ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL OF SCIENCE ENGINEERING AND TECHNOLOGY

TOTAL LEAST SQUARES MATCHING OF 3D SURFACES

PhD THESIS

Umut AYDAR

Department of Geomatics Engineering

Geomatics Engineering Programme

APRIL 2014

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3 BOYUTLU YÜZEYLERİN TOPLAM EN KÜÇÜK KARELER YÖNTEMİ İLE EŞLEŞTİRİLMESİ

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Umut AYDAR, a PhD student of ITU Graduate School Of Science Engineering And Technology student ID 501072613, successfully defended the thesis entitled "**TOTAL LEAST SQUARES MATCHING OF 3D SURFACES**", which he prepared after fulfilling the requirements specified in the associated legislations, before the jury whose signatures are below.

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To my family,

FOREWORD

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February 2014 Umut AYDAR
(Geomatic Engineer)

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ABBREVIATIONS

EIV	**:** Errors In Variables
GUI	**:** Graphical User Interface
ICP	**:** Iterative Closest Point (Yinelemeli En Yakın Nokta)
LS	**:** Least Squares (En Küçük Kareler (EKK))
MGH	**:** Modified Gauss-Helmert
TLS	**:** Total Least Squares (Toplam En Küçük Kareler (TEKK))

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TOTAL LEAST SQUARES MATCHING OF 3D SURFACES

SUMMARY

Co-registration of point clouds of partially scanned objects is the first step of the 3D modeling workflow. The aim of co-registration is to merge the overlapping point clouds by estimating the spatial transformation parameters. In computer vision and photogrammetry domain one of the most popular methods is the ICP (Iterative Closest Point) algorithm and its variants. ICP and other variants of it generally use closed-form solution techniques. These types of solution techniques do not give statistical quality measures of the estimated parameters.

There exist the 3D Least Squares (LS) matching methods as well. The co-registration methods commonly use the least squares (LS) estimation method in which the unknown transformation parameters of the (floating) search surface is functionally related to the observation of the (fixed) template surface. Here, the stochastic properties of the search surfaces are usually omitted. This omission is expected to be minor and does not disturb the solution vector significantly. However, the a posteriori covariance matrix will be affected by the neglected uncertainty of the function values of the search surface. This causes deterioration in the realistic precision estimates. In this thesis, we propose a method where the stochastic properties of both the observations and the parameters are considered under an errors-in-variables (EIV) model.

The mathematical model of the applied method is based on the non-linear Gauss-Helmert Model. In the model, Lagrange Multipliers are eliminated and thus it is called Modified Gauss-Helmert (MGH) by Kanatani. From this aspect, it is very appropriate solution method for large data sets. The geometrical relation between data sets is established by rigid body transformation. Objective of the model is to estimate the optimal transformation parameters of rigid-body transformation by minimizing the Mahalonobis distance in the sense of Maximum Likelihood. Since the functional model is non-linear, the system is linearized with respect to the unknowns and observations. The solution is obtained iteratively.

The stochastic properties of search surface elements are taken into account in MGH model, whilst this is ignored in the classical Gauss-Markov (GM) model. On the other hand, if the covariance matrices are defined as isotropic and homogeneous for both template and search surfaces, the MGH model gives almost the same results with the GM model. Therefore, we define anistropic and inhomogeneous covariance matrices for both data sets and compare the results with the LS method.

The most important and critical part of the registration algorithms is correspondence search. We used point-to-point and point-to-plane error metrics together for correspondence search with some rejection strategies.

3 BOYUTLU YÜZEYLERİN TOPLAM EN KÜÇÜK KARELER YÖNTEMİ İLE EŞLEŞTİRİLMESİ

ÖZET

3 Boyutlu modelleme işleminin ilk işlemi parçalar halinde elde edilmiş olan nokta bulutlarının birleştirilmesidir. Eşleştirme işleminde amaç, farklı istasyonlardan taranarak elde edilmiş olan ve herbiri lokal bir koordinat sisteminde tanımlı yüzey parçaları arasındaki dönüşüm parametrelerinin hesaplanması ve bu iki yüzeyin tek bir koordinat sistemine dönüştürülmesidir. Bu işlem temelde iki koordinat sistemi arasında belirlenen ortak noktalar yardımıyla bir koordinat dönüşümüdür. 3 Boyutlu yüzey eşleştirme algoritmaları içerisinde en popüler olan ve ticari yazılımlarda da sıklıkla tercih edilen yöntem Yinelemeli En Yakın Nokta (Iterative Closest Point-ICP) olarak adlandırılan yöntemdir. Yöntem üç kısımdan oluşmaktadır. Bunlardan ilki, iki veri seti içerinde (hedef ve arama verisi) karşılıklı ortak noktaların bulunmasıdır. Daha sonra bulunan ortak noktalar yardımıyla dönüşüm parametreleri hesaplanır ve son işlemde dönüşüm uygulanarak arama verisi hedef verisine yaklaştırılır. İşlem, her iki veri setindeki dönüşüm parametreleri arasındaki fark belirlenen eşik değere ulaşana kadar yinelemeli olarak tekrar eder. ICP, kullanılan eşlenik nokta arama yöntemine göre temelde ikiye ayrılmaktadır. İlk yöntem Besl ve McKay tarafından 1992 yılında önerilen, hedef verisinde yer alan bir noktaya karşılık arama verisinde en küçük Öklid uzaklığına sahip olan noktayı bulan ve bu iki noktayı eşlenik kabul eden yöntemdir. Chen ve Medioni tarafından 1992 yılında önerilen yöntemde ise, hedef verisinde bulunan bir noktanın eşleniği olarak, bu noktanın arama verisi üzerinde en yakın olduğu yüzey üzerinde; ve noktadan yüzeye olan normal doğrultusunun bu yüzeye değdiği nokta olarak kabul edilmektedir. Daha sonra bu iki algoritmayı temel alan çeşitli ICP algoritmaları önerilmiştir. ICP ve türevlerinde genellikle kapalı form çözümler uygulanmaktadır. Bu tür çözümler hesaplanan parametrelerin güvenilirliği ve doğruluğu hakkında istatistik bilgi içermemektedirler.

Diğer bir önemli eşleştirme algoritması En Küçük Kareler yöntemi ile 3 Boyutlu yüzey eşleştirmesidir. Oluşturulan gözlem denklemleri ile hedef ve arama yüzeyleri arasında fonksiyonel ilişki kurulur. En Küçük Kareler yöntemine göre parametre kestirimi yönteminde arama verisinin stokastik özellikleri gözardı edilir. Bu durumun sonuç vektörünü etkilemeyeceği düşünülmektedir. Ancak oluşan belirsizlik kestirim sonrası elde eldilen kovaryans matrisinde etkisini gösterecek ve gerçekçi olmayan parametre kestirim doğruluklarının ortaya çıkmasına neden olacaktır. Daha gerçekçi sonuçlar elde edilebilmesi için, hedef verisi ile birlikte arama verisinin de stokastik özelliklerini gözönünde bulunduran bir çözüm yöntemi uygulanmalıdır. Bu tez çalışmasında, hatalı değişkenler (errors-in-variables) olarak ifade edilebilecek modele uygun bir çözüm yöntemi kullanılarak yüzey birleştirilmesi önerilmektedir.

Önerilen yöntemin matematiksel modeli lineer olmayan modifiye edilmiş Gauss-Helmert (MGH) modeldir. Yöntemin önemli bir avantajı Lagrange çarpanlarının normal denklemlerden elimine edilmiş olmasıdır. Optimizasyon problemlerinde koşul denklemleri için tanıtılan Lagrange çarpanları ile birlikte her iterasyonda çözülmesi gereken normal denklemler matrisinin boyutları artmaktadır. Bu durum hesap yükünü oldukça artırmaktadır. Modelde iki veri seti arasındaki ilişki 6 parametreli benzerlik dönüşümü ile sağlanmakta ve noktalar arasındaki Mahalonobis uzaklıkları minimize edilmektedir. Fonksiyonel model lineer olmadığı için sistem bilinmeyenler ve gözlem denklemlerine göre kısmi türevler alınarak lineerleştirilir. Lineer olmayan denklem sisteminin çözümü için iyi seçilmiş başlangıç yaklaşık değerlerine ihtiyaç duyulmaktadır. Aksi takdirde fonksiyonun yerel minimuma yakınsama riski ortaya çıkmaktadır.

Çalışmada kullanılan modifiye edilmiş Gauss-Helmert modelinin kullanılabilmesi için her iki veriye ait stokastik özelliklerin de bilinmesine gereksinim vardır. Genellikle algılayıcı sistemler tarafından elde edilmiş 3 boyutlu veri setleri için isotropic ve homojen varyans-kovaryans matrisleri kullanılmaktadır. Eğer her iki veri seti için de kovaryans matrisleri eşit ve birim matris olarak alınırsa, klasik en küçük kareler yöntemi bu tip problemlerin çözümünde optimum sonucu vermektedir. Ancak belirtilen ağırlık modeli gerçekçi değildir. Algılayıcılar ile elde edilmiş olan veri setlerinde herbir noktanın doğruluk değeri farklıdır. Öyle ki algılayıcıya yakın olan ya da algılayıcının bakış açısının dik olduğu noktalar daha yüksek doğruluklu olarak ölçülecektir. Algılayıcı ile nokta arasındaki açı ve mesafe arttıkça noktanın algılanma doğruluğu azalır. Öte yandan, yine bir sensör kullanılarak elde edilmiş arama verisinin hatasız kabul edilmesi de gerçekçi değildir. Bu hususlardan yola çıkılarak, her iki veri için de mümkün olduğunca gerçekçi bir öncül kovaryans matris tanımı yapılmasına ihtiyaç duyulmaktadır. Tarayıcı sistemlerin kalibrasyonu ve hata kaynakları çalışmaları incelendiğinde veri doğruluğunu etkileyen farklı etkenlerin olduğu görülmektedir. Bunlar arasında sıcaklık ve nem koşulları, kenar etkisi, yüzeylerin yansıtıcı özellikleri, laser ışınının yansıma açısı vb. gibi etkiler öne çıkmaktadır. Kovaryans tanımlamasında belirtilen faktörlerden sadece yansıma açısı ele alınmış, diğerlerinin etkileri gözardı edilmiştir. Tarayıcıdan çıkan laser ışınının izi yansıma açısının sıfır olduğu (yüzeye dik durum) durumlarda tam bir daire oluşturuken, açı arttıkça izin elipse döndüğü gözlenmektedir. Bu da algılayıcıya geri dönen lazer ışınının sinyal kalitesini azaltarak özellikle mesafe ölçümlerinde olumsuz etkiye sebep olmaktadır. Bazı çalışmalarda yansıma açısının mesafe doğruluğuna ters orantılı olarak etki ettiği belirtilmektedir. Öte yandan lazer tarayıcı sistemlerin açı ve mesafe ölçme doğrulukları bilinmektedir. Çalışmada bu değerler ve yansıma açısı etkisi kullanılarak her bir noktaya ait öncül kovaryans değerleri hedef ve arama verileri çin ayrı olarak hesaplanmış ve modele bu matrisler girdi olarak sokulmuştur. Farklı veri setleri kullanılarak yapılan testler sonucunda, isotropik ve homojen noise tanımlaması yapılan veri setleri ile yapılan dönüşümlerde En Küçük Kareler (EKK) yöntemi (Gauss-Markov) ve Toplam En Küçük Kareler (TEKK) yöntemlerinin (Modified Gauss-Helmert) parametre kestirim doğrulukları bakımından çok benzer sonuçlar verdiği gözlemlenmiştir. Farklı ağırlıklar kullanılarak yapılan testlerde ise TEKK yöntemi ile daha gerçekçi sonuçlar elde edilebilmektedir.

Eşleştirme işleminde en önemli sorun nokta eşleşmelerinin sağlanmasıdır. Eşlenik noktaların ne derece doğru belirlendiği parametre kestirim doğruluklarını ve sonuç vektörünü de etkileyebilecek bir faktördür. Öte yandan, eşlenik nokta arama işlemi, tüm birleştirme algoritmalarında en fazla zaman alan ve hesap yüküne sahip olan kısımdır. Noktalar arası en yakın Öklid uzaklıklarını baz alan eşleştirme yöntemi noktalar ile yüzey arasında eşleştirme yöntemine göre daha hızlı olmakla birlikte; ikinci yöntemde daha iyi yakınsama değerleri elde edilebilmektedir. Eşleştirme işlemini hızlandırmak amacıyla bazı veri yapıları (kdtree, octree vb.) ve eşik değer tanımlamaları kullanılmaktadır. Bu çalışmada nokta eşleşmeleri, yukarıda bahsedilen iki temel yöntemin birlikte kullanılması ile elde edilmektedir. Ayrıca eşleştirme algoritması için bir takım eşik değer tanımlamaları ve koşullar kullanılmıştır. Buna göre her iki yüzey verisinde sınır noktalarda kalan noktalar eşleşme öncesi elenmektedir. Yine iki nokta arası en kısa uzaklığın belirlenen eşik değerden büyük çıkması durumunda da noktalar elenmekte ve eşlenik olarak kabul edilmemektedir. Bir diğer kısıt ise noktanın yüzey normali doğrultusunda projeksiyonu sonucu ilgili üçgenin sınırları içerisinde kalması şartıdır. Tüm belirtilen işlemler ve koşullar sonucu eşlenik olarak kabul edilen noktalar sıralı olarak kaydedilmekte ve koordinat dönüşümleri bu noktalar kullanılarak yapılmaktadır.

1. INTRODUCTION

1.1 Motivation

Since laser scanners as a direct measurement device or cameras for image acqusition are line-of-sight devices, any kind of object has to be acquired partially and with an overlap for 3D modelling purpose. Hence, partially acquired data are obtained in their coordinate sytem uniquely. As a result of this situation, a merging problem of all those data arises in order to create a full 3D objects. Aforementioned problem can basically be defined as a coordinate transformation and is generally termed as registration or alignment in literature.

The objective of registration is to merge the overlapping point clouds by estimating the spatial transformation parameters for each partial surface view. There are three main solution for the problem. The first one is to use special targets provided by the producing companies. These special shaped and sized targets are placed into the scanning area in an appropriate distribution. These targets must be defined and scanned individually during the process. All these placing, defining and scanning procceses of these targets increase the scanning time and labor naturally. Another solution is to run the scanning campaign based on a geodetic infrastructure which is possible nowadays with the new generation laser scanners. These new generation scanners provide the flexibility of using well-known geodetic measurement methods like traversing, resection or intersection. By one of these methods, the each scan position is defined and by this way the direct registration of all scans are obtained. But, as it is stated above, a pre-defined geodetic infrastructure is required for this solution which is again a factor increasing the scanning time, labor and number of stuff. Moreover, measurement errors of the geodetic system and positioning errors of the scanner on a known point will definitely affect the registration accuracy.

The third alternative for registration problem is so-called surface based registration techniques. The basic principle of all these techniques is to calculate the 3D rigid body transformation parameters by the help of sufficient number of corresponding point pairs in two data sets. Once the establishment of the correspondences are set, the transformation is calculated. The correspondence search differs in diverse studies in literature in terms of objective function to be minimized. These tecniques requires more post-processing job in office environment on the contrary of other two mentioned before.

1.2 Aim of the Research

Surface based registration techniques have been studied extensively for years by the researchers from many disciplines like Geospatial sciences (Photogrammetry, Geodesy etc.), Computer vision (Human vision, autonomous robot systems, virtual reality), Medical sciences (3D medical imaging), Mechanical engineering (reverse engineering) etc. in which 3D models and 3D modeling techniques are frequently used. It is still an active area of research and studies especially are focused on accuracy assessments, automatisation, effective correspondence search algorithms and robust optimization techniques.

The most popular surface based registration technique is the Iterative Closest Point (ICP) which is introduced by Besl and McKay (1992). After the introduction of ICP, it became a popular and dominant technique and the researchers have proposed many variants. It is really an effective and easily applicable method by its straight forward mathematical formulation. Despite the popularity of the ICP, it has some disadvantages such as the inability of accuracy assesment of transformation parameters. ICP based algorithms generally use closed-form solutions for the estimation of transformation parameters. The closed-form solutions cannot fully consider the statistical accuracy assesment of the estimated parameters.

Another powerfull and adaptive method for the registration problem is the 3D least squares surface matching proposed by Gruen and Akca in 2005. The method is the extension and adaptation of mathematical model of Least Squares 2D image matching for the 3D surface matching problem. Calculation of the transformation parameters is based on the solution of a system of non-linear equations where coordinates of points in one system (template) are regarded as obsevation vector and the transformation parameters are regarded as unknown vector. The parameter estimation is achieved using the Generalized Gauss-Markov model. But, in the functional model of classical Least Squares method, only the observation vector elements are assumed as erroneous while design matrix elements are treated as error-free. The stochastic properties of the elements in design matrix are ignored. In other words, for the registration case, it is assumed that the search surface points are not erroneous while the points in template data are contaminated by random errors. If stochastic properties of the search surface elements are ignored the solution vector is expected to be minor. However, the a posteriori covariance matrix is affected by the neglected uncertainty of the search surface. This situation naturally causes unrealistic precision estimation of unknown parameters.

On the other hand, another assumption in almost all registration algorithms is the homogeneous and isotropic noise for both data sets. In fact this assumption is incorrect and unrealistic for 3-D data acquired by 3-D sensing such as stereovision and laser/ultrasonic range finders, because the accuracy is usually different between the depth direction and the direction orthogonal to it, resulting in an inhomogeneous and anisotropic noise distribution depending on the position, orientation, and type of the sensor (Kanatani 2012). Besides, data sets acquired with different sensors or data sets acquired in different times also have inhomogeneous and anisotropic noise level. In the case of a registration process (data fusion, change detection monitoring etc.) of this kind of data, the stochastic properties should be taken into consideration for an optimal, realistic solution.

As a consequence, a new approach which takes into account the shortcomings mentioned above should be adapted for the registration problem of 3D surface patches. For this reason, in this study we propose a method where the stochastic properties of both the observations and the parameters are considered under an errors-in-variables (EIV) model. This thesis study aims to investigate the results of a 3D registration problem in terms of statistical behaviors under an EIV model by using an appropriate and optimal solution model for large data sets.

2. LITERATUR REVIEW ON 3D SURFACE MATCHING

3D object modeling plays an important role for many applications from reverse engineering to creating the real-world models for virtual reality, architecture or deformation analysis. In the last decade, laser scanners had an utmost importance for 3D object modeling due to their ability of providing reliable 3D data very fast and directly. Since the range scanners are line-of-sight instruments, in many cases an object has to be scanned from different standpoints to be able to cover the whole object. As a result, separate point clouds, which are in their own local co-ordinate systems uniquely, are obtained. In order to form a 3D model, these point clouds have to be merged in one co-ordinate system. This process is called alignment or registration. Various methods were proposed and the studies in this area are still in progress especially in computer vision discipline including the most popular Iterative Closest Point (ICP) algorithm and its variants.

According to ICP algorithm proposed by Besl and McKay (1992), each point in one data set is paired with the closest point in the other data set to form correspondence pairs. To minimize squared distance between points in each correspondence, point to point error metric is used. The process of this iterative algorithm can be clarifed with 3 main steps; 1. Pairing each point of one data set to the closest point in the other data set, 2. Computing the motion that minimises the mean square error between the paired points, 3. Appliying the motion to one data set and updating the mean square error (D. Chetverikov, 19991). This process is iterated until the convergence is achieved.

Chen and Medioni (1991) proposed a similar iterative scheme using a different pairing procedure based on surface normal vector. To minimize square distance between points in each correspondence, point to plane error metric is used. Unlike other techniques, this technique is usually solved using standard nonlinear least squares methods and each iteration is faster than other techniques. However, this formulation is only applicable to points on surfaces.

Rusinkiewicz and Levoy (2001) give an update of the variants of the ICP algorithm. They compare several ICP variant characteristics according to their convergence speed and effects on the perfomance of ICP algorithm. They introduce the concept of normal-space-directed sampling, and show that it improves the convergence in scenes involving sparse, small-scale surface features.

Despite the popularity of the ICP, there are some disadvantageous aspects of it in terms of accuracy assesment of transformation parameters. One another powerful and adaptive method for the registration problem is the 3D least squares surface matching proposed by Gruen and Akca, in 2005. The method is the extension and adaptation of mathematical model of Least Squares 2D image matching for the 3D surface matching problem. The transformation parameters of the search surfaces are estimated with respect to a template surface. The solution is achieved when the sum of the squares of the 3D spatial (Euclidean) distances between the surfaces are minimized. The parameter estimation is achieved using the Generalized Gauss-Markov model (Akca, 2010). At this model, the points on the template surface are considered as observations, contaminated by random errors, while the search surface points are assumed as error-free. Here, and also in the ICP methods, the stochastic properties of the search surfaces are usually omitted. This causes deterioration in the realistic precision estimates. More details on this issue can be found in Gruen (1985), Maas (2002), Gruen and Akca (2005), Kraus et al. (2006), and Akca (2010). These algorithms consider the noise as coming from one measurement only, while both surface measurements are in fact corrupted by noise. In order to overcome this undesirable situation and to obtain more realistic precision estimates, another approach which takes the stochastic properties of the elements of design matrix into consideration should be applied. The problem can be solved by using an estimaton technique which is called as total least squares (TLS), in the sense of Errors-in-Variables model (EIV). Markovsky and Huffel (2007) outlines the different solution methods and application areas of EIV model in detail. Ramos and Verriest (1997) proposed to use the total least squares approach for the registration of m-D data. In their study, they use a mixed solution which is the combination of Least squares and Total Least Squares methods for the registration of 2D medical images. However, they do not give any information about the precision of the transformation parameters. Akyılmaz (2007) uses Total Least Squares method for coordinate transformation in Geodetic applications. Since the author uses a closed-form solution method in this study, there is not any information about precision of estimated parameters as well. A mathematical model is given by Neitzel (2007) where an iterative Gauss-Helmert type of adjustment model with the linearized condition equations is adopted. However, in this method the size of the normal equations to be solved increases dramatically depending on the number of conjugate points, since each point introduces three more Lagrange multipliers into the normal equations. Thus, the larger the number of conjugate points requires the greater the normal equations to be solved.

3. MATHEMATICAL MODEL OF TOTAL LEAST SQUARES MATCHING

3.1 Functional Model

Suppose we have two 3D data sets acquired by a range scanner from adjacent standpoints. The task is to calculate the transformation parameters that bring these two data sets into the best alignment. Let A_m and B_n (m=1,…,M and n=1,…,N),are template and source point clouds respectively, which are defined in a local coordinate system and need to be merged into a common coordinate system. The geometric relationship is established by a six parameters 3D rigid-body transformation such that (3.1);

$$A = R(B) + T \tag{3.1}$$

Where R is an orthogonal rotation matrix and T is a translation vector.

We need to find some common points in both data sets in order to apply (3.1). We establish the correspondence by using two different error metrics in combination which is given in Chapter 3.4 in detail. Assume that $a_i(\mathrm{x,y,z})$ and $b_i(\mathrm{x,y,z})$ (i=1,……..,G) are corresponding point pairs which are the subset of original data sets A_m and B_n. The functional model with respect to the errors-in-variable model is formed as (3.2);

$$\mathrm{a_i}\left(\mathrm{x,y,z}\right) + \mathrm{e_i}\left(\mathrm{x,y,z}\right) = \mathrm{b_i}\left(\mathrm{x,y,z}\right) + \mathrm{E_i}\left(\mathrm{x,y,z}\right) \tag{3.2}$$

Thus the observation equations of rigid-body transformation in EIV model is written as (3.3);

$$a_i + e_i = t + R(b_i + E_i) \tag{3.3}$$

Where e_i and E_i are true error vectors introduced to the template and search data respectively, with the assumptions (3.4);

$$
\begin{aligned}
e_i &\sim N(0, Q_{xx}[a_i]) \\
E_i &\sim N(0, Q_{xx}[b_i]
\end{aligned}
\tag{3.4}
$$

If we apply this model to 3D rigid-body transformation, the mathematical model is established as (3.5);

$$l + v_x = (A + v_A)\beta \tag{3.5}$$

where v_x is the $n \times 1$ residual vector of observations and v_A is an $n \times m$ error matrix of the corresponding elements of design matrix. The elements of both v_x and v_A are independent and conforming the normal distributed with zero mean. Once a minimisation of $[\widetilde{v_A}; \widetilde{v_x}]$ is found, then any β satisfying $(A + \widetilde{v_A})\beta = 1 + \widetilde{v_x}$ is the solution vector of the problem by total least squares.

3.2 Stochastic Model

Stochastic model defines the variance and covariances of observations. The standard deviations of the estimated transformation parameters and the correlations between them may give useful information concerning the stability of the system and quality of the data content (Gruen, 1985a) The covariance matrix consists of two components as a positive definite symmetric matrix Q_{ll}, also called the cofactor matrix (Mikhail & Ackermann, 1976) being an initial covariance matrix, giving the structure of Q_{xx}, and the multiplicative variance factor σ_0 to be estimated (Foerstner, 2000). Cofactor matrix Q_{ll} is defined as the inverse of the weight matrix as (3.6);

$$Qll = P^{-1} = \begin{vmatrix} \frac{\sigma_i^2}{\sigma_0^2} & & & \\ & \frac{\sigma_i^2}{\sigma_0^2} & & \\ & & \ddots & \\ & & & \frac{\sigma_i^2}{\sigma_0^2} \end{vmatrix} \tag{3.6}$$

Where σ_0 is the a-priori standard deviation of unit weight and σ_i is the a-priori standard deviation of the observation i, (i = 1, 2,...,….G).

In classical Least Squares approach, the stochastic properties of the elements in design matrix is ignored. In other words, it is assumed that the search surface points are not erroneous while the points in template data are contaminated by random errors.

On the other hand, the noise is assumed to be homogeneous and isotropic for both data sets in almost all registration algorithms.

The least square solution is optimal for this kind of a model. However, this model is not realistic in many situations (Ohta and Kanatani, 1998).

- The noise is often neither isotropic nor identical (Kanazawa et. al., 1995). 3D data acquired by 3D sensing such as stereovision and laser/ultrasonic range finders, because the accuracy is usually different between the depth direction and the direction orthogonal to it, resulting in an inhomogeneous and anisotropic noise distribution depending on the position, orientation, and type of the sensor (Kanatani 2012).

- Erroneous and error-free analysis does not make sense between data sets, if they are both acquired by a range scanner. It is so clear that if one is affected from noise, the other one has to be as well.

Taking these considerations into account, we treat both coordinate components of template and search data as observations and therefore they are both affected by random errors. From this point view, we define different covariance matrices $Q_{xx}[a_i]$ and $Q_{xx}[b_i]$ for template and search data, respectively.

Ohta and Kanatani, (1998) shows that if $Q_{xx}[a_i] = Q_{xx}[b_i] = I$ where I is identity matrix, 3.16 reduces to the least squares method. This proves that the Least Square method is optimal for isotropic and homogenous noise even if both data sets contain errors.

The studies on calibration of laser scanners (Lichti, 2007) and laser scanner accuracy assessment (Boehler, 2003) show that there are many factors affecting the individual point precision of terrestrial laser scanners like instrument's range and angular accuracy, geometric factors (e.g. edge effect and incidence angle), atmospheric conditions (humidity, temperature etc.) and radiometric effects (surface reflectivity etc.). In our implementation, we define a covariance matrix for each individual point by using the range and angular accuracy of the scanner and incidence angle. Since the cartesian coordinates are derived quantities, we can transform back them into the spherical coordinates (ρ=range, θ=horizontal angle and φ=vertical angle) basically by using the equations (3.7);

$$\begin{aligned}
\rho_i &= \sqrt{x_i^2 + y_i^2 + z_i^2} \\
\theta_i &= arctan(\frac{y_i}{x_i}) \\
\varphi_i &= arctan(\frac{z_i}{\sqrt{x_i^2 + y_i^2}}) \\
r_i &= [\rho_i \, \theta_i \, \varphi_i]
\end{aligned} \qquad \textbf{(3.7)}$$

Thus we can write the Cartesian coordinates of a point P_i in terms of spherical coordinates as (3.8);

$$P_i = \left[\left\{ \begin{array}{c} P_x = \rho_i \, cos(\theta_i) cos(\varphi_i) \\ P_y = \rho_i \, cos(\varphi_i) sin(\theta_i) \\ P_z = \rho_i \, sin(\varphi_i) \end{array} \right\} \right] \qquad \textbf{(3.8)}$$

The 3×3 covariance matrix of point P_i is obtained from propagating the precision of the original spherical observables, which are typically provided by the manufacturer (Grant, 2012). The variances of these spherical observables are $\sigma_{P_i}, \sigma_{\theta_i}, \sigma_{\varphi_i}$ respectively. Precision values of spherical coordinates provided by the vendor are (3.9);

$$\sum_r = \begin{bmatrix} \sigma_{P_i}{}^2 & & \\ & \sigma_{\theta_i}{}^2 & \\ & & \sigma_{\varphi_i}{}^2 \end{bmatrix} \tag{3.9}$$

The second factor affecting the point accuracy is the incidence angle α, which is defined as the angle between the laser beam and local surface normal on observed point. Simply it could be explained as the orientation of the local surface with respect to the scanner position. Soudarissanane et al. (2011) explain that the incidence angle and range accuracy are inversely proportional. Therefore, we update our range measurement accuracy by dividing the range variance by cosine of the incidence angle. By replacing the first element in (3.9) with $\left(\frac{\sigma_{P_i}}{cos(\alpha)}\right)^2$, we obtain a new precision variance matrix for point $r_i, \sum r_{new}$. If we apply the error propagation law, we obtain the covariance matrix for point P_i by using the (3.10);

$$Q_{xx} = F\sum r_{new} F^T \tag{3.10}$$

Where F is the Jacobian matrix of P_i with respect to ρ_i, θ_i and φ_i respectively (3.11).

$$F = \begin{bmatrix} \cos(\varphi)\cos(\theta) & -\rho\cos(\varphi)\sin(\theta) & -\rho\cos(\theta)\sin(\varphi) \\ \cos(\varphi)\sin(\theta) & \rho\cos(\varphi)\cos(\theta) & -\rho\sin(\varphi)\sin(\theta) \\ \sin(\varphi) & 0 & \rho\cos(\varphi) \end{bmatrix} \tag{3.11}$$

3.3 Proposed Modified Gauss-Helmert Model

The generalized total least squares solution of the 3D-similarity transformation by introducing the quaternions as the representation of the rotation matrix×scale factor $S = sR$ based on iteratively linearized Gauss-Helmert model has been presented by Akyilmaz (2011). However, this model requires the solution of a normal matrix, which includes the corresponding terms for transformation parameters as well as the Lagrange multipliers, thus yielding a larger size of system of equations to be solved at each iteration with increasing number of the identical points of the transformation problem. Following the idea of Akyilmaz (2011), Kanatani and Niitsuma (2012) has developed a new computational scheme for 3D-similarity transformation which they call Modified Iterative Gauss-Helmert model by reducing the so-called Lagrange multipliers and hence the size of the normal matrix is dramatically reduced. In other words, the unknowns to be solved at each iteration are equal to seven, i.e. the number of transformation parameters. This kind of reduction provides advantage, especially in terms of computational cost. In original publication of Kanatani and Niitsuma in 2012, the model is applied as 7 parameters similarity transformation for the registration of range images which takes the scale factor into consideration. However, if the case is the registration of point cloud data, it is possible to ignore the scale factor, since the scale does not vary between data sets. Therefore, in our study, we modified the model by eliminating the scale factor in order to apply 6 parameters rigid-body transformation. In quaternion algebra, the scaled rotatiton matrix is defined as (3.12);

$$S = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1 q_2 - q_0 q_3) & 2(q_1 q_3 + q_0 q_2) \\ 2(q_2 q_1 + q_0 q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2 q_3 - q_0 q_1) \\ 2(q_3 q_1 - q_0 q_2) & 2(q_3 q_2 + q_0 q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix} \quad \textbf{(3.12)}$$

This matrix represents a rotation if q is normalized to unit norm (Kanatani, 1990). If q is not restricted to a unit vector, the square norm $\|q\|^2$ represents the scale changes (Kanatani, 1990). For normalising procedure, the following constraint has to be introduced (3.13).

$$q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1 \quad \textbf{(3.13)}$$

Thus, the final estimate of rotation matrix is orthogonal and free of scale factor. If an arbitrary parameter of quaternion in (3.13) is written in terms of other remaning ones and substituted into the system of observation equations, the new system of equations would have 6 unknown parameters (three linear parameters defining the translation and three parameters for rotation). By rearranging the (3.13) we can write (3.14);

$$N = q_0 = \sqrt{1 - q_1^2 - q_2^2 - q_3^2} \quad \textbf{(3.14)}$$

Substituting (3.14) into (3.12) we eliminate one parameter and obtain the pure rotation matrix as (3.15);

$$\tilde{S} = \begin{bmatrix} -2q_2^{\ 2} - 2q_3^{\ 2} + 1 & 2q_1q_2 - 2q_3N & 2q_2N + 2q_1q_3 \\ 2q_3N + 2q_1q_2 & -2q_1^{\ 2} - 2q_3^{\ 2} + 1 & 2q_2q_3 - 2q_1N \\ 2q_1q_3 - 2q_2N & 2q_1N + 2q_2q_3 & -2q_1^{\ 2} - 2q_2^{\ 2} + 1 \end{bmatrix} \tag{3.15}$$

In so-called model, let a_i and b_i are the corresponding pairs of observed points $\left(i = 1, \ldots, M\right)$; $Q_{xx}\left[a_i\right]$and $Q_{xx}\left[b_i\right]$ are normalized covariance matrices; and $\overline{a_i}$ and $\overline{b_i}$ are the true positions of a_i and b_i respectively. Then the optimal estimation of the rigid-body transformation parameters R (rotation) and T (translation) in the sense of Maximum Likelihood is to minimize the Mahalanobis distance given as follows (3.16).

$$J = \frac{1}{2}\sum_{i=1}^{M}\left(a_i - \bar{a}_i\right)^T Q_{xx}^{\ -1}\left[a_i\right]\left(a_i - \bar{a}_i\right) + \frac{1}{2}\sum_{i=1}^{M}\left(b_i - \bar{b}_i\right)^T Q_{xx}^{\ -1}\left[b_i\right]\left(b_i - \bar{b}_i\right) \tag{3.16}$$

Where the observation equations are defined with respect to $\overline{a_i}$ and $\overline{b_i}$ as **(3.17)**;

$$\bar{a}_i = \tilde{S}\overline{b_i} + t \tag{3.17}$$

The total error vector is defined as (3.18);

$$e_i = a_i - \tilde{\mathrm{S}}b_i - t \tag{3.18}$$

And the weigth matrix is defined as the inverse of the covariances of observed coordinates (3.19).

$$W_i = (\tilde{S}Q_{xx}\left[a_i\right]\tilde{S}' + Q_{xx}\left[b_i\right])^{-1}$$

$$with \qquad V_iW_i = I \tag{3.19}$$

$$V_i = (\tilde{S}Q_{xx}\left[a_i\right]\tilde{S}' + Q_{xx}\left[b_i\right])$$

If the Lagrange Multipliers λ are introduced into (3.16) as constraint (3.17), we have (3.20);

$$J=\frac{1}{2}\sum_{i=1}^{M}\left(a_i-\bar{a}_i\right)^T Q_{xx}{}^{-1}\left[a_i\right]\left(a_i-\bar{a}_i\right)...$$
$$...+\frac{1}{2}\sum_{i=1}^{M}\left(b_i-\bar{b}_i\right)^T Q_{xx}{}^{-1}\left[b_i\right]\left(b_i-\bar{b}_i\right)-\sum_{i=1}^{M}\lambda_i^T\left(\bar{a}_i-\tilde{S}\bar{b}_i-t\right) \quad \textbf{(3.20)}$$

The solution of the optimization problem of (3.20) is given with respect to Gauss-Helmert model. The defined problem is a non-linear system of equations and can be solved with an iterative approach. Therefore the approximate values a_i^0, b_i^0, q and t are required for the unknowns.Thus we can write (3.21);

$$\bar{a}_i=a_i^{(0)}+\Delta\bar{a}_i \qquad \bar{b}_i=b_i^{(0)}+\Delta\bar{b}_i$$
$$\bar{q}=q+\Delta q \qquad \bar{t}=t+\Delta t \quad \textbf{(3.21)}$$

Where $\Delta\bar{a}_i$, $\Delta\bar{b}_i$, Δq and Δt are corrections to the approximations.

If (3.21) substituted into (3.17) and expanded into Taylor Series, we obtain **(3.22)**;

$$b_i^{(0)}+\Delta\bar{b}_i=\tilde{S}(a_i^{(0)}+\Delta\bar{a}_i)+\sum_{i=0}^{3}\Delta q_i\frac{\partial\tilde{S}}{\partial q_i}a_i^{(0)}+t+\Delta t \quad \textbf{(3.22)}$$

Substituting (3.21) and (3.22) into (3.20), the objective function reads (3.23);

$$\tilde{J}=\frac{1}{2}\sum_{i=1}^{M}((\mathrm{a}_i-a_i^{(0)}-\Delta\bar{a}_i)^{\mathrm{T}}Q_{xx}\left[a_i\right]^{-1}(\mathrm{a}_i-a_i^{(0)}-\Delta\bar{a}_i))...$$
$$...+\frac{1}{2}\sum_{i=1}^{N}((\mathrm{b}_i-b_i^{(0)}-\Delta\bar{b}_i)^{\mathrm{T}}Q_{xx}\left[b_i\right]^{-1}(\mathrm{b}_i-b_i^{(0)}-\Delta\bar{b}_i))... \quad \textbf{(3.23)}$$
$$...-\sum_{i=1}^{N}(\lambda_i^{\mathrm{T}}(\mathrm{b}_i^{(0)}+\Delta\bar{b}_i-\tilde{S}(\mathrm{a}_i^{(0)}+\Delta\bar{a}_i)-\sum_{i=0}^{3}\Delta q_i\frac{\partial\tilde{S}}{\partial q_i}a_i^{(0)}-t-\Delta t))$$

Setting the partial derivatives of (3.23) to zero yields (3.24), (3.25);

$$\frac{\partial\tilde{J}}{\partial\Delta\bar{a}_{\mathrm{i}}}=-Q_{xx}\left[a_i\right]^{-1}(a_i-a_i^{(0)}-\Delta\bar{a}_{\mathrm{i}})+\tilde{\mathrm{S}}^{\mathrm{T}}\lambda_i=0 \qquad \textbf{(a)}$$
$$\frac{\partial\tilde{J}}{\partial\Delta\bar{b}_{\mathrm{i}}}=-Q_{xx}\left[b_i\right]^{-1}(\mathrm{b}_i-b_i^{(0)}-\Delta\bar{b}_{\mathrm{i}})-\lambda_i=0 \qquad \textbf{(b)} \quad \textbf{(3.24)}$$

$$\frac{\partial \tilde{J}}{\partial q_i} = \sum_{i=1}^{M} (\lambda_i, \frac{\partial \tilde{S}}{\partial q_i} a_i^{(0)}) = 0 \qquad \textbf{(a)}$$
$$\frac{\partial \tilde{J}}{\partial \Delta t} = \sum_{i=1}^{M} \lambda_i = 0 \qquad \textbf{(b)} \qquad \textbf{(3.25)}$$

Differentiating (3.15) with respect to quaternion parameters q_i, $i = 1,2,3...$

$$\frac{\partial \tilde{S}}{\partial q_i} = 2Q_i$$
$$Q_1 = \begin{bmatrix} 0 & q_2+(q_1q_3)/N & q_3-(q_1q_2)/N \\ q_2-(q_1q_3)/N & -2q_1 & (q_1^2/N)-N \\ q_3+(q_1q_2)/N & N-q_1^2/N & -2q_1 \end{bmatrix}$$
$$Q_2 = \begin{bmatrix} -2q_2 & q_1+(q_2q_3)/N & N-(q_2^2/N) \\ q_1-(q_2q_3)/N & 0 & q_3+(q_1q_2)/N \\ (q_2^2/N)-N & q_3-(q_1q_2)/N & -2q_2 \end{bmatrix} \qquad \textbf{(3.26)}$$
$$Q_3 = \begin{bmatrix} -2q_3 & (q_3^2/N)-N & q_1-(q_2q_3)/N \\ N-(q_3^2/N) & -2q_3 & q_2+(q_1q_3)/N \\ q_1+(q_2q_3)/N & q_2-(q_1q_3)/N & 0 \end{bmatrix}$$

We define a 3×3 matrix as follows (3.27);

$$U_i^{(0)} = 2(Q_1 a_i^{(0)}\ Q_2 a_i^{(0)}\ Q_3 a_i^{(0)}) \qquad \textbf{(3.27)}$$

By rearranging the (3.24a) and (3.24b) and substituting these equations into (3.22), we obtain (3.28);

$$2\sum_{i=0}^{3} \Delta q_i Q_i a_i^{(0)} + \Delta t - (\tilde{S} Q_{xx}\left[a_i\right]\tilde{S}^{T} + Q_{xx}\left[b_i\right]\lambda_i) = e_i \qquad \textbf{(3.28)}$$

and from (3.25a) we obtain (3.29);

$$\sum_{i=1}^{M} U_i^{(0)T} \lambda_i = 0 \qquad \textbf{(3.29)}$$

By using the matrices U_i^0 and V_i, (3.28) can be rewritten as (3.30);

$$U_i^0 \Delta q + \Delta t - V_i \lambda_i = e_i \qquad \textbf{(3.30)}$$

(3.29) and (3.30) are linear equations defined by λ_i, Δq and Δt. By solving these system of linear equations, we obtain $\overline{q}$, $\overline{t}$, $\overline{a_i}$ and $\overline{b_i}$ which are the optimal solution for the constrained non-linear optimization problem given in (3.20). However, these systems of equations still contain the Lagrange Multipliers which dramatically increase the size of normal equations which is $(3M+6)$. Eliminating these multipliers reduces the memory usage in computers. Memory usage becomes more important when the corresponding point number M increases. As a result of the elimination of Lagrange Multipliers, the size of normal equations becomes equal to the number of unknowns, i.e. 6.

From (3.30), λ_i can be expressed as (3.31);

$$\lambda_i = W_i(U_i^{(0)}\Delta q + \Delta t - e_i) \tag{3.31}$$

If (3.31) is substituted into (3.24a), (3.24b), (3.25a) and (3.25b) we have linear equations expressed by only Δq and Δt. Parameters are then estimated by the iterative solution of the following normal equations (3.32).

$$\begin{pmatrix} \sum_{i=1}^{M} U_i^{(0)^T} W_i U_i^{(0)} & \sum_{i=1}^{M} U_i^{(0)^T} W_i \\ \sum_{i=1}^{M} W_i U_i^{(0)} & \sum_{i=1}^{M} W_i \end{pmatrix} \begin{pmatrix} \Delta q \\ \Delta t \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{M} U_i^{(0)^T} W_i e_i \\ \sum_{i=1}^{M} W_i e_i \end{pmatrix} \tag{3.32}$$

The algorithm can be summarized as

Step1 Provide initial guess for q, t and assign $J_0 = \infty$

Step2 Compute the rotation matrix $\tilde{S}$ in (3.15)

Step3 Compute the vectors e_i and matrices W_i in (3.18 and 3.19)

Step4 Compute $a_i^{(0)}$ as; $a_i^{(0)} = a_i + V_0[a_i]\tilde{S}^T W_i e_i$

Step5 Check for the J. If $J \approx J_0$ stop. Else $J_0 \leftarrow J$

Step6 Compute the matrices Q_i and $U_i^{(0)}$ in (3.26 and 3.27)

Step7 Solve for the 6-D linear equation in (3.32)

Step8 Update $a_i^{(0)}, q$ and t as follows

$q \leftarrow q + \Delta q$, $t \leftarrow t + \Delta t$

3.4 Surface Representation

Surface generation by using laser scanning is typically a sampling process of real world objects. 3D scanners collect sampling data from object surfaces depending on the defined resolution. Collected data is generally unorganized and recorded as an ASCII file including header information which contains the number of horizontal or vertical laser profiles in scan order. Although this data set is collected topologically during the scanning process, the recorded ASCII file does not reflect the topological relationships of points. To be able to create surface mesh from this data set, which best approximates the object surface, the collected data must be re-organized in accordance with the object topology. The topology is established by the help of the scan line order which is read from the header information of ASCII output file. As an example of an output file, let $P(x_a, \mathrm{y}_a, z_a)$ is a $(m \times n, 3)$ raw scan file which $a = 1, 2, \ldots., m \times n$ and m, n are row and column numbers, respectively. We create an $M_{m,n}$ empty matrix, and fill the matrix M by using the output file with respect to the point indexes. The matrix M contains point indexes and reflects the object topology. By using the M matrix, vertex list of meshes is created and then surface geometry is basically established by fitting triangular patches to the 3 neighbouring points. The surface is represented by simply structured meshes as composite of planar patches in non-parametric implicit form (Figure 3.1).

$$\boxed{\begin{array}{l} m \times n \\ \begin{pmatrix} 1 & x_1 & y_1 & z_1 \\ 2 & x_2 & y_2 & z_2 \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ m \times n & x_{m \times n} & y_{m \times n} & z_{m \times n} \end{pmatrix} \end{array}} \qquad \begin{pmatrix} 1 & \cdots & \cdots & \cdots & m \\ m+1 & m+2 & & & 2 \times m \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \ddots & \vdots \\ n & \cdots & \cdots & \cdots & m \times n \end{pmatrix}$$

(a) **(b)**

Figure 3.1 : (a) Raw data file (b)Topologically ordered point cloud.

3.5 Correspondence Search

Correspondence search is the most critical part of all registration algorithms. The success of a registration method depends on how correct correspondences were established between two data sets. False matches cause to incorrect results. In order to prevent false matches, different type of constraints and conditions can be introduced to the search algorithm. The correspondence search in this study is guided by using two well-known methods. Besl and McKay (1992) introduced the point-to-point search in their original ICP paper. According to this method, each available point in template surface is matched with the closest point in search surface. Then, the sum of the squared distances between the points in each correspondence pair is minimized. In point-to-point search, all points in template data are enforced to match with a point in search data. This enforcement is not realistic due the occluded or non-overlapping areas in search surface. The procedure usually ends up with some false point matches. Chen and Medioni (1991) introduced an alternative error metric called point-to-plane algorithm. In point-to-plane error metric, the sum of the squared distances between each point in template data and the tangent plane at its corresponding destination point in search data is minimized. Due to the large search area and heavy mathematical computations, point-to-plane error metric is more computationaly expensive than point-to-point version. On the other hand, the researchers have observed significantly better convergence rates with point-to-plane (Rusinkiewicz, 2001). Considering the advantageous and disadvantageous parts of these two algorithms, a mixed search procedure provides more stable and fast results. In our implementation, these two methods were used together in a combination, in order to benefit from the advantageous parts of them. We also introduced some conditions and constraints. The point-to-plane search was accelerated significantly by using a kd-tree nearest neighbor searcher. Our correspondence search procedure can be summarized as follow:

- Run point-to-point search: It is called as coarse matching.
- Mesh the search data.
- Use coarse matching to narrow the search area for point-to-plane.
- Run point-to-plane search: It is called as fine matching.

3.5.1 Boundary condition

Points located at the boundaries of point clouds are generally assumed as erroneous because of the edge effect and incidence angle. During the scanning procces of an edge, even the laser beam is well focused on the object, while some of the spot of the laser beam returns from the edge, other part may return from the behind scene or may not return back to the sensor. This situation leads erroneous points. Especially, a systematic effect can be observed when cylindrical and spherical targets are observed (Lichti et. al., 2002). In our implementation, we introduce a boundary condition which detects the points in boundaries. Thus, we exclude these points from correspondence search. With this condition, it is possible to detect the points not only at boundaries, but also in occluded areas. The conditon is capable to find the outliers as well. According to the introduced condition, 8 neighbouring points of a point are investigated and nodata values are counted. No data means, if there is not any return from the object in response to the sent laser beam, the value of this laser beam is recorded as zero in raw data. We classify a point as boundary point or outlier, if %25 of the neighbours of the point consists of nodata values. Figure 2 shows an example of the detected boundary points.

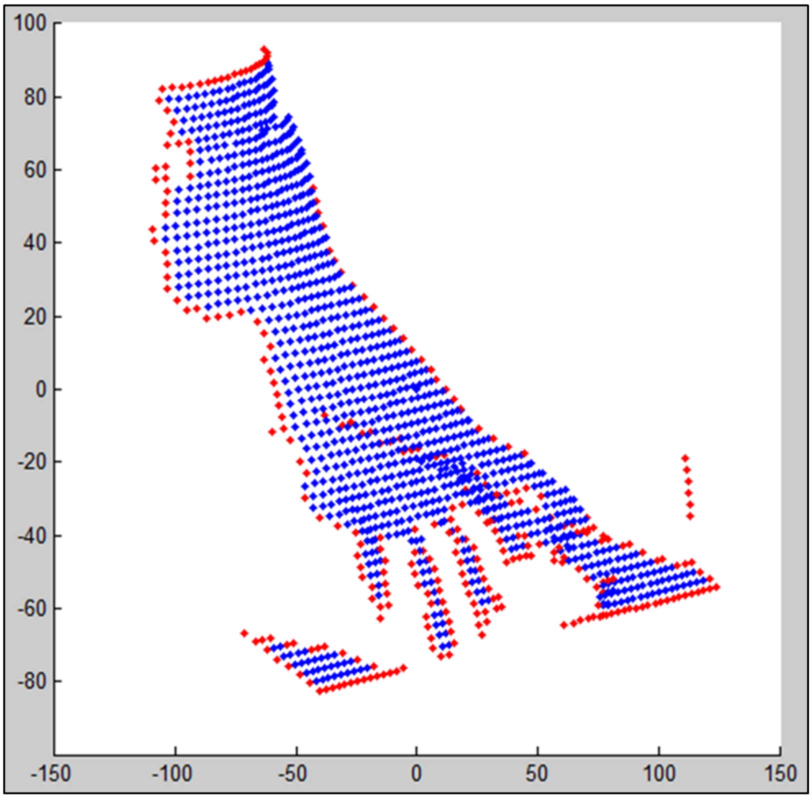


Figure 3.2 : Boundary points and outliers.

3.5.2 Maximum allowable distance threshold

Maximum allowable distance is a pre-defined and dynamically changing threshold value at each iteration. Point pairs whose Euclidian distance exceeds the threshold are eliminated from matching even if these two points have the minimum distance value. By this way, any outliers or spurious points are detected and eliminated in early phase of iterations. The threshold should be defined high enough at the beginning of the iterations in order to let some coarse pairings and it should decrease through the iterations. This kind of a threshold provides a coarse to fine matching flow. The maximum allowable distance threshold is defined as $k \times m_0$ in our implementation in such a way that it will be updated at the beginning of each iteration. m_0 is defined as the sum of the square root of Euclidian distances at the end of each iteration where k is a constant value defined by the user. Selection of k depends on how close two data sets were pre-aligned. If pre-alignment is good, then a small value should be selected for k value for correct matching, otherwise a greater value should be assigned for k in order to prevent elimination of many correct correspondences. In our implementation we choose 2.5 for k value for the first iteration as it is proposed in Masuda et al., (1996).

3.5.3 Search area limitation

In point-to-plane error metric, the sum of the squared distances between each point in template data and the tangent plane at its corresponding destination point in search data is minimized. If there is not any spatial partitioning techniques (kd-tree, boxing etc.) employed or any restrictions introduced for correspondence search, the algorithm performs distance calculation for a candidate point using the all available surface patches in search data. In our implementation we introduce a constrain which allows correspondence search in a limited area. The coarse match point which is found by the point-to-point search is used for outlining the limited area. Consequently the procedure is followed by the point-to-plane search where the fine matching point is found. The fine matching point is searched within the 6 neighbouring triangles which are fictitiously formed around the coarse match point.

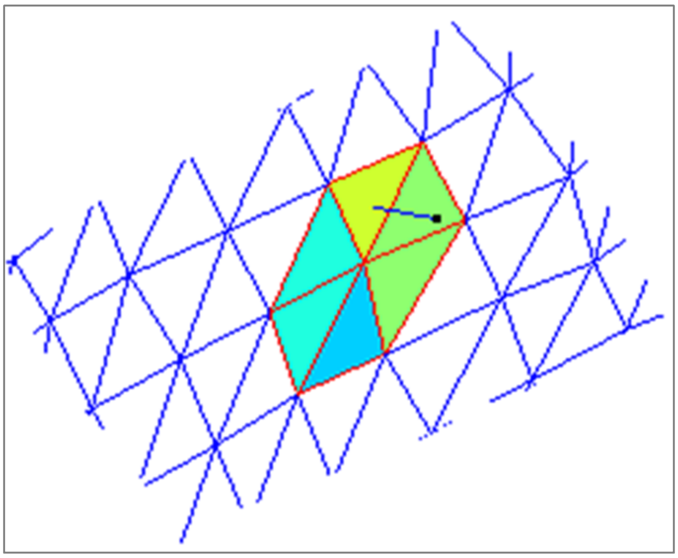


Figure 3.3 : Limited search area for point-to-plane correspondence search.

3.5.4 Point-in-polygon test

In computational geometry, the point-in-polygon test is used to determine whether a point lies inside a polygon or not. We use this test in our implementation as the final check for fine matching. With respect to this condition, a point which has the minimum point to plane distance must be inside the related polygon when it is projected onto the surface through the surface normal.

Our strategy for point-in-polygon test is as follows;

Let $P_0(\mathrm{x}, \mathrm{y}, \mathrm{z})$ be the candidate point for correspondence from template data and $P_1\ P_2\ P_3$ is one of the available triangle in search data.

Step I : We first calculate the distance d between point P_0 and the triangle $P_1\ P_2\ P_3$ by using the point to plane distance formula **(3.33)**.

$$d = \frac{Ax + By + Cz + D}{\sqrt{A^2 + B^2 + C^2}} \qquad \textbf{(3.33)}$$

Where A, B, C and D are plane parameters.

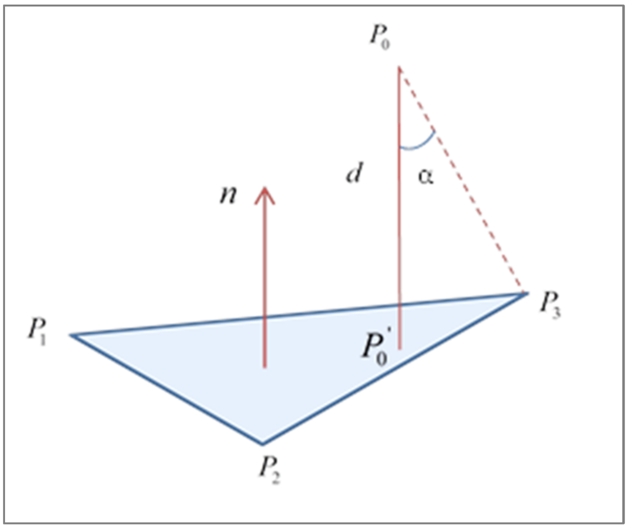


Figure 3.4 : The distance of point P_0 to triangle $P_1\ P_2\ P_3$.

Step II: Second step is the projection of P_0 onto surface through the surface normal. Let P_0 be the projected point which it is calculated from **(3.34)**;

$$P_0^{'} = P_0 - d \times \frac{n}{|n|} \tag{3.34}$$

Where n is the surface normal.

Step III: In this step we check whether point P_0' lies inside the triangle. For this purpose, we define the vectors $P_0' P_j$ $(j = 1, 2, 3)$ and calculate the angles (Figure 2.5) between each pairs of vectors.

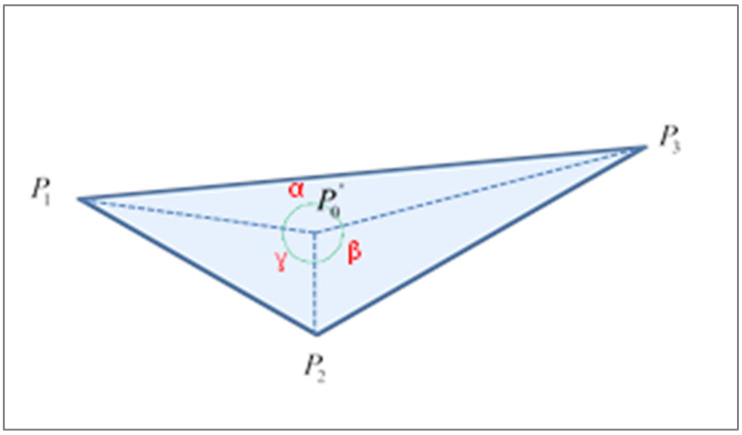


Figure 3.5 : Angles between each pair of vectors.

The angles α , β and γ can be calculated from **(3.35)**;

$$\cos\alpha = \frac{\overrightarrow{P_0P_1} \cdot \overrightarrow{P_0P_3}}{\left|\overrightarrow{P_0P_1}\right|\left|\overrightarrow{P_0P_3}\right|} \quad \cos\beta = \frac{\overrightarrow{P_0P_2} \cdot \overrightarrow{P_0P_3}}{\left|\overrightarrow{P_0P_2}\right|\left|\overrightarrow{P_0P_3}\right|} \quad \cos\gamma = \frac{\overrightarrow{P_0P_1} \cdot \overrightarrow{P_0P_2}}{\left|\overrightarrow{P_0P_1}\right|\left|\overrightarrow{P_0P_2}\right|} \qquad \textbf{(3.35)}$$

Where '•' denotes the dot product.

After calculation of the angles, we check the sum of the angles in order to determine whether the point is inside or outside of the triangle or it is on one of the edge of the triangle. Fig 3.6 shows the three possibilities of the point position with respect to triangle. The decision procedure is as shown below. In our code we use a pre-defined threshold value τ in order to prevent floating point error.

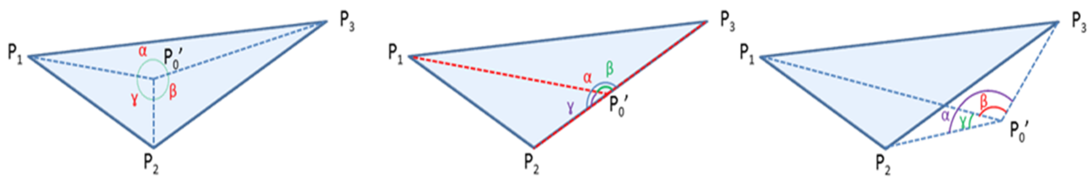


Figure 3.6 : Point-triangle relationship.

$if \ \ (\alpha + \beta - 360) \leq \tau$

P_0' is inside the triangle and it is counted as the fine correspondent point

else if $(\alpha + \beta - 360) \leq \tau \ \&\& \ ((\alpha + \beta - 180^\circ) = (\lambda - 180^\circ)) \leq \tau$

P_0' is on the edge and it is still counted as the fine correspondent point

else

P_0' is outside the triangle and it is excluded from matching.

3.6 Convergence Behavior and Computational Cost

The mathematical model of the Modified Gauss-Helmert is non-linear and therefore, it requires linearization of the objective function. Like all non-linear models, this model also needs good approximations of unknown parameters to ensure the convergence. There is always a danger of converging to local minima in case of poor approximation values.

In our implementation, we use some stopping criteria in order to check the convergence. For this purpose, we define threshold values for both angles and translations. When the change in transformation parameters between successive iterations fell below the defined threshold, it is assumed that the convergence is ensured.

$$|dc_i| < d_i \quad , \quad dc_i \in \{dt_x, dt_y, dt_z, dq_1, dq_2, dq_3\}$$

In case of long iteration numbers due to bad approximation values, we also define a maximum iteration number threshold in order to prevent false convergence.

We conducted diverse tests by using different stochastic structures. Our purpose is to investigate the differences in results of proposed Modified Gauss-Helmert model and classical Least Squares method at first step. In addition, the second group of tests aim to show the differences between isotropic and anisotropic noise conditions in Modified Gauss-Helmert model. As a result of these tests, we see different convergence behaviors (Figure 3.7, Figure 3.8, Figure 3.9). Change in parameters in MGH model becomes sharply at first iterations and continues smoothly. In contrast, changes in parameters in Least Squares are more smooth and steady through the iterations. We observe that the best convergence rates are achieved in MGH model with introduced anisotropic covariance matrices.

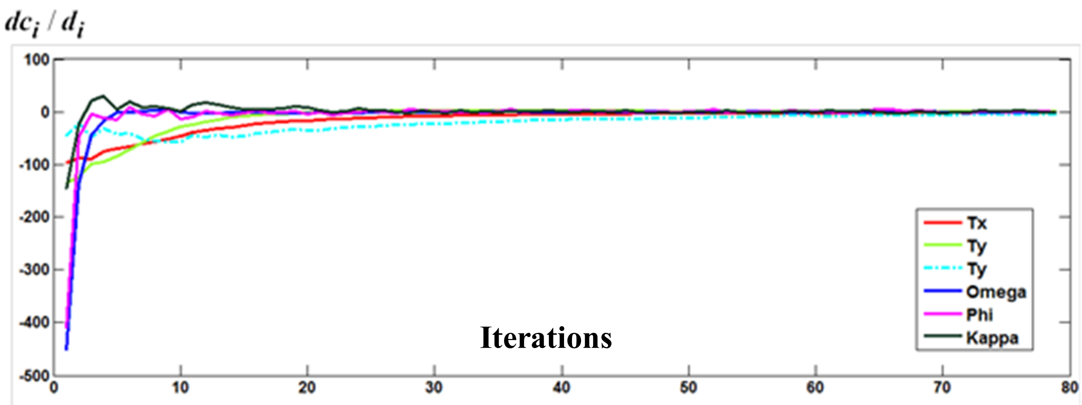


Figure 3.7 : Convergence behaviour of least-squrares matching.

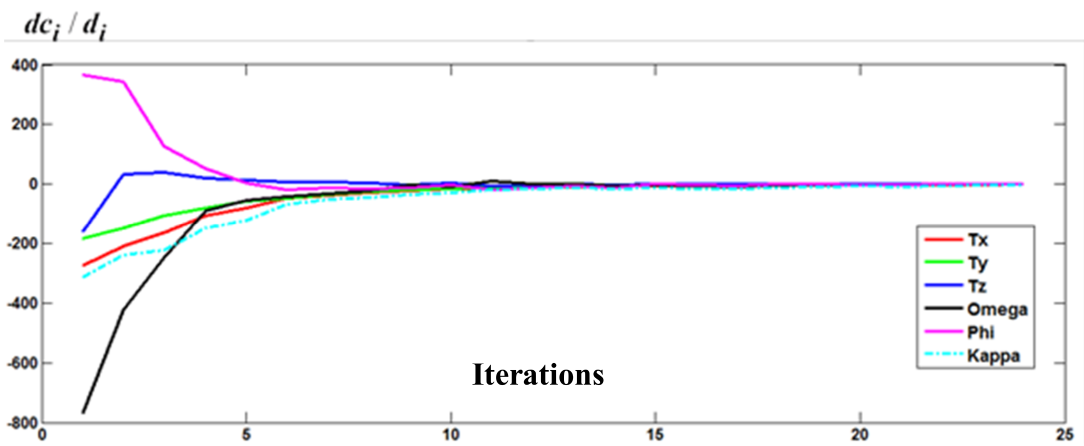


Figure 3.8 : Convergence behaviour of total least-squares with anisotropic and inhomogeneous covariance matrix.

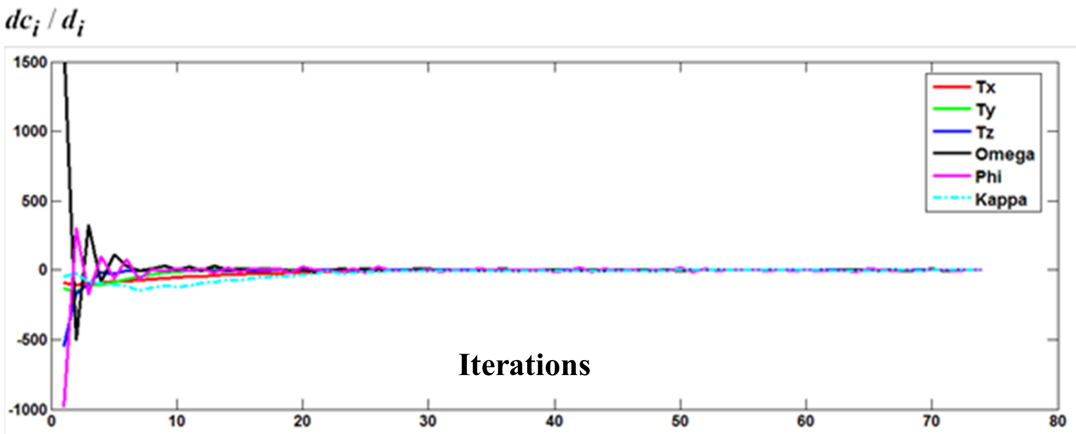


Figure 3.9 : Convergence behaviour of total least-squares with isotropic and homogeneous covariance matrix.

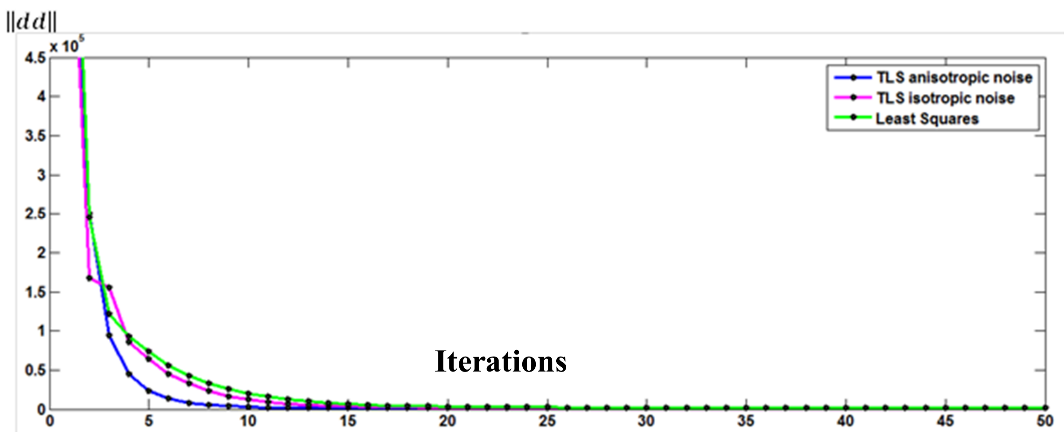


Figure 3.10 : Convergence with respect to sum of the squared distances

The objective of almost all surface based ICP algorithms is to minimize the sum of the squared distances between the points in template data and the tangent plane at its corresponding destination point in search data. Although this parameter is not a stopping criteria for our implementation, we still calculate this value in order to observe the convergence behavior in terms of it (Figure 3.10). The convergence behavior with respect to sum of the squared distances is again better for MGH model with introduced anisotropic covariance matrix case.

Computational cost is directly proportional with the size of the data. The most time consuming part of the process is correspondence search. Correspondence search are speeded up by using search trees and partitioning techniques which is given in detail in section (3.5)

Parameter estimation part takes a short time as well as the less iteration number, which are usually about 4-5 iterations. Lagrange Multipliers are eliminated in MGH model. Thus, the size of normal equations decreases dramatically and reduces the memory usage in computers. Therefore, the MGH model is a very appropriate solution for large data sets comparing to the other TLS models in literature (Fellus 2005, Akyılmaz 2007, and Nietzel 2010).

3.7 Internal Quality Control

The reliability and quality of estimated parameters can be controlled by investigating the a-posteriori covariance matrix. We calculate the a-posteriori standard deviation of unit weight by (3.36);

$$\widehat{\sigma_0} = \sqrt{\frac{e^T.W.e}{r}} \tag{3.36}$$

With redundancy $r = n - u$.

Where e is total error vector and W total weight matrix defined in equations 3.18 and 3.19 and T stands for transpose.

A-posteriori standard deviation of individual parameters is given by (3.37);

$$\sigma_i = \widehat{\sigma_0}\sqrt{q_{ii}} \tag{3.37}$$

where q_{ii} is the i th diagonal element of the cofactor matrix Q_{ll}.

$$\widehat{Q_{ll}} = \begin{pmatrix} \sum_{i=1}^{M} U_i^{(0)^T} W_i U_i^{(0)} & \sum_{i=1}^{M} U_i^{(0)^T} W_i \\ \sum_{i=1}^{M} W_i U_i^{(0)} & \sum_{i=1}^{M} W_i \end{pmatrix}^{-1} \quad \textbf{(3.38)}$$

As we define the rotation matrix by quaternions in our implementation, first three diagonal elements of co-factor matrix Q_{ll} includes the cofactors of estimated quaternion parameters (3.38). At this step we prefer to define the standard deviations in terms of Euler angles in order to make the comparison easier with the results of Gauss-Markov model. The relationship between Euler angles and quaternions is given as (3.39);

$$\begin{bmatrix} \omega \\ \varphi \\ \kappa \end{bmatrix} = \begin{bmatrix} \arctan \dfrac{2(q_0 q_1 + q_2 q_3)}{1 - 2(q_1^2 + q_2^2)} \\ \arcsin(2(q_0 q_2 - q_3 q_1)) \\ \arctan \dfrac{2(q_0 q_2 + q_1 q_2)}{1 - 2(q_2^2 + q_3^2)} \end{bmatrix} \quad \textbf{(3.39)}$$

By applying the error propagation rule to (3.39), we obtain the standard deviations of Euler angles Omega, Phi and Kappa, which are the rotations about X, Y and Z axes respectively.

4. EXPERIMENTAL RESULTS

All experiments were carried out by using a self-developed programme in MATLAB Computing Language environment. For the simplicity we name this programme as TLS3d throughout this section. We also coded the classical Least Squares in MATLAB as well which we call LS shortly. Our purpose is to investigate the results of proposed MGH model for the registration problem and make comparisons between the results obtained by LS.

We conducted several tests by using diverse laser scanning data acquired by different devices, which have alternate working principles, accuracy specifications and noise characteristics. Besides, we used some synthetic data in order to control the results of programmes.

In all experiments, initial approximations were provided beforehand by the help of external software (Leica cyclone, LS3D of Akca (2010)) since we did not create a Graphical User Interface (GUI) for selecting some control points on both data. The developed software can handle the rotational differences about three axes up to 20˚, but it is not that much capable about the translation vector.

The experiments in TLS3d can be classified into three cases in terms of the introduced initial covariance matrix. The first group of tests are conducted by using the same covariance matrices and isotropic noise for all sets. The second tests are carried out by using different covariance matrices for template and search surfaces with isotropic noise and the final group of tests are with different covariance matrices with anisotropic noise as explained in section 3.2

4.1 Old Historical House

The first experiment is the registration of two scans of an old historical house. Data was acquired by Leica C10 (Leica Geosystems) scanner. Scanning campaign was carried out for the "Historical Kars City" project which aims obtaining the façade relieves and street silhouette of old-town in Kars City. The "House" data were cropped from main street data which includes millions of points. The house was scanned from two consecutive stations which are about 30 m away from each other. First scan station is very close to the building and viewing angle is nearly at normal position to the façade. But the second station is very far away from the façade and the incidence angles of laser beams are very high. The average point spacing is 1cm for scan station 1 while it increases up to 6cm for scan station 2. In this test, we used both TLS3d and LS for matching process. In TLS3d, we introduced anisotropic and non-homogeneous covariance matrices for template and search surfaces. The covariance matrices were calculated by using the equation 3.10. We use the accuracy specifications of the scanner given by the manufacturer which is 12″ for horizontal and vertical angle measurements and 4mm for range. The matching is successful for both cases. Obtained results are given in Table (4.1)

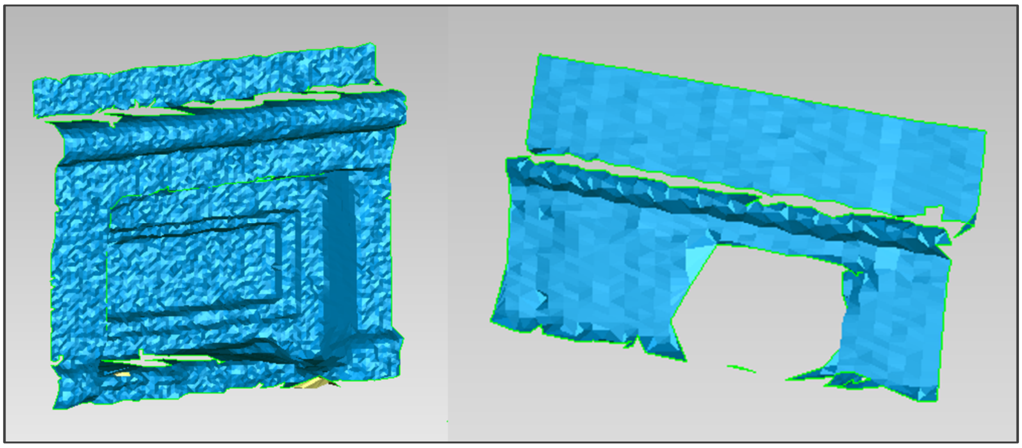


Figure 4.1 : Template and search surface Old Historical House

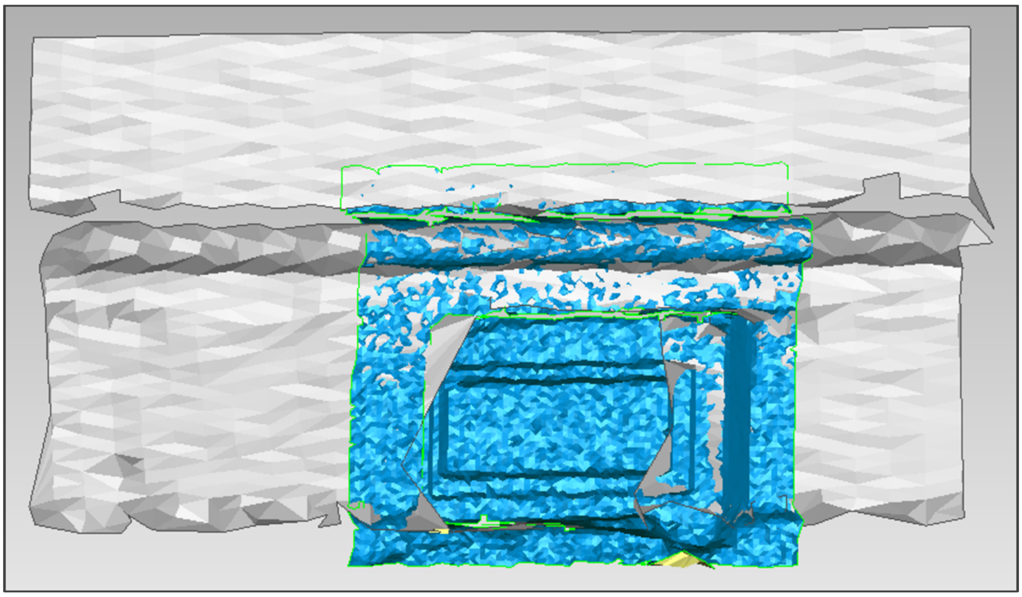


Figure 4.2 : Old Historical House-registration results with TLS3d

Table 4.1 : Numerical results of 'old historical house' data

Model	No. of Matched Points	σ_0 (mm)	σ_{T_x} (mm)	σ_{T_y} (mm)	σ_{T_z} (mm)	σ_ω (grad)	σ_φ (grad)	σ_κ (grad)
TLS3d	3694	1.167388	0.005322	0.001579	0.002460	0.000226	0.000174	0.000585
LS	3974	0.008962	0.004673	0.001549	0.006432	0.000619	0.000568	0.000521

Stopping criteria was assigned as 0.0001 m for translation vector and 0.001^g for rotation parameters in both methods. Results show that, accuracy of estimated parameters is more realistic in TLS. Moreover, it would not be incorrect to say that TLS3d produces more accurate and reliable results. The final sigma naught value is a statistically derived quantity and shows the amount of observation residuals. This value does not give any information about the accuracy, but we use sigma naught in $\sigma_i = \widehat{\sigma_0}\sqrt{q_{ii}}$ in order to calculate the a-posteriori sigma_i values of individual parameters. By checking the sigma_i values in Table 4.1, we see more or less equal values although there are high differences between sigma naught values for both methods. This situation indicates that the obtained a-posteriori cofactor matrix in TLS3d is much smaller than the matrix in LS.

4.2 Building Façade

The second experiment is carried out by using 3D overlapping surface patches in Figure 4.3 of a historical building. The data was acquired by Leica C-10 time-of-flight type of laser scanner. Average point spacing of original data is 1.5 cm. But data was resampled in order to decrease the number of points and the resampled data has point spacing of about 3 cm. These pre-aligned data sets were registered in TLS3d by introducing anisotropic and non-homogeneous covariance matrix for template and search surfaces. Since scan positions are close to each other as well as the façade, and scanning is made at nearly normal position to the façade, incidence angle and distance do not perturb the covariance matrix so much. As a result of this the final covariance matrices were calculated as almost the same for both data sets. The numerical results of this test are given in Table 4.2.

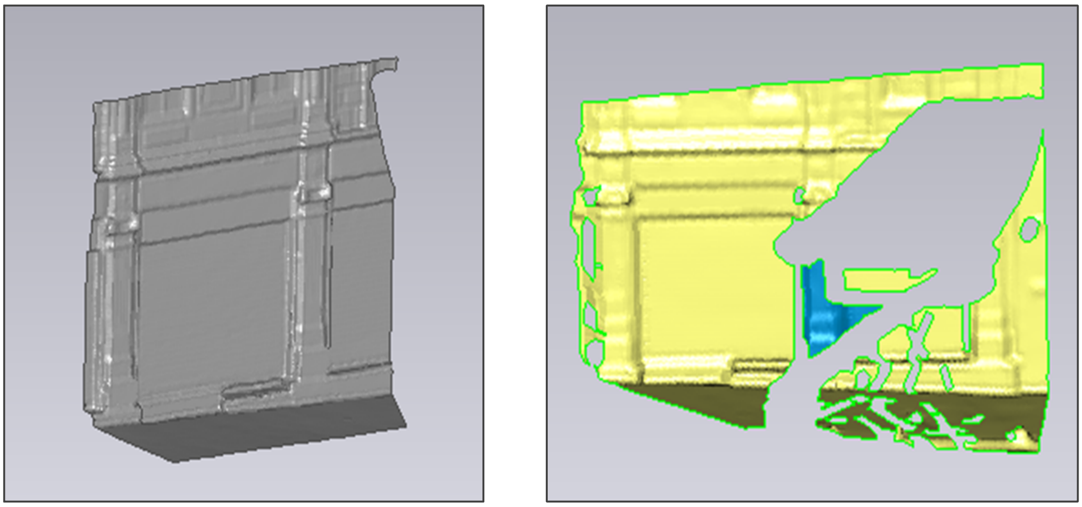


Figure 4.3 : Template and search surface of Building Façade

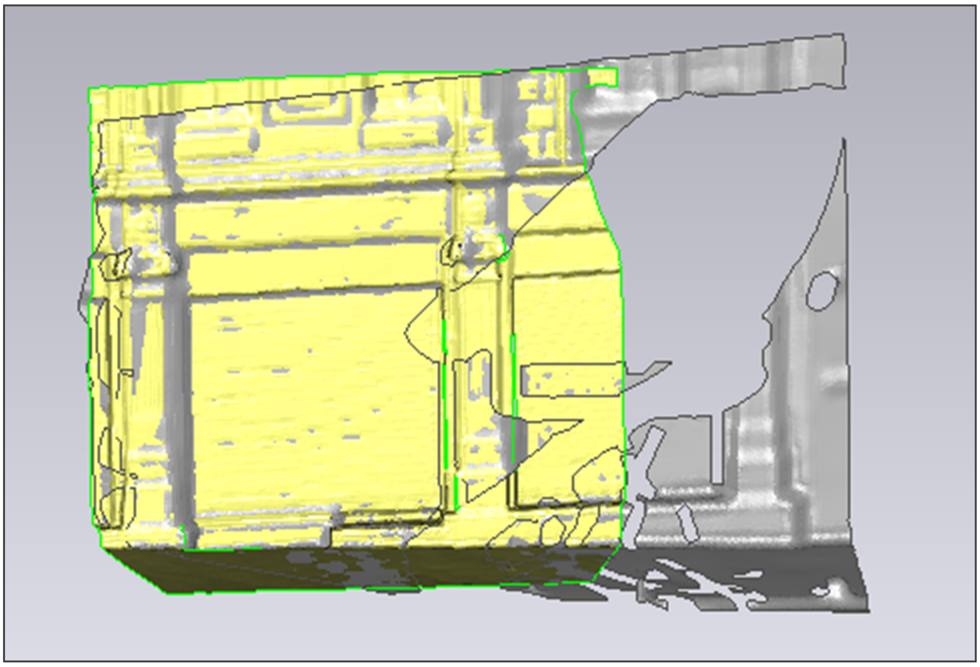


Figure 4.4 : Building Façade-Registration results

Table 4.2 : Numerical results of 'building façade' data

Model	No. of Matched Points	σ_0 (mm)	σ_{T_x} (mm)	σ_{T_y} (mm)	σ_{T_z} (mm)	σ_ω (grad)	σ_φ (grad)	σ_κ (grad)
TLS3d	14209	2.41	0. 695	1.044	0.783	0.000060	0.000027	0.000070

4.3 Pyramid

The third experiment is the registration of two scans of a small pyramid shape object. The half of the pyramid was scanned by Leica C10 scanner from close range. The average point spacing is 4 mm. This data set can be considered as difficult for point to plane search because the object is consist of three planar surfaces which means it does not contain sufficient surface characteristics in terms of the orientation of the surface normal. Moreover, the worst initial approximations (~18˚ rotation about z axis) among the all tests were introduced for this data. Registration process is successful. The results of the test are given in Table 4.3.

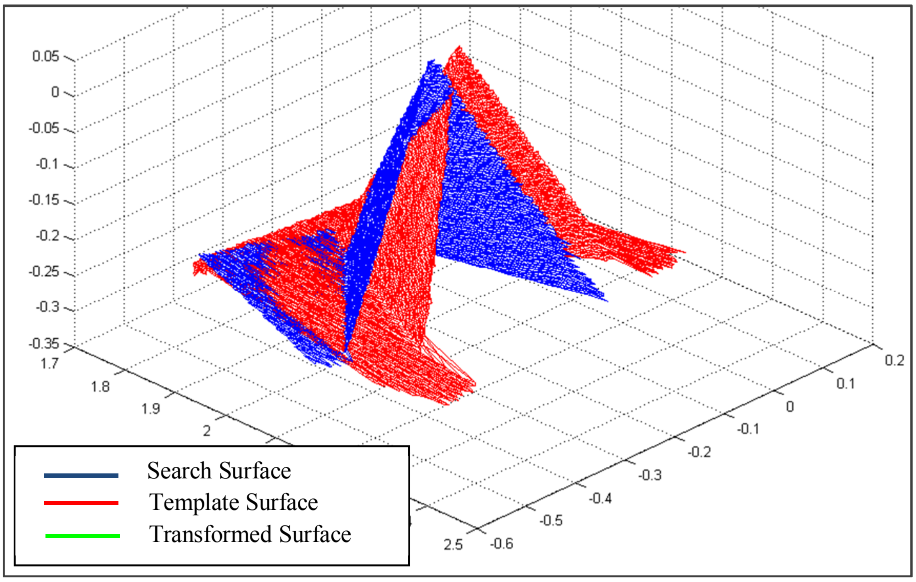


Figure 4.5 : Pyramid- template and search data before registration.

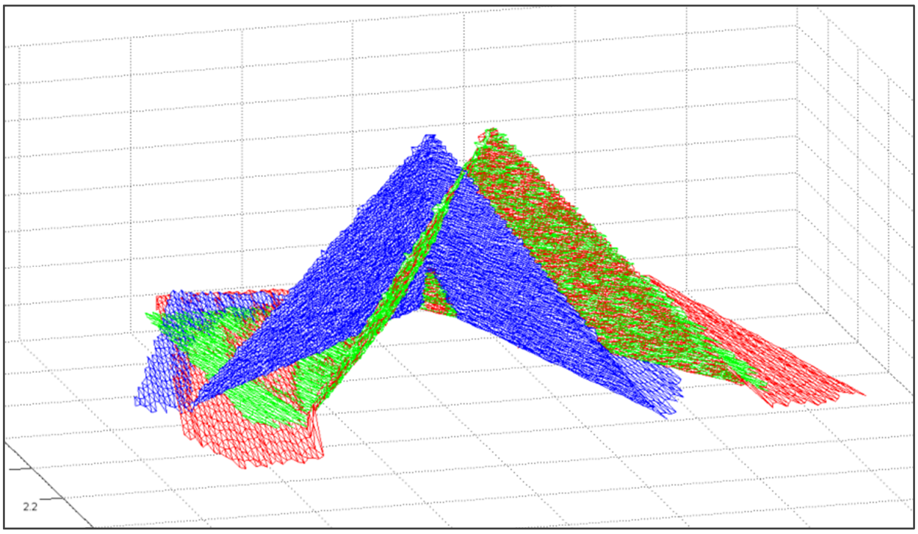


Figure 4.6 : Pyramid- registration results.

Table 4.3 : Numerical results of 'pyramid' data

Model	No. of Matched Points	σ_0 (m)	σ_{T_x} (m)	σ_{T_y} (m)	σ_{T_z} (m)	σ_ω (grad)	σ_φ (grad)	σ_κ (grad)
TLS3d	4960	0.144437	0.000045	0.000241	0.000247	0.000086	0.000124	0.000119

4.4 Oil Storage Tank

The fourth experiment is the registration of two partial scans of an oil storage tank. The data were acquired by Leica C10 through the project of monitoring the structural deformations of oil storage tanks. Our purpose is to check the performance of the algorithm for the registration of relatively difficult data sets. The outer shell of the tank is very smooth and the shapes of the surfaces are concave. This is a typical problematic data set especially for point to plane search algorithm, because the orientation of surface normal do not show any differences. Although the scan positions are not too far from each other, viewing angle of the scanner is very narrow for search data. As a result of this configuration, the average point spacing differs for template and search surfaces which are 3 cm and 9 cm respectively. Sampling density on search data is very low with respect to the template surface. Sampling density impacts the registration accuracy. Theoretically when surfaces are scanned with low resolution, worse results are obtained. As surfaces have less points, points correspondences are found with low precision (Salvi et al., 2007). Furthermore, the overlap is relatively small between two data sets which is approximately % 25. The obtained results of the registration are given in Table 4.4. In spite of this challenging data set, the matching is successful. The accuracy in depth direction is relatively better than lateral direction as it is expected.

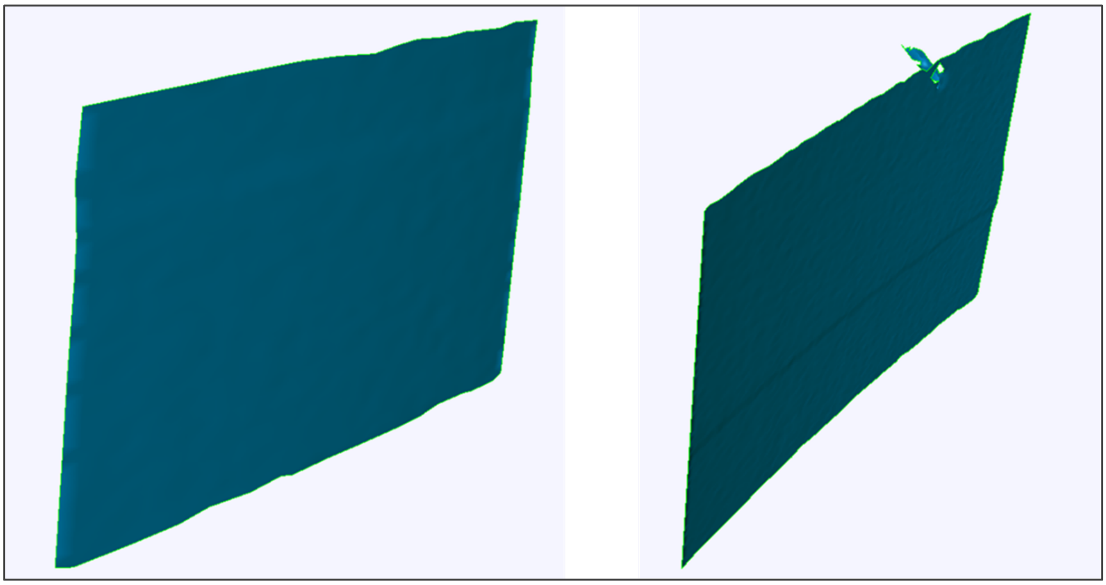


Figure 4.7 : (a) Search surface, (b) Template surface of Oil Tank.

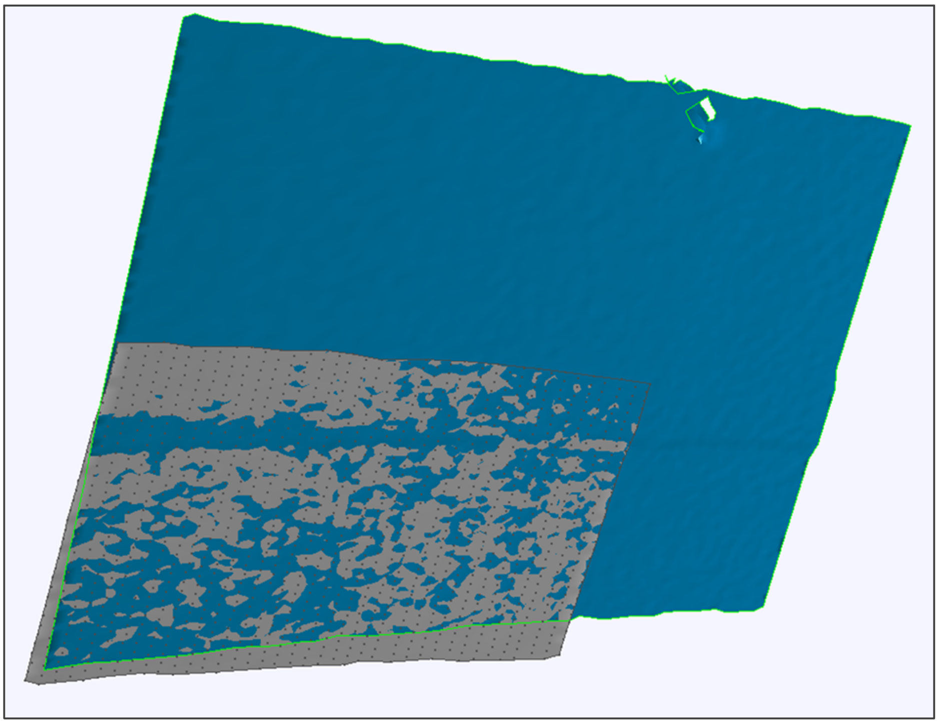


Figure 4.8 : Registration result

Table 4.4 : Numerical results of 'oil storage tank' data

Model	No. of Matched Points	σ_0 (m)	σ_{T_x} (m)	σ_{T_y} (m)	σ_{T_z} (m)	σ_ω (grad)	σ_φ (grad)	σ_κ (grad)
TLS3d	2728	0.152669	0.000279	0.000592	0.000809	0.000036	0.000039	0.000023

4.5 MATLAB Surface Data

The fifth experiment is the registration of a synthetic data. This surface data is actually very famous among the MATLAB users since it illustrates in help files of MATLAB for surface plot feature. The data is very appropriate for matching algorithms with its varying surface characteristics (peaks, slopes and flat regions). It is also very similar to the real point cloud data structure since it is formed as a grid data. For our test, we created a template surface which contains 625 points. Then we added Gaussian noise to the template data and applied transformation with known rotation angles and translation vector. Finally we added Gaussian noise again to the transformed data which represents the search surface. Since we did not occlude any parts from any data, we have full overlap between template and search surfaces. Introduced noise is anisotropic for both surfaces; however it is homogeneous within the individual sets. We used both LS and TLS3d for matching process in this test. Results are given in Table 4.5. Registration is successful for both cases. The numerical results show that the difference between theoretical precisions of estimated parameters is minor and thus insignificant. This test proves that these two methods do not differ from each other even if both data sets contain noise.

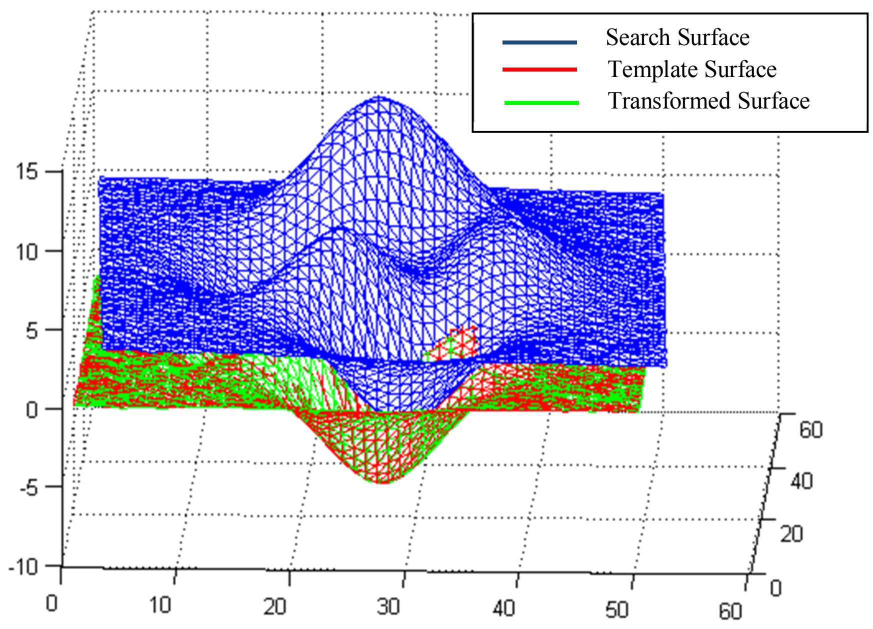


Figure 4.9 : Registration with TLS3D

Table 4.5 : Numerical results of 'matlab surface' data

Model	No. of Matched Points	σ_0 (unitless)	σ_{T_x} (unitless)	σ_{T_y} (unitless)	σ_{T_z} (unitless)	σ_ω (degree)	σ_φ (degree)	σ_κ (degree)
TLS3d	587	0.022706	0.002680	0.002639	0.004481	0.000447	0.000457	0.000336
LS	600	0.032329	0.002263	0.002252	0.003756	0.000189	0.000188	0.000137

4.6 Weary Heracles Statue

The sixth experiment is the registration of two partial surfaces which is a part of "Weary Heracles" statue. The data has been scanned by Breuckmann optoTOP-HE coded structured light system. Original data set comprises approximately 250000 points individually with 0.5 mm point spacing. Both *'Weary Heracles Statute'* data in this experiment and *'Surface patch'* data in experiment 4.7 are kindly provided by Dr. Devrim AKCA. In this test we resampled the data set and decreased the number of points since our implementation produces the results very slowly with this kind of intense data. Resampled data consist of approximately 50000 points with 1 mm point spacing. The matching process was carried out by using both LS and TLS3d with homogeneous and isotropic noise conditions. The obtained results are given in Table 4.6. Results show similarity with the results of MATLAB surface data experiment. Theoretical precisions of estimated parameters do not differ significantly. Figure 4.10 shows the 3D comparison of two methods by using the 3D compare module of Geomagic Studio (Raindrop Geomagic, Inc.) software. Patterns of the residuals show similarity for both methods.

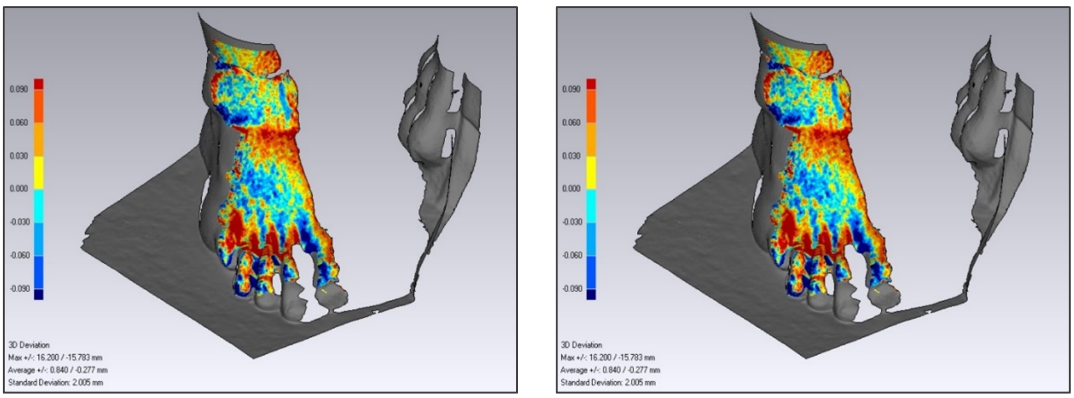


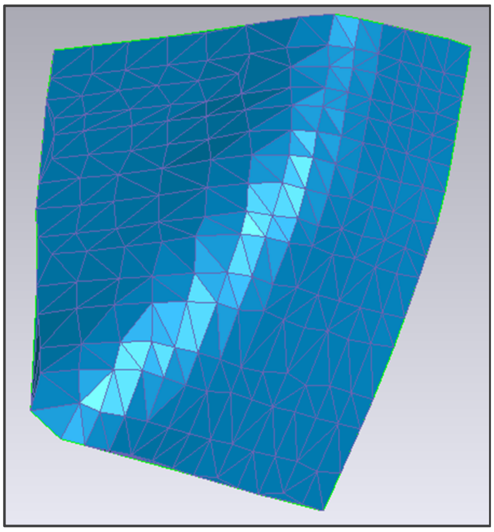
Figure 4.10 : (a) Residuals after TLS estimation (b) Residuals after LS estimation.

Table 4.6 : Numerical results of 'weary heracles' data

Model	No. of Matched Points	σ_0 (mm)	σ_{T_x} (mm)	σ_{T_y} (mm)	σ_{T_z} (mm)	σ_ω (grad)	σ_φ (grad)	σ_κ (grad)
TLS3d	36941	0.0292	0.00034	0.00040	0.00028	0.000006	0.000006	0.000004
LS	36855	0.0408	0.00033	0.00040	0.00028	0.000006	0.000006	0.000004

4.7 Surface Patch

The seventh experiment is the matching of two surface patches Figure 3. The data is acquired by an IMAGER 5003 terrestrial laser scanner (Zoeller+Fröchlich). Average point spacing is 1 cm. Obtained numerical results for two different registrations are given in Table 4.7. In this experiment also, although the theoretical precisions changes slightly, differences are very minor and insufficient for a decision making for the comparison purpose of two methods.



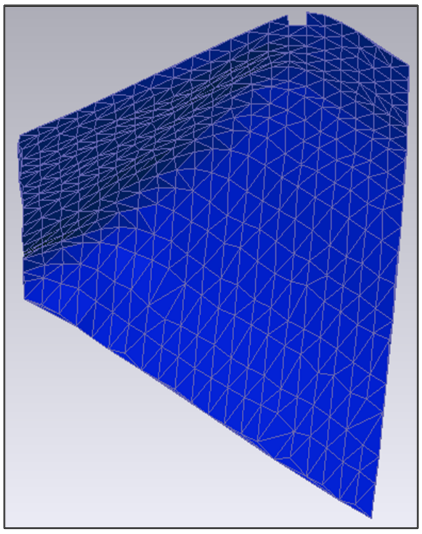


Figure 4.11 : (a) is the template and (b) is the search surface.

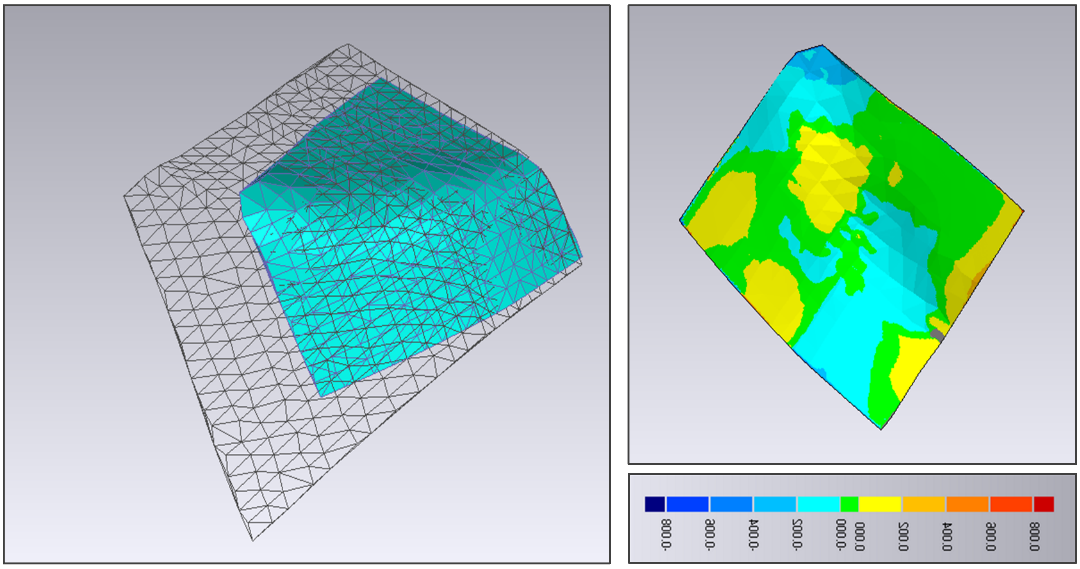


Figure 4.12 : (a)Registration with TLS3d ans (b) shows the residuals.

Table 4.7 : Numerical results of ‘surface patch’ data

Model	No. of Matched Points	σ_0 (mm)	σ_{T_x} (mm)	σ_{T_y} (mm)	σ_{T_z} (mm)	σ_ω (grad)	σ_φ (grad)	σ_κ (grad)
TLS3d	219	0.000733	0.008513	0.007517	0.001344	0.001327	0.001481	0.001191
LS	224	0.001010	0.008177	0.007144	0.001278	0.001262	0.001424	0.001138

4.8 Wave Data

The eighth experiment is the registration process by using synthetic data. The data is called ‘wave data’ and is commonly used by many researchers in order to test their registration algorithms (Salvi et. al., 2007, Rusinkiewiz, 2001, Grant, 2012). The data was downloaded from the MATLAB registration toolbox (Salvi et al., 2007). Transformation was simulated by applying rotation and transformation to data. Then we created anisotropic independent Gaussian noise by using the covariance definition given in equation 3.10. Obtained noises were added to both template and search surfaces. Figure 4.13 shows introduced noise vectors.

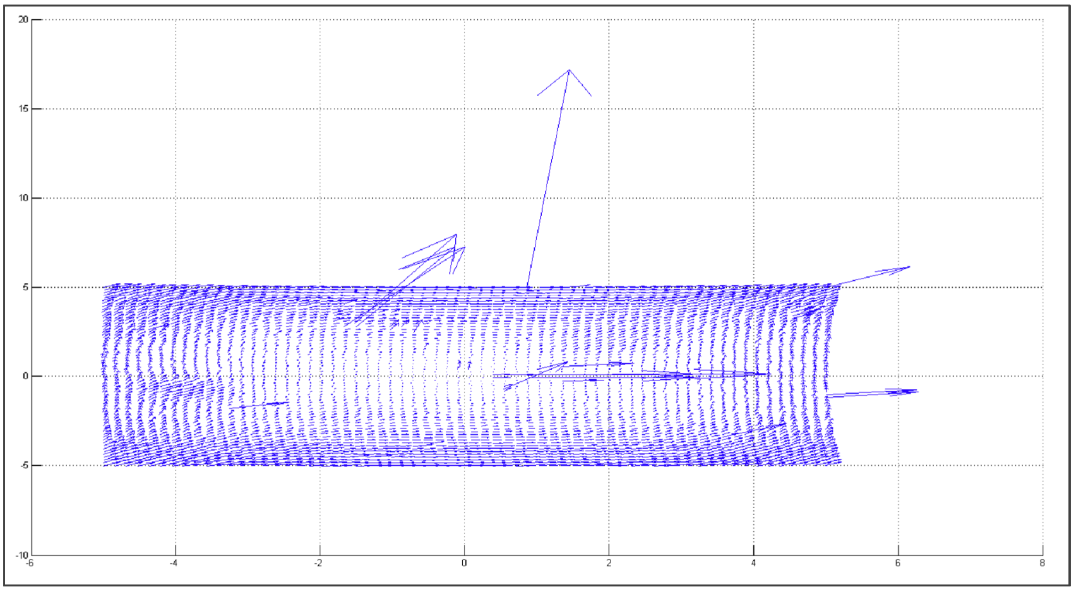


Figure 4.13 : Introduced noise vectors

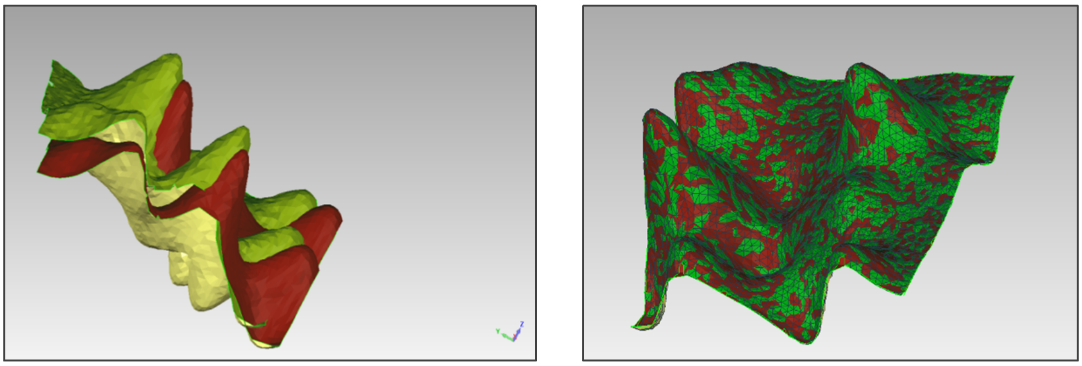


Figure 4.14 : (a)Template and search data, (b) Template and transformed surface.

Table 4.8 : Numerical results of ‘wave’ data

Model	No. of Matched Points	σ_0 (mm)	σ_{T_x} (mm)	σ_{T_y} (mm)	σ_{T_z} (mm)	σ_ω (degree)	σ_φ (degree)	σ_κ (degree)
TLS3d	3635	0.006377	0.000151	0.000151	0.000150	0.000101	0.000101	0.000076
LS	3907	0.008996	0.000145	0.000145	0.000144	0.000047	0.000047	0.000035
TLS3d_anis.	3634	5.014204	0.000326	0.000343	0.000152	0.000047	0.000034	0.000071

The registration process was carried out for three times. In first run, we used TLS3d without introducing the pre-defined covariance matrices as input. In second run, we use LS and for the third run we use again TLS3d by introducing the covariance matrices as input. Registration is successful for all runs. Numerical results of the tests are given in Table 4.8. Results are quite similar and consistent with the other experiments. Results do not change in significant level for LS and TLS3d without using a-priori covariance definition. But at the last run, the effects of the introduced anisotropic and non-homogeneous covariance matrices are observed. The obtained a-posteriori covariance matrices are smaller than the other two runs. In the view of these results, it is hard to state that TLS3d produces more correct results, but it would not be incorrect to say that it produces more reliable and realistic estimation values.

5. CONCLUSIONS

5.1 Summary

In this thesis, we analyzed the results of a 3D registration problem in terms of statistical behaviors by using so called Total Least Squares method within an EIV model. On the contrary of classical Least Squares approach, EIV model accounts the measurement errors as well as stochastic properties of independent variables.

The mathematical model of the applied method is based on the non-linear Gauss-Helmert Model. The model is adapted by Kanatani (2012) by eliminating the Lagrange Multlipliers and it is called as Modified Gauss-Helmert Model. The model is formulated for seven parameters similarity transformation in (Kanatani 2012). We made a small modification at the model by eliminating the scale factor to be able to use it for 6 parameters rigid-body transformation. There are diverse solution methods for EIV model given in details in Chapter 2. The MGH is an advantegous model in terms of computational cost within the other methods since the Lagrange Multipliers are eliminated from the normal equations and hence, the size of the matrix to be solved dramatically decreases to the number of unknown parameters. Therefore, we choose this model since it is the most appropriate one especially for large data sets, like point cloud.

The objective of the model is to estimate the optimal transformation parameters of rigid-body transformation by minimizing the Mahalonobis distance in the sense of Maximum Likelihood. Mahalonobis distance differs from Euclidean distance in that it takes into consideration the correlation between data sets. If unitary covariance matrices are introduced for both data sets, then the Mahalonobis distance becomes equal to the Euclidean distance.

In the classical Gauss-Markov model, the observation equations are established with the assumption that only the template surface elements are observations and contaminated by random errors. In fact, this is unrealistic and the search surface elements also contain errors. The stochastic properties of search surface elements are taken into account in MGH model, whilst this is ignored in the classical Gauss-Markov model. On the other hand, if the covariance matrices are defined as isotropic and homogeneous for both template and search surfaces, the MGH model gives almost the same results with the Gauss-Markov model. Therefore, we defined anistropic and non-homogeneous covariance matrix for both data sets individually. The correspondence search is the most critical and time consuming part of the registration process. We used two well-known variants of ICP (Besl and McKay, 1992, Chen and Medioni, 1991) in combination for correspondence search. Our registration algorithm contains some conditions and thresholds in order to prevent false matches and speed up the process.

5.2 Conclusions

We applied MGH model for the registration problem of 3D surfaces. The relationship between the surfaces is established by rigid body transformation.

All range measurement data are acquired by a sensor. Thus, it is not realistic to ignore the stochastic properties of search data and treat template data as only the erroneous one. On the other hand, the noise characteristics cannot be isotropic and homogeneous for these data sets. Actually, if the a-priori covariance matrices are introduced as homogeneous for both data sets, the classical Least Square method provides the optimal solution. But this is not realistic as well, because of the existence of many impacts affecting the individual point accuracy such as angular and range accuracy of the sensors, incidence angle of the laser beam, temperature, surface reflectivity, etc. Taking these facts into account, we introduced anisotropic, inhomogeneous a-priori covariance matrices for both data sets. Our experiments were carried out in two groups. In the first group of experiments we used homogeneous and isotropic noise model and we obtained the results of both LS and TLS methods. The numerical results show that the difference between theoretical precisions of estimated parameters is minor and thus insignificant. Therefore, it can be stated that both LS and TLS methods do not differ from each other in the case of this kind of noise definition. Second group of experiments are conducted with the anisotropic and inhomogeneous noise definition and the solution is obtained from TLS. The obtained a-posteriori covariance matrices are smaller than the ones in the first group. In the view of these results, it is hard to state that TLS solution within

EIV model produces more correct results for solution vekctor, but it would not be incorrect to say that it produces more reliable and realistic accuracy values of estimated parameters.

Surface representation is basically established as structured meshes with implicit planar surface patches in our implementation. Although it is a very basic structure and represents the object surface quite well, it produces some spurious triangles with longer edges and narrow angles especially in curved parts of the surface. As a result of this situation, some possible candidate points are eliminated from matching. Some complex meshing strategies (Delaunay Triangulation, TIN etc.) could possibly prevent this problem. Although the combined search algorithm lasts long due to the introduced conditions and natural slowness of point-to-point search, it works quite well and finds correct point pairs which was our first priority.

Since our model is a non-linear system of equations, it requires good approximation values for the parameters. Our implementation is successful for rotational differences up to 20˚, but the translation vector should be well approximated for the convergence. One of our observations is that the convergence in the direction of surface normal is faster than lateral directions. Since we define equal stopping criteria for same types of parameters (rotations and translations), sometimes we were faced with the oscillations in some parameters, but thorough the convergence they turn to be smooth.

5.3 Future Work

The overall of this thesis study is successful and the results are satisfactory to conclude the thesis. However, the author thinks that some further improvements and extensions would give more robust results.

The calculated anisotropic and inhomogeneous covariance matrices are block diagonal. By this definition, we ignore the correlation between the observations. A full covariance matrix that contains the correlations, can give better results in a-posteriori covariance matrix.

Due to the ignorance of some scanner errors (temperature, edge effect, object reflectance etc.), our covariance definition is relatively optimistic. A covariance matrix definition which accounts for the whole significant scanner errors, would undoubtly give more realistic results.

Our solution model evaluates the Euclidian distance. The model can be extended by introducing some other parameters and conditions (intensity, geometric conditions etc.). By this way, obtaining more stable and robust results could be possible.

Our implementation is built on MATLAB computing language which is very user friendly and flexible software. We took the advantage of using some built in functions in toolboxes. However, the capability of the software of processing especially large data sets is limited in terms of speed and requires so much virtual memory in computers. This situation limited us in using larger data sets. Using a different programming language could produce faster results and give the opportunity of working with larger data sets which provides the flexibility of analysing the results in higher degree of freedom level.

REFERENCES

Abdelhafiz, A. (2009). Integrating Digital Photogrammetry and Terrestrial Laser Scanning, (PhD Thesis), Institute for Geodesy and Photogrammetry, Technical University Braunschweig, Germany.

Akca, D. (2010). Co-registration of Surfaces by 3D Least Squares Matching. *Photogrammetric Engineering & Remote Sensing*. Vol. 76, no. 3, pp. 307-318.

Akca, D. (2010). Least Squares 3D Surface Matching, (PhD Thesis), Swiss Federal Institute of Technology Zurich, Switzerland.

Akyılmaz, O. (2007) Total least squares solution of coordinate transformation. Surv Rev 39(303):68–80

Akyılmaz, O. (2011). Solution of the Heteroscedastic Datum Transformation Problems, *1st International Workshop on the Quality of Geodetic Observation and Monitoring Systems (QuGOMS)*, Munich, Germany, April 13-15.

Alexa, M., Behr, J., Cohen-Or, D., Fleishman, S., Levin, D., Silva, C. T. (2001). Point Set Surfaces, *IEEE Visualization*, San Diego, USA, October 21-26.

Arun, K. S., Huang, T. S., Blostein, S. D. (1987). Least-Squares Fitting of Two 3-D point Sets. *IEEE Transactions on Pattern Analysis and Machine Intelligence*. Vol. 9, no 5, pp. 698-699.

Bae, K. H., Belton, D., Lichti, D. D. (2009). A Closed-Form Expression of the Positional Uncertainty for 3D Point Clouds. *IEEE Transiction on Pattern Analysis and Machine Intelligence*. Vol 31, no 4, pp 577-590.

Besl, P. J. and McKay, N. D. (1992). A Method for Registration of 3-D Shapes. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, Vol 14, no 2, pp 239-256.

Blais, G. and Levne, M. D. (1993). Registering Mutiview Range Data to Create 3D Computer Objects, *Centre for Intelligent Machines Report*, **TR-CIM-93-16**, , McGill University, Montreal, Quebec, Canada.

Chen, Y. and Medioni, G. (1991). Object Modeling by Registration of Multiple Range Images, *Proceedings of the 1991 IEEE International Conference on Robotics and Automation* (pp. 2724- 2729). Sacremento, California.

Chetverikov, D. (1991). Fast Neighborhood Search in Planar Point Sets, *Pattern Recognition Letters,* **12**, 409-412.

Dorai, C., Wang, G., Jain, A. K., Mercer, C. (1996). Registration and Integration of Multiple Object Views for 3d Model Construction. *IEEE*

Transactions on Pattern Analysis and Machine Intelligence, Vol 20, no 1, pp 15-16.

Eggert, D. W., Lorusso, A., Fisher, R. B. (1997). Estimating 3-D Rigid Body Transformations: A Comparison of Four Major Algorithms, *Machine Vision and Applications* **9**, 272-290.

Estepar, R. S. J., Brun, A., Westin, C. F. (2007). Robust Generalized Total Least Squares Iteratıve Closest Point Registration. Signal Processing, Vol 87, pp 2283-2302.

Felus, Y., Schaffrin, B. (2005) Performing similarity transformations using the errors-in-variables-model. In: Proceedings of theASPRS Meeting, Washington, DC, May 2005, on CD

Förstner, W. (2000). On Weighting and Choosing Constraints for Optimally reconstructing the Geometry of Image Triplets, *Proceedings of the 6th European Conference on Computer Vision* (pp 669-684). London, UK.

Grant, D., Bethel, J., Crawford, M. (2012). Point-to-Plane Registration of Terrestrial Laser Scans, *ISPRS Jpurnal of Photogrammetry and Remote Sensing* **72**, 16-12.

Gruen, A., (1985a). Adaptive least squares correlation: a powerful image matching technique. South African Journal of Photogrammetry, Remote Sensing and Cartography 14 (3), 175-187.

Gruen, A. and Akca, D. (2005). Least Squares 3D Surface Matching, ISPRS Panoramic Photogrammetry Workshop, Dresden, Germany, February 19-22.

Kanatani, K. (1990). Group-Theoretical Methods in Image Understanding, Springer, Berlin, Germany, 1990.

Kanatani, K. and Nitsuma, H. (2012). Optimal Computation of 3-D Similarity: Gauss-Newton vs. Gauss-Helmert, *Computational Statistics & Data Analysis,* **56,** 4470-4483.

Kanazawa, Y., Kanatani, K. (1995). Reliability of 3-D reconstruction by stereo vision, IEICE Trans. Inf. Syst., E78-D, 1301–1306.

Lichti, D. D. (2007). Error Modelling, calibration and Analysis of an AM-CW Terrestrial Laser Scanner System, *ISPRS Journal of Photogrammetry & Remote Sensing*, **61,** 307-324.

Low, K. L. (2004). Linear Least-Square Optimization for Point-to-Plane ICP Surface Registration, *University of North Caroline Department of Computer Science Technical Report*, **TR04-004,** Chapel Hill.

Markovsky I. and Huffel, S. V. (2007). Overview of Total Least Squares Methods. Signal Process, Vol 87, no 19, pp 2283-2302.

Masuda, T., Sakaue, K., Yakoya, N. (1996). Registration and Integration of Multiple Range Images for 3D Model Construction*, IEEE Proceedings of ICPR* (pp. 879- 883). Vienna, Austria.

Matei, B. and Meer, P. (1999). Optimal Rigid Motion Estimation and Performance Evaluation with Bootstrap, *Proceedings of the 5th European Conference on Computer Vision* (pp 339 345). Freiburg, Germany.

Neitzel, F. (2010). Generalization of Total Least-Squares on Examples of Unweighted and Weighted 2D Similarity Transformation, *J Geod*, **84**, 751-762.

Neitzel, F. and Petrovic, S. (2008). Total Least Squares (TLS) im Kontext der Ausgleichung Nach Kleinsten Quadraten am Beispiel der Ausgleichenden Geraden. *Zeitschrift für Geodasie, Geoinformation und Landmanagement*, Vol 133, no 3, pp 141-148.

Neugebauer, P.J. (1997). Reconstruction of Real-World Objects via Simultaneous registration and Robust Combination of Multiple Range Images. *International Journal of Shape Modeling*, Vol 3, no 1&2, pp 71-90.

Ohta, N. and Kanatani, K. (1998). Optimal Estimation of Three- Dimensional Rotation and Reliability Evaluation, *Proceedings of the 5th European Conference on Computer Vision* (pp 175-187). Freiburg, Germany.

Park, S. Y. and Subbarao, M. (2003). An Accurate and First Point-to-Plane Registration Technique, *Pattern Recognition Letters,* **14** 2967-2976.

Pennec, X. and Ayache, N. (1996). Randomness and Geometric Features in Computer Vision*, Epidaure Project*, San Fransisco, California, June 10-20.

Pennec, X. and Thirion, J. P. (1997). A Framework for Uncertainty and Validation 3-D Registration Methods based on Points and Frames. *International Journal of Computer Vission*, Vol 25, no 3, pp 203-229.

Pottmann, H., Leopoldseder, S., Hofer, M. (2004). Registration without ICP. *Computer Vision and Image Understanding*, Vol 95, no 1, pp 54-71.

Pulli, K. (1999). Multiview Registration for Large Data Sets, *Proceedings of the International Conference on 3D Digital Imaging and Modeling* (pp 160-168).

Ramos, J. A. and Verriest, E. I. (1997). Total least Squares Fitting of Two Point Stes in m-D, *Proceeding of the 36th Conference on Decision & Control* (pp 5048-5053). San Diego, California.

Rusinkiewicz, S. and Levoy, M. (2001). Efficient Variants of the ICP Algorithm, *Third International Conference on 3-D Digital Imaging and Modeling Proceeding* (pp 145-152). Quebec, Canada.

Salvi, J., Matabosch, C., Fofi, D., Forest, J. (2007). A Review of Recent Range Image Registration Methods wirh Accuracy Evaluation. Image and Vission Computing, Vol 25, no 2, pp 578-596.

Soudarissanane, S., Lindenbergh, R., Menenti, M., Teunissen, P. (2011). Scanning Geometry: Influencing Factor on the Quality of Terrestrial Laser Scanning Points*, ISPRS Journal of Photogrammetry and Remote Sensing*, **66**, 389-399.

Tsakiri, M., Lichti, D., Pfeifer N. (2006). Terrestrial Laser Scanning for Deformation Monitoring, *12th FIG Symposium*, Baden, May 22-24.

Weng, J., Huang, T., S., Ahuja, N. (1989). Motion and Structure from Two Perspective Views: Algorithms, Error Analysis, and Error Estimation. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, Vol 11, no 5, pp 451-476.

Westfeld, P., Maas H., Pust, O., Kitzhofer, J., Brücker, C. (2010). 3D Least Squares Matching for Volumetric Velocimetry Data Processing, *15th International Symposium on Applications of Laser Techniques to Fluid Mechanics,* Lisbon, Portugal, July 5-8.

Wu, J. (2012). Cell-Based Deformation Monitoring via 3D Point Clouds, *PhD Thesis*, A la Faculte de L'Environnement Naturel, Ecole Polytechnique Federale de Lausanne, Switzerland.

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International Publications (SCI)

- Duran Z., **Aydar U.**, Digital modeling of world's first known length reference unit: The Nippur cubit rod, Journal of Cultural Heritage, *Journal of Cultural Heritage*, Volume **13**, Issue 3, July–September 2012, Pages 352–356

National Publications

- **Aydar U.**, Avşar E. Ö., Kaya Ş., Bozkurtoğlu E., Şeker D. Z., Yüzeylerin Pürüzlülük Açısının Lazer Tarayıcılar Yardımıyla Belirlenmesi, *HKM Jeodezi, Jeoinformasyon ve Arazi Yönetimi Dergisi*, 2011/2 Özel Sayısı, S. 33-38

International Conference Papers

- Kaya, S., Bozkurtoglu E., Avsar E. O., **Aydar U.**, Seker, D. Z., Determination of Roughness Angles of Surfaces Using Laser Scanner. *FIG Working Week – Bridging the Gap Beetween Cultures*, 18–22 May 2011, Marrakech, Morocco.
- Arpacı K. E., Ozyaşar M., Duran Z., Avsar E. O., **Aydar U.**,Webbased Presentation of Indoor Modeling by Means of Photogrammetry, *XXIIIrd*

International CIPA Symposium, 12-16 September 2011, Prague, Czech Republic

- Avsar E. O., **Aydar U.**, Seker D. Z., Education on Cultural Heritage Documentation and Terrestrial Photogrammetry at ITU Department of Geomatics, *XXIIIrd International CIPA Symposium*, 12-16 September 2011, Prague, Czech Republic
- **Aydar, U.**, Akyol, O., Duran, Z., 2011. A low-cost laser scanning system design, *XXIIIrd International CIPA Symposium*, 12-16 September 2011, Prague, Czech Republic
- Duran, Z., **Aydar, U.**, 2008. Nippur Uzunluk Ölçme Aletinin 3B Modellenmesi ve Görselleştirilmesi, *Türkiye Ulusal Fotogrametri ve Uzaktan Algılama Birliği V. Teknik Sempozyumu*, Ankara, Türkiye
- Duran, Z., **Aydar, U.**, 2008. Measurement and 3D Modelling of an Ancient Measuring Device: Nippur Cubit Rod,*XXIst Congress of International Society for Photogrammetry and Remote Sensing*,3-11 Jul 2008 Beijing, CHINA
- **Aydar, U.,** Avşar, E.O., Altan, M.O., 2007. Obtaining Façade Plan Of A Historical Building With Orthorectification Of Single Images Gathered By Mobile Phones And Digital Cameras, *XXIth CIPA Symposium*, Athens, Greece
- Avşar, E. Ö., **Aydar, U.**, Seker, D.Z., 2006. Virtual Reconstruction of a Semi Destroyed Historical Bridge, *5th Turkish-German Joint Geodetic Days*, 29-31 March, Berlin, Germany.
- Seker, D.Z., Avşar, E. Ö., **Aydar, U.**, 2006. Modeling of Historical Bridges Using Photogrammetry and Virtual Reality, *FIG XXIII Congress and XXIX General Assembly - Shaping the Change*, 8-13 October 2006, Munich, Germany.

National Conference Papers

- Çelik, M.F., Küçükosman, G., Yüksel, E., Duran, Z., **Aydar, U.**, Avşar, Ö. 2013. Lazer Tarayıcı Yardımıyla 3B İç Mekan Modeli Oluşturulması ve CBS Ortamına Aktarılması, *TMMOB Coğrafi Bilgi Sistemleri Kongresi*, 11-13 Kasım 2013, Ankara 11-13 Kasım 2013 Ankara, Türkiye
- Akay, S. S., Uzun, G., Duran, Z., Avşar, E.Ö., **Aydar, U.**, 2012. Lazer Tarama Yöntemi İle Elde Edilen Üç Boyutlu İç Mekân Modellerinin Coğrafi Bilgi Sistemlerinde Kullanım Olanakları, *IV. Uzaktan Algılama ve Coğrafi Bilgi Sistemleri Sempozyumu (UZAL-CBS 2012)*, 16-19 Ekim 2012 Zonguldak
- Uzun, G., Akay, S. S., Duran, Z., **Aydar, U.**, Avşar, E.Ö. 2012. Üç Boyutlu İç Mekân Modelinin Yersel Lazer Tarama Yöntemi İle Elde Edilmesi Örneği, HKMO-Mühendislik Ölçmeleri STB Komisyonu, *6. Mühendislik Ölçmeleri Sempozyumu*, 3-5 Ekim 2012, Afyon Kocatepe Üniversitesi- Afyonkarahisar
- Avşar, E. Ö., **Aydar, U.**, Duran, Z., Arpacı, K. E., 2009. Digital Fotogrametri Tekniği Yardımıyla İç Mekan Modellemesi ve Web Tabanlı Sunulması, TMMOB Harita ve Kadastro Mühendisleri Odası, *12. Türkiye Harita Bilimsel ve Teknik Kurultayı,* 11–15 Mayıs 2009, Ankara, Türkiye

- Duran, Z., **Aydar, U.**, 2009. Nippur Uzunluk Ölçme Aletinin 3B Modellenmesi ve Görselleştirilmesi, *Türkiye Ulusal Fotogrametri ve Uzaktan Algılama Birliği V. Teknik Sempozyumu*, 4-6 Şubat 2009, MTA, Ankara.

- Altan, O., Külür, S., Toz, G., Şeker, D.Z., Demirel, H., Duran, Z., Akçay, Ö. Avşar, E.Ö., **Aydar, U.**, 2008. Tarihi Kent Dokusunun Korunmasında Fotogrametrinin Kullanım Olanakları, *Kent Yönetimi, İnsan ve Çevre Sorunları'08 Sempozyumu*, 02-06 Kasım 2008, İstanbul.

- Vatansever, C., Yardımcı, N., Avşar, E. Ö., **Aydar, U.**, Şeker, D. Z., 2007. İnce Çelik Levhalı Perdede Levhanın Düzlemdışı Hareketlerinin Fotogrametri İle Belirlenmesi, *Türkiye Ulusal Fotogrametri ve Uzaktan Algılama Birliği IV. Teknik Sempozyumu,* 5-7 Haziran 2007, İstanbul, Türkiye

PUBLICATIONS/PRESENTATIONS ON THE THESIS

- **Aydar, U.**, Altan, O., 2014. Total Least Squares Registration of 3D Surfaces, *International Journal of Environment and Geoinformatics* Volume **1**, Issue 1, March 2014. *(accepted).*
- **Aydar, U.**, Akca, D., Altan, O., Akyilmaz, O., 2013, Total Least Squares Registration of 3D Surfaces, *ISPRS Laser Scanning Workshop 2013*, Antalya, Turkey
- **Aydar, U.**, Altan, M.O., Akyilmaz, O., Akca, D., 2012. Co-registration of 3D point clouds by using an error-in- variables model. *The 22th ISPRS Congress, Melbourne*, Australia, August 25 – September 1. International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences, vol. XXXIX, part B5, pp. 151-155.