

ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL OF SCIENCE
ENGINEERING AND TECHNOLOGY

**TYPE-2 FUZZY MODEL INVERSION METHODS
AND FUZZY MODEL BASED CONTROLLER DESIGN**

Ph.D.THESIS

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Department of Control Engineering

Control and Automation Engineering Programme

APRIL 2012

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İSTANBUL TEKNİK ÜNİVERSİTESİ ★ FEN BİLİMLERİ ENSTİTÜSÜ

**TİP-2 BULANIK MODELLERİN TERSİNİN ALINMASI VE
BULANIK MODEL TABANLI KONTROLÖR TASARIMI**

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“Pain is temporary. It may last a minute, or an hour, or a day, or a year, but eventually it will subside and something else will take its place. If I quit, however, it lasts forever. That surrender, even the smallest act of giving up, stays with me.”
Lance Armstrong

FOREWORD

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ABBREVIATIONS

BB-BC	: The Big Bang-Big Crunch
BBBC-T1FMIC	: BB-BC based Type-1 Fuzzy Model Inverse Controller
BBBC-T2FMIC	: BB-BC based Type-2 Fuzzy Model Inverse Controller
CSTR	: Continuous Stirred Tank Reactor
FLS	: Fuzzy Logic System
FLS	: Fuzzy Logic System
FOPDT	: First-Order-Plus-Time Delay Model
FOU	: Footprint of Uncertainty
GAUSS2MF	: Gaussian Combinational Membership Function
IAE	: Integral Absolute Error
IMC	: Internal Model Control
IMC-PID	: Internal Model Control based PID
ISE	: Integral Square Error
IT2-D-FMIC	: Interval Type-2 Decomposable Fuzzy Model Inverse Controller
IT2FLS	: Interval Type-2 Fuzzy Logic System
IT2FM	: Interval Type-2 Fuzzy Model
IT2-FSM	: Interval Type-2 Fuzzy Submodel
KM	: Karnik-Mendel Type Reduction
NIMC	: Nonlinear Internal Model Control
OPT-PID	: Optimization Based PID
RMSE	: Root Mean Square Error
SISO	: Single Input-Single Output
T1FLS	: Type-1 Fuzzy Logic System
T1FM	: Type-1 Fuzzy Model
T1FMIC	: Type-1 Fuzzy Model Inverse Controller
T1FS	: Type-1 Fuzzy Set
T1-TS-FM	: Type-1 Takagi-Sugeno Fuzzy Model Inverse Controller
T1-TS-FMIC	: Type-1 Takagi-Sugeno Fuzzy Model Inverse Controller
T2FLS	: Type-2 Fuzzy Logic System
T2FM	: Type-2 Fuzzy Model
T2FMIC	: Type-2 Fuzzy Model Inverse Controller
T2FS	: Type-2 Fuzzy Set
T2-TS-FM	: Type-2 Takagi-Sugeno Fuzzy Model Inverse Controller
T2-TS-FMIC	: Type-2 Takagi-Sugeno Fuzzy Model Inverse Controller
TV	: Total Variation
VAF	: Variance Accounted For

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TYPE-2 FUZZY MODEL INVERSION METHODS AND FUZZY MODEL BASED CONTROLLER DESIGN

SUMMARY

In recent years, fuzzy logic systems have been attracting research interest and are successfully implemented in control design and modelling problems. On the other hand, researchers demonstrated that there may be limitations in the ability of ordinary fuzzy logic systems to model the uncertainties and nonlinearities. It has been demonstrated that type-2 fuzzy logic systems are much more powerful tools than ordinary (type-1) fuzzy logic systems to represent highly nonlinear and/or uncertain systems. The concept of type-2 fuzzy sets is a generalization of ordinary (type-1) fuzzy sets and it was first introduced by Zadeh. As a consequence, type-2 fuzzy logic systems have been applied in various areas especially in modeling and control of complex nonlinear systems.

The use of inverse system model as a controller might be an efficient way in controlling non-linear systems. It is also a known fact that fuzzy logic modeling is a powerful tool in representing nonlinear systems. Therefore, inverse fuzzy model can be used as a controller for controlling nonlinear plants. An inverse controller based on a type-2 fuzzy model is shown to be an efficient way to control nonlinear/uncertain processes. Although, inverse fuzzy model controllers may produce perfect control in perfect model match case and/or non-existence of disturbances, this open loop control would not be sufficient in the case of modeling mismatches or disturbances. Therefore, the inverse controller and fuzzy model are embedded into a closed loop nonlinear internal model control scheme to provide an effective closed loop control performance.

Even though various fuzzy inversion methods have been devised for type-1 fuzzy logic systems up to now, there does not exist any method for type-2 fuzzy logic systems. In this thesis, three inversion methodologies will be proposed for interval type-2 fuzzy models.

In the first proposed inversion method, the inverse fuzzy control signal generation is handled as an optimization problem and solved at each sampling time in an online manner. Since an online implementation of an optimization algorithm is needed at each sampling time, the BB-BC optimization algorithm is used due to its simplicity and high convergence speed. Since the inverse signal is calculated iteratively, the proposed method does not require any invertibility conditions and is not limited to certain structures of the fuzzy model.

Secondly, an inversion method for interval type-2 Takagi-Sugeno fuzzy logic system has been proposed. In this strategy, local linearization of type-2 TS fuzzy model is performed so as to obtain a local model at a certain operating point. Then, the inverse fuzzy model signal is calculated directly using this local model via simple calculations for type-2 TS fuzzy model. This inversion strategy is based on simple

manipulation of the antecedent and consequence parts of the fuzzy model. The proposed method is limited to interval type-2 fuzzy models which are piecewise affine according to the inversion variable.

Finally, an analytical methodology has been developed to form the inverse of a certain class of interval type-2 fuzzy models with singleton consequents. The proposed inversion methodology consists of two main steps, decomposing a certain class of interval type-2 fuzzy model into submodels and then finding the inverse of each possible activated interval type-2 fuzzy submodel. In this context, the decomposition property has been extended and generalized to interval type-2 fuzzy logic sets. The analytical formulation of the interval type-2 fuzzy submodel output is tried to be reached for the inverse solution. However, the type-2 fuzzy model output cannot be presented in a closed form due to the Karnik-Mendel type reduction method. Therefore, an iterative algorithm is proposed based on an analytical methodology to form the inverse controller exactly. Finally, the inverse interval type-2 fuzzy model is obtained through inversion of each fuzzy submodel. The proposed method is limited to decomposable interval type-2 fuzzy models.

For all three type-2 fuzzy model inversion methods, simulation studies have been performed where the beneficial sides are shown clearly.

The control of pH in a continuous stirred tank reactors are complex industrial process with dominant nonlinearities. Since the pH process inherits severe nonlinearities, uncertainties and high sensitivity near the neutralization point, it is a benchmark process to evaluate control performances. Therefore, a real-time control study is performed on an experimental pH neutralization process in order to demonstrate the feasibility and effectiveness of the proposed inversion methodologies. The effectiveness of the inverse type-2 fuzzy model controller structures are tried to be shown by making comparisons both with the PID control strategies and inverse type-1 fuzzy model controller structures. Firstly, the type-1 and interval type-2 fuzzy models of the pH processes are obtained since inverse fuzzy controllers are model based design strategies. It has been illustrated that type-2 fuzzy sets are superior to type-1 fuzzy sets in the presence of uncertainties and strong nonlinearities within systems to be modeled or controlled. As a consequence, the inverse type-2 fuzzy model controllers have improved the closed-loop and disturbance rejection performances as shown through the real-time control of pH neutralization process. Experimental results demonstrate the superiority of the inverse type-2 fuzzy model controller structure compared to the inverse type-1 fuzzy model controller and conventional PID control structures.

In this Ph.D.thesis, various type-2 fuzzy model inversion methods are proposed and the proposed inverse type-2 fuzzy model control structures are proven to be an effective method in controlling processes which inherit nonlinear dynamics and uncertainties.

TİP-2 BULANIK MODELLERİN TERSİNİN ALINMASI VE BULANIK MODEL TABANLI KONTROLÖR TASARIMI

ÖZET

Bulanık sistemler, matematik modelden bağımsız tasarlanabilme özelliğinden dolayı birçok araştırmacının ilgisini çekmiştir. Özellikle kontrolör tasarımı ve doğrusal olmayan sistemleri modellenmesinde başarıyla uygulanmaktadır. Öte yandan, araştırmacılar belirsizlikler ve doğrusal olmayan özellikleri ifade etmek için sıradan (tip-1) bulanık kümelerin yeteneklerinde sınırlamalar olabileceğini gösterdi. Tip-2 bulanık küme kavramı tip-1 bulanık kümeler olarak bilinen geleneksel bulanık küme kavramının bir genişlemesi olarak Zadeh tarafından önerilmiştir. Tip-2 bulanık mantık kümelerinin en önemli özelliği belirsizlikleri üyelik fonksiyonlarında daha iyi ifade edebilme özelliğine sahip olmalarıdır ve bu özelliklerinden dolayı, modellenecek veya kontrol edilecek sistemlerde, belirsizliklerin ve doğrusal olmayan karakteristiklerin olduğu durumlarda tercih edilmektedir. Bu nedenle, tip-2 bulanık mantık sistemlerin kullanım alanı gün geçtikçe artmaktadır. Tip-2 bulanık mantık sistemin yapısı tip-1 bulanık mantık sistem yapısına çok benzerdir. Tek fark tip-2 bulanık mantık sistemlerindeki çıkış işlem bloğunda tip indirgeme mekanizmasının olmasıdır. Doğal olarak, tip-2 bulanık mantık sistemlerin çıkarım mekanizmaları tip-1 bulanık mantık sistemlerinden daha fazla karmaşık olduğundan dolayı hesaplama zamanları daha fazladır. Bu nedenle araştırmacılar, aralık değerli tip-2 bulanık sistemler olarak adlandırılan özel bir tip-2 bulanık mantık sistemlerini önerdi. Aralık değerli tip-2 bulanık sistemler, EĞER-O HALDE şeklindeki bulanık kurallardan oluşmaktadır. Tip-2 bulanık mantık sistemlerin kural yapısındaki öncül ve/veya sonuç önermeleri aralık değerli tip-2 bulanık kümeleriyle ifade edilmektedir. Sistemin çıkışını hesaplayabilmek için, ilk önce bulanıklaştırma bloğunda keskin girişler tip-2 bulanık kümelerle dönüştürülürler. Daha sonra, çıkarım mekanizması tanımlanmış kuralları kullanarak giriş değerlerini tip-2 bulanık değerlerine dönüştürür. Elde edilen tip-2 bulanık küme çıkışları, tip indirgeme mekanizması ile tip-1 bulanık kümelerle dönüştürülür. Tip azaltılması işlemi ile elde edilen kümeler durulaştırıcı mekanizması ile keskin çıkışlara dönüştürülür. Aralık değerli tip-2 bulanık mantık sistemler, özellikle doğrusal olmayan sistemlerin modellenmesi ve kontrolünde başarıyla uygulanmıştır.

Doğrusal olmayan sistemlerin modellenmesi için en etkili yöntemlerden biri bulanık modellemesidir. İdeal modellenmiş bir sistemin tersi kullanılarak yapılacak kontrolün mükemmel sonuç vereceği açıktır. Bu bağlamda bulanık model tersine dayalı doğrusal olmayan bulanık kontrolörler tasarlanabilir. Modelleme hatasının ve bozucuların olmadığı ideal durumda sürekli hal hatasız mükemmel bir kontrol sağlanır. Diğer taraftan, pratikte modelleme hatalarından ve bozuculardan kaçınmak imkânsızdır. Bu hatalarının çıkış üzerindeki etkisini ortadan kaldırmak amacıyla, bulanık modelin tersi doğrusal olmayan içsel model kontrol yapısında kullanılmaktadır. Bu yapının sağladığı en önemli avantaj ise bozuculara ve modelleme hatalarına karşı dayanıklı olmasıdır.

Tip-1 bulanık modeller için birden çok ters bulanık model tekniği önerilmiştir. Literatürde bulanık modelin tersinin bulunmasında dört ana yöntem bulunmaktadır. Önerilen ilk yöntem sadece tekli yapıdaki bulanık modellere uygulanabilmektedir. Mevcut olan ikinci yöntemde ise ayrıştırabilen bulanık modellerin tersi alınabilirken, üçüncü yöntemde ise ters bulanık model yinelemeli olarak elde edilmektedir. Dördüncü yöntemde ise, ölçülen veriden yapay sinir ağları aracılığıyla doğrudan tip-1 bulanık modelin tersinin elde edilir. Tip-1 bulanık sistemleri için çeşitli ters alma yöntemleri önerilmiş olsa da, tip-2 bulanık sistemleri için herhangi bir yöntem önerilmemiştir. Bu doktora tez çalışmasında, tip-2 bulanık modeller için üç tane ters alma yöntemi önerilmiştir. Önerilen bütün yöntemlerde Karnik-Mendel tip indirgemesi tercih edilmiştir.

Birinci ters alma yönteminde, yeni bir evrimsel arama yöntemi olan Büyük Patlama ve Büyük Çöküş (BP-BÇ) tabanlı yeni bir ters bulanık model elde etme yöntemi önerilmiştir. Kullanılan bu yeni evrimsel arama yöntemin çalışma prensibi kısaca şöyle özetlenebilir; düzenli bir durumdan kaotik bir duruma dönüşümler gerçekleştirilerek karmaşık optimizasyon problemin optimal çözümünün bulunmasıdır. BP-BÇ optimizasyon yöntemin yüksek bir yakınsama hızına ve düşük bir hesaplama zamanına sahip olduğundan bu evrimsel algoritmanın çevrimiçi gerçeklemesi mümkündür. Büyük Patlama Büyük Çöküş optimizasyon yöntemi kullanılarak referans işareti ile bulanık model çıkışı arasındaki hatayı minimize edecek kontrol işareti her örnekleme zamanında üretilebilir. Anlatılan bu yapı, ters bulanık model kontrolörün üreteceği işaretleri taklit etmektedir. Ters bulanık model kontrol işareti üretimi bir optimizasyon problemi olarak ele alındığından dolayı herhangi bir koşul gerektirmez ve sadece bazı bulanık model yapıları ile sınırlı değildir. Ancak, önerilen yöntemi Tek girişli-Tek çıkışlı (TGTC) süreçler ile sınırlıdır.

Önerilen ikinci ters alma metodu ise, aralık değerli tip-2 Takagi-Sugeno bulanık mantık sistemler için geliştirilmiştir. Bu yöntem, Babuska tarafından önerilen tip-1 Takagi-Sugeno bulanık model ters alma yönteminin tip-2 bulanık mantık sistemlere genelleştirilmesidir. İlk önce, her örnekleme zamanında elle alınan aralık değerli tip-2 bulanık model anlık olarak doğrusallaştırılır. Daha sonra, elde edilen anlık bulanık modelin öncül ve sonuç kısımlarında yapılan basit matematik işlemler ile ters bulanık model oluşturulmaktadır. Bu yöntem ile sadece Takagi-Sugeno yapısındaki tip-2 bulanık modellerin tersi birebir bulanabilir. Önerilen yöntem TGTC süreçler ile sınırlıdır.

Önerilen üçüncü ters alma yönteminde ise, ters tip-2 bulanık model alma işlemi sadece belirli yapıdaki aralık değerli tip-2 bulanık modeller için gerçekleştirilmektedir. Elle alınan tip-2 bulanık model, ayrıştırabilen tekli yapıda olması durumunda, tip-2 bulanık modelin tersi matematiksel olarak birebir bulanabilir. Bu ters alma yöntemi iki temel adımdan oluşmaktadır. Önce, sistemin bulanık modeli bulanık alt modellere ayrıştırılır. Bu bağlamda, tip-1 bulanık mantık kümeleri için önerilmiş olan ayrışma özelliği tip-2 bulanık mantık kümeleri için genişletilmiş ve genelleştirilmiştir. Daha sonra, tip-2 bulanık model ters alma işlemi gerçekleştirmek için sadece ateşlenen bulanık alt modellerin tersi alınır. Bu amaçla, tip-2 bulanık alt modelin çıkışının analitik formülasyonu elde edilmeye çalışılmıştır. Ancak, tip-2 bulanık alt model çıkışı Karnik-Mendel tip indirgeme yöntemi tercih edildiğinden dolayı kapalı formda ifade edilememektedir. Bu nedenle, analitik bir yöntem dayalı ancak yinelemeli bir tip-2 bulanık model ters alma metodu önerilmiştir. Ateşlenen her tip-2 bulanık alt modelin tersinin alınmasıyla ters tip-2 bulanık model elde edilir. Tip-2 bulanık model ters alma işlemi sonucunda 3 tane

durumla karşılanabilir. Bunlar çözümün olmaması, tek bir veya birden çok çözümün olması durumlarıdır. Eğer tip-2 bulanık model tersi alınabilir ise bir çözümün varlığından kesinlikle söz edebiliriz. Birden çok çözümün olması durumunda, ele aldığımız sistemin fiziksel kısıtlamalarında göz önüne alarak uygun çözüm seçilmelidir. Göz önüne alınabilecek ölçütler arasında minimum ya da maksimum enerji de bulunmaktadır. Dikkat edilmesi gereken bir diğer husus ise, önerilen yönteminin TGTÇ süreçleri ile sınırlı olmasıdır.

Her üç tip-2 bulanık mantık sistem ters alma yöntemleri için benzetim çalışmaları yapılmış ve yüksek başarımlar elde edilmiştir.

Önerilen ters tip-2 bulanık model kontrolör yapıların uygulanabilirliğini göstermek ve başarımını incelemek için gerçek zamanda bir pH süreç sistemi üzerinde uygulanmıştır. pH nötralizasyon süreci iyon derişimlerinden kaynaklanan güçlü doğrusal olmayan dinamikler içermekte ve de nötr noktasında yüksek duyarlılık göstermektedir. Bu özelliklerinden dolayı önerilen yöntemler literatürde mevcut olan ters tip-1 bulanık kontrolör ve de klasik PID kontrolör yapılarıyla karşılaştırılmıştır. Ters bulanık model kontrol tekniklerin uygulayabilmek için ilk önce pH sürecinin tip-1 ve tip-2 bulanık modelleri elde edilmiştir. Bu amaçla, bir sistem tanıma deneyi yapılmıştır. Elde edilen bulanık modeller incelendiğinde, sürecin doğrusal olmayan dinamikler en iyi ifade eden modelin tip-2 bulanık model olduğu görülmüştür. Gerçek zamanlı kontrol uygulamalarında, pH süreci nötr (7pH) noktasında yüksek duyarlılık gösterdiğinden dolayı, uygulanan kontrol yapıları bu nokta etrafında karşılaştırılmıştır. Bu amaçla, referans değerleri sırasıyla 6, 8 ve 7 pH olarak seçilmiştir. Uygulanan kontrol yapılarının gürbüzlüğü test etmek amacıyla, 7 pH çalışma noktasında bozucu performansları incelenmiştir. Ayrıca, tip-2 bulanık modelleri en büyük dezavantajı tip azaltma/durulaştırma çıkarım mekanizması olduğundan dolayı tip-1 ve tip-2 ters bulanık model kontrol yapıların hesaplama zamanları incelenmiştir. Söz konusu sürece, tip-1 ve önerilen tip-2 ters bulanık model kontrol yapıları uygulandığında, en iyi geçici hal cevabını veren yöntemlerin ters tip-2 bulanık model kontrol yapıları olduğu gözlenmiştir. Üstelik önerilen kontrol yapıları, sistemi diğer kontrol yapılarına kıyasla, daha az enerji uygulayarak, daha kısa bir sürede referans işaretine yakınsatmıştır. Ayrıca, önerilen yeni ters tip-2 bulanık model kontrol yapısının çıkış bozucularının kötü etkilerini diğer kontrolör yapılarına nazaran daha kısa zamanda ortadan kaldırdığı gözlenmiştir. Deneysel sonuçlar, ters tip-1 bulanık model kontrolör ve klasik PID kontrol yapıları ile karşılaştırıldığında ters tip-2 bulanık model kontrolör yapıların üstünlüğünü göstermektedir.

Bu tez çalışmasında önerilen ters tip-2 bulanık model kontrol yapılarının, özellikle doğrusal olmayan dinamikleri olan süreçlerin kontrol edilmesinde etkili bir yöntem olduğu anlaşılmıştır.

1. INTRODUCTION

Fuzzy logic systems (FLSs), which were introduced by Zadeh (1975), are well known for their ability to model linguistics and system uncertainties. It is a very well known fact that fuzzy models can smoothly integrate a priori knowledge with information obtained from process data (Babuska, 1998; Abonyi 2003). Therefore, Ordinary fuzzy logic systems are powerful tools in representing nonlinear and/or uncertain systems. The nonlinearities/ uncertainties of the process can be modelled using fuzzy membership functions (Abonyi 2003; Celikyilmaz and Türksen 2009). Due to this ability, fuzzy systems are very effective in modelling and control of complex nonlinear systems whose complete mathematical models are not available. Since most of the systems and processes are characterized by uncertainties and nonlinearities, various fuzzy modeling and control strategies have been successfully implemented in many problems (Eksin et al. 2001, Babuska et al.,2002; Guzelkaya et al. 2003; Precup et. al, 2004; Fuente et al. 2006; Genc et al. 2009; Ahn et. al, 2009; Chen et. al, 2009; Karasakal et. al, 2011).

However, researchers demonstrated that there might be limitations in the ability of ordinary fuzzy logic systems in the ability of to model the uncertainties and nonlinearities (Hagras, 2004). This limitation is mainly caused because of the fact that, the membership grade for each input value is a crisp value. In recent years, it has been shown that type-2 fuzzy sets are more suitable in circumstances where it is difficult to determine the accurate membership function for a fuzzy set (Karnik et al., 1999). The concept of type-2 fuzzy sets is a generalization of ordinary (type-1) fuzzy sets and was first introduced by Zadeh (1975). Mendel, (2000) stated that fuzzy logic systems that are described with at least one type-2 fuzzy set are named as type-2 fuzzy logic systems. The major contribution of type-2 fuzzy logic systems is the footprint of uncertainty that models the uncertainties/nonlinearities in a secondary membership function. In (Liang et al., 2000 and Coupland et al., 2007), it has been illustrated that type-2 fuzzy sets are much more powerful to handle uncertainties and nonlinearities compared to type-1 fuzzy logic systems. On the other hand, the major

problem with type-2 fuzzy logic system is the computational complexity in the type reduction operations (Liang and Mendel, 2000; Wu and Mendel, 2002; Mendel et al., 2006). Therefore, to overcome this problem, Liang et al. (2000) and Mendel (2000) proposed a special type of type-2 fuzzy sets called interval type-2 fuzzy sets (IT2-FSs).

In IT2-FSs, a type-reduction mechanism is used to map the interval type-2 fuzzy set into a type-1 fuzzy set and afterwards routine defuzzification procedure is accomplished to obtain a crisp quantity (Liang and Mendel, 2000; Mendel, 2000; Wu and Mendel, 2011). However, researchers still concern the computational cost of calculating the output of interval type-2 fuzzy systems. Here, the type reduction mechanism is the main cause of the complexity issue. The most commonly used algorithm for type reduction is called Karnik-Mendel (KM) and it is firstly presented in (Wu and Mendel, 2007). In this method, the type reduced set is formed after determination of the optimal switching points. However, since these important switching points cannot be predetermined as explicit functions of the inputs the KM algorithm has to calculate the type reduced set of the type-2 fuzzy system iteratively (Liang and Mendel, 2000; Wu and Mendel, 2007). Consequently, the computation of the type reduced set becomes a bottleneck for interval type-2 fuzzy logic systems. Therefore, several approximations of the KM algorithm have been proposed (Begian et al., 2008; Du and Ying, 2010; Nie and Tan, 2010; Wu and Tan, 2005; Wu and Mendel, 2002). Nevertheless, it has been shown that the alternative type reduction methods cannot capture the features of the KM algorithm (Wu, 2011).

Interval type-2 fuzzy logic systems have been successfully used in controller design (Kheireddine et al., 2007, Lin et al, 2009). In (Pan et al., 2011), a H_∞ tracking-based direct adaptive interval type-2 fuzzy controller for a class of perturbed uncertain affine nonlinear systems has been proposed. In addition, Al-khazraji et al., (2011) introduced a type-2 fuzzy sliding mode control structure for nonlinear systems. Furthermore, Lin et al, (2010) introduced a state observer based indirect adaptive interval type-2 fuzzy controller. In addition to these approaches, Liao et al., (2010) suggested type-2 fuzzy model based model predictive control structures. Moreover, in order to show the feasibility of type-2 fuzzy logic systems, real-time control applications such as liquid-level process control (Wu and Tan, 2006); autonomous mobile robots (Martinez et al., 2009); plants control (Castillo et al., 2005); bioreactor

control (Galluzzo et al., 2008); pH control (Kumbasar et al., 2011a; Kumbasar et al., 2011b) have been handled and implemented successfully.

In the fuzzy logic literature, various type-1 fuzzy model inversion methods have been proposed. Babuska (1998) stated that the exact inverse of a fuzzy model can be attained for a certain class of type-1 fuzzy models. It has been shown that, the type-1 inversion is only valid when process is modeled as singleton fuzzy type where the input domain is defined with %50 overlapped triangular membership functions. However, it is required that the fuzzy model must satisfy some invertibility conditions. When these conditions are not satisfied the fuzzy model must be decomposed into invertible parts (Babuska, 1998). Moreover, Boukezzoula et al. (2007) proposed an inversion method in which the fuzzy model is decomposed into fuzzy meshes and the inverse of the global fuzzy system is obtained through inversion of each fuzzy mesh. The inverse fuzzy model has been obtained through the principle of rule-by-rule inversion. In (Abonyi et al., 1999), the inverse model is obtained directly through mapping the output and input data of the process via fuzzy logic approach. Furthermore, iterative fuzzy model inversion methods have also been proposed by researchers which do not require any invertibility conditions to be satisfied (Kumbasar, et al., 2008a; Kumbasar, et al. 2011c).

In this thesis, three inversion methodologies are proposed for interval type-2 fuzzy models. The Karnik - Mendel type reduction method is preferred for all three inversion methods due to the inconsistency and the adaptiveness (novelty) features in achieving such a goal. On the other hand, it is very well-known fact that Karnik - Mendel type-reduction method increases the computational complexity. In the first proposed method, the inverse fuzzy control signal generation is handled as an optimization problem and the problem is tried to be solved within each sampling period of time in an online manner (Kumbasar, et al. 2011a; Kumbasar, et al. 2011d). Secondly, an inversion method for interval type-2 Takagi-Sugeno fuzzy logic system has been proposed in which the calculation of inverse model is done based on simple manipulations of the antecedent and consequence parts of the fuzzy model (Kumbasar, et al. 2011b). Finally, an analytical methodology has been developed to form the inverse of a certain class of interval type-2 fuzzy models with singleton consequents (Kumbasar, et al. 2011e; Kumbasar, et al. 2011f).

In the first proposed interval type-2 fuzzy model inversion, the output of the inverse type-2 fuzzy model signal is generated as a solution of an optimization problem (Kumbasar, et al. 2011a; Kumbasar, et al. 2011d). The design methodology introduced here is based on a recursive optimization procedure that searches for an optimal inverse model signal at every sampling time. Since the task of optimization should be accomplished in between two sampling periods the use of a fast optimization algorithm becomes essential (Kumbasar, et al. 2011a; Kumbasar, et al. 2011d). Therefore, a new evolutionary computation algorithm named as Big Bang Big Crunch (BB-BC) proposed by Erol and Eksin (2006) is used for that purpose. The leading advantage of BB-BC is the high convergence speed and the low computation time. The working principle of this method can be explained as the transformation of a convergent solution to a chaotic state and then back to a single tentative solution point. Recently, BB-BC algorithm has found applications in many areas where the computational time and convergence time is important. Especially where the optimization problem must be solved in relatively small sampling times such as in fuzzy model inversion (Kumbasar, et al. 2008a), fuzzy model adaptation (Kumbasar, et al. 2008b) the efficiency of this evolutionary algorithm has been demonstrated. Moreover, in the optimization of highly nonlinear engineering problems such as controller design (Dogan and Istefanopulos, 2007; İplikci, 2010), design of space trusses (Camp, 2007), and size reduction of space trusses (Kaveh and Talatahari, 2009), this algorithm has been preferred because of its high convergence speed. In that respect, the BB-BC optimization algorithm is both appropriate and feasible for online implementations. In this study, the BB-BC optimization algorithm is used to generate the optimal control signal so as to minimize the error between the reference input signal and the output of the model of the system within every sampling period of time. Since the type-2 fuzzy model inverse signal obtained as a result of an optimization problem, the presented methodology does not require any invertibility conditions or any limitations on the form of the type-2 fuzzy model (Kumbasar, et al. 2011a).

The second proposed inversion method is for interval type-2 Takagi-Sugeno fuzzy models (Kumbasar, et al. 2011b). In this strategy, local linearization of type-2 TS fuzzy model is performed so as to obtain a local model at a certain operating point. Then, the inverse type-2 interval TS fuzzy model has been obtained through the

principle of permutation of the antecedent and consequence parts. The calculations are simple and straightforward. The proposed methodology is a generalization of the type-1 TS fuzzy model inversion proposed by Babuska (1998). It is important to note here that the developed inversion strategy is based on a pure analytic method. However, the proposed method is still restricted to SISO processes (Kumbasar, et al. 2011b).

Thirdly, an exact inversion method for a certain class of interval type-2 fuzzy models has been proposed (Kumbasar, et al. 2011e; Kumbasar, et al. 2011f; Kumbasar, et al. 2011g). The proposed inversion methodology consists of two main steps, decomposing a certain class of interval type-2 fuzzy model into submodels and then finding the inverse of each possible activated interval type-2 fuzzy submodel. At first, the decomposition property for type-1 fuzzy logic systems has been extended and generalized to interval type-2 fuzzy logic systems (Kumbasar, et al. 2011f). According to this property, the input space is divided into several appropriate subspaces and a type-2 fuzzy logic system is consequently decomposed into interval type-2 fuzzy subsystems. Secondly, the analytical formulation of the type-2 fuzzy subsystem output is directly used to set up a routine to find the inverse solution. However, the type reduced set of the type-2 fuzzy subsystem output cannot be presented in a closed form, since the Karnik-Mendel type reduction is preferred. To overcome this bottleneck, the switching points of the type-reduced set are initialized and then the analytical formulation of the inverse interval type-2 fuzzy subsystem output is explicitly driven. After this initial setting, the analytical formulation of the inverse interval type-2 fuzzy logic subsystem output is explicitly driven and the inverse solutions are calculated (Kumbasar, et al. 2011g). Then, these initialized switching points are checked whether they are the optimal ones or not for the calculated inverse solutions. Finally, the inverse interval type-2 fuzzy model is obtained through inversion of each fuzzy submodel. The developed inversion strategy determines either an exact unique solution or exact multiple solutions if there exist. It is important to note here that the developed inversion strategy is based on a pure analytical method (Kumbasar, et al. 2011e; Kumbasar, et al. 2011f). As a consequence, the exactness of the obtained solutions is guaranteed. However, the proposed method is still restricted to SISO processes.

Internal Model Control (IMC) is one of the mostly used control strategy in process control area because of simplicity in design and disturbance rejection properties (Garcia et al., 1982). Due to this design simplicity and effectiveness of the IMC, various strategies have been developed to design PID controllers based on this principle (Rivera et al., 1986; Morari et al., 1989; Horn et al., 1996; Lee et al., 1998; Skogestad et al., 2003). Moreover, the internal model control principle has been extended to design other approaches for nonlinear systems (Economou et al., 1986). In this context, several Nonlinear Internal Model Control (NIMC) schemes have been proposed since it is robust against disturbances and model mismatches (Economou et al., 1986; Brown et al. 1997; Narayanan et al., 1997; Rivals and Personnaz, 2000). In the NIMC scheme, the most difficult task is to find a process model which will represent the nonlinear and/or uncertain systems especially for practical chemical processes. Economou et al., (1986) have demonstrated that the main controller can directly be obtained by model inversion in their scheme. However, even if the model matches the process perfectly, the inverse of the nonlinear model may not be obtained analytically. The inversion of nonlinear models is generally obtained by numerical methods in spite of fact that sufficient and necessary conditions are not guaranteed (Economou et al., 1986). In order to overcome this problem, researchers proposed NIMC structures where the main controller design is based on type-1 fuzzy model inversion (Boukezzoula et al., 2003; Kumbasar et al., 2008a; Oblak et al., 2010; Kumbasar et al., 2011a).

In this study, the proposed inverse interval type-2 fuzzy models are embedded into the nonlinear internal model control structures as the main controllers (Kumbasar et al., 2011a; Kumbasar et al., 2011b; Kumbasar et al., 2011e). Since the inverse interval type-2 fuzzy model is calculated exactly, an offset-free control performance is guaranteed. Moreover, the model mismatches and disturbances will be compensated due to the nonlinear internal model control structure. The effectiveness of the proposed control structures are shown on simulation studies and the closed loop control performances are compared with a type-1 fuzzy model inverse controller structures (Kumbasar et al., 2011a; Kumbasar et al., 2011b; Kumbasar et al., 2011e).

Moreover, in order to show the feasibility of the iterative type-2 fuzzy model and interval type-2 TS fuzzy model inversion methods, a real-time control study has been performed (Kumbasar et al., 2011a; Kumbasar et al., 2011b). The effectiveness of the

proposed methods is demonstrated on a pH neutralization experimental set-up which exhibits a highly nonlinear characteristic (Wright and Kravaris, 1991). The control performances are compared with a type-1 fuzzy model inverse controllers and conventional PID control structures. The servo and disturbance rejection performances of the structures have been compared based on defined performance measures.

This paper is organized in 7 sections excluding the conclusion section. In section 2, the interval type-2 fuzzy model structure is briefly explained. In section 3, the general fuzzy model based inverse controller design strategy is explained. In section 4, the proposed iterative type-2 fuzzy model inversion strategy is explained in detail and a simulation study is performed to show control performance. In section 5, an existing fuzzy model inversion method for type-1 TS fuzzy logic systems is generalized to type-2 fuzzy logic systems and the newly obtained interval type-2 fuzzy model based fuzzy inverse controller is discussed and a simulation study is performed to show control performance. In Section 6, the proposed decomposable interval type-2 fuzzy model inversion and inverse controller design methods are explained in detail. In section 7, a real time control study is performed on a pH neutralization process (G.U.N.T pH value Control Trainer RT-552). Within this section, the type-1 and type-2 fuzzy modeling approaches are given. In addition, the performances of the fuzzy models are compared and discussed. Then, the proposed iterative type-2 fuzzy model control and interval type-2 Takagi-Sugeno fuzzy model control structures are implemented and tested on the pH neutralization process. The proposed inverse type-2 controllers are compared with type-1 fuzzy model control and conventional control structures. The servo and disturbance rejection performances of the inverse fuzzy controllers and conventional PID controllers are compared with each other based on defined performance measures within this section. Finally, the results are evaluated and discussed in the conclusion section.

2. INTERVAL TYPE-2 FUZZY LOGIC SETS AND SYSTEMS

2.1 Interval Type-2 Fuzzy Logic Sets

Type-2 fuzzy sets are generalized forms of those of type-1 fuzzy sets. Mathematically, type-2 fuzzy sets are not easy to describe like type-1 fuzzy sets. A type-2 fuzzy set, denoted as $\tilde{A}$, is characterized by a type-2 membership function $\mu_{\tilde{A}}(x, u)$, where $x \in X$ and $u \in J_x \subseteq [0, 1]$, i.e.:

$$\tilde{A} = \left\{ ((x, u), \mu_{\tilde{A}}(x, u)) \mid \forall x \in X, \forall u \in J_x \subseteq [0, 1] \right\} \quad (2.1)$$

in which $0 \leq \mu_{\tilde{A}}(x, u) \leq 1$. For a continuous universe of discourse, $\tilde{A}$ can be expressed as

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x, u) / (x, u), J_x \subseteq [0, 1] \quad (2.2)$$

where $\int$ denotes union over all admissible x and u . J_x is referred to as the primary membership of x , and $\mu_{\tilde{A}}(x, u)$ is a type-1 fuzzy set known as the secondary set. The uncertainty in the primary membership of a type-2 fuzzy set $\tilde{A}$ is defined by a region named footprint of uncertainty (FOU) which is also the union of all primary memberships. When $\mu_{\tilde{A}}(x, u) = 1$ for $\forall u \in J_x \subseteq [0, 1]$, an interval type-2 fuzzy set is obtained (IT2 FS). Examples of trapezoidal and gaussian type-2 fuzzy sets are given in Figure 2.1a and Figure 2.1b, respectively. It can be described in terms of an upper membership function $\bar{\mu}_{\tilde{A}}(x)$ and a lower membership function $\underline{\mu}_{\tilde{A}}(x)$. The primary membership J_x is also given in Figure 2.1a and Figure 2.1b, and its associated possible secondary membership functions can be trapezoidal, interval, etc. When the interval secondary membership function that is illustrated in Figure 2.1c is taken, an interval type-2 fuzzy set is obtained (Mendel, 2000, Mendel et al., 2006).

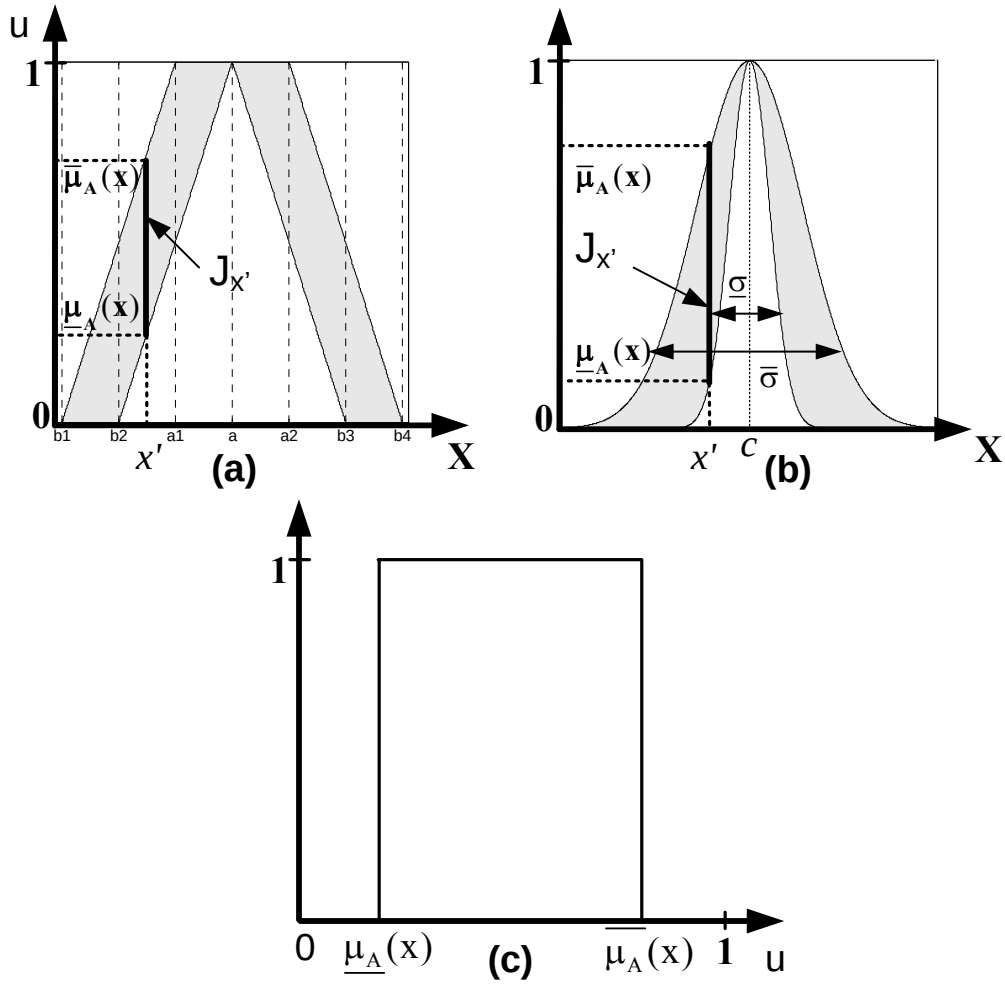


Figure 2.1 : Illustration of (a) a type-2 fuzzy trapezoidal membership function (b) a type-2 fuzzy gaussian membership function (c) an interval secondary membership function.

2.2 Interval Type-2 Fuzzy Logic Systems

A fuzzy logic system described with at least one type-2 fuzzy set is called a type-2 fuzzy logic system (T2FLS). A block diagram of a T2FLS, that is a special fuzzy logic system, is given in Figure 2.2. Similar to a type-1 FLS, a type-2 FLS includes type-2 fuzzifier, rule-base, inference engine, and substitutes the defuzzifier by the output processor.

A T2FLS is characterized by IF-THEN rules, but its antecedent and/or consequent fuzzy sets are defined by type-2 fuzzy sets. The type-2 fuzzy set outputs are then processed by the type-reducer which combines the output sets which leads to type-1 fuzzy sets called the type-reduced set. The defuzzifier can then defuzzify the type-

reduced type-1 fuzzy outputs to produce crisp outputs (Mendel, 2000, Mendel et al., 2006).

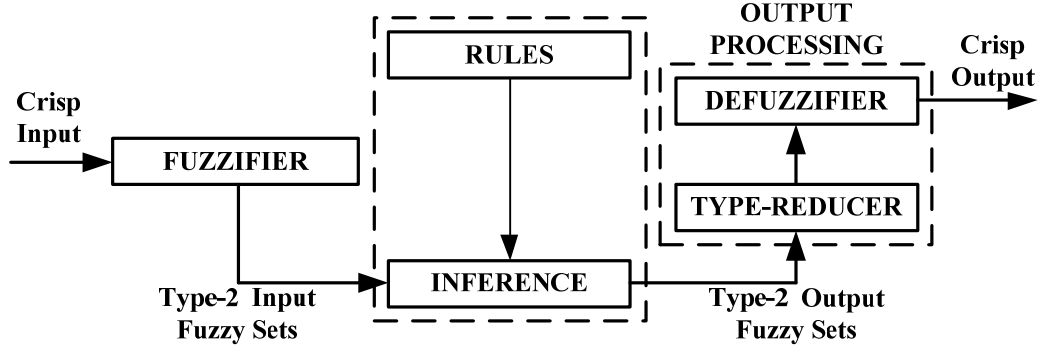


Figure 2.2 : Block diagram of type-2 fuzzy logic system.

According to the rule-base formation, there are three possible structures for building such FLCs as classified in Table 2.1. The most general case is called Model I and this structure has type-2 fuzzy sets in antecedents, and type-1 fuzzy sets in consequents. There are two special cases of Model I: Model II with crisp numbers (singletons) in the consequents; Model III type-1 fuzzy sets in antecedents and in consequents (Mendel, 2000).

Table 2.1: Classification of Type-2 FLCs.

Consequent MF	Antecedent MF	
	Type-1 Fuzzy Set	Type-2 Fuzzy Set
Type-0 Fuzzy Set (Crisp)	Type-1 FLC	MODEL II
Type-1 Fuzzy Set (Interval)	MODEL III	MODEL I

A type-2 fuzzy rule can be defined as following:

$$r_j : \text{IF } x_1 \text{ is } \tilde{A}_1 \text{ and } x_2 \text{ is } \tilde{A}_2 \text{ and } \dots \text{ and } x_n \text{ is } \tilde{A}_n \text{ THEN } y \text{ is } C^j, \quad j = 1, 2, \dots, N \quad (2.3)$$

where $\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n$ are type-2 fuzzy sets on the universe of discourses $x_1, x_2, \dots, x_n$, while C^j is the consequent interval sets of the j^{th} IF-THEN rule, N is the total number of rules.

$$\begin{aligned} \tilde{\mu}_1 &= [\underline{\mu}_1, \bar{\mu}_1] \\ \tilde{\mu}_2 &= [\underline{\mu}_2, \bar{\mu}_2] \\ &\vdots \\ \tilde{\mu}_n &= [\underline{\mu}_n, \bar{\mu}_n] \end{aligned} \quad (2.4)$$

where $\underline{\mu}_j$ and $\bar{\mu}_j$ are the firing sets of the lower membership function and upper membership function, respectively. The firing sets inherit the uncertainties of the antecedents and affect the interval consequent sets. The total firing set for each rule is defined as:

$$f^j = \begin{bmatrix} \underline{f}^j & \bar{f}^j \end{bmatrix}$$

where

$$\begin{aligned} \underline{f}^j &= \underline{\mu}_1(x_1) \cap \underline{\mu}_2(x_2) \cap \dots \cap \underline{\mu}_n(x_n) \\ \bar{f}^j &= \bar{\mu}_1(x_1) \cap \bar{\mu}_2(x_2) \cap \dots \cap \bar{\mu}_n(x_n) \end{aligned} \quad (2.5)$$

The consequent C^j of the rule r_j is also an interval set, defined as:

$$C^j = [c_l^j, c_r^j] \quad (2.6)$$

$y^* = [y_l, y_r]$ is the crisp output of type-2 fuzzy logic system (T2FLS). It inherits the uncertainty of the output of a type-2 FLS due to antecedent or consequent parameter uncertainties. y_l and y_r can be calculated using a type reduction method (Mendel, 2000, Mendel et al., 2006). Then, the type reduced fuzzy set for an interval type-2 FLS can then be expressed as:

$$Y_{TR} = [y_l(x), y_r(x)] \quad (2.7)$$

where Y_{TR} is an interval type-1 fuzzy set determined by its two end points $y_l(x)$ and $y_r(x)$. The output of the T2FLS can be obtained by using the average value of $y_l(x)$ and $y_r(x)$. So the crisp output of T2FLS is calculated using

$$y^*(x) = \frac{y_r(x) + y_l(x)}{2} \quad (2.8)$$

2.3 Type Reduction Methods

In this subsection, the proposed type-reduction methods for IT2-FLS will be explained. The most commonly used type reduction and defuzzification method is the iterative Karnik and Mendel algorithm (Karnik and Mendel, 1999). Moreover, five approximations to the Karnik-Mendel method have proposed in order to reduce the computational complexity and to represent the output in a closed loop form. Wu

(2011) pointed out two features of the Karnik-Mendel type reduction method; namely, inconsistency and adaptiveness. Inconsistency means that the upper and lower membership functions of the same type-2 fuzzy set may or may not be used simultaneously in computing the type-reduced set. The adaptiveness has the meaning that the bounds of the type-reduced interval set change as the inputs change. It has been stated that the alternative five approximations (Begian et al., 2008; Du and Ying, 2010; Nie and Tan, 2010; Wu and Tan, 2005; Wu and Mendel, 2002) to the KM algorithms cannot simultaneously capture inconsistency and adaptiveness features (Wu, 2011). These two features provide the model to handle the uncertainties that may appear within systems much better. For this reason, the Karnik-Mendel type reduction method is preferred in this thesis.

2.3.1 Karnik- Mendel type reduction method

To be able to calculate the two end points of the type reduced set (y_l and y_r), first c_r^j and c_l^j are re-ordered such that $c_l^1 \leq c_l^2 \leq \dots \leq c_l^N$, $c_r^1 \leq c_r^2 \leq \dots \leq c_r^N$ and the corresponding weights $\bar{f}$ and $\underline{f}$ are matched, respectively.

Let us first define y_l and y_r in a compact form:

$$y_l = \min_{\forall \theta^j \in [\bar{f}^j, \underline{f}^j]} \left\{ \frac{\sum_{j=1}^N \theta^j C^j}{\sum_{i=1}^N \theta^j} \right\} \quad (2.9)$$

$$y_r = \max_{\forall \theta^j \in [\bar{f}^j, \underline{f}^j]} \left\{ \frac{\sum_{j=1}^N \theta^j C^j}{\sum_{i=1}^N \theta^j} \right\} \quad (2.10)$$

It should be noted that $\forall \theta^j \in [0, 1]$, $C^j = [c_l^j, c_r^j]$ and $\theta^j = [\underline{f}^j, \bar{f}^j]$ in (2.9) and (2.10).

Let us define

$$y(\theta^1, \dots, \theta^N) = \frac{\sum_{j=1}^N \theta^j C^j}{\sum_{i=1}^N \theta^j} \quad (2.11)$$

and differentiating y with respect to θ^k , we find that

$$\frac{\partial y(\theta^1, \dots, \theta^N)}{\partial \theta^K} = \frac{C^k - y(\theta^1, \dots, \theta^N)}{\sum_{i=1}^N \theta^i} \quad (2.12)$$

From (2.12) it can be observed that

$$\frac{\partial y(\theta^1, \dots, \theta^N)}{\partial \theta^K} \begin{cases} \geq 0 & \text{if } C^k \geq y(\theta^1, \dots, \theta^N) \\ < 0 & \text{if } C^k < y(\theta^1, \dots, \theta^N) \end{cases} \quad (2.13)$$

This gives the direction in which θ^k should be changed to reach the minimum or the maximum; that is, it provides an information on what to do on the value of $y(\theta^1, \dots, \theta^N)$ (decrease or increase). Thus, from (2.13), the following meta-rules can be extracted:

- (i) if $C^k \geq y(\theta^1, \dots, \theta^N)$ $y(\theta^1, \dots, \theta^N)$ increases(decreases) as θ^K increases(decreases)
- (ii) if $C^k < y(\theta^1, \dots, \theta^N)$ $y(\theta^1, \dots, \theta^N)$ increases(decreases) as θ^K decreases(increases)

From (i), to calculate possible minimum value of $y(\theta^1, \dots, \theta^N)$

$$\begin{cases} \text{if } C^k \geq y(\theta^1, \dots, \theta^N) & \theta^K = \bar{f} \\ \text{if } C^k < y(\theta^1, \dots, \theta^N) & \theta^K = \underline{f} \end{cases} \Rightarrow \theta^j = \begin{cases} \bar{f}^j & i \leq K \\ \underline{f}^j & i > K+1 \end{cases} \quad (2.14)$$

From (ii), to calculate possible maximum value of $y(\theta^1, \dots, \theta^N)$

$$\begin{cases} \text{if } C^k \leq y(\theta^1, \dots, \theta^N) & \theta^K = \underline{f} \\ \text{if } C^k > y(\theta^1, \dots, \theta^N) & \theta^K = \bar{f} \end{cases} \Rightarrow \theta^j = \begin{cases} \underline{f}^j & i \leq K \\ \bar{f}^j & i > K+1 \end{cases} \quad (2.15)$$

Defining $y_r^{(R)}$ and $y_l^{(L)}$, for $1 \leq L, R \leq N-1$, as:

$$y_l^{(L)} = \frac{\sum_{j=1}^L \bar{f}^j c_l^j + \sum_{j=L+1}^N \underline{f}^j c_l^j}{\sum_{j=1}^L \bar{f}^j + \sum_{j=L+1}^N \underline{f}^j} \quad (2.16)$$

$$y_r^{(R)} = \frac{\sum_{j=1}^R \underline{f}^j c_r^j + \sum_{j=R+1}^N \bar{f}^j c_r^j}{\sum_{j=1}^R \underline{f}^j + \sum_{j=R+1}^N \bar{f}^j} \quad (2.17)$$

Then $y_l(x)$ is the minimum of all $y_l^{(L)}(x)$, and $y_r(x)$ is the maximum of all $y_r^{(R)}(x)$, i.e.

$$\begin{aligned}
 y_l(x) &= \min_{1 \leq L \leq N-1} \{ y_l^{(L)}(x) \} = y_l^{(L^*(x))}(x) \\
 &= \frac{\sum_{j=1}^{L^*(x)} \bar{f}^j c_l^j + \sum_{j=L^*(x)+1}^N \underline{f}^j c_l^j}{\sum_{j=1}^{L^*(x)} \bar{f}^j + \sum_{j=L^*(x)+1}^N \underline{f}^j}
 \end{aligned} \tag{2.18}$$

where

$$L^*(x) = \arg \min_{1 \leq L \leq N-1} \{ y_l^{(L)}(x) \} \tag{2.19}$$

and

$$\begin{aligned}
 y_r(x) &= \max_{1 \leq R \leq N-1} \{ y_r^{(R)}(x) \} = y_r^{(R^*(x))}(x) \\
 &= \frac{\sum_{j=1}^{R^*(x)} \underline{f}^j c_r^j + \sum_{j=R^*(x)+1}^N \bar{f}^j c_r^j}{\sum_{j=1}^{R^*(x)} \underline{f}^j + \sum_{j=R^*(x)+1}^N \bar{f}^j}
 \end{aligned} \tag{2.20}$$

where

$$R^*(x) = \arg \max_{1 \leq R \leq N-1} \{ y_r^{(R)}(x) \} \tag{2.21}$$

The iterative Karnik and Mendel algorithm can be used to obtain the points $R^*(x)$ and $L^*(x)$ which is given in Table 2.2 (Mendel, 2000).

2.3.2 The Alternative type reduction methods

As mentioned in subsection 2.3, five approximations to the KM type-reducer exist. These methods are explained briefly. All the approximations are proposed in order to represent the T2-FLS output in a closed loop form. However, as it has been stated in (Wu, 2011), these reduction methods cannot simultaneously capture the features of the Karnik-Mendel type reduction. On the other hand, they reduce the computational complexity.

Table 2. 2: Calculation of the two end points of the type reduced set.

Steps	The Karnik Mendel Algorithm for computing y_l	The Karnik Mendel Algorithm for computing y_r
1.	Sort c_l^j ($j=1,2,\dots,N$) in increasing order such that $c_l^1 \leq c_l^2 \leq \dots \leq c_l^N$. Match the corresponding weights $\bar{f}, \underline{f}$. (with their index corresponds to the renumbered c_l^j)	Sort y_r^j ($i=1, 2,\dots,N$) in increasing order such that $c_r^1 \leq c_r^2 \leq \dots \leq c_r^N$. Match the corresponding weights $\bar{f}, \underline{f}$. (with their index corresponds to the renumbered c_r^j)
2.	Initialize f_j by setting: $f_j = \frac{\bar{f}_j + \underline{f}_j}{2}, \quad j=1,2,\dots,N$ Compute $y = \frac{\sum_{j=1}^N f_j c_l^j}{\sum_{j=1}^N f_j}$	Initialize f_j by setting: $f_j = \frac{\bar{f}_j + \underline{f}_j}{2}, \quad j=1,2,\dots,N$ Compute $y = \frac{\sum_{j=1}^N f_j c_r^j}{\sum_{j=1}^N f_j}$
3.	Find the switch point L ($1 \leq L \leq N-1$) such that $c_l^j \leq y \leq c_l^{j+1}$	Find the switch point R ($1 \leq R \leq N-1$) such that $c_r^j \leq y \leq c_r^{j+1}$
4.	Set $f_j = \begin{cases} \bar{f}^j & j \leq L \\ \underline{f}^j & j > L \end{cases}$ and compute $y' = \frac{\sum_{j=1}^N f_j c_l^j}{\sum_{j=1}^N f_j}$	Set $f_j = \begin{cases} \bar{f}^j & j \leq R \\ \underline{f}^j & j > R \end{cases}$ and compute $y' = \frac{\sum_{j=1}^N f_j c_r^j}{\sum_{j=1}^N f_j}$
5.	Check if $y=y'$. If not go to step 3 and set $y' = y$. If yes stop and set $y=y'$	Check if $y=y'$. If not go to step 3 and set $y' = y$. If yes stop and set $y=y'$

2.3.2.1 The uncertainty bound method

In this type-reduction mechanism proposed by (Wu and Mendel, 2002), the inner and outer bound sets for the type-reduced set of an IT2-FLS is derived. The bound sets estimates the uncertainty contained in the output of the IT2-FLS.

The UB type-reducer computes the type reduced set as follows:

$$y_l = \frac{y_l + \bar{y}_l}{2} \quad (2.22)$$

$$y_r = \frac{y_r + \bar{y}_r}{2} \quad (2.23)$$

where

$$\bar{y}_l = \min\{\underline{y}^{(0)}, \underline{y}^{(N)}\} \quad (2.24)$$

$$\underline{y}_r = \max\{\bar{y}^{(0)}, \bar{y}^{(N)}\} \quad (2.25)$$

$$\underline{y}_l = \bar{y}_l - \frac{\sum_{n=1}^N (\bar{f}^n - \underline{f}^n) \sum_{n=1}^N \underline{f}^n (\underline{y}^n - \underline{y}^l) \sum_{n=1}^N \bar{f}^n (\underline{y}^N - \underline{y}^n)}{\sum_{n=1}^N \bar{f}^n \sum_{n=1}^N \underline{f}^n \sum_{n=1}^N \underline{f}^n (\underline{y}^n - \underline{y}^l) + \sum_{n=1}^N \bar{f}^n (\underline{y}^N - \underline{y}^n)} \quad (2.26)$$

$$\bar{y}_r = \underline{y}_r - \frac{\sum_{n=1}^N (\bar{f}^n - \underline{f}^n) \sum_{n=1}^N \bar{f}^n (\bar{y}^n - \bar{y}^l) \sum_{n=1}^N \underline{f}^n (\bar{y}^N - \bar{y}^n)}{\sum_{n=1}^N \bar{f}^n \sum_{n=1}^N \underline{f}^n \sum_{n=1}^N \bar{f}^n (\bar{y}^n - \bar{y}^l) + \sum_{n=1}^N \underline{f}^n (\bar{y}^N - \bar{y}^n)} \quad (2.27)$$

in which

$$\begin{aligned} \underline{y}^{(0)} &= \frac{\sum_{n=1}^N \underline{f}^n \underline{y}^n}{\sum_{n=1}^N \underline{f}^n} & \underline{y}^{(N)} &= \frac{\sum_{n=1}^N \bar{f}^n \underline{y}^n}{\sum_{n=1}^N \bar{f}^n} \\ \bar{y}^{(N)} &= \frac{\sum_{n=1}^N \underline{f}^n \bar{y}^n}{\sum_{n=1}^N \underline{f}^n} & \bar{y}^{(0)} &= \frac{\sum_{n=1}^N \bar{f}^n \bar{y}^n}{\sum_{n=1}^N \bar{f}^n} \end{aligned} \quad (2.28)$$

2.3.2.2 The Wu-Tan method

Wu and Tank (2005) proposed a closed form type reduction method based on the equivalent type-1 membership grades. The idea is to first find the equivalent type-1 fuzzy membership grade, to replace the each firing interval as follows:

$$\mu_j(x_n) = \bar{\mu}_j(x_n) - h_j^n(x) [\bar{\mu}_j(x_n) - \underline{\mu}_j(x_n)] \quad (2.29)$$

where $h_j^n(x)$ is a function of the in inputs x , and varies for each IT2-FS. Then the output can be calculated as:

$$y = \frac{\sum_{n=1}^N f^n y^n}{\sum_{n=1}^N f^n} \quad (2.30)$$

2.3.2.3 The Nie-Tan method

In the proposed method by Nie and Tan (2010), the IT2-FLS output is calculated as:

$$y = \frac{\sum_{n=1}^N (\underline{f}^n + \bar{f}^n) y^n}{\sum_{n=1}^N (\underline{f}^n + \bar{f}^n)} \quad (2.31)$$

It can be clearly seen that, NT method is special case of the WT method when $h_j^n(x) = 0.5$.

2.3.2.4 The Du-Ying method

In this method (Du and Ying, 2010), the crisp outputs obtained by all possible combinations of the lower and upper firing strengths is calculated

$$y_m = \frac{\sum_{n=1}^N f^{n*} y^n}{\sum_{n=1}^N f^{n*}}, \quad m = 1, 2, \dots, 2^N \quad (2.32)$$

where $f^{n*} \in \{\underline{f}^n, \bar{f}^n\}$. The final output value is the average of these, i.e.,

$$y = \frac{1}{2^N} \sum_{m=1}^{2^N} y_m \quad (2.33)$$

2.3.2.5 The Begian-Melek-Mendel method

Begian et al., (2008) proposed another closed form type reduction method, as follows:

$$y = \alpha \frac{\sum_{n=1}^N \underline{f}^n y^n}{\sum_{n=1}^N \underline{f}^n} + \beta \frac{\sum_{n=1}^N \bar{f}^n y^n}{\sum_{n=1}^N \bar{f}^n} \quad (2.34)$$

where α and β are adjustable coefficients.

3. FUZZY INVERSE MODEL CONTROLLER DESIGN

The simplest way to design a model based controller for nonlinear processes is to set up the inverse model of the system as the controller (Economou et al., 1986). In theory, the advantage of using an inverse model as controller is that it provides zero error in dynamic transients and steady state since it inherits the process dynamics (Babuska, 1998). Since it is a model based this control design strategy, consider a stable single input single output (SISO) IT2-FM which is described by the following equation:

$$y(k+1) = f(x(k), u(k)) \quad (3.1)$$

where $x(k)$ is the state vector, $y(k+1)$ is the output, $u(k)$ is the manipulated variable and $f(\cdot)$ represents the fuzzy mapping. The objective of inverse model is to calculate the input signal such that the system output at the next sampling time will eventually become equal to the desired reference signal denoted as $y_{des}(k+1)$. This objective can be formally stated as follows:

$$u(k) = f^{-1}(x(k), y_{des}(k+1)) \quad (3.2)$$

However, it is usually difficult to find the inverse function analytically.

In this thesis, three inversion methodologies will be presented for interval type-2 fuzzy models. In the first proposed method, the inverse fuzzy model signal generation is handled as an optimization problem the problem is tried to be solved within each sampling period of time in an online manner (Kumbasar, 2011a). Secondly, an inversion method for interval type-2 Takagi-Sugeno fuzzy logic system has been proposed in which the calculation of inverse model is done based on simple manipulations of the antecedent and consequence parts of the fuzzy model (Kumbasar, 2011b). Finally, an analytical methodology has been developed to form the inverse of a certain class of interval type-2 fuzzy models with singleton consequents (Kumbasar, 2011e). Based on the proposed inversion methodologies, the inverse controllers can be easily formed using the fuzzy model of the process which is illustrated in Figure 3.1.

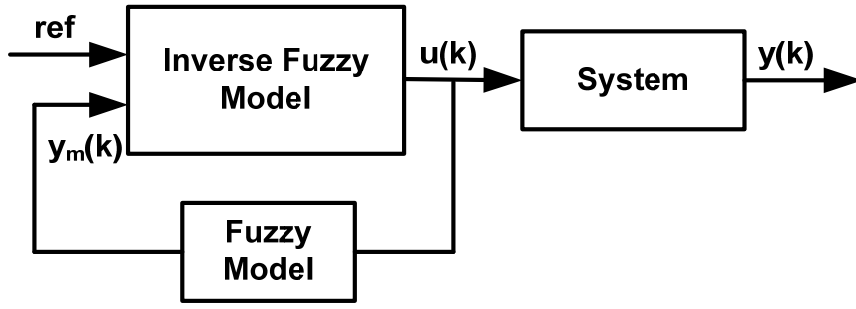


Figure 3.1 : Illustration of fuzzy model inverse controller in an open loop fashion.

Since the type-2 fuzzy models are more powerful to represent process dynamics compared to type-1 fuzzy models, an inverse type-2 model based controller should naturally provide a more effective control performance than type-1 inverse controller. In section 4, three type-2 fuzzy model inversion methods will be proposed and it will be illustrated that model inversion is exact, i.e. $y(k+1)=y_{des}(k+1)$. Therefore, if the type-2 fuzzy model matches the process perfectly, the process output would converge to the desired set point. In achieving such an aim, the Karnik–Mendel type reduction method is preferred due to the inconsistency and the adaptiveness features of this type reduction method (Wu, 2011).

Even though inverse model controllers may produce perfect control while operating in an open loop fashion, it will not be sufficient in the case of modeling mismatches or disturbances. Thus, the control of a nonlinear process within a nonlinear internal model control (NIMC) scheme which is illustrated in Figure 3.2 is implemented to provide an effective control performance (Kumbasar et al., 2011a). Since the inverse controller will inherit the dynamics of the process, it will possess perfect control with zero error in steady state (Economou et al., 1986). In the case of a model mismatch or disturbance, the difference of the process output and fuzzy model will be fed back in order to compensate the errors (Kumbasar et al., 2011a; Kumbasar et al., 2011b). Moreover, a robustness filter in the NIMC scheme is used to filter the noise and to stabilize the control loop by reducing the sensitivity gain (Boukezzoula et al., 2007). The robustness filter is chosen as follows:

$$R(s) = \frac{1}{T_f s + 1} \quad (3.3)$$

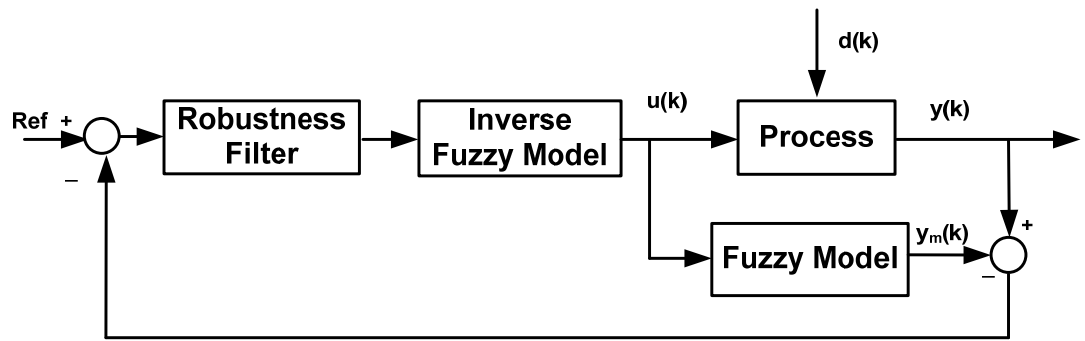


Figure 3.2 : Inverse fuzzy model based NIMC scheme.

4. ITERATIVE TYPE-2 FUZZY MODEL INVERSION AND CONTROLLER DESIGN

In this section, a type-2 fuzzy model inversion method is proposed (Kumbasar et al., 2011a; Kumbasar et al., 2011d). In this structure, the output of the inverse type-2 fuzzy model signal is generated as a solution of an optimization problem. For this optimization problem BB-BC algorithm is used since an online implementation of this evolutionary algorithm is feasible at each sampling time due to the simplicity and high convergence speed of the algorithm (Erol and Eksin, 2006). In order to show the closed loop performance of the proposed iterative inverse controller a simulation study has been performed. The fuzzy model and its inverse are embedded into NIMC scheme to obtain a robust performance against modeling mismatches or disturbances that might occur over the system (Kumbasar et al., 2011a; Kumbasar et al., 2011d). The control performance is compared with an iterative type-1 fuzzy model inverse control structure proposed by (Kumbasar et al., 2011d).

4.1 Big Bang–Big Crunch Optimization

The Big Bang–Big Crunch (BB-BC) optimization method consists of two main steps. The first step is the “Big Bang” phase where candidate solutions are randomly distributed over the search space and the next step is the “Big Crunch” phase where a contraction procedure calculates the center of mass for the population. The initial Big Bang population is randomly generated over the entire search space just like the other evolutionary search algorithms. All subsequent “Big Bang” phases are randomly distributed about the center of mass or the best fit individual in a similar fashion (Erol and Eksin, 2006).

After the “Big Bang” phase, a contraction procedure is applied as the “Big Crunch” phase to form a center or a representative point for further “Big Bang” operations. In this phase, the contraction operator takes the current positions of each candidate solution in the population and its associated cost function value and computes a center of mass. The center of mass can be computed as:

$$x_c = \frac{\sum_{i=1}^N \frac{1}{f^i} x_i}{\sum_{i=1}^N \frac{1}{f^i}} \quad (4.1)$$

where x_c is the position of the center of mass, x_i is the position of the candidate, f^i is the cost function value of the i^{th} candidate, and N is the population size. Instead of the center of mass, the best fit individual can also be chosen as the starting point in the “Big Bang” phase. The new generation for the next iteration “Big Bang” phase is normally distributed around x_c . The new candidates around the center of mass are calculated by adding or subtracting a normal random number whose value decreases as the iterations elapse. This can be formalized as:

$$x^{\text{new}} = x_c + \frac{r\alpha(x_{\max} - x_{\min})}{k} \quad (4.2)$$

where r is random number; α is a parameter limiting the size of the search space, $x_{\max}$ and $x_{\min}$ are the upper and lower limits; and k is the iteration step. In (Erol and Eksin, 2006) the working principle of this evolutionary method is explained as to transform a convergent solution to a chaotic state which is a new set of solutions. The pseudo code is given in Table 4.1.

In order to improve the computational efficiency and performance of the general BB-BC algorithm, an elitist strategy introduced in (Camp, 2007) is applied in this BB-BC optimization. In this modification, positions of new candidate solutions at the beginning of each Big Bang are normally distributed around a new point located between the center of mass and the best solution.

$$x^{\text{new}} = \beta x_c + (1-\beta)x_{\text{best}} + \frac{r\alpha(x_{\max} - x_{\min})}{k} \quad (4.3)$$

where β is the parameter controlling the influence of the global best solution x_{best} on the location of new candidate solutions. This modification of generating the new solution can be viewed as to an elitist strategy, where the best solution influences the direction of the search, and consequently potentially improve the overall search performance.

Table 4.1 : BB-BC optimization algorithm.

Step 1 (Big Bang Phase)

An initial generation of N candidates is generated randomly in the search space.

Step 2

The cost function values of all the candidate solutions are computed.

Step 3 (Big Crunch Phase)

The center of mass is calculated. Either the best fit individual or the center of mass is chosen as the point of Big Bang Phase.

Step 4

New candidates are calculated around the new point calculated in Step 3 by adding or subtracting a random number whose value decreases as the iterations elapse.

Step 5

Return to Step 2 until stopping criteria has been met.

4.2 Inverse Type-2 Fuzzy Model Controller Design

The simplest way to design a model based controller for processes is to set up the inverse model of the system as the controller (Economou, 1986). The objective of inverse control is to calculate the control input such that the system output at the next sampling time will eventually become equal to the desired reference state signal, $x_d(k+1)$. This can be formally stated as follows:

$$u(k)=f^{-1}(x(k), x_d(k+1)) \quad (4.4)$$

It is usually difficult to find the inverse function analytically. Thus, numerical optimization methods should be utilized to find the control input $u(k)$ in a recursive manner. For this reason, in this proposed control structure, the inverse type-2 fuzzy model control signal generation is handled as an optimization problem that can be stated as minimizing the performance function J given in (4.5) while the system output from any initial state $x(0)=x_0$ is to be driven to a desired state $x(k)=x_d(k)$ (Kumbasar et al., 2011a; Kumbasar et al., 2011d). The chosen performance function J for optimality purposes to be minimized at every sampling time is given as:

$$J=\min_{u(k)} \left\| (e^T(k+1)e(k+1)) \right\| \quad (4.5)$$

where $e(k+1)=x(k+1)-x_d(k+1)$ is the error at time $k+1$. This optimization problem yields us the optimal inverse model signal or the best approximation which can be solved via any optimization algorithm such as particular swarm optimization, genetic algorithm, ant colony optimization, etc. Since an online implementation of an optimization algorithm is needed at each sampling time, the BB-BC optimization

algorithm is used due to its simplicity and high convergence speed. In Figure 4.1, the proposed BB-BC based type-2 fuzzy model inverse controller (BBBC-T2FMIC) is presented (Kumbasar et al., 2011a; Kumbasar et al., 2011d).

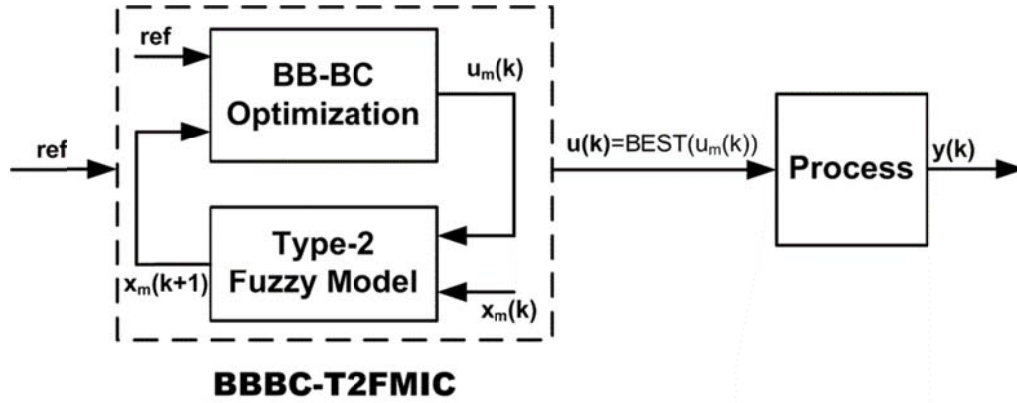


Figure 4.1 : BB-BC based type-2 fuzzy model inverse controller scheme.

In the case of existence of a transport delay in the process, a predicted fuzzy model should be used to eliminate effect of the time delay. If the process has no time delay, then the normal or unpredicted fuzzy model would be used in the optimization procedure.

4.3 Simulation Study

In the simulation study, the type-2 fuzzy model and its inverse as the primary controller are embedded into a nonlinear internal model control structure to provide an effective and robust control performance. In order to show the effectiveness of the proposed BBBC-T2FMIC the level control of a spherical tank is considered (Agrawal and Lakshminarayan, 2003). This real time application case is designed in order to demonstrate that the inverse fuzzy model control signal can be generated within a short sampling time even though the time constant of the system to be controlled is relatively small. It has been observed that the system output still converges to the desired reference successfully since the required computations are accomplished within the time limits. Therefore, the applicability of proposed methods to a fast system has been demonstrated and underlined. The inverse controller closed loop performance will be compared with the BB-BC based type-1 fuzzy model inverse controller (BBBC-T1FMIC) proposed by Kumbasar et al., (2008a). In order to make a fair comparison of the BBBC-T2FMIC scheme with the BBBC-T1FMIC scheme two performance measures are considered to be:

i) Integral Absolute Error (IAE), which is defined as:

$$\text{IAE} = \int_0^{\infty} |r(t) - y(t)| dt \quad (4.6)$$

ii) Total Variation (TV) of the control input (Skogestad, 2003), which is defined as:

$$\text{TV} = \sum_{i=1}^{\infty} |u_{i+1} - u_i| \quad (4.7)$$

A schematic diagram of the tank level system is illustrated in Figure 4.2.

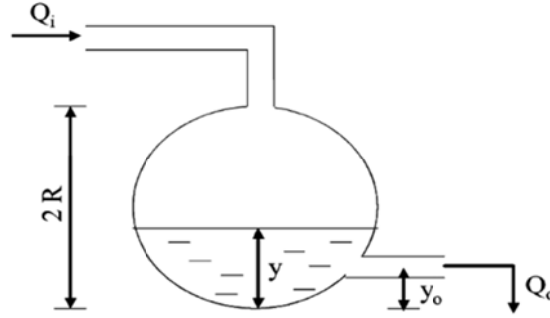


Figure 4.2 : Illustration of the spherical tank system-1.

The differential equation related to the tank level system is given as:

$$Q_i(t-d) - Q_o = (\pi - \pi(R-y)^2) \frac{dy}{dt} \quad (4.8)$$

where R is the radius of the spherical tank, Q_i is the inlet volumetric flow rate, and Q_o is the outlet flow rate. The delay from the manipulated input Q_i to the controlled output y is indicated by d . The outlet flow rate Q_o is related to the level y via the Bernoulli equation:

$$Q_o = \sqrt{2g(y-y_o)} \quad (4.9)$$

where g represents the gravitation constant and y_o represents the height of the outlet pipe as measured from the base of the column. The parameters of the process are given in Table 4.2. The nominal operating value of y is taken as 0.5 m. At this nominal operating value, the output y will respond faster than the case when y is close to 1. Moreover it is assumed the system output inherits noise.

Table 4.2 : The parameters of the spherical tank system-1.

Radius of tank [m]	$R = 1\text{ m}$
Delay from Q_i to y	$d = 0\text{ s}$
Gravity acceleration [m/s^2]	$g = 9.81\text{ m/s}^2$
Height of the output pipe [m]	$y_o = 0.1\text{ m}$
Inlet volumetric flow rate [m^3/s]	$Q_i(t) : 0\text{--}6\text{ m}^3/\text{s}$
Outlet volumetric flow rate [m^3/s]	$Q_o(t)$
Height of liquid level [m]	y

Through empirical studies, the parameters for the evolutionary algorithm α and β have chosen as 1 and 0.25, respectively, such that the optimal solution can be provided for BBBC-T1FMIC and BBBC-T2FMIC design problems. The population size and the number of iterations for the BB-BC optimization method have been chosen as 20.

At first, the effectiveness of type-2 fuzzy modeling will be demonstrated. Therefore, the type-1 fuzzy model (T1FM) and type-2 fuzzy model (T2FM) of the process have been obtained and compared. The state vector for the type-1 and type-2 fuzzy model has been chosen as ($T_s=0.05\text{s}$) $x(k+1)=[y(k), Q_i(k)]$. In order to represent the nonlinearities of spherical tank better, the membership functions of the type-1 fuzzy model is defined with 4 type-1 gauss functions while the membership functions of the type-2 fuzzy model are defined with 4 type-2 gauss functions. To check the obtained type-1 and type-2 fuzzy models, a test signal which is illustrated in Fig.6a is applied. The fuzzy models are simulated against the model output which is illustrated in Fig. 6b. In order make a fair comparison between the fuzzy models, a performance index is defined as follows:

$$\text{IAE} = \int_0^{200} |y_p(t) - y_{\text{FM}}(t)| dt \quad (4.10)$$

where $y_p(t)$ is the process output and $y_{\text{FM}}(t)$ is the fuzzy model output. The performance values obtained from (20) for the T1FM and T2FM are found as 6.915 and 5.824, respectively. Moreover, it has been demonstrated from Figure 4.3 that the type-2 fuzzy model represents the defined nonlinear process better than the type-1 fuzzy model.

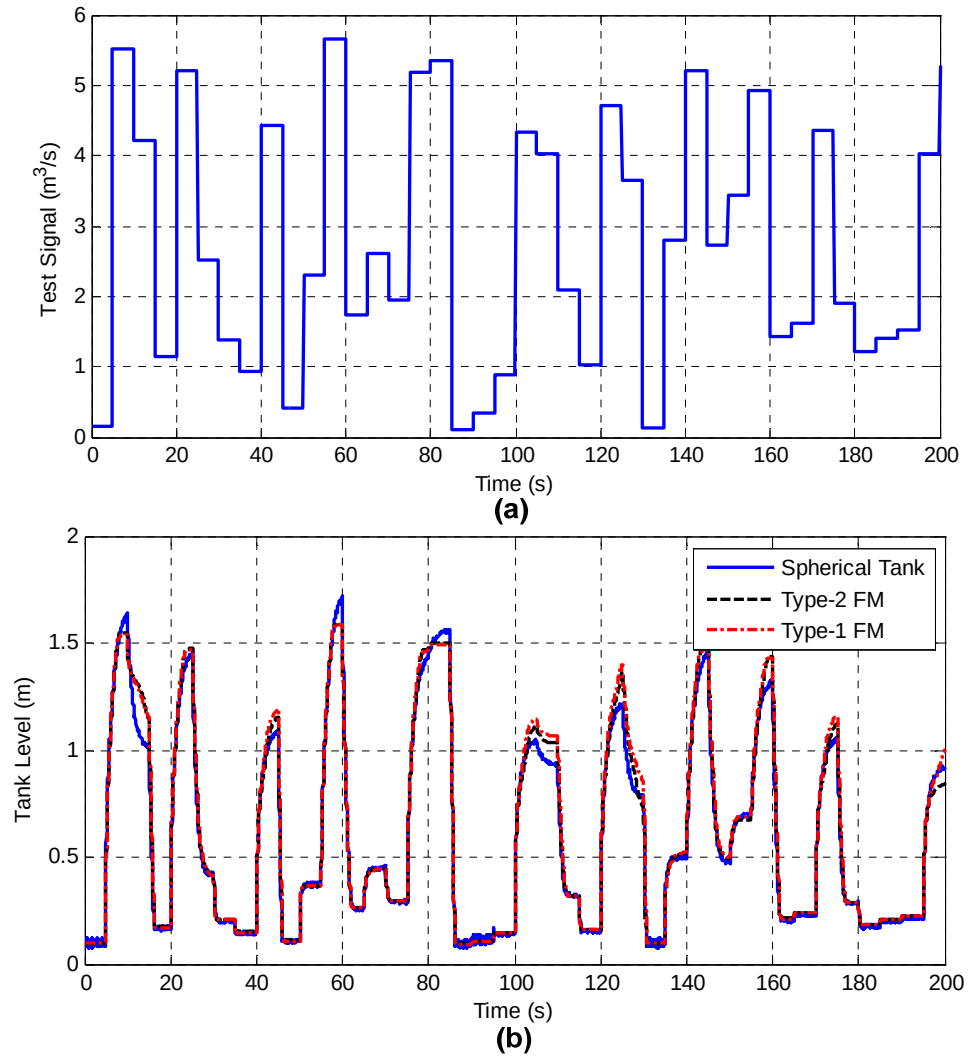


Figure 4.3 : Illustration of (a) test signal (b) spherical tank process output, type-1 and type-2 fuzzy model outputs.

First of all, since the nonlinearity is related to the level of the tank, a servo performance of the BBBC-T1FMIC and BBBC-T2FMIC structures have been compared. It has been illustrated in Figure 4.4 that the BBBC-T2FMIC scheme provides better transient state performances than the BBBC-T1FMIC under varying reference signals. Moreover, the IAE value of BBBC-T2FMIC structure is less than BBBC-T1FMIC controller structure, as it can be seen in Table 4.3. Additionally, the BBBC-T2FMIC scheme has a low value of TV that shows that it has the smoothest control signal.

Table 4.3 : Closed loop control performance comparison of the spherical tank system-1.

	Servo Performance Simulation		Disturbance Rejection Simulation	
	IAE	TV	IAE	TV
BBBC-T1FMIC	0.3893	2.9776	0.1429	1.9860
BBBC-T2FMIC	0.3134	2.4816	0.1176	1.8944

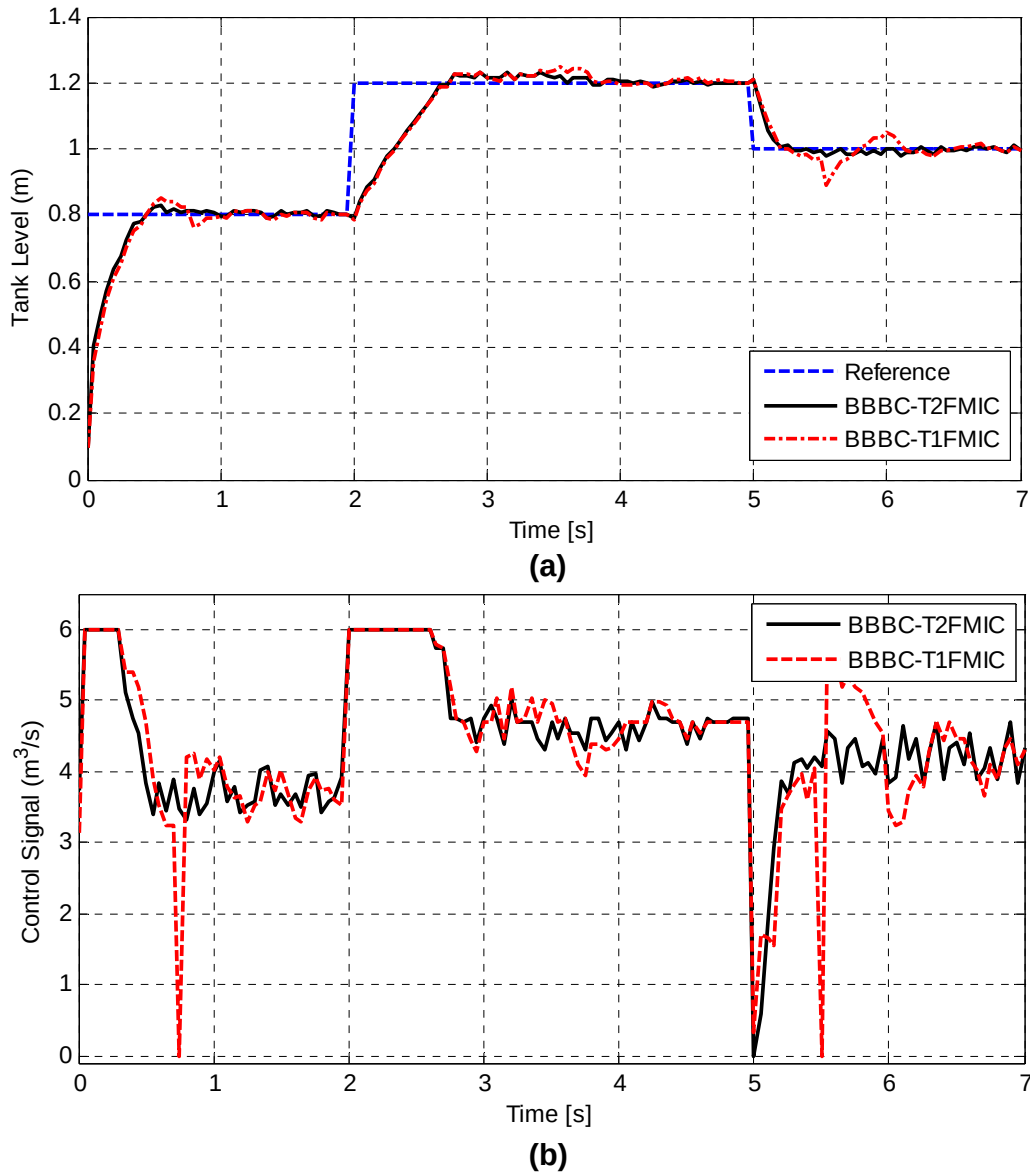


Figure 4.4 : Illustration of (a) the spherical tank system-1 outputs (b) the iterative inverse control signals for varying reference values.

Secondly, the case of a disturbance rejection performance of the BBBC-T2FMIC and BBBC-T1FMIC scheme has been examined. First of all, a reference signal of “1m” has been applied. After the process converged to the desired tank level, a step output

disturbance with the magnitude of “0.3” and a step input disturbance with the magnitude of “2” have been applied in 1th and 3th seconds, respectively. As it can be seen from the Figure 4.5 the proposed control structure compensated very effectively the input/output disturbances in a short period of time compared to the BBBC-T1FMIC structure. Moreover, the control signal of the BBBC-T2FMIC scheme is smoother and therefore has a lower TV value. Also, the IAE value is the better.

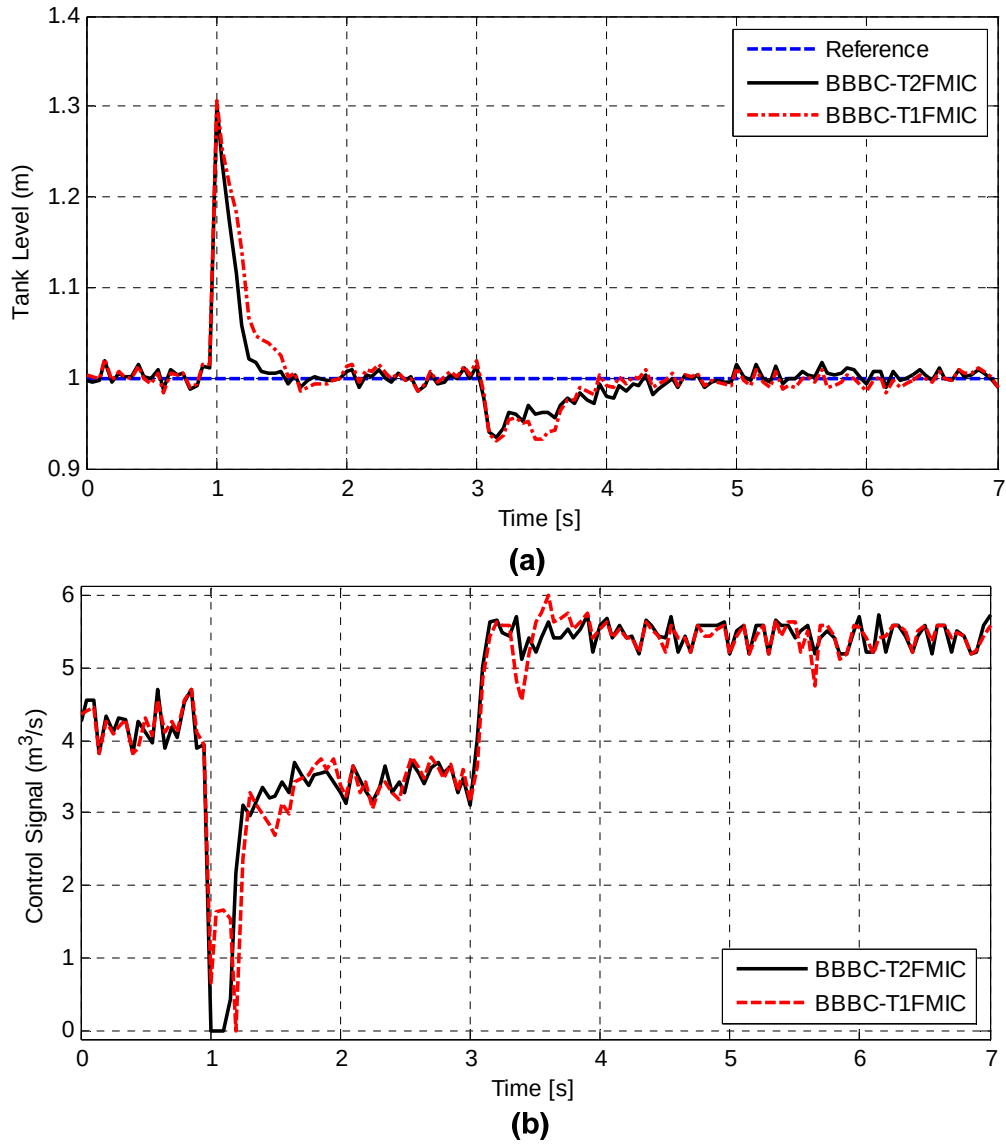


Figure 4.5 : Illustration of (a) the spherical tank system-1 outputs (b) the iterative inverse control signals for input and output disturbances.

4.4 Conclusions and Comments

In this chapter, a new iterative type-2 fuzzy model inversion technique based on the BB-BC optimization is developed which does not require any invertibility conditions or any limitations on the form of the fuzzy process model. In this new approach, the inverse type-2 fuzzy model control signal generation is handled as an optimization problem. Since the BB-BC optimization algorithm has a high convergence speed and low computational time, the optimal inverse type-2 fuzzy model control signal can be generated within each sampling time in an on line fashion. The low computational time and high convergence properties of this optimization algorithm make it possible to calculate the inverse model output signal within two sampling periods. A simulation example is presented to show the superiority of the approach. In the simulation studies, the proposed inverse control structure is implemented on a spherical tank system which can be considered as a fast and practically non-linear system. The proposed BBBC-T2FMIC structure has been compared with the BBBC-T1FMIC structure. Since the type-2 fuzzy models map the nonlinearities and uncertainties better than the type-1 fuzzy models, the BBBC-T2FMIC ameliorate the overall performance of the system with respect to disturbance rejection and set-point following much better. Moreover, it should be emphasized that the BB-BC optimization algorithm that has been used in the proposed type-2 fuzzy model inverse control structures is both appropriate and feasible for online implementations even in the case of system possessing fast dynamics.

5. INTERVAL TYPE-2 TAKAGI-SUGENO FUZZY MODEL INVERSION AND CONTROLLER DESIGN

In this chapter, an inversion method for interval type-2 Takagi-Sugeno fuzzy logic system has been proposed (Kumbasar et al., 2011b). The inverse type-2 interval TS fuzzy model has been obtained through the principle of rule-by-rule inversion by permutation of the antecedent and consequence parts. The calculations are simple and straightforward. The proposed methodology is a generalization of the type-1 TS fuzzy model inversion proposed by Babuska (1998). It is important to note here that the developed inversion strategy is based on a pure analytic method (Kumbasar et al., 2011b). However, the proposed method is still restricted to SISO processes. A simulation study has been performed to illustrate the effectiveness of the proposed inversion method.

5.1 Inverse Type-2 Fuzzy Model Controller Design

Generally, it is difficult, to find inverse fuzzy function in an analytical form, only some special forms can be calculated such as type-1 TS fuzzy model (Babuska, 1998). In this strategy, local linearization of type-1 TS fuzzy model is performed so as to obtain a local model at a certain operating point. Then, the inverse fuzzy model signal is calculated directly using this local model via simple calculations for type-1 TS fuzzy model. This inversion strategy is based on simple manipulation of the antecedent and consequence parts of the fuzzy model. This simple and straightforward methodology can be generalized to the inversion of interval type-2 TS fuzzy models (Kumbasar et al., 2011b). In this context, let us assume that a type-2 fuzzy model of a SISO process is available and its rules are given as:

$$\begin{aligned}
 r_j : & \text{IF } y(k) \text{ is } \tilde{A}_1 \text{ and } y(k-1) \text{ is } \tilde{A}_2 \text{ and } \dots \text{ and } y(kn_y + 1) \text{ is } \tilde{A}_{n_y} \\
 & \text{and } u(k - n_d + 1) \text{ is } \tilde{B}_1 \text{ and } \dots u(k - n_d - n_u + 2) \text{ is } \tilde{B}_{n_u}, \\
 & \text{THEN } y_j(k+1) \text{ is } w_j = \tilde{p}_0^j + \tilde{p}_1^j x(k) + \tilde{p}_2^j u(k), \quad j = 1, 2, \dots, N
 \end{aligned} \tag{5.1}$$

where $\tilde{A}_1 \dots \tilde{A}_j$ and $\tilde{B}_1 \dots \tilde{B}_j$ are type-2 fuzzy sets, $\tilde{p}_0^j$, $\tilde{p}_1^j$ and $\tilde{p}_2^j$ consequent type-2 fuzzy sets, n_d transport delay, n_y past outputs, n_u past inputs.

For the sake of simplicity, it is assumed that $n_y = n_u = N$ and $n_d = 1$ (assuming no transport delay). Thus, the defined fuzzy sets could then be represented as one multi-dimensional state type-2 fuzzy set $\tilde{X} = \tilde{A}_1 \times \dots \times \tilde{A}_N \times \tilde{B}_1 \times \dots \times \tilde{B}_N$. Substituting $\tilde{B}$ for $\tilde{B}_1$, the rule base structure can then be redefined as follows:

$$r_j : \text{IF } x(k) \text{ is } \tilde{X} \text{ and } u(k) \text{ is } \tilde{B}, \text{ THEN } y(k+1) \text{ is } w_j(k+1) = \tilde{p}_0^j + \tilde{p}_1^j x(k) + \tilde{p}_2^j u(k) \quad (5.2)$$

Therefore, the type-2 fuzzy model output $y(k+1)$ can be calculated as the weighted average of the linear consequents in the individual rules:

$$y(k+1) = \frac{\sum_{j=1}^N \tilde{f}_j (\tilde{p}_0^j + \tilde{p}_1^j x(k) + \tilde{p}_2^j u(k))}{\sum_{j=1}^N \tilde{f}_j} \quad (5.3)$$

If the parameters $\tilde{p}_0^j$, $\tilde{p}_1^j$ and $\tilde{p}_2^j$ are assumed to be frozen at a certain operating point $(x(k), u(k))$, then type-2 fuzzy model can be represented as the following uncertain linear time invariant discrete model:

$$y(k+1) = \tilde{p}_0^* + \tilde{p}_1^* x(k) + \tilde{p}_2^* u(k) \quad (5.4)$$

where $\tilde{p}_0^*$, $\tilde{p}_1^*$ and $\tilde{p}_2^*$ can be calculated using the following relations:

$$\begin{aligned} \tilde{p}_0^* &= \frac{\sum_{j=1}^N \tilde{f}_j \cdot \tilde{p}_0^j}{\sum_{j=1}^N \tilde{f}_j} & \tilde{p}_1^* &= \frac{\sum_{j=1}^N \tilde{f}_j \cdot \tilde{p}_1^j}{\sum_{j=1}^N \tilde{f}_j} \\ \tilde{p}_2^* &= \frac{\sum_{j=1}^N \tilde{f}_j \cdot \tilde{p}_2^j}{\sum_{j=1}^N \tilde{f}_j} \end{aligned} \quad (5.5)$$

Since, here, the type-2 fuzzy logic systems are considered to be of interval type, the calculated $\tilde{p}_0^*$, $\tilde{p}_1^*$ and $\tilde{p}_2^*$ type-2 consequent sets possess upper and lower bounds (e.g. $\tilde{p}_0^* = [\underline{p}_0^* \quad \bar{p}_0^*]$).

Consequently, (5.4) can be reformulated for left and right values of the type-2 fuzzy model as follows:

$$y_r(k+1) = \bar{p}_0^* + \bar{p}_1^* x(k) + \bar{p}_2^* u_r(k) \quad (5.6)$$

$$y_l(k+1) = \underline{p}_0^* + \underline{p}_1^* x(k) + \underline{p}_2^* u_l(k) \quad (5.7)$$

Since the objective of inverse model is to force the systems output to reach the desired input signal value ($y_{des}(k+1)$) at the next sampling time then naturally (5.6) and (5.7) can be stated as:

$$u_r(k) = f^{-1}(x(k), y_{des}(k+1)) \quad (5.8)$$

$$u_l(k) = f^{-1}(x(k), y_{des}(k+1)) \quad (5.9)$$

Thus, the inverse type-2 TS fuzzy model law can be formulated as:

$$u_r(k) = (y_{des}(k+1) - \bar{p}_0^* - \bar{p}_1^* x(k)) / \bar{p}_2^* \quad (5.10)$$

$$u_l(k) = (y_{des}(k+1) - \underline{p}_0^* - \underline{p}_1^* x(k)) / \underline{p}_2^* \quad (5.11)$$

where $\bar{p}_0^*, \bar{p}_1^*, \bar{p}_2^*, \underline{p}_0^*, \underline{p}_1^*$ and $\underline{p}_2^*$ are updated at each sampling time. Thus, the crisp output of inverse interval TS-T2FLS is:

$$u(k) = \frac{u_l(k) + u_r(k)}{2} \quad (5.12)$$

If there exists any time delay in the system then the method should be applied after n_d steps of prediction have been performed, similar to what has to be done for the case of the type-1 TS fuzzy model inversion (Babuska, 1998).

The proposed interval type-2 fuzzy model inversion algorithm is presented in Table 5.1 (Kumbasar et al., 2011b). The inversion method has been embedded in the original Karnik-Mendel type reduction /defuzzification algorithm.

Table 5.1 : Calculation of the left and right inverse type-2 fuzzy model signals.

Step	Calculation of the Left Inverse Type-2 Fuzzy Model Signal	Calculation of the Right Inverse Type-2 Fuzzy Model Signal
1.	Sort $\underline{w}_j$ ($j=1,2,\dots,N$) in increasing order such that $\underline{w}_1 \leq \underline{w}_2 \leq \dots \leq \underline{w}_N$. Match the corresponding weights f_j, p_0^j, p_1^j, p_2^j (with their index corresponds to the renumbered $\underline{w}_j$)	Sort $\overline{w}_j$ ($i=1,2,\dots,N$) in increasing order such that $\overline{w}_1 \leq \overline{w}_2 \leq \dots \leq \overline{w}_N$. Match the corresponding weights f_j, p_0^j, p_1^j, p_2^j (with their index corresponds to the renumbered $\overline{w}_j$)
2.	Initialize f_j by setting $f_j = \frac{\overline{f}_j + f_j}{2}, \quad j=1,2,\dots,N$ Compute $y = \frac{\sum_{j=1}^N f_j \underline{w}_j}{\sum_{j=1}^N f_j}$	Initialize f_j by setting $f_j = \frac{\overline{f}_j + f_j}{2}, \quad j=1,2,\dots,N$ Compute $y = \frac{\sum_{j=1}^N f_j \overline{w}_j}{\sum_{j=1}^N f_j}$
3.	Find the switch point L ($1 \leq L \leq N-1$) such that $\underline{w}_L \leq y \leq \underline{w}_{L+1}$	Find the switch point R ($1 \leq R \leq N-1$) such that $\overline{w}_R \leq y \leq \overline{w}_{R+1}$
4.	Set $w_j = \begin{cases} \overline{w}_j & j \leq L \\ \underline{w}_j & j > L \end{cases}$ and compute $y' = \frac{\sum_{j=1}^N f_j \overline{w}_j}{\sum_{j=1}^N f_j}$	Set $w_j = \begin{cases} \underline{w}_j & j \leq R \\ \overline{w}_j & j > R \end{cases}$ and compute $y' = \frac{\sum_{j=1}^N f_j \underline{w}_j}{\sum_{j=1}^N f_j}$
5.	Check if $y=y'$. If not go to step 3 and set $y' = y$. If yes stop and set $y = y'$ $\underline{p}_0^* = \frac{\sum_{j=1}^N f_j \cdot \underline{p}_0^j}{\sum_{j=1}^N f_j} \quad \underline{p}_1^* = \frac{\sum_{j=1}^N f_j \cdot \underline{p}_1^j}{\sum_{j=1}^N f_j}$ $\underline{p}_2^* = \frac{\sum_{j=1}^N f_j \cdot \underline{p}_2^j}{\sum_{j=1}^N f_j}$ $u_1(k) = (y_{des}(k+1) - \underline{p}_0^* - \underline{p}_1^* x(k)) / \underline{p}_2^*$	Check if $y=y'$. If not go to step 3 and set $y' = y$. If yes stop and set $y = y'$ $\overline{p}_0^* = \frac{\sum_{j=1}^N f_j \cdot \overline{p}_0^j}{\sum_{j=1}^N f_j} \quad \overline{p}_1^* = \frac{\sum_{j=1}^N f_j \cdot \overline{p}_1^j}{\sum_{j=1}^N f_j}$ $\overline{p}_2^* = \frac{\sum_{j=1}^N f_j \cdot \overline{p}_2^j}{\sum_{j=1}^N f_j}$ $u_r(k) = (y_{des}(k+1) - \overline{p}_0^* - \overline{p}_1^* x(k)) / \overline{p}_2^*$

5.1 Simulation Study

To illustrate the effectiveness of the proposed inversion method, a simple example with two inputs and one output interval type-2 Takagi-Sugeno fuzzy model is presented. The inputs are represented by three Gaussian shape type-2 membership functions which are illustrated in Figure 5.1. The output is represented by type-2 linear function sets. The consequent parameters $\bar{p}_0^j, \bar{p}_1^j$ and $\bar{p}_2^j$ of the interval type 2 fuzzy input sets are tabulated in Table 5.2.

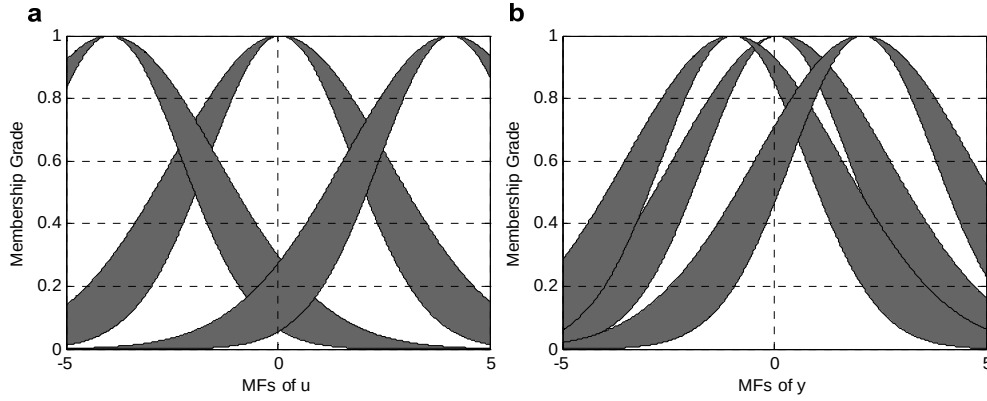


Figure 5.1 : Antecedent membership functions of the type-2 TS fuzzy models.

Table 5.2 : Consequent parameters of the type-2 TS fuzzy models.

Rule Number	$\bar{p}_0$	$\underline{p}_0$	$\bar{p}_1$	$\underline{p}_1$	$\bar{p}_2$	$\underline{p}_2$
r_1	-0.0002	-0.0001	0.9953	0.2429	0.1147	0.0171
r_2	0.0004	0.0002	0.9353	0.6429	0.2048	0.1571
r_3	0.0003	0.0002	0.9253	0.7429	0.1461	0.0571

Since the inputs and outputs are defined with type-2 fuzzy sets; this is the most general type-2 Takagi-Sugeno fuzzy system (Mendel, 2000). The rule base of the model consists of three Takagi Sugeno type rules as following:

$$r_j : \text{IF } y(k) \text{ is } \tilde{A}_j \text{ and } u(k) \text{ is } \tilde{B}_j , \\ \text{THEN } y(k+1) \text{ is } w^j = \bar{p}_0^j + \bar{p}_1^j y(k) + \bar{p}_2^j u(k), j = 1, 2, 3$$

In this simulation study, the proposed inversion method has been tested whether it can force the system output to the desired reference signal in an open loop fashion as illustrated in Figure 5.2.

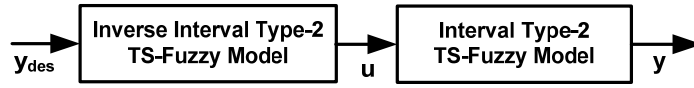


Figure 5.2 : Interval type-2 T-S fuzzy model inversion scheme.

Two different reference signals have been applied, a sinusoidal signal ($y_{des1} = \sin(t)$) and a uniform distributed random number within the interval $[-2, 2]$ (y_{des2}). The tracking error between the fuzzy model output and reference signal for both cases are illustrated in Figure 5.3. It can be concluded that the type-2 fuzzy system inversion is exact since the amplitude of the error is around 10^{-6} for S_{des1} and 10^{-3} for S_{des2} .

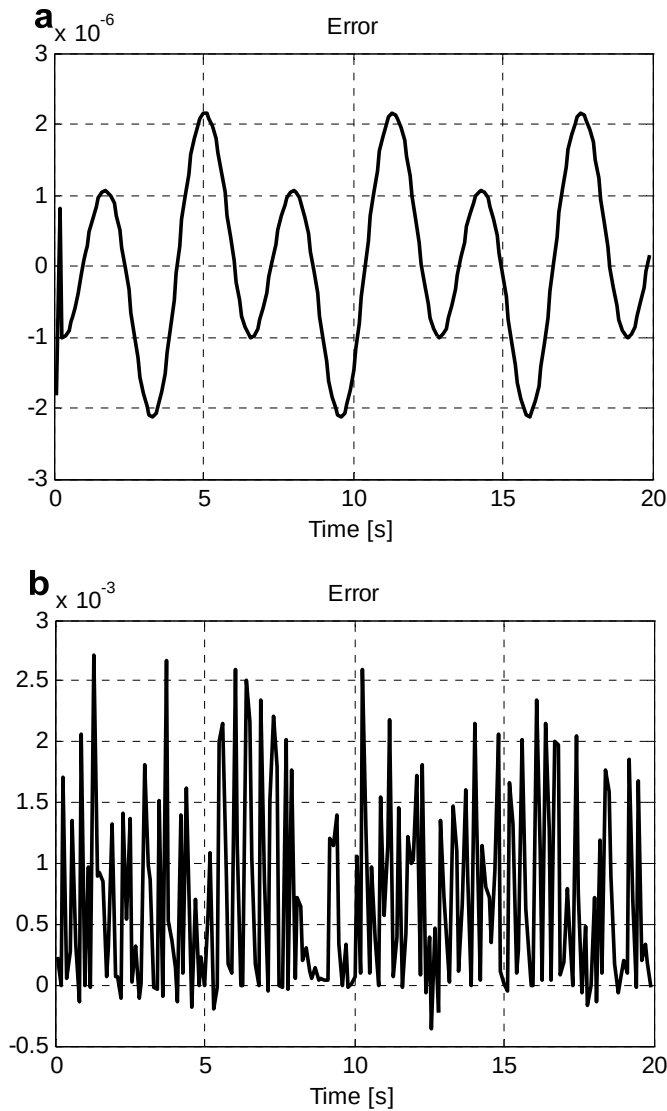


Figure 5.3 : Illustration of the tracking error in the case of (a) a sinusoidal reference signal (b) an uniform distributed random reference signal for the interval type-2 T-S fuzzy model inversion scheme.

5.2 Conclusions and Comments

In this chapter, a systematic method has been presented to find or form the inverse model of interval type-2 Takagi-Sugeno fuzzy systems. The inversion technique developed and generalized for a class of type-2 fuzzy systems consists of simple permutation of the antecedent and consequence parts based on a purely analytical methodology. The effectiveness and the performance of the method has been shown on a simulation example in which the output of an interval type-2 Takagi-Sugeno fuzzy system has been forced to follow certain desired reference signals in an open loop fashion. Inspecting the tracking error values between the desired input reference signal and the system output signal, it can easily be concluded that exact inversion of interval type-2 Takagi-Sugeno fuzzy system has been accomplished. However, the proposed method is still restricted to SISO processes

6. DECOMPOSABLE INTERVAL TYPE-2 FUZZY MODEL INVERSION AND CONTROLLER DESIGN

In this section, an analytical inverse controller design methodology has been proposed for a certain class of interval type-2 fuzzy models (IT2-FM) (Kumbasar et al., 2011e). At first, the interval type-2 fuzzy model is decomposed into interval type-2 fuzzy submodels (Kumbasar et al., 2011f). Then, the analytical formulation of the type-2 fuzzy submodel output is directly used to set up a routine to find the inverse solution. However, the type reduced set of the type-2 fuzzy submodel output cannot be presented in a closed form, since the Karnik-Mendel type reduction is preferred. To overcome this bottleneck, the switching points of the type reduced set are initialized and then the analytical formulation of the inverse interval type-2 fuzzy subsystem output is explicitly driven (Kumbasar et al., 2011g). Afterwards, the candidate inverse solutions can be calculated under certain conditions. If the conditions are satisfied the interval type-2 fuzzy submodel can be inverted exactly. Finally, the inverse interval type-2 fuzzy model controller is obtained through inversion of each fuzzy submodel (Kumbasar et al., 2011e).

In order to illustrate the decomposition property and decomposable interval type-2 fuzzy model inversion two illustrative examples are presented for each concept. Moreover two simulation studies are presented to show the exactness and the closed loop control performance of the proposed inversion method.

6.1 Decomposition of an Interval Type-2 Fuzzy Model

The decomposition property is a simple constructive procedure which is first introduced for type-1 fuzzy models (Zeng et al., 1996) and generalized for type-2 fuzzy logic systems (Kumbasar et al., 2011f). Based on this property, a type-2 fuzzy model is decomposed into several fuzzy submodels. The decomposition property facilitates the inversion methodology significantly which will be explained in detail in the Subsection 6.2.

Let us consider an interval T2 FM expressed as follows:

$$\begin{aligned} R^{(i_1, i_2, \dots, i_{n_y}, j_1, j_2, \dots, j_{n_u})} : & \text{ IF } y(k+n_d-1) \text{ is } \tilde{A}_1^{i_1} \text{ and } \dots y(k+n_d-p) \text{ is } \tilde{A}_p^{i_p} \text{ and} \\ & \dots \text{ and } y(k+n_d-n_y) \text{ is } \tilde{A}_{n_y}^{i_{n_y}} \text{ and } u(k) \text{ is } \tilde{B}_1^{j_1} \text{ and } \dots \text{ and } u(k-q+1) \text{ is } \tilde{B}_q^{j_q} \\ & \dots \text{ and } u(k-n_u+1) \text{ is } \tilde{B}_{n_u}^{j_{n_u}}, \text{ THEN } y(k+n_d) \text{ is } C_{(i_1, i_2, \dots, i_{n_y}, j_1, j_2, \dots, j_{n_u})} \end{aligned}$$

where n_y is total number of past outputs, while n_u is total number of past inputs and n_d represents the transport delay. $\tilde{A}_p^{i_p}$ are interval type-2 fuzzy sets describing the input variables $y(k+n_d-p)$ ($p=1, \dots, n_y$) and $\tilde{B}_q^{j_q}$ are interval type-2 fuzzy sets describing the variables $u(k-q+1)$ ($q=1, \dots, n_u$) and the consequent interval set is

$$C_{(i_1, i_2, \dots, i_{n_y}, j_1, j_2, \dots, j_{n_u})} = \left[\underline{c}_{(i_1, i_2, \dots, i_{n_y}, j_1, j_2, \dots, j_{n_u})}, \bar{c}_{(i_1, i_2, \dots, i_{n_y}, j_1, j_2, \dots, j_{n_u})} \right].$$

Considering N_p membership functions for describing $y(k+n_d-p)$ ($i_p \in I_p = \{1, 2, \dots, N_p\}$) and M_q membership functions for describing $u(k-q+1)$ ($j_q \in J_q = \{1, 2, \dots, M_q\}$), then the complete rule base is composed of $N = \prod_{p=1, \dots, n} N_p \cdot \prod_{q=1, \dots, f} M_q$ rules with the corresponding index set as $I = I_1 \times I_2 \times \dots \times I_{n_y} \times J_1 \times J_2 \times \dots \times J_{n_u}$.

In this study, each input domain is partitioned with interval type-2 trapezoidal fuzzy sets. The interval T2FM uses fully overlapping trapezoidal sets in the sense of upper and lower fuzzy membership functions. Thus, it is guaranteed that a crisp value of $y(k+n_d-p)$ belongs to $\tilde{A}_p^{i_p}$ and $\tilde{A}_p^{i_{p+1}}$ while a crisp value of $u(k-q+1)$ belongs to $\tilde{B}_q^{j_q}$ and $\tilde{B}_q^{j_{q+1}}$ type-2 fuzzy sets. Then, the property which the upper and lower fuzzy membership functions must satisfy is defined as:

$$\begin{aligned} \bar{\mu}_{\tilde{A}_p^{i_p}} + \underline{\mu}_{\tilde{A}_p^{i_{p+1}}} &= 1 & \underline{\mu}_{\tilde{A}_p^{i_p}} + \bar{\mu}_{\tilde{A}_p^{i_{p+1}}} &= 1 \\ \underline{\mu}_{\tilde{B}_q^{j_q}} + \bar{\mu}_{\tilde{B}_q^{j_{q+1}}} &= 1 & \bar{\mu}_{\tilde{B}_q^{j_q}} + \underline{\mu}_{\tilde{B}_q^{j_{q+1}}} &= 1 \end{aligned} \quad (6.1)$$

However, it should be noted that the decomposition property is not limited to trapezoidal fuzzy sets, any interval type-2 membership functions can be used for decomposition as long as the property in (6.1) is satisfied. This property has been illustrated in Figure 6.1 for a certain universe of discourse.

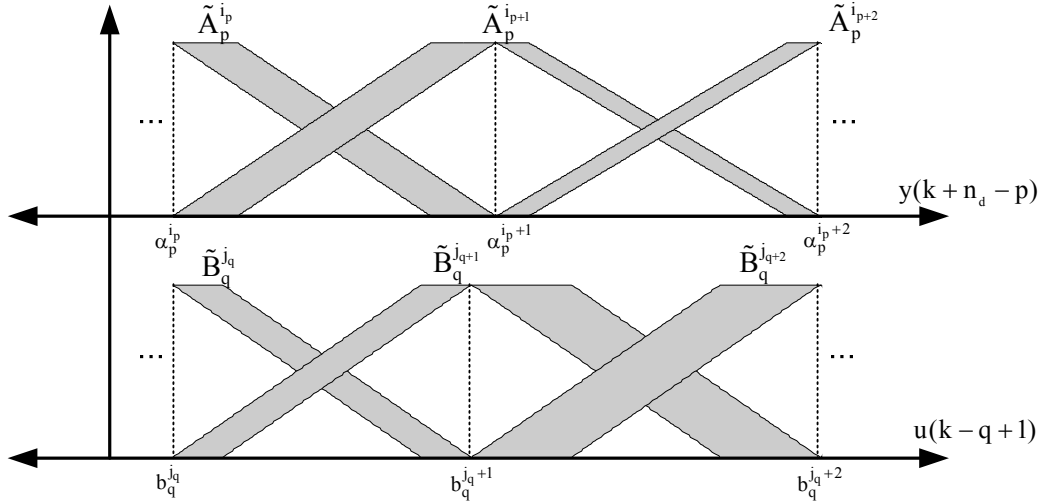


Figure 6.1 : Input Membership functions of the IT2-FM for $y(k + n_d - p)$ and $u(k - q + 1)$.

As a consequence, each partitioned universe of discourse of $y(k + n_d - p)$ can be viewed as the union of the intervals defined by two consecutive modal values of the lower membership functions of $\tilde{A}_k^{i_k}$ and $\tilde{A}_k^{i_{k+1}}$.

$$\tilde{A}_p = \bigcup_{i_p=1, \dots, N_p-1} [\alpha_p^{i_p}, \alpha_p^{i_{p+1}}] \quad (6.2)$$

where $\alpha_p^{i_p}$ represents the modal value of the lower membership function $\tilde{A}_p^{i_p}$. Similarly for each partitioned universe of discourse of $u(k - q + 1)$, $\tilde{B}_q$ can be represented as:

$$\tilde{B}_q = \bigcup_{j_q=1, \dots, M_q-1} [b_q^{j_q}, b_q^{j_{q+1}}] \quad (6.3)$$

where $b_q^{j_q}$ represents the modal value of the lower membership function $\tilde{B}_q^{j_q}$.

Then, the multi-dimensional discourse $\tilde{X} = \tilde{A}_1 \times \dots \times \tilde{A}_p \times \dots \times \tilde{B}_1 \times \dots \times \tilde{B}_q$ is then the union of each $\tilde{X}^{i_1, i_2, \dots, i_p, j_1, j_2, \dots, j_q}$ defined as:

$$\tilde{X}^{i_1, i_2, \dots, i_p, j_1, j_2, \dots, j_q} = [\alpha_1^{i_1}, \alpha_1^{i_2}] \times \dots \times [\alpha_p^{i_p}, \alpha_p^{i_{p+1}}] \times [b_1^{j_1}, b_1^{j_2}] \times \dots \times [b_q^{j_q}, b_q^{j_{q+1}}] \quad (6.4)$$

$$i_p = 1, \dots, N_p - 1, j_q = 1, \dots, M_q - 1$$

As a consequence, the interval type-2 fuzzy model can be decomposed into $\prod_{p=1,\dots,n_y} (N_p - 1) \cdot \prod_{q=1,\dots,n_u} (M_q - 1)$ interval type-2 fuzzy submodels (IT2-FSM).

The IT2-FSMs are composed of the following $2^{n_u+n_y}$ rules:

$$\begin{aligned}
 R_{(v_1, \dots, v_{n_y}, w_1, \dots, w_{n_u})}^{(i_1, i_2, \dots, i_{n_y}, j_1, j_2, \dots, j_{n_u})} : & \text{ IF } y(k) \text{ is } \tilde{A}_1^{i_1+v_1} \dots \text{ and } y(k+1-p) \text{ is } \tilde{A}_p^{i_p+v_p} \dots \\
 & \text{ and } y(k+1-n_y) \text{ is } \tilde{A}_{n_y}^{i_{n_y}+v_{n_y}} \text{ and } u(k) \text{ is } \tilde{B}_1^{j_1+w_1} \text{ and } \dots \\
 & \text{ and } u(k-q+1) \text{ is } \tilde{B}_q^{j_q+w_q} \dots \text{ and } u(k-n_u+1) \text{ is } \tilde{B}_{n_u}^{j_{n_u}+w_{n_u}} \quad (6.5) \\
 & \text{ THEN } y(k+1) \text{ is } C_{(i_1+v_1, \dots, i_{n_y}+v_{n_y}, j_1+w_1, \dots, j_{n_u}+w_{n_u})}; \\
 & v_p, w_q \in \{0, 1\}, p = 1, \dots, n_y, q = 1, \dots, n_u
 \end{aligned}$$

In this structure, $(i_1, i_2, \dots, i_{n_y}, j_1, j_2, \dots, j_{n_u})$ refer to the IT2-FSM to which the rule belongs while $(v_1, \dots, v_{n_y}, w_1, \dots, w_{n_u})$ are the indices of the rules of IT2-FSM. Therefore, the internal type-2 fuzzy model can be viewed as a collection of fuzzy submodels (Kumbasar et al., 2011f).

6.1.1 Illustrative example-1

An IT2 FLS with two inputs (x_1, x_2) and one output (y) is considered in order to provide a more vivid and easier illustration and explanation of this concept (Kumbasar et al., 2011f). The rule base of this illustrative example is given in Table 6.1 and each input domain consists of three IT2 FSs as illustrated in Figure 6.2a. According to the decomposition property, the rule base is partitioned in the four subsystems as shown in Figure 6.2b.

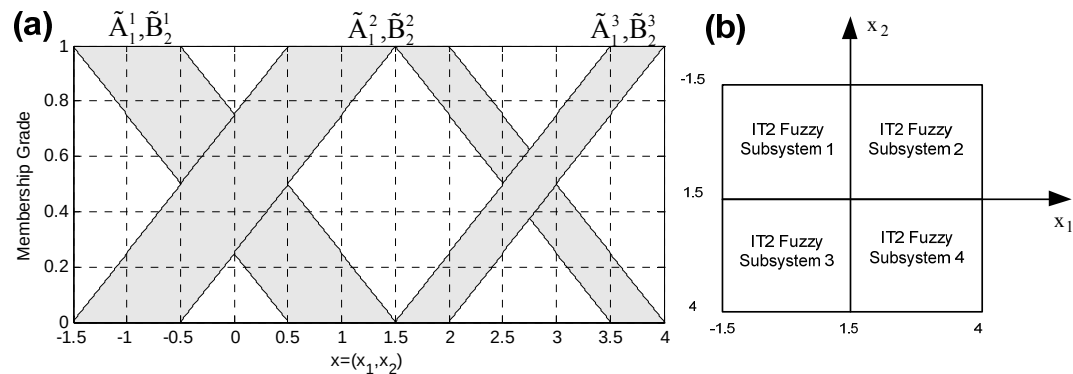


Figure 6.2 : Illustrations of (a) membership functions IT2 for the inputs (x_1, x_2) (b) IT2 fuzzy subsystems of the illustrative example-1.

The input-output mapping of the interval type-2 fuzzy logic system is illustrated in Figure 6.3.

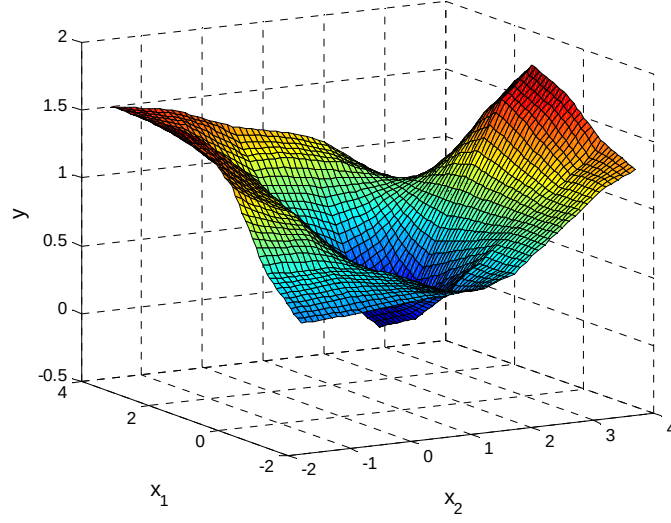


Figure 6.3 : Illustrations of the input-output mapping of the illustrative example-1.

For an input vector $x=(x_1,x_2)=(0.3,3)$, the activated rules are spanned by IT2 Fuzzy Subsystem 3.

Table 6.1: The rulebase of the IT2 FLS for the illustrative example-1.

x_1	x_2		
	$\tilde{B}_1^1$	$\tilde{B}_1^2$	$\tilde{B}_1^3$
$\tilde{A}_1^1$	$C^1 = [\underline{c}_1, \overline{c}_1] = [0.3, 0.5]$	$C^2 = [\underline{c}_2, \overline{c}_2] = [0.4, 0.6]$	$C^3 = [\underline{c}_3, \overline{c}_3] = [1.2, 1.3]$
$\tilde{A}_1^2$	$C^4 = [\underline{c}_4, \overline{c}_4] = [1.4, 1.6]$	$C^5 = [\underline{c}_5, \overline{c}_5] = [-0.1, 0]$	$C^6 = [\underline{c}_6, \overline{c}_6] = [1.7, 1.8]$
$\tilde{A}_1^3$	$C^7 = [\underline{c}_7, \overline{c}_7] = [1.4, 1.6]$	$C^8 = [\underline{c}_8, \overline{c}_8] = [1.1, 1.2]$	$C^9 = [\underline{c}_9, \overline{c}_9] = [0.6, 0.4]$

The firing membership values of the six IT2 FSs are:

$$\begin{aligned}
[\underline{\mu}_{\tilde{A}_1^1}(x_1), \overline{\mu}_{\tilde{A}_1^1}(x_1)] &= [0.00, 0.00] & [\underline{\mu}_{\tilde{B}_1^1}(x_1), \overline{\mu}_{\tilde{B}_1^1}(x_2)] &= [0.05, 0.55] \\
[\underline{\mu}_{\tilde{A}_1^2}(x_1), \overline{\mu}_{\tilde{A}_1^2}(x_1)] &= [0.25, 0.50] & [\underline{\mu}_{\tilde{B}_2^1}(x_1), \overline{\mu}_{\tilde{B}_2^1}(x_2)] &= [0.45, 0.95] \\
[\underline{\mu}_{\tilde{A}_1^3}(x_1), \overline{\mu}_{\tilde{A}_1^3}(x_1)] &= [0.50, 0.75] & [\underline{\mu}_{\tilde{B}_3^1}(x_1), \overline{\mu}_{\tilde{B}_3^1}(x_2)] &= [0.00, 0.00]
\end{aligned}$$

Upon applying the Karnik-Mendel type reduction algorithm , the switching points for the IT2 Fuzzy Subsystem 3 and the overall IT2 Fuzzy system are calculated as

$L^* = 1, R = 3^*$ and $L^* = 1, R = 6^*$, respectively. The type reduced set $(y_{lo}(x), y_{ro}(x))$ of the overall IT2 fuzzy systems is:

$$y_{lo} = \frac{\bar{f}^1 \underline{c}_1 + \bar{f}^2 \underline{c}_2 + \bar{f}^3 \underline{c}_3 + \bar{f}^4 \underline{c}_4 + \bar{f}^5 \underline{c}_5 + \bar{f}^6 \underline{c}_6 + \bar{f}^7 \underline{c}_7 + \bar{f}^8 \underline{c}_8 + \bar{f}^9 \underline{c}_9}{\bar{f}^1 + \bar{f}^2 + \bar{f}^3 + \bar{f}^4 + \bar{f}^5 + \bar{f}^6 + \bar{f}^7 + \bar{f}^8 + \bar{f}^9}$$

$$= 0.4750$$

and

$$y_{ro} = \frac{\underline{f}^1 \bar{c}_1 + \underline{f}^2 \bar{c}_2 + \underline{f}^3 \bar{c}_3 + \underline{f}^4 \bar{c}_4 + \underline{f}^5 \bar{c}_5 + \underline{f}^6 \bar{c}_6 + \underline{f}^7 \bar{c}_7 + \underline{f}^8 \bar{c}_8 + \underline{f}^9 \bar{c}_9}{\underline{f}^1 + \underline{f}^2 + \underline{f}^3 + \underline{f}^4 + \underline{f}^5 + \underline{f}^6 + \underline{f}^7 + \underline{f}^8 + \underline{f}^9}$$

$$= 1.5235$$

Consequently, the crisp output $(y_o(x))$ of the T2-FLS

$$y_o = \frac{y_{lo} + y_{ro}}{2} = 0.9993$$

The type reduced set $(y_{ld}(x), y_{rd}(x))$ of the IT2 fuzzy subsystem is:

$$y_{ld} = \frac{\bar{f}^2 \underline{c}_2 + \bar{f}^3 \underline{c}_3 + \bar{f}^5 \underline{c}_5 + \bar{f}^6 \underline{c}_6}{\bar{f}^2 + \bar{f}^3 + \bar{f}^5 + \bar{f}^6}$$

$$= 0.4750$$

and

$$y_{rd} = \frac{\underline{f}^2 \bar{c}_2 + \underline{f}^3 \bar{c}_3 + \underline{f}^5 \bar{c}_5 + \underline{f}^6 \bar{c}_6}{\underline{f}^2 + \underline{f}^3 + \underline{f}^5 + \underline{f}^6}$$

$$= 1.5235$$

Consequently, the crisp output $y_d(x)$ of the IT2 fuzzy subsystem:

$$y_d = \frac{y_{ld} + y_{rd}}{2} = 0.9993$$

The crisp output of the activated IT2-fuzzy logic subsystem is calculated as 0.9993 which is equal to the calculated crisp output of the overall interval type-2 fuzzy logic system. Moreover, Figure 6.4 shows the input-output mapping of IT2 FLS according to the input (x_1) while x_2 is fixed to the value of 3. It can be clearly seen that for $\forall x_1 \in [-1.5, 1.5]$ only IT2 Fuzzy subsystem-3 is activated while for $\forall x_1 \in [1.5, 4]$

only IT2 Fuzzy subsystem-4 is activated. It has been illustrated that an IT2-FLS can be decomposed into IT2 fuzzy subsystems (Kumbasar et al., 2011f).

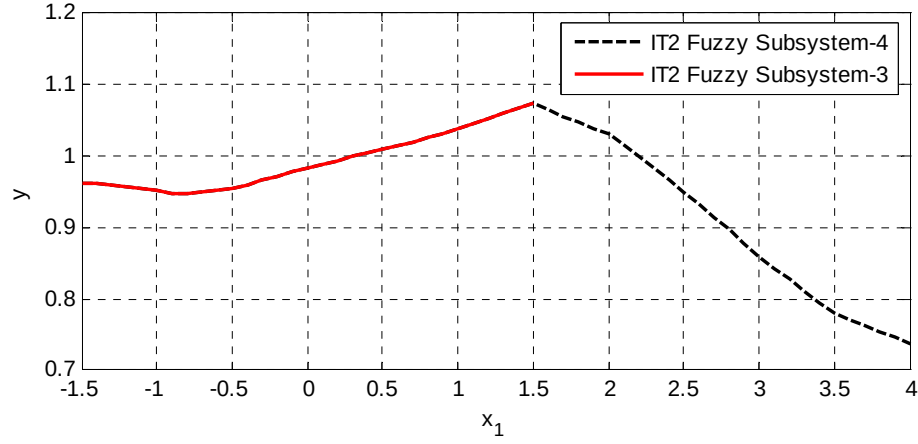


Figure 6.4 : Two dimensional input-output mapping of IT2-FLS of illustrative example-1.

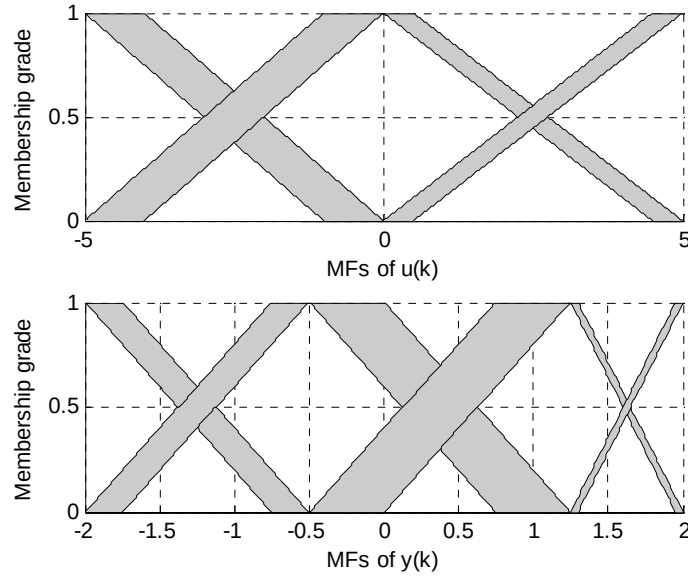
6.1.2 Illustrative example-2

Let us consider a stable SISO IT2-FM with two inputs ($y(k), u(k)$) and one output ($y(k+1)$) to illustrate the decomposition property. The rule base of this illustrative example is given in Table 6.2.

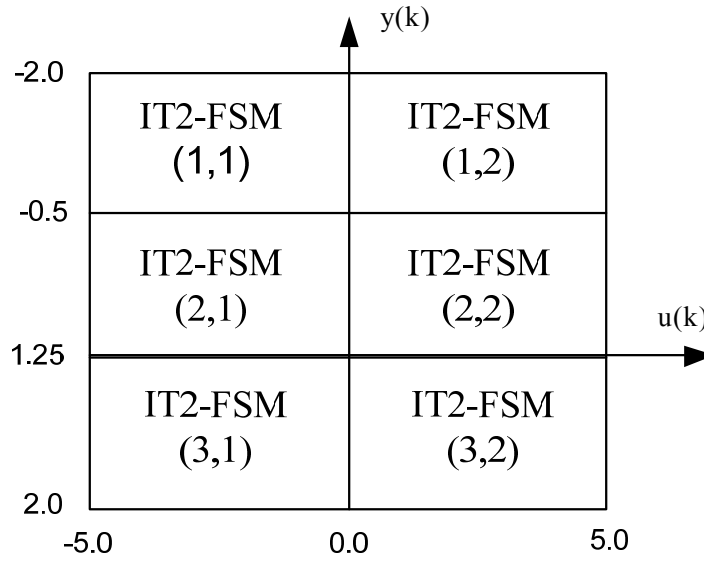
Table 6.2 : The rulebase of the IT2-FM for the illustrative example-2.

$y(k)$	$u(k)$		
	$\tilde{B}_1^1$	$\tilde{B}_1^2$	$\tilde{B}_1^3$
$\tilde{A}_1^1$	$C^1 = [\underline{c}_1, \overline{c}_1] = [-2.0, -1.8]$	$C^2 = [\underline{c}_2, \overline{c}_2] = [-1.6, -1.1]$	$C^3 = [\underline{c}_3, \overline{c}_3] = [-1.7, -1.0]$
$\tilde{A}_1^2$	$C^4 = [\underline{c}_4, \overline{c}_4] = [-1.8, 0.0]$	$C^5 = [\underline{c}_5, \overline{c}_5] = [-0.7, -0.4]$	$C^6 = [\underline{c}_6, \overline{c}_6] = [0.0, 0.6]$
$\tilde{A}_1^3$	$C^7 = [\underline{c}_7, \overline{c}_7] = [0.6, 0.7]$	$C^8 = [\underline{c}_8, \overline{c}_8] = [0.3, 0.5]$	$C^9 = [\underline{c}_9, \overline{c}_9] = [1.5, 1.6]$
$\tilde{A}_1^4$	$C^{10} = [\underline{c}_{10}, \overline{c}_{10}] = [1.2, 1.6]$	$C^{11} = [\underline{c}_{11}, \overline{c}_{11}] = [1.4, 1.5]$	$C^{12} = [\underline{c}_{12}, \overline{c}_{12}] = [1.8, 2.1]$

The input membership functions are illustrated in Figure 6.5a. According to the decomposition property, the rule base is partitioned in the six subsystems as shown in Figure 6.5b.



(a)



(b)

Figure 6.5 : Illustrations of (a) the IT2-FSs of the inputs $(y(k), u(k))$ (b) the decomposed IT2-FM for the illustrative example-2.

If the IT2-FM is excited with an input signal $(u(k))$ which is illustrated in Figure 6.6, then only three IT2-FSMs are activated which are IT2-FSM (2,2), (3,2) and (1,1). For instance, for $\forall y(k) \in [-0.5, 1.25]$ and $\forall u(k) \in [0, 5]$ only IT2-FSM (2, 2) is activated while for $\forall y(k) \in [-2, -0.5]$ and $\forall u(k) \in [-5, 0]$ only IT2-FSM (1, 1) is

activated. It has been illustrated that a certain class of IT2-FM can be decomposed into IT2-FSMs (Kumbasar et al., 2011e).

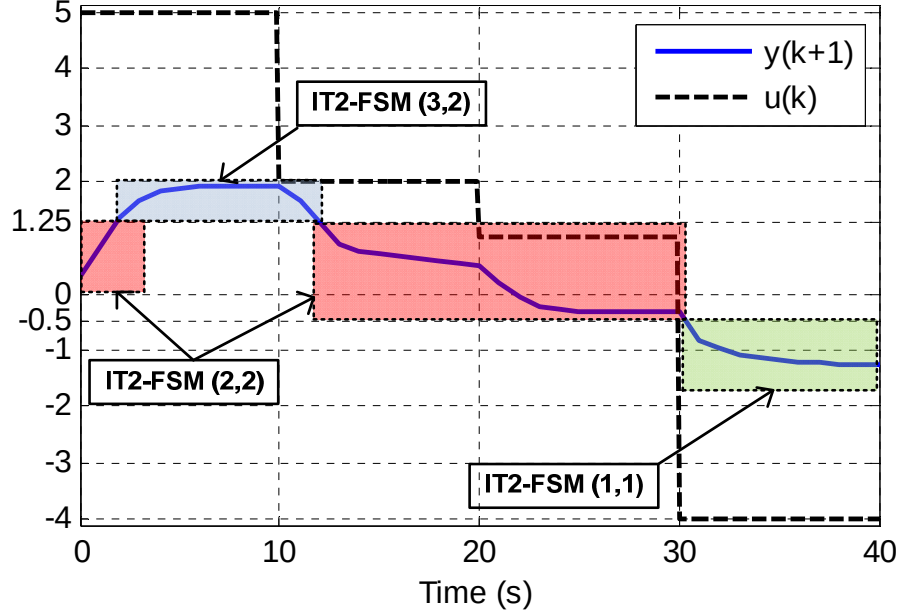


Figure 6.6 : Illustrations of the activated IT2-FSMs of the illustrative example-2.

6.2 Inverse Interval Type-2 Fuzzy Model Controller Design

It is a known fact that, the objective of the inverse control is to calculate an input signal such that the model output (y^*) will become equal to the desired reference signal denoted as y_{des} . To be able to accomplish such a goal, the type reduced set (y_l and y_r) of an IT2-FM are defined as follows:

$$\begin{aligned} y_l &= y^* - \gamma \\ y_r &= y^* + \gamma \end{aligned} \quad (6.6)$$

where γ represents the uncertainty between the defuzzified output to one of the end points of the type reduced set and $\gamma \geq 0$. If γ is equal to zero then the T2-FM will reduce to a T1-FM (Kumbasar et al., 2011g). Figure 6.7 illustrates the type reduced set, the output uncertainty parameter (γ) and the defuzzified output.

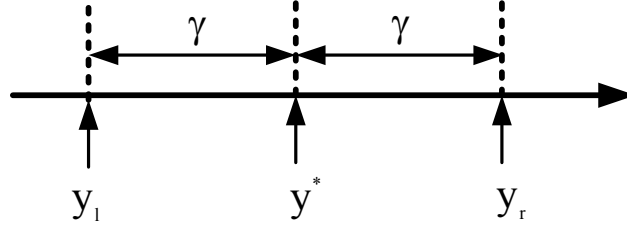


Figure 6.7 : Illustrations of the type reduced set $[y_l(x), y_r(x)]$, output uncertainty parameter (γ) and the defuzzified output $(y^*(x))$.

Therefore, since the output uncertainty parameter (γ) represents the uncertainty between the defuzzified output to one of the end points of the type reduced set, the desired reference values of the type reduced set are must be defined as follows:

$$y_{des_l} = y_{des} - \gamma \quad (6.7)$$

$$y_{des_r} = y_{des} + \gamma \quad (6.8)$$

Here, γ is an unknown parameter and will vary for each input set.

As it has been illustrated in the previous section, for any input vector the IT2-FM can be decomposed to IT2- FSMs with $2^{n_u+n_y}$ rules. Therefore, the proposed inversion methodology will be explained on an IT2- FSM which consists of $N=2^{n_u+n_y}$ rules with interval singleton consequents. For the sake of simplicity, it is assumed there is no time delay ($n_d = 1$). Then, the rules are defined in the form as:

$$\begin{aligned} R_{(i_1, \dots, i_{n_y}, j_1, \dots, j_{n_u})}^{(v_1, \dots, v_{n_y}, w_1, \dots, w_{n_u})} : & \text{ IF } y(k) \text{ is } \tilde{A}_1^{i_1+v_1} \dots \text{ and } y(k+1-p) \text{ is } \tilde{A}_p^{i_p+v_p} \dots \\ & \text{ and } y(k+1-n_y) \text{ is } \tilde{A}_{n_y}^{i_{n_y}+v_{n_y}} \text{ and } u(k) \text{ is } \tilde{B}_1^{j_1+w_1} \text{ and } \dots \\ & \text{ and } u(k-q+1) \text{ is } \tilde{B}_q^{j_q+w_q} \dots \text{ and } u(k-n_u+1) \text{ is } \tilde{B}_{n_u}^{j_{n_u}+w_{n_u}} \\ & \text{ THEN } y(k+1) \text{ is } C_{(i_1+v_1, \dots, i_{n_y}+v_{n_y}, j_1+w_1, \dots, j_{n_u}+w_{n_u})}; \\ & v_p, w_q \in \{0, 1\}, p = 1, \dots, n_y, q = 1, \dots, n_u \end{aligned}$$

where each input domain is represented with two interval type-2 trapezoidal membership functions ($i_p \in I_p = \{1, 2\}, j_q \in J_q = \{1, 2\}$) and the consequent interval set is $C_j = [\underline{c}_j, \overline{c}_j]; j = 1, \dots, N$ ($N=2^{n_u+n_y}$).

The firing strengths for each input are defined as follows:

$$\begin{aligned}
\mu_{\tilde{A}_1^1} &= [\underline{\mu}_{\tilde{A}_1^1}, \bar{\mu}_{\tilde{A}_1^1}] & \mu_{\tilde{B}_1^1} &= [\underline{\mu}_{\tilde{B}_1^1}, \bar{\mu}_{\tilde{B}_1^1}] \\
\mu_{\tilde{A}_1^2} &= [\underline{\mu}_{\tilde{A}_1^2}, \bar{\mu}_{\tilde{A}_1^2}] & \mu_{\tilde{B}_1^2} &= [\underline{\mu}_{\tilde{B}_1^2}, \bar{\mu}_{\tilde{B}_1^2}] \\
&\vdots & & \vdots \\
\mu_{\tilde{A}_{n_y}^2} &= [\underline{\mu}_{\tilde{A}_{n_y}^2}, \bar{\mu}_{\tilde{A}_{n_y}^2}] & \mu_{\tilde{B}_{n_u}^2} &= [\underline{\mu}_{\tilde{B}_{n_u}^2}, \bar{\mu}_{\tilde{B}_{n_u}^2}]
\end{aligned} \tag{6.10}$$

The total firing set for each rule is defined as:

$$f^j = \begin{bmatrix} \underline{f}^j & \bar{f}^j \end{bmatrix}; j = 1, \dots, N=2^{n_u+n_y} \tag{6.11}$$

where

$$\begin{aligned}
\underline{f}^j &= \underline{\mu}_{\tilde{A}_1^{i_1}} \cap \dots \cap \underline{\mu}_{\tilde{A}_{n_u}^{i_2}} \cap \underline{\mu}_{\tilde{B}_1^{j_1}} \cap \dots \cap \underline{\mu}_{\tilde{B}_{n_y}^{j_2}} \\
\bar{f}^j &= \bar{\mu}_{\tilde{A}_1^{i_1}} \cap \dots \cap \bar{\mu}_{\tilde{A}_{n_u}^{i_2}} \cap \bar{\mu}_{\tilde{B}_1^{j_1}} \cap \dots \cap \bar{\mu}_{\tilde{B}_{n_y}^{j_2}}
\end{aligned} \tag{6.12}$$

For the sake of simplicity, it is assumed that the interval consequent set is in order such that $\underline{c}_1 \leq \underline{c}_2 \leq \dots \leq \underline{c}_N$, $\bar{c}_1 \leq \bar{c}_2 \leq \dots \leq \bar{c}_N$ and the corresponding weights $\bar{f}$ and $\underline{f}$ are matched. Reminding that $u(k)$ is the manipulated variable and the corresponding membership grades are $\mu_{B_1^1} = [\underline{\mu}_{B_1^1}, \bar{\mu}_{B_1^1}]$, $\mu_{B_1^2} = [\underline{\mu}_{B_1^2}, \bar{\mu}_{B_1^2}]$. Equating y_{des_l} to y_l from (2.16) and y_{des_r} to y_r from (2.17)

$$y_{des_l} = y_{des} - \gamma = \frac{\sum_{j=1}^{L^*} \bar{f}^j \underline{c}_j + \sum_{j=L^*+1}^N \underline{f}^j \bar{c}_j}{\sum_{j=1}^{R^*} \bar{f}^j + \sum_{j=L^*+1}^N \underline{f}^j} \tag{6.13}$$

$$y_{des_r} = y_{des} + \gamma = \frac{\sum_{j=1}^{R^*} \underline{f}^j \bar{c}_j + \sum_{j=R+1}^N \bar{f}^j \underline{c}_j}{\sum_{j=1}^{R^*} \underline{f}^j + \sum_{j=R+1}^N \bar{f}^j} \tag{6.14}$$

After some mathematical manipulations, (6.13) and (6.14) can be reformulated as:

$$\psi_1^L(\cdot) \bar{\mu}_{B_1^1} + \psi_2^L(\cdot) \bar{\mu}_{B_1^2} + \psi_3^L(\cdot) \underline{\mu}_{B_1^2} + \psi_4^L(\cdot) \underline{\mu}_{B_1^1} = 0 \tag{6.15}$$

$$\psi_1^R(\cdot) \underline{\mu}_{B_1^1} + \psi_2^R(\cdot) \underline{\mu}_{B_1^2} + \psi_3^R(\cdot) \bar{\mu}_{B_1^2} + \psi_4^R(\cdot) \bar{\mu}_{B_1^1} = 0 \tag{6.16}$$

where $\psi_i^L(.)$ and $\psi_i^R(.)$ are functions of a possible combination of the following parameters: $\underline{\mu}\tilde{A}_1^1, \bar{\mu}\tilde{A}_1^1, \dots, \underline{\mu}\tilde{A}_{n_y}^2, \bar{\mu}\tilde{A}_{n_y}^2, \underline{\mu}\tilde{B}_2^1, \bar{\mu}\tilde{B}_2^1, \dots, \underline{\mu}\tilde{B}_{n_u}^2, \bar{\mu}\tilde{B}_{n_u}^2, y_{des}, \underline{c}_j, \bar{c}_j, g, R^*, L^*$ for $i=1, \dots, 4$.

Combing (6.1), (6.15) and (6.16),

$$\underbrace{\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ \psi_1^L(.) & \psi_2^L(.) & \psi_3^L(.) & \psi_4^L(.) \\ \psi_1^R(.) & \psi_2^R(.) & \psi_3^R(.) & \psi_4^R(.) \end{bmatrix}}_A x = \underbrace{\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}}_b \quad (6.17)$$

where $x = [\bar{\mu}\tilde{B}_1^1 \quad \bar{\mu}\tilde{B}_1^2 \quad \underline{\mu}\tilde{B}_1^2 \quad \underline{\mu}\tilde{B}_1^1]^T$ is the solution set.

When an input signal activates the IT2-FSM, the desired reference value, i.e. y_{des} and the corresponding consequent interval parameters $(\underline{c}_j, \bar{c}_j)$ are known; while the membership grades of the measured states $(\underline{\mu}\tilde{A}_1^1, \bar{\mu}\tilde{A}_1^1, \dots, \underline{\mu}\tilde{A}_{n_y}^2, \bar{\mu}\tilde{A}_{n_y}^2, \underline{\mu}\tilde{B}_2^1, \bar{\mu}\tilde{B}_2^1, \dots, \underline{\mu}\tilde{B}_{n_u}^2, \bar{\mu}\tilde{B}_{n_u}^2)$ are calculated. The only unknown parameters are γ , L^* and R^* . Since calculating L^* and R^* depends on the solution set $x = [\bar{\mu}\tilde{B}_1^1 \quad \bar{\mu}\tilde{B}_1^2 \quad \underline{\mu}\tilde{B}_1^2 \quad \underline{\mu}\tilde{B}_1^1]^T$, these parameters cannot be predetermined.

Therefore, each combination of $L, R \in \{1, \dots, N-1\}$ ($N=2^{n_u+n_y}$) should be tested iteratively. At each iteration, it will be assumed that the initialized values are the extrema points (L^* , R^*) of (2.19) and (2.21), respectively. After initializing L and R , the solution set can be found via following simple linear matrix equation:

$$x = A^{-1}b \quad (6.18)$$

such that $x \in [0, 1]$ and where $\forall x_i \in x$ is function of γ .

Lemma 1

For certain L and R values, the defined A matrix in (6.17) is invertible if and only if $\gamma > 0$ and the universe of discourse each input variable is fully partitioned in the sense the upper and lower fuzzy membership functions (Kumbasar et al., 2011f).

Proof:

(i) Since the input membership functions are type-2 fuzzy sets, and because of the definition of the IT2 FSs the membership values of the upper and lower membership functions will be always be different $\underline{\mu}_{\tilde{B}_q}^i \neq \overline{\mu}_{\tilde{B}_q}^i, \underline{\mu}_{\tilde{A}_p}^j \neq \overline{\mu}_{\tilde{A}_p}^j$ and likewise since the output consequents are interval sets $\underline{c}_j \neq \overline{c}_j$.

(ii) Since the universe of discourse each input variable is fully partitioned in the sense the upper and lower fuzzy membership function, the upper and lower fuzzy membership functions satisfy the following property for each input set given in (6.1).

(iii) From (i) it can be concluded that for all $\psi_i^L(.)$ in (6.15), the following condition satisfies $\psi_m^L(.) \neq \psi_n^L(.) \quad \forall m \neq n, m, n = 1 \dots 4$ while for all $\psi_i^R(.)$ in (6.16), the following condition satisfies, $\psi_m^R(.) \neq \psi_n^R(.) \quad \forall m \neq n, m, n = 1 \dots 4$. Naturally all $\psi_i^L(.)$ and $\psi_i^R(.)$ get distinct values.

(iv) From (i) and since the Karnik-Mendel type reducer computes always the two distinct end points of the type reduced interval set for $\gamma > 0$, the coefficients of (6.15) and (6.16) will be distinct, i.e., $\psi_m^R(.) \neq \psi_n^L(.)$.

Therefore, in the light of (ii), (iii) and (iv) stated above the rank of the A matrix will always be in full rank for certain L and R values. $\square$

The following assertions can be made on the existence of inverse solution according to the output uncertainty parameter (γ) in three cases:

Case 1: If the desired set value is not in the defined universe of discourse then the output uncertainty parameter γ does not exist then an inverse solution cannot be found.

Case 2: If $\gamma > 0$ and there does not exist any γ which satisfies $\forall x_i(\gamma) \in [0,1]$ then an inverse solution cannot be found for certain L and R values.

Case 3: If $\gamma > 0$ and there exists γ which satisfies $\forall x_i(\gamma) \in [0,1]$, it must be checked whether the solution set ($x(\gamma)$) lies on the defined type-2 fuzzy membership functions. Then, a candidate inverse solution set can be calculated for the certain L and R values.

For **Case 3**, the membership functions of manipulated variable ($u(k)$) are partitioned in three regions as it has been illustrated in Figure 6.8. As a consequence, γ must satisfy at least one of the following equalities.

$$(a) \quad \bar{\mu}_{\tilde{B}_1^1}(\gamma) = 1 \quad \underline{\mu}_{\tilde{B}_1^2}(\gamma) = 0 \quad \underline{\mu}_{\tilde{B}_1^1}(\gamma) \geq \beta \quad \bar{\mu}_{\tilde{B}_1^2}(\gamma) \leq 1 - \beta \quad (6.19a)$$

$$(b) \quad \bar{\mu}_{\tilde{B}_1^2}(\gamma) - \underline{\mu}_{\tilde{B}_1^2}(\gamma) = 1 - \beta \quad \bar{\mu}_{\tilde{B}_1^1}(\gamma) - \underline{\mu}_{\tilde{B}_1^1}(\gamma) = 1 - \beta \quad (6.19b)$$

$$(c) \quad \bar{\mu}_{\tilde{B}_1^2}(\gamma) = 1 \quad \underline{\mu}_{\tilde{B}_1^1}(\gamma) = 0 \quad \underline{\mu}_{\tilde{B}_1^2}(\gamma) \geq \beta \quad \bar{\mu}_{\tilde{B}_1^1}(\gamma) \leq 1 - \beta \quad (6.19c)$$

where β is the limiting membership grade of $\tilde{B}_1^1$ and $\tilde{B}_1^2$; z_1, z_2, z_3, z_4 are the limiting points of each region as shown in Figure 6.8.

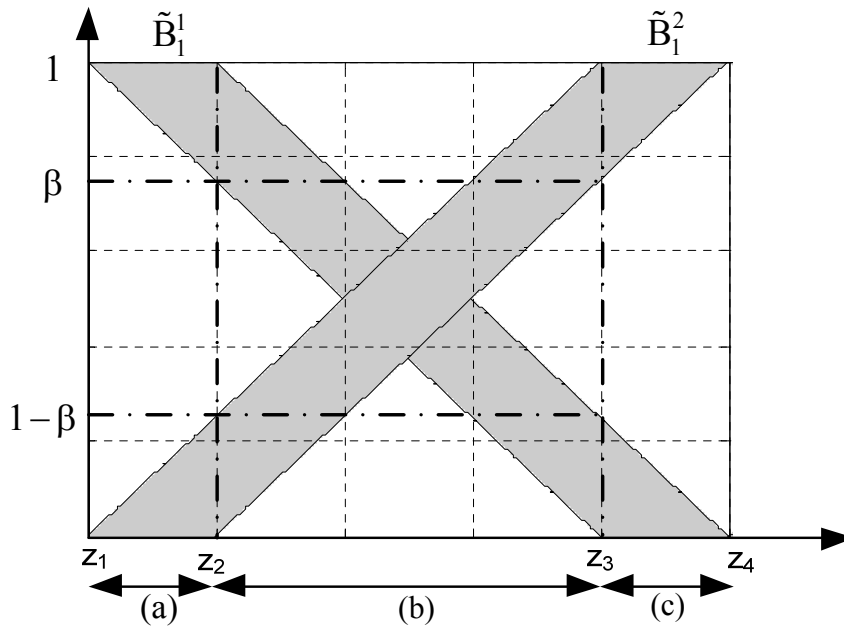


Figure 6.8 : Illustrations of the partitioned IT2-FS of the manipulated variable.

Moreover, from many possible candidate solutions the inverse solution set(s) is/are the one(s) which satisfies the conditions in Lemma-2.

Lemma 2

If the following conditions are satisfied then the certain L and R are the solutions which minimize/ maximize (2.19) and (2.21), respectively.

$$a) \text{ If } L^* = \arg \min_{1 \leq L \leq N-1} \{ y_1^{L*} \} \text{ then } y_1^{L*}(x) < y_1^L \text{ for } \forall L (L \neq L^*).$$

$$b) \text{ If } R^* = \arg \max_{1 \leq R \leq N-1} \{ y_r^{R*} \} \text{ then } y_r^{R*} > y_r^R \text{ for } \forall R (R \neq R^*).$$

Proof: The Karnik-Mendel type reduction algorithm has been proven to converge monotonically and there exists only one solution set of (L, R) which minimize/maximize (2.19) and (2.21), respectively (Mendel and Liu, 2007; Wu and Mendel, 2011). $\square$

If there exists γ^* which satisfies conditions of Lemma-2, then defuzzify the solution set $x(\gamma^*)$ and obtain the crisp value of the manipulated variable ($u(k)$). Otherwise, the calculated candidate solution set $x(\gamma^*)$ is not an inverse solution set.

While obtaining the inverse solution set from the proposed iterative method based on an analytical methodology, two important issues must be considered which are the existence of an inverse solution and the possibility of multiple solutions (Boukezzoula et al., 2007). Reminding that during the proposed inversion procedure, the two end points of the type reduced set are equated as $y_l = y_{des} - \gamma$ and $y_r = y_{des} + \gamma$, it may be observed that $y_{des} - \gamma^* \notin y_l$ and $y_{des} + \gamma^* \notin y_r$ for any combination of L and R values ($L, R \in \{1, \dots, N-1\}$). Then the desired reference is not achievable and naturally no inverse solution will exist. In this case, this problem can simply be solved by reducing reference signal until an inverse solution exists. Another issue about calculation of the inverse solution is the possibility of the existence of multiple inverse solutions. In this case, a selection criterion should be defined such as minimum/maximum energy or variation of the manipulated variable (Kumbasar, et al. 2011f).

As it has been illustrated in subsection 6.1, a decomposable interval type-2 fuzzy model with N_p and M_q membership functions for describing $y(k + n_d - p)$ and $u(k - q + 1)$ can be decomposed to $S = \prod_{p=1, \dots, n} (N_p - 1) \cdot \prod_{q=1, \dots, f} (M_q - 1)$ IT2-FSMs.

Based on this fact, the inversion methodology for an IT2-FSM can be generalized and naturally the general IT2-FM inversion can be accomplished. To achieve such a goal, the inversion method must be applied to each possible activated interval type-2 fuzzy submodel. Consequently, as the number of inputs ($n_u + n_y$) and membership functions ($N_p + M_q$) increase, the computational complexity will increase which is mainly caused of the initialization procedure in the proposed inversion method. On the other hand, since the inversion is obtained analytically, the exactness of this method is guaranteed. The general IT2-FM inversion algorithm is given in Table 6.3

(Kumbasar et al., 2011e; Kumbasar et al., 2011f). If there exists any time delay in the system then the method should be applied after n_d steps of prediction have been performed, similar to what has to be done for the case of the type-1 TS fuzzy model inversion (Babuska, 1998).

Table 6.3 : Inversion algorithm of decomposable interval type-2 fuzzy models.

Step 1	Decompose the IT2 FM, $S = \prod_{p=1,\dots,n} (N_p - 1) \cdot \prod_{q=1,\dots,f} (M_q - 1)$ IT2 fuzzy submodels. Determine the possible activated IT2 Fuzzy submodels.
Step 2	Calculate interval the firing strength for each rule: f^j Sort c_j ($j=1,2,\dots,N$) in increasing order. Match the corresponding weights $\bar{f}^j, \underline{f}^j$, Sort $\bar{c}_j$ ($i=1, 2\dots N$) in increasing order. Match the corresponding weights $\bar{f}^j, \underline{f}^j$ Set $L_0=1$ and $R_0=1$ or $L_0=\frac{N}{2.4}$ and $R_0=\frac{N}{1.7}$ (Enhanced Karnik Mendel initialization (Wu and Mendel, 2009))
Step 3	Equate $y_l = y_{des} - \gamma$, $y_r = y_{des} + \gamma$ and obtain the A Matrix in (6.17). Find the candidate inverse solution set $x(\gamma)$ from $x = A^{-1}b$ such that $x(\gamma) \in [0,1]$.
Step 4	Check for each partitioned region of the membership functions of manipulated variable via (6.19a-b-c) whether the inverse solution set lies on the defined IT2-FS for γ . If not go to Step 2 and change L/R value, otherwise calculate the inverse solution set $x(\gamma^*)$
Step 5	Replace the calculated the solution set $x(\gamma^*)$ in (2.16) and (2.17) and check the conditions of Lemma-2. If both conditions are satisfied $x(\gamma^*)$ is an inverse solution, then defuzzify the solution and obtain the crisp value of the manipulated variable. Otherwise change L/R value go to Step 2
Step 6	If there does not exist any solution, the desired reference value is not achievable. In the case of multiple inverse solutions, a selection is done basis of additional criteria. Apply the inverse solution to the IT2-FM.

6.2.1 Illustrative example-3

In this example, the proposed interval type-2 fuzzy system inversion based on an analytical methodology is illustrated. The rule base of this example is given in Table 6.4, and each input domain consists of two IT2 FSs as illustrated in Figure 6.9 (Wu, 2011).

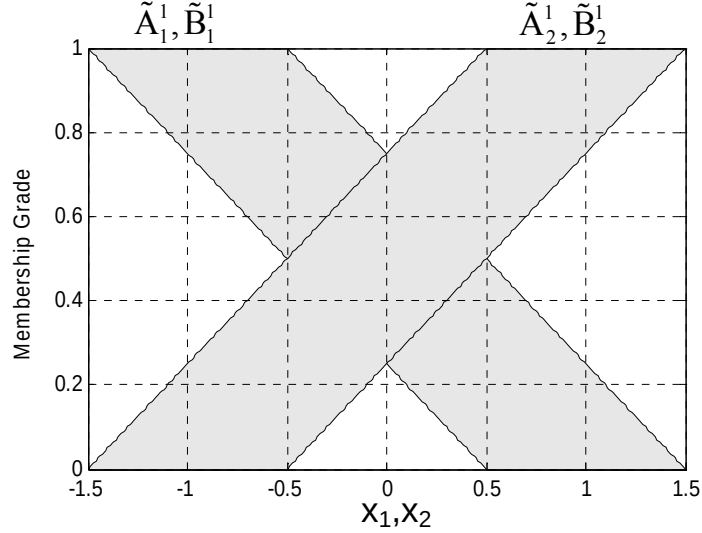


Figure 6.9 : IT2 Membership functions for the inputs $x=(x_1, x_2)$ in illustrative example-3.

Table 6.4 : Rulebase and consequents of the IT2 FLS in illustrative example-3.

x_2	x_1	
	$\tilde{B}_1^1$	$\tilde{B}_2^1$
$\tilde{A}_1^1$	$C_1 = [\underline{c}_1, \overline{c}_1] = [-1, -0.9]$	$C_2 = [\underline{c}_2, \overline{c}_2] = [-0.6, -0.4]$
$\tilde{A}_2^1$	$C_3 = [\underline{c}_2, \overline{c}_2] = [0.4, 0.6]$	$C_4 = [\underline{c}_4, \overline{c}_4] = [0.9, 1]$

Three distinct cases are examined depending on x_2 and desired reference values. In the first case $x_2 = -0.3$ and the desired setpoint is $y_{des} = -0.0463$. For the first case the membership values for $x_2 = -0.3$ are:

$$\begin{aligned} [\underline{\mu}\tilde{A}_1^1 \quad \underline{\mu}\tilde{A}_2^1] &= [0.9 \quad 0.4] \\ [\underline{\mu}\tilde{A}_1^1 \quad \underline{\mu}\tilde{A}_2^1] &= [0.6 \quad 0.1] \end{aligned}$$

The desired reference values are defined as $y_{des_l}(x) = (-0.0463) - \gamma$ and $y_{des_r}(x) = (-0.0463) + \gamma$. Let us examine the case of $L=1, R=3$ as example. For this L and R values, the linear equation set can be found as:

$$\underbrace{\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ \psi_1^L(.) & \psi_2^L(.) & \psi_3^L(.) & \psi_4^L(.) \\ \psi_1^R(.) & \psi_2^R(.) & \psi_3^R(.) & \psi_4^R(.) \end{bmatrix}}_A \mathbf{x} = \underbrace{\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}}_b \quad (6.20)$$

where $\psi_1^L(.) = -\frac{9}{10} - \frac{9}{10} \left(-\frac{463}{10000} - \gamma \right)$, $\psi_2^L(.) = 0$, $\psi_3^L(.) = -\frac{3}{20} + \frac{1}{2} \left(\frac{463}{10000} + \gamma \right)$,

$\psi_4^L(.) = \frac{1}{25} + \frac{1}{10} \left(\frac{463}{10000} + \gamma \right)$, $\psi_1^R(.) = 0$, $\psi_2^R(.) = \frac{3}{5} - \frac{3}{5} \left(-\frac{463}{10000} + \gamma \right)$,

$\psi_3^R(.) = -\frac{4}{25} - \frac{2}{5} \left(-\frac{463}{10000} + \gamma \right)$ and $\psi_4^R(.) = -\frac{3}{10} + \frac{1}{2} \left(\frac{463}{10000} - \gamma \right)$.

The solution set $\mathbf{x}(\gamma)$ can be found from $\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$ such that $\mathbf{x}(\gamma) \in [0,1]$. The A matrix is always invertible if $\gamma > 0$ (Lemma-1). After checking for each partitioned region of the membership functions via **(6.19a)-(6.19b)-(6.19c)** it is determined that only **(6.19c)** is satisfied for $\gamma = 0.6709$. As consequence there is only one possible

inverse solution which is $\mathbf{x}(\gamma) = [\underline{\mu}\tilde{\mathbf{B}}_1^1 \quad \underline{\mu}\tilde{\mathbf{B}}_1^2 \quad \underline{\mu}\tilde{\mathbf{B}}_1^2 \quad \underline{\mu}\tilde{\mathbf{B}}_1^1]^T = [0.45 \quad 1 \quad 0.55 \quad 0]^T$.

After replacing $\mathbf{x}(\gamma)$ in **(2.16)** and **(2.17)**, the conditions given in Lemma-2 must be checked. For $\gamma = 0.6709$, $L^*=1$ and $R^*=3$, the values of y_1^{L*} and y_r^{R*} are calculated as -0.7169 and 0.6244, respectively. It can easily be determined that for $\forall L$ ($L \neq 1$) y_1^{L*} is the minimum of y_1^L while $\forall R$ ($R \neq 3$) that y_r^{R*} is the maximum of y_r^R . So for $\gamma^* = 0.6709$, $L^*=1$ and $R^*=3$, the conditions in Lemma-2 are satisfied. Therefore $\mathbf{x}(\gamma^*)$ is an inverse solution, and the corresponding crisp value is calculated as $x_1=0.6$. In Table 6.5, the cases of different values of L and R are presented (Kumbasar et al., 2011g). It can be clearly seen that only for $\gamma^* = 0.6709$, $L^*=1$ and $R^*=3$ an inverse solution exists.

In the second case ($x_2=0.4$, $y_{des} = 0.0781$), the existence of multiple inverse solutions has been encountered. As it has been illustrated in Table 6.5, the proposed inversion procedure generates three inverse solutions for the manipulated variable (x_1). All solutions will force the system output to the desired setpoint. However, in this case an extra selection criterion such as minimum/maximum energy is needed to choose one of the solutions.

In the third case, if $y_{des} = 0.8$ and $x_2=0.4$, an inverse solution does not exist for any (R, L) combination.

Table 6.5 : The inverse solution sets for all possible values of L and R.

L	R	For $x_2=-0.3$			For $x_2=0.4$		
		γ^* Values (Lemma-1)	Conditions of Lemma-2	Inverse Solution	γ^* Values (Lemma-1)	Conditions of Lemma-2	Inverse Solution
1	1	$\gamma^* = \{\}$	Not Satisfied	No Solution	$\gamma^* = \{\}$	Not Satisfied	No Solution
1	2	$\gamma^* = 0.6579$	Not Satisfied	No Solution	$\gamma^* = 0.6413$ $\gamma^* = 0.4505$ $\gamma^* = 0.5748$	Not Satisfied Not Satisfied Not Satisfied	No Solution
1	3	$\gamma^* = 0.6709$	Satisfied	$x_1 = 0.6$	$\gamma^* = 0.4981$	Not Satisfied	No Solution
2	1	$\gamma^* = \{\}$	Not Satisfied	No Solution	$\gamma^* = \{\}$	Not Satisfied	No Solution
2	2	$\gamma^* = 0.6515$	Not Satisfied	No Solution	$\gamma^* = 0.6455$ $\gamma^* = 0.4693$ $\gamma^* = 0.4930$	Not Satisfied Satisfied Satisfied	No Solution $x_1 = -1.4329$ $x_1 = -1.3081$
2	3	$\gamma^* = 0.6523$	Not Satisfied	No Solution	$\gamma^* = 0.5288$	Satisfied	$x_1 = -1.1527$
3	1	$\gamma^* = \{\}$	Not Satisfied	No Solution	$\gamma^* = \{\}$	Not Satisfied	No Solution
3	2	$\gamma^* = \{\}$	Not Satisfied	No Solution	$\gamma^* = \{\}$	Not Satisfied	No Solution
3	3	$\gamma^* = \{\}$	Not Satisfied	No Solution	$\gamma^* = \{\}$	Not Satisfied	No Solution

6.2.2 Illustrative example-4

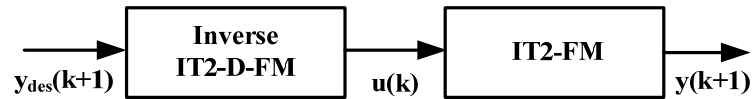


Figure 6.10 : Interval type-2 decomposable fuzzy model inverse controller in an open loop scheme.

In order to validate the proposed generalized inversion strategy, a simulation study is performed on the presented IT2 –FM given in subsection 6.1.2 (Kumbasar et al., 2011e). The desired set-point trajectory that should be tracked by the IT2-FM is chosen as $y_{des} = 0.4 + 0.6\sin(k)$. The general scheme of the inversion scheme is illustrated in Figure 6.10. In the case of existence of multiple inverse solutions, the minimum energy criterion is used for selection. According the minimum energy criteria inverse solution set of $u(k)$ is plotted in Figure 6.11a. As it can be clearly

seen in Figure 6.11b, the output produced by the IT2-FM converges to the desired reference trajectory.

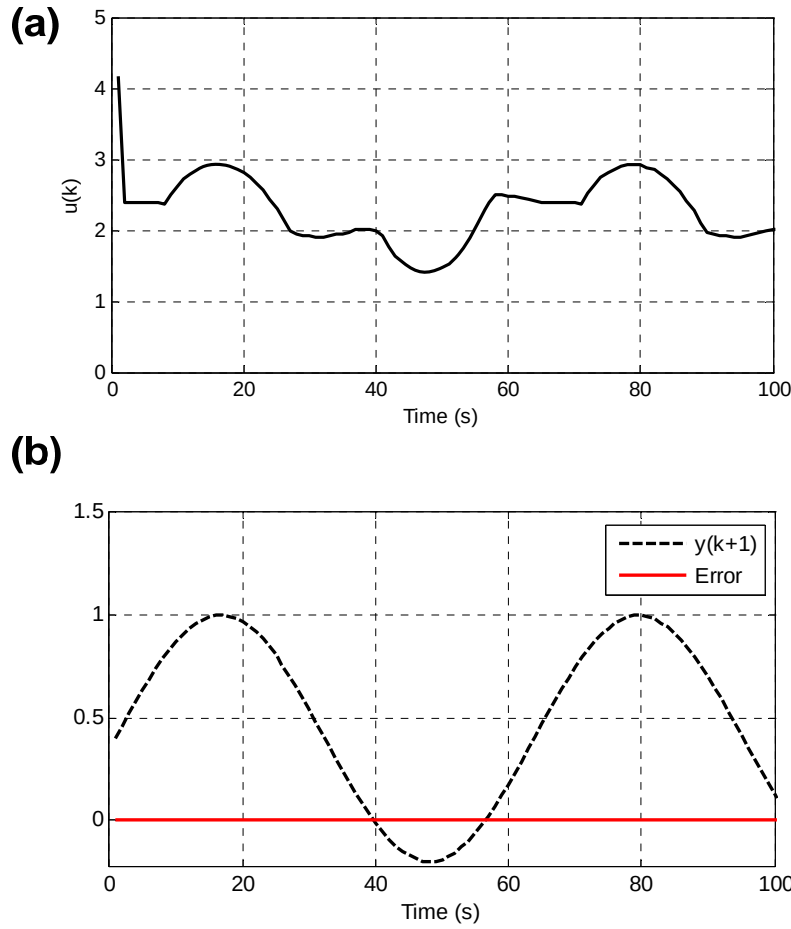


Figure 6.11 : Illustrations of (a) the calculated inverse control signal ($u(k)$) (b) the IT2-FM output ($y(k+1)$) and the tracking error.

6.3 Simulation Study-1

In order to show the exactness of the proposed inversion strategy, a simulation study is performed on the presented IT2 –FLS given in subsection 6.1.1 (Kumbasar et al., 2011f). The shared variable x_2 is a random number uniformly distributed in $[0, 2]$. The desired setpoint trajectory that should be tracked by the IT2-FLS is chosen as $S_{des}=0.5+0.2*\sin(k)$. The general scheme of the inversion scheme is illustrated in Figure 6.12. According the maximum and minimum energy criteria two inverse solution sets of x_1 are plotted in Figure 6.13. The exactness is validated by randomly selecting one of the all possible generated inverse solutions which is plotted in Figure 6.14a. As it can be clearly seen in Figure 6.14b, the output produced by the fuzzy

system converges to the desired setpoint trajectory, i.e. $y = S_{des}$. This confirms that the inversion is exact.

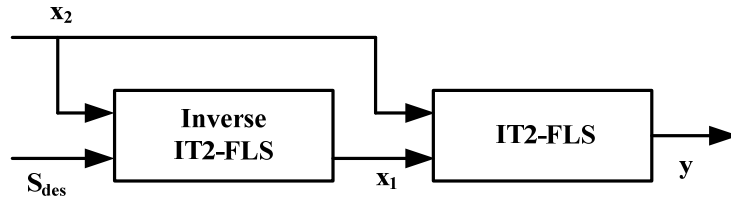


Figure 6.12 : Interval type-2 decomposable fuzzy logic system inversion scheme.

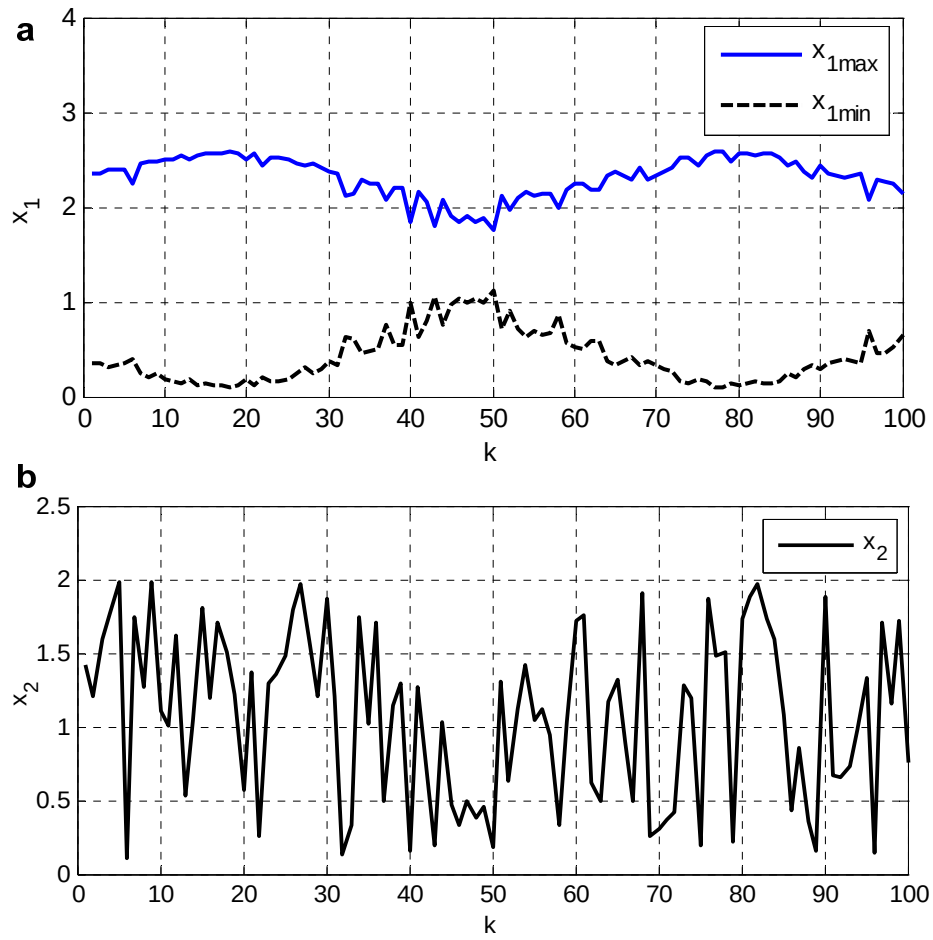


Figure 6.13 : Illustrations of (a) calculated inverse solutions (x_1) (b) shared variable x_2 .

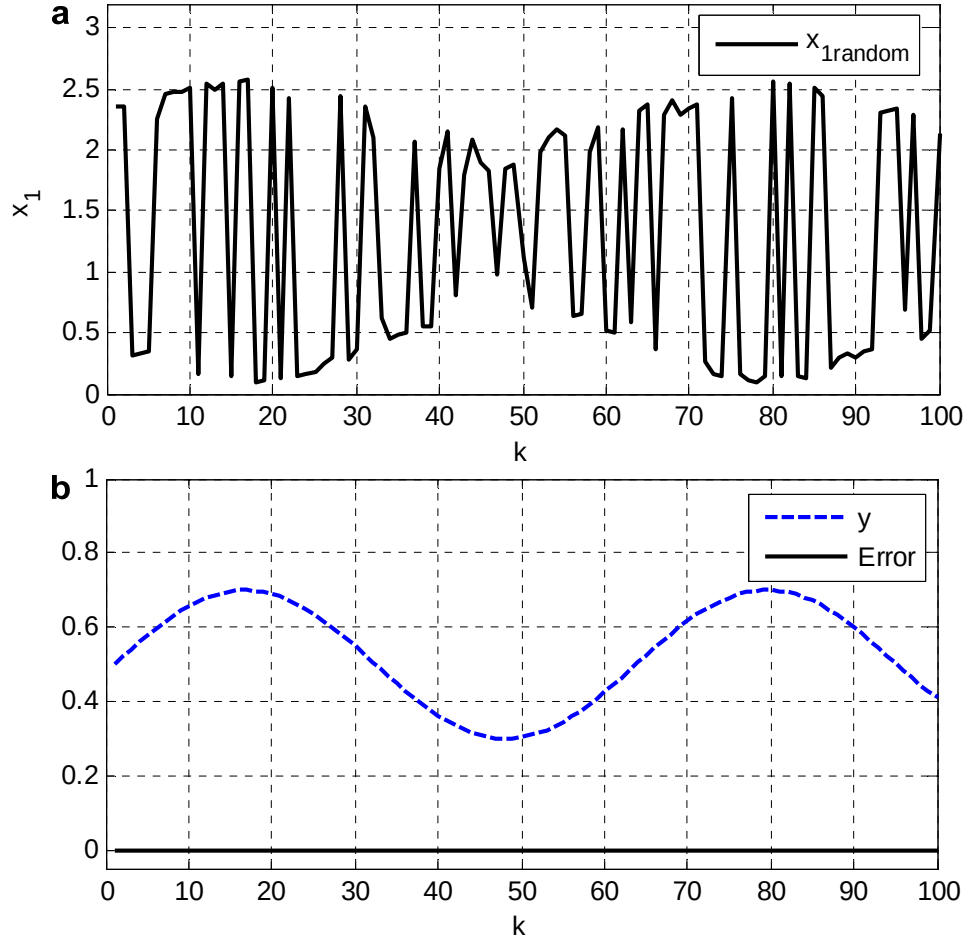


Figure 6.14 : Illustrations of (a) randomly selected inverse solution (x_1) (b) decomposable IT2-FLS output (y) and the tracking error.

6.4 Simulation Study-2

In the simulation study, two nonlinear processes are considered in order to show the closed loop control performance of the proposed interval type-2 decomposable fuzzy model inverse controller (IT2-D-FMIC) (Kumbasar et al., 2011e). The handled processes are the spherical tank system and the two cascaded tank system. These two systems are chosen because of their nonlinear/uncertain characteristics and different types of order. The spherical tank is a first order nonlinear system where as the two cascaded tank system is a second order nonlinear system. The inverse interval type-2 controller closed loop performance will be compared with the inverse type-1 controller (T1FMIC) structures proposed by Boukezzoula et al., (2003) and Babuska (1998). These inverse type-1 fuzzy model controllers are determined on a pure analytical technique and it has been proven that the type-1 fuzzy model inversion is

exact for both proposed methodologies. Therefore, the abbreviation “T1FMIC” will represent those two type-1 fuzzy model inverse controllers in the simulation studies. Since the developed IT2-D-FMIC is an exact analytical inversion method the comparison made with other exact T1FMIC structures. However, both control strategies will be examined and compared in the sense of computational time, explicitly. Three performance measures are considered for comparison. Two of these performance measures are selected from the classical transient system response criteria; namely, the settling time (T_s), and the overshoot (%OS) while the third performance measure is considered as Integral Square Error (ISE), which is defined as:

$$ISE = \int_0^{\infty} (r(t) - y(t))^2 dt \quad (6.21)$$

In the simulation studies, the servo performance and the disturbance rejection performances of control schemes have been examined and compared via the defined performance measures. Moreover, the computational times of the inverse control strategies are compared and commented. The time needed for experiment runs was measured on a personal computer with an Intel Pentium Dual Core E7500 2.93 MHz processor, 2.99 GB RAM, and software package Matlab/SIMULINK R2007A.

Since both the T1FMIC and T2FMIC structures are model based control strategies, the type-1 and interval type-2 fuzzy models of the handled processes must be obtained. The identification of the type-1 fuzzy model (T1-FM) is accomplished via the ANFIS/ Matlab toolbox. The antecedent membership functions of the type-1 fuzzy rules are defined with %50 overlapping triangular membership functions and the consequent parts are defined with crisp singleton parameters. The identification related to the interval type-2 fuzzy model (IT2-FM) has been accomplished as an extension of T1-FM. The antecedent and consequent parameters of the interval type-2 fuzzy model are optimized via the Genetic algorithm such that property (6.1) is satisfied and the Root Mean Square Error (RMSE) is minimized. The RMSE is also as a performance measure in order make a fair comparison between the fuzzy models, and defined as:

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{k=1}^n (y_p - y_{FM})^2} \quad (6.22)$$

where y_p is the process output and y_{FM} is the fuzzy model output.

6.4.1 The spherical tank system

A schematic diagram of the spherical tank system-2 (Faccin and Trierweiler, 2004) is illustrated in Figure 6.15.

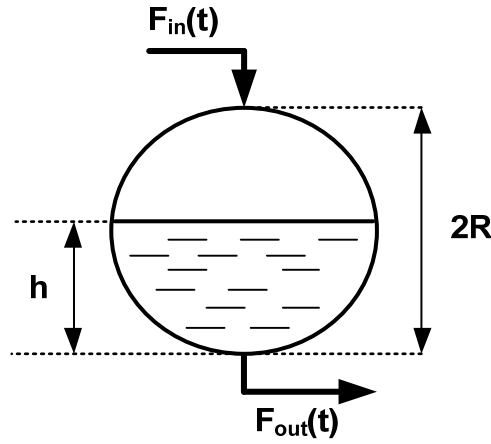


Figure 6.15 : Illustrations of the spherical tank system-2.

The differential equation related to the tank level system (Faccin and Trierweiler, 2004) is given as:

$$F_{in}(t-d) - F_{out}(t) = (\pi - \pi(R-h)^2) \frac{dh}{dt} \quad (6.23)$$

where R is the radius of the spherical tank, $F_{in}(t)$ is the inlet volumetric flow rate, and K is the outlet capacity coefficient. The delay from the manipulated input F_{in} to the controlled output h is indicated by d . The outlet flow rate $F_{out}(t)$ is related to the level h via the following equation

$$F_{out}(t) = K\sqrt{h} \quad (6.24)$$

In this study, $d=0$ s, $R=12.5$ cm and $K=2$ L.min⁻¹.cm^{1/2} are used as process parameters while the inlet flow range is 0-10 L/min (Faccin and Trierweiler, 2004). The initial condition of the tank level is taken as 1cm. Moreover, it is assumed that, the system output inherits noise to make the simulation study more realistic.

6.4.1.1 The comparison T1-FM and IT2-FM representation performances of the spherical tank system

An identification experiment is accomplished to obtain fuzzy models. For that purpose, a multi-signal is used as input signal of the process and data set with the sampling time of 10 s has been collected. The regression vector for both the type-1 and interval type-2 fuzzy model is constructed by considering $h(k)$ and $F_{in}(k)$. The antecedent membership functions of the type-1 fuzzy rules are defined with four triangular membership functions and the consequent part is defined with sixteen crisp singleton parameters. To be fair, a similar structure is used for the IT2-FM. The inputs of the type-2 fuzzy model are represented by four interval type-2 trapezoidal membership functions, and the consequent part is defined with sixteen interval parameters to match the nonlinearities and uncertainties of the spherical tank process enhanced.

A verification input signal is applied to the spherical tank process, T1-FM and IT2-FM. For the validation data, the calculated performance values for the IT2-FM is $RMSE = 0.6417$ while the T1-FM has the $RMSE = 0.3147$ on the same data set. Moreover, it has been illustrated in Figure 6.16 that the IT2-FM represents the nonlinear process much better than the type-1 fuzzy model.

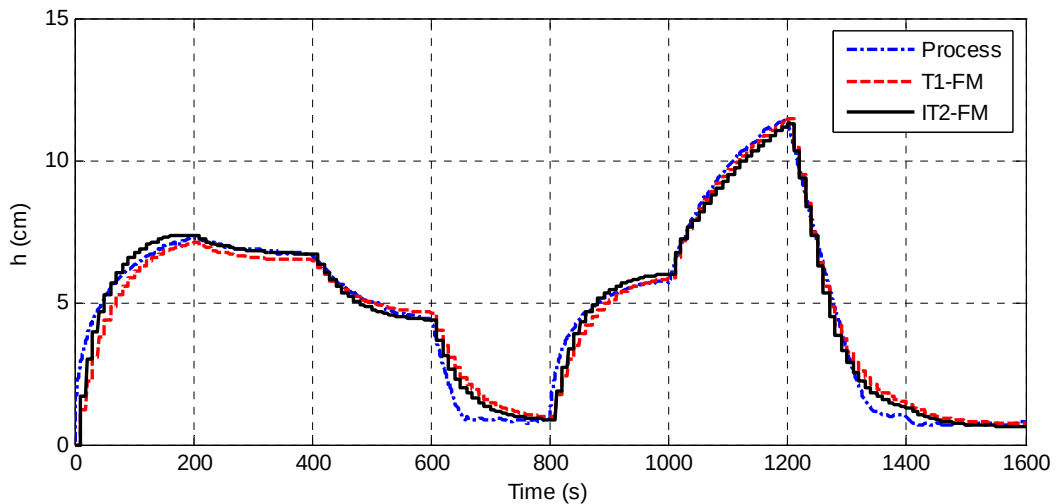


Figure 6.16 : Illustrations of the spherical tank system-2, type-1 and interval type-2 fuzzy model outputs.

6.4.1.2 The performance evaluation of the T1FMIC and IT2-D-FMIC schemes for the spherical tank system

In this subsection, the servo and disturbance rejection performances of T1FMIC and the proposed IT2-D-FMIC structures have been examined. The time constant (T_f) of the robustness filter of the nonlinear internal model control scheme has been chosen as 20 second for both two inverse control schemes. In the servo performance study, since the nonlinearity is related to the level of the tanks, the desired references have been chosen as 8 cm, 14 cm, and 10 cm in successive order.

Table 6.6 : Servo performance comparison of the inverse fuzzy controllers for the spherical tank system-2.

	1-8 cm			8-14 cm			14-10 cm		
	ISE	T_s	OS	ISE	T_s	OS	ISE	T_s	OS
T1FMIC	3729.8	60s	%6	10376	90s	%3	1810.0	70s	%6
IT2-D-FMIC	3214.8	30s	%1	9131.3	70s	%0	1050.5	25s	%0

It has been illustrated in Figure 6.17 that the IT2-D-FMIC scheme provides better transient state performances than the T1FMIC scheme under varying reference values. Moreover, the performance values of IT2-D-FMIC structure are less than T1FMIC controller structure, as it can be clearly seen in Table 6.6. For instance, The IT2-D-FMIC structure reduces the overshoot to 0%; it decreases the setting time about 64% and ISE value about 46% when it is compared to the T1FMIC structure for the reference value variation from 14 cm to 10 cm. The similar comments can be made for the other two reference value variations.

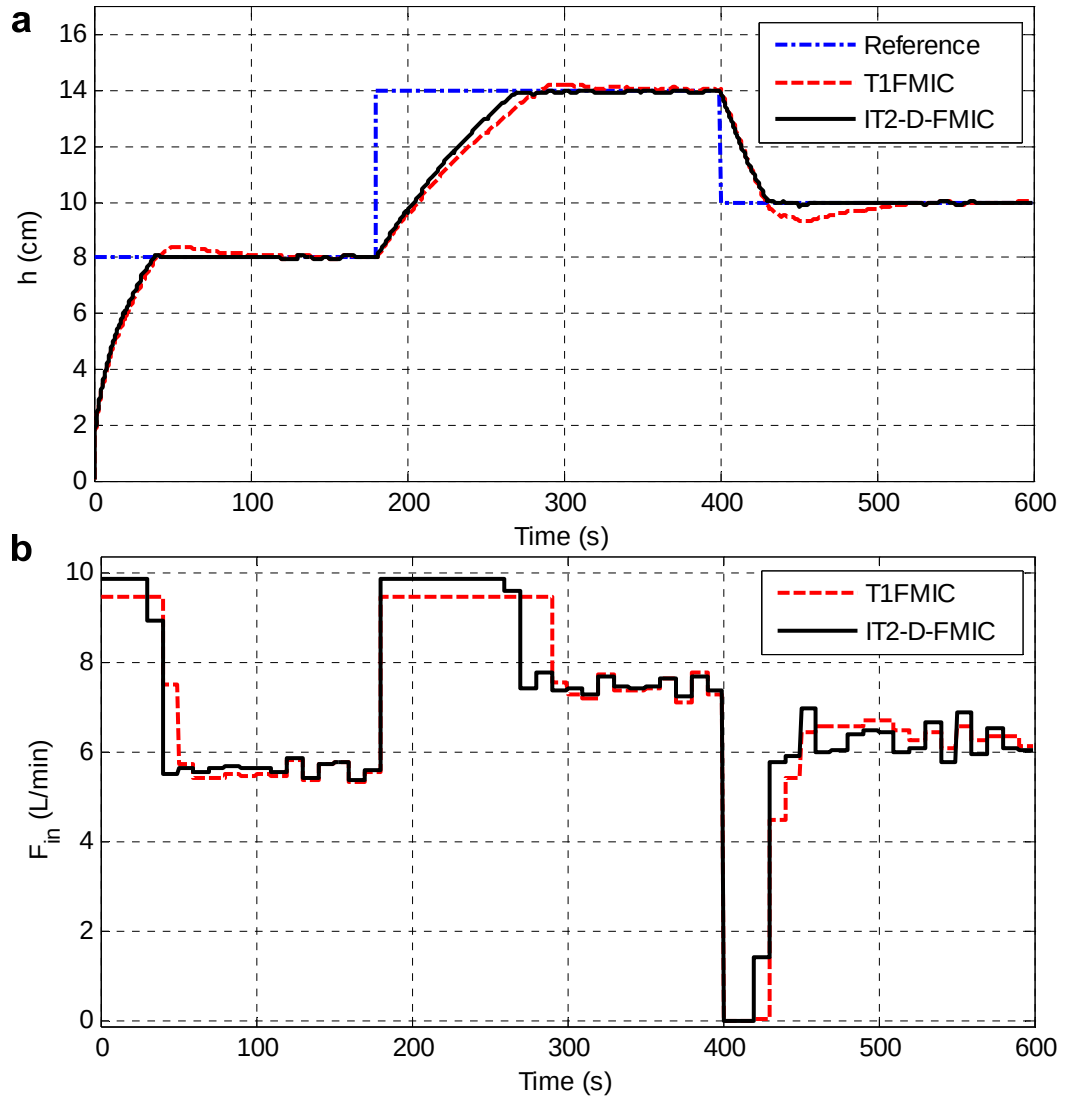


Figure 6.17 : Illustrations of (a) the spherical tank system-2 outputs (b) the control signals of the inverse fuzzy controller schemes for varying reference values.

The disturbance rejection performances of the IT2-D-FMIC and T1FMIC controllers have also been examined. First, a reference signal of 10 cm has been applied and when the process converged to this reference value, a step output disturbance with a magnitude of 2 cm has been applied in the 50th second. The calculated performance value for the IT2-D-FMIC is $ISE = 192.07$ while the T1FMIC has correspondingly $ISE = 235.83$. As it can be seen from the Figure 6.18, the proposed control structure compensated very effectively the output disturbance in a short period of time compared to the T1FMIC structure.

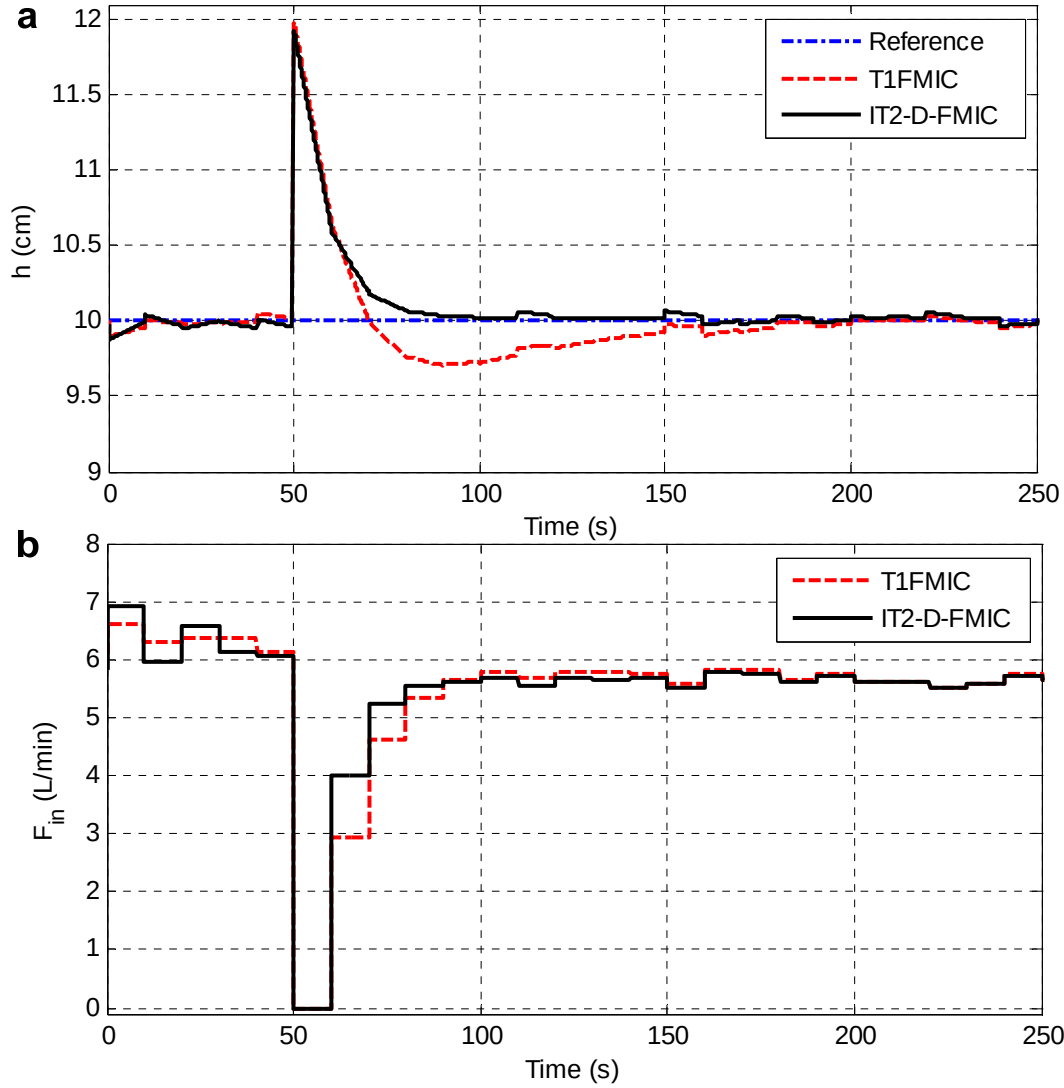


Figure 6.18 : Illustrations of (a) the spherical tank system-2 outputs (b) the control signals of the inverse fuzzy controller schemes for the output disturbance.

Moreover, the computational times of the inverse control strategies are compared for the servo performance study. As it can be observed in Table 6.7; the maximum, the average and the minimum computational times are given. It can be clearly observed that, the average computational time of the IT2-D-FMIC is almost 15 times longer than the T1FMIC's (Babuska, 1998) and 96 times bigger than the T1FMIC's (Boukezzoula et al., 2007). Although the average computational time of the IT2-D-FMIC is relatively long compared to the T1FMICs', it has been illustrated in the servo and disturbance rejection performance studies that IT2-D-FMIC scheme

outperforms the T1FMIC scheme. However, the proposed inverse controller structure is still feasible for the handled nonlinear process since the sampling time is 10s.

Table 6.7 : Computational time comparison of the inverse fuzzy controllers for the spherical tank system-2.

	Computational Time		
	Maximum	Average	Minimum
T1FMIC (Babuska, 1998)	54.92ms	10.82ms	2.291ms
T1FMIC (Boukezzoula et al., 2007)	3.215ms	1.673ms	1.364ms
IT2-D-FMIC (Kumbasar et al., 2011e)	164.1ms	155.2ms	151.3ms

6.4.2 The two cascaded tank process

A two-cascaded tank process is illustrated in Figure 6.19. The first tank (above – “Tank 2”) is a straight tank with a cross sectional area A_2 . The second tank (below – “Tank 1”) is an inclined tank. The cross-sectional area is related to the level of the tank. The angle of its sidewall is β , and the width of Tank 1 is L .

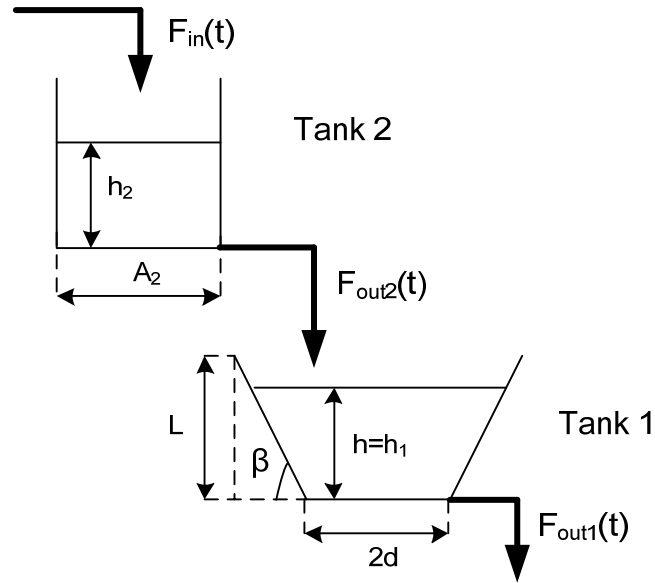


Figure 6.19 : Illustration of the two-cascaded tank process.

The differential equation related to the tank level system is given as:

$$\begin{aligned} \frac{dh_1}{dt} &= \frac{-\gamma_1 \sqrt{2gh_1} + \gamma_2 \sqrt{2gh_2}}{2(d + h_1 \tan \beta) L} \\ \frac{dh_2}{dt} &= -\frac{\gamma_2}{A_2} \sqrt{2gh_2} + \frac{1}{A_2} F_{in}(t) \end{aligned} \quad (6.26)$$

The system output is defined as:

$$h = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \quad (6.27)$$

The process parameters are taken as $A_2=85 \text{ cm}^2$, $\gamma_1=0.3539 \text{ cm}^2$, $\gamma_2=0.3027 \text{ cm}^2$, $L=20 \text{ cm}$, $\beta=5.1^\circ$, $d=3.32 \text{ cm}$, the acceleration of gravity $g=9.8 \text{ ms}^{-2}$ and the initial levels are $h_{10}=0.5\text{cm}$, $h_{20}=0.5\text{cm}$ while the inlet flow range is $0\text{-}100 \text{ cm}^3/\text{s}$ (Wang et al., 2003).

6.4.2.1 The comparison T1-FM and IT2-FM representation performances of the two cascaded tank system

Similar to the previous simulation study, an identification experiment is accomplished in which a multi-signal is used as input signal of the process. The input and output data set has been collected with the sampling time of 5 s. The process is approximated for both the type-1 and type-2 fuzzy models in the following:

$$h(k+1) = f(h(k), h(k-1), u(k))$$

The identified T1-FM is composed of sixty four fuzzy rules where the antecedent membership functions are defined with four triangular membership functions. Similarly, the identified IT2-FM is composed of sixty four fuzzy rules in which the inputs are represented by four interval type-2 trapezoidal membership functions.

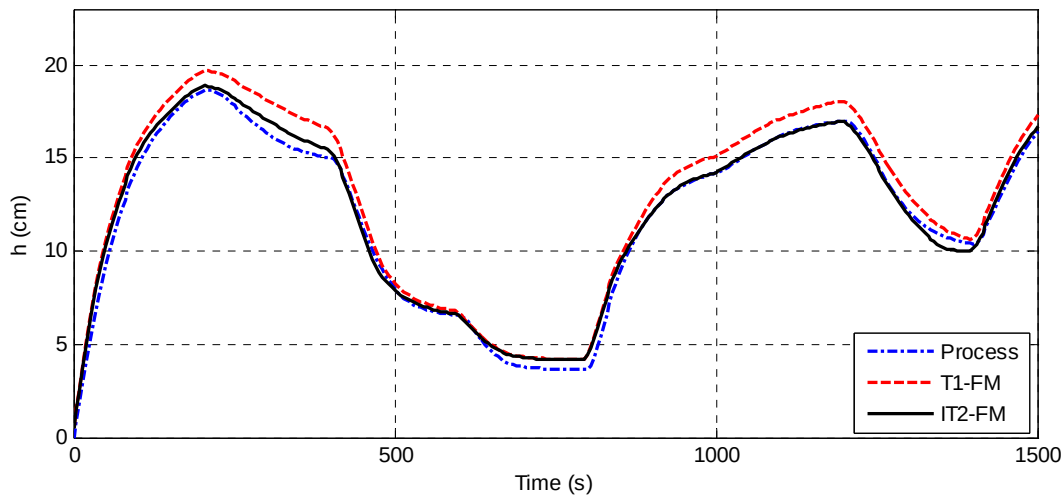


Figure 6.20 : Illustrations of the two cascaded tank process output, type-1 and interval type-2 fuzzy model outputs.

A test signal is applied for validation purposes of the T1-FM and IT2-FM. For the validation data set, the calculated performances values for the T1-FM are RMSE= 0.9582 while the IT2-FM has correspondingly RMSE= 0.4331. Thus, it can be concluded that IT2-FM represents the nonlinear cascade two-tank process much better than the T1-FM. Furthermore, the illustration in Figure 6.20 confirms this fact.

6.4.2.2 The performance evaluation of the T1FMIC and IT2-D-FMIC structures for the cascaded two tank system

In this subsection, the servo and disturbance rejection performances of the proposed IT2-D-FMIC and T1FMIC structures have been examined. The time constant (T_f) of the robustness filter of the nonlinear internal model control scheme has been chosen as 15 seconds for both of the methods. Since the nonlinearity is related to the levels of the cascaded tanks, the desired reference values have been chosen 12 cm, 10 cm, and 14 cm in successive order.

Table 6.8 : Servo performance comparison of the inverse fuzzy controllers for the two cascaded tank process.

	0.5-12 cm			12-10 cm			10-14 cm		
	ISE	T_s	OS	ISE	T_s	OS	ISE	T_s	OS
T1FMIC	524.8	55s	%6	100.1	20s	%2	87.88	55s	%6
IT2-D-FMIC	502.3	40s	%1	99.91	20s	%0	56.12	35s	%3

As it can be clearly seen in Table 6.8, the performance values of IT2-D-FMIC structure are less than T1FMIC controller structure. For instance, The IT2-D-FMIC structure reduces the overshoot to 3% and it decreases the setting time about 37% and ISE value about 36% when it is compared to the T1FMIC scheme for reference value variation from 10 cm to 14 cm. On the other hand, almost identical performance values have been calculated for the reference value variation from 12cm to 10cm. The servo performances of the inverse controllers are illustrated in Figure 6.21.

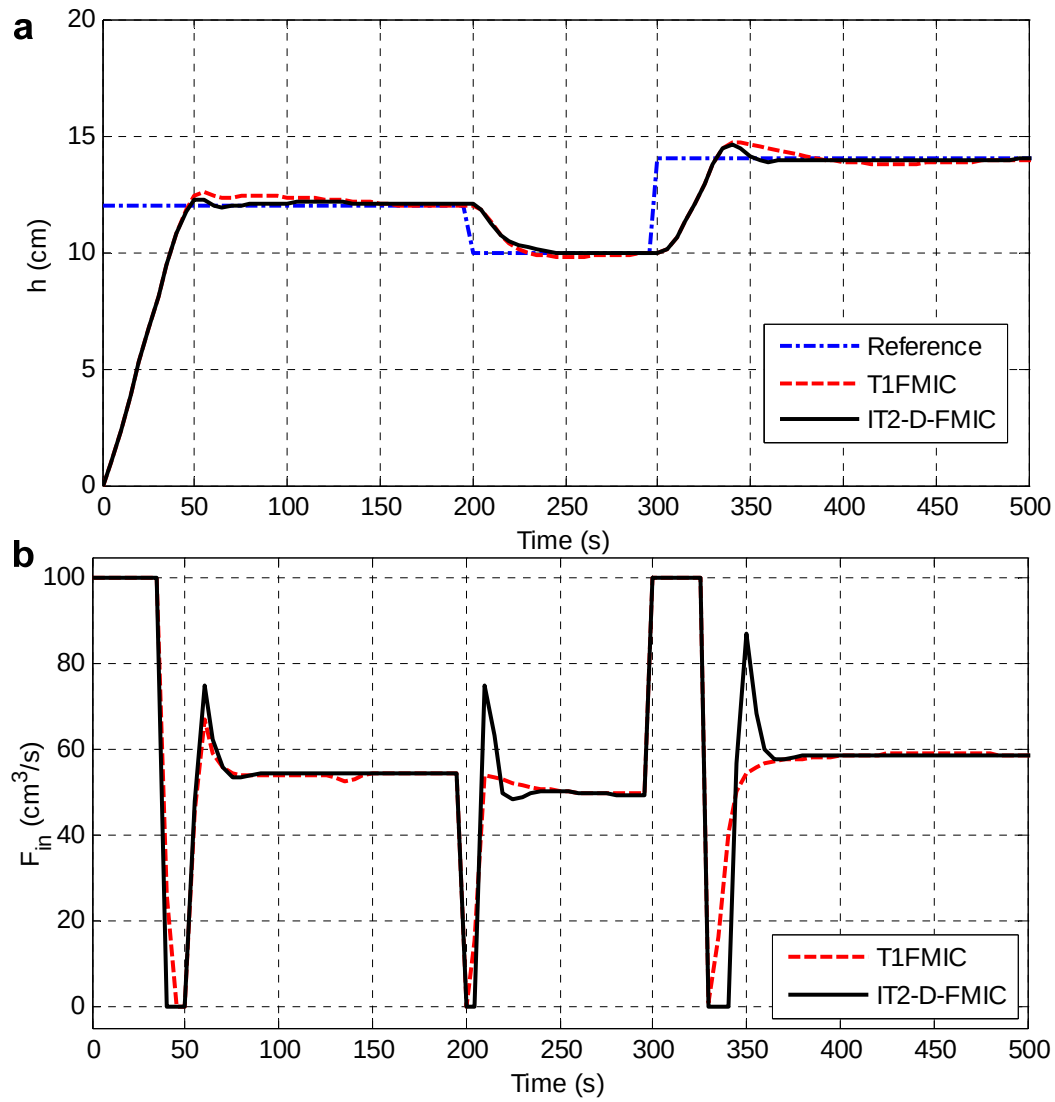


Figure 6.21 : Illustration of (a) the two cascaded tank process outputs (b) the control signals of the inverse fuzzy controllers for varying reference values.

Secondly, the disturbance rejection performances of the IT2-D-FMIC and T1FMIC controllers have been examined. A reference signal of 12 cm has firstly been applied and after the process settled down to this value, a step output disturbance with a magnitude of 2 cm has been applied in 50th second. The calculated performance value for the T1FMIC is ISE= 15.78 while the IT2-D-FMIC has correspondingly ISE= 10.31. As it can be seen from the Figure 6.22 the proposed control structure compensated the output disturbance very effectively in a short time compared to the T1FMIC structure.

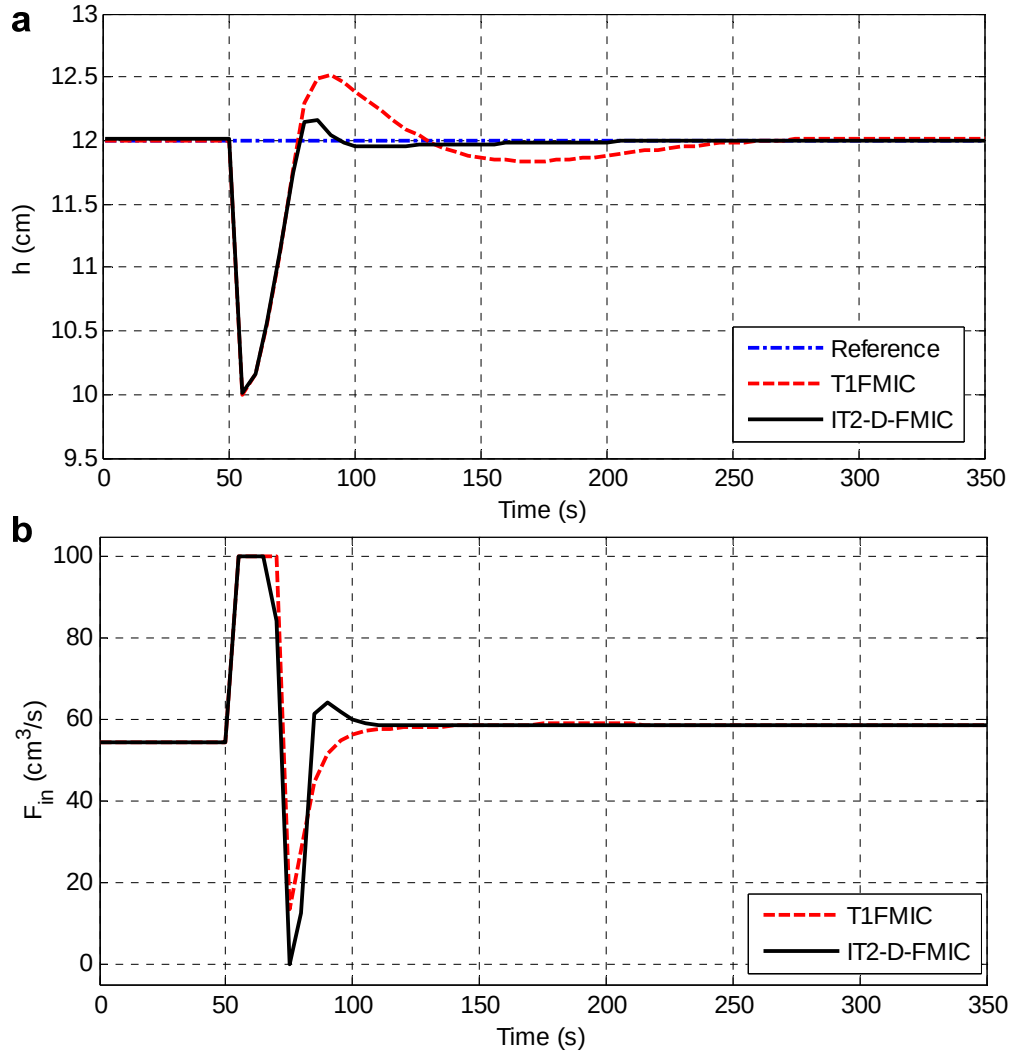


Figure 6.22 : Illustrations of (a) the two cascaded tank process outputs (b) the control signals of the inverse fuzzy controllers for the output disturbance.

Similar to the previous simulation study, the computational times of the inverse control strategies are compared for the servo performance study. Parallel results to the previous simulation study have been obtained and given in Table 6.9. It can be clearly observed that, the average computational time of the IT2-D-FMIC is longer than the T1FMIC's proposed by Babuska, (1998) and the T1FMIC's proposed by Boukezzoula et al., (2007). However, since the sampling time is 5s, the proposed IT2-D-FMIC structure is still feasible. As it has been illustrated in the servo and disturbance rejection performance studies that IT2-D-FMIC scheme outperforms the T1FMIC scheme.

Table 6.9 : Computational time comparison of the inverse fuzzy controllers for the two cascaded tank process.

	Computational Time		
	Maximum	Average	Minimum
T1FMIC (Babuska, 1998)	49.22ms	11.21ms	3.320ms
T1FMIC (Boukezzoula et al., 2007)	3.563ms	2.931ms	2.029ms
IT2-D-FMIC (Kumbasar et al., 2011e)	2108ms	1753ms	599.5ms

6.5 Conclusions and Comments

In this section, a systematic method based on analytical methodology has been presented to find the inverse model of decomposable interval type-2 fuzzy models with interval singleton consequents. At first, the decomposition property presented for type-1 fuzzy logic systems has been extended and generalized for type-2 fuzzy logic systems. Then, the handled type-2 fuzzy model is decomposed into IT2 FSMs based on the generalized decomposition property for interval type-2 fuzzy logic systems. Two examples are presented in order to illustrate this property. Then, the analytical formulation of the type-2 fuzzy submodel output is directly used to find the inverse solution set. However, the type reduced set of the type-2 fuzzy logic submodel cannot be presented in a closed form, due to the Karnik-Mendel type reduction method. To overcome this bottleneck, the switching points of the type reduced set are initialized. After this setting, the analytical formulation of the inverse interval type-2 fuzzy submodel output is explicitly driven and the candidate inverse solutions are calculated. For each initialization, it has been shown that the candidate inverse solution set must satisfy certain conditions. It has been proven in Lemma-1 that the candidate inverse solution set only exist under certain conditions, namely the existence of the output uncertainty parameter (γ) and $\forall x_i(\gamma) \in [0,1]$ for certain L and R values. If these conditions are satisfied, then the certain switching points (L, R) must be checked whether they are the optimal ones or not for the calculated inverse solution set ($x(\gamma)$). For this purpose, two conditions (Lemma-2) must be checked which are extracted from the meta-rules of the Karnik-Mendel type reduction method. If the conditions are satisfied the interval type-2 fuzzy submodel can be inverted and an inverse solution set can be calculated. Finally, the inverse of the interval type-2 fuzzy model is obtained through inversion of each possible activated IT2 FSM. The developed inversion strategy determines either an exact unique solution or exact multiple solutions if there exist. It is important to note here

that the developed inversion strategy is based on a pure analytical method. Two illustrative examples are presented in order to demonstrate the inversion strategy.

Moreover, simulation studies have been performed to show the exactness and the closed loop control performance of the proposed inversion method. The exactness of the method have been shown on a simulation study in which the output of an interval type-2 fuzzy system has been forced to follow a certain desired reference signal in an open loop fashion. Inspecting the tracking error values between the desired input reference signal and the system output, it can easily be concluded that inversion of interval type-2 fuzzy logic system is exact.

The closed loop control performance of the proposed inversion method has been shown on simulation studies in which the exact inverse interval type-2 fuzzy model is implemented in a nonlinear internal model control scheme as the main controller. The proposed inverse control structure has been implemented on two nonlinear/uncertain processes which are a spherical tank process and a two cascaded nonlinear tank process, respectively. The effectiveness of the proposed IT2-D-FMIC method has been shown by comparing the results obtained via the T1FMIC methods based on predefined performance measures. The servo and disturbance rejection performances of all methods are investigated and examined. The IT2-D-FMIC method provides lower performance measure values compared to the T1FMIC method for both nonlinear processes. However, the computational time of the IT2-D-FMIC is relatively bigger than the T1FMIC's which is mainly caused because of the initialization procedure of switching points in the proposed inversion method. However, the results of the control applications demonstrated that the IT2-D-FMIC is superior to T1FMIC since type-2 fuzzy models are more effective to map high nonlinearities and uncertainties. Besides, since the sampling time for each handled nonlinear process is relatively higher than the computational time of IT2-D-FMIC, the control application of the proposed inverse controller is feasible.

Future work will focus on reducing the computational time of IT2-D-FMIC structure in order to be able to control nonlinear/uncertain process with relatively small sampling times. Since the IT2-FM can be decomposed in IT2 FSMs, each submodel inversion can be computed simultaneously in parallel. Therefore, implementing the proposed control structure in parallel computer architecture will significantly decrease the computation time, and make it feasible for real-time control applications

with relatively small sampling times. Besides, preferring an alternative type-reduction method may significantly reduce the computational complexity of the inversion method since the initialization procedure of switching points will not be needed. However, two important criteria must be considered while extending for other types of IT2-FLS namely, the decomposability property and the handled fuzzy systems must have interval/crisp singleton consequents.

7. REAL-TIME CONTROL STUDY

The pH control problem is commonly encountered in the chemical process and wastewater management. Neutralization processes are complex industrial process with dominant nonlinearities. The main complexity of the process consists of the nonlinearity reflected in the S-shaped gain curve of the system. The pH process inherits nonlinearity and high sensitivity near the neutralization point (Pishvaie and Shahrokhi, 2006). Therefore, the control of the neutralization process has been studied for several years but still remains to be a challenging problem (Wright and Kravaris, 1991).

In this section, a real-time control study has been performed in order to show the feasibility of the BBBC- type-2 fuzzy model inverse control (BBBC-T2FMIC) and interval type-2 Takagi-Sugeno fuzzy model inverse control (T2-TS-FMIC) structures. The effectiveness's of the proposed type-2 fuzzy model inverse control (T2FMIC) methods are demonstrated on a pH neutralization experimental set-up and compared with type-1 fuzzy model inverse control (T1FMIC) structures and conventional PID control strategies.

7.1 The GUNT pH Value Control Trainer RT 552

The pH Value Control Trainer RT 552 provides a comprehensive experimental introduction to the fundamentals of pH process control. A caustic solution is added to fresh water by way of a metering pump and the value of this solution is measured as pH_1 . Then, a basic solution is then added as a neutralizing agent via a second metering pump and the pH value is measured as pH_2 after the chemical reaction occurred in the pipeline. A digital industrial controller tries to adjust this pH_2 value to a set point value by controlling flow rate of the second metering pump. Figure 7.1 illustrates the general view of the apparatus and the schematic working principles.

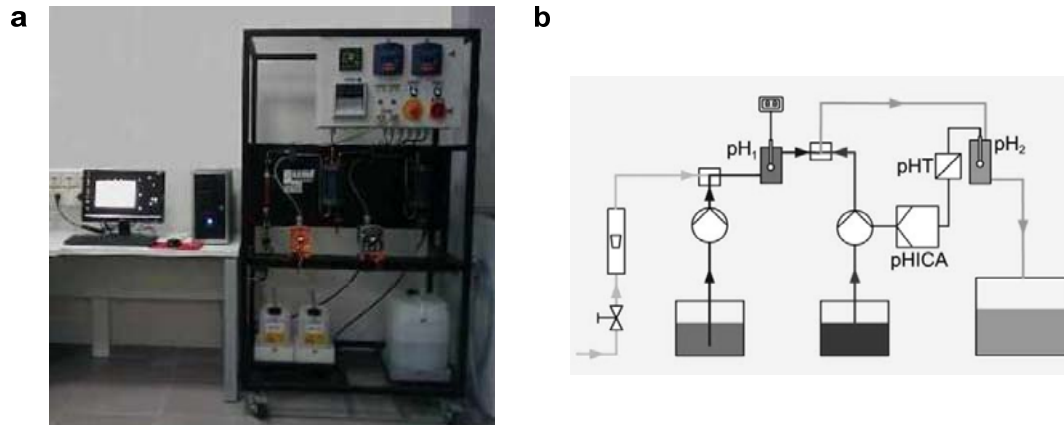


Figure 7.1: (a) A general view of RT552- pH Process Set, (b) Schematic working principle of the RT 552- pH Process Set

The process reaction consists of a strong acid (H-Cl) and strong base (Na-OH) as a titrating stream. The operating conditions and the parameters of the neutralization process are given in Table 7.1. For the simplicity the acetic acid stream, F_a , is considered to be constant at its nominal value while the base flow rate is considered as the manipulated variable (F_b). Therefore, the process can be described as the sodium hydroxide stream as input (F_b) and the pH (pH_2) as output.

Table 7.1 : Parameters of the pH neutralization process.

Symbols	Description	Value
V	Volume of the CSTR	0.8 l
F_a	Flow rate of the influent stream	1 l/s
F_b	Flow rate of the titrating stream	0-2.1 ml/s (%0 -100)
C_a	Concentration of the influent stream	$6.3096 \cdot 10^{-4} \text{ M}$
C_b	Concentration of the titrating stream	$13 \cdot 10^{-4} \text{ M}$

7.2 Fuzzy Modelling of the pH Process

In this section, the effectiveness of type-2 fuzzy modelling will be demonstrated. Therefore, the type-1 and type-2 fuzzy models of the neutralization process have been obtained and compared in terms of predefined cost functions. An identification experiment is applied to GUNT RT-552. For that purpose, a multi-signal (three random signal generators with different frequencies and amplitudes) is used as input signal of the process and data set of 3000 samples with the sampling time of 2 s has been collected (Kumbasar et al., 2011a; Kumbasar et al., 2011c). The regression

vector for both the type-1 and type-2 fuzzy model is constructed by considering $pH(k)$ and $Fb(k)$ values and the process is then approximated as:

$$pH(k+1) = f(Fb(k), pH(k)) \quad (7.1)$$

The identification of the fuzzy models has been accomplished in two stages. In the forward pass, the antecedent parameters are fixed while the consequent parameters are adjusted via the RLS algorithm. In the backward pass, this time the consequent parameters are fixed, the antecedent parameters of the membership functions are determined via BB-BC optimization algorithm. The identification procedure described above is accomplished in an offline manner.

7.2.1 The Type-1 fuzzy model of the pH neutralization process

To map the nonlinearities of the process better, the antecedent membership functions of the type-1 fuzzy model are defined with seven Gaussian combinational membership functions (Gauss2mf) which are a combination of two gaussian membership functions determining the shape of the left and right curves. A Gauss2mf is defined as follows:

$$f(x; \sigma_1, c_1, \sigma_2, c_2) = \begin{bmatrix} e^{-\frac{(x-c_1)^2}{2\sigma_1^2}} & x \leq c_1 \\ 1 & c_1 \leq x \leq c_2 \\ e^{-\frac{(x-c_2)^2}{2\sigma_2^2}} & c_2 \leq x \end{bmatrix} \quad (7.2)$$

The first function, specified by σ_1 and c_1 , determines the shape of the left curve. The second function specified by σ_2 and c_2 determines the shape of the right curve. The antecedent parameters of the type-1 fuzzy model (T1FM) are tabulated in Table 7.2 (Kumbasar et al., 2011a). The consequent part is defined with seven Takagi-Sugeno type rules in the following format:

$$\begin{aligned} r_j : & \text{IF } F_b(k) \text{ is } A_j \text{ and } pH(k) \text{ is } B_j, \\ & \text{THEN } pH(k+1) \text{ is } w^j = p_0^j + p_1^j F_b(k) + p_2^j pH(k), \quad j = 1, \dots, 7 \end{aligned}$$

The consequent parameters of obtained fuzzy rules are tabulated in Table 7.3.

Table 7.2 : Antecedent parameters of the type-1 fuzzy model .

	A₁	A₂	A₃	A₄	A₅	A₆	A₇
σ_1	5.7979	2.7795	26.6039	0.0000	5.8143	7.3802	1.9521
c_1	86.000	67.0000	49.0000	-50.000	86.0000	86.0000	43.0000
σ_2	15.0113	10.4055	9.2543	9.5435	15.0256	0.0000	1.3679
c_2	88.0000	74.0000	53.0000	19.0000	88.0000	150.000	49.0000
	B₁	B₂	B₃	B₄	B₅	B₆	B₇
σ_1	0.3609	0.0000	0.9497	1.4569	0.6101	1.9630	0.5809
c_1	4.3081	1.4691	6.9619	6.7150	7.4981	8.1763	8.6725
σ_2	0.0627	2.9904	0.9217	1.0125	0.7826	0.7021	0.0000
c_2	5.7594	4.3187	7.3981	6.8275	7.6094	8.2719	11.7353

Table 7.3 : Consequent parameters of the type-1 fuzzy model.

Rule	p₀	p₁	p₂
r ₁	-0.69630	0.00410	1.07160
r ₂	-0.03799	0.00127	0.99534
r ₃	-0.00274	0.00098	0.99209
r ₄	-0.02017	0.00072	0.99478
r ₅	-0.09533	0.00130	1.00230
r ₆	-0.01852	0.00140	0.99150
r ₇	-0.15299	-0.00295	1.03200

7.2.2 Type-2 fuzzy model of the pH neutralization process

Since type-2 FLS is an extension of type-1 FLS and in order to reduce the complexity of identifying the type-2 fuzzy model (T2FM), the initial conditions of the antecedent and consequent parameters are chosen the same as the optimal results found for the type-1 fuzzy model. Moreover, to match the nonlinearities of the process enhanced, the inputs are represented by seven Gaussian combinational type-2 membership functions. After the estimation of the antecedent and consequents, the Type-2 T-S fuzzy model rules can be expressed as follows:

$$r_j : \text{IF } F_b(k) \text{ is } \tilde{A}_j \text{ and pH}(k) \text{ is } \tilde{B}_j, \\ \text{THEN pH}(k+1) \text{ is } w^j = \tilde{p}_0^j + \tilde{p}_1^j F_b(k) + \tilde{p}_2^j \text{pH}(k), \quad j = 1, \dots, 7$$

The antecedent and the consequent parameters of the interval type-2 fuzzy set are given in Table 7.4 and Table 7.5, respectively (Kumbasar et al., 2011a).

Table 7.4 : Antecedent parameters of the type-2 fuzzy model.

	A₁	A₂	A₃	A₄	A₅	A₆	A₇
$\overline{\sigma_1}$	5.7979	10.7795	46.8000	0.0000	15.8143	17.3802	1.9521
$\underline{\sigma_1}$	2.7700	2.7800	26.6039	0.0000	5.8900	0.3702	1.9121
$\overline{c_1}$	86.0000	67.0000	49.0000	-50.000	86.0000	86.000	43.000
$\underline{c_1}$	86.0000	67.0000	49.0000	-50.000	80.0000	86.000	43.000
$\overline{\sigma_2}$	15.0113	50.055	19.2543	9.9435	15.0256	0.000	1.3679
$\underline{\sigma_2}$	0.0113	4.9055	9.3300	9.5420	0.0260	0.000	1.2679
$\overline{c_2}$	88.0000	74.0000	53.0000	20.0000	88.0000	150.000	49.000
$\underline{c_2}$	88.0000	70.0000	50.0000	19.0000	88.0000	150.000	49.000
	B₁	B₂	B₃	B₄	B₅	B₆	B₇
$\overline{\sigma_1}$	0.3609	0.0000	0.9497	1.9569	0.6101	10.963	0.5809
$\underline{\sigma_1}$	0.0609	0.0000	0.1497	1.4564	0.6101	1.9630	0.0009
$\overline{c_1}$	4.3081	1.4691	6.9619	6.7150	7.4981	8.1763	8.6725
$\underline{c_1}$	4.3081	1.4691	6.9619	6.7150	7.4981	8.1763	8.6725
$\overline{\sigma_2}$	0.2627	2.9904	2.9217	1.0125	0.7826	0.7021	0.0000
$\underline{\sigma_2}$	0.0627	2.9904	0.9217	0.0125	0.7826	0.0021	0.0000
$\overline{c_2}$	5.7594	4.3187	7.3981	6.8275	7.6094	8.2719	11.7353
$\underline{c_2}$	5.7594	3.3187	7.3981	6.8275	4.6094	8.2719	5.7353

Table 7.5 : Consequent parameters of the type-2 fuzzy model.

Rule	p₀	p₁	p₂
r ₁	-0.69629	0.00410	1.07170
r ₂	-0.03799	0.00127	0.99550
r ₃	-0.00274	0.00098	0.99225
r ₄	-0.02017	0.00072	0.99494
r ₅	-0.09533	0.00130	1.00250
r ₆	-0.01852	0.00140	0.99166
r ₇	-0.15299	-0.00295	1.03220

7.2.3 Performance comparison of the type-1 and type-2 fuzzy models

To check the obtained type-1 and type-2 fuzzy models, a uniform random test signal which is illustrated in Figure 7.2a is applied. The fuzzy models are simulated against the model output which is illustrated in Figure 7.2b. In order make a fair comparison between the fuzzy models, two performance indexes are defined.

i) Root Mean Square Error (RMSE), which is defined as:

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{k=1}^n (y_p - y_{FM})^2} \quad (7.4)$$

ii) Variance Accounted For (VAF) (Babuska, 1998), which is defined as:

$$\text{VAF} = \%100 \times \left[1 - \frac{\text{var}(y_p - y_{FM})}{\text{var}(y_p)} \right] \quad (7.5)$$

where y_p is the process output and y_{FM} is the fuzzy model output. The variance accounted for index is often used to assess the quality of a model, by comparing the true output with the output of the model. If two signals are equal then the VAF value is %100, otherwise the VAF value is lower (Babuska, 1998).

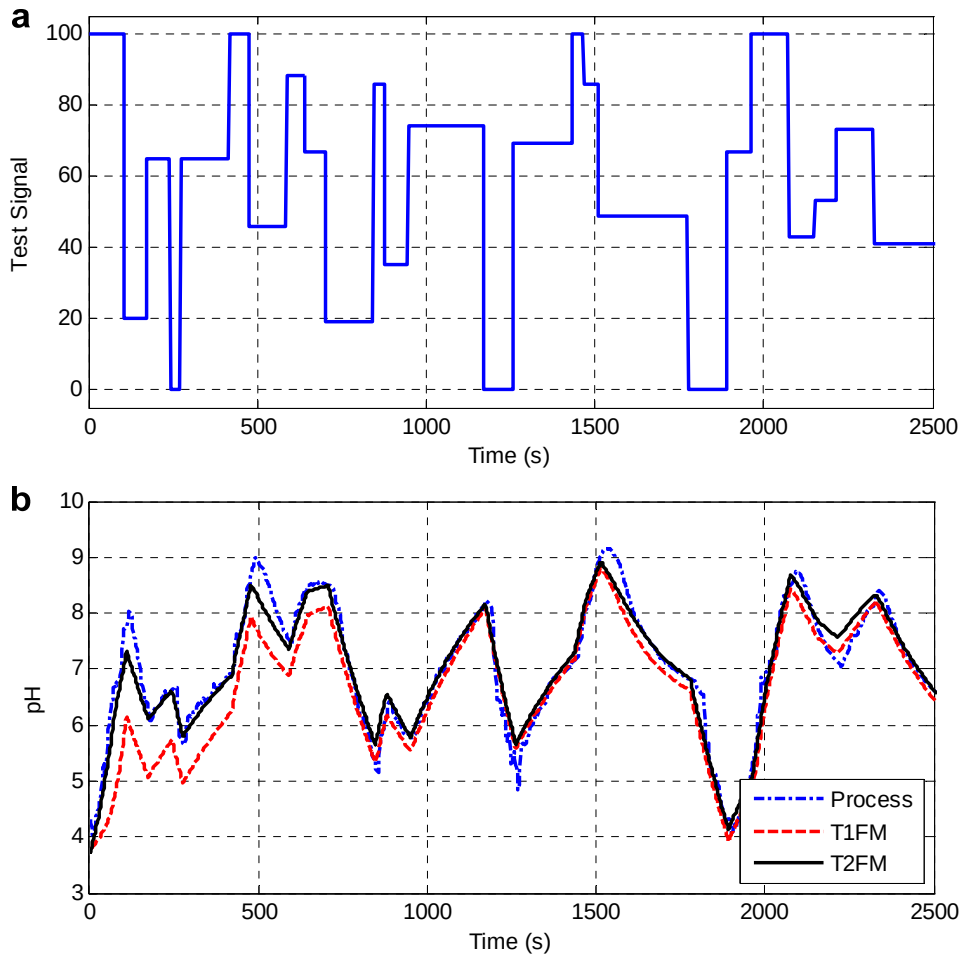


Figure 7.2 : Illustration of (a) test signal (b) pH neutralization process output, type-1 and type-2 fuzzy model outputs.

The performance values obtained from (7.4) and (7.5) are tabulated in **Table 7.6**. It can be clearly seen that the T2FM has a higher VAF value, and lower RMS value compared to the T1FM. Thus, it can be concluded that T2FM is better than T1FM in the presence of uncertainties and strong nonlinearities. Here, it has been illustrated in Figure 7.2b that the T2FM represents the nonlinear neutralization process much better than the type-1 fuzzy model.

Table 7.6: Comparison of the fuzzy models.

Fuzzy Model Type	RMSE	VAF
T1FM	0.5998	83.0993
T2FM	0.2634	94.5942

7.3 Control Applications on the pH Neutralization Process

It is a known fact that neutralization processes are complex industrial process with dominant nonlinearities and the main complexity of such processes consist of the nonlinearity reflected in the S-shaped gain curve of the system. Here, the proposed T2FMIC methods are implemented on the pH neutralization experimental setup and its effectiveness is tried to be shown by making comparisons both with the PID control strategies and T1FMIC methods. In order to make the comparisons fair, four performance measures are considered. Two of these performance measures are selected from the classical transient system response criteria; namely, the settling time (T_s), and the overshoot (%OS) while the other performance measures are considered as Integral Absolute Error (IAE) (4.6) and Total Variation (TV) of the control input (4.7).

In the real time control application studies, the servo performance of the control structures are compared especially around the pH value of 7, since the major difficulty of controlling pH neutralization processes is the nonlinear S-shaped pH titration curve. Thus, the desired reference trajectories have been chosen as 6, 8, and 7 pH values, in this successive order. In addition, the disturbance rejection performances of control schemes have been examined around the pH value of 7. Moreover, since the main drawback of type-2 fuzzy models is caused by type reduction/ defuzzification inference mechanism, the computational time is examined for both the T1FMIC and T2FMIC methods. The time needed for experiment runs was measured and compared on a personal computer with an Intel Pentium Dual

Core E7500 2.93 MHz processor, 2.99 GB RAM, and software package Matlab/SIMULINK R2007A.

7.3.1 Performance evaluation of the conventional PID controllers

In this subsection, at first two conventional PID controllers are designed based on a first-order-plus-time delay model (FOPDT). As it is well-known, high order systems or nonlinear process can be represented by first order plus dead time systems that corresponds to (Astrom and Hagglund, 2007; Skogestad, 2003):

$$G(s) = \frac{K e^{-Ls}}{\tau s + 1} \quad (7.6)$$

The transfer function of a conventional PID controller can be given as:

$$PID(s) = K_c \left(1 + \frac{1}{\tau_i s} + \tau_d s \right) \quad (7.7)$$

where K_c is the proportional gain, τ_i the integral gain, τ_d the derivative gain.

When the pH experimental setup is modelled as FOPDT, the model parameters are obtained based on the step response method as follows (Astrom and Hagglund, 2007): $K=0.1168$, $\tau=258.75s$, $L=4s$.

In the first PID design strategy, the controller parameters of the conventional PID controller are optimized so as to minimize the IAE performance index via the BB-BC optimization algorithm. The optimum values of these parameters are found as $K_c=70.58$, $\tau_i=96.62$, $\tau_d=1.39$. In the second PID control design strategy, the PID controller parameters are tuned based on IMC design procedure (Morari and Zafiriou, 1989). In this method, the PID structure inherits a low pass filter:

$$PID(s) = K_c \left(1 + \frac{1}{\tau_i s} + \tau_d s \right) \left(\frac{1}{1 + \tau_f s} \right) \quad (7.8)$$

where controller parameters are defined as:

$$K_c = \frac{\tau + 0.5L}{K(\tau_c + L)}; \tau_i = \tau + 0.5L; \tau_d = \frac{0.5L\tau}{\tau + 0.5L}; \tau_f = \frac{0.5L\tau_c}{\tau_c + L} \quad (7.9)$$

In this design procedure, the desired closed-loop time constant τ_c is the design parameter (Morari and Zafiriou, 1989). For the pH experimental setup on which the real time applications are done, the time constant τ_c is chosen as 20 seconds so as to obtain moderate closed loop system response making comprise between IAE, overshoot and settling time. Then, the controller parameters are calculated as $K_c=92.94$, $\tau_i=234.67$, $\tau_D=2.20$, $\tau_f=1.66$. This design parameter τ_c is chosen equal to the design filter parameter in (3.3) of T2FMIC and T1FMIC schemes so as to be fair in comparison. Both the optimization based PID (OPT-PID) and the IMC based PID (IMC-PID) controllers are implemented as the discrete-time versions obtained with the bilinear transform.

The servo performances of the conventional PID control structures are illustrated in Figure 7.3. It can be clearly observed that the performances of the control structures vary for each operating point since the linear controllers are designed for a certain operating point. The performance measures values in Table 7.7 confirm this fact (Kumbasar et al., 2011a).

Table 7.7 : Servo performance comparison of the conventional PID controllers for pH neutralization process

		IAE	Ts	OS	TV
4-6 pH	OPT-PID	138.3	192s	%12	139.4
	IMC-PID	113.8	131s	%3	249.6
6-8 pH	OPT-PID	76.2	152s	%15	347.7
	IMC-PID	74.5	152s	%10	399.9
8-7 pH	OPT-PID	30.4	72s	% 6	347.8
	IMC-PID	29.7	58s	%12	277.9

In order to evaluate the disturbance rejection performance of the PID control strategies, at first a reference signal of “7 pH” has been applied and after the process converged to the desired pH value, a unit step output disturbance has been applied in 50th second. As it can be seen from the Figure 7.4, the OPT-PID compensated the disturbance while the IMC- PID controller was not able to compensate it. The performance measures are tabulated in Table 7.8.

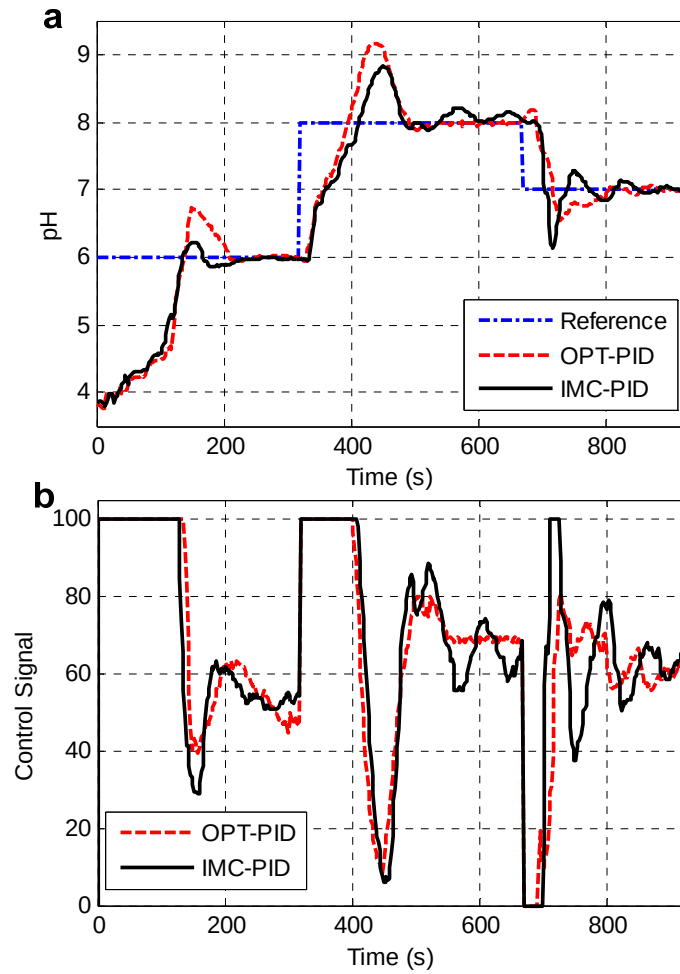


Figure 7.3 : Illustration of (a) the pH neutralization process outputs (b) the PID control signals for varying reference values.

Table 7.8 : Disturbance rejection performance comparison of the conventional PID controllers for pH neutralization process.

	IAE	TV
OPT-PID	34.2	325.9
IMC-PID	Unstable System Response	

As it can be seen from the Figure 7.4, the OPT-PID compensated the disturbance while the IMC- PID controller was not able to compensate it. The performance measures are tabulated in Table 7.8.

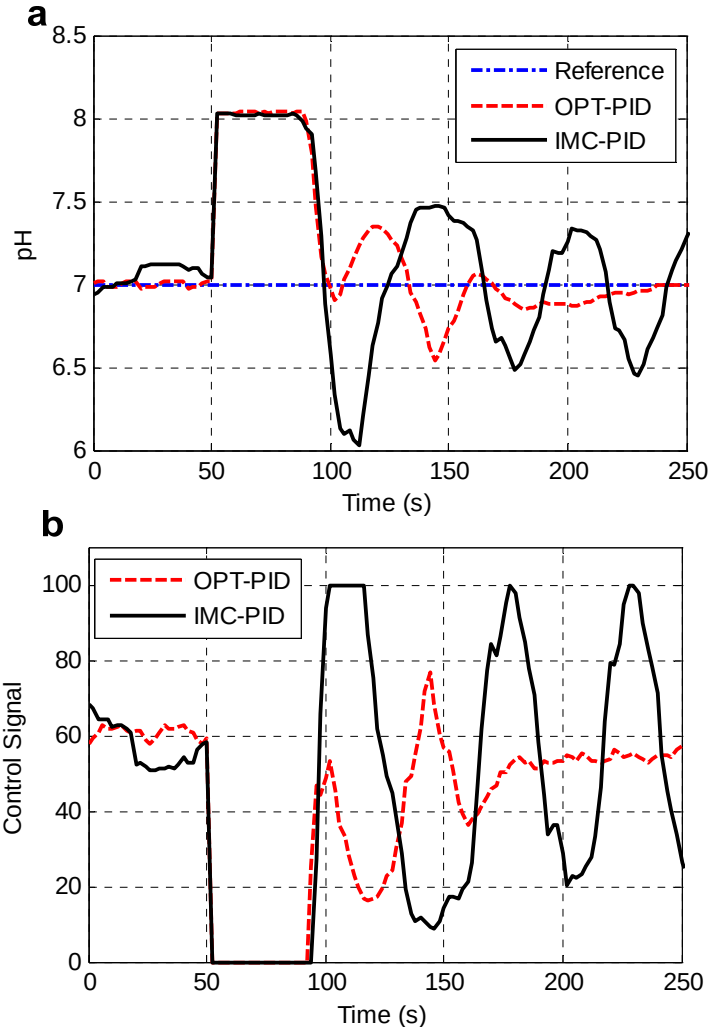


Figure 7.4 : Illustration of (a) the pH neutralization process output (b) the PID control signals for output disturbances for conventional PID controllers.

7.3.2 Performance evaluation of the iterative inverse fuzzy controllers

Here, the proposed iterative BBBC-T2FMIC structure is implemented on pH neutralization experimental setup and its effectiveness is tried to be shown by making comparisons with the BBBC-T1FMIC method which is proposed in (Kumbasar, 2008a; Kumbasar, 2011c). Through empirical studies, the parameters for the evolutionary BB-BC optimization algorithm α and β have chosen as 1 and 0.25, respectively in both BBBC-T1FMIC and BBBC-T2FMIC methods. The population size and the number of iterations for the BB-BC optimization method have been chosen as 20. Moreover, the time constant (T_f) of the feedback filter of the internal model control scheme has been chosen as 20 second for both of the methods.

In the servo performance study, the BBBC-T2FMIC scheme provides better transient state performances then the BBBC- T1FMIC scheme as it has been illustrated in

Figure 7.5. Moreover, the performance values of BBBC-T2FMIC structure are less than BBBC-T1FMIC structure, as it can be seen in Table 7.9. Additionally, the BBBC-T2FMIC scheme has a low value of TV that shows that it has the smoothest control signal (Kumbasar et al., 2011a).

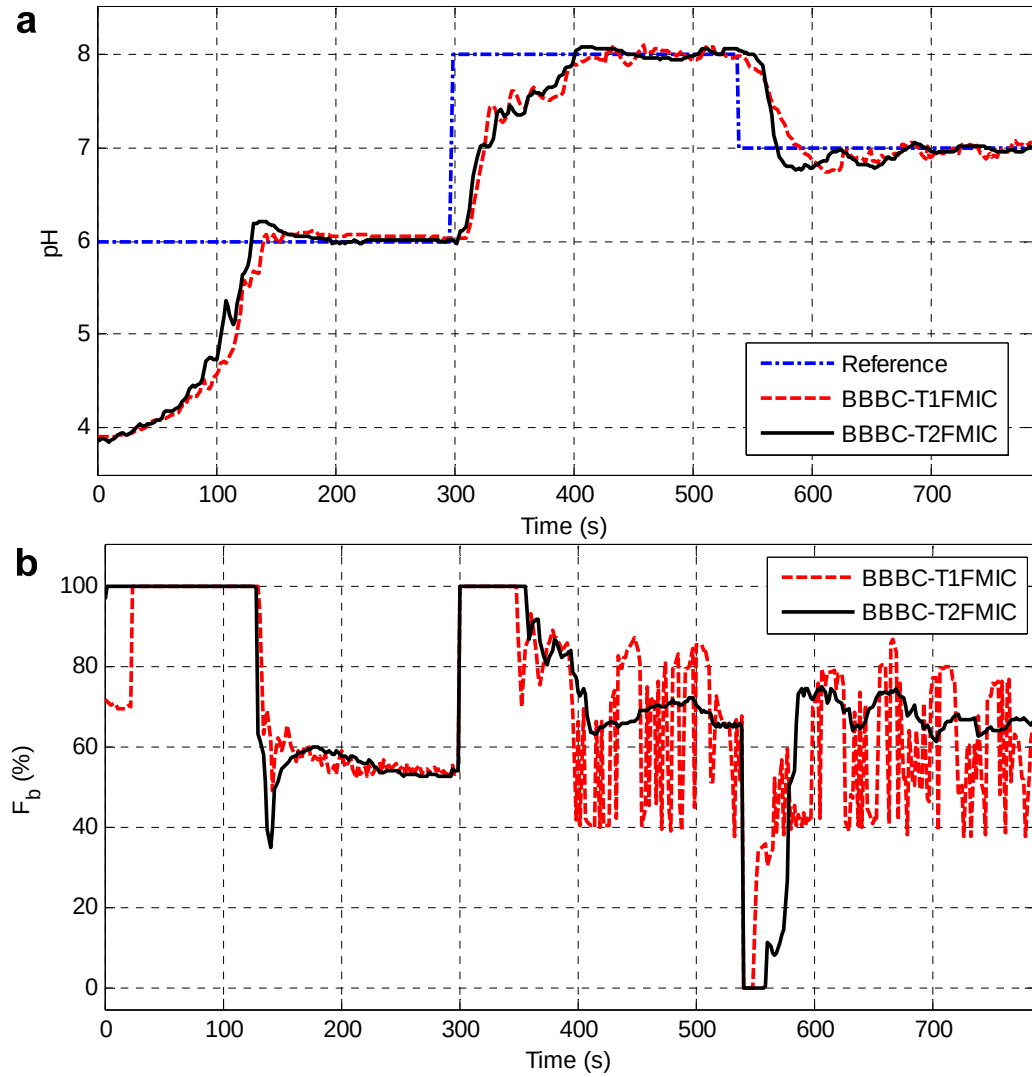


Figure 7.5 : Illustration of (a) the pH neutralization process outputs (b) the iterative inverse fuzzy model control signals for varying reference values.

Table 7.9 : Servo performance comparison of the iterative inverse controllers for pH neutralization process.

		IAE	Ts	OS	TV
4-6 pH	BBBC-T1FMIC	113.2	130s	%0	239.9
	BBBC-T2FMIC	106.0	125s	%2	115.8
6-8 pH	BBBC-T1FMIC	45.5	92s	%0	1190.5
	BBBC-T2FMIC	43.7	75s	%0	146.4
8-7 pH	BBBC-T1FMIC	22.6	35s	%4	1489.6
	BBBC-T2FMIC	22.5	28s	%3	232.5

The disturbance rejection performance of the BBBC-T2FMIC and BBBC-T1FMIC scheme has been examined. First of all, a reference signal of “7 pH” has been applied. After the process converged to the desired pH value, a unit step output disturbance has been applied in 50th second. As it can be seen from the Figure 7.6 the proposed control structure compensated very effectively the output disturbance in a short period of time compared to the BBBC-T1FMIC structure. Moreover, the control signal of the BBBC-T2FMIC scheme is smoother and therefore has a lower TV value, as it can be seen in Table 7.10 (Kumbasar et al., 2011a).

Moreover, the computational time is examined for both the BBBC-T1FMIC and BBBC-T2FMIC. To make a fair comparison, the initial population, the random number (“r” which is needed in Big Bang phase for generating the new population) and the initial population are randomly initialized for both controllers. Moreover, the population size and the number of iterations for the BB-BC optimization method have been chosen as 20. The average computational time for computing the optimal control signal is for the BBBC-T2FMIC is 0.16s while for BBBC-T1FMIC is 0.08s. Although the computational time of the BBBC-T1FMIC is 2 times lower, it has been illustrated in the servo and disturbance rejection performance studies that BBBC-T2FMIC scheme outperforms the BBBC-T1FMIC scheme (Kumbasar et al., 2011a).

Table 7.10 : Disturbance rejection performance comparison of the iterative inverse controllers for pH neutralization process.

	IAE	TV
BBBC-T1FMIC	31.1	386.9
BBBC-T2FMIC	26.6	185.2

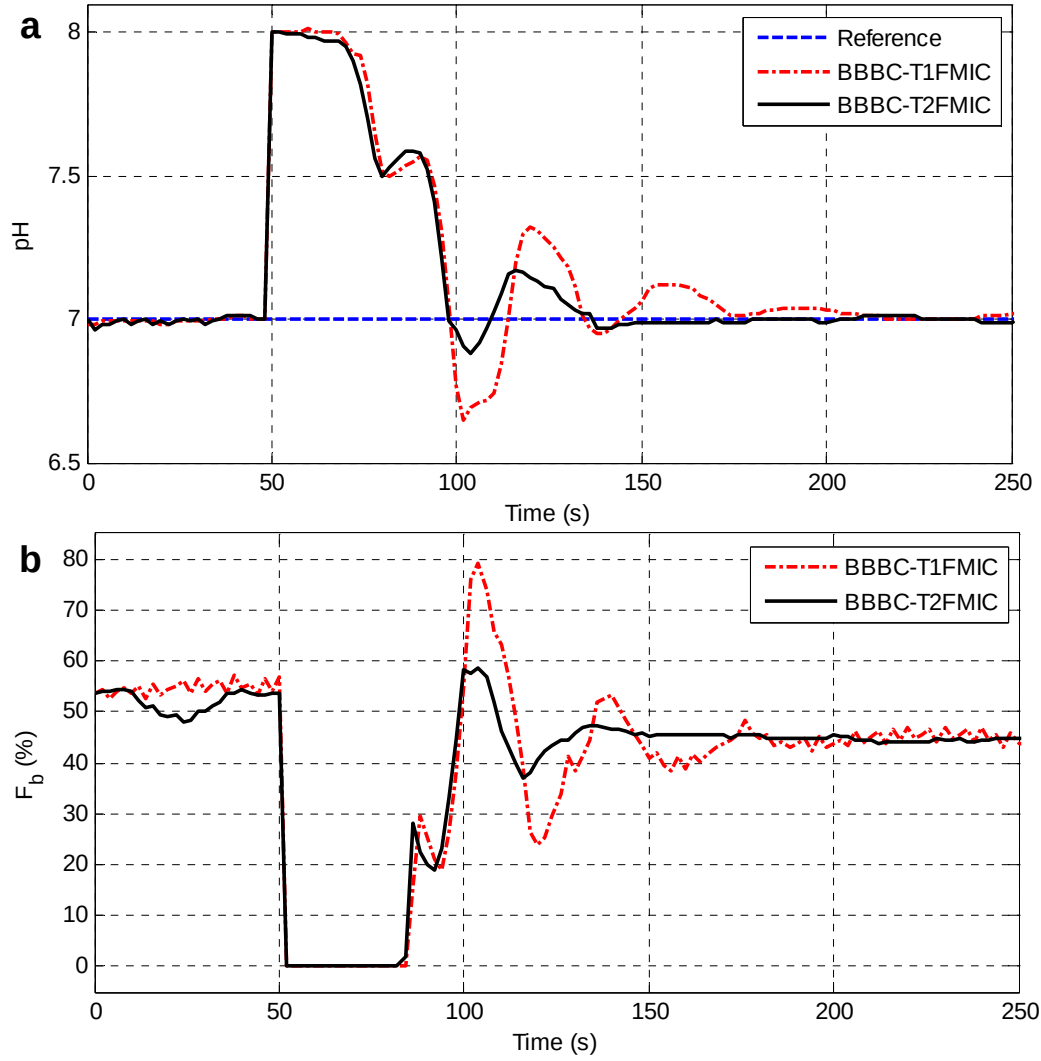


Figure 7.6 : Illustration of (a) the pH neutralization process outputs (b) the for iterative inverse fuzzy model control signals for output disturbance.

7.3.3 Performance evaluation of the inverse T-S fuzzy controllers

In this subsection, the effectiveness and the superiority of the proposed interval type-2 Takagi-Sugeno inverse controller (T2-TS-FMIC) method over the interval type-1 Takagi-Sugeno inverse controller (T1-TS-FMIC) is tried to be shown. The time constant (T_f) of the robustness filter of the nonlinear internal model control scheme has been set as 20 second via some empirical try outs for both the T2-TS-FMIC and the T1-TS-FMIC structures (Kumbasar et al., 2011b).

Since the major difficulty of controlling pH neutralization processes is the nonlinear S-shaped pH titration curve, a servo performance of the T1-TS-FMIC and T2-TS-FMIC structures have been compared. The desired reference trajectories have been chosen as 6, 8, and 7 pH values, respectively. It has been illustrated in Figure 7.7 that

the T2-TS-FMIC scheme provides better transient state performances then the T1-TS-FMIC scheme under varying set points. Moreover, the IAE value of T2-TS-FMIC structure is less than T1-TS-FMIC controller structure, as it can be seen in Table 7.11. Additionally, the T2-TS-FMIC scheme has a low TV value that shows that it has the smoothest control signal (Kumbasar et al., 2011b).

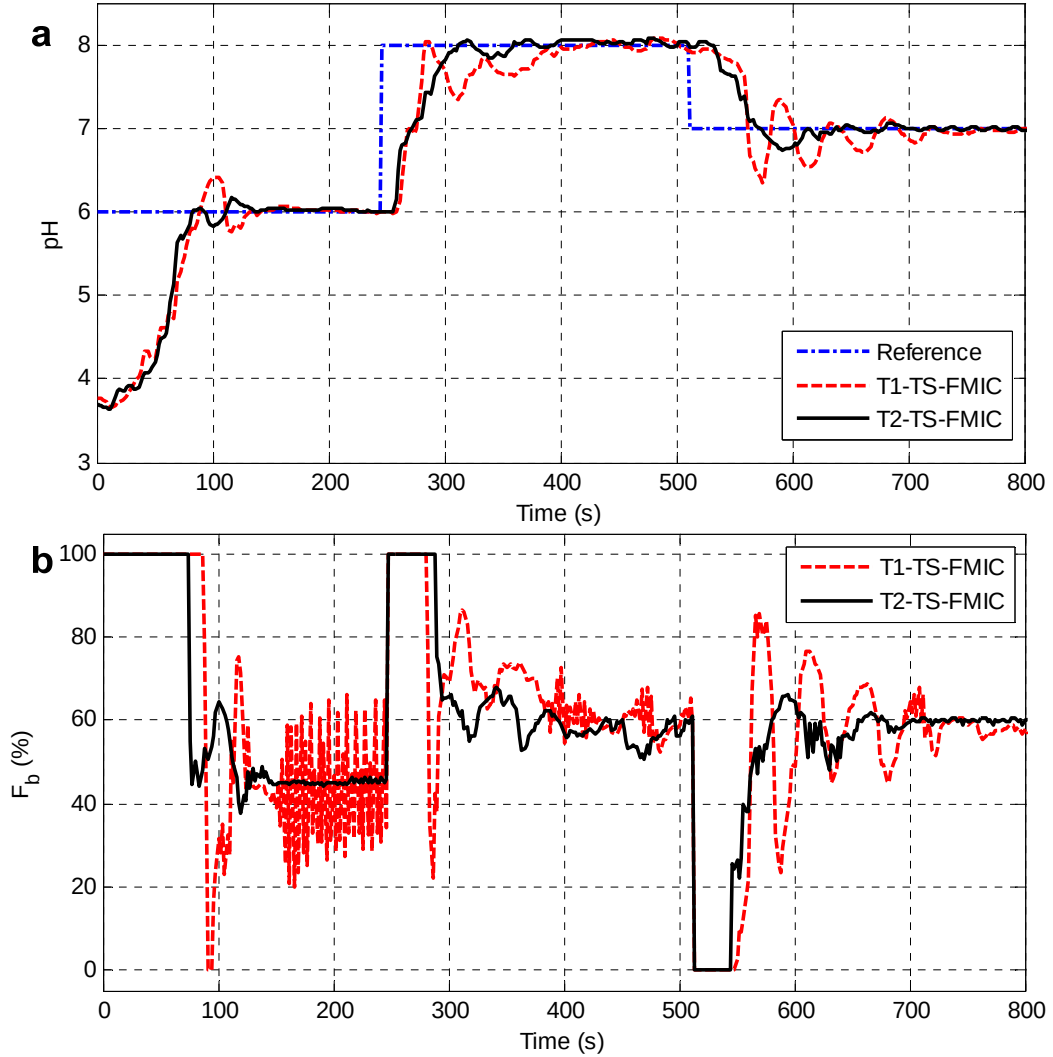


Figure 7.7 : Illustration of (a) the pH neutralization process outputs (b) the inverse Takagi-Sugeno fuzzy model control signals for varying reference values.

Secondly, the disturbance rejection performance of the T2-TS-FMIC and T1-TS-FMIC scheme has been examined. First of all, a reference signal of “7 pH” has been applied. After the process converged to the set point value, a unit step output disturbance has been applied in 50th second. As it can be seen from the Figure 7.8 the proposed T2-TS-FMIC structure compensated very effectively the output disturbance in a short period of time compared to the T1-TS-FMIC structure. Moreover, the

control signal of the T2-TS-FMIC scheme is smoother and therefore has a lower TV value. Also, the ITSE value is almost %46 the better, as it can be seen in Table 7.12 (Kumbasar et al., 2011b).

Table 7.11 : Servo performance comparison of the Takagi-Sugeno fuzzy inverse controllers for the pH neutralization process.

		IAE	Ts	OS	TV
4-6 pH	T1-TS-FMIC	76.3	108s	%6	1612
	T2-TS-FMIC	72.1	72s	%2	184.3
6-8 pH	T1-TS-FMIC	45.8	94s	%0	546.3
	T2-TS-FMIC	34.7	76s	%0	199.2
8-7 pH	T1-TS-FMIC	39.1	110s	%10	546.3
	T2-TS-FMIC	25.5	46s	%3	309.5

Table 7.12 : Disturbance rejection performance comparison of the Takagi-Sugeno fuzzy inverse controllers for the pH neutralization process.

	IAE	TV
T1-TS-FMIC	25.1506	793.8
T2-TS-FMIC	13.7857	170.3

Moreover, the computational time is examined for both the T1-TS-FMIC and T2-TS-FMIC. For the servo performance study the total computational time of the T2-TS-FMIC is 0.3246s while for T1-TS-FMIC it is 0.2154s, whereas the average computational times are 8.115ms and 5.384ms for the T2-TS-FMIC and T1-TS-FMIC, respectively. This means almost 50-percent deceleration due to type reduction/ defuzzification inference mechanism of T2FLS structure. Although the total computational time of the T2-TS-FMIC is relatively high compared to the T1-TS-FMIC's, it has been illustrated in the servo and disturbance rejection performance studies that T2-TS-FMIC scheme outperforms the T1-TS-FMIC scheme (Kumbasar et al., 2011b).

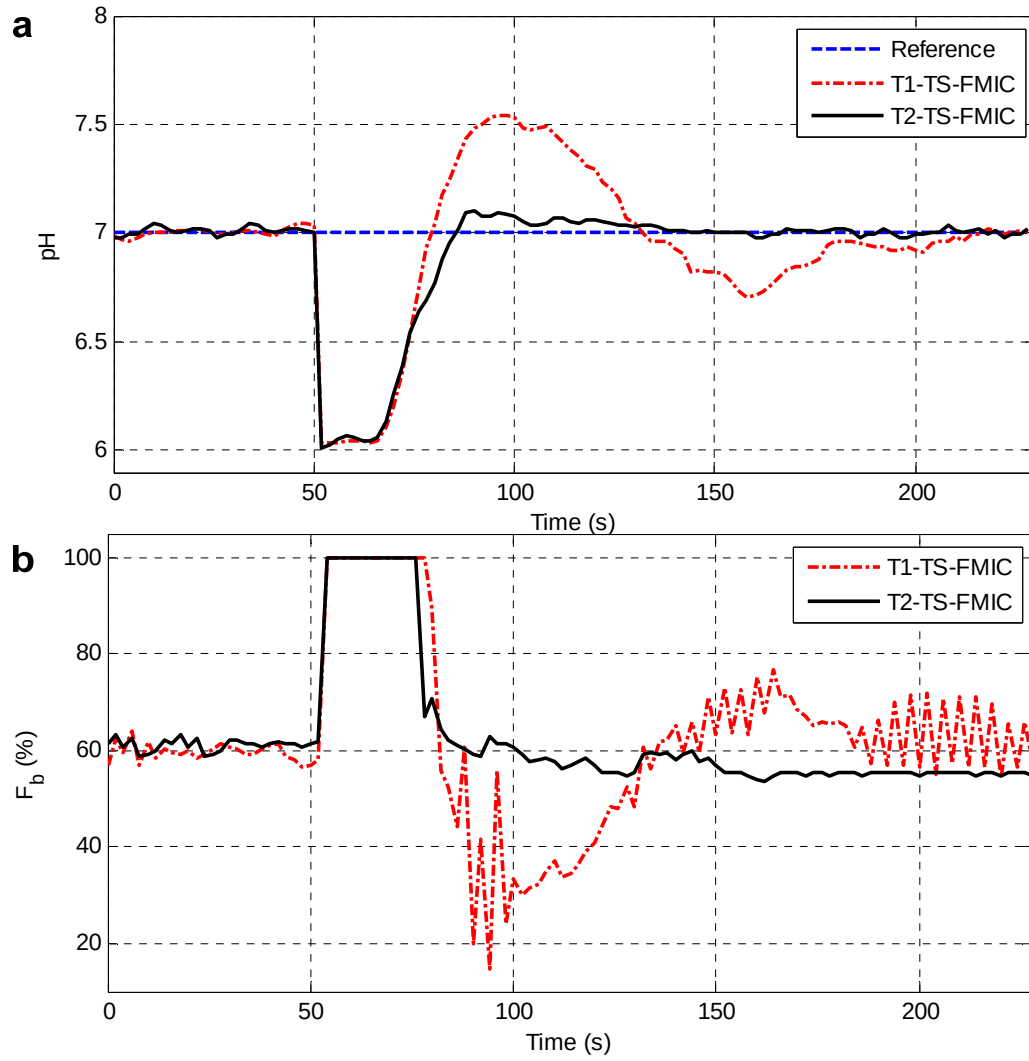


Figure 7.8 : Illustration of (a) the pH neutralization process outputs (b) the inverse Takagi-Sugeno fuzzy model control signals for output disturbances

7.4 Conclusions and Comments

The real-time control study is focused on modeling and control of nonlinear processes and here pH neutralization process is chosen as the application area which is one of the most challenging nonlinear systems in industry. For this reason, firstly the obtained type-1 and type-2 fuzzy models are compared using a uniform random test signal. Root Mean Square Error and Variance Accounted For performance measure indices are used to make a fair comparison between the two fuzzy models. It can be clearly seen that the type-2 fuzzy model has a higher VAF value, and lower RMS value compared to the type-1 fuzzy model. Thus, it can be concluded that type-2 fuzzy model is superior compared to the type-1 fuzzy model in the presence of

uncertainties and strong nonlinearities. That is, the type-2 fuzzy model represents the nonlinear neutralization process much better than the type-1 fuzzy model.

In order to show the effectiveness of the proposed BBBC-T2FMIC (Kumbasar et al., 2011a) and T2-TS-FMIC (Kumbasar et al., 2011b) methods, a comparison study is made with conventional controllers and the BBBC-T1FMIC and T1-TS-FMIC methods proposed by Kumbasar et al., (2008a) and Babuska (1998), respectively. The control performances of the proposed T2FMICs and the other methods are compared based on four performance measures which are overshoot, settling time, IAE, and TV of the control input.

Firstly, the transient performances are investigated under the reference trajectories of 6, 8, and 7 pH values, varying in this successive order. The proposed BBBC-T2FMIC and T2-TS-FMIC schemes provides better performance values compared to conventional PID controllers and the T1FMIC scheme while using a smoother control signal. Since it is a known fact that the pH process inherits nonlinearity and high sensitivity near the neutralization point, reference variation from 6-8 pH are inspected in detail for all of the implemented control structures. The performance values are tabulated in Table 7.13. All the inverse fuzzy control structures reduce the overshoot to 0% and decrease the settling time when they are compared to the PID control structures. Both BBBC-T2FMIC and T2-TS-FMIC schemes outperform BBBC-T1FMIC and T1-TS-FMIC schemes.

Table 7.13 : Performance comparison of the implemented controllers for reference variation 6-8pH.

	6-8 pH			
	IAE	Ts	OS	TV
OPT-PID	76.2	152s	%15	347.7
IMC-PID	74.5	152s	%10	399.9
BBBC-T1FMIC	45.5	92s	%0	1190.5
BBBC-T2FMIC	43.2	78s	%0	146.4
T1-TS-FMIC	45.8	94s	%0	546.3
T2-TS-FMIC	34.7	76s	%0	199.2

For instance, the T2FMIC structures decrease the setting time about 20%. It has also decreased TV value that shows that they have the smoothest control signal. As it can be clearly seen, the T2-TS-FMIC and BBBC-T2FMIC structures have almost identical performance values in the sense of settling time and OS. On the other hand,

BBBC-T2FMIC scheme has a lower TV value while the T2-TS-FMIC has a lower IAE value. Similar comments can be made for the other two reference value variations and disturbance rejection performances. On the other hand, the average computational times of the T2FMICs are almost two times longer than that of T1FMICs, the cause of which is the type reduction/ defuzzification inference existing within the T2FLS structure. However, the results of the real time control applications demonstrated that the T2FMICs are superior to T1FMICs since type-2 fuzzy models are more effective to map high nonlinearities and uncertainties. Besides, since the sampling time could be chosen as 2s for the pH experimental set-up, the real time application of the proposed inverse controller was still feasible.

8. CONCLUSIONS AND RECOMMENDATIONS

In this Ph.D. thesis, three type-2 fuzzy model inversion methodologies are proposed for inverse controller design. In achieving such a goal, the Karnik - Mendel type reduction method is preferred for all the three inversion methods due to the inconsistency and the adaptiveness features.

Firstly, an iterative type-2 fuzzy model inversion technique based on the BB-BC optimization is developed. In this new approach, the inverse type-2 fuzzy model control signal generation is handled as an optimization problem. Since the BB-BC optimization algorithm has a high convergence speed and low computational time, the optimal inverse type-2 fuzzy model control signal can be generated within each sampling time in an on line fashion. The effectiveness and the performance of the method have been shown on both simulation and real-time control studies.

Secondly, a systematic method has been presented to find or form the inverse model of interval type-2 Takagi-Sugeno fuzzy systems for identification, measurement, and control applications. The inversion technique developed and generalized for a class of type-2 fuzzy systems consists of simple permutation of the antecedent and consequence parts based on a purely analytical methodology. The effectiveness and the performance of the method have been shown on both simulation and real-time control studies.

Thirdly, an analytical inverse controller design methodology has been proposed for a certain class of interval type-2 fuzzy models with singleton consequents. The proposed inversion methodology consists of two main steps, decomposing a certain class of interval type-2 fuzzy model into submodels and then finding the inverse of each possible activated interval type-2 fuzzy submodel. To be able to accomplish such a goal, the switching points of the type reduced set are initialized in order to drive the analytical formulation of the inverse interval type-2 fuzzy subsystem output explicitly. Therefore, an iterative algorithm is proposed based on an analytical methodology to form the inverse controller exactly under some conditions. Finally,

the inverse of the interval type-2 fuzzy model is obtained through inversion of each possible activated IT2 FSMs. The exactness and the closed loop control performance of the inversion method have been shown on simulation studies.

It is important to note here that the developed inversion methods and inverse control strategies are restricted to stable SISO processes. However, the proposed inversion method can easily be extended to interval type-2 fuzzy logic systems with alternative type reduction mechanisms. Preferring an alternative type-reduction method may significantly reduce the computational complexity of the inversion method. On the other hand, there are some limitations and constraints of the inversion methods for the interval type-2 Takagi-Sugeno fuzzy model and decomposable interval type-2 fuzzy model inversion methods. For the interval type-2 Takagi-Sugeno fuzzy model inversion, the method is limited to fuzzy models which are piecewise affine according to the inversion variable. Similarly, the decomposable interval type-2 fuzzy model inversion method is only valid for a certain class of interval type-2 fuzzy models with singleton consequents. Moreover, there exist invertibility conditions to be satisfied in this inversion methodology. However, it is important to note here that both developed inversion and inverse control methods are based on a pure analytical method and the exactness is guaranteed. Whereas, the proposed iterative type-2 fuzzy model inversion does not require any invertibility conditions and is not limited to certain type-2 fuzzy model structures since the inversion is solved as an optimization problem. On the other hand, the exactness of the inversion is not guaranteed.

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List of Publications and Patents:

A) International Journal Papers:

- Karasakal, O., Güzelkaya M., Eksin İ., Yesil E., **Kumbasar, T.**, 2011. Normalized Acceleration Based Online Rule Weighting of Fuzzy PID Controllers, Engineering Applications of Artificial Intelligence, (Under Review).
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