U}$	:	$714{,}19 \int_0^{D_f} a\, dD$
$\mathbf{V}$	:	$714{,}19 \int_0^{D_f} a^{1/2}\, dD$
W	:	bit weight, 1000 lb
$\overline{W}$	:	equivalent 7 7/8 in. bit weight $= \frac{7{,}88W}{H}$
a	:	$0{,}928125D^2 + 6D + 1$
$a_1 - a_8$	:	exponents in the penetration rate equation
d_b	:	bit diameter
f_1	:	$\frac{1}{A_f}\frac{dT}{dD} = \frac{am}{i}$
f_2	:	$\frac{1}{A_f C_f}\frac{dF}{dD} = \frac{\overline{W}^k\, rma^{1-p}}{i}$
f_3	:	$\frac{S}{A_f}\frac{dB_x}{dD} = \frac{Nam}{Li}$
$f_{a_1} - f_{a_8}$	:	functions defining effect of various drilling variables
g_p	:	is the pore pressure in pounds per gallon

i	: $N + 4{,}348*10^{-5}*N^{3}$
i'	: $1 + 13{,}044*10^{-5}*N^{2}$
k	: exponent on weight in drilling rate equation
m	: $1359{,}1 - 714{,}19*\log\overline{W}$
$\overline{m}$	: $\frac{m}{714{,}19}$
m'	: $\frac{-310{,}169}{\overline{W}}$
p	: exponent on a in drilling rate equation
r	: $\left[e^{-100/N^2}N^{0{,}428} + 0{,}2N\left(1 - e^{-100/N^2}\right)\right]$ (for hard formations)
r	: $\left[e^{-100/N^2}N^{0{,}75} + 0{,}5N\left(1 - e^{-100/N^2}\right)\right]$ (for soft formations)
r'	: $\left[e^{-100/N^2}\left[\frac{0{,}428N^{0{,}428}}{N} + \frac{200N^{0{,}428}}{N^3} - \frac{40}{N^2} - 0{,}2\right]\right] + 0{,}2$ (hard formations)
r'	: $\left[e^{-100/N^2}\left[\frac{0{,}75N^{0{,}75}}{N} + \frac{200N^{0{,}75}}{N^3} - \frac{100}{N^2} - 0{,}5\right]\right] + 0{,}5$ (soft formations)
t_b	: bit life, hours
t_c	: nonrotating time during bit run, hours
t_t	: time of tripping opeations
λ	: exponent expressing effect of rotary speed on drilling rate
ρ_c	: mud weight, ppg
σ	: effect of weight on bearing wear
τ_H	: formation abrasiveness constant

LIST OF TABLES

Page

LIST OF FIGURES

Page

DEVELOPING DRILLING OPTIMIZATION PROGRAM FOR GALLE AND WOODS METHOD

SUMMARY

Drilling procedures of the oil and gas wells have always been highly priced and passing through on to deep water offshore drilling which brought together increasing investment made driling optimization much more important. Those investments necessitate more than one hunderd million dollars nowadays. For this purpose drilling optimization programs have been developed which considers best combinations of the driling parameters such as weight on bit, rotary speed, hydraulics etc. during the pre – drilling planning or real time drilling operations. In order to get the proper answer to above mentioned issues lots of work and study have been done and literature riview section presents some of them. Drilling optimization methods mostly used by industry are given in groups generally:

- Multiple Regression Method
- Drill off Test Method
- Galle and Woods Analytical Method

In this study, Galle and Woods method prefered because it provides analytical solutions. Most of the derivations of this method are produced. However, interpolation function is also generated and used successfully for the parameter L which is the function of the bit weight has unknown physical meaning. Using those interpolation function during the calculation procedures, error percentage is reduced from %15 to less than %1.

Besides usage of this model with the proposed manner is very difficult and very complicated as shown and explained in the fifth chapter. Providing combined graphical solutions also increases the probability of false reading of parameters that are very sensitive during calculations.

To reduce the above mentioned difficulties, a user friendly program is developed in Delphi. The results obtained from the developed user friendly program are compared with the results presented in the literature and reasonable outcomes are achieved.

SONDAJ OPTİMİZASYONU PROGRAMININ GALLE VE WOODS METODU İÇİN GELİŞTİRİLMESİ

ÖZET

Petrol ve doğal gaz kuyularının sondajı her zaman maliyetli olmuştur. Karadan derin deniz sularının sondajına geçişte maliyet farkları ve yüksek yatırım miktarları sondaj optimizasyonlarının önemini daha da artırmıştır. Bu miktarlar günümüzde yüz milyon ABD dolarını geçmektedir. Bu amaçla sondaj optimizasyonunda en iyi sondaj parametreleri (matkap yükü, dönme hızı, hidrolik vb.) kombinasyonlarını hem sondaj öncesi planlamada hem de sondaj operasyonları sırasında gerçek zamanda veren programlar geliştirilmiştir. Bu konuda yapılmış çok sayıda literatür çalışması vardır. Endüstri tarafından kullanılan sondaj optimizasyon modelleri genel olarak aşağıdaki gruplar halinde verilebilir:

- Regresyon Metodu
- Drill – Off Test Metodu
- Galle ve Woods Analitik Metodu

Bu çalışmada Galle ve Woods modeli analitik bir çözüm yöntemi olması nedeniyle tercih edilmiştir. Ancak, bu modelin önerildiği şekliyle kullanılması beşinci bölümde gösterildiği gibi çok zor ve karmaşıktır. Çözüm yönteminin grafiksel olması, okuma hatalarının yapılma olasılığını yükseltmektedir. Çözümde grafikten okunan değerlerin kullanılması ve sonuçların bu okumalara çok hassas olması yöntemin kolayca uygulanabilirliğine engel oluşturmaktadır. Diğer taraftan, bu modele ait denklemlerin türetimleri literatürde verilmemiştir ve kaynak olarak gösterilen çalışmalara artık ulaşılamamaktadır. Bu nedenle modele ait tüm denklem türetimleri bu çalışmada yeniden yapılmış ve çalışma ekinde verilmiştir. Ayrıca modelde matkap yüküne bağlı bir fonksiyon şeklinde tanımlanan L parametresi için Newtonian interpolasyon yöntemi kullanılarak, yeni bir yaklaşım geliştirilmiştir. Geliştirilen bu yeni yaklaşım sonucu, L parametresi hesaplarında oluşan hata payı % 15'den %1'in altına indirilmiştir.

Yukarıda belirtilen zorlukları azaltmak (grafiksel çözümden bağımsız bir hale getirmek) ve modelin kolay kullanılabilirliğini sağlamak amacıyla Delphi yazılım dilinde Galle ve Woods yöntemi ile sondaj optimizasyonu yapan bir program kodu geliştirilmiştir. Geliştirilen bu program literatürde verilen veriler ile test edilmiş ve sonuçlar karşılaştırılmıştır. Programdan ve literatürden elde edilen sonuçların birbirleriyle aynı oldukları gösterilmiştir.

1. INTRODUCTION

Since the beginning of the petroleum industry, optimization of the planning procedure, drilling operation, drill string and casing design and etc. have been taken into consideration. As time goes by those optimization techniques become much more important because of increasing cost of the operations such as considering hiring, operating, and service expenditures differs from onshore and offshore and costs about one million to tenth millions even more. The oil and gas companies have been interesting in the cost effective capital. One of the main parts of the operations together with the exploration, production, and etc. is the drilling. And drilling operation makes quiet up the investment. Consequently it should be evaluated carefully because of the cost it generates. Once a drilling system is established, there are only few parameters that can be changed due to the system limitations. The most important variables affecting the drilling operations to be cost-effective have been identified and studied include (1) bit type, (2) formation characteristics, (3) drilling fluid properties, (4) bit operating conditions (bit weight and rotary speed), (5) bit tooth wear, and (6) bit hydraulics (Bourgoyne et. al, 1991). However, with the variation encountered in formation characteristics the optimum value of drilling parameters changes and during drilling only few operation parameters can be changed to their optimum values. Thus, parameters are defined as alterable and unalterable parameters; formation characteristics and depth are determined as unalterable parameters, and bit type, drilling fluid properties, bit tooth wear, bit hydraulics and weight on bit (WOB) and rotary speed (RPM) are determined as alterable. After the drilling operation started there are left only three, namely weight on bit, rotary speed and hydraulic parameters which are mostly alterable during the drilling operation. In general, two parameters mostly WOB and RPM are the most important among others that influence the drilling rate and drilling cost at most.

1.1 Purpose of the Thesis

This thesis represents a new approach for the selection of optimum combinations of rotary speed and bit weight to minimize total drilling cost and predicting approximate bit life remained. Since the drilling operation started the only parameters to control and to make some alterations are the applied weight on bit and rotary speed which affect the drilling cost, drilling rate, and wear rate of bit tooth. One of the main objectives of drilling operation is realizing the operation as much as possible for maximum savings. Another main factor is maximum penetration rate. But in this thesis cost effective drilling operation preferred and for those purpose the method proposed by Galle E.M. and Woods H.B. "The Best Constant Weight and Rotary Speed for Rock Bits" selected (Galle E.M. and Woods H.B.,1963) . The following important question is proposed: "Do optimum combinations of bit weight and rotary speed exist which will minimize the cost to drill specified depth intervals?" A mathematical analysis of the cost to drill any depth interval is the basis of the method selected and the best combination of bit weight and rotary speed which minimize total drilling cost is concerned in this thesis. Another important variable and parameter which is concerned in this work is the forecasting bit dullness and the remaining life of the bit. Besides derivation of the analytical solution is not available in the literature. The complete mathematical derivation of the model is given in this study.

1.2 Background

The comprehensive mathematical models of the well drilling process have made it possible to estimate quantitatively the effect of certain key parameters involved. With the prime objective of reducing the cost of drilling a well, the researchers investigated various optimization procedures and how they could be used, in conjunction with the mathematical model selected, to achieve this reduction. The present knowledge of the drilling process restricted the number of parameters to be optimized to two, namely, the weight on the bit and the rotary speed. Rotary drilling is a complex subject involving many variables, some of which may be altered and others not. Those factors that may be controlled are mud properties, hydraulics, bit type, weight on bit and rotary speed. Unaltered variables include such things as rock properties, formation to be drilled and depth. A complete description of how these variables affect rotary drilling and how they interact is available in the literature (Lummus, 1970). However, this thesis is concerned with only two of the most important controllable variables, weight on bit and rotary speed. It is assumed that the other factors have already been optimized. Although many investigations have taken place over the years, in 1958 Speer was the first to propose a comprehensive method for determining optimum drilling techniques. He demonstrated empirically the interrelationships of penetration rate, weight on bit, rotary speed, hydraulic horsepower and formation drillability (Speer, 1958). Speer combined these five relationships into a chart for determining of field test data. Further developments of a comprehensive model were introduced by different authors.

In this thesis, Galle and Woods model is concerned. The assumptions made by Galle and Woods will apply to any optimization procedure based on their equations. The Galle and Woods equations are defined as follow:

$$\frac{dF}{dT} = \frac{C_f \bar{W}^k r(N)}{a(D)^p} \tag{1}$$

$$\frac{dD}{dT} = \frac{1}{A_f} \frac{i(N)}{a(D)m(\bar{W})} \tag{2}$$

$$\frac{dB_x}{dT} = \frac{N}{SL(\bar{W})} \tag{3}$$

The above three formulas are representing the drilling rate, dulling rate and bearing life, respectively. In these equations the formation drillability factor, C_f , the formation abrasiveness factor, A_f, and the drilling fluid factor, S, are functions of bit type, hydraulics, drilling fluid and formation.

In original work done by Galle and Woods the following three procedures and eight original cases were evaluated;

Procedures:

1) The best combination of constant weight and rotary speed
2) The best constant weight for any given rotary speed
3) The best constant rotary speed for any given weight

The first has application where rig flexibility permits the use of any weight or rotary speed, the second where rig limitations or vibration problems dictate the rotary speed that must be used, and the third where crooked hole conditions dictate the maximum weight used.

Cases:

i) Teeth limit bit life
ii) Bearings limits bit life
iii) Bearings and teeth wear out simultaneously
iv) Drilling rate limits economical bit life cases
v) Drilling rate and bearings limit bit life simultaneously
vi) Drilling rate and teeth limit bit life simultaneously
vii) Drilling rate, teeth and bearings limit bit life simultaneously
viii) Neither drilling rate, nor teeth, nor bearings limit bit life

Cases i, ii, and iv were thoroughly investigated and case iii was found to have only a limited range of significance and obtained by linear interpolation of cases i and ii. In this thesis the following three procedures and two of eight original cases are considered;

Procedures:

1) The best combination of constant weight and rotary speed
2) The best constant weight for any given rotary speed
3) The best constant rotary speed for any given weight

Cases:

i) Teeth limit bit life
ii) Drilling rate limits economical bit life cases

It is believed and assumed that teeth limitation and drilling rate limitations are among the most considered and influenced cases and theoretical and mathematical considerations are given to only these cases.

2. OPTIMIZATION

The word optimum, meaning "best", is synonymous with "most" or "maximum" in one case and with "least" or "minimum" in another. The term, optimize, means to achieve the optimum, and optimization refers to the act of optimizing. Thus, optimization theory encompasses the quantitative study of optima and methods for finding them.

There is no such thing as a "true" optimum drilling program; invariably compromises must be made because of limitations beyond our control that result in something less than optimum (Lummus, 1970).

In general terms, an optimization problem consists in selecting from among a set of feasible alternatives, one which is optimal according to a given criterion (Dano, 1975).

The optimization term in this thesis are considered as the drilling procedure, which the best constant weight and rotary speed together with another controllable drilling parameters yield the penetration rate with the minimum drilling cost.

2.1 Purpose of Optimization

In petroleum industry, highest expenses are encountered during drilling operations. Since the produced hydrocarbons from present reservoirs are becoming far from meeting the demands, major oil companies begin spending enormous budgets for the recovery and exploration of new oil and gas reserves. The drilling costs increase drastically because most of the search for new reserves is conducted in offshore locations or hard-to-reach depths. It is only possible for a drilling operation to be successful, safe, and economic with comprehensive drilling program and design. The aim of this study is to conduct a mathematical optimization of the drilling parameters which are thought to have a high influence on rate of penetration, and to determine the necessary drilling conditions analytically in order to minimize the drilling cost. Throughout this study, weight on bit, rotation speed, and the bit wear are assumed to have a direct impact on rate of penetration. An analytical drilling cost definition is introduced using the developed rate of penetration equation, and optimization of the drilling parameters in order to satisfy the minimum drilling cost condition is achieved by applying certain mathematical methods.

2.2 Optimization of Alterable Drilling Parameters

The drilling variables can be classified as alterable or unalterable, as shown in Table 2.1. Classification is not strict, as some of the unalterable ones may be altered by a change in the alterable ones. For example, a change in mud type may allow for a change in bit type, resulting in a faster penetration rate through a particular formation. There is considerable interdependence among the alterable variables. For instance, mud viscosity and fluid loss are considerably influenced by the type and amount of solids.

Of course during the drilling operation only some alterable variables could be changed for a better drilling procedure, penetration rate, and mainly for a maximum cost and time savings.

Table 2.1: Alterable and unalterable variables (Iqbal F., 2008).

Alterable	**Unalterable**
Mud type	Weather
Solids content	Location
Viscosity	Rig condition
Fluid loss	Rig flexibility
Density	Corrosive borehole gases
	Bottom hole temperature
Hydraulics	Round Trip Time
Pump pressure	Rock Properties
Jet velocity	Characterisitc hole preoblems
Circulating rate	Water availability
Annular velocity	Formation to be drilled
	Crew efficiency
Bit Type	Depth
Weight on bit	
Rotary speed	

2.3 Drilling Mud

The first step in putting together an optimized drilling program should be to plan a detailed mud program. The drilling fluid is the single most important factor affecting drilling rate. Selection of the best mud for a particular area will allow use of optimum hydraulics to clean the bit and hole and enable effective implementation of optimum weight-rotary speed relationships to drill faster and to properly wear out the bit. In this study, it is assumed that the optimum drilling mud is carefully selected. In general, the higher the solid content or density of the mud, the lower the penetration rate and the higher the cost.

2.4 Hydraulics

Optimum hydraulics is the proper balance of the hydraulic elements that will adequately clean the bit and borehole with minimum horsepower. The elements are flow rate, which sets annular velocity and pressure losses in the system; pump pressure, which sets jet velocity through nozzles; flow rate-pump horsepower relationship, which sets hydraulic horsepower at bit; and the drilling fluid, which determines the pressure losses and cuttings transport rate. To achieve optimum hydraulics, these elements must work in the proper ratios. These ratios are sometimes hard to define. For example, the proper balance between flow rate and annular velocity depends on bit cleaning, erosion of borehole in turbulent flow, and lost circulation problems. Optimum jet velocity depends on formation characteristics, mud solids, WOB, bit type, annular velocity limitations. Decisions on defining the proper balance between the hydraulic elements make this one of the most difficult phases of drilling optimization. However, successful hydraulics programs can be prepared by first considering two factors: bit cleaning and hole cleaning. In general, drilling rate increases with increasing hydraulic level.

2.5 Bit Selection

To do a good job of selecting bits for drilling a particular well, the engineer must have a working knowledge of the types of bits available from the major bit manufacturers and how best to use these bits in drilling formations ranging from very soft to very hard, considering such problems as deviation, solids content of the mud, hole gauge and lost circulation. A comprehensive bit correlation chart, continually updated to include new bits, is therefore, a starting point in selecting the proper bits for drilling a well. It is also important that the engineer have both qualitative and quantitative descriptions of bit wear from nearby control wells in order to do a good job of selecting bits for the proposed well. It can be surmised from the foregoing remarks that complete information on bit wear from control wells is an absolute necessity in planning a comprehensive bit selection program.

2.6 Weight On Bit

The weight applied to the bit has a major effect on both penetration rate and the life of the bit. Thus, the determination of the best weight is one of the problems faced by the drilling engineer. When drilling, weight is applied to the cutters so the rock is penetrated. Up to certain limits the more weight applied the faster the bit will drill. If too much weight is applied, the cutters may become completely buried (known as bit flounder) and weight will be taken by the cones or bit body as depicted in Figure 2.1. This will reduce rate of penetration (ROP) and rapidly wear the cones. Increasing weight will also accelerate wear on bearings and cutters.

Deviation is also affected by WOB. In a vertical borehole with a build bottom hole assembly (BHA), increasing weight will deflect the well path from vertical. (Devereux S.,1998).

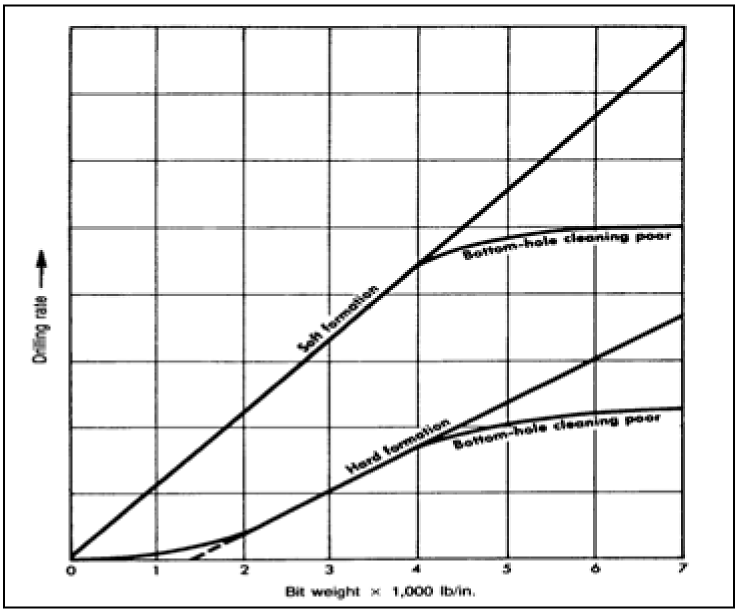

Figure 2.1: Effect of bit weight on penetration rate (Moore,1986)

2.7 Rotary Speed

Increasing rotary speed (RPM) will increase rate of penetration (ROP) up to a point where the cutters are moving too fast to penetrate the formation before they move on. Excess RPM will cause premature bearing failure or may cause PDC or diamond cutters to overheat.

Deviation is also affected by RPM. Higher rotary speeds tend to stabilize the directional tendencies of rotary BHAs. A rotary BHA has a tendency to turn to the right; this tendency is weaker at higher rotary speed. Rotary speeds that cause string vibrations must be avoided. At higher rotary speeds that kind of problems occurs and it should be prevented (Figure 2.2.). It is considered that the rotary speed also affects the bearing life. The bearing life decreses faster at higher rotary speeds.

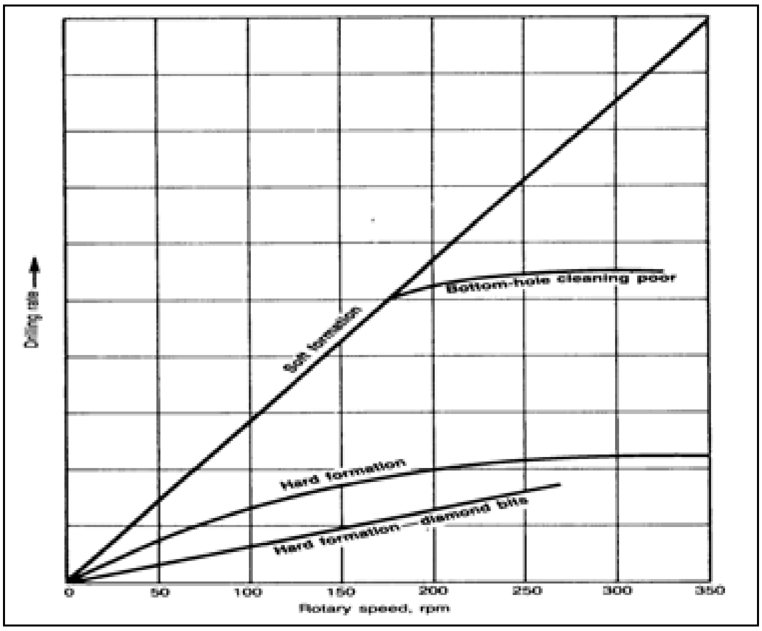

Figure 2.2: Effect of rotary speed on penetration rate (Moore, 1986)

3. OPTIMIZATION METHODS

There are many optimization methods or techniques are available for industry today. And methods for computing the optimum bit weight and rotary speed combinations for achieving minimum cost are also available. All of these methods require the use of mathematical models to define the effect of bit weight and rotary speed on penetration rate and bit wear. Most of the techniques available are based on the cost per foot analysis. The cost per foot for various assumed bit weights and rotary speeds can be computed using penetration rate and bit wear models. One of the models is investigated deeply and this thesis is based on that approach and significance of the other models stressed, as well. Brief explanations and usage of the models are given in this section.

3.1 Galle and Woods Method

In this section of the thesis, only stepwise calculation procedure of the work done by Galle and Woods (Galle E.M. and Woods H.B., 1963) with a table which is for some parameters is underlined but detailed procedures and cases are given on the next section.

The following steps outline and illustrate the calculation procedures which is presented on the original work.

Step 1. Get a record of the bit's performance and wear data

Step 2. Record bit size (d) and type: 8.75 " OSC

Step 3. Record hourly operational costs (Copr): 50,00 $/hr

Step 4. Record bit cost (Cb): 200,00 $

Step 5. Record the depth in (10000 feet) and depth out (10180 feet)

Step 6. Record the round trip time (tt) for the bit: 6 hrs

Step 7. Record the bit footage (F): 180 feet

Step 8. Record rotating time (T) of the bit 12 hrs

Step 9. Record the weight on bit (1000's lb): 35

Step 10. Record the rotary speed (N): 100 rpm

Step 11. Record the tooth dullness (D): 6/8

Step 12. Record the bearing wear (B): 4/8

Step 13. Calculate the equivalent weight on bit as 1000’s lb (W)

$W = 7{,}875/\text{step } 2 * \text{step } 9 = 7{,}875/8{,}75 * 35 = 31{,}5$

Step 14. Using the result from step 13, obtain the weight on bit parameter (m) from Table 3.1 $m = 0{,}404$

Step 15. Using the result from step 13, obtain the bearing life parameter (L) from Table 3.1 $L = 2316$

Step 16. Using the result from step 10, obtain the rotary speed parameter (i) from Table 3.1 $i = 143$

Step 17. Using the result from step 10, obtain the rotary speed parameter (r) from Table 3.1 $r = 31{,}8$

Step 18. Using the result from step 11, obtain the tooth dullness parameter (U) from Table 3.1 $U = 1834$

Step 19. Using the result from step 11, obtain the tooth dullness parameter (V) from Table 3.1 $V = 967$

Step 20. Calculate the formation abrasiveness coefficient (A)

$A = (T * i)/(m * U) = (\text{step } 8 * \text{step } 16)/(\text{step } 14 * \text{step } 18) = (12 * 143)/(0{,}404 * 1834) = 2{,}32$

Step 21. Calculate the formation drillability coefficient (C)

$C = (F * i)/(A * r * W * m * V) =$

$= (\text{step } 7 * \text{step } 16)/(\text{step } 20 * \text{step } 17 * \text{step } 13 * \text{step } 14 * \text{step } 19) =$

$= (180 * 143)/(2{,}32 * 31{,}8 * 31{,}5 * 0{,}404 * 967) = 0{,}0284$

Step 22. Calculate the drilling fluid coefficient (S)

$S = (T * N)/(B * L) = (\text{step } 8 * \text{step } 10)/(\text{step } 12 * \text{step } 15) = (12 * 100)/(4/8 * 2316) = 1{,}04$

Step 23. Calculate the cost parameter (G)

$G = Cb/Copr + tt = \text{step } 4/ \text{step } 3 + \text{step } 6 = 200/50 + 6 = 10$

Step 24. Calculate the normalized chart coefficient (A_n)

$A_n = G/A = \text{step } 23/\text{step} 20 = 10/2{,}32 = 4{,}31$

Step 25. Calculate the normalized chart coefficient (S_n)

$S_n = S/A = \text{step } 22/\text{step } 20 = 1{,}04/2{,}32 = 0{,}45$

Step 26. Go to the “tornado” chart. Figure 3.1 (Mitchel, 1995)

Step 27. Draw a small "dot" with a pencil where the value on the two axes intersect. Note the value on the abscissa, S_n, is 0,45 and the value on the ordinate, A_n, is 4,31. The "dot" on the "tornado" chart is shown for the example in these steps. The location of the "dot" will change for another example.

Step 28. If, as in this example, the "dot" is drawn in the "Teeth Limit Bit Life" region, then and only then a horizontal line is drawn to the left until it intersects the first vertically curved line. A new "dot" is drawn at that point and it is the "dot" to be used. The above has been illustrated in the "tornado" chart.

Step 29. Record the new value of S_n which is the value on the abscissa directly below the new "dot" and call the new value S_n' 0,425. If the dot is drawn in the envelope, then it is the "dot" to be used. In this instance, S_n is changed.

Step 30. With the "dot" drawn in step 28, record the cost per foot parameter (K): 0,00267.

Step 31. With the "dot" drawn in step 28, record the optimal equivalent weight on bit (W): 49,5 1000's lb.

Step 32. With the "dot" drawn in step 28, record the optimal rotary speed ($N^{'}$): 96 RPM.

Step 33. With the "dot" drawn in step 28, record Df': 8/8.

Step 34. Compute optimal weight on bit (WOB'): 55 1000's lbs (49,5*8,75/7,875)

Step 35. Using the result from step 31, obtain L' from Table 3.1

L = 1084

Step 36. Compute the expected rotation time (t): 11,13 hours.

t' = (Sn' * L' * A)/N' = (step 29 * step 35 * step 20)/step 32 =

= (0,425 * 1084 * 2,32)/96 = 11,13 hours

Step 37. Compute the expected bit footage (F'): 224 feet.

F' = C * (A * An + t')/K = step 21 * (step 20 * step 24* + step 36)/step 30 =

= 0,284 * (2,32 * 4,31 + 11,13)/0,00267 = 224 feet

Step 38. Compute expected cost per foot (C/F)' 4,70 $/ft.

(C/F') = K * Copr/C = step 30 * step 3/ step 21 = 0,00267 * 50/0,0284 = 4,70 $/ft

Step 39. Compute actual drilling cost per foot (C/F): 6,11 $/ft.

(C/F) = [step 4 + (step 3 * step 6) + (step 3 * step 8)]/step 7 =

= [200 + (50*6) + (50*12)]/180 = 6,11$/ft.

Step 40. Compare the savings and percent savings.

Savings = [C/F – (C/F)'] * F' = [step 39 – step38] * step 37 = (6,11 – 4,70) * 224 = $ 315,84

% Savings = [C/F – (C/F)'] * 100/(C/F)' = [6,11 – 4,70] * 100/4,70 = 30%

It is obvious that the calculation procedure is very complicated and after carefully selection of appropriate "tornado" charts is very important then using it is another difficulty. In order to make complicated calculation procedure a user friendly useful computer simulator written in Delphi (Version 7.0) and detailed calculation is discussed extensively in the next chapter.

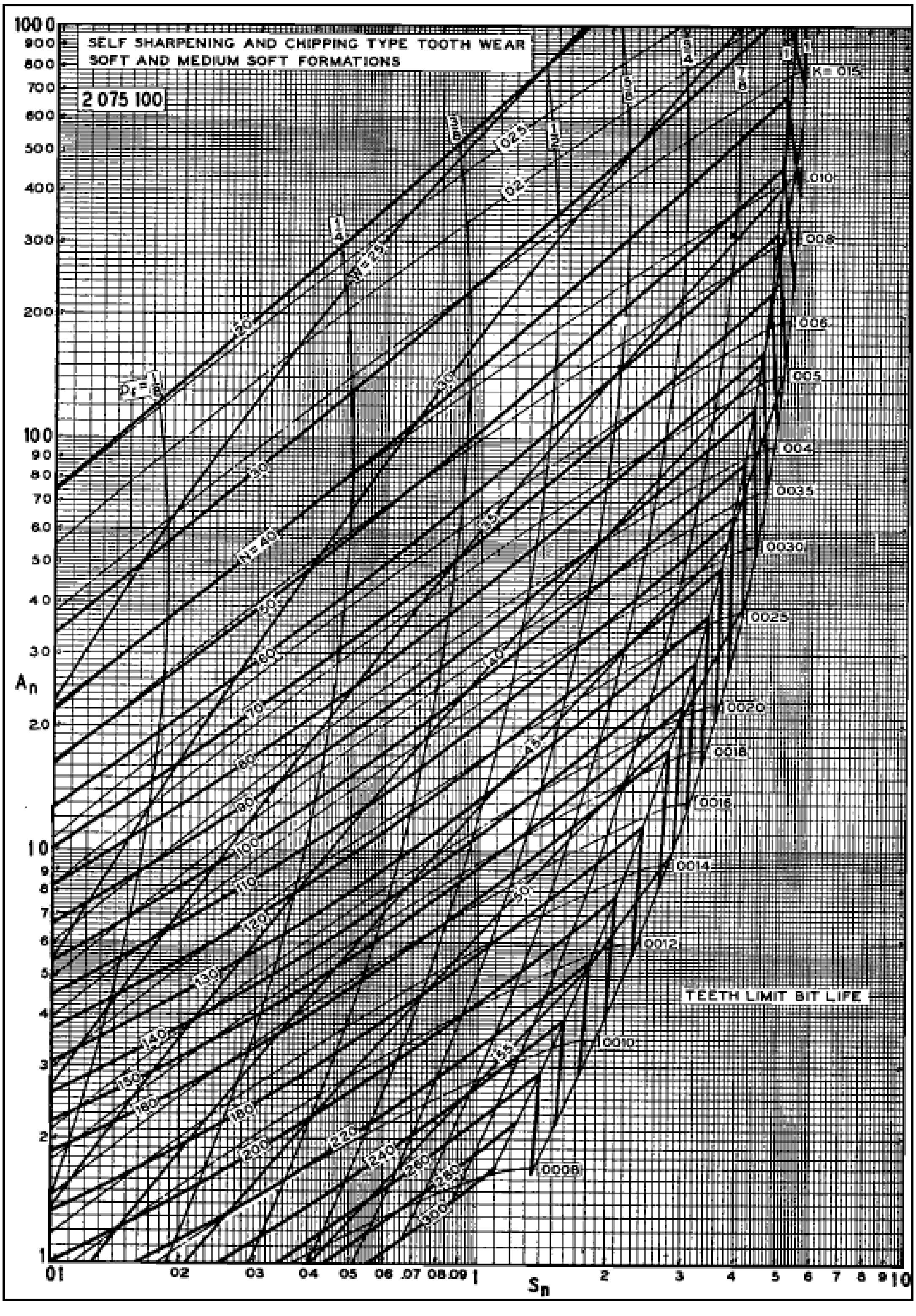

Figure 3.1: The best combination of weight and revolutions per minute for given A_n and S_n (Galle E.M. and Woods H.B., 1963)

Table 3.1: Functions used in calculations **(**Galle E.M. and Woods H.B., 1963)

N	i	r (2 043 100)	r (2 075 060 / 2 075 100)
10	10	2 25	5 2
15	15	3 12	7 6
20	20	3 69	9 6
25	26	4 12	11 4
30	31	4 47	13 0
35	37	4 77	14 6
40	43	5 04	16 2
45	49	5 29	17 6
50	55	5 52	19 0
55	62	5 73	20 4
60	69	5 94	21 8
65	77	6 13	23 1
70	85	6 32	24 4
75	93	6 50	25 7
80	102	6 67	27 0
85	112	6 84	28 2
90	122	7 00	29 4
95	132	7 15	30 6
100	143	7 31	31 8
105	155	7 45	33 0
110	168	7 60	34 1
115	181	7 74	35 3
120	195	7 87	36 4
125	210	8 01	37 5
130	226	8 14	38 7
135	242	8 26	39 8
140	259	8 39	40 8
145	278	8 51	41 9
150	297	8 63	43 0
160	338	8 87	45 1
170	384	9 09	47 2
180	434	9 31	49 3
190	488	9 53	51 3
200	548	9 73	53 3
225	720	10 23	58 2
250	929	10 69	63 0
275	1179	11 13	67 6
300	1474	11 54	72 2
325	1818	11 94	76 6
350	2214	12 32	81 0
375	2668	12 68	85 3
400	3183	13 03	89 5

$\bar{W}$	$\bar{m}$	L	$\bar{W}$ 6
5	1 204	13239	2 61
6	1 124	12279	2 93
7	1 057	11376	3 22
8	999	10532	3 48
9	948	9745	3 74
10	903	9016	3 98
11	861	8360	4 22
12	823	7758	4 44
13	789	7207	4 66
14	756	6702	4 87
15	726	6240	5 08
16	698	5840	5 27
17	672	5440	5 47
18	647	5080	5 66
19	624	4750	5 85
20	601	4439	6 03
21	580	4170	6 21
22	560	3920	6 39
23	541	3680	6 56
24	522	3470	6 73
25	505	3270	6 90
26	488	3080	7 06
27	471	2910	7 22
28	455	2770	7 38
29	440	2630	7 54
30	425	2496	7 70
31	411	2370	7 85
32	397	2260	8 00
33	384	2160	8 15
34	371	2060	8 29
35	358	1963	8 44
36	346	1880	8 58
37	334	1800	8 73
38	323	1725	8 87
39	311	1650	9 01
40	300	1578	9 15
41	290	1515	9 28
42	279	1460	9 41
43	269	1400	9 55
44	259	1340	9 68
45	249	1288	9 81
46	240	1240	9 94
47	230	1195	10 07
48	221	1150	10 19
49	212	1105	10 32
50	204	1063	10 45
51	195	1025	10 58
52	186	988	10 70
53	178	953	10 82
54	170	918	10 93
55	162	884	11 05
56	154	853	11 17
57	147	823	11 29
58	139	794	11 42
59	132	766	11 53
60	124	739	11 65
61	117	714	11 78
62	110	689	11 90
63	103	665	12 01
64	096	642	12 12
65	090	620	12 22
66	083	599	12 34
67	076	578	12 46
68	070	558	12 57
69	064	538	12 68
70	057	520	12 79
71	051	502	12 89
72	045	484	13 00
73	039	467	13 10
74	033	450	13 21
75	027	434	13 32
76	022	418	13 43
77	016	403	13 54
78	010	388	13 65
79	005	373	13 76

D	U	V
0	0	0
1/8	123	105
1/4	316	236
3/8	581	389
1/2	920	563
5/8	1337	756
3/4	1834	967
7/8	2413	1194
4/4	3078	1437

3.2 Drill off Test Method

Minimum cost drilling (MCD) requires a quantitative evaluation of the variables involved. A solution for minimum-cost drilling assuming constant bit weight and rotary speed over the entire bit life has been programmed for use in computing MCD schedules. This solution is subject to certain limiting assumptions such as;

- Drilling cost is the summation of bit cost, rotating cost, connection cost and hoisting cost.
- Diamond bits are excluded.
- Bit life is limited by either bearing failure or tooth wear, or by a combination of operational factors that make it cheaper to pull an incompletely consumed bit.
- Circulating hydraulics are adequate and do not limit drilling rate.
- Bit weight considerations exclude hole deviation.
- Drilling rate is a function of only bit weight, rotary speed, and degree of tooth dullness; that is, the effects of pressure, lithology, fluid property, hydraulics and drill string dynamics are ignored.

In equation form, the expressions are as follows:

Drilling Rate:

$$\frac{dy}{dt} = \frac{K * (W - M) * N^{\lambda}}{(1 + C_2 H_t)} \qquad \mathbf{(3.1)}$$

W: bit weight, thousands of pounds

M: bit weight extrapolated to zero drilling rate

λ: exponent expressing effect of rotary speed on drilling rate

C_2: constant

H_t: normalized tooth height, equal to zero for a sharp tooth and one for a fully worn tooth.

The value of M is derived from five spot drill – off test as follows;

$$M = 0{,}50 * \left[W_2 - R_2 * \left(\frac{W_2 - W_3}{R_2 - R_3}\right) + W_3 - R_3 * \left(\frac{W_3 - W_4}{R_3 - R_4}\right)\right] \qquad \mathbf{(3.2)}$$

and λ as follows;

$$\lambda = 0{,}50 \left[\frac{\ln\left(\frac{R_3}{R_2}\right)}{\ln\left(\frac{N_3}{N_2}\right)} + \frac{\ln\left(\frac{R_4}{R_5}\right)}{\ln\left(\frac{N_4}{N_5}\right)} \right] \quad \mathbf{(3.3)}$$

The formation drillability factor, *K*, is calculated at the each test spot and the average value of drillability factor as follows;

$$K = 0{,}25 \left[\left(\frac{R_2}{(W_2 - M){N_2}^{\lambda}} \right) + \left(\frac{R_3}{(W_3 - M){N_3}^{\lambda}} \right) + \left(\frac{R_4}{(W_4 - M){N_4}^{\lambda}} \right) + \left(\frac{R_5}{(W_5 - M){N_5}^{\lambda}} \right) \right] \quad \mathbf{(3.4)}$$

Hence, M, R, W, N, and λ factors are known then the K could be calculated as follows;

$$K = \left(\frac{R}{(W - M)N^{\lambda}} \right) \quad \mathbf{(3.5)}$$

Bearing Wear

The rate of bearing wear is directly proportional to the rate of rotation and bit weight raised to the power σ:

$$B_1 = \left(\frac{B}{NW^{\sigma}} \right) \quad \mathbf{(3.6)}$$

B: normalized bearing wear, equal to zero for new bearings and one for fully-consumed bearings

σ: effect of weight on bearing wear

The weight exponent, σ, relates bearing wear rate to bit weight, and has been determined experimentally. A value of 1.5 was observed for common drilling fluids.

Tooth Wear

The effects of rotary speed, bit weight, and tooth wear on tooth wear rate are presented in the following relation:

$$T_1 = \left(\frac{-D_1 * W + D_2 * \left(1 + \frac{C_1}{2}\right)}{0{,}001 * A_f (P * N + Q N_3)} \right) \tag{3.7}$$

Where, A_f is the formation abrasiveness factor

C_1, Q and P are listed parameters in Table 3.2

D_1 and D_2 are listed parameters in Table 3.3 or could be calculated as follows, in inches

$D_1 = 0{,}2708 * D^{-0{,}6207}$

$D_2 = 2{,}7057 * D^{0{,}3817}$

Table 3.2: Tooth wear parameters for three cone rock bits (Moore, 1986)

	Rock Bit Make and Type						Values of Parameter		
Group No.	Hughes	Chicago Pneumatic	Security	Smith	Reed	Globe	P	Q	C_1
1	OSC-3	ES1C	S3	DT3	Y-T3	SS3C	2.5	1.088×10^{-4}	7
2	OSC-1	ES1	S4	DTZ	YT-1	S3C	2.0	0.870×10^{-4}	6
3	OSC	ES2 ES3	S6	KZP	YT	M3C	2.0	0.870×10^{-4}	6
4	OWV	EMW	M4N	SVZ	YS-1	—	1.5	0.653×10^{-4}	5
5	OW	EM3	M4	C2	YS	MH3C	1.2	0.522×10^{-4}	4
6	OWS		M5				1.2	0.522×10^{-4}	4
7	OWC	EH-1	M4L	T2	YM	MHT3C	0.9	0.392×10^{-4}	3
8	W7	HH1	H7	C4	YH	—	0.65	0.283×10^{-4}	2
9	W7R	EH2	H7W	4W	YHW	—	0.5	0.218×10^{-4}	2

Table 3.3: Bit size parameters (Moore, 1986)

Bit OD (in.)	D_1*	D_2*
6¼	.088	5.50
6¾	.083	5.61
7⅞	.074	5.94
8⅝	.071	6.11
9⅝	.066	6.38
9⅞	.065	6.44
10¾	.062	6.68
12¼	.058	7.15

*Based on a destruction weight of 10,000 lb/in. of bit diameter.

The values of the parameters shown in Table 3.2 and Table 3.3 were derived by empirically fitting published data and from personal communication with research personnel of the Hughes Tool Co.(F. S. Young, Jr., 1969)

Drilling Cost

Cost per foot equation is given as,

$$F_c = \left(\frac{C_b + C_r * (R_t + TT + S_t + C)}{F_t}\right) \quad (3.8)$$

Where,

C_b: bit cost, $

C_r: rig cost, $/hr

R_t: rotating time, hr

TT: trip time, hr

S_t: connection time, hr

F_t: footage drilled, ft.

3.3 Regression Method

One of the most accepted and used model is the complete mathematical drilling model proposed by Bourgoyne, Jr. A. T. and Young, Jr. F.S. (Bourgoyne and Young, 1974) known as multiple regression method.

They proposed using eight functions to model the effect of most of the drilling variables. They defined the following relations in their model.

$$ROP = f_{a_1} * f_{a_2} * f_{a_3} * f_{a_4} * f_{a_5} * f_{a_6} * f_{a_7} * f_{a_8} \quad \mathbf{(3.9)}$$

Where f_{a_1} to f_{a_8} expresses the different normalized effects on ROP such as rock drillability, operational parameters and bit wear. In the a_1 to a_8 are experimental model constants. $f_{a_1}$is the effect of rock drillability which is proportional with formation rock strength and is given by:

$$f_{a_1} = e^{2.303a_1} \quad \mathbf{(3.10)}$$

The second term is the depth effect given as;

$$f_{a_2} = e^{2.303a_2(10000-D)} \quad \mathbf{(3.11)}$$

Where D is depth in feet. The third term is the effect pore pressure has on ROP where overpressure will increase ROP and f_{a_3} is given as;

$$f_{a_3} = e^{2.303a_3D^{0,69}(g_p-9)} \quad \mathbf{(3.12)}$$

Where g_p is the pore pressure in pounds per gallon equivalent. The fourth term is the effect of overbalance on ROP caused by mud weight increase.

$$f_{a_4} = e^{2.303a_4D(g_p-\rho_c)} \quad \mathbf{(3.13)}$$

Where ρ_c is mud weight in pounds per gallon. The fifth term is the effect on ROP caused by changing the weight on bit.

$$f_{a_5} = \left[\frac{\left(\frac{W}{d_b}\right) - \left(\frac{W}{d_b}\right)_t}{4 - \left(\frac{W}{d_b}\right)_t} \right]^{a_5} \quad (3.14)$$

Where, W is the weight on bit, d_b is the bit diameter. The sixth term is the effect of rotary speed on ROP.

$$f_{a_6} = \left(\frac{N}{60}\right)^{a_6} \quad (3.15)$$

Where N is rotary speed. The seventh term is the effect of bit wear on ROP.

$$f_{a_7} = e^{-a_7 h} \quad (3.16)$$

Where h gives the amount of bit wear for a bit. The last term is the jet impact force effect which includes the effect of bit hydraulics on ROP.

$$f_{a_8} = \left(\frac{F_j}{1000}\right)^{a_8} \quad (3.17)$$

Simple analytical expression for the best constant bit weight and rotary speed were derived by the authors for the case in which tooth wear limits bit life. Cost per foot equation is given as;

$$C_f = \frac{C_f}{\Delta D}\left(\frac{C_b}{C_f} + t_t + t_c + t_b\right) \quad (3.18)$$

Substituting equation for t_b and for ΔD given below;

$$\int_0^{t_b} dt = J_2 * \tau_H * \int_0^{h_f} (1 + H_2 h) dh \qquad (3.19)$$

$$dD = J_2 * J_1 * \tau_H * e^{-a_7 h} * (1 + H_2 h) dh \qquad (3.20)$$

and taking $\frac{\partial C_f}{[\partial(\frac{W}{d})]} = 0$ and solving yields

$$\left(\frac{C_b}{C_f} + t_t + t_c\right)\left[a_5 - \left[\frac{\left(\frac{W}{d_b}\right) - \left(\frac{W}{d_b}\right)_t}{\left(\frac{W}{d_b}\right)_{max} - \left(\frac{W}{d_b}\right)_t}\right]\right] + a_5 J_2 \tau_H \int_0^{h_f} (1 + H_2 h) dh = 0 \qquad (3.21)$$

$$\left(\frac{C_b}{C_f} + t_t + t_c\right) * \left(1 - \frac{H_1}{a_6}\right) + J_2 * \tau_H * \int_0^{h_f} (1 + H_2 h) dh = 0 \qquad (3.22)$$

Solving these two equations simultaneously for (W/d_b) gives the following expression for optimum bit weight.

$$\left(\frac{W}{d_b}\right)_{max} = \frac{a_5 * H_1 * \left(\frac{W}{d_b}\right)_{max} + a_6 * \left(\frac{W}{d_b}\right)_t}{a_5 * H_1 + a_6} \qquad (3.23)$$

If the optimum bit weight predicted by this equation is greater than the flounder bit weight, then the flounder bit weight must be used for the optimum. The optimum bit life is obtained by;

$$t_b = \left(\frac{C_b}{C_f} + t_t + t_c\right) * \left(\frac{H_1}{a_6} - 1\right) \quad \mathbf{(3.24)}$$

The optimum rotary speed N_{opt} is obtained as follows;

$$N_{opt} = 60 * \left[\frac{\tau_H}{t_b} \frac{\left(\frac{W}{d_b}\right) - \left(\frac{W}{d_b}\right)_t}{4 - \left(\frac{W}{d_b}\right)_t}\right]^{\frac{1}{H_1}} \quad \mathbf{(3.25)}$$

For the case where bit life is limited by bearing wear or penetration rate, such simple expression for the optimum conditions have not been found and the construction of a cost per foot table is the best approach.

Because of the similarity among the parameters some minor changes are done on the parameters used in original work presented by Bourgoyne, Jr. A. T. and Young, Jr. F.S. (Bourgoyne and Young, 1974) and those changed parameters are given in the abbreviations.

In order to determine the function constants from a_1 to a_8 offset well data are required and to determine those eight parameters at least 30 wells data are needed. Depth points must be selected from the shale zones and 30 different shale zones can also be used from a single well.

4. LITERATURE RIVIEW

An extensive literature review was carried out on the subject to find possible points that will need some research and could result in future new research topics. Because of the great number of articles in these areas, this literature review is limited to the most representative and/or well known work.

John W. Speer, in 1958, published a report that developed a simple method for determining the combination of weight on bit, rotary speed and hydraulic horsepower which produces minimum drilling cost. Empirical relationships are developed to show the influence on penetration rate of weight on bit, rotary speed and hydraulic horsepower. Optimum weight on bit is shown in relation to formation drillability, and optimum rotary speed is related to weight on bit. These relationships are then combined into a chart for determining optimum drilling techniques from a minimum of field test data.

It appears that the first analytical approach to drilling optimization was published by Moore, (Moore 1959). He presented a paper about the factors that affect the drilling rate. The importance of this fine work resides in the fact that it was the first one to analyze the importance of both mechanical and hydraulic parameters on the drilling rate from a very accurate and systematic point of view. Although at present there are more accurate and reliable models to describe the drilling process, this work represented a true advance in drilling technology when it was published.

Graham and Muench, in 1959, used what is sometimes called the “graphical” approach, together with the more realistic drilling equations, to calculate optimum combinations of weight and speed to bearing failure. In their paper, cost per foot is computed vs. weight for various depths at fixed speed. This is repeated for various speeds until optimum is found for each depth.

Cunningham (1960) presented laboratory data which show relationships between rock-bit bearing life and rotary speed, drilling rate and rotary speed, rock-bit tooth wear and rotary speed. Calculations of lowest costs using the optimum constant rotary speed for a given bit weight are given for drilling medium hard to hard abrasive formations where cutting structure limits bit life. In his work the specific objectives were to determine:

1. The effect of rotary speed on bit bearing life
2. The effect of rotary speed on drilling rate
3. The effect of rotary speed on tooth wear for a bit drilling in an abrasive formation
4. The approximate rotary speed that minimizes cost in some specific instances.

Galle and Woods (1960) presented a pioneer work that created a major breakthrough in drilling technology, mainly when referring to optimization aspects. Necessary conditions for the optimal variable weight speed path are found using classical calculus of variations with integrated drilling equations acting as constraints. The paper defines a model with a drilling rate equation that is a function of weight on bit, rotary speed, type of formation, and bit tooth.

In 1962, Billington and Blenkarn used the equations and techniques developed by Galle and Woods to optimize the variable weight schedule when speed is fixed by rig limitations. In their paper theoretical charts for use in calculating the optimum constant-speed and variable weight program to obtain minimum drilling cost are presented.

Galle and Woods (1963) followed the similar procedures that they used in their early 1960 paper. They presented procedures for determining: the best combination of constant weight and rotary speed; the best constant weight for any given rotary speed; and the best constant rotary speed for any given weight. For each of these procedures, they presented eight cases considering a combination of bit teeth and bearings life, and drilling rate limits economical bit life. They established empirical equations for the effects of weight on bit, rotary speed, and cutting structure dullness on drilling rate, rate of tooth wear and bearing life.

Young (1968) found a solution for minimum drilling cost assuming constant bit weight and rotary speed over the entire bit life, using basic equations for drilling rate, bit bearing wear, bit teeth wear and cost per foot. This solution was programmed for use in an on site computer system.

Reed (1972) developed a method to find the best combination of weight on bit and rotary speed in two cases, constant or variable parameters, in order to have the least cost per foot. His method agreed very well with results from previous papers, but it was considered to be more precise because the equations were solved in a more rigorous way using a Monte Carlo scheme.

Wilson and Bentsen (1972) in their published paper developed three methods of varying complexity. The first method seeks to minimize the cost per foot drilled during a bit run. The second method minimizes the cost of a selected interval, and the third method minimizes the cost over a series of intervals. It was found that each of the methods gave a worthwhile cost saving and that the saving increased as the complexity of the method increased.

Bourgoyne and Young (1974) developed a mathematical model, using a multiple regression analysis technique of detailed drilling data, to describe the drilling rate based on formation depth, formation strength, formation compaction, pressure differential across the bottom hole, bit diameter and bit weight, rotary speed, bit wear and bit hydraulics. As a function of these eight parameters, a mathematical model was developed in order to find the best constant weight on bit, rotary speed and optimum hydraulics for a single bit run in order to achieve minimum cost per foot. The method also predicts the drilling hours and bit wear. They considered that more emphasis had been placed on the collection of detailed drilling data to aid in the selection of improved drilling practices. Thus, the constants that appear in their model could be determined from a multiple regression analysis of field data.

Warren (1984) defined a new model to explain rate of penetration when using roller-cone bits that includes the effect of both the initial chip formation and cuttings removal process. The strategy was to develop an initial basic model that will be refined by addition of a more varied set of test conditions every time that new data are added.

Burgess and Lesso (1985) presented a paper that their work extends the torque model proposed by Warren for soft formation milled tooth bits to show how the effects of tooth wear can be taken into account. The method is intended for formations like shales that are drilled by a gouging and scraping action. An interpretation technique, based on the model, is developed. It is called the Mechanical Efficiency Log (MEL). It uses time averaged values of penetration rate, rotation speed, and measurements while drilling (MWD) values of torque and weight on bit.

Winters, Warren and Onya (1987) developed a model, which relates roller bit penetration rates to the bit design, the operating conditions, and the rock mechanics. Rock ductility is defined as a major influence on bit performance. Cone offset is recognized as an important design feature for drilling ductile rock. The model relates these factors to predict the drilling response of each bit under reasonable combinations of operating conditions. Field data obtained with roller cone bits can be interpreted to generate a rock strength log. The rock strength log can be used in conjunction with the bit model to predict and interpret the drilling response of roller cone bits.

Ohara (1989) presented a method of bit selection based on previous work by Mason and also developed a new equation for rate of penetration that takes into account the following parameters: weight on bit, rotary speed, differential pressure at the bottom of hole, depth, bit jet force, compressive strength, teeth wear, and bit diameter. In this paper, a set of coefficients was defined according the above parameters and the formations being drilled. The coefficients were determined based on field data from four wells and used to find optimum parameters to minimize drilling costs in a fifth well. The results presented were very good. A computer program was developed in order to speed up the process of determining the formation coefficients and the optimum parameters.

Jardine (1990) in his published paper describes the development of processing techniques to extract roller cone bit wear information from near-bit force and acceleration measurements. Laboratory data have been collected for bits in a variety of wear states operating in a small atmospheric drilling machine and under the simulated downhole conditions provided by a fullscale drilling test station. This has given information on haw bit cone speed increases as the wear process progressively reduces the cone radius. The improved understanding of the nature of the bit signature has led to the development of novel processing techniques for the detection of bit wear from near bit measurements.

Maidla and Shiniti Ohara (1991) state that a computer program was developed for the simultaneous selection of a roller cutter bit, bit bearing, weight on bit, and drillstring rotation that minimizes drilling cost per foot for a single bit run. Two drilling models were tested with data from five wells located offshore Alagoas, Brazil. Results show that the rate of penetration of the fifth well can be predicted with coefficients calculated from the four previous wells, resulting cost savings.

Bonet, Cunha and Prado (1993) analyzed the drilling cost not for the operation of one single bit as usual, but for the operation of an entire drilling phase, from its initial to final depth, in homogeneous formations. The main objective of this work was to find the optimum drilling parameters for each bit used during the phase, the number of bits to be used and the depth where each bit will be changed. A computer program was developed to simplify the use of the method.

Kuru and Wojtanowicz (1993) presented a new methodology in drilling optimization using a dynamic programming the dynamic drilling strategy. This strategy employs a two-stage optimization procedure, locally for each drill bit, and globally for the whole well, and generates an optimum bit program for the whole well. The program includes distribution of bit footage along the well paths, depths of tripping operations, bit control algorithms for all bits, and the optimum number of bits per well.

Hareland and Rampersad (1994) presented a new approach to predicting the performance of full hole and core drag bits. The model is based on theoretical considerations of single cutter rock interaction, lithology coefficients and bit wear. Several new modeling features are introduced, these include "equivalent bit radius" and "dynamic cutter action", "lithology coefficients" and "cutter wear". The model is applicable to all types of drag bits (Natural Diamond Bits), Polycrystalline Diamond Compact Bits (PDC) any Geoset Bits with correct cutter geometrical description. The model is useful for pre planning, day to day and post drilling analysis, as well as drilling optimization. The advantages of this model include, optimization of operating parameters, optimization of bit parameters, and support of a total drilling system for penetration rate, solids control and hydraulics optimization.

Barragan, Santos and Maidla (1997) presented a paper where the main objective was to show that drilling optimization by well phase (multiple bit runs) is more economical than optimization by single bit runs. They developed a method based on as heuristic approach to seek the optimum conditions using Monte Carlo Simulation and specially developed numerical algorithms. This method does not depend on a particular drilling model and has been tested with several models.

Umran S. et al.(1997) laid the project out for TPAO and for that purpose developed computer programming considering drilling optimization methods mostly used by industry.

Bilgesu, Altmis, Ameri, Mohaghegh and et al. (1998) presented a new methodology to predict the wear for three cone bits under varying operating conditions. In this approach, six variables (weight on bit, rotary speed, pump rate, formation hardness, bit type and torque) were studied over a range values. A simulator was used to generate drilling data to eliminate errors coherent to field measurements. The data generated was used to establish the relationship between complex patterns. A three-layer artificial neural network was designed and trained with measured data. This method incorporates computational intelligence to define the relationship between the variables. Further, it can be used to estimate the rate of penetration and formation characteristics. In this study, the value of 0,997 was obtained by the model as the correlation coefficient between the predicted and measured bearing wear and both wear values.

Clegg and Barton (2006) presented a new set of performance indices for PDC bits. These are derived from a sophisticated mathematical model and describe performance in terms of:

- ROP, Rate of Penetration, or how fast the bit will drill for a given Weight on Bit (WOB)
- Durability, how resistant the bit is to abrasive wear
- Stability, how resistant the bits is to lateral vibration
- Steerability, how the bit responds to side forces and therefore how steerable it is on Rotary Steerable Systems.

Once the relative importance of each index is established, the optimal bit for the specific application can be selected. The paper presents results from pilot studies and demonstrates a scientific and rational approach to bit selection. Also the approach gives not only improved but also more consistent and reliable results.

Iqbal (2008) in his paper presents the algorithm, calculations and optimization procedure, which does not require any sophistication and can be applied to an ordinary drilling rig. It is therefore, recommended in this paper that a simple optimization process with no additional costs will increase the efficiency without adding any real cost at large. In this method the well is divided into Lithological sections and each interval is optimized separately. A direct search technique is used to determine the number of bits and optimal values of W and N, required drilling each interval at minimum cost.

Rashidi, Hareland and Nygaard (2008) established a method for evaluating real time bit wear and to create a field tool that can assist in the decision when to pull the bit. However, two main methods of optimizing drilling are mechanical specific energy (MSE) and inverted rate of penetration (ROP) models. In their paper they presented that both methods can help to optimize the drilling operation by analyzing drilling variables like weight on bit and rotary speed.

Salakhov, Yamaliev and Dubinsky (2008) proposed a method that the primary object is to evaluate the current conditions of the drilling system and suggest modifying values of main drilling control parameters to optimize the efficiency of the drilling in whole, while reducing the probability of premature wear of the drill bit. The fundamental theory behind the proposed approach is based on some elements of fractal analysis as well as artificial neural networks (NN). In their case, the “system” consists of the drilling components (mud, drill bit, etc.), as well as the formation being drilled. Drilling control parameters include both the parameters adjustable in real time, such as hook load, RPM, or mud flow rate.

Eren and Ozbayoglu (2010) in their paper presented that the objective of optimizing drilling parameters in real time is to arrive to a methodology that considers past drilling data and predicts drilling trend advising optimum drilling parameters in order to save drilling costs and reduce the probability of encountering problems. The linear drilling rate of penetration model previously introduced by Bourgoyne and Young (1974) that is based on multiple regression analysis has been utilized in real time to:

- Achieve coefficients of multiple regression specific to formation
- Have a rate of penetration vs. depth prediction as a function of certain drilling parameters
- Determine optimum drilling parameters specific to the formation being drilled.

5. DERIVATION OF THE GALLE AND WOODS OPTIMIZATION METHOD

In this chapter of the thesis the derivations and properties of the method presented by Galle and Woods are given in general state.

5.1 Objective of the Approach

Over the past years lots of drilling models have been proposed for the optimization of the rotary drilling process. Empirical relations for the effect of rotary speed, weight on bit and cutting structure dullness on drilling rate, rate of tooth wear, and bearing life have been established (Galle and Woods, 1960). In the selected teeth limit bit life case and drilling rate limits economical bit life case which are considered in this study. We have four equations with six variables *N*, $\overline{W}$, D_f, *K*, A_n, and S_n. Explanation relating to the parameters are on the next part of the chapter. It is possible to eliminate any three and express any one of the remaining in terms of the other two. Specifically *N*, $\overline{W}$, D_f, and *K* may be expressed in terms of A_n and S_n. In general each case defines a region in the $A_n - S_n$ plane. Actually all the parameters except *N*, $\overline{W}$, D_f can be determined using *N*, $\overline{W}$, D_f parameters which should be given or known from the previous field data. Where, *N*, is the rotary speed, $\overline{W}$, is the normalized weight parameters, and D_f, is bit dullness parameter. *N* and $\overline{W}$ parameters are always available during the drilling operation but D_f is not available since it has been evaluated from the past field data or from the previous bit record assuming the change in the property of lithology is negligible. Using those evaluated parameters in the equations and determining best constant weight, rotary speed which yields minimum cost per foot results are another objective as well. In order to make calculation procedures less time consuming, complicated, out of tornado charts and visually available nonlinear numerical programming developed in Delphi (version 7.0) as of another purpose of this study.

5.2 Background of the Program

The equations used are the same as in the Galle and Woods model which calculates the best constant weight on bit and rotary speed. During this calculation procedure there are some parameters that make the calculations in some degree difficulty because of the some previous data necessity. Sometimes it is difficult to correctly interpret values of the previous data, specially, such as dulling degree of the bit used. In general it depends on the person who evaluated the grading of bit dullness. Parameters which are very important during the calculation period such as A_f, C_f and D_f are the abrasiveness, formation drillability and bit dullness parameters respectively. In order to estimate the abrasiveness, formation drillability parameters which are the functions of D_f, bit dull condition, N, rotary speed, and $\overline{W}$, normalized weight on bit should be known.

Only three procedures and two cases out of original three procedures and eight cases which are given in the Galle and Woods model are considered in this work comprehensively. It is believed that teeth limit and penetration rate limit bit life cases have much more influence on drilling operation. Full derivatives of the cases are given in Appendix.

In this thesis also in the numerical programming simulator the following procedures are presented:

1. The best combination of constant weight and rotary speed,
2. The best constant weight for any given rotary speed,
3. The best constant rotary speed for any given weight

The following cases are considered, for each of the procedures listed above as well:

Case 1: Teeth limit bit life.

Case 2: Penetration rate limits economical bit life

Drilling rate equation

$$\frac{dF}{dT} = \frac{C_f W^k r}{a^p} \qquad (5.1)$$

wherein:

k = 1.0 (for most formations except very soft formations)

= 0.6 (for very soft formations)

p = 0.5 (for self sharpening or chipping type bit tooth wear)

r, is essentially rotary speed to a fractional power and *a*, is a function of dullness. Drilling rate increases with drillability (C_f), weight and rotary speed and decreases with dullness. In this equation the effects of bit type, hydraulics, drilling fluid and formation are all included in the drillability constant C_f.

Rate of dulling equation

$$\frac{dD}{dT} = \frac{1}{A_f}\,\frac{i}{am} \qquad (5.2)$$

i, is a quantity that increases with rotary speed, *a*, increses with dullness, and *m*, decreases with increase in weight. A_f – decreases with increase of formation abrasiveness. Thus the rate of wear increases as abrasiveness, weight, and rotary speed increase, and decreases as dullness increases. In this equation the effect of bit type, hydraulics, drilling fluid and formation are all included in the abrasiveness constant, A_f.

Bearing life equation

$$B = S * \frac{L}{N} \qquad (5.3)$$

The symbol *L*, is used to denote a decreasing function with increasing weight. The other quantities are explained under Abbreviations section.

Denoting the fraction of bearing life expended by B_x, then in time *T:*

$$B_x = \frac{T}{B} = \frac{TN}{SL} \tag{5.4}$$

This applies only if weight and rotary speed are constant during the time *T*. Bearing life decreases with increases in weight and rotary speed, and increases with the drilling fluid factor *S*. The value of *S* for any given drilling fluid will change with different bit types containing bearings of different capacity.

Terms are introduced in this section without definition, defined in Abbreviations section.

Using calculus mathematics for the cases and procedures the following equations are obtained as a result.

5.2.1 The Best Constant Weight and Rotary Speed

The best constant weight and rotary speed equation for teeth limit and penetration rate limit bit life as respectively:

$$K = \frac{\frac{\partial P_1}{\partial N}}{\frac{\partial P_2}{\partial N}} = \frac{\frac{\partial P_1}{\partial \overline{W}}}{\frac{\partial P_2}{\partial \overline{W}}} = \frac{\frac{i'}{i}\int_0^1 a\,dD}{r\overline{W}^k\left(\frac{i'}{i} - \frac{r'}{r}\right)\int_0^1 a^{1-p}\,dD}$$
$$= \frac{\frac{m'}{m}\int_0^1 a\,dD}{r\overline{W}^k\left[\frac{m'}{m} + \frac{k}{\overline{W}}\right]\int_0^{D_f} a^{1-p}\,dD} \tag{5.5}$$

and

$$\frac{\frac{\partial P_1}{\partial N}}{\frac{\partial P_2}{\partial N}} = \frac{\frac{\partial P_1}{\partial \overline{W}}}{\frac{\partial P_2}{\partial \overline{W}}} = \frac{\frac{\partial P_1}{\partial D_f}}{\frac{\partial P_2}{\partial D_f}} = \frac{\frac{i}{i}\int_0^{D_f} a\,dD}{r\overline{W}^k\left(\frac{i}{i} - \frac{r}{r}\right)\int_0^{D_f} a^{1-p}\,dD}$$
$$= \frac{\frac{m}{m}\int_0^{D_f} a\,dD}{r\overline{W}^k\left[\frac{m}{m} + \frac{k}{\overline{W}}\right]\int_0^{D_f} a^{1-p}\,dD} = \frac{(a^p)_{D_f}}{\overline{W}^k r} \tag{5.6}$$

5.2.2 The Best Constant Weight for Any Given Rotary Speed

Best constant weight for any given rotary speed equation for teeth limit and penetration rate limit bit life as respectively:

$$\frac{A_n + \frac{m^{'}}{i}\int_0^1 a dD}{\frac{m\overline{W}^k r}{i}\int_0^1 a^{1-p} dD} = \frac{\frac{m^{'}}{m}\int_0^1 a dD}{r\overline{W}^k\left[\frac{m^{'}}{m} + \frac{k}{\overline{W}}\right]\int_0^{D_f} a^{1-p} dD} = K \qquad (5.7)$$

and

$$\frac{A_n + \frac{m}{i}\int_0^{D_f} a dD}{\frac{m\overline{W}^k r}{i}\int_0^{D_f} a^{1-p} dD} = \frac{\frac{m}{m}\int_0^{D_f} a dD}{r\overline{W}^k\left[\frac{m}{m} + \frac{k}{\overline{W}}\right]\int_0^{D_f} a^{1-p} dD} = \frac{(a^p)_{D_f}}{\overline{W}^k r} = K \qquad (5.8)$$

if we solve equation (5.7) for "r", gives the best constant weight for any given rotary speed:

$$r = \frac{\frac{m^{'}}{m}\int_0^1 a dD}{K\overline{W}^k\left[\frac{m^{'}}{m} + \frac{k}{\overline{W}}\right]\int_0^{D_f} a^{1-p} dD} \qquad (5.9)$$

5.2.3 The Best Constant Rotary Speed for Any Given Weight

Best constant rotary speed for any given weight equation for teeth limit and penetration rate limit bit life as respectively:

$$\frac{A_n + \frac{m^{'}}{i}\int_0^1 a dD}{\frac{m\overline{W}^k r}{i}\int_0^1 a^{1-p} dD} = \frac{\frac{i^{'}}{i}\int_0^1 a dD}{r\overline{W}^k\left(\frac{i^{'}}{i} - \frac{r^{'}}{r}\right)\int_0^1 a^{1-p} dD} = K \qquad (5.10)$$

and

$$\frac{A_n + \frac{m}{i}\int_0^{D_f} a dD}{\frac{m\bar{W}^k r}{i}\int_0^{D_f} a^{1-p} dD} = \frac{\frac{i}{i}\int_0^{D_f} a dD}{r\bar{W}^k(\frac{i}{i} - \frac{r}{r})\int_0^{D_f} a^{1-p} dD} = K \qquad (\mathbf{5.11})$$

if we solve for "$\bar{W}^k$", gives the best constant rotary speed for any given weight:

$$\bar{W}^k = \frac{\frac{i'}{i}\int_0^1 a dD}{Kr(\frac{i'}{i} - \frac{r'}{r})\int_0^1 a^{1-p} dD} \qquad (\mathbf{5.12})$$

5.2.4 Selection of Proper Set of Graphs

There are 3 sets of graphs each identified by a 7 digit number. The proper set to use is determined by the formation being drilled.

The sets are:

2 075 060: For very soft formations where hydraulics may limit drilling rate to some extent.

2 075 100: For soft and medium soft formations such as shales and redbeds.

2 043 100: For medium hard and hard formations such as limes, dolomites, and sands.

The first digit in the number denotes the type of tooth wear obtained on the dull bit. The number 2 is for self-sharpening or chipping type wear. Although there are other types of wear, they occur so seldom that they are not considered in their paper. The first 3 digit group denotes the response of drilling rate to rotary speed. The 3 digit groups 043 and 075 are used when drilling rate varies as the 043 or 075 power of rotary speed, respectively. The second 3 digit group denotes the response of drilling rate to weight, with 100 and 060 being used when drilling rate varies with weight to the 1.0 or 0.60 power, respectively.

5.2.5 Calculation of Formation Constants

Calculation of formation constants A_f and C_f from a constant weight and rotary speed bit run:

A_f is a measure of the abrasiveness of the formation. A very abrasive formation have a low for A_f. C_f is a measure of the drillability of the formation. Slow drilling formations will have a low value for C_f. It will be necessary to have the following performance information on a bit run at constant weight and rotary speed in the formation under consideration and in a similar drilling fluid:

Bit size, in

Rig cost, $/hr

Bit cost, $

Trip time per 1000 ft, hr

Depth, ft

Formation: Use appropriate graphs

Footage, F_f, ft

Rotating time, T_f, hr

Weight, W, lb

Rotary speed, N

Dull condition, D

Bearing condition, B_{xf}

Calculate the equivalent bit weight using Eq. (5.13):

$$\overline{W} = 7{,}88 * \frac{W}{H} \qquad \mathbf{(5.13)}$$

Using the preceding values on N, $\overline{W}$, and D read or could be calculate the values i, r, $\overline{m}$, L, U, and V from the Table 3.1**.** Also read $\overline{W^{0,6}}$ when using graph 2 075 060.

Calculate formation abrasiveness and drillability parameters from Eq. (5.14) and Eq. (5.15) respectively:

$$A_f = \frac{T_f * i}{U\overline{m}} \qquad \mathbf{(5.14)}$$

and

$$C_f = \frac{F_f * i}{A_f * r * V * \overline{W} * \overline{m}} \qquad (5.15)$$

When using (2 075 100) for soft and medium soft formations or (2 043 100) for medium hard and hard formations

$$C_f = \frac{F_f * i}{A_f * r * V * \overline{W^{o,6}} * \overline{m}} \qquad (5.16)$$

When using (2 075 060) for very soft formations

Calculation of drilling fluid constant S from a constant weight and rotary speed bit run:

Bearing life is affected by weight, rotary speed, and drilling fluid. A high value of S means a good drilling with respect to bearing life.

Determination the value of S from:

$$S = \frac{T_f * N}{B_{xf} * L} \qquad (5.17)$$

Calculation of A_n and S_n

Calculate:

$$A = \frac{Bit\ cost}{Rig\ cost\ per\ hour} + Trip\ time, hour \qquad (5.18)$$

$$A_n = \frac{A}{A_f} \qquad (5.19)$$

$$S_n = \frac{S}{A_f} \qquad (5.20)$$

The calculation procedures and calculated parameters shown below are also considered in the Delphi and given under Comparison headings which also compare the original previous field data as input with the computed optimized results in different colors as well.

Calculation of total rotating time,

$$Total\ rotating\ time = \left[\frac{S_n{}^* * L}{N}\right] * A_f \qquad \mathbf{(5.21)}$$

Calculation of cost per foot

$$Cost\ per\ foot = \frac{K\ (rig\ cost\ per\ hour)}{C_f} \qquad \mathbf{(5.22)}$$

Calculation of total footage

$$F_f = \frac{A_n + \frac{S_n{}^* * L}{N}}{K} * A_f * C_f \qquad \mathbf{(5.23)}$$

Instruction for using graphs which are given in original study done by Galle and Woods shows how boring and impractical they are. Developing Delphi programming for optimization method sets us free of using complicated graphs and relieved calculation procedures sufficiently.

There are several set of graphs published for this purpose. The particular graph to use from this set will depend upon the limitations of the drilling rig and whether teeth, bearings, or drilling rate limit the life of the bit when run according to these procedures. The proper way to use each type of graph will be discussed in the succeeding paragraphs. However, the usage of the graphs are very difficult and complex.

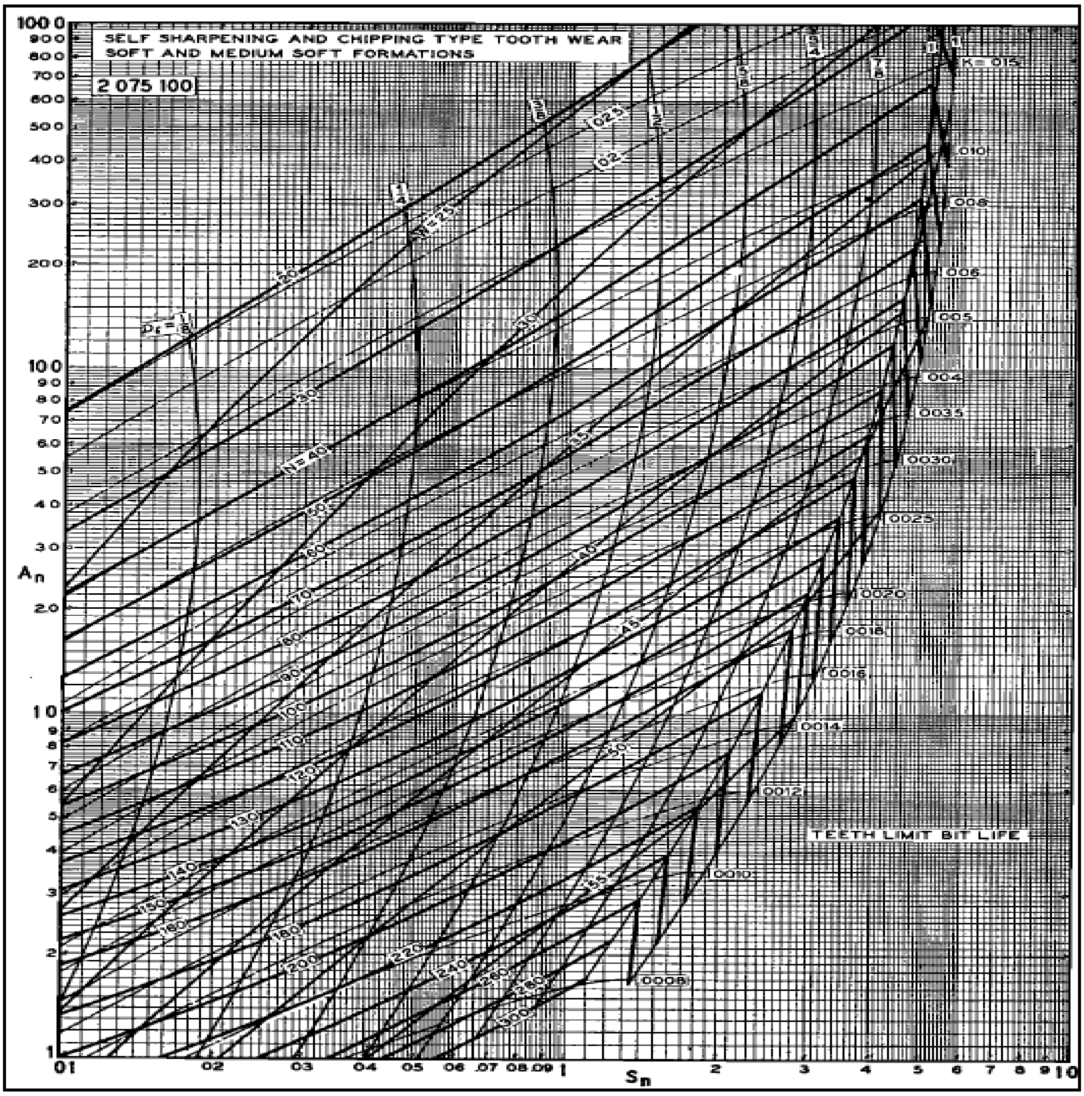

Figure 5.1: The best combination of weight and rotary speed for given A_n and S_n **(**Galle E.M. and Woods H.B., 1963)

5.2.6 The Best Combination of Constant Weight and Rotary Speed

Use Figure 5.1 for the best combination of constant weight and rotary speed. Find the intersection of the A_n and S_n lines and read $\overline{W}$, N, K, and D_f. If the $A_n S_n$ intersection falls in the teeth or drilling rate limit bit life region, use the intersection of the A_n line and the right-hand boundary of the curves. Call the value of S_n read at this point $S_n{}^*$.

5.2.7 The Best Weight for Given A_n and RPM

Use Figure 5.2 for the best weight for given A_n and RPM (Teeth Limit Bit Life). Find the intersection of the A_n , line and the rpm lines representing the rotary speeds available on the rig. Read the values of $S_n{}^*$, *K*, *and* $\overline{W}$ at these intersections. Only those rotary speeds which give a value of $S_n{}^*$ less than S_n.

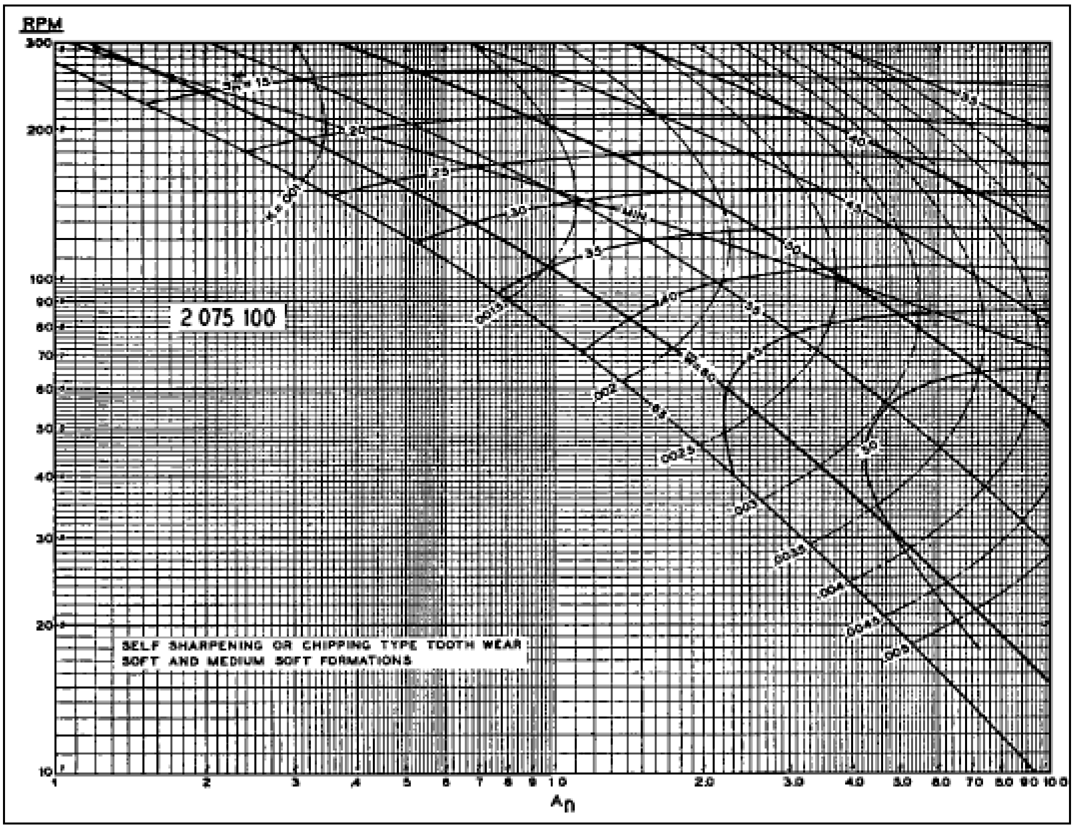

Figure 5.2: The best weight for given A_n and RPM **(**Galle E.M. and Woods H.B., 1963)

The best rotary speed to use is the one which gives the minimum value of *K*. The very best combination of weight and rotary speed for any value of A_n, is that given by the intersection of the A_n, line and the line marked "MIN". For those rotary speeds for which $S_n{}^*$ is greater than S_n .The bearings will wear out before the teeth. If the A_n and RPM intersection falls outside the domain to the lower left, the bit must be pulled because of drilling rate. In either case, use the graph titled, Weight for Given A_n, S_n , and Rotary Speed (Bearings or Drilling Rate Limit Bit Life). Figure 5.3.

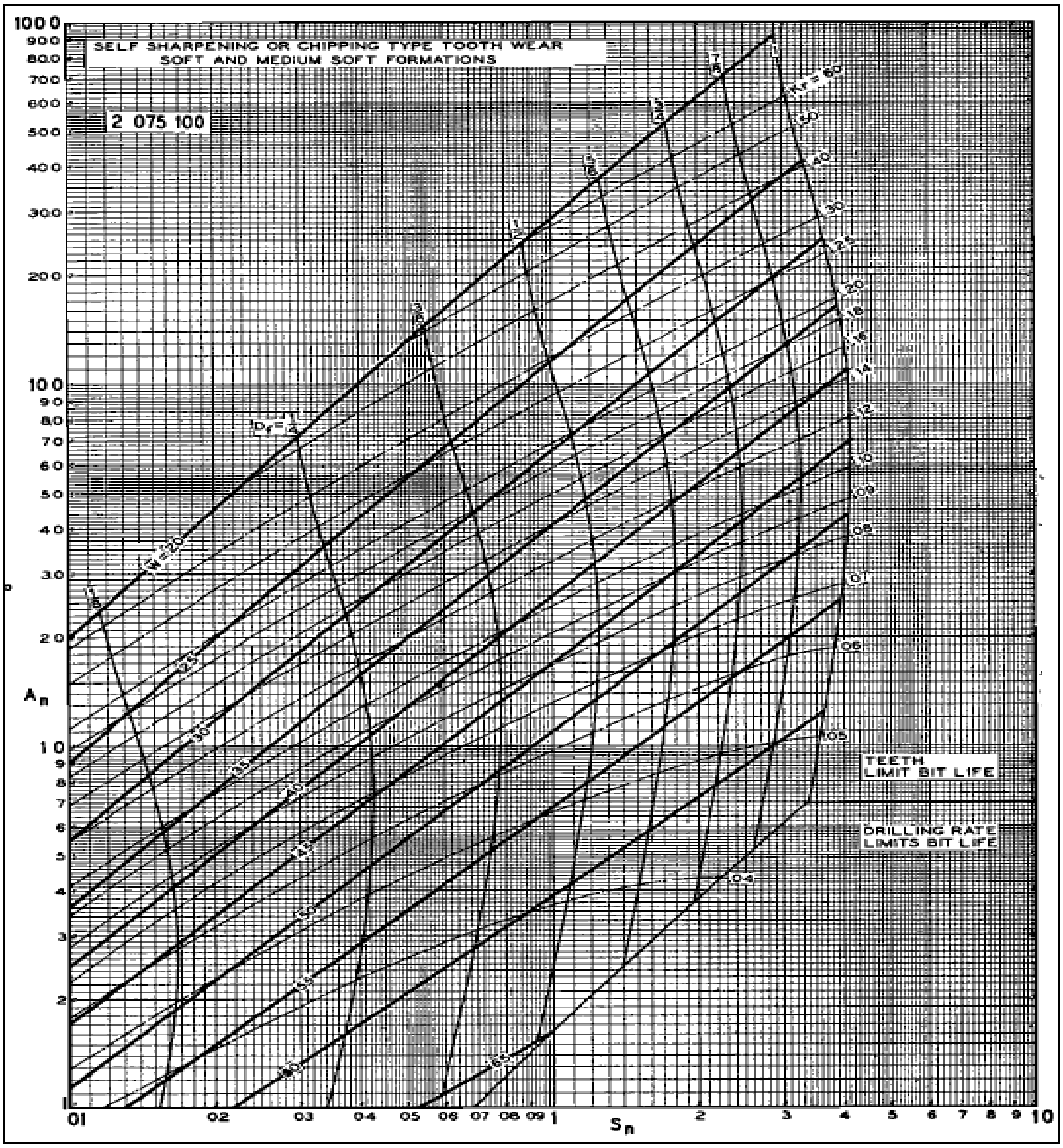

Figure 5.3: Weight, *Kr*, and D_f for given A_n, S_n and rotary speed **(**Galle E.M. and Woods H.B., 1963)

5.2.8 The Best RPM for Given A_n and Weight

Use Figure 5.4 for the best RPM for Given A_n**,** and Weight (Teeth Limit Bit Life). Find the intersection of the A_n line and the value of $\overline{W}$ representing the maximum weight that can be run because of crooked-hole or limited number of drill collars. Read the values of S_n, *K* and *N* at this intersection on Figure 5.4. If the value of S_n, is less than $S_n{}^*$ bearings will limit the life of the bit.

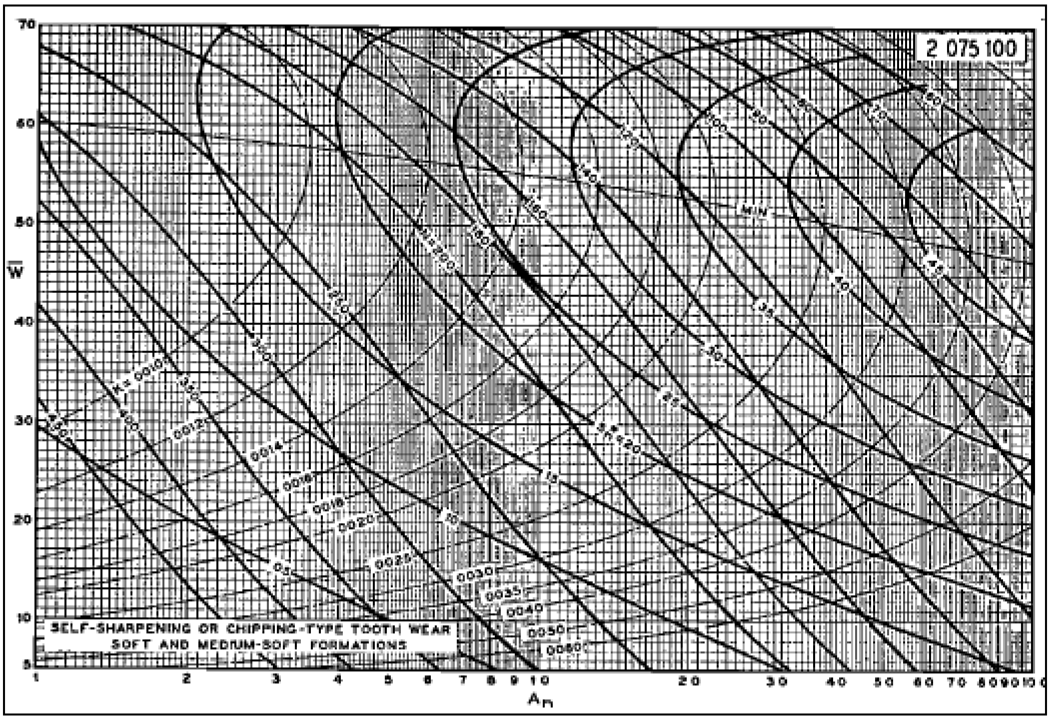

Figure 5.4: The best RPM for given A_n and weight **(**Galle E.M. and Woods H.B., 1963)

If the A_n, $\overline{W}$, intersection falls outside the domain to the lower left, the bit must be pulled because of drilling rate. In either case, use the graph titled, Rotary Speed for Given A_n and S_n, and Weight (Bearings or Drilling Rate Limit Bit Life). On rigs that do not have separate rotary drives, it will be necessary to select a rotary speed as near to the calculated value as possible. Particularly for low weight, the calculated rotary speed may be quite high. In such case, use a rotary speed as high as for good judgment deems safe.

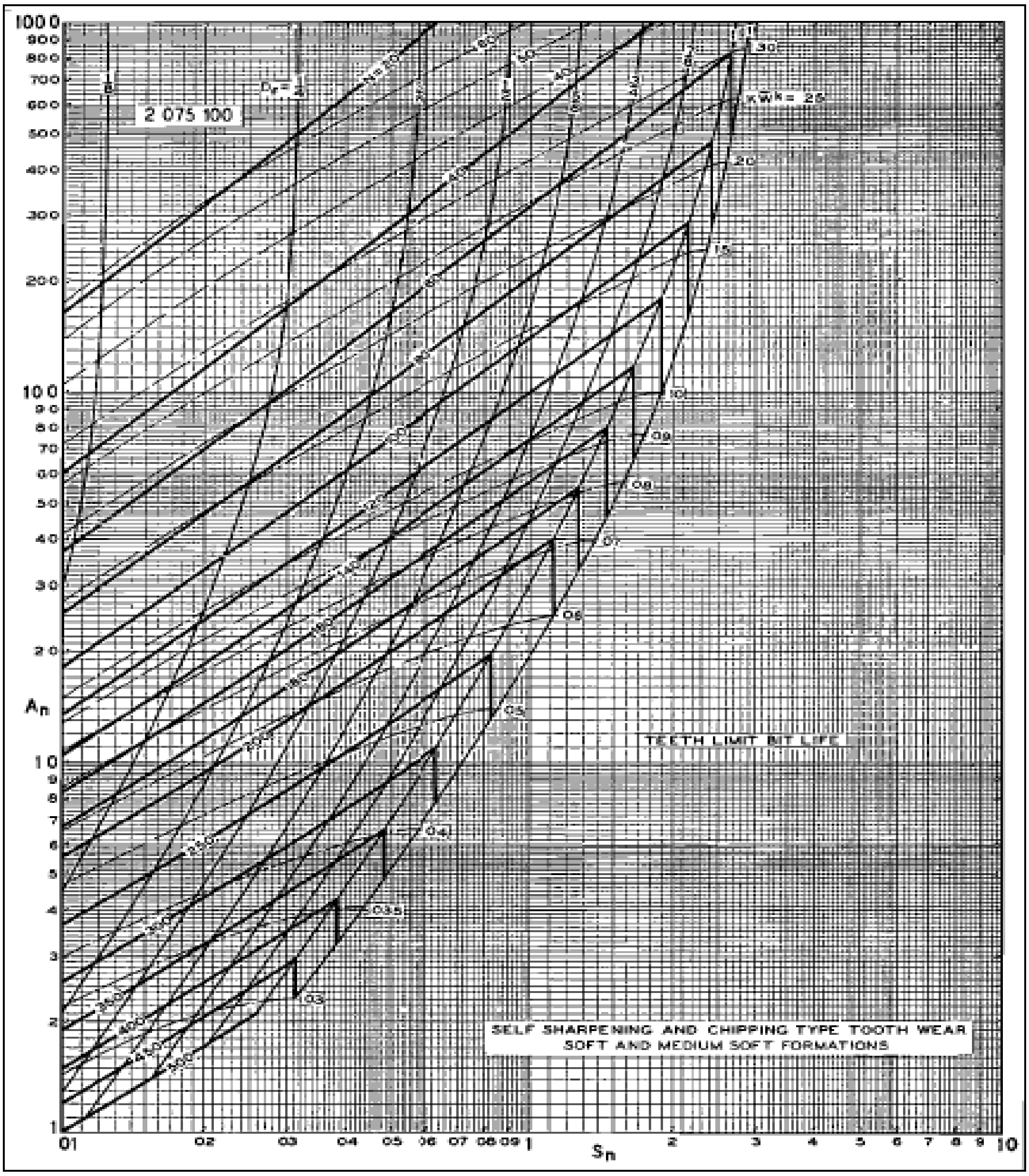

Figure 5.5: The best RPM for given A_n, S_n and weight **(**Galle E.M. and Woods H.B., 1963)

It is obvious that using tornado charts is very complicated, confusing and the graphs given in this thesis is the only set of graphs for 2 075 100 and there are two more set of graphs for 2 075 060 and 2 043 100 as well. When you chose the wrong set of graphs the results and calculated values of parameters will also give inappropriate outcome which if trying to use those values will probably jeopardize drilling operation and cause cost damages which increase cost per foot also undesirable results.

6. DEVELOPMENT OF THE PROGRAM AND APPLICATION

On the various articles published in the literature, it can be found that many different mathematical models were used to explain the drilling process. These models were principally used to assist bit selection and to optimize the mechanical weight on bit and rotary speed parameters and hydraulic parameters in an attempt to reduce drilling costs.

A computer code for Galle and Woods drilling optimization method has been developed in Delphi (Version 7.0) in order to determine optimum weight on bit and rotary speed values. The well data obtained from the original work done by Galle and Woods is used here together with bit and rig cost information, rotary speed, weight on bit, tooth and bearing wear parameters to calculate the best weight on bit and rotary speed. The reason of the chosen data is the lack of the sufficient or complete data available in literature. Another point of the chosen data set is it provides comparison between the obtained results of the program and results of the Galle and Woods study. In order to get the desired level of bit dullness the code also calculates the expected cost per foot, footage and drilling time.

This kind of optimization model has been developed by Serpen U. et. al. in 1997 in Fortran; however, their code is not user friendly. In this thesis, the similar computer code for the Galle and Woods method has been written in Delphi. Also some additional changes have been made on the new program, particularly defining function L. The Delphi code provides not only applied weight and rotary speed on bit but also the results of the intermediate calculation procedures such as optimized footage, optimized rotating time and one of the main parameters of the drilling operation is the drilling cost. General appearance of the developed program is shown in Figure 6.1.

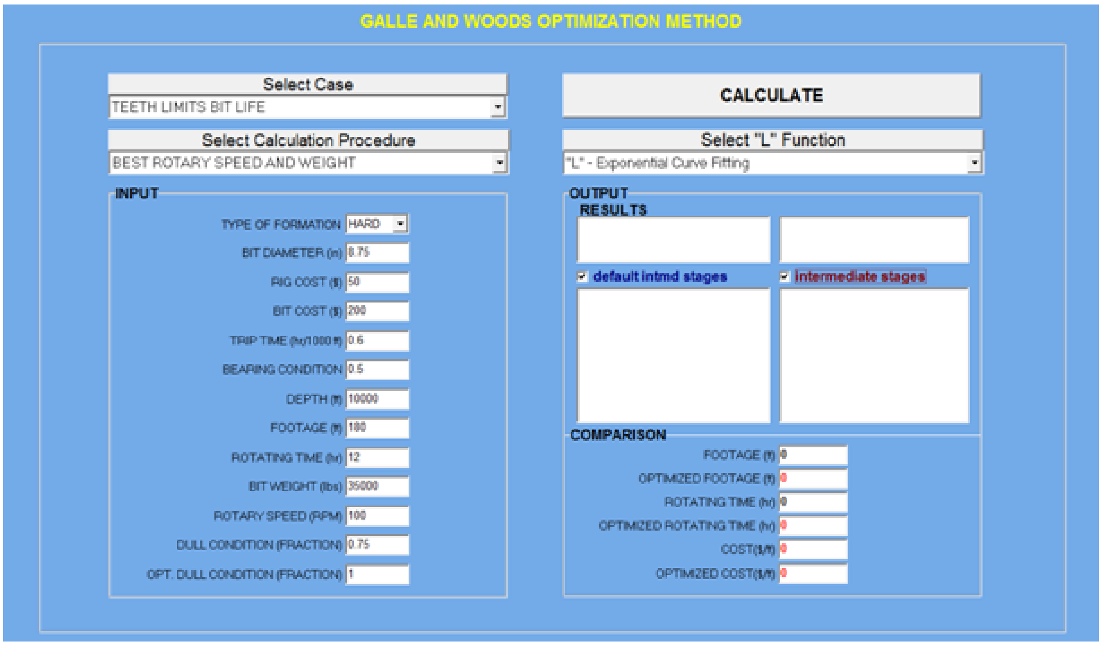

Figure 6.1: General appearance of the program

The background of the program and calculation procedures are given in the previous part of the chapter in detail and in this part application of the program will be the main target.

At first, previous well and bit records data should be inserted along with its formation characteristics such as soft, medium or hard formation information to the input frame. Desired dull condition should also be inserted in order to get the proper results for the wearing fraction of the bit.

Then it comes to select the case which two of them are considered; teeth limit bit life and penetration rate limits bit life. After those selection, it is time to select desired optimization procedures which are (1) the best combination of constant rotary speed and weight, (2) the best constant rotary speed for any given weight, and (3) the best constant weight for any given rotary speed.

6.1 Developed Weight Function for Parameter L

Another important parameter also the function of weight is L which considered here and different equations and functions are generated to define this parameter correctly which its function is not given in the study done by Galle and Woods. For that reason comprehensive study and research also done to get the right function L in order to use in the program. Serpen U. et. al. used this parameter read from the table given by

Galle and Woods. At least three kind of functions are found and used for parameter L in this thesis; one is exponential function (Mitchel B., 1995), the other one is equation developed by D. Wilson and R. Bentsen (D.W and R.B, 1972) and another is developed in this study using Newtonian interpolation that are given as Equation (6.1), (6.2), (6.3) and Table 6.1 respectively.

$$L = \exp\left(8{,}9609 + 0{,}90393 * \ln(\overline{W}) - 0{,}36297 * \ln(\overline{W}^2)\right) \qquad \mathbf{(6.1)}$$

$$L = \frac{21338{,}87}{(1+0{,}031187 * \overline{W})^{3{,}2258}} \qquad \mathbf{(6.2)}$$

and

Table 6.1: Newtonian interpolation constants for function L

		bo	**b1**	**b2**	**b3**	**b4**	**b5**	**b6**	**b7**
index	**X**	**Y**	**1. degree**	**2. degree**	**3. degree**	**4. degree**	**5. degree**	**6. degree**	**7. degree**
1	5	13239	-699,9	20,145	-0,394	0,0055	-5,5E-05	3,12E-07	1,07E-09
11	15	6240	-297	8,315	-0,171	0,0027	-3,7E-05	3,88E-07	
21	25	3270	-130,7	3,16	-0,060	0,0009	-1,3E-05		
31	35	1963	-67,5	1,355	-0,021	0,0002			
41	45	1288	-40,4	0,7	-0,010				
51	55	884	-26,4	0,39					
61	65	620	-18,6						
71	75	434							

After the interpolation the resulted function is very complicated and the 7th degree polynomial equation presented as Equation (6.3). The comparison and error percentage are made between original values of parameter L and D.W and R.B equation, Newtonian function and exponential function values obtained for L and given in Table 6.2.

$$
\begin{aligned}
L = &\ bo \\
&+ b1*(Xn-X1) \\
&+ b2*(Xn-X1)*(Xn-X2) \\
&+ b3*(Xn-X1)*(Xn-X2)*(Xn-X3) \\
&+ b4*(Xn-X1)*(Xn-X2)*(Xn-X3)*(Xn-X4) \\
&+ b5*(Xn-X1)*(Xn-X2)*(Xn-X3)*(Xn-X4)*(Xn-X5) \\
&+ b6*(Xn-X1)*(Xn-X2)*(Xn-X3)*(Xn-X4)*(Xn-X5)*(Xn-X6) \\
&+ b7*(Xn-X1)*(Xn-X2)*(Xn-X3)*(Xn-X4)*(Xn-X5)*(Xn-X6)*(Xn-X7)
\end{aligned} \tag{6.3}
$$

Table 6.2: Newtonian interpolation resulted values

L – Tabulated	Weight (1000 lb)	Newtonian Interpolation (NI)	Error (%) of NI	Exponential (EXP)	Error (%) of EXP	David Wilson and Ramon Bentsen	Error (%) of DW& RB
13239	5	13239,00	0,00	13036,67	-1,53	13966,21	5,49
12279	6	12249,70	-0,24	12274,09	-0,04	12912,03	5,16
11376	7	11337,79	-0,34	11446,70	0,62	11959,70	5,13
10532	8	10497,88	-0,32	10626,00	0,89	11097,31	5,37
9745	9	9724,87	-0,21	9844,74	1,02	10314,60	5,85
9016	10	9013,92	-0,02	9116,54	1,12	9602,65	6,51
8360	11	8360,50	0,01	8445,49	1,02	8953,73	7,10
7758	12	7760,31	0,03	7830,87	0,94	8361,07	7,77
7207	13	7209,32	0,03	7269,68	0,87	7818,76	8,49
6702	14	6703,74	0,03	6757,94	0,83	7321,62	9,25
6240	15	6240,00	0,00	6291,37	0,82	6865,08	10,02
5840	16	5814,77	-0,43	5865,75	0,44	6445,13	10,36
5440	17	5424,94	-0,28	5477,10	0,68	6058,20	11,36
5080	18	5067,58	-0,24	5121,75	0,82	5701,15	12,23
4750	19	4739,99	-0,21	4796,39	0,98	5371,16	13,08
4439	20	4439,63	0,01	4498,01	1,33	5065,75	14,12
4170	21	4164,16	-0,14	4223,94	1,29	4782,70	14,69
3920	22	3911,40	-0,22	3971,79	1,32	4520,01	15,31
3680	23	3679,34	-0,02	3739,44	1,62	4275,91	16,19
3470	24	3466,11	-0,11	3524,99	1,58	4048,79	16,68
3270	25	3270,00	0,00	3326,74	1,74	3837,21	17,35
3080	26	3089,44	0,31	3143,19	2,05	3639,89	18,18
2910	27	2922,98	0,45	2973,01	2,17	3455,65	18,75
2770	28	2769,31	-0,02	2814,98	1,62	3283,44	18,54
2630	29	2627,21	-0,11	2668,04	1,45	3122,30	18,72
2496	30	2495,59	-0,02	2531,22	1,41	2971,37	19,05
2370	31	2373,46	0,15	2403,65	1,42	2829,85	19,40
2260	32	2259,90	0,00	2284,57	1,09	2697,03	19,34
2160	33	2154,12	-0,27	2173,26	0,61	2572,26	19,09

Table 6.2: Continued

1963	35	1963,00	0,00	1971,53	0,43	2344,54	19,44
1880	36	1876,42	-0,19	1880,01	0,00	2240,55	19,18
1800	37	1795,11	-0,27	1794,08	-0,33	2142,52	19,03
1725	38	1718,61	-0,37	1713,33	-0,68	2050,04	18,84
1578	40	1578,38	0,02	1565,80	-0,77	1880,19	19,15
1515	41	1513,97	-0,07	1498,36	-1,10	1802,16	18,95
1460	42	1452,97	-0,48	1434,74	-1,73	1728,32	18,38
1400	43	1395,12	-0,35	1374,66	-1,81	1658,39	18,46
1340	44	1340,19	0,01	1317,88	-1,65	1592,13	18,82
1288	45	1288,00	0,00	1264,17	-1,85	1529,28	18,73
1240	46	1238,36	-0,13	1213,34	-2,15	1469,65	18,52
1195	47	1191,11	-0,33	1165,18	-2,50	1413,03	18,25
1150	48	1146,10	-0,34	1119,52	-2,65	1359,23	18,19
1105	49	1103,21	-0,16	1076,20	-2,61	1308,09	18,38
1063	50	1062,32	-0,06	1035,08	-2,63	1259,43	18,48
1025	51	1023,31	-0,16	996,00	-2,83	1213,12	18,35
988	52	986,07	-0,19	958,86	-2,95	1169,02	18,32
953	53	950,51	-0,26	923,52	-3,09	1126,99	18,26
918	54	916,52	-0,16	889,88	-3,06	1086,92	18,40
884	55	884,00	0,00	857,84	-2,96	1048,69	18,63
853	56	852,86	-0,02	827,30	-3,01	1012,21	18,66
823	57	823,01	0,00	798,18	-3,02	977,37	18,76
794	58	794,35	0,04	770,39	-2,97	944,09	18,90
766	59	766,79	0,10	743,86	-2,89	912,28	19,10
739	60	740,25	0,17	718,52	-2,77	881,85	19,33
714	61	714,64	0,09	694,30	-2,76	852,74	19,43
689	62	689,87	0,13	671,15	-2,59	824,88	19,72
665	63	665,89	0,13	648,99	-2,41	798,20	20,03
642	64	642,61	0,10	627,79	-2,21	772,64	20,35
620	65	620,00	0,00	607,48	-2,02	748,13	20,67
599	66	598,01	-0,16	588,02	-1,83	724,64	20,98
578	67	576,63	-0,24	569,38	-1,49	702,11	21,47
558	68	555,85	-0,39	551,49	-1,17	680,48	21,95
538	69	535,70	-0,43	534,34	-0,68	659,72	22,62
520	70	516,24	-0,72	517,88	-0,41	639,78	23,03
502	71	497,56	-0,88	502,07	0,01	620,62	23,63
484	72	479,79	-0,87	486,89	0,60	602,21	24,42
467	73	463,12	-0,83	472,30	1,13	584,50	25,16
450	74	447,75	-0,50	458,27	1,84	567,47	26,10
434	75	434,00	0,00	444,78	2,49	551,09	26,98
418	76	422,20	**1,01**	431,81	**3,30**	535,32	**28,07**
403	77	412,79	**2,43**	419,32	**4,05**	520,13	**29,07**
388	78	406,27	**4,71**	407,30	**4,98**	505,51	**30,29**
373	79	403,23	**8,10**	395,73	**6,09**	491,42	**31,75**

At the results of the using Newtonian interpolation the error percent between the tabulated values of L is less than zero and it also observed that at the yellow marked points which are selected for interpolation calculation the error percent is zero which indicate well matching. There are also red marked points exist where error percentages started increasing is the indication of the non linear function. The eight percent error observed for 79000 lb weight which has a 403,231 for Newtonian interpolation regarding to 373 for tabulated value. The error at this point is high because the Newtonian Interpolation does not consider the points when are out of data sequence. If the data fall in the data range, the interpolation works well; but if it fallls out of the data range like 79000 lbs in this example, it requires an extrapolation method to predict it. The figure describing this error is given in Figure 6.2.

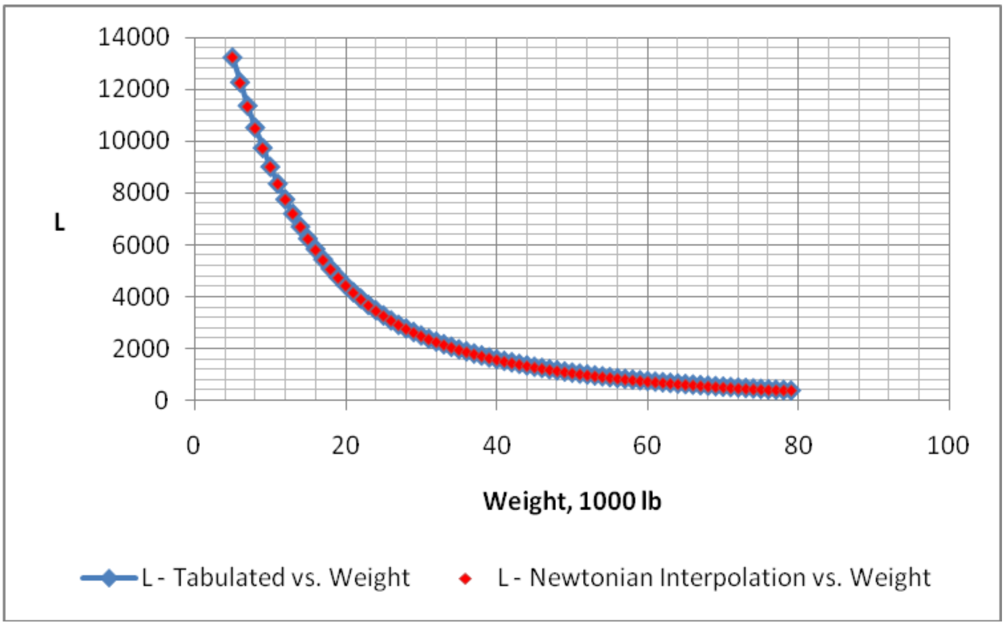

Figure 6.2: Comparison of L tabulated and L Newtonian interpolation

At the results of the exponential equation the error percent between the tabulated values of L and the exponential is about 6 percent and less. It also observed that at the yellow marked points which are selected for comparison of calculated exponential values and calculated interpolation values. The error percent is between 0,817 percent and 3,050 percent. There are also red marked points exist where error percentages started increasing is the indication of the non linear function. Approximately six percent error observed for 79 000 lb weight which has a 395,727 for exponential L value regarding to 373 for tabulated L value. The figure describing this error is given Figure 6.3. so far.

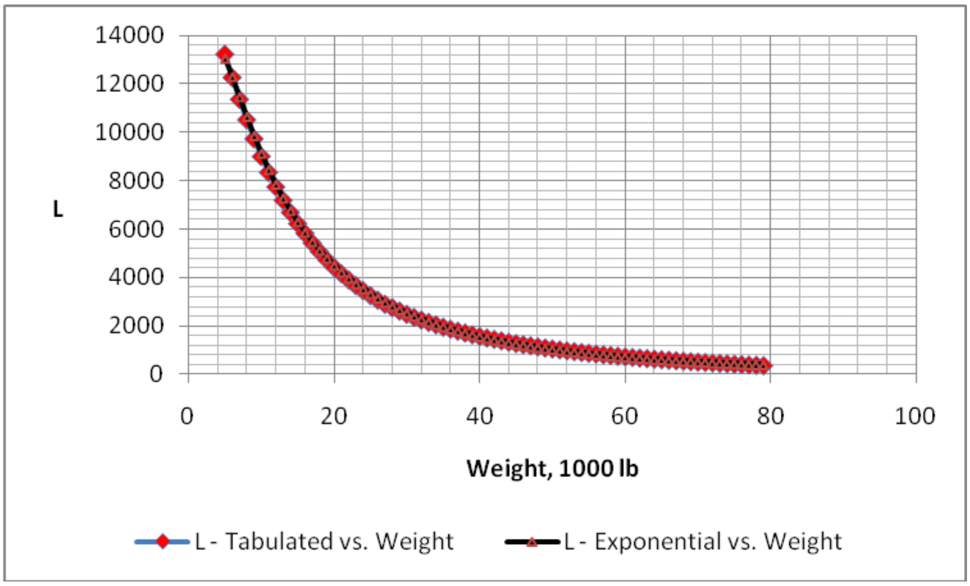

Figure 6.3: Comparison of L tabulated and L exponential

At the results of the David Wilson and Ramon Bentsen equation the error percent between the tabulated values of L and the D.W. and R. B. is about 32 percent and less. It also observed that at the yellow marked points which are selected for comparison of calculated values and calculated interpolation values. The error percent is between 5,5 percent and 27 percent. There are also red marked points exist where error percentages started increasing is the indication of the non linear function. Approximately 32 percent error observed for 79 000 lb weight which has a 492,42 for D.W. and R. B. L value regarding to 373 for tabulated L value. Though it is not clear but the figure describing this error is given Figure 6.4. so far.

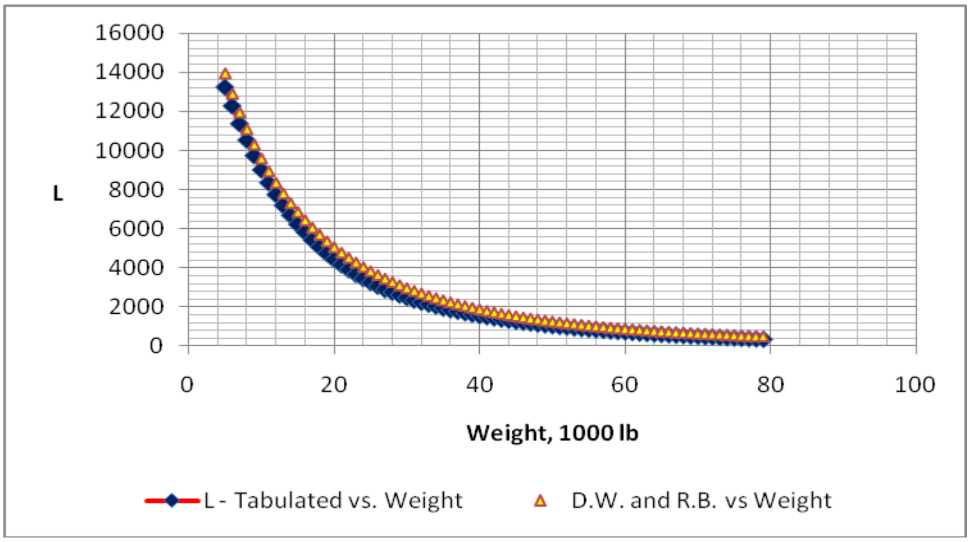

Figure 6.4: Comparison of L tabulated and L D.W. and R. B.

6.2 Application of the Program

Both of the Newtonian and exponential functions are used in the numerical program and could be used as an optional choice. Because of the excess error percentage of D.W. and R. B. equation this one is not considered in the developed program. There is slight difference between original and optimized parameters values.

Because of the insufficient available field data, only the data given in Galle and Woods work is used and those data are shown below in Table 6.3.

Table 6.3: Well data used for calculation in program

Case: Teeth Limit Bit Life	Value	Units
Bit	8,75	in.
Rig Cost	50	$/hr
Bit Cost	200	$
Trip Time	0,6	hr/1000 ft
Depth	10	1000ft
Formation	Firm shale	
Use Graphs	2 075 100	
Footage, Ff	180	ft
Rotating Time, Tf	12	hr
Weight, W	35	1000 lbm
Rotary Speed, N	100	rpm
Dull Condition	0,75	
Bearing Condition	0,5	

After all the necessary information is inserted the program could be run and the optimized weight and rotary speed for the minimum drilling cost could be calculated. Because of the lack of the sufficient well data comparisons made between the original results given in Galle and Woods study which used tornado charts and program results. The results of the program is reasonable but there is small difference exits and its supposed that differences are due to parameter L which is the function of weight and exact equation for L parameter did not given. The optimized best constant weight and rotary speed using default data for teeth limits bit life case and best rotary speed and weight procedure in the study and optimized parameters compared and the difference for weight and rotary speed is %0,545 and %1,042 respectively.

Table 6.4: Comparison results for weight and rotary speed

Optimized parameters	**G – W**	**Model Program**	**Error, %**
Weight, W	55	55,3	-0,545
Rotary Speed, N	96	95	1,042

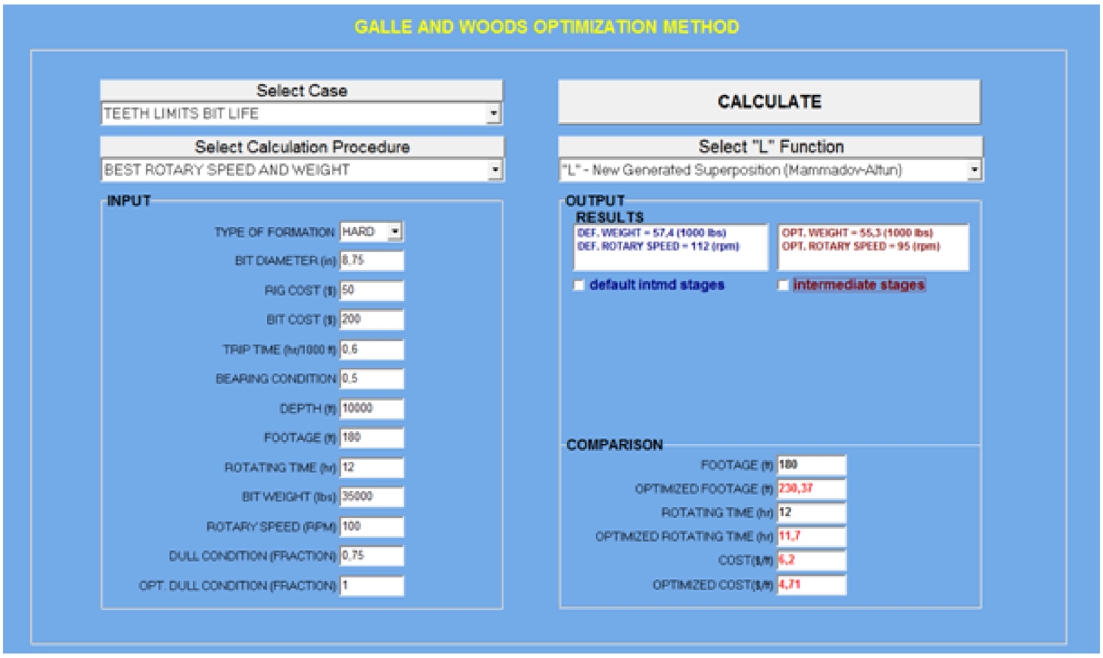

Figure 6.5: Model program results for default well data

Clicking on the “default intermediate (intmd) stages” and “ intermediate stages” signs simulator automatically shows calculated parameters such as – wbar, a, a_f, a_n, c_f, s, s_n, k, lcrv1 in different colors which are used during the optimization process and are shown in Figure 6.6.

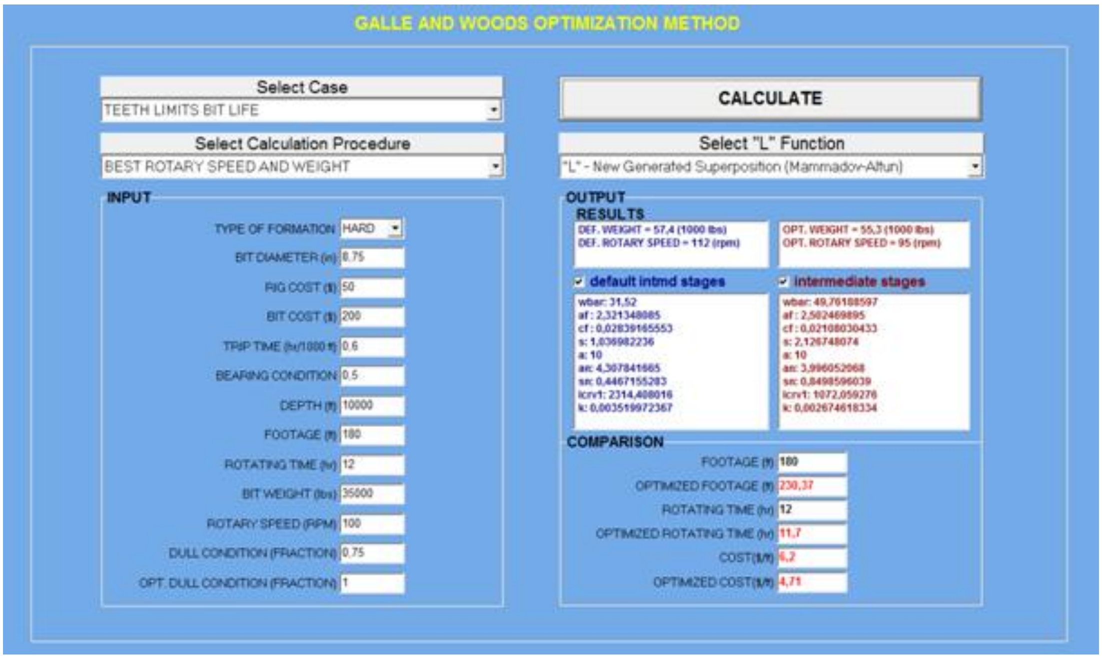

Figure 6.6: Model program results for default well data with intermediate stages

The comparison made for intermediate stages parameters using L Newtonian interpolation and L exponential equations in order to see how far those parameters are affected and are given in Table 6.5.

Tabel 6.5: Comparison for intermediate parameters using function L

Intermediate Parameters	Exponential (EXP)	Newtonian Interpolation (NS)	Error (%) between EXP vs. NI
Wbar	31,520	31,520	0,000
af	2,321	2,321	0,000
cf	0,028	0,028	0,000
S	1,025	1,037	1,167
A	10,000	10,000	0,000
An	4,308	4,308	0,000
Sn	0,442	0,447	1,167
U	1834,052	1834,052	0,000
V	966,545	966,545	0,000
lcrv1	2341,728	2314,408	-1,180

The obvious influence of the L functions used during the calculation is seen on three parameters which are fluid parameter s, dimensionless fluid parameter s_n, and L parameter itself shown as *lcrv1*. The percentage for those parameters are 1,16, 1,16 and 1,18 respectively s, s_n and *lcrv1*.

6.3 Comparison of Optimized Drilling Parameters

There is also comparison window which shows the results of the footage drilled, rotating time and cost for previous well and also optimized footage drilled, rotating time and cost for the desired dull condition and shown in Figure 6.7.

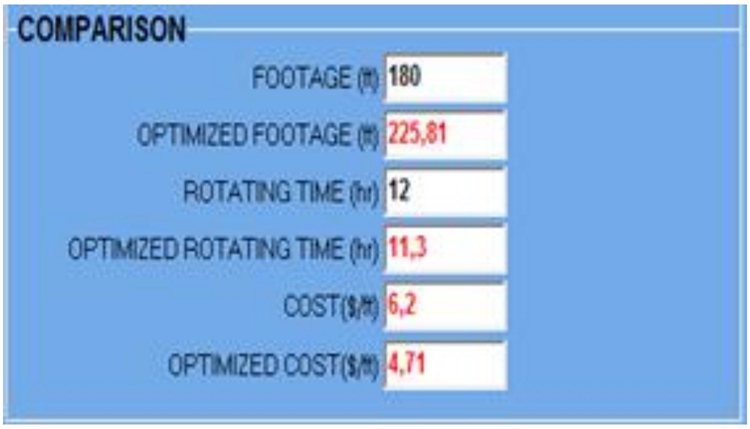

Figure 6.7: Comparison of footage drilled, rotating time and cost

Compared parameters shown in comparison window makes it easy to see how to choose the right set of operating values for optimal drilling process and gives possibility to make quick judgment during the operation. It is seen for the optimized weight 55000 lbs and rotary speed 96 rpm it would take only 11,3 hr to drill 225,81 ft for 4,71 $/ft. The difference between the original study optimized values and simulator optimized values is given in Table 6.6.

Table 6.6: Comparison of footage drilled, rotating time and cost

Optimized parameters	G. and W.	Model Program	Error, %
Total rotating time, hr	11,13	11,7	4,872
Cost per foot, $/ft	4,72	4,71	-0,212
Total footage, ft	223	225,81	1,244

Thus, it is seen that the difference between the Galle and Woods and model program is below %5 and negligible. The match between the Galle and Woods and model program is excellent. Thus, the developed program can be used in drilling optimization purpose confidently.

Table 6.7: Improvement made in Galle and Woods method

G - W	Total rotating time, hr	12	Cost per foot, $/ft	6,72	Total footage, ft	180
	Opt. rotating time, hr	11,13	Opt. Cost per foot, $/ft	4,72	Opt.Total footage, ft	223
Improvements, %		7,25		29,76		-23,89

As it is show in Table 6.7., the improvements obtained after optimization are %7,25 for total rotating time, %29,76 for cost per foot and %23,89 for total footage drilled in Galle and Woods method.

Tbale 6.8: Improvements made in model program

Model Program	Total rotating time, hr	12	Cost per foot, $/ft	6,72	Total footage, ft	180
	Opt. rotating time, hr	11,3	Opt. Cost per foot, $/ft	4,71	Opt.Total footage, ft	225,81
Improvements, %		5,83		29,91		-25,45

After the model program run it is seen that the results were satisfactory regarding Galle and Woods method. The %5,88 gain was obtained for total rotating time, %29,91 for cost per foot and %25,45 for total footage drilled.

In general using model program provides much more straightforwardness for getting better idea about the optimization procedures during predrilling planing and real time drilling.

6.4 Effect of Some Drilling Parameters on the Optimization Procedure

In order to investigate the level of importance of the drilling parameters on the result of drilling optimization and to evaluate their influence to each other some up to date synthetic data is given in Table 6.9 for a range of input variables are used in the program.

Table 6.9: Synthetic data

DATA		
Teeth Limit Bit Life		**Units**
Bit	8,75	in.
Rig Cost	**10000**	$/hr
Bit Cost	**2000**	$
Trip Time	0,6	hr/1000 ft
Depth	**10**	1000ft
Formation	Firm shale	
Use Graphs	2 075 100	
Footage, Ff	**200**	ft
Rotating Time, Tf	12	hr
Weight, W	35	1000 lbm
Rotary Speed, N	100	rpm
Desired Dull Condition (Df)	1	
Dull Condition	0,75	
Bearing Condition	0,5	

To see the effect of yellow and bold marked parameters on the cost of the operations the program is run. The obtained results are listed in Table 6.10 and given in Figure 6.8.

Table 6.10: Default synthetic data results

Default	
Opt. Weight, 1000 lbs	58
Opt. RPM	117
Opt. Footage, ft	213,61
Opt. Rotating Time, hr	8,32
Cost, $/ft	1120
Opt. Cost, $/ft	633

It is seen how far the prices of the drilling equipment influence the drilling operations.

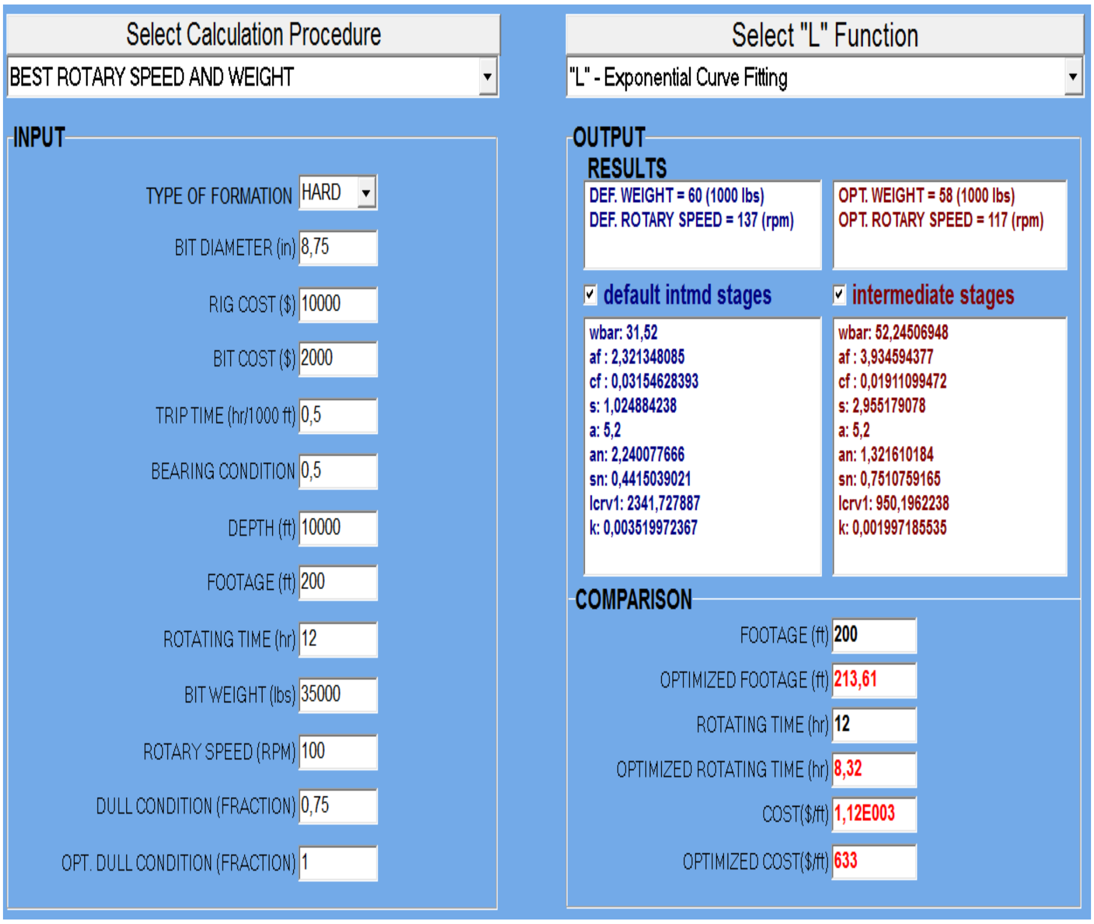

Figure 6.8: Default synthetic data results

To see the effect of the rig cost the rig rental cost is increased from 10000$ to 50000$ and the result is given in Table 6.11.

Table 6.11: Effect of the rig cost

Rig Cost, $	**50000**
Opt. Weight, 1000 lbs	58
Opt. RPM	118
Opt. Footage, ft	212
Opt. Rotating Time, hr	8,22
Cost, $/ft	5580
Opt. Cost, $/ft	3130

The outcome of the increased rig cost upgraded the cost from 633$ to 3130$ for the optimized cost. Rig cost has an anermous influence on the drilling cost and must be considered carefully. On the other hand, it is vital to realize the necessities of the optimization that resulted in approximately %50 of cost savings in this example in terms of optimized and not optimized costs when the rig costs are high. Note that if the rental cost is increased five folds, the operational cost increases %395.

The next considered parameter is the bit cost which its price increased from 2000$ to 10000$ and the results are given in Table 6.12.

Table 6.12: Effect of the bit cost

Bit Cost, $	**10000**
Opt. Weight, 1000 lbs	57,4
Opt. RPM	112
Opt. Footage, ft	221
Opt. Rotating Time, hr	8,87
Cost, $/ft	1120
Opt. Cost, $/ft	672

The consequence of the increased bit cost upgraded the cost from 633$ to 672$ for the optimized cost which require close attention. Note that the the bit coast increased five folds; however, the operational cost increased only %6,2. It is obvious that the bit cost has not as much influence on the drilling operational cost as the rig rental cost.

The next considered parameter is the depth reached which increased from 10000 ft to 20000 ft and the results are given in Table 6.13.

Table 6.13: Effect of the depth

Depth, ft	**20000**
Opt. Weight, 1000 lbs	55
Opt. RPM	94
Opt. Footage, ft	251
Opt. Rotating Time, hr	11,4
Cost, $/ft	1120
Opt. Cost, $/ft	861

The outcome of the increased depth upgraded the cost from 633$ to 861$ for the optimized cost which is quiet enough to consider carefully. It is clear that trip time influence is also important factor on the drilling cost. When the trip depth is doubled, the drilling cost increased %36 indicating that hoisting system should also optimized. The next considered parameter is the footage drilled per bit which increased from 200 ft to 500 ft and the results are given in Table 6.14.

Table 6.14: Effect of the footage

Footage, ft	**500**
Opt. Weight, 1000 lbs	58
Opt. RPM	117
Opt. Footage, ft	534
Opt. Rotating Time, hr	8,32
Cost, $/ft	446
Opt. Cost, $/ft	253

The obtained result was very satisfactory. In general when we drill more footage with per bit, the operational cost dramatically decreases. In this example, the optimizad cost decreased from 633$ to 253$ which provide drastic savings. In other words, when the footage per bit increases 2.5 folds, the drilling cost decreases %60 indicating that the footage drilled per bit is much more important rather than the bit cost itself.

As a result it is observed from the sensitivity of parameter analysis that the effect of parameters considered for the cost of the drilling operation is require qiute close care in order to get cost effective drilling operations. In other words, the cost effective drilling operations require to use optimization models like Galle and Woods given in this study. It is possible to reduce the drilling operational costs considerably with the Galle and Woods optimization method.

7. CONCLUSION AND RECOMMENDATIONS

Galle and Woods drilling optimization method is revisited. Mathematical derivation of the method not available in the literature is showed and explained.

The best constant weight and rotary speed, the best constant weight for any given rotary speed and the best rotary speed for any given weight procedures for teeth limit bit life and penetration rate limits economical bit life cases have been considered in the model program for the field application.

For each procedure of selected cases the drilling cost, footage drilled, rotating time may be calculated at a specified bit dullness. The optimized drilling results can be compared with those of none optimized drilling conditions.

Tabulated L parameters given in the original manuscript are defined by employing 7^{th} degree Newton interpolation technique and error of L values between the table values and interpolation values is reduced less than %1 at all points. In addition, function L can be used to calculate for any WOB values that are not tabulated in the original work.

Numerical programming code for the Galle and Woods method has been written by using Delphi model programming. The model program provide convenience for the observation of the direct alterations made on the input data and provide to see effect of change of each parameters, as well.

Using model program provides much more straightforwardness for getting better idea about the optimization procedures during pre-drilling planing.

The improvements obtained for optimized parameters using the data provided in the original manuscript are %5,88 for total rotating time, %29,91 for cost per foot and %25,45 for total footage drilled.

The effect of parameters on the drilling cost is investigated by synthetic data sets. The sensitivity analysis reveals that the rig rental cost, footage drilled per bit, and the trip depth are the main parameters that have the considerable influence on the drilling costs. It is also interesting that the bit coast has less effect on the drilling cost than that of the others.

It is shown that optimized parameters particularly weight on bit and rotary speed can cut the costs about %50 compare to the results obtained from the non optimized drilling conditions.

It has been seen that implementation of the optimization procedures is very effective. At the same time, without the engineering supervision those procedures should not be used in field applications.

The model can be developed for PDC bits by eliminating bearing life parameter.

The model can further be developed by taking into consideration of hydraulic parameter.

The model can be developed for bit bearing life limits the penetration rate, as well.

New teeth dullness equation can be developed and used in the model.

Phisical meaning of the function L should be clarified.

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APPENDICES

At the Appendix part mathematical background of this thesis for the following procedures will be presented:

1. The best combination of constant weight and rotary speed,
2. The best constant weight for any given rotary speed,
3. The best constant rotary speed for any given weight

The following three cases are considered, for each of the above mentioned procedures listed:

Case 1: Teeth limit bit life.

Case 2: Drilling rate limits economical bit life.

Case 3: Bearings limit bit lifeş

Drilling rate equation

$$\frac{dF}{dT} = \frac{C_f W^k r}{a^p} \qquad \textbf{(A.1)}$$

Wherein:

k = 1.0 (for most formations except very soft formations)

= 0.6 (for very soft formations)

p = 0.5 (for self sharpening or chipping type bit tooth wear)

r, is essentially rotary speed to a fractional power and *a*, is a function of dullness. Drilling rate increases with drillability (C_f), weight and rotary speed and decreases with dullness. In this equation the effects of bit type, hydraulics, drilling fluid and formation are all included in the drillability constant C_f.

Rate of dulling equation

$$\frac{dD}{dT} = \frac{1}{A_f} \frac{i}{am} \qquad (\mathbf{A.2})$$

i, is a quantity that increases with rotary speed, *a*, increses with dullness, and *m*, decreases with increase in weight. A_f – decreases with increase of formation abrasiveness. Thus the rate of wear increases as abrasiveness, weight, and rotary speed increase, and decreases as dullness increases. In this equation the effect of bit type, hydraulics, drilling fluid and formation are all included in the abrasiveness constant A_f.

Bearing life equation

$$B = S * \frac{L}{N} \qquad (\mathbf{A.3})$$

The symbol *L*, is used to denote a decreasing function with increasing weight. The other quantities are explained under Abbreviations.

Denoting the fraction of bearing life expended by B_x, then in time *T:*

$$B_x = \frac{T}{B} = \frac{TN}{SL} \qquad (\mathbf{A.4})$$

This applies only if weight and rotary speed are constant during the time *T*.

Bearing life decreases with increases in weight and rotary speed, and increases with the drilling fluid factor *S*. The value of *S* for any given drilling fluid will change with different bit types containing bearings of different capacity.

Optimization procedures of the cases.

Terms are introduced in this section without definition, they are defined in Abbreviations section.

The three equations (A.1), (A.2), and (A.3) may be combined to obtain

$$\frac{dT}{dD} = A_f f_1 \qquad \textbf{(A.5)}$$

$$\frac{dF}{dD} = C_f A_f f_2 \qquad \textbf{(A.6)}$$

$$\frac{dB_x}{dD} = \frac{A_f}{S} f_3 \qquad \textbf{(A.7)}$$

f_1 , f_2 and f_3 are functions of *N*, $\overline{W}$, and *D*.

The cost per foot (expressed in rig hours per foot) of a bit run is

$$C = \frac{A + T_f}{F_f} \qquad \textbf{(A.8)}$$

or

$$C = \frac{A + A_f \int_0^{D_f} f_1 \, dD}{C_f A_f \int_0^{D_f} f_2 \, dD} \qquad \textbf{(A.9)}$$

Since

$T_f = \int_0^{D_f} \frac{dT}{dD} dD$, and $F_f = \int_0^{D_f} \frac{dF}{dD} dD$.

Similarly,

$$B_{xf} = \int_0^{D_f} \frac{dB_x}{dD} dD \qquad \textbf{(A.10)}$$

and

$$B_{xf} = \frac{A_f}{S} \int_0^{D_f} f_3 \, dD \qquad \textbf{(A.11)}$$

Equations (A.9) and (A.11) may be put into dimensionless form and if we divide both sides of equations by A_f, then

$$C = \frac{A/A_f + A_f/A_f \int_0^{D_f} f_1\, dD}{C_f A_f/A_f \int_0^{D_f} f_2\, dD} = \frac{A_n \int_0^{D_f} f_1\, dD}{C_f \int_0^{D_f} f_2\, dD} \quad \textbf{(A.12)}$$

denote A_n as $A_n = A/A_f$ and multiply by C_f, then

$$C * C_f = \frac{A_n + \int_0^{D_f} f_1\, dD}{\int_0^{D_f} f_2\, dD} = K \quad \textbf{(A.13)}$$

$$B_{xf} = \frac{A_f/A_f}{S/A_f} \int_0^{D_f} f_3\, dD = \frac{1}{S_n} \int_0^{D_f} f_3\, dD \quad \textbf{(A.14)}$$

We will minimize K subject to restrictions

$$D_f \leq 1, \int_0^{D_f} f_3\, dD \leq S_n, \frac{dK}{dD_f} \leq 0 \quad \textbf{(A.15)}$$

The two cases will be considered:

Case 1.

$$D_f = 1, \int_0^{D_f} f_3\, dD \leq S_n, \frac{dK}{dD_f} \leq 0 \quad \textbf{(A.16)}$$

Case 2.

$$D_f \leq 1, \int_0^{D_f} f_3\, dD \leq S_n, \frac{dK}{dD_f} = 0 \quad \textbf{(A.17)}$$

Letting

$$P_1 = \int_0^{D_f} f_1 \, dD, \qquad P_2 = \int_0^{D_f} f_2 \, dD, and \;\; P_3 = \int_0^{D_f} f_3 \, dD \tag{A.18}$$

$$K = \frac{A_n + P_1}{P_2} \tag{A.19}$$

And from the equation (A.14) since

$$B_x \le 1, \;\; S_n \ge P_3 \tag{A.20}$$

Minimizing relations and restrictions for the cases are (Sokolnikoff and et. al., 1941):

Case 1.

$$\frac{\partial K}{\partial N} = \frac{\partial K}{\partial \overline{W}} = 0 \qquad D_f = 1, \; S_n \ge \; P_3 \, , \frac{\partial K}{\partial D_f} \; \le 0 \tag{A.21}$$

Case 2.

$$\frac{\partial K}{\partial N} = \frac{\partial K}{\partial \overline{W}} = 0 \qquad D_f \le 1, S_n \ge \; P_3 \, , \frac{\partial K}{\partial D_f} = 0 \tag{A.22}$$

Using the minimizing relations (A.21) and (A.22) along with the equation (A.19) and the definitions for P_1, P_2, and P_3, the following minimizing equations may be derived.

Case 1.

$$K = \frac{\frac{\partial P_1}{\partial N}}{\frac{\partial P_2}{\partial N}} = \frac{\frac{\partial P_1}{\partial \overline{W}}}{\frac{\partial P_2}{\partial \overline{W}}} \tag{A.23}$$

Case 2.

$$K = \frac{\frac{\partial P_1}{\partial N}}{\frac{\partial P_2}{\partial N}} = \frac{\frac{\partial P_1}{\partial \overline{W}}}{\frac{\partial P_2}{\partial \overline{W}}} = \frac{\frac{\partial P_1}{\partial D_f}}{\frac{\partial P_2}{\partial D_f}} \qquad (\mathbf{A.24})$$

Since $P_1 = \frac{m}{i}\int_0^{D_f} adD$, $P_2 = \frac{m\overline{W}^k r}{i}\int_0^{D_f} a^{1-p}dD$, and $P_3 = \frac{Nm}{Li}\int_0^{D_f} adD$,

the following derivatives may be written with respect to $N, \overline{W}, D_f$:

$$\frac{\partial P_1}{\partial N} = -\frac{i'm}{i^2}\int_0^{D_f} adD \qquad (\mathbf{A.25})$$

$$\frac{\partial P_2}{\partial N} = \left[\frac{ir' - ri'}{i^2}\right] m\overline{W}^k \int_0^{D_f} a^{1-p}dD \qquad (\mathbf{A.26})$$

$$\frac{\partial P_3}{\partial N} = \left[\frac{i - Ni'}{i^2}\right]\frac{m}{L}\int_0^{D_f} adD \qquad (\mathbf{A.27})$$

$$\frac{\partial P_1}{\partial \overline{W}} = \frac{m'}{i}\int_0^{D_f} adD \qquad (\mathbf{A.28})$$

$$\frac{\partial P_2}{\partial \overline{W}} = [m'\overline{W}^k + km\overline{W}^{k-1}]\frac{r}{i}\int_0^{D_f} a^{1-p}dD \qquad (\mathbf{A.29})$$

$$\frac{\partial P_3}{\partial \overline{W}} = \left[\frac{Lm' - mL'}{L^2}\right]\frac{N}{i}\int_0^{D_f} adD \qquad (\mathbf{A.30})$$

$$\frac{\partial P_1}{\partial D_f} = \frac{m}{i}(a)_{D_f} \qquad (\mathbf{A.31})$$

$$\frac{\partial P_2}{\partial D_f} = \frac{m\overline{W}^k r}{i}(a^{1-p})_{D_f} \qquad (\mathbf{A.32})$$

$$\frac{\partial P_3}{\partial D_f} = \frac{Nm}{Li}(a)_{D_f} \tag{A.33}$$

If we substitute these equations into the minimizing equations (A.23) the following equations are obtained.

Case 1. Teeth limit bit life

1. The procedure of best combination of constant weight and rotary speed

$$K = \frac{\frac{\partial P_1}{\partial N}}{\frac{\partial P_2}{\partial N}} = \frac{\frac{\partial P_1}{\partial \overline{W}}}{\frac{\partial P_2}{\partial \overline{W}}} \tag{A.23}$$

If we take left hand side of equation (A.23) and make simplification we will get following equations:

$$\frac{\frac{\partial P_1}{\partial N}}{\frac{\partial P_2}{\partial N}} = \frac{-\frac{i'm}{i^2}\int_0^1 adD}{\left[\frac{ir' - ri'}{i^2}\right] m\overline{W}^k \int_0^1 a^{1-p} dD} = \frac{-i' \int_0^1 adD}{(ir' - ri')\overline{W}^k \int_0^1 a^{1-p} dD} \tag{A.34}$$

If the both numerator and denominator are divided by “-ir”,

$$\frac{\frac{\partial P_1}{\partial N}}{\frac{\partial P_2}{\partial N}} = \frac{-i' \int_0^1 adD}{(ir' - ri')\overline{W}^k \int_0^1 a^{1-p} dD} = \frac{\frac{i'}{ir}\int_0^1 adD}{(\frac{i'}{i} - \frac{r'}{r})\overline{W}^k \int_0^1 a^{1-p} dD}$$

$$= \frac{\frac{i'}{i}\int_0^1 adD}{r\overline{W}^k(\frac{i'}{i} - \frac{r'}{r}) \int_0^1 a^{1-p} dD} \tag{A.35}$$

Right hand side of equation (A.23) will be simplified, and following equations will be obtained:

$$\frac{\frac{\partial P_1}{\partial \overline{W}}}{\frac{\partial P_2}{\partial \overline{W}}} = \frac{\frac{m^{'}}{i}\int_0^1 a dD}{[m^{'}\overline{W}^k + km\overline{W}^{k-1}]\frac{r}{i}\ \int_0^1 a^{1-p} dD}$$

$$= \frac{m' \int_0^1 a dD}{[m^{'}\overline{W}^k + km\overline{W}^{k-1}] r \int_0^1 a^{1-p} dD} \qquad \textbf{(A.36)}$$

If we divide both numerator and denominator of equation (A.36) by "m" and take the "$\overline{W}^k r$" out of the parenthesis, we obtain:

$$\frac{\frac{\partial P_1}{\partial \overline{W}}}{\frac{\partial P_2}{\partial \overline{W}}} = \frac{\frac{m^{'}}{m}\int_0^1 a dD}{r\overline{W}^k \left[\frac{m^{'}}{m} + \frac{k}{\overline{W}}\right]\ \int_0^1 a^{1-p} dD} \qquad \textbf{(A.37)}$$

If we substitute equations (A.35) and (A.37) into the equation (A.23) following equation will be obtained:

$$K = \frac{\frac{\partial P_1}{\partial N}}{\frac{\partial P_2}{\partial N}} = \frac{\frac{\partial P_1}{\partial \overline{W}}}{\frac{\partial P_2}{\partial \overline{W}}} = \frac{\frac{i^{'}}{i}\int_0^1 a dD}{r\overline{W}^k (\frac{i^{'}}{i} - \frac{r^{'}}{r}) \int_0^1 a^{1-p} dD}$$

$$= \frac{\frac{m^{'}}{m}\int_0^1 a dD}{r\overline{W}^k \left[\frac{m^{'}}{m} + \frac{k}{\overline{W}}\right]\ \int_0^1 a^{1-p} dD} \qquad \textbf{(A.38)}$$

We can write equation (A.38) in the following form:

$$\frac{\frac{i^{'}}{i}\int_0^1 a dD}{r\overline{W}^k (\frac{i^{'}}{i} - \frac{r^{'}}{r}) \int_0^1 a^{1-p} dD} = \frac{\frac{m^{'}}{m}\int_0^1 a dD}{r\overline{W}^k \left[\frac{m^{'}}{m} + \frac{k}{\overline{W}}\right]\ \int_0^1 a^{1-p} dD}$$

or

$$\frac{\frac{i'}{i}}{(\frac{i'}{i}-\frac{r'}{r})}=\frac{\frac{m'}{m}}{\left[\frac{m'}{m}+\frac{k}{\bar{W}}\right]} \qquad (\mathbf{A.39})$$

If we divide left hand side by $(\frac{i'}{i})$ and right hand side by $(\frac{m'}{m})$ of equation (A.39), then the next equality will be obtained:

$$\left[-\frac{ir'}{i'r}=\frac{km}{\bar{W}m'}\right] \qquad (\mathbf{A.40})$$

Left hand side and right hand side of equation (A.40) are function of N and W respectively.

2. The procedure of best constant weight for any given rotary speed:

$$\frac{A_n+\frac{m'}{i}\int_0^1 adD}{\frac{m\bar{W}^k r}{i}\int_0^1 a^{1-p}dD}=\frac{\frac{m'}{m}\int_0^1 adD}{r\bar{W}^k\left[\frac{m'}{m}+\frac{k}{\bar{W}}\right]\int_0^1 a^{1-p}dD}=K \qquad (\mathbf{A.41})$$

Equation (A.41) is taken from equation (A.38), and if we solve equation (A.41) for "r", gives the best constant weight for any given rotary speed:

$$r=\frac{\frac{m'}{m}\int_0^1 adD}{K\bar{W}^k\left[\frac{m'}{m}+\frac{k}{\bar{W}}\right]\int_0^1 a^{1-p}dD} \qquad (\mathbf{A.42})$$

3. The procedure of best constant rotary speed for any given weight:

$$\frac{A_n+\frac{m'}{i}\int_0^1 adD}{\frac{m\bar{W}^k r}{i}\int_0^1 a^{1-p}dD}=\frac{\frac{i'}{i}\int_0^1 adD}{r\bar{W}^k(\frac{i'}{i}-\frac{r'}{r})\int_0^1 a^{1-p}dD}=K \qquad (\mathbf{A.43})$$

Equation (A.43) is taken from equation (A.38), and if we solve for "$\overline{W}^k$", gives the best constant rotary speed for any given weight:

$$\overline{W}^k = \frac{\frac{i'}{i}\int_0^1 a dD}{Kr(\frac{i'}{i} - \frac{r'}{r})\int_0^1 a^{1-p} dD} \qquad \textbf{(A.44)}$$

Case 2. Drilling rate limits economical bit life.

If we take into consideration of the derivatives from (A.25) to (A.33) and put into the (A.24) we will get started above – mentioned procedures for the Case 2.

1. The procedure of best combination of constant weight and rotary speed

$$K = \frac{\frac{\partial P_1}{\partial N}}{\frac{\partial P_2}{\partial N}} = \frac{\frac{\partial P_1}{\partial \overline{W}}}{\frac{\partial P_2}{\partial \overline{W}}} = \frac{\frac{\partial P_1}{\partial D_f}}{\frac{\partial P_2}{\partial D_f}} \qquad \textbf{(A.24)}$$

If we take left hand side of equation (A.24) and make simplification we will get following equations:

$$\frac{\frac{\partial P_1}{\partial N}}{\frac{\partial P_2}{\partial N}} = \frac{-\frac{i'm}{i^2}\int_0^{D_f} a dD}{\left[\frac{ir' - ri'}{i^2}\right] m\overline{W}^k \int_0^{D_f} a^{1-p} dD} = \frac{-i'\int_0^{D_f} a dD}{(ir' - ri')\overline{W}^k \int_0^{D_f} a^{1-p} dD} \qquad \textbf{(A.45)}$$

If the both numerator and denominator are divided by "-ir",

$$\frac{\frac{\partial P_1}{\partial N}}{\frac{\partial P_2}{\partial N}} = \frac{-i'\int_0^{D_f} a dD}{(ir' - ri')\overline{W}^k \int_0^{D_f} a^{1-p} dD} = \frac{\frac{i'}{ir}\int_0^{D_f} a dD}{(\frac{i'}{i} - \frac{r'}{r})\overline{W}^k \int_0^{D_f} a^{1-p} dD}$$

$$= \frac{\frac{i'}{i}\int_0^{D_f} a dD}{r\overline{W}^k(\frac{i'}{i} - \frac{r'}{r})\int_0^{D_f} a^{1-p} dD} \qquad \textbf{(A.46)}$$

The middle part of equation (A.24) will be simplified, and following equations will be obtained:

$$\frac{\frac{\partial P_1}{\partial \overline{W}}}{\frac{\partial P_2}{\partial \overline{W}}} = \frac{\frac{m'}{i}\int_0^{D_f} a dD}{[m'\overline{W}^k + km\overline{W}^{k-1}]\frac{r}{i}\ \int_0^{D_f} a^{1-p} dD}$$

$$= \frac{m'\int_0^{D_f} a dD}{[m'\overline{W}^k + km\overline{W}^{k-1}]r\int_0^{D_f} a^{1-p} dD} \qquad \textbf{(A.47)}$$

If we divide both numerator and denominator of (A.47) by "m" and take the "$\overline{W}^k r$" out of the parenthesis, we obtain:

$$\frac{\frac{\partial P_1}{\partial \overline{W}}}{\frac{\partial P_2}{\partial \overline{W}}} = \frac{\frac{m'}{m}\int_0^{D_f} a dD}{r\overline{W}^k\left[\frac{m'}{m} + \frac{k}{\overline{W}}\right]\ \int_0^{D_f} a^{1-p} dD} \qquad \textbf{(A.48)}$$

Now we take left hand side of (A.24) and will put appropriate derivatives into it then make simplifications.

$$\frac{\frac{\partial P_1}{\partial D_f}}{\frac{\partial P_2}{\partial D_f}} = \frac{\frac{m}{i}(a)_{D_f}}{\frac{m\overline{W}^k r}{i}\ (a^{1-p})_{D_f}} = \frac{\frac{m}{i}(a)_{D_f}}{\frac{m\overline{W}^k r}{i}\ (a)_{D_f}(a^{-p})_{D_f}} = \frac{(a^p)_{D_f}}{\overline{W}^k r} \qquad \textbf{(A.49)}$$

If we substitute equations (A.46), (A.48) and (A.49) into the equation (A.24) following equation will be obtained:

$$K = \frac{\frac{\partial P_1}{\partial N}}{\frac{\partial P_2}{\partial N}} = \frac{\frac{\partial P_1}{\partial \overline{W}}}{\frac{\partial P_2}{\partial \overline{W}}} = \frac{\frac{\partial P_1}{\partial D_f}}{\frac{\partial P_2}{\partial D_f}} = \frac{\frac{i'}{i}\int_0^{D_f} a dD}{r\overline{W}^k(\frac{i'}{i} - \frac{r'}{r})\int_0^{D_f} a^{1-p} dD}$$

$$= \frac{\frac{m'}{m}\int_0^{D_f} a dD}{r\overline{W}^k\left[\frac{m'}{m} + \frac{k}{\overline{W}}\right]\ \int_0^{D_f} a^{1-p} dD} = \frac{(a^p)_{D_f}}{\overline{W}^k r} \qquad \textbf{(A.50)}$$

We can write equation (A.50) in the following form:

$$\frac{\frac{i'}{i}\int_0^{D_f} a dD}{r\overline{W}^k(\frac{i'}{i}-\frac{r'}{r})\int_0^{D_f} a^{1-p} dD} = \frac{\frac{m'}{m}\int_0^{D_f} a dD}{r\overline{W}^k\left[\frac{m'}{m}+\frac{k}{\overline{W}}\right]\int_0^{D_f} a^{1-p} dD} = \frac{(a^p)_{D_f}}{\overline{W}^k r} \quad \textbf{(A.51)}$$

If we divide left hand side by $(\frac{i'}{i})$ and middle part by $(\frac{m'}{m})$ then make some mathematical treatment we will get the next form of (A.51).

$$\frac{\frac{i'}{i}\int_0^{D_f} a dD}{(\frac{i'}{i}-\frac{r'}{r})\int_0^{D_f} a^{1-p} dD} = \frac{\frac{m'}{m}\int_0^{D_f} a dD}{\left[\frac{m'}{m}+\frac{k}{\overline{W}}\right]\int_0^{D_f} a^{1-p} dD} = (a^p)_{D_f} \quad \textbf{(A.52)}$$

Thus,

$$-\frac{ir'}{i'r} = \frac{km}{\overline{W}m'} = \frac{\int_0^{D_f} a dD}{(a^p)_{D_f}\int_0^{D_f} a^{1-p} dD} - 1 \quad \textbf{(A.53)}$$

2. The procedure of best constant weight for any given rotary speed:

$$\frac{A_n + \frac{m'}{i}\int_0^{D_f} a dD}{\frac{m\overline{W}^k r}{i}\int_0^{D_f} a^{1-p} dD} = \frac{\frac{m'}{m}\int_0^{D_f} a dD}{r\overline{W}^k\left[\frac{m'}{m}+\frac{k}{\overline{W}}\right]\int_0^{D_f} a^{1-p} dD} = \frac{(a^p)_{D_f}}{\overline{W}^k r} = K \quad \textbf{(A.54)}$$

Equation (A.54) is another form of equation (A.24), and if we consider (A.53) which is also another simplified form of (A.24) solve for "r", gives the best constant weight for any given rotary speed:

$$\frac{km}{\overline{W}m'} = \frac{\int_0^{D_f} a dD}{(a^p)_{D_f}\int_0^{D_f} a^{1-p} dD} - 1 \quad \textbf{(A.55)}$$

3. The procedure of best constant rotary speed for any given weight:

$$\frac{A_n + \frac{m'}{i}\int_0^{D_f} a dD}{\frac{m\overline{W}^k r}{i}\int_0^{D_f} a^{1-p} dD} = \frac{\frac{i'}{i}\int_0^{D_f} a dD}{r\overline{W}^k(\frac{i'}{i} - \frac{r'}{r})\int_0^{D_f} a^{1-p} dD} = K \qquad \textbf{(A.56)}$$

Equation(A.56) is another form of equation(A.24), and if we consider (A.53) which is also another simplified form of (A.24) solve for "$\overline{W}^k$", gives the best constant weight for any given rotary speed:

$$-\frac{ir'}{i'r} = \frac{\int_0^{D_f} a dD}{(a^p)_{D_f}\int_0^{D_f} a^{1-p} dD} - 1 \qquad \textbf{(A.57)}$$

Case 3. Bearings limit bit life.

Minimizing relations and restrictions for the bearing limits bit life case are follows as:

$$\frac{\partial K}{\partial N}\frac{\partial P_3}{\partial D_f} - \frac{\partial K}{\partial D_f}\frac{\partial P_3}{\partial N} = 0 \qquad \textbf{(A.58)}$$

$$\frac{\partial K}{\partial \overline{W}}\frac{\partial P_3}{\partial D_f} - \frac{\partial K}{\partial D_f}\frac{\partial P_3}{\partial \overline{W}} = 0$$

$$D_f \leq 1, S_n \geq P_3, \frac{\partial K}{\partial D_f} \leq 0$$

Using the minimizing relations and (A.58) along with the definitions for P_1, P_2, and P_3, the following minimizing equations may be derived.

Case 3.

$$K = \frac{\frac{\partial P_1}{\partial N}\frac{\partial P_3}{\partial D_f} - \frac{\partial P_1}{\partial D_f}\frac{\partial P_3}{\partial N}}{\frac{\partial P_2}{\partial N}\frac{\partial P_3}{\partial D_f} - \frac{\partial P_2}{\partial D_f}\frac{\partial P_3}{\partial N}} = \frac{\frac{\partial P_1}{\partial \overline{W}}\frac{\partial P_3}{\partial D_f} - \frac{\partial P_1}{\partial D_f}\frac{\partial P_3}{\partial \overline{W}}}{\frac{\partial P_2}{\partial \overline{W}}\frac{\partial P_3}{\partial D_f} - \frac{\partial P_2}{\partial D_f}\frac{\partial P_3}{\partial \overline{W}}} \qquad \textbf{(A.59)}$$

At first the left hand side of the equation (A.59) will be considered for mathematical evaluation.

$$K = \frac{\frac{\partial P_1}{\partial N}\frac{\partial P_3}{\partial D_f} - \frac{\partial P_1}{\partial D_f}\frac{\partial P_3}{\partial N}}{\frac{\partial P_2}{\partial N}\frac{\partial P_3}{\partial D_f} - \frac{\partial P_2}{\partial D_f}\frac{\partial P_3}{\partial N}} \qquad \textbf{(A.60)}$$

The mathematical simplifications are made on the numerator of the equation (60).

$$\begin{aligned}\frac{\partial P_1}{\partial N}\frac{\partial P_3}{\partial D_f} - \frac{\partial P_1}{\partial D_f}\frac{\partial P_3}{\partial N} \\ &= -\frac{i'm}{i^2}\int_0^{D_f} adD * \frac{Nm}{Li}(a)_{D_f} \\ &- \frac{m}{i}(a)_{D_f}\left[\frac{i - Ni'}{i^2}\right]\frac{m}{L}\int_0^{D_f} adD \qquad \textbf{(A.61)}\end{aligned}$$

$$\frac{\partial P_1}{\partial N}\frac{\partial P_3}{\partial D_f} - \frac{\partial P_1}{\partial D_f}\frac{\partial P_3}{\partial N} = -\frac{m^2}{Li^2}(a)_{D_f}\int_0^{D_f} adD \qquad \textbf{(A.62)}$$

The simplifications are made on the denominator of the equation (A.59)

$$\begin{aligned}\frac{\partial P_2}{\partial N}\frac{\partial P_3}{\partial D_f} - \frac{\partial P_2}{\partial D_f}\frac{\partial P_3}{\partial N} \\ &= \left[\frac{ir' - ri'}{i^2}\right]m\overline{W}^k\int_0^{D_f} a^{1-p}dD * \frac{Nm}{Li}(a)_{D_f} \\ &- \left[\frac{i - Ni'}{i^2}\right]\frac{m}{L}\int_0^{D_f} adD * \frac{m\overline{W}^k r}{i}(a^{1-p})_{D_f} \\ &= \frac{m^2}{Li^2}N\overline{W}^k r(a)_{D_f}\left[\left(\frac{r'}{r} - \frac{i'}{i}\right)\int_0^{D_f} a^{1-p}dD \right. \\ &\left. - \frac{1}{(a^p)_{D_f}}\left(\frac{1}{N} - \frac{i'}{i}\right)\int_0^{D_f} adD\right] \qquad \textbf{(A.63)}\end{aligned}$$

If we combine the equations (A.62) and (A.63) in the equation (A.60) we will get the following equation.

$$K = \frac{\frac{m^2}{Li^2}(a)_{D_f}\int_0^{D_f} a dD}{\frac{m^2}{Li^2}N\bar{W}^k r(a)_{D_f}\left[\left(\frac{i'}{i}-\frac{r'}{r}\right)\int_0^{D_f} a^{1-p} dD - \frac{1}{(a^p)_{D_f}}\left(\frac{i'}{i}-\frac{1}{N}\right)\int_0^{D_f} a dD\right]} \qquad \textbf{(A.64)}$$

The final form of the left hand side of the equation (A.60) is given as:

$$K = \frac{\frac{1}{N}\int_0^{D_f} a dD}{\bar{W}^k r\left[\left(\frac{i'}{i}-\frac{r'}{r}\right)\int_0^{D_f} a^{1-p} dD - \frac{1}{(a^p)_{D_f}}\left(\frac{i'}{i}-\frac{1}{N}\right)\int_0^{D_f} a dD\right]} \qquad \textbf{(A.65)}$$

Now the right hand side of the equation (A.59) will be taken into consideration.

$$K = \frac{\frac{\partial P_1}{\partial \bar{W}}\frac{\partial P_3}{\partial D_f} - \frac{\partial P_1}{\partial D_f}\frac{\partial P_3}{\partial \bar{W}}}{\frac{\partial P_2}{\partial \bar{W}}\frac{\partial P_3}{\partial D_f} - \frac{\partial P_2}{\partial D_f}\frac{\partial P_3}{\partial \bar{W}}} \qquad \textbf{(A.66)}$$

At first the mathematical evaluation will be done on the nominator of the right hand side.

$$\begin{aligned} \frac{\partial P_1}{\partial \bar{W}}\frac{\partial P_3}{\partial D_f} - \frac{\partial P_1}{\partial D_f}\frac{\partial P_3}{\partial \bar{W}} &= \frac{m'}{i}\int_0^{D_f} a dD * \frac{Nm}{Li}(a)_{D_f} - \frac{m}{i}(a)_{D_f} \\ &* \left[\frac{Lm' - mL'}{L^2}\right]\frac{N}{i}\int_0^{D_f} a dD \\ &= \frac{Nmm'}{Li^2}(a)_{D_f}\left[1-(1-\left(\frac{mL'}{m'L}\right)\right]\int_0^{D_f} a dD \end{aligned} \qquad \textbf{(A.67)}$$

The mathematical evaluation will be done on the denominator.

$$\begin{aligned} \frac{\partial P_2}{\partial \bar{W}}\frac{\partial P_3}{\partial D_f} - \frac{\partial P_2}{\partial D_f}\frac{\partial P_3}{\partial \bar{W}} &= \left[m'\bar{W}^k + km\bar{W}^{k-1}\right]\frac{r}{i}\int_0^{D_f} a^{1-p} dD * \frac{Nm}{Li}(a)_{D_f} \\ &- \frac{m\bar{W}^k r}{i}(a^{1-p})_{D_f} * \left[\frac{Lm' - mL'}{L^2}\right]\frac{N}{i}\int_0^{D_f} a dD \end{aligned} \qquad \textbf{(A.68)}$$

If both the equation (A.67) and (A.68) are combined in equation (A.66) then the next mathematical equation is obtained.

$$K = \frac{\frac{\partial P_1}{\partial \overline{W}}\frac{\partial P_3}{\partial D_f} - \frac{\partial P_1}{\partial D_f}\frac{\partial P_3}{\partial \overline{W}}}{\frac{\partial P_2}{\partial \overline{W}}\frac{\partial P_3}{\partial D_f} - \frac{\partial P_2}{\partial D_f}\frac{\partial P_3}{\partial \overline{W}}}$$

$$= \frac{\frac{Nmm'}{Li^2}(a)_{D_f}\left[1 - (1 - \left(\frac{mL'}{m'L}\right)\right]\int_0^{D_f} adD}{\frac{Nm\overline{W}^k r}{Li^2}(a)_{D_f}\left[\left[m' + \frac{km}{\overline{W}}\right]\int_0^{D_f} a^{1-p}dD - \frac{1}{(a^p)_{D_f}}\left[\frac{Lm'' - mL'}{L}\right]\int_0^{D_f} adD\right]}$$

After some mathematical simplification the resulted equation is obtained.

$$K = \frac{\frac{L'}{L}\int_0^{D_f} adD}{\overline{W}^k r\left[\left[\frac{m'}{m} + \frac{k}{\overline{W}}\right]\int_0^{D_f} a^{1-p}dD - \frac{1}{(a^p)_{D_f}}\left[\frac{m'}{m} - \frac{L'}{L}\right]\int_0^{D_f} adD\right]} \qquad \mathbf{(A.69)}$$

If the equations (A.65) and (A.69) are brought together then the final form of the equation for case 3 is obtained.

$$K = \frac{\frac{L'}{L}\int_0^{D_f} adD}{\overline{W}^k r\left[\left[\frac{m'}{m} + \frac{k}{\overline{W}}\right]\int_0^{D_f} a^{1-p}dD - \frac{1}{(a^p)_{D_f}}\left[\frac{m'}{m} - \frac{L'}{L}\right]\int_0^{D_f} adD\right]}$$

$$= \frac{\frac{1}{N}\int_0^{D_f} adD}{\overline{W}^k r\left[\left(\frac{i'}{i} - \frac{r'}{r}\right)\int_0^{D_f} a^{1-p}dD - \frac{1}{(a^p)_{D_f}}\left(\frac{i'}{i} - \frac{1}{N}\right)\int_0^{D_f} adD\right]} \qquad \mathbf{(A.70)}$$

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