

ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL OF SCIENCE
ENGINEERING AND TECHNOLOGY

**PRESSURE AND TEMPERATURE BEHAVIORS OF SINGLE-PHASE
WATER GEOTHERMAL RESERVOIRS UNDER VARIOUS
PRODUCTION/INJECTION SCHEMES**

Ph.D. THESIS

Yıldırar PALABIYIK

Department of Petroleum and Natural Gas Engineering

Petroleum and Natural Gas Engineering Programme

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Thesis Advisor: Prof. Dr. Mustafa ONUR

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BASINÇ VE SICAKLIK DAVRANIŞLARI**

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To my beloved family,

FOREWORD

In recent years, due to an increased interest in renewable energy sources, appropriate evaluation of geothermal energy resources, modeling studies and field management of these resources in Turkey and the world have gained a significant importance. Turkey's geothermal capacity is increasing day by day with new explorations and investments. Although the geothermal energy is considered as one of the renewable and sustainable energy sources, it can readily become an exhaustible source by improper planning, assessment, and management practices. Reservoir simulation and parameter estimation methods are not only useful but also necessary tools for appropriate evaluation of the geothermal energy sources. However, studies regarding evaluation of geothermal reservoirs, particularly the studies focusing on pressure and temperature behavior of these reservoirs are scarce in the literature. In this study, a coupled non-isothermal reservoir-wellbore simulator that is capable of simulating both pressure and temperature behaviors of single-phase liquid-dominated geothermal systems is presented. The developed model and the simulator during the course of this study can be extended to use for history matching, parameter estimation, and prediction purposes in future studies. The study contributes to the geothermal energy literature by providing new approaches and insights to the modeling and behaviors of the pressure and temperature of the geothermal wells and reservoirs.

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TABLE OF CONTENTS

	<u>Page</u>
FOREWORD.....	ix
TABLE OF CONTENTS.....	xi
LIST OF TABLES.....	xv
LIST OF FIGURES.....	xvii
NOMENCLATURE.....	xxv
SUMMARY.....	xxix
ÖZET.....	xxxii
1. INTRODUCTION.....	1
1.1 Purpose of Thesis.....	1
1.2 Literature Review.....	1
1.3 Methodology.....	6
2. EFFECTS OF PRESSURE AND TEMPERATURE ON WATER	
PROPERTIES.....	9
2.1 Effects of Pressure and Temperature on Water Density.....	9
2.2 Effects of Pressure and Temperature on Water Viscosity.....	11
2.3 Effects of Pressure and Temperature on Isothermal Water Compressibility....	11
2.4 Effects of Pressure and Temperature on Thermal Expansion Coefficient of	
Water.....	13
2.5 Effects of Pressure and Temperature on Joule-Thomson Coefficient.....	14
3. DERIVATIONS OF EQUATIONS FOR NON-ISOTHERMAL MODEL.....	17
3.1 Derivation of Model Equations.....	17
3.1.1 Derivation and discretization of mass conservation equation.....	17
3.1.2 Derivation and discretization of energy conservation equation.....	24
3.1.3 Initial and boundary conditions.....	29
3.1.3.1 Pressure conditions.....	29
3.1.3.2 Temperature conditions.....	31
3.1.4 Derivation and discretization of wellbore equations.....	32
3.2 2D (r - z) Grid System.....	34
3.3 Time Step Selection.....	35
3.4 Solution Finite Difference Equations.....	37
3.5 Note on Wellbore and Sandface Pressure and Temperature.....	41
3.6 Sensitivities of the Solutions to the Number of Grid Blocks.....	42
3.6.1 Constant-rate injection case with a finite wellbore approach.....	42
3.6.2 Constant-rate production case for a fully penetrating well.....	44
3.6.3 Constant-rate production case for a limited-entry well.....	51
3.6.4 Constant-rate injection case for a limited-entry well.....	54
4. VERIFICATION OF THE MODEL.....	61
4.1 Verification of the Model with TOUGH2.....	61
4.1.1 Case 1: Constant-rate injection case for a fully penetrating well.....	62
4.1.2 Case 2: Constant-rate production case for a fully penetrating well.....	62
4.2 Case 3: Verification of the Model with an Analytical Solution.....	67

5. PRESSURE AND TEMPERATURE BEHAVIORS.....	71
5.1 Production Case.....	71
5.1.1 Effect of porosity and flow rate on the sandface pressure and temperature.....	72
5.1.2 Effect of permeability on the sandface pressure and temperature.....	76
5.1.3 Effect of rock thermal conductivity on the sandface pressure and temperature.....	79
5.1.4 Effect of rock heat capacity on the sandface pressure and temperature.....	81
5.1.5 Effect of skin on the sandface pressure and temperature.....	83
5.1.6 Effect of variable-rate on the sandface pressure and temperature.....	85
5.1.7 Effect of limited-entry length on the sandface pressure and temperature.....	88
5.1.8 Effect of high pressure and high temperature on the limited-entry well..	91
5.2 Injection Case.....	96
5.2.1 Effect of permeability on pressure and temperature behaviors.....	96
5.2.2 Effect of outer reservoir boundary with constant-pressure on sandface pressure and temperature behaviors.....	101
5.2.3 Effect of rock heat capacity on the sandface pressure and temperature.....	104
5.2.4 Effect of heat transfer to strata.....	108
5.2.4.1 Effect of heat transfer to strata on a thick reservoir.....	108
5.2.4.2 Effect of heat transfer to strata on a thin reservoir.....	110
5.2.5 Effect of a finite wellbore approach on pressure and temperature behaviors.....	113
5.2.6 Effect of gravity on pressure and temperature behaviors.....	117
5.2.7 Effect of rock thermal conductivity and flow rate on the sandface pressure and temperature behaviors.....	121
5.2.8 Effect of porosity and flow rate on the sandface pressure and temperature behaviors.....	123
5.2.9 Effect of skin on the sandface pressure and temperature.....	124
5.2.10 Effect of limited-entry length on the sandface pressure and temperature behaviors.....	126
5.2.11 Effect of different injection temperatures on pressure and temperature behaviors.....	128
5.2.12 Packer-probe application to an anisotropy case.....	132
5.2.13 Constant-pressure injection case.....	145
6. CONCLUSIONS AND RECOMMENDATIONS.....	149
6.1 Conclusions.....	149
6.1.1 Development and verification of the model.....	149
6.1.2 Effect of rate.....	150
6.1.3 Effect of permeability.....	151
6.1.4 Effect of skin.....	151
6.1.5 Effect of porosity.....	151
6.1.6 Effect of rock thermal conductivity.....	152
6.1.7 Effect of constant-pressure outer boundary.....	152
6.1.8 Effect of heat transfer to strata.....	152
6.1.9 Effect of gravity.....	152
6.1.10 Effect of limited-entry length.....	152
6.1.11 Effect of different injection temperature.....	153

6.1.12 Effect of anisotropy.....	153
6.1.13 Effect of skin and permeability at constant-pressure injection.....	153
6.1.14 Sensitivities of rock parameters for the finite wellbore approach.....	153
6.1.15 Sensitivities of rock parameters for the high pressure and high temperature reservoir.....	154
6.1.16 Sensitivities of rock parameters for anisotropic reservoirs.....	154
6.2 Recommendations for Future Works.....	154
6.2.1 Analytical equations/expressions for pressure and temperature at the wellbore, sandface and along the observation points.....	154
6.2.2 2D sensitivity maps.....	155
6.2.3 Automated parameter estimation problem (Inverse problem).....	155
6.2.4 Effects of brine salinity and/or dissolved gas.....	155
6.2.5 Segmented wellbore coupled with the reservoir model.....	156
REFERENCES.....	157
APPENDICES.....	161
Appendix A.....	163
Appendix B.....	183
CURRICULUM VITAE.....	195

LIST OF TABLES

	<u>Page</u>
Table 3.1 : Input data for reservoir systems	43
Table 4.1 : Input data for 2D (r - z) reservoir systems, cases 1 and 2.....	62
Table 4.2 : Input data for 1D (r) reservoir for the analytical case, case 3.....	68
Table 5.1 : Parameters used in modeling skin zone near the wellbore.	83

LIST OF FIGURES

	<u>Page</u>
Figure 2.1 : A comparison of density computed by Equation 2.1 with that computed by COWAT program as a function of pressure at a constant temperature of 353.15 K.....	10
Figure 2.2 : A comparison of density computed by Equation 2.1 with that computed by COWAT program as a function of temperature at a constant pressure of 6894.8 kPa.....	10
Figure 2.3 : Water Viscosity as a function of temperature at constant pressure, computed by TOUGH2 correlation.....	12
Figure 2.4 : Water Viscosity as a function of pressure at constant temperature, computed by TOUGH2 correlation.....	12
Figure 2.5 : Isothermal water compressibility as a function of pressure at constant temperature, computed by the simulator developed.....	13
Figure 2.6 : Isobaric thermal expansion coefficient as a function of temperature at constant temperature, computed by the simulator developed.....	14
Figure 2.7 : Constant enthalpy lines of 2×10^5 and $H = 6 \times 10^5$ J/kg plotted on T/p graph for pure water.....	16
Figure 3.1 : Schematic of a grid block in the system of r - z axis.....	19
Figure 3.2 : 3D view of wellbore/reservoir model with single cell.....	30
Figure 3.3 : 2D (r - z) cross-sectional view of wellbore/reservoir model with single cell.....	30
Figure 3.4 : Schematic of a grid system constructed in r - z axis.....	36
Figure 3.5 : Time step points equally spaced apart on the logarithmic time axis.....	37
Figure 3.6 : Time step points on Cartesian time axis.....	37
Figure 3.7 : Schematic of r - z grid system with 2D coordinate and grid block indexing for an $N_r = 3$ and $N_z = 5$ grid system.....	39
Figure 3.8 : Jacobian matrix for 2D grid system of $N_r = 3$ and $N_z = 5$	41
Figure 3.9 : Comparison of wellbore and sandface pressure behaviors as a function of time for 1D (r) model during constant-rate injection period.....	44
Figure 3.10 : Plot of delta pressure and Bourdet derivative of wellbore and sandface as a function of elapsed time for 1D (r) model during constant-rate injection period.....	45
Figure 3.11 : Comparison of wellbore and sandface temperature behaviors as a function of time for 1D (r) model during constant-rate injection period.....	46
Figure 3.12 : Schematic of the grid block structure with an extra cell in the r -direction.....	46
Figure 3.13 : Comparison of wellbore and sandface temperature behaviors with extra grid as a function of time for 1D (r) model during constant-rate injection period.....	47

Figure 3.14 :	Sensitivity of the sandface pressure solutions to the number of grid blocks in the z -direction for 2D (r - z) model during constant-rate production from fully penetrating well.....	47
Figure 3.15 :	Plot of delta pressure and Bourdet derivative at the sandface in the z -direction for 2D (r - z) model during constant-rate production from fully penetrating well.....	48
Figure 3.16 :	Sensitivity of the sandface pressure solutions to the number of grid blocks in the r -direction for 2D (r - z) model during constant-rate production from fully penetrating well.....	49
Figure 3.17 :	Plot of delta pressure and Bourdet derivative at the sandface in the r -direction during constant-rate production from fully penetrating well.....	49
Figure 3.18 :	Sensitivity of the sandface temperature solutions to the number of grid blocks in the z -direction for 2D (r - z) model during constant-rate production from fully penetrating well.....	50
Figure 3.19 :	Sensitivity of the sandface temperature solutions to the number of grid blocks in the r -direction for 2D (r - z) model during constant-rate production from fully penetrating well.....	50
Figure 3.20 :	Schematic of the grid block structure for the constant-rate production case with limited-entry well.....	51
Figure 3.21 :	Sensitivity of the sandface pressure solutions to the number of grid blocks in the z -direction for 2D (r - z) model with limited-entry well during constant-rate production.....	52
Figure 3.22 :	Plot of delta pressure and Bourdet derivative at the sandface in the z -direction for 2D (r - z) model during constant-rate production from limited-entry well.....	52
Figure 3.23 :	Sensitivity of the sandface pressure solutions to the number of grid blocks in the r -direction for 2D (r - z) model with the limited-entry well during constant-rate production period.....	53
Figure 3.24 :	Plot of delta pressure and Bourdet derivative at the sandface in the r -direction for 2D (r - z) model during constant-rate production from limited-entry well.....	53
Figure 3.25 :	Sensitivity of the sandface temperature solutions to the number of grid blocks in the z -direction for 2D (r - z) model with the limited-entry well during constant-rate production period.....	54
Figure 3.26 :	Sensitivity of the sandface temperature solutions to the number of grid blocks in the r -direction for 2D (r - z) model with the limited-entry well during constant-rate production period.....	55
Figure 3.27 :	Sensitivity of the sandface pressure solutions to the number of grid blocks in the z -direction for 2D (r - z) model with the limited-entry well during constant-rate injection period.....	55
Figure 3.28 :	Plot of delta pressure and Bourdet derivative at the sandface in the z -direction for 2D (r - z) model during constant-rate injection from limited-entry well.....	56
Figure 3.29 :	Sensitivity of the sandface pressure solutions to the number of grid blocks in the r -direction for 2D (r - z) model with the limited-entry well during constant-rate injection period.....	57
Figure 3.30 :	Plot of delta pressure and Bourdet derivative at the sandface in the r -direction for 2D (r - z) model during constant-rate injection from limited-entry well.....	57

Figure 3.31 : Sensitivity of the sandface temperature solutions to the number of grid blocks in the z -direction for 2D (r - z) model with the limited-entry well during constant-rate injection period.....	58
Figure 3.32 : Sensitivity of the sandface temperature solutions to the number of grid blocks in the r -direction for 2D (r - z) model with the limited-entry well during constant-rate injection period.....	58
Figure 4.1 : Comparison of pressure behaviors along the radial distance in reservoir for the model and TOUGH2 during constant-rate injection period, case 1.....	63
Figure 4.2 : Comparison of temperature behaviors along radial distance in reservoir for the model and TOUGH2 during constant-rate injection period, case 1.....	63
Figure 4.3 : Comparison of sandface pressure behaviors for the model and TOUGH2 as a function of time during constant-rate injection period, case 1.....	64
Figure 4.4 : Comparison of sandface temperature behaviors for the model and TOUGH2 as a function of time during constant-rate injection period, case 1.....	64
Figure 4.5 : Plot of delta pressure and Bourdet derivative at the sandface for the model and TOUGH2 as a function of time during constant-rate injection.....	65
Figure 4.6 : Comparison of sandface pressure behaviors for the model and TOUGH2 as a function of time during constant-rate production period, case 2.....	66
Figure 4.7 : Comparison of sandface temperature behaviors for the model and TOUGH2 as a function of time during constant-rate production period, case 2.....	66
Figure 4.8 : Plot of delta pressure and Bourdet derivative at the sandface for the model and TOUGH2 as a function of time during constant-rate production.....	67
Figure 4.9 : Comparison of temperature behaviors for the model and analytical solution at $r = 100$ m as a function of time during constant-rate injection period.....	69
Figure 5.1 : Schematic of a fully penetrating production well.....	72
Figure 5.2 : Effect of reservoir porosity on the sandface pressure drop of a fully penetrating well producing at a constant-rate.....	73
Figure 5.3 : Effect of flow rate on the sandface pressure drop of a fully penetrating well producing at a constant-rate.....	74
Figure 5.4 : Effect of flow rate on the sandface temperature drop of a fully penetrating well producing at a constant-rate.....	74
Figure 5.5 : Effect of flow rate on the sandface temperature of a fully penetrating well producing at a constant-rate.....	75
Figure 5.6 : Effect of reservoir permeability on the sandface pressure drop of a fully penetrating well producing at a constant-rate.....	77
Figure 5.7 : Plots of delta pressure and derivative of delta pressure as a function of elapsed time showing effect of permeability during constant-rate production.....	77
Figure 5.8 : Effect of reservoir permeability on the sandface temperature of a fully penetrating well producing at a constant-rate.....	78

Figure 5.9 : Effect of rock thermal conductivity on the sandface pressure drop of a fully penetrating well producing at a constant-rate.....	79
Figure 5.10 : Effect of rock thermal conductivity on the sandface pressure drop of a fully penetrating well producing at a constant-rate.....	80
Figure 5.11 : Effect of rock thermal conductivity on the sandface temperature of a fully penetrating well producing at a constant-rate.....	80
Figure 5.12 : Effect of rock heat capacity on the sandface pressure drop of a fully penetrating well producing at a constant-rate.....	81
Figure 5.13 : Effect of rock heat capacity on the sandface pressure drop of a fully penetrating well producing at a constant-rate.....	82
Figure 5.14 : Effect of rock heat capacity on the sandface temperature of a fully penetrating well producing at a constant-rate.....	82
Figure 5.15 : Cartesian plot of production–build-up pressure response of a fully penetrating well in the reservoir with different skin factors.....	84
Figure 5.16 : Semi-log plot of production–build-up pressure response of a fully penetrating well in the reservoir with different skin factor.....	84
Figure 5.17 : Cartesian plot of production–build-up temperature response of a fully penetrating well in the reservoir with different skin factors.....	86
Figure 5.18 : Semi-log plot of production–build-up temperature response of a fully penetrating well in the reservoir with different skin factors.....	86
Figure 5.19 : Sandface pressure response of fully penetrating production well at variable-rate.....	87
Figure 5.20 : Sandface temperature response of fully penetrating production well at variable-rate.....	88
Figure 5.21 : Schematic of a limited-entry production well.....	89
Figure 5.22 : Effect of the open interval length on the sandface pressure behaviors during constant-rate production.....	89
Figure 5.23 : Plot of Bourdet derivative as a function of elapsed time for effect of the limited-entry length.....	90
Figure 5.24 : Effect of the open interval length on the sandface temperature behaviors during constant-rate production.....	91
Figure 5.25 : Sandface pressure behavior of a limited-entry well producing at a high constant-rate in a reservoir with high initial pressure ($p^0 = 30000$ kPa) and initial temperature ($T^0 = 453.15$ K).....	93
Figure 5.26 : Plots of delta pressure and Bourdet derivative of delta pressure as a function of elapsed time for effect of the limited-entry length.....	93
Figure 5.27 : Sandface temperature behavior of a limited-entry well producing at a high constant-rate in a reservoir with high initial pressure ($p^0 = 30000$ kPa) and initial temperature ($T^0 = 453.15$ K).....	94
Figure 5.28 : The sensitivities of the sandface pressure to various reservoir parameters for the limited-entry well case.....	95
Figure 5.29 : The sensitivities of the sandface temperature to various rock parameters for the limited-entry production well.....	96
Figure 5.30 : Schematic of a fully penetrating injection well.....	97
Figure 5.31 : Sandface pressure behaviors of a fully penetrating well in the reservoir with different permeability values during constant-rate injection.....	97
Figure 5.32 : Plots of delta pressure and derivative of delta pressure as a function of elapsed time showing effect of permeability during constant-rate injection.....	98

Figure 5.33 :	Pressure behaviors of a fully penetrating well along radial distance in the reservoir with different permeability values as a function of time during constant-rate injection.....	100
Figure 5.34 :	Sandface temperature behaviors of a fully penetrating well in the reservoir with different permeability values during constant-rate injection.....	100
Figure 5.35 :	Temperature behaviors of a fully penetrating well along radial distance in the reservoir with different permeability values as a function of time during constant-rate injection.....	101
Figure 5.36 :	Top view of a huge edge aquifer surrounding the reservoir.....	101
Figure 5.37 :	Plots of delta pressure and derivative of delta pressure as a function of elapsed time showing effect of constant-pressure, outer reservoir boundary condition during constant-rate injection.....	102
Figure 5.38 :	Plots of delta pressure and derivative of delta pressure as a function of elapsed time showing effect of constant-pressure, outer reservoir boundary condition during very long injection period.....	105
Figure 5.39 :	Effect of constant-pressure, outer reservoir boundary on the sandface temperature behaviors of a fully penetrating well in the reservoir with different permeability values during constant-rate injection.....	106
Figure 5.40 :	Sandface pressure behaviors of a fully penetrating well in the reservoir with different values of rock heat capacity during constant-rate injection.....	106
Figure 5.41 :	Plots of delta pressure and derivative of delta pressure as a function of elapsed time showing effect of rock heat capacity during constant-rate injection.....	107
Figure 5.42 :	Sandface temperature behaviors of a fully penetrating well in the reservoir with different values of rock heat capacity during constant-rate injection.....	107
Figure 5.43 :	Plot of temperature vs depth of reservoir, overburden and underburden at the initial conditions.....	108
Figure 5.44 :	Effect of heat transfer from reservoir to adjacent strata on the sandface pressure behavior of a fully penetrating well during constant-rate injection.....	109
Figure 5.45 :	Effect of heat transfer from reservoir to adjacent strata on the sandface temperature behavior of a fully penetrating well during constant-rate injection.....	109
Figure 5.46 :	Plot of temperature vs depth of reservoir, overburden and underburden at the initial conditions for a thin reservoir ($h = 10$ m).....	110
Figure 5.47 :	Effect of heat transfer from reservoir to adjacent strata on the sandface pressure behavior of a fully penetrating well during constant-rate injection for a thin reservoir ($h = 10$ m).....	111
Figure 5.48 :	Effect of heat transfer from reservoir to adjacent strata on the sandface temperature behavior of a fully penetrating well during constant-rate injection for a thin reservoir ($h = 10$ m).....	111
Figure 5.49 :	Schematic of reservoir, overburden and underburden strata for a thin reservoir ($h = 10$ m) with the top, center and bottom layers of the reservoir.....	112

Figure 5.50 : Plots of delta pressure and Bourdet derivative as a function of elapsed time showing effect of heat transfer from reservoir to adjacent strata on the pressure behavior at $r = 20$ m during constant-rate injection for a thin reservoir ($h = 10$ m).....	113
Figure 5.51 : Effect of heat transfer from reservoir to adjacent strata on the temperature behavior at $r = 20$ m for center, top and bottom layers of a fully penetrating well during constant-rate injection for a thin reservoir ($h = 10$ m).....	114
Figure 5.52 : The sensitivities of the porosity to the wellbore and sandface pressures for 1D (r) injection well at a constant-rate.....	115
Figure 5.53 : The sensitivities of the other rock parameters to the wellbore and sandface pressures for 1D (r) injection well at a constant-rate.....	115
Figure 5.54 : The sensitivities of the rock parameters to the wellbore and sandface temperatures for 1D (r) injection well at a constant-rate.....	116
Figure 5.55 : Schematic of the gridblock structure for the constant-rate injection case with gravity effect for limited-entry well.....	117
Figure 5.56 : Sandface pressure behaviors of a limited-entry well in the reservoir with gravity effect during constant-rate (1 kg/s) injection.....	118
Figure 5.57 : Sandface temperature behaviors of a limited-entry well in the reservoir with gravity effect during constant-rate (1 kg/s) injection.....	119
Figure 5.58 : 2D (r - z) temperature distribution map in the reservoir at 1000 days for a limited-entry well without gravity during constant-rate (1 kg/s) injection.....	120
Figure 5.59 : 2D (r - z) temperature distribution map in the reservoir at 1000 days for a limited-entry well with gravity during constant-rate (1 kg/s) injection.....	120
Figure 5.60 : Effect of rock thermal conductivity on the sandface pressure behavior as a function of time with rate effect during injection and fall-off periods.....	122
Figure 5.61 : Effect of rock thermal conductivity on the sandface temperature behavior as a function of time with rate effect during injection and fall-off periods.....	122
Figure 5.62 : Effect of porosity on the sandface pressure behaviors as a function of time during injection and fall-off periods.....	123
Figure 5.63 : Effect of porosity on the sandface temperature behaviors as a function of time during injection and fall-off periods.....	124
Figure 5.64 : Effect of skin factor on the sandface pressure behaviors during injection and fall-off periods.....	125
Figure 5.65 : Effect of skin factor on the sandface temperature behaviors during injection and fall-off periods.....	126
Figure 5.66 : Schematic of limited-entry injection well.....	127
Figure 5.67 : Effect of the open interval length on the sandface pressure behaviors during injection and fall-off periods.....	127
Figure 5.68 : Effect of the open interval length on the sandface temperature behaviors during injection and fall-off periods.....	128
Figure 5.69 : Effect of injection fluid temperature on pressure change and its Bourdet derivative for the injection period for a limited-entry well in an isotropic reservoir.....	129

Figure 5.70 : Effect of injection temperature of injected water on the sandface temperature behavior of a limited-entry well during injection period.....	133
Figure 5.71 : Schematic of a probe configuration of a packer-probe wireline formation tester for a vertical well.....	133
Figure 5.72 : Schematic of the gridblock structure used for the packer-probe application to an anisotropic reservoir.....	134
Figure 5.73 : Probe pressure comparisons of the three different permeability cases during injection and fall-off periods in the Cartesian plot.....	135
Figure 5.74 : Probe pressure comparisons of the three different permeability cases during injection and fall-off periods in the semi-log plot	135
Figure 5.75 : Probe temperature comparisons of the three different permeability cases during injection and fall-off periods in the Cartesian plot.....	136
Figure 5.76 : Probe temperature comparisons of the three different permeability cases during injection and fall-off periods in the semi-log plot	136
Figure 5.77 : 2D (r - z) temperature distribution map for the case with $k_r = 10$ md, $k_z = 100$ md at the end of injection period (10 days).....	138
Figure 5.78 : 2D (r - z) temperature distribution map for the case with $k_r = 10$ md, $k_z = 100$ md at the end of fall-off period (20 days).....	138
Figure 5.79 : 2D (r - z) temperature distribution map for the case with $k_r = 100$ md, $k_z = 100$ md at the end of injection period (10 days).....	139
Figure 5.80 : 2D (r - z) temperature distribution map for the case with $k_r = 100$ md, $k_z = 100$ md at the end of fall-off period (20 days).....	139
Figure 5.81 : 2D (r - z) temperature distribution map for the case with $k_r = 1000$ md, $k_z = 100$ md at the end of injection period (10 days).....	140
Figure 5.82 : 2D (r - z) temperature distribution map for the case with $k_r = 1000$ md, $k_z = 100$ md at the end of fall-off period (20 days).....	140
Figure 5.83 : Sensitivities of the probe pressures to the rock parameters during injection and fall-off for the reservoir with $k_r = 10$ md, $k_z = 100$ md..	141
Figure 5.84 : Sensitivities of the probe pressures to the rock parameters during injection and fall-off for the reservoir with $k_r = 100$ md, $k_z = 100$ md	142
Figure 5.85 : Sensitivities of the probe pressures to the rock parameters during injection and fall-off for the reservoir with $k_r = 1000$ md, $k_z = 100$ md	142
Figure 5.86 : Sensitivities of the probe temperatures to the rock parameters during injection and fall-off for the reservoir with $k_r = 10$ md, $k_z = 100$ md	143
Figure 5.87 : Sensitivities of the probe temperatures to the rock parameters during injection and fall-off for the reservoir with $k_r = 100$ md, $k_z = 100$ md	144
Figure 5.88 : Sensitivities of the probe temperatures to the rock parameters during injection and fall-off for the reservoir with $k_r = 1000$ md, $k_z = 100$ md	145
Figure 5.89 : Effect of reservoir permeability on the sandface temperature of a fully penetrating well producing at a constant-pressure injection.....	146
Figure 5.90 : Effect of skin factor on the sandface temperature of a fully penetrating well producing at a constant-pressure injection.....	147

Figure B.1 : 2D (r - z) schematic of heat flow in porous media containing reservoir, overburden and underburden strata during an injection activity.....	183
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NOMENCLATURE

Capital Letters

A	:Cross-sectional area, m ²
C	:Specific heat capacity, J/kg-K
D	:Distance measured from a certain reference (datum) in the direction of gravity, m
H	:Specific enthalpy, J/kg
J	:Jacobian matrix
M	:Mobility ratio
N	:Total number of grid blocks (cells)
$\tilde{N}_{cyc}$	:Number of log-cycles
$\tilde{N}_{pts}$	:Number of time step points for each log-cycle
$\tilde{N}_{tp}$	:Total number of time step points
R	:Time ratio
S	:Skin factor
$\underline{S}$	:Sensitivity coefficient, Pa or K
T	:Temperature, K
T_1	:Reservoir temperature at any time and location, K
T_{1D}	:Dimensionless reservoir temperature at any time and location
T_2	:Overburden/underburden temperature at any time and location, K
T_{2D}	:Dimesionless overburden/underburden temperature at any time and location
U	:Specific internal energy, J/kg
V	:Volume, m ³

Small Letters

b	:Half of reservoir thickness, m
c	:Compressibility, Pa ⁻¹
f	:Residual vector
g	:Acceleration of gravity, m/s ²
h	:Reservoir thickness, m

h_1	:Top of open interval of limited-entry well, m
h_2	:Bottom of open interval of limited-entry well, m
h_w	:Open interval length to flow, m
$\tilde{k}$	:Absolute permeability, m ²
$\tilde{k}_s$	:Permeability of skin zone, m ²
p	:Pressure, Pa
p_D	:Dimensionless pressure
q	:Flow rate, kg/s or m ³ /s
$\tilde{q}$	:Thermal conduction term based on Fourier's law, J/m ² -s
r	:Radial distance, m
r_D	:Dimensionless radial distance
r_e	:Reservoir radius, m
r_s	:Radius of skin zone, m
r_w	:Well radius, m
s	:Single gridblock index
t	:Time, s
t_b	:Beginning of run time, s
t_D	:Dimensionless time
t_e	:End of run time, s
v	:Velocity, m/s
w	:Primary unknown variable vector
x	:Direction in x-coordinate
y	:Direction in y-coordinate
z	:Vertical distance, m
z_D	:Dimensionless vertical distance

Greek Characters

α	:Geometric factor
β	:Isobaric thermal expansion coefficient, K ⁻¹
γ	:Fluid pressure gradient, Pa/m
ψ	:Primary variable in the sensitivity coefficient formula
Γ	:Transmissibility term related to conduction in energy balance equation
δ	:Difference operator
Δ	:Difference operator
∇	:Gradient operator

θ	:Angular direction
Θ	:Dimensionless variable
κ	:Rock parameter in the sensitivity coefficient formula
ζ	:A factor to compute logarithmic time step
λ	:Thermal conductivity, J/m-s-K
μ	:Viscosity, Pa-s
μ_{JT}	:Joule-Thomson coefficient, K/Pa
ρ	:Density, kg/m ³
ϕ	:Porosity
Ω	:Transmissibility term related to convection in mass balance equation

Subscripts

o	:Reference condition
d	:datum
e	:Outer boundary of reservoir
g	:Time step index
H	:Constant-enthalpy condition
i	:Grid block index for radial direction
inj	:Injection
j	:Grid block index for theta direction
k	:Grid block index for vertical direction
l	:Grid block index related to vertical direction for wellbore
m	:Index for Primary unknown variable vector
n	:Residual Index
N	:Total number of grid blocks (cells)
p	:Constant-pressure condition
pr	:Production
r	:Radial direction
s	:Solid rock phase
sf	:Sandface
t	:Total system
T	:Constant-temperature condition
w	:Water
x	:Index for enthalpy
y	:Index for enthalpy
wf	:Wellbore

z :Vertical direction

Superscripts

a :Convergence exponent

0 :Initial condition

n :Time step

r :Iteration step

T :Temperature

w :Well

Signs

$\bullet$:Divergence operator (Scalar product)

$\rightarrow$:Vector

PRESSURE AND TEMPERATURE BEHAVIORS OF SINGLE-PHASE WATER GEOTHERMAL RESERVOIRS UNDER VARIOUS PRODUCTION/INJECTION SCHEMES

SUMMARY

The integration of production data such as pressure, rate, etc., through history matching has become common place throughout the petroleum and geothermal industries. The investigation of temperature data in addition to pressure for the purpose of reservoir characterization has recently attracted the attention of various researchers. It has been shown that temperature measurements in addition to pressure measurements aid in reservoir characterization. History matching of temperature measurements in geothermal applications requires the usage of a forward model which is capable of simulating the temperature behavior not only inside the reservoir but also inside or at the sandface along the wellbore as temperature measurements can be made inside or at the sandface along the wellbore in a producing or a shut-in well.

In this study, a non-isothermal numerical reservoir simulator (or forward model) that is capable of simulating both pressure and temperature behaviors of single-phase liquid-dominated geothermal systems is presented. The model is based on solving the mass and energy balance equations for the reservoir. Furthermore, the model is also capable of simulating heat losses from the reservoir to the adjacent overburden and underburden strata enabling realistic simulations of temperature to be made in the well. All equations are solved in a fully-implicit manner using the well-known Newton's method for handling the non-linearity. The model is 2D (r - z) cylindrical and hence provides realistic descriptions of sandface pressure and temperature behaviors. Using the model, the transient behavior of especially temperature and various sensitivities of formation and well properties on the pressure and temperature responses are studied.

The example cases considered in this study were used to investigate the effects of various reservoir parameters such as permeability, skin factor, porosity, thermal conductivity and rock heat capacity on pressure and temperature behaviors and the sensitivities of pressure and temperature, especially sandface temperature, to changes in these parameters. The effects of the reservoir parameters on temperature response were quantified through the use of sensitivity coefficients, which are defined as the derivatives of temperature with respect to the natural logarithm of the parameter of interest, were calculated and compared. Various synthetic examples have been designed to investigate the sensitivity of temperature to the aforementioned parameters. Based on the results of these synthetic case studies, it has been found that, depending on the production or injection mode (constant-rate production, or shut-in following production or fall-off following injection), the sandface temperature shows sensitivity to parameters like porosity, permeability, rock thermal conductivity, rock density and rock heat capacity. It has also been found that skin (due to damage or stimulation around the well) and permeability have significant effects on sandface temperature at constant pressure injection case.

These results also guide the possible future works to be conducted on the inverse problem solutions and parameter estimation studies by history matching because the sensitivity coefficients provide information which of the fluid and heat flow parameters can be better determined from temperature measurements.

TEK-FAZ SU İÇEREN JEOTERMAL REZERVUARLARIN ÇEŞİTLİ ÜRETİM/ENJEKSİYON DURUMLARINDAKİ BASINÇ VE SICAKLIK DAVRANIŞLARI

ÖZET

Tarihsel çakıştırmayla üretim verisinin birleşimi petrol ve jeotermal endüstrisinde olağan bir durum haline gelmiştir. Rezervuarların karakterizasyonu için, son zamanlarda basınç verilerine ek olarak sıcaklık verilerinin araştırılması birçok araştırmacının dikkatini çekmektedir. Yapılan araştırmalar göstermektedir ki basınç ölçümlerine ek olarak sıcaklık ölçümleri rezervuar tanımlamasında katkı sağlamaktadır. Jeotermal uygulamalardaki sıcaklık ölçümlerine yapılan tarihsel çakıştırma, sadece rezervuar içerisindeki sıcaklık davranışını değil, aynı zamanda sıcaklık ölçümleri kuyu üretim veya kapama dönemindeyken kuyu içerisinde veya kuyu yüzeyinde yapılabildiğinden, kuyu içi veya kuyu yüzeyindeki sıcaklık davranışını da modelleyebilen ileri bir modelin kullanımını gerektirir.

Bu çalışmada, tek-faz sıvı hakim jeotermal sistemlerin hem basınç hem de sıcaklık davranışlarını modelleyebilen izotermal olmayan sayısal bir rezervuar simülatörü (ileri model) sunulmaktadır. Model, rezervuar için kütle ve enerji denge denklemlerinin birlikte çözme esasına dayanmaktadır. Geliştirilen model, kuyuda yapılacak gerçekçi sıcaklık simülasyonlarını gerçekleştirerek rezervuardan alt ve üst bitişik tabakalara olan ısı kayıplarını da modelleyebilmektedir. Bütün denklemler, doğrusal olmayan koşulların üstesinden gelmek için iyi bilinen Newton yöntemi kullanılarak tamamen kapalı bir halde çözülmektedir. Model, iki boyutlu (r - z) silindiriktir ve bu sebeple, kuyu yüzeyi basınç ve sıcaklık davranışlarının gerçekçi tanımlamalarını sağlamaktadır. Bu modeli kullanarak, özellikle sıcaklığın üzerine birçok formasyon ve kuyu özelliğinin duyarlılığı çalışılmaktadır.

Bu çalışmada gerçekleştirilen örnek uygulamalar, geçirgenlik, zar faktörü, gözeneklilik, ısı iletkenlik ve kayaç ısı kapasitesi gibi çeşitli rezervuar parametrelerinin basınç ve sıcaklık davranışlarına etkileriyle özellikle kuyu yüzeyi sıcaklık davranışlarının bu parametrelerin değişimine olan duyarlılıklarını araştırmak için kullanılmıştır. Rezervuar parametrelerinin sıcaklık davranışı üzerine etkileri, ilgili parametrenin doğal logaritmasına göre sıcaklığın türevi olarak tanımlanan duyarlılık katsayıları vasıtasıyla sayısallaştırılarak hesaplanmış ve karşılaştırılmıştır. Sıcaklığın söz konusu parametrelere olan duyarlılığını araştırmak için çeşitli sentetik örnekler tasarlanmıştır. Bu sentetik örnek çalışmaların sonuçlarına bağlı olarak, üretim veya enjeksiyon durumu (sabit debide üretim veya üretimi izleyen kapama dönemi ile sabit debide enjeksiyon veya enjeksiyonu izleyen düşüm dönemi) için, kuyu yüzeyi sıcaklığının gözeneklilik, geçirgenlik, kayaç ısı iletkenliği, kayaç yoğunluğu ve kayaç ısı kapasitesi gibi parametrelere duyarlılığı olduğu bulunmuştur. Zar faktörü (kuyuya yakın civarda oluşan hasar veya canlandırma nedeniyle) ve geçirgenliğin sabit basınçta enjeksiyon durumu için kuyu yüzeyi sıcaklığına önemli etkisi olduğu gözlemlenmiştir.

Bu sonuçlar, duyarlılık katsayılarının akışkan ve ısı akışı parameterlerinin sıcaklık ölçümlerinden daha iyi belirlenebileceği gerçeği nedeniyle ters problem çözümleri ve tarihsel karşılaştırmayla parametre tahmin çalışmaları ile ilgili olası gelecek çalışmalara da rehberlik etmektedir.

1. INTRODUCTION

Temperature measurements, though routinely recorded in well test applications, are usually ignored in reservoir characterization. However, the investigation of temperature data for the purpose of reservoir characterization has recently attracted the attention of various researchers. Temperature measurements in addition to pressure data have been shown to aid in reservoir characterization. The use of temperature measurements for geothermal reservoir characterization requires a forward model which is capable of simulating the temperature behavior of a geothermal system. The main objective of this study is to further investigate the temperature behavior (at a sandface well and observation points along the wellbore) of single-phase water geothermal systems for the reservoir characterization, especially under constant and variable-rate production and injection scenarios by a non-isothermal single-phase simulator developed during the course of this study.

1.1 Purpose of Thesis

The purpose of this study is to develop a 2D (r - z) non-isothermal radial flow simulator that is capable of predicting both pressure and temperature behaviors of liquid-dominated geothermal reservoirs containing single-phase water and then perform sensitivity study with respect to various petrophysical properties (such as porosity, permeability, skin, thermal conductivity and rock heat capacity) for investigating the information content of the wellbore pressure and temperature responses. Such a sensitivity study may reveal which petrophysical parameters can be reliably estimated from temperature and pressure data in cases where parameter estimation methods are employed for history matching.

1.2 Literature Review

In the literature, temperature and pressure behaviors of geothermal reservoirs subject to production of hot geothermal water or injection of cold water are usually predicted by using two different models; distributed models and lumped-parameter models. The

lumped-parameter models, assuming isothermal flow behavior, have been proposed by Grant et al., (1982), Axelsson, (1989), Alkan and Satman, (1990), Sarak et al., (2005) and Tureyen et al., (2007). Onur et al. (2008) have proposed a non-isothermal lumped-parameter model which enables one to predict both pressure and temperature behaviors of a liquid-dominated geothermal reservoir which is idealized as a single-closed or recharged tank. Onur et al. (2008) have shown that if both average reservoir temperature and average reservoir pressure data are used together in history-matching, one could determine reservoir parameters such as reservoir bulk volume and porosity. Then, Tureyen et al. (2009) have extended the model proposed by Onur et al. (2008) to the reservoir systems that can be represented by multiple tanks. However, since lumped-parameter modeling was used in these studies, transient behavior of pressure and temperature along with sensitivities of formation and well properties on the pressure and temperature responses were not investigated.

Although the lumped-parameter models are efficient models in view of practicability and time, distributed analytical, semi-analytical, and numerical models take into account spatial variations in addition to temporal changes of pressure/temperature. For instance, it is possible to model properties like heterogeneities, different reservoir geometries, complex well geometries and multiphase flow with the numerical models. In petroleum and geothermal engineering literature, there are various studies solving the equations based on mass and heat flow balances that take into account spatial variations of rock/fluid properties in reservoir.

The initial studies were performed in 1950s, 1960s and 1970s. Some important ones that may be related to the scope of this study have been made by Lauwerier (1955), Ramey (1962), Whiting and Ramey (1969), Atkinson and Ramey (1977), Lippmann et al. (1977), Coats (1977), Crookston et al. (1977) and Toronyi and Farouq Ali (1977). Atkinson and Ramey (1977) presented the analytical solutions for several 1D problems to predict temperature behavior in the reservoir under various simplifying assumptions (e.g. rock/fluid properties independent of pressure and temperature, uniform fluid velocity inside the reservoir, etc.). In other studies, usually, emphasis was put on solving terms with fully or partially implicit 3D numerical models containing non-linear mass and energy balance equations with source/sink. In these studies, pressure and temperature distributions in reservoir were mainly investigated for production and injection cases. Furthermore, in these studies, emphasis was not given to a more

realistic modeling of 2D (r - z) model and to directly handle the flow towards the wellbore more realistically.

The first study that considered pressure and temperature behaviors in wellbore that produced single-phase geothermal water was made by Miller (1980). In this study, pressure and temperature changes in wellbore were calculated by using mass, energy and momentum balance equations. However, in that study, flow in reservoir was assumed to be isothermal, and only mass balance equation was taken into consideration. Non-linear equation sets were solved numerically by using Newton's method. It is important to note that in this thesis, non-isothermal flow is taken into account in the reservoir.

In general, the models based on analytical and semi-analytical solutions, under more restricted assumptions, approximately calculate pressure and temperature behaviors in wellbore and reservoir in comparison with numerical models. For instance, most of the analytical and semi-analytical models predicting pressure and temperature behaviors near the wellbore ignore conductive heat transfer in reservoir in comparison with convective heat transfer. These models consider that flow and thermal parameters (such as fluid and rock densities, viscosity, fluid and rock heat capacities, fluid adiabatic coefficient, isobaric thermal expansion coefficient, Joule-Thomson coefficient) related to rock and fluid are independent of pressure and temperature. Furthermore, these solutions consider only the cases with constant rate production from reservoir or constant rate injection into the reservoir. Under these assumptions, it is analytically possible to solve mass (pressure for single-phase flow) balance equation independent of heat flow equation. The latest studies focusing on the analytical and semi-analytical solutions on this topic have been presented by Ramazanov et al. (2010), Duru and Horne (2010a), and Duru and Horne (2011). Ramazanov et al. (2010) used the method of characteristics to calculate temperature behavior, whereas Duru and Horne (2010a, 2011) modeled pressure and temperature behaviors by using the method of operator-splitting and time stepping. Both solutions are semi-analytical because they require time stepping to evaluate the solutions. The important advantage of the method presented by Duru and Horne (2010a, 2011) in comparison with the model of Ramazanov et al. (2010) is that Duru and Horne (2010a, 2011) consider both effects of the conductive and convective heat transfers in reservoir, whereas Ramazanov et al. (2010) considered only the convective heat

transfer in the reservoir. In these studies, it has been claimed that reservoir permeability and porosity along with skin zone can be estimated from temperature measurements. Furthermore, Duru and Horne (2010a) have shown that production rate data from temperature data could be predictable. However, as Ramazanov et al. (2010) have also mentioned, the conclusions above can only be valid for the cases where the wellbore storage effects could be negligible and flow rates are very high. Therefore, they have recommended that the more general numerical solutions should be used to obtain the more accurate temperature solutions. In another study, Muradov and Davies (2011) have proposed a temperature model for a horizontal well-producing a low compressibility fluid. The asymptotic, analytical solution for early times has been derived, and the solution's derivation process involved solving three separate sub-problems: (1) the temperature change during the transient fluid flow period, (2) the Joule-Thomson effect driven temperature change, and (3) the temperature distribution disturbance due to the transient fluid expansion process.

As mentioned before, in this study, pressure and temperature behaviors of single-phase water geothermal reservoirs are studied by developing a 2D (r - z) numerical model removing the limitations in analytical and numerical models mentioned above. Studies of App (2008) and Sui et al. (2008a, b) can be given as the examples of numerical models considered in the literature regarding the topic of this study. App (2008) has developed a transient, 1D radial model coupling the conservation of mass and energy equations for predicting the pressure and temperature behaviors of a system that has the components oil, connate water and rock. In that study, the pressure and temperature behaviors under non-isothermal conditions in the reservoir due to Joule-Thomson expansion of reservoir fluids have been presented. Sui et al. (2008a) developed a 2D (r - z) radial simulator to study temperature behavior for the case of single-phase liquid flow in 2D stratified systems. In the study, the improved energy balance equation has been formulated in a general way to contain the effect like thermal expansion, while the temporal and spatial variations of pressure required in energy balance equation have been calculated from the mass (pressure) balance equation developed under the assumptions of isothermal flow and slightly compressible fluid. This assumption is essentially the one used in the semi-analytical solutions of Ramazanov et al. (2010) and Duru and Horne (2010a). Hence, these solutions cannot rigorously model the pressure and temperature behaviors under non-isothermal conditions. The most

important finding of Sui et al. (2008a) was that in stratified systems, the wellbore temperature is sensitive to the radius and permeability of damage zone near the wellbore. In their second study, Sui et al. (2008b) presented an algorithm for an inverse solution formulated as a non-linear least-squares regression problem to estimate permeability (region outside damage zone), porosity, radius and permeability of damage zone by history matching observed temperature and pressure data. Sui et al. (2008b) reported that the related parameters could reliably be estimated by history matching temperature data if noise in temperature measurements is not very high.

Another study regarding the information content of transient temperature data has been conducted by Duru and Horne (2010b). In this study, inverse solutions of permeability and porosity distributions in reservoir were investigated by history matching to temperature data with the method of Ensemble Kalman Filter (EnKF) by using a forward model based on coupled numerical solution of mass and energy balance equations for a 3D (x - y - z) system. The main conclusion of the study was that temperature data contain more information about porosity distribution than that of permeability distribution. In another study, App and Yoshioka (2011) have evaluated the impact of reservoir permeability on sandface temperatures using a dimensionless analysis. App and Yoshioka (2011) have claimed that at low permeability, the sandface temperature change is small under high drawdown conditions while the sandface temperature changes become larger at high permeability.

As mentioned, the main objective of this study is to investigate the behaviors and sensitivities of the pressure and temperature at the production/injection locations at the wellbore, sandface, and observation points along the wellbore for a single-phase liquid geothermal reservoir. Based on this investigation, the specific contributions of this study to the science of petroleum and geothermal engineering can be stated as follows:

- A new 2D (r - z) fully implicit numerical model is presented by using more general forms of mass and energy balance equations derived during the course of this study.
- The mass and energy balance equations presented in this study are original and based on implicit forms of pressure and temperature formulations providing realistic solutions to the problem of interest.

- New insights about the temperature behavior of a geothermal well represented by measurements at the sandface and observation points along the wellbore are presented for various different modes of production (constant-rate/constant-pressure production/injection including build-up and fall-off periods) and of outer reservoir boundary conditions (no-flow and recharge boundaries). To the best of our knowledge, such behaviors have not presented and discussed elsewhere before and are additional original contributions of this study.
- Sensitivity investigations in terms of sensitivity coefficients of wellbore or sandface pressure and especially temperature data to various reservoir parameters are performed and performed for non-isothermal flow conditions, various outer reservoir boundary conditions as well as for various modes of production; constant-rate or constant-pressure production/injection/build-up/fall-off periods.
- The sensitivity coefficients of temperature data to various reservoir parameters are presented. Such coefficients help one to understand the information content of the temperature in the sense that which of the parameters could be more reliably determined by history matching of temperature and/or pressure data.

1.3 Methodology

Numerical models split the reservoir domain into many cells and apply the mass and/or energy conservation laws on each cell. The resulting equations are then solved using numerical techniques such as the finite-difference methods.

In this study, a numerical simulation approach has been taken first. A two-dimensional (r - z) numerical model that rigorously accounts for mass, semi-empirical momentum of Darcy's equation, and energy balances to include convection and conduction heat losses to the adjacent strata is developed by using a finite-difference method. The model simulates pressure and temperature transients resulting from production of hot water and/or reinjection of low temperature (or "cold") water into reservoir and enables to handle variable production and injection rate histories. The non-linear mass and energy balance equations are solved by using the well-known Newton's method in a fully implicit manner. The results of the model are compared and verified by using the results of the well-known geothermal reservoir simulator TOUGH2 developed by

Pruess et al. (1999) and an analytical solution developed by Lauwerier (1955) and modified by Satman (2003).

Then, several synthetic test cases considering various modes of production/injection as well as different outer reservoir boundary conditions in the r and z -directions have been designed to investigate the behaviors of pressure and temperature and their sensitivities to formation and well properties. by using the developed numerical simulator. Sensitivities of temperature to various reservoir/well parameters for for are investigated for both homogeneous and heterogeneous reservoir systems to understand the information content of the temperature and pressure data under non-isothermal conditions. Only the synthetic test cases have been considered to demonstrate the methodology proposed in this study because real field data are currently not available.

2. EFFECTS OF PRESSURE AND TEMPERATURE ON WATER PROPERTIES

The effects of pressure and temperature on the water parameters such as density, viscosity, isothermal compressibility, isobaric thermal expansion coefficient and Joule-Thomson coefficient are investigated in this section.

2.1 Effects of Pressure and Temperature on Water Density

Water density can simply be calculated with the help of the following approximate Equation 2.1 by assuming constant (i.e, not changing significantly with pressure and temperature) isothermal water compressibility (c_w) and isobaric thermal expansion coefficient (β_w):

$$\rho_w = \rho_w(p_0, T_0) [1 + c_w(p - p_0) - \beta_w(T - T_0)] \quad (2.1)$$

In the above equation, ρ_w , p_0 and T_0 represent water density, reference pressure and reference temperature, respectively. However, in the investigation performed in this study, it is shown that values of water density obtained from Equation 2.1 could dramatically differ from values of water density obtained from the steam tables (IFC, 1967 and SME, 2006), especially for the cases where temperature significantly changes at constant pressure.

Figure 2.1 shows a comparison of water density computed by Equation 2.1 with that computed by COWAT subroutine written by O'Sullivan (1990) as a function of pressure with pressure at a constant temperature of 353.15 K. The subroutine COWAT is also used by the well-known geothermal simulator TOUGH2. As can be seen from Figure 2.1, the density computed by Equation 2.1 perfectly matches to the density computed by COWAT program. Figure 2.2 shows a comparison of density computed by Equation 2.1 with that computed by COWAT program as a function of temperature at a constant pressure of 6894.8 kPa.

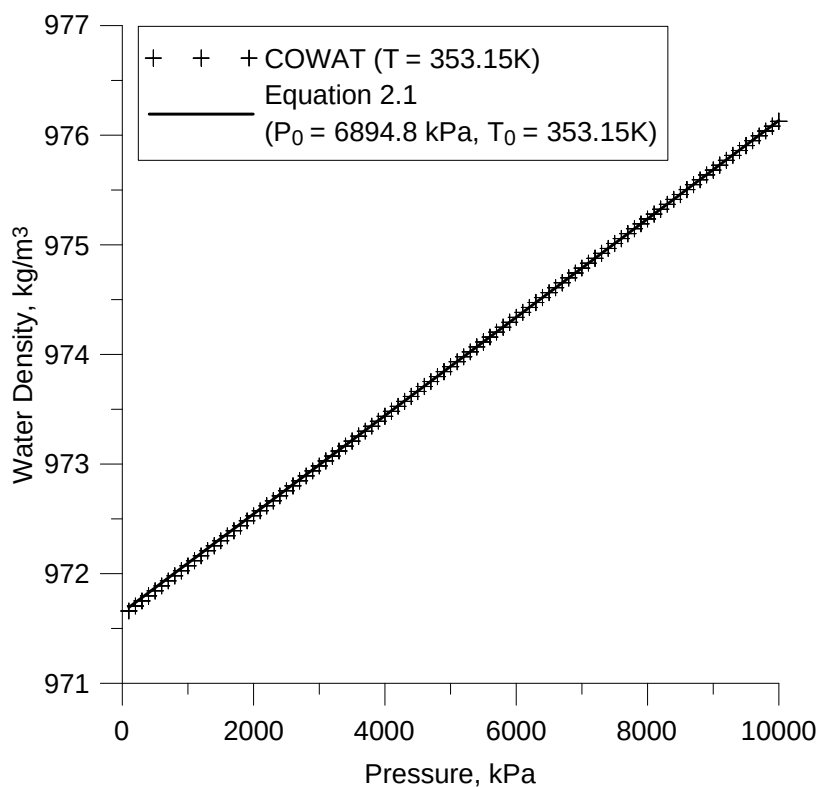


Figure 2.1: A comparison of density computed by Equation 2.1 with that computed by COWAT program as a function of pressure at a constant temperature of 353.15 K.

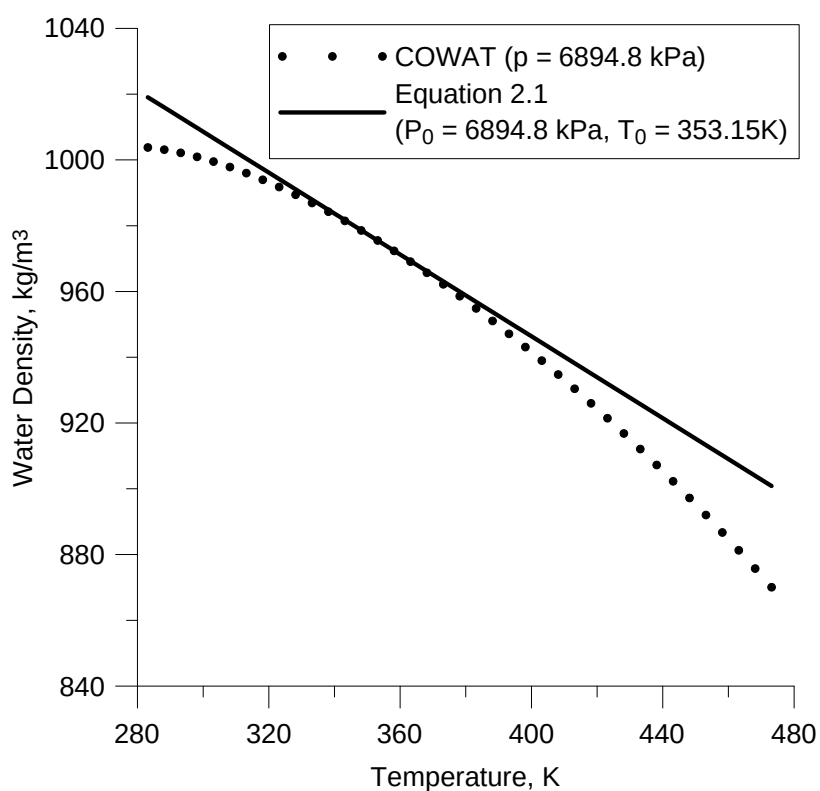


Figure 2.2: A comparison of density computed by Equation 2.1 with that computed by COWAT program as a function of temperature at a constant pressure of 6894.8 kPa.

As is seen from Figure 2.2, the two behaviors are significantly different from each other except at and around the reference temperature T_0 where Equation 2.1 is approximated. The reason for the significant changes observed in density with temperature at a constant pressure is that c_w and β_w values are strongly affected by the temperature change unlike the pressure change. Therefore, in this study, density is calculated by using COWAT program prepared by making use of steam tables instead of Equation 2.1.

2.2 Effects of Pressure and Temperature on Water Viscosity

Here, the change of water viscosity with pressure and temperature is investigated. The viscosity routine in the TOUGH2 simulator is used for this investigation. Figure 2.3 shows water viscosity computed by TOUGH2 correlation as a function of temperature at constant pressure. According to this figure, it can be seen that water viscosity slightly changes with pressure at constant temperature. Figure 2.4 shows water viscosity computed by TOUGH2 correlation as a function of pressure at constant temperature. In Figure 2.4, y-axis on the left hand side represents the viscosity scale at 423.15K while y-axis on the right hand side represents the viscosity scale at 323.15K. As can be seen from this figure, water viscosity significantly changes with temperature at constant pressure. Furthermore, it can be seen from the figures that, as expected, viscosity exponentially decreases with increasing temperature at constant pressure while it linearly increases with increasing pressure at constant temperature. Consequently, it can be stated that it is a quite realistic to treat viscosity of water as a constant independent of pressure.

2.3 Effects of Pressure and Temperature on Isothermal Water Compressibility

In Equation 2.2, c_w represents isothermal water compressibility.

$$c_w = \frac{1}{\rho_w} \left(\frac{\partial \rho_w}{\partial p} \right)_T \quad (2.2)$$

In the simulator developed in this study, Equation 2.2 is used to compute the values of isothermal water compressibility. Here, the effects of pressure and temperature on water compressibility are investigated.

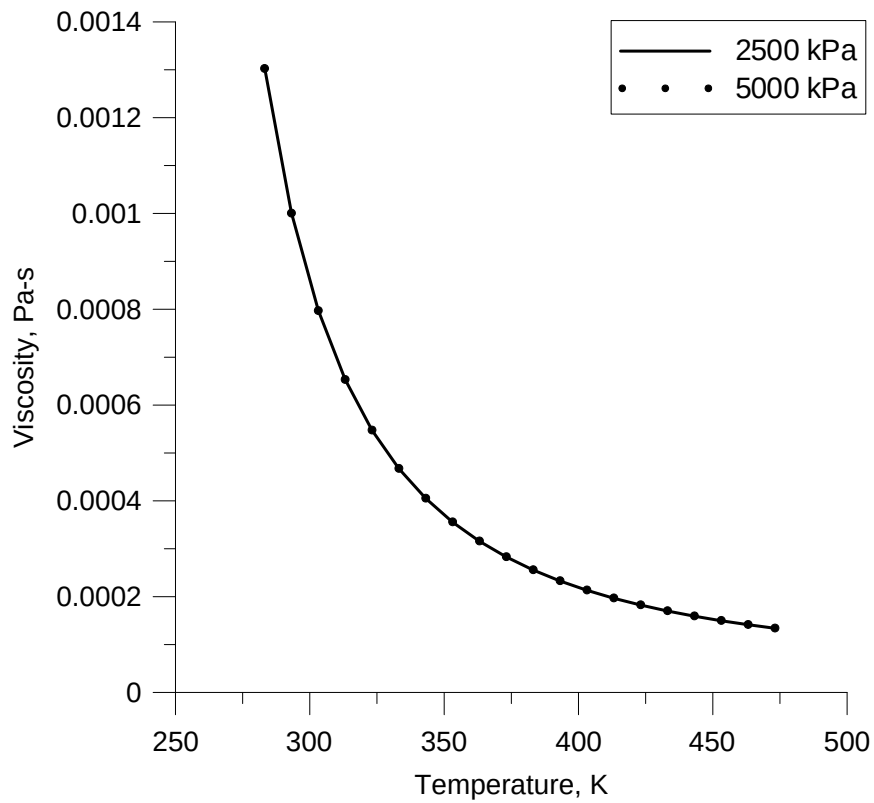


Figure 2.3: Water viscosity as a function of temperature at constant pressure, computed by TOUGH2 correlation.

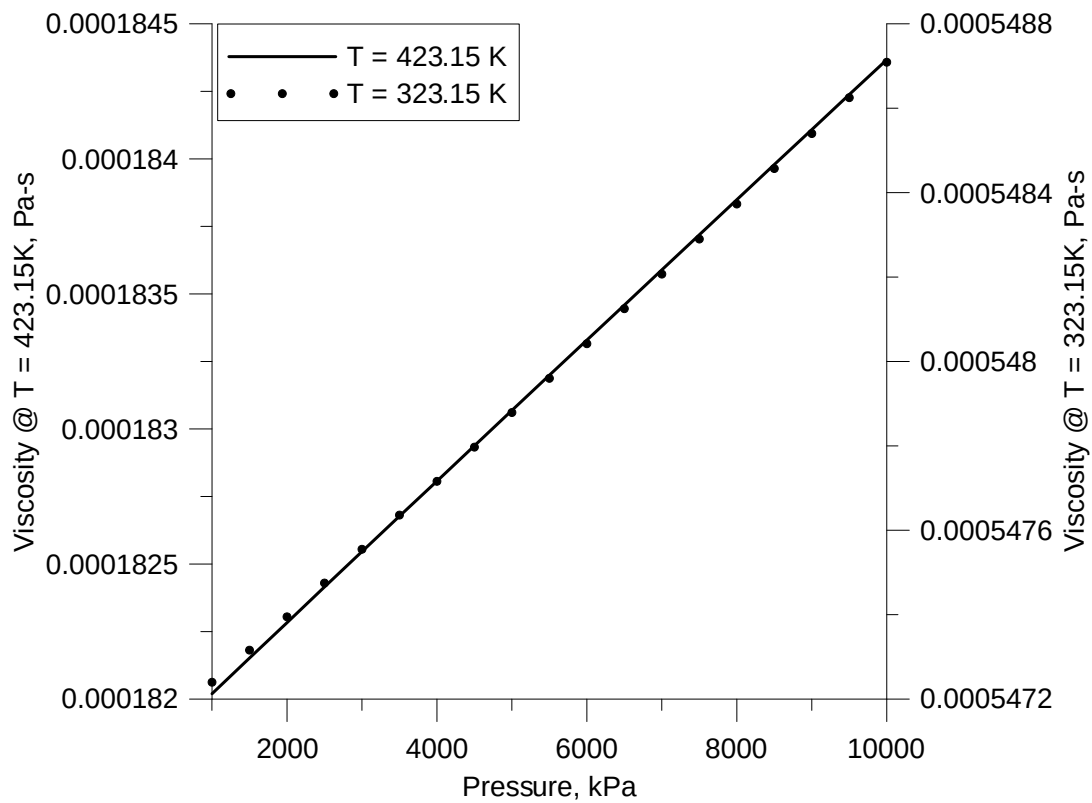


Figure 2.4: Water viscosity as a function of pressure at constant temperature, computed by TOUGH2 correlation.

The values of water density are computed by using COWAT program. Figure 2.5 shows the change of isothermal water compressibility computed by the program as a function of pressure. From Figure 2.5, it can be stated that the isothermal water compressibility slightly changes with pressure. It can be seen that these changes are linear.

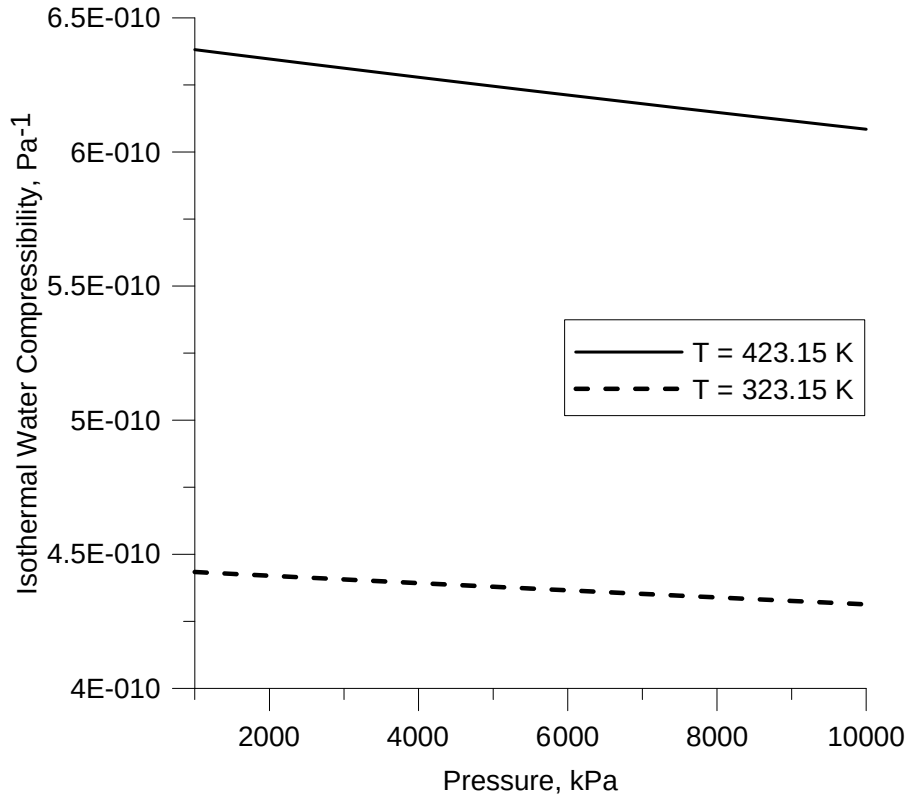


Figure 2.5: Isothermal water compressibility as a function of pressure at constant temperature, computed by the simulator developed.

2.4 Effects of Pressure and Temperature on Thermal Expansion Coefficient of Water

In Equation 2.3, β_w represents the isobaric thermal expansion coefficient of water.

$$\beta_w = -\frac{1}{\rho_w} \left(\frac{\partial \rho_w}{\partial T} \right)_p \quad (2.3)$$

In the simulator developed in this study, Equation 2.3 is used to compute the values of the isobaric thermal expansion coefficient. Here, the effects of pressure and temperature on isobaric thermal expansion coefficient are investigated. Values of water density are computed by using COWAT program.

Figure 2.6 shows isobaric thermal expansion coefficient computed as a function of temperature by the simulator. From Figure 2.6, it can be observed that the isobaric thermal expansion coefficient significantly increases with increasing temperature. It can also be seen from Figure 2.6 that this behavior exhibits some deviations from linearity especially at low pressure condition.

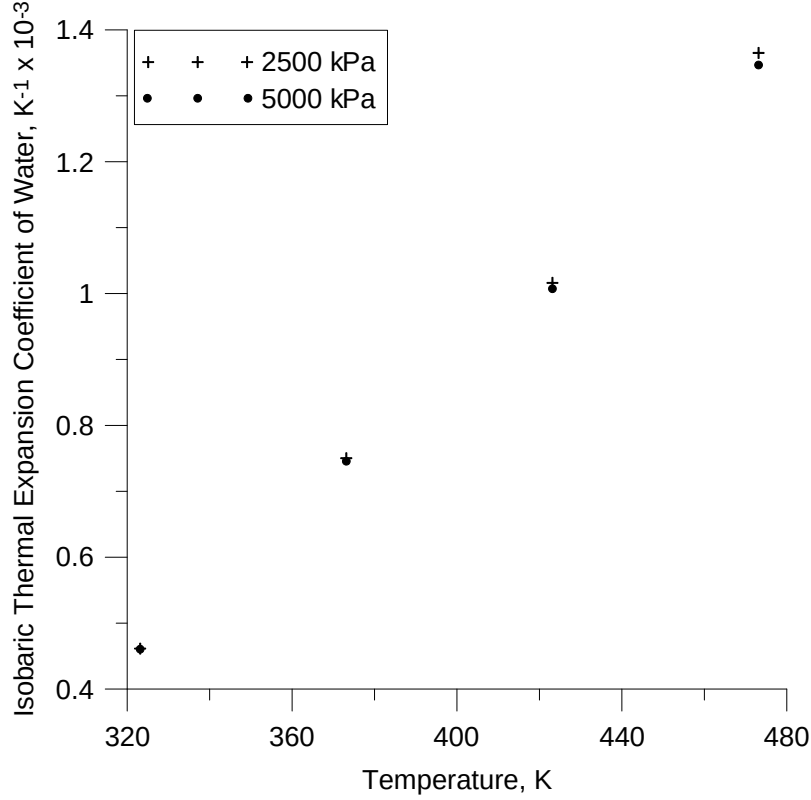


Figure 2.6: Isobaric thermal expansion coefficient as a function of temperature at constant temperature, computed by the simulator developed.

2.5 Effects of Pressure and Temperature on Joule-Thomson Coefficient

By definition, Joule-Thomson coefficient of a fluid is defined as the change in temperature for a given change in pressure during an isenthalpic, or constant enthalpy process (Moore, 1972):

$$\mu_{JT} = \left(\frac{\partial T}{\partial p} \right)_H \quad (2.4)$$

As can be seen from Equation 2.4, the slope of a plot of pressure (on x-axis) versus temperature (on y-axis) at a constant enthalpy gives the Joule-Thomson coefficient. Katz (1972) uses the term “frictional heating” to describe temperature increases from Joule-Thomson expansion of fluid. Friction creates a pressure drop that results in an

increase in temperature due to Joule-Thomson expansion of the fluid; flow in a pipeline is such an isenthalpic process. However, as to be noted later, flow in a geothermal reservoir is in general not an isenthalpic process, but it is combination of various processes that can result in enthalpy changes in the reservoir.

Figure 2.7 shows the lines of constant enthalpy values at 2×10^5 and 6×10^5 J/kg plotted on a T/p graph. This figure was constructed by using COWAT program and a code written for this part of the study. In Figure 2.7, y-axis on the left hand side represents the temperature scale for $H = 6 \times 10^5$ J/kg while y-axis on the right hand side represents the temperature scale for $H = 2 \times 10^5$ J/kg. From Figure 2.7, it can be seen that (i) T decreases almost linearly with p at constant enthalpy for water and hence, T can be well approximated by a straight line as a function of p at constant enthalpy for water; (ii) the absolute values of the slopes (which are equal to the Joule-Thomson coefficients) of the straight lines decrease with increasing enthalpy. As a result of item (i), the temperature increases while the pressure decreases because the slopes of the equations (Joule-Thomson coefficients) are negative. Hence, this trend indicates heating in geothermal water. On the other hand, a decrease in the absolute value of Joule-Thomson coefficient of water with increasing enthalpy indicates that degree of heating in geothermal water also slightly decreases. For instance, from Figure 2.7, for a pressure drop of 10000 kPa at the constant enthalpies of $H = 2 \times 10^5$ and 6×10^5 J/kg, increases in temperature will be about 1.6 K (or 1.6°C), and 1.5 K (or 1.5°C), respectively. These numbers indicate that heating (or equivalently increase in temperature due to decrease in pressure; i.e., during production) or cooling (or equivalently decrease in temperature due to increase in pressure, i.e., during injection) in a single-phase water geothermal reservoir due to Joule-Thomson may not be so significant.

As a final remark, it is worth noting that flow in a geothermal reservoir is not an isenthalpic process. Enthalpy is lost or gained in a reservoir through heat transfer to the cap and base strata and from the enthalpy related to the mass from production or injection volumes, as to be discussed in the next chapter.

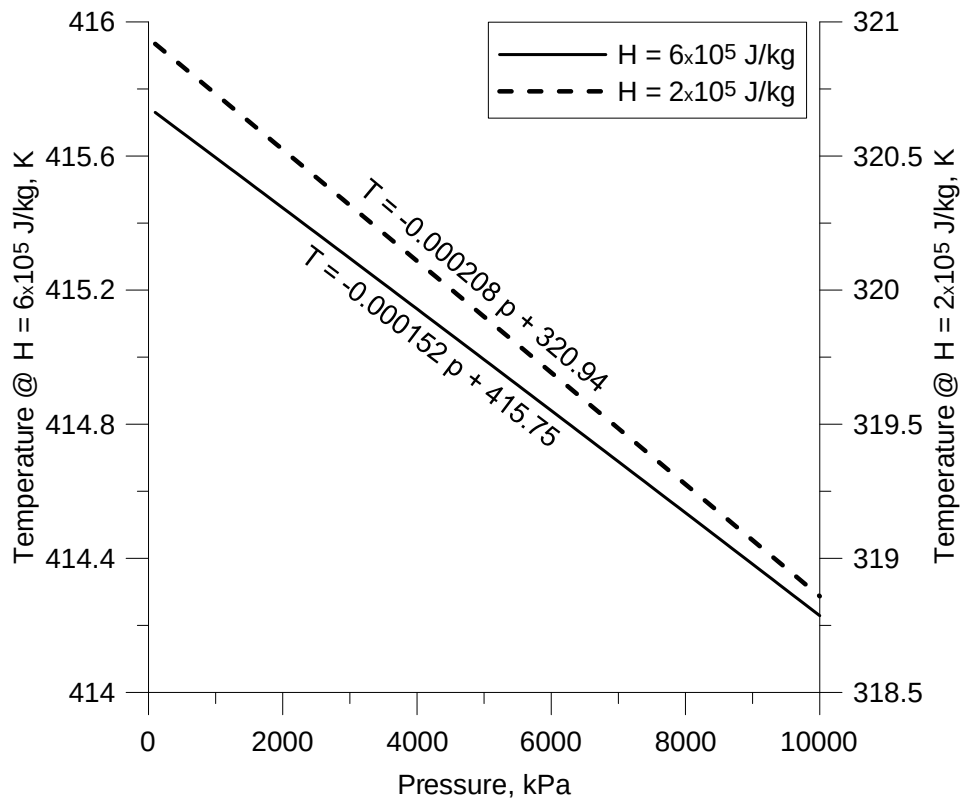


Figure 2.7: Constant enthalpy lines of $H = 2 \times 10^5$ and $H = 6 \times 10^5$ J/kg plotted on T/p graph for pure water.

3. DERIVATIONS OF EQUATIONS FOR NON-ISOTHERMAL MODEL

In this section, derivations of the mass and energy balance equations constituting the basis of the non-isothermal model developed in this study are presented. These equations are given for both the reservoir and the wellbore.

In this study, non-isothermal flow of single-phase water in a geothermal reservoir is considered. Such flow can be modeled by considering only two phases; mobile water and stationary solid rock and solving both mass and energy balances on each of these phases. It should be pointed out that all derivations to be given below consider the international system of unit (SI).

3.1 Derivation of Model Equations

In this subsection, from starting the generalized conservation equations, derivation and discretization of mass and energy balances for 2D (r - z) reservoir system are given. The detailed derivations are given in Appendix A.

3.1.1 Derivation and discretization of mass conservation equation

As mentioned previously, single-phase non-isothermal flow of water is considered in the porous media. The generalized continuity equation for mass conservation of water phase in the r - θ - z coordinates is given by:

$$\frac{\partial(\phi\rho_w)}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r}(\rho_w r v_r) + \frac{1}{r} \frac{\partial}{\partial \theta}(\rho_w v_\theta) + \frac{\partial}{\partial z}(\rho_w v_z) = 0 \quad (3.1)$$

Since the terms in the θ -direction are ignored,

$$\frac{\partial(\phi\rho_w)}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r}(\rho_w r v_r) + \frac{\partial}{\partial z}(\rho_w v_z) = 0 \quad (3.2)$$

In Equation 3.2, ρ_w represents water density and ϕ represents porosity of the reservoir. It is important to note that the derivation of Equation 3.2 considers that the z -direction increases from top to bottom (Figure 3.1).

Darcy's law is used for the velocity terms in Equation 3.2 in r and z -directions as follows:

$$v_r = -\frac{k_r}{\mu_w} \frac{\partial p}{\partial r} \quad (3.3)$$

$$v_z = -\frac{k_z}{\mu_w} \left(\frac{\partial p}{\partial z} - \gamma_w \frac{\partial D}{\partial z} \right) \quad (3.4)$$

In Equations 3.3 and 3.4, k_r and k_z represent permeabilities in r and z -directions, respectively. In Equation 3.4, D represents distance measured as positive from a certain reference (datum) in the direction of gravity and $\gamma_w (= \rho_w g)$ represents specific gravity of water.

Using Darcy's law in Equation 3.2 gives:

$$\frac{\partial(\rho_w \phi)}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[\rho_w r \frac{k_r}{\mu_w} \frac{\partial p}{\partial r} \right] - \frac{\partial}{\partial z} \left[\rho_w \frac{k_z}{\mu_w} \left(\frac{\partial p}{\partial z} - \gamma_w \frac{\partial D}{\partial z} \right) \right] \quad (3.5)$$

The following discretization is used for the left hand side of Equation 3.5:

$$\frac{\partial(\rho_w \phi)}{\partial t} = \frac{(\rho_w \phi)_{i,k}^{n+1} - (\rho_w \phi)_{i,k}^n}{\Delta t} \quad (3.6)$$

In the above equation, the superscript n refers to the present time and $n+1$ refers to the time at which the solutions will be determined. Δt represents the time from n to $n+1$. The subscript i represents the reservoir gridblock index in the r -direction and the subscript k represents the reservoir gridblock index in the z -direction.

In this method, the standard central finite-difference method is used for the discretization of the conservation equations. The following gridblock representation can be considered for this approach (In Figure 3.1, (r_i, θ_j, z_k) is the geometric center of the i, j, k gridblock):

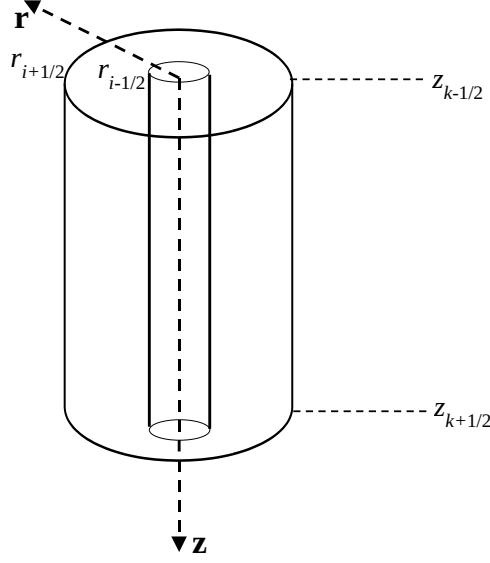


Figure 3.1: Schematic of a gridblock in the system of r - z axis.

Now, if the first term on right hand side of Equation 3.5 is considered,

$$\left\{ \frac{1}{r} \frac{\partial}{\partial r} \left[\rho_w r \frac{k_r}{\mu_w} \frac{\partial p}{\partial r} \right] \right\}_{r=r_i, z=z_k} = \frac{1}{r_{i,k}} \frac{\left(r \rho_w \frac{k_r}{\mu_w} \right)_{i+\frac{1}{2},k} \left(\frac{\partial p}{\partial r} \right)_{i+\frac{1}{2},k} - \left(r \rho_w \frac{k_r}{\mu_w} \right)_{i-\frac{1}{2},k} \left(\frac{\partial p}{\partial r} \right)_{i-\frac{1}{2},k}}{r_{i+\frac{1}{2},k} - r_{i-\frac{1}{2},k}} \quad (3.7)$$

Similarly, the second term on the right hand side of Equation 3.5 is considered as:

$$\left\{ \frac{\partial}{\partial z} \left[\rho_w \frac{k_z}{\mu_w} \left(\frac{\partial p}{\partial z} - \gamma_w \frac{\partial D}{\partial z} \right) \right] \right\}_{r=r_i, z=z_k} = \frac{\left(\rho_w \frac{k_z}{\mu_w} \right)_{i,k+\frac{1}{2}} \left(\frac{\partial p}{\partial z} - \gamma_w \frac{\partial D}{\partial z} \right)_{i,k+\frac{1}{2}} - \left(\rho_w \frac{k_z}{\mu_w} \right)_{i,k-\frac{1}{2}} \left(\frac{\partial p}{\partial z} - \gamma_w \frac{\partial D}{\partial z} \right)_{i,k-\frac{1}{2}}}{z_{i,k+\frac{1}{2}} - z_{i,k-\frac{1}{2}}} \quad (3.8)$$

Now, if the spatial derivatives of pressure at the grid block boundaries are considered, the following difference approximations are used:

$$\left(\frac{\partial p}{\partial r} \right)_{i+\frac{1}{2},k} = \frac{p_{i+1,k} - p_{i,k}}{r_{i+1,k} - r_{i,k}} \quad (3.9)$$

$$\left(\frac{\partial p}{\partial r}\right)_{i-\frac{1}{2},k} = \frac{p_{i,k} - p_{i-1,k}}{r_{i,k} - r_{i-1,k}} \quad (3.10)$$

$$\left(\frac{\partial p}{\partial z} - \gamma_w \frac{\partial D}{\partial z}\right)_{i,k+\frac{1}{2}} = \frac{p_{i,k+1} - p_{i,k}}{z_{i,k+1} - z_{i,k}} - \gamma_{w,i,k+\frac{1}{2}} \frac{z_{i,k+1} - z_{i,k}}{z_{i,k+1} - z_{i,k}} \quad (3.11)$$

$$\left(\frac{\partial p}{\partial z} - \gamma_w \frac{\partial D}{\partial z}\right)_{i,k+\frac{1}{2}} = \frac{p_{i,k} - p_{i,k-1}}{z_{i,k} - z_{i,k-1}} - \gamma_{w,i,k-\frac{1}{2}} \frac{z_{i,k} - z_{i,k-1}}{z_{i,k} - z_{i,k-1}} \quad (3.12)$$

Using Equations 3.6 - 3.12 in Equation 3.5 gives (Details of the derivations can be found in Appendix A):

$$\begin{aligned} & \frac{(\rho_w \phi)_{i,k}^{n+1} - (\rho_w \phi)_{i,k}^n}{\Delta t} \\ &= \frac{1}{r_{i,k}} \frac{1}{\left(r_{i+\frac{1}{2},k} - r_{i-\frac{1}{2},k}\right)} \left(r \rho_w \frac{k_r}{\mu_w}\right)_{i+\frac{1}{2},k} \left(\frac{1}{r_{i+1,k} - r_{i,k}}\right) (p_{i+1,k} - p_{i,k}) \\ & - \frac{1}{r_{i,k}} \frac{1}{\left(r_{i+\frac{1}{2},k} - r_{i-\frac{1}{2},k}\right)} \left(r \rho_w \frac{k_r}{\mu_w}\right)_{i-\frac{1}{2},k} \left(\frac{1}{r_{i,k} - r_{i-1,k}}\right) (p_{i,k} - p_{i-1,k}) \\ & + \frac{1}{\left(z_{i,k+\frac{1}{2}} - z_{i,k-\frac{1}{2}}\right)} \left(\rho_w \frac{k_z}{\mu_w}\right)_{i,k+\frac{1}{2}} \left(\frac{1}{z_{i,k+1} - z_{i,k}}\right) \left[p_{i,k+1} - p_{i,k} - \gamma_{w,i,k+\frac{1}{2}} (z_{i,k+1} - z_{i,k})\right] \\ & - \frac{1}{\left(z_{i,k+\frac{1}{2}} - z_{i,k-\frac{1}{2}}\right)} \left(\rho_w \frac{k_z}{\mu_w}\right)_{i,k-\frac{1}{2}} \left(\frac{1}{z_{i,k} - z_{i,k-1}}\right) \left[p_{i,k} - p_{i,k-1} - \gamma_{w,i,k-\frac{1}{2}} (z_{i,k} - z_{i,k-1})\right] \end{aligned} \quad (3.13)$$

Since the above equation is per unit volume, both sides of the equation need to be multiplied by the bulk volume of the grid block:

$$\begin{aligned}
& \pi \Delta z_{i,k} \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \frac{(\rho\phi)_{i,k}^{n+1} - (\rho\phi)_{i,k}^n}{\Delta t} \\
&= \frac{1}{r_{i,k}} \pi \Delta z_{i,k} \left(r_{i+\frac{1}{2},k} + r_{i-\frac{1}{2},k} \right) \left(r \rho_w \frac{k_r}{\mu_w} \right)_{i+\frac{1}{2},k} \left(\frac{1}{r_{i+1,k} - r_{i,k}} \right) (p_{i+1,k} - p_{i,k}) \\
&- \frac{1}{r_{i,k}} \pi \Delta z_{i,k} \left(r_{i+\frac{1}{2},k} + r_{i-\frac{1}{2},k} \right) \left(r \rho_w \frac{k_r}{\mu_w} \right)_{i-\frac{1}{2},k} \left(\frac{1}{r_{i,k} - r_{i-1,k}} \right) (p_{i,k} - p_{i-1,k}) \quad (3.14) \\
&+ \pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \left(\rho_w \frac{k_z}{\mu_w} \right)_{i,k+\frac{1}{2}} \left(\frac{1}{z_{i,k+1} - z_{i,k}} \right) \left[p_{i,k+1} - p_{i,k} \right. \\
&- \left. \gamma_{w,i,k+\frac{1}{2}} (z_{i,k+1} - z_{i,k}) \right] \\
&- \pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \left(\rho_w \frac{k_z}{\mu_w} \right)_{i,k-\frac{1}{2}} \left(\frac{1}{z_{i,k} - z_{i,k-1}} \right) \left[p_{i,k} - p_{i,k-1} \right. \\
&- \left. \gamma_{w,i,k-\frac{1}{2}} (z_{i,k} - z_{i,k-1}) \right]
\end{aligned}$$

Now, the transmissibilities can be defined as follows:

$$\Omega_{r,i+\frac{1}{2},k} = \frac{1}{r_{i,k}} \pi \Delta z_{i,k} \left(r_{i+\frac{1}{2},k} + r_{i-\frac{1}{2},k} \right) \left(r \rho_w \frac{k_r}{\mu_w} \right)_{i+\frac{1}{2},k} \left(\frac{1}{r_{i+1,k} - r_{i,k}} \right) \quad (3.15)$$

$$\Omega_{r,i-\frac{1}{2},k} = \frac{1}{r_{i,k}} \pi \Delta z_{i,k} \left(r_{i+\frac{1}{2},k} + r_{i-\frac{1}{2},k} \right) \left(r \rho_w \frac{k_r}{\mu_w} \right)_{i-\frac{1}{2},k} \left(\frac{1}{r_{i,k} - r_{i-1,k}} \right) \quad (3.16)$$

$$\Omega_{z,i,k+\frac{1}{2}} = \pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \left(\rho_w \frac{k_z}{\mu_w} \right)_{i,k+\frac{1}{2}} \left(\frac{1}{z_{i,k+1} - z_{i,k}} \right) \quad (3.17)$$

$$\Omega_{z,i,k-\frac{1}{2}} = \pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \left(\rho_w \frac{k_z}{\mu_w} \right)_{i,k-\frac{1}{2}} \left(\frac{1}{z_{i,k} - z_{i,k-1}} \right) \quad (3.18)$$

Equation 3.14 can be rewritten using the above definitions of the transmissibilities as follows:

$$\begin{aligned}
& \pi \Delta z_{i,k} \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \frac{(\rho_w \phi)_{i,k}^{n+1} - (\rho_w \phi)_{i,k}^n}{\Delta t} \\
&= \Omega_{r,i+\frac{1}{2},k} (p_{i+1,k} - p_{i,k}) - \Omega_{r,i-\frac{1}{2},k} (p_{i,k} - p_{i-1,k}) \\
&+ \Omega_{z,i,k+\frac{1}{2}} \left[p_{i,k+1} - p_{i,k} - \gamma_{w,i,k+\frac{1}{2}} (z_{i,k+1} - z_{i,k}) \right] \\
&- \Omega_{z,i,k-\frac{1}{2}} \left[p_{i,k} - p_{i,k-1} - \gamma_{w,i,k-\frac{1}{2}} (z_{i,k} - z_{i,k-1}) \right]
\end{aligned} \tag{3.19}$$

If a fully implicit approach is used, then all pressure and pressure dependent terms on the right-hand side of Equation 3.19 are evaluated at the present time level t_{n+1} and hence, Equation 3.19 can be expressed as:

$$\begin{aligned}
& \pi \Delta z_{i,k} \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \frac{(\rho_w \phi)_{i,k}^{n+1} - (\rho_w \phi)_{i,k}^n}{\Delta t} \\
&= \Omega_{r,i+\frac{1}{2},k}^{n+1} (p_{i+1,k}^{n+1} - p_{i,k}^{n+1}) - \Omega_{r,i-\frac{1}{2},k}^{n+1} (p_{i,k}^{n+1} - p_{i-1,k}^{n+1}) \\
&+ \Omega_{z,i,k+\frac{1}{2}}^{n+1} \left[p_{i,k+1}^{n+1} - p_{i,k}^{n+1} - \gamma_{w,i,k+\frac{1}{2}}^{n+1} (z_{i,k+1} - z_{i,k}) \right] \\
&- \Omega_{z,i,k-\frac{1}{2}}^{n+1} \left[p_{i,k}^{n+1} - p_{i,k-1}^{n+1} - \gamma_{w,i,k-\frac{1}{2}}^{n+1} (z_{i,k} - z_{i,k-1}) \right]
\end{aligned} \tag{3.20}$$

It is important to note that the transmissibility Ω and the accumulation term $(\rho_w \phi)$ are both functions of pressure and temperature.

The Hawkins relationship (Hawkins, 1956) is used for the treatment of the skin factor due to damage or stimulation as follows:

$$S = \left(\frac{k_r}{k_s} - 1 \right) \ln \left(\frac{r_s}{r_w} \right) \tag{3.21}$$

Hence, the permeability that would mimic a skin factor of S would be obtained by:

$$k_s = \frac{k_r}{\frac{S}{\ln \left(\frac{r_s}{r_w} \right)} + 1} \tag{3.22}$$

In Equation 3.22, k_r is the permeability in the r -direction. Here, skin is treated as a permeability altered zone around wellbore where permeability of this zone is computed by Eq. 3.22 and this permeability is assigned to all the gridblocks in this skin zone. In the applications considered in this study, the permeability value of this zone is assigned to the first two gridblocks around wellbore.

For any reservoir grid block i, k , the mass balance equation is given as follows:

$$\begin{aligned}
& \pi \Delta z_{i,k} \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \frac{(\rho_w \phi)_{i,k}^{n+1} - (\rho_w \phi)_{i,k}^n}{\Delta t} \\
&= \Omega_{r,i+\frac{1}{2},k}^{n+1} (p_{i+1,k}^{n+1} - p_{i,k}^{n+1}) - \Omega_{r,i-\frac{1}{2},k}^{n+1} (p_{i,k}^{n+1} - p_{i-1,k}^{n+1}) \\
&+ \Omega_{z,i,k+\frac{1}{2}}^{n+1} \left[p_{i,k+1}^{n+1} - p_{i,k}^{n+1} - \gamma_{w,i,k+\frac{1}{2}}^{n+1} (z_{i,k+1} - z_{i,k}) \right] \\
&- \Omega_{z,i,k-\frac{1}{2}}^{n+1} \left[p_{i,k}^{n+1} - p_{i,k-1}^{n+1} - \gamma_{w,i,k-\frac{1}{2}}^{n+1} (z_{i,k} - z_{i,k-1}) \right]
\end{aligned} \tag{3.23}$$

It is assumed that a total of N_r and N_z number of gridblocks is available in the r and z -directions, respectively (Details of the derivations can be found in Appendix A).

The final word on mass balance will be regarding the evaluation of the terms in the transmissibilities at the $\pm 1/2$ boundaries. Harmonic averaging is used for the permeability in the r -direction. In the r -direction, the harmonic averaging is computed as follows:

$$k_{r,i+\frac{1}{2},k} = \frac{k_{r,i+1,k} k_{r,i,k} \ln \left(\frac{r_{i+1,k}}{r_{i,k}} \right)}{k_{r,i,k} \ln \left(\frac{r_{i+1,k}}{r_{i+\frac{1}{2},k}} \right) + k_{r,i+1,k} \ln \left(\frac{r_{i+\frac{1}{2},k}}{r_i} \right)} \tag{3.24}$$

Harmonic averaging is also used for the z -direction by using the following relationship:

$$k_{z,i,k+\frac{1}{2}} = \frac{(\Delta z_{i,k} + \Delta z_{i,k+1}) k_{z,i,k} k_{z,i,k+1}}{k_{z,i,k} \Delta z_{i,k+1} + k_{z,i,k+1} \Delta z_{i,k}} \tag{3.25}$$

In the r -direction, the standard arithmetic averaging is used for the fluid properties:

$$\left(\frac{\rho_w}{\mu}\right)_{i+\frac{1}{2},k} = \frac{\left(\frac{\rho_w}{\mu}\right)_{i,k} + \left(\frac{\rho_w}{\mu}\right)_{i+1,k}}{2} \quad (3.26)$$

Similarly, for the z -direction:

$$\left(\frac{\rho_w}{\mu}\right)_{i,k+\frac{1}{2}} = \frac{\left(\frac{\rho_w}{\mu}\right)_{i,k} + \left(\frac{\rho_w}{\mu}\right)_{i,k+1}}{2} \quad (3.27)$$

3.1.2 Derivation and discretization of energy conservation equation

Various forms of energy conservation equation are given in the literature. In this study, the form of energy conservation equation also named as “thermal energy equation” in Bird et al. (1960) is used to derive energy conservation equation for reservoir system. In addition, it is assumed that a local thermal equilibrium between solid rock and fluid at a location given in reservoir is available, that is, two systems (both solid rock and fluid) at the same location have the same temperature value.

The thermal energy balance equation is given by Bird et al. (1960) in terms of internal energy:

$$\frac{\partial}{\partial t}(\rho U) - (\nabla \bullet \rho_w H_w \vec{v}_w) - (\nabla \bullet \vec{q}) = 0 \quad (3.28)$$

In Equation 3.28, “ ∇ ” represents the gradient operator and the symbol “ $\bullet$ ” represents scalar product. In Equation 3.28, U represents the specific internal energy of the system. U_w represents the specific internal energy of water. The first term on the left hand side of Equation 3.28 defines temporal accumulation of internal energy per unit reservoir volume. The second term represents heat transfer transported with fluid flow due to convection. The third term defines rate of change in internal energy due to conduction.

It should be kept in mind that the heat is stored in both the rock and the fluid and local thermal equilibrium is assumed. Hence, the accumulation term of the above equation needs to further be modified:

$$\frac{\partial}{\partial t} [\phi \rho_w U_w + (1-\phi) \rho_s C_{p,s} T] = -(\nabla \bullet \rho_w H_w \vec{v}_w) - (\nabla \bullet \tilde{q}) \quad (3.29)$$

According to Equation 3.29, it can be seen that this method evaluates the internal energy terms implicitly (they are not expressed in terms of pressure, temperature or enthalpy). Since the above equation is per unit volume, both sides of the equation need to be multiplied by the volume of the grid block as follows:

$$\begin{aligned} & \pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \Delta z_k \frac{\partial}{\partial t} [\phi \rho_w U_w + (1-\phi) \rho_s C_{p,s} T] \\ &= -\pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \Delta z_k (\nabla \bullet \rho_w H_w \vec{v}_w) \\ & \quad - \pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \Delta z_k (\nabla \bullet \tilde{q}) \end{aligned} \quad (3.30)$$

Now, the first term on the right hand side of Equation 3.30 is first considered. Actually, this looks very much like the term in mass balance $\nabla \bullet \rho \vec{v}$. The discretization of the mass balance equation has been given previously. The only difference is that now there will be the internal energy term multiplying the transmissibility. Hence, the discretization of the first term on the right hand side of Equation 3.30 will have the following form (Details of the derivations can be found in Appendix A):

$$\begin{aligned} & -\pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \Delta z_k (\nabla \bullet \rho_w U_w \vec{v}_w) = \\ & \Omega_{r,i+\frac{1}{2},k} H_{w,i+\frac{1}{2},k} (p_{i+1,k} - p_{i,k}) - \Omega_{r,i-\frac{1}{2},k} H_{w,i-\frac{1}{2},k} (p_{i,k} - p_{i-1,k}) \\ & + \Omega_{z,i,k+\frac{1}{2}} H_{w,i,k+\frac{1}{2}} \left[p_{i,k+1} - p_{i,k} - \gamma_{w,i,k+\frac{1}{2}} (z_{i,k+1} - z_{i,k}) \right] \\ & - \Omega_{z,i,k-\frac{1}{2}} H_{w,i,k-\frac{1}{2}} \left[p_{i,k} - p_{i,k-1} - \gamma_{w,i,k-\frac{1}{2}} (z_{i,k} - z_{i,k-1}) \right] \end{aligned} \quad (3.31)$$

At this point, it is worth noting that when the internal energies at the grid block boundaries are evaluated, upwinding will be performed. In other words, the flow direction will be checked to figure out which block pressure and temperature should be used to determine the internal energy.

For the r -direction,

If $p_{i+1,k} > p_{i,k}$, then $U_{w,i+1,k}$ is used.

If $p_{i+1,k} < p_{i,k}$, then $U_{w,i-1,k}$ is used.

For the z -direction,

If $p_{i,k+1} > p_{i,k} + \gamma_{w,i,k+\frac{1}{2}}(z_{i,k+1} - z_{i,k})$, then $U_{w,i,k+1}$ is used.

If $p_{i,k+1} < p_{i,k} + \gamma_{w,i,k+\frac{1}{2}}(z_{i,k+1} - z_{i,k})$, then $U_{w,i,k-1}$ is used.

Now, the second term on the right hand side of Equation 3.30 will be considered. This is the conduction term that can be evaluated as follows:

$$-\pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \Delta z_k (\nabla \bullet \tilde{q}) = -\pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \Delta z_k \left[\frac{1}{r} \frac{\partial}{\partial r} (r \tilde{q}_r) + \frac{\partial}{\partial z} (\tilde{q}_z) \right] \quad (3.32)$$

At this point, Fourier's law of heat conduction will be required to be recalled which can be given as:

$$\tilde{q}_r = -\lambda_t \frac{\partial T}{\partial r} \quad (3.33)$$

$$\tilde{q}_z = -\lambda_t \frac{\partial T}{\partial z} \quad (3.34)$$

In Equations 3.33 and 3.34, λ represents thermal conductivity. $\lambda_t [= \phi \lambda_w + (1 - \phi) \lambda_s]$ is the thermal conductivity of the total reservoir system which is a porosity weighted average of solid rock and fluid thermal conductivities (λ_w represents thermal conductivity of water and λ_s represents thermal conductivity of solid rock phase). while $\tilde{q}$ is the heat conduction term based on Fourier's law ($\tilde{q}_r$ represents the heat conduction in the r -direction and $\tilde{q}_z$ represents the heat conduction in the z -direction).

If Equations 3.33 and 3.34 are used in Equation 3.32 and evaluating in block i, k , it can be obtained (Details of the derivations can be found in Appendix A):

$$\begin{aligned}
& -\pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \Delta z_k (\nabla \bullet \tilde{q}) \\
& = \pi \left(r_{i+\frac{1}{2},k} + r_{i-\frac{1}{2},k} \right) \Delta z_k \frac{1}{r_{i,k}} (r\lambda_t)_{i+\frac{1}{2},k} \frac{1}{r_{i+1,k} - r_{i,k}} (T_{i+1,k} - T_{i,k}) \\
& - \pi \left(r_{i+\frac{1}{2},k} + r_{i-\frac{1}{2},k} \right) \Delta z_k \frac{1}{r_{i,k}} (r\lambda_t)_{i-\frac{1}{2},k} \frac{1}{r_{i,k} - r_{i-1,k}} (T_{i,k} - T_{i-1,k}) \\
& \pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \lambda_{t,i,k+\frac{1}{2}} \frac{1}{z_{i,k+1} - z_{i,k}} (T_{i,k+1} - T_{i,k}) \\
& - \pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \lambda_{t,i,k-\frac{1}{2}} \frac{1}{z_{i,k} - z_{i,k-1}} (T_{i,k} - T_{i,k-1})
\end{aligned} \tag{3.35}$$

where

$$\left(\frac{\partial T}{\partial r} \right)_{i+\frac{1}{2},k} = \frac{T_{i+1,k} - T_{i,k}}{r_{i+1,k} - r_{i,k}} \tag{3.36}$$

$$\left(\frac{\partial T}{\partial r} \right)_{i-\frac{1}{2},k} = \frac{T_{i,k} - T_{i-1,k}}{r_{i,k} - r_{i-1,k}} \tag{3.37}$$

$$\left(\frac{\partial T}{\partial z} \right)_{i,k+\frac{1}{2}} = \frac{T_{i,k+1} - T_{i,k}}{z_{i,k+1} - z_{i,k}} \tag{3.38}$$

$$\left(\frac{\partial T}{\partial z} \right)_{i,k-\frac{1}{2}} = \frac{T_{i,k} - T_{i,k-1}}{z_{i,k} - z_{i,k-1}} \tag{3.39}$$

Now, if the “conduction” transmissibilities are defined as follows:

$$\Gamma_{r,i+\frac{1}{2},k} = \pi \left(r_{i+\frac{1}{2},k} + r_{i-\frac{1}{2},k} \right) \Delta z_k \frac{1}{r_{i,k}} (r\lambda_t)_{i+\frac{1}{2},k} \frac{1}{r_{i+1,k} - r_{i,k}} \tag{3.40}$$

$$\Gamma_{r,i-\frac{1}{2},k} = \pi \left(r_{i+\frac{1}{2},k} + r_{i-\frac{1}{2},k} \right) \Delta z_k \frac{1}{r_{i,k}} (r\lambda_t)_{i-\frac{1}{2},k} \frac{1}{r_{i,k} - r_{i-1,k}} \tag{3.41}$$

$$\Gamma_{z,i,k+\frac{1}{2}} = \pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \lambda_{t,i,k+\frac{1}{2}} \frac{1}{z_{i,k+1} - z_{i,k}} \tag{3.42}$$

$$\Gamma_{z,i,k-\frac{1}{2}} = \pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \lambda_{t,i,k-\frac{1}{2}} \frac{1}{z_{i,k} - z_{i,k-1}} \tag{3.43}$$

Using Equations 3.40-3.43 in Equation 3.35, it can be obtained:

$$\begin{aligned}
& -\pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \Delta z_k (\nabla \bullet \tilde{q}) = \Gamma_{r,i+\frac{1}{2},k} (T_{i+1,k} - T_{i,k}) - \Gamma_{r,i-\frac{1}{2},k} (T_{i,k} - T_{i-1,k}) \\
& + \Gamma_{z,i,k+\frac{1}{2}} (T_{i,k+1} - T_{i,k}) - \Gamma_{z,i,k-\frac{1}{2}} (T_{i,k} - T_{i,k-1})
\end{aligned} \tag{3.44}$$

Now, using Equations 3.31 and 3.44 in Equation 3.30 and also using a forward differencing scheme to handle the accumulation term with fully-implicit approach gives:

$$\begin{aligned}
& \pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \Delta z_k \frac{\left(\left[\phi \rho_w U_w + (1-\phi) \rho_s C_{p,s} T \right]_{i,k}^{n+1} - \left[\phi \rho_w U_w + (1-\phi) \rho_s C_{p,s} T \right]_{i,k}^n \right)}{\Delta t} \\
& = \Omega_{r,i+\frac{1}{2},k}^{n+1} H_{w,i+\frac{1}{2},k}^{n+1} (p_{i+1,k}^{n+1} - p_{i,k}^{n+1}) - \Omega_{r,i-\frac{1}{2},k}^{n+1} H_{w,i-\frac{1}{2},k}^{n+1} (p_{i,k}^{n+1} - p_{i-1,k}^{n+1}) \\
& + \Omega_{z,i,k+\frac{1}{2}}^{n+1} H_{w,i,k+\frac{1}{2}}^{n+1} \left[p_{i,k+1}^{n+1} - p_{i,k}^{n+1} - \gamma_{w,i,k+\frac{1}{2}}^{n+1} (z_{i,k+1} - z_{i,k}) \right] \\
& - \Omega_{z,i,k-\frac{1}{2}}^{n+1} H_{w,i,k-\frac{1}{2}}^{n+1} \left[p_{i,k}^{n+1} - p_{i,k-1}^{n+1} - \gamma_{w,i,k-\frac{1}{2}}^{n+1} (z_{i,k} - z_{i,k-1}) \right] \\
& + \Gamma_{r,i+\frac{1}{2},k}^{n+1} (T_{i+1,k}^{n+1} - T_{i,k}^{n+1}) - \Gamma_{r,i-\frac{1}{2},k}^{n+1} (T_{i,k}^{n+1} - T_{i-1,k}^{n+1}) \\
& + \Gamma_{z,i,k+\frac{1}{2}}^{n+1} (T_{i,k+1}^{n+1} - T_{i,k}^{n+1}) - \Gamma_{z,i,k-\frac{1}{2}}^{n+1} (T_{i,k}^{n+1} - T_{i,k-1}^{n+1})
\end{aligned} \tag{3.45}$$

Finally, it should be noted that the above equation does not include a term considering heat transfer from reservoir to adjacent overburden and underburden strata. In this method, heat transfer from reservoir to adjacent strata is modelled by assigning different temperature values (temperature gradients) to overburden and underburden strata in z-direction (Details of this approach are given in Section 5.2.4 of Chapter 5). The main motivation to apply such an approach is itself of the nature. As is known, in an idealized geothermal structure, a natural temperature gradient is available underground and this gradient is generally linear from the surface to the top of the reservoir. Inside the reservoir, temperature approximately stabilizes at the original geothermal reservoir temperature along the whole thickness of the reservoir and another temperature gradient occurs from the bottom of the reservoir to the downwards.

3.1.3 Initial and boundary conditions

Initial and boundary conditions must be defined to solve the equations given above individually in terms of pressure and temperature.

Here, the classical wellbore/reservoir model assuming that the “well” can be represented by a single cell is considered to simulate the wellbore and sandface pressure and temperature as a function of time. In other words, wellbore is considered as a single cell from reservoir part where production/injection takes place to the point in which single-phase flow is dominated at a certain height in the wellbore (Figures 3.2 and 3.3). Then, the wellbore pressure and temperature are calculated combining the mass and energy balance equations on this single cell with mass and energy balance equations of reservoir. Here, since single-phase liquid flow is considered, the part of interval where this flow condition can be valid is adjacent part to reservoir interval open to production/injection of the wellbore (Figures 3.2 and 3.3). Hence, the single cell wellbore/reservoir model considered here is simple to simulate changes of wellbore and sandface pressure and temperature. However, it is worth noting that the wellbore/reservoir modeling considered here may not be sufficient for measurements of pressure and temperature performed at much higher elevations (or depths) than the reservoir interval open to production/injection. In the evaluation of such measurements, a wellbore model connecting the wellbore segmented to the reservoir is required (Miller, 1980; Livescu et al., 2010; Ramazanov et al. 2010). However, such models are not in the scope of this study.

3.1.3.1 Pressure conditions

The initial condition for pressure is as follows:

$$p(r, z, t = 0) = p^0(r, z) \quad (3.46)$$

Boundary conditions for pressure are as follows:

$$\left(\frac{\partial p}{\partial z} - \gamma_w \frac{\partial D}{\partial z} \right)_{r, z=0} = 0, \quad r_w \leq r \leq r_e \quad (3.47)$$

$$\left(\frac{\partial p}{\partial z} - \gamma_w \frac{\partial D}{\partial z} \right)_{r, z=h} = 0, \quad r_w \leq r \leq r_e \quad (3.48)$$

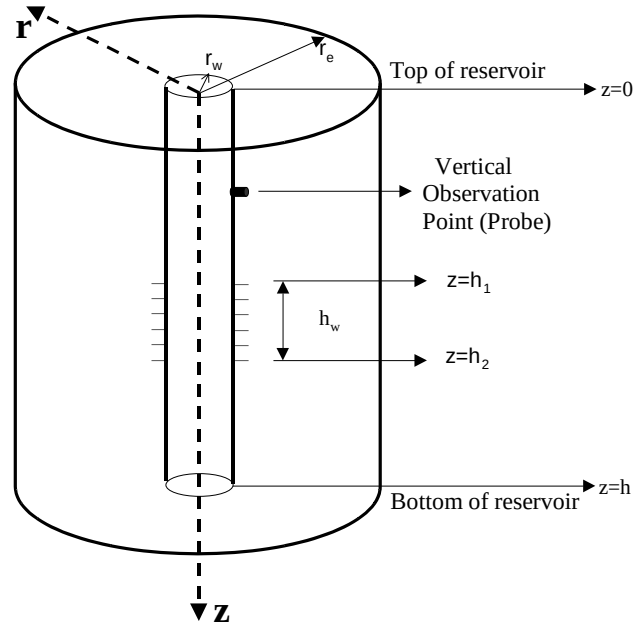


Figure 3.2: 3D view of wellbore/reservoir model with single cell.

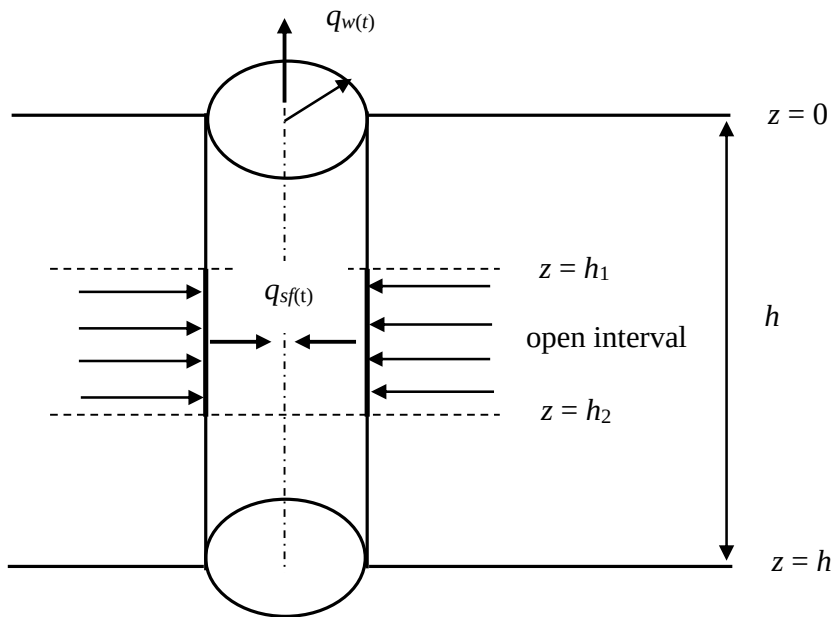


Figure 3.3: 2D (r - z) cross-sectional view of wellbore/reservoir model with single cell.

$$\left(\frac{\partial p}{\partial r} \right)_{r=r_e, z} = 0, \quad 0 \leq z \leq h, \quad (3.49)$$

$$\left(\frac{\partial p}{\partial r} \right)_{r=r_w, z} = 0, \quad 0 \leq z \leq h_1, \quad h_2 \leq z \leq h \quad (3.50)$$

In the equations given above, z is the vertical coordinate axis and $z = 0$ and $z = h$ represent top and bottom of reservoir, respectively. r_w and r_e represent well and reservoir radius in radial direction, respectively. γ_w indicates fluid gradient due to acceleration in the z -direction:

$$\gamma_w = \rho_w g \quad (3.51)$$

Here, it is assumed that reservoir is horizontal, that is, it is not inclined. Therefore, acceleration is effective only in vertical direction (z -direction; Figure 3.2).

In Equation 3.46, p^0 represents initial pressure in the system and can change with location (r, z) in the most common form. Initial pressure can be taken independent of location (equal at all locations) for the cases that acceleration effect in z direction is negligible. In this case, p^0 can be taken constant for all (r, z) values. Initial pressure must be the same on a given r - θ plane, but it must change with z coordinate for more realistic cases so that acceleration effect is not ignored. For this case, p^0 cannot be equal anymore for all locations and changes with z (or D) coordinate as follows:

$$p^0(r, z) = p^0(r, z^0) + \gamma(D - D^0) = p_d^0 + \gamma(D - D^0) \quad (3.52)$$

Here, D represents elevation from a selected datum that initial pressure value $p_d^0 (= p^0(r, D^0))$ is known. If the datum is chosen as the top of the reservoir, D^0 will be taken as zero according to coordinate system used in Figure 3.2. Equation 3.52 is used to derive initial pressure distribution in the simulator.

3.1.3.2 Temperature conditions

As initial condition for temperature, a condition similar to that of pressure can be used as follows:

$$T(r, z, t = 0) = T^0(r, z) \quad (3.53)$$

For example, as boundary conditions for temperature, it can be assumed that temperature gradient is constant at the boundaries of top ($z = 0$) and bottom ($z = h$) of reservoir in the z -direction

$$\left(\frac{\partial T}{\partial z} \right)_{r,z=0 \& h} = \gamma_z^T, \quad r_w \leq r \leq r_e \quad (3.54)$$

and similarly, the following expression can be used as boundary condition for temperature in the r -direction.

$$\left(\frac{\partial T}{\partial r} \right)_{r=r_w \& r_e} = \gamma_r^T, \quad 0 \leq z \leq h \quad (3.55)$$

Here, γ_r^T and γ_z^T represent temperature gradients in the r and z -directions. If it is desired, it can also be possible to model as closed boundaries to heat flow by equating these gradients to zero. Dirichlet type boundary conditions can be considered as another boundary conditions for temperature (temperature is the boundary condition that is specified at the boundaries):

$$T(r = r_e, z) = T_e, \quad 0 \leq z \leq h \quad (3.56)$$

3.1.4 Derivation and discretization of wellbore equations

At first, modeling the wellbore pressure is taken into account. The set of N number of equations for mass conservation has been derived from mass balance on reservoir grid blocks. If the wellbore pressure is desired to be computed, one more equation is needed to be added to the system of equations. The well is considered to be a tank which is connected to grid blocks at locations where the well is open to flow (well will only make connections to grid blocks that are located within the vicinity of the well).

Mass balance on the well tank yields:

$$\left[\begin{array}{c} \text{Mass Accumulation} \\ \text{Rate in Well} \end{array} \right] - \left[\begin{array}{c} \text{Mass Contribution Rate} \\ \text{from Connecting Gridblocks} \end{array} \right] + \left[\begin{array}{c} \text{Production or Injection Rate} \\ \text{from Well} \end{array} \right] = 0 \quad (3.57)$$

Spatial discretization on Equation 3.57 gives:

$$V^w \frac{d\rho_w^w}{dt} - \sum_{l=1}^{N_c} \Omega_{r_w, k_l} \left(p_{1, k_l} - p_{wf} - \gamma_{w, 1, k_l} (z_{1, k_l} - z_{wf}) \right) + q = 0 \quad (3.58)$$

where

$$\Omega_{r_w, k_l} = \frac{1}{r_{1, k_l}} \pi \Delta z_{1, k_l} (r_{3/2, k_l} + r_{w, k_l}) \left(r \rho_w \frac{k}{\mu_w} \right)_{r_w, k_l} \left(\frac{1}{r_{1, k_l} - r_{w, k_l}} \right) \quad (3.59)$$

In Equation 3.58, V^w represents volume section open to flow of well along thickness of the reservoir. N_c represents number of connections. p_{wf} represents the wellbore pressure. q represents production or injection rate. l represents grid block index open to flow which is adjacent to the well in the z -direction.

Temporal discretization on the accumulation term in Equation 3.58 also yields:

$$V^w \left(\frac{\rho_w^{w, n+1} - \rho_w^{w, n}}{\Delta t} \right) - \sum_{l=1}^{N_c} \Omega_{r_w, k_l} \left(p_{1, k_l} - p_{wf} - \gamma_{w, 1, k_l} (z_{1, k_l} - z_{wf}) \right) + q = 0 \quad (3.60)$$

When modeling the wellbore temperature, a similar approach is used to that of modeling the wellbore pressure, given at the beginning of subsection. It is assumed that the well is a tank and energy balance is applied to this tank.

Energy balance on the well tank yields:

$$\left[\begin{array}{c} \text{Energy Accumulation} \\ \text{Rate in Well} \end{array} \right] - \left[\begin{array}{c} \text{Energy Contribution Rate} \\ \text{from Connecting Gridblocks} \\ \text{through Convection} \end{array} \right] - \left[\begin{array}{c} \text{Energy Contribution Rate} \\ \text{from Connecting Gridblocks} \\ \text{through Conduction} \end{array} \right] + \left[\begin{array}{c} \text{Energy Contribution Rate} \\ \text{due to Production or Injection} \end{array} \right] = 0 \quad (3.61)$$

Spatial discretization on Equation 3.61 gives:

$$V^w \frac{d}{dt} (\rho_w^w U_w^w) - \sum_{l=1}^{N_c} \Omega_{r_w, k_l} H_y \left(p_{1, k_l} - p_{wf} - \gamma_{w, 1, k_l} (z_{1, k_l} - z_{wf}) \right) - \sum_{l=1}^{N_c} \Gamma_{r_w, k_l} (T_{1, k_l} - T_{wf}) + q H_x = 0 \quad (3.62a)$$

where

$$\Gamma_{r_w, k_l} = \pi (r_{3/2, k_l} + r_{w, k_l}) \Delta z_{k_l} \frac{1}{r_{1, k_l}} (r \lambda_t)_{r_w, k_l} \frac{1}{r_{1, k_l} - r_{w, k_l}} \quad (3.62b)$$

In Equation 3.62a, H_x becomes H_w for the enthalpy of water produced and becomes $H_{w,inj}$ for enthalpy of the injected water. N_c represents number of connections. T_{wf} represents the wellbore temperature.

In Equation 3.62a,

$$H_y = H_w(p_1, T_1) \quad \text{if} \quad p_1 > p_{wf} + \gamma_{w,1,k_l} (z_{1,k_l} - z_{wf}) \quad (3.63a)$$

$$H_y = H_w(p_{wf}, T_{wf}) \quad \text{if} \quad p_1 < p_{wf} + \gamma_{w,1,k_l} (z_{1,k_l} - z_{wf}) \quad (3.63b)$$

Temporal discretization on the accumulation term in Equation 3.62a also yields:

$$V^w \frac{(\rho_w^w U_w^w)^{n+1} (\rho_w^w U_w^w)^n}{\Delta t} - \sum_{l=1}^{N_c} \Omega_{r_w, k_l} H_{w, r_w, k_l} (p_{1, k_l} - p_{wf}) - \sum_{l=1}^{N_c} \Gamma_{r_w, k_l} (T_{1, k_l} - T_{wf}) \quad (3.64)$$

$$+ qH_x = 0$$

3.2 2D (r-z) Grid System

Finite difference method is used in construction of 2D (r-z) numerical simulator. For both direction, “block centered” grid system that is commonly used in these type of simulators in the literature is used as grid system. Two different methods are taken into consideration while constructing the grid points in the r -axis. The first one of those is “block centered grid system” divided by geometrical intervals that is suggested by MacDonald and Coats (1970). As described above, construction of grid points in r -axis is known as MacDonald-Coats’ method in the literature. In this method, when number of N_r gridblocks is given, grid points in the r -direction are divided to intervals geometrically:

$$r_{i+1} = \alpha r_i \quad i = 0, 1, \dots, N_r \quad (3.65)$$

Here,

$$r_0 = \frac{r_w \ln \alpha}{(\alpha - 1)} \quad (3.66)$$

$$\alpha = \left(\frac{r_e}{r_w} \right)^{1/N_r} \quad (3.67)$$

Grid block boundaries are computed from the following equation:

$$r_{i+1/2} = \frac{r_{i+1} - r_i}{\ln\left(\frac{r_{i+1}}{r_i}\right)} = \frac{r_{i+1} - r_i}{\ln \alpha} \quad i = 0, 1, \dots, N_r \quad (3.68)$$

If MacDonald-Coats' method defined by Equations 3.65-3.68 is used, in the r -direction, grid blocks having smaller length are constructed surroundings of the well. In contrary, grid blocks having larger length are constructed while moving away the well.

In z -axis, block centered grid system is also used. Grid points in this direction are determined by the defined block lengths, Δz_k . When number of N_z grid blocks and lengths are given, grid block boundaries in z -axis are defined as follows:

$$z_{1/2} = 0 \quad (3.69)$$

$$z_{k+1/2} = z_{k-1/2} + \Delta z_k \quad k = 1, 2, \dots, N_z \quad (3.70)$$

$$z_{N_z+1/2} = h \quad (3.71)$$

Here, while $z_{1/2} = 0$ represents the uppermost boundary of the reservoir in z -axis, h represents total reservoir thickness. Grid points are defined as follows:

$$z_k = z_{k-1/2} + \frac{\Delta z_k}{2} \quad k = 1, 2, \dots, N_z \quad (3.72)$$

z_k is half of sum of $z_{k-1/2}$ and $z_{k+1/2}$ and is expressed as follows:

$$z_k = \frac{(z_{k-1/2} + z_{k+1/2})}{2} \quad (3.73)$$

General view of grid system used in r - z -axis is shown in Figure 3.4.

3.3 Time Step Selection

In this section, it is described how time steps are generated where the solutions are evaluated. The basic idea of the algorithm used to generate time steps is that it takes the uniform number of steps for each log-cycle.

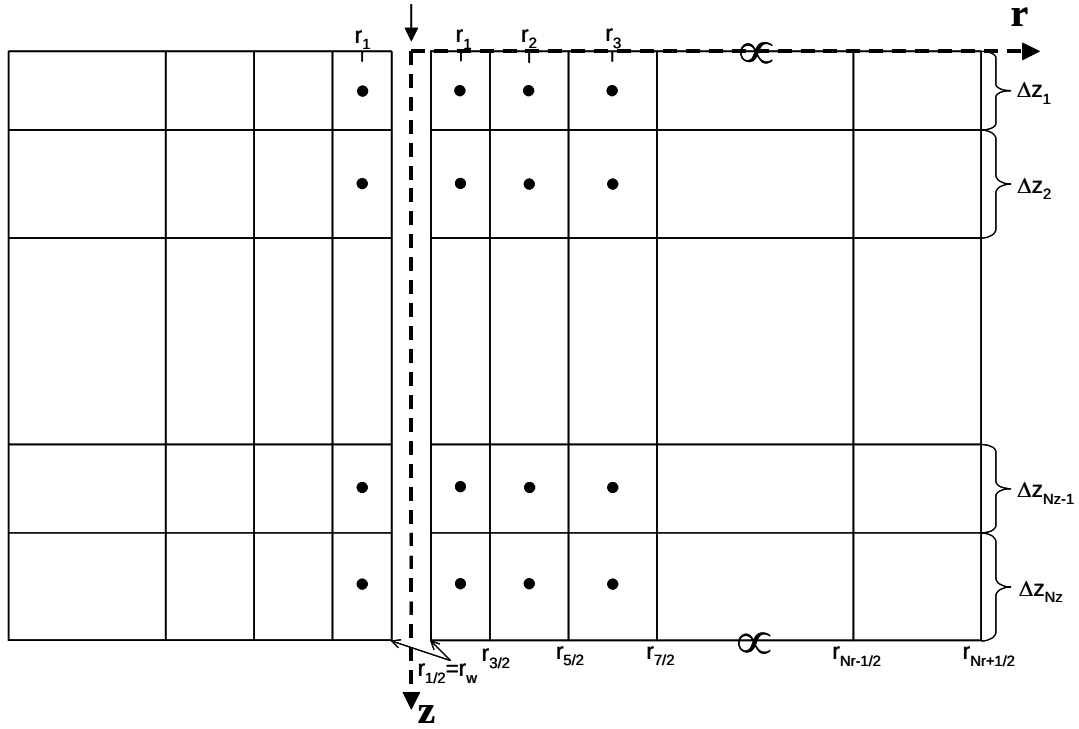


Figure 3.4: Schematic of a grid system constructed in r - z axis.

The following algorithm is used for time step selection.

- At first, the following ratio is defined:

$$R = \frac{t_e}{t_b} \quad (3.74)$$

where t_b represents the beginning of time and t_e represents the end of time.

- Number of log-cycles is determined.

$$\tilde{N}_{cyc} = \text{int}(\log R) + 1 \quad (3.75)$$

where $\tilde{N}_{cyc}$ represents the number of log-cycles and “int” represents integer.

- Total number of points to use is determined.

$$\tilde{N}_{tp} = \tilde{N}_{cyc} \tilde{N}_{pts} \quad (3.76)$$

where $\tilde{N}_{tp}$ represents the total number of time step points and $\tilde{N}_{pts}$ represents time step points for each log-cycle.

- A factor is computed for logarithmic time step.

$$\zeta = R^{\frac{1}{\tilde{N}_p}} \quad (3.77)$$

$$\tilde{N}_{tp} = \tilde{N}_{tp} + 1 \quad (3.78)$$

where $\tilde{N}_{tp}$ represents the total number of time step points and $\tilde{N}_{pts}$ represents time step points for each log-cycle.

- Finally, time steps are generated using the following equation.

$$t_g = t_{g-1}\zeta \quad (3.79)$$

where the subscript g represents the time step index.

As a result of this algorithm, time points that are equally spaced apart on the logarithmic time axis are generated as can be seen from Figure 3.5. On a Cartesian time axis, the time steps are small at first; however, they get larger as time increases as can be seen from Figure 3.6.

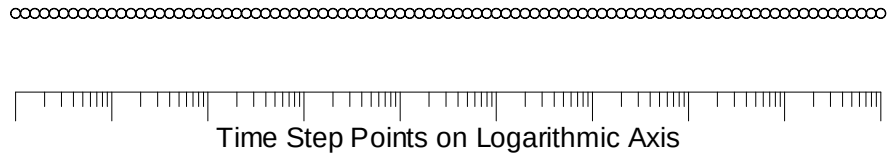


Figure 3.5: Time step points equally spaced apart on the logarithmic time axis.

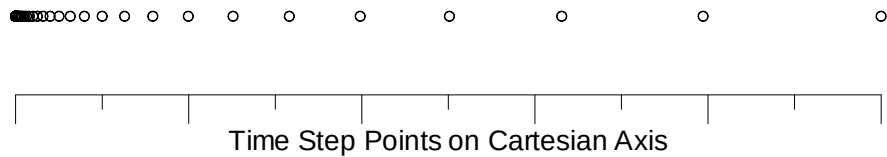


Figure 3.6: Time step points on Cartesian time axis.

3.4 Solution of Finite Difference Equations

Finite difference equations obtained are solved by Newton's method as they are derived as fully implicit and are non-linear.

They have $2N + 2$ equations with $2N + 2$ unknowns, where N is equal to $N = N_r \times N_z$, and the total number of the unknowns are the pressure and temperature vectors that are defined as:

$$\vec{p}^{n+1} = \begin{bmatrix} p_{wf}^{n+1} \\ p_1^{n+1} \\ p_2^{n+1} \\ \vdots \\ p_N^{n+1} \end{bmatrix} \quad (3.80)$$

$$\vec{T}^{n+1} = \begin{bmatrix} T_{wf}^{n+1} \\ T_1^{n+1} \\ T_2^{n+1} \\ \vdots \\ T_N^{n+1} \end{bmatrix} \quad (3.81)$$

In Equations 3.80 and 3.81, a grid block indexing based on the “standard” ordering method for the unknowns (pressure and temperature) is used. Hence, the 2D coordinate (i, k) can be related to single grid block indices by the following formula:

$$s = i + (k - 1) \times N_r \quad (3.82)$$

for $i = 1, 2, \dots, N_r$ and $k = 1, 2, \dots, N_z$.

In Figure 3.7, the grid system is considered where $N_r = 3$ and $N_z = 5$ for a limited entry vertical well (the bold numbers denote the grid block indices and thick horizontal lines show the open interval). According to the following figure, fluid enters from 4th, 7th, 10th and 13th grid blocks into the well (for production case) while it enters from well into the reservoir (for injection case).

In the Newton’s procedure, the primary variables (basic unknown variable vector) are ordered as:

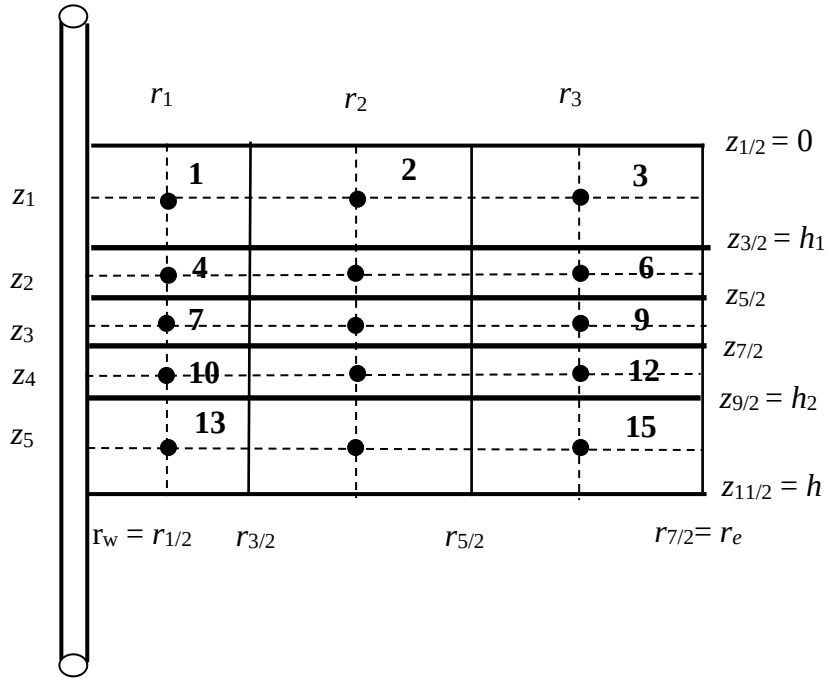


Figure 3.7: Schematic of r - z grid system with 2D coordinate and grid block indexing for an $N_r = 3$ and $N_z = 5$ grid system.

$$\vec{w}^{r+1} = \begin{bmatrix} p_{wf}^{r+1} \\ T_{wf}^{r+1} \\ p_1^{r+1} \\ T_1^{r+1} \\ p_2^{r+1} \\ T_2^{r+1} \\ \vdots \\ p_N^{r+1} \\ T_N^{r+1} \end{bmatrix} \quad (3.83)$$

$\vec{w}$ is a $2N+2$ dimensional vector and superscript r is iteration level where $N = N_r \times N_z$.

The Newton's procedure is described as follows:

$$J(\vec{w}^r) \delta \vec{w}^{r+1} = -\vec{f}(\vec{w}^r) \quad (3.84)$$

where

$$\vec{w}^{r+1} = \vec{w}^r + \delta \vec{w}^{r+1} \quad (3.85)$$

$\vec{f}(\vec{w}^r)$ is the $2N+2$ dimensional residual vector (obtained by equating each discrete equation to zero). “J” is the $(2N+2) \times (2N+2)$ Jacobian matrix to be constructed as:

$$J(\vec{w}^r) = \begin{bmatrix} \frac{\partial f_1(\vec{w}^r)}{\partial w_1} & \frac{\partial f_1(\vec{w}^r)}{\partial w_2} & \dots & \frac{\partial f_1(\vec{w}^r)}{\partial w_{2N+2}} \\ \frac{\partial f_2(\vec{w}^r)}{\partial w_1} & \frac{\partial f_2(\vec{w}^r)}{\partial w_2} & \dots & \frac{\partial f_2(\vec{w}^r)}{\partial w_{2N+2}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_{2N+2}(\vec{w}^r)}{\partial w_1} & \frac{\partial f_{2N+2}(\vec{w}^r)}{\partial w_2} & \dots & \frac{\partial f_{2N+2}(\vec{w}^r)}{\partial w_{2N+2}} \end{bmatrix} \quad (3.86)$$

Equations 3.84 and 3.85 are solved by taking an initial guess vector $\vec{w}^0$ for a time step until convergence. For convergence test, the following criteria is used:

$$\frac{\|\vec{w}^{r+1} - \vec{w}^r\|_2}{\|\vec{w}^r\|_2 + 10^{-14}} \leq 10^{-a} \quad (3.87)$$

where a is an integer and is taken as 5.

The elements of the Jacobian are computed numerically as follows:

$$\frac{\partial f_n(\vec{w}^r)}{\partial w_m} = \frac{f_n(w_m + \varepsilon) - f_n(w_m - \varepsilon)}{2\varepsilon} \quad (3.88)$$

For the cases considered in this thesis, the parameter ε for pressure and temperature is taken as 10^{-2} and 10^{-3} , respectively.

The Jacobian matrix can be constructed for the example grid system. Here, a specified production case is assumed. For this example case, there are 32 primary variables and the Jacobian matrix is 32 by 32. The structure of the Jacobian matrix is shown in Figure 3.8. p and T in Figure 3.8 denote the non-zero entries while p_{wf} and T_{wf} denote the connections to wellbore pressure and temperature. As can be seen from Figure 3.8, the matrix is non-symmetric and sparse. So, to solve this matrix problem, “direct” or “iterative” algorithms depending on the size of the matrix can be used. In this study, a direct method based on Gauss-Jordan elimination method named as “gaussj” given by Press et al. (1992) in Numerical Recipes was used.

simulator TOUGH2. It should be noted that the well model used in TOUGH2 is not completely understood. This is the reason why the sandface pressure and temperature have been chosen to work. In order to eliminate all effects of wellbore accumulation (both mass and energy), the volume of the wellbore has been chosen to set to a very small number (approximately $3 \times 10^{-4} \text{ m}^3$). This number has been chosen as a result of a trial and error procedure. The same approach (unless otherwise stated) is also considered in Chapter 5.

3.6 Sensitivities of the Solutions to the Number of Grid Blocks

In this section, the sensitivity of the solutions of the numerical model developed here to the number of gridblocks used in the r and z directions is investigated. Such an investigation is helpful to determine the appropriate grid system to be used in the application cases. Here, some of these investigations are shown for four different cases. The first one is for an injection case with a finite wellbore in the 1D (r) grid system. The second one is for a fully penetrating production well (line sink approach) in the 2D (r - z) grid system. The third one is for a limited-entry production well (line sink approach) in the 2D (r - z) grid system. The last one is for a limited-entry injection well (line source approach) in the 2D (r - z) grid system.

In the cases in this chapter, the model is a closed (no-flow and no-heat flux) reservoir system at the outer boundary (at $r = r_e$) of the reservoir. Heat transfer from reservoir to adjacent overburden and underburden strata (at $z = 0$ and $z = h$) is ignored for these cases. Anisotropy in permeability has not been considered in the reservoir. Gravity effect is ignored in all cases performed. Well is treated as a source or sink where its volume (V^w) is assumed so small (approximately $3 \times 10^{-4} \text{ m}^3$) unless otherwise stated. Hence, wellbore storage effect is ignored and it is assumed that geothermal water is pure water (no dissolved solids inside the water). The pertinent reservoir parameters used in the cases in this chapter are given in Table 3.1.

3.6.1 Constant-rate injection case with a finite wellbore approach

In this case, sensitivities of the solutions to the number of gridblocks in r -direction are investigated for the finite wellbore approach for a constant-rate injection case. The parameters used in the injection case are the same as in Table 3.1, except that the wellbore volume (V^w) is approximately taken as 3.14 m^3 .

Table 3.1 : Input data for reservoir systems.

Parameter	Value
P^0 , kPa	10000
T^0 , K (°C)	413.15 (140)
r_w , m	0.1
r_e , m	1000
h , m	100
V^w , m ³	3.14×10^{-4}
ϕ	0.2
k , m ² (md)	1×10^{-13} (100)
c_s , 1/kPa	2.9×10^{-7}
β_s , 1/K	1×10^{-5}
$C_{p,s}$, J/kg-K	1000
ρ_s , kg/m ³	2650
λ_t , J/m-s-K	2.92

The other difference in comparison with that of the next cases is that the grid system is 1D (r) instead of 2D (r - z). This case contains a constant mass rate of 1 kg/s injection with an injection temperature of 333.15K.

Figure 3.9 shows wellbore and sandface pressure behaviors with different number of gridblocks in the r -direction as a function of time. As can be seen from this figure, the wellbore and the sandface pressure behaviors exhibit an excellent agreement and pressure values do not change with different number of gridblocks in the r -direction. Since the simulator uses a wellbore model with a single cell, wellbore storage effects are not observed so much and the wellbore pressure signal is directly reflected to the sandface.

Figure 3.10 shows delta pressures of wellbore ($\Delta p = p_{wf} - p^0$) and sandface with Bourdet derivatives of those for the same case. Bourdet derivative is defined as the derivative of the pressure change with respect to the natural logarithm of time; that is, $\Delta p' = d\Delta p / d \ln t$ (Bourdet et al., 1989). Figure 3.10 also supports the agreement between the wellbore and sandface pressure behaviors.

On the other hand, Figure 3.11 shows wellbore and sandface temperature behaviors with different number of grid blocks in the r -direction as a function of time. According to Figure 3.11, unlike the wellbore and sandface pressures, the wellbore and the sandface temperatures exhibit some differences in their behaviors. Since temperature front (signal) does not suddenly reach the sandface in comparison with that of pressure, the wellbore temperature decreases faster than the sandface temperature.

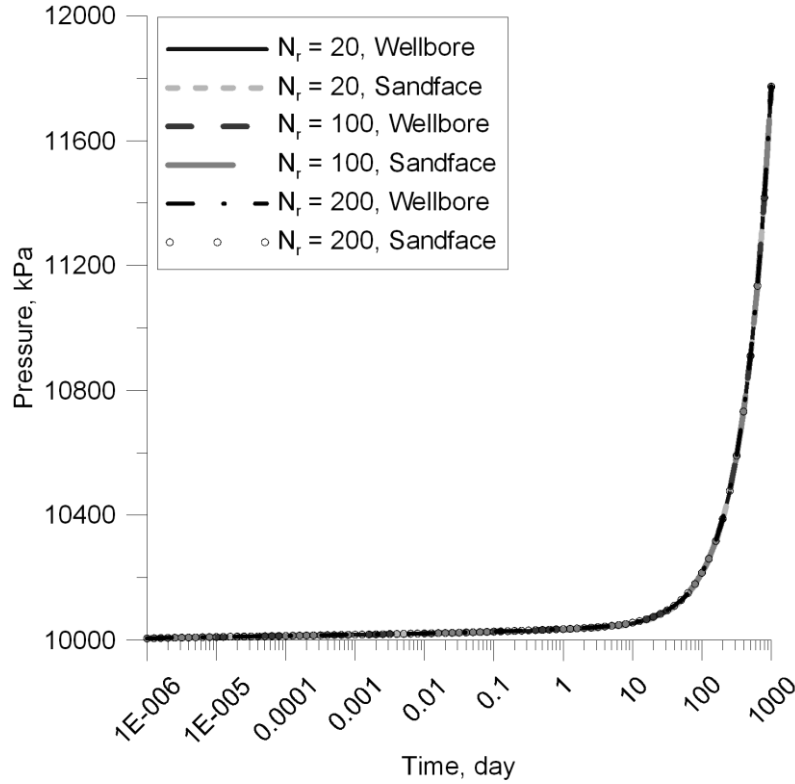


Figure 3.9: Comparison of wellbore and sandface pressure behaviors as a function of time for 1D (r) model during constant-rate injection period.

Here, there is another important point. The sandface temperature is affected by the number of grid blocks in the r -direction while the wellbore temperature is not affected from it. As can be seen from Figure 3.11, at least 100 grid blocks in the r -direction need to be used to model the sandface temperature accurately. However, an extra thin grid next to the well can simply be added to avoid the problem of using many grid blocks in the r -direction. This is depicted in Figure 3.12. The size of the extra grid in the r -direction is taken as 0.006 m. When this extra grid is used, reasonable accuracies of sandface temperature can be reached while keeping the number of grid blocks in the r -direction significantly lower than 100. This is shown in Figure 3.13. For the cases given in Chapter 5, the same approach is used.

3.6.2 Constant-rate production case for a fully penetrating well

In this case, a production is performed during 1000 days at a constant-rate of 1 kg/s from a fully penetrating well. The parameters used in this case are the same as in Table 3.1. In this case and the next cases in this chapter, the grid block structure with an extra cell in the r -direction similar to that shown in Figure 3.12 is used.

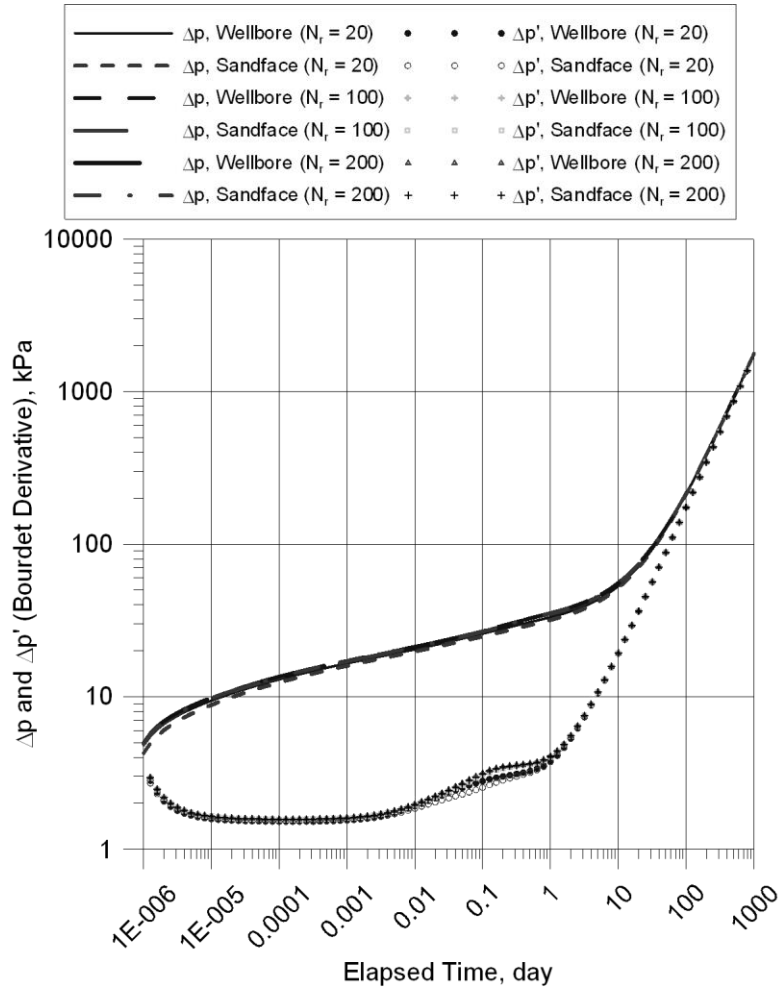


Figure 3.10: Plot of delta pressure and Bourdet derivative of wellbore and sandface as a function of elapsed time for 1D (r) model during constant-rate injection period.

Firstly, the sensitivity of the solutions to the number of grid blocks in the z -direction is investigated by fixing the number of grid blocks in the r -direction. Secondly, the sensitivity of the solutions to the number of gridblocks in the r -direction is investigated by fixing the number of grid blocks in the z -direction.

Figure 3.14 shows the sandface pressure (in the middle of the interval open to flow) behaviors with different number of grid blocks in the z -direction as a function of time. As can be seen from Figure 3.14, the sandface pressure behaviors do not change with number of grid blocks in the z -direction. Figure 3.15 shows delta pressures at the sandface with Bourdet derivatives of those for the same case. Figure 3.15 also supports the pressure behaviors observed in Figure 3.14. As a consequence, it can obviously be seen that number of grid blocks used in the z -direction is not very important for this type of case.

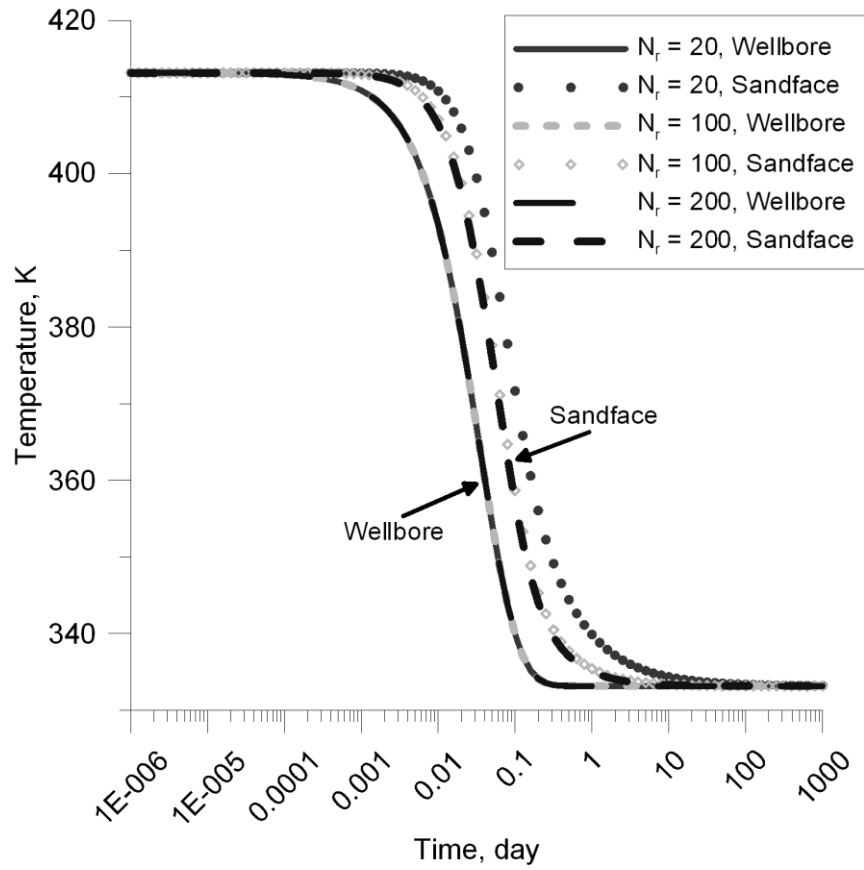


Figure 3.11: Comparison of wellbore and sandface temperature behaviors as a function of time for 1D (r) model during constant-rate injection period.

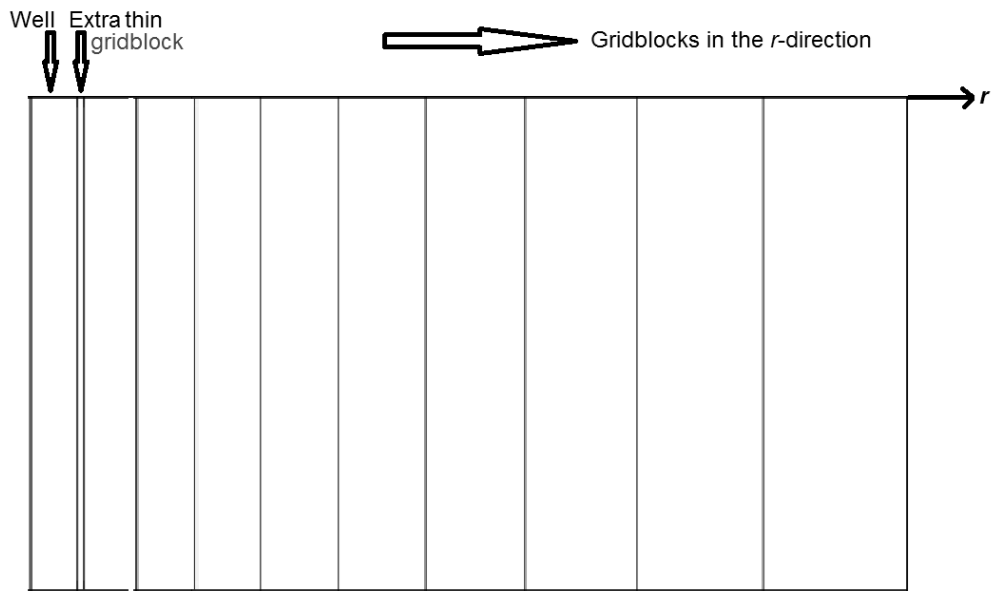


Figure 3.12: Schematic of the grid block structure with an extra cell in the r -direction.

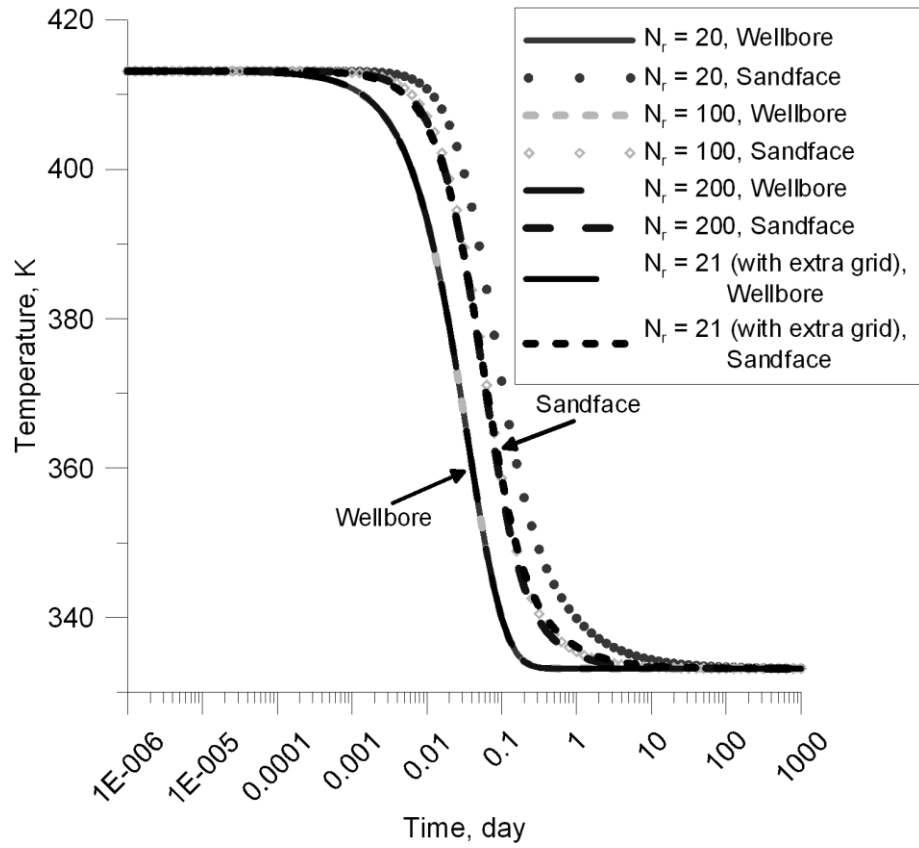


Figure 3.13: Comparison of wellbore and sandface temperature behaviors with extra grid as a function of time for 1D (r) model during constant-rate injection period.

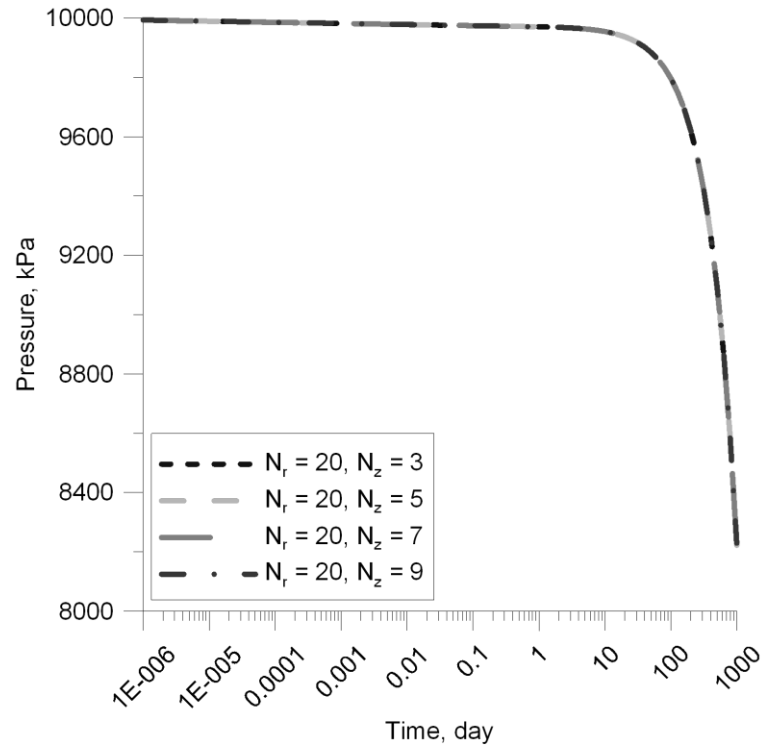


Figure 3.14: Sensitivity of the sandface pressure solutions to the number of gridblocks in the z-direction for 2D (r - z) model during constant-rate production from fully penetrating well.

Figure 3.16 shows the sandface pressure behaviors with different number of grid blocks in the r -direction as a function of time. As can be seen from Figure 3.16, the sandface pressure is not affected so much by the number of grid blocks in the r -direction.

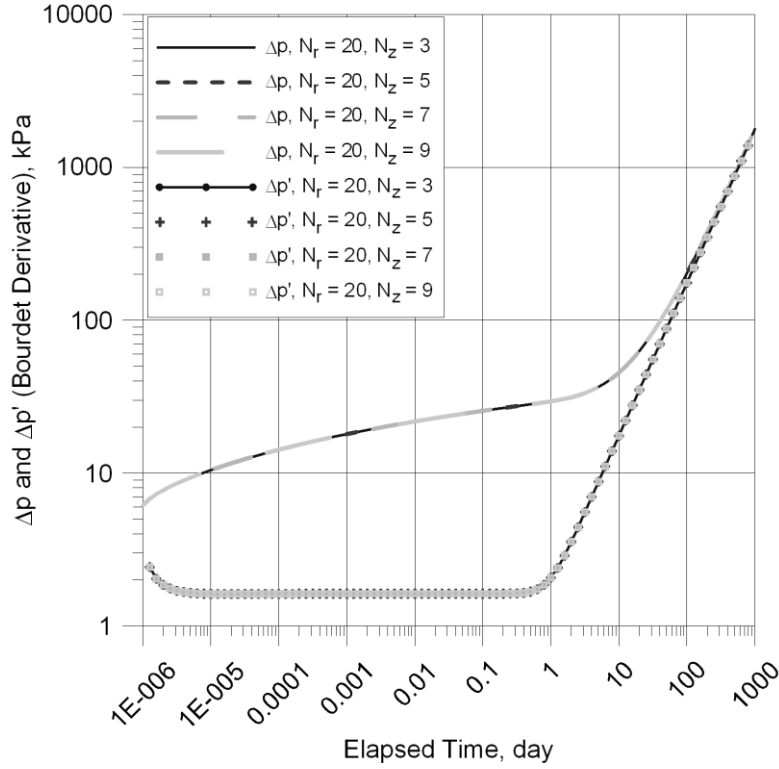


Figure 3.15: Plot of delta pressure and Bourdet derivative at the sandface in the z -direction for 2D (r - z) model during constant-rate production from fully penetrating well.

Figure 3.17 shows delta pressures at the sandface with Bourdet derivatives of those for the same case. Figure 3.17 supports the pressure behaviors observed in Figure 3.16 except that the behaviors are different in the case with $N_r = 5$, $N_z = 9$. Hence, it can be observed from Figure 3.17 that using low number of grid blocks (e.g., $N_r = 5$) in the r -direction is not sufficient to model the pressure behaviors of this type of case.

Figures 3.18 and 3.19 show the sandface temperature behaviors with different number of grid blocks in the z -direction and in the r -direction as a function of time, respectively. As can be seen from Figure 3.18, the sandface temperature is not affected so much by the number of grid blocks in the z -direction. Figure 3.19 is not also affected so much by the number of grid blocks in the r -direction except that the behaviors are different in the case with $N_r = 5$, $N_z = 9$ because number of grid blocks used in the r -direction is not sufficient for this case.

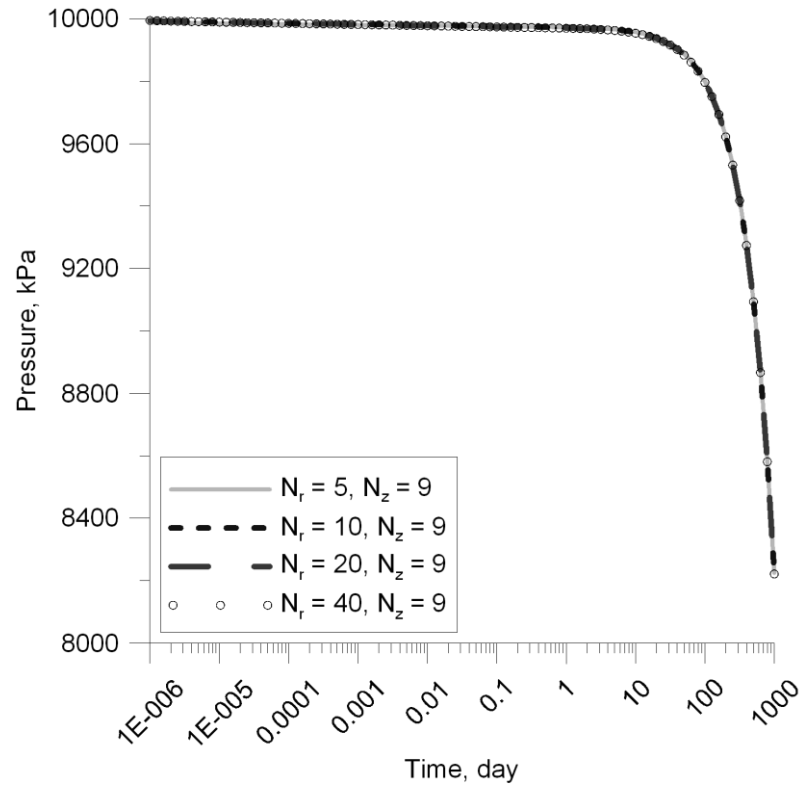


Figure 3.16: Sensitivity of the sandface pressure solutions to the number of gridblocks in the r -direction for 2D (r - z) model during constant-rate production from fully penetrating well.

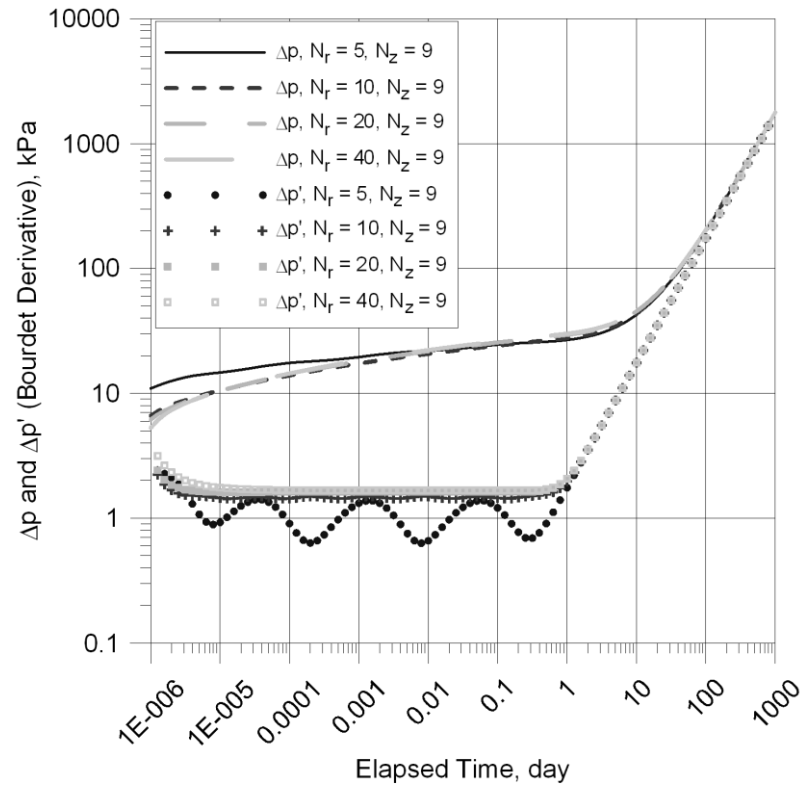


Figure 3.17: Plot of delta pressure and Bourdet derivative at the sandface in the r -direction during constant-rate production from fully penetrating well.

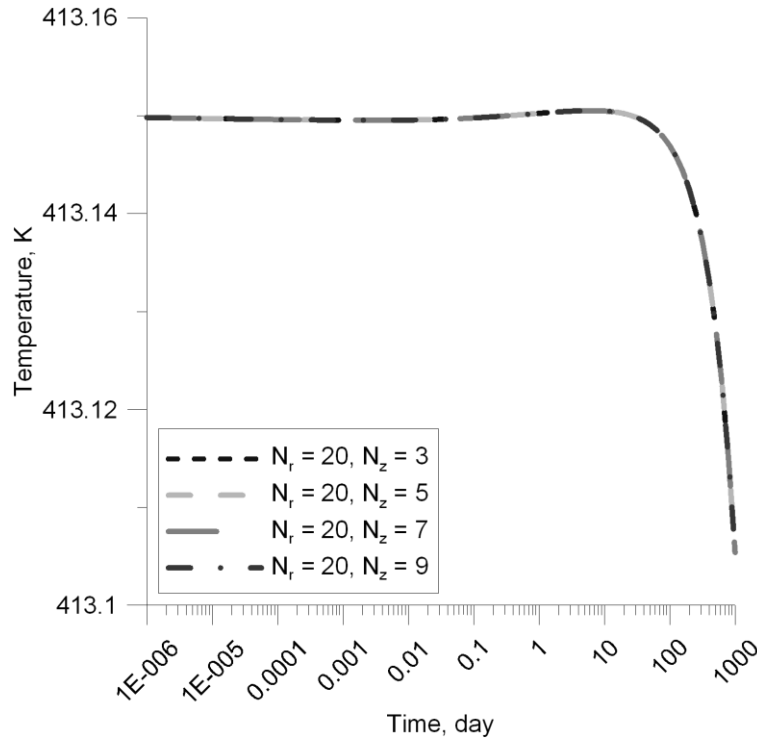


Figure 3.18: Sensitivity of the sandface temperature solutions to the number of gridblocks in the z -direction for 2D (r - z) model during constant-rate production from fully penetrating well.

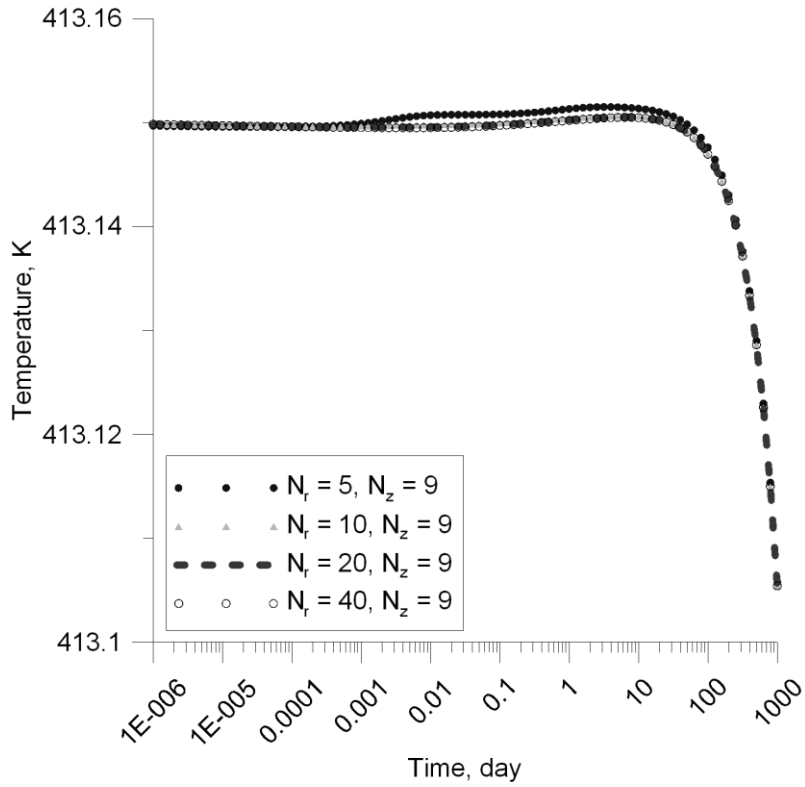


Figure 3.19: Sensitivity of the sandface temperature solutions to the number of gridblocks in the r -direction for 2D (r - z) model during constant-rate production from fully penetrating well.

3.6.3 Constant-rate production case for a limited-entry well

The parameters used in this case are the same as the previous case. This case contains a limited-entry well, interval length open to flow is 1 m, and the open interval to flow is in the middle of the reservoir in the z -direction. Figure 3.20 shows a schematic of the grid structure of this case. As can be seen from Figure 3.20, logarithmic grids are used in the z -direction to model the limited-entry case.

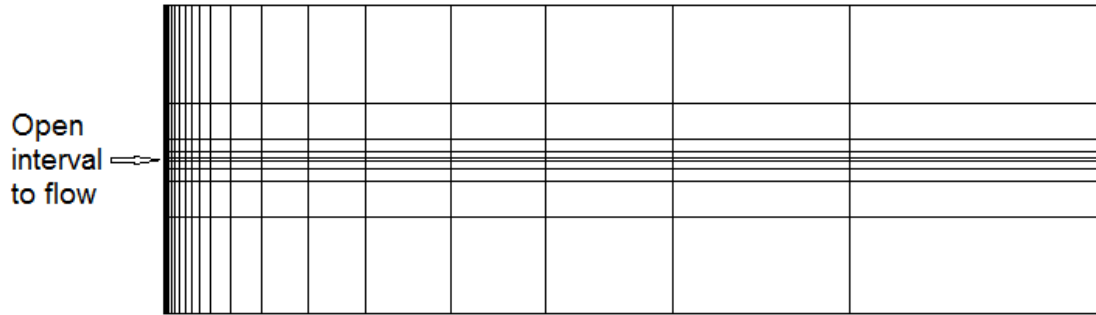


Figure 3.20: Schematic of the gridblock structure for the constant-rate production case with limited-entry well.

Figure 3.21 shows the sandface pressure (in the middle of the interval open to flow) behaviors with different number of gridblocks in the z -direction as a function of time. As can be seen from Figure 3.21, the sandface pressure behaviors of the cases with 3, 5 and 7 gridblocks in the z -direction are significantly different from those of the cases with 9 and 11 grid blocks in the z -direction. From here, it is possible to say that 9 or 11 grid blocks in the z -direction can be used for such cases. Figure 3.22 shows delta pressures at the sandface with Bourdet derivatives of those for the same case. Figure 3.22 also supports the pressure behaviors observed in Figure 3.21.

Figure 3.23 shows the sandface pressure behaviors with different number of grid blocks in the r -direction as a function of time. As can be seen from Figure 3.23, the sandface pressure behaviors of the case with 5 grid blocks in the r -direction are significantly different from those of the other ones. According to Figure 3.23, it is possible to say that the grid systems $N_r = 20$, $N_z = 9$ or $N_r = 40$, $N_z = 9$ can be used for such cases. Figure 3.24 shows delta pressures at the sandface with Bourdet derivatives of those for the same case. Figure 3.24 also supports the pressure behaviors observed in Figure 3.23.

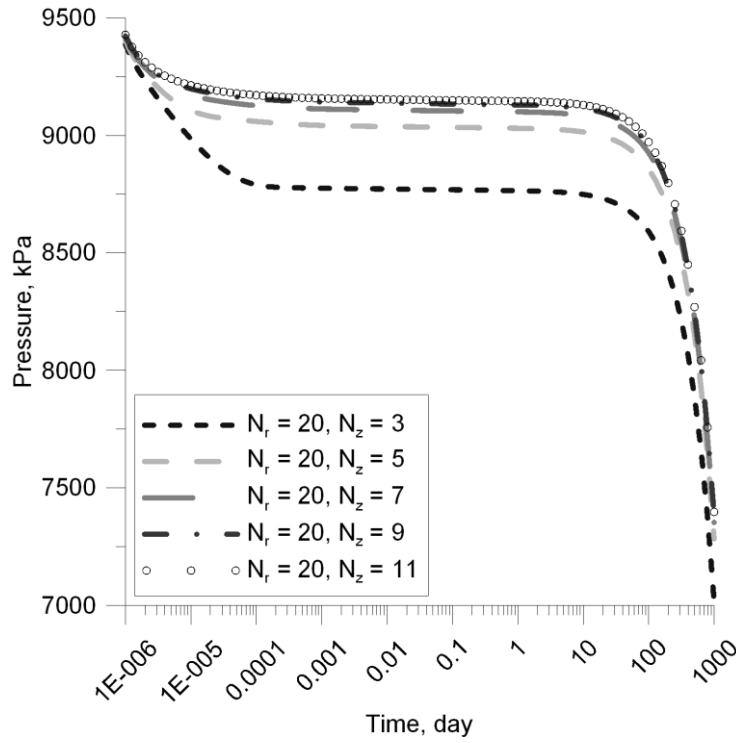


Figure 3.21: Sensitivity of the sandface pressure solutions to the number of grid blocks in the z -direction for 2D (r - z) model with limited-entry well during constant-rate production.

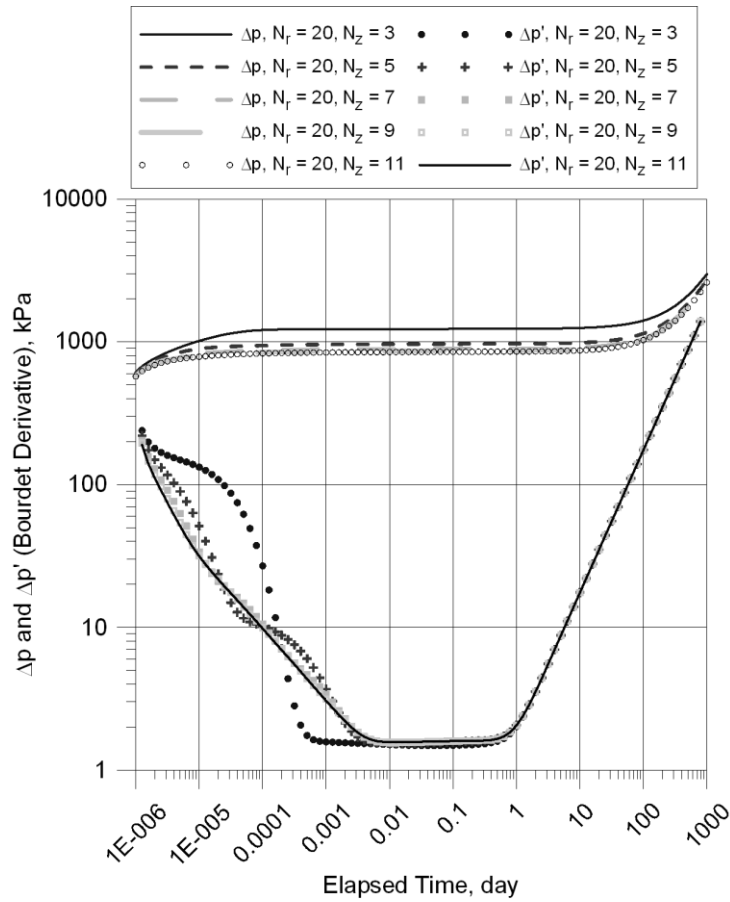


Figure 3.22: Plot of delta pressure and Bourdet derivative at the sandface in the z -direction for 2D (r - z) model during constant-rate production from limited-entry well.

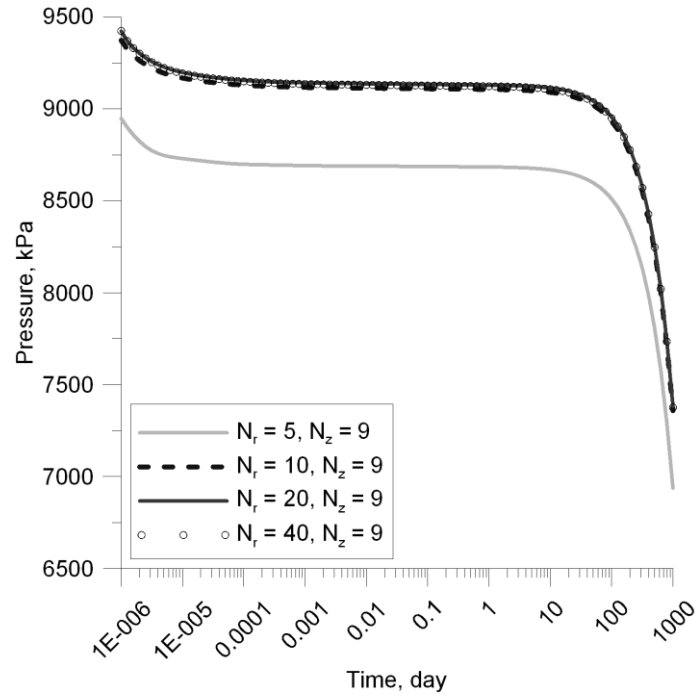


Figure 3.23: Sensitivity of the sandface pressure solutions to the number of grid blocks in the r -direction for 2D (r - z) model with the limited-entry well during constant-rate production period.

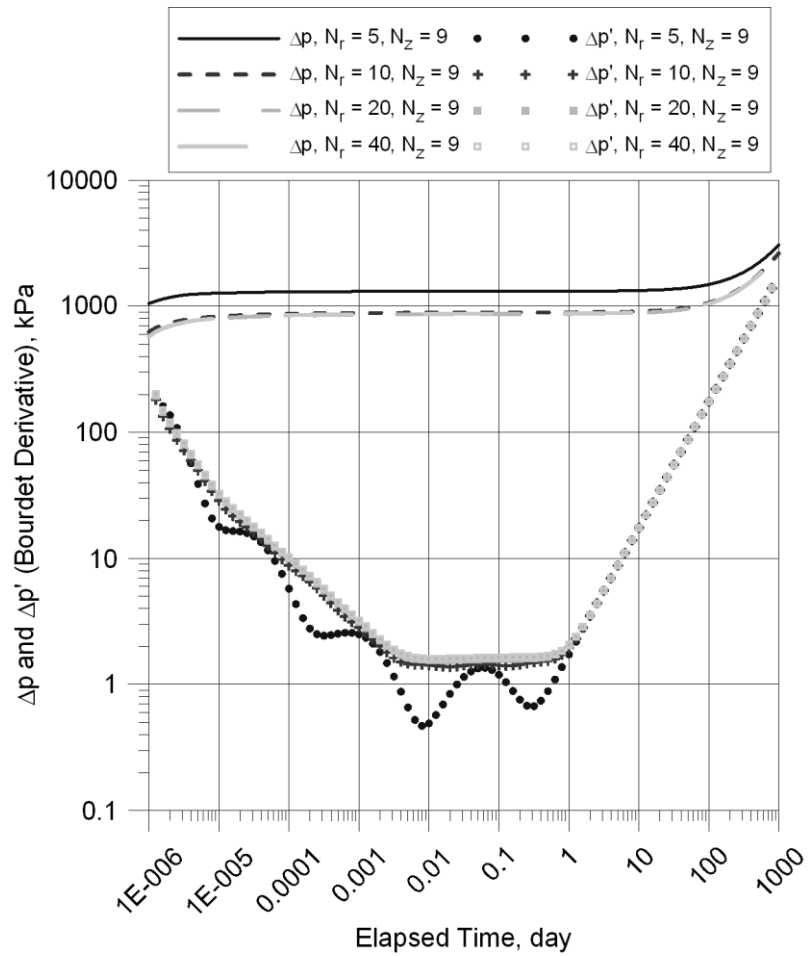


Figure 3.24: Plot of delta pressure and Bourdet derivative at the sandface in the r -direction for 2D (r - z) model during constant-rate production from limited-entry well.

Figure 3.25 shows the sandface temperature behaviors with different number of grid blocks in the z -direction as a function of time for the same case. As can be seen from Figure 3.25, the sandface temperature behaviors of the cases with 3, 5 and 7 grid blocks in the z -direction are different from those of the cases with 9 and 11 grid blocks in the z -direction. From here, it is possible to say that 9 or 11 grid blocks in the z -direction can be used for such cases.

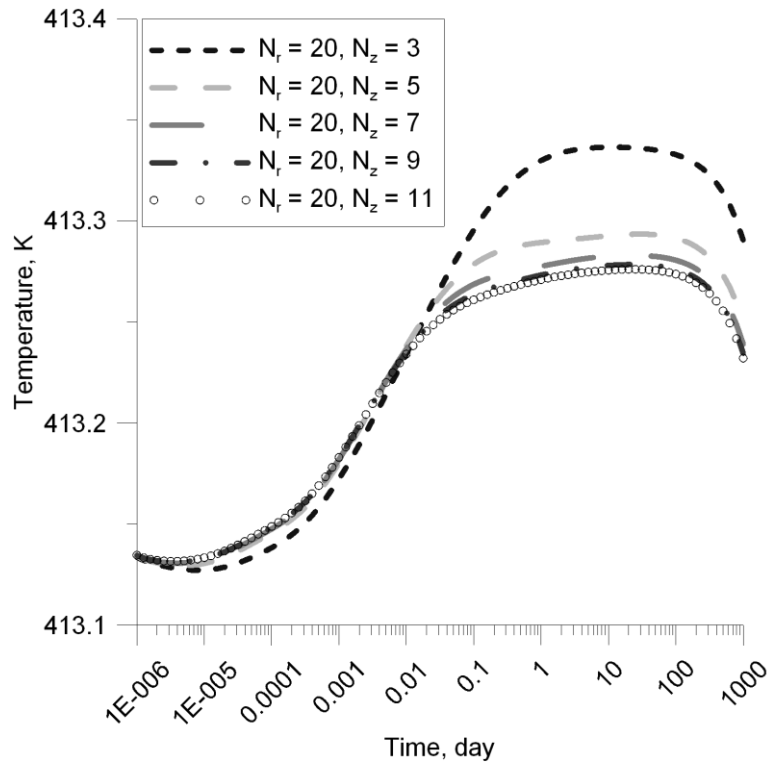


Figure 3.25: Sensitivity of the sandface temperature solutions to the number of grid blocks in the z -direction for 2D (r - z) model with the limited-entry well during constant-rate production period.

Figure 3.26 shows the sandface temperature behaviors with different number of grid blocks in the r -direction as a function of time for the same case. As can be seen from Figure 3.26, the sandface temperature behaviors of the cases with 5 and 10 grid blocks in the r -direction are different from those of the cases with 20 and 40 grid blocks in the r -direction. From here, it is possible to say that the grid systems $N_r = 20$, $N_z = 9$ or $N_r = 40$, $N_z = 9$ can be used for such cases.

3.6.4 Constant-rate injection case for a limited-entry well

The parameters used in this case are the same as the previous case. This case contains a constant mass rate of 1 kg/s injection with an injection temperature of 333.15K. The grid structure of this case is also the same as the previous case.

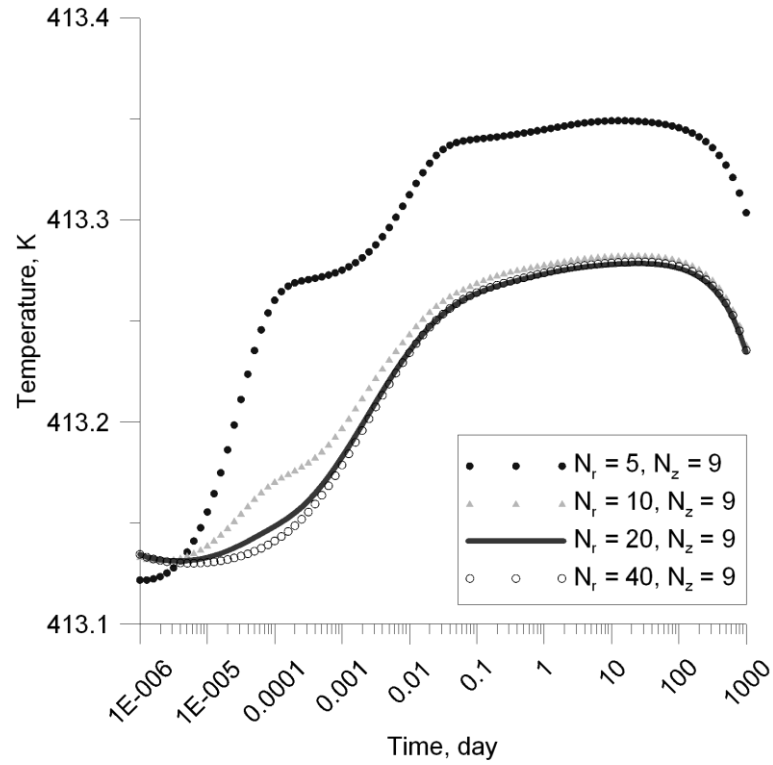


Figure 3.26: Sensitivity of the sandface temperature solutions to the number of grid blocks in the r -direction for 2D (r - z) model with the limited-entry well during constant-rate production period.

Figure 3.27 shows the sandface pressure (in the middle of the interval open to flow) behaviors with different number of grid blocks in the z -direction as a function of time.

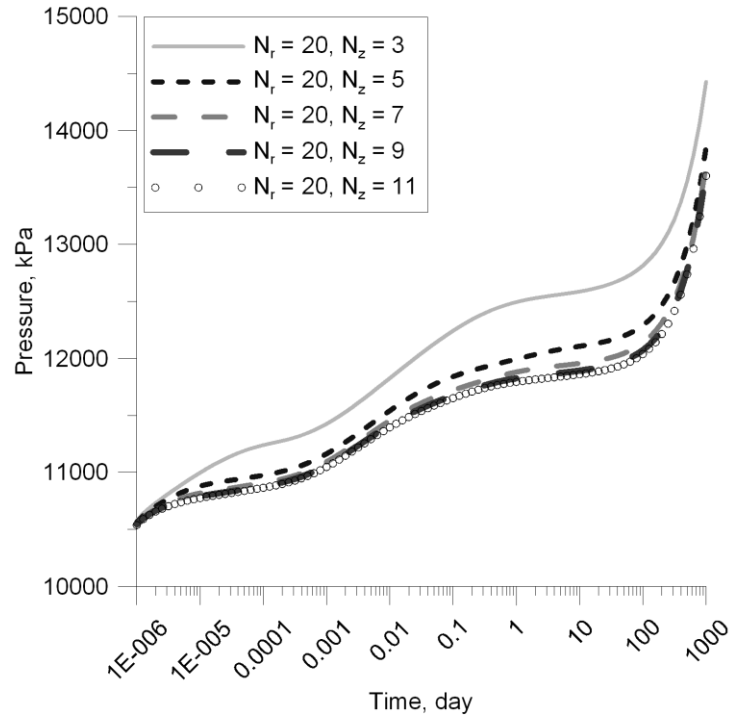


Figure 3.27: Sensitivity of the sandface pressure solutions to the number of grid blocks in the z -direction for 2D (r - z) model with the limited-entry well during constant-rate injection period.

As can be seen from Figure 3.27, the sandface pressure behaviors of the cases with 3, 5 and 7 grid blocks in the z -direction are significantly different from those of the cases with 9 and 11 grid blocks in the z -direction. From here, it is possible to say that 9 or 11 gridblocks in the z -direction can be used for such cases. Figure 3.28 shows delta pressures at the sandface with Bourdet derivatives of those for the same case. Figure 3.28 also supports the pressure behaviors observed in Figure 3.27.

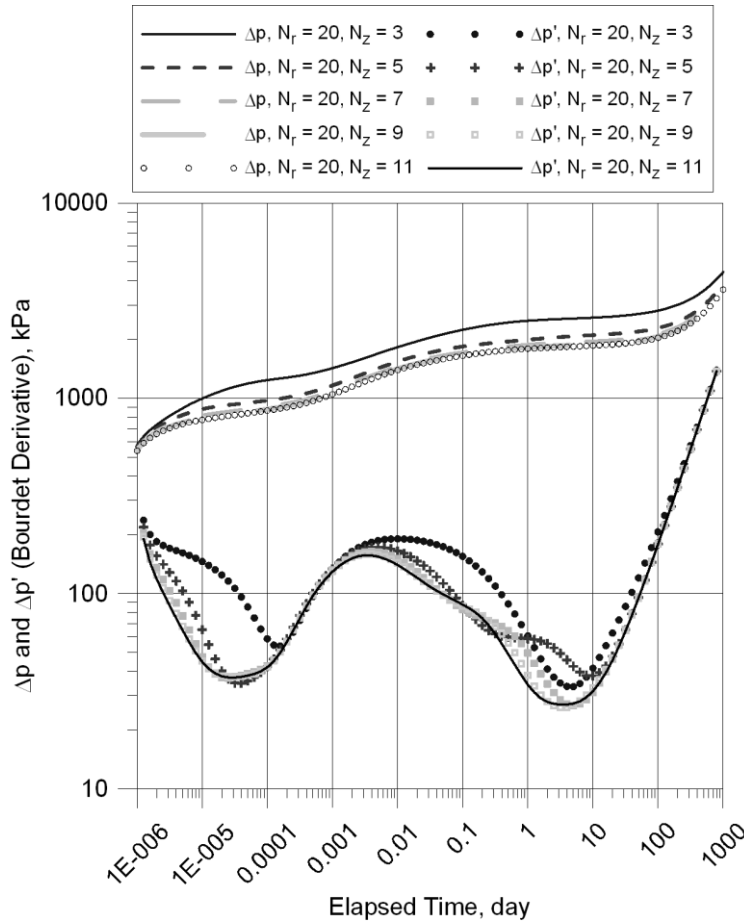


Figure 3.28: Plot of delta pressure and Bourdet derivative at the sandface in the z -direction for 2D (r - z) model during constant-rate injection from limited-entry well.

Figure 3.29 shows the sandface pressure behaviors with different number of grid blocks in the r -direction as a function of time. As can be seen from Figure 3.29, the sandface pressure behaviors of the case with 5 grid blocks in the r -direction are significantly different from those of the other ones. According to Figure 3.29, it is possible to say that the grid systems $N_r = 20$, $N_z = 9$ or $N_r = 40$, $N_z = 9$ can be used for such cases. Figure 3.30 shows delta pressures at the sandface with Bourdet derivatives of those for the same case. Figure 3.30 also supports the pressure behaviors observed in Figure 3.29.

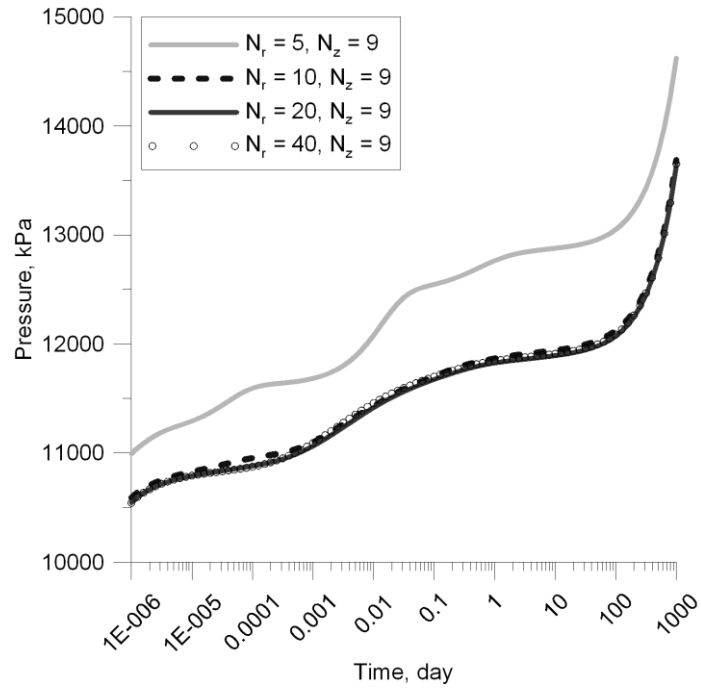


Figure 3.29: Sensitivity of the sandface pressure solutions to the number of grid blocks in the r -direction for 2D (r - z) model with the limited-entry well during constant-rate injection period.

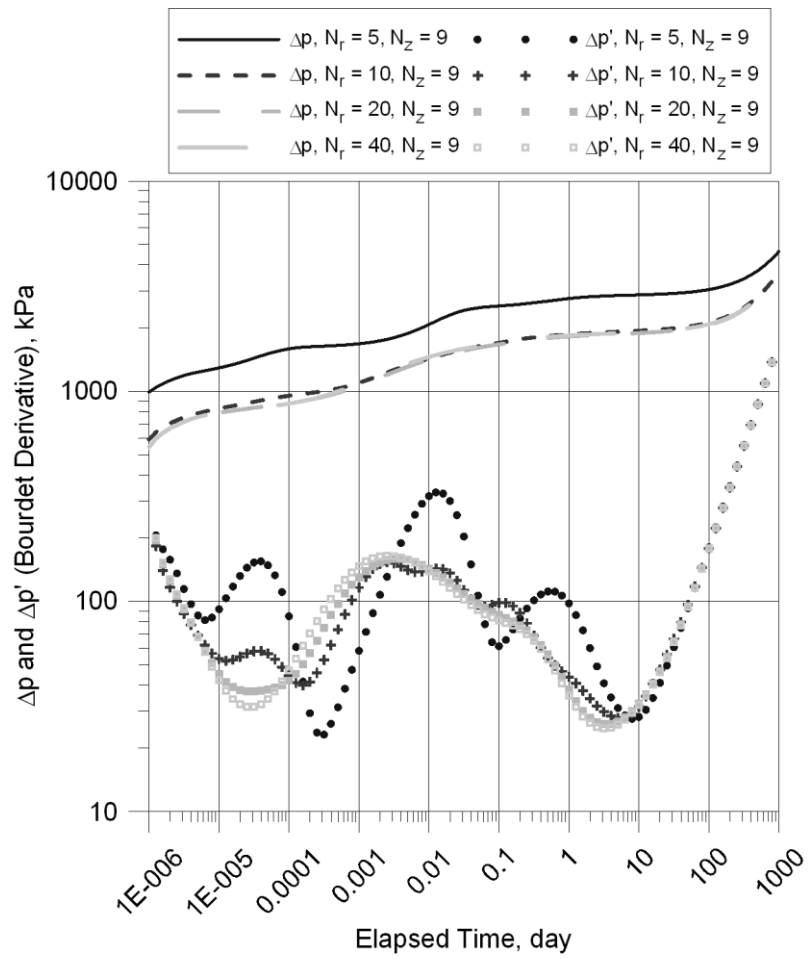


Figure 3.30: Plot of delta pressure and Bourdet derivative at the sandface in the r -direction for 2D (r - z) model during constant-rate injection from limited-entry well.

Figures 3.31 and 3.32 show the sandface temperature behaviors with different number of grid blocks in the z -direction and in the r -direction as a function of time, respectively.

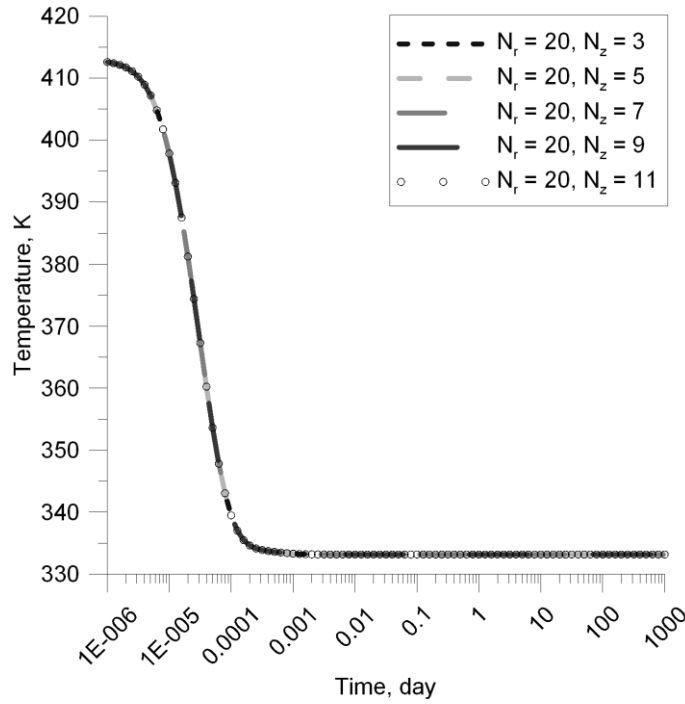


Figure 3.31: Sensitivity of the sandface temperature solutions to the number of grid blocks in the z -direction for 2D (r - z) model with the limited-entry well during constant-rate injection period.

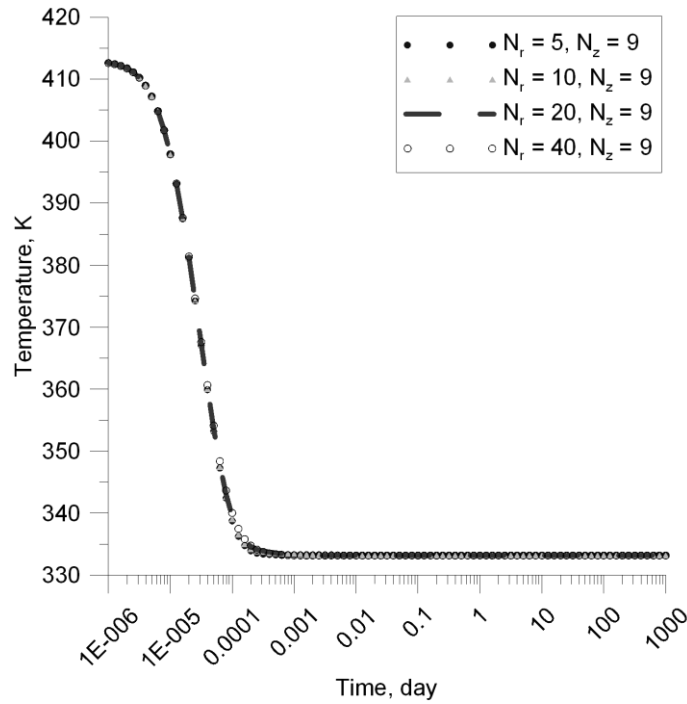


Figure 3.32: Sensitivity of the sandface temperature solutions to the number of grid blocks in the r -direction for 2D (r - z) model with the limited-entry well during constant-rate injection period.

As can be seen from these figures, the sandface temperature behaviors do not change so much from number of gridblocks in the r and z -direction for this case.

4. VERIFICATION OF THE MODEL

In this section, firstly, the verification of the model developed is conducted through comparison with the well-known reservoir simulator TOUGH2 (Pruess et al., 1999) using different scenarios of production and injection. Secondly, a comparison is performed with the 1D (r) analytical solution developed by Lauwerier (1955) in Cartesian coordinates and then modified by Satman (2003) in cylindrical coordinates for a constant-rate injection case.

4.1 Verification of the Model with TOUGH2

Here, the developed model has been verified in a 2D (r - z) closed (no-flow and no-heat flux) reservoir system at the outer boundary (at $r = r_e$) of the reservoir. Heat transfer from reservoir to adjacent overburden and underburden strata (at $z = 0$ and $z = h$) is ignored for these cases. However, at least for the cases considered in this chapter, it has been observed that heat transfer to adjacent strata does not have any significant effect on pressure and temperature behaviors (to be discussed in Chapter 5). Anisotropy in permeability has not been considered in the reservoir for the verification cases. Gravity effect is ignored in all cases performed. In the verification and application cases, well is treated as a source or sink where its volume (V^w) is assumed so small (approximately $3 \times 10^{-4} \text{ m}^3$) for both simulators (the model developed and TOUGH2) unless otherwise stated. Hence, wellbore storage effect is ignored and it is assumed that geothermal water is pure water (no dissolved solids inside the water). Salinity affects the magnitudes of fluid properties (e.g., density, viscosity, etc.) and not the rock properties. Hence, it is expected to be seen some slight changes in the pressure and temperature when water contains some dissolved solids. However, it is not expected changes in the behaviors of pressure and temperature transients and would be similar to those given here and in the next chapter for pure water.

4.1.1 Case 1: Constant-rate injection for a fully penetrating well

This scenario (Case 1) contains a constant mass rate of 1 kg/s injection from a fully penetrating vertical well with an injection temperature of 333.15K. The other pertinent reservoir parameters used in this case are given in Table 4.1.

Table 4.1 : Input data for 2D (r - z) reservoir systems, cases 1 and 2.

Parameter	Value
N_r	20
N_z	9
P^0 , kPa	10000
T^0 , K (°C)	413.15 (140)
r_w , m	0.1
r_e , m	1000
h , m	100
V^w , m ³	3×10^{-4}
ϕ	0.2
k , m ² (md)	1×10^{-13} (100)
c_s , 1/kPa	2.9×10^{-7}
β_s , 1/K	1×10^{-5}
$C_{p,s}$, J/kg-K	1000
ρ_s , kg/m ³	2650
λ_t , J/m-s-K	2.92

Figures 4.1 and 4.2 show pressure and temperature behaviors along the radial distance in reservoir for the model and TOUGH2, respectively. As can be seen from the figures, both pressure and temperature behaviors from the two different simulators exhibit an excellent agreement for all time periods (from 2×10^{-5} days to 1000 days).

Figures 4.3 and 4.4 show sandface pressure and temperature (in the middle of the interval open to flow) behaviors as a function of time for model and TOUGH2, respectively. As can be seen from these figures, both pressure and temperature behaviors from both different simulators exhibit a very good agreement. Figure 4.5 shows delta pressures at the sandface with Bourdet derivatives of those for the same case. Figure 4.5 also supports the pressure behaviors observed in Figure 4.3.

4.1.2 Case 2: Constant-rate production for a fully penetrating well

Here, a production scenario with a constant mass rate of 1 kg/s (case 2) has been performed from a fully penetrating vertical well during 1000 days. The pertinent reservoir parameters used in this case are the same as Table 4.1.

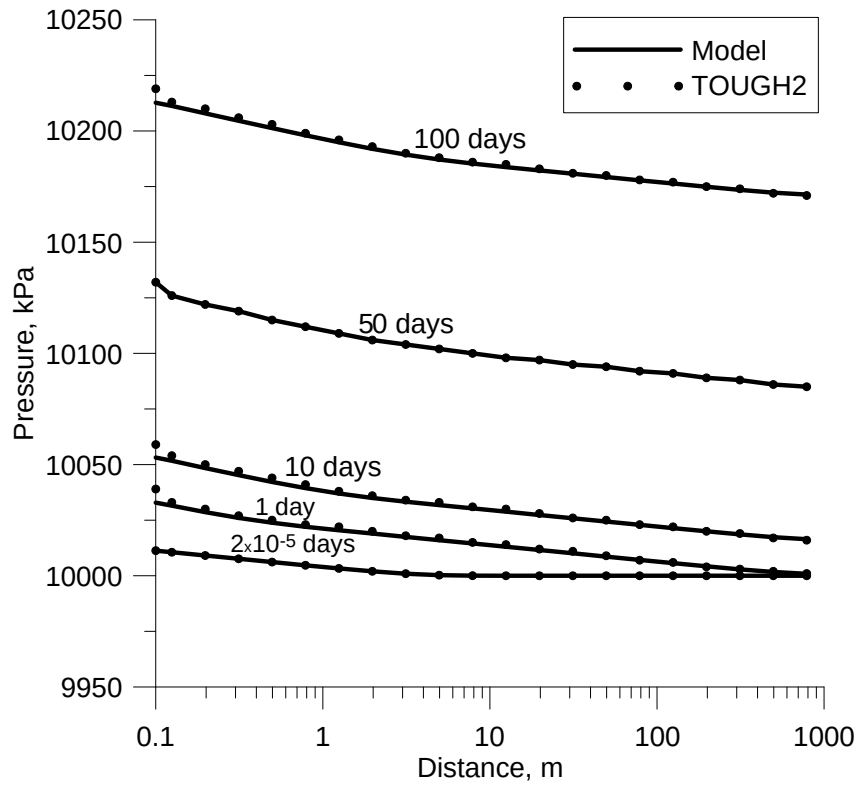


Figure 4.1: Comparison of pressure behaviors along the radial distance in reservoir for the model and TOUGH2 during constant-rate injection period, case 1.

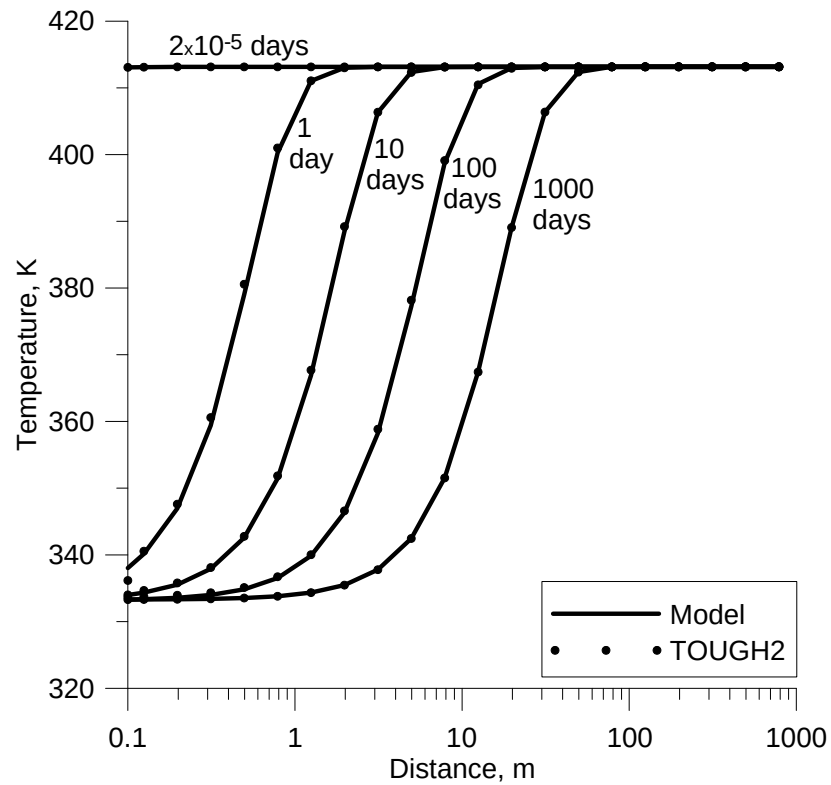


Figure 4.2: Comparison of temperature behaviors along radial distance in reservoir for the model and TOUGH2 during constant-rate injection period, case 1.

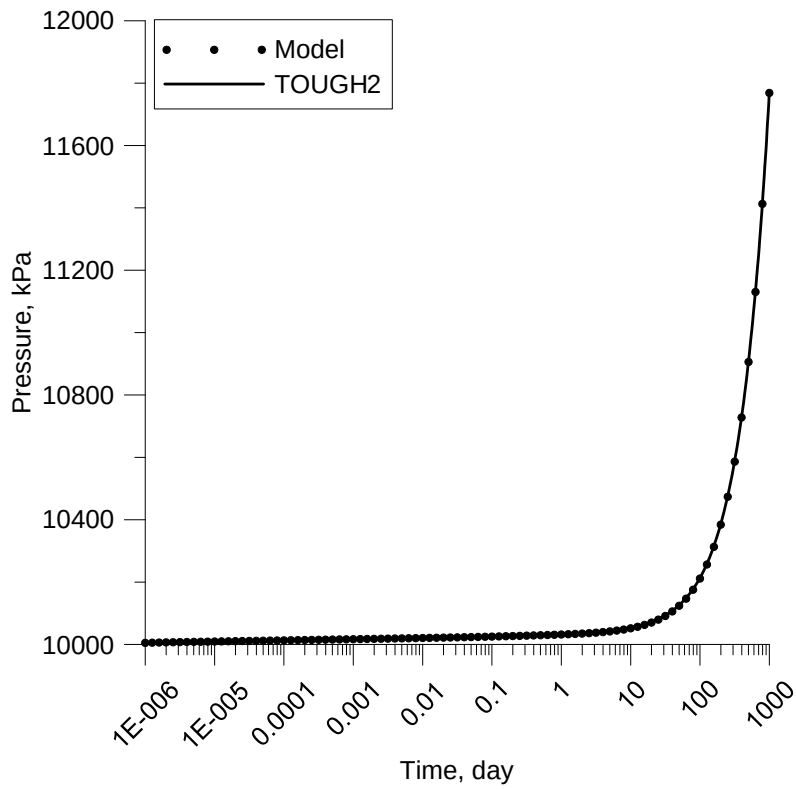


Figure 4.3: Comparison of sandface pressure behaviors for the model and TOUGH2 as a function of time during constant-rate injection period, case 1.

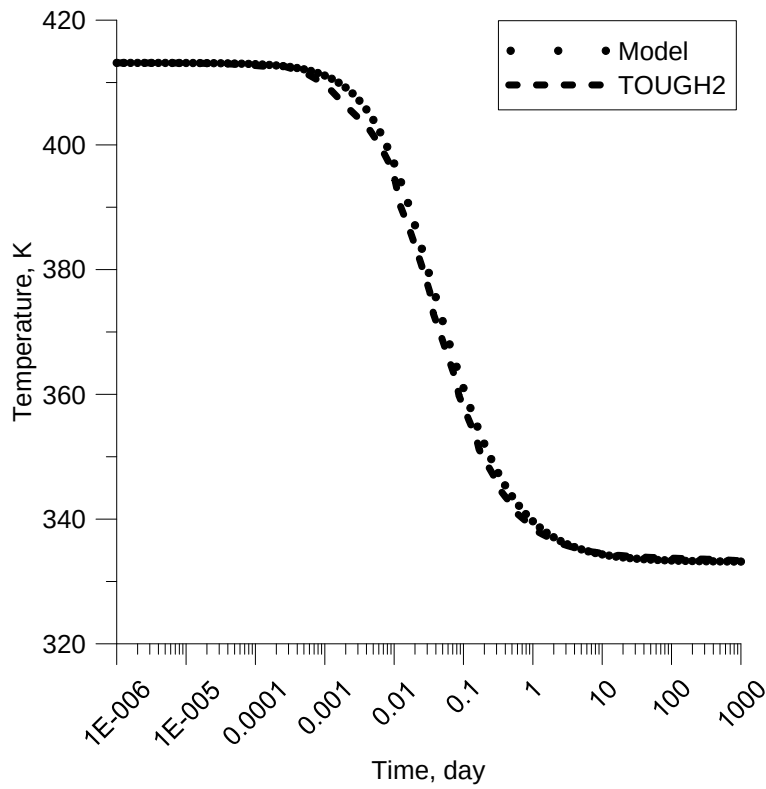


Figure 4.4: Comparison of sandface temperature behaviors for the model and TOUGH2 as a function of time during constant-rate injection period, case 1.

Figures 4.6 and 4.7 show sandface pressure and temperature (in the middle of the interval open to flow) behaviors as a function of time for model and TOUGH2, respectively. As can be seen from these figures, both pressure and temperature behaviors from both different simulators exhibit an excellent agreement. Figure 4.8 shows delta pressures at the sandface with Bourdet derivatives of those for the same case. Figure 4.8 also supports the pressure behaviors observed in Figure 4.6.

Finally, if Figures 4.6 and 4.7 are carefully inspected, it can be seen that pressure and temperature behaviors exhibit similar trends for production case, i.e., pressure and temperature decrease slightly with time during the infinite-acting period (i.e., the effect of the no-flow and no-heat flux boundary is not felt at the sandface of the well) and then pressure and temperature rapidly decreases when the effect of no-flow and no-heat flux boundary at $r = r_e$ are felt at the sandface. The effect of no-flow and no-heat boundary felt at the sandface for this reservoir system is around 2 days. On the other hand, as expected, the pressure and temperature behaviors for the injection case (case 1) in Figures 4.3 and 4.4 exhibit different trends than the corresponding behaviors for the production case (case 2) in Figs. 4.6 and 4.7.

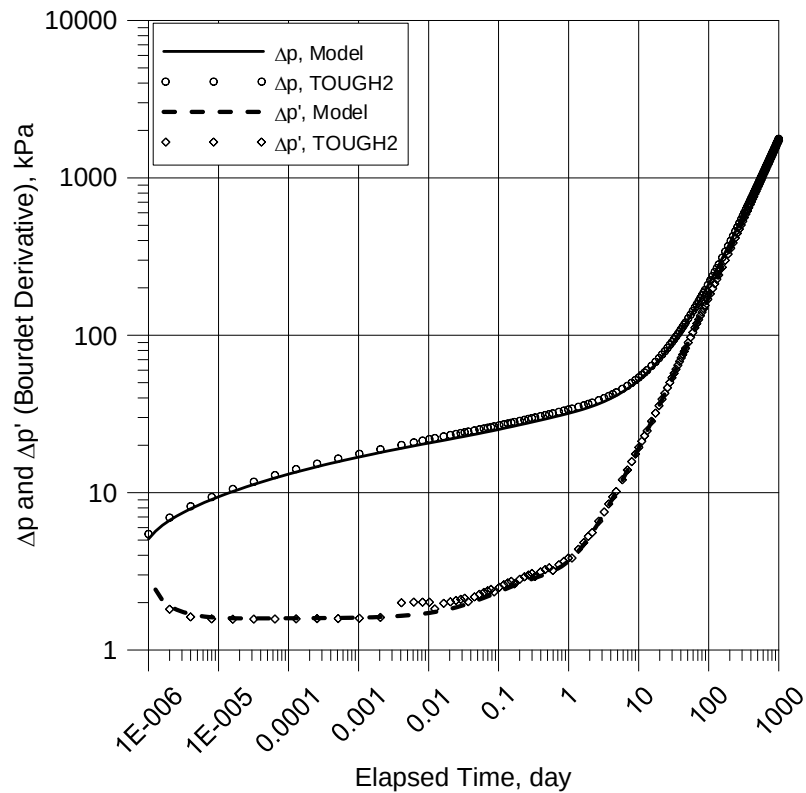


Figure 4.5: Plot of delta pressure and Bourdet derivative at the sandface for the model and TOUGH2 as a function of time during constant-rate injection.

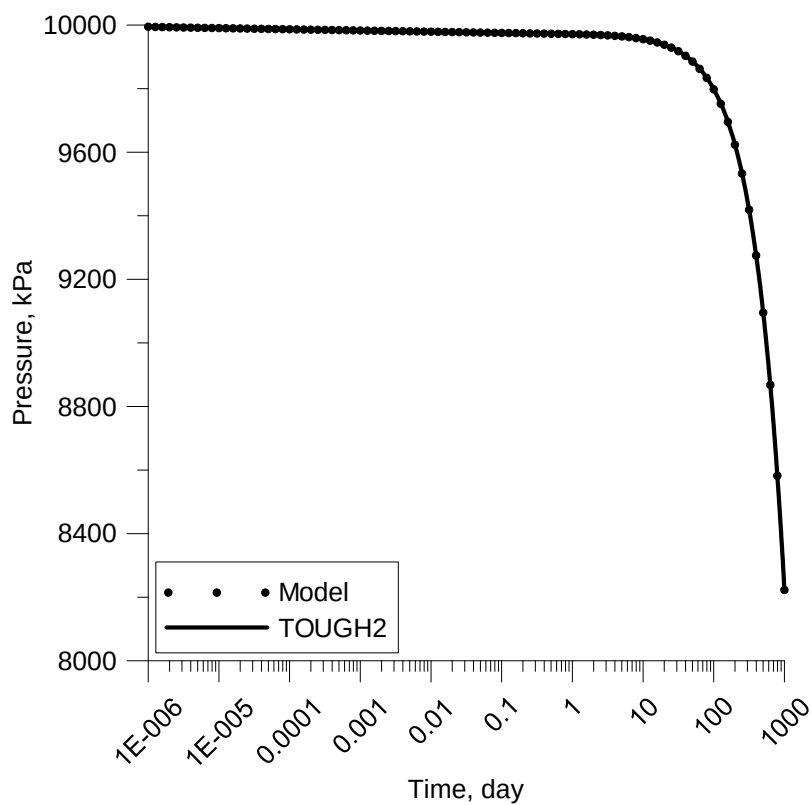


Figure 4.6: Comparison of sandface pressure behaviors for the model and TOUGH2 as a function of time during constant-rate production period, case 2.

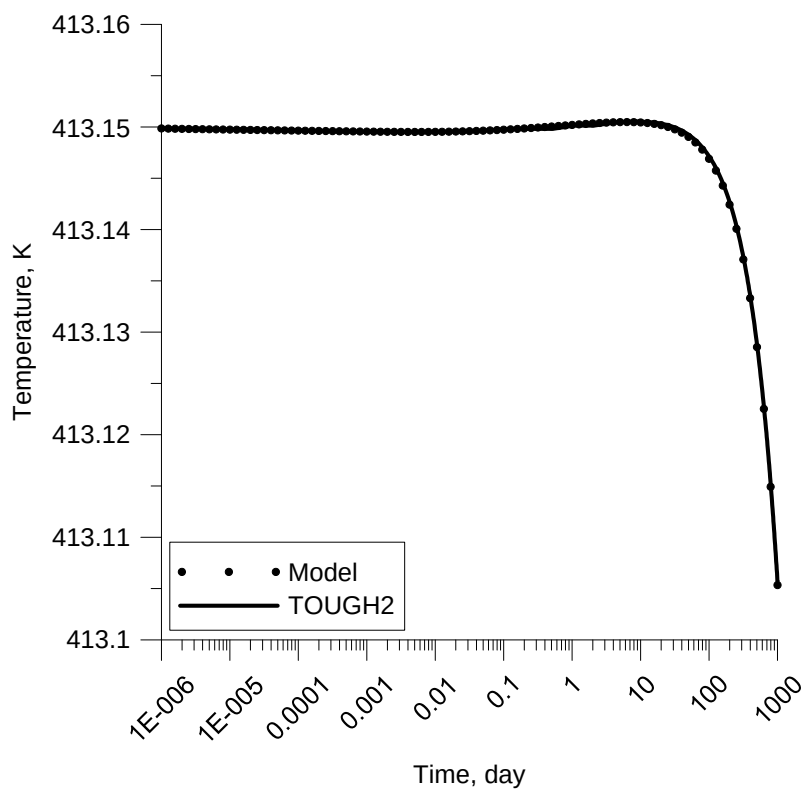


Figure 4.7: Comparison of sandface temperature behaviors for the model and TOUGH2 as a function of time during constant-rate production period, case 2.

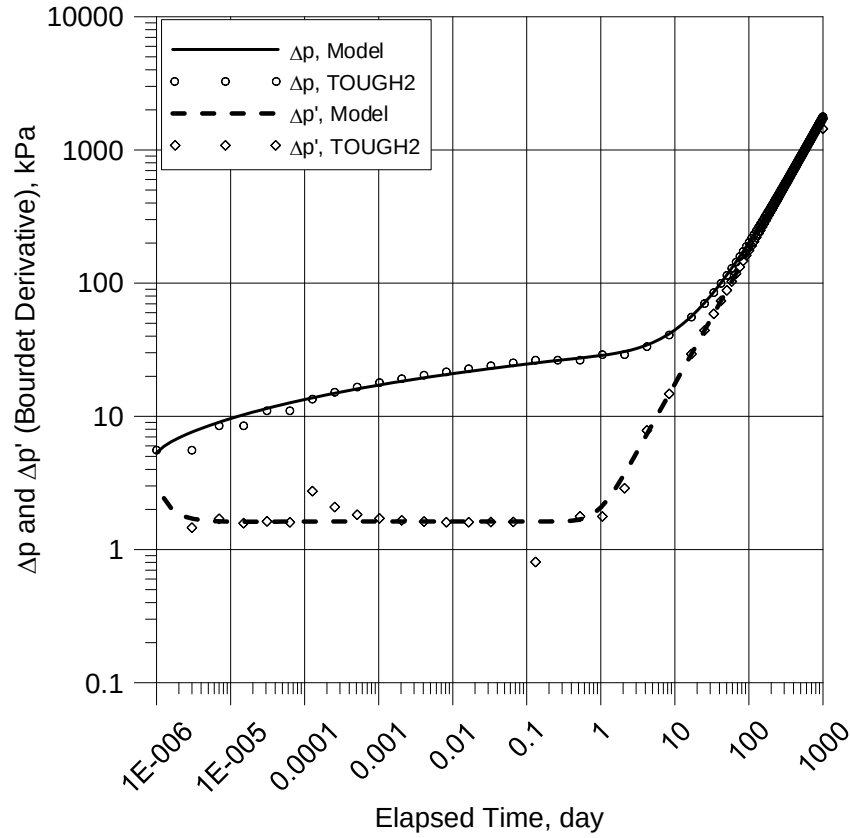


Figure 4.8: Plot of delta pressure and Bourdet derivative at the sandface for the model and TOUGH2 as a function of time during constant-rate production.

In the injection case, convection is the dominant process and the magnitudes of temperature changes are much bigger than that of the production case. Consequently, temperature behavior does not follow the trend of pressure behavior unlike the production case. Conversely, in production case, trend of temperature behavior is dependent on the pressure behavior because of the fluid expansion. The details of these behaviors will be discussed in Chapter 5.

4.2 Case 3: Verification of the Model with an Analytical Solution

In this subsection, a comparison is performed with a 1D (r) analytical solution for convective heat transport in porous media. This study was developed by Lauwerier (1955) in the Cartesian coordinates and modified by Satman (2003) in cylindrical coordinates for a constant-rate injection case. The detailed equations of this analytical solution are presented in Appendix B. The main assumptions of the solution are as follows:

- Reservoir temperature is only dependent on the distance r due to convection.
- Conduction is considered only in the z -direction at reservoir-overburden and reservoir-underburden interfaces.
- Conduction in the r -direction is neglected in the reservoir, adjacent overburden and underburden strata.
- Convection in the r -direction is neglected in the adjacent overburden and underburden strata.
- Density of water and solid rock with heat capacity of water and solid rock are assumed constant, so it is assumed that these parameters do not change with temperature as a function of time.

This injection case is performed at a constant volumetric rate of $0.0417 \text{ m}^3/\text{s}$ ($3600 \text{ m}^3/\text{day}$) during 1500 days for a 1D (r) reservoir. The other pertinent reservoir parameters used in this case are given in Table 4.2. It should be noted that in the analytical solution, the product of density and specific heat capacity for rock and water as well as the thermal conductivity are treated as constant at their values given in Table 4.2. The product of the density and specific heat capacity (ρC_p) is usually referred to as the thermal capacity.

Table 4.2: Input data for 1D (r) reservoir for the analytical case, case 3.

Parameter	Value
T^0 , K (°C)	375.15 (102)
T_{inj} , K (°C)	333.15 (60)
h , m	24
$\rho_s C_{p,s}$, J/m ³ -K	2.728×10^6
$\rho_w C_{p,w}$, J/m ³ -K	4.2×10^6
λ_t , J/m-s-K	2

Figure 4.9 shows the temperature behavior at $r = 100 \text{ m}$ as a function of time for the model and the analytical solution. As can be seen from the figure, temperature behaviors from the two different models exhibit the similar trends, though there are some small differences between the two models, particularly at earlier times. These differences may be caused by the different assumptions made by two different approaches. For instance, the analytical solution assumes that some reservoir parameters such as heat capacity of water and density are constant (independent of temperature and pressure).

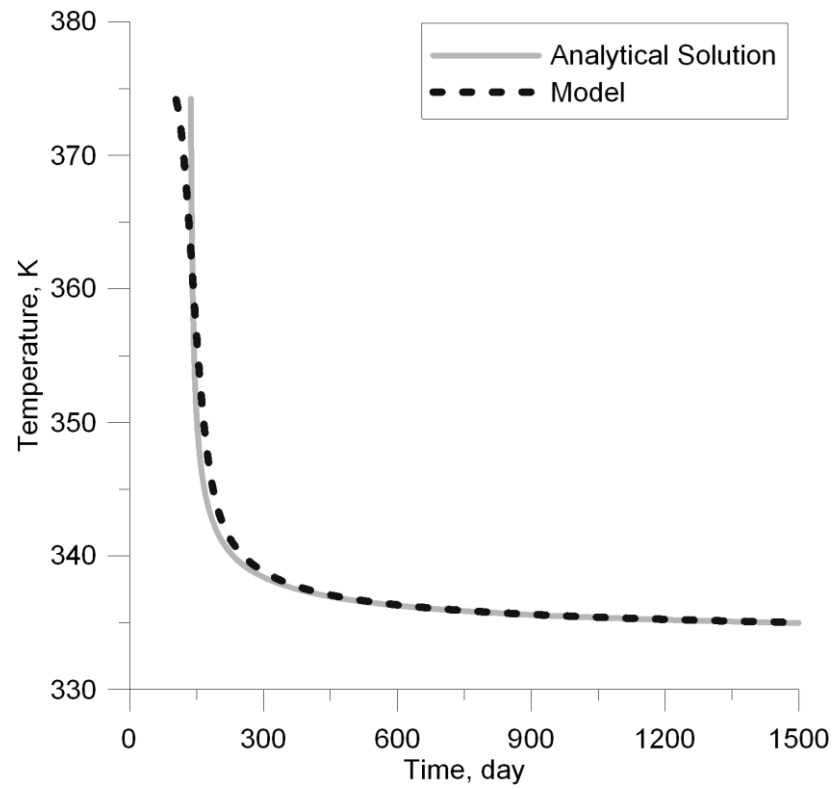


Figure 4.9: Comparison of temperature behaviors for the model and analytical solution at $r = 100$ m as a function of time during constant-rate injection period.

In the numerical model, it is considered that such parameters are rigorously treated as a function of temperature and pressure. Nevertheless, the differences in the temperatures between the numerical model and the analytical solution are quite small, which further validates the model.

5. PRESSURE AND TEMPERATURE BEHAVIORS

In this section, various synthetic case studies simulated with the developed model in the 2D (r - z) reservoir system for predicting both pressure and temperature behaviors of geothermal reservoirs containing single-phase water are presented. The following conditions are valid for all the application cases throughout this chapter unless otherwise stated. The grid system where $N_r = 21$ and $N_z = 9$ containing an extra grid as discussed in Chapter 3 is used in the application cases. In the application cases, well is treated as a source or sink where its volume (V^w) is assumed so small (approximately $3.14 \times 10^{-4} \text{ m}^3$). Hence, wellbore storage effect is ignored and it is assumed that geothermal water is pure water (no dissolved solids inside the water). Production and injection cases have been modeled for a system with a closed outer boundary. A closed outer boundary condition in the r -direction indicates no-flow and no-heat flux (insulated) at the outer boundary at $r = r_e$ of the reservoir. Heat transfer (loss and/or gain) to adjacent strata (overburden at $z = 0$ and underburden at $z = h$) is not taken into consideration. Reservoir temperature is constant (413.15K) at the initial conditions. Anisotropy is not taken into consideration (horizontal and vertical permeabilities are the same). Sensitivities of pressure and temperature responses to rock parameters like porosity, permeability, skin factor, rock thermal conductivity and heat capacity are investigated. The effects of well completion (fully penetrating or limited-entry) are also considered. Effects of different intervals for a well having limited-entry and effects of different injection temperatures on pressure and temperature behaviors for a limited-entry well are also discussed. Parameters in Table 4.1 given in Chapter 4 are also used in this chapter.

5.1 Production Case

In this subsection, different constant-rate production and production-buildup cases are discussed for fully penetrating and limited-entry wells. Additionally, a variable-rate production case is also discussed for a fully penetrating well.

5.1.1 Effect of porosity and flow rate on the sandface pressure and temperature

In the first case, the sandface pressure and temperature behaviors in periods of 200 days at constant flow rates of 1 kg/s and 5 kg/s are observed from a fully penetrating well. Parameters in Table 4.1 given in Chapter 4 are also used in this case. The other reservoir and well properties mentioned at the beginning of Chapter 5 are also valid for this case. Figure 5.1 shows schematic of a fully penetrating production well.

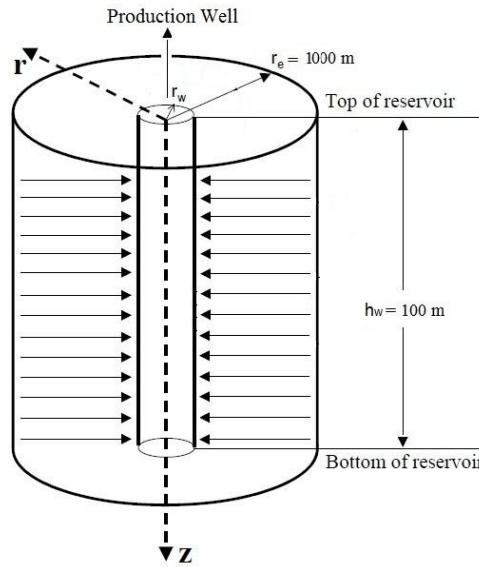


Figure 5.1: Schematic of a fully penetrating production well.

Figure 5.2 shows the log-log diagnostic plots of delta pressure at the sandface (in the middle of the interval open to flow) and derivative of delta pressure of a fully penetrating well in reservoirs with different porosity values during constant-rate production of 200 days. According to Figure 5.2, pressure drop increases as porosity decreases. This is an expected result because a reservoir with higher porosity has more fluid to store and support the pressure drop caused by withdrawal of a certain amount of fluid from the reservoir and so, much pressure drop occurs in a reservoir with lower porosity to supply the same amount of fluid production. For closed systems, the following approximation can be used: $\Delta p_1/\Delta p_2 = \phi_1/\phi_2$. If this approximation is used for this case, it can be found that $\Delta p_{0.2}/\Delta p_{0.04} = 0.2/0.04 = 5$. As is also seen from Figure 5.2, pressure drop observed in the reservoir with lower porosity (0.04) is approximately 5 times higher than that of the higher one (0.2). Moreover, as can be seen from Figure 5.2, the radial flow (exhibited by a constant value of the Bourdet

derivative; Bourdet et al., 1989) is observed at early times (for instance, approximately until 1 day as can be seen from the derivative behavior of the reservoir producing at a constant-rate of 1 kg/s with $\phi = 0.20$) for the reservoirs. After the radial flow period, derivative line with +1 slope indicating that the reservoir outer boundary effects are felt is observed. The pseudosteady-state flow regime (pressure change is constant with changing time) is observed during this period (up to 200 days).

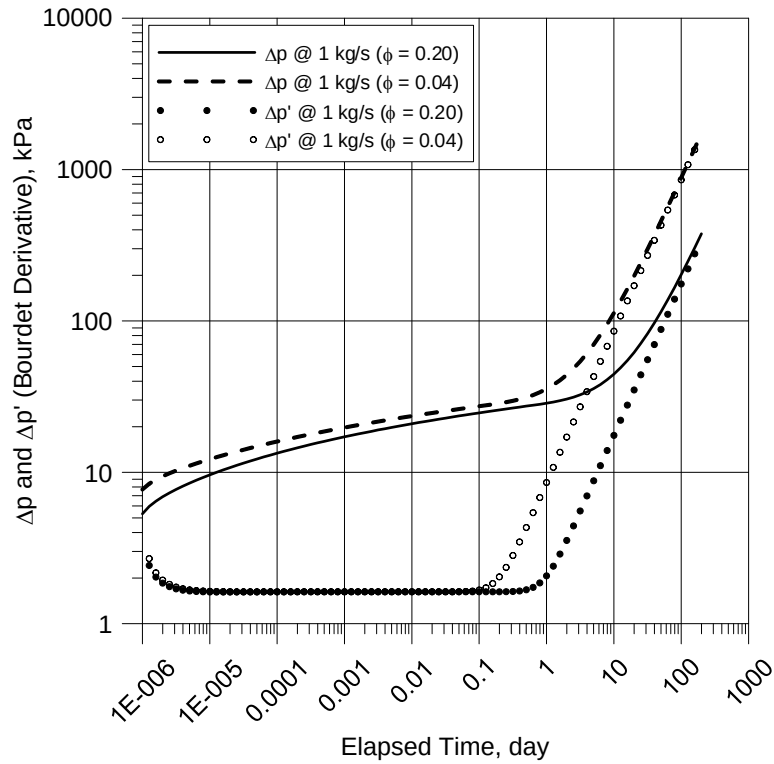


Figure 5.2: Effect of reservoir porosity on the sandface pressure drop of a fully penetrating well producing at a constant-rate.

Figure 5.3 shows the log-log diagnostic plots of delta pressure and derivative of delta pressure as a function of elapsed time for the same case in view of rate effect. If Figure 5.3 is inspected, much more pressure drop occurs in a higher rate (5 kg/s) because much more amount of fluid is withdrawn from the reservoir in contrast to that in a lower rate (1 kg/s).

Figure 5.4 shows the sandface temperature behaviors of a fully penetrating well in reservoirs with different porosity values. According to Figure 5.4, temperature has higher values as porosity decreases. This is caused by the fact that the reservoir with a lower porosity has a higher heat content than that of reservoir having higher porosity.

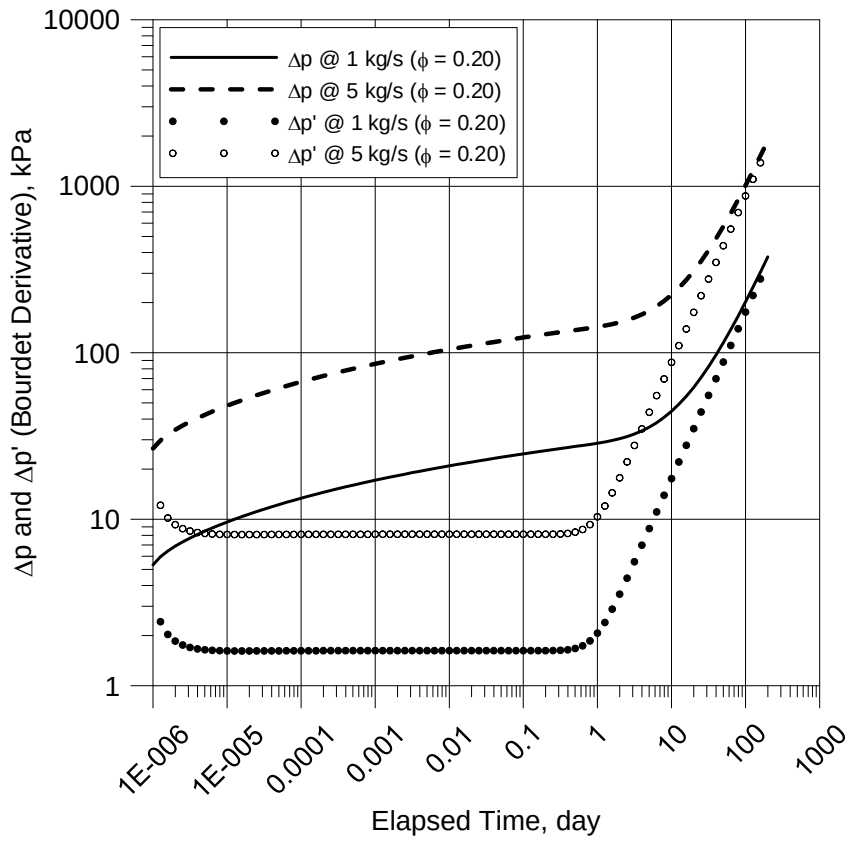


Figure 5.3: Effect of flow rate on the sandface pressure drop of a fully penetrating well producing at a constant-rate.

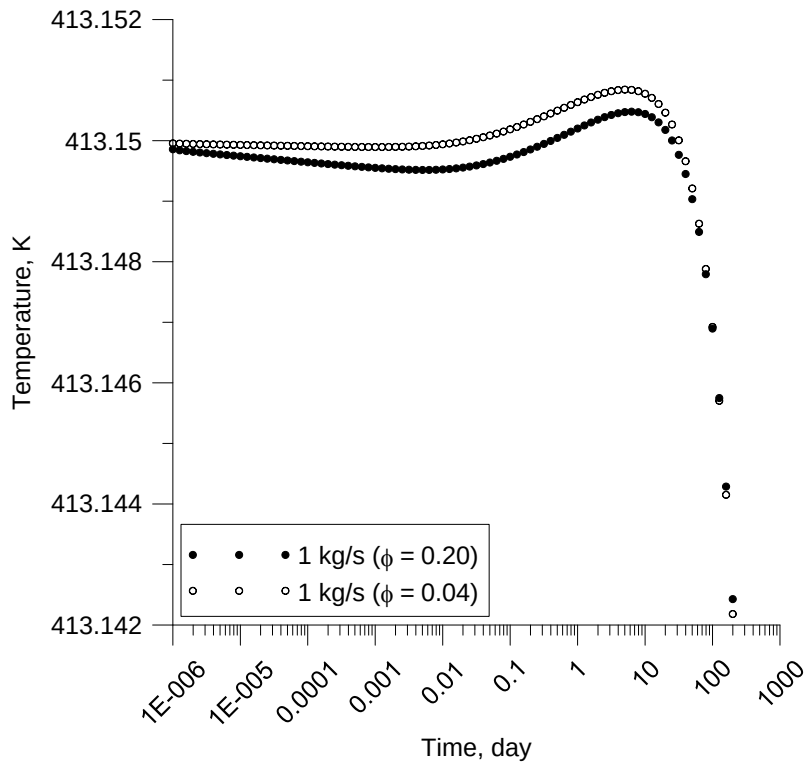


Figure 5.4: Effect of reservoir porosity on the sandface temperature of a fully penetrating well producing at a constant-rate.

As is known, most of the heat content in a reservoir (approximately 90% of heat content) is stored in rock while much less amount of heat content is stored in water. During production period, sandface temperature decreases at very early and late times of production due to expansion of the fluid volume nearby the well region and expansion of the entire reservoir fluid volume at late times. In other times, a temperature increase is observed in geothermal water with Joule-Thomson effect in a relationship with the pressure drop occurred (see Figure 2.7). At late times, high pressure drops in a relationship with expansion of the entire reservoir fluid volume surpass Joule-Thomson effect and so, a cooling period resumes in geothermal water again.

Figure 5.5 shows the sandface temperature behaviors for the same case in view of rate effect. As can be seen from Figure 5.5, more temperature changes occur in higher rate (5 kg/s) with changing porosity in comparison with that in lower rate (1 kg/s) and magnitude of changes observed in the sandface temperature due to production increase as the production rate increases.

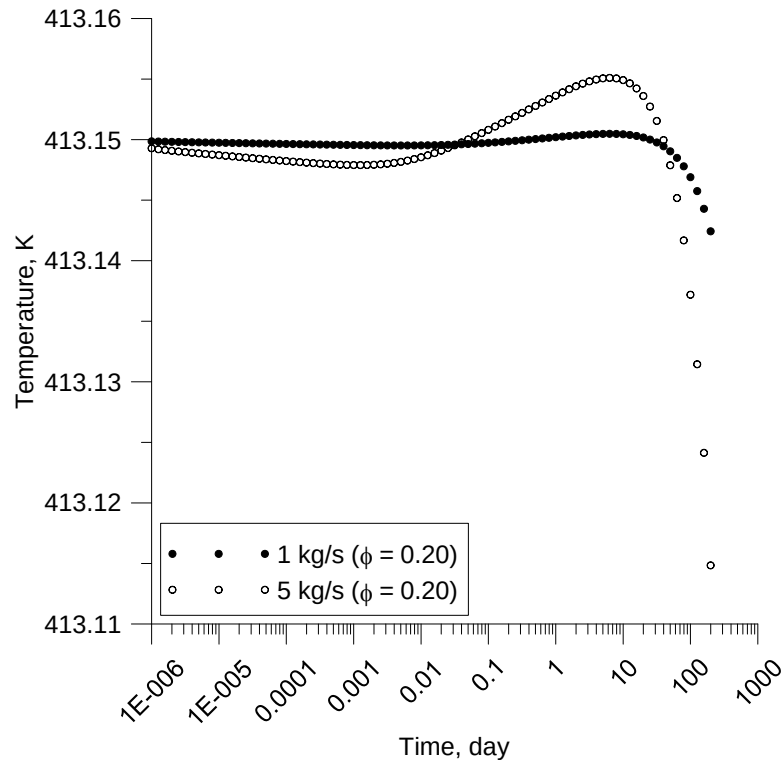


Figure 5.5: Effect of flow rate on the sandface temperature of a fully penetrating well producing at a constant-rate.

5.1.2 Effect of permeability on the sandface pressure and temperature

Shown in Figure 5.6 are the sandface pressure (in the middle of the interval open to flow) behaviors of a fully penetrating well in the reservoir with different permeability values at a constant-rate of 1 kg/s during production of 1000 days. Parameters in Table 4.1 given in Chapter 4 are also used in this case. The other reservoir and well properties mentioned at the beginning of Chapter 5 are also valid for this case. As can be seen from Figure 5.6, the highest pressure drop is observed in the reservoir with the lowest permeability ($k = 1 \times 10^{-15} \text{ m}^2$) especially at early time during drawdown. This is an expected result because resistance to flow increases with decreasing permeability and creates an additional pressure drop. For higher permeability cases ($k = 1 \times 10^{-14} \text{ m}^2$, $k = 5 \times 10^{-14} \text{ m}^2$, $k = 1 \times 10^{-13} \text{ m}^2$ and $k = 5 \times 10^{-13} \text{ m}^2$), pressure drop is much less than that of the lowest permeability case because resistance to flow is lower for these cases. In addition, it can also be seen from Figure 5.6 that pressure behavior does not significantly change for the high permeability cases whereas pressure behavior in the reservoirs where permeability is very low is significant. Figure 5.7 shows the log-log diagnostic plots of delta pressure and derivative of delta pressure as a function of elapsed time for the same case. Figure 5.7 supports the pressure behaviors observed in Figure 5.6. As can be seen from Figure 5.7, the reservoir with the lowest permeability ($k = 1 \text{ md}$) has the highest derivative values during radial flow because the highest pressure drop is observed in this reservoir. Moreover, as can be seen from Figure 5.7, at first, the radial flow (exhibited by a constant value of the Bourdet derivative; Bourdet et al., 1989) is observed (for instance, approximately until 100 days as can be seen from the derivative behavior of the reservoir with the lowest permeability) for the reservoirs. After the radial flow period, the pseudosteady-state flow regime is observed (up to 1000 days).

Figure 5.8 shows the sandface temperature behaviors for the same case. Temperature variations increase with decreasing permeability value. As is seen from Figure 5.8, the greatest variations in temperature behaviors are observed especially for the lowest permeability ($k = 1 \times 10^{-15} \text{ m}^2$) case. For this case, at early time, temperature drop occurs due to the fluid expansion process near the small volume around the well.

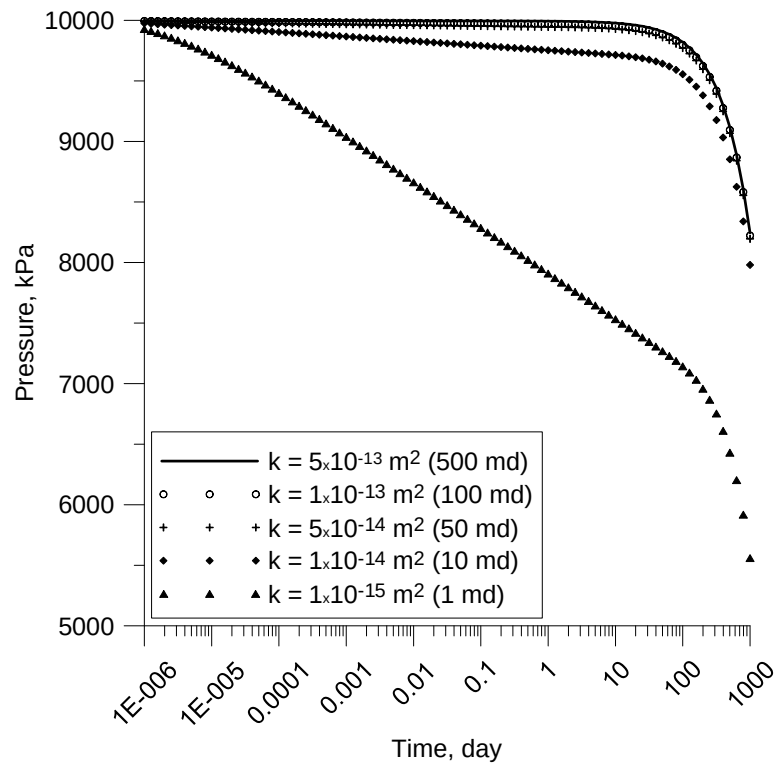


Figure 5.6: Effect of reservoir permeability on the sandface pressure drop of a fully penetrating well producing at a constant-rate.

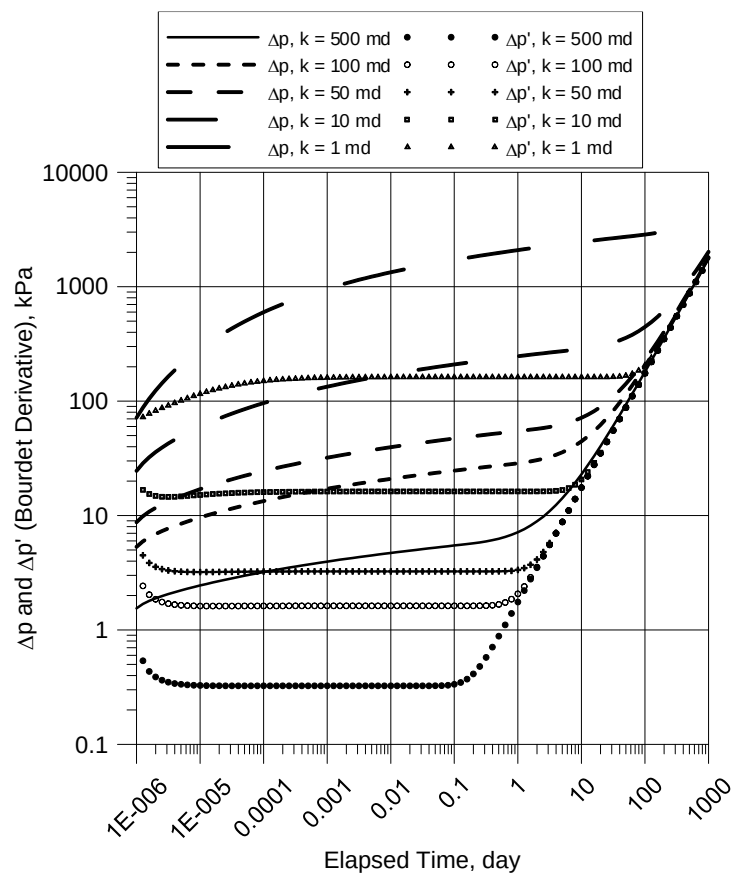


Figure 5.7: Plots of delta pressure and derivative of delta pressure as a function of elapsed time showing effect of permeability during constant-rate production.

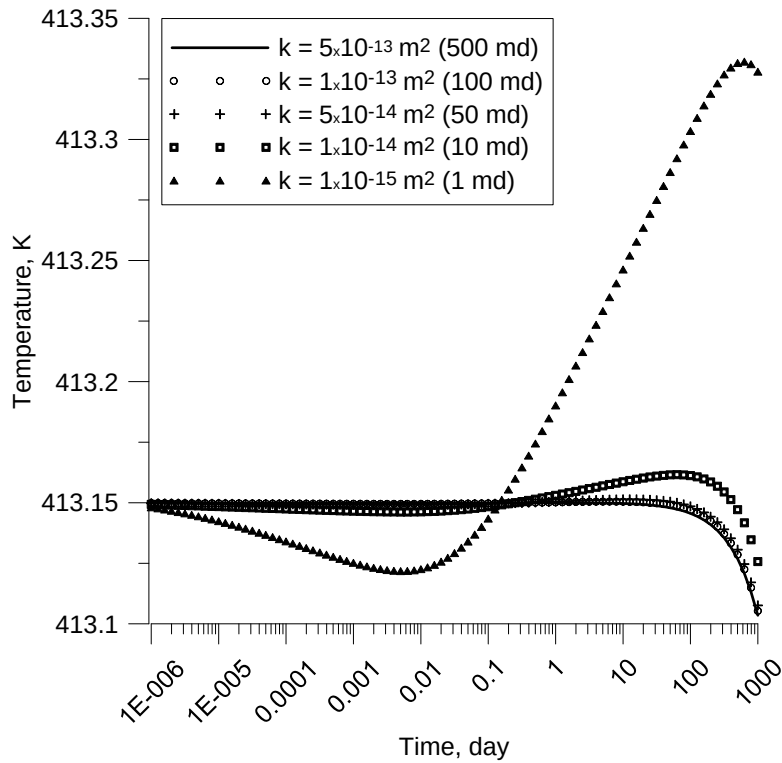


Figure 5.8: Effect of reservoir permeability on the sandface temperature of a fully penetrating well producing at a constant rate.

During this period, heating effect, or equivalently increase in temperature, due to Joule-Thomson fluid expansion also exists, but temperature drop due fluid expansion dominates over the temperature increase (heating) due to Joule-Thomson expansion as the pressure drop is quite small during early time and hence the net effect is the temperature drop at the sandface. This is related to the pressure drop at early time (approximately until 0.01 day). Then, the main cause of temperature increase (heating) observed in the plots is Joule-Thomson effect until the pseudosteady-state flow (approximately until 100 days). Finally, the last temperature drop occurs due to large pressure drops occurring as a result of the expansion of the whole reservoir fluid volume during the pseudosteady-state flow (until 1000 days) to meet the production rate at the well. Note that the reservoir system is closed to fluid and heat flow and hence, during the pseudo-steady state flow reservoir acts a close tank system. So, having large pressure drops due to removal of mass reduces temperature at the sandface.

From here, it may be possible to conclude that the effect of reservoir permeability on the temperature may be more important than that of reservoir porosity previously discussed for such a production case if the magnitudes of the values are considered.

5.1.3 Effect of rock thermal conductivity on the sandface pressure and temperature

Figure 5.9 shows the effect of rock thermal conductivity on the sandface pressure behavior of a fully penetrating well producing at a constant-rate. Parameters in Table 4.1 given in Chapter 4 are also used in this case. The other reservoir and well properties mentioned at the beginning of Chapter 5 are also valid for this case. According to Figure 5.9, rock thermal conductivity does not have any effect on pressure behavior for this case.

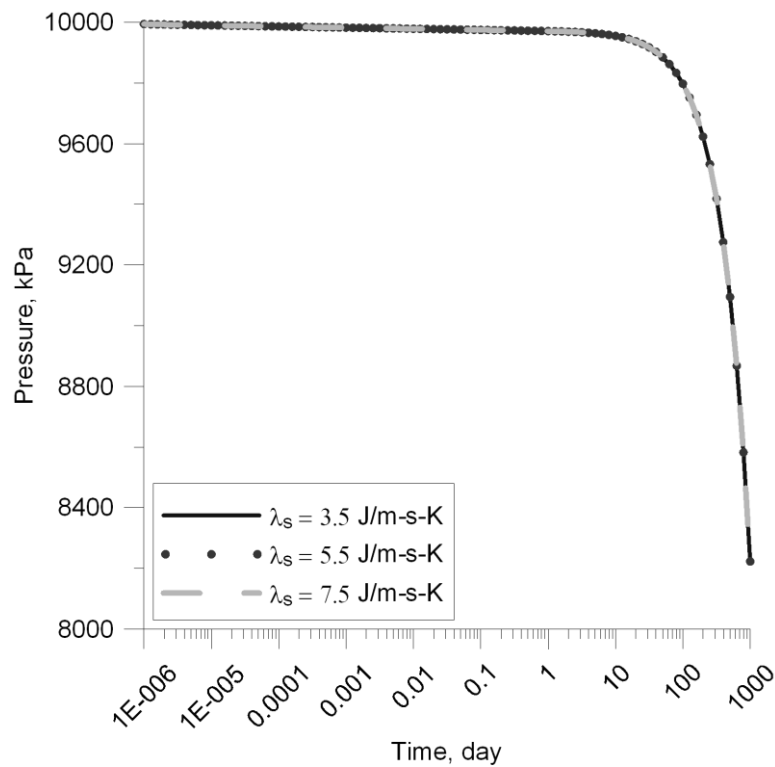


Figure 5.9: Effect of rock thermal conductivity on the sandface pressure drop of a fully penetrating well producing at a constant-rate.

Figure 5.10 shows the log-log diagnostic plots of delta pressure and derivative of delta pressure as a function of elapsed time for the same case. Figure 5.10 supports the pressure behaviors observed in Figure 5.9.

Figure 5.11 shows temperature behaviors for the same case. It can be seen from Figure 5.11 that rock thermal conductivity does not also have any effect on temperature behavior like the pressure behavior. Figures 5.8-5.10 indicate that conduction has no significant effect on constant-rate production case.

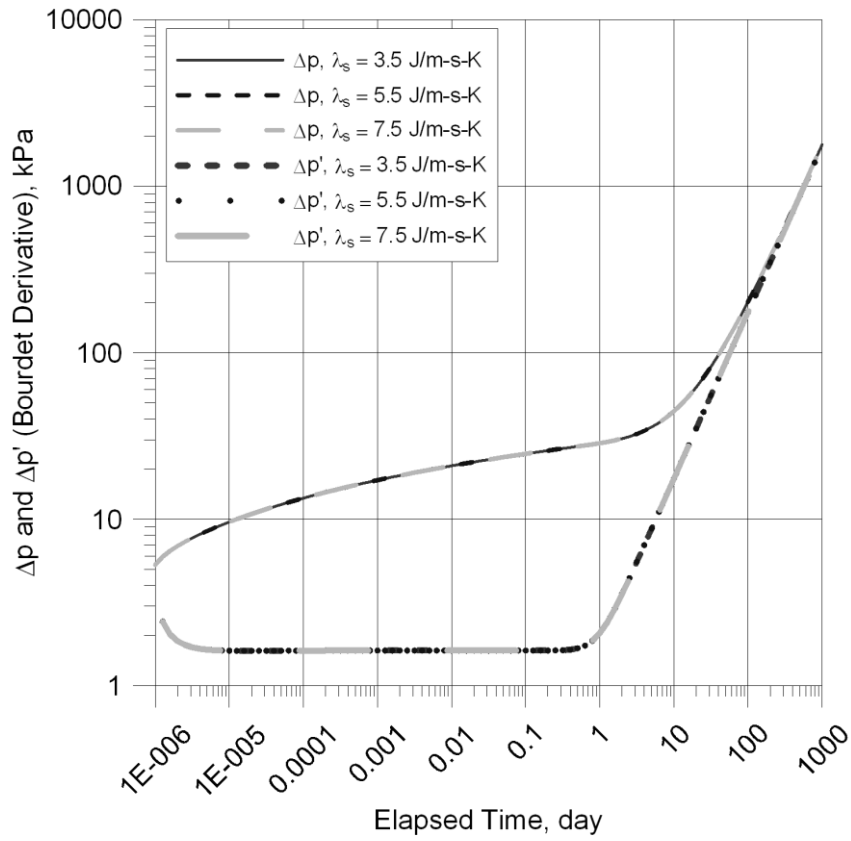


Figure 5.10: Effect of rock thermal conductivity on the sandface pressure drop of a fully penetrating well producing at a constant-rate.

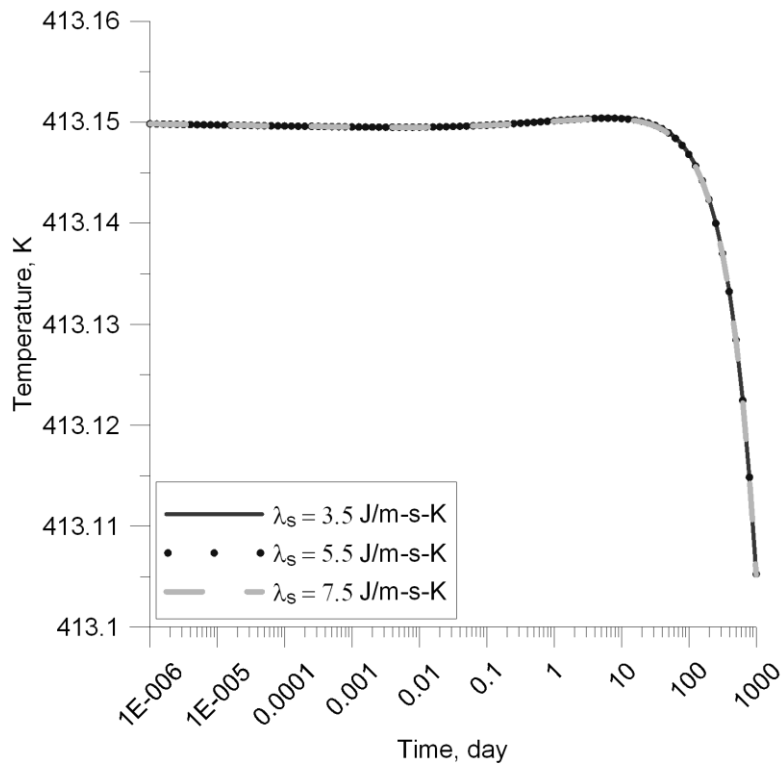


Figure 5.11: Effect of rock thermal conductivity on the sandface temperature of a fully penetrating well producing at a constant-rate.

This is expected because convection is more dominant mechanism during constant-rate production or in general during production period.

5.1.4 Effect of rock heat capacity on the sandface pressure and temperature

Figure 5.12 shows the effect of rock heat capacity on the sandface pressure behavior. Parameters in Table 4.1 given in Chapter 4 are also used in this case. The other reservoir and well properties mentioned at the beginning of Chapter 5 are also valid for this case.

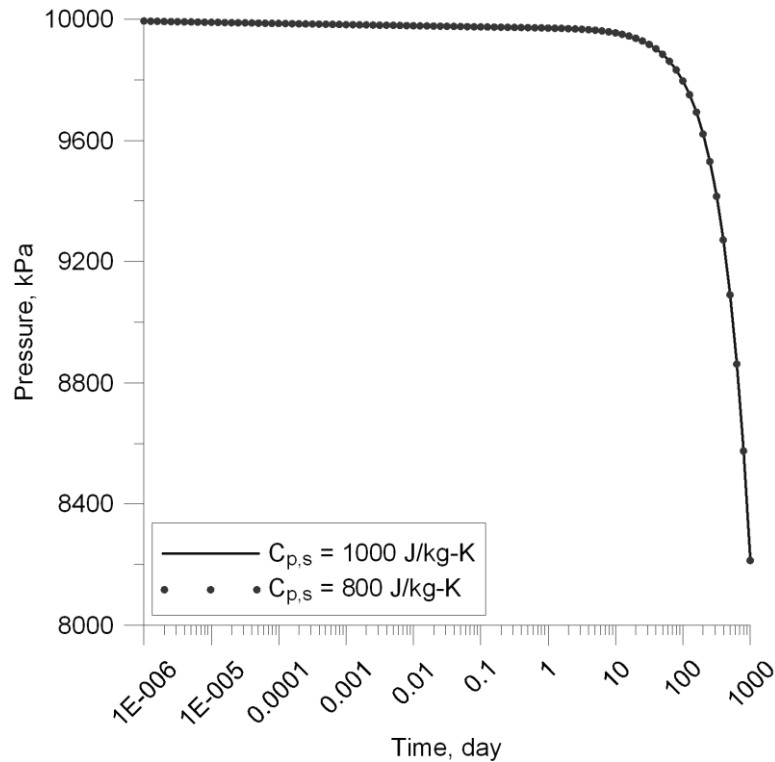


Figure 5.12: Effect of rock heat capacity on the sandface pressure drop of a fully penetrating well producing at a constant-rate.

Figure 5.13 shows the log-log diagnostic plots of delta pressure and derivative of delta pressure as a function of elapsed time for the same case. Fig. 5.13 shows the effect of the same parameter on the sandface temperature of a fully penetrating well producing at a constant-rate.

Based on Figure 5.14, the sandface temperature is slightly lower for a reservoir with a small rock heat capacity than that for a reservoir with a large rock heat capacity during constant-rate production. This is an expected result since reservoirs having higher rock heat capacity have higher heat content than that of reservoir having lower rock heat capacity.

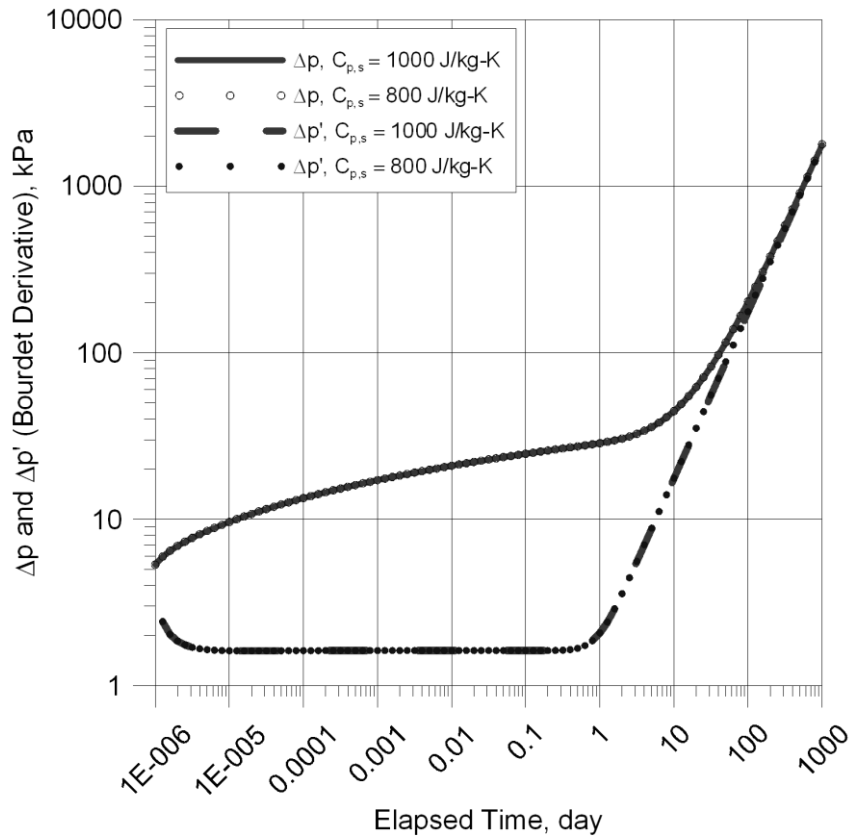


Figure 5.13: Effect of rock heat capacity on the sandface pressure drop of a fully penetrating well producing at a constant-rate.

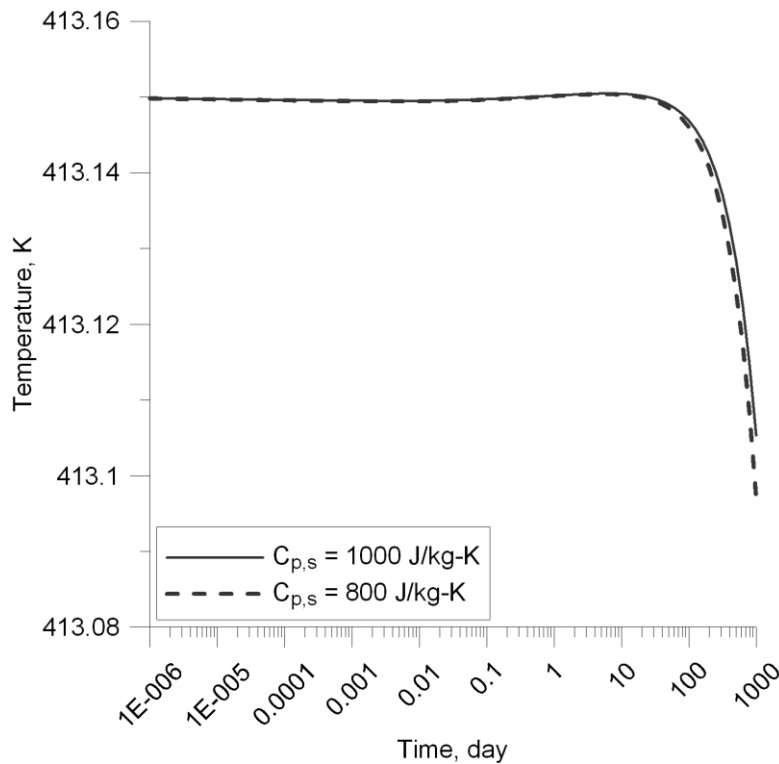


Figure 5.14: Effect of rock heat capacity on the sandface temperature of a fully penetrating well producing at a constant-rate.

The results of these figures show that rock heat capacity does not have a significant effect on the sandface pressure behavior, but it has some effect on the sandface temperature (see Figure 5.14).

5.1.5 Effect of skin on the sandface pressure and temperature

Figure 5.15 shows production–build-up pressure responses of a fully penetrating well in the reservoir with different skin factors ($S = 0, 5$ and 20). In this case, firstly, a drawdown of 1 day at constant-rate of 1 kg/s is performed, and then the well is shut-in during 1 day. When geothermal water is produced at flow rate of 1 kg/s, pressure follows the expected trend. Pressure decreases sharply at early times (transient flow period) and then, pressure decreases linearly with time as can be seen from Figure 5.16. After the well is shut-in, the build-up period commences. Pressure suddenly starts to increase at early times of the build-up period and the amount of pressure increase decreasingly continues up to end of the period. Eventually, at the end of the period (2 days), pressure approximates to initial reservoir pressure. This is a typical build-up behavior in respect of pressure. The parameters used in modeling skin zone near the wellbore are given in Table 5.1. Other parameters in Table 4.1 given in Chapter 4 are also used in this case. The other reservoir and well properties mentioned at the beginning of Chapter 5 are also valid for this case.

Table 5.1: Parameters used in modeling skin zone near the wellbore.

Parameter	Value		
S	0	5	20
r_w , m	0.1	0.1	0.1
r_s , m	0.1512	0.1512	0.1512
k_r , m ²	1×10^{-13}	1×10^{-13}	1×10^{-13}
k_s , m ²	1×10^{-13}	7.5×10^{-15}	2×10^{-15}

Permeability of skin zone (k_s) is used near the wellbore (the first two gridblocks in r -direction) along fully penetrating pay zone in z -direction. Other gridblocks have the same absolute permeability ($k_r = k_z = k = 1 \times 10^{-13}$ m²) in the r and z -directions because anisotropy is not considered in the reservoir. As can be seen from Figures 5.14, increasing the skin factor increases the pressure drop during the drawdown period. This is an expected result because resistance to flow increases with increasing skin and creates an additional pressure drop.

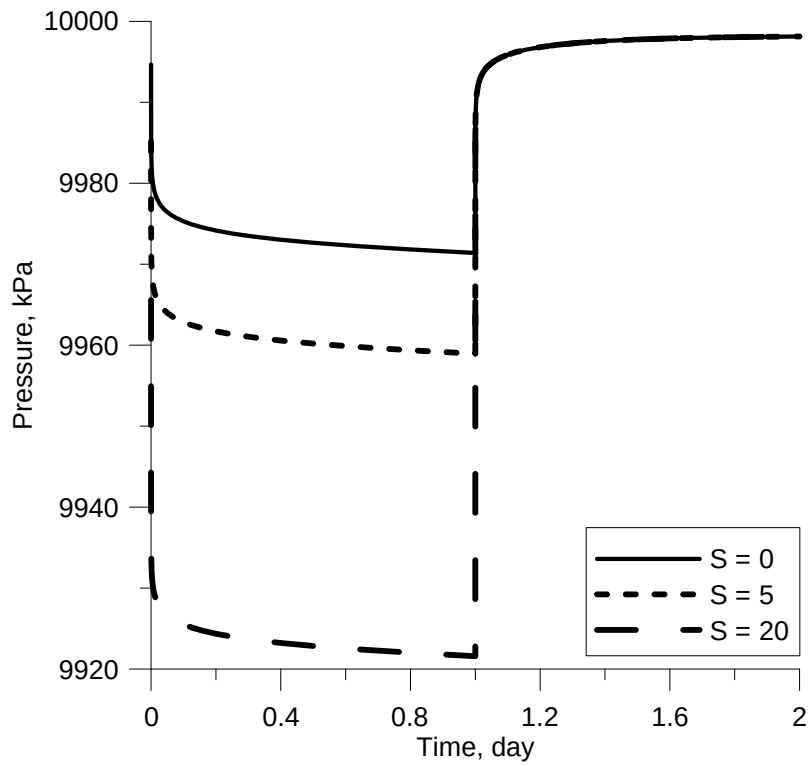


Figure 5.15: Cartesian plot of production-build-up pressure response of a fully penetrating well in the reservoir with different skin factors.

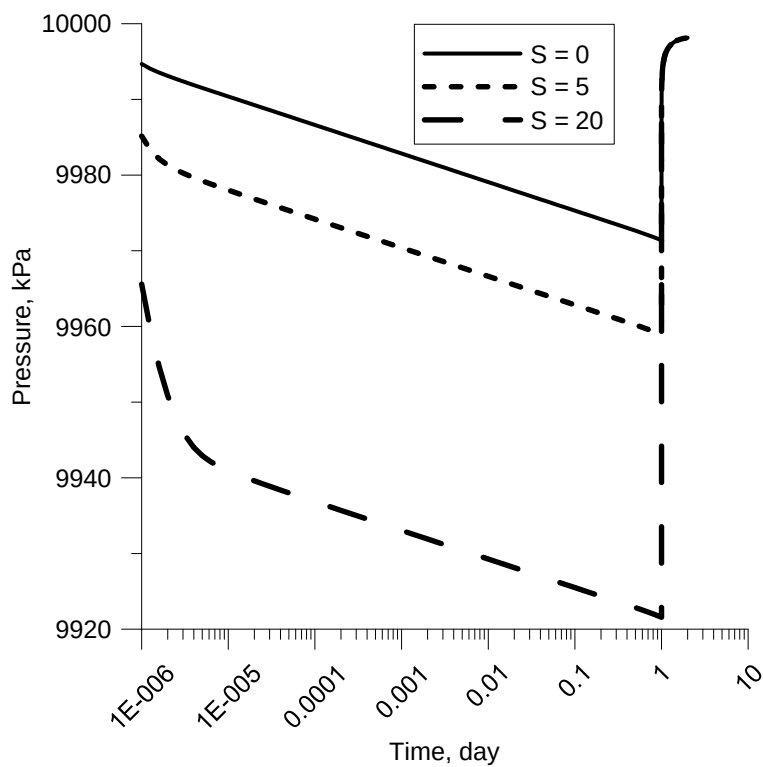


Figure 5.16: Semi-log plot of production-build-up pressure response of a fully penetrating well in the reservoir with different skin factors.

On the other hand, it should be noted that the build-up period is independent of skin because there is no wellbore storage effects. This is why all the cases exhibit the same pressure response during the build-up period.

Figures 5.17 and 5.18 show Cartesian and semi-log plot of production–build-up sandface temperature responses for the same case, respectively. According to Figure 5.18, if drawdown period is assessed, temperature drop is higher with increasing skin at early time. This may be caused by expansion of more fluid (additional pressure drop) at early time. Then, the cause of temperature increase in the three plots until end of drawdown is Joule-Thomson effects. At the beginning of the build-up, a temperature spike (a sharp increase in temperature) is observed for all skin cases due to compression of fluid. This is caused by the fact that sudden rate drop occurs (from 1 kg/s to 0 kg/s). Then, in contrast to drawdown period, temperature decreases realize to balance these sudden temperature increases. App (2010) has also expressed that transient pressure behavior is responsible for creating either a heating or cooling effect at the wellbore immediately after a rate change and these temperature effects are transient in nature and decay rapidly.

5.1.6 Effect of variable-rate on the sandface pressure and temperature

In this case, production periods with 3 different flow rates of 1 day duration are performed. Flow rates are 1 kg/s, 0.5 kg/s and 2 kg/s, respectively. The parameters in Table 4.1 given in Chapter 4 are also used in this case. The other reservoir and well properties mentioned at the beginning of Chapter 5 are also valid for this case. Figure 5.19 shows the sandface pressure response of a fully penetrating well for variable-rate case. A similar behavior can be observed with constant-rate case for the first flow rate period of 1 kg/s.

Pressure decreases sharply at early times (transient flow period) and then, pressure decreases linearly with time up to end of the first flow rate period (1 day). After the rate is changed to 0.5 kg/s, a sudden pressure increase occurs due to decreasing flow rate and then, pressure slightly decreases up to 2 days. At the final flow period at a rate of 2 kg/s, an opposite pressure relationship is observed. A sudden pressure decrease occurs due to increasing flow rate and then, pressure linearly decreases up to end of the drawdown (2 days).

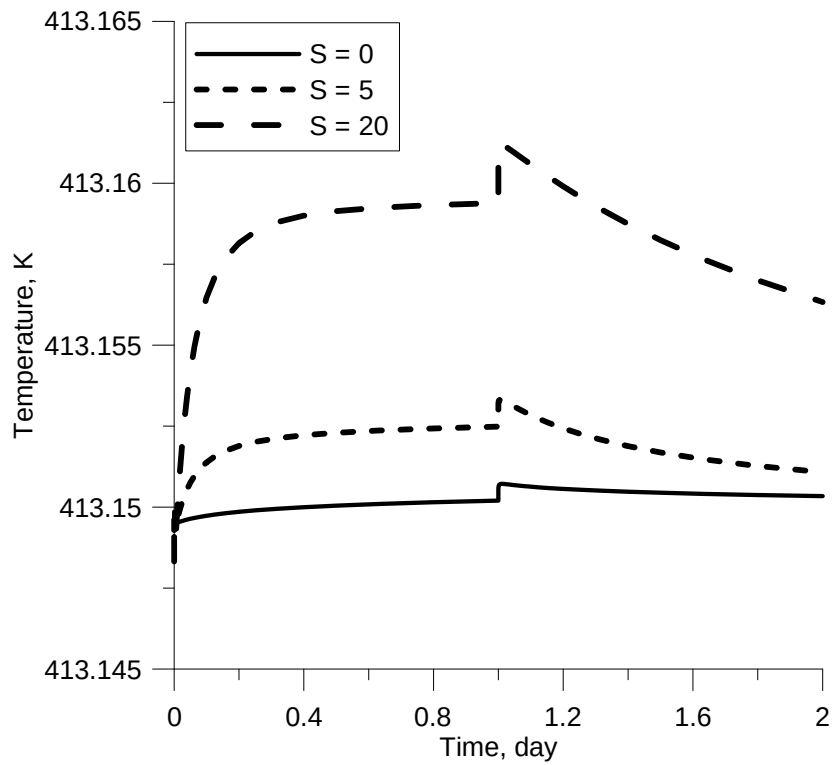


Figure 5.17: Cartesian plot of production-build-up temperature response of a fully penetrating well in the reservoir with different skin factors.

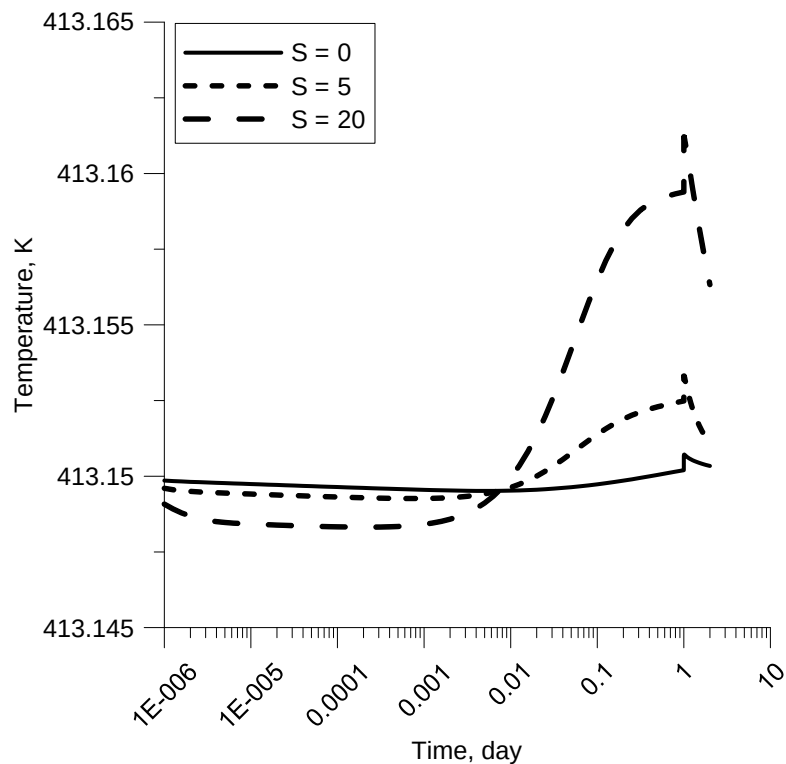


Figure 5.18: Semi-log plot of production-build-up temperature response of a fully penetrating well in the reservoir with different skin factors.

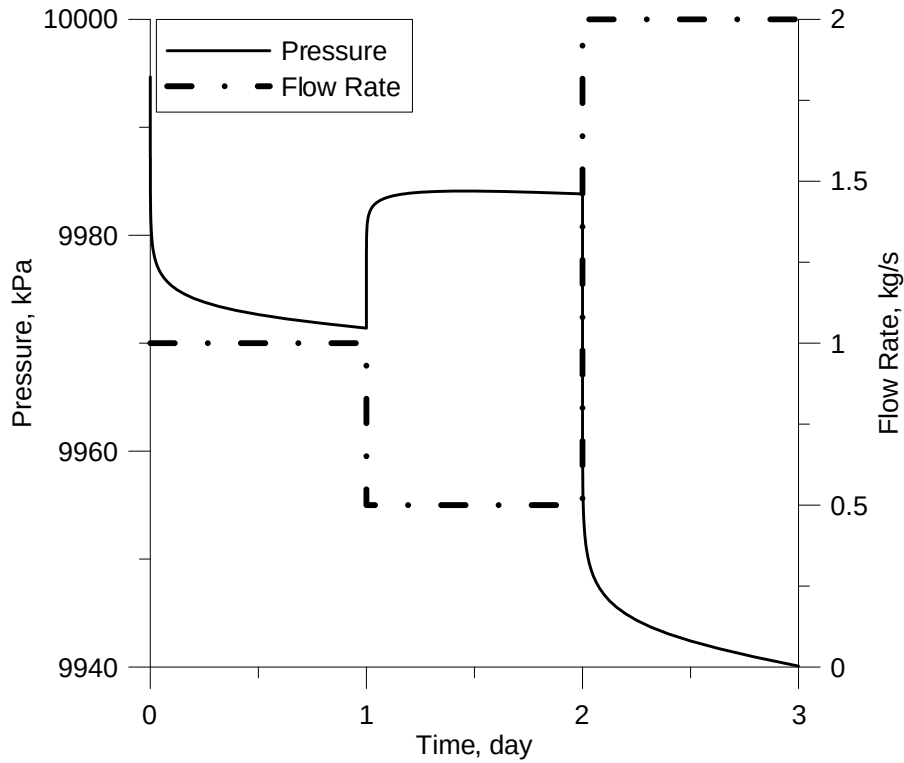


Figure 5.19: Sandface pressure response of fully penetrating production well at variable-rate.

Figure 5.20 shows the sandface temperature response of a fully penetrating well for variable-rate case. According to Figure 5.20, a sudden temperature drop occurs due to the expansion of fluid in a relationship with suddenly increasing flow rate (from 0 kg/s to 1 kg/s) at early time and then, temperature increases because of Joule-Thomson effects up to end of the first flow rate period (1 day). After the rate is changed to 0.5 kg/s, a temperature spike is observed due to the compression of fluid in a relationship with decreasing flow rate (from 1 kg/s to 0.5 kg/s). Then, a temperature drop realizes to balance this sudden temperature increase arising from the compression of fluid up to 2 days. At the final flow period at a rate of 2 kg/s, temperature behavior is similar to the first flow period.

In fact, these results indicate that if such sharp changes in temperature with change in rate can be detected, one can be able to determine the time points where the rate changes occur and even estimate flow rate by history matching as noted by the previous researchers, e.g., see Duru and Horne (2010a). Since the flow rate is two times greater (2 kg/s) than that (1 kg/s) of the first flow period, firstly, the temperature drop and secondly, the temperature increase realize approximately two times greater than those of the first flow period.

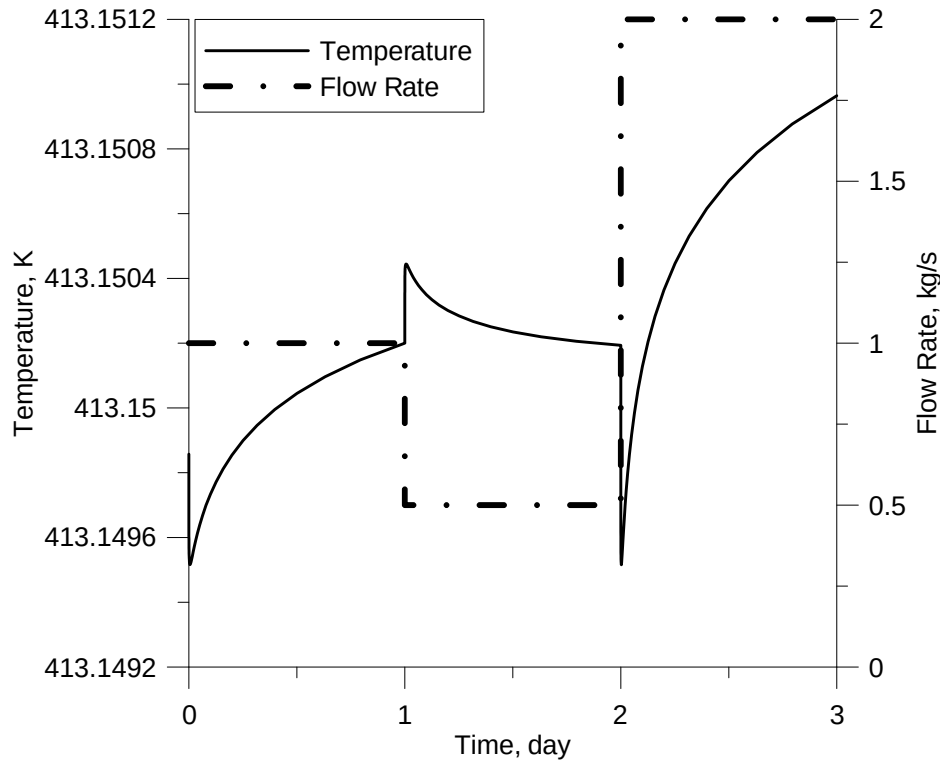


Figure 5.20: Sandface temperature response of fully penetrating production well at variable-rate.

5.1.7 Effect of limited-entry length on the sandface pressure and temperature behaviors

In this subsection, the effect of the limited-entry length on the sandface pressure and temperature is investigated for constant-rate production case. The only difference of this case from the constant-rate production case with fully penetrating well is the production time (1000 days instead of 200 days) here. The parameters in Table 4.1 given in Chapter 4 are also used in this case. The other reservoir and well properties mentioned at the beginning of Chapter 5 are also valid for this case.

Since structure of the grid system used in this case is the same as the limited-entry production case given in Chapter 3, it is not illustrated again, here. In this case, open interval lengths to flow are considered to be in the middle of the reservoir in the z -direction. Figure 5.21 shows schematic of limited-entry production well.

Figure 5.22 shows the effect of the length of the open interval ($h_w = 1$ m and $h_w = 10$ m) on the sandface pressure response in comparison with the sandface response for the fully penetrating well case ($h_w = h = 100$ m).

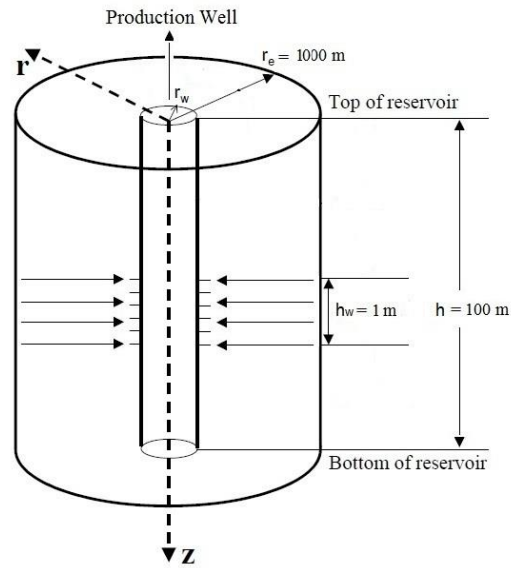


Figure 5.21: Schematic of a limited-entry production well.

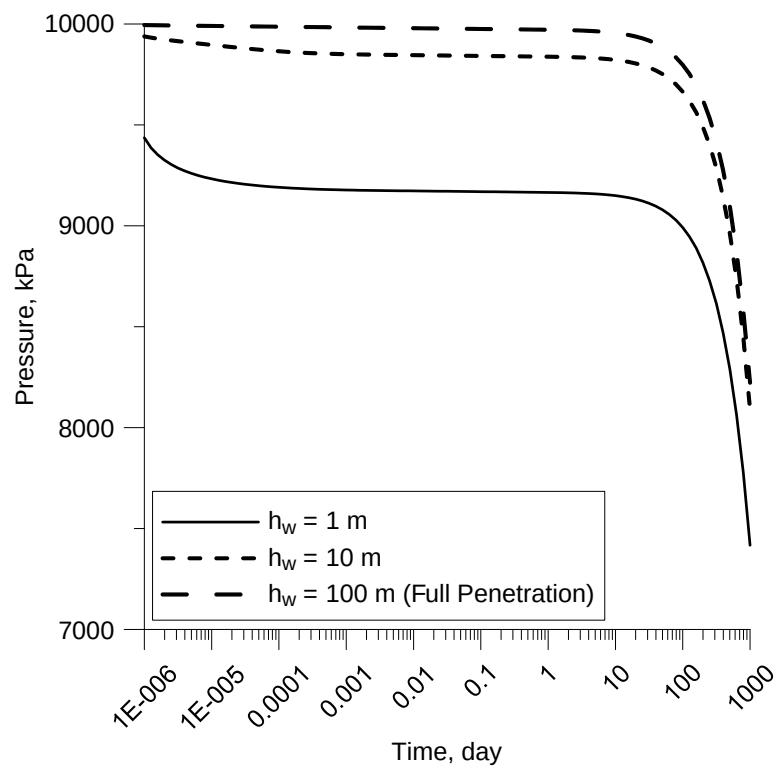


Figure 5.22: Effect of the open interval length on the sandface pressure behaviors during constant-rate production.

In these cases, differently from the fully penetrating well, more pressure drops occur due to the area open to flow is much narrower (especially for the case that $h_w = 1$ m). At early time, very sudden pressure drops are observed because of the limited-entry. It should be noted that limited-entry well provides an additional (pseudo) skin effect

and this results in larger pressure drop as seen from Figure 5.22 when h_w decreases. Figure 5.23 shows the log-log diagnostic plot of derivative of delta pressure as a function of elapsed time to demonstrate the effect of the limited-entry length. As can be seen from Figure 5.23, differently from fully penetrating well ($h_w = 100$ m), the spherical flow (flow regime that has the derivative line with $-1/2$ slope) is observed at early times (approximately until 0.01 days) due to the limited-entry effect ($h_w = 1$ m). After this period, the radial flow (flow regime that the derivative is flat) is observed lately (approximately between 0.01 days and 1 day) in comparison with that of fully penetrating well because of the limited-entry effect. After the radial flow period, derivative line with $+1$ slope indicating that boundary effects are felt is observed. Pseudosteady-state flow regime (pressure change is constant with changing time) is observed during this period (up to 1000 days).

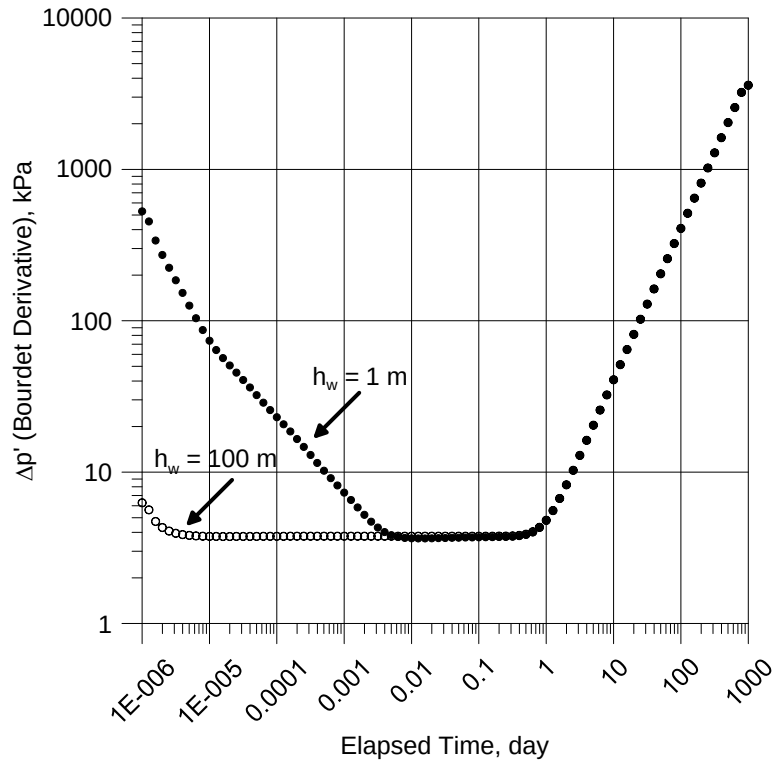


Figure 5.23: Plot of Bourdet derivative as a function of elapsed time for effect of the limited-entry length.

Figure 5.24 shows variations in temperature behaviors for the same case. This behaviors are similar to those of the case that temperature behaviors are observed with decreasing permeability effect as previously explained. According to Figure 5.24, changes in temperature become more significant with decreasing open interval length of the limited-entry well.

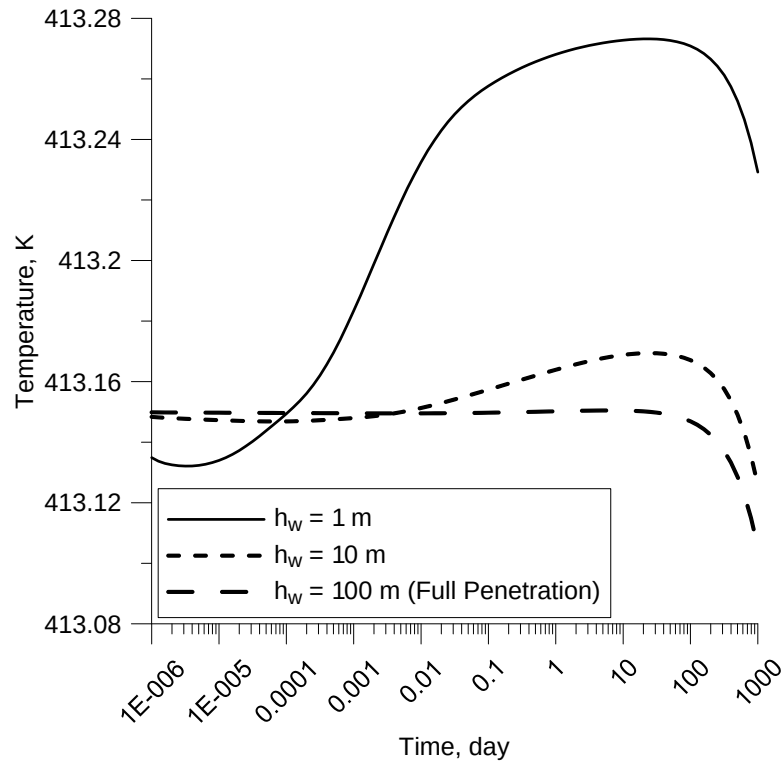


Figure 5.24: Effect of the open interval length on the sandface temperature behaviors during constant-rate production.

As is seen from Figure 5.24, the greatest variations in temperature behaviors are observed especially for the lowest open interval length ($h_w = 1$ m). For this case, at early time, temperature drop occurs due to the fluid expansion process in a relationship with the pressure drop at early time, and then, a temperature increase is caused by Joule-Thomson effects until the pseudosteady-state flow. Finally, a temperature drop is observed due to the pressure drop during pseudosteady-state flow (until 1000 days).

5.1.8 Effect of high pressure and high temperature on the limited-entry well

In this case, the sandface pressure and temperature behaviors (in the middle of the interval open to flow) in a period of 150 days at a constant flow rate of 60 kg/s is observed for the limited-entry well ($h_w = 1$ m) and differently from the previous limited-entry case, initial reservoir pressure is 30000 kPa; initial reservoir temperature is 453.15K (180°C) and permeability is $3.16 \times 10^{-13} \text{ m}^2$ (316 md). This permeability is chosen as the geometric mean of an anisotropic reservoir that has $k_r = 100$ md and $k_z = 1000$ md or $k_r = 1000$ md and $k_z = 100$ md.

Other reservoir and well properties given in Table 4.1 in Chapter 4 and mentioned at the beginning of Chapter 5 are also valid for this case. Furthermore, sensitivities of

various rock parameters to the sandface pressure and temperature behaviors are investigated by using the plots of sensitivity coefficients as a function of time. The grid system used in this case is the same as the previous case and open interval lengths to flow are considered to be in the middle of the reservoir in the z -direction.

Figure 5.25 shows the sandface pressure behaviors for this case ($h_w = 1$ m). Figure 5.26 shows the log-log diagnostic plots of delta pressure and derivative of delta pressure as a function of elapsed time. As can be seen from Figure 5.26, the spherical flow (flow regime that has the derivative line with $-1/2$ slope) is observed at early times (approximately until 0.001 days) due to the limited-entry effect ($h_w = 1$ m). After this period, the radial flow (flow regime that the derivative is flat) is observed earlier (approximately between 0.001 days and 0.1 days) than that of the previous limited-entry case. In this case, more pressure drop occurs due to a higher flow rate than that of the previous case. Pressure behavior is also very similar to the previous case. Firstly, very sudden pressure drop is observed at early time because of the limited-entry (spherical flow). After this period, the derivative stabilizes during the radial flow. After the radial flow period, the reservoir outer boundary effects are observed and pseudosteady-state flow regime is observed during this period (up to 150 days).

Figure 5.27 shows variations in temperature behavior for the same case. Since the temperature behavior observed is similar to that of the previous case, it is not explained in detail again, here.

However, it is important to note that for this case, there is a larger drawdown than that of the previous case with $h_w = 1$ m (compare Figures 5.22 and 5.25). Hence, having larger drawdown causes more changes in sandface temperature increase. The main difference of this case from the previous one is that a temperature change created by the fluid expansion and Joule-Thomson effects becomes 0.5°C (it was approximately 0.14°C in the previous case) due to the effect of high flow rate rather than that of high temperature if the temperature drop realized during pseudosteady-state flow period is not taken into consideration.

At this point, for this limited-entry well case, we would like to investigate the sensitivities of the sandface pressure and temperature to various reservoir parameters by making use of the plots of sensitivity coefficients as a function of time.

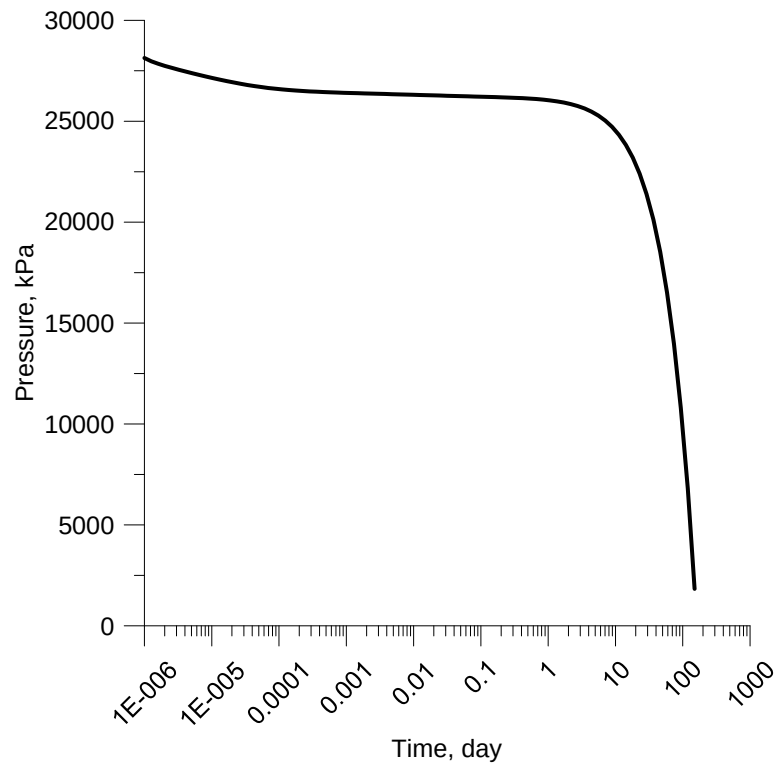


Figure 5.25: Sandface pressure behavior of a limited-entry well producing at a high constant-rate in a reservoir with high initial pressure ($p^0 = 30000$ kPa) and initial temperature ($T^0 = 453.15\text{K}$).

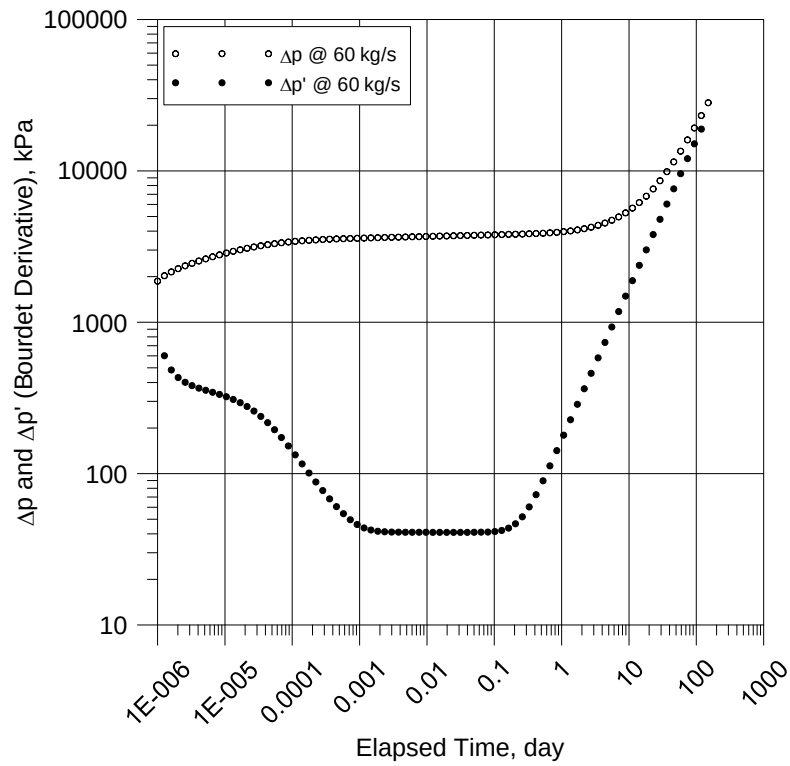


Figure 5.26: Plots of delta pressure and Bourdet derivative of delta pressure as a function of elapsed time for effect of the limited-entry length.

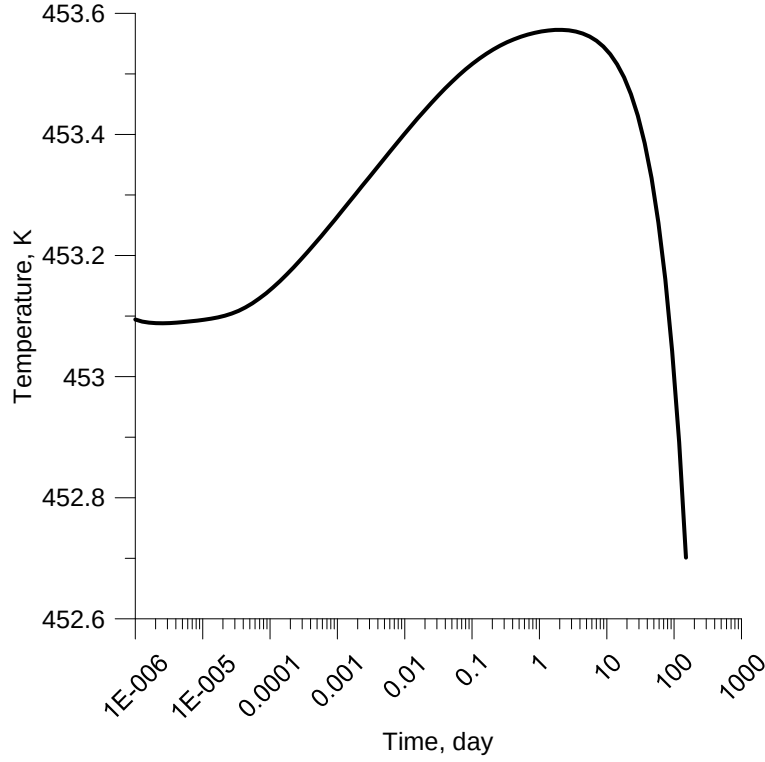


Figure 5.27: Sandface temperature behavior of a limited-entry well producing at a high constant-rate in a reservoir with high initial pressure ($p^0 = 30000$ kPa) and initial temperature ($T^0 = 453.15$ K).

Sensitivity coefficient ($\underline{S}$) of a primary variable (ψ) with respect to the natural logarithm of a model parameter (κ) is as follows:

$$\underline{S} = \left(\frac{\partial \psi}{\partial \ln \kappa} \right) = \kappa \left(\frac{\partial \psi}{\partial \kappa} \right) \quad (5.1)$$

In Equation 5.1, the primary variable ψ represents pressure (p) or temperature (T) in the cases and the model parameter κ represents the rock parameter (porosity, permeability, rock thermal conductivity, rock density and rock heat capacity) of which sensitivity will be investigated.

Figure 5.28 shows the sensitivities of the sandface pressure to the various rock parameters as a function of time. It can be seen from Figure 5.28 that the rock parameter showing the greatest sensitivity to the pressure is porosity for this case. After the porosity, horizontal permeability has an important sensitivity to the pressure. This is an expected result because the pressure investigated is the pressure at the sandface where the open interval to flow is. Hence, the sensitivity to horizontal permeability is much greater than that to vertical permeability. On the other hand, the

sandface pressure shows some sensitivity to rock density and rock heat capacity as a function of time while it is not sensitive to the rock thermal conductivity for this case.

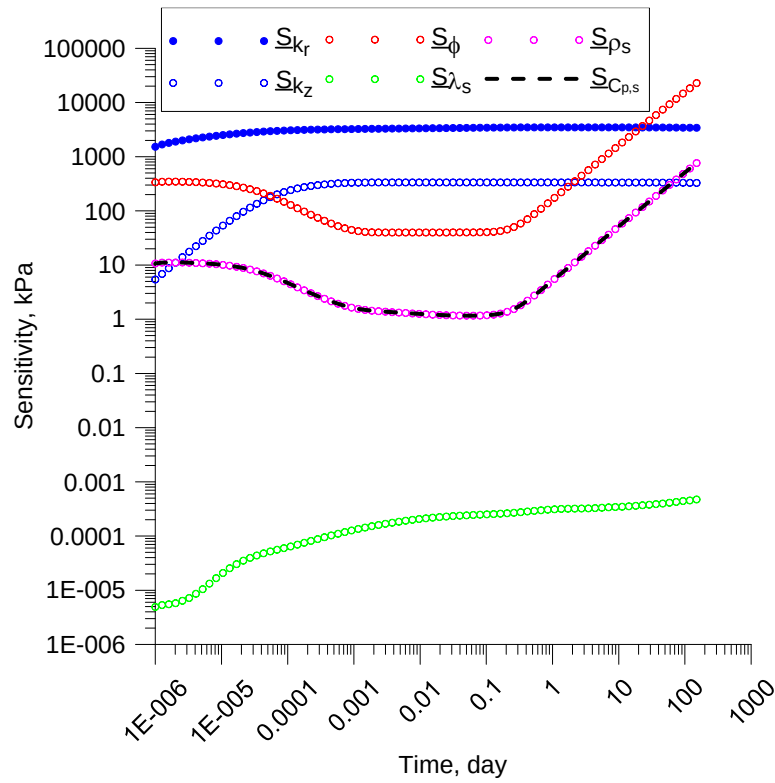


Figure 5.28: The sensitivities of the sandface pressure to various reservoir parameters for the limited-entry well case.

Figure 5.29 shows the sensitivities of the sandface temperature to rock parameters as a function of time for the same case. As can be seen from Figure 5.29, the sensitivities of the temperature to various reservoir parameters are not so much in respect of the order of magnitude. Among the parameters, rock density and rock heat capacity have the greatest sensitivities. After those, the sensitivity to k_r is greater than that to k_z . Additionally, the sensitivities to horizontal permeability and vertical permeability are negative. It means that the temperature decreases with increasing permeability or the temperature increases with decreasing permeability. On the other hand, temperature shows some sensitivity to porosity, but very little sensitivity to the rock thermal conductivity for this case.

5.2 Injection Case

In this subsection, different constant-rate injection and injection–fall-off cases are discussed for fully penetrating and limited-entry wells.

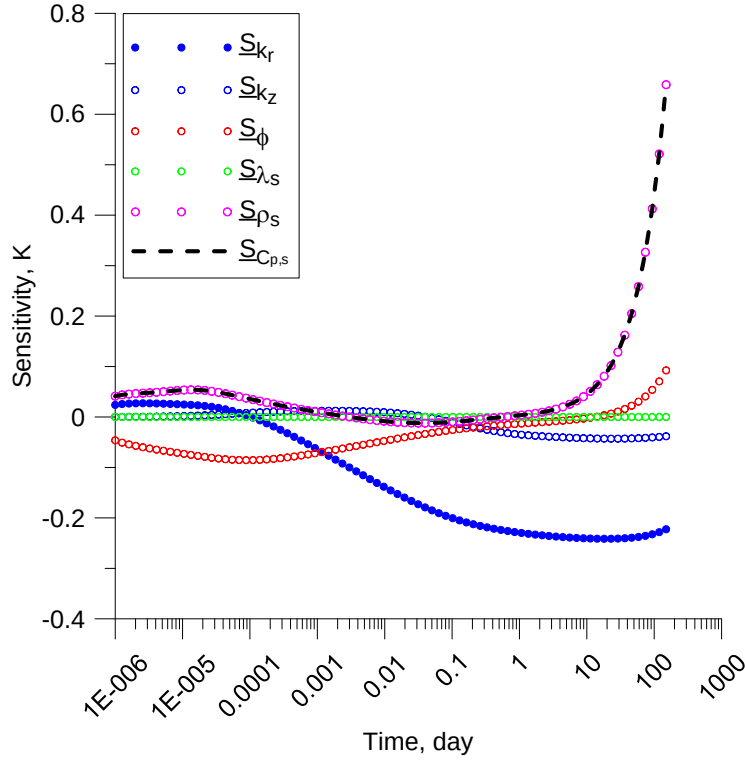


Figure 5.29: The sensitivities of the sandface temperature to various rock parameters for the limited-entry production well.

5.2.1 Effect of permeability on pressure and temperature behaviors

In this case, colder water with $T_{inj} = 333.15\text{K}$ (60°C) is injected from fully penetrating well into the reservoir with $T^0 = 413.15\text{K}$ (140°C) at a constant-rate of 1 kg/s during 1000 days. The other reservoir and well properties given in Table 4.1 in Chapter 4 and mentioned at the beginning of Chapter 5 are also valid for this case. Figure 5.30 shows a schematic picture of a fully penetrating injection well.

Shown in Figure 5.31 are the sandface pressure (in the middle of the interval open to flow) behaviors of a fully penetrating well in the reservoir with different permeability values during constant-rate injection of 1000 days. As can be seen from Figure 5.31, the highest pressure increase is observed in the reservoir with the lowest permeability ($k = 1 \times 10^{-15}\text{ m}^2$) especially at early time. This is an expected result because resistance to flow increases with decreasing permeability and creates an additional pressure increase (pseudo-skin) in contrast to the production case. If Figure 5.31 is carefully inspected, the pressure behaviors follow an inverse trend in comparison with those of the production case.

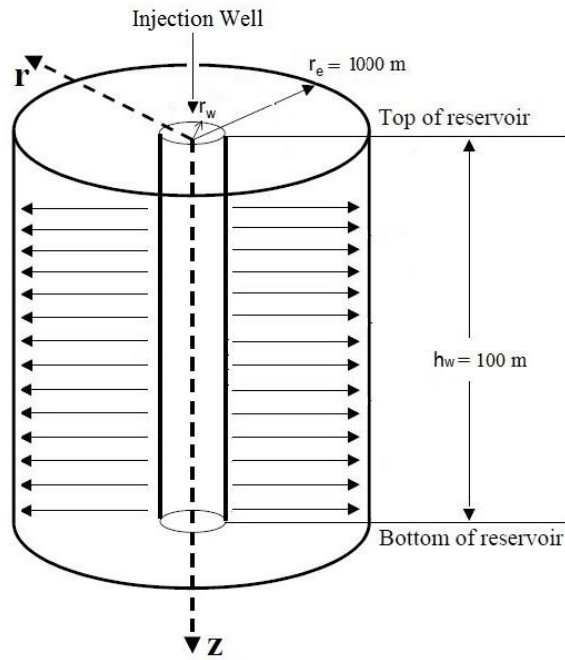


Figure 5.30: Schematic of a fully penetrating injection well.

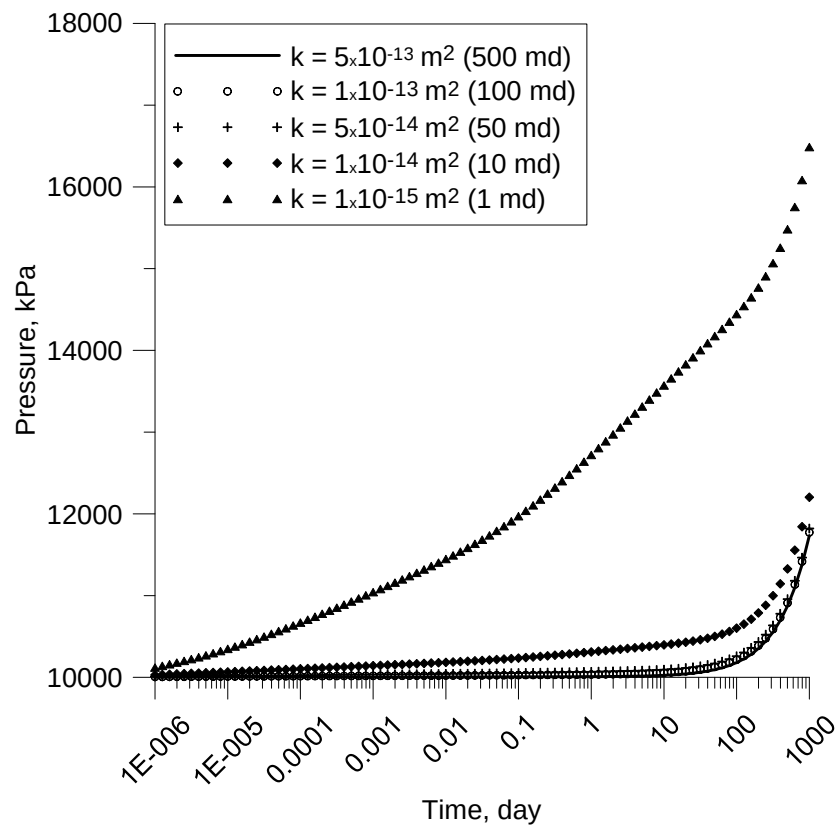


Figure 5.31: Sandface pressure behaviors of a fully penetrating well in the reservoir with different permeability values during constant-rate injection.

For higher permeability cases ($k = 1 \times 10^{-14}$, $k = 5 \times 10^{-14}$, $k = 1 \times 10^{-13}$ and $k = 5 \times 10^{-13}$ m²), pressure increase is much less than that of the lowest permeability case because resistance to flow is much lower for these cases and this situation makes flow easy at the sandface. Figure 5.32 shows the log-log diagnostic plots of delta pressure and Bourdet derivative of delta pressure as a function of elapsed time for the same case. Figure 5.32 supports the pressure behaviors observed in Figure 5.31. As can be seen from Figure 5.32, the reservoir with the lowest permeability ($k = 1$ md) has the highest derivative values because the highest pressure increase is observed in this reservoir like the production case.

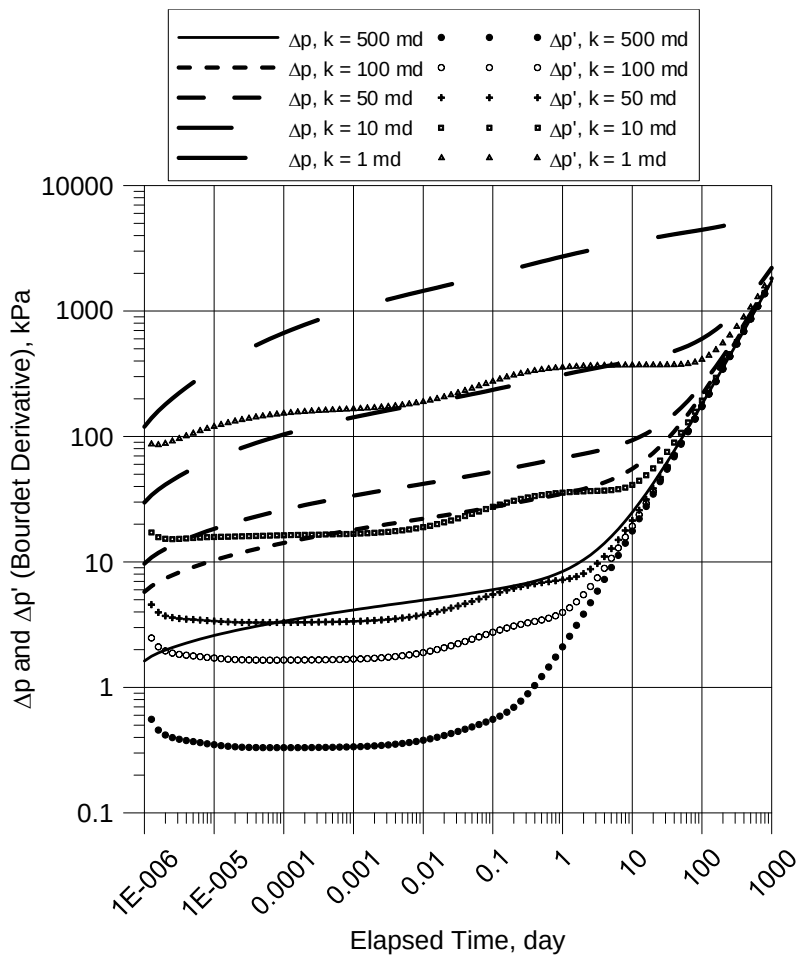


Figure 5.32: Plots of delta pressure and derivative of delta pressure as a function of elapsed time showing effect of permeability during constant-rate injection.

However, in this case, there is a difference from the production case. Here, colder fluid with higher viscosity and density than those of the reservoir fluid is injected into the reservoir (this effect will be discussed in detail in Section 5.2.11 of Chapter 5). Hence, this causes that the derivative behaviors of the radial flows observed in the reservoirs is not as stabilized as in the production case. They have slightly curvature behaviors.

As can be seen from Figure 5.32, two different radial flow periods are observed with decreasing permeability. These behaviors are observed for the reservoir with the lowest permeability ($k = 1$ md) in the most obvious way. For this reservoir, at first, the first radial flow is observed approximately between 0.0001 days and 0.01 days, and then, the second radial flow is observed approximately between 1 day and 100 days. This is caused by the fact that the original reservoir fluid with 413.15K (140°C) dominates the flow during the first radial flow period. On the other hand, cold water zone dominates the flow during the second radial flow period by moving of colder fluid zone further into the reservoir. Consequently, the pseudosteady-state flow regime is observed at the latest in the reservoir with the lowest permeability due to the fact that pressure signal reaches the outer boundary at the latest in the reservoir with the lowest permeability.

Figure 5.33 shows pressure behaviors of a fully penetrating well along radial distance in the reservoir with different permeability values as a function of time during constant-rate injection. Figure 5.33 also shows that pressure behavior is significantly affected from changing permeability for this case.

Figure 5.34 shows temperature behaviors as a function of time for the same case. As can be seen from this figure, temperature behavior is not affected so much from changing permeability for this case.

Figure 5.35 shows temperature behaviors of a fully penetrating well along radial distance in the reservoir with different permeability values as a function of time during constant-rate injection. Figure 5.35 also verifies that permeability does not have effect even the temperature distribution inside the reservoir. This case is really same as the water saturation profile that would be obtained from a water injection problem into an oil reservoir (single-layer) at a constant-rate injection. In that case, it can be shown that saturation profile is independent of permeability.

Hence, this is related to the fact that convection is more dominant than conduction and the problem becomes hyperbolic (wave equation) like the Buckley-Leverett problem. On the other hand, it should be noted that the problem is parabolic in the production case.

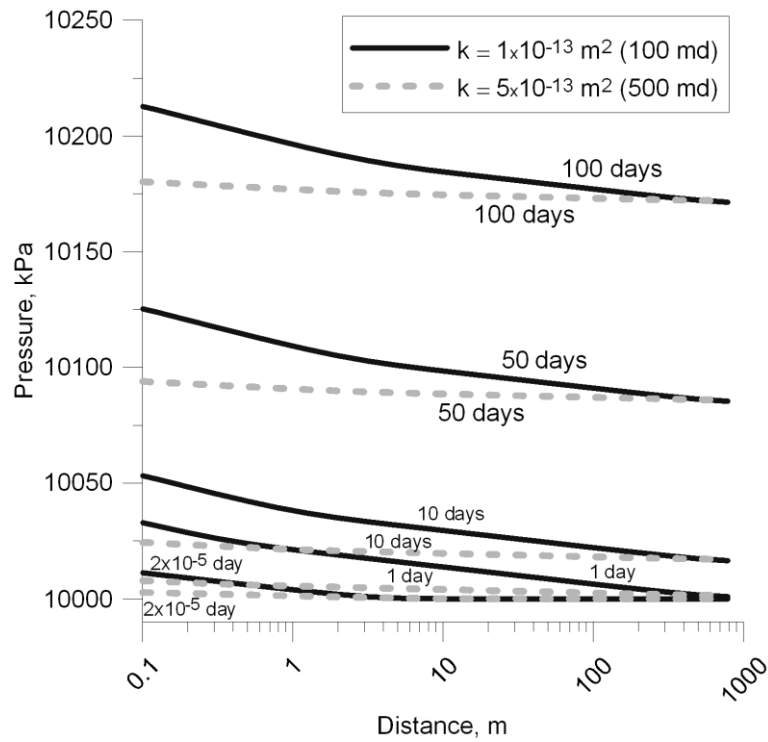


Figure 5.33: Pressure behaviors of a fully penetrating well along radial distance in the reservoir with different permeability values as a function of time during constant-rate injection.

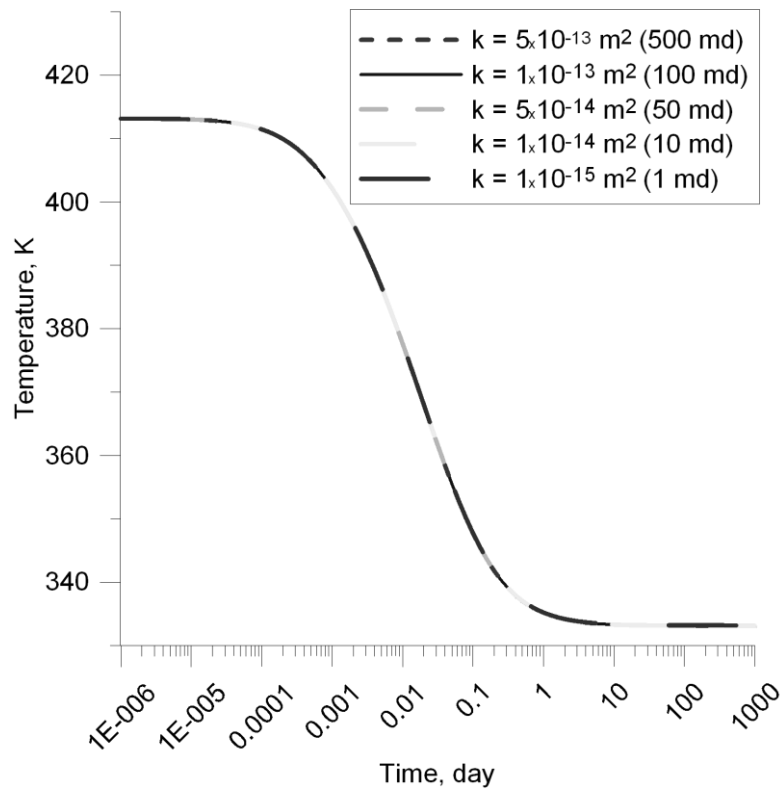


Figure 5.34: Sandface temperature behaviors of a fully penetrating well in the reservoir with different permeability values during constant-rate injection.

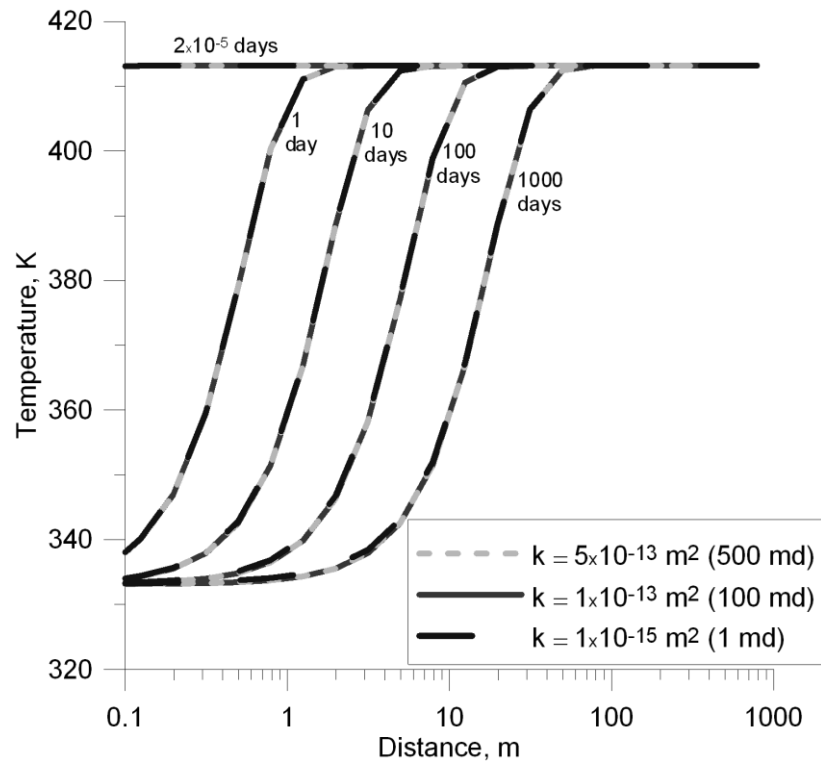


Figure 5.35: Temperature behaviors of a fully penetrating well along radial distance in the reservoir with different permeability values as a function of time during constant-rate injection.

5.2.2 Effect of outer reservoir boundary with constant-pressure on sandface pressure and temperature behaviors

Now the previously same case is considered again. However this time, it is considered that the reservoir is surrounded by a huge edge aquifer as shown in Figure 5.36.

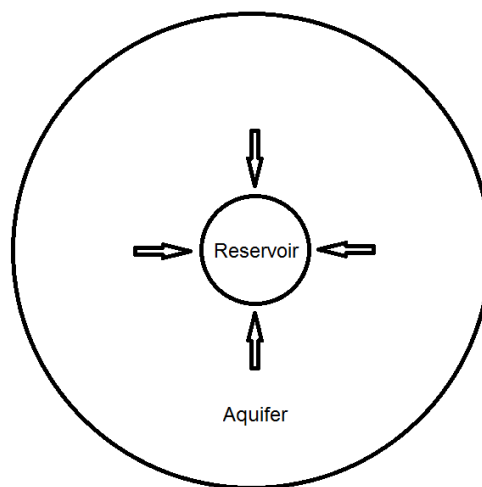


Figure 5.36: Top view of a huge edge aquifer surrounding the reservoir.

The bulk volume of the edge aquifer is 10^{99} m^3 and its initial temperature is 413K (140°C) which is the same as the initial reservoir temperature. A constant-pressure outer boundary condition is created by using a huge edge aquifer surrounding the reservoir. The other parameters used is the same as the previous case.

Figure 5.37 shows the log-log diagnostic plots of delta pressure and derivative of delta pressure of a fully penetrating well in the reservoir with different permeability values during constant-rate injection of 1000 days at no-flow and constant-pressure outer boundary conditions. As can be seen from Figure 5.37, a colder fluid with higher viscosity and density than those of the reservoir fluid is injected into the reservoir like the previous case. Pseudosteady-state flow is observed for the no-flow cases after the outer boundary effects are felt while derivative displays the unit slope line which indicates a sealed outer boundary. For the constant-pressure outer boundary cases, a constant derivative is observed with an increasing pressure. This is caused by the fact that the cold zone in the reservoir is still moving (see Figure 5.35) and the new injected cold fluid encounters with the cold fluid zone previously created in the reservoir.

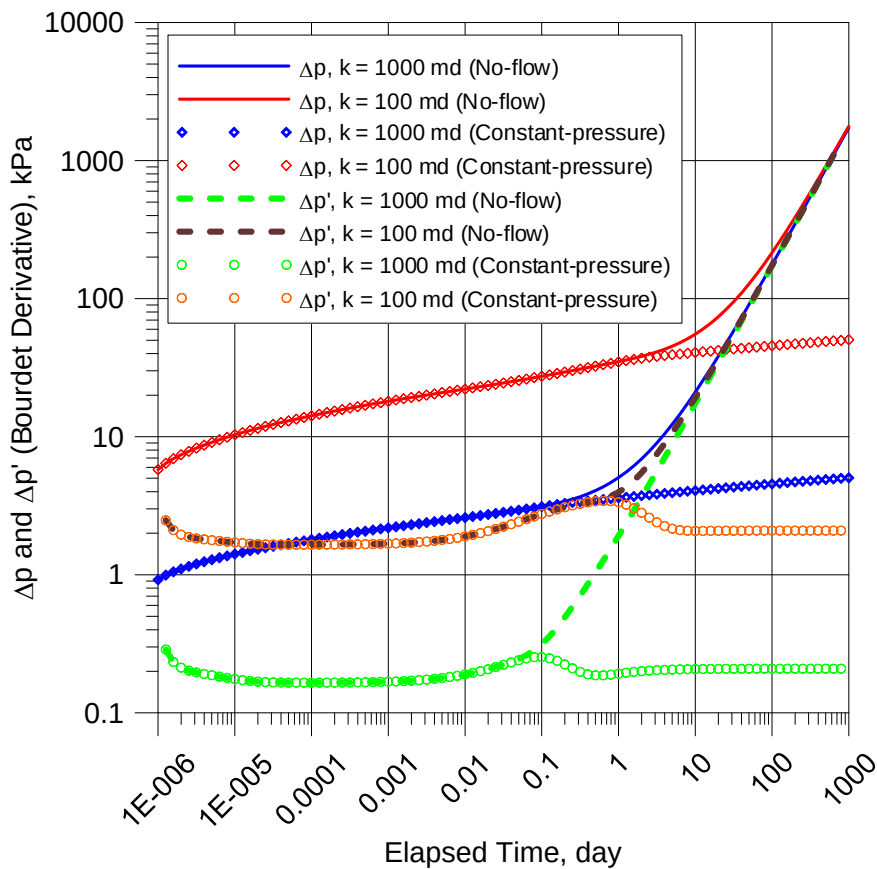


Figure 5.37: Plots of delta pressure and derivative of delta pressure as a function of elapsed time showing effect of constant-pressure, outer reservoir boundary condition during constant-rate injection.

Since it is difficult to displace the cold fluid with higher viscosity and density (i.e., the resistance increases to injection), the steady-state flow is not observed for the constant-pressure outer boundary cases during 1000 days unless the cold water zone reaches the reservoir-aquifer boundary.

For the constant-pressure outer boundary cases, it is interesting to note that a zero-slope (or constant) derivative line is observed at late time, and the value of this late-time zero-slope line is higher than that of the early-time zero-slope derivative line reflecting the hot water (or reservoir fluid) mobility. In a similar problem, injection of cold water into a hot oil reservoir having constant-pressure outer boundary, Bratvold and Horne (1990) observed the same radial flow behaviors as shown in Figure 37 and derived a mathematical equation for the late-time zero-slope line, which can be expressed as follows:

$$p_D' = 0.5(1 - M) = 0.5 \left(1 - \frac{\mu_{w,res}}{\mu_{w,inj}} \right) \quad (5.2)$$

In Equation 5.2, p_D' represents dimensionless pressure derivative based on the injected water mobility, M represents the mobility ratio. In our case, M represents the ratio of mobility of injected (cold) water to the mobility of the reservoir (hot) water. As the case considered here is a single-phase water in a homogeneous reservoir, then M in Equation 5.2 just represents the ratio of viscosity of reservoir fluid to viscosity of injected fluid, and hence the second equality of Equation 5.2 follows. If the viscosity values are replaced into the second equality of Equation 5.2, the dimensionless pressure-derivative value for the late-time solution can be calculated as follows:

$$p_D' = 0.5 \left(1 - \frac{0.000199}{0.000469} \right) \quad (5.3)$$

$$p_D' = 0.288 \quad (5.4)$$

The dimensionless pressure-derivative p_D' in the left-hand side of Equation 5.2 is defined by the following equation in SI units:

$$p_D' = \frac{2\pi kh}{q\mu_{w,inj}} \Delta p' \quad (5.5)$$

where $\Delta p'$ is the derivative of pressure with respect to natural logarithm of time, i.e., $\Delta p' = d\Delta p / d \ln t$. If the values of input parameters [$k = 1 \times 10^{-13} \text{ m}^2$, $h = 100 \text{ m}$, $q = 1.017 \times 10^{-3} \text{ m}^3/\text{s}$, $\mu_{w, inj} = 0.000469 \text{ Pa.s}$, and $\Delta p' = 2090.6 \text{ Pa} = 2.09 \text{ kPa}$, where $\Delta p'$ is read from the pressure-derive zero-slope line for $k = 100 \text{ md}$ (constant-pressure) in Figure 5.37] are used in Equation 5.5, the dimensionless-pressure derivative can be calculated as follows:

$$p_D' = \frac{2 \times \pi \times 10^{-13} \times 100}{1.017 \times 10^{-3} \times 0.000469} \times 2090.6 \quad (5.6)$$

$$p_D' = 0.275 \quad (5.7)$$

which agree well with that predicted by Equation 5.2 (or Equation 5.3).

Figure 5.38 shows the log-log diagnostic plots of delta pressure and derivative of delta pressure for the same case at constant-pressure outer boundary conditions. However this time, a very long injection period (10^9 days) is performed to be able to see the steady-state flow regime although it is practically impossible to realize this application for such a case. As can be seen from Figure 5.38, if injection time was long enough, cold water zone would reach the reservoir-aquifer boundary. Consequently, pressure would not change with time and steady-state flow regime would be observed.

Figure 5.39 shows the sandface temperature behaviors during 1000 days for the same case. As can be seen from Figure 5.39, constant-pressure, outer boundary does not affect the temperature behavior. This is caused by the fact that cold temperature front does not reach the reservoir outer boundary and so, aquifer cannot feel any temperature change. As is known, the pressure signal is transmitted faster throughout the reservoir than that of temperature.

5.2.3 Effect of rock heat capacity on the sandface pressure and temperature behaviors

In this case, colder water with $T_{inj} = 333.15\text{K}$ (60°C) is injected from fully penetrating well into the reservoir with $T^0 = 413.15\text{K}$ (140°C) at a constant-rate of 1 kg/s during 1000 days. The other reservoir and well properties given in Table 4.1 in Chapter 4 and mentioned at the beginning of Chapter 5 are also valid for this case.

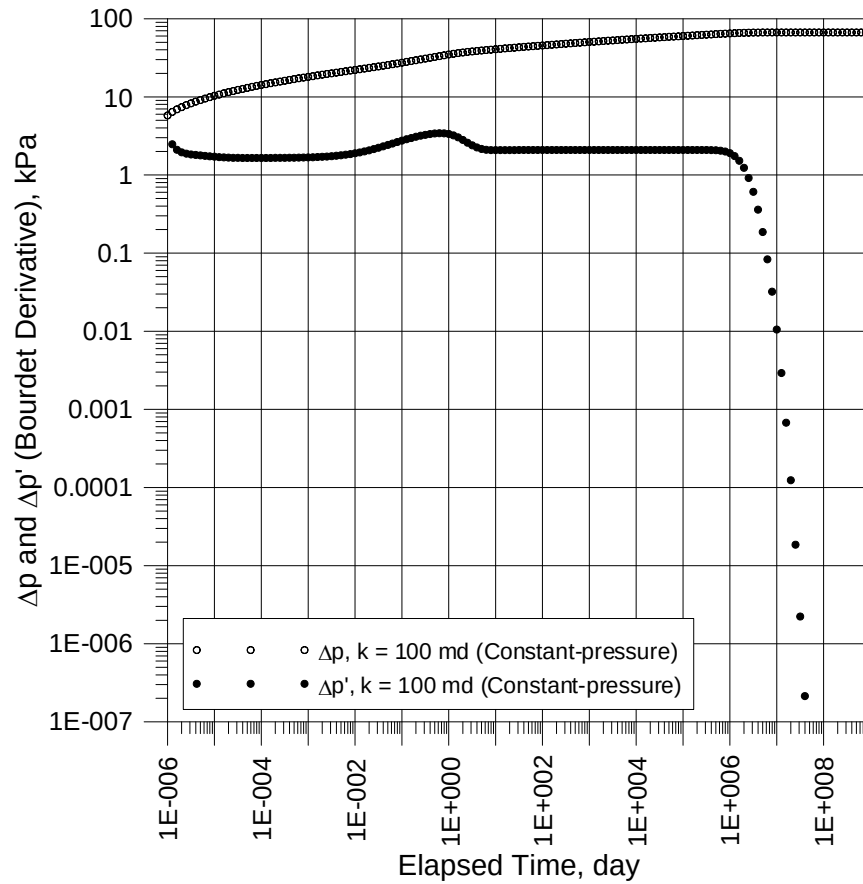


Figure 5.38: Plots of delta pressure and derivative of delta pressure as a function of elapsed time showing effect of constant-pressure, outer reservoir boundary condition during very long injection period.

Figure 5.40 shows pressure behaviors of a fully penetrating well as a function of time in the reservoir with different values of rock heat capacity for this case. According to Figure 5.40, rock heat capacity does not have any effect on pressure behavior like the constant-rate production case as expected. Figure 5.41 also supports this behavior.

Figure 5.42 shows temperature behaviors of a fully penetrating well as a function of time in the reservoir with different values of rock heat capacity for the same case. According to Figure 5.42, temperature drop in the reservoir with higher rock heat capacity is lesser because fluid in the reservoir with higher rock heat capacity is more heated by the reservoir rock than that of fluid in the reservoir with lower rock heat capacity when colder water is injected into reservoir. This is also an expected result like the constant-rate production case since reservoirs with higher rock heat capacity have higher heat content than that of reservoir with lower rock heat capacity.

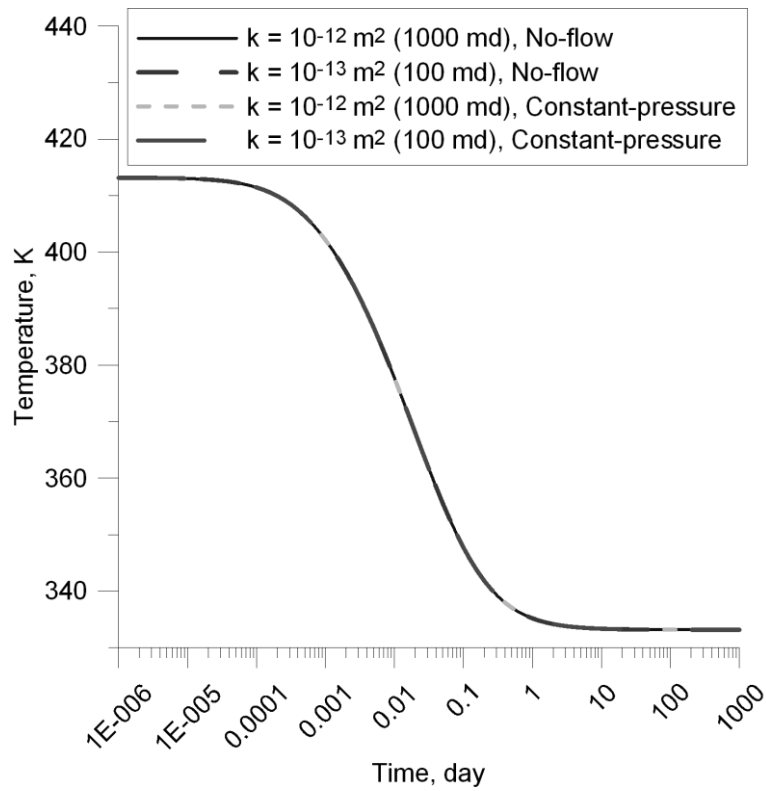


Figure 5.39: Effect of constant-pressure, outer reservoir boundary on the sandface temperature behaviors of a fully penetrating well in the reservoir with different permeability values during constant-rate injection.

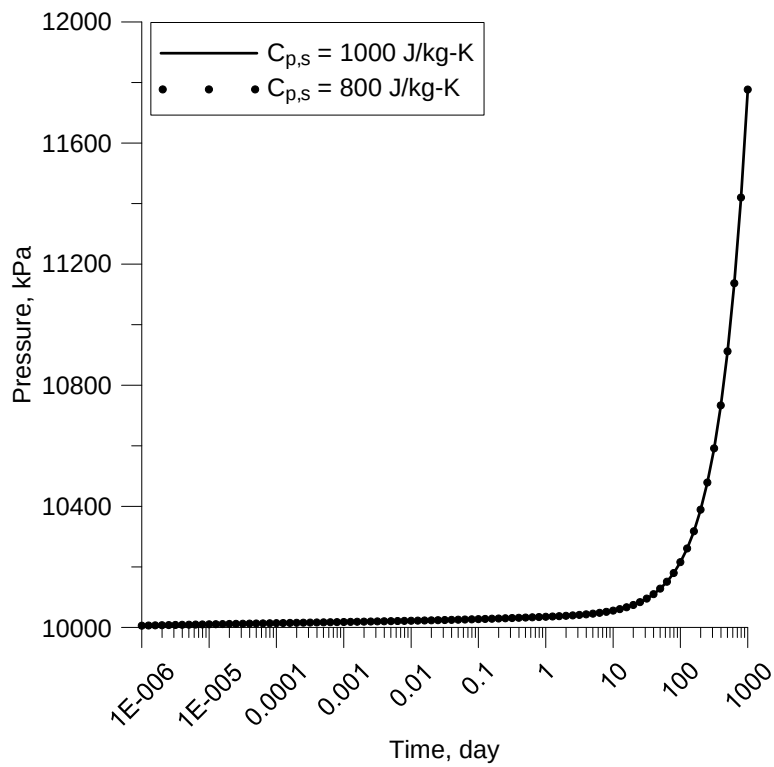


Figure 5.40: Sandface pressure behaviors of a fully penetrating well in the reservoir with different values of rock heat capacity during constant-rate injection.

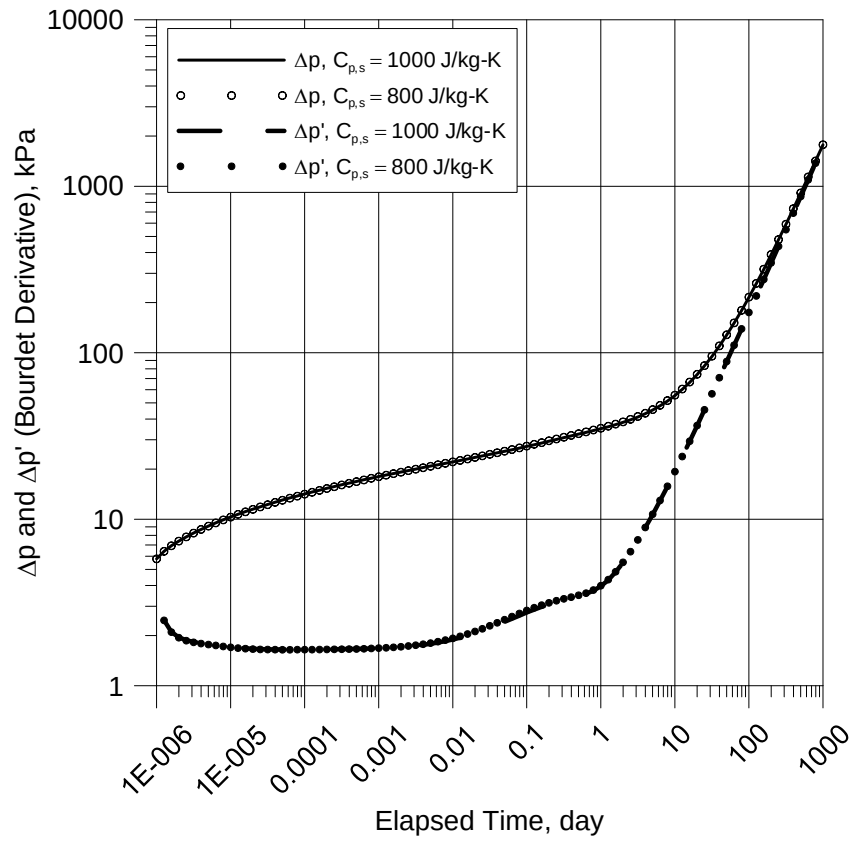


Figure 5.41: Plots of delta pressure and derivative of delta pressure as a function of elapsed time showing effect of rock heat capacity during constant-rate injection.

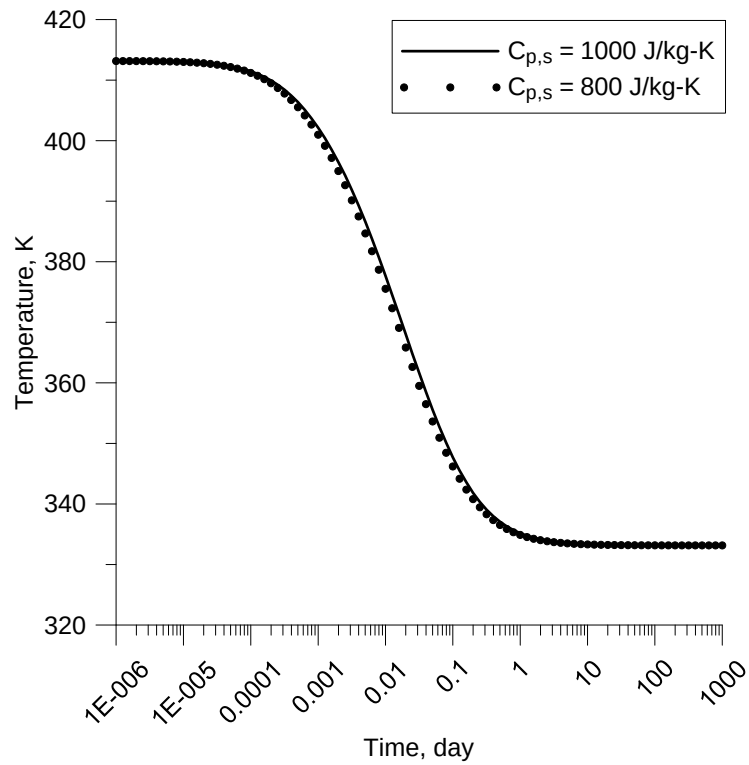


Figure 5.42: Sandface temperature behaviors of a fully penetrating well in the reservoir with different values of rock heat capacity during constant-rate injection.

5.2.4 Effect of heat transfer to strata

In this section, the same fully penetrating well case is considered. However, linear temperature gradients are assigned to overburden and underburden strata to simulate conductive heat transfer in the z-direction. Two types of cases are considered for this case as thick ($h = 100$ m) and thin ($h = 10$ m) reservoirs, respectively.

5.2.4.1 Effect of heat transfer to strata on a thick reservoir

Figure 5.43 shows the plot of temperature vs depth of reservoir, overburden and underburden strata for the thick reservoir at the initial conditions. Reservoir temperature is constant (413.15K) at the initial conditions.

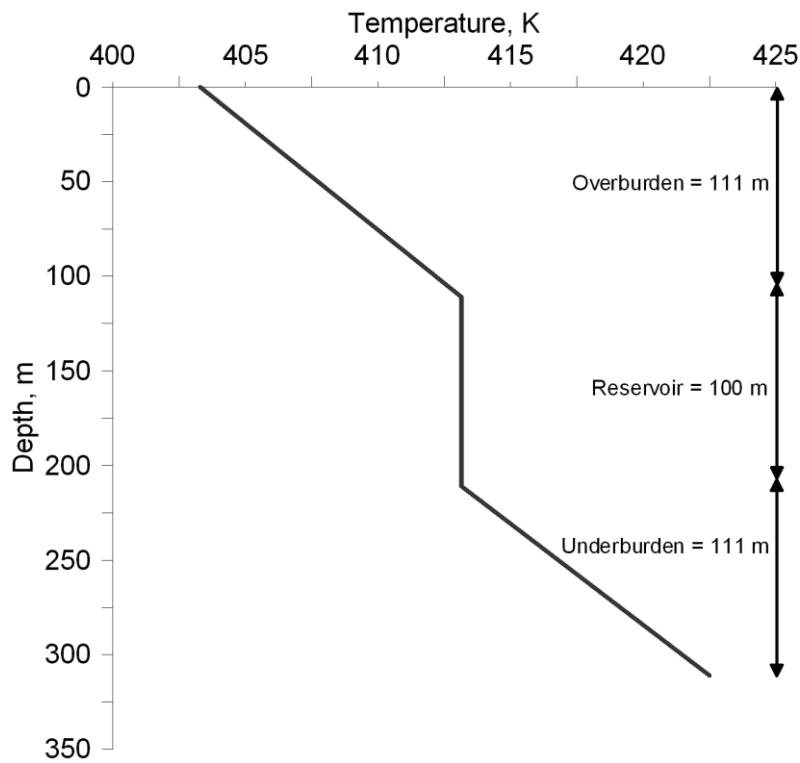


Figure 5.43:Plot of temperature vs depth of reservoir, overburden and underburden at the initial conditions.

Figures 5.44 and 5.45 show effect of conductive heat transfer from reservoir to overburden and underburden adjacent strata on the sandface pressure and temperature (in the middle of the interval open to flow) behaviors of a fully penetrating well as a function of time in the z-direction for the same case. According to the results of Figures 5.44 and 5.45, heat transfer from reservoir to adjacent strata does not have any effect on the sandface pressure and temperature behaviors.

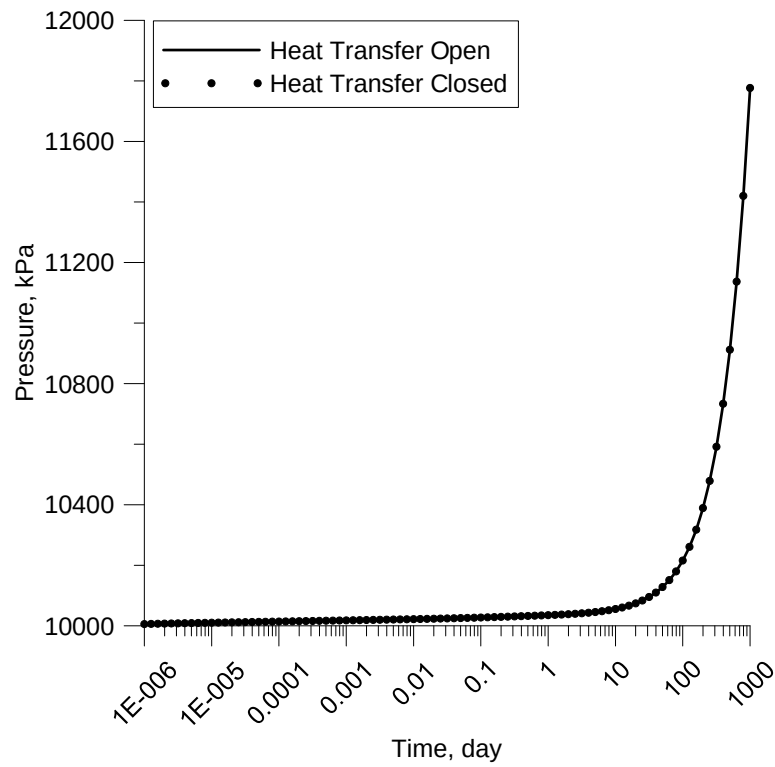


Figure 5.44: Effect of heat transfer from reservoir to adjacent strata on the sandface pressure behavior of a fully penetrating well during constant-rate injection.

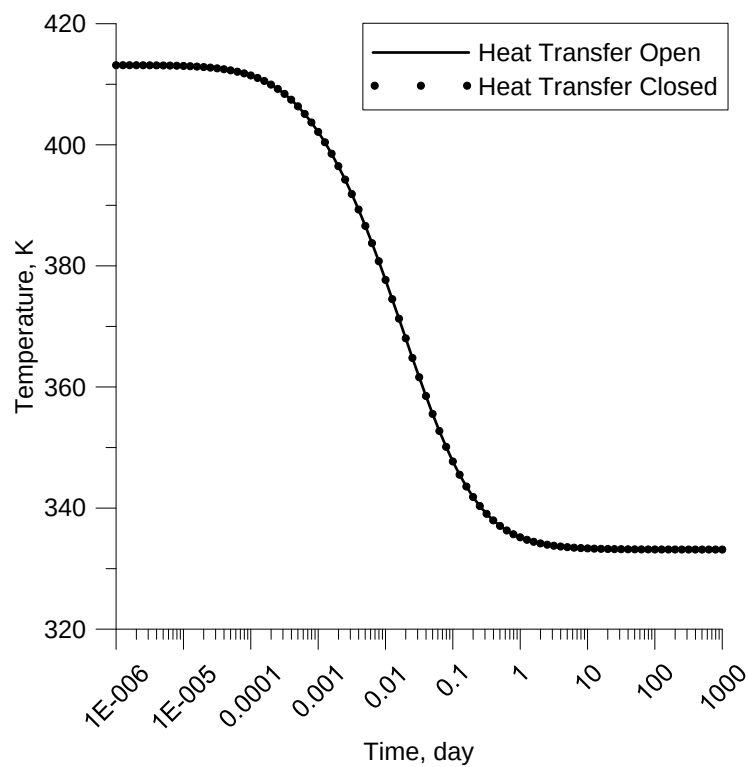


Figure 5.45: Effect of heat transfer from reservoir to adjacent strata on the sandface temperature behavior of a fully penetrating well during constant-rate injection.

This is an expected result because the reservoir is quite thick and hence heat losses are not significant and this does not lead to significant enthalpy (or temperature) changes in the reservoir as compared to enthalpy changes due to convection in the reservoir.

5.2.4.2 Effect of heat transfer to strata on a thin reservoir

Here, the effects of heat losses to strata on pressure and temperature behaviors of the reservoir are investigated for the same case if the reservoir is thin. Figure 5.46 shows the plot of temperature vs depth of reservoir, overburden and underburden strata for a thinner reservoir ($h = 10$ m) at the initial conditions. The purpose of this investigation is that conductive heat transfer to adjacent strata may be more effective on especially temperature behavior than that of the thicker reservoir ($h = 100$ m).

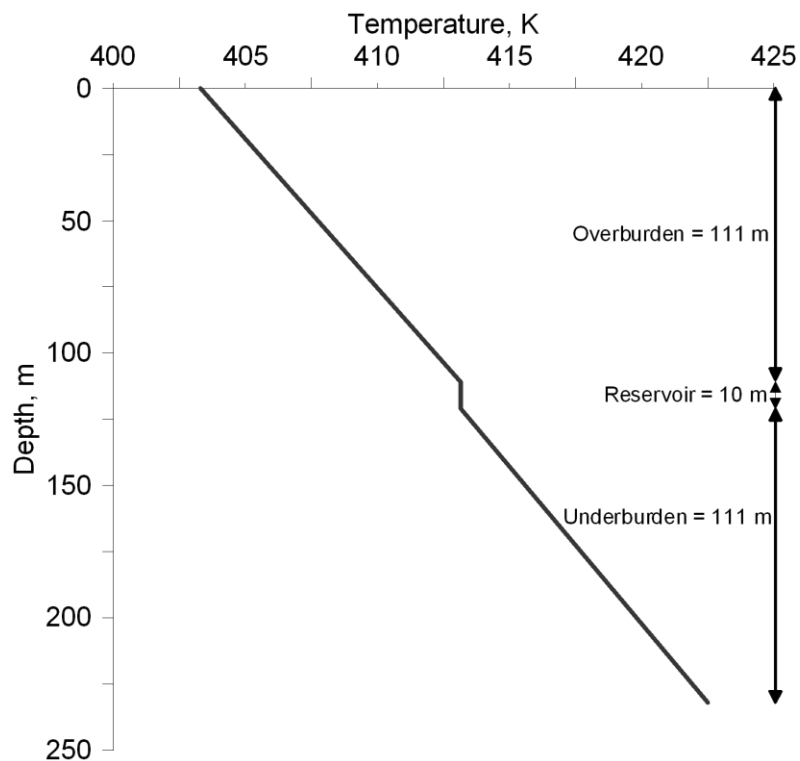


Figure 5.46:Plot of temperature vs depth of reservoir, overburden and underburden at the initial conditions for a thin reservoir ($h = 10$ m).

Figures 5.47 and 5.48 show effect of conductive heat transfer from reservoir to overburden and underburden adjacent strata on the sandface pressure and temperature (in the middle of the interval open to flow) behaviors of a fully penetrating well as a function of time in the z -direction for the same case. According to the results of Figures 5.47 and 5.48, heat transfer from reservoir to adjacent strata does not have any effect on the sandface pressure and temperature behaviors like the previous case.

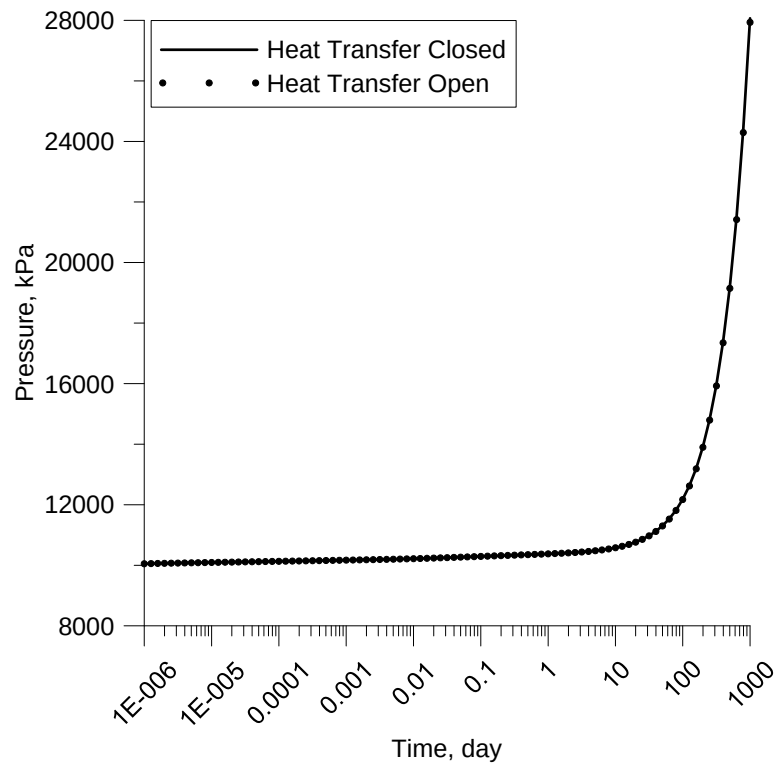


Figure 5.47: Effect of heat transfer from reservoir to adjacent strata on the sandface pressure behavior of a fully penetrating well during constant-rate injection for a thin reservoir ($h = 10$ m).

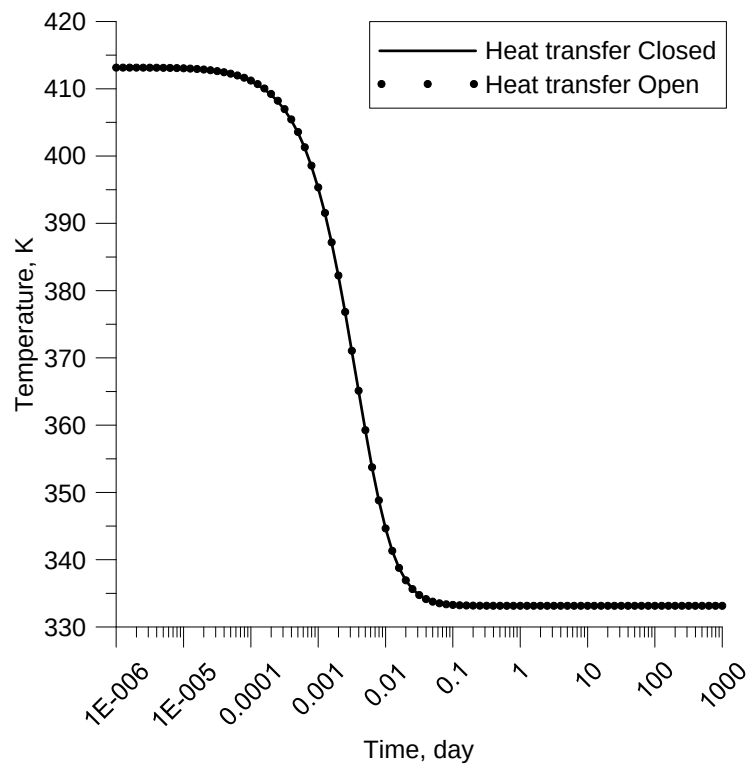


Figure 5.48: Effect of heat transfer from reservoir to adjacent strata on the sandface temperature behavior of a fully penetrating well during constant-rate injection for a thin reservoir ($h = 10$ m).

The difference of this case from the previous one is that the sandface temperature reaches the injection temperature much faster (approximately at 0.1 days) than that (approximately at 10 days) of the previous case because of the fact that this reservoir is thin.

Figure 5.49 shows a schematic of reservoir, overburden and underburden strata for a thin reservoir ($h = 10$ m) with the center, top and bottom layers of the reservoir.

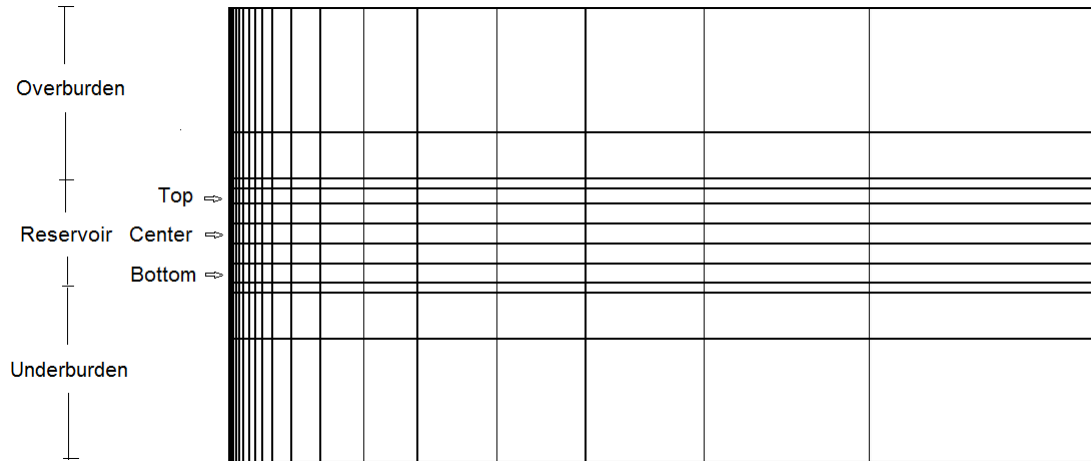


Figure 5.49: Schematic of reservoir, overburden and underburden strata for a thin reservoir ($h = 10$ m) with the top, center and bottom layers of the reservoir.

Figure 5.50 shows the log-log diagnostic plots of delta pressure and derivative of delta pressure as a function of elapsed time showing effect of heat transfer from reservoir to adjacent strata on the pressure behavior at $r = 20$ m during constant-rate injection for a thin reservoir ($h = 10$ m) for the same case. As can be seen from Figure 5.50, heat transfer from reservoir to adjacent strata does not have any effect on the pressure behavior at $r = 20$ m like as is seen in the sandface pressure behavior.

Figure 5.51 shows effect of conductive heat transfer from reservoir to overburden and underburden adjacent strata on the temperature behaviors of the center, top and bottom layers of the reservoir at $r = 20$ m as a function of time for the same case. As can be seen from Figure 5.51, the temperature behaviors at the center, top and bottom layers do not change if heat transfer is closed. On the other hand, changes in the temperature behaviors are observed due to the fact that this reservoir is thinner than that of the previous case if heat transfer is open. The lowest cooling effect is observed at the temperature of the bottom layer of the reservoir because overburden temperatures are greater than those of underburden and the greatest conductive heat contribution comes

from the underburden strata. As expected, the highest cooling effect occurs in the temperature of the center layer because the less amount of heat by conduction is transferred to this layer from underburden and overburden adjacent strata in comparison with those of the other layers. These results indicate the locations of temperature become important in thin reservoirs.

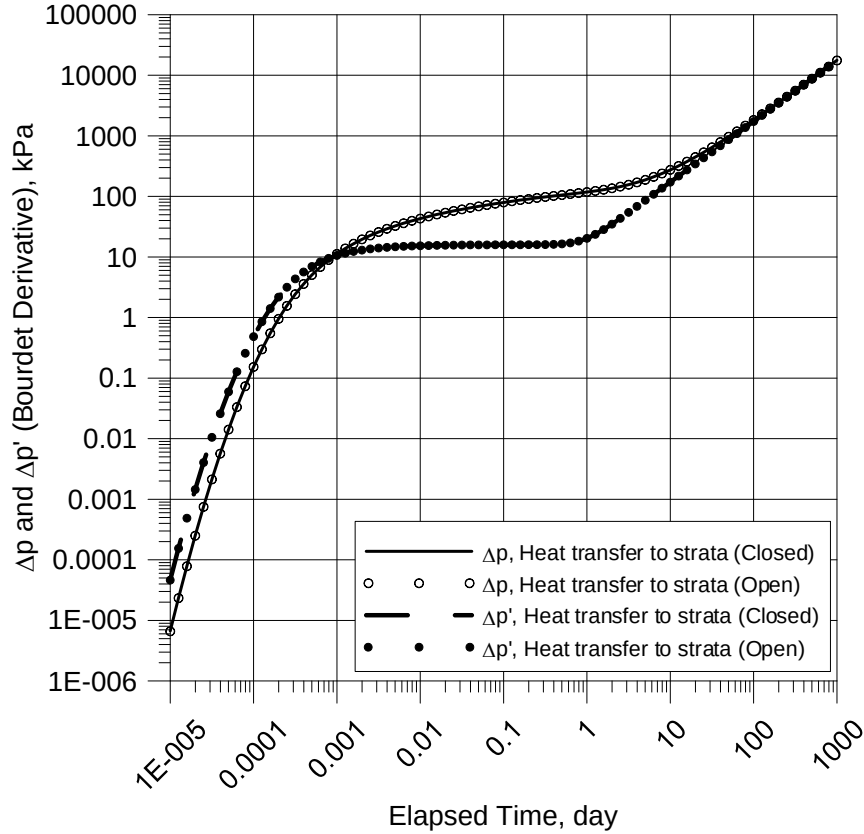


Figure 5.50: Plots of delta pressure and Bourdet derivative as a function of elapsed time showing effect of heat transfer from reservoir to adjacent strata on the pressure behavior at $r = 20$ m during constant-rate injection for a thin reservoir ($h = 10$ m).

5.2.5 Effect of a finite wellbore approach on pressure and temperature behaviors

This is the same case given in Section 3.6.1 of Chapter 3. Since the wellbore and sandface pressure and temperature behaviors have been discussed in detail in Chapter 3, they are not explained here again.

Figure 5.52 shows the sensitivities of wellbore and sandface pressures to porosity for 1D (r) injection case are investigated for the finite wellbore approach for a constant-rate injection case. On the other hand, Figure 5.53 shows the sensitivities of wellbore and sandface pressures to the other rock parameters (permeability, rock thermal conductivity, rock density and rock heat capacity).

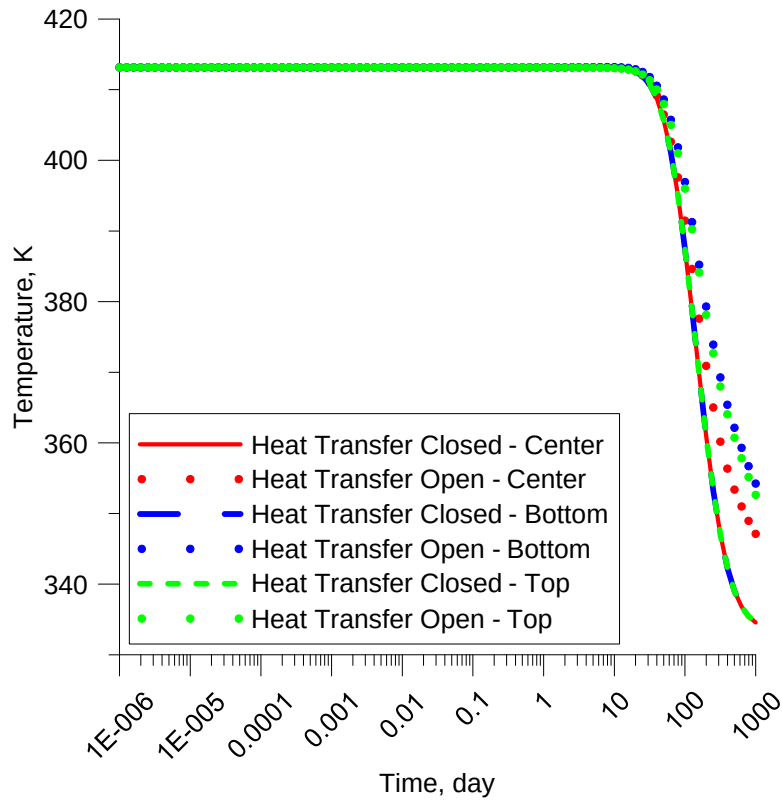


Figure 5.51: Effect of heat transfer from reservoir to adjacent strata on the temperature behavior at $r = 20$ m for center, top and bottom layers of a fully penetrating well during constant-rate injection for a thin reservoir ($h = 10$ m).

In Figures 5.52 and 5.53, “wf” represents wellbore while “sf” represents sandface. It can be seen from Figures 5.52 and 5.52 that the rock parameter showing the greatest sensitivity to the pressure is porosity for this case. After the porosity, permeability has some sensitivity to the pressure. Pressure shows some sensitivity to rock density and rock heat capacity depending on the pressure behavior as a function of time while the pressure shows a little sensitivity to the rock thermal conductivity for this case. Pressure sensitivities are not affected from wellbore or sandface in respect of the location of the pressure.

Figure 5.54 shows the sensitivities of the rock parameters to the wellbore and sandface temperature for the same case. As can clearly be seen from Figure 5.54, the wellbore temperature is not sensitive to any rock parameter while the sandface temperature shows some sensitivities to the rock parameters. At this point, it can be concluded that the sandface temperature should be used to investigate the sensitivities rather than the wellbore temperature for a such type of wellbore-reservoir model. Rock thermal conductivity has the greatest sensitivities.

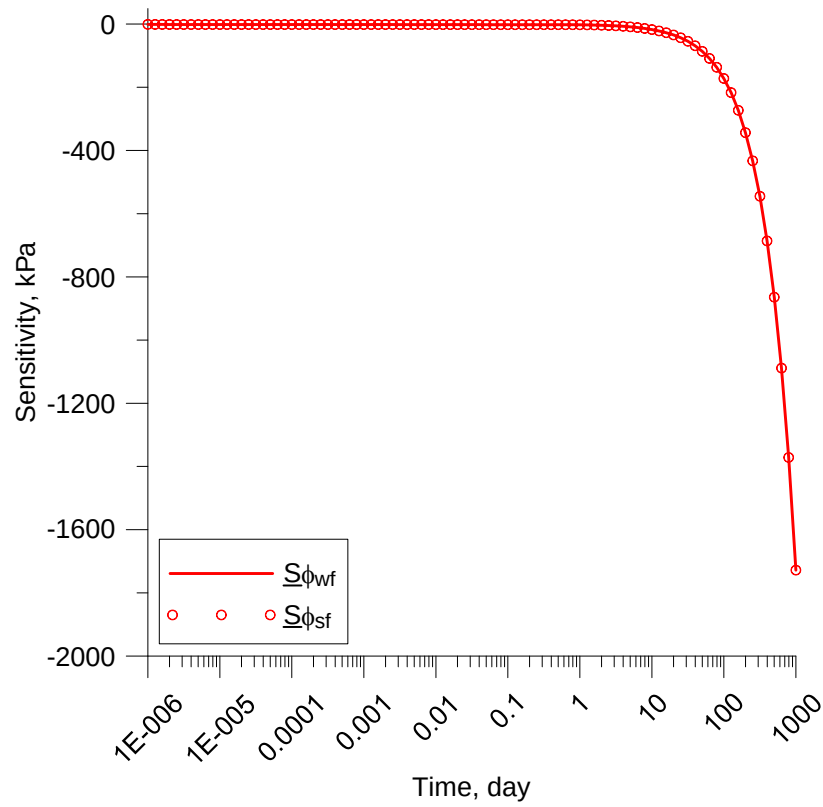


Figure 5.52: The sensitivities of the porosity to the wellbore and sandface pressures for 1D (r) injection well at a constant-rate.

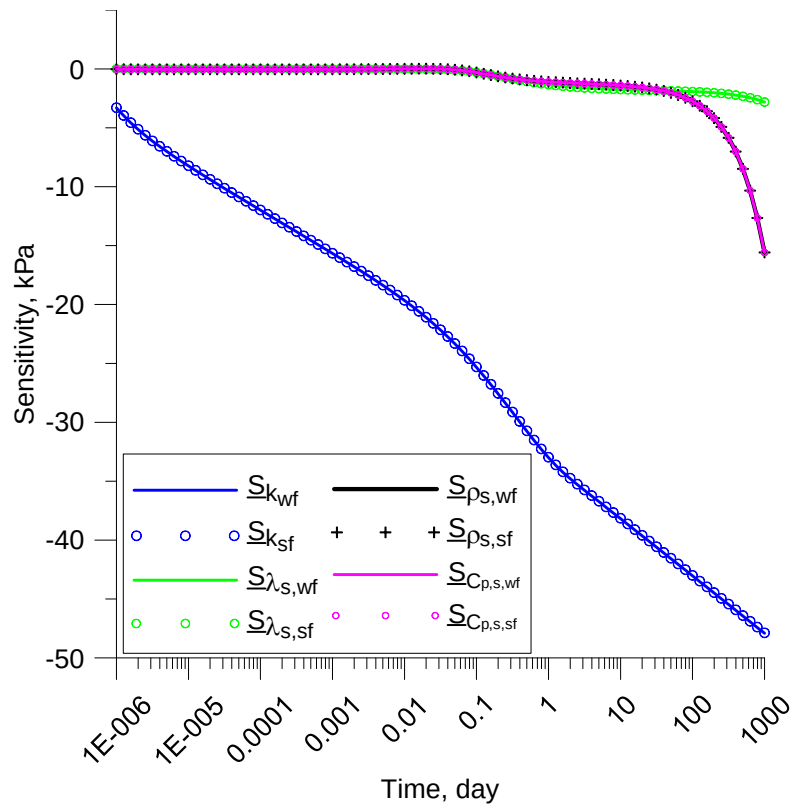


Figure 5.53: The sensitivities of the other rock parameters to the wellbore and sandface pressures for 1D (r) injection well at a constant-rate.

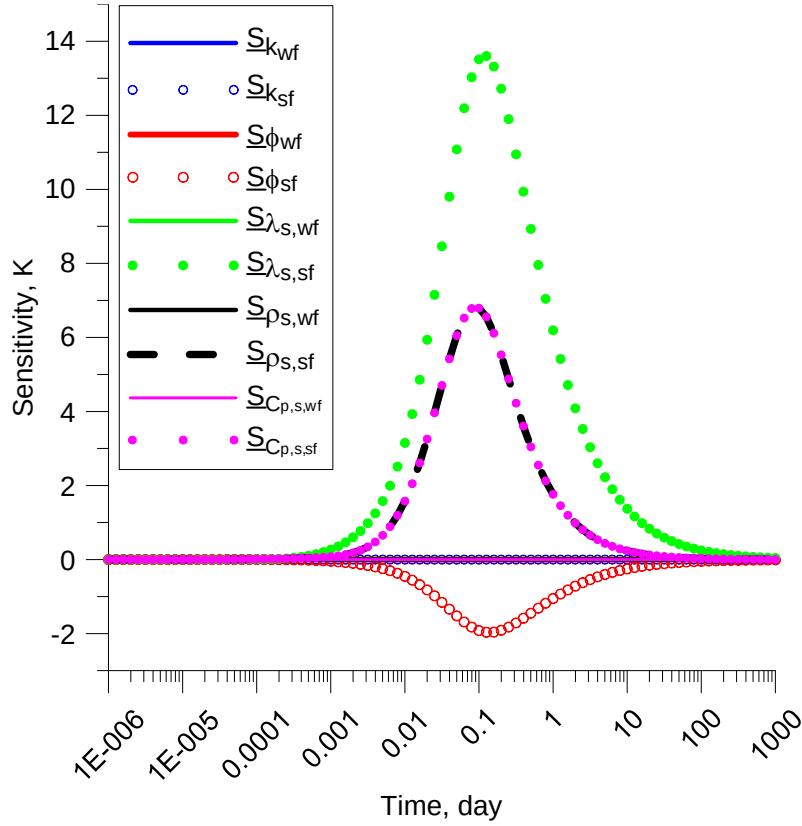


Figure 5.54: The sensitivities of the rock parameters to the wellbore and sandface temperatures for 1D (r) injection well at a constant-rate.

After the rock thermal conductivity, rock density and rock heat capacity have important sensitivities. These parameters show positive sensitivities. It means that the temperature increases with increasing rock thermal conductivity, rock density or rock heat capacity. On the other hand, the temperature shows some sensitivity to porosity. Sensitivity to porosity is negative. This is an expected result because temperature increases with decreasing porosity due to increasing rock volume fraction. If the trends of the sensitivities of the parameters are carefully inspected, it can be seen that these trends are related to the sandface temperature behaviors. If Figure 3.13 in Chapter 3 is recalled, temperature roughly remains unchanged at early time, and so there is no sensitivities at early time. Then, the sensitivities of the parameters starts increasing with decreasing temperature until the sandface temperature approximates to the injection temperature. The sensitivities show a peak while the sandface temperature approximates the injection temperature. After reaching the injection temperature, another period starts that the sandface temperature approximately remains unchanged. Hence, the sensitivities approximate to zero again.

5.2.6 Effect of gravity on pressure and temperature behaviors

Up to this point, gravity effect has been ignored in all cases performed. Here, at first, gravity effects on the sandface pressure and temperature behaviors (in the middle of the interval open to flow) are investigated.

In this case, colder water with $T_{inj} = 333.15\text{K}$ (60°C) is injected from limited-entry well into the reservoir with $T^0 = 413.15\text{K}$ (140°C) at a constant-rate of 1 kg/s during 1000 days. The other reservoir and well properties given in Table 4.1 in Chapter 4 and mentioned at the beginning of Chapter 5 are also valid for this case. The reference layer is taken as the center of the middle layer. However, an initialization process in respect of the pressure distribution is required in the z -direction before starting the injection to apply a realistic case with the gravity effect.

When the initialization begins, initial reservoir pressure is 10000 kPa in all locations of the reservoir. For this purpose, firstly, the case has been run at a constant-rate of 0 kg/s (no-production and no-injection) during 10^6 days to achieve a gravitational (non-uniform) pressure distribution in the z -direction. This distribution represents the initial pressure conditions required to initiate the injection case. Figure 5.55 shows the grid system (uniform grids in the z -direction) and the initial pressure distribution used in this case.

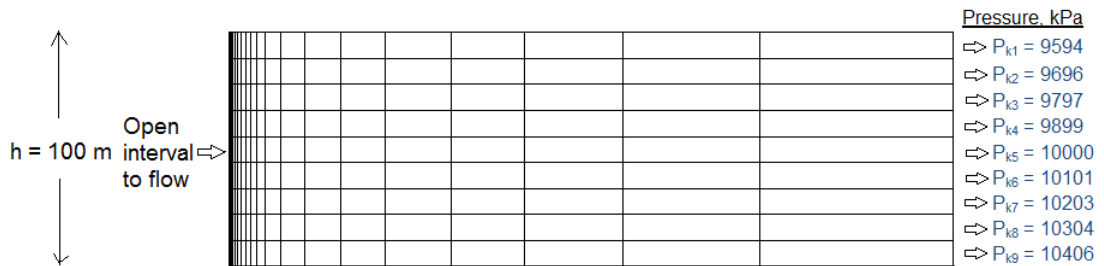


Figure 5.55: Schematic of the gridblock structure for the constant-rate injection case with gravity effect for limited-entry well.

Figures 5.56 shows the gravity effect on the sandface pressure behaviors of a limited-entry injection well as a function of time. As can be seen from Figure 5.56, the sandface pressure in the other layers excluding that in the center layer (open interval layer) remains approximately near the pressure about of 10000 kPa because of the uniform initial pressure distribution (no-gravity effect) until the pseudosteady-state flow begins. After this period, the pressures in the other layers increase. The sandface pressure in the center layer follows the expected pressure increase trend as previously

observed. The sandface pressure in the center layer does not change with gravity because it is the reference layer and initial pressure is the same (10000 kPa) in this layer for both two different cases. On the other hand, the sandface pressures in the adjacent upper layer of the center layer and top layer take lower values than the others while the sandface pressures in the lower layer adjacent to the center layer and bottom layer take higher values with the gravity effect.

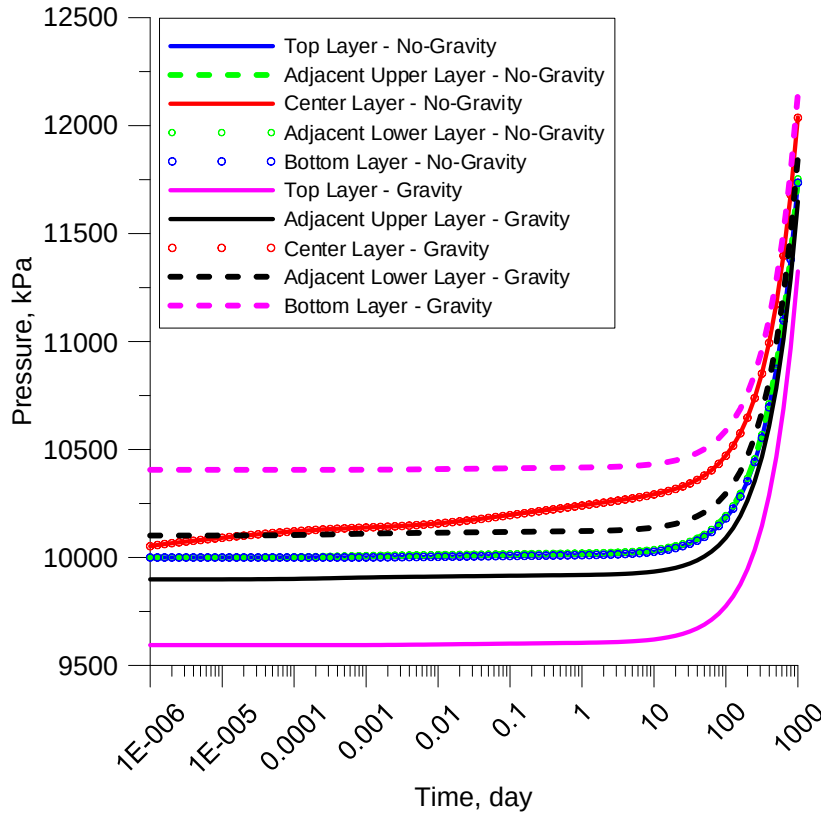


Figure 5.56: Sandface pressure behaviors of a limited-entry well in the reservoir with gravity effect during constant-rate (1 kg/s) injection.

Figure 5.57 shows the gravity effect on the sandface temperature behaviors for the same case. As can be seen from Figure 5.57, the sandface temperature does not change in the z-direction like the pressure if the gravity effect is not active. Similarly, the sandface temperature in the center layer does not change with gravity because it is the reference layer and initial pressure is the same (10000 kPa) in this layer for both two different cases. On the other hand, even if its effect is not so clear, the sandface temperature in the upper layer adjacent to the center layer with gravity takes higher values than that of the upper layer adjacent to the center layer with no-gravity and that of the upper layer adjacent to the center layer with gravity. Additionally, the sandface temperature at the top layer with gravity takes higher values than that of top layer with

no-gravity and that of the bottom layer with gravity because the layers at the bottom of the reference layer cools more than the other ones with the effect of gravity. The reason is that the layers at the bottom of the reference layer cools more with the effect of gravity is that the injected water has higher density and hence tends to go the bottom layers due to gravity as it is cooler than the reservoir fluid.

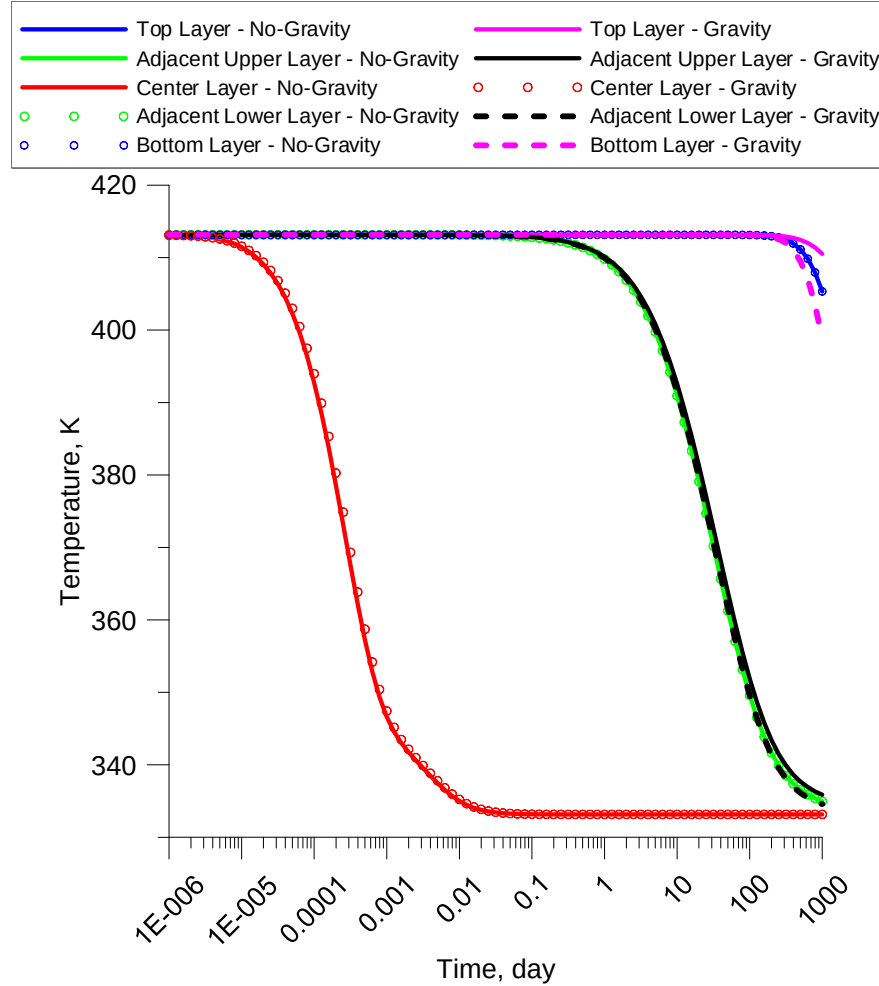


Figure 5.57: Sandface temperature behaviors of a limited-entry well in the reservoir with gravity effect during constant-rate (1 kg/s) injection.

Figures 5.58 and 5.59 show the 2D (r - z) temperature distribution maps without gravity and with gravity for this case at 1000 days, respectively. As can be seen from Figures 5.58 and 5.59, the case without gravity exhibits a symmetrical temperature distribution in the z -direction while the case with gravity exhibits a slightly non-symmetrical temperature distribution in the z -direction. According to Figure 5.59, in the case with gravity, the reservoir temperatures are lower in the lower portion of the reservoir with the effect of the gravity and temperature signal cannot reach the upper boundary of the

reservoir. Furthermore, spherical flow is clearly observed in Figures 5.58 and 5.59 with the effect of the limited-entry.

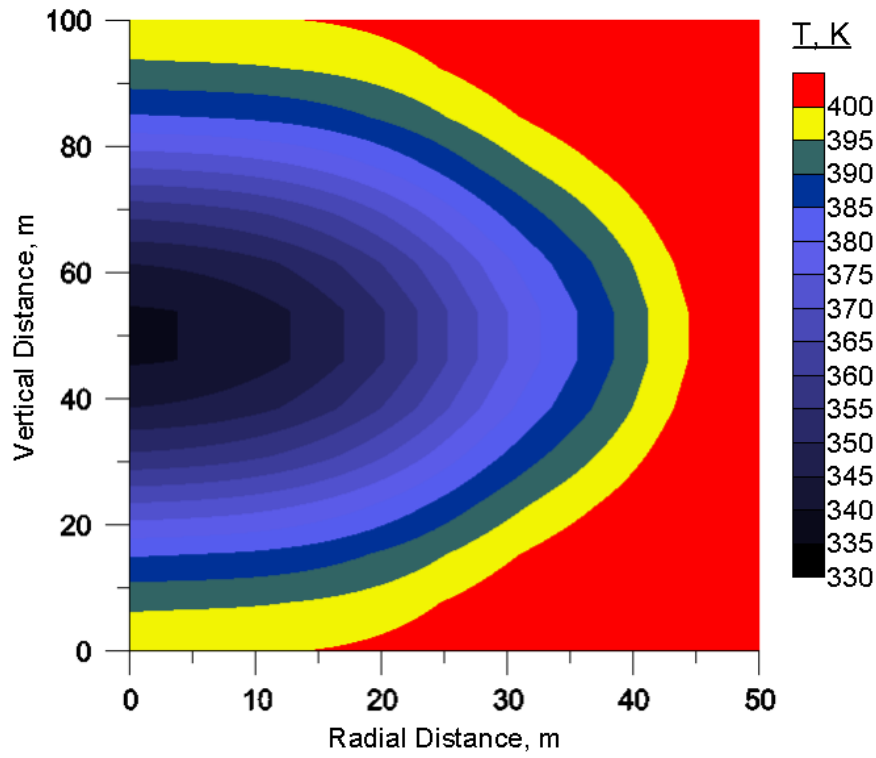


Figure 5.58: 2D (r - z) temperature distribution map in the reservoir at 1000 days for a limited-entry well without gravity during constant-rate (1 kg/s) injection.

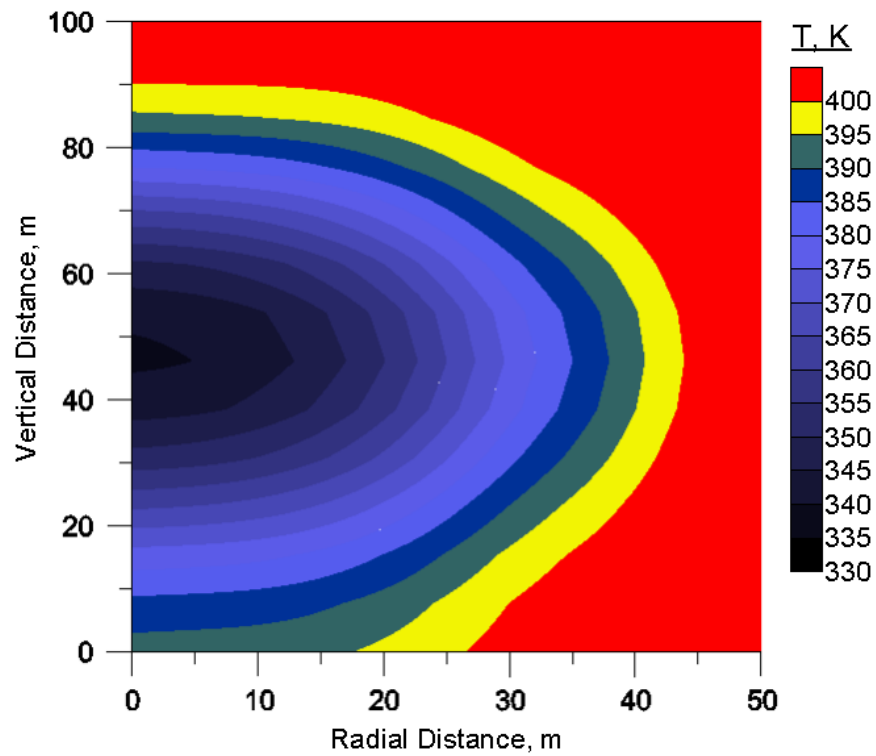


Figure 5.59: 2D (r - z) temperature distribution map in the reservoir at 1000 days for a limited-entry well with gravity effect at constant-rate (1 kg/s) injection.

5.2.7 Effect of rock thermal conductivity and flow rate on the sandface pressure and temperature behaviors

In this case, the effects of the rock thermal conductivity to the sandface pressure and temperature are investigated for an injection–fall-off case. From this point throughout the thesis, gravity is ignored in all cases.

In this case, injection cases are performed at constant mass rates of 1 kg/s and 10 kg/s during 1 day and then, the well is shut-in during another 1 day period. The other reservoir and well properties given in Table 4.1 in Chapter 4 and mentioned at the beginning of Chapter 5 are also valid for this case.

Figure 5.60 shows the sandface pressure behaviors (in the middle of the interval open to flow) of a fully penetrating well in the reservoir with different values of rock thermal conductivity during injection-falloff cases. According to Figure 5.60, the effect of rock thermal conductivity on pressure behavior is negligible. If pressure behaviors are inspected in respect of rate effect, much more pressure increase occurs in higher rate (10 kg/s) because much more amount of fluid is injected into the reservoir in contrast to that in lower rate (1 kg/s). A compression effect at the sandface is created by the injection of higher amount of fluid and this causes pressure increase at higher rate. It should be noted that the pressure of the reservoir with higher injection rate stabilizes at a higher pressure than that with lower rate at the end of the fall-off, but this is expected because the average reservoir pressure is higher in higher injection rate case and hence the late-time fall-off pressure stabilizes at this higher reservoir pressure.

Figure 5.61 shows the sandface temperature behaviors for the same case. In contrast to pressure behavior, the temperature is significantly affected by change of the rock thermal conductivity during both injection and fall-off periods. It can be seen from Figure 5.61 that the temperature drop decreases as the rock thermal conductivity increases during injection period. On the other hand, the temperature increases for all cases during the fall-off period because fall-off is a conduction dominated process. Besides, as can be seen from Figure 5.61, much more temperature drops occur in higher rate (10 kg/s) in the injection period because convection becomes more dominant over conduction as the injection rate increases and hence the sandface temperature approaches injection temperature faster.

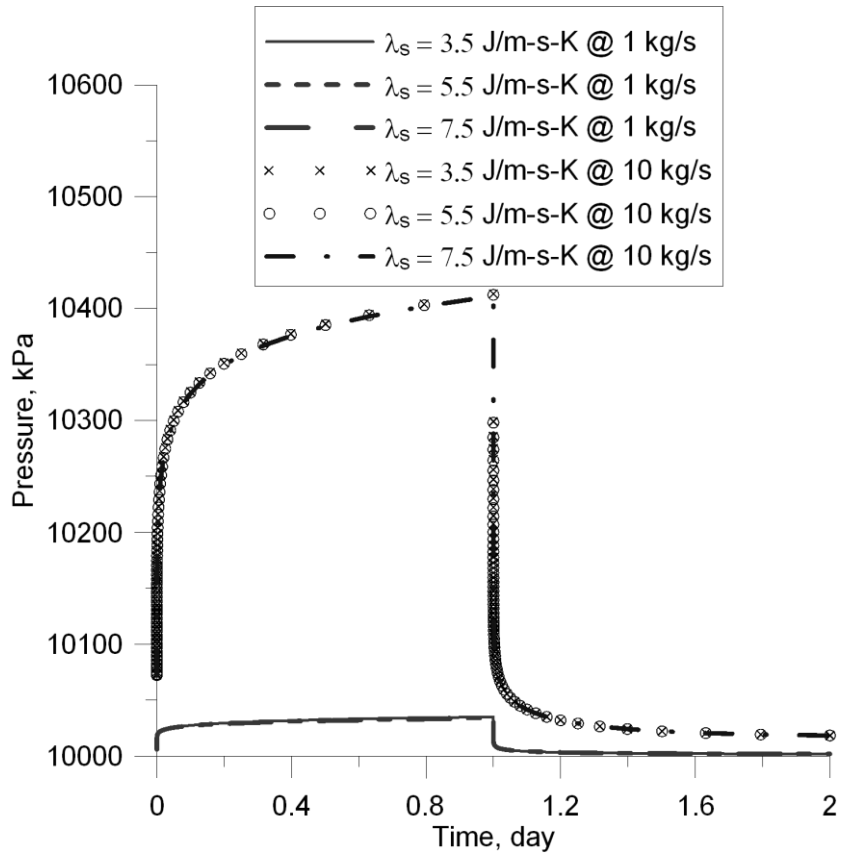


Figure 5.60: Effect of rock thermal conductivity on the sandface pressure behavior as a function of time with rate effect during injection and fall-off periods.

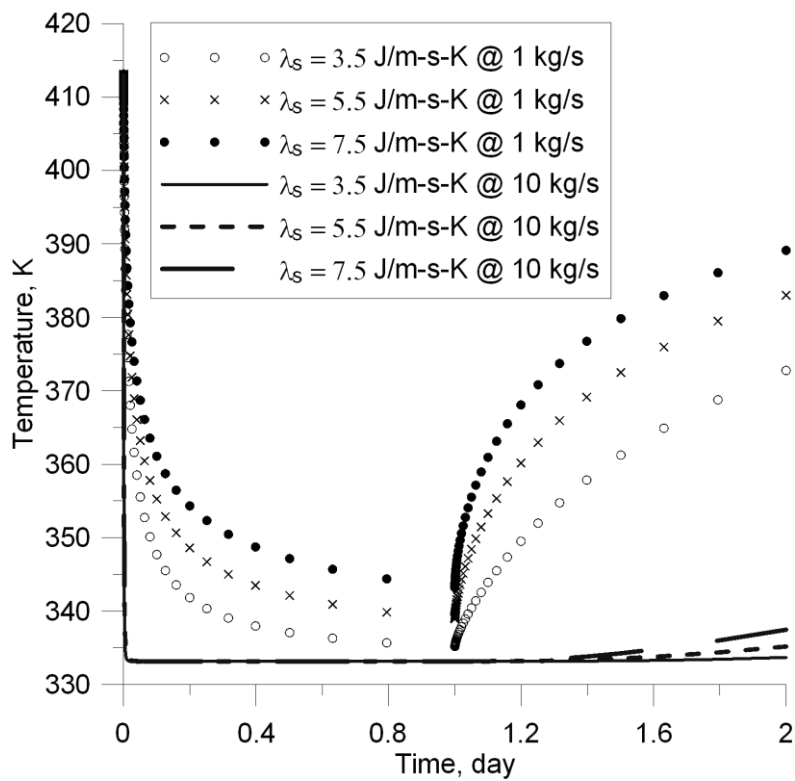


Figure 5.61: Effect of rock thermal conductivity on the sandface temperature behavior as a function of time with rate effect during injection and fall-off periods.

5.2.8 Effect of porosity and flow rate on the sandface pressure and temperature behaviors

In this case, an injection is performed at constant mass rate of 1 kg/s during 1 day and then, the well is shut in for another 1 day period. The other reservoir and well properties given in Table 4.1 in Chapter 4 and mentioned at the beginning of Chapter 5 are also valid for this case.

Figure 5.62 presents the sandface pressure behaviors of a fully penetrating well in the reservoir with different porosity values during an injection–falloff case. As expected, pressure has higher values during injection and fall-off periods for lower porosity because the reservoir with lower porosity has higher diffusivity.

Figure 5.63 shows the behavior of the sandface temperature for the same case. The temperature for the reservoir with lower porosity is higher than that with higher porosity during injection and fall-off periods. The effect of porosity on the temperature becomes more pronounced especially for the fall-off period because the effect of conduction on the temperature becomes comparable with the effect of convection on the temperature during fall-off period.

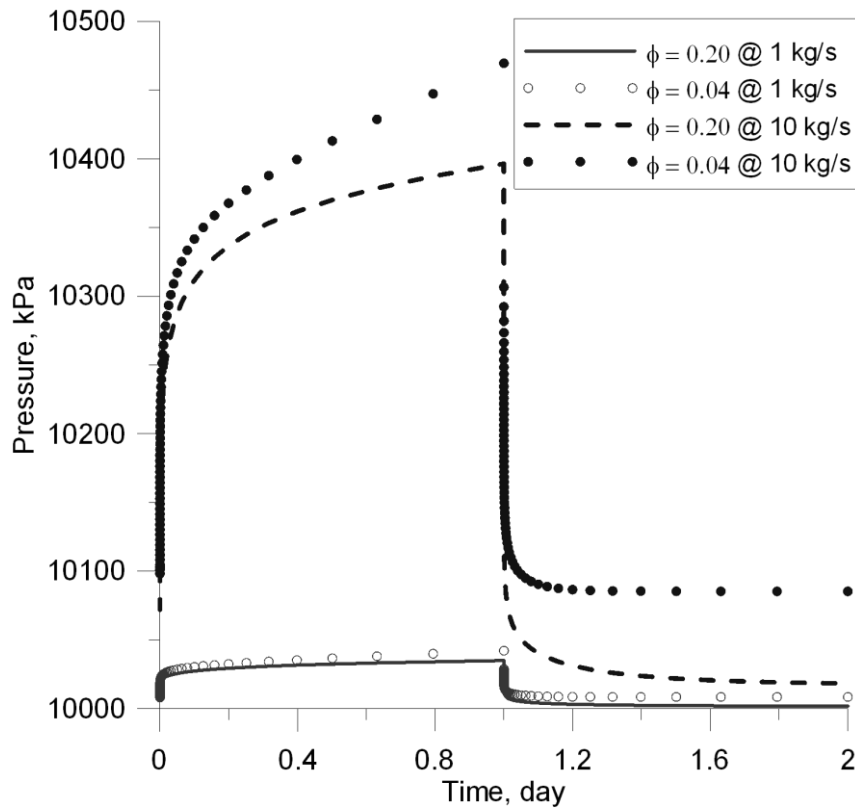


Figure 5.62: Effect of porosity on the sandface pressure behaviors as a function of time during injection and fall-off periods.

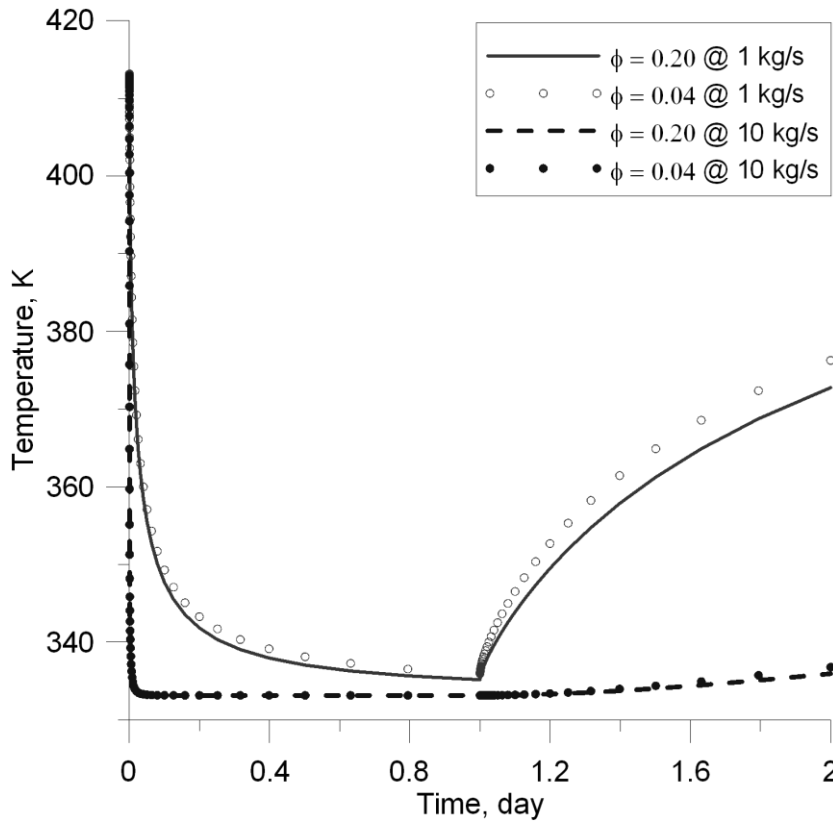


Figure 5.63: Effect of porosity on the sandface temperature behaviors as a function of time during injection and fall-off periods.

Furthermore, as noted in the derivation of the total energy balance equation in Chapter 3 and Appendix A, the total thermal conductivity [$\lambda_t = \phi\lambda_w + (1-\phi)\lambda_s$] of the reservoir changes as the porosity changes. The total thermal conductivity will also increase with decreasing porosity value since the rock thermal conductivity is much greater than that of water. At this point, it can be understood that the effect of porosity on temperature may be important especially at fall-off period if noise on temperature measurements is not so much. Moreover, as can also be seen from Figure 5.63, much more temperature drops occur in higher rate (10 kg/s) in the injection period like the previous case. Similarly, during fall-off, the temperature for the reservoir where water is injected at a higher rate is lower than that of the reservoir where water is injected at a lower rate. The reservoir with a low porosity has a high temperature as expected.

5.2.9 Effect of skin on the sandface pressure and temperature

Figure 5.64 shows the sandface pressure behaviors of a fully penetrating well in the reservoirs with different skin factor values during an injection–falloff case.

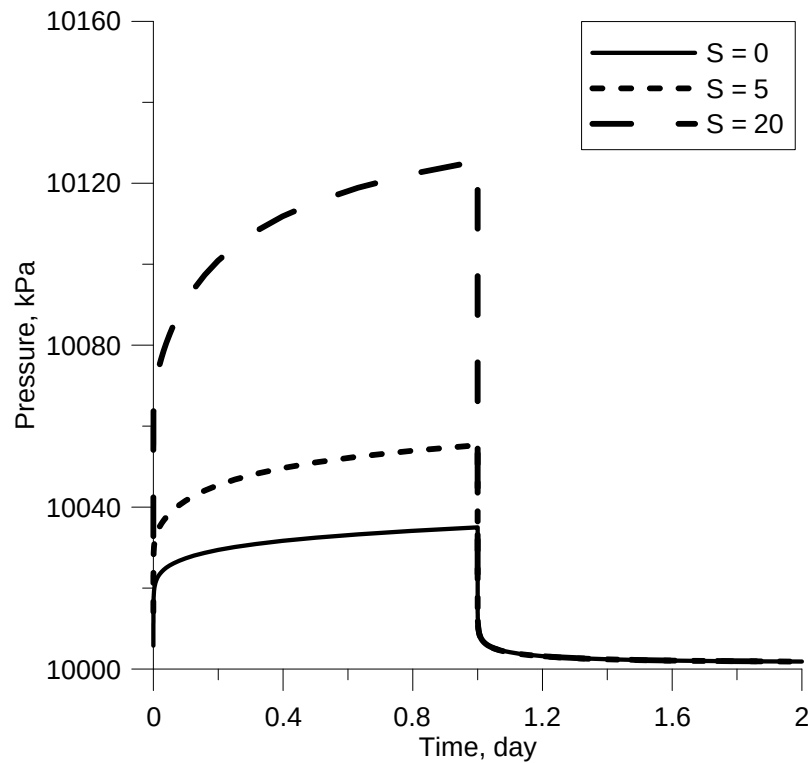


Figure 5.64: Effect of skin factor on the sandface pressure behaviors during injection and fall-off periods.

The reservoir and well properties given in Table 4.1 in Chapter 4 and mentioned at the beginning of Chapter 5 are also valid for this case. Increasing skin increases the sandface pressure because flow here is from well into the reservoir and resistance to flow due to increasing skin creates an additional pressure increase near the well at injection period. This is an expected behavior. All cases exhibit similar pressure drop behaviors by approximating to the initial reservoir pressure at fall-off period because fall-off pressure data are independent of skin if there are not wellbore storage effects.

Figure 5.65 shows the sandface temperature behaviors for the same case. It should be noted that temperature is not affected from skin during constant-rate injection and fall-off periods. This is an expected result because injection is at a constant-rate. No matter what the skin is the same amount of water (hence the same amount of heat) will enter the reservoir. If the injection was performed at constant pressure, the injection rate would become variable. Hence, different amounts of heat would have been injected into the reservoirs with different skins for such a case.

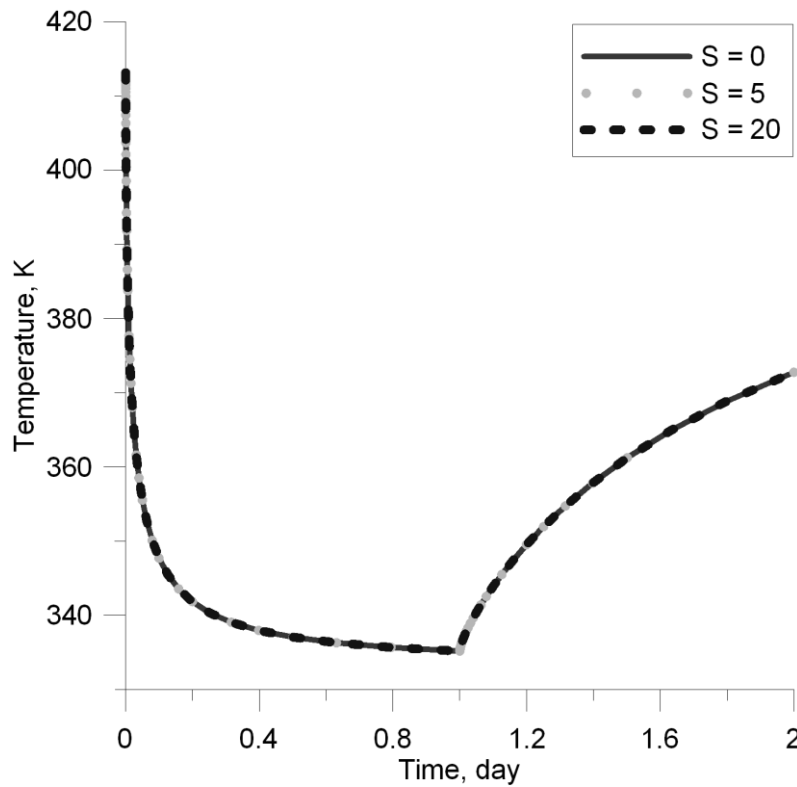


Figure 5.65: Effect of skin factor on the sandface temperature behaviors during injection and fall-off periods.

5.2.10 Effect of limited-entry length on the sandface pressure and temperature behaviors

In this subsection, the effects of the limited-entry lengths on the sandface pressure and temperature are investigated for injection-falloff cases. In this case, open interval lengths to flow are considered to be in the middle of the reservoir in z -direction like the previous cases. The other reservoir and well properties given in Table 4.1 in Chapter 4 and mentioned at the beginning of Chapter 5 are also valid for this case. Figure 5.66 shows a schematic of the limited-entry injection well.

Figure 5.67 shows the sandface pressure behaviors for the wells with different open intervals to flow (from 1 m to 100 m) during an injection-falloff case. According to Figure 5.67, pressure increases as the open interval length decreases. This is an expected behavior because the limited-entry creates a resistance to flow and hence pressure increases at a constant-rate injection period. All cases exhibit the similar pressure behaviors by approximating to the initial reservoir pressure at fall-off period.

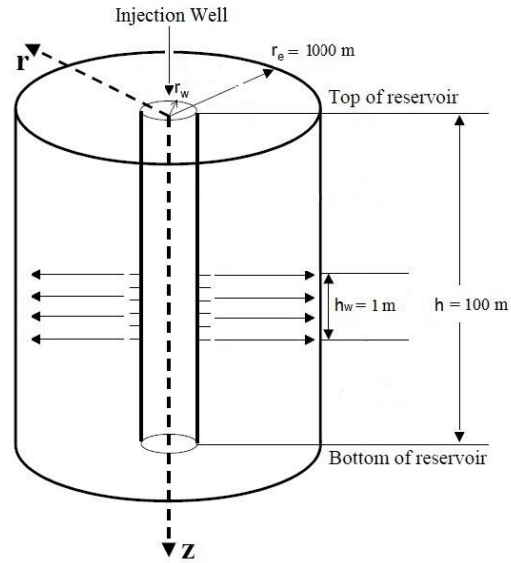


Figure 5.66: Schematic of limited-entry injection well.

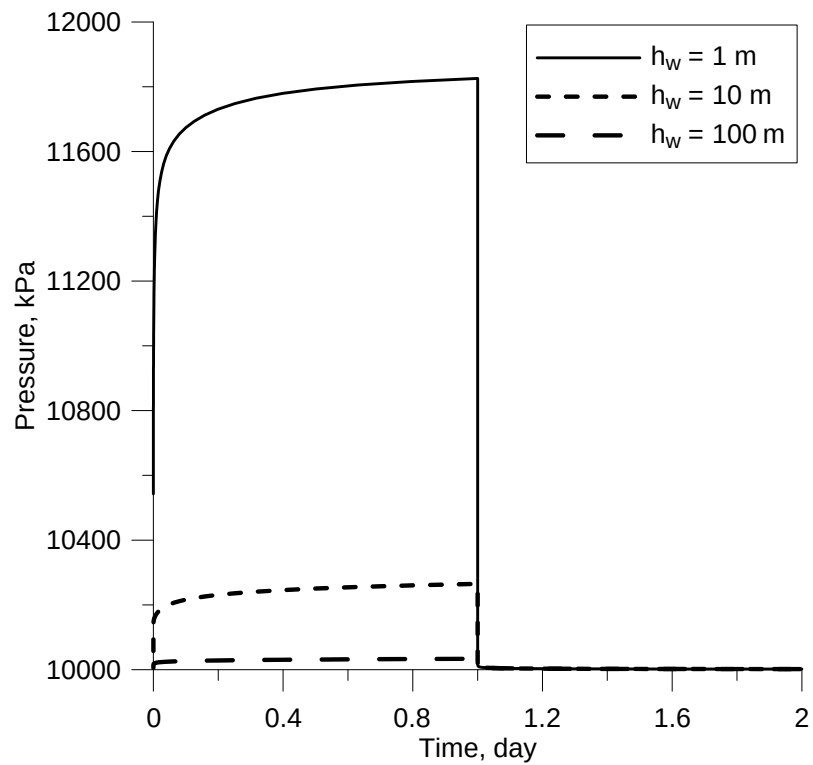


Figure 5.67: Effect of the open interval length on the sandface pressure behaviors during injection and fall-off periods.

Figure 5.68 gives the effects of the open interval length (h_w) on temperature. At this point, it is important to note that the temperature is measured halfway through the thickness of the reservoir. For all cases of limited-entry lengths, the rate is kept constant and the same. However, it should be noted that when the limited-entry length

is increased, lesser fluid is passed through the point where the gauge is placed. This results in less cooling. Differences in temperature behavior after the shut-in period are due to the characteristics of the invaded (cool) zone created during injection.

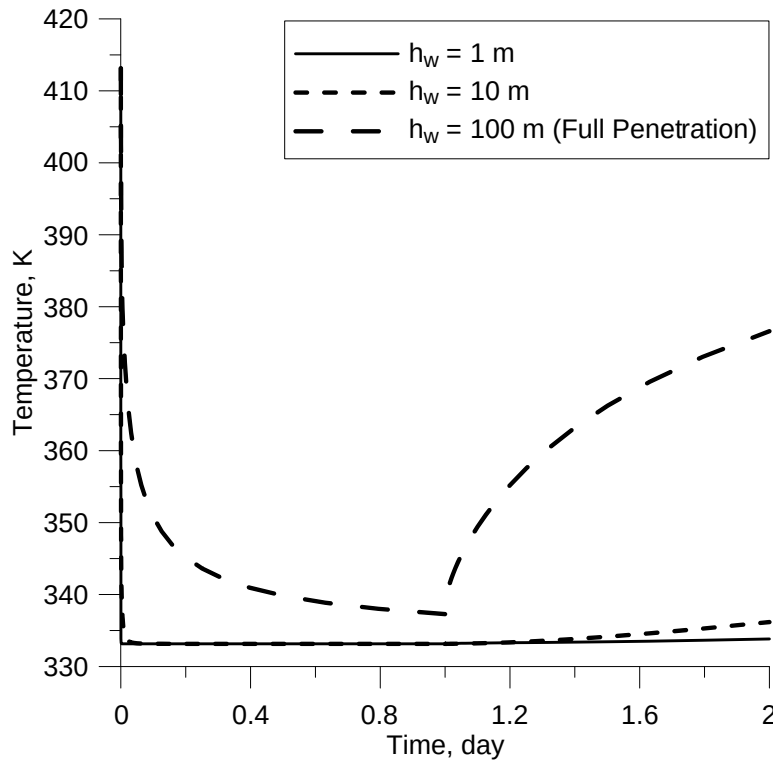


Figure 5.68: Effect of the open interval length on the sandface temperature behaviors during injection and fall-off periods.

5.2.11 Effect of different injection temperatures on pressure and temperature behaviors

In this case, the effects of different injection temperatures on the sandface pressure and temperature are investigated for a limited-entry well at constant-rate injection during 1000 days. In this case, open interval lengths to flow ($h_w = 1$ m) are considered to be in the middle of the reservoir in the z -direction like the previous cases. Constant-rate injection is performed at 5 different temperature values that are equal to 333.15K (60°C), 353.15K (80°C), 373.15K (100°C), 393.15K (120°C) and 413.15K (140°C), respectively. The other reservoir and well properties given in Table 4.1 in Chapter 4 and mentioned at the beginning of Chapter 5 are also valid for this case.

Figure 5.69 presents the effect of different injected water temperature on the log-log plots of pressure change and its Bourdet derivative for the same case. For all cases presented in Figure 5.69, the initial reservoir temperature is the same and equal to 413.15K (140°C) and the injection rate is the same and equal to 1 kg/s. It can be seen

from Figure 5.69 that pressure and derivative responses significantly change with decreasing injected temperature from $T_{inj} = 140^{\circ}\text{C}$, which is the original reservoir temperature, to $T_{inj} = 60^{\circ}\text{C}$. This is mainly caused by differences in the viscosity and density of the injected cold water and the original hot reservoir fluid that are reflected on the transient temperature and pressure behavior by the injected cold water on the original hot reservoir water in a porous/permeable medium.

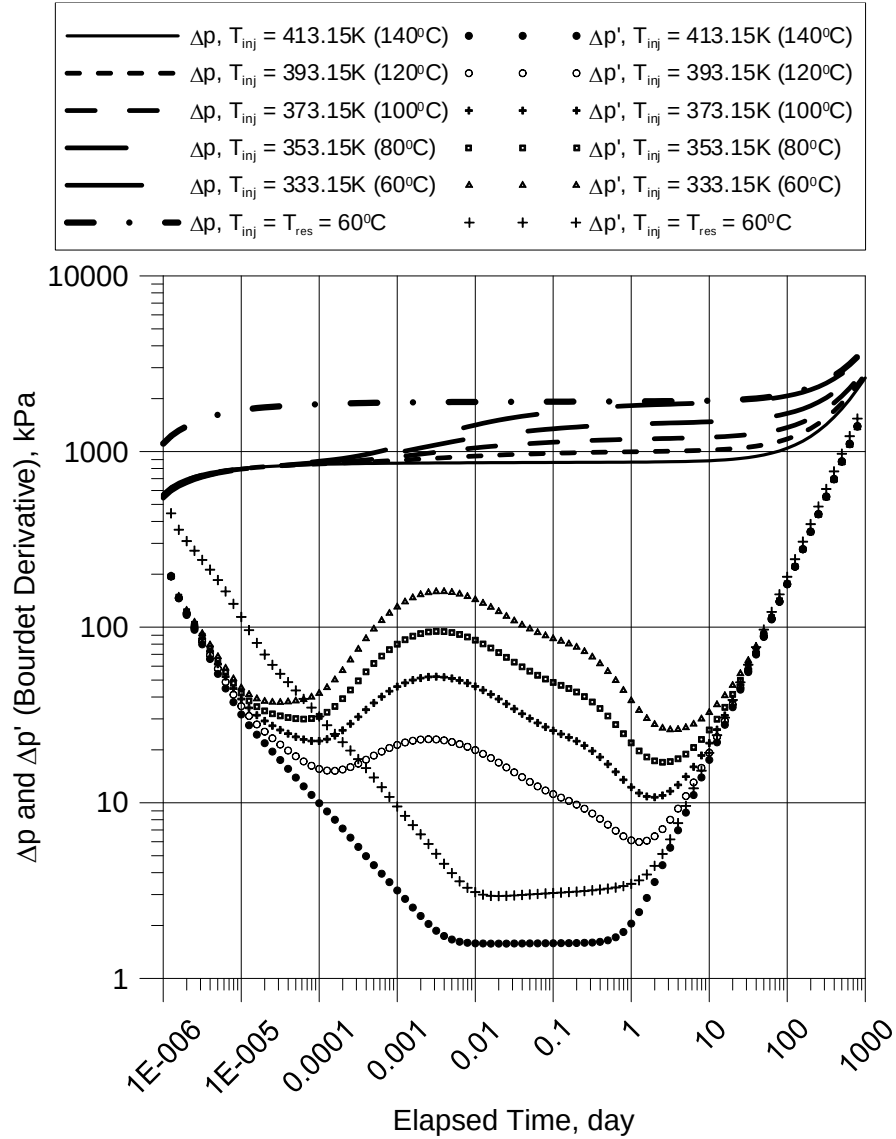


Figure 5.69: Effect of injection fluid temperature on pressure change and its Bourdet derivative for the injection period for a limited-entry well in an isotropic reservoir.

It is worth noting that the injection problem is a moving boundary value problem where the temperature front propagates into the reservoir with time (by injection of cold water at the well). On the other hand, the fall-off problem is almost a stationary boundary where the temperature front or distribution established in the reservoir at the onset of

shut-in (or at the end of injection) is approximately stationary (i.e., the temperature front does not move) at all times during the falloff period. Hence, at a given fixed time in an injection test, it would be expected that there is a cold-injected water bank close to the wellbore, a warm water bank ahead of this zone, and an outer hot water zone not influenced by injection.

These zones are clearly seen in 2D (r - z) temperature distribution maps shown in Figures 5.58 and 5.59. As can also be seen, temperature at a given fixed point in the z -direction changes gradually not like step in these zones along the r -direction, though the temperature change in the transition zone (warm water zone) from inner cold to outer hot water zone seems to be quite steep (see Figure 5.35). It should be noted that at 1000 days of injection, the transition zone extends from 2 to 50 m in the r -direction, while the cold water zone extends from the wellbore to 2 m and the hot water zone extends from 50 to 1000 m, which is the outer reservoir boundary. So, the temperature front moves quite slow and injected water is only a very small portion of the reservoir.

Although not shown here, the temperature along the z -direction at given fixed r behind the hot water zone shows almost a normal distribution; the lowest temperature around the open interval where the water is injected and then increasing asymptotically to the hot water temperature towards the top and bottom boundaries of the reservoir.

Pressure derivative behavior shown in Figure 5.69 exhibits various flow regimes at early, intermediate and late times, controlled by the changes of fluid properties of the water caused by injection of water into a hot water zone. For comparison purposes, two cases have also been included where the injected water and initial reservoir water temperatures are the same; one case with $T_{inj} = T^0 = 60^\circ\text{C}$ and the other case $T_{inj} = T^0 = 140^\circ\text{C}$. It should be noted that these two cases correspond essentially to single-phase isothermal flow conditions. As expected and also known from the single-phase isothermal pressure transient theory of limited-entry wells, see for example Kuchuk et al. (2010), pressure derivatives for both cases exhibit spherical flow ($-1/2$ slope line on log-log plot), infinite-acting radial flow (zero slope line on log-log plot), and pseudosteady-state (pss) flow ($+1$ slope line on log-log plot). The pressure derivatives (and also pressure changes) for the case where $T_{inj} = T^0 = 140^\circ\text{C}$ are lower than those corresponding to the case where $T_{inj} = T^0 = 60^\circ\text{C}$ during spherical and infinite-acting radial flow regimes because the density and viscosity (or mobility) for the hot water injected/reservoir case are lower than those for the cold water injected/reservoir case,

and hence the resistance to injection in the case of injecting hot water is less as compared to injecting cold water into the reservoir. In other words, we can inject with a less pressure change in the case where hot water is injected into a hot water reservoir. The same is also true for pressure changes and derivatives during pss flow; but the difference pressure changes and particularly pressure derivatives between the two cases the during the pss is small because pressure change and derivative responses at the well during pss flow is mainly controlled by the reciprocal of the porosity-compressibility product which is slightly lower for the low temperature case (see Figure 2.5).

For the cases of Figure 5.69 where the injected water temperature is lower than the initial reservoir temperature, we see quite wild pressure derivative behaviors particularly during the infinite-acting radial flow. As expected, at very early times, pressure changes and pressure-derivative exhibit the same spherical flow based on the properties of the hot reservoir water (at 140°C) as at early times the volume of injected water is quite small, and the temperature front is almost near the wellbore, and hence the nearby well region is mainly occupied by hot water. However, as time progresses, the injected water front moves further deep into the reservoir both in the r and z -directions, for instance as shown in Figure 5.35 in the r -direction, and the changes in the viscosity (or mobility) particularly near the temperature front in the r and z -directions become large and such sharp changes in viscosity (or mobility) cause wild pressure derivative behaviors; increasing first and decreasing like heterogeneous reservoir with increasing permeability in the r -direction, depending on the injected water temperature. It should be noted that the cooler the injection temperature the higher the contrast in the viscosities (or mobilities) of the injected and reservoir water. Clearly, the pressure derivatives during infinite-acting radial flow for the cases where the temperature of the injected water is different from the initial reservoir temperature do not exhibit any discernible flow regimes, for example they do not exhibit a constant line like the isothermal cases. As noted above, pressure derivatives at the intermediate times or during infinite-acting flow increase first and decrease before they exhibit the same unit-slope line due to pss flow. The main reason for the derivative increase first and then decreases before they exhibit the same unit-slope line is that the pressure propagates much faster from the temperature front and hence the viscosity (or mobility) of the hot reservoir zone starts to largely influence the pressure derivative as

time progresses. In the time interval from the end of the spherical flow to the beginning of the pss flow, pressure derivative is primarily influenced by the mobility of the water, and to second degree by the compressibility in the hot water (or uninvasion) zone. As also noted by Bratvold and Horne (1990), the pressure change and the pressure derivative during the pss flow at constant injection rate is mainly controlled by the properties, mainly the porosity-compressibility product, of the hot water zone. This is also evident from the results shown in Figure 5.69. It should be noted that pressure-derivative for all cases of water injection at temperature less than the initial reservoir temperature exhibits the pss flow controlled by the properties of the initial reservoir water at the temperature of 140°C. As a final remark, it should be noted that as the injected water temperature decreases, the time to reach the pss flow increases. This is expected because increasing the injected water temperature increases the mobility of the water and hence the pressure propagates faster in such system and the outer boundary effect is felt earlier at the well.

Figure 5.70 shows the sandface temperature behaviors for the same case. According to Figure 5.70, temperature decreases faster with decreasing injection temperature during injection period as expected. Besides, the limited-entry effect also accelerates this temperature drop.

5.2.12 Packer-probe application to an anisotropy case

In this case, the pressure and temperature behaviors are investigated at an observation (probe) point along the wellbore due to excitation from a source/sink location (represented by a dual-packer) as shown schematically in Figure 5.71. As can be seen from Figure 5.71, the pay zone has been located near the bottom of the reservoir.

Such tests (acquiring pressure and temperature measurements at the probe and dual-packer locations) are routinely applied within the context of wireline formation testing in industry to determine existence of vertical interference in the formation, estimate horizontal and vertical permeabilities and initial formation pressure and temperatures, last but not least obtain in-situ fluid samples for PVT analysis.

Figure 5.72 shows the grid system (non-uniform grids in the z-direction) used in this case.

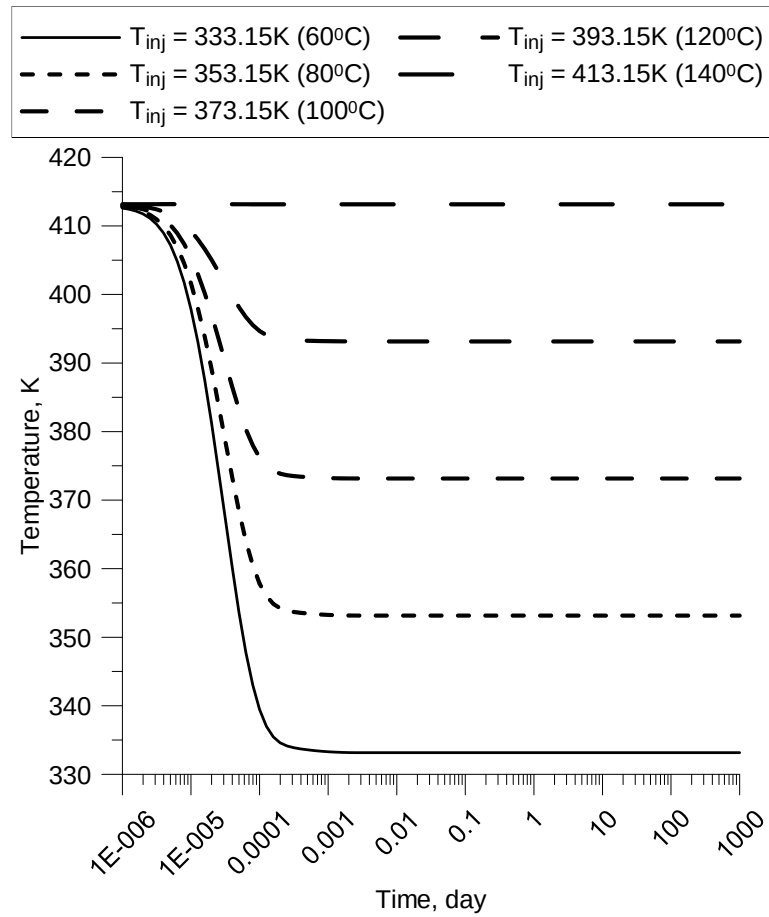


Figure 5.70: Effect of injection temperature of injected water on the sandface temperature behavior of a limited-entry well during injection period.

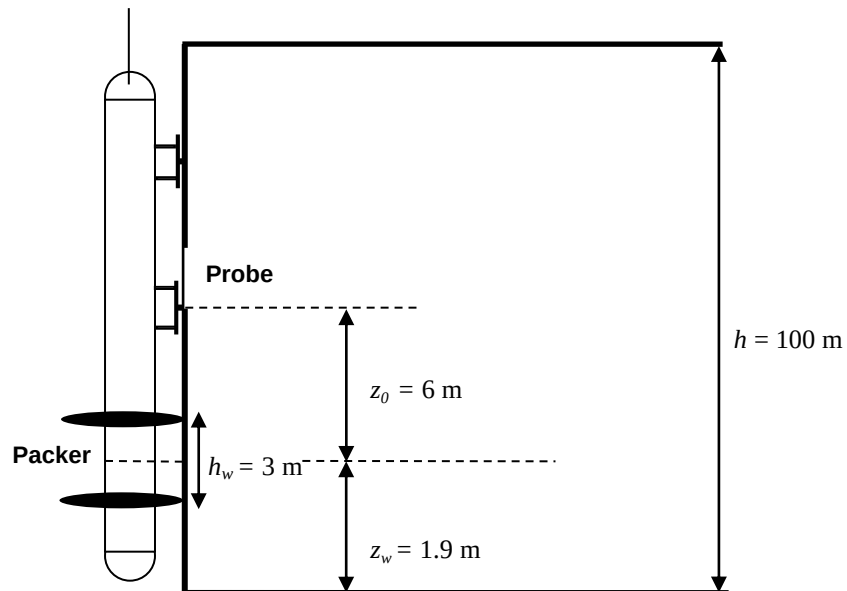


Figure 5.71: Schematic of a probe configuration of a packer-probe wireline formation tester for a vertical well.

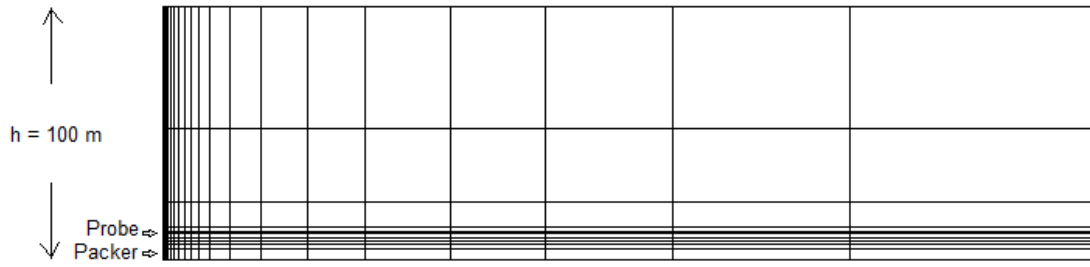


Figure 5.72: Schematic of the gridblock structure used for the packer-probe application to an anisotropic reservoir.

For this investigation, an injection-falloff test containing an injection period of 10 days and a fall-off (shut-in) period of another 10 days. In these cases, colder water with $T_{inj} = 333.15\text{K}$ (60°C) is injected into the reservoir with $T = 413.15\text{K}$ (140°C) at a constant-rate of 1 kg/s during injection period like the previous cases.

Firstly, three different permeability cases are evaluated. In the first one, horizontal permeability is $k_r = 10$ md and vertical permeability $k_z = 100$ md (anisotropy ratio = 0.1). In the second one, horizontal permeability is $k_r = 1000$ md and vertical permeability $k_z = 100$ md (anisotropy ratio = 10). In the third one, horizontal and vertical permeabilities are the same (100 md and anisotropy ratio = 1). The other parameters used are the same as the previous case. Furthermore, sensitivities of various rock parameters to the sandface pressure and temperature behaviors are investigated by using the plots of sensitivity coefficients as a function of time.

Figures 5.73 and 5.74 show the probe pressure comparisons of the three different permeability cases during injection and fall-off periods in Cartesian and semi-log plots, respectively. According to Figures 5.73 and 5.74, the case with $k_r = 10$ md, $k_z = 100$ md has the greatest pressure values than the other ones. This is an expected result because the vertical interference effect in this case is more than the other cases and pressure increase is much more in this case due to the fact that k_r has the lowest value. Hence, pressure increase decreases with increasing horizontal permeability because resistance to flow decreases. As expected, pressure drop in the case with the lowest k_r occurs at the latest in comparison with the other cases during fall-off.

Figures 5.75 and 5.76 show the probe temperature comparisons of the three different permeability cases during injection and fall-off periods in Cartesian and semi-log plots, respectively.

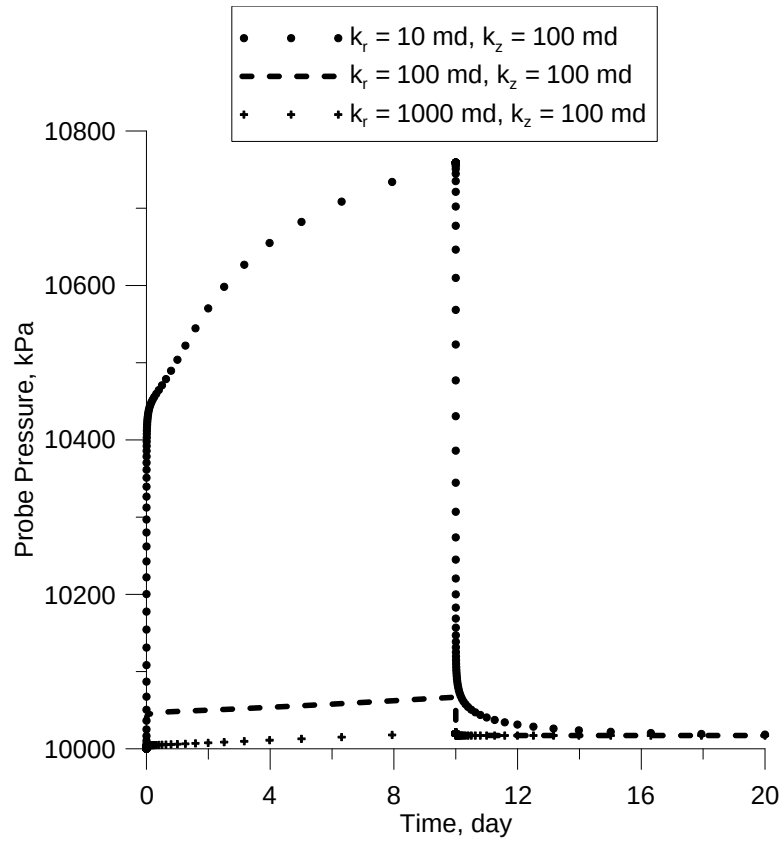


Figure 5.73: Probe pressure comparisons of the three different permeability cases during injection and fall-off periods in the Cartesian plot.

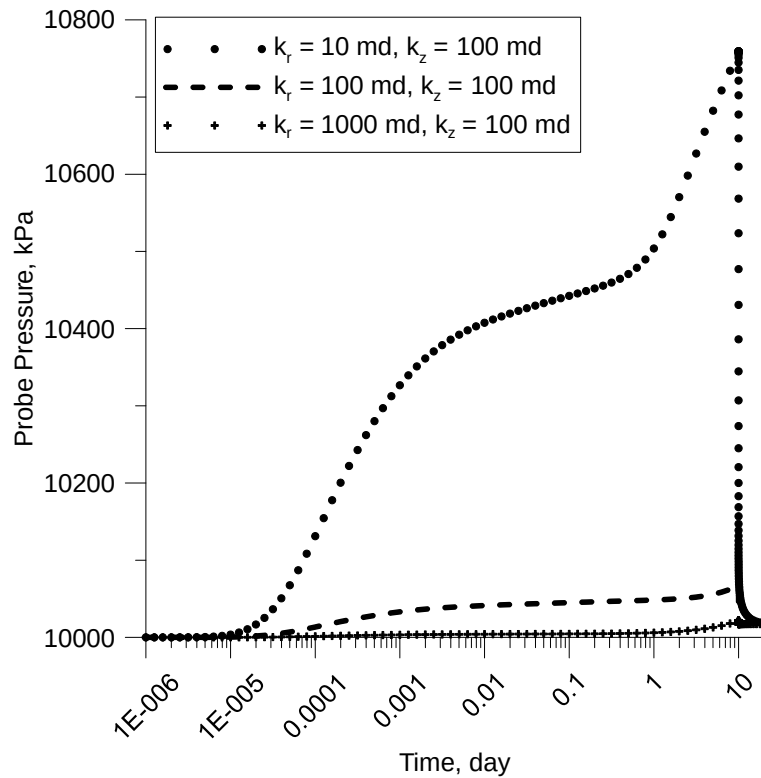


Figure 5.74: Probe pressure comparisons of the three different permeability cases during injection and fall-off periods in the semi-log plot.

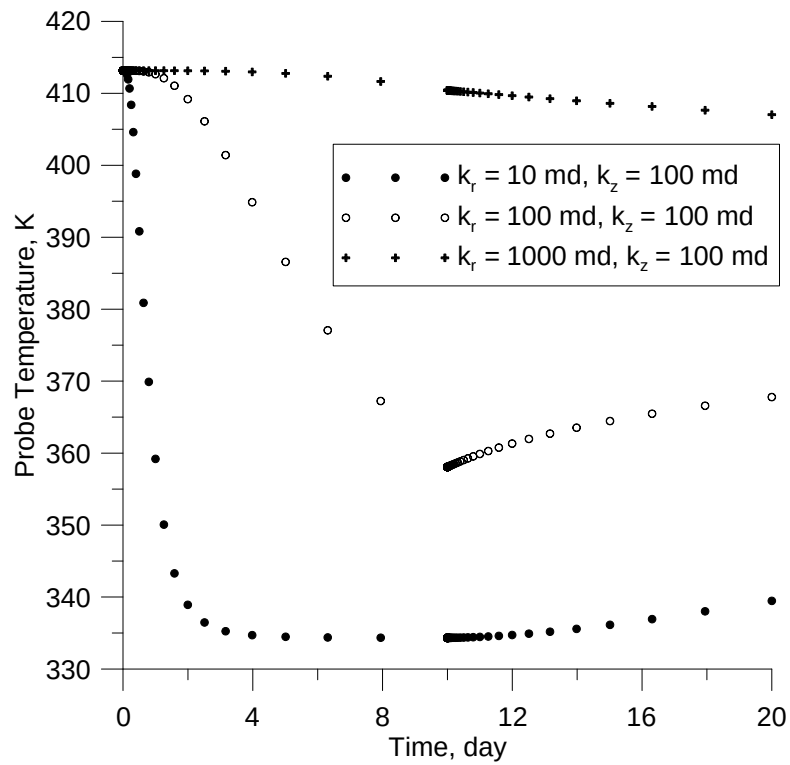


Figure 5.75: Probe temperature comparisons of the three different permeability cases during injection and fall-off periods in the Cartesian plot.

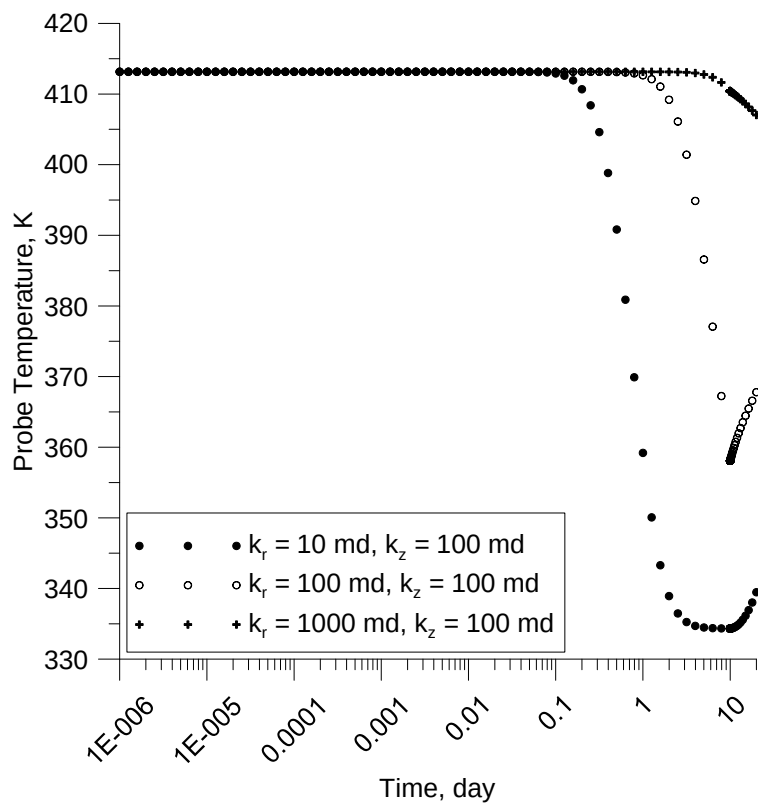


Figure 5.76: Probe temperature comparisons of the three different permeability cases during injection and fall-off periods in the semi-log plot.

It can be seen that from Figures 5.75 and 5.76 that the vertical interference effects are observed among the cases in respect of probe temperature. The greatest temperature drop is observed in the case with $k_r = 10$ md, $k_z = 100$ md while the smallest one is observed in the case with $k_r = 1000$ md, $k_z = 100$ md. This is why the case with $k_r = 10$ md, $k_z = 100$ md has the smallest horizontal permeability and has the biggest vertical interference effect than the others. Hence, cold injected water creates much more temperature drop at the probe. Since conduction is the dominant process during fall-off, the probe temperature in the reservoir with $k_r = 1000$ md, $k_z = 100$ md goes on decreasing during fall-off until a certain amount of time because the cold front reaches the probe at the latest for this case.

Figures 5.77 and 5.78 show 2D (r - z) temperature distribution maps at the end of the injection (10 days) and fall-off (20 days) periods for the reservoir with $k_r = 10$ md, $k_z = 100$ md, respectively. As can be seen from Figure 5.77, temperature can propagate faster in the z -direction during injection for this case because vertical permeability is higher than horizontal permeability.

On the other hand, as can be seen from Figure 5.78, the hotter zone in the reservoir increases the temperature of the cooled zone by conduction during fall-off.

Figures 5.79 and 5.80 show 2D (r - z) temperature distribution maps at the end of the injection (10 days) and fall-off (20 days) periods for the reservoir with $k_r = 100$ md, $k_z = 100$ md, respectively. As can be seen from Figure 5.79, temperature can propagate both in the r -direction and in the z -direction during injection for this case because vertical permeability and horizontal permeability are the same (isotropic reservoir).

On the other hand, as is previously seen in Figure 5.78, Figure 5.80 also shows that the hotter zone in the reservoir increases the temperature of the cooled zone by conduction during fall-off.

Figures 5.81 and 5.82 show 2D (r - z) temperature distribution maps at the end of the injection (10 days) and fall-off (20 days) periods for the case with $k_r = 1000$ md, $k_z = 100$ md, respectively. In contrast to Figures 5.77 and 5.78, temperature can propagate faster in the r -direction during injection for this case because horizontal permeability is higher than vertical permeability.

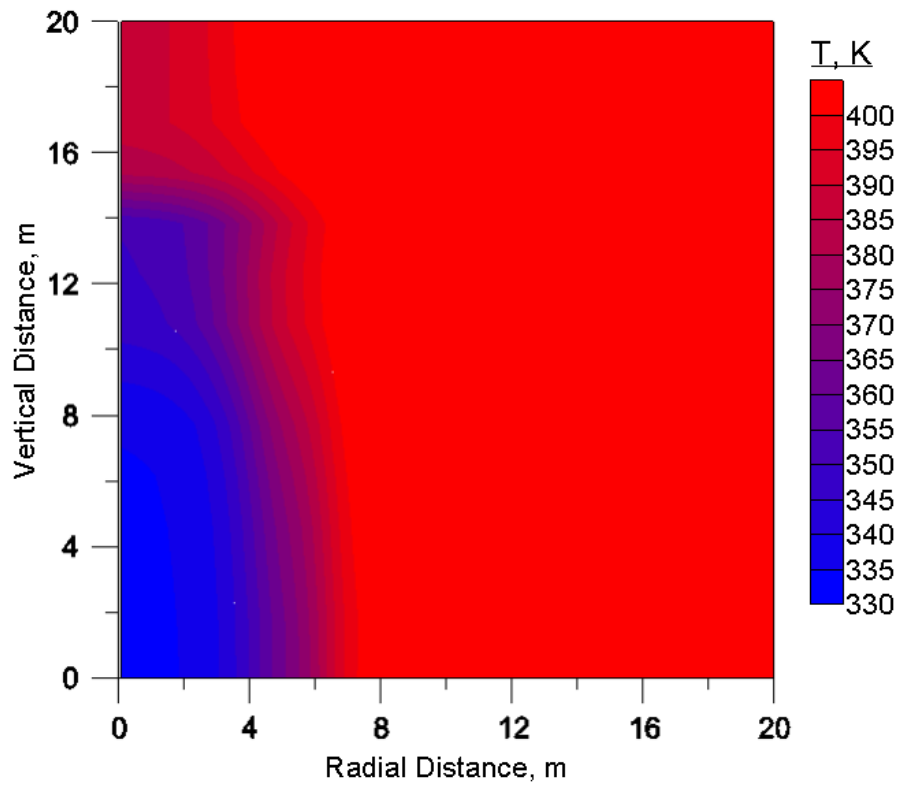


Figure 5.77: 2D (r - z) temperature distribution map for the case with $k_r = 10$ md, $k_z = 100$ md at the end of injection period (10 days).

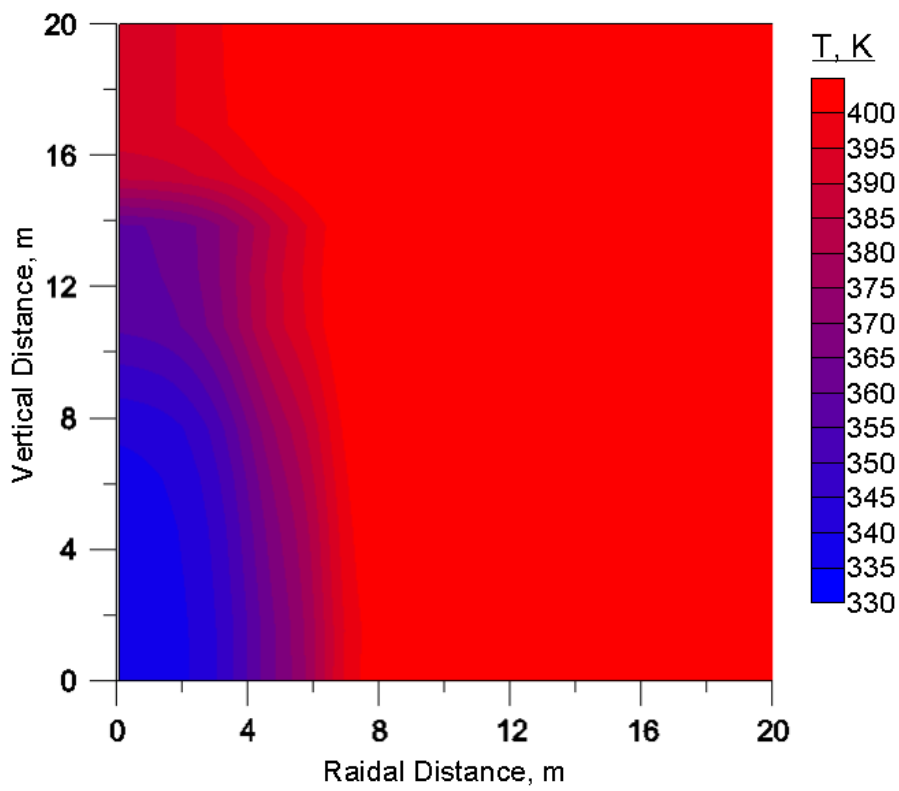


Figure 5.78: 2D (r - z) temperature distribution map for the case with $k_r = 10$ md, $k_z = 100$ md at the end of fall-off period (20 days).

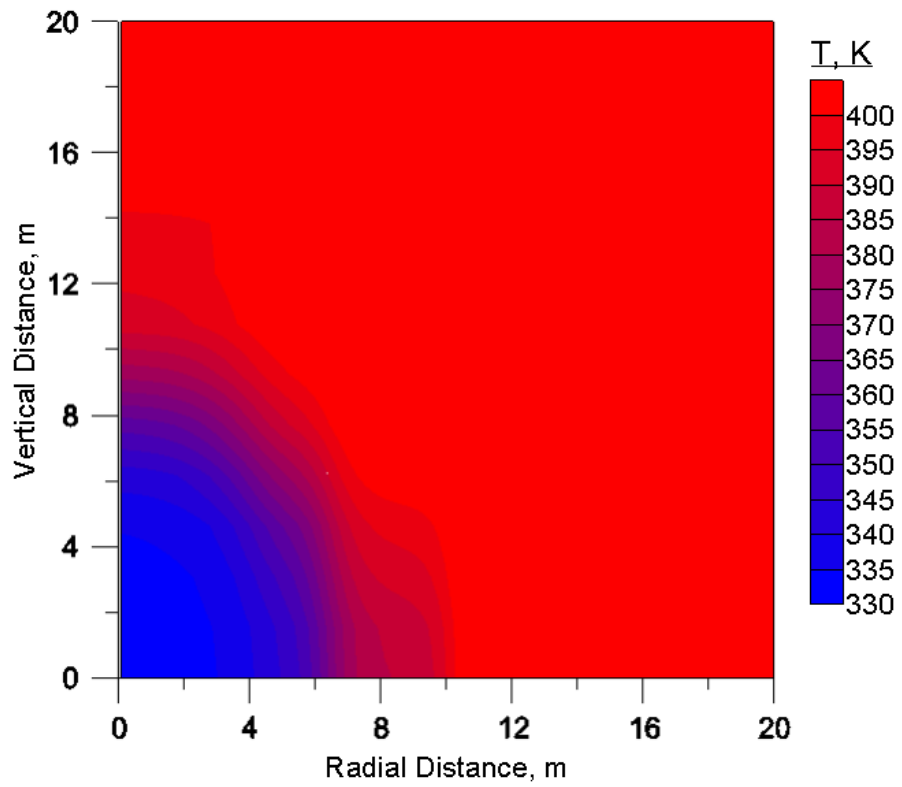


Figure 5.79: 2D (r - z) temperature distribution map for the case with $k_r = 100$ md, $k_z = 100$ md at the end of injection period (10 days).

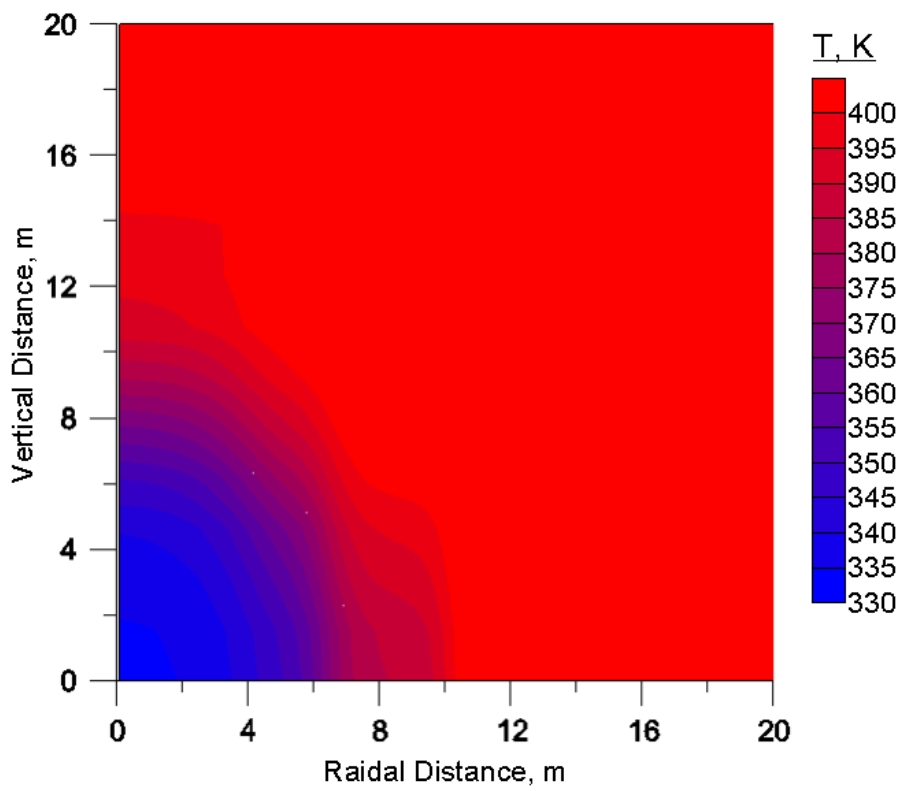


Figure 5.80: 2D (r - z) temperature distribution map for the case with $k_r = 100$ md, $k_z = 100$ md at the end of fall-off period (20 days).

On the other hand, as is previously seen in Figures 5.78 and 5.80, Figure 5.82 also shows that the hotter zone in the reservoir increases the temperature of the cooled zone by conduction during fall-off.

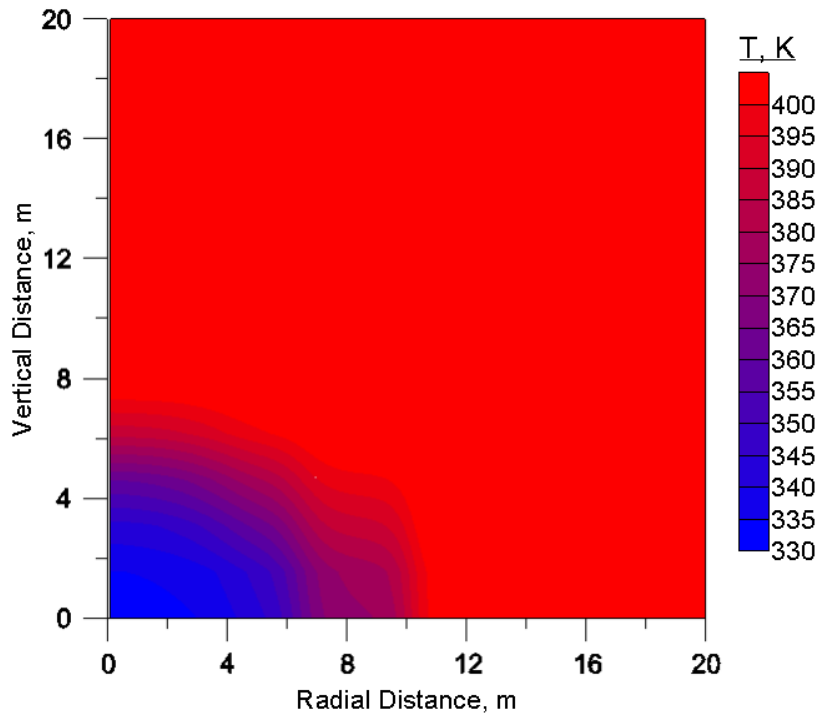


Figure 5.81: 2D (r - z) temperature distribution map for the case with $k_r = 1000$ md, $k_z = 100$ md at the end of injection period (10 days).

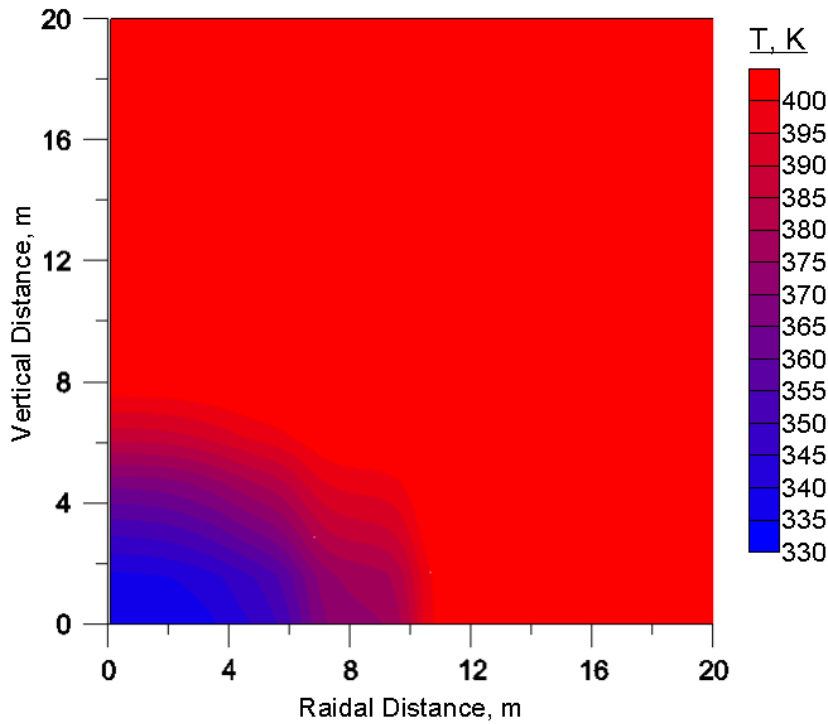


Figure 5.82: 2D (r - z) temperature distribution map for the case with $k_r = 1000$ md, $k_z = 100$ md at the end of fall-off period (20 days).

In the sensitivity investigations for this case, horizontal and vertical permeability values are taken as $k_r = 316$ md and $k_z = 316$ md, respectively.

Figure 5.83 shows the sensitivities of probe pressures to various rock parameters for the reservoir with $k_r = 10$ md, $k_z = 100$ md. As can be seen from Figure 5.83, probe pressure shows the greatest sensitivity to k_r for this case. On the other hand, k_z has positive sensitivity to the probe pressure. It means that the probe pressure increases with increasing vertical permeability. After k_r , porosity, k_z , rock density and rock heat capacity has some sensitivity to the probe pressure. Probe pressure has not a significant sensitivity to rock thermal conductivity for this case.

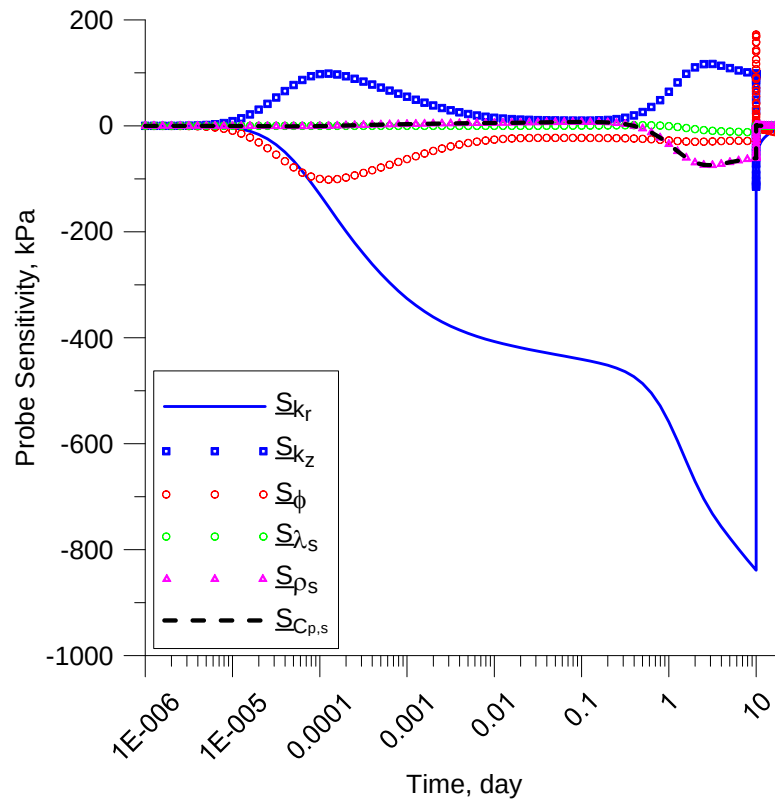


Figure 5.83: Sensitivities of the probe pressure to the rock parameters during injection and fall-off for the reservoir with $k_r = 10$ md, $k_z = 100$ md.

Figure 5.84 shows the sensitivities of probe pressures to various rock parameters for the reservoir with $k_r = 100$ md, $k_z = 100$ md. As can be seen from Figure 5.84, probe pressure shows the greatest sensitivity to k_r as in the previous case. Differently from the previous case, the sensitivity values in this case are lower due to increasing horizontal permeability.

Figure 5.85 shows the sensitivities of probe pressures to various rock parameters for the reservoir with $k_r = 1000$ md, $k_z = 100$ md.

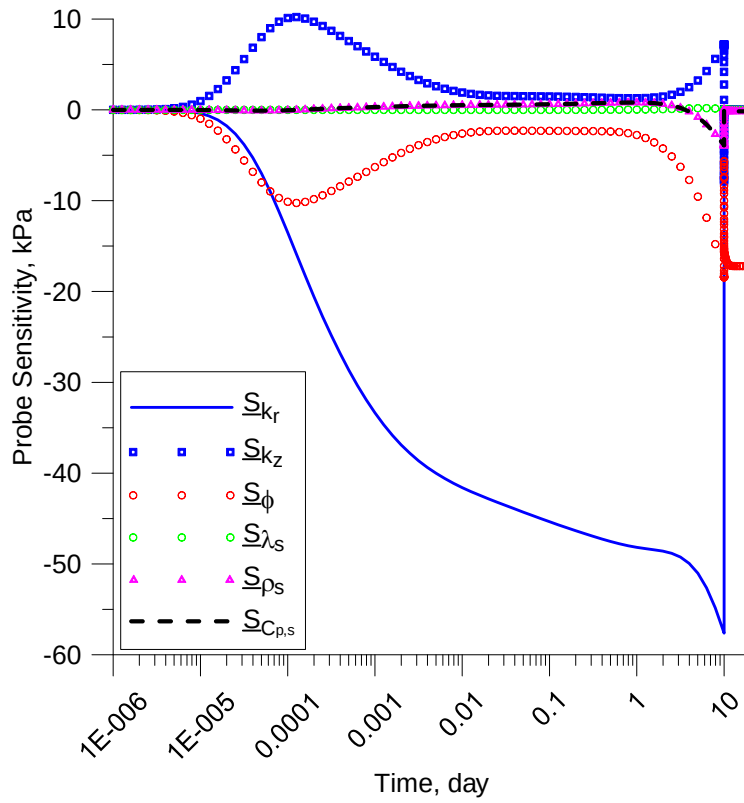


Figure 5.84: Sensitivities of the probe pressures to the rock parameters during injection and fall-off for the reservoir with $k_r = 100$ md, $k_z = 100$ md.

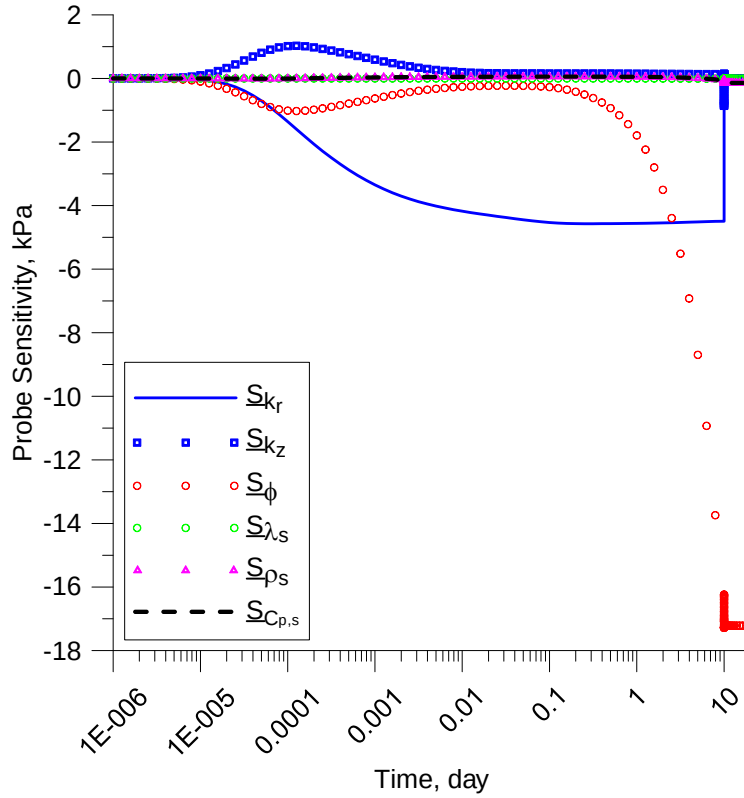


Figure 5.85: Sensitivities of the probe pressures to the rock parameters during injection and fall-off for the reservoir with $k_r = 1000$ md, $k_z = 100$ md.

As can be seen from Figure 5.85, differently from the previous cases, the probe sensitivity to horizontal and vertical permeability in this case is much lower because resistance to flow in the r -direction decreases due to increasing horizontal permeability.

Figure 5.86 shows the sensitivities of the probe temperatures to various rock parameters for the reservoir with $k_r = 10$ md, $k_z = 100$ md. At early time, any of the parameters does not show any sensitivity to the probe temperature during injection until the cold front reaches the probe (approximately 0.1 days). Then, the sensitivities of the parameters starts increasing with decreasing probe temperature. As can be seen from Figure 5.86, probe temperature shows the greatest sensitivity to k_r and k_z . These parameters have almost the same sensitivities. However, horizontal permeability has a positive sensitivity while vertical permeability has a negative sensitivity. After the horizontal and vertical permeabilities, the probe temperature has important sensitivities to rock density and rock heat capacity. The probe temperature is more sensitive to rock thermal conductivity than those to others at fall-off period. Probe temperature shows the lowest sensitivity to porosity during injection for this case.

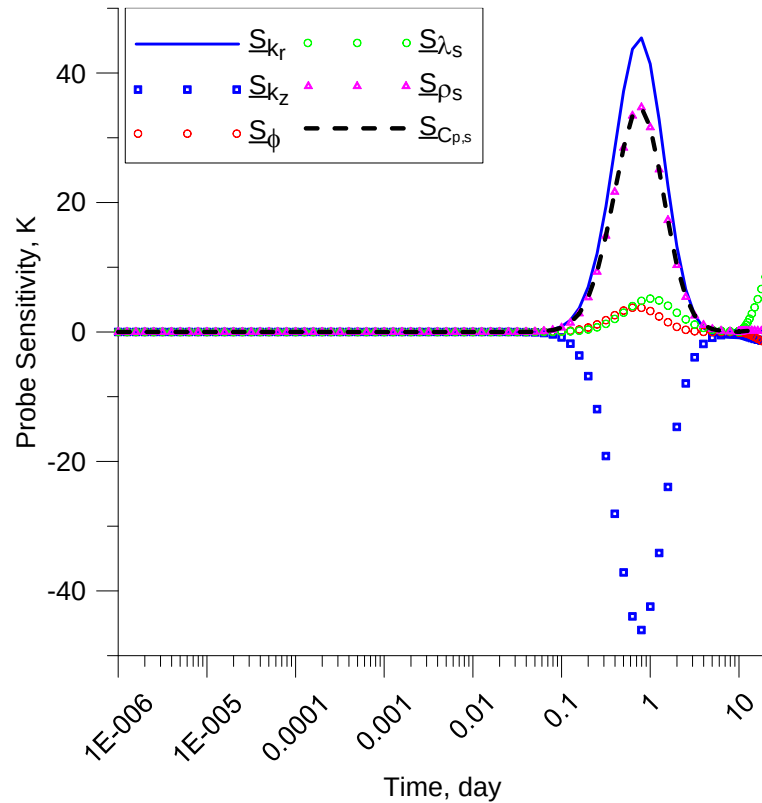


Figure 5.86: Sensitivities of the probe temperatures to the rock parameters during injection and fall-off for the reservoir with $k_r = 10$ md, $k_z = 100$ md.

Figure 5.87 shows the sensitivities of the probe temperatures to various rock parameters for the reservoir with $k_r = 100$ md, $k_z = 100$ md. Any of the parameters does not show any sensitivity to the probe temperature during injection until the cold front reaches the probe (approximately 1 day) as in the previous case. Differently from the previous case, the sensitivities of the parameters starts increasing later than those of the previous case due to increasing horizontal permeability.

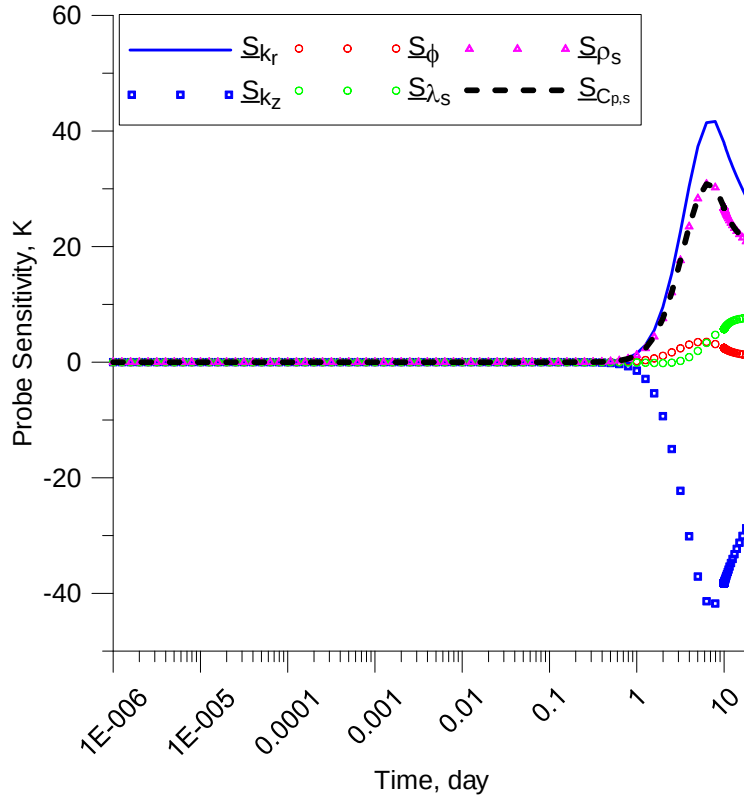


Figure 5.87: Sensitivities of the probe temperatures to the rock parameters during injection and fall-off for the reservoir with $k_r = 100$ md, $k_z = 100$ md.

Figures 5.88 shows the sensitivities of the probe temperatures to various rock parameters for the reservoir with $k_r = 1000$ md, $k_z = 100$ md. At early time, any of the parameters does not show any sensitivity to the probe temperature during injection until the cold front reaches the probe (approximately 2 days) as in the previous case. In this case, the cold front reaches the probe at the latest than the previous cases due to the fact that this reservoir has the highest horizontal permeability. Differently from the previous cases, the probe sensitivity to horizontal and vertical permeability in this case is lower than the other cases because resistance to flow in the r -direction decreases due to increasing horizontal permeability.

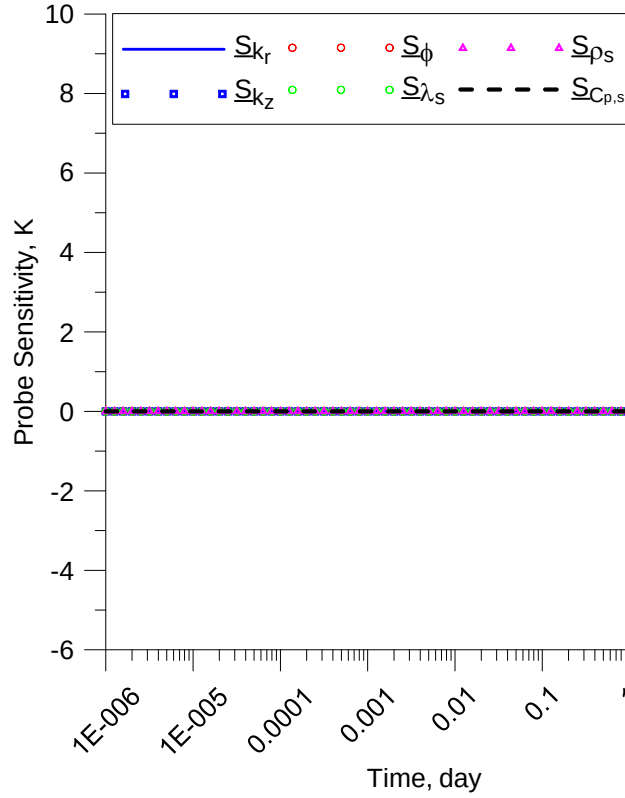


Figure 5.88: Sensitivities of the probe temperatures to the rock parameters during injection and fall-off for the reservoir with $k_r = 1000$ md, $k_z = 100$ md.

5.2.13 Constant-pressure injection case

In this case, an injection of 1000 days at constant bottom-hole pressure of 10000 kPa is performed for a fully penetrating well. Here, differently from the constant-rate injection case, colder water is injected into the reservoir at constant-pressure instead of constant-rate and the initial reservoir pressure is 5000 kPa. This means that injection rate changes, whereas the bottom-hole pressure remains constant at 10000 kPa.

Figure 5.89 shows the sandface temperature behaviors of a fully penetrating well in the reservoir with different permeability values during constant-pressure injection of 1000 days. According to Figure 5.89, it can be seen that permeability has a significant effect on temperature at constant pressure injection in contrast to the constant-rate injection case. This is simply due to the fact that the injection rate becomes variable at constant bottom-hole pressure injection, and the amount of rate that will enter into reservoir is controlled by reservoir permeability. Hence, different amounts of heat are injected into the reservoir for different permeabilities. The least cooling is observed for the reservoir with the lowest permeability. This is an expected result because the

least amount of colder fluid at the lowest rate is injected into the reservoir with the lowest permeability.

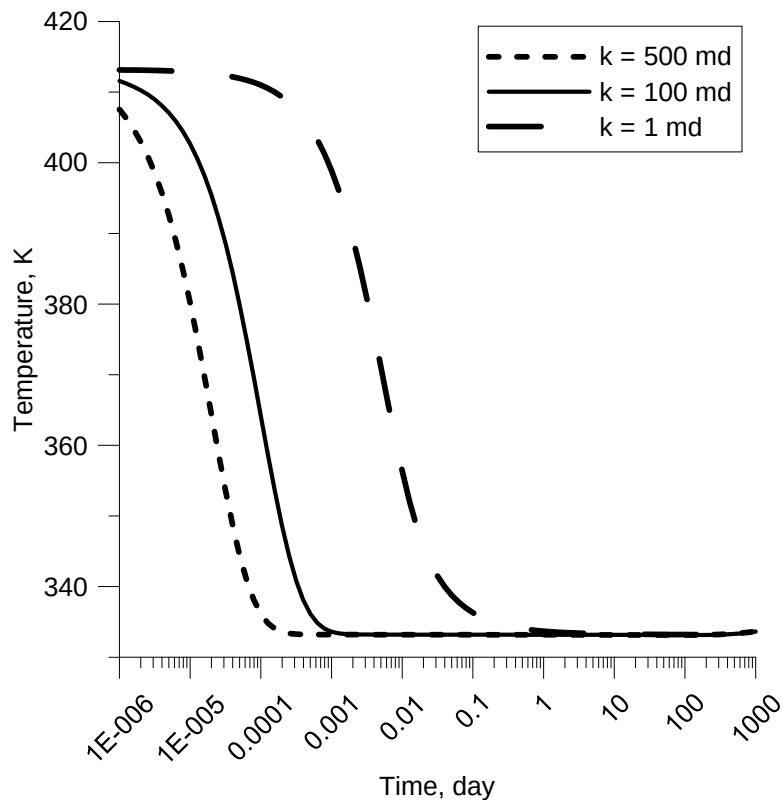


Figure 5.89: Effect of reservoir permeability on the sandface temperature of a fully penetrating well producing at a constant-pressure injection.

Figure 5.90 shows the sandface temperature behaviors of a fully penetrating well in the reservoir with different skin zones during constant pressure injection of 1000 days. According to Figure 5.90, it can be seen that skin has a significant effect on temperature at constant pressure injection in contrast to the constant-rate injection case. This is simply due to the fact that the injection rate becomes variable at constant bottom-hole pressure injection, and the amount of rate that will enter into reservoir is controlled by skin. Hence, different amounts of heat are injected for different skin zones. The least cooling is observed in the reservoir with the highest skin due to the similar reasons also valid for the previous permeability case.

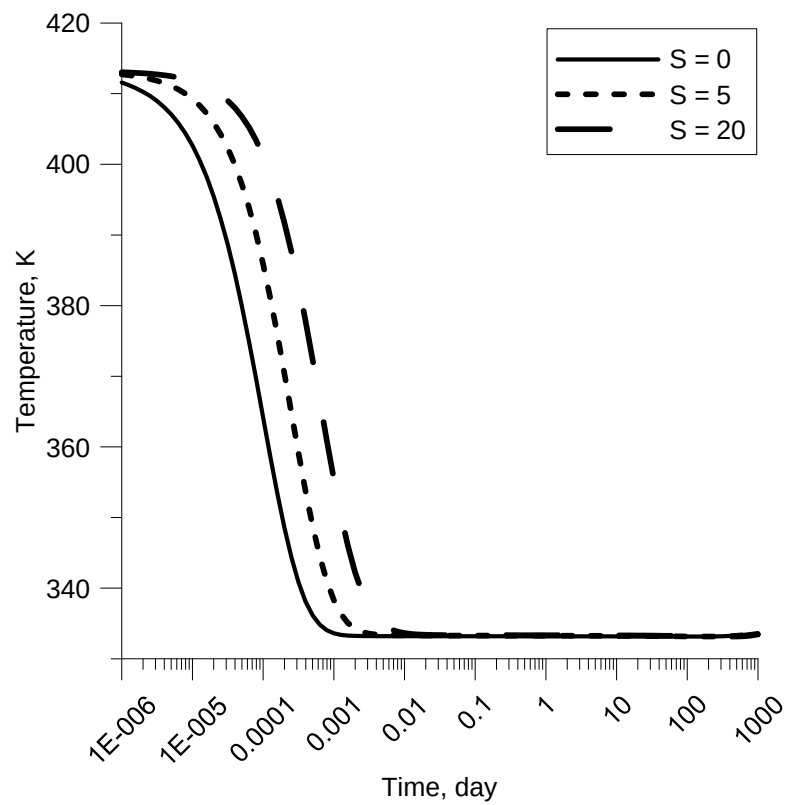


Figure 5.90: Effect of skin factor on the sandface temperature of a fully penetrating well producing at a constant-pressure injection.

6. CONCLUSIONS AND RECOMMENDATIONS

The conclusions obtained from this study and the recommendations for a future work that might be considered are given in this chapter.

6.1 Conclusions

In this section, the conclusions obtained from this study are presented.

6.1.1 Development and verification of the model

In this study, a 2D (r - z) non-isothermal single-well simulator capable of modeling pressure and temperature behaviors in the wellbore, at the sandface and observation points along the wellbore for a single phase liquid-dominated geothermal reservoirs has been developed. The simulator can also model heat transfer from reservoir to adjacent overburden and underburden strata and include the effect of gravity on pressure and temperature behavior of the well and the geothermal reservoir. In addition, different types of the well completion such as fully-penetrating and limited-entry with different modes of production (constant-rate production/injection, constant-pressure injection, build-up, and fall-off) can be simulated. Different types of outer reservoir boundary conditions; such as no-flow and no-heat flux or no-flow, but recharge due to huge edge aquifer surrounding the reservoir at $r = r_e$ as well as no-flow and no-heat flux reservoir top and bottom boundaries; can all be considered.

Various cases simulating different modes of production and injection activities of a well, including build-up and fall-off, and variable production/injection rate sequences have been performed for verification of the simulator developed. It has been observed that the pressure and temperature behaviors of the model as functions of time and distance exhibit very good agreements with those of the well-known thermal simulator TOUGH2. Additionally, the model has been also verified and compared with an analytical solution for a constant-rate injection case. As a result, the verification of the 2D (r - z) non-isothermal simulator developed has been achieved successfully.

By the use of the simulator developed and verified, an extensive investigation on the effects of all the aforementioned conditions of the well production and outer reservoir boundary conditions; i.e., type of well completion such as a fully-penetrating and limited-entry wells, type of production mode at the well, and type of outer reservoir boundary condition, on the pressure and temperature behaviors in the wellbore, at the sandface, and observation points along the wellbore as well as inside the reservoir for a single-phase liquid (water) geothermal reservoir has been presented in this study.

6.1.2 Effect of rate

In general, during production period, it has been found that the sandface temperature decreases at very early times due to expansion of the fluid volume nearby wellbore region in response to production at the well. Afterwards, as the pressure drop continues to increase, the sandface temperature starts to increase because the decrease in the temperature due to fluid expansion nearby wellbore region is overcome by heating effect of the Joule-Thomson fluid expansion at the intermediate times. At late times, if the outer reservoir boundary is no-flow and no-heat flux, the sandface temperature decreases at a much higher rate than that of the early time due to a larger pressure drop that occurs in response to production by the expansion of the entire reservoir fluid volume during the pseudo-steady state flow that occurs at late production times.

In summary, it was found that the sandface temperature decreases at very early and late times of production due to expansion of the fluid volume nearby the well region at early times and expansion of the entire reservoir fluid volume at late times. The temperature increase seen at the intermediate times is due to Joule-Thomson heating. This behavior of the sandface temperature is true regardless of the well completion type; i.e., it is valid for both fully-penetrating and limited-entry wells.

It was also found that for a constant-rate production case, the magnitude of changes observed in the sandface temperature increases as the production rate increases. However, magnitude of changes in temperature due to production are normally small on the order of 1K (or 1°C) even with very large values of production rate such as 60 kg/s considered in this study.

Production-build-up and variable-rate production cases have shown that a sudden temperature spike occurs at a time point where the rate is changed. Such a spike in temperature is due to expansion (caused by increase in flow rate) or compression

(caused by decrease in flow rate) of the fluid volume at a time point where the rate is decreased (e.g. at the moment of build-up when the production rate is decreased to zero). On the contrary, it was observed that the temperature suddenly decreases due to expansion of the fluid volume at a time point where the rate is increased (e.g. at the start of production or at a time point where the rate is increased). The magnitude of this sudden temperature change increases as the magnitude of the rate change increases.

As to the effect of rate on sandface pressure, it has been found that increasing production rate decreases the sandface pressure as expected. As the temperature changes are normally small with production from a reservoir, the behavior of sandface pressure to production from a non-isothermal water geothermal reservoir is quite similar to that of the sandface pressure to production from a isothermal geothermal reservoir model.

6.1.3 Effect of permeability

It was found that for the constant-rate production case, at the effects of fluid expansion and Joule-Thomson on the sandface temperature for the reservoirs with low permeability are more significant than those for the reservoirs with moderate and high permeabilities.

It was found that permeability does not affect the sandface temperature for the constant-rate injection case.

6.1.4 Effect of skin

It was found that increasing the skin factor at well increases the effects of fluid expansion and Joule-Thomson on the sandface temperature like the permeability.

It was observed that skin does not affect the sandface temperature for the constant-rate injection case like the permeability.

6.1.5 Effect of porosity

It was found that reservoir porosity has no significant effect on the sandface temperature for the constant-rate production case.

It was observed that the porosity has some effect on the sandface temperature for the injection-fall-off case at especially fall-off period due to the effect of conduction in the reservoir.

6.1.6 Effect of rock thermal conductivity

It was found that the rock thermal conductivity has an important effect on the sandface temperature for the injection-fall-off case at especially fall-off period because conduction starts to become important in fall-off as the temperature front established at the onset of fall-off period is almost stationary.

6.1.7 Effect of constant-pressure outer boundary

It was observed that a huge aquifer surrounding the geothermal reservoir does not normally affect the sandface temperature for the constant-rate injection case. If the injection period is quite long, then some effect of constant-pressure outer boundary may be observable on the sandface temperature at late times of injection.

6.1.8 Effect of heat transfer to strata

It was observed that heat transfer from the reservoir to adjacent overburden and underburden strata has an important effect on the temperature in the reservoir at late times for the “thin” reservoirs during the constant-rate injection.

6.1.9 Effect of gravity

2D (r - z) temperature distribution maps and the sandface temperature plots as a function of time have shown that the reservoir temperatures decrease faster in the lower portion of the reservoir with the gravity effect during the constant-rate injection from a limited-entry well.

6.1.10 Effect of limited-entry length

It was found that the effects of fluid expansion (temperature drop) and Joule-Thomson (temperature increase) create more temperature variations with decreasing limited-entry length for the constant-rate production case.

The sandface temperature decreases with decreasing limited-entry length during the injection period for injection-falloff case because more fluid is passed through the point where the gauge is placed if limited-entry length decreases.

6.1.11 Effect of different injection temperature

The sandface pressure behaviors significantly change (or increase) with decreasing injection temperature during constant-rate injection from a limited-entry well because colder injected water has higher viscosity (or mobility) and density than that of hotter original reservoir water (this results in pressure increase with the compression effect).

6.1.12 Effect of anisotropy

2D (r - z) temperature distribution maps and the probe temperature plots as a function of time have shown that cold front moves further in vertical direction for the case with $k_r = 10$ md, $k_z = 100$ md in comparison with the cases with $k_r = 100$ md, $k_z = 100$ md and $k_r = 1000$ md, $k_z = 100$ md for an injection-falloff case.

6.1.13 Effect of skin and permeability at constant-pressure injection

It was observed that, unlike the constant-rate injection case, the sandface temperature behavior changes at constant-pressure injection. This is because injection rate varies with changing skin and permeability at constant pressure injection. Hence, different amounts of heat in different rates are injected into the reservoir.

6.1.14 Sensitivities of rock parameters for the finite wellbore approach

Sensitivity plots as a function time at constant-rate injection for a 1D (r) reservoir have shown that the sensitivities of the wellbore and sandface pressure to all rock parameters are the same. The sandface/wellbore pressure shows the greatest sensitivity to the porosity, and the second largest sensitivity to permeability.

Sensitivity plots for the same case have also shown that the wellbore temperature is not sensitive to any rock parameter. From here, it can be concluded that the sandface temperature should be used to investigate the sensitivities rather than the wellbore temperature. The sandface temperature shows the greatest sensitivity to the rock thermal conductivity. After the rock thermal conductivity, rock density and rock heat capacity have important sensitivities.

The above conclusion is indeed an original and important conclusion in that it indicates that the importance of the use of wireline formation tester tools like packer-probe and probe-probe to have pressure and measurements at the sandface of the production interval and/or along observation points at the sandface along the wellbore.

6.1.15 Sensitivities of rock parameters for the high pressure and high temperature reservoir

Sensitivity plots as a function of time at a high constant-rate production from a limited-entry well for the high pressure and high temperature reservoir have shown that the sensitivities of the sandface pressure show the greatest sensitivity to the porosity. After the porosity, pressure has some sensitivity to the horizontal permeability.

Sensitivity plots for the same case have also shown that the sensitivities of the sandface temperature shows the greatest sensitivity to the rock density and rock heat capacity and the second greatest sensitivity to horizontal permeability. However, magnitudes of these sensitivities are quite low ranging from -0.25K and 0.7K.

6.1.16 Sensitivities of rock parameters for anisotropic reservoirs

Sensitivity plots as a function time for a packer-probe case during injection-falloff from anisotropic reservoirs have shown that probe temperature shows the greatest sensitivity to horizontal and vertical permeabilities for low and moderate horizontal permeabilities during injection and fall-off while it shows the greatest sensitivity to rock density and rock heat capacity for high horizontal permeabilities.

6.2 Recommendations for Future Works

Based on the study conducted, the following recommendations are given to further improve and expand the work given here and also guide research directions for future works to be performed related to the topic of this study:

6.2.1 Analytical equations/expressions for pressure and temperature at the wellbore, sandface and along the observation points

In this study, we considered investigation and understanding of the pressure and temperature behaviors during production, injection, build-up, and fall-off periods mainly by interpreting the results of the numerical simulator qualitatively. For aiding the analysis of pressure/temperature transient tests to be conducted under non-isothermal conditions for parameter estimation e.g., spherical-flow, radial-flow, pseudosteady-state flow regimes, it is recommended that some analytical expressions for wellbore, sandface and observation points along the sandface p and T be derived or obtained from the related work done in the literature. This is particularly important

to understand the information content of temperature data at early times such storage dominated flow period, spherical flow around limited-entry wells, at intermediate times such infinite-acting radial flow, and late-times where the outer reservoir boundary starts to influence pressure at the well.

6.2.2 2D sensitivity maps

2D (r - z) maps of sensitivity coefficients describing the sensitivities of pressure and temperature to such as the rock parameters permeability, porosity, rock thermal conductivity, rock density and rock heat capacity can be constructed to investigate the spatial sensitivity variations.

6.2.3 Automated parameter estimation problem (Inverse problem)

The existing simulator should be extended to perform automated reservoir parameter estimation based on pressure and/or temperature data. For this purpose, one can consider by using the method of Ensemble-Kalman Filter and/or gradient type methods such as Levenberg-Marquardt. Such a study will precisely delineate the information content of pressure and temperature data under isothermal conditions in the presence of noise in pressure and/or temperature measurements. It should be noted that in our study, we studied the information content of pressure and temperature data solely in terms of sensitivity coefficients. Although these sensitivities give clue about the information content of pressure/temperature data, they alone are not sufficient to claim that the parameters showing sensitivity on pressure and temperature can be estimated reliably and uniquely in the presence of noise in p and T measurements.

6.2.4 Effects of brine salinity and/or dissolved gas

The simulator developed assumes the geothermal water's dissolved solid content and gas content are zero; i.e., water does not contain any impurities. Typically, geothermal water (referred to as brine) contains some level of impurities in terms of dissolved minerals, salts, etc and dissolved gas such as carbon dioxide (CO_2). Such impurities affect the fluid and rheological properties (density, viscosity, etc.). Hence, it is recommended that the effects of such impurities in brine in terms of salinity and/or concentration of CO_2 be incorporated into the simulator and be studied on pressure and temperature behaviors of the well, sandface and inside the reservoir.

6.2.5 Segmented wellbore coupled with the reservoir model

The simulator developed in this work treats the well (or wellbore) as a single cell or tank having infinite conductivity if gravity is ignored along the z -direction and neglects pressure and temperature losses/gains due to non-isothermal and friction effects in the z -direction. This assumption is quite valid if the measurement locations of p and T in and along the wellbore are in the close vicinity of the open interval subject to production/injection and/or the net pay zone thickness is quite large. If the measurement location(s) is (are) far above the open interval, and the pay thickness is large, then wellbore and sandface pressures and temperatures predicted by our single-cell wellbore may not be very accurate. Hence, it is recommended that a more rigorous discretized (or segmented) wellbore-reservoir model integrating a segmented wellbore (in the z -direction) from bottom-hole to the top of the pay zone or to the wellhead, coupled with a discretized reservoir model be developed for further modeling and investigating the wellbore/sandface p and T behaviors more realistically.

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APPENDICES

Appendix A : Detailed Derivations of Mass and Energy Balance Equations

Appendix B : Detailed Derivations of Analytical Solution Developed by
Lauwerier (1955) and Modified by Satman (2003)

Appendix A : Detailed Derivations of Mass and Energy Balance Equations

For this work, non-isothermal flow is considered in the porous media. The generalized continuity equation for mass conservation in the r - θ - z coordinates is given as follows:

$$\frac{\partial(\phi\rho_w)}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r}(\rho_w r v_r) + \frac{1}{r} \frac{\partial}{\partial \theta}(\rho_w v_\theta) + \frac{\partial}{\partial z}(\rho_w v_z) = 0 \quad (\text{A.1})$$

Since the terms in the θ -direction are ignored,

$$\frac{\partial(\phi\rho_w)}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r}(\rho_w r v_r) + \frac{\partial}{\partial z}(\rho_w v_z) = 0 \quad (\text{A.2})$$

At this point, it is important to note that the derivation of Equation A.1 considers that the z -direction increases from top to bottom. Throughout this study, the SI (system of international) units will be used.

Darcy's law will be used for the velocity terms in Equation A.2 in r and z -directions as follows:

$$v_r = -\frac{k_r}{\mu_w} \frac{\partial p}{\partial r} \quad (\text{A.3})$$

$$v_z = -\frac{k_z}{\mu_w} \left(\frac{\partial p}{\partial z} - \gamma_w \frac{\partial D}{\partial z} \right) \quad (\text{A.4})$$

In Equations A.3 and A.4, k_r ve k_z represent permeabilities in the r and z -directions, respectively, and μ_w represents water viscosity. In Equation A.4, D represents distance measured from a certain reference (datum) in the direction of gravity and $\gamma_w = \rho_w g$ represents specific gravity and gradient of water.

Using Darcy's law in the continuity equation gives:

$$\frac{\partial(\rho_w \phi)}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[\rho_w r \frac{k_r}{\mu_w} \frac{\partial p}{\partial r} \right] - \frac{\partial}{\partial z} \left[\rho_w \frac{k_z}{\mu_w} \left(\frac{\partial p}{\partial z} - \gamma_w \frac{\partial D}{\partial z} \right) \right] \quad (\text{A.5})$$

The following discretization is used for the left hand side of Equation A.5:

$$\frac{\partial(\rho_w \phi)}{\partial t} = \frac{(\rho_w \phi)_{i,k}^{n+1} - (\rho_w \phi)_{i,k}^n}{\Delta t} \quad (\text{A.6})$$

In the above equation, the superscript n refers to the present time and $n+1$ refers to the time at which the solutions will be determined. Δt represents the time from n to $n+1$. The subscript i represents the reservoir gridblock index in the r -direction and the subscript k represents the reservoir gridblock index in the z -direction.

Now, if the first term on the right hand side of Equation A.5 is considered,

$$\left\{ \frac{1}{r} \frac{\partial}{\partial r} \left[\rho_w r \frac{k_r}{\mu_w} \frac{\partial p}{\partial r} \right] \right\}_{\substack{r=r_i \\ z=z_k}} = \frac{1}{r_{i,k}} \frac{\left(r \rho_w \frac{k_r}{\mu_w} \frac{\partial p}{\partial r} \right)_{i+\frac{1}{2},k} - \left(r \rho_w \frac{k_r}{\mu_w} \frac{\partial p}{\partial r} \right)_{i-\frac{1}{2},k}}{r_{i+\frac{1}{2},k} - r_{i-\frac{1}{2},k}} \quad (\text{A.7})$$

Rearranging the above equation gives:

$$\begin{aligned} & \left\{ \frac{1}{r} \frac{\partial}{\partial r} \left[\rho_w r \frac{k_r}{\mu_w} \frac{\partial p}{\partial r} \right] \right\}_{\substack{r=r_i \\ z=z_k}} \\ &= \frac{1}{r_{i,k}} \frac{\left(r \rho_w \frac{k_r}{\mu_w} \right)_{i+\frac{1}{2},k} \left(\frac{\partial p}{\partial r} \right)_{i+\frac{1}{2},k} - \left(r \rho_w \frac{k_r}{\mu_w} \right)_{i-\frac{1}{2},k} \left(\frac{\partial p}{\partial r} \right)_{i-\frac{1}{2},k}}{r_{i+\frac{1}{2},k} - r_{i-\frac{1}{2},k}} \end{aligned} \quad (\text{A.8})$$

Similarly, the second term on the right hand side of Equation A.5 is considered as:

$$\begin{aligned} & \left\{ \frac{\partial}{\partial z} \left[\rho_w \frac{k_z}{\mu_w} \left(\frac{\partial p}{\partial z} - \gamma_w \frac{\partial D}{\partial z} \right) \right] \right\}_{\substack{r=r_i \\ z=z_k}} = \\ & \frac{\left[\rho_w \frac{k_z}{\mu_w} \left(\frac{\partial p}{\partial z} - \gamma_w \frac{\partial D}{\partial z} \right) \right]_{i,k+\frac{1}{2}} - \left[\rho_w \frac{k_z}{\mu_w} \left(\frac{\partial p}{\partial z} - \gamma_w \frac{\partial D}{\partial z} \right) \right]_{i,k-\frac{1}{2}}}{z_{i,k+\frac{1}{2}} - z_{i,k-\frac{1}{2}}} \end{aligned} \quad (\text{A.9})$$

Rearranging the above equation gives:

$$\begin{aligned}
& \left\{ \frac{\partial}{\partial z} \left[\rho_w \frac{k_z}{\mu_w} \left(\frac{\partial p}{\partial z} - \gamma_w \frac{\partial D}{\partial z} \right) \right] \right\}_{r=r_i}^{z=z_k} \\
&= \frac{\left(\rho_w \frac{k_z}{\mu_w} \right)_{i,k+\frac{1}{2}} \left(\frac{\partial p}{\partial z} - \gamma_w \frac{\partial D}{\partial z} \right)_{i,k+\frac{1}{2}} - \left(\rho_w \frac{k_z}{\mu_w} \right)_{i,k-\frac{1}{2}} \left(\frac{\partial p}{\partial z} - \gamma_w \frac{\partial D}{\partial z} \right)_{i,k-\frac{1}{2}}}{z_{i,k+\frac{1}{2}} - z_{i,k-\frac{1}{2}}} \quad (\text{A.10})
\end{aligned}$$

Now, if pressure derivatives at the grid block boundaries are considered,

$$\left(\frac{\partial p}{\partial r} \right)_{i+\frac{1}{2},k} = \frac{p_{i+1,k} - p_{i,k}}{r_{i+1,k} - r_{i,k}} \quad (\text{A.11})$$

$$\left(\frac{\partial p}{\partial r} \right)_{i-\frac{1}{2},k} = \frac{p_{i,k} - p_{i-1,k}}{r_{i,k} - r_{i-1,k}} \quad (\text{A.12})$$

$$\left(\frac{\partial p}{\partial z} - \gamma_w \frac{\partial D}{\partial z} \right)_{i,k+\frac{1}{2}} = \frac{p_{i,k+1} - p_{i,k}}{z_{i,k+1} - z_{i,k}} - \gamma_{w,i,k+\frac{1}{2}} \frac{z_{i,k+1} - z_{i,k}}{z_{i,k+1} - z_{i,k}} \quad (\text{A.13})$$

$$\left(\frac{\partial p}{\partial z} - \gamma_w \frac{\partial D}{\partial z} \right)_{i,k-\frac{1}{2}} = \frac{p_{i,k} - p_{i,k-1}}{z_{i,k} - z_{i,k-1}} - \gamma_{w,i,k-\frac{1}{2}} \frac{z_{i,k} - z_{i,k-1}}{z_{i,k} - z_{i,k-1}} \quad (\text{A.14})$$

Combining Equations A.6-A.14 in Equation A.5 gives:

$$\begin{aligned}
& \frac{(\rho_w \phi)_{i,k}^{n+1} - (\rho_w \phi)_{i,k}^n}{\Delta t} \\
&= \frac{1}{r_{i,k}} \frac{\left(r \rho_w \frac{k_r}{\mu_w} \right)_{i+\frac{1}{2},k} \left(\frac{p_{i+1,k} - p_{i,k}}{r_{i+1,k} - r_{i,k}} \right) - \left(r \rho_w \frac{k_r}{\mu_w} \right)_{i-\frac{1}{2},k} \left(\frac{p_{i,k} - p_{i-1,k}}{r_{i,k} - r_{i-1,k}} \right)}{r_{i+\frac{1}{2},k} - r_{i-\frac{1}{2},k}} \\
& \quad + \frac{\left(\rho_w \frac{k_z}{\mu_w} \right)_{i,k+\frac{1}{2}} \left(\frac{p_{i,k+1} - p_{i,k}}{z_{i,k+1} - z_{i,k}} - \gamma_{w,i,k+\frac{1}{2}} \frac{z_{i,k+1} - z_{i,k}}{z_{i,k+1} - z_{i,k}} \right) - \left(\rho_w \frac{k_z}{\mu_w} \right)_{i,k-\frac{1}{2}} \left(\frac{p_{i,k} - p_{i,k-1}}{z_{i,k} - z_{i,k-1}} - \gamma_{w,i,k-\frac{1}{2}} \frac{z_{i,k} - z_{i,k-1}}{z_{i,k} - z_{i,k-1}} \right)}{z_{i,k+\frac{1}{2}} - z_{i,k-\frac{1}{2}}} \quad (\text{A.15})
\end{aligned}$$

Rearranging Equation A.15 gives:

$$\begin{aligned}
& \frac{(\rho_w \phi)_{i,k}^{n+1} - (\rho_w \phi)_{i,k}^n}{\Delta t} \\
&= \frac{1}{r_{i,k}} \frac{1}{\left(r_{i+\frac{1}{2},k} - r_{i-\frac{1}{2},k}\right)} \left[\left(r \rho_w \frac{k_r}{\mu_w}\right)_{i+\frac{1}{2},k} \left(\frac{p_{i+1,k} - p_{i,k}}{r_{i+1,k} - r_{i,k}}\right) \right. \\
&\quad \left. - \left(r \rho_w \frac{k_r}{\mu_w}\right)_{i-\frac{1}{2},k} \left(\frac{p_{i,k} - p_{i-1,k}}{r_{i,k} - r_{i-1,k}}\right) \right] \\
&\quad + \frac{1}{\left(z_{i,k+\frac{1}{2}} - z_{i,k-\frac{1}{2}}\right)} \left[\left(\rho_w \frac{k_z}{\mu_w}\right)_{i,k+\frac{1}{2}} \left(\frac{p_{i,k+1} - p_{i,k}}{z_{i,k+1} - z_{i,k}} - \gamma_{w,i,k+\frac{1}{2}} \frac{z_{i,k+1} - z_{i,k}}{z_{i,k+1} - z_{i,k}}\right) \right. \\
&\quad \left. - \left(\rho_w \frac{k_z}{\mu_w}\right)_{i,k-\frac{1}{2}} \left(\frac{p_{i,k} - p_{i,k-1}}{z_{i,k} - z_{i,k-1}} - \gamma_{w,i,k-\frac{1}{2}} \frac{z_{i,k} - z_{i,k-1}}{z_{i,k} - z_{i,k-1}}\right) \right]
\end{aligned} \tag{A.16}$$

Further rearranging the above equation gives:

$$\begin{aligned}
& \frac{(\rho_w \phi)_{i,k}^{n+1} - (\rho_w \phi)_{i,k}^n}{\Delta t} \\
&= \frac{1}{r_{i,k}} \frac{1}{\left(r_{i+\frac{1}{2},k} - r_{i-\frac{1}{2},k}\right)} \left(r \rho_w \frac{k_r}{\mu_w}\right)_{i+\frac{1}{2},k} \left(\frac{1}{r_{i+1,k} - r_{i,k}}\right) (p_{i+1,k} - p_{i,k}) \\
&\quad - \frac{1}{r_{i,k}} \frac{1}{\left(r_{i+\frac{1}{2},k} - r_{i-\frac{1}{2},k}\right)} \left(r \rho_w \frac{k_r}{\mu_w}\right)_{i-\frac{1}{2},k} \left(\frac{1}{r_{i,k} - r_{i-1,k}}\right) (p_{i,k} - p_{i-1,k}) \\
&\quad + \frac{1}{\left(z_{i,k+\frac{1}{2}} - z_{i,k-\frac{1}{2}}\right)} \left(\rho_w \frac{k_z}{\mu_w}\right)_{i,k+\frac{1}{2}} \left(\frac{1}{z_{i,k+1} - z_{i,k}}\right) \left[p_{i,k+1} - p_{i,k} - \gamma_{w,i,k+\frac{1}{2}} (z_{i,k+1} - z_{i,k})\right] \\
&\quad - \frac{1}{\left(z_{i,k+\frac{1}{2}} - z_{i,k-\frac{1}{2}}\right)} \left(\rho_w \frac{k_z}{\mu_w}\right)_{i,k-\frac{1}{2}} \left(\frac{1}{z_{i,k} - z_{i,k-1}}\right) \left[p_{i,k} - p_{i,k-1} - \gamma_{w,i,k-\frac{1}{2}} (z_{i,k} - z_{i,k-1})\right]
\end{aligned} \tag{A.17}$$

Since the above equation is per unit volume, both sides of it need to be multiplied by the bulk volume of the grid block:

$$\begin{aligned}
& \pi \Delta \mathbf{z}_{i,k} \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \frac{(\rho_w \phi)_{i,k}^{n+1} - (\rho_w \phi)_{i,k}^n}{\Delta t} \\
&= \frac{1}{r_{i,k}} \frac{\pi \Delta \mathbf{z}_{i,k} \left(r_{i+\frac{1}{2},k} - r_{i-\frac{1}{2},k} \right) \left(r_{i+\frac{1}{2},k} + r_{i-\frac{1}{2},k} \right)}{\left(r_{i+\frac{1}{2},k} - r_{i-\frac{1}{2},k} \right)} \left(r \rho_w \frac{k_r}{\mu_w} \right)_{i+\frac{1}{2},k} \left(\frac{1}{r_{i+1,k} - r_{i,k}} \right) \begin{pmatrix} p_{i+1,k} \\ -p_{i,k} \end{pmatrix} \\
&- \frac{1}{r_{i,k}} \frac{\pi \Delta \mathbf{z}_{i,k} \left(r_{i+\frac{1}{2},k} - r_{i-\frac{1}{2},k} \right) \left(r_{i+\frac{1}{2},k} + r_{i-\frac{1}{2},k} \right)}{\left(r_{i+\frac{1}{2},k} - r_{i-\frac{1}{2},k} \right)} \left(r \rho_w \frac{k_r}{\mu_w} \right)_{i-\frac{1}{2},k} \left(\frac{1}{r_{i,k} - r_{i-1,k}} \right) \begin{pmatrix} p_{i,k} \\ -p_{i-1,k} \end{pmatrix} \\
&+ \frac{\pi \Delta \mathbf{z}_{i,k} \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right)}{\left(z_{i,k+\frac{1}{2}} - z_{i,k-\frac{1}{2}} \right)} \left(\rho_w \frac{k_z}{\mu_w} \right)_{i,k+\frac{1}{2}} \left(\frac{1}{z_{i,k+1} - z_{i,k}} \right) \begin{bmatrix} p_{i,k+1} - p_{i,k} \\ -\gamma_{w,i,k+\frac{1}{2}} (z_{i,k+1} - z_{i,k}) \end{bmatrix} \\
&- \frac{\pi \Delta \mathbf{z}_{i,k} \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right)}{\left(z_{i,k+\frac{1}{2}} - z_{i,k-\frac{1}{2}} \right)} \left(\rho_w \frac{k_z}{\mu_w} \right)_{i,k-\frac{1}{2}} \left(\frac{1}{z_{i,k} - z_{i,k-1}} \right) \begin{bmatrix} p_{i,k} - p_{i,k-1} \\ -\gamma_{w,i,k-\frac{1}{2}} (z_{i,k} - z_{i,k-1}) \end{bmatrix}
\end{aligned} \tag{A.18}$$

$$\begin{aligned}
& \pi \Delta \mathbf{z}_{i,k} \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \frac{(\rho \phi)_{i,k}^{n+1} - (\rho \phi)_{i,k}^n}{\Delta t} \\
&= \frac{1}{r_{i,k}} \pi \Delta \mathbf{z}_{i,k} \left(r_{i+\frac{1}{2},k} + r_{i-\frac{1}{2},k} \right) \left(r \rho_w \frac{k_r}{\mu_w} \right)_{i+\frac{1}{2},k} \left(\frac{1}{r_{i+1,k} - r_{i,k}} \right) (p_{i+1,k} - p_{i,k}) \\
&- \frac{1}{r_{i,k}} \pi \Delta \mathbf{z}_{i,k} \left(r_{i+\frac{1}{2},k} + r_{i-\frac{1}{2},k} \right) \left(r \rho_w \frac{k_r}{\mu_w} \right)_{i-\frac{1}{2},k} \left(\frac{1}{r_{i,k} - r_{i-1,k}} \right) (p_{i,k} - p_{i-1,k}) \\
&+ \pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \left(\rho_w \frac{k_z}{\mu_w} \right)_{i,k+\frac{1}{2}} \left(\frac{1}{z_{i,k+1} - z_{i,k}} \right) \begin{bmatrix} p_{i,k+1} - p_{i,k} \\ -\gamma_{w,i,k+\frac{1}{2}} (z_{i,k+1} - z_{i,k}) \end{bmatrix} \\
&- \pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \left(\rho_w \frac{k_z}{\mu_w} \right)_{i,k-\frac{1}{2}} \left(\frac{1}{z_{i,k} - z_{i,k-1}} \right) \begin{bmatrix} p_{i,k} - p_{i,k-1} \\ -\gamma_{w,i,k-\frac{1}{2}} (z_{i,k} - z_{i,k-1}) \end{bmatrix}
\end{aligned} \tag{A.19}$$

Now, the transmissibilities in the r and z -directions can be defined as follows:

$$\Omega_{r,i+\frac{1}{2},k} = \frac{1}{r_{i,k}} \pi \Delta z_{i,k} \left(r_{i+\frac{1}{2},k} + r_{i-\frac{1}{2},k} \right) \left(r \rho_w \frac{k_r}{\mu_w} \right)_{i+\frac{1}{2},k} \left(\frac{1}{r_{i+1,k} - r_{i,k}} \right) \quad (\text{A.20})$$

$$\Omega_{r,i-\frac{1}{2},k} = \frac{1}{r_{i,k}} \pi \Delta z_{i,k} \left(r_{i+\frac{1}{2},k} + r_{i-\frac{1}{2},k} \right) \left(r \rho_w \frac{k_r}{\mu_w} \right)_{i-\frac{1}{2},k} \left(\frac{1}{r_{i,k} - r_{i-1,k}} \right) \quad (\text{A.21})$$

$$\Omega_{z,i,k+\frac{1}{2}} = \pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \left(\rho_w \frac{k_z}{\mu_w} \right)_{i,k+\frac{1}{2}} \left(\frac{1}{z_{i,k+1} - z_{i,k}} \right) \quad (\text{A.22})$$

$$\Omega_{z,i,k-\frac{1}{2}} = \pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \left(\rho_w \frac{k_z}{\mu_w} \right)_{i,k-\frac{1}{2}} \left(\frac{1}{z_{i,k} - z_{i,k-1}} \right) \quad (\text{A.23})$$

Equation A.19 can be rewritten using the definitions of the transmissibilities as follows:

$$\begin{aligned} & \pi \Delta z_{i,k} \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \frac{(\rho_w \phi)_{i,k}^{n+1} - (\rho_w \phi)_{i,k}^n}{\Delta t} \\ &= \Omega_{r,i+\frac{1}{2},k} (p_{i+1,k} - p_{i,k}) - \Omega_{r,i-\frac{1}{2},k} (p_{i,k} - p_{i-1,k}) \\ &+ \Omega_{z,i,k+\frac{1}{2}} \left[p_{i,k+1} - p_{i,k} - \gamma_{w,i,k+\frac{1}{2}} (z_{i,k+1} - z_{i,k}) \right] \\ &- \Omega_{z,i,k-\frac{1}{2}} \left[p_{i,k} - p_{i,k-1} - \gamma_{w,i,k-\frac{1}{2}} (z_{i,k} - z_{i,k-1}) \right] \end{aligned} \quad (\text{A.24})$$

If a fully-implicit approach is used, Equation A.24 can be written as:

$$\begin{aligned} & \pi \Delta z_{i,k} \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \frac{(\rho_w \phi)_{i,k}^{n+1} - (\rho_w \phi)_{i,k}^n}{\Delta t} \\ &= \Omega_{r,i+\frac{1}{2},k}^{n+1} (p_{i+1,k}^{n+1} - p_{i,k}^{n+1}) - \Omega_{r,i-\frac{1}{2},k}^{n+1} (p_{i,k}^{n+1} - p_{i-1,k}^{n+1}) \\ &+ \Omega_{z,i,k+\frac{1}{2}}^{n+1} \left[p_{i,k+1}^{n+1} - p_{i,k}^{n+1} - \gamma_{w,i,k+\frac{1}{2}}^{n+1} (z_{i,k+1} - z_{i,k}) \right] \\ &- \Omega_{z,i,k-\frac{1}{2}}^{n+1} \left[p_{i,k}^{n+1} - p_{i,k-1}^{n+1} - \gamma_{w,i,k-\frac{1}{2}}^{n+1} (z_{i,k} - z_{i,k-1}) \right] \end{aligned} \quad (\text{A.25})$$

At this point, it is important to note that the transmissibilities and the accumulation terms are both functions of pressure and temperature.

The Hawkins relationship (Hawkins, 1956) is used for the treatment of the skin factor due to damage as follows:

$$S = \left(\frac{k_r}{k_s} - 1 \right) \ln \left(\frac{r_s}{r_w} \right) \quad (\text{A.26})$$

Hence, the permeability that would mimic a skin factor of S would be obtained by:

$$k_s = \frac{k_r}{\frac{S}{\ln \left(\frac{r_s}{r_w} \right)} + 1} \quad (\text{A.27})$$

In Equation A.27, k_r represents the homogeneous r -directional reservoir permeability assigned to the skin zone concentric to the wellbore with a radius r_s .

For any reservoir grid block i, k , the mass balance equation is given as follows:

$$\begin{aligned} & \pi \Delta z_{i,k} \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \frac{(\rho_w \phi)_{i,k}^{n+1} - (\rho_w \phi)_{i,k}^n}{\Delta t} \\ &= \Omega_{r,i+\frac{1}{2},k}^{n+1} (p_{i+1,k}^{n+1} - p_{i,k}^{n+1}) - \Omega_{r,i-\frac{1}{2},k}^{n+1} (p_{i,k}^{n+1} - p_{i-1,k}^{n+1}) \\ &+ \Omega_{z,i,k+\frac{1}{2}}^{n+1} \left[p_{i,k+1}^{n+1} - p_{i,k}^{n+1} - \gamma_{w,i,k+\frac{1}{2}}^{n+1} (z_{i,k+1} - z_{i,k}) \right] \\ &- \Omega_{z,i,k-\frac{1}{2}}^{n+1} \left[p_{i,k}^{n+1} - p_{i,k-1}^{n+1} - \gamma_{w,i,k-\frac{1}{2}}^{n+1} (z_{i,k} - z_{i,k-1}) \right] \end{aligned} \quad (\text{A.28})$$

It is assumed that a total of N_r and N_z number of grid blocks is available in the r and z -directions, respectively.

If $i = 1$,

$$\begin{aligned}
& \pi \Delta z_{1,k} \left(r_{\frac{3}{2},k}^2 - r_{w,k}^2 \right) \frac{(\rho_w \phi)_{1,k}^{n+1} - (\rho_w \phi)_{1,k}^n}{\Delta t} \\
&= \Omega_{r,\frac{3}{2},k}^{n+1} (p_{2,k}^{n+1} - p_{1,k}^{n+1}) - \Omega_{r,w,k}^{n+1} (p_{1,k}^{n+1} - p_{w,k}^{n+1}) \\
&+ \Omega_{z,1,k+\frac{1}{2}}^{n+1} \left[p_{1,k+1}^{n+1} - p_{1,k}^{n+1} - \gamma_{w,1,k+\frac{1}{2}}^{n+1} (z_{1,k+1} - z_{1,k}) \right] \\
&- \Omega_{z,1,k-\frac{1}{2}}^{n+1} \left[p_{1,k}^{n+1} - p_{1,k-1}^{n+1} - \gamma_{w,1,k-\frac{1}{2}}^{n+1} (z_{1,k} - z_{1,k-1}) \right]
\end{aligned} \tag{A.29}$$

where

$$\Omega_{r,w,k} = \frac{1}{r_{1,k}} \pi \Delta z_{1,k} \left(r_{\frac{3}{2},k}^2 + r_{w,k}^2 \right) \left(r \rho_w \frac{k_r}{\mu} \right)_{1,k} \left(\frac{1}{r_{1,k} - r_{w,k}} \right) \tag{A.30}$$

If $i = N_r$, the transmissibility at the outer boundary will simply be set to be zero,

$$\Omega_{r,i+\frac{1}{2},k}^{n+1} = 0 :$$

$$\begin{aligned}
& \pi \Delta z_{N_r,k} \left(r_e^2 - r_{i-\frac{1}{2},k}^2 \right) \frac{(\rho_w \phi)_{N_r,k}^{n+1} - (\rho_w \phi)_{N_r,k}^n}{\Delta t} = -\Omega_{r,N_r-\frac{1}{2},k}^{n+1} (p_{N_r,k}^{n+1} - p_{N_r-1,k}^{n+1}) \\
&+ \Omega_{z,N_r,k+\frac{1}{2}}^{n+1} \left[p_{N_r,k+1}^{n+1} - p_{N_r,k}^{n+1} - \gamma_{w,N_r,k+\frac{1}{2}}^{n+1} (z_{N_r,k+1} - z_{N_r,k}) \right] \\
&- \Omega_{z,N_r,k-\frac{1}{2}}^{n+1} \left[p_{N_r,k}^{n+1} - p_{N_r,k-1}^{n+1} - \gamma_{w,N_r,k-\frac{1}{2}}^{n+1} (z_{N_r,k} - z_{N_r,k-1}) \right]
\end{aligned} \tag{A.31}$$

If $k = 1$, then the transmissibility at the outer boundary will simply be set to be zero,

$$\Omega_{z,i,k-\frac{1}{2}}^{n+1} = 0 :$$

$$\begin{aligned}
& \pi \Delta z_{i,1} \left(r_{i+\frac{1}{2},1}^2 - r_{i-\frac{1}{2},1}^2 \right) \frac{(\rho_w \phi)_{i,1}^{n+1} - (\rho_w \phi)_{i,1}^n}{\Delta t} \\
&= \Omega_{r,i+\frac{1}{2},1}^{n+1} (p_{i+1,1}^{n+1} - p_{i,1}^{n+1}) - \Omega_{r,i-\frac{1}{2},1}^{n+1} (p_{i,1}^{n+1} - p_{i-1,1}^{n+1}) \\
&+ \Omega_{z,i,\frac{3}{2}}^{n+1} \left[p_{i,2}^{n+1} - p_{i,1}^{n+1} - \gamma_{w,i,\frac{3}{2}}^{n+1} (z_{i,2} - z_{i,1}) \right]
\end{aligned} \tag{A.32}$$

If $k = N_z$, then the transmissibility at the outer boundary will simply be set to be zero,

$$\Omega_{z,i,k+\frac{1}{2}}^{n+1} = 0 ,$$

$$\begin{aligned} & \pi \Delta z_{i,N_z} \left(r_{i+\frac{1}{2},N_z}^2 - r_{i-\frac{1}{2},N_z}^2 \right) \frac{(\rho_w \phi)_{i,N_z}^{n+1} - (\rho_w \phi)_{i,N_z}^n}{\Delta t} \\ &= \Omega_{r,i+\frac{1}{2},N_z}^{n+1} (p_{i+1,N_z}^{n+1} - p_{i,N_z}^{n+1}) - \Omega_{r,i-\frac{1}{2},N_z}^{n+1} (p_{i,N_z}^{n+1} - p_{i-1,N_z}^{n+1}) \\ & \quad - \Omega_{z,i,N_z-\frac{1}{2}}^{n+1} \left[p_{i,N_z}^{n+1} - p_{i,N_z-1}^{n+1} - \gamma_{w,i,N_z-\frac{1}{2}}^{n+1} (z_{i,N_z} - z_{i,N_z-1}) \right] \end{aligned} \quad (\text{A.33})$$

The final word on mass balance will be regarding the evaluation of the terms in the transmissibilities at the $\pm 1/2$ boundaries.

Harmonic averaging is used for the permeability. In the r -direction, the harmonic averaging is computed as follows:

$$k_{r,i+\frac{1}{2},k} = \frac{k_{r,i+1,k} k_{r,i,k} \ln \left(\frac{r_{i+1,k}}{r_{i,k}} \right)}{k_{r,i,k} \ln \left(\frac{r_{i+1,k}}{r_{i+\frac{1}{2},k}} \right) + k_{r,i+1,k} \ln \left(\frac{r_{i+\frac{1}{2},k}}{r_i} \right)} \quad (\text{A.34})$$

Harmonic averaging is used for the z -direction as well using the following relationship:

$$k_{z,i,k+\frac{1}{2}} = \frac{(\Delta z_{i,k} + \Delta z_{i,k+1}) k_{z,i,k} k_{z,i,k+1}}{k_{z,i,k} \Delta z_{i,k+1} + k_{z,i,k+1} \Delta z_{i,k}} \quad (\text{A.35})$$

In the r -direction, standard arithmetic averaging is used for the fluid properties:

$$\left(\frac{\rho_w}{\mu} \right)_{i+\frac{1}{2},k} = \frac{\left(\frac{\rho_w}{\mu} \right)_{i,k} + \left(\frac{\rho_w}{\mu} \right)_{i+1,k}}{2} \quad (\text{A.36})$$

Similarly, for the z-direction:

$$\left(\frac{\rho_w}{\mu}\right)_{i,k+\frac{1}{2}} = \frac{\left(\frac{\rho_w}{\mu}\right)_{i,k} + \left(\frac{\rho_w}{\mu}\right)_{i,k+1}}{2} \quad (\text{A.37})$$

The thermal energy balance equation is given by Bird et al. (1960) in terms of internal energy:

$$\frac{\partial}{\partial t}(\rho U) - (\nabla \bullet \rho_w H_w \vec{v}_w) - (\nabla \bullet \tilde{q}) = 0 \quad (\text{A.38})$$

It should be kept in mind that the heat is stored in both the rock and the fluid and local thermal equilibrium is assumed. Hence, the accumulation term of the above equation needs to further be modified:

$$\frac{\partial}{\partial t}[\phi \rho_w U_w + (1-\phi) \rho_s C_{p,s} T] = -(\nabla \bullet \rho_w H_w \vec{v}_w) - (\nabla \bullet \tilde{q}) \quad (\text{A.39})$$

Since Equation A.39 is given per unit volume, each side of the equation is first multiplied by the bulk volume of the gridblock.

$$\begin{aligned} & \pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \Delta z_k \frac{\partial}{\partial t} [\phi \rho_w U_w + (1-\phi) \rho_s C_{p,s} T] \\ &= -\pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \Delta z_k (\nabla \bullet \rho_w H_w \vec{v}_w) \\ & \quad - \pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \Delta z_k (\nabla \bullet \tilde{q}) \end{aligned} \quad (\text{A.40})$$

Now, the first term on the right hand side of Equation A.40 is first considered. Actually, this looks very much like the term in mass balance $\nabla \bullet \rho \vec{v}$. The discretization of the mass balance equation has been given previously. The only difference is that now there will be the internal energy term multiplying the transmissibility. Hence, the discretization of the first term on the right hand side of Equation A.40 will have the following form:

$$\begin{aligned}
& -\pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \Delta z_k (\nabla \bullet \rho_w U_w \bar{v}_w) = \\
& \Omega_{r,i+\frac{1}{2},k} H_{w,i+\frac{1}{2},k} (p_{i+1,k} - p_{i,k}) - \Omega_{r,i-\frac{1}{2},k} H_{w,i-\frac{1}{2},k} (p_{i,k} - p_{i-1,k}) \\
& + \Omega_{z,i,k+\frac{1}{2}} H_{w,i,k+\frac{1}{2}} \left[p_{i,k+1} - p_{i,k} - \gamma_{w,i,k+\frac{1}{2}} (z_{i,k+1} - z_{i,k}) \right] \\
& - \Omega_{z,i,k-\frac{1}{2}} H_{w,i,k-\frac{1}{2}} \left[p_{i,k} - p_{i,k-1} - \gamma_{w,i,k-\frac{1}{2}} (z_{i,k} - z_{i,k-1}) \right]
\end{aligned} \tag{A.41}$$

where

$$\Omega_{r,i+\frac{1}{2},k} = \frac{1}{r_{i,k}} \pi \Delta z_{i,k} \left(r_{i+\frac{1}{2},k} + r_{i-\frac{1}{2},k} \right) \left(r \rho_w \frac{k_r}{\mu_w} \right)_{i+\frac{1}{2},k} \left(\frac{1}{r_{i+1,k} - r_{i,k}} \right) \tag{A.41a}$$

$$\Omega_{r,i-\frac{1}{2},k} = \frac{1}{r_{i,k}} \pi \Delta z_{i,k} \left(r_{i+\frac{1}{2},k} + r_{i-\frac{1}{2},k} \right) \left(r \rho_w \frac{k_r}{\mu_w} \right)_{i-\frac{1}{2},k} \left(\frac{1}{r_{i,k} - r_{i-1,k}} \right) \tag{A.41b}$$

$$\Omega_{z,i,k+\frac{1}{2}} = \pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \left(\rho_w \frac{k_z}{\mu_w} \right)_{i,k+\frac{1}{2}} \left(\frac{1}{z_{i,k+1} - z_{i,k}} \right) \tag{A.41c}$$

$$\Omega_{z,i,k-\frac{1}{2}} = \pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \left(\rho_w \frac{k_z}{\mu_w} \right)_{i,k-\frac{1}{2}} \left(\frac{1}{z_{i,k} - z_{i,k-1}} \right) \tag{A.41d}$$

At this point, it is important to note that when the internal energies at the grid block boundaries are evaluated, upwinding will be performed. In other words, the flow direction will be checked to figure out which grid block pressure and temperature should be used to determine the internal energy.

For the r -direction,

If $p_{i+1,k} > p_{i,k}$, then $U_{w,i+1,k}$ is used.

If $p_{i+1,k} < p_{i,k}$, then $U_{w,i-1,k}$ is used.

For the z-direction,

If $p_{i,k+1} > p_{i,k} + \gamma_{w,i,k+\frac{1}{2}}(z_{i,k+1} - z_{i,k})$, then $U_{w,i,k+1}$ is used.

If $p_{i,k+1} < p_{i,k} + \gamma_{w,i,k+\frac{1}{2}}(z_{i,k+1} - z_{i,k})$, then $U_{w,i,k-1}$ is used.

Now, the second term on the right hand side of Equation A.40 will be considered. This is the conduction term. In cylindrical coordinates, this term can be evaluated as follows:

$$\begin{aligned} & -\pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \Delta z_k (\nabla \cdot \tilde{q}) \\ & = -\pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \Delta z_k \left[\frac{1}{r} \frac{\partial}{\partial r} (r \tilde{q}_r) + \frac{1}{r} \frac{\partial}{\partial \theta} (\tilde{q}_\theta) + \frac{\partial}{\partial z} (\tilde{q}_z) \right] \end{aligned} \quad (\text{A.42})$$

Since the heat flow in the theta direction will be ignored, Equation A.42 will reduce to:

$$-\pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \Delta z_k (\nabla \cdot \tilde{q}) = -\pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \Delta z_k \left[\frac{1}{r} \frac{\partial}{\partial r} (r \tilde{q}_r) + \frac{\partial}{\partial z} (\tilde{q}_z) \right] \quad (\text{A.43})$$

At this point, Fourier's law of heat conduction will be required to be recalled which states:

$$\tilde{q}_r = -\lambda_t \frac{\partial T}{\partial r} \quad (\text{A.44})$$

$$\tilde{q}_z = -\lambda_t \frac{\partial T}{\partial z} \quad (\text{A.45})$$

If Equations A.44 and A.45 are used in Equation A.43, it can be obtained:

$$\begin{aligned} & -\pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \Delta z_k (\nabla \cdot \tilde{q}) \\ & = \pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \Delta z_k \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \lambda_t \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial z} \left(\lambda_t \frac{\partial T}{\partial z} \right) \right] \end{aligned} \quad (\text{A.46})$$

Evaluating Equation A.46 in grid block i, k , it can be obtained:

$$\begin{aligned}
& -\pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \Delta z_k (\nabla \bullet \tilde{q}) \\
& = \pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \Delta z_k \frac{1}{r_{i,k}} \frac{\left(r \lambda_t \frac{\partial T}{\partial r} \right)_{i+\frac{1}{2},k} - \left(r \lambda_t \frac{\partial T}{\partial r} \right)_{i-\frac{1}{2},k}}{r_{i+\frac{1}{2},k} - r_{i-\frac{1}{2},k}} \\
& \quad + \pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \Delta z_k \frac{\left(\lambda_t \frac{\partial T}{\partial z} \right)_{i,k+\frac{1}{2}} - \left(\lambda_t \frac{\partial T}{\partial z} \right)_{i,k-\frac{1}{2}}}{z_{i,k+\frac{1}{2}} - z_{i,k-\frac{1}{2}}}
\end{aligned} \tag{A.47}$$

If the following approximations are made

$$\left(\frac{\partial T}{\partial r} \right)_{i+\frac{1}{2},k} = \frac{T_{i+1,k} - T_{i,k}}{r_{i+1,k} - r_{i,k}} \tag{A.48}$$

$$\left(\frac{\partial T}{\partial r} \right)_{i-\frac{1}{2},k} = \frac{T_{i,k} - T_{i-1,k}}{r_{i,k} - r_{i-1,k}} \tag{A.49}$$

$$\left(\frac{\partial T}{\partial z} \right)_{i,k+\frac{1}{2}} = \frac{T_{i,k+1} - T_{i,k}}{z_{i,k+1} - z_{i,k}} \tag{A.50}$$

$$\left(\frac{\partial T}{\partial z} \right)_{i,k-\frac{1}{2}} = \frac{T_{i,k} - T_{i,k-1}}{z_{i,k} - z_{i,k-1}} \tag{A.51}$$

it can be obtained:

$$\begin{aligned}
& -\pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \Delta z_k (\nabla \bullet \tilde{q}) \\
& = \pi \left(r_{i+\frac{1}{2},k} + r_{i-\frac{1}{2},k} \right) \Delta z_k \frac{1}{r_{i,k}} (r\lambda_t)_{i+\frac{1}{2},k} \frac{1}{r_{i+1,k} - r_{i,k}} (T_{i+1,k} - T_{i,k}) \\
& - \pi \left(r_{i+\frac{1}{2},k} + r_{i-\frac{1}{2},k} \right) \Delta z_k \frac{1}{r_{i,k}} (r\lambda_t)_{i-\frac{1}{2},k} \frac{1}{r_{i,k} - r_{i-1,k}} (T_{i,k} - T_{i-1,k}) \\
& \pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \lambda_{t,i,k+\frac{1}{2}} \frac{1}{z_{i,k+1} - z_{i,k}} (T_{i,k+1} - T_{i,k}) \\
& - \pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \lambda_{t,i,k-\frac{1}{2}} \frac{1}{z_{i,k} - z_{i,k-1}} (T_{i,k} - T_{i,k-1})
\end{aligned} \tag{A.52}$$

Now, if conduction transmissibilities are defined as follows:

$$\Gamma_{r,i+\frac{1}{2},k} = \pi \left(r_{i+\frac{1}{2},k} + r_{i-\frac{1}{2},k} \right) \Delta z_k \frac{1}{r_{i,k}} (r\lambda_t)_{i+\frac{1}{2},k} \frac{1}{r_{i+1,k} - r_{i,k}} \tag{A.53}$$

$$\Gamma_{r,i-\frac{1}{2},k} = \pi \left(r_{i+\frac{1}{2},k} + r_{i-\frac{1}{2},k} \right) \Delta z_k \frac{1}{r_{i,k}} (r\lambda_t)_{i-\frac{1}{2},k} \frac{1}{r_{i,k} - r_{i-1,k}} \tag{A.54}$$

$$\Gamma_{z,i,k+\frac{1}{2}} = \pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \lambda_{t,i,k+\frac{1}{2}} \frac{1}{z_{i,k+1} - z_{i,k}} \tag{A.55}$$

$$\Gamma_{z,i,k-\frac{1}{2}} = \pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \lambda_{t,i,k-\frac{1}{2}} \frac{1}{z_{i,k} - z_{i,k-1}} \tag{A.56}$$

Using Equations A.53-A.56 in Equation A.52 the following equation can be obtained:

$$\begin{aligned}
& -\pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \Delta z_k (\nabla \bullet \tilde{q}) = \Gamma_{r,i+\frac{1}{2},k} (T_{i+1,k} - T_{i,k}) - \Gamma_{r,i-\frac{1}{2},k} (T_{i,k} - T_{i-1,k}) \\
& + \Gamma_{z,i,k+\frac{1}{2}} (T_{i,k+1} - T_{i,k}) - \Gamma_{z,i,k-\frac{1}{2}} (T_{i,k} - T_{i,k-1})
\end{aligned} \tag{A.57}$$

Now, using Equations A.41 and A.57 in Equation A.40 and also using a forward differencing scheme to handle the accumulation term gives:

$$\begin{aligned}
& \pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \Delta z_k \frac{\left(\left[\phi \rho_w U_w + (1-\phi) \rho_s C_{p,s} T \right]_{i,k}^{n+1} - \left[\phi \rho_w U_w + (1-\phi) \rho_s C_{p,s} T \right]_{i,k}^n \right)}{\Delta t} \\
&= \Omega_{r,i+\frac{1}{2},k} H_{w,i+\frac{1}{2},k} (p_{i+1,k} - p_{i,k}) - \Omega_{r,i-\frac{1}{2},k} H_{w,i-\frac{1}{2},k} (p_{i,k} - p_{i-1,k}) \\
&+ \Omega_{z,i,k+\frac{1}{2}} H_{w,i,k+\frac{1}{2}} \left[p_{i,k+1} - p_{i,k} - \gamma_{w,i,k+\frac{1}{2}} (z_{i,k+1} - z_{i,k}) \right] \\
&- \Omega_{z,i,k-\frac{1}{2}} H_{w,i,k-\frac{1}{2}} \left[p_{i,k} - p_{i,k-1} - \gamma_{w,i,k-\frac{1}{2}} (z_{i,k} - z_{i,k-1}) \right] \\
&+ \Gamma_{r,i+\frac{1}{2},k} (T_{i+1,k} - T_{i,k}) - \Gamma_{r,i-\frac{1}{2},k} (T_{i,k} - T_{i-1,k}) \\
&+ \Gamma_{z,i,k+\frac{1}{2}} (T_{i,k+1} - T_{i,k}) - \Gamma_{z,i,k-\frac{1}{2}} (T_{i,k} - T_{i,k-1})
\end{aligned} \tag{A.58}$$

A fully-implicit approach will give:

$$\begin{aligned}
& \pi \left(r_{i+\frac{1}{2},k}^2 - r_{i-\frac{1}{2},k}^2 \right) \Delta z_k \frac{\left(\left[\phi \rho_w U_w + (1-\phi) \rho_s C_{p,s} T \right]_{i,k}^{n+1} - \left[\phi \rho_w U_w + (1-\phi) \rho_s C_{p,s} T \right]_{i,k}^n \right)}{\Delta t} \\
&= \Omega_{r,i+\frac{1}{2},k}^{n+1} H_{w,i+\frac{1}{2},k}^{n+1} (p_{i+1,k}^{n+1} - p_{i,k}^{n+1}) - \Omega_{r,i-\frac{1}{2},k}^{n+1} H_{w,i-\frac{1}{2},k}^{n+1} (p_{i,k}^{n+1} - p_{i-1,k}^{n+1}) \\
&+ \Omega_{z,i,k+\frac{1}{2}}^{n+1} H_{w,i,k+\frac{1}{2}}^{n+1} \left[p_{i,k+1}^{n+1} - p_{i,k}^{n+1} - \gamma_{w,i,k+\frac{1}{2}}^{n+1} (z_{i,k+1} - z_{i,k}) \right] \\
&- \Omega_{z,i,k-\frac{1}{2}}^{n+1} H_{w,i,k-\frac{1}{2}}^{n+1} \left[p_{i,k}^{n+1} - p_{i,k-1}^{n+1} - \gamma_{w,i,k-\frac{1}{2}}^{n+1} (z_{i,k} - z_{i,k-1}) \right] \\
&+ \Gamma_{r,i+\frac{1}{2},k}^{n+1} (T_{i+1,k}^{n+1} - T_{i,k}^{n+1}) - \Gamma_{r,i-\frac{1}{2},k}^{n+1} (T_{i,k}^{n+1} - T_{i-1,k}^{n+1}) \\
&+ \Gamma_{z,i,k+\frac{1}{2}}^{n+1} (T_{i,k+1}^{n+1} - T_{i,k}^{n+1}) - \Gamma_{z,i,k-\frac{1}{2}}^{n+1} (T_{i,k}^{n+1} - T_{i,k-1}^{n+1})
\end{aligned} \tag{A.59}$$

If $i = 1$ for energy balance of the reservoir grid blocks,

$$\begin{aligned}
& \pi \left(r_{\frac{3}{2},k}^2 - r_{w,k}^2 \right) \Delta z_k \frac{\left(\left[\phi \rho_w U_w + (1-\phi) \rho_s C_{p,s} T \right]_{1,k}^{n+1} - \left[\phi \rho_w U_w + (1-\phi) \rho_s C_{p,s} T \right]_{1,k}^n \right)}{\Delta t} \\
&= \Omega_{r,\frac{3}{2},k}^{n+1} H_{w,\frac{3}{2},k}^{n+1} \left(p_{2,k}^{n+1} - p_{1,k}^{n+1} \right) - \Omega_{r_w,k}^{n+1} H_{w,\frac{1}{2},k}^{n+1} \left(p_{1,k}^{n+1} - p_{wf,k}^{n+1} \right) \\
&+ \Omega_{z,1,k+\frac{1}{2}}^{n+1} H_{w,1,k+\frac{1}{2}}^{n+1} \left[p_{1,k+1}^{n+1} - p_{1,k}^{n+1} - \gamma_{w,1,k+\frac{1}{2}}^{n+1} \left(z_{1,k+1} - z_{1,k} \right) \right] \\
&- \Omega_{z,1,k-\frac{1}{2}}^{n+1} H_{w,1,k-\frac{1}{2}}^{n+1} \left[p_{1,k}^{n+1} - p_{1,k-1}^{n+1} - \gamma_{w,1,k-\frac{1}{2}}^{n+1} \left(z_{1,k} - z_{1,k-1} \right) \right] \\
&+ \Gamma_{r,\frac{3}{2},k}^{n+1} \left(T_{2,k}^{n+1} - T_{1,k}^{n+1} \right) - \Gamma_{r_w,k}^{n+1} \left(T_{1,k}^{n+1} - T_{wf,k}^{n+1} \right) \\
&+ \Gamma_{z,1,k+\frac{1}{2}}^{n+1} \left(T_{1,k+1}^{n+1} - T_{1,k}^{n+1} \right) - \Gamma_{z,1,k-\frac{1}{2}}^{n+1} \left(T_{1,k}^{n+1} - T_{1,k-1}^{n+1} \right)
\end{aligned} \tag{A.60}$$

where

$$\Gamma_{r,w,k} = \pi \left(r_{\frac{3}{2},k} + r_{w,k} \right) \Delta z_k \frac{1}{r_{1,k}} (r \lambda_t)_{1,k} \frac{1}{r_{1,k} - r_{w,k}} \tag{A.61}$$

If $i = N_r$, the following transmissibilities will be set to zero, $\Omega_{r,N_r+\frac{1}{2},k}^{n+1} = \Gamma_{r,N_r+\frac{1}{2},k}^{n+1} = 0$

$$\begin{aligned}
& \pi \left(r_{e,k}^2 - r_{N_r-\frac{1}{2},k}^2 \right) \Delta z_k \frac{\left(\left[\phi \rho_w U_w + (1-\phi) \rho_s C_{p,s} T \right]_{N_r,k}^{n+1} - \left[\phi \rho_w U_w + (1-\phi) \rho_s C_{p,s} T \right]_{N_r,k}^n \right)}{\Delta t} \\
&= \Omega_{r,N_r-\frac{1}{2},k}^{n+1} H_{w,N_r-\frac{1}{2},k}^{n+1} \left(p_{N_r,k}^{n+1} - p_{N_r-1,k}^{n+1} \right) \\
&+ \Omega_{z,N_r,k+\frac{1}{2}}^{n+1} H_{w,N_r,k+\frac{1}{2}}^{n+1} \left[p_{N_r,k+1}^{n+1} - p_{N_r,k}^{n+1} - \gamma_{w,N_r,k+\frac{1}{2}}^{n+1} \left(z_{N_r,k+1} - z_{N_r,k} \right) \right] \\
&- \Omega_{z,N_r,k-\frac{1}{2}}^{n+1} H_{w,N_r,k-\frac{1}{2}}^{n+1} \left[p_{N_r,k}^{n+1} - p_{N_r,k-1}^{n+1} - \gamma_{w,N_r,k-\frac{1}{2}}^{n+1} \left(z_{N_r,k} - z_{N_r,k-1} \right) \right] \\
&- \Gamma_{r,N_r-\frac{1}{2},k}^{n+1} \left(T_{N_r,k}^{n+1} - T_{N_r-1,k}^{n+1} \right) + \Gamma_{z,N_r,k+\frac{1}{2}}^{n+1} \left(T_{N_r,k+1}^{n+1} - T_{N_r,k}^{n+1} \right) - \Gamma_{z,N_r,k-\frac{1}{2}}^{n+1} \left(T_{N_r,k}^{n+1} - T_{N_r,k-1}^{n+1} \right)
\end{aligned} \tag{A.62}$$

If $k = 1$, the following transmissibilities will be set to zero, $\Omega_{r,i,\frac{1}{2}}^{n+1} = \Gamma_{r,i,\frac{1}{2}}^{n+1} = 0$

$$\begin{aligned}
& \pi \left(r_{i+\frac{1}{2},1}^2 - r_{i-\frac{1}{2},1}^2 \right) \Delta z_1 \frac{\left(\left[\phi_w U_w + (1-\phi) \rho_s C_{p,s} T \right]_{i,1}^{n+1} - \left[\phi_w U_w + (1-\phi) \rho_s C_{p,s} T \right]_{i,1}^n \right)}{\Delta t} \\
&= \Omega_{r,i+\frac{1}{2},1}^{n+1} H_{w,i+\frac{1}{2},1}^{n+1} (p_{i+1,1}^{n+1} - p_{i,1}^{n+1}) - \Omega_{r,i-\frac{1}{2},1}^{n+1} H_{w,i-\frac{1}{2},1}^{n+1} (p_{i,1}^{n+1} - p_{i-1,1}^{n+1}) \\
&+ \Omega_{z,i,\frac{3}{2}}^{n+1} H_{w,i,\frac{3}{2}}^{n+1} \left[p_{i,2}^{n+1} - p_{i,1}^{n+1} - \gamma_{w,i,\frac{3}{2}}^{n+1} (z_{i,2} - z_{i,1}) \right] \\
&+ \Gamma_{r,i+\frac{1}{2},1}^{n+1} (T_{i+1,1}^{n+1} - T_{i,1}^{n+1}) - \Gamma_{r,i-\frac{1}{2},1}^{n+1} (T_{i,1}^{n+1} - T_{i-1,1}^{n+1}) + \Gamma_{z,i,\frac{3}{2}}^{n+1} (T_{i,2}^{n+1} - T_{i,1}^{n+1})
\end{aligned} \tag{A.63}$$

If $k = N_z$, the following transmissibilities will be set to zero, $\Omega_{r,i,N_z+\frac{1}{2}}^{n+1} = \Gamma_{r,i,N_z+\frac{1}{2}}^{n+1} = 0$

$$\begin{aligned}
& \pi \left(r_{i+\frac{1}{2},N_z}^2 - r_{i-\frac{1}{2},N_z}^2 \right) \Delta z_{N_z} \frac{\left(\left[\phi_w U_w + (1-\phi) \rho_s C_{p,s} T \right]_{i,N_z}^{n+1} - \left[\phi_w U_w + (1-\phi) \rho_s C_{p,s} T \right]_{i,N_z}^n \right)}{\Delta t} \\
&= \Omega_{r,i+\frac{1}{2},N_z}^{n+1} H_{w,i+\frac{1}{2},N_z}^{n+1} (p_{i+1,N_z}^{n+1} - p_{i,N_z}^{n+1}) - \Omega_{r,i-\frac{1}{2},N_z}^{n+1} H_{w,i-\frac{1}{2},N_z}^{n+1} (p_{i,N_z}^{n+1} - p_{i-1,N_z}^{n+1}) \\
&- \Omega_{z,i,N_z-\frac{1}{2}}^{n+1} H_{w,i,N_z-\frac{1}{2}}^{n+1} \left[p_{i,N_z}^{n+1} - p_{i,N_z-1}^{n+1} - \gamma_{w,i,N_z-\frac{1}{2}}^{n+1} (z_{i,N_z} - z_{i,N_z-1}) \right] \\
&+ \Gamma_{r,i+\frac{1}{2},N_z}^{n+1} (T_{i+1,N_z}^{n+1} - T_{i,N_z}^{n+1}) - \Gamma_{r,i-\frac{1}{2},N_z}^{n+1} (T_{i,N_z}^{n+1} - T_{i-1,N_z}^{n+1}) - \Gamma_{z,i,N_z-\frac{1}{2}}^{n+1} (T_{i,N_z}^{n+1} - T_{i,N_z-1}^{n+1})
\end{aligned} \tag{A.64}$$

The set of N number of equations for mass conservation has been derived from mass balance on reservoir grid blocks. If the wellbore pressure is desired to be computed, one more equation is needed to be added. The well is considered to be a tank which is connected to grid blocks at locations where well is open to flow (well will only make connections to grid blocks that are located within the vicinity of the well).

Mass balance on the well tank yields:

$$\left[\begin{array}{c} \text{Mass Accumulation} \\ \text{Rate in Well} \end{array} \right] - \left[\begin{array}{c} \text{Mass Contribution Rate} \\ \text{from Connecting Gridblocks} \end{array} \right] + \left[\begin{array}{c} \text{Production or Injection Rate} \\ \text{from Well} \end{array} \right] = 0 \tag{A.65}$$

Spatial discretization on Equation A.65 gives:

$$V^w \frac{d\rho_w^w}{dt} - \sum_{l=1}^{N_c} \Omega_{r_w, k_l} \left(p_{1, k_l} - p_{wf} - \gamma_{w, 1, k_l} (z_{1, k_l} - z_{wf}) \right) + q = 0 \quad (\text{A.66})$$

where

$$\Omega_{r_w, k_l} = \frac{1}{r_{1, k_l}} \pi \Delta z_{1, k_l} \left(r_{3/2, k_l} + r_{w, k_l} \right) \left(r \rho_w \frac{k}{\mu_w} \right)_{r_w, k_l} \left(\frac{1}{r_{1, k_l} - r_{w, k_l}} \right) \quad (\text{A.67})$$

In Equation A.66, V^w represents volume section open to flow of well along thickness of the reservoir. N_c represents number of connections. p_{wf} represents the wellbore pressure. q represents production or injection rate. l represents grid block index open to flow which is adjacent to the well in the z -direction.

Temporal discretization on the accumulation term in Equation A.66 also yields:

$$V^w \left(\frac{\rho_w^{w, n+1} - \rho_w^{w, n}}{\Delta t} \right) - \sum_{l=1}^{N_c} \Omega_{r_w, k_l} \left(p_{1, k_l} - p_{wf} - \gamma_{w, 1, k_l} (z_{1, k_l} - z_{wf}) \right) + q = 0 \quad (\text{A.68})$$

When modeling the wellbore temperature, a similar approach is used to that of modeling the wellbore pressure, given at the beginning of subsection. It is assumed that the well is a tank and energy balance is applied to this tank.

Energy balance on the well tank yields:

$$\left[\begin{array}{c} \text{Energy Accumulation} \\ \text{Rate in Well} \end{array} \right] - \left[\begin{array}{c} \text{Energy Contribution Rate} \\ \text{from Connecting Gridblocks} \\ \text{through Convection} \end{array} \right] - \left[\begin{array}{c} \text{Energy Contribution Rate} \\ \text{from Connecting Gridblocks} \\ \text{through Conduction} \end{array} \right] + \left[\begin{array}{c} \text{Energy Contribution Rate} \\ \text{due to Production or Injection} \end{array} \right] = 0 \quad (\text{A.69})$$

Spatial discretization on Equation A.69 gives:

$$V^w \frac{d}{dt} (\rho_w^w U_w^w) - \sum_{l=1}^{N_c} \Omega_{r_w, k_l} H_y \left(p_{1, k_l} - p_{wf} - \gamma_{w, 1, k_l} (z_{1, k_l} - z_{wf}) \right) - \sum_{l=1}^{N_c} \Gamma_{r_w, k_l} (T_{1, k_l} - T_{wf}) + q H_x = 0 \quad (\text{A.70})$$

where

$$\Gamma_{r_w, k_l} = \pi \left(r_{3/2, k_l} + r_{w, k_l} \right) \Delta z_{k_l} \frac{1}{r_{1, k_l}} (r \lambda_t)_{r_w, k_l} \frac{1}{r_{1, k_l} - r_{w, k_l}} \quad (\text{A.71})$$

In Equation A.70, H_x becomes H_w for the enthalpy of water produced and becomes $H_{w, inj}$ for enthalpy of the injected water. N_c represents number of connections. T_{wf} represents the wellbore temperature.

In Equation A.70,

$$H_y = H_w(p_1, T_1) \quad \text{if} \quad p_1 > p_{wf} + \gamma_{w, 1, k_l} (z_{1, k_l} - z_{wf}) \quad (\text{A.72a})$$

$$H_y = H_w(p_{wf}, T_{wf}) \quad \text{if} \quad p_1 < p_{wf} + \gamma_{w, 1, k_l} (z_{1, k_l} - z_{wf}) \quad (\text{A.72b})$$

Temporal discretization on the accumulation term in Equation A.70 also yields:

$$V^w \frac{(\rho_w^w U_w^w)^{n+1} (\rho_w^w U_w^w)^n}{\Delta t} - \sum_{l=1}^{N_c} \Omega_{r_w, k_l} H_{w, r_w, k_l} (p_{1, k_l} - p_{wf}) - \sum_{l=1}^{N_c} \Gamma_{r_w, k_l} (T_{1, k_l} - T_{wf}) \quad (\text{A.73})$$

$$+ qH_x = 0$$

Appendix B : Detailed Derivations of Analytical Solution Developed by Lauwerier (1955) and Modified by Satman (2003)

An analytical solution of heat flow in porous media during an injection activity has been proposed by Lauwerier (1955). This reservoir system is depicted as follows:

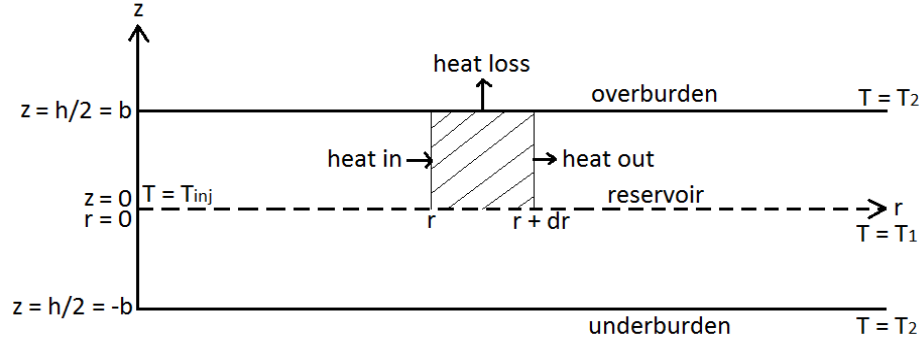


Figure B.1: 2D (r - z) schematic of heat flow in porous media containing reservoir, overburden and underburden strata during an injection activity.

The main assumptions of the model are as follows:

- T_1 is assumed constant at interval of flow in any cross section along vertical distance z .
- T_1 is only dependent on distance r due to convection in reservoir.
- Conduction in only z -direction is considered at reservoir-overburden and/or reservoir-underburden interfaces.
- Conduction in the r -direction is neglected in reservoir, overburden and underburden layers.
- Convection in the r -direction is neglected in overburden and underburden layers.

Here, derivations of analytical solution in cylindrical coordinates are given. From starting generalized energy balance equation of Bird et al. (1984), the following heat flow equation can be expressed as follows:

$$\underbrace{\frac{\partial}{\partial t}(\phi \rho_w U_w)}_{\text{accumulation term}} + \underbrace{\nabla \cdot (\rho_w U_w \vec{v}_w)}_{\text{convection term}} - \underbrace{\nabla \cdot (\lambda_t \nabla T_2)}_{\substack{\text{conduction term} \\ \text{(from Fourier's law)}}} = 0 \quad (\text{B.1})$$

Firstly, we can write surface areas in the r -direction and z -direction and bulk volume of a control volume in cylindrical coordinates (see Figure 3.1) as follows:

$$A_{r-\frac{\Delta r}{2}} = 2\pi \left(r - \frac{\Delta r}{2} \right) \Delta z \quad (\text{B.2})$$

$$A_{r+\frac{\Delta r}{2}} = 2\pi \left(r + \frac{\Delta r}{2} \right) \Delta z \quad (\text{B.3})$$

$$A_{z+\frac{\Delta z}{2}} = 2\pi \frac{\left[\left(r + \frac{\Delta r}{2} \right)^2 - \left(r - \frac{\Delta r}{2} \right)^2 \right]}{2} = \pi \left(r^2 + r\Delta r + \frac{\Delta r^2}{4} - r^2 + r\Delta r - \frac{\Delta r^2}{4} \right) \quad (\text{B.4})$$

$$A_{z+\frac{\Delta z}{2}} = 2\pi r \Delta r \quad (\text{B.5})$$

$$V_b = 2\pi r \Delta r \Delta z \quad (\text{B.6})$$

If we write convection term in the r -direction over Δt ,

$$\left[\begin{array}{c} \text{Net Energy In} \\ \text{by Convection} \end{array} \right] = \left[\begin{array}{c} \text{Energy In} \\ \text{by Convection} \end{array} \right] - \left[\begin{array}{c} \text{Energy Out} \\ \text{by Convection} \end{array} \right] \quad (\text{B.7})$$

$$\left[\begin{array}{c} \text{Net Energy In} \\ \text{by Convection} \end{array} \right] = \left[\left\{ \left[\left(\rho_w U_w v_{w,r} \right)_{r-\frac{\Delta r}{2}} \right] 2\pi \left(r - \frac{\Delta r}{2} \right) \Delta z \right\} - \left\{ \left[\left(\rho_w U_w v_{w,r} \right)_{r+\frac{\Delta r}{2}} \right] 2\pi \left(r + \frac{\Delta r}{2} \right) \Delta z \right\} \right] \Delta t \quad (\text{B.8})$$

$$\left[\begin{array}{c} \text{Net Energy In} \\ \text{by Convection} \end{array} \right] = \left[\left[\left(r \rho_w U_w v_{w,r} \right)_{r-\frac{\Delta r}{2}} \right] - \left[\left(r \rho_w U_w v_{w,r} \right)_{r+\frac{\Delta r}{2}} \right] \right] 2\pi \Delta z \Delta t \quad (\text{B.9})$$

If we write conduction term in the z -direction over Δt ,

$$\left[\begin{array}{c} \text{Net Heat Transfer} \\ \text{to Adjacent Strata} \\ \text{by Conduction} \end{array} \right] = \left(\lambda_t \frac{\partial T_2}{\partial z} \right)_{z+\frac{\Delta z}{2}} 2\pi r \Delta r \Delta t \quad (\text{B.10})$$

If we write energy accumulation term over Δt ,

$$\left[\begin{array}{c} \text{Energy} \\ \text{Accumulation} \end{array} \right] = \left[\begin{array}{c} \text{Energy In} \\ \text{over } \Delta t \end{array} \right] - \left[\begin{array}{c} \text{Energy Out} \\ \text{over } \Delta t \end{array} \right] \quad (\text{B.11})$$

$$\begin{bmatrix} \text{Energy} \\ \text{Accumulation} \end{bmatrix} = \begin{bmatrix} \left[\left(\phi \rho_w U_w + (1-\phi) \rho_s U_s \right) \right]_{t+\Delta t} \\ - \left[\left(\phi \rho_w U_w + (1-\phi) \rho_s U_s \right) \right]_t \end{bmatrix} 2\pi r \Delta r \Delta z \quad (\text{B.12})$$

Replacing Equations B.9, B.10 and B.12 into Equation B.7 yields

$$\begin{aligned} & \left[\left(\phi \rho_w U_w + (1-\phi) \rho_s U_s \right) \right]_{t+\Delta t} - \left[\left(\phi \rho_w U_w + (1-\phi) \rho_s U_s \right) \right]_t \Big] 2\pi r \Delta r \Delta z \\ &= - \left[\left[\left(r \rho_w U_w v_{w,r} \right)_{r-\frac{\Delta r}{2}} \right] - \left[\left(r \rho_w U_w v_{w,r} \right)_{r+\frac{\Delta r}{2}} \right] \right] 2\pi \Delta z \Delta t \\ &+ \left(\lambda_t \frac{\partial T_2}{\partial z} \right)_{z+\frac{\Delta z}{2}} 2\pi r \Delta r \Delta t \end{aligned} \quad (\text{B.13})$$

Dividing the above equation by $2\pi r \Delta r \Delta t$ and rearranging gives

$$\begin{aligned} & \Delta z \left(\frac{\left[\left(\phi \rho_w U_w + (1-\phi) \rho_s U_s \right) \right]_{t+\Delta t} - \left[\left(\phi \rho_w U_w + (1-\phi) \rho_s U_s \right) \right]_t}{\Delta t} \right) \\ &+ \frac{\Delta z}{r} \left(\frac{\left(r \rho_w U_w v_{w,r} \right)_{r+\frac{\Delta r}{2}} - \left(r \rho_w U_w v_{w,r} \right)_{r-\frac{\Delta r}{2}}}{\Delta r} \right) - \left(\lambda_t \frac{\partial T_2}{\partial z} \right)_{z+\frac{\Delta z}{2}} = 0 \end{aligned} \quad (\text{B.14})$$

Taking limit $\Delta r \rightarrow 0$ and $\Delta t \rightarrow 0$ produces

$$\Delta z \left(\frac{\partial \left[\left(\phi \rho_w U_w + (1-\phi) \rho_s U_s \right) \right]}{\partial t} \right) + \frac{\Delta z}{r} \left(\frac{\partial \left(r \rho_w U_w v_{w,r} \right)}{\partial r} \right) - \left(\lambda_t \frac{\partial T_2}{\partial z} \right)_{z+\frac{\Delta z}{2}} = 0 \quad (\text{B.15})$$

We know that

$$U_w = C_{p,w} T_1 \quad (\text{B.16})$$

$$U_s = C_{p,s} T_1 \quad (\text{B.17})$$

Using the above relationships in Equation B.15 gives

$$\Delta z \left(\frac{\partial \left[\left(\phi \rho_w C_{p,w} T_1 + (1-\phi) \rho_s C_{p,s} T_1 \right) \right]}{\partial t} \right) + \frac{\Delta z}{r} \left(\frac{\partial \left(r \rho_w C_{p,w} T_1 v_{w,r} \right)}{\partial r} \right) - \left(\lambda_t \frac{\partial T_2}{\partial z} \right)_{z+\frac{\Delta z}{2}} = 0 \quad (\text{B.18})$$

In radial coordinates, velocity changes in the r -direction due to varying surface area in contrast to the case in Cartesian coordinates. Hence, since flow rate, differently from velocity, can be considered as constant in the r -direction, velocity term in the above equation can be written in terms of flow rate as follows:

$$\Delta z \left(\frac{\partial \left[\left(\phi \rho_w C_{p,w} T_1 + (1-\phi) \rho_s C_{p,s} T_1 \right) \right]}{\partial t} \right) + \frac{\Delta z}{r} \left(\frac{\partial \left(r \rho_w C_{p,w} T_1 \frac{q}{A_r} \right)}{\partial r} \right) - \left(\lambda_t \frac{\partial T_2}{\partial z} \right)_{z+\frac{\Delta z}{2}} = 0 \quad (\text{B.19})$$

$$\Delta z \left(\frac{\partial \left[\left(\phi \rho_w C_{p,w} T_1 + (1-\phi) \rho_s C_{p,s} T_1 \right) \right]}{\partial t} \right) + \frac{\Delta z}{r} \left(\frac{\partial \left(r \rho_w C_{p,w} T_1 \frac{q}{2\pi r \Delta z} \right)}{\partial r} \right) - \left(\lambda_t \frac{\partial T_2}{\partial z} \right)_{z+\frac{\Delta z}{2}} = 0 \quad (\text{B.20})$$

$$\Delta z \left(\frac{\partial \left[\left(\phi \rho_w C_{p,w} T_1 + (1-\phi) \rho_s C_{p,s} T_1 \right) \right]}{\partial t} \right) + \frac{\rho_w C_{p,w} q}{2\pi r} \left(\frac{\partial T_1}{\partial r} \right) - \left(\lambda_t \frac{\partial T_2}{\partial z} \right)_{z+\frac{\Delta z}{2}} = 0 \quad (\text{B.21})$$

Rearranging reduces the above equation to the following form.

$$b \rho_1 C_1 \frac{\partial T_1}{\partial t} + \frac{\rho_w C_{p,w} q}{2\pi r} \frac{\partial T_1}{\partial r} - \lambda_2 \left(\frac{\partial T_2}{\partial z} \right)_{z=b} = 0 \quad (\text{B.22})$$

where

$$b = \Delta z \quad (\text{B.23})$$

$$\rho_1 C_1 = \phi \rho_w C_{p,w} + (1 - \phi) \rho_s C_{p,s} \quad (\text{B.24})$$

$$\lambda_2 = \lambda_t = \phi \lambda_w + (1 - \phi) \lambda_s \quad (\text{B.25})$$

Here, initial and boundary conditions are given.

- T_2 : Initial temperature of overburden and underburden at $r > 0$, $-b < z < b$
- T_1 : Reservoir temperature at any r , t
- T_{inj} : Injection temperature
- From $t = 0$ to t ,
 $T_{inj} < T_1$ at $r = 0$, $-b < y < b$ (line well in the reservoir) while injecting water of T_{inj} .
- Conduction occurs at reservoir/overburden and reservoir/underburden interfaces ($r > 0$, $z = b$ and $r > 0$, $z = -b$).

Multiplying Equation B.22 by $\frac{b}{\lambda_2}$ yields

$$\frac{b^2 \rho_1 C_1}{\lambda_2} \frac{\partial T_1}{\partial t} + \frac{b \rho_w C_{p,w} q}{2\pi r \lambda_2} \frac{\partial T_1}{\partial r} - b \left(\frac{\partial T_2}{\partial z} \right)_{z=b} = 0 \quad (\text{B.26})$$

Initial and boundary conditions for reservoir are as follows:

Initial condition:

$$t = 0 \Rightarrow T_1 = T^0 \text{ (Initial reservoir temperature)} \quad (\text{B.27})$$

Boundary conditions:

$$r = 0, t > 0 \Rightarrow T_1 = T_{inj} \text{ (Injection temperature)} \quad (\text{B.28})$$

$$z = b, t > 0 \Rightarrow T_1 = T_2 \text{ (Interface temperature)} \quad (\text{B.29})$$

If conduction equation is expressed for overburden and underburden,

$$\rho_2 C_2 \frac{\partial T_2}{\partial t} = \lambda_2 \frac{\partial^2 T_2}{\partial z^2} \quad (\text{B.30})$$

$$\frac{\rho_2 C_2}{\lambda_2} \frac{\partial T_2}{\partial t} = \frac{\partial^2 T_2}{\partial z^2} \quad (\text{B.31})$$

Initial and boundary conditions for overburden and underburden are as follows:

Initial condition:

$$t = 0 \Rightarrow T_2 = T^0 \text{ (Initial reservoir temperature)} \quad (\text{B.32})$$

Boundary conditions:

$$z \rightarrow \infty, t > 0 \Rightarrow T_2 = T^0 \text{ (Initial reservoir temperature)} \quad (\text{B.33})$$

$$z = b, t > 0 \Rightarrow T_2 = T_1 \text{ (Interface temperature)} \quad (\text{B.34})$$

Now dimensionless variables for Equations B.26 can be defined as follows:

$$\frac{b^2 \rho_1 C_1}{\lambda_2} \frac{\partial T_1}{\partial t} = \frac{\partial T_{1D}}{\partial t_D} \Rightarrow t_D = \frac{\lambda_2 t}{\rho_1 C_1 b^2} \quad (\text{B.35})$$

$$\frac{b \rho_w C_{p,w} q}{2\pi r \lambda_2} \frac{\partial T_1}{\partial r} = \frac{\partial T_{1D}}{\partial r_D} \Rightarrow r_D = \frac{2\lambda_2 \pi r^2}{\rho_w C_{p,w} q b} \quad (\text{B.36})$$

$$b \left(\frac{\partial T_2}{\partial z} \right)_{z=b} = \left(\frac{\partial T_{2D}}{\partial z_D} \right)_{z=b} \Rightarrow z_D = \frac{z}{b} \quad (\text{B.37})$$

Now a dimensionless variable for Equations B.31 can be defined as follows:

Firstly, the left hand side (LHS) of Equation B.31 is evaluated,

$$\frac{\rho_2 C_2}{\lambda_2} \frac{\partial T_2}{\partial t} = \frac{\partial T_{2D}}{\partial t_D} \Rightarrow \frac{\rho_2 C_2}{\lambda_2} \frac{\partial T_2}{\partial t} = \frac{\rho_1 C_1 b^2}{\lambda_2} \frac{\partial T_{2D}}{\partial t} \Rightarrow LHS = \frac{\rho_2 C_2}{\rho_1 C_1 b^2} \quad (\text{B.38})$$

Secondly, the right hand side (RHS) of Equation B.31 is evaluated,

$$\frac{\partial^2 T_2}{\partial z^2} = \frac{\partial^2 T_{2D}}{\partial z_D^2} \Rightarrow \frac{\partial^2 T_2}{\partial z^2} = \frac{1}{b^2} \frac{\partial^2 T_{2D}}{\partial z_D^2} \Rightarrow RHS = \frac{1}{b^2} \quad (\text{B.39})$$

Equating Equation B.38 to Equation B.39 gives

$$\frac{\rho_2 C_2}{\rho_1 C_1 b^2} = \frac{1}{b^2} \Rightarrow \Theta = \frac{\rho_1 C_1}{\rho_2 C_2} \quad (\text{B.40})$$

Finally, all dimensionless variables are as follows:

$$T_{1D} = \frac{T_1 - T^0}{T_{inj} - T^0} \quad (\text{B.41})$$

$$T_{2D} = \frac{T_2 - T^0}{T_{inj} - T^0} \quad (\text{B.42})$$

$$r_D = \frac{2\lambda_2 \pi r^2}{\rho_w C_{p,w} q b} \quad (\text{B.43})$$

$$z_D = \frac{z}{b} \quad (\text{B.44})$$

$$t_D = \frac{\lambda_2 t}{\rho_1 C_1 b^2} \quad (\text{B.45})$$

$$\Theta = \frac{\rho_1 C_1}{\rho_2 C_2} \quad (\text{B.46})$$

Using dimensionless variables above Equation B.26 becomes

$$\frac{\partial T_{1D}}{\partial t_D} + \frac{\partial T_{1D}}{\partial r_D} - \left(\frac{\partial T_{2D}}{\partial z_D} \right)_{z_D=1} = 0 \quad (\text{B.47})$$

Equation B.31 becomes

$$\frac{1}{\Theta} \frac{\partial T_{2D}}{\partial t_D} = \frac{\partial^2 T_{2D}}{\partial z_D^2} \quad (\text{B.48})$$

From this point, our new initial and boundary conditions become

Initial condition:

$$t_D = 0, T_{1D} = T_{2D} = 0 \text{ at any } r_D, z_D \quad (\text{B.49})$$

Boundary conditions:

$$r_D = 0, T_{1D} = 1 \text{ at } t_D > 0, -1 < z_D < 1 \quad (\text{B.50})$$

$$z_D = 1, T_{1D} = T_{2D} \text{ at any } r_D, t_D \quad (\text{B.51})$$

$$z_D \rightarrow \infty, T_{2D} = 0 \text{ at any } r_D, t_D \quad (\text{B.52})$$

Taking Laplace Transformations (LTs) of Equations B.47 and B.48 with respect to t_D , we can define the following LTs,

$$L[T_{1D}(r_D, t_D)] = u(r_D, \hat{p}) \quad (\text{B.53})$$

$$L[T_{2D}(z_D, t_D)] = v(z_D, \hat{p}) \quad (\text{B.54})$$

where $\hat{p}$ represents the Laplace transform variable with respect to dimensionless time t_D . Performing LT for Equation B.47,

$$\int_0^\infty e^{-\hat{p}t_D} \frac{\partial T_{1D}}{\partial t_D} dt_D + \int_0^\infty e^{-\hat{p}t_D} \frac{\partial T_{1D}}{\partial r_D} dt_D - \int_0^\infty e^{-\hat{p}t_D} \left(\frac{\partial T_{2D}}{\partial z_D} \right)_{z_D=1} dt_D \quad (\text{B.55})$$

Evaluating the first term in Equation B.55,

$$\int_0^\infty e^{-\hat{p}t_D} \frac{\partial T_{1D}}{\partial t_D} dt_D = \hat{p}u - T_{1D}(z_D, 0) \quad (\text{B.56})$$

Evaluating the second term in Equation B.55,

$$\int_0^\infty e^{-\hat{p}t_D} \frac{\partial T_{1D}}{\partial r_D} dt_D = \frac{\partial}{\partial r_D} \int_0^\infty e^{-\hat{p}t_D} T_{1D} dt_D = \frac{\partial u(r_D, \hat{p})}{\partial r_D} = \frac{du}{dr_D} \quad (\text{B.57})$$

Using Equations B.56 and B.57 in Equation B.55 gives

$$\hat{p}u - T_{1D}(z_D, 0) + \frac{du}{dr_D} - \left(\frac{dv}{dz_D} \right)_{z_D=1} = 0 \quad (\text{B.58})$$

Using Equation B.49 in Equation B.58 yields

$$\hat{p}u + \frac{du}{dr_D} - \left(\frac{dv}{dz_D} \right)_{z_D=1} = 0 \quad (\text{B.59})$$

Performing LT for Equation B.48,

$$\frac{1}{\Theta} \int_0^\infty e^{-\hat{p}t_D} \frac{\partial T_{2D}}{\partial t_D} dt_D = \int_0^\infty e^{-\hat{p}t_D} \left(\frac{\partial^2 T_{2D}}{\partial z_D^2} \right) dt_D \quad (\text{B.60})$$

Evaluating RHS of Equation B.60,

$$\int_0^\infty e^{-\hat{p}t_D} \left(\frac{\partial^2 T_{2D}}{\partial z_D^2} \right) dt_D = \frac{\partial^2}{\partial z_D^2} \int_0^\infty e^{-\hat{p}t_D} T_{1D} dt_D = \frac{\partial^2 v(z_D, \hat{p})}{\partial z_D^2} = \frac{d^2 v}{dz_D^2} \quad (\text{B.61})$$

Evaluating LHS of Equation B.60,

$$\frac{1}{\Theta} \int_0^\infty e^{-\hat{p}t_D} \frac{\partial T_{2D}}{\partial t_D} dt_D = \frac{1}{\Theta} (\hat{p}v - T_{2D}(z_D, 0)) \quad (\text{B.62})$$

Using Equations B.61 and B.62 in Equation B.60 gives

$$\frac{1}{\Theta} [\hat{p}v - T_{2D}(z_D, 0)] = \frac{d^2 v}{dz_D^2} \quad (\text{B.63})$$

Using Equation B.49 in Equation B.63 yields

$$\frac{1}{\Theta} \hat{p}v = \frac{d^2 v}{dz_D^2} \quad (\text{B.64})$$

If we apply the second LTs to Equations B.59 and B.64 with respect to r_D , respectively, we can obtain the following equations:

$$\hat{p}\bar{u} + \hat{s}\bar{u} - u(0) - \left(\frac{d\bar{v}}{dz_D} \right)_{z_D=1} = 0 \quad (\text{B.65})$$

$$\frac{1}{\Theta} \hat{p}\bar{v} = \frac{d^2\bar{v}}{dz_D^2} \quad (\text{B.66})$$

where

$$L[u(r_D, \hat{p})] = \bar{u}(\hat{s}, \hat{p}) \quad (\text{B.67})$$

$$L[v(z_D, \hat{p})] = \bar{v}(\hat{s}, \hat{p}) \quad (\text{B.68})$$

Using Equation B.50 in Equation B.65 gives

$$\hat{p}\bar{u} + \hat{s}\bar{u} - \frac{1}{\hat{p}} - \left(\frac{d\bar{v}}{dz_D} \right)_{z_D=1} = 0 \quad (\text{B.69})$$

$$(\hat{p} + \hat{s})\bar{u} - \frac{1}{\hat{p}} - \left(\frac{d\bar{v}}{dz_D} \right)_{z_D=1} = 0 \quad (\text{B.70})$$

where

$$L[T_{1D}(r_D, t_D)] = u(0, \hat{p}) = L[1] = \frac{1}{\hat{p}} \quad (\text{B.71})$$

Equation B.66 can be solved as an ordinary differential equation as follows:

$$B^2 - \frac{\hat{p}}{\Theta} = 0 \quad \Rightarrow \quad B = \mp \sqrt{\frac{\hat{p}}{\Theta}} \quad (\text{B.72})$$

From here, the following relationship can be written as

$$\bar{v} = C_1^* e^{-\sqrt{\frac{\hat{p}}{\Theta}} z_D} + C_2^* e^{\sqrt{\frac{\hat{p}}{\Theta}} z_D} \quad (\text{B.73})$$

Using Equation B.52 in Equation B.73 gives the following relationship because C_2^* must be zero to satisfy the boundary condition.

$$\bar{v} = C_1^* e^{-\sqrt{\frac{\hat{p}}{\Theta}} z_D} \quad (\text{B.74})$$

Using Equation B.51, the following equation can be written as

$$\bar{u}(\hat{s}, \hat{p}) = \bar{v}(\hat{s}, \hat{p}) \text{ at } z_D = 1 \text{ for all } \hat{s}, \hat{p} \quad (\text{B.75})$$

From here, Equation B.74 can also be expressed as

$$\bar{u} = C_1^* e^{-\sqrt{\frac{\hat{p}}{\Theta}} z_D} \quad (\text{B.76})$$

$$C_1^* = \bar{u} e^{\sqrt{\frac{\hat{p}}{\Theta}} z_D} \quad (\text{B.77})$$

Using Equation B.77 in Equation B.74 gives

$$\bar{v} = \bar{u} e^{-(z_D - 1)\sqrt{\frac{\hat{p}}{\Theta}}} \quad (\text{B.78})$$

Taking the derivative of $\bar{v}$ with respect to z_D gives

$$\frac{\partial \bar{v}}{\partial z_D} = -\bar{u} \sqrt{\frac{\hat{p}}{\Theta}} e^{-(z_D - 1)\sqrt{\frac{\hat{p}}{\Theta}}} \quad (\text{B.79})$$

$$\left(\frac{\partial \bar{v}}{\partial z_D} \right)_{z_D=1} = -\bar{u} \sqrt{\frac{\hat{p}}{\Theta}} \quad (\text{B.80})$$

Using Equation B.80 in Equation B.70 yields

$$(\hat{p} + \hat{s})\bar{u} - \frac{1}{\hat{p}} + \bar{u} \sqrt{\frac{\hat{p}}{\Theta}} = 0 \quad (\text{B.81})$$

$$\bar{u} \left(\hat{p} + \hat{s} + \sqrt{\frac{\hat{p}}{\Theta}} \right) = \frac{1}{\hat{p}} \quad (\text{B.82})$$

$$\bar{u} = \frac{1}{\hat{p} \left(\hat{p} + \hat{s} + \sqrt{\frac{\hat{p}}{\Theta}} \right)} \quad (\text{B.83})$$

Taking the first inverse LT of Equation B.83 with respect to $\hat{s}$ gives

$$\bar{u} = \frac{1}{\hat{p}} e^{\left[-\left(\hat{p} + \sqrt{\frac{\hat{p}}{\Theta}} \right) r_D \right]} \quad (\text{B.84})$$

$$\bar{u} = \frac{e^{-\hat{p}r_D} e^{-r_D \sqrt{\frac{\hat{p}}{\Theta}}}}{\hat{p}} \quad (\text{B.85})$$

Taking the second inverse LT of Equation B.85 with respect to $\hat{p}$ yields

$$T_{1D}(r_D, t_D) = \text{erfc} \left(\frac{r_D}{2\sqrt{\Theta(t_D - r_D)}} \right) \quad (\text{B.86})$$

or using the definition given by Equation B.41 for dimensionless temperature gives

$$T_1(r_D, t_D) = T^0 + (T_{inj} - T^0) \text{erfc} \left(\frac{r_D}{2\sqrt{\Theta(t_D - r_D)}} \right) \quad (\text{B.87})$$

where $t_D > r_D$ or T_1 can also be expressed as functions of r and t using the definitions given by Equations B.43-B.46 for the other dimensionless variables as follows:

$$T_1(r, t) = T^0 + (T_{inj} - T^0) \text{erfc} \left(\frac{\lambda_2 \pi r^2}{\rho_w C_{p,w} q b \sqrt{\frac{\lambda_2 t}{\rho_2 C_2 b^2} - \frac{2 \lambda_2 \pi r^2 \rho_1 C_1}{\rho_w C_{p,w} q b \rho_2 C_2}}} \right) \quad (\text{B.88})$$

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