

**MECHANICS AND DYNAMICS OF MILLING
WITH GENERALIZED GEOMETRY**

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FOREWORD

Milling operations are commonly used in many different industries. A major problem associated with milling is chatter vibrations. Chatter leads to the exponential growth of chip thickness, and therefore cutting forces in an unstable manner until the cutter either jumps out of the cut, or tool breakage occurs. Apart from causing chipping at the cutting edge, chatter vibrations also result in very poor surface finishes. Hence, appropriate selection of cutting conditions plays an important role in avoiding chatter vibrations, extending tool life and obtaining a good surface finish quality. The mechanics and dynamics of milling operations conducted with generalized cutter shapes are studied in this thesis. The study leads to the prediction of cutting forces, torque, power and chatter vibration free spindle speeds and depths of cut which are all important in obtaining high productivity and acceptable part quality. The thesis also provides a generalized parametric representation of complex cutter geometry, such as variable pitch angles between the successive teeth, bull nose inserts, helical ball end mills, tapered helical ball end mills and inserted end mills, which are used in aerospace and die machining industry. The critical chatter stability border, which is characterized by the prediction of borderline stable axial depth of cuts and spindle speeds, are predicted analytically.

The mathematical and simulation models presented in this thesis are all verified experimentally. The models are coded in a PC program, and released to various companies who support the research financially at The University of British Columbia. The feedback received from the users in the companies is incorporated into the research. This thesis is a compilation of the research work done under the supervision of Prof. Dr. Yusuf Altıntaş, at the University of British Columbia, Department of Mechanical Engineering, Manufacturing Automation Laboratory.

I wish to express my sincere appreciation to my research supervisor, Prof. Dr. Yusuf Altıntaş, for his support, guidance and encouragement throughout my research at the University of British Columbia. I would like to thank my co-supervisor Assistant Prof. Dr. Muzaffer Erten, for his support at Istanbul Technical University. I would also like to thank the Mechanical Engineering Faculty of Istanbul Technical University and Tınçel Foundation for their financial support that enabled me to pursue my research abroad. In addition I also wish to thank my colleagues at the Manufacturing Automation Laboratory at UBC for their assistance and patience. This research was funded by Natural Sciences and Engineering Council (NSERC) of Canada, Boeing and General Motors Corporations under the grant numbers (NSERC, CRD, NSERC Operating, NSERC Strategic, Boeing, GM). Finally, I would like to thank to my family and relatives who have supported me throughout my academic career. I dedicate this work to them.

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LIST OF SYMBOLS

a, b	: Rectangular insert dimensions, insert width and length
a_{lim}	: Critical depth of cut
D	: Diameter of cutter
db	: Differential projected edge length
dS	: Differential cutting edge length
f_n	: Spindle speed frequency
F_{ij}, F_{rj}, F_{aj}	: Tangential, radial and axial force acting on j th flute
F_x, F_y, F_z	: Force components in X, Y and Z direction
g	: Unit step function
h	: Valid cutting edge height from tool tip along the Z axis
$h(\phi_j)$	: Instantaneous chip thickness
i	: Helix angle
I_r	: Insert center radius from tool center
I_z	: Insert Z location from tool center
j	: Tooth (flute) number ($j = 0, 1, \dots, N-1$)
k	: Integer number of full vibration waves
K_{tc}, K_{rc}, K_{ac}	: Milling shear component of cutting force coefficients in tangential, radial and axial directions
K_{te}, K_{re}, K_{ae}	: Milling edge component of cutting force coefficients in tangential, radial and axial directions
$lead$	: Lead value for constant lead
M	: Intersection point of the first line and the arc
M_r	: Radius of the intersection point of the first line and the corner arc
M_z	: z coordinate of the intersection point of the first line and the corner arc
N	: Intersection point of the arc and the second line
N	: Number of flutes
n	: Spindle speed
N_r	: Radius of the intersection point of the second line and the corner arc
N_z	: z coordinate of the intersection point of the second line and the corner arc
O	: The point at the tool tip
p_x, p_y, p_z	: X, Y and Z coordinate for a point on the insert cutting edge with respect to insert coordinate system
$r(z)$	: Radius of the cutter at z level

$\vec{r}(z)$	: Vector from tool center to the cutting edge point
$R, \theta_s, \theta_e, o_f$	: Convex triangular insert dimensions, arc radius, arc starting angle and ending angle from y the offset value of the arc center from center of insert,
R_r	: Radial distance of corner radius to the center
r_t	: Chip thickness ratio
$\mathbf{R}_x(\beta)$	: Insert rotation matrix about X axis with axial rake angle (β)
$\mathbf{R}_y(\delta)$	: Insert rotation matrix about Y axis with lead angle (δ)
R_z	: z level of corner radius center
$\mathbf{R}_z(\varphi)$	: Insert rotation matrix about Z axis with index angle (φ)
S	: Intersection point of the second line and h level
s_p	: Initializing value at the tool tip
s_t	: Average feed rate per tooth
s_{ij}	: Feed rate for j th tooth
T	: Spindle period
T_j	: Tooth passing period
$\mathbf{T}_M$	: Total transformation matrix for the insert
O, O'	: Cutter and insert center points
V_{IC}	: Insert center position vector form cutter center
V_{CE}	: Cutting edge position vector form insert center
V_{CER}	: Cutting edge position vector form insert center after rotation
V_p	: Cutting edge position vector form cutter center
X, Y, Z	: Cutter coordinate system
u, v, w	: Insert local coordinate system
x, y	: Dynamic displacements in feed X and normal Y directions
$z_{j,1}(\phi_j), z_{j,2}(\phi_j)$	: Axial integration limits
η	: Chip flow angle
ψ	: Cutting edge position angle on the XY plane
Λ	: Eigenvalue of stability equation
δ	: Insert rotation in X axis with δ lead angle and positive direction is in CW
β	: Insert rotation in Y axis with β axial rake angle and positive direction is in CW for inserted cutters
β	: Slope angle for the first line of cutter profile with respect to vertical line for solid end mills
φ	: Insert rotation in Z axis with φ lag angle and positive direction is in CCW
ε	: Phase shift between present and previous vibration waves
α	: Slope angle for the first line of cutter profile with respect to horizontal line
κ	: The angle between cutting edge normal and Z axis

ϕ	: Total angular rotation for cutting edge on the XY plane
$\Delta\phi_p$	: Pitch nonuniformity angle
ψ_{1e}	: Final lag angle for the first line (at point M)
ψ_{1s}	: Starting lag angle for the first line (at the tool tip)
ψ_{2e}	: Final lag angle for the second line (at point S)
ψ_{2s}	: Starting lag angle for the second line (at point N)
ψ_{ae}	: Final lag angle for the corner arc line (at point N)
ψ_{as}	: Starting lag angle for the corner arc line (at point M)
ω_c	: Chatter frequency
ϕ_j	: Rotation angle of cutting edge j
Ω	: Angular velocity of the spindle ($\Omega = 2\pi n / 60$)
ε	: Phase shift between present and previous vibration waves
β_j	: Wave length
β_n	: Normal friction angle at the rake face
α_n	: Normal rake angle
ϕ_n	: Normal shear angle at the shear zone
$\phi_{p,j}$	: Pitch angle for tooth j
$\Delta\phi_p$	: Pitch nonuniformity angle
ϕ_{st}, ϕ_{ex}	: Entry and exit angles of cut
$\Delta x, \Delta y$	: Regenerative displacements in X and Y direction
$[\Phi]$	: Transfer function matrix
$[A]$	: Directional dynamic milling force coefficient matrix
$[T]$	: Transformation matrix for the cutting forces
$\{\Delta d\}$	: Dynamic displacement vector
$\{B\}$	: Time delay vector
$\{F\}$	: Regenerative dynamic force vector

MECHANICS AND DYNAMICS OF MILLING

WITH GENERALIZED GEOMETRY

SUMMARY

Milling operations are commonly used in many different industries. A major problem associated with milling is chatter vibrations. Chatter leads to the exponential growth of chip thickness, and therefore cutting forces in an unstable manner until the cutter either jumps out of the cut, or tool breakage occurs. Apart from causing chipping at the cutting edge, chatter vibrations also result in very poor surface finishes. Hence, appropriate selection of cutting conditions plays an important role in avoiding chatter vibrations, extending tool life and obtaining a good surface finish quality.

The mechanics and dynamics of milling operations conducted with generalized cutter shapes are studied in this thesis. The objective of geometric modeling of milling cutters is to design a milling simulation system, which can handle a variety of different end mills for improved cutter design or process planning in industry. The study leads to the prediction of chip thickness, cutting forces, torque, power, tool vibrations, workpiece vibrations and chatter vibration free spindle speeds and depths of cut which are all important in obtaining high productivity and acceptable part quality.

The cutting forces in milling are dependent upon the cutting edge geometry, work material properties, cutting speed, feed rate, depth of cut and radial width of cut. By using the exact kinematics of milling motion that traces a trochoidal path, the exact chip thickness removed by a point on the cutting tool edge is modeled analytically. The cutting forces are proportional to the chip thickness as well as the material properties and tool geometry. When the forces are summed along the cutting edge, the total forces acting on the spindle and part are predicted. The thesis also provides a generalized parametric representation of complex cutter geometry, such as variable pitch angles between the successive teeth, bull nose inserts, helical ball end mills and tapered helical ball end mills which are used in aerospace and die machining industry.

In this study, there are three main objectives, which are;

- Developing generalized geometric model for arbitrary end mills and investigation of mechanics and dynamics,
- Developing generalized geometric model for inserted (indexable) cutters and investigation of mechanics and dynamics,

- Analytical prediction of chatter stability lobes as well as an analytical design technique for chatter free variable pitch cutters.

Generalized end mill geometry envelope is defined in a similar manner to the parametric representation used by APT and CAD/CAM software systems. The formulation of cutting edge coordinates on the cutter is presented. It is shown that a vast variety of helical end mill geometries can be designed using the proposed geometric model. The modeled cutting edge is broken into small segments which may have varying cutting constants. As application examples, case studies involving helical ball end, tapered helical ball end, and bull nose cutters are presented. It is experimentally shown that the developed model can be used in predicting cutting forces, vibrations, dimensional surface finish, as well as chatter stability lobes for any cutter geometry within the defined envelope. Mechanics and dynamics of this geometry were investigated using a technique similar to one presented for ball end milling by Altıntaş and Lee.

The end mill geometry is modeled by helical flutes wrapped around a parametric envelope. The coordinates of a cutting edge point along the parametric helical flute are mathematically expressed. The cutter is divided into small disk elements, enabling the mechanics and dynamics of milling to be solved for each element. The chip thickness at each cutting point is evaluated by using the true kinematics of milling, including the structural vibrations of both the cutter and workpiece. By integrating the process along each cutting edge which is in contact with the workpiece, the cutting forces, vibrations, dimensional surface finish and chatter stability lobes for an arbitrary end mill are predicted. The predicted and measured cutting forces, surface roughness and stability lobes for ball, helical tapered ball, and bull nosed end mills are provided to illustrate the viability of the proposed generalized end mill analysis.

Inserted cutters are widely used in roughing and finishing of parts. The insert geometry and distribution of inserts on the cutter body vary significantly in industry depending on the application. A generalized mathematical model of inserted cutters geometry were developed for the purpose of predicting cutting forces, vibrations, dimensional surface finish and stability lobes in milling. The edge geometry is defined in the local coordinate system of each insert, and placed and oriented on the cutter body using the cutter's global coordinate system. The cutting edge locations are defined mathematically, and used in predicting the chip thickness distribution along the cutting zone. Each insert may have a different geometry, such as rectangular, convex triangular or a mathematically definable edge. Each insert can be placed on the cutter body mathematically by providing the coordinates of the insert center with respect to the cutter body center. The inserts can be oriented by rotating them around the cutter body, thus each insert may be assigned to have different lead and axial rake angles. By solving the mechanics and dynamics of cutting at each edge point, and integrating them over the contact zone, it is shown that the milling process can be predicted for any inserted cutter. A sample of inserted cutter modeling and analysis examples are provided with experimental verifications. A new variable cutting coefficient model along the z direction was applied to the inserted cutters by using average cutting coefficients method, which is presented in detail by Budak et. al. The dynamic investigations of inserted cutter were done by using time domain and analytical solution proposed by Altıntaş and Budak.

The thesis also considers the vibrations produced during milling. The structural dynamic model of the cutter is modeled as a multi-degree of freedom system but is lumped at the tool tip. The equation of motion of the flexible cutter is added to the evaluation of the chip thickness. The cutting forces and surface finish generated during milling with vibrations are predicted in time domain. The vibrations of the tool and workpiece are calculated based on the dynamic behaviour of the machine-tool, the workpiece dynamics and the cutting forces. By analyzing the exponential growth of vibrations, the stability of the cutting operation is investigated. The presence of the chatter vibrations, or chatter stability, is predicted by linearizing the dynamic milling and solving the eigenvalue problem analytically. The critical chatter stability border, which is characterized by the prediction of borderline stable axial depths of cut and spindle speeds, are predicted analytically. Analytical solutions are proven with time domain solutions.

An analytical prediction of stability lobes for milling cutters with variable pitch angles is also presented in this thesis by modifying uniform pitch cutters stability lobe solution, which is previously developed by Altıntaş and Budak. This method requires the cutting constants, number of teeth, and transfer function of the cutter mounted on the machine tool as inputs to a chatter stability expression. The stability is formulated by transforming time varying directional cutting constants into time invariant constants. Constant regenerative time delay in uniform cutters is transformed into non-uniform multiple regenerative time delay for variable pitch cutters. The chatter free axial depth of cut is solved from the eigenvalues of the stability expression, whereas the spindle speed is identified from regenerative phase delays. The proposed technique has been verified with extensive cutting tests and time domain simulations. Shirase and Altıntaş' static model of cutting forces and chip load generation for variable pitch cutters are used.

The practical use of the analytical solution of variable pitch cutters is demonstrated by an optimal tooth spacing design application, which increases the chatter free depths of cut significantly. The main idea in designing a cutter with different pitch angles is to obtain different phase angles between successive surface waves which enable a larger depth of cut to be achieved, without chatter, compared to the uniform pitch case. The pitch angles are chosen in such a manner that the axial depth of cut is maximized for a given spindle speed. The developed method gives all the possible pitch angles at a specified speed range where it is tuned to provide minimum regeneration for a given set of machine tool dynamics, i.e. machine - spindle - cutter assembly.

Variable pitch cutter design was implemented for a case of cutting Al356 with a four fluted cutter at a spindle speed of 5000 rev/min, which gives optimum tool life for the workpiece and the cutter, as 70° - 110° - 70° - 110° instead of 90° - 90° - 90° - 90° . As a result, the stable maximum depth of cut increased more than three times under the same cutting conditions, but only changing the cutter pitch angles. This improvement was verified with experiments and simulations in frequency and time domain.

The mathematical and simulation models presented in this thesis are all verified experimentally. Results such as cutting forces, workpiece and spindle vibrations, stability lobes, chatter frequencies and surface finish profiles were obtained through frequency domain analyses and time domain simulations, which were in close agreement. A laser displacement sensor, accelerometers, force dynamometers,

impact hammers with force sensor, an acoustic microphone, and a laser surface scanner were used to measure input and output variables of the milling simulations. The experimental and simulated results were very close to each other under these carefully controlled conditions. Throughout all this work, more than two thousands cutting tests were done to verify the theory, which is developed in the thesis. Some of the cutting tests were repeated several times to obtain accurate results. Some of the tests results, which gave wrong outcomes, were not given in the thesis. The simulations incorporating the theoretical work developed in this thesis, can be an quite useful tool to identifying the optimum cutting conditions for improved productivity.

The models are coded in a-PC program, and released to various companies who support the research financially at The University of British Columbia – Manufacturing Automation Laboratory. The feedback received from the users in the companies is incorporated into the research.



GENELLEŞTİRİLMİŞ TAKIM GEOMETRİSİ İLE FREZELEME MEKANİĞİNİN VE DİNAMİĞİNİN İNCELENMESİ

ÖZET

Frezeleme işlemleri birçok farklı sanayi dalında yaygın olarak kullanılır. Frezeleme ile ilgili önemli bir problem tırlama titreşimidir. Tırlama, takımın kararsız bir şekilde iş parçasından ayrılmasına veya talaş kalınlığının ve bundan dolayı kesme kuvvetlerinin üstel olarak büyümesine ve takımın kırılmasına sebep olur. Kesme ağzında talaş birikmesinin dışında, tırlama titreşimleri sonucu yüzey kalitesi çok kötüleşir. Bu nedenle kesme şartlarının uygun seçimi tırlama titreşimlerini yok etmede, takım ömrünün uzamasında ve iyi bir yüzey kalitesi elde etmede önemli bir rol oynar.

Bu tezde, genelleştirilmiş takım geometrisi ile frezeleme işleminin mekanik ve dinamik üzerinde çalışıldı. Freze takımlarının geometrik olarak modellenmesinin amacı, endüstride geliştirilmiş takım tasarımı ve işlem planlaması için farklı parmak freze takımlarını kapsayabilen bir frezeleme simülasyon sisteminin tasarlanmasıdır. Bu çalışma ile, yüksek verimlilik ve uygun parça kalitesi elde edilmesinde önemli olan talaş kalınlığı, kesme kuvvetleri, moment, güç, takım titreşimleri, parça titreşimleri, ve tırlama titreşimlerinin meydana gelmediği kesme hızı ve kesme derinlikleri tayin edilebilmektedir.

Frezelemedeki kesme kuvvetleri, kesme ağzı geometrisine, iş parçası malzeme özelliklerine, kesme hızına, ilerlemeye, kesme derinliğine ve kesme genişliğine bağlıdır. Kendi eksenini etrafında dönen ve trokoid bir yörünge üzerinde hareket eden freze takımının gerçek kinematiği kullanılarak takımın kesme ağzı üzerindeki bir nokta tarafından kaldırılan gerçek talaş kalınlığı analitik olarak modellenmektedir. Kesme kuvvetleri talaş kalınlığına bağlı olduğu gibi, malzeme özellikleri ile ve takım geometrisine de bağlıdır. Kuvvetler kesme ağzı boyunca toplanarak, iş parçasında ve iş milinde oluşan toplam kuvvetler bulunabilir. Bu çalışma ile ayrıca, havacılık ve kalıp sanayiinde kullanılmakta olan değişken açılı, plaketsel, helisel küresel kafalı ve helisel konik küresel kafalı parmak frezeler gibi kompleks geometriye sahip takımların parametrik tasarımında imkan verilmektedir.

Bu çalışmada üç ana amaç vardır, bunlar:

- Her türlü parmak freze takımı için genelleştirilmiş geometrik modelin geliştirilmesi, mekanik ve dinamik açılarından incelenmesi,

- Plaketli takımlar için genelleştirilmiş geometrik modelin geliştirilmesi, mekanik ve dinamik açılardan incelenmesi,
- Değişken taksimatlı takımların tırlama titreşimleri meydana getirmeyecek şekilde analitik yöntemle tasarımı ve tırlama kararlılık eğrilerinin analitik olarak hesaplanması.

Genelleştirilmiş parmak freze dış geometrisi zarf formu APT ve CAD/CAM programlama sistemleri tarafından kullanılan parametrik gösterime benzer bir yöntemle tanımlanmıştır. Takım üzerindeki kesici ağız koordinatlarının formülasyonu geliştirildi. Verilen geometrik modelin kullanımı ile, çok çeşitli parmak freze geometrilerinin tasarlanabildiği gösterilmiştir. Modellenen kesme ağızı değişen kesme sabitlerine sahip küçük parçacıklara ayrıldı. Uygulama örnekleri olarak, helisel küresel başlı parmak freze, konik helisel parmak freze ve köşesi yuvarlatılmış takımları kapsayan çalışmalar verilmiştir. Geliştirilen modelin, tanımlanan zarf geometrisi içinde herhangi bir takım geometrisi için kesme kuvvetlerinin, titreşimlerin, boyutsal yüzey pürüzlülüğünün, hatta tırlama kararlılık eğrilerinin belirlenmesinde kullanılabildiği deneysel olarak da gösterildi. Altıntaş ve Lee tarafından küresel parmak freze için verilmiş olan yöntemle benzer bir teknik kullanılarak bu geometri mekanik ve dinamik açıdan incelenmiştir.

Parmak freze geometrisi, parametrik zarf geometrisi etrafına helisel kanalların sarılması ile modellendi. Parametrik helisel kanal boyunca kesme ağızı her noktasının koordinatları matematiksel olarak ifade edildi. Takım, mekanik ve dinamik açıdan incelenebilmek için küçük elemanlara bölündü. Herbir kesme noktasında talaş kalınlığı değerleri, takım ve iş parçasının yapısal titreşimleri dahil edilerek ve frezelemenin gerçek kinematikleri kullanılarak belirlendi. İş parçası ile temasta olan her kesme ağızı boyunca integrasyonla, genel bir parmak freze için kesme kuvvetleri, titreşimler, boyutsal yüzey kalitesi ve tırlama kararlılık eğrileri belirlendi. Küresel başlı, helisel konik küresel başlı ve köşesi yuvarlatılmış parmak freze için hesaplanan ve ölçülen kesme kuvvetleri, yüzey pürüzlülüğü ve kararlılık eğrileri verilerek genelleştirilmiş parmak freze analizinin uygulanabilirliği gösterilmiştir.

Plaketli takımlar ince ve kaba talaş kaldırılması aşamalarında yaygın olarak kullanılmaktadır. Plaket geometrileri ve takım gövdesi üzerindeki plaketlerin dağılımı endüstrideki uygulamalara bağlı olarak farklılıklar gösterir. Plaketli takım geometrisi için genelleştirilmiş matematiksel model, frezelemedeki kesme kuvvetlerinin, titreşimlerinin, boyutsal yüzey kalitesinin ve kararlılık eğrilerinin hesaplanabilmesi amacıyla geliştirildi. Kesici ağız geometrisi her plaket için yerel koordinat sisteminde tanımlandı ve takım genel koordinat sistemi kullanılarak takım gövdesi üzerinde konumlandırılmıştır. Kesme ağızı konumları matematiksel olarak tarif edildi ve kesme bölgesi boyunca talaş kalınlığı değişiminin belirlenmesinde kullanıldı. Her plaket prizmatik, üçgensel dış bükey vb. veya matematiksel olarak tanımlanabilen farklı bir geometriye sahip olabilir. Plaket merkezlerinin koordinatları takım gövdesi merkezine göre belirlenerek, her plaket takım gövdesi üzerine matematiksel olarak yerleştirilebilir. Plaketler takım gövdesi üzerinde döndürülerek, her plaket farklı eğim ve eksenel talaş açılara sahip olacak şekilde yerleştirilebilir. Ağızın her noktasında kesme işlemi mekanik ve dinamik açıdan incelendi ve temas alanı boyunca integre edilerek, frezeleme işleminin herhangi bir plaketli takım için hesaplanabildiği gösterilmiştir. Plaketli takım modeli örneklerin analizi ile deneysel olarak doğrulandı. Budak ve arkadaşları tarafından verilmiş olan ortalama kesme

sabitleri metodu kullanılarak, Z eksenı boyunca yeni bir deęiřken kesme sabitleri modeli, plaketli takımlara uygulandı. Plaketli takımların dinamik aıdan incelenmesi, daha nceden Altıntař ve Budak tarafından nerilen analitik zm ve zamana baęlı zmlerin kullanılmasıyla yapılmıřtır.

alıřmada ayrıca frezeleme sırasında meydana gelen titreřimler de gz nne alınmıřtır. Takımın yapısal dinamik modeli ok serbestlik dereceli bir sistem olarak takım ucundan modellendi. Esnek takımın hareket denklemleri talař kalınlıęının belirlenmesine dahil edildi. Frezeleme iřlemi sırasında ortaya ıkan yzey kalitesi ve kesme kuvvetleri zamana baęlı olarak hesaplandı. İř parası ve takım titreřimleri, iř parası ve tezgahın dinamik davranıřları ve kesme kuvvetlerine gre hesaplandı. Titreřimlerin stel olarak bymesi analiz edilerek kesme iřleminin kararlılıęı arařtırıldı. Tırlama titreřimleri veya tırlama kararlılıęı, dinamik frezelemenin lineer analizinin yapılması ve analitik olarak zdeęer probleminin zmlenmesiyle hesaplandı. Kararlı kesme derinlikleri ve kesme hızı sınır izgisini karakterize eden kritik tırlama kararlılık sınırı analitik olarak belirlendi. Analitik zmler zamana baęlı zmlerle ispatlandı.

Tezde ayrıca, daha nce Altıntař ve Budak tarafından verilmiř olan sabit taksimatlı parmak freze kararlılık eęrisi zm geliřtirilerek, deęiřken taksimatlı freze takımları iin kararlılık eęrilerinin analitik olarak belirlenmiřtir. Bu yntem, tırlama kararlılıęı ifadesinde kesme sabitlerini, kesme aęız sayısını ve tezgah zerine takılan takımın transfer fonksiyonuna ihtiya gstermektedir. Kararlılık, zamanla deęiřen, yne baęlı kesme sabitlerinin, zamanla deęiřmeyen sabitlere dnřtrlmesiyle formle edilmiřtir. Eřit taksimatlı takımlardaki sabit kesme gecikmesi, deęiřken taksimatlı takımlar iin ok sayıda deęiřken gecikmesine dnřtrlmřtr. Kararlı kesme derinlięi, kararlılık eřitlięinden bulunur, oysa kesme hızı, kesmedeki faz farklarından bulunmaktadır. nerilen yntem ok sayıda kesme testi ve zamana baęlı simülasyonlarla doęrulanmıřtır. alıřmada, Shirase ve Altıntař'ın deęiřken taksimatlı takımlar iin talař yk ve kesme kuvvetleri statik modeli kullanılmıřtır.

Deęiřken taksimatlı takımların analitik zmlerinin pratik kullanımı, tırlamanın ortaya ıkmadıęı kesme derinlięi sınırını olduka ykselten en uygun kesici aęız tasarımı uygulaması ile gsterilmiřtir. Farklı aęız aılı bir takımın tasarlanmasındaki ana fikir, sabit taksimatlıdaki durumla karřılařtırıldıęında yksek kesme derinliklerine ulařabilen yzey dalgaları arasında farklı faz aısı oluřturulmasıdır. Taksimat aıları, verilen bir devir sayısı iin kesme derinlięini maksimum yapacak bir řekilde seilmiřtir. Geliřtirilen yntem, verilen tezgah-iř mili-takım grubu dinamikleri iin minimum faz farkını oluřturan ve gz nne alınan kesme hızında mmkn olan btn aęız aılarını vermektedir.

5000 dev/dak 'daki kesme hızında drt aęızlı bir takımla Al356 malzemesinin kesilmesi rneęi takım ve iř parası iin optimum takım mrn veren deęiřken taksimatlı takımların tasarımı yapılarak 90°-90°-90°-90° yerine 70°-110°-70°-110° bulunmuřtur. iin yapıldı. Sonuta, kararlı maksimum kesme derinlięi, sadece takım aılarının deęiřtirilmesiyle aynı kesme řartları altında,  katından fazla arttırılmıřtır. Bu iyileřme, deneylerle ve zamana ve frekansa baęlı zmlerle ispatlanmıřtır.

Bu tezde verilen simlasyon ve matematiksel modellerin hepsi deneysel olarak doęrulanmıřtır. Kesme kuvvetleri, takım ve iř parası titreřimleri, kararlılık eęrileri, tırlama frekansları ve yzey przllę profilleri gibi deęerleri, frekansa baęlı

analizler ve zamana bağılı çözümlerden gerçeğe yakın olarak elde edildi. Frezeleme simülasyonlarının giriş ve çıkış değerlerinin ölçülmesinde bir lazer yer değiştirme algılayıcısı, ivme ölçücüler, kuvvet ölçücüler, darbe algılayıcı çekiçler, bir mikrofon ve bir lazer yüzey tarayıcısı kullanılmıştır. Deneysel sonuçlarla simülasyon sonuçları, dikkatlice kontrol edilen şartlar altında bir birine oldukça yakın çıkmıştır. Bu tezde verilen teorileri doğrulamak için bütün çalışma boyunca ikibinden fazla kesme testi yapılmıştır. Doğru sonuçların alınabilmesi için bazı kesme testleri defalarca tekrar edilmiştir. Yanlış değerler veren bazı test sonuçları tezde verilmemiştir. Tezde geliştirilen teorik çalışmanın simülasyonlarla birleştirilmesi imalat hızını artırmak üzere en uygun kesme şartlarının belirlenmesinde çok yararlı olacaktır.

Modeller için bir bilgisayar programı hazırlanmış ve British Columbia Üniversitesi – İmalat Otomasyonu Laboratuvarı'nda araştırmayı maddi olarak destekleyen çeşitli firmalara (Boeing, General Motors, Pratt&Whitney) verilmiştir. Bu firmalardaki kullanıcılardan alınan tavsiye ve eleştiriler araştırmaya dahil edilmiştir.



1. INTRODUCTION

Milling operations are an important part of the many manufacturing processes in industry. Cylindrical end milling, ball end or complex end milling for die machining, taper ball end milling of aircraft jet engine turbine blades, indexable end milling, and face milling of automotive components are typical milling operations used in industry. The cutters used in these operations usually remove excess amounts of material from a single workpiece. The cutting geometry in most of these milling operations is very complex due to the cutting edge geometry. Furthermore, most parts usually have tight dimensional tolerances, which have to be satisfied during machining. From the manufacturer's point of view, productivity and quality of a finished workpiece, and the cost of the cutting process are all-important factors. Chatter is a significant problem in rough machining, where a larger depth of cut is involved. In order to maintain stability during such cutting operations, the vibration map of the cutting process has to be identified for the given cutter geometry, workpiece material, cutting conditions and dynamics; this is necessary for a productive cutting operation. In this thesis, methods have been developed for optimizing the cutting process for any tool geometry, based on the mechanic and dynamic behavior of the machine and the workpiece system. These methods allow high production rates to be reached without chatter vibrations, while satisfying the end product quality and tolerance constraints.

General tool geometries for solid, indexable and variable pitch cutters, and the associated mechanics and dynamics are studied in this work. Mathematical models for cutter geometries and cutting edge positions on the cutter are developed for different cutter types. Stability lobes are calculated in frequency and time domain to find the optimum spindle speeds and depths of cut that maximize the production while maintaining stability of the cutting operation. Variable pitch cutter design techniques were developed to shift the stability pockets in order to enable higher depths of cut to be reached at a specified spindle speed while maintaining cutting

process constraints such as tool wear. A number of different cutters and workpiece materials were used to verify the theory. Close agreement between simulation and experimental results were recorded.

In Chapter 2, a review of the existing research in milling mechanics and dynamics is presented. Kinematic models of milling given in the literature are discussed. Generally, these methods are based on the motions of the cutter and workpiece together, and the chip thickness. The geometric models for solid and inserted cutters presented in the literature are reviewed. Different milling force models developed by different authors for the dynamic analysis of milling are discussed. The existing literature on chatter stability is also presented in this chapter. Finally, geometric, kinematic and dynamic analyses of variable pitch cutters are given.

In Chapter 3, the mechanics of milling are described in detail. A milling operation is a periodic process, which is dependent upon spindle speed, number of flutes and pitch angles. The chip thickness geometry along the depth of cut and cutter rotation angle is defined. Identification of cutting coefficients is discussed from mechanics and mechanistic models' points of view. Orthogonal to oblique cutting transformation models are classified as metal cutting mechanics models. On the other hand, exponential force and linear edge force models are defined as mechanistic models. The milling dynamics are presented in time and frequency domains separately. The chip and surface finish generation mechanisms are included in the time domain section.

Generalized tool geometry for solid cutters is described in Chapter 4. The geometry of the envelope is defined in a similar way to the parametric representations used by APT (Automatically Programmed Tools) and CAD/CAM (Computer Aided Design/Computer Aided Manufacturing) software systems for general tool geometry. The cutting edge geometry is created on the envelope by wrapping a helix around the cutter. A methodology for cutting coefficient identification is given for different cutter geometry and workpiece material cases. Average cutting coefficients were obtained for bull nose cutter and workpiece materials Aluminum alloy Al7075, Titanium Ti_6Al_4V and Cast iron GGG70. The orthogonal cutting coefficients were obtained from the average cutting coefficient model. The simulation and

experimental results are compared and chatter stability lobes are verified by experimental results.

The geometric modeling of inserted cutters is presented in Chapter 5. The cutter geometry and insert geometry are defined separately. A point on the insert is defined with respect to the insert center location and orientation. The variable cutting coefficients were obtained along the depth of cut. The mechanics and dynamic models of milling used in this particular work are summarized and the details of the process models are referred to the relevant literature. Prediction of the cutting forces, vibrations, dimensional surface finish, and chatter stability were demonstrated using two different inserted helical end mills. Simulation and experimental results are presented.

The objective of geometric modeling of milling cutters is to design a milling simulation system, which can handle a variety of different end mills for improved cutter design and process planning for industry.

Optimum design and analysis of variable pitch cutters is presented in Chapter 6. Analytical stability lobe prediction for milling cutters with variable pitch angles is achieved by modifying the stability lobe prediction approach for regular pitch cutters. The stability is formulated by using a transformation of time varying directional coefficients to frequency domain. The chatter free axial depth of cut is obtained from the solution of the eigenvalues in the stability expression. The spindle speed is identified from regenerative phase delays. The stability lobes are verified with the time domain simulations and experimental cutting tests. For the practical use of the variable pitch cutters, optimum variable pitch cutter design methods are developed in frequency domain, which gives a large chatter free depth of cut. Two different approaches are suggested. In the first one, different possibilities of pitch angles are scanned to obtain the highest stable depth of cut at a given spindle speed. The second approach is an empirical algorithm that uses a given chatter frequency. An application is presented for a given workpiece material, tool geometry and set of system dynamics. The higher order terms in the stability analysis were also considered. The surface roughness properties of different cases were studied. The simulation results were verified with the results of the experimental cutting tests.

In Chapter 7, the thesis is concluded with a brief summary of contributions and future research recommendations.

In this study, there are three main objectives, which are;

- Developing generalized geometric model for arbitrary end mills and investigation of mechanics and dynamics,
- Developing generalized geometric model for inserted (indexable) cutters and investigation of mechanics and dynamics,
- Analytical prediction of chatter stability lobes as well as an analytical design technique for chatter free variable pitch cutters.

In the generalized geometric model for arbitrary end mills, the envelope is defined in a similar manner to the parametric representation used by APT and CAD/CAM softwares. Mathematical models of the envelope geometry and cutting edge positions were developed. These models were presented for each zones of cutter body in general cases. Mechanics and dynamics of this geometry were investigated using a technique similar to the one given by Altıntaş and Lee for ball end milling.

In the generalized geometric model for inserted cutters, a model for the definition of cutting edge positions and design models of the multiple inserted in a matrix form were developed. The cutting edge positions were defined by a vector, which extends from cutter body center to the cutting edge at any given level z . In this model, two different inserts were considered. A new variable cutting coefficient model along the z direction was applied to the inserted cutters by using average cutting coefficients method, which is presented by Budak et. al. The dynamic investigations of inserted cutter were carried out using time domain and analytical solutions proposed by Altıntaş and Budak.

A solution method based on the analytical stability lobes for variable pitch cutters, was developed by modifying uniform pitch cutters stability lobe solution, which is previously developed by Altıntaş and Budak. The static model of cutting forces and chip load generation model for variable pitch cutters are given by Shirase and Altıntaş. The stability formulation developed previously for regular pitch cutters is extended to irregular pitch cutters in this study. In addition, an analytical method of

selecting optimum pitch angles was developed. This method gives all possible pitch angles at a specified speed range, where it is tuned to provide minimum regeneration for a given set of machine tool dynamics, i.e. spindle-cutter assembly.



2. LITERATURE SURVEY

2.1 Overview

Mechanics and dynamics of end milling are highly interdisciplinary. Metal cutting, structural dynamics and stability of chatter vibration topics are all involved in the analysis. In this chapter, a literature review of the research in milling that pertains to this thesis is presented. Kinematics of milling is first discussed, including models of cutter and workpiece motion and chip thickness. Next, a summary of milling tool geometry is given. Then milling force models and chatter stability analysis are discussed. Finally, previous variable pitch cutter research found in milling literature is presented.

2.2 Kinematics of Milling

Milling is an intermittent multi-point cutting operation in which a rotating cutter feeds into the workpiece. Unlike the steady state turning operation, which has a constant chip thickness, milling has a variable chip thickness during each spindle revolution. Milling operations can be classified into two categories: peripheral and face milling. In peripheral milling there are two different cases, which are up and down milling (or climb milling) (Figure 2.1).

In up milling, the chip thickness increases as the cutting edge enters into the workpiece. It is the opposite in down milling, where the chip thickness decreases as the cutting edge moves in the workpiece. The cutting edge moves against the feed direction in up milling, in contrast to the down milling operation. The cutting forces are proportional to chip thickness. Earlier milling mechanics research were about chip formation mechanisms and required spindle power estimation [1-7]. The actual shape of an undeformed chip is formed as the cutting edge traverses a trochoidal path

proven by Martellotti [8,9]. Due to sudden changes in the chip thickness, the cutting forces jump from a small value to a high one in down milling.

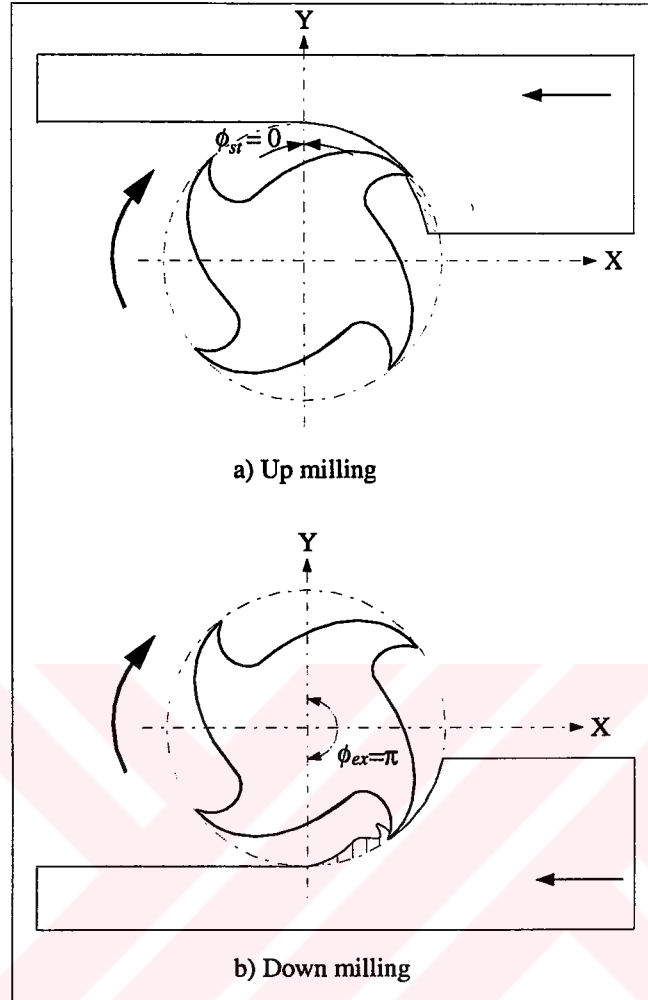


Figure 2.1. Up and down milling for peripheral milling.

Assuming that the cutter and workpiece are stationary during one tooth passing period, the varying chip thickness can approximately be written as,

$$h(\phi) = s_f \sin \phi$$

where s_f and ϕ are the feed rate and the immersion angle of the cutting edge. This sinusoidal equation is very commonly used in literature to calculate the cutting forces. For a more accurate solution, Altıntaş and his co-workers [10-12] used a simulation program that digitizes the cutting surface and calculates the exact chip thickness by considering the true kinematics of dynamic milling. The cutting forces, vibrations and chatter depend on the chip thickness. Hence, the accuracy of chip

thickness estimation is very important in order to obtain reliable predictions of the other physical values.

2.3 Tool Geometry

Milling cutters usually have more than one cutting edge and helical flute. There are two main types of milling cutters: solid and inserted. The solid cutters are usually one piece and flutes are created by grinding helical grooves on to the cutter body. The inserted cutters have a main body and hard inserts mounted on the body. The main difference between solid cutters and inserted cutters is the cutting edges. Cutting edges of solid cutters are a part of the cutter and they are continuous, while inserted cutter edges are not. This is very important in grinding of cutters since solid cutters dimensions change with the grinding process. However, for inserted cutters, the same dimensions can be realized repeatedly by replacing worn inserts with new ones. Some of the standard cutting tools can be seen in Figure 2.2. Cutter geometry depends on the workpiece cutting profile, the material type of the workpiece, and the type of milling operation such as rough or finish milling. Solid carbide tools are usually used for low chip load and finish milling, whereas the inserted (indexable) cutter can remove more material with multiple inserts.

Cutters have two main geometries, which are the body envelope and the edge geometry. The body envelope geometry can be cylindrical, taper, ball end or bull nose. The edge geometry can be straight flute, helical or wavy cutting edge.

The cutting force directions mainly depend on tool and cutting edge geometry. Usually one cutting edge or more can cut in different points at the same time depending on the depth and width of the cut. Consequently, the total cutting forces are obtained by summing up the acting forces on the cutting edges in cut.

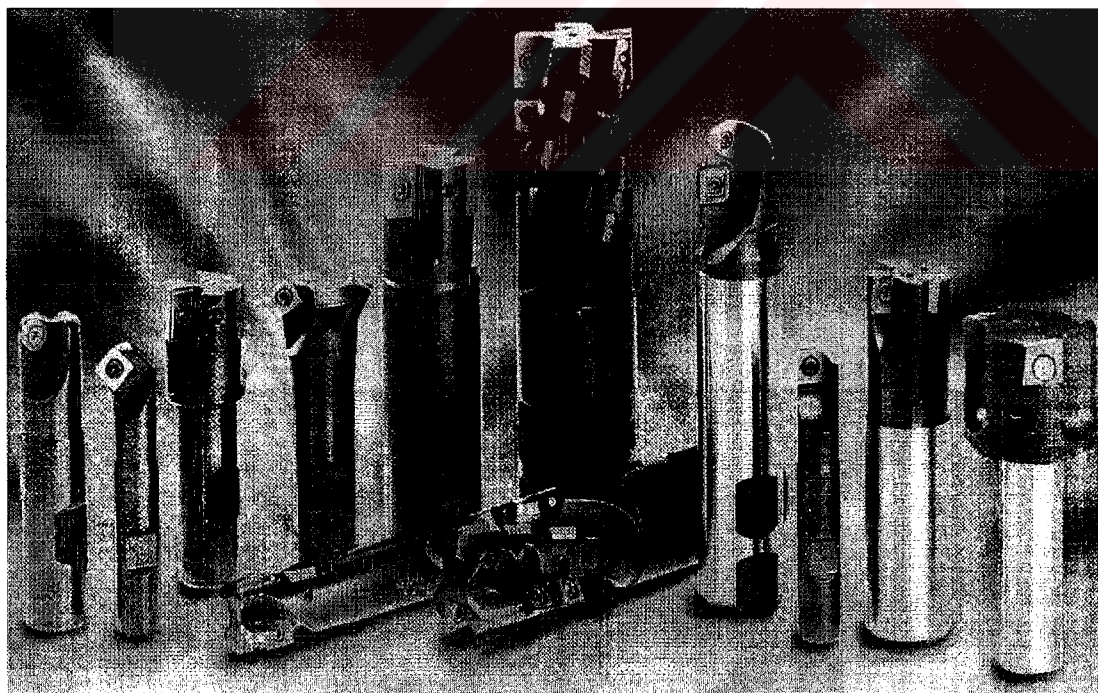
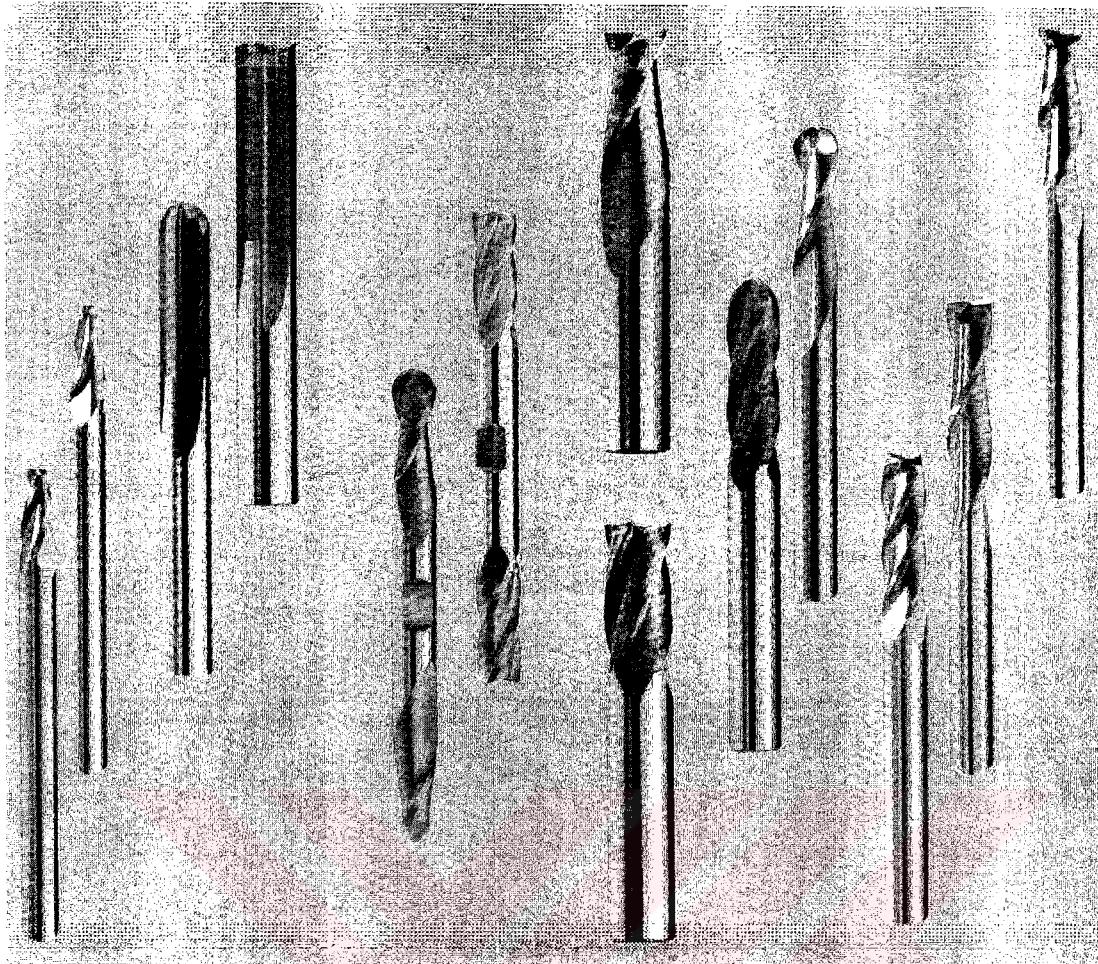


Figure 2.2. Milling solid and inserted (indexable) cutters (Kennametal standard cutting tools).

2.3.1 Solid End Mills

A variety of helical end mill geometries are used in milling operations. Simple cylindrical helical end mills are used in peripheral milling of prismatic parts. Straight and helical ball end mills are widely used in machining sculptured dies and surfaces of the parts used in aerospace industry. Bull nose cutters produce a surface finish with a rounded inside edge. Tapered helical end mills are used in five axis machining of jet engine compressors, and form cutters are used to open complex profiles such as turbine blade carrier rings. A classical approach in the literature has been to develop milling mechanics models for each cutter shape, e.g., mechanics and dynamics models were developed individually for face [13], cylindrical [14-18], ball end [19-22] and tapered ball end mills [22]. Ehmann et al. [23] summarized the past researches in mechanics and dynamics of milling, which were mainly specific to standard end milling and face milling cutters. On the other hand, a generalized mechanics and dynamics model that can be used to analyze any cutter geometry is required for analyzing a variety of end mill shapes used in manufacturing industry.

The geometry of a milling cutter has two major components. The envelope, which is the outer geometry of the cutter, is used in generating NC tool paths on CAD/CAM systems. This envelope is also used in identifying the intersection between the cutter and the workpiece geometry, which is required in simulating the material removal process and in dynamically updating the blank geometry for graphical NC tool path verification [24-26]. The second component is the cutting edge geometry along the flutes, which is required in analyzing the mechanics and dynamics of the milling process. The prediction of the cutting forces and vibrations requires these coordinates, as well as rake, helix, and clearance angles of the cutting edge point on the flute [17].

2.3.2 Inserted (Indexable) Cutters

Inserted face and end milling cutters are widely used in industry. Face milling cutters have evenly or unevenly spaced inserts, and are used in removing excess material from the face of parts such as transmission boxes, engine blocks or machine tool columns and beds. Inserts are placed in both radial and axial directions on the indexed end mills. Indexed cutters are used in removing massive amounts of material

from the periphery of parts such as aircraft spars, airframe support structures and rough pocketing of dies and molds. The distribution of inserts may be regular or arbitrary depending on the application. In order to improve the surface finish quality, maintain chatter stability and balance the cutting force on the cutter body, inserts with various shapes can be placed at various locations on the milling cutter body.

The industry requires a generalized mathematical model, which allows the analysis of all inserted milling cutters used in practice [27]. However, the research in the past mainly focused on the modeling of mechanics and dynamics of special cutter geometry. Fu et al. [13] provided a comprehensive modeling of inserted face-milling cutters. They modeled rectangular inserts with corner radius, and introduced equivalent lead and rake angles, which are related to experimentally identified cutting constants. Fu et al.'s work [13] became a basis for a number of successive mechanistic models in milling, and led to improved static and dynamic models [28]. The mechanistic approach allowed the integration of milling mechanics into the optimization of cutting conditions during tool path planning on CAD/CAM systems [25,26,29]. The cutting forces are assumed to be proportional to the instantaneous chip area in most face milling cases, but the variation of cutting pressure along the curved cutting edge is also considered in some of the more rigorous models [30]. Spieawak [18] presented an analytical method integrating the spindle tilt, runout and eccentricity in identifying the chip thickness. Choudhury and Mathew investigated the effect of non-uniform insert pitch for face milling [31]. However, a generalized handling of the mechanics and dynamics of inserted end mills has not received similar attention. The majority of the research up to now focused on the accurate identification of chip load in static and dynamic milling and identification of cutting constants for a single insert.

2.4 Modeling of Milling Forces

Milling cutting force models can be classified into two categories, which are *mechanistic* and *mechanics of milling* models. In *mechanistic models*, cutting coefficients contain cutter geometry, helix angle, rake angle, relief angle, workpiece material, uncut chip thickness, chip width and other variables. Hence, the cutting force coefficients will be valid for a specific cutter geometry and workpiece material.

This *model* has been studied by many researchers in milling [14,32-34]. In *mechanics of cutting models*, coefficients are functions of shear stress, shear angle and friction angle [17,35]. This model uses an orthogonal cutting database in determining force coefficients without calibration tests. Therefore, it is very useful for general application to different cutters. However, orthogonal cutting database preparation takes considerable effort and time proportional to the coverage of the database. This model is very practical in optimizing the tool geometry in process planning stage.

The simplest milling force model is the average rigid force model, which assumes that the average power consumed, torque, tangential cutting force and the dimensional error on the machine surface are proportional to the material removal rate [36,37]. Sawin [3] and Salomon [4] developed the first expressions for pulsating cutting forces in milling. A relationship was found based on the assumption of a cylindrical straight tool cutter that cutting pressure varies as an exponential function of chip thickness.

$$F_t = K_s ah \quad (2.1)$$

where K_s is cutting pressure, a is chip width and h is the chip thickness. K_s is $K_s = Ch^x$, where C and x are determined experimentally. Sabberwal [38] and Koeigsberger [39] developed the first complete model in this group by extending this model to predict cutting forces with helical cutters.

The average cutting pressure coefficient and constant edge force coefficients have also become widely used [40]. Tlusty and McNeil [41], Kline et al. [33], Sutherland and Devor [14] and Montgomery and Altıntaş [10] contributed in this area. The model used in this thesis was originally proposed by Zorev [42] and developed by Budak and Altıntaş [17] into a practical formulation as,

$$\left. \begin{aligned} F_t &= K_{te}a + K_{tc}ah \\ F_r &= K_{re}a + K_{rc}ah \\ F_a &= K_{ae}a + K_{ac}ah \end{aligned} \right\} \quad (2.2)$$

where a , and h are depth of cut and chip thickness respectively. Subindices (c) and (e) represent shear and edge force components, respectively. K_{te} , K_{re} , K_{ae} , K_{tc} , K_{rc} and K_{ac} are force coefficients determined from experimental cutting forces in the X, Y and Z directions at various feed rates.

2.5 Chatter Stability

Productivity in end and face milling is reduced by chatter vibrations. Because of these vibrations, finished surface quality is degraded and the material has to be removed at slower rates. There are some possible ways to reduce these vibrations:

- Increase the stiffness of machine tool-part structures
- Reduce the spindle speed to increase process damping
- Select a spindle speed from stability charts
- Dynamically vary the spindle speed during the cutting process

Not all of these options are plausible all the time. Generally, once a milling machine is purchased, it is not possible to increase its stiffness. As far as the part is concerned, it may not be possible to modify the part geometry to increase its stiffness. Any reduction in spindle speed reduces the material removal rate, hence reduced feed rate. Therefore, the most practical and efficient option to avoid chatter vibrations is to select a spindle speed such that the depth of cut is a maximum and the cutting process is stable. Unlike aluminum, materials such as titanium or hardened die-mold alloys cannot be cut at high speeds without accelerating the tool wear. Thus, in selecting a suitable spindle speed for such material types, tool wear needs to be taken into account as well as chatter vibrations. Radulescu [43] and Altıntaş [44] noted that at the present technological level it is not realizable to avoid chatter vibrations by using spindle speed variations, which requires spindle drives with large frequency bandwidth and torque delivery. Therefore, this study proposes a novel technique for the design and analysis of variable pitch cutters, avoiding chatter vibrations in a given depth of cut and feed rate.

Ever since the pioneering works of Thusty [45,46] and Tobias [47] in this area in 1960's, chatter vibrations have been the main topic of many researches. In the model developed by Tobias, the cutting forces vary because of the dynamic variations in chip thickness and cutting velocity. The variations in chip thickness occur due to either outer modulations (or wave removing), which are surface modulations left behind in the previous pass, or inner modulations (or wave cutting), which are the vibrations of the tool towards the cut surface. The cutting velocity variations occur because of tool vibrations in the cutting direction. According to Tobias, the total damping in the system consists of structural damping, damping due to the chip thickness and cutting velocity variations. Tobias based the stability criteria on the total damping of the system. In Thusty's stability analysis, he showed that the cutting system has better stability against mode coupling chatter mechanism. In his model, regeneration mechanism is the most important mode of the chatter. The phase difference between the outer and the inner modulations determines the amount of dynamic variations in chip thickness and is a function of cutting conditions and the dynamic characteristics of the system. He introduced a process damping mechanism generated by the flank contact.

The rotating nature, multiple cutting teeth and coupled multi degree of freedom system complicate the stability analysis of the milling process. Due to rotation of the cutter, directional coefficients (direction cosines) change periodically in each cycle. In order to overcome this problem, Thusty [45,46,48] used his orthogonal cutting-stability formula, assuming an average direction and number of flutes engaged in cut. No theoretical basis is given for these approximations. Thus, his directional coefficients were constants, calculated for the assumed average direction. The accuracy of this model is not predictable. In later studies, Thusty et al [41,49-51] and other authors [10,52-55] used time domain simulations to solve the stability problem of milling process. Although the accuracy was improved, time domain simulations were computationally prohibitive. In order to obtain a milling stability diagram, many simulations needed to be performed at different spindle speeds and depths of cut. The milling of a flexible work piece exhibits even bigger challenges for time domain simulations, as the dynamic properties of the workpiece change during the cut. Hence, the chatter simulations need to be repeated at different locations along the feed direction.

One of the major sources of non-linearity in chatter is the loss of contact between the cutting tooth and the workpiece. According to Tlustý [41], this type of non-linearity can be handled best in time domain simulations. However, this phenomenon is not important in predicting the initial stages of chatter [56]. In order to improve computational speed, Opitz et al [57-59] used average coefficients in their model rather than the periodic coefficients. The average values were based on the contact duration for a cutting tooth. By doing this, they were able to reduce the model to a single degree-of-freedom system.

The work of Sridhar et al [60] is the first comprehensive theoretical analysis of milling stability. The dynamic milling forces for a straight tooth cutter are formulated. Their stability algorithm is based on the evaluation of the system's state transition matrix. In [43,44,46], important developments on the dynamics of end and face milling are reported. Minis et al. [62,63] implemented the theory of periodic differential equations to milling dynamics. In their formulation, the Nyquist stability criterion is used to determine the stability limits. The algorithm evaluates the eigenvalues at increasing depths of cut, until the stability limit is reached. The Nyquist criterion is also used as the stability limit in the work of Lee et al [53,64-66].

Altıntaş and Budak [67-71] developed a novel theory, which allows analytical prediction of chatter stability in milling with regular pitch cutters. They have proven the method both experimentally and using time domain simulations. Altıntaş and Budak's method is adapted to variable pitch cutters in this thesis.

2.6 Variable Pitch Cutters

The idea of using variable pitch cutters for chatter suppression is not new. Stability of milling with irregular tooth pitch cutters was formerly studied by Slavicek [72], Vanherck [73], and Opitz et al. [74].

Slavicek [72] adopted Tlustý's regular pitch cutter theory to obtain stability lobes for face milling with irregular pitch cutters. In his work, he assumed the cutter to have an infinite radius and to possess two different pitch angles which were repeated one after the other, with the same number of total occurrences. Optimum pitch angle calculation was based on utilizing phase shift angles, such that half of the phase shift

difference between two pitch angles had to be 90 degrees, or its integer multiple, for the limit of depth of cut to reach its maximum value. For the best case, this value corresponded to twice the depth of cut reached using regular pitch cutters. Slavicek has not recommended, in this scheme, a phase shift value to yield optimum results. He only gave the optimum difference value between two-phase shift angles.

Vanherck [73], augmented Slavicek's [72] cutter design approach by increasing the number of different pitch angles that can occur repeatedly. He experimented with nonlinear pitch angle variations, as well as linear ones, and also attempted to increase stability by using nonparallel cutting edges. Similar to Slavicek, Vanherck also assumed the cutter to have an infinite radius. Using the theory he developed, he was able to obtain depths of cut more than four times, larger than the depths of cut achieved with regular pitch cutters. He recommended 90-degree phase angle to reach the maximum stability and 180 degree phase angle difference between two pitches for linear pitch evolution. He did not give any details on how to calculate nonlinear pitch cutters and nonparallel cutting edges. Varterasian [54] also reported experimental results with randomly spaced pitch angles.

Other cutter design techniques for chatter suppression include having serrated cutting edges [75] or alternating helix angles [73,76]. Wu investigated the selection of optimal pitch angles using numerical simulation [77]. Tlustý, Ismail and Zaton [78] used chatter theory for orthogonal cutting and numerical simulations in analyzing the influence of variable pitch angles, serrated edges and alternating helix angles in stability. However, none of the past work treated chatter stability of variable pitch cutters theoretically. This is mainly due to the lack of analytical modeling of chatter stability in milling, which has time varying excitation amplitude and directions due to periodicity and intermittent behavior of chip formation. Budak and Altıntaş [67-71] developed analytical prediction of chatter stability in milling with regular pitch cutters.

In this study, the chatter stability theory developed by Altıntaş and Budak [67-69] is extended to cover variable pitch cutters. The dynamics of milling with variable pitch angles have been formulated, and an analytical technique for predicting the stability lobes has been derived. Furthermore, an analytical identification of optimal pitch angle distribution between the teeth is presented. The predicted stability lobes have

been verified with an extensive amount of milling tests. It has been shown that the material removal rate, i.e. the productivity, at a prescribed cutting speed range can be increased significantly by selecting a cutter with analytically identified pitch angles.



3. MECHANICS OF MILLING

3.1 Introduction

The milling operation is a periodic process that changes with spindle speed, number of flutes and pitch angles. In a milling operation, a usually prismatic workpiece is clamped on the machine table and moves towards the cutter, which is held in a rotating spindle with a chuck. The position of the tool with respect to the workpiece determines the radial immersion type, which can be slotting, up or down milling. If the cutter diameter is entirely in the workpiece, the cutting type is slotting.

The chip thickness changes with immersion angle and feed rate. The cutting forces are related to chip thickness and cutting edge geometry. In milling, the cutting process is a multi-point process. Thus, the cutting operation is discontinuous for every immersion and every axial level of cut according to cutting edge geometry.

3.2 Mechanics of Milling

In the peripheral milling, the feed direction is along the positive X and the cut surface is perpendicular to the Y axis. The Z direction is along the cutter axis and its reference point is at the tool tip. Chip generation and also cutting forces highly depend on cutter immersion angles into the workpiece, feed rate, depth of cut, number of flutes, cutter envelope geometry, helix angle, cutting edge geometry, spindle speed and the direction of rotation. The cutting forces on the cutting edge appear in tangential, radial and axial directions. The cutting force directions can be seen in Figure 3.1.

The cutting forces in X, Y and Z directions are given as follows,

$$\left. \begin{aligned} F_{xj}(\phi) &= -F_{tj} \cos \phi - F_{rj} \sin \phi \\ F_{yj}(\phi) &= F_{tj} \sin \phi - F_{rj} \cos \phi \\ F_{zj}(\phi) &= F_{aj} \end{aligned} \right\} \quad (3.1)$$

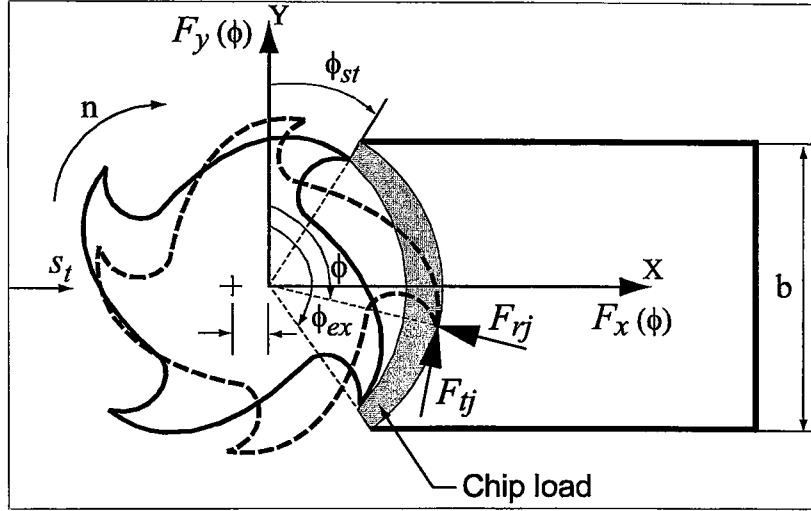


Figure 3.1. Milling cutting force directions and immersion.

The tangential, radial and axial cutting force directions are different for every immersion angle. During the milling operation, only parts of the cutting edges are engaged in cutting along the cutter axis.

The total feed, normal and axial cutting forces can be formulated as,

$$\left. \begin{aligned} F_x &= \sum_{j=1}^N F_{xj}(\phi_j) \\ F_y &= \sum_{j=1}^N F_{yj}(\phi_j) \\ F_z &= \sum_{j=1}^N F_{zj}(\phi_j) \end{aligned} \right\} \quad (3.2)$$

The cutting edge engages with the workpiece and a cutting force is generated when the immersion angle is $\phi_{st} \leq \phi_j \leq \phi_{ex}$. Cutting torque and power can be obtained using tangential force.

$$\begin{aligned}
T_c &= \frac{D}{2} \cdot \sum_{j=1}^N F_{tj}(\phi_j) \\
P_t &= (\pi D n) \cdot \sum_{j=1}^N F_{tj}(\phi_j)
\end{aligned}
\tag{3.3}$$

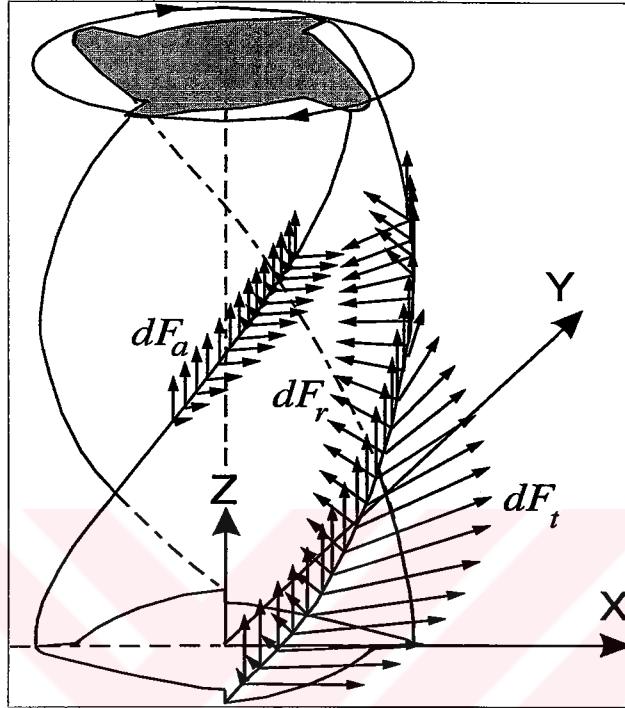


Figure 3.2. Cutting forces along the cutting edges [79].

3.3 Identification of Cutting Constants

Cutting forces act on the cutting edge. In milling, the cutter envelope and cutting edge geometry are usually very complex. In the mechanistic model, cutting coefficients include the effects of the tool geometry and cutting edge geometry. This method is usually more practical, however, these cutting coefficients can only be used for that particular cutter geometry. In the mechanics of milling approach, the effects of cutter and tool geometry are not explicitly included in the cutting coefficients.

Armarego et al. [80-83] used the oblique cutting model for milling force prediction. His method uses an orthogonal cutting database in determining the milling force coefficients without a calibration test.

3.3.1 Orthogonal to Oblique Cutting Transformation

In orthogonal cutting operation, cutting velocity is perpendicular to the cutting edge and it is uniform with width of cut. Therefore it can be investigated as a two-dimensional operation. In an oblique cutting operation, the cutting velocity is not perpendicular to the cutting edge and it is a three-dimensional cutting operation. The oblique cutting operation has very complex geometry. To simplify the three-dimensional oblique cutting geometry, the geometrical and kinematics transformations are usually used from two-dimensional orthogonal cutting geometry [17]. Illustration of orthogonal and oblique cutting geometries can be seen in Figure 3.3.

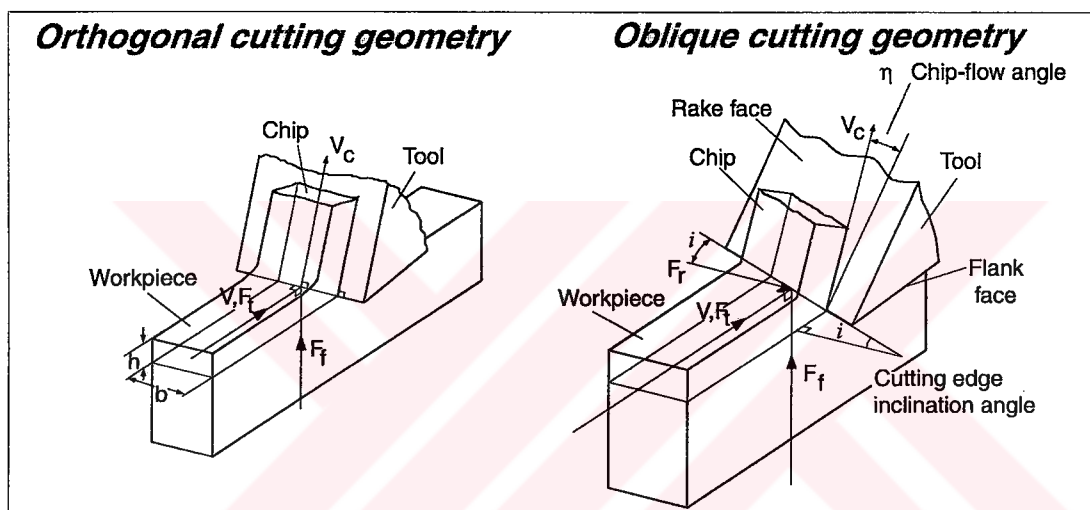


Figure 3.3. Orthogonal and Oblique cutting geometry [79].

In orthogonal cutting, the main parameters are shear stress (τ) on the shear plane, shear angle (ϕ) and friction angle (β). The shear angle is an angle between the direction of the cutting speed (V) and the shear plane. The shear plane is assumed to be a very thin plane. Shear stress is assumed to have uniform distribution on the shear plane. The friction angle is defined on the rake face. The cutting edge is assumed to be sharp and without any chamfer or radius. There are two cutting force components, which are feed force and cutting force. The feed force is in the direction of uncut chip thickness. The tangential cutting force is in the direction of the cutting velocity. The shear stress can be found as follow [56],

$$\tau_s = \frac{F_t \cos \phi_c - F_f \sin \phi_c}{b \frac{h}{\sin \phi_c}} \quad (3.4)$$

where b is the width of cut and h is the uncut chip thickness. The shear angle is found from geometry as a function of rake angle (α_r) and the chip compression ratio (r_c) or chip length ratio (r_l).

$$\phi_c = \tan^{-1} \left[\frac{r_c \cos \alpha_r}{1 - r_c \sin \alpha_r} \right] = \tan^{-1} \left[\frac{r_l \cos \alpha_r}{1 - r_l \sin \alpha_r} \right] \quad (3.5)$$

The chip ratios are,

$$\left. \begin{aligned} r_c &= \frac{h}{h_c} \\ r_l &= \frac{l}{l_c} \end{aligned} \right\} \quad (3.6)$$

where h , h_c are uncut chip thickness and chip thickness, respectively. Similarly l , l_c are uncut chip length and chip length, respectively. The friction angle can be found from the tangential and feed forces as follows,

$$\beta_a = \alpha_r + \tan^{-1} \left(\frac{F_f}{F_t} \right) \quad (3.7)$$

Using the resultant the force, the tangential and feed forces can be found as follows,

$$\left. \begin{aligned} F_t &= F \cos(\beta_a - \alpha_r) \\ F_f &= F \sin(\beta_a - \alpha_r) \end{aligned} \right\} \quad (3.8)$$

The cutting forces (F_t , F_f) can be found as a function of chip thickness, width of cut, shear stress, shear angle, friction angle and rake angle.

$$\left. \begin{aligned} F_t &= bh \left[\tau_s \frac{\cos(\beta_a - \alpha_r)}{\sin \phi_c \cos(\phi_c + \beta_a - \alpha_r)} \right] \\ F_f &= bh \left[\tau_s \frac{\sin(\beta_a - \alpha_r)}{\sin \phi_c \cos(\phi_c + \beta_a - \alpha_r)} \right] \end{aligned} \right\} \quad (3.9)$$

Tangential (K_t) and feed (K_f) cutting force coefficients are defined as follows,

$$\left. \begin{aligned} K_t &= \tau_s \frac{\cos(\beta_a - \alpha_r)}{\sin \phi_c \cos(\phi_c + \beta_a - \alpha_r)} \\ K_f &= \tau_s \frac{\sin(\beta_a - \alpha_r)}{\sin \phi_c \cos(\phi_c + \beta_a - \alpha_r)} \end{aligned} \right\} \quad (3.10)$$

The total cutting forces can be found as a function of chip thickness (h), width of cut (b) and cutting constants (K_{tc}) and (K_{fc}).

$$\left. \begin{aligned} F_t &= K_{tc}bh + K_{te}b \\ F_f &= K_{fc}bh + K_{fe}b \end{aligned} \right\} \quad (3.11)$$

Cutting coefficients K_{tc} and K_{fc} are the shearing components. The edge components are K_{te} and K_{fe} .

The cutting velocity vector is inclined at an acute angle i to the plane normal to the cutting edge in oblique cutting. The friction angle (β) in the normal plane is given by,

$$\tan \beta_n = \tan \beta \cos \eta_c \quad (3.12)$$

where η_c is the chip flow angle. The normal shear angle in the normal plane is given by,

$$\phi_n = \tan^{-1} \left[\frac{r_l \cos \alpha_n}{1 - r_l \sin \alpha_n} \right] = \tan^{-1} \left[\frac{r_l (\cos \eta_c / \cos i) \cos \alpha_n}{1 - r_l (\cos \eta_c / \cos i) \sin \alpha_n} \right] \quad (3.13)$$

The following equation applies to the oblique cutting process [35,83].

$$\tan(\phi_n + \beta_n) = \frac{\cos \alpha_n \tan i}{\tan \eta_c - \sin \alpha_n \tan i} \quad (3.14)$$

In oblique cutting, the cutting forces are,

$$\left. \begin{aligned} F_t &= K_{tc}bh + K_{te}b \\ F_r &= K_{rc}bh + K_{re}b \\ F_a &= K_{ac}bh + K_{ae}b \end{aligned} \right\} \quad (3.15)$$

The cutting coefficients can be expressed in terms of the variables used in the analysis of cutting mechanics as follows [35]:

$$\left. \begin{aligned} K_{tc} &= \frac{\tau_s}{\sin \phi_n} \frac{\cos(\beta_n - \alpha_n) + \tan \eta_c \sin \beta_n \tan i}{\sqrt{\cos^2(\phi_n + \beta_n - \alpha_n) + \tan^2 \eta_c \sin^2 \beta_n}} \\ K_{rc} &= \frac{\tau_s}{\sin \phi_n \cos i} \frac{\sin(\beta_n - \alpha_n)}{\sqrt{\cos^2(\phi_n + \beta_n - \alpha_n) + \tan^2 \eta_c \sin^2 \beta_n}} \\ K_{ac} &= \frac{\tau_s}{\sin \phi_n} \frac{\cos(\beta_n - \alpha_n) \tan i - \tan \eta_c \sin \beta_n}{\sqrt{\cos^2(\phi_n + \beta_n - \alpha_n) + \tan^2 \eta_c \sin^2 \beta_n}} \end{aligned} \right\} \quad (3.16)$$

Oblique cutting forces can be obtained from the orthogonal cutting process by evaluating the shear angle (ϕ_c), average friction angle (β_n) and shear stress (τ_s) from cutting tests [17,79]. The feed (F_f) and tangential (F_t) forces can be easily obtained from orthogonal cutting and the friction angle can be found using equation (3.7). From equation (3.14), the shear angle can also be calculated using the friction angle value. Finally, equation (3.4) can be used to find the shear stress. The cutting coefficients can be calculated using equation (3.16). To calculate the cutting coefficients, the following assumptions should be made:

- The normal shear angle in oblique cutting (ϕ_n) is equal to the orthogonal shear angle (ϕ_c).
- The rake angle in orthogonal cutting (α_r) is equal to the normal rake angle (α_n).
- The chip flow angle (η) is equal to the oblique angle (i) by taking Stabler's chip flow rule [75].
- For a given speed, chip load and tool-workpiece material pair, the friction angle (β_a) and shear stress (τ_s) are the same in orthogonal and oblique cutting operations.

3.3.2 Mechanistic Models

There are two different main mechanistic models: the exponential force coefficient model and the linear edge force model [84]. In the exponential model, the cutting forces are proportional to the chip thickness. Although Merchant [56] in 1944 modeled the linear relation between the chip thickness and the cutting forces in orthogonal cutting, the proportional relationship between the uncut chip thickness and the tangential milling force component was reported by Koenigsberger and Sabberwal [85] in 1961. Tlustý and McNeil [45] and Kline et al. [33] have extended this model to include a radial force component and have been widely used in milling force analysis [14,34,86,87]. Masuko [88], Albrecht [89] and Zorev [42], independently, proposed cutting models which consider the edge forces. For zero helix cutters, the linear edge force model was used by Armarego and Epp [40] in formulating the milling forces.

If the cutting force coefficients are calibrated accurately, exponential and linear edge force models give acceptable force predictions. The linear edge force model has linear force coefficients and a better physical interpretation of the cutting process. This is the advantage of the linear edge force model. However, the edge forces have to be predicted by experiment, as there is no acceptable model for the prediction of the edge forces. Also, the plough and flank edge forces may not be completely independent as suggested by the linear edge force model. However, this can be used as an acceptable first order approximation.

The exponential and linear force models were also worked on by Budak et al. [17,84]. Some explanations about these force models are provided in the next section.

3.3.2.1 Exponential Force Coefficient Model

The elemental tangential dF_{tj} , radial dF_{rj} and axial dF_{aj} cutting forces acting on the flute j of a rigid end mill are shown in Figure 3.4, and given by

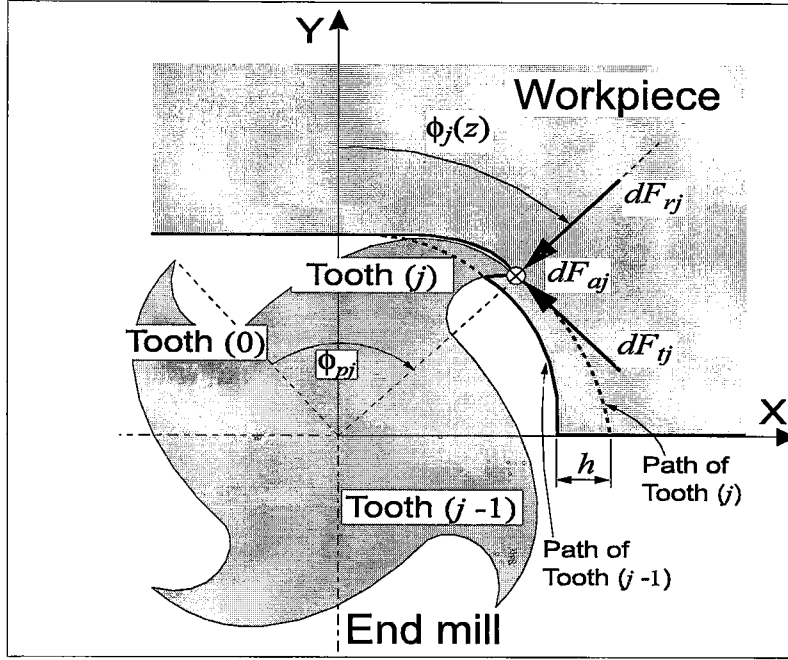


Figure 3.4. Milling cutting force directions and coordinate system

$$\left. \begin{aligned} dF_{tj}(\phi, z) &= K_t h_j(\phi, z) db \\ dF_{rj}(\phi, z) &= K_r dF_{tj}(\phi, z) \\ dF_{aj}(\phi, z) &= K_a dF_{tj}(\phi, z) \end{aligned} \right\} \quad (3.17)$$

where ϕ is the immersion angle measured from the Y axis. The uncut chip thickness, $h_j(\phi, z)$, can be approximated as,

$$h_j(\phi, z) = s_t \sin \phi_j(z) \quad (3.18)$$

where s_t is the feed per tooth and $\phi_j(z)$ is the immersion angle for flute j at the level of z . Due to the helix, the immersion for flute j changes along the axial direction z ,

$$\phi_j(z) = \phi + \sum_{n=0}^{j-1} \phi_{pn} - \psi(z) \quad (3.19)$$

where $\psi(z)$ is the lag angle at the level of z . The cutting parameters K_t , K_r and K_a are expressed as exponential functions of the average chip thickness per flute period, h_a , as follows,

$$\left. \begin{aligned} K_t &= K_T h_a^{-p} \\ K_r &= K_R h_a^{-q} \\ K_a &= K_A h_a^{-s} \end{aligned} \right\} \quad (3.20)$$

where constants K_T , K_R , K_A , p , q and s are determined experimentally for a tool-workpiece material pair. The average chip thickness h_a is expressed as,

$$h_a = \frac{a \int_{\phi_{st}}^{\phi_{ex}} s_t \sin \phi d\phi}{a(\phi_{ex} - \phi_{st})} \quad (3.21)$$

$$h_a = s_t \frac{\cos \phi_{st} - \cos \phi_{ex}}{(\phi_{ex} - \phi_{st})}$$

where a is the axial depth of cut and ϕ_{st} and ϕ_{ex} are the entrance and exit angles respectively. The elemental cutting forces in the feed (X), normal (Y) and axial (Z) directions can be expressed,

$$\left. \begin{aligned} dF_{xj}(\phi, z) &= -dF_{ij}(\phi, z) \cos \phi_j(z) - dF_{rj}(\phi, z) \sin \phi_j(z) \\ dF_{yj}(\phi, z) &= dF_{ij}(\phi, z) \sin \phi_j(z) - dF_{rj}(\phi, z) \cos \phi_j(z) \\ dF_{zj}(\phi, z) &= dF_{aj}(\phi, z) \end{aligned} \right\} \quad (3.22)$$

Substituting equations (3.17) and (3.18) in (3.22), cutting force intensities for flute j are obtained as,

$$\left. \begin{aligned} \frac{dF_{xj}(\phi, z)}{dz} &= -K_t s_t [\cos \phi_j(z) + K_r \sin \phi_j(z)] \sin \phi_j(z) \\ \frac{dF_{yj}(\phi, z)}{dz} &= K_t s_t [\sin \phi_j(z) - K_r \cos \phi_j(z)] \sin \phi_j(z) \\ \frac{dF_{zj}(\phi, z)}{dz} &= K_t K_a s_t \sin \phi_j(z) \end{aligned} \right\} \quad (3.23)$$

The total cutting force contributed by flute j can be found by integrating the intensities along the in-cut portion of the flute as follows:

$$\left. \begin{aligned} F_{xj}(\phi, z) &= \frac{K_t s_t}{4k_i} \left[-\cos 2\phi_j(z) + K_r (2\phi_j(z) - \sin 2\phi_j(z)) \right]_{z_{j,1}(\phi)}^{z_{j,2}(\phi)} \\ F_{yj}(\phi, z) &= \frac{-K_t s_t}{4k_i} \left[2\phi_j(z) - \sin 2\phi_j(z) + K_r \cos 2\phi_j(z) \right]_{z_{j,1}(\phi)}^{z_{j,2}(\phi)} \\ F_{zj}(\phi, z) &= \frac{-K_t K_a s_t}{k_i} \left[\cos \phi_j(z) \right]_{z_{j,1}(\phi)}^{z_{j,2}(\phi)} \end{aligned} \right\} \quad (3.24)$$

where $z_{j,1}(\phi)$ and $z_{j,2}(\phi)$ are lower and upper limits of the in-cut portion of the flute j . Total cutting forces on the cutter are found by summing the forces contributed by all the flutes which are in-cut.

$$\left. \begin{aligned} F_x(\phi) &= \sum_{j=0}^{N-1} F_{xj}(\phi) \\ F_y(\phi) &= \sum_{j=0}^{N-1} F_{yj}(\phi) \\ F_z(\phi) &= \sum_{j=0}^{N-1} F_{zj}(\phi) \end{aligned} \right\} \quad (3.25)$$

3.3.2.2 Linear Edge Force Model

The general geometry of the end milling operation and coordinate system is shown in (Figure 3.4). The elemental tangential, dF_t , radial dF_z and axial dF_a cutting forces acting on flute j of an "ideal" system (with a rigid cutter and zero eccentricity in the cutter axis of rotation) are shown in (Figure 3.4) and given as follows,

$$\left. \begin{aligned} dF_{tj}(\phi, z) &= [K_{te} + K_{tc} h_j(\phi, z)] dz \\ dF_{rj}(\phi, z) &= [K_{re} + K_{rc} h_j(\phi, z)] dz \\ dF_{aj}(\phi, z) &= [K_{ae} + K_{ac} h_j(\phi, z)] dz \end{aligned} \right\} \quad (3.26)$$

where $h_j(\phi, z) = s_{tj} \sin \phi_j(z)$ is the uncut chip thickness, dz is the differential length in cutter axis, and s_{tj} is the feed rate for j th tooth of variable pitch end mills. On a conventional tool, s_{tj} is constant for all the flutes. ϕ is the immersion (or tool rotation) angle measured clockwise from the positive Y axis to a reference tooth $j=0$, which has an immersion angle of ϕ at its tip ($z=0$). The local immersion of a differential element of flute j at the axial coordinate z is,

$$\phi_j(z) = \phi + \sum_{n=0}^{j-1} \phi_{p,n} - \psi(z) \quad (3.27)$$

where $\phi_{p,n}$ is the spacing angle, i.e. the pitch angle, between two cutting edges. The feed rate of flute j can be expressed as a function of the pitch angles as,

$$s_{ij} = \frac{Ns_i \phi_{pj}}{2\pi} \quad (3.28)$$

where,

$$s_i = \sum_{j=0}^{N-1} \frac{s_{ij}}{N} \quad (3.29)$$

where (Ns_i) (mm/rev) is the linear translation of the whole cutter in one revolution of the spindle. At an axial depth z , the helix lag angle of the differential element on flute j from its tip ($z=0$) is $k_i = \tan i/R$, while i and R are the helix angle and the cutter radius, respectively. The cutting forces in equation (3.26) are modeled in terms of two main components, K_{te} , K_{re} and K_{ae} *edge force* components due to touching at the cutting edge, and K_{tc} , K_{rc} and K_{ac} *cutting force* components due to shearing at the shear plane and the friction at the rake face. The edge and cutting components are represented by on-a-unit-area-of-cut basis. The *milling force coefficients* K_{te} , K_{re} , K_{ae} , K_{tc} , K_{rc} and K_{ac} can be obtained from either an orthogonal cutting data base using oblique transformation or average cutting forces measured at various feed rates in slot milling tests [17,84].

The cutting forces contributed by flute j are found by substituting equation (3.26) into (3.22), and integrating along the in-cut portion of flute j ,

$$\left. \begin{aligned}
F_{xj}(\phi, z) &= \frac{R}{\tan i} \left[K_{te} \sin \phi_j(z) - K_{re} \cos \phi_j(z) + \right. \\
&\quad \left. + \frac{S_{ij}}{4} (K_{rc} (2\phi_j(z) - \sin 2\phi_j(z)) - K_{tc} \cos 2\phi_j(z)) \right]_{z_{j,1}(\phi)}^{z_{j,2}(\phi)} \\
F_{yj}(\phi, z) &= -\frac{R}{\tan i} \left[-K_{te} \sin \phi_j(z) - K_{re} \cos \phi_j(z) + \right. \\
&\quad \left. + \frac{S_{ij}}{4} (K_{tc} (2\phi_j(z) - \sin 2\phi_j(z)) + K_{rc} \cos 2\phi_j(z)) \right]_{z_{j,1}(\phi)}^{z_{j,2}(\phi)} \\
F_{zj}(\phi, z) &= \frac{R}{\tan i} \left[K_{ae} \phi_j(z) - s_{ij} K_{ac} \cos \phi_j(z) \right]_{z_{j,1}(\phi)}^{z_{j,2}(\phi)}
\end{aligned} \right\} \quad (3.30)$$

In the above equation, $z_{j,1}(\phi)$ and $z_{j,2}(\phi)$ are the lower and upper limits of the flute j . Total cutting forces on the cutter at immersion ϕ are found by summing forces contributed by all flutes which are engaged in cutting [34].

By the integration of the cutting forces in equation (3.30) for one revolution, the average cutting forces can be found as follows,

$$\left. \begin{aligned}
\overline{F_x} &= -K_{te} S + K_{re} T - \frac{S_t}{4} (-K_{tc} P + K_{rc} Q) \\
\overline{F_y} &= -K_{te} T - K_{re} S + \frac{S_t}{4} (-K_{tc} Q + K_{rc} P) \\
\overline{F_z} &= -\frac{aN}{2\pi} K_{ae} (\phi_{ex} - \phi_{st}) + s_t K_{ac} T
\end{aligned} \right\} \quad (3.31)$$

In the above equation P, Q, S and T are,

$$\left. \begin{aligned}
P &= \frac{aN}{2\pi} [\cos 2\phi]_{\phi_{st}}^{\phi_{ex}} \\
Q &= \frac{aN}{2\pi} [2\phi - \sin 2\phi]_{\phi_{st}}^{\phi_{ex}} \\
S &= \frac{aN}{2\pi} [\sin \phi]_{\phi_{st}}^{\phi_{ex}} \\
T &= \frac{aN}{2\pi} [\cos \phi]_{\phi_{st}}^{\phi_{ex}}
\end{aligned} \right\} \quad (3.32)$$

where a , ϕ_{st} and ϕ_{ex} are the axial depth of cut and the start and exit angles of the cut respectively. The parameters P, Q, S, and T in equation (3.32) are constant in each cutting test for different feed rates. Each of the average cutting forces is a linear

function of feed rate (s_f). The offset is contributed by the edge forces component. Therefore the cutting forces and feed rate linear equation can be written as follows by separating the edge forces,

$$\left. \begin{aligned} \overline{F_x} &= \overline{F_{xe}} + s_f \overline{F_{xc}} \\ \overline{F_y} &= \overline{F_{ye}} + s_f \overline{F_{yc}} \\ \overline{F_z} &= \overline{F_{ze}} + s_f \overline{F_{zc}} \end{aligned} \right\} \quad (3.33)$$

Using equations (3.31) and (3.33), the cutting force coefficients can be obtained as [17],

$$\left. \begin{aligned} K_{te} &= -\frac{\overline{F_{xe}}S + \overline{F_{ye}}T}{S^2 + T^2} & K_{tc} &= 4\frac{\overline{F_{xc}}P + \overline{F_{yc}}Q}{P^2 + Q^2} \\ K_{re} &= \frac{K_{te}S + \overline{F_{xe}}}{T} & K_{rc} &= \frac{K_{tc}P - 4\overline{F_{xc}}}{Q} \\ K_{ae} &= -\frac{2\pi}{aN} \frac{\overline{F_{ze}}}{(\phi_{ex} - \phi_{st})} & K_{ac} &= \frac{\overline{F_{zc}}}{T} \end{aligned} \right\} \quad (3.34)$$

3.4 Time Domain Dynamics of Milling

Milling system dynamics can be considered in the X and Y directions as two orthogonal degrees of freedom as shown in Figure 3.5. Cutter dynamics contain the entire machine structure and the cutter. During the cutting operation, the workpiece and the cutter vibrate together. The orthogonal modes of the structure in the X and Y directions can be written as,

$$\begin{aligned} \ddot{x}(t) + 2\zeta_x \omega_{nx} \dot{x}(t) + \omega_{nx}^2 x(t) &= \frac{\omega_{nx}^2}{k_x} F_x(t) \\ \ddot{y}(t) + 2\zeta_y \omega_{ny} \dot{y}(t) + \omega_{ny}^2 y(t) &= \frac{\omega_{ny}^2}{k_y} F_y(t) \end{aligned} \quad (3.35)$$

where $(\omega_{nx}, \omega_{ny})$, (ζ_x, ζ_y) and (k_x, k_y) are the natural frequencies, structural damping ratios and stiffnesses in feed (x) and normal (y) directions (Figure 3.5), respectively. $F_x(t)$ and $F_y(t)$ are the dynamic cutting forces. The equations of

motion (3.35) are evaluated at each discrete time interval Δt using recursive, discrete equivalents of the continuous differential equation. The discrete time domain solution of (3.35) in the X and Y directions is,

$$\begin{aligned} x(t) &= \frac{F_x(t) + 2F_x(t - \Delta t) + F_x(t - 2\Delta t) - A_x x(t - \Delta t) - B_x x(t - 2\Delta t)}{C_x} \\ y(t) &= \frac{F_y(t) + 2F_y(t - \Delta t) + F_y(t - 2\Delta t) - A_y y(t - \Delta t) - B_y y(t - 2\Delta t)}{C_y} \end{aligned} \quad (3.36)$$

where,

$$\begin{aligned} m_x &= \frac{k_x}{(2\pi\omega_{nx})^2} & ; & \quad m_y = \frac{k_y}{(2\pi\omega_{ny})^2} \\ c_x &= 4\pi\zeta_x m_x \omega_{nx} & ; & \quad c_y = 4\pi\zeta_y m_y \omega_{ny} \\ A_x &= -8\frac{m_x}{dt^2} + 2k_x & ; & \quad A_y = -8\frac{m_y}{dt^2} + 2k_y \\ B_x &= 4\frac{m_x}{dt^2} - 2\frac{c_x}{dt} + k_x & ; & \quad B_y = 4\frac{m_y}{dt^2} - 2\frac{c_y}{dt} + k_y \\ C_x &= 4\frac{m_x}{dt^2} + 2\frac{c_x}{dt} + k_x & ; & \quad C_y = 4\frac{m_y}{dt^2} + 2\frac{c_y}{dt} + k_y \end{aligned} \quad (3.37)$$

If residue values are used ($R_x = R_{rx} + iR_{ix}$; $R_y = R_{ry} + iR_{iy}$) instead of dynamic parameters, the discrete time domain solution is,

$$\begin{aligned} x(t) &= \frac{C_{1x}F_x(t) + C_{2x}F_x(t - \Delta t) + C_{3x}F_x(t - 2\Delta t) - C_{4x}x(t - \Delta t) - C_{5x}x(t - 2\Delta t)}{C_{0x}} \\ y(t) &= \frac{C_{1y}F_y(t) + C_{2y}F_y(t - \Delta t) + C_{3y}F_y(t - 2\Delta t) - C_{4y}y(t - \Delta t) - C_{5y}y(t - 2\Delta t)}{C_{0y}} \end{aligned} \quad (3.38)$$

where,

$$\left. \begin{aligned}
C_{0x} &= \frac{4}{dt^2} + 4 \frac{\dot{\zeta}_x \omega_{nx}}{dt} + (\zeta_x \omega_{nx})^2 + \omega_{nx}^2; C_{0y} = \frac{4}{dt^2} + 4 \frac{\dot{\zeta}_y \omega_{ny}}{dt} + (\zeta_y \omega_{ny})^2 + \omega_{ny}^2 \\
C_{1x} &= 2R_{rx} \zeta_x \omega_{nx} - 2R_{ix} \omega_{nx} + 4 \frac{R_{rx}}{dt}; C_{1y} = 2R_{ry} \zeta_y \omega_{ny} - 2R_{iy} \omega_{ny} + 4 \frac{R_{ry}}{dt} \\
C_{2x} &= 4R_{rx} \dot{\zeta}_x \omega_{nx} - 4R_{ix} \dot{\omega}_{nx}; C_{2y} = 4R_{ry} \dot{\zeta}_y \omega_{ny} - 4R_{iy} \dot{\omega}_{ny} \\
C_{3x} &= -\frac{8}{dt^2} + 2(\zeta_x \omega_{nx})^2 + 2\omega_{nx}^2; C_{3y} = -\frac{8}{dt^2} + 2(\zeta_y \omega_{ny})^2 + 2\omega_{ny}^2 \\
C_{5x} &= \frac{4}{dt^2} - 4 \frac{\dot{\zeta}_x \omega_{nx}}{dt} + (\zeta_x \omega_{nx})^2 + \omega_{nx}^2; C_{5y} = \frac{4}{dt^2} - 4 \frac{\dot{\zeta}_y \omega_{ny}}{dt} + (\zeta_y \omega_{ny})^2 + \omega_{ny}^2
\end{aligned} \right\} \quad (3.39)$$

In the time domain simulation, the following main steps can be pursued:

- Along the Z axis (axial direction), the cutter and workpiece are divided into equal number of elements.
- The cutting surface in the radial direction at each level is digitized with equal number of points by a discrete rotation angle.
- Using exact kinematics of milling for each point on the cutting edge, the rigid body motion is calculated. It is subtracted from the surface point left in the previous tooth period to calculate the current chip thickness. The regenerative chip thickness can be calculated by including static or dynamic displacements of the cutting edge points.
- According to the cutting force model, the cutting force coefficients are evaluated for each local chip thickness. The cutting force model can be the linear average model, the exponential force model, the orthogonal cutting database using oblique transformation model [17], or another one.
- The cutting forces in the radial (F_r), tangential (F_t) and axial (F_a) directions are evaluated for each cutting edge point which is in contact with the material. They are summed and resolved in the X, Y and Z directions (F_x, F_y, F_z).
- The vibrations are obtained by applying the cutting forces on the structure.

- The cutting edge positions are updated by calculating the edge deflection because of the vibration.
- According to the cutting edge position on the workpiece, the surface is updated.
- The solution is repeated by rotating the cutter in small angular increments.

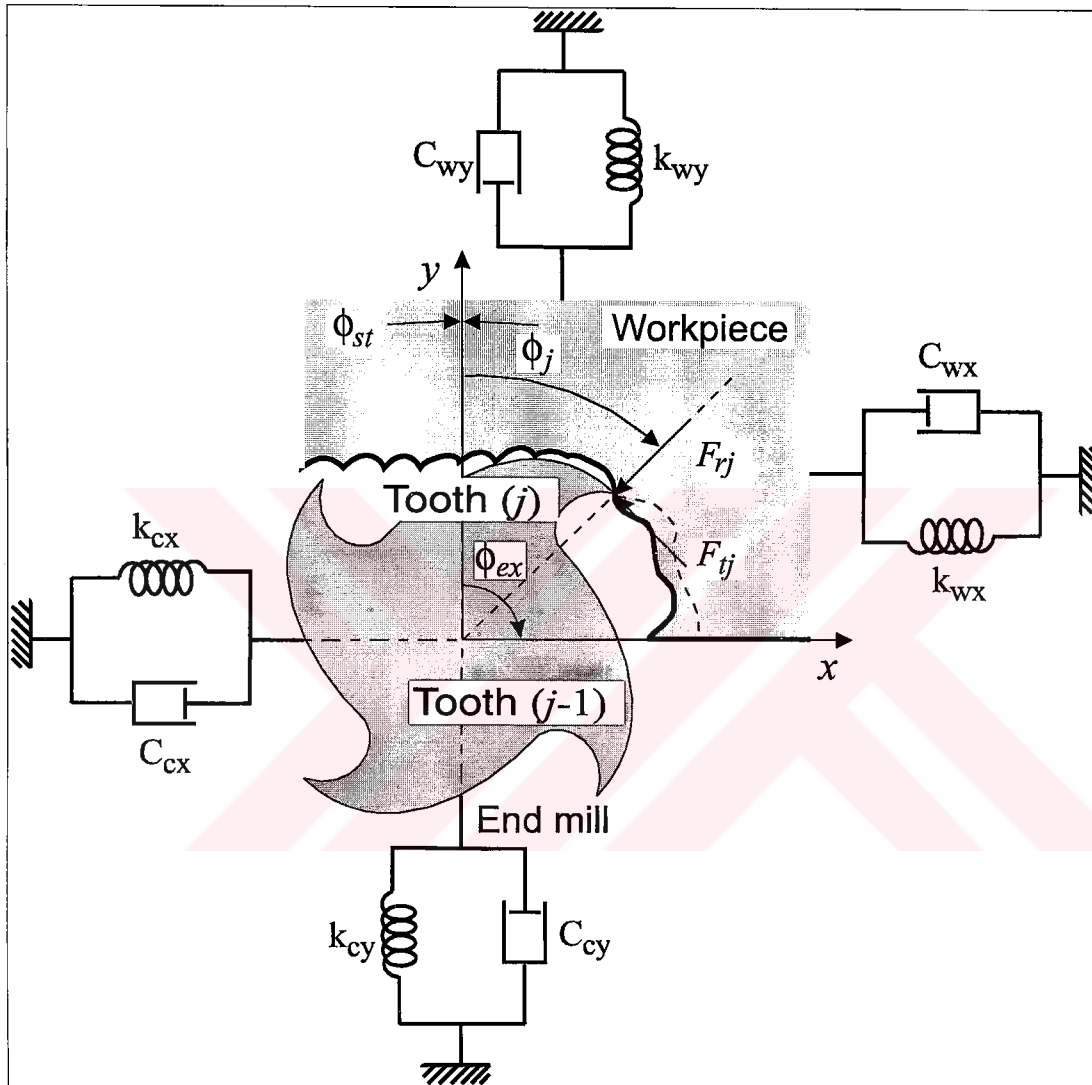


Figure 3.5. Two degree of freedom end milling system with workpiece dynamics.

3.4.1 Kinematics of Chip and Surface Generation

During cutting operation the cutter and the workpiece experience vibrations which affect the chip thickness. Using a sinusoidal model for the prediction of the chip thickness gives only an approximate value. To determine the exact chip thickness, the cutter and the workpiece surfaces are digitized at each discrete time interval.

The exact trochoidal rigid body motion of the cutter was first described by Martellotti [4]. This model allows the effects of a flexible workpiece and a flexible cutter to be easily considered. In this model, it is possible to include dynamic chip thickness and changing radial immersion because of the vibrations and deflection.

The cutter and the workpiece surface are divided into a number of small levels. The cutter rotation is in the discrete domain and feed motion is into the workpiece. After each step, if the cutting edge has engaged with the workpiece, the workpiece points are updated.

There are two main methods available to update the angular and radial position of the cutting edge, if the tooth is submerged in the workpiece. Montgomery and Altıntaş [10,59] used a surface array, which is updated with the instantaneous angular position of the tooth at each time step (Figure 3.6a). The radial position of each point is taken as either the radius of the cutter, if the tooth is submerged in the workpiece, or as the radial position of the surface left by the previous tooth.

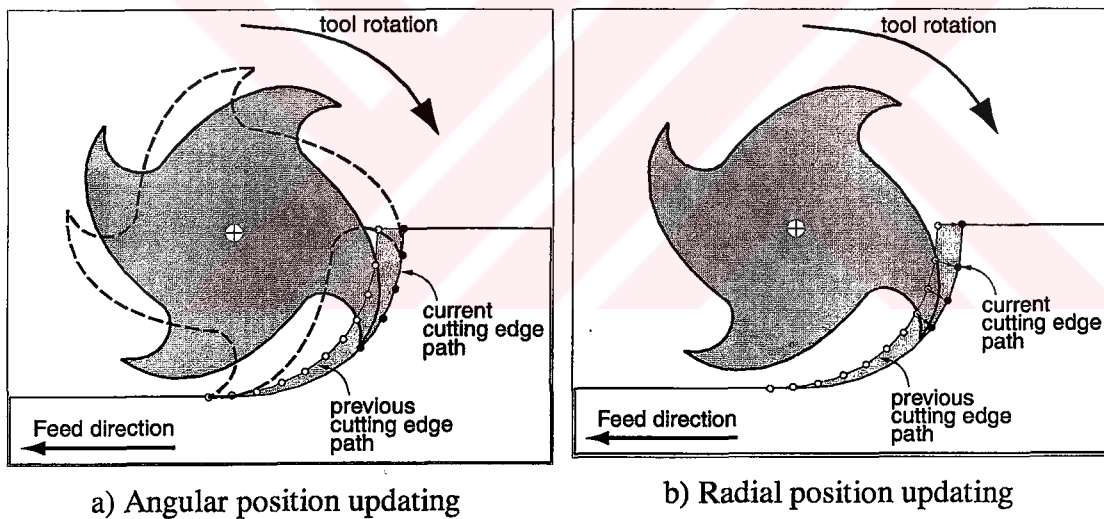


Figure 3.6. Surface tracking methods.

Lee and Altıntaş [11] used an evenly distributed array of points on the surface, which were updated by moving them radially outwards as the flute cut into the workpiece (Figure 3.6b). This method gives much better results for complex geometry cutters such as ball end, taper and general mills, where the radius of the cutter approaches zero at the tip of the cutter. However, this method has some limitations for small radial widths of cut and long simulation runs. The radial motion of the points, as they are updated at the cutting edge in combination with the feed in the X direction,

causes the points to drift in an accumulating manner and the immersion angles to vary with time. Campomanes and Altıntaş [12] combined Montgomery's method for the arc surface, and Lee's method for maintaining the history of the finished surfaces.

The cutter and workpiece are divided into a number of levels in the axial direction and rotational increments in feed direction. If the cutter and workpiece are rigid, the position of the cutter center is stationary. The workpiece surface points are calculated by assuming a rigid tool and a rigid workpiece. Then the dynamics of the machine and cutting tool system are included in the model of the cutting operation.

The transformations from Cartesian to Polar coordinates are given as follows,

$$\left. \begin{aligned} r &= \sqrt{(x - \Delta x)^2 + (y - \Delta y)^2} \\ \phi &= \frac{\pi}{2} - \tan^{-1} \left(\frac{y - \Delta y}{x - \Delta x} \right) \end{aligned} \right\} \quad (3.40)$$

where,

$$\left. \begin{aligned} \Delta x &= \Delta x_{cutter} - \Delta x_{workpiece} \\ \Delta y &= \Delta y_{cutter} - \Delta y_{workpiece} \end{aligned} \right\} \quad (3.41)$$

The position of a point on the cutting edge in Cartesian coordinates are calculated as follows:

$$\left. \begin{aligned} x_j &= R \cdot \sin \phi_j \\ y_j &= R \cdot \cos \phi_j \end{aligned} \right\} \quad (3.42)$$

where R , ϕ_j are the distance from the point on the cutting edge to the cutter center and angular position of the cutting edge respectively. The relative displacements of the cutter and the workpiece can be seen in Figure 3.7. If the cutting edges have runout (R_r), it will affect the chip thickness and surface finish. In this case, the cutting edge position can be calculated as follows,

$$\left. \begin{aligned} x_j &= (R + R_r) \cdot \sin \phi_j \\ y_j &= (R + R_r) \cdot \cos \phi_j \end{aligned} \right\} \quad (3.43)$$

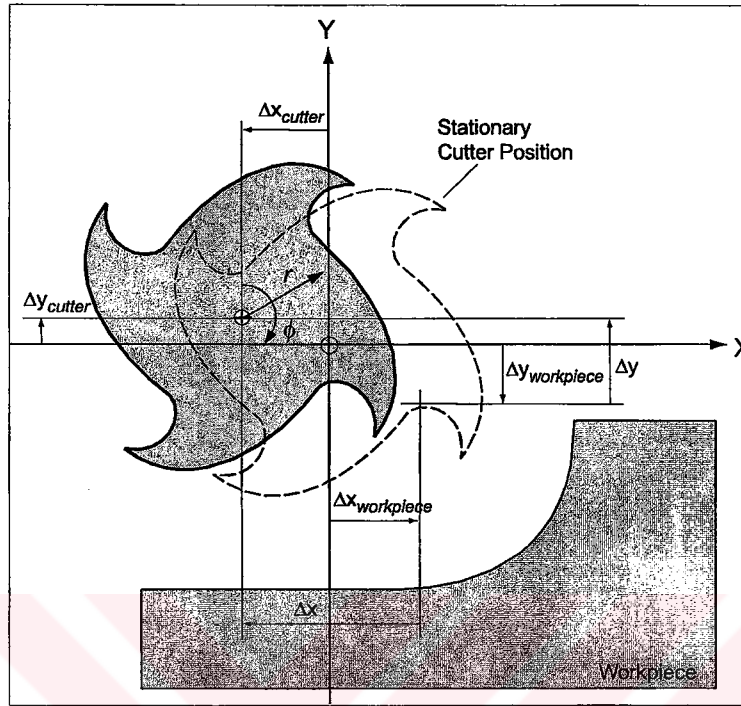


Figure 3.7. Tool and workpiece relative displacements [12].

3.4.2 Simulation of Chatter in Time Domain

There are two main mechanisms that cause chatter. These are mode coupling and regeneration of waviness. *Mode coupling* is a mechanism of self-excitation that can only be associated with situations where the relative vibration between the tool and the workpiece exists simultaneously in at least two directions in the plane of the orthogonal cutting. *Regeneration of waviness* is possible in almost all machining operations, because the tool removes the chip from a surface which was produced by the tool in the preceding pass, i.e., the surface produced in milling by the preceding tooth of the cutter. Also, if there is relative vibration between the tool and the workpiece, waviness is generated on the cut surface [48].

There are some important effects in chatter, especially at low spindle speeds. The main effect is *process damping*. Due to the sharpness of the cutting edge, part of the chip is forced to go under the tool and causes interference on the tool clearance flank (Figure 3.8).

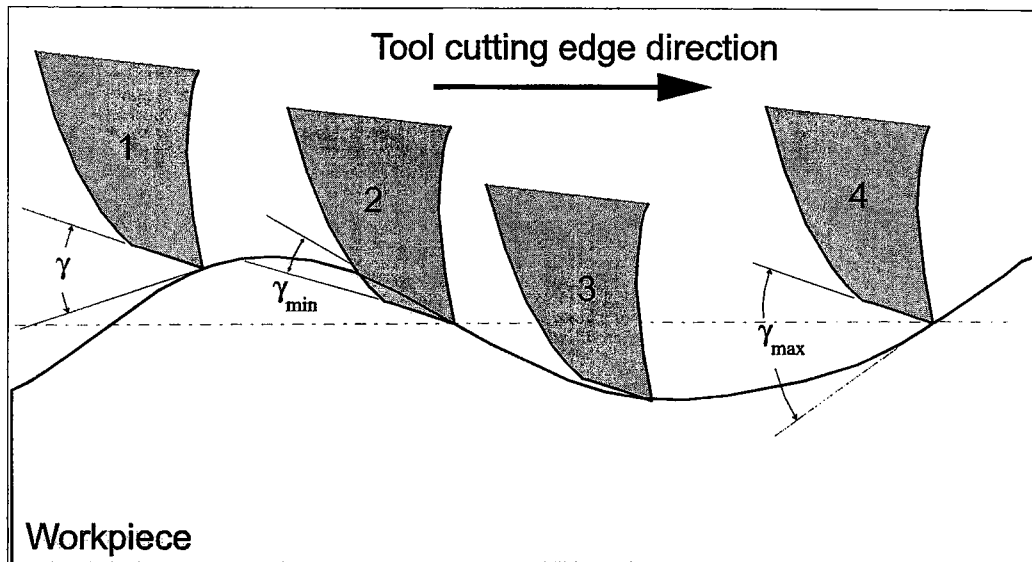


Figure 3.8. Mechanism of damping in the cutting process [48].

Several researchers worked on increasing the stability. The velocity principle (penetration rate) is attributed to Tobias [47]. Sisson and Kegg [90] describe the process damping using physical quantities such as chatter frequency, clearance angles and cutting edge radius. Some other researchers [59,91,92] describe process damping by explaining the variation of the relief angle of the cutting tool because of the vibrations.

Cutting edge position depends on the system dynamics, static deflection of the cutter and chip load. If the previous and current cutting edge paths are parallel to each other and there is no vibration, that means chip load is instantaneously constant. If both vibration wave paths are parallel to each other, there is forced vibration. The chip thickness is constant and does not cause chatter vibration. If there is a phase shift between the two paths, the chip thickness changes significantly. This causes a chatter vibration, which can grow until the cutter jumps out from the workpiece (Figure 3.9).

The dynamic chip thickness for the cutting edge engaged with workpiece along the depth of cut is calculated to predict the chatter stability. The system vibrations become unstable if there is chatter. However, the maximum vibration limit is reached when the cutting tool jumps out of the workpiece.

In time domain, the stability lobes can be found as the axial depth of cut limit versus a range of spindle speeds using the following steps:

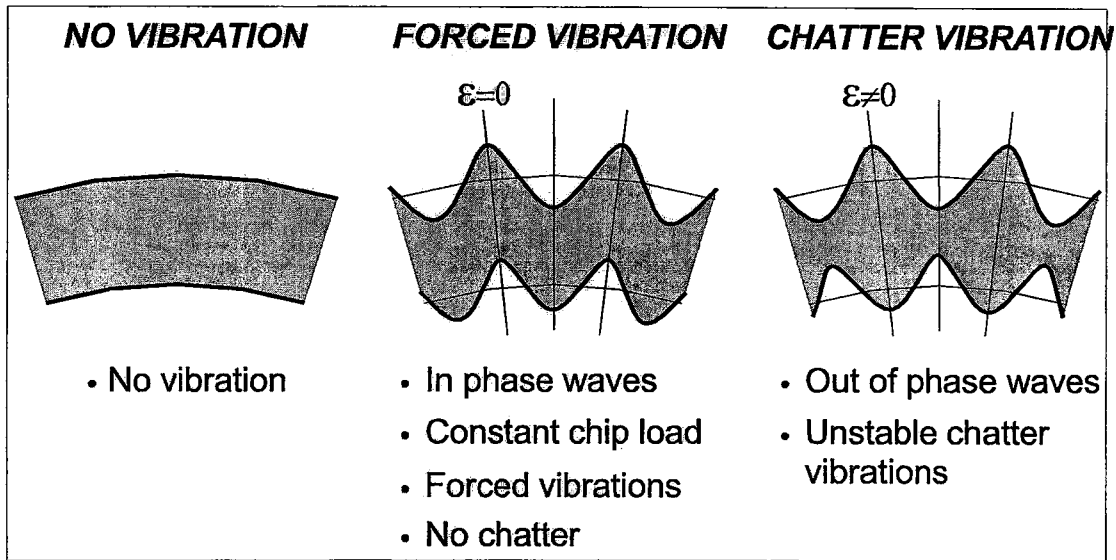


Figure 3.9. Vibration types.

- Cutting tool geometry (i.e. number of flutes, type of the pitch angle distribution and dimensions of the cutter), feed rate, cutter orientation and immersion, workpiece material and cutting coefficients, machine and workpiece dynamics, are specified.
- Starting axial depth of cut (a) and spindle speed range and step size are specified.
- The maximum static uncut chip thickness, $h_{static,max}$, is stored by running a static time domain simulation for a given spindle speed. This is equal to the feed per tooth (s_t) or uniform pitch end mill cutters with no runout.
- By including the dynamics of the workpiece and the cutter, another time domain simulation is run, until a steady-state chatter condition is reached. The largest dynamic chip thickness of all cutting points on the cutter, $h_{dynamic,max}$, is stored for the last few revolutions of the simulation.
- The chatter parameter (ξ) is calculated based on $\xi = \frac{h_{dynamic,max}}{h_{static,max}}$.
- If ξ is bigger than a predetermined limit (1.1 is used for the time domain simulation in this thesis), it is assumed that there is chatter, otherwise there is no chatter vibration.

- There are two possibilities for the system stable or unstable; a_{min} is set to the current value of depth of cut (a), if the cutting process is stable. Static and dynamic simulations are repeated until chatter occurs by doubling the depth of cut. a_{max} is set to the depth of cut value when chatter occurred. If the cutting process is unstable, a_{max} is set to the current value of depth of cut. Static and dynamic simulations are repeated until the process becomes stable by halving the depth of cut. a_{min} is put to the stable depth of cut, a .
- The simulation is repeated until the range from a_{min} to a_{max} is found to be smaller than the set tolerance.

3.5 Stability of Milling in Frequency Domain

The chatter model in frequency domain was developed by Altıntaş and Budak [67-69]. The effect of feed rate is studied by Campomanes [12], who showed that large feed rates affect the chatter stability at low cutting speeds. He developed an analytical model of the dynamic milling system with the influence of feed on the regenerative phase shift, and analytically solved the stability of the system in the frequency domain.

The cutting forces dynamically vibrate the system in the X and Y directions. The dynamic displacements are x and y in the feed (X) and normal (Y) directions, respectively (Figure 3.5). The dynamic displacements affect the rotating tooth number (j) in the radial direction, hence the displacement for the tooth number (j) can be expressed as:

$$v_j = -x \sin \phi_j - y \cos \phi_j \quad (3.44)$$

where ϕ_j is the angular position of tooth j (Figure 3.5), and measured clockwise from the positive Y axis as :

$$\phi_j(t) = \Omega t + \psi_j \quad (3.45)$$

where ψ_j is tooth lag (location) angle and $\Omega (= 2\pi n / 60)$ is the spindle rotational velocity in radians per second.

The total chip thickness has two parts as static and dynamic. The static part is due to rigid body motion of the cutter, which is $(s_t \sin \phi_j)$. The dynamic chip thickness is evaluated by taking the difference between the previous (v_{j-1}) and current (v_j) tooth radial displacement. The feed rate (s_t) is constant for a regular pitch cutter, however it varies for variable pitch cutters from one tooth to another. The chip thickness can be written as (Figure 3.10),

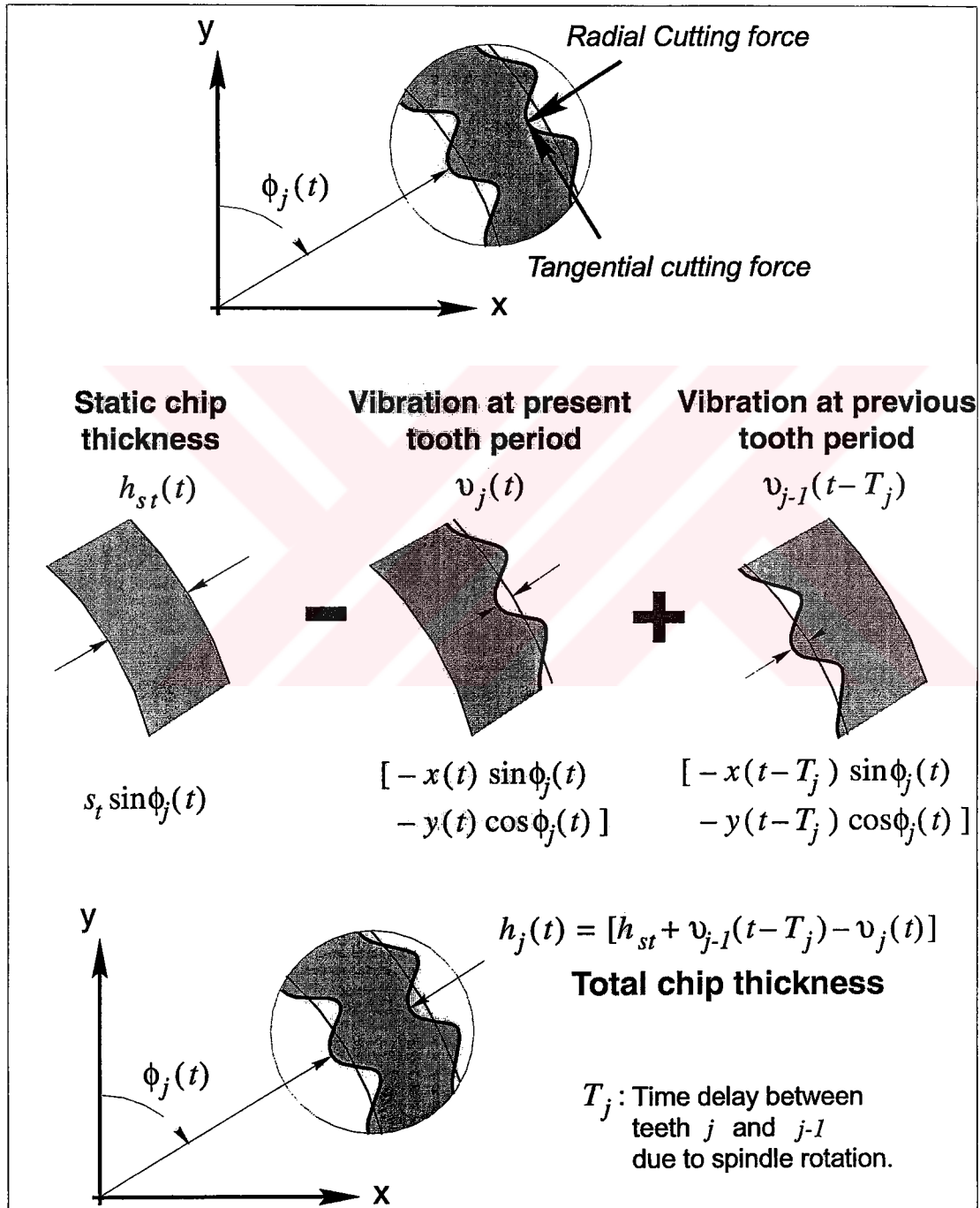


Figure 3.10. Dynamic chip thickness.

$$h_j(t) = g(\phi_j) \left[s_t \sin \phi_j + (v_{j-1}(t - T_j) - v_j(t)) \right] \quad (3.46)$$

where $g(\phi_j)$ is a unit step function that gives one when the tooth is touching (in-cut) the workpiece between the immersion angle limits ϕ_{st} and ϕ_{ex} , otherwise gives zero when the tooth is not touching (out of cut) the workpiece. The $g(\phi_j)$ function can be given as,

$$g(\phi_j) = \begin{cases} 1 & \text{if } \phi_{st} < \phi_j < \phi_{ex} \quad \text{in cut} \\ 0 & \text{if } \phi_j < \phi_{st} \text{ or } \phi_j > \phi_{ex} \quad \text{out of cut} \end{cases} \quad (3.47)$$

The static chip thickness does not affect the wave regeneration mechanism on the total chip thickness. Hence the static part can be subtracted from total chip thickness. The dynamic chip thickness can be expressed in the X-Y coordinate system as,

$$h_j(t) = g(\phi_j) [\Delta x_j \sin \phi_j + \Delta y_j \cos \phi_j] \quad (3.48)$$

where Δx_j and Δy_j are the displacements in the previous tooth pass in the X and Y directions, respectively.

$$\left. \begin{aligned} \Delta x_j &= x_j - x_{j-1} \\ \Delta y_j &= y_j - y_{j-1} \end{aligned} \right\} \quad (3.49)$$

The cutting forces in the tangent (F_{tj}) and radial (F_{rj}) directions for tooth j are proportional to the axial depth of cut (a) and the chip thickness (h),

$$\left. \begin{aligned} F_{tj} &= K_t a h(\phi_j) \\ F_{rj} &= K_r F_{tj} \end{aligned} \right\} \quad (3.50)$$

where K_t is the cutting pressure coefficient in the tangential direction and K_r is a ratio between the radial and tangential cutting coefficients. The force components in the X and Y directions are:

$$\left. \begin{aligned} F_{xj} &= -F_{tj} \cos \phi_j - F_{rj} \sin \phi_j \\ F_{yj} &= F_{tj} \sin \phi_j - F_{rj} \cos \phi_j \end{aligned} \right\} \quad (3.51)$$

The total cutting force in the X and Y directions can be found by summing the elemental cutting forces acting on the cutter for each tooth.

$$\left. \begin{aligned} F_x &= \sum_{j=0}^{N-1} F_{xj}(\phi_j) \\ F_y &= \sum_{j=0}^{N-1} F_{yj}(\phi_j) \end{aligned} \right\} \quad (3.52)$$

Substituting the chip thickness (3.48) and the forces (3.50) into (3.51), and rearranging the resulting expression in matrix form is,

$$\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = \frac{1}{2} a K_t \begin{bmatrix} a_{xx} & a_{xy} \\ a_{yx} & a_{yy} \end{bmatrix} \begin{Bmatrix} \Delta x \\ \Delta y \end{Bmatrix} \quad (3.53)$$

where the parameters a_{xx} , a_{xy} , a_{yx} and a_{yy} are the directional milling force coefficients which are time dependent. They are given as follow,

$$\left. \begin{aligned} a_{xx} &= \sum_{j=0}^{N-1} -g(\phi_j) [\sin 2\phi_j + K_r (1 - \cos 2\phi_j)] \\ a_{xy} &= \sum_{j=0}^{N-1} -g(\phi_j) [(1 + \cos 2\phi_j) + K_r \sin 2\phi_j] \\ a_{yx} &= \sum_{j=0}^{N-1} g(\phi_j) [(1 - \cos 2\phi_j) - K_r \sin 2\phi_j] \\ a_{yy} &= \sum_{j=0}^{N-1} g(\phi_j) [\sin 2\phi_j - K_r (1 + \cos 2\phi_j)] \end{aligned} \right\} \quad (3.54)$$

Equation (3.53) can be expressed in the time domain in matrix form as:

$$\{F(t)\} = \frac{1}{2} a K_t [A(t)] \{\Delta(t)\} \quad (3.55)$$

The milling forces $[A(t)]$ are periodic at tooth passing frequency for regular pitch cutters or at spindle rotating frequency for variable pitch cutters. This can be expanded into a Fourier series. Hence, the axial depth of cut limit can be calculated in frequency domain. Details of frequency domain calculations will be given in Chapter 6. The chatter block diagram is illustrated and explained thoroughly in Figure 3.11 by Altıntaş [71,79].

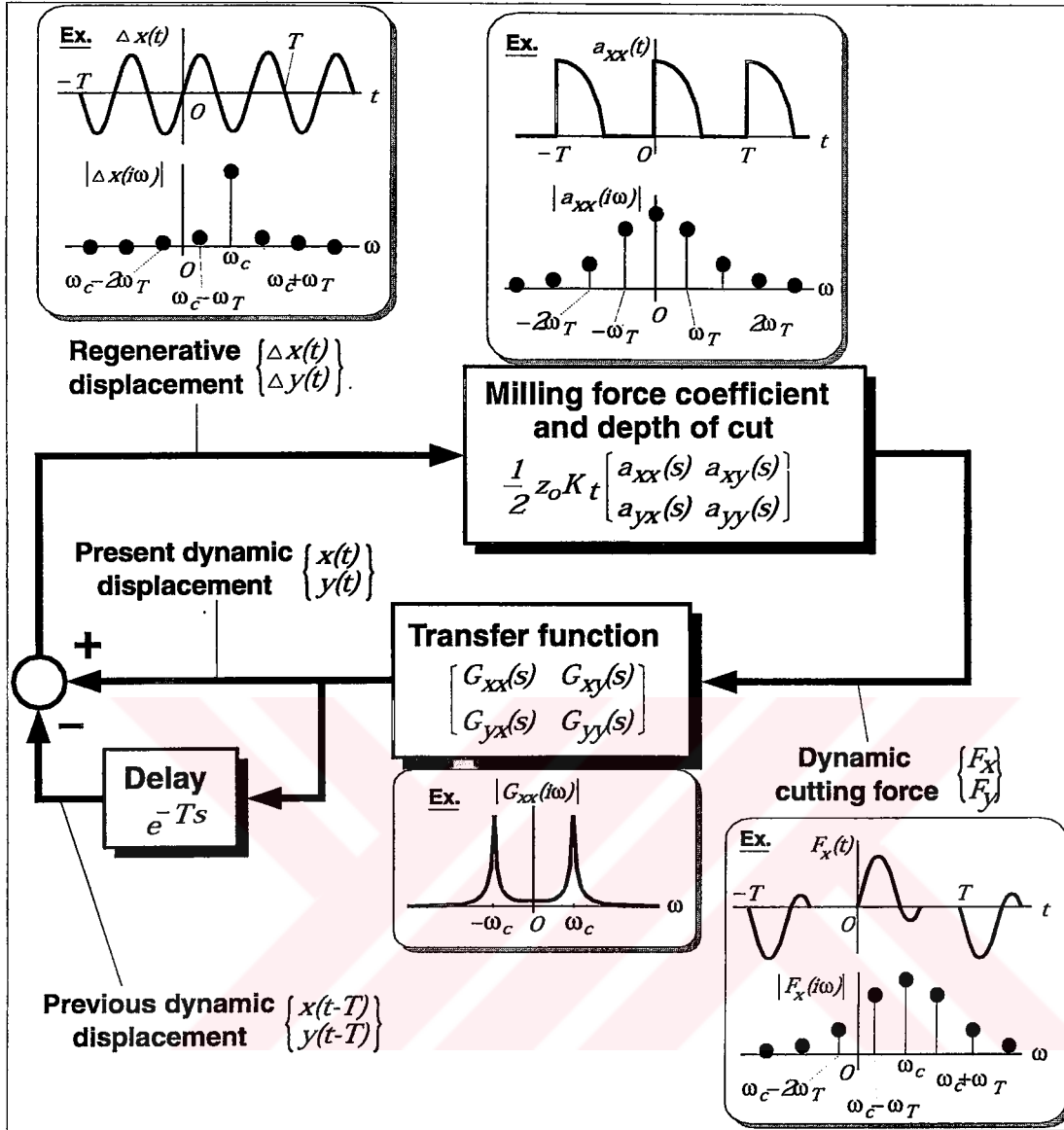


Figure 3.11. Block diagram of chatter stability [71].

3.6 Simulation of Stability Lobes for Time and Frequency Domain

Chatter stability lobes show the stable and unstable areas of a cutting operation. In milling, there are three variables for a given material and cutter geometry. They are axial depth of cut, width of cut (or cutter entrance and exit angles to the workpiece) and spindle speed. These three parameters should be selected at optimum values so that the machine does not have chatter vibrations, and maximum speed and axial depth of cut are realized. Certainly there is a tool wear problem at very high speeds, especially for hard workpiece materials. This is another research area.

Time domain stability lobe simulation takes very long, in comparison to the analytical solution. Analytical stability lobe calculation is a direct solution, but the time domain stability lobe calculation is an iterative method that tries all cutting conditions and finds stable points. The time domain simulation period also depends on the natural frequencies of the system, depth of cut and spindle speed. Large depth of cut, high vibration frequency and low spindle speed increase the computational time of time domain stability lobe simulation. They affect the number of layers and step size of the rotation.

The analytical and time domain calculation results are given in Figure 3.12. One mode is taken in X and Y directions. Natural frequencies, damping ratios and stiffness are $\omega_{nx} = 500$ Hz $\omega_{ny} = 700$ Hz, $\zeta_x = \zeta_y = 0.05$, $k_x = k_y = 10^7$ N/m, respectively. The workpiece is assumed to be rigid. The workpiece material is P20 mold steel which has $K_{tc} = 3391.0$ and $K_{rc} = 1947.0$ N/mm².

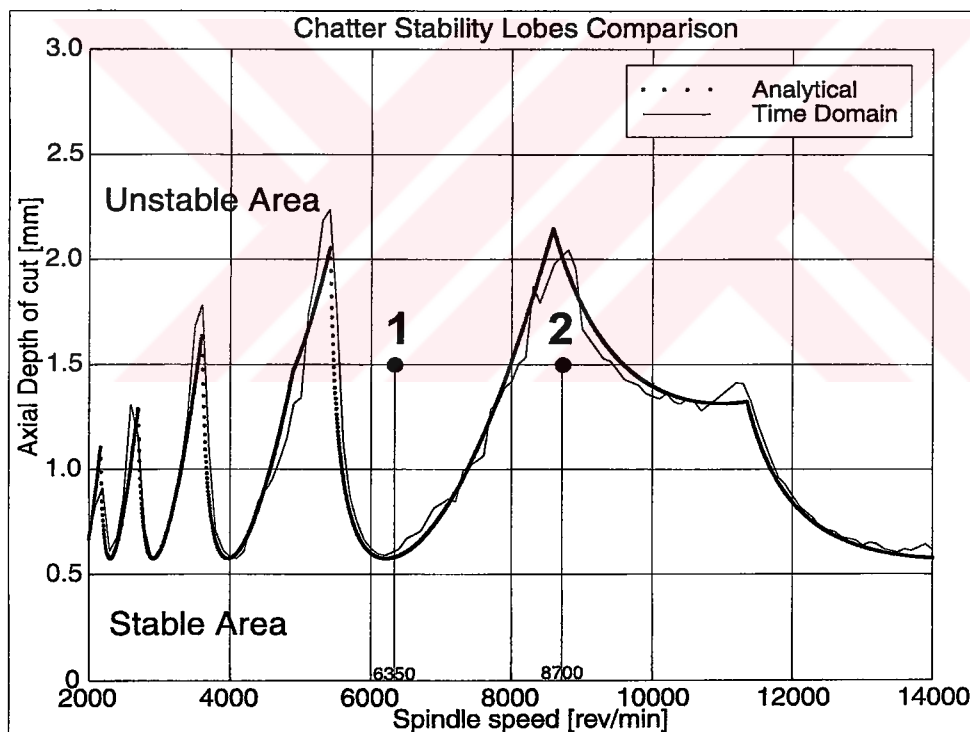


Figure 3.12. Time domain and analytical stability lobes. Half immersion down milling with 4 flute cylindrical end mill, 20.0 mm diameter, $K_{tc} = 3391.0$ MPa, $K_r = 0.5$.

Single time domain simulations were done for stable and unstable points, which were at 6350 (point 1) and 8700 (point 2) rev/min for 1.5 mm depth of cut and 0.05 mm/tooth feed rate. The speeds 6350 and 8700 rev/min were stable and unstable points respectively, according to the generated stability lobes (Figure 3.12). Cutting

forces and tool tip vibrations are shown in Figure 3.13 and Figure 3.14 for 20 revolutions by choosing the same cutting conditions. At 6350 rev/min and 1.5 mm depth of cut, the cutting process was unstable which can be seen from the cutting forces and tool tip vibrations. There are large vibrations at this point. However, there was no chatter vibration at 8700 rev/min for the same depth of cut. Tool vibration FFTs (Figure 3.15) show that there were two dominant vibration frequencies i.e. 752 Hz, which is the chatter frequency at 6350 rev/min spindle speed and 580 Hz, which is the tooth passing frequency at 8700 rev/min.

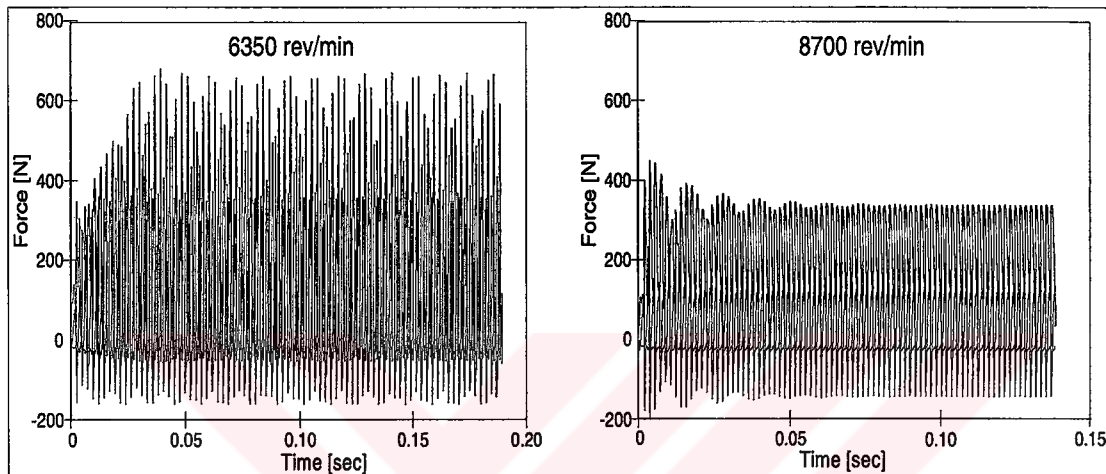


Figure 3.13. Cutting forces at 1.5 mm depth of cut for 6350 and 8700 rev/min

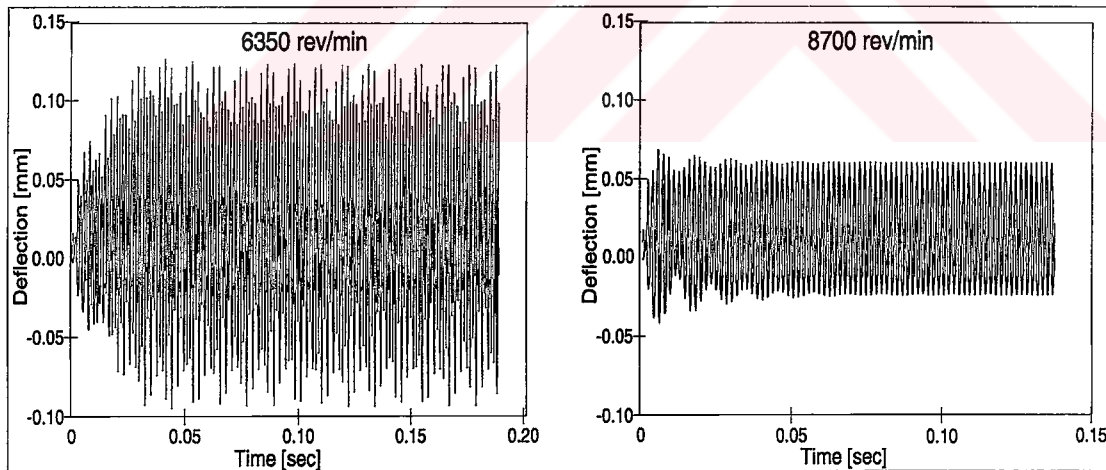


Figure 3.14. Tool vibrations at 1.5 mm depth of cut for 6350 and 8700 rev/min

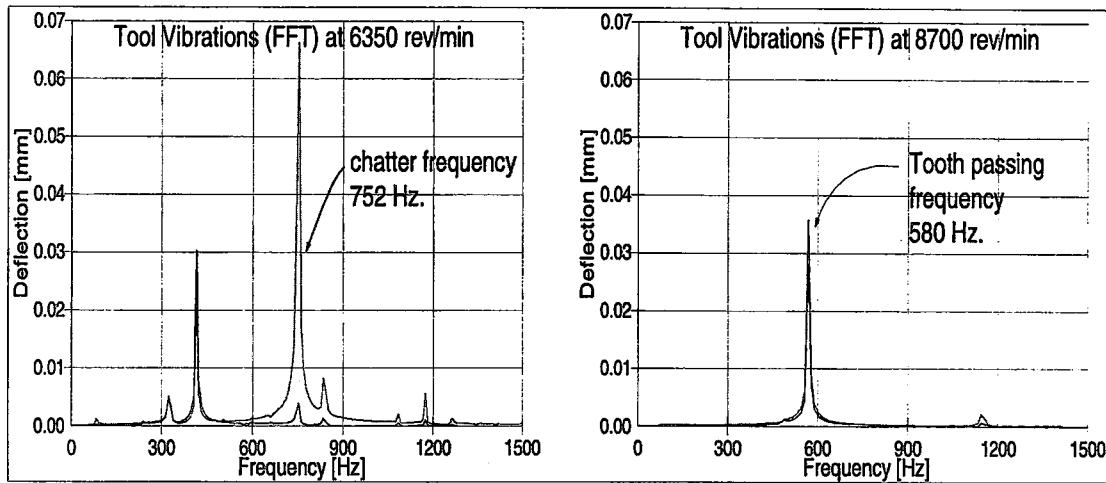


Figure 3.15. FFT of tool vibrations at 1.5 mm depth of cut for 6350 and 8700 rev/min

3.7 Runout Model in Designing Arbitrary Cutter Geometry

Runout is the cutting edge position difference between the different teeth, with respect to the cutter center (Figure 3.16). Grinding errors for the flutes, curvature on the cutter body, error on chuck and errors on the spindle can affect runout. If the cutter does not have any runout, the instantaneous chip thickness will be the same for every tooth. Otherwise the instantaneous chip thickness will differ among different teeth. In some cases where runout is sufficiently large, some teeth may not even engage in cutting at all. Runout for a single fluted cutter is not defined. This is because the cutter diameter is determined by the only cutting edge. The effects of axial offset runout on force and geometry are investigated in detail by Kline and DeVor [85].

Runout does not affect the chatter phenomenon when all the cutting edges have chip load. If the runout is greater than the feed rate, some of the teeth will not cut. The number of flutes in cutting will be different and this will affect the stability of the cutting process. As an example, Al7075 is selected as the workpiece material. The cutting coefficients are $K_{tc}=796.08$, $K_{rc}=168.83$, $K_{ac}=222.04$ [N/mm²], $K_{te}=27.71$, $K_{re}=30.80$, $K_{ae}=1.40$ [N/mm]. The cylindrical end mill used has 4 flutes and a diameter of 3/4 inch (Figure 3.17). The cutting conditions were half immersion down milling at 0.050 mm/tooth feed rate. The workpiece was assumed to be rigid and the cutting tool dynamics are given in Table 3.1. The stability lobes were first obtained for these cutting conditions without runout. Then, the stability of the cutting process

was re-analyzed for the case of 0.100 mm runout on one of the flutes. In the first simulation the pitch angles were 90° - 90° - 90° - 90° . In the second simulation one of the cutting edges was not cutting because of the large runout. The pitch angles became 90° - 180° - 90° because one of the flutes was not touching the surface. The difference between the stability lobes for these two cases can be seen in Figure 3.18.

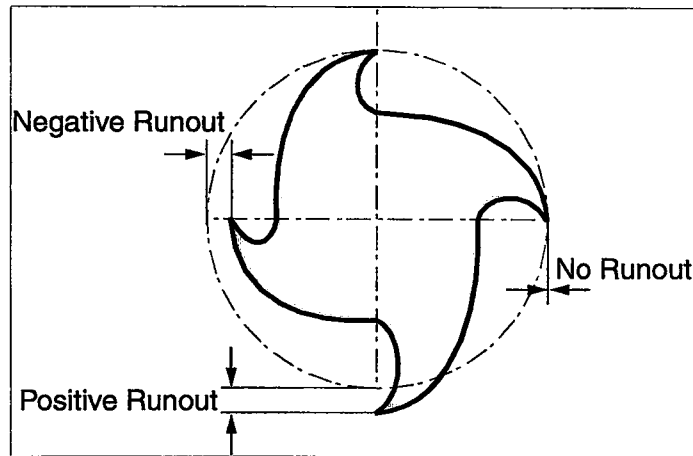


Figure 3.16. Runout definition.

Table 3.1. The dynamic parameters for the cutter.

Direction	Mode	ω_n [Hz]	ζ [%]	Stiffness [N/m]
X	1	500	5.0	$1.0 \cdot 10^{-7}$
Y	1	700	5.0	$1.0 \cdot 10^{-7}$

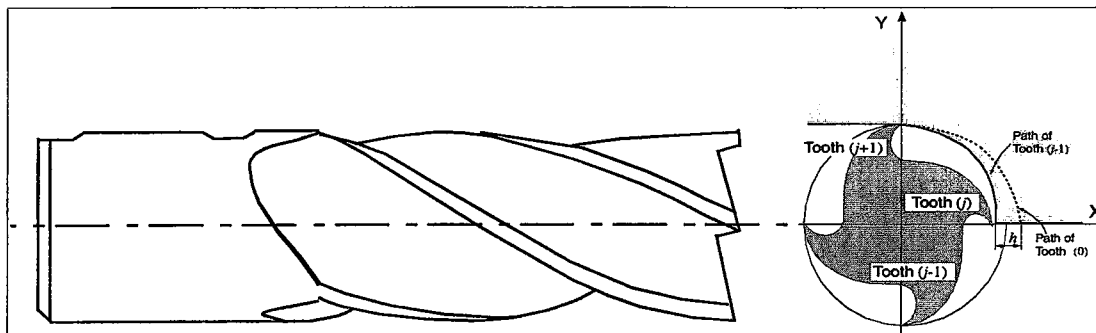


Figure 3.17. Larger runout than feed rate.

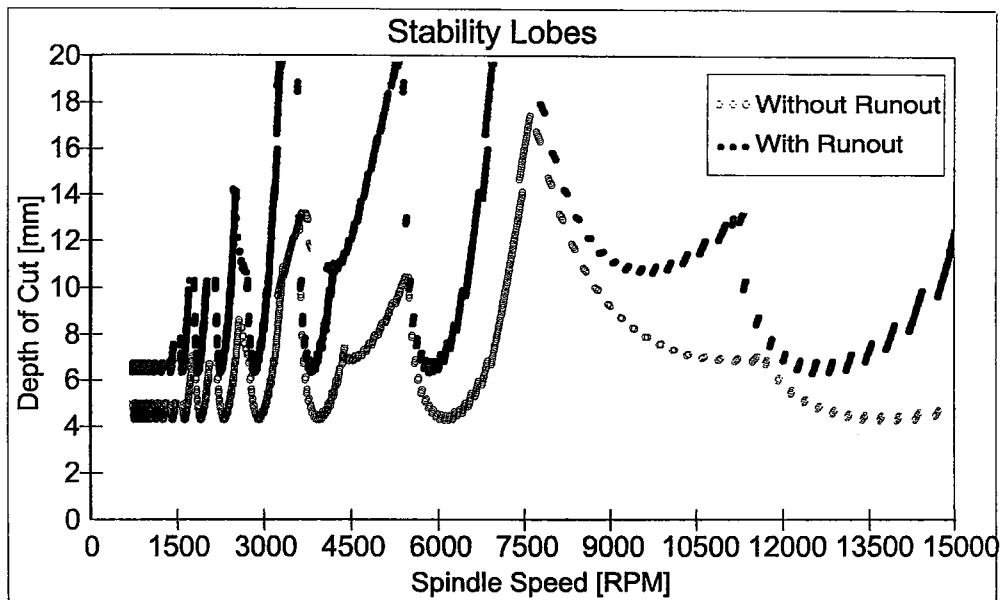


Figure 3.18. Stability lobes difference for large runout when one of the flutes is not cutting. Feed rate is 0.050 mm/tooth, runout is 0.100 mm for one of the teeth on the four-fluted cutter.

In the model, exact chip load kinematics is used. The exact position of the cutting edge on the workpiece is known in the discrete (geometric) domain. The cutting edge difference is added to the simulation program for every discrete level and tooth.

3.7.1 Sample Design Example

Using the developed runout model, different cutter geometries can be defined for the envelope of a given cutter. The cutter is divided into small discrete levels and the diameter difference is measured for every level and every cutting edge according to an existing geometry. This geometry can be a cylindrical cutter. This method can be used for defining very complex cutters, some of which are used in specific industrial applications and may have sine waves or other profiles on the cutting edge. Smaller increments along the cutter axis can be selected for complex geometries. An example can be seen in Figure 3.19. Linear interpolation is used for the runout between the measurement levels.

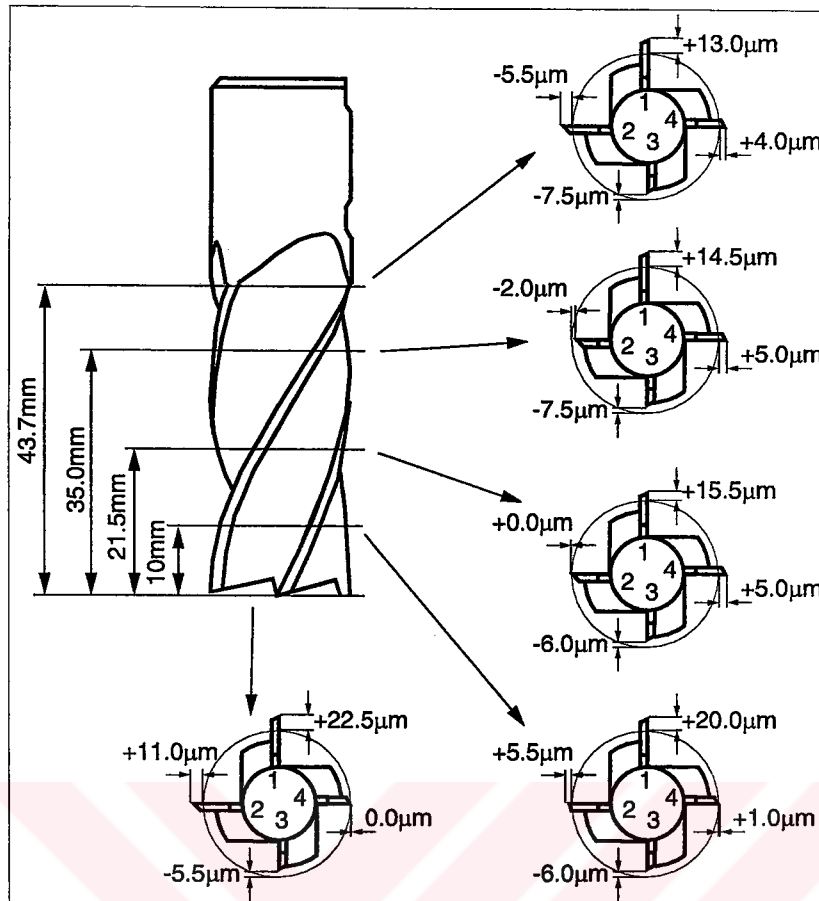


Figure 3.19. Runout measurements for different levels.

3.7.2 Simulation and Experimental Results

Using the runout model, a cylindrical ball end mill cutter has been tested (Figure 3.20). In the model the cutter is assumed to be cylindrical and the dimensional differences between the assumed cylindrical cutter and measured cutter are taken as runout. The runout values are measured for all the teeth at every 1.0 mm increment level. The imaginary cylindrical cutter is assumed to have a 1.0 inch diameter. The runout measurements are given in Table 3.2.

The experimental and simulated cutting force results are given in Figure 3.21. In this model there is no local geometrical cutting edge information. Therefore the average cutting coefficients have been used.

Table 3.2. Runout values for Ball End mill with respect to cylindrical end mill. Radius is 6.0 mm.

Level No:	Level from tool tip [mm]	Flute 1 Runout [mm]	Flute 2 Runout [mm]	Level No:	Level from tool tip [mm]	Flute 1 Runout [mm]	Flute 2 Runout [mm]
1	0.00	-6.000	-6.000	17	3.20	-0.693	-0.693
2	0.20	-4.465	-4.465	18	3.40	-0.593	-0.593
3	0.40	-3.846	-3.846	19	3.60	-0.501	-0.501
4	0.60	-3.385	-3.385	20	3.80	-0.418	-0.418
5	0.80	-3.007	-3.007	21	4.00	-0.343	-0.343
6	1.00	-2.683	-2.683	22	4.20	-0.276	-0.276
7	1.20	-2.400	-2.400	23	4.40	-0.217	-0.217
8	1.40	-2.148	-2.148	24	4.60	-0.166	-0.166
9	1.60	-1.921	-1.921	25	4.80	-0.121	-0.121
10	1.80	-1.715	-1.715	26	5.00	-0.084	-0.084
11	2.00	-1.528	-1.528	27	5.20	-0.054	-0.054
12	2.20	-1.357	-1.357	28	5.40	-0.030	-0.030
13	2.40	-1.200	-1.200	29	5.60	-0.013	-0.013
14	2.60	-1.056	-1.056	30	5.80	-0.003	-0.003
15	2.80	-0.925	-0.925	31	6.00	0.0	0.0
16	3.00	-0.804	-0.804	-	-	-	-

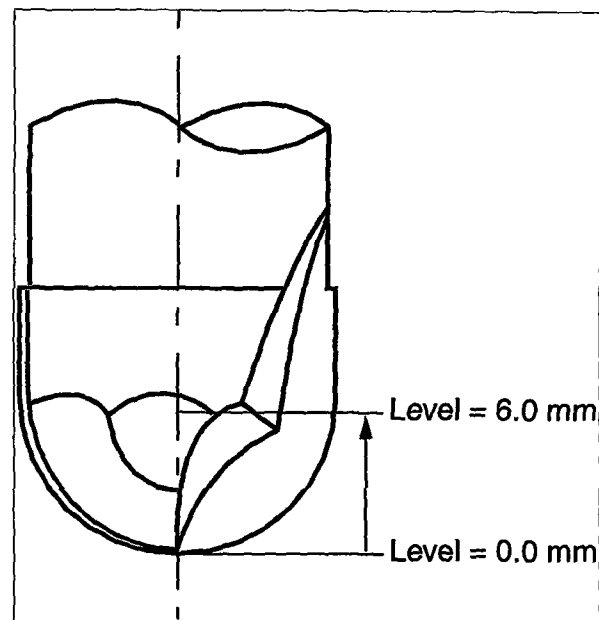


Figure 3.20. Cutter geometry for runout model.

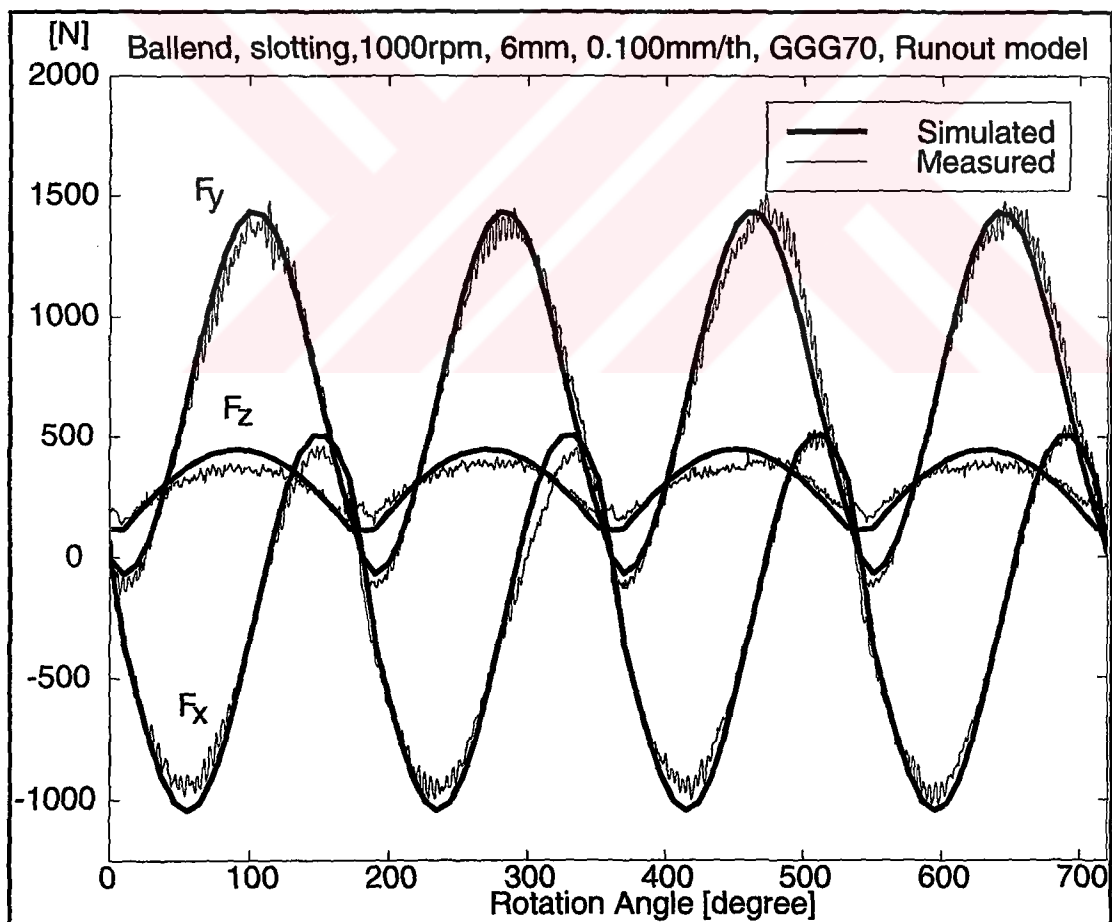


Figure 3.21. Experimental and simulated cutting forces using runout model. Cutting conditions: slotting, 0.100 mm/tooth feed rate, 6.0 mm depth of cut. Material is GGG70. Ball end mill 6.0 mm radius. Spindle speed is 1000 rev/min.

3.8 Summary

The mechanics of milling are given for general applications. Cutting coefficient identification techniques were explained for orthogonal and milling operations. The time and frequency domain simulations based on exact kinematics of milling were discussed. The runout model is developed, which allows any cutting edge profile to be imposed on a reference cutter shape, while simulating different runout values for each flute throughout the cutter axis.



4. GENERALIZED GEOMETRIC MODEL OF HELICAL END MILLING CUTTERS

4.1 Introduction

A generalized model of end mill geometry, cutting flutes and modeling of the cutting force system are introduced in this chapter. Generalized end mill geometry envelope is defined in a similar manner to the parametric representation used by APT [93] and CAD/CAM software systems. The formulation of cutting edge coordinates on the cutter is presented. It is shown that a vast variety of helical end mill geometries can be designed using the proposed geometric model. The modeled cutting edge is broken into small segments which may have varying cutting constants. As application examples, case studies involving helical ball end, tapered helical ball end, and bull nose cutters are presented. It is experimentally shown that the developed model can be used in predicting cutting forces, vibrations, dimensional surface finish, as well as chatter stability lobes for any cutter geometry within the defined envelope.

A variety of helical end mill geometries are used in industry. Helical cylindrical, helical ball, taper helical ball, bull nosed and special purpose end mills are widely used in the aerospace, automotive and die machining industries. While the geometry of each cutter can be different, the mechanics and dynamics of the milling process at each cutting edge are common. The end mill geometry is modeled by helical flutes wrapped around a parametric envelope. The coordinates of a cutting edge point along the parametric helical flute are mathematically expressed. The cutter is divided into small disk elements, enabling the mechanics and dynamics of milling to be solved for each element. The chip thickness at each cutting point is evaluated by using the true kinematics of milling, including the structural vibrations of both the cutter and workpiece. By integrating the process along each cutting edge which is in contact with the workpiece, the cutting forces, vibrations, dimensional surface finish and

chatter stability lobes for an arbitrary end mill are predicted. The predicted and measured cutting forces, surface roughness and stability lobes for ball, helical tapered ball, and bull nosed end mills are provided to illustrate the viability of the proposed generalized end mill analysis.

The objective of geometric modeling of milling cutters is to design a milling simulation system, which can handle a variety of different end mills for improved cutter design or process planning in industry.

4.2 Analytical Design of End Mill Geometry

APT and CAD/CAM systems define the envelope of milling cutters by seven geometric parameters [93]:

$$\text{CUTTER/ } D, R, R_r, R_z, \alpha, \beta, h$$

where the cutter parameters D , R , R_r , R_z , α , β and h are shown in Figure 4.1. The generalized parametric statement can define a variety of face and helical end mill shapes used in industry as shown in Figure 4.2. These seven geometric parameters are independent of each other, but some geometric constraints between them hold to ensure the reliability of the resulting profile.

The restrictions for general tool geometry design are:

- The only parameters that can take negative values are R_r and β . All other parameters must either be positive or zero,
- R_r , R_z and R must be chosen such that the resultant corner arc intersects both the first line and the second line in a manner that none of the tool surface normals in the final tool configuration intersect,
- D , h and α must be chosen such that no inconsistency will arise from their interaction,
- α and β must be chosen such that their sum is less than 90° . If β is negative, α must be less than 90° ,

- D and h must be greater than zero.

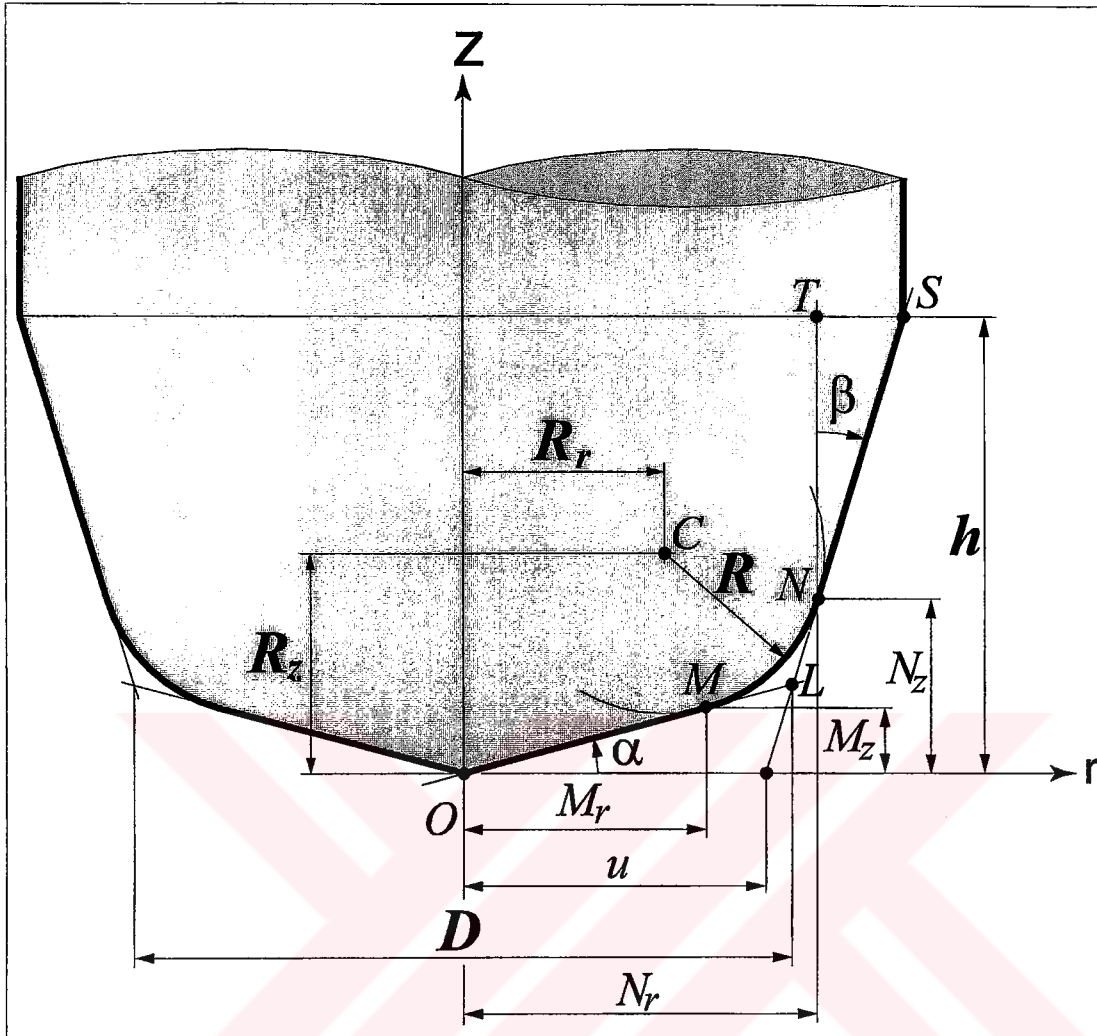


Figure 4.1. General tool geometry.

Different tool parameters are given as an example (Figure 4.2):

- To define a cylindrical end mill:

$$D \neq 0, R = 0, R_r = D/2, R_z = 0, \alpha = \beta = 0, h \neq 0$$

- To define a ball end mill:

$$D \neq 0, R = R_z = D/2, R_r = 0, \alpha = \beta = 0, h \neq 0$$

- To define a bull nose end mill:

$$D \neq 0, R = R_z \neq 0, R_r \neq 0, \alpha = \beta = 0, h \neq 0$$

- To define a taper end mill:

$$D \neq 0, R = 0, R_r = D/2, R_z = 0, \alpha = 0, \beta \neq 0, h \neq 0$$

- To define a taper ball end mill:

$$D \neq 0, R = R_z \neq 0, R_r = 0, \alpha = 0, \beta \neq 0, h \neq 0$$

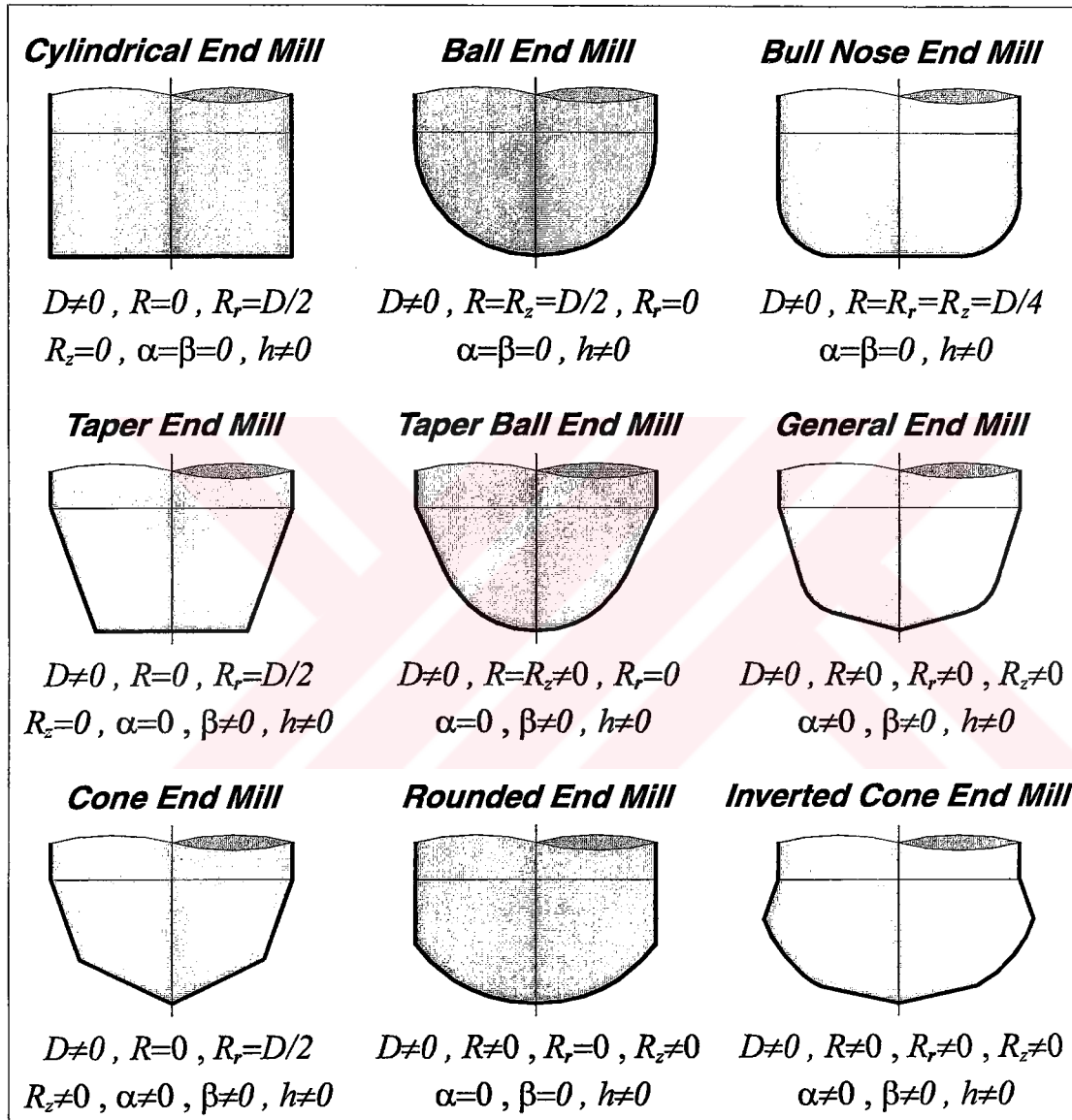


Figure 4.2. End mill shapes.

A helical cutting edge is wrapped around the end mill envelope as shown in Figure 4.3. The mechanics of cutting require the identification of coordinates, the local cutting edge geometry, chip load, and the three differential cutting forces (dF_a , dF_r , dF_t) at cutting points (i.e. P in Figure 4.3) along the cutting edge. Point P has

elevation z , radial distance $r(z)$ on XY plane, axial immersion angle $\kappa(z)$ and radial lag angle $\psi(z)$. The axial immersion angle is defined as the angle between the cutter axis and normal of the helical cutting edge at point P (Figure 4.3). The radial lag angle is the angle between the line, which connects the point P to the cutter tip on the XY plane and the cutting edge tangent at the tip of the cutter. The coordinates of P are defined by vector $\vec{r}(z)$ in cylindrical coordinates.

The periphery of the milling cutter is divided into three zones (Figure 4.1):

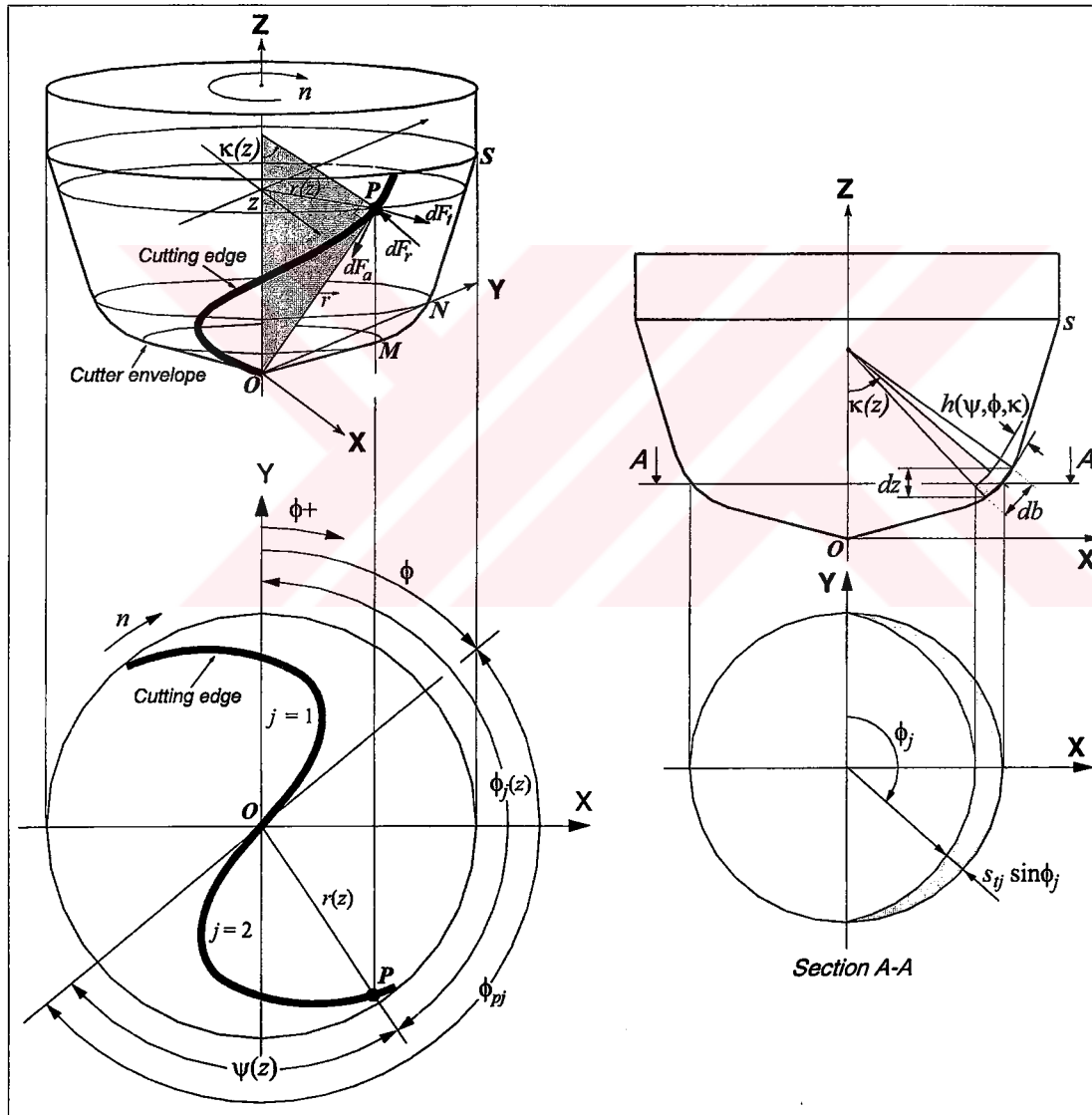


Figure 4.3. Geometric model of the general end mill.

$$\left. \begin{aligned}
r(z) &= \frac{z}{\tan \alpha} && \text{for zone } OM \\
r(z) &= \sqrt{R^2 - (R_z - z)^2} + R_r && \text{for zone } MN \\
r(z) &= u + z \tan \beta, \quad u = \frac{D}{2}(1 - \tan \alpha \tan \beta) && \text{for zone } NS
\end{aligned} \right\} \quad (4.1)$$

where $r(z)$ is the radius of the cutter at elevation z and u is the distance between the cutter tip and the point at which the NS line intersects the XY plane.

An arc with the center at point C , radial offset R_r and radius R , is tangent to or intersects the taper lines OL and LS at points M and N , respectively (Figure 4.1). The radial and axial offsets of points M and N from the cutter axis and tip are found, respectively, as:

$$\left. \begin{aligned}
M_r &= \frac{R_z \tan \alpha + R_r + \sqrt{(R^2 - R_r^2) \tan^2 \alpha + 2R_z R_r \tan \alpha - R_z^2 + R^2}}{\tan^2 \alpha + 1} \\
M_z &= M_r \tan \alpha \quad \text{for } 0 \leq \alpha < 90
\end{aligned} \right\} \quad (4.2)$$

and

$$\left. \begin{aligned}
N_r &= \frac{(R_r - u) \tan \beta + R_z - \sqrt{(R^2 - R_z^2) \tan^2 \beta + 2R_z (R_r - u) \tan \beta - (R_r - u)^2 + R^2}}{\tan^2 \beta + 1} \\
N_z &= u + N_r \tan \beta \quad \text{for } \beta < 90
\end{aligned} \right\} \quad (4.3)$$

In Figure 4.1, the lines OM and SN are not necessarily tangent to the corner arc at the points M and N . If the lines are tangent to the arc, the outer surface of the cutter will be continuous throughout the three segments, otherwise the surface will be discontinuous along with the helix angle.

The radial offset at elevation z and the axial immersion angle in the three zones are (Figure 4.1 and Figure 4.3),

$$\left. \begin{aligned}
r(z) &= \frac{z(\psi)}{\tan \alpha} && \text{for along line } OM \\
r(z) &= R_r + \sqrt{R^2 - (R_z - z(\psi))^2} && \text{for along arc } MN \\
r(z) &= N_r + (z(\psi) - N_z) \tan \beta && \text{for along line } NS
\end{aligned} \right\} \quad (4.4)$$

The angle between the cutting edge normal and Z axis is $\kappa(z)$:

$$\left. \begin{aligned}
\kappa(z) &= \alpha && \text{for along line } OM \\
\kappa(z) &= \sin^{-1} \left(\frac{r(z) - R_r}{R} \right) && \text{for along arc } MN \\
\kappa(z) &= \frac{\pi}{2} - \beta && \text{for along line } NS
\end{aligned} \right\} \quad (4.5)$$

4.2.1 Generalized Geometric Model of the Helical Cutting Edges

Helical flutes can be wrapped around the cutters as shown in Figure 4.4. A vector drawn from cutter tip (O) to any point (P) on a helical flute can be expressed as (Figure 4.3),

$$\left. \begin{aligned}
\vec{r}_j &= x_j \vec{i} + y_j \vec{j} + z_j \vec{k} \\
&= r(\phi_j) (\sin \phi_j \vec{i} + \cos \phi_j \vec{j}) + z(\phi_j) \vec{k}
\end{aligned} \right\} \quad (4.6)$$

where ϕ_j is the radial immersion angle of point P on flute number j . The radial immersion angle (ϕ_j) varies as a function of rotation angle, flute position, and the local helix angle at point P . The first flute ($j=1$) is considered to be a reference edge, and its rotation angle at the elevation $z=0$ is ϕ . The immersion angle for flute j at axial location z is expressed as:

$$\phi_j(z) = \phi + \sum_{n=1}^j \phi_{p,n} - \psi(z) \quad (4.7)$$

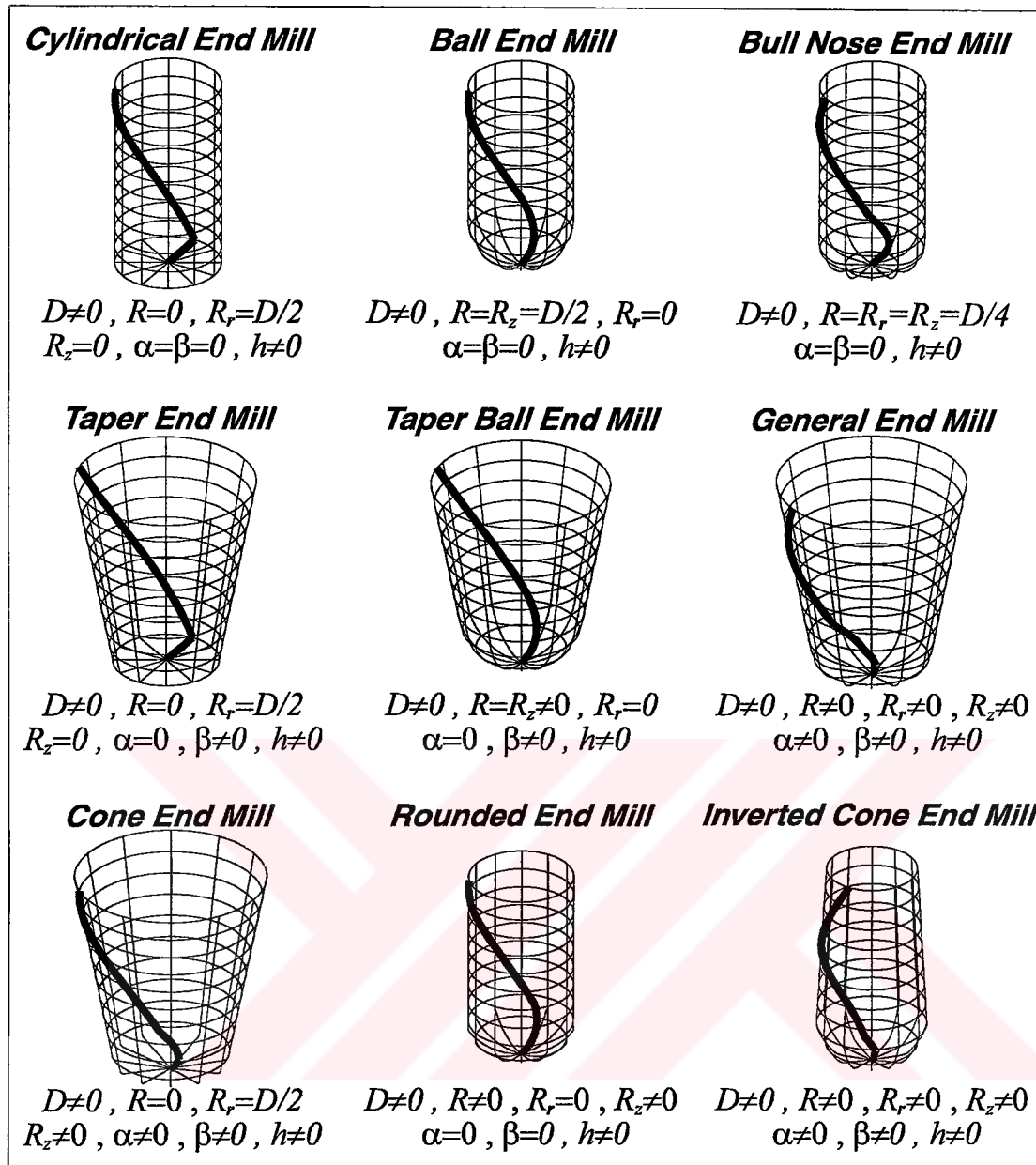


Figure 4.4. Helical cutting edges on the tool.

where $\phi_{p,n}$ is the pitch angle between the preceding flutes. It should be noted that this general formulation is valid for variable pitch cutters as well. The radial lag angle $\psi(z)$ is due to the local helix angle i . Since the cutter diameter can be different along the axis, the helix and thus lag angles vary along the flute as well, and they are evaluated in the following subsection.

An infinitesimal length (dS) of a helical cutting edge segment can be given as follows,

$$\left. \begin{aligned} dS &= |dr| = \sqrt{r^2(\phi) + (r'(\phi))^2 + (z'(\phi))^2} d\phi \\ r'(\phi) &= \frac{dr(\phi)}{d\phi} \\ z'(\phi) &= \frac{dz(\phi)}{d\phi} \end{aligned} \right\} \quad (4.8)$$

The chip thickness changes with both radial (ϕ_j) and axial immersion (κ) as the following,

$$\left. \begin{aligned} h(\phi_j) &= s_{ij} \sin \phi_j \sin \kappa \\ s_{ij} &= \frac{\text{feedrate [mm/min]} \phi_{pj}}{n [\text{rev/min}] 2\pi} \end{aligned} \right\} \quad (4.9)$$

where s_{ij} and ϕ_{pj} are the feed per tooth and pitch angle for tooth j , respectively. Note that the effective feed for each tooth may be different when variable pitch cutters are used. The lag angle $\psi(z)$ has a different expression for each zone along the flute, which is wrapped around general cutter geometry.

Zone OM ($z \leq M_z$): The helix angle is assumed to be constant at this usually small cone part, i.e. $i(z) = i_o$. Ramaraj [22] gives the differential equation and its solution for a helical flute spiraled on a cone as,

$$\left. \begin{aligned} \frac{dr}{d\psi} - \frac{\cos \alpha}{\tan i_o} r &= 0 \\ r &= e^{(\cos \alpha / \tan i_o) \psi} \end{aligned} \right\} \quad (4.10)$$

Since the spiral radius is zero at the tool tip, the lag angle condition becomes $\psi(0) = -\infty$. Instead, the simulations were started with a small radial offset from the tip, i.e. $r_s = M_r / 20$, which gives a starting lag angle of $\psi_{1s} = \ln r_s \tan i_o / \cos \alpha$. The variation of lag angle along the helical flute is evaluated from equation (4.10),

$$\psi(z) = \frac{\ln(z \cot \alpha) \tan i_o}{\cos \alpha} - \psi_{1s} \quad (4.11)$$

The final lag angle at the point M becomes

$$\psi_{1e} = \frac{\ln M_r \tan i_o}{\cos \alpha} - \psi_{1s} \quad (4.12)$$

If the cone zone (*OM*) does not exist (i.e. bull nose and ball end cutters), α , ψ_{1s} and ψ_{1e} will be zero.

Arc Zone MN ($M_z < z \leq N_z$): Due to changing radial offset from the cutter axis, the helix angle varies along the flute for constant lead cutters as,

$$i(z) = \tan^{-1} \left[\frac{(r(z) - R_r) \tan i_o}{R} \right] \quad (4.13)$$

Since the arc is not a full quarter due to tangency to the cone, there is a discontinuity on the helix angle at point *M*. This leads to the following lag angle expression,

$$\psi(z) = \frac{(R + z - R_z) \tan i_o}{R} - \psi_{as} + \psi_{1e} \quad (4.14)$$

where ψ_{1e} and ψ_{as} are the final lag angles at point *M* formed by the cone and arc, respectively. The lag angle (ψ_{as}) formed by the arc at point *M* is,

$$\psi_{as} = \frac{(R + M_z - R_z) \tan i_o}{R} \quad (4.15)$$

The final lag angle at the end point *N* of the arc is:

$$\psi_{ae} = \frac{(R + N_z - R_z) \tan i_o}{R} - \psi_{as} + \psi_{1e} \quad (4.16)$$

If the arc is missing from the cutter geometry $R = 0$, $\rightarrow \psi_{as} = 0$, $\psi_{ae} = \psi_{1e}$.

Taper Zone NS ($N_z < z$): The taper zone of the cutters is ground either with a constant lead or constant helix. The constant lead, which leads to variable helix angle along the flute, is preferred by the cutter grinders in order to save material during re-grinding operation. However, cutting mechanics are more uniform with constant helix cutters, which require varying lead. Both methods are modeled here.

Constant helix: The helix angle is constant, and the lag angle changes along the flute:

$$\begin{aligned}
 i(z) &= i_o \\
 \psi(z) &= \frac{\ln(N_r - (N_z - z)\tan\beta)\tan i_o}{\sin\beta} - \psi_{2s} + \psi_{ae} \quad \text{if } \beta \neq 0 \\
 \psi(z) &= \frac{(z - N_z)\tan i_o}{N_r} - \psi_{2s} + \psi_{ae} \quad \text{if } \beta = 0
 \end{aligned} \quad (4.17)$$

where ψ_{ae} and ψ_{2s} are the final lag angle at the point N from the arc and the initial lag angle generated by the taper zone (NS) at point N , respectively. The initial value ψ_{2s} is,

$$\begin{aligned}
 \psi_{2s} &= \frac{\ln(N_r)\tan i_o}{\sin\beta} \quad \text{if } \beta \neq 0 \\
 \psi_{2s} &= 0 \quad \text{if } \beta = 0
 \end{aligned} \quad (4.18)$$

The final value ψ_{2e} at point S is,

$$\begin{aligned}
 \psi_{2e} &= \frac{\ln(N_r - (N_z - h)\tan\beta)\tan i_o}{\sin\beta} - \psi_{2s} + \psi_{ae} \quad \text{if } \beta \neq 0 \\
 \psi_{2e} &= \frac{(h - N_z)\tan i_o}{N_r} + \psi_{ae} \quad \text{if } \beta = 0
 \end{aligned} \quad (4.19)$$

Constant Lead: The lead of the helical flute is constant, and the helix varies along the flute. The nominal helix (i_s) and lead ($lead$) of the tapered flute is defined at point N ,

$$i_s = \tan^{-1} \left[\frac{2\pi N_r}{lead \cdot \cos\beta} \right] \quad (4.20)$$

which leads to a variable helix expression,

$$i(z) = \tan^{-1} \left[\frac{(\psi - \psi_{ae}) r(z)}{z - N_z} \right] \quad (4.21)$$

The variation of the lag angle $\psi(z)$ is given by,

$$\psi(z) = \frac{(z - N_z) \tan i_s}{N_r} + \psi_{ae} \quad \text{for } N_z \leq z \leq h \quad (4.22)$$

4.2.2 Finding z [$z(\psi)$] Value For a Given Lag Angle [ψ]

The $z(\psi)$ value can be found from $\psi(z)$ equations given before. The solutions are different at different zones.

Zone OM ($z \leq M_z$): The helix angle is constant at this zone $i(z) = i_o$. By taking z from equation (4.11),

$$z(\psi) = \frac{e^{\left[\frac{(\psi + \psi_{1s}) \cos \alpha}{\tan i_o} \right]}}{\cot \alpha} \quad (4.23)$$

Arc Zone MN ($M_z < z \leq N_z$): From equation (4.14),

$$z(\psi) = \frac{(\psi + \psi_{as} - \psi_{1e})R}{\tan i_o} + R_z - R \quad (4.24)$$

Taper Zone NS ($N_z < z$):

Constant helix: The helix angle is constant, and the lag angle changes along the flute. From equation (4.17):

$$\left. \begin{aligned} z(\psi) &= \frac{(\psi + \psi_{2s} - \psi_{ae})N_r}{\tan i_o} + N_z & \text{if } \beta = 0 \\ z(\psi) &= \frac{e^{\left[\frac{(\psi + \psi_{2s} - \psi_{ae}) \sin \beta}{\tan i_o} \right]} - N_r}{\tan \beta} + N_z & \text{if } \beta \neq 0 \end{aligned} \right\} \quad (4.25)$$

Constant Lead: The lead of the helical flute is constant, and the helix varies along the flute. From equation (4.22),

$$z(\psi) = \frac{(\psi - \psi_{ae})N_r}{\tan i_s} + N_z \quad (4.26)$$

4.3 Modeling of Cutting Forces

The differential tangential (dF_t), radial (dF_r) and axial (dF_a) cutting forces acting on infinitesimal cutting edge segment are given by Altıntaş and Lee [20],

$$\left. \begin{aligned} dF_t &= K_{te} dS + K_{tc} h(\phi, \kappa) db \\ dF_r &= K_{re} dS + K_{rc} h(\phi, \kappa) db \\ dF_a &= K_{ae} dS + K_{ac} h(\phi, \kappa) db \end{aligned} \right\} \quad (4.27)$$

where $h(\phi, \kappa)$ is the uncut chip thickness normal to the cutting edge and varies with the position of the cutting point and cutter rotation. Subindices (c) and (e) represent shear and edge force components, respectively. The edge cutting coefficients K_{te} , K_{re} and K_{ae} are constants and relate to the cutting edge length dS given in equation (4.8). The shear force coefficients K_{tc} , K_{rc} , and K_{ac} are identified either mechanistically from milling tests conducted at a range of feed rates [13,21] or a set of orthogonal cutting tests using an oblique transformation method presented by Budak et al. [17] in 1996. db ($db = dz / \sin \kappa$) is the projected length of an infinitesimal cutting flute in the direction of the cutting velocity. The chip thickness $h(\phi, \kappa)$ is evaluated using the true kinematics of milling by Martellotti [4] in 1945 as well as the vibration of both the cutter and workpiece. The cutter is rotated at the spindle speed and the workpiece is fed with the given feed using a small discrete time interval. The positions of cutting points along the flute are evaluated using the geometric model. The location of the same flute point on the cut surface is identified using both the rigid body kinematics as well as structural displacements of the cutter and workpiece. The dynamic chip thickness is evaluated by subtracting the present coordinate of the cutting point from the previous surface generated by the preceding tooth. The mathematical model and the procedure to evaluate dynamic chip load is well explained for helical cylindrical and ball end mills by Montgomery and Altıntaş

[10] in 1991 and Altıntaş and Lee [20] in 1998. Once the chip load is identified and cutting constants are evaluated for the local edge geometry, the cutting forces in the Cartesian coordinate system can be evaluated as,

$$\{dF_{xyz}\} = [T] \{dF_{ra}\} \quad (4.28)$$

or

$$\begin{bmatrix} dF_x \\ dF_y \\ dF_z \end{bmatrix} = \begin{bmatrix} -\sin \phi \sin \kappa & -\cos \phi & -\sin \phi \cos \kappa \\ -\cos \phi \sin \kappa & \sin \phi & -\cos \phi \cos \kappa \\ -\cos \kappa & 0 & -\sin \kappa \end{bmatrix} \begin{bmatrix} dF_r \\ dF_t \\ dF_a \end{bmatrix} \quad (4.29)$$

The total cutting forces for the rotational position ϕ can be found by integrating equation (4.29) along the axial depth of cut for all cutting flutes which are in contact with the workpiece.

$$\left. \begin{aligned} F_x(\phi) &= \sum_{j=1}^{N_f} F_{xj}[\phi(z)] \\ F_y(\phi) &= \sum_{j=1}^{N_f} F_{yj}[\phi(z)] \\ F_z(\phi) &= \sum_{j=1}^{N_f} F_{zj}[\phi(z)] \end{aligned} \right\} \quad (4.30)$$

where N_f is the number of flutes on the cutter. The differential cutting forces can be evaluated as follows,

$$\left. \begin{aligned} F_{xj}[\phi(z)] &= \int_{z_1}^{z_2} \begin{bmatrix} -dF_{rj} \sin \phi_j \sin \kappa_j & -dF_{tj} \cos \phi_j & -dF_{aj} \sin \phi_j \cos \kappa_j \end{bmatrix} dz \\ F_{yj}[\phi(z)] &= \int_{z_1}^{z_2} \begin{bmatrix} -dF_{rj} \cos \phi_j \sin \kappa_j & dF_{tj} \sin \phi_j & -dF_{aj} \cos \phi_j \cos \kappa_j \end{bmatrix} dz \\ F_{zj}[\phi(z)] &= \int_{z_1}^{z_2} \begin{bmatrix} -dF_{rj} \cos \phi_j & & -dF_{aj} \sin \kappa_j \end{bmatrix} dz \end{aligned} \right\} \quad (4.31)$$

where z_1 and z_2 are the contact boundaries of the flute within the cut and can be found from the geometric model of each zone. The cutter is axially digitized with small disk elements with a uniform differential height of dz . The differential cutting forces are calculated along the full contact length for all flutes which are in-cut, and

digitally summed to find the total cutting forces $F_x(\phi)$, $F_y(\phi)$ and $F_z(\phi)$ at a given rotation angle $\phi = \Omega \cdot dt$ where $\Omega(\text{rad/s})$ is the spindle speed and dt is the differential time interval for digital integration.

The average cutting coefficients can be found from equation (4.27) and (4.29). If the equation (4.27) is substituted into the equation (4.29), the following equation is obtained,

$$\begin{bmatrix} dF_x \\ dF_y \\ dF_z \end{bmatrix} = \begin{bmatrix} -\sin \phi \sin \kappa & -\cos \phi & -\sin \phi \cos \kappa \\ -\cos \phi \sin \kappa & \sin \phi & -\cos \phi \cos \kappa \\ -\cos \kappa & 0 & -\sin \kappa \end{bmatrix} \begin{bmatrix} K_{re} dS + K_{rc} h(\phi, \kappa) db \\ K_{le} dS + K_{lc} h(\phi, \kappa) db \\ K_{ae} dS + K_{ac} h(\phi, \kappa) db \end{bmatrix} \quad (4.32)$$

This can be separated into two different parts: cutting and edge forces. The cutting force part is,

$$[dF_c] = [T][K_c] h(\phi, \kappa) db \quad (4.33)$$

The edge force part is,

$$[dF_e] = [T][K_e] dS \quad (4.34)$$

Total cutting forces are given,

$$[dF_e] + [dF_c] = [T] \{ [K_e] dS + [K_c] h(\phi, \kappa) db \} \quad (4.35)$$

The cutting coefficients can be calculated from these equations as follows,

$$[K_{lc} \quad K_{rc} \quad K_{ac}]^T = [T]^{-1} [dF_c] \frac{1}{h(\phi, \kappa) db} \quad (4.36)$$

The edge force part is,

$$[K_{le} \quad K_{re} \quad K_{ae}]^T = [T]^{-1} [dF_e] \frac{1}{dS} \quad (4.37)$$

where $[T]^{-1}$ is,

$$[T]^{-1} = \frac{1}{2\cos^2 \kappa - 1} \begin{bmatrix} \sin \phi \sin \kappa & \cos \phi \sin \kappa & -\cos \kappa \\ -\cos \phi & \sin \phi & 0 \\ -\sin \phi \cos \kappa & -\cos \phi \cos \kappa & \sin \kappa \end{bmatrix} \quad (4.38)$$

The structural dynamics of both the cutter and the workpiece are measured at the tool tip, and the modes are identified using experimental modal analysis technique. The dynamic chip thickness, cutting forces, vibrations, surface finish, torque and moments generated by the interaction of cutting forces and structural dynamics are simulated in time domain using a technique similar to one presented for ball end milling by Altıntaş and Lee [20] in 1998. The chatter stability lobes are predicted using both the time domain simulations and the frequency domain analytical chatter stability prediction method [20,67,71].

4.4 Cutting Coefficients

4.4.1 Identification of Cutting Coefficients Along the Cutting Edge

Average cutting coefficients are calculated for material types Al7075 and Ti₆Al₄V, using a bull nose cutter (Figure 4.5). A hydraulic chuck has been used for static cutting force measurement because of its rigidity. Transfer function measurement parameters of the hydraulic chuck with an attached bull nose cutter (Figure 4.5) are given in Table 4.1 (Figure 4.6). There is no mode below 500 Hz.

4.4.1.1 Average Cutting Coefficients for Al-7075:

The average cutting forces were evaluated from more than 50 cutting tests for different depths of cut and feed rates. The average edge and cutting forces can be seen in Figure 4.7. The cutting tests were done from 0.5 mm up to 4.5 mm depth of cut and feed rates were selected from 0.025 mm/th up to 0.200 mm/th. Because the CNC Machining Center did not have enough torque to handle high feed rates at large depths of cut, tests could not be conducted for all feed rates after a 2.5 mm depth of cut.

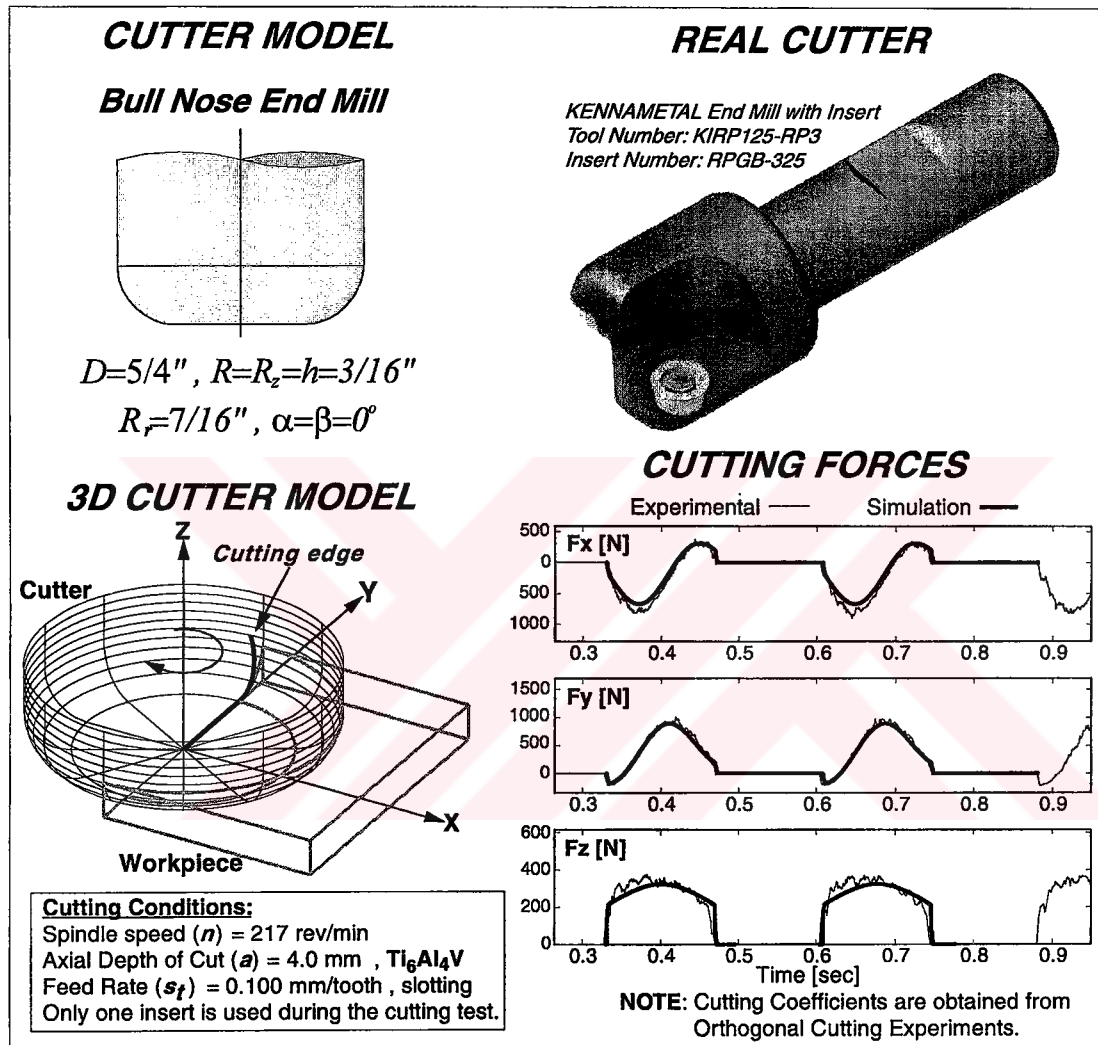
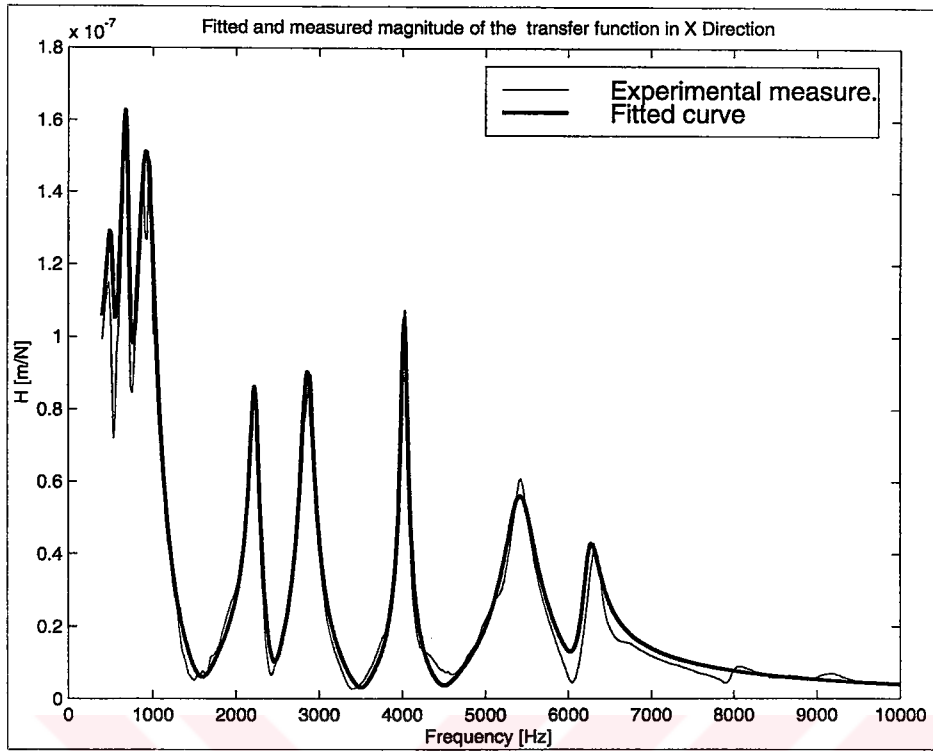


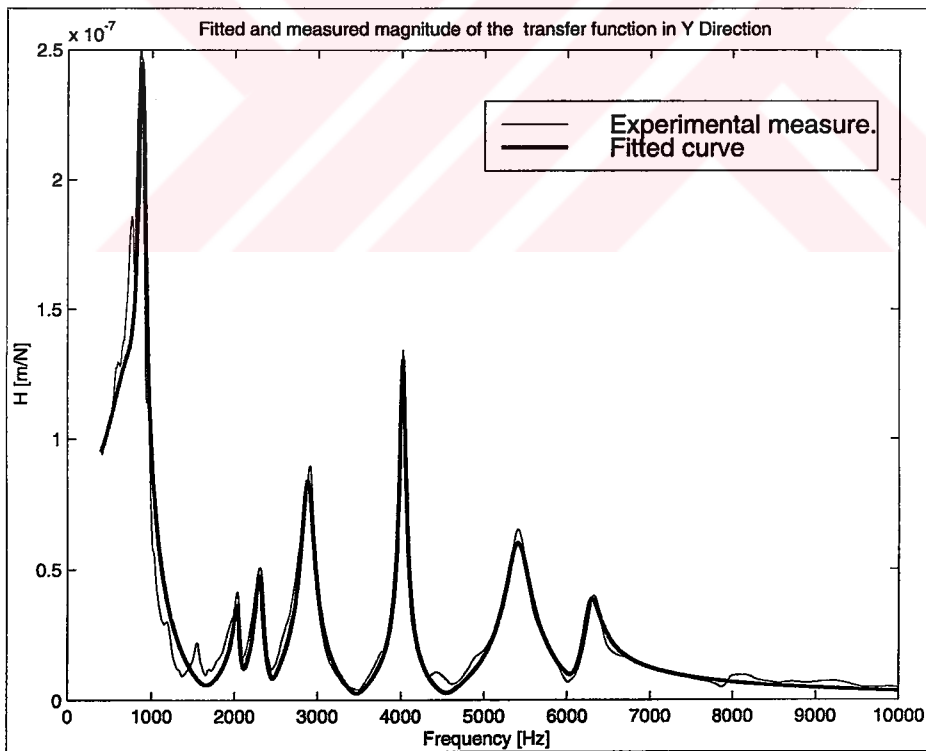
Figure 4.5. General tool geometry application for inserted end mill (See Table 4.4 for the orthogonal cutting coefficients).

Table 4.1. Measured modal parameters of the bull nose inserted cutter on a machining center with a hydraulic chuck.

Direction	Mode	ω_n [Hz]	ζ [%]	Mode Residue [m/N]
X	1	516.19	9.54	$(-10.115 - i 13.811) \cdot 10^{-6}$
	2	689.32	7.45	$(1.3455 - i 32.578) \cdot 10^{-6}$
	3	932.56	13.0	$(7.9385 - i 110.33) \cdot 10^{-6}$
	4	2233.04	2.73	$(-3.4317 - i 31.198) \cdot 10^{-6}$
	5	2863.24	2.90	$(0.40901 - i 46.275) \cdot 10^{-6}$
	6	4024.26	1.01	$(0.81921 - i 26.280) \cdot 10^{-6}$
	7	5413.30	3.59	$(-14.787 - i 67.282) \cdot 10^{-6}$
	8	6251.76	1.69	$(9.8471 - i 22.171) \cdot 10^{-6}$
Y	1	880.37	5.19	$(-28.977 - i 49.567) \cdot 10^{-6}$
	2	785.21	28.6	$(65.683 - i 92.650) \cdot 10^{-6}$
	3	2034.77	1.83	$(0.56967 - i 6.6662) \cdot 10^{-6}$
	4	2308.09	2.05	$(0.92997 - i 12.694) \cdot 10^{-6}$
	5	2883.98	2.62	$(-6.2268 - i 39.260) \cdot 10^{-6}$
	6	4020.51	0.82	$(0.64399 - i 27.111) \cdot 10^{-6}$
	7	5403.18	2.77	$(-10.720 - i 55.490) \cdot 10^{-6}$
	8	6274.37	1.67	$(3.1361 - i 21.641) \cdot 10^{-6}$



a) X Direction



b) Y Direction

Figure 4.6. Transfer function of the system from tool tip with Hydraulic Chuck, (See Table 4.1 for parameters)

dF_c values were calculated from equation (4.33) by taking K_c as an identity matrix. The total transformation matrix $[T]$, chip load h and projected length db value have been calculated. This value contains cutting conditions and parameters of the tool geometry. The calculation was repeated for every depth of cut. The edge forces were calculated from the experiments for different depths of cut and feed rates. The average cutting forces were calculated and linear equations were fit for every depth of cut (Figure 4.7). The edge forces were estimated by extrapolating the fit linear equation to zero feed rate. The pure cutting forces were calculated by subtracting the edge forces from the average forces.

The average cutting and edge force coefficients were calculated from equations (4.36) and (4.37) for every axial depth of cut. The cutting and edge coefficients can be seen in Figure 4.8. These coefficients appear to be independent of the depth of cut. Therefore, the average coefficients were taken (see Table 4.2). Comparison between experimental and simulated cutting forces are presented in Figure 4.9.

Table 4.2. The average cutting and edge force cutting coefficients for Al7075 (see Figure 4.5 for the geometry of the tool).

Depth of cut [mm]	K_{tc} [N/mm ²]	K_{rc} [N/mm ²]	K_{ac} [N/mm ²]	K_{te} [N/mm]	K_{re} [N/mm]	K_{ae} [N/mm]
0.5	1364.42	983.18	29.54	18.66	24.44	0.99
1.0	1304.46	863.82	0.74	19.06	24.85	0.65
1.5	1224.83	769.58	23.66	18.96	24.64	1.44
2.0	1260.45	736.20	51.90	19.91	26.02	2.15
2.5	1306.02	715.45	29.86	19.01	27.11	2.07
3.0	1317.84	667.22	70.98	22.67	37.24	4.17
3.5	1346.92	763.13	75.24	18.57	26.08	2.41
4.0	1380.46	786.38	84.11	20.94	28.68	2.56
4.5	1367.87	692.93	96.42	22.08	32.32	4.15
Average	1319.41	788.83	48.751	19.65	26.77	2.05

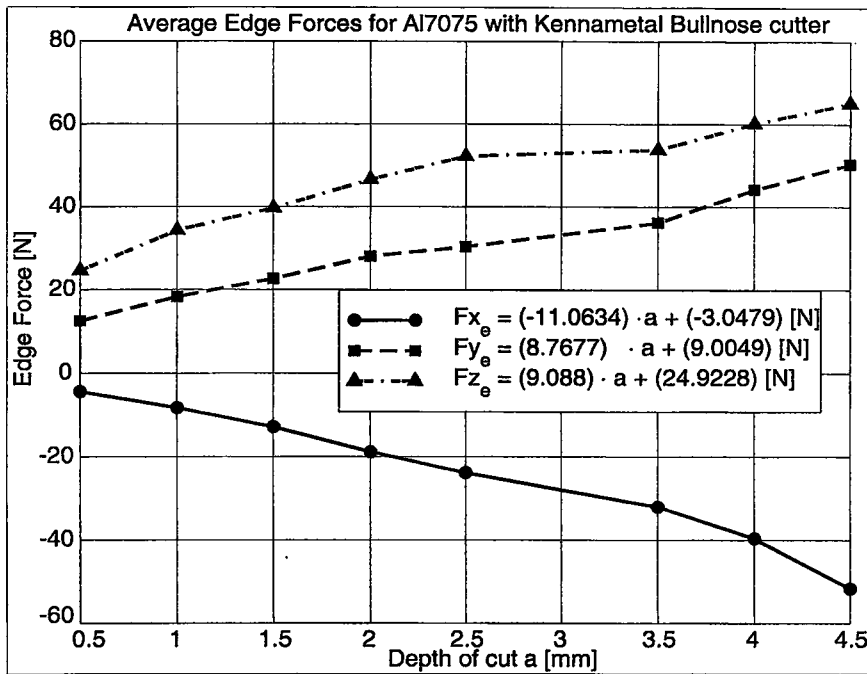
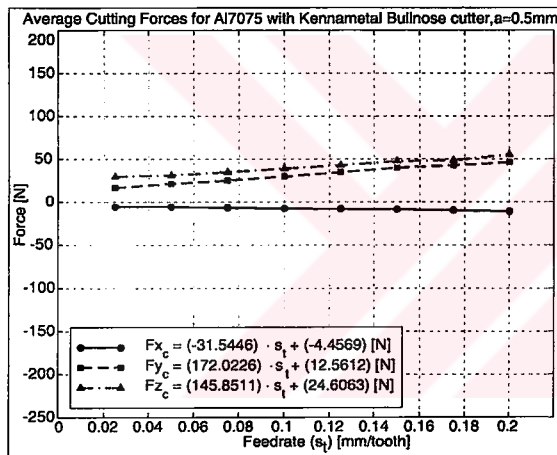
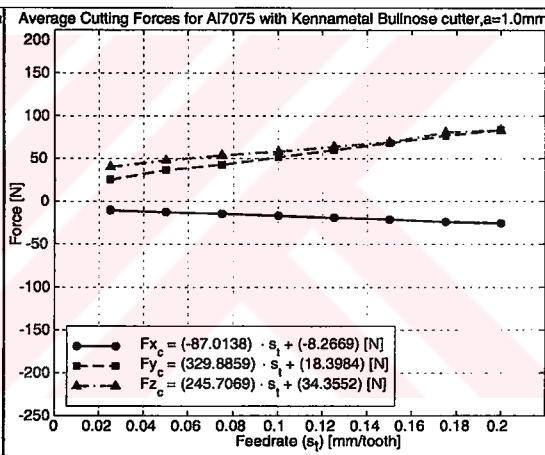


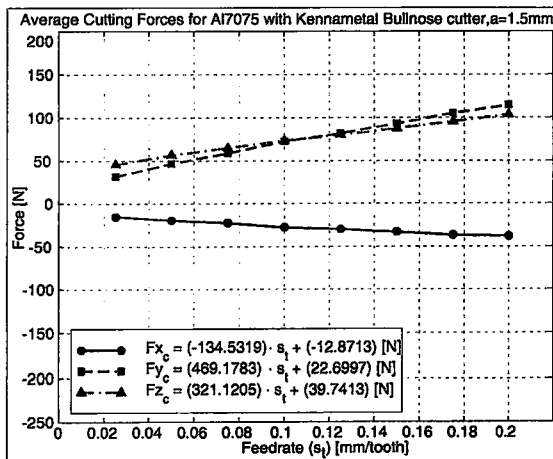
Figure 4.7 a) Edge forces



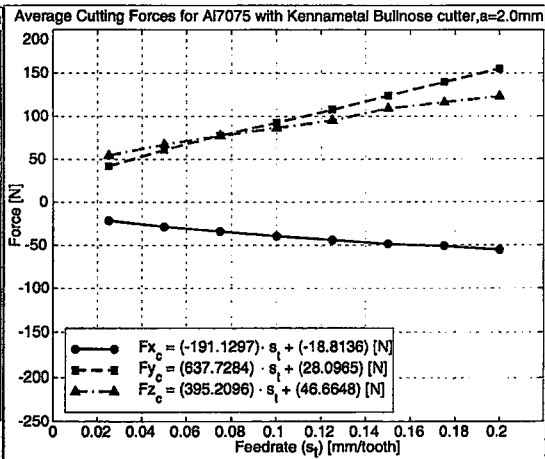
b) Cutting forces, axial depth 0.5 mm



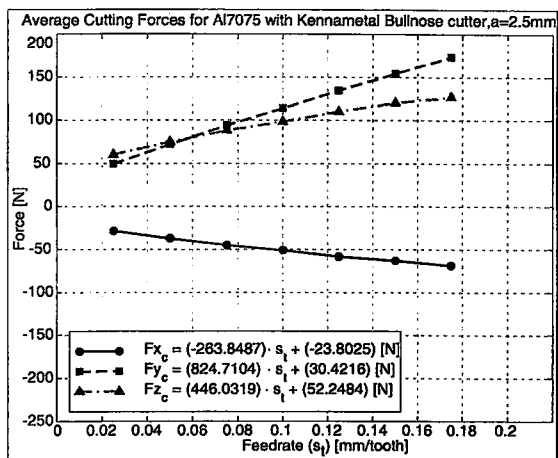
c) Cutting forces, axial depth 1.0 mm



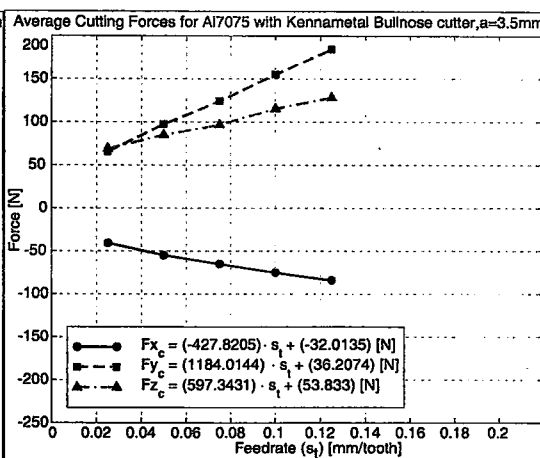
d) Cutting forces, axial depth 1.5 mm



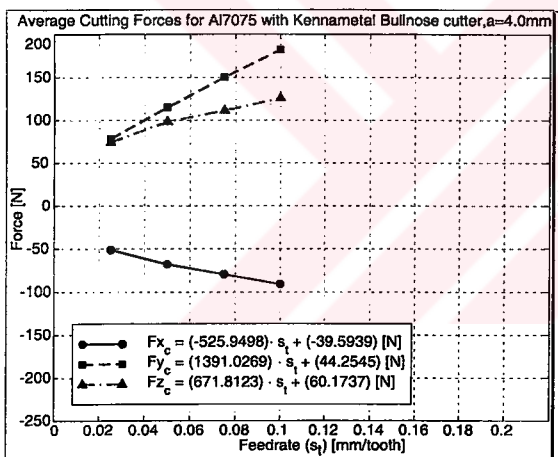
e) Cutting forces, axial depth 2.0 mm



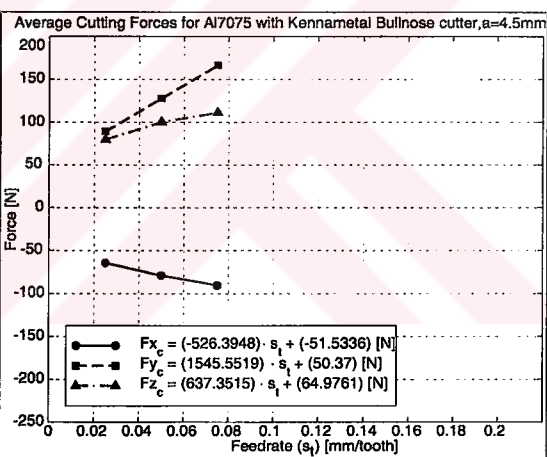
f) Cutting forces, axial depth 2.5 mm



g) Cutting forces, axial depth 3.5 mm



h) Cutting forces, axial depth 4.0 mm



i) Cutting forces, axial depth 4.5 mm

Figure 4.7. Average cutting forces for Al7075 and Kennametal Bull nose carbide cutter (See Figure 4.5 for tool geometry).

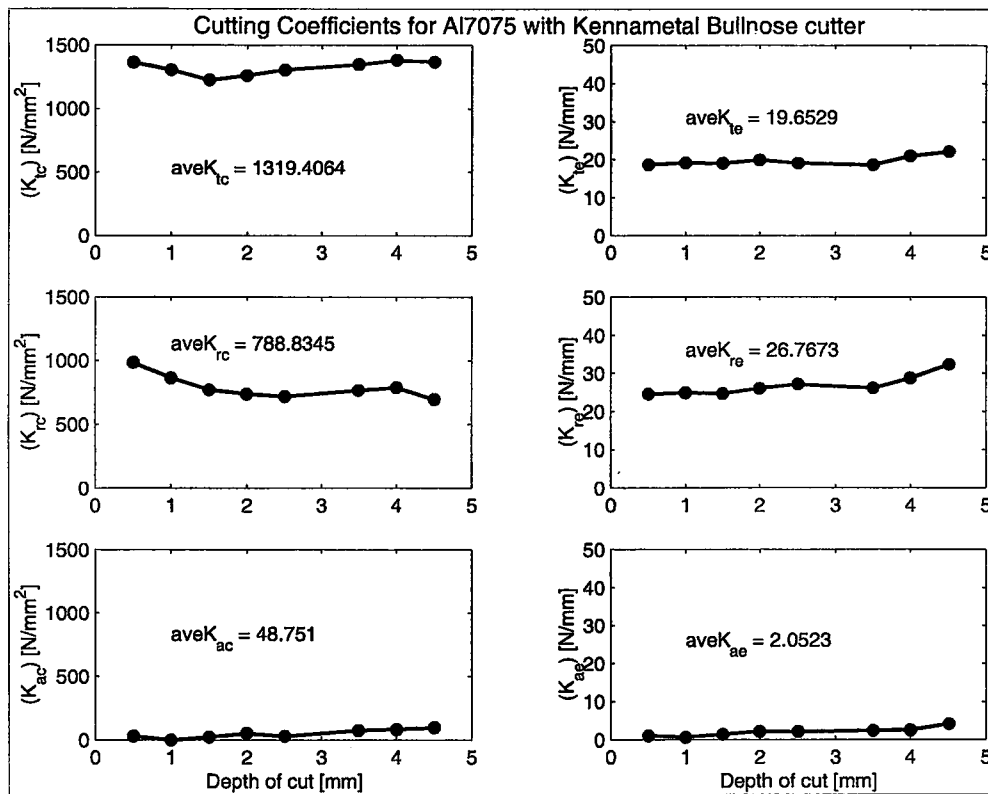


Figure 4.8. Average cutting coefficients for Al7075 (See Table 4.2)

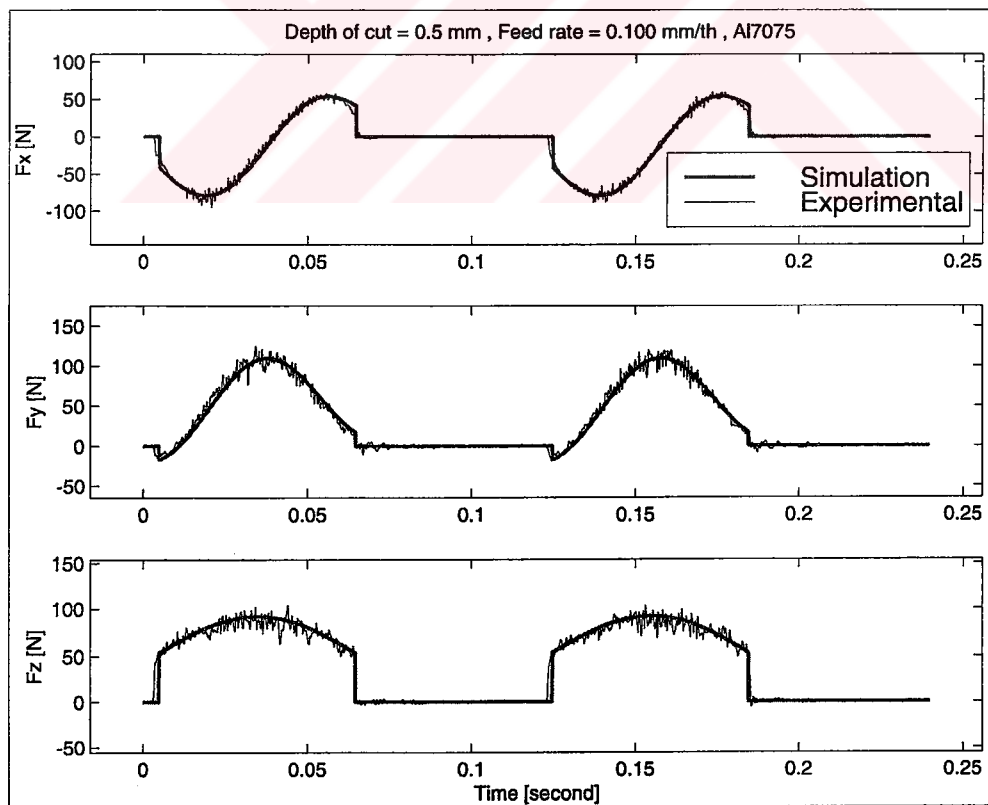


Figure 4.9 a) Depth of cut 0.5 mm, feed rate 0.100 mm/th

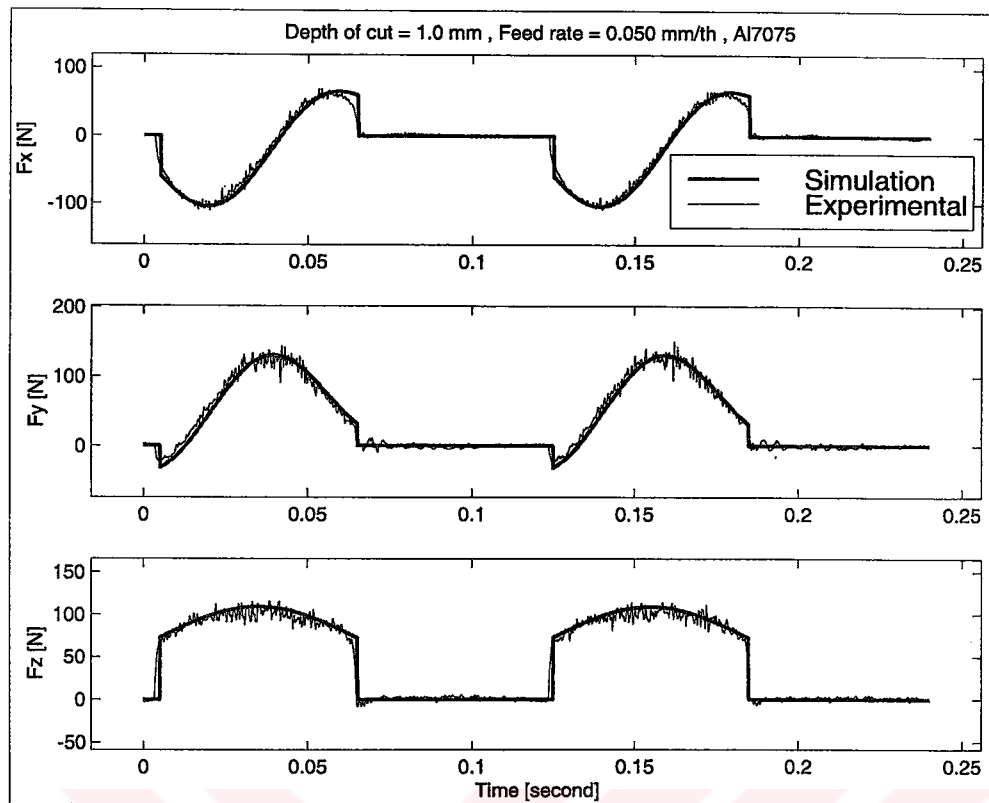


Figure 4.9 b) Depth of cut 1.0 mm, feed rate 0.050 mm/th

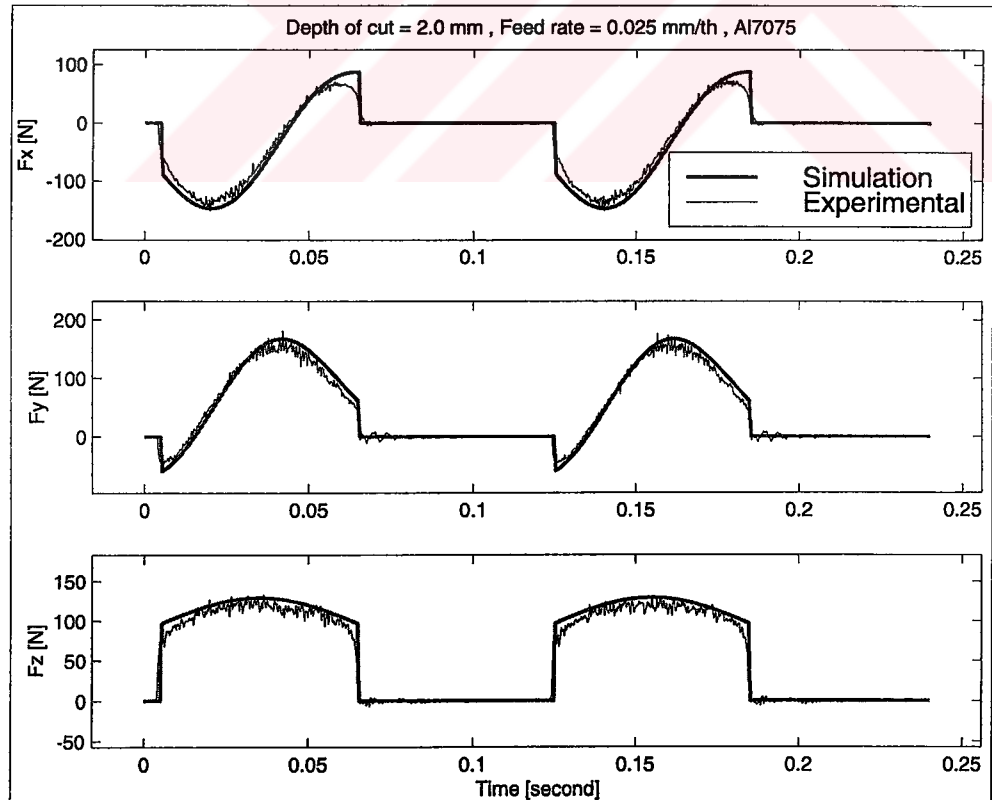


Figure 4.9 c) Depth of cut 2.0 mm, feed rate 0.025 mm/th

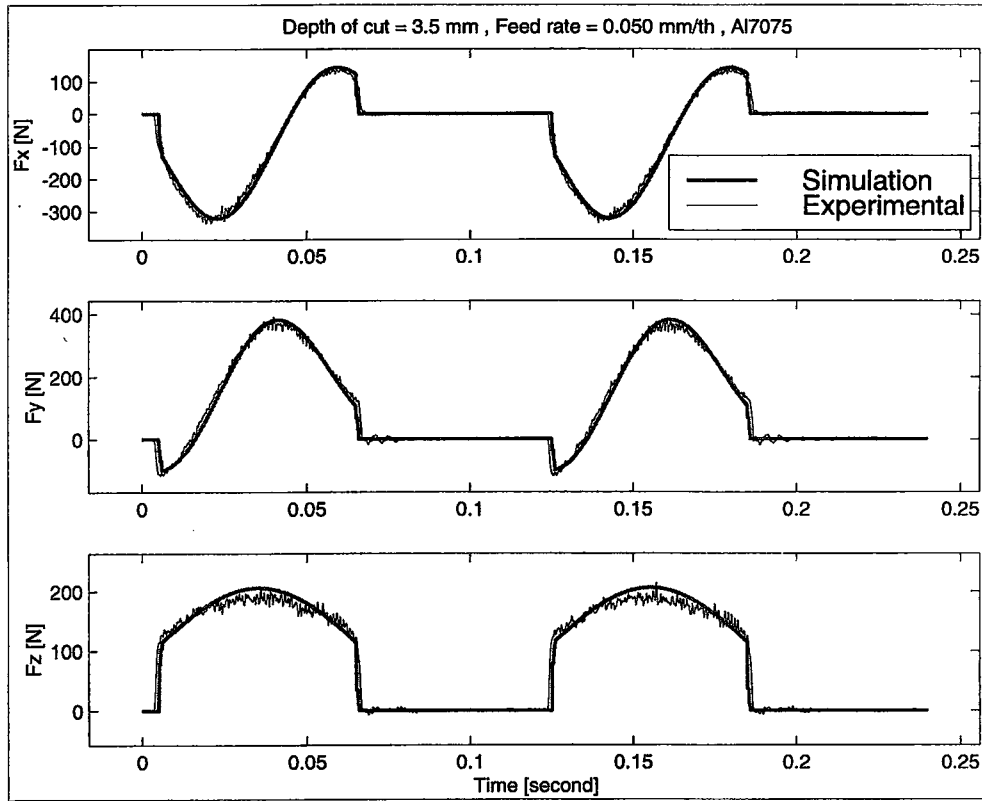


Figure 4.9 d) Depth of cut 3.5 mm, feed rate 0.050 mm/th

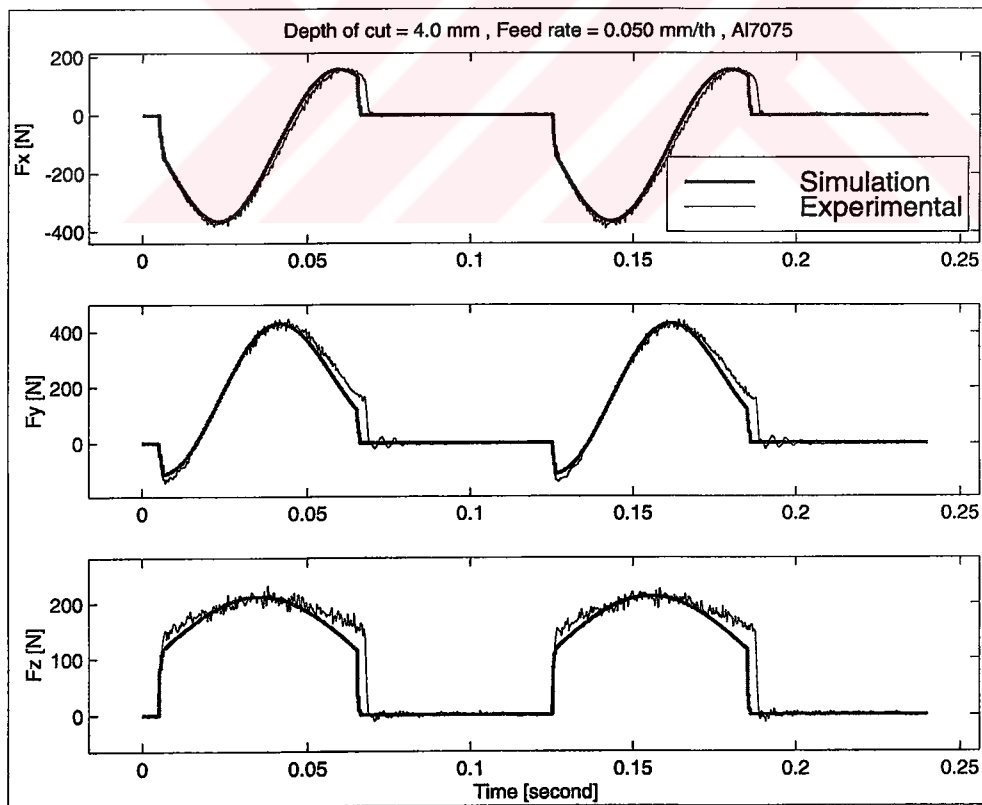
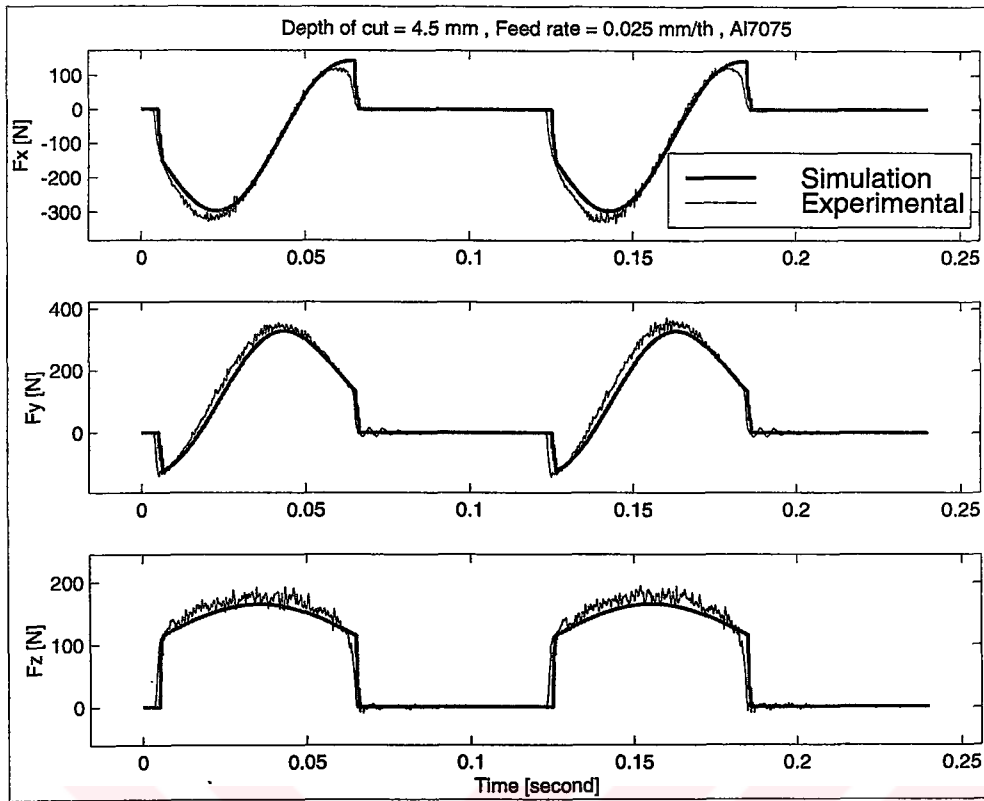


Figure 4.9 e) Depth of cut 4.0 mm, feed rate 0.050 mm/th



f) Depth of cut 4.5 mm, feed rate 0.025 mm/th

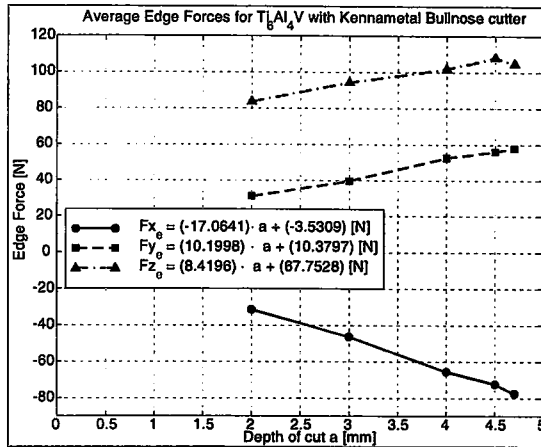
Figure 4.9. Cutting forces for Kennametal Insert Bull Nose End mill, $D=31.75\text{mm}$, $R=4.7625\text{mm}$, $R_r=11.1125\text{mm}$, $R_z=4.7625\text{mm}$, $\alpha=0^\circ$, $\beta=0^\circ$, $h=4.7625\text{mm}$, rake angle 0° , helix angle 5° , constant helix, 1 insert, insert material coated carbide (See Figure 4.5 for tool geometry). Workpiece material Al7075, cutting type slotting, spindle speed 500rpm, (See Table 4.2 for cutting coefficients).

4.4.1.2 Average Cutting Coefficients for $\text{Ti}_6\text{Al}_4\text{V}$:

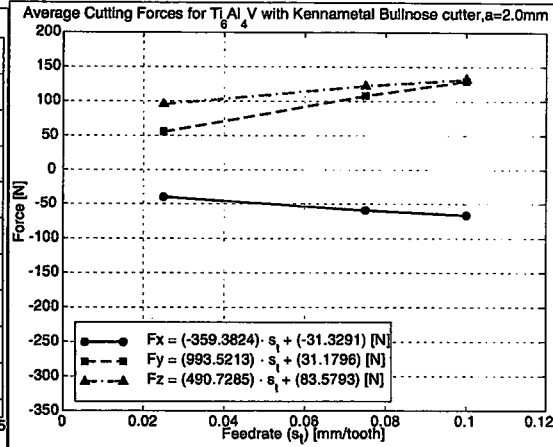
The average cutting coefficients were calculated for $\text{Ti}_6\text{Al}_4\text{V}$ material and a bull nose cutter using the same method, which has been explained in the previous section. 17 cutting tests were done for different depths of cut and feed rates. The average cutting forces can be seen in Figure 4.10.

The cutting and edge coefficients can be seen in Figure 4.11. The average coefficients are independent of the depth of cut (see

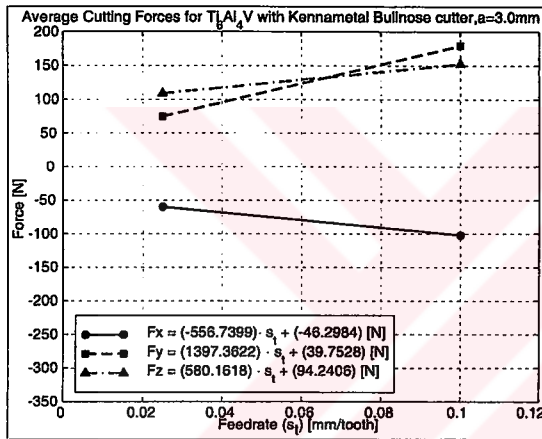
Table 4.3). Cutting force comparisons for experimental and simulation results are given in Figure 4.12.



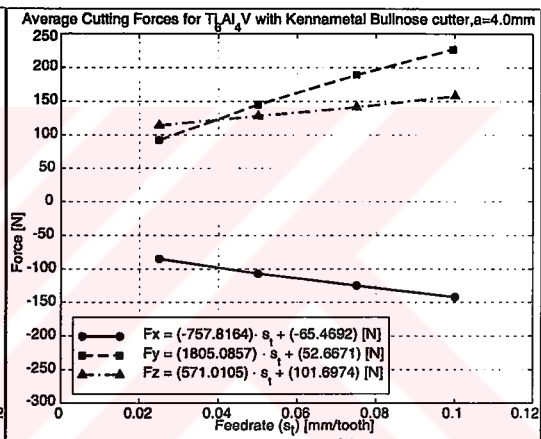
a) Edge forces



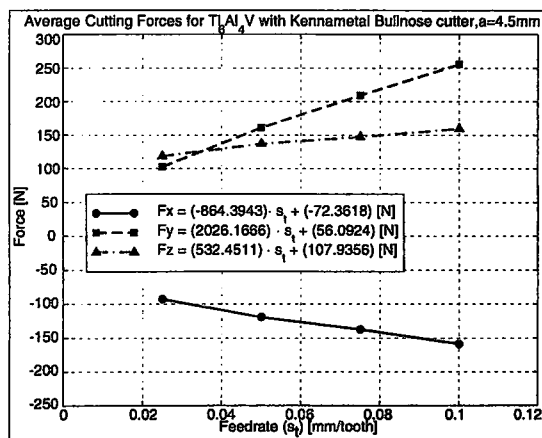
b) Cutting forces, axial depth 2.0 mm



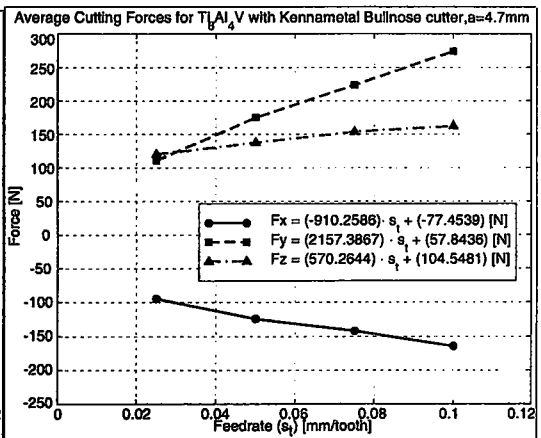
c) Cutting forces, axial depth 3.0 mm



d) Cutting forces, axial depth 4.0 mm



e) Cutting forces, axial depth 4.5 mm



f) Cutting forces, axial depth 4.7 mm

Figure 4.10. Average cutting forces for Ti_6Al_4V (See Table 4.3 for cutting coefficients), (See Figure 4.5 for tool geometry).

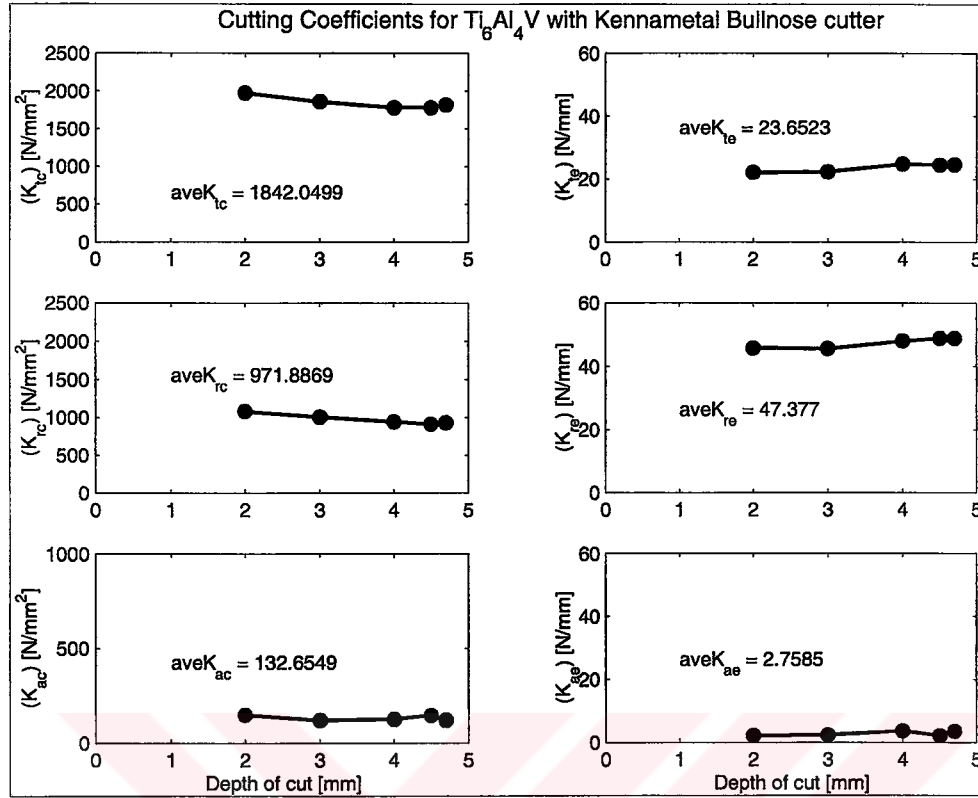


Figure 4.11. Average cutting coefficients for Ti_6Al_4V , (See Table 4.3).

Table 4.3. The average cutting and edge force cutting coefficients for Ti_6Al_4V (see Figure 4.5 for the geometry of the tool).

Depth of cut [mm]	K_{tc} [N/mm ²]	K_{rc} [N/mm ²]	K_{ac} [N/mm ²]	K_{te} [N/mm]	K_{re} [N/mm]	K_{ae} [N/mm]
1.0	2331.15	1809.77	75.72	19.56	40.60	3.73
2.0	1972.88	1076.27	148.20	22.03	45.76	2.14
3.0	1857.58	1005.97	120.76	22.29	45.56	2.36
4.0	1782.14	940.24	127.13	24.79	47.94	3.63
4.5	1782.61	909.83	146.91	24.55	48.82	2.14
4.7	1815.04	927.12	120.27	24.60	48.81	3.53
Average	1842.05	971.89	132.65	23.65	47.38	2.76

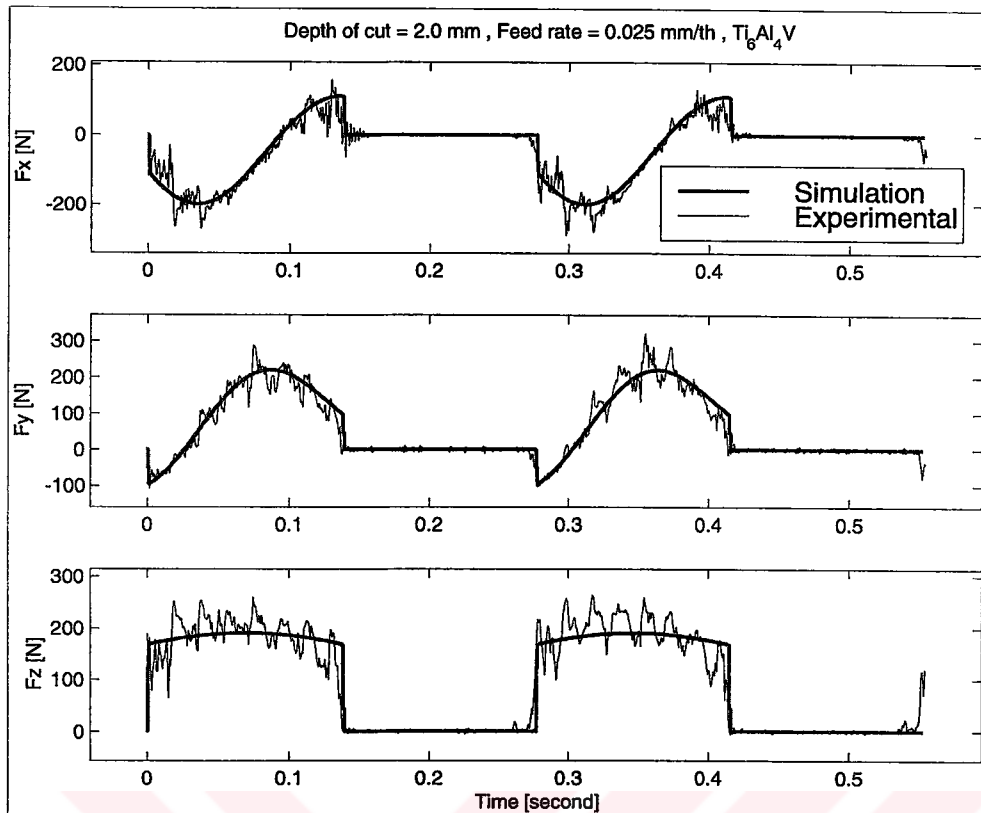


Figure 4.12 a) Depth of cut 2.0 mm, feed rate 0.025 mm/th

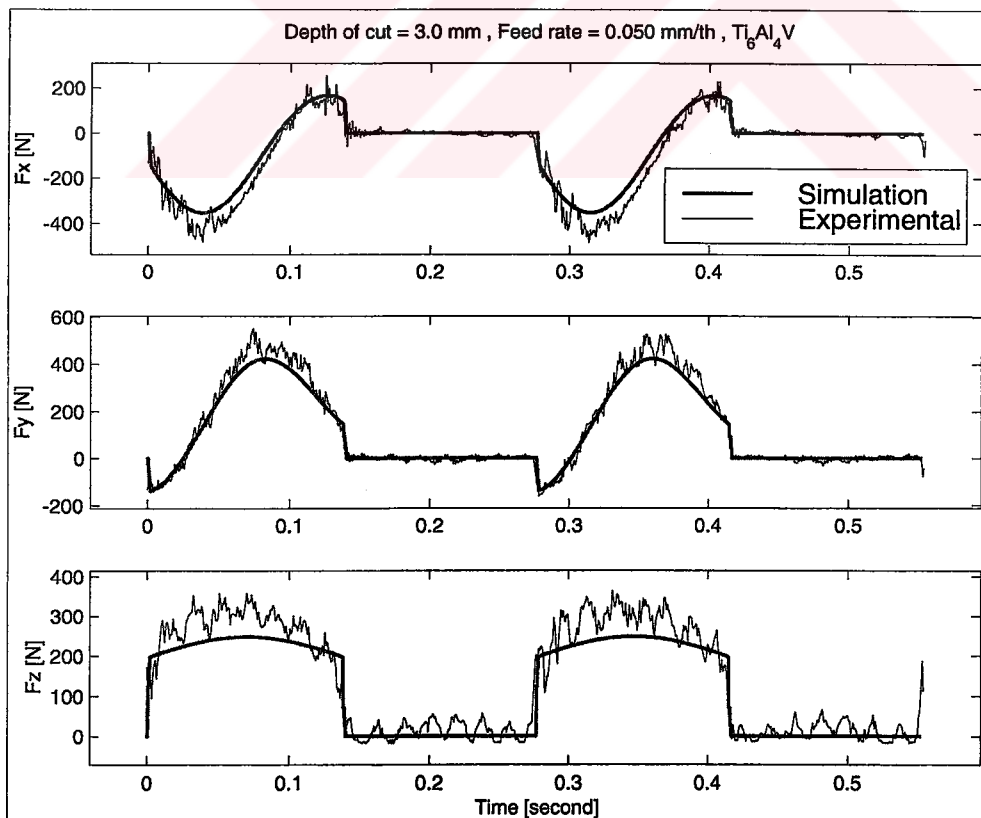


Figure 4.12 b) Depth of cut 3.0 mm, feed rate 0.050 mm/th

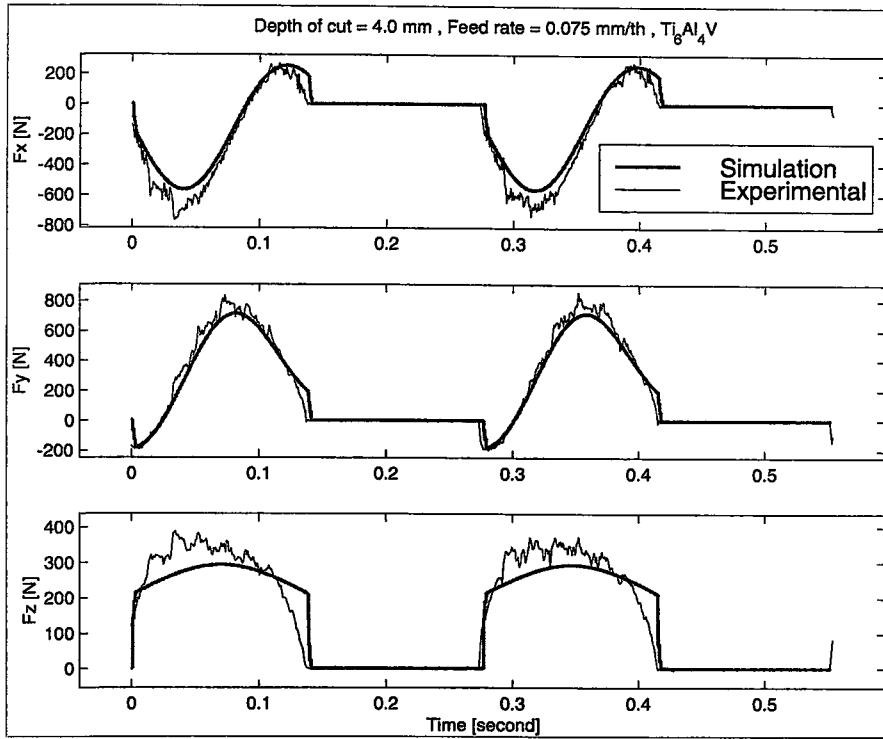
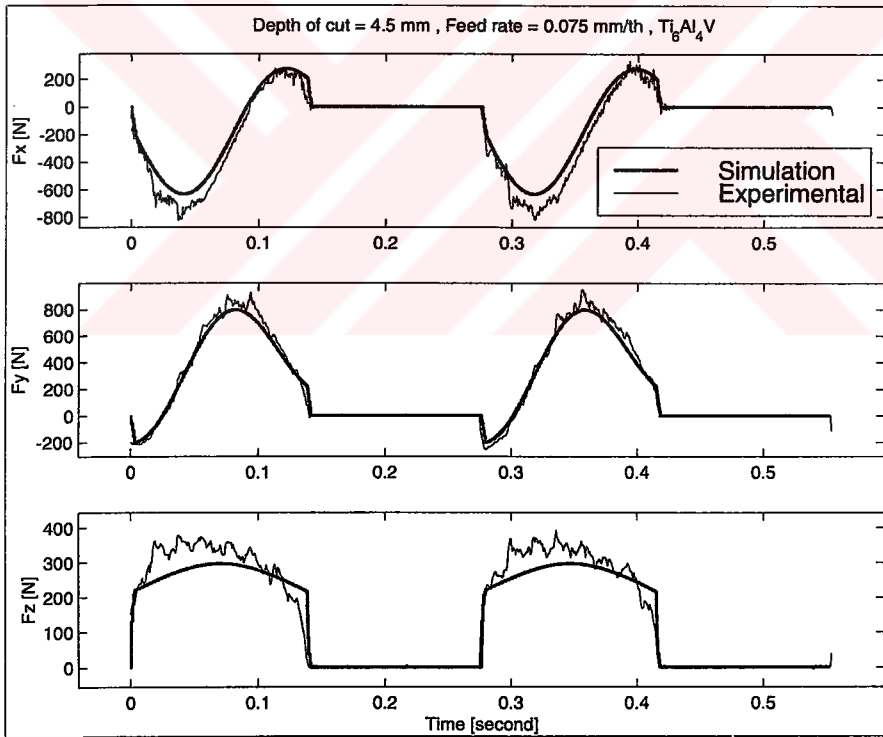


Figure 4.12 c) Depth of cut 4.0 mm, feed rate 0.075 mm/th



d) Depth of cut 4.5 mm, feed rate 0.075 mm/th

Figure 4.12. Cutting forces for Kennametal Insert Bull Nose End mill, $D=31.75\text{mm}$, $R=4.7625\text{mm}$, 0mm $R_r=11.1125\text{mm}$, $R_z=4.7625\text{mm}$, $\alpha=0^\circ$, $\beta=0^\circ$, $h=4.7625\text{mm}$, rake angle 0° , helix angle 5° , constant helix, 1 insert, insert material coated carbide (See Figure 4.5 for tool geometry). Workpiece material $\text{Ti}_6\text{Al}_4\text{V}$, cutting type slotting, spindle speed 500rpm, (See Table 4.3 for cutting coefficients).

4.4.2 Cutting Parameters From Orthogonal Cutting

The general tool geometry has been tested for different cutter geometries. Ti₆Al₄V has been used as workpiece material. The workpiece material cutting coefficients have been calculated from orthogonal cutting tests conducted at the Manufacturing Automation Laboratory [17]. The cutting coefficients for Ti₆Al₄V are given in Table 4.4. One of the cutting force comparisons for an experiment and a simulation is given in Figure 4.5.

Table 4.4. Orthogonal Cutting Coefficients for Ti₆Al₄V (τ :shear stress, β :friction angle [deg], α :rake angle [deg], t :cutting thickness [mm] , r_o : chip thickness ratio) [17].

$\tau_s = 613 \text{ [N/mm}^2\text{]}$ $\beta_a = 19.1 + 0.29 \alpha$
$r_t = r_o t^a$ $r_o = 1.75 - 0.028 \alpha$ $a = 0.331 - 0.0082 \alpha$
$K_{te} = 24 \text{ [N/mm]}$ $K_{re} = 43 \text{ [N/mm]}$

4.4.3 Orthogonal Cutting Parameters From Average Cutting Forces

The average cutting coefficients were calculated earlier for a bull nose cutter and the workpiece materials Al7075 (Table 4.2) and Ti₆Al₄V (

Table 4.3). The cutter helix and rake angles were 0° and 5° degree, respectively.

Orthogonal cutting parameters can be estimated using orthogonal to oblique transformation equations (3.16). In these equations, there are three unknown parameters: shear angle (ϕ_c), average friction angle (β_n) and shear stress (τ_s), if the cutting coefficients are known.

There are two main principles, which are The Maximum Shear and Minimum Energy Principles. The shear stress appears in the direction of maximum shear stress according to The Maximum Shear Principle, which was proposed by Krystof [94]. The following equation is given,

$$\phi_n = \frac{\pi}{4} - (\beta_n - \alpha_n) \quad (4.39)$$

The shear stress appears in the direction of minimum energy according to the Minimum Energy Principle, which was proposed by Merchant [95]. The following equation is given,

$$\phi_n = \frac{\pi}{4} - \frac{\beta_n - \alpha_n}{2} \quad (4.40)$$

The friction angle was estimated as $\beta_n = 30.89^\circ$ for Al7075 and bull nose cutter using average cutting coefficients in Table 4.2. According to the Maximum Shear Stress Principle, the shear angle (ϕ_n) and shear stress (τ_s) were calculated as 14.11° and 264.32MPa, respectively. According to the Minimum Energy Principle, the shear angle and shear stress were calculated as 29.55° and 373.89 MPa, respectively. Some experimental and simulation results are presented in Figure 4.13 for the bull nose cutter and Al7075 workpiece material by using orthogonal parameters from the maximum shear stress principle. The same orthogonal parameters were used to simulate the cylindrical end mill (Figure 4.14). The Maximum Shear Stress Principle gave better results than the Minimum Energy Principle in this application.

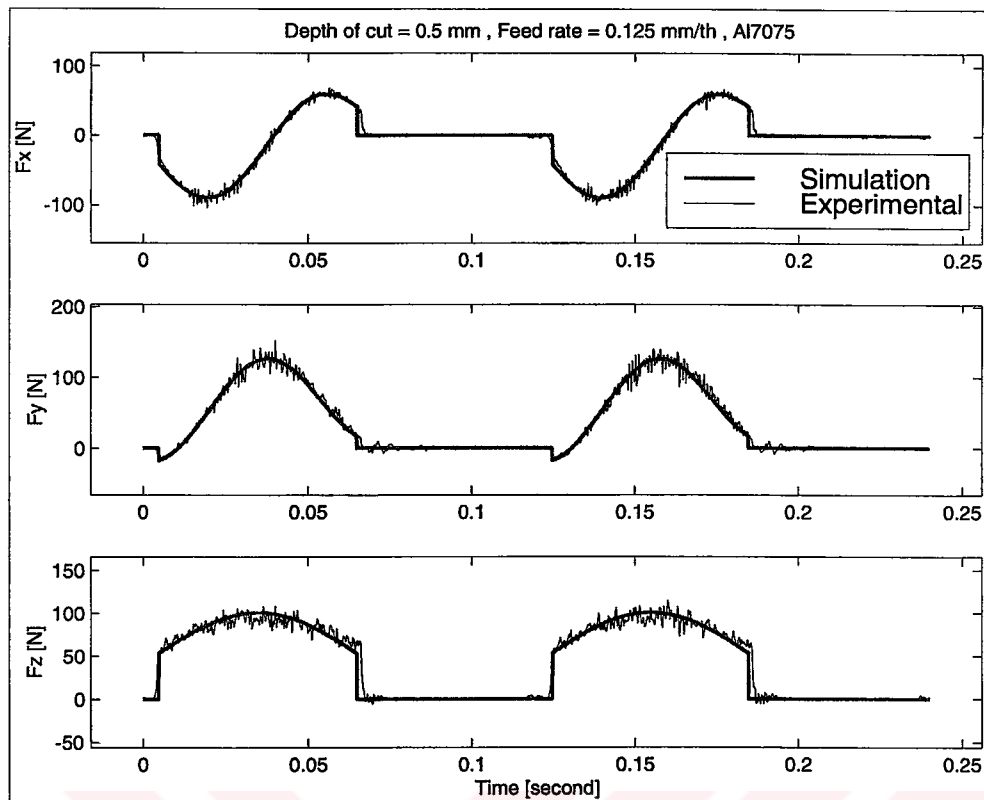


Figure 4.13 a) Depth of cut 0.5 mm, feed rate 0.0125 mm/th

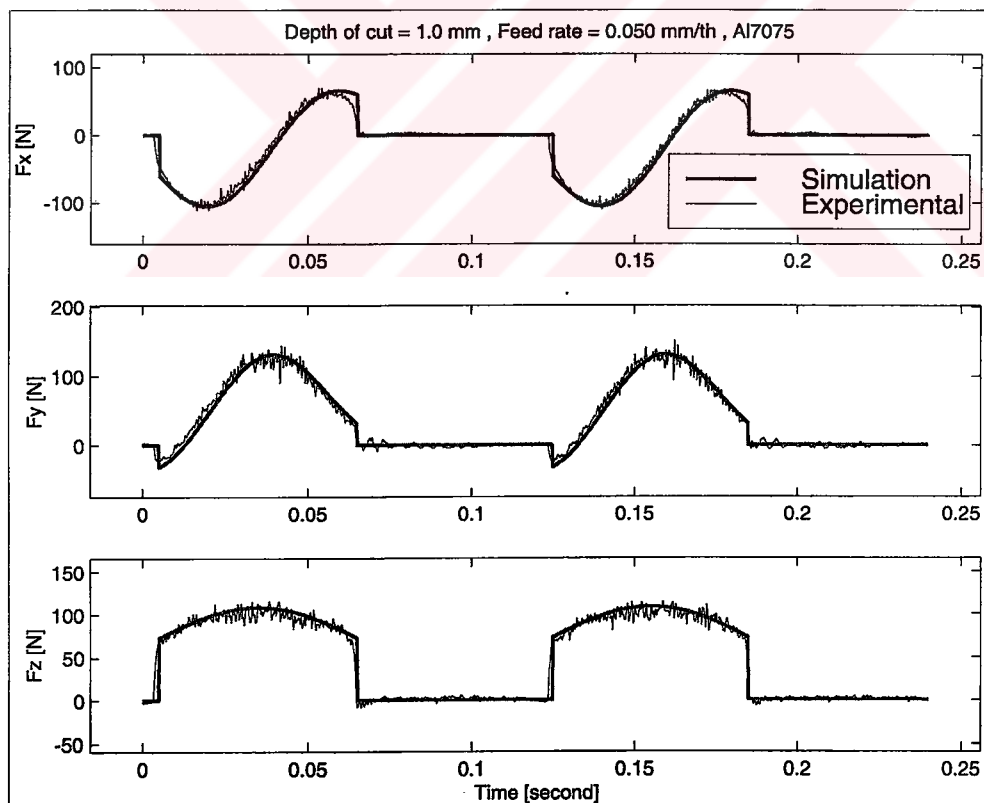


Figure 4.13 b) Depth of cut 1.0 mm, feed rate 0.050 mm/th

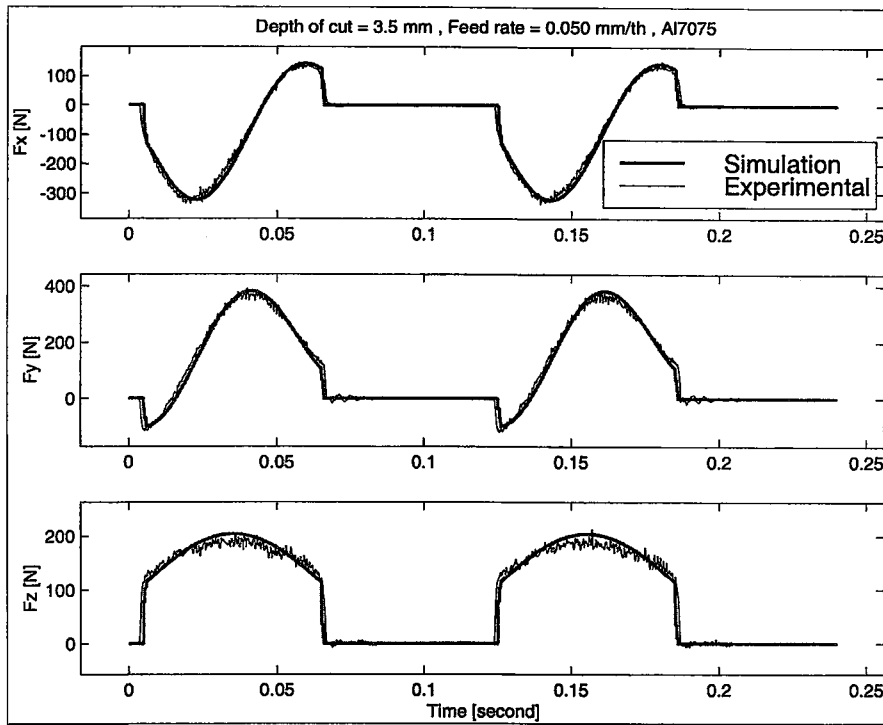
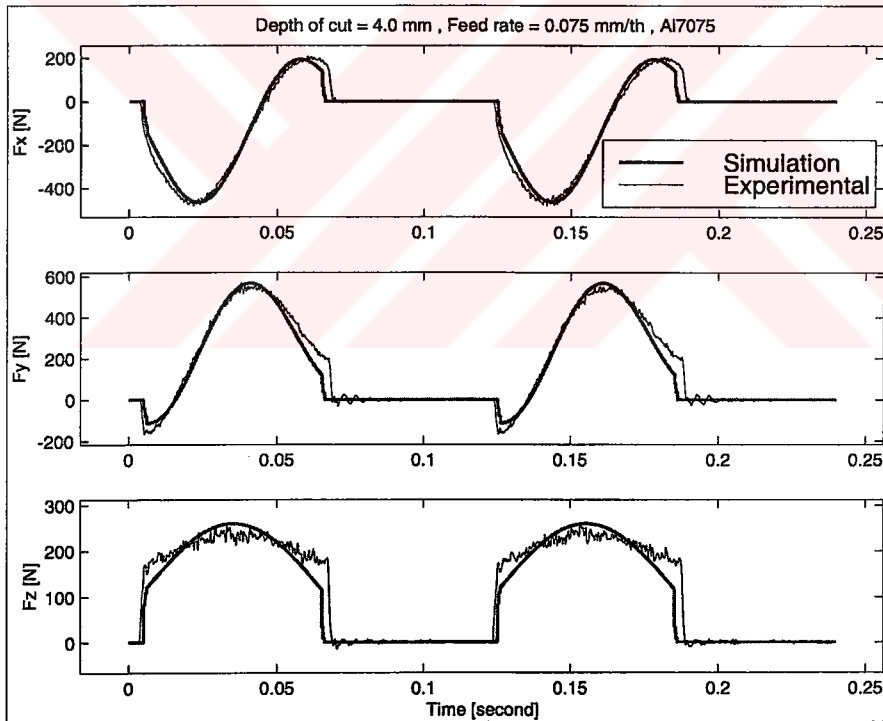
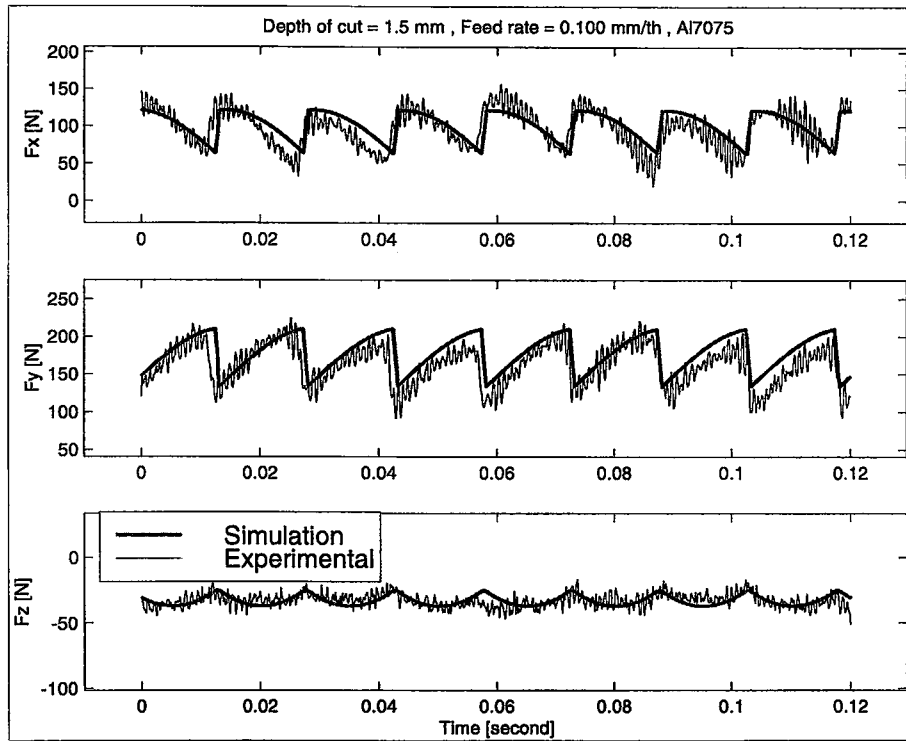


Figure 4.13 c) Depth of cut 3.5 mm, feed rate 0.050 mm/th

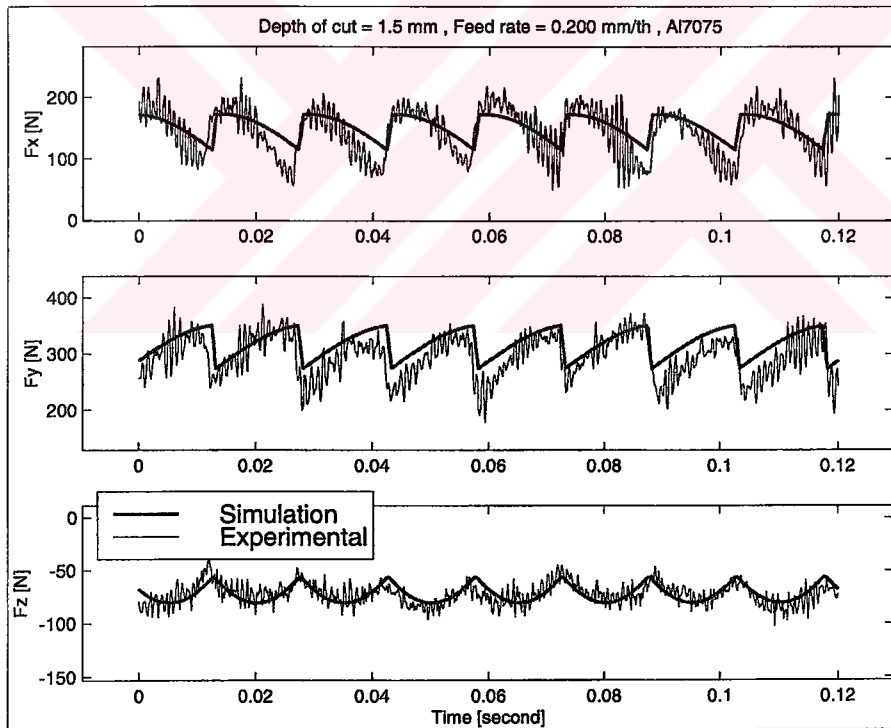


d) Depth of cut 4.0 mm, feed rate 0.075 mm/th

Figure 4.13. Cutting forces for a Bull nose cutter, Orthogonal parameters found from average force coefficients (See Table 4.5 for cutting coefficients) according to Maximum Shear Stress Principle. $D=31.75\text{mm}$, $R=4.7625\text{mm}$, $R_r=11.1125\text{mm}$, $R_z=4.7625\text{mm}$, $\alpha=0^\circ$, $\beta=0^\circ$, $h=4.7625\text{mm}$, rake angle 0° , helix angle 5° , constant helix, 1 flute, (See Figure 4.5 for tool geometry). Workpiece material Al7075, cutting type slotting, spindle speed 500rpm.



a) Depth of cut 1.5 mm, feed rate 0.100 mm/th



b) Depth of cut 1.5 mm, feed rate 0.200 mm/th

Figure 4.14. Cutting forces for Cylindrical End Mill, Orthogonal parameters found from average force coefficients (See Table 4.5 for cutting coefficients) according to Maximum Shear Stress Principle. $D=19.05\text{mm}$, $R=0\text{mm}$, $R_r=0\text{mm}$, $R_z=0\text{mm}$, $\alpha=0^\circ$, $\beta=0^\circ$, $h=38.0\text{mm}$, rake angle 10° , helix angle 30° , constant helix, 4 flutes, workpiece material Al7075, cutting type slotting, spindle speed 1000rpm.

Table 4.5. Orthogonal cutting coefficient parameters. These results were obtained for a 19.05 mm cylindrical end mill, helix angle 30°, 4 flute, rake angle 10°, axial depth of cut 2.0mm, material Ti₆Al₄V.

	Minimum Energy Principle (See Table 4.3)	Maximum Shear Stress Principle (See Table 4.3)	From Orthogonal Cutting Test (See Table 4.4)
Friction Angle (β_n) [deg]	16.69	16.69	19.1
Shear Angle (ϕ_n) [deg]	36.65	28.31	($s_t=0.025\text{mm/th}$) 27.37 ($s_t=0.050\text{mm/th}$) 33.07 ($s_t=0.075\text{mm/th}$) 36.67 ($s_t=0.100\text{mm/th}$) 39.32
Shear Stress (τ_s) [N/mm ²]	590.50	549.85	613.0
K_{tc} [N/mm²]	1682.1	1682.1	($s_t=0.025\text{mm/th}$) 1905.1 ($s_t=0.050\text{mm/th}$) 1774.9 ($s_t=0.075\text{mm/th}$) 1746.1 ($s_t=0.100\text{mm/th}$) 1748.1
K_{rc} [N/mm²]	529.48	529.48	($s_t=0.025\text{mm/th}$) 599.75 ($s_t=0.050\text{mm/th}$) 558.75 ($s_t=0.075\text{mm/th}$) 549.67 ($s_t=0.100\text{mm/th}$) 550.31
K_{ac} [N/mm²]	618.15	618.15	($s_t=0.025\text{mm/th}$) 700.08 ($s_t=0.050\text{mm/th}$) 652.22 ($s_t=0.075\text{mm/th}$) 641.66 ($s_t=0.100\text{mm/th}$) 642.37
K_{te} [N/mm]	29.31	29.31	24.0
K_{re} [N/mm]	44.67	44.67	43.0
K_{ae} [N/mm]	1.12	1.12	3.0

The method was applied to a cylindrical end mill used in cutting Ti₆Al₄V. The results were compared with The Minimum Energy and Maximum Shear Stress Principles, and orthogonal cutting data. The results can be seen in Table 4.5. The friction angle difference between orthogonal cutting force data and average cutting parameters is 2.41° (12.6%). The shear stress difference is 22.5 MPa (3.7%) for the Minimum Energy Principle and 63.15 MPa (10.3%) for the Maximum Shear Stress Principle. In the orthogonal cutting data, shear angle changes with chip thickness, but this is constant in the average cutting method. This method gave very close values to orthogonal parameters. This is not an exact solution, but if the cutting operation is compared, the milling cutting tests are easier and shorter than the orthogonal cutting tests.

4.5 Simulation and Experimental Results

More than 300 cutting tests with various cutter geometry and material were conducted using the generalized method presented here. Samples of results for helical and inserted end mills are presented here to demonstrate the flexibility of the proposed model.

4.5.1 Tapered Helical Ball End Mill:

These cutters are mainly used in five axis milling of jet engine compressors made of Titanium Ti₆Al₄V alloys. Because of large axial depths of cut and poor machinability of Titanium, the chip loads are small and the cutting speed is low in order to avoid shank breakage and edge chipping. Chatter vibrations are also most frequently experienced due to heavy cuts with slender end mills. The proposed model is applied to the design and analysis of tapered cutters for an aircraft jet engine company. The objective was to optimize the cutting tool geometry for strength, and to identify the chatter stability lobes to avoid self-excited vibrations during milling compressors and integral bladed rotors. The cutting constants for Ti₆Al₄V are identified using an orthogonal to oblique cutting transformation method given in Budak et al. [68,69] in 1996. One particular cutter geometry generated by the proposed geometric model is given in Figure 4.15. Although the tests were conducted at various depths of cut and feed rates, two sample predictions and experimental validations are given in Figure

4.16. The tests cover both the ball end and tapered zones. The higher axial depths of cut produced severe chatter vibrations on the machining center used, due to the lack of rigidity of its spindle. However, the five axis machining centers used in the industrial production of such parts are rigid enough to deliver the required cutting conditions. Even though the orthogonal cutting database is used, the predicted and measured cutting forces are in satisfactory agreement.

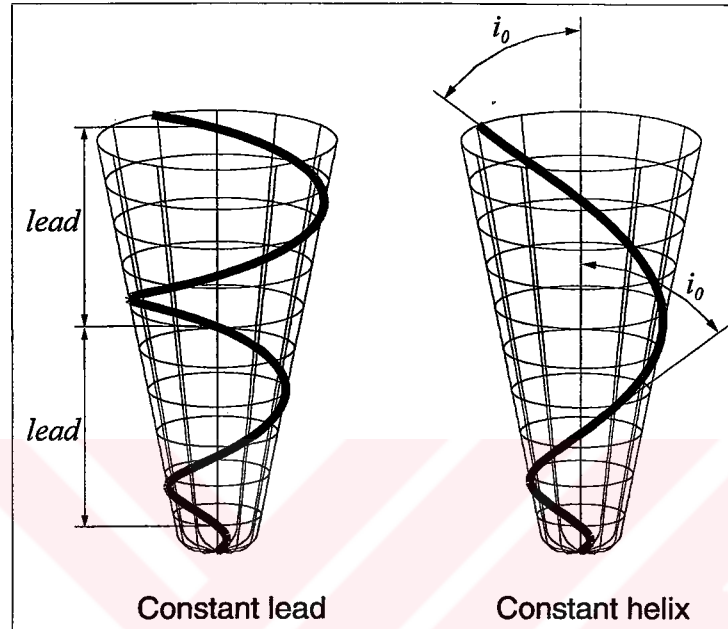


Figure 4.15. Taper helical ball end mill. Constant lead and constant helix on the taper.

4.5.2 Ball End Cutters:

Ball end mills are used mostly in the die and mold machining industry. The surface finish, static form errors, chatter vibrations, and tool life are the main constraints in ball end milling of dies and molds. The prediction of ball end milling forces and cutting constants (by Yucesan and Altıntaş [21] in 1996), chatter stability (by Altıntaş et al. [71] in 1999), and surface finish (by Altıntaş and Lee [20] in 1998) were presented before using specific geometric models of ball end mills. Slot ball end milling tests and predicted cutting forces are shown in Figure 4.17. The cutter material was WC coated with TiAlON, the work material was GGG70 spheroidal graphite cast iron with 251-283HB hardness, and the cutting conditions are given in Figure 4.17.

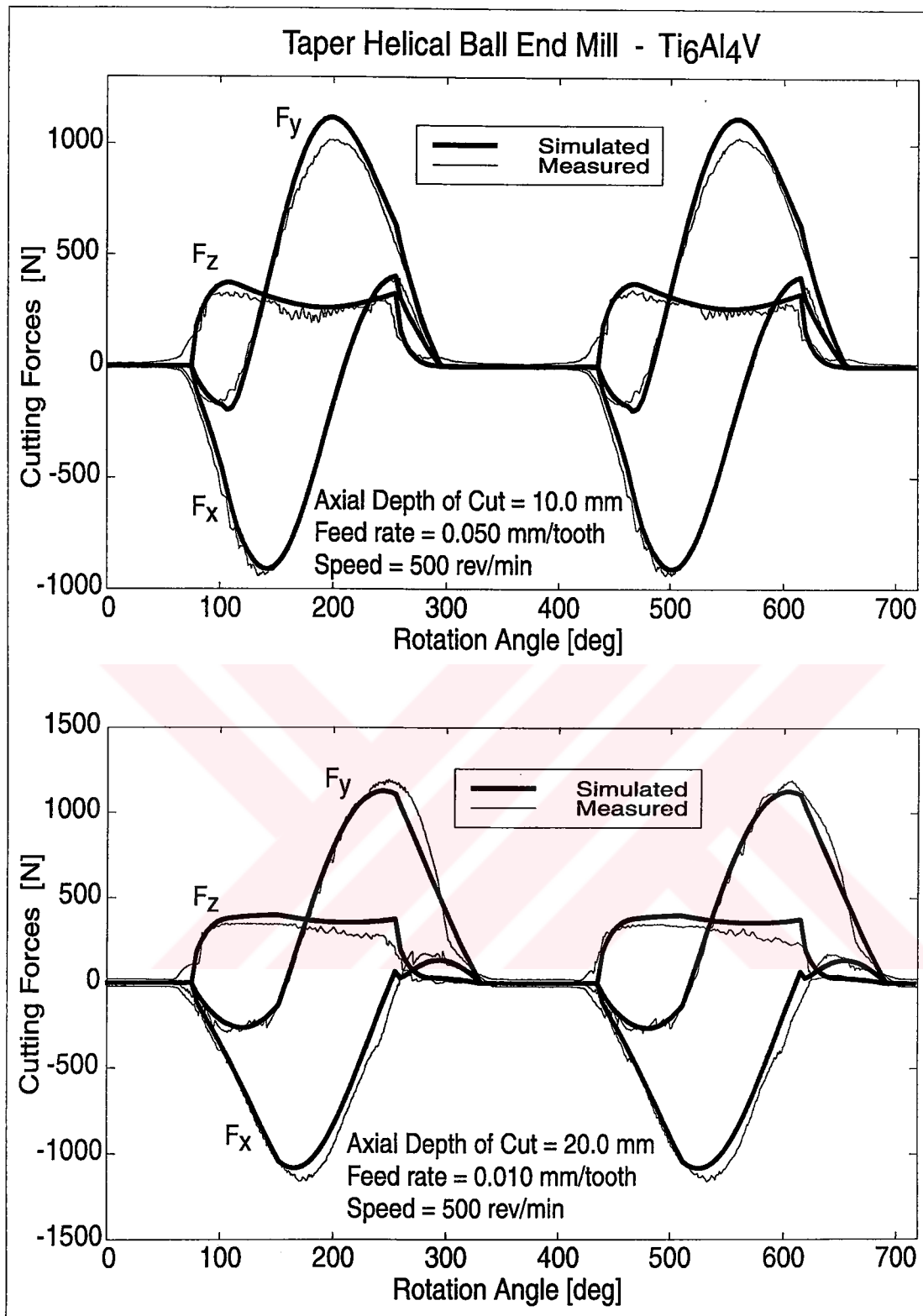


Figure 4.16. Measured and predicted cutting forces for slot cutting. Cutting conditions: rake angle = 10° , relief angle = 10° , tool type is taper helical ball end mill, $D = 6.0$ mm, $R = 3.0$ mm, $R_r = 0$ mm, $R_z = 3.0$ mm, $\alpha = 0^\circ$, $\beta = 4.0^\circ$, $h = 38.0$ mm, $lead = 105.0$ mm, $N_f = 1$ flute, carbide cutter mill (See Table 4.4 for the cutting coefficients).

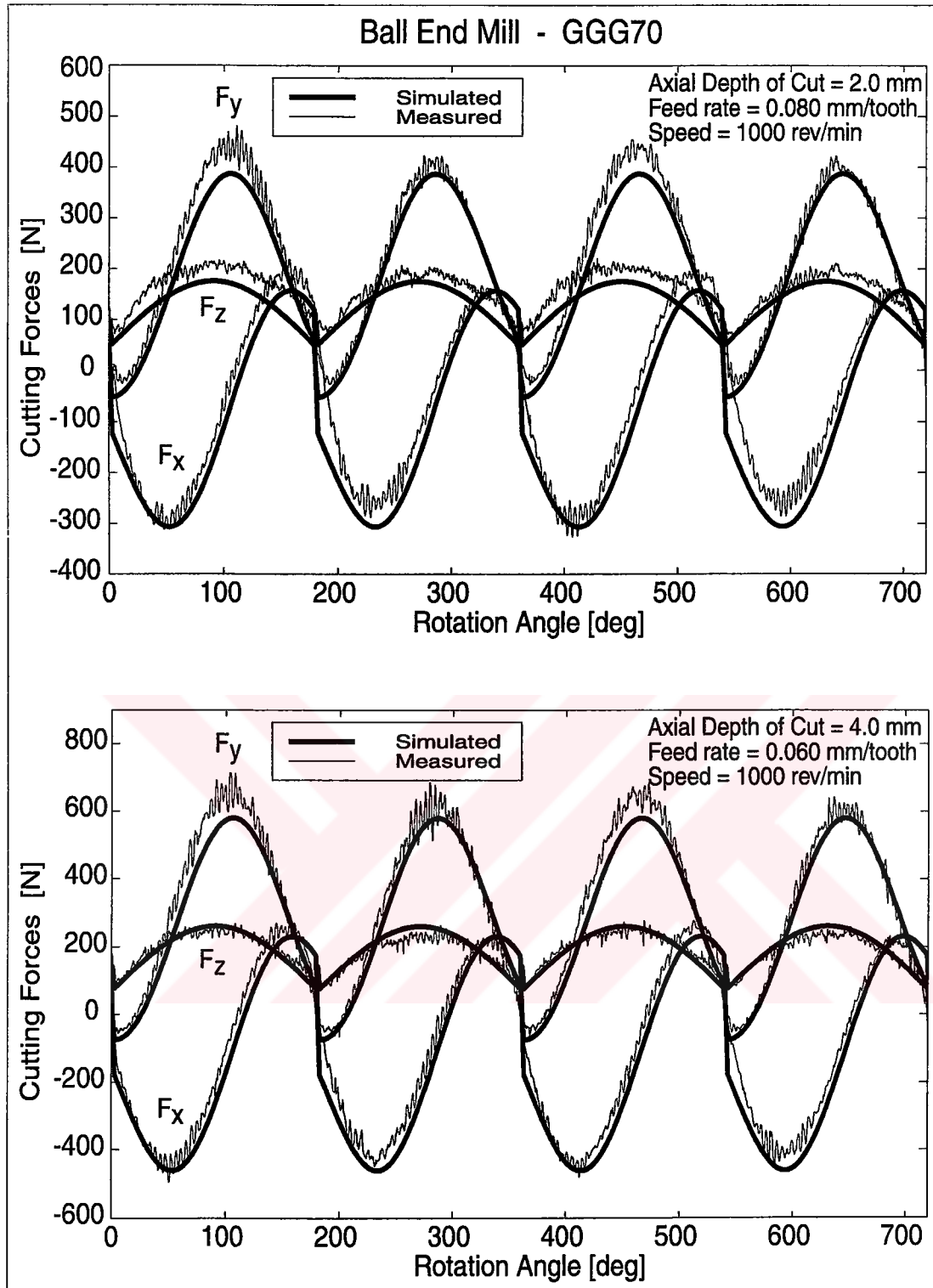


Figure 4.17. Measured and predicted cutting forces for slot cutting. Cutting conditions: rake angle = 10° , relief angle = 0° , tool type is ball end mill, $D = 12.0$ mm, $R = R_r = 6.0$ mm, $R_z = 0$ mm, $\alpha = 0^\circ$, $\beta = 0^\circ$, $h = 6.0$ mm, $N_f = 2$ flutes, cutter material was WC coated with TiAlON, workpiece material GGG70 spheroidal graphite, Cutting coefficients: $K_{tc}=2172.1$, $K_{rc}=848.90$, $K_{ac}=-725.07$ N/mm², $K_{te}=17.29$, $K_{re}=7.79$, $K_{ae}=-6.63$ N/mm.

4.5.3 Bull Nosed Cutter:

A bull nosed cutter with two coated circular inserts was used in milling Titanium Ti_6Al_4V and Aluminum Al-7075 alloys. The cutter was first designed using the proposed general geometric model as shown in Figure 4.5. The cutting forces were predicted for slot milling of Ti_6Al_4V with one insert. The orthogonal cutting database was used in evaluating the average cutting constants. The simulated and measured cutting forces were found to be in good agreement as shown in Figure 4.5 and Figure 4.18. The experimental is given in Figure 4.19. The same cutter was also tested on Al-7075. The transfer function of the cutter attached to a taper 40 spindle with a mechanical chuck is given in Table 4.6. The transfer function model has the following structure:

$$\frac{x}{F} = [\Phi_{xx}(s)] = \sum_{k=1}^K \frac{[R_{1x} + R_{2x}s]_k}{s^2 + 2\zeta_{x,k}\omega_{x,k}s + \omega_{x,k}^2}$$

where x and F are the vibration displacement and force in the feed direction, respectively. $\zeta_{x,k}$ and $\omega_{x,k}$ are the damping ratio and natural frequency for mode k , and K is the number of modes. The modal parameters were evaluated from estimated complex mode residues $(\sigma_k \pm i\nu_k)$ as $R_{1x,k} = 2(\zeta_{x,k}\omega_{nx,k}\sigma_k - \omega_{dx,k}\nu_k)$, $R_{2x,k} = 2\sigma_k$. Similar terminology is used in the normal direction Y. The chatter stability lobes of the system are predicted both in time and frequency domain (see Figure 4.21). The analytical stability solution is accurate, computationally fast, and agrees well with the stability lobes predicted by the time consuming, iterative time domain solutions [67]. The stability diagram indicates chatter free milling speed at 14000 rev/min with an axial depth of cut 4.7 mm. The same axial depth is predicted to produce significant chatter at the lower spindle speed of 9500 rpm. The comparison of simulated and experimentally measured cutting forces, surface finishes, forced and chatter vibration frequencies indicate the validity of the proposed model. At 14000 rpm spindle speed, there is no chatter, the cutting forces have regular static pulsation at tooth passing frequency, and the surface finish is smooth, as predicted. At 9500 rpm, there is chatter at 1448 Hz which coincides to the second bending mode of the spindle in the feed direction. The force magnitudes are at least twice as large as the chatter free machining test, and the surface is quite wavy.

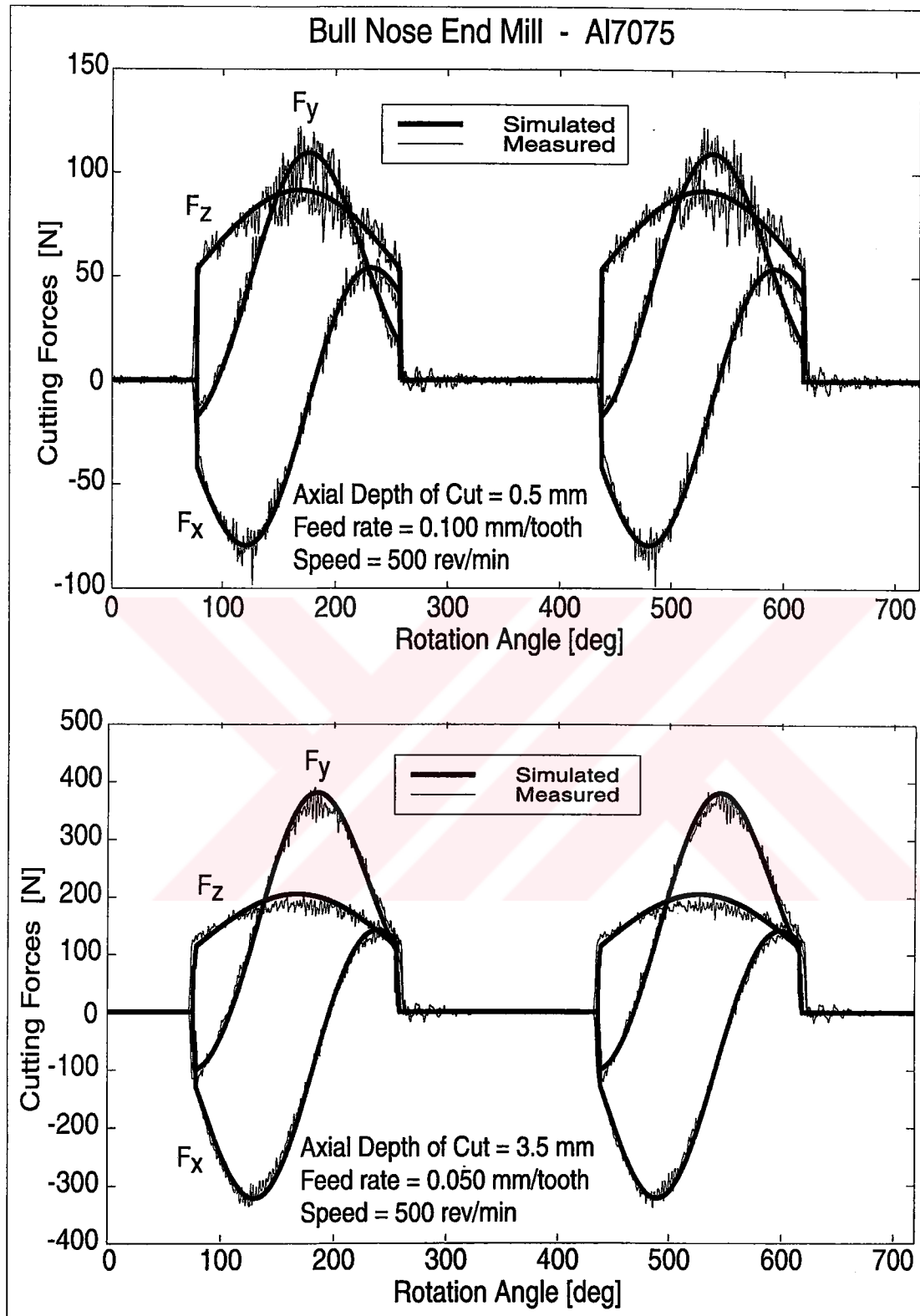


Figure 4.18. Measured and predicted cutting forces for slot cutting. (See Figure 4.5 for tool geometry). $N_f = 1$ flute, (See Table 4.2 for cutting coefficients).

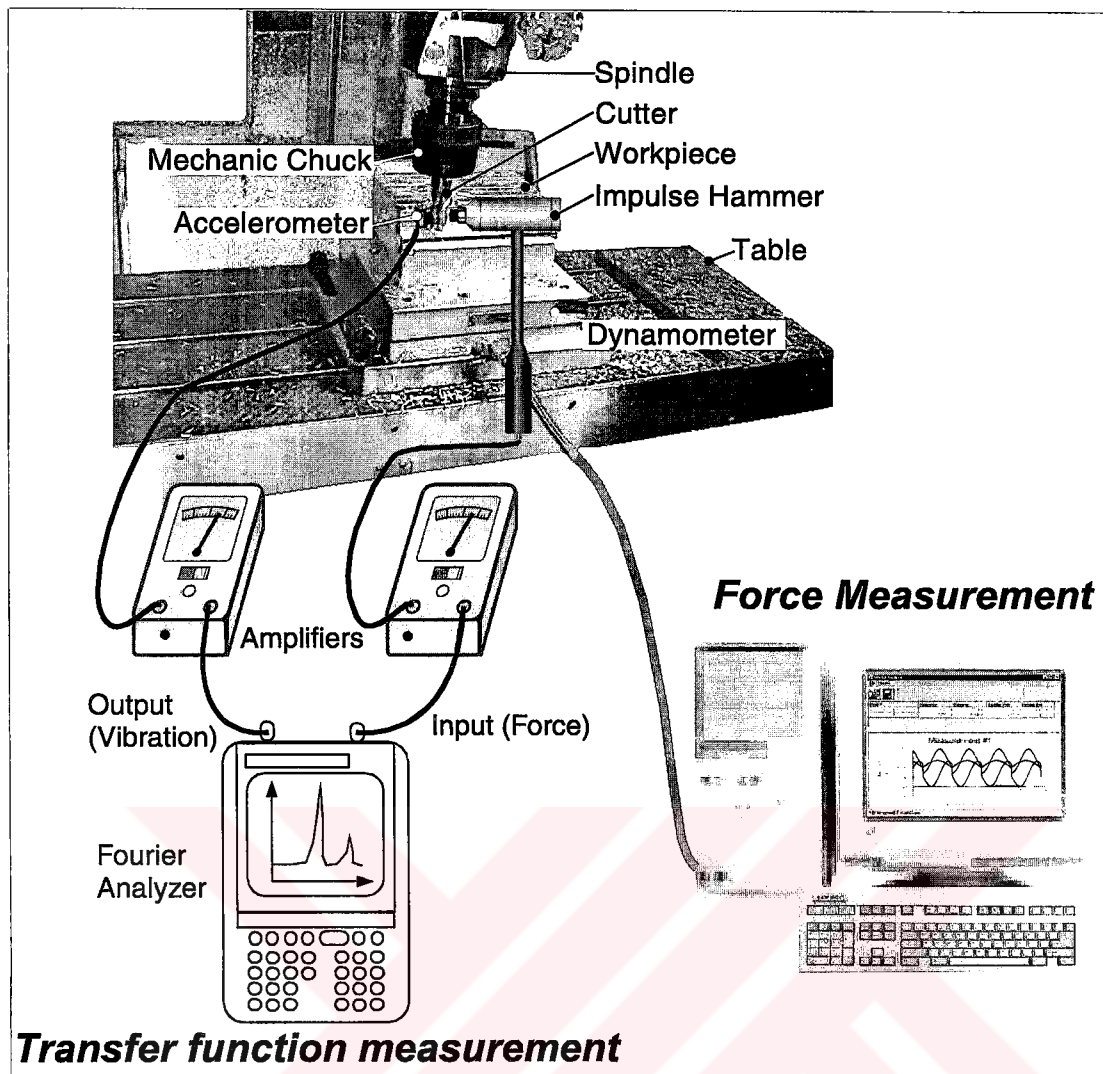
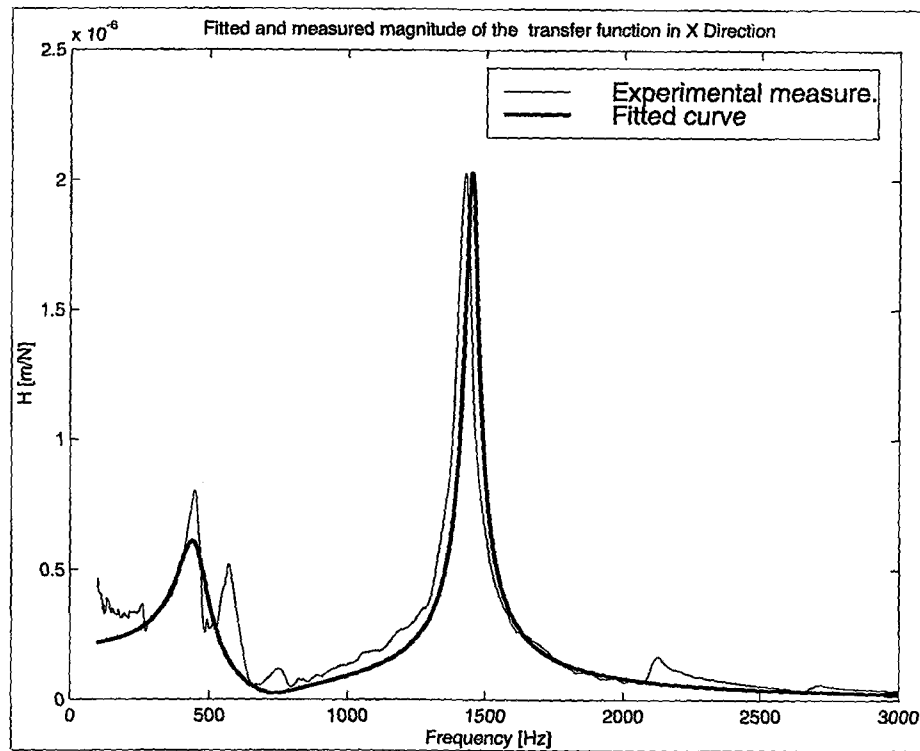


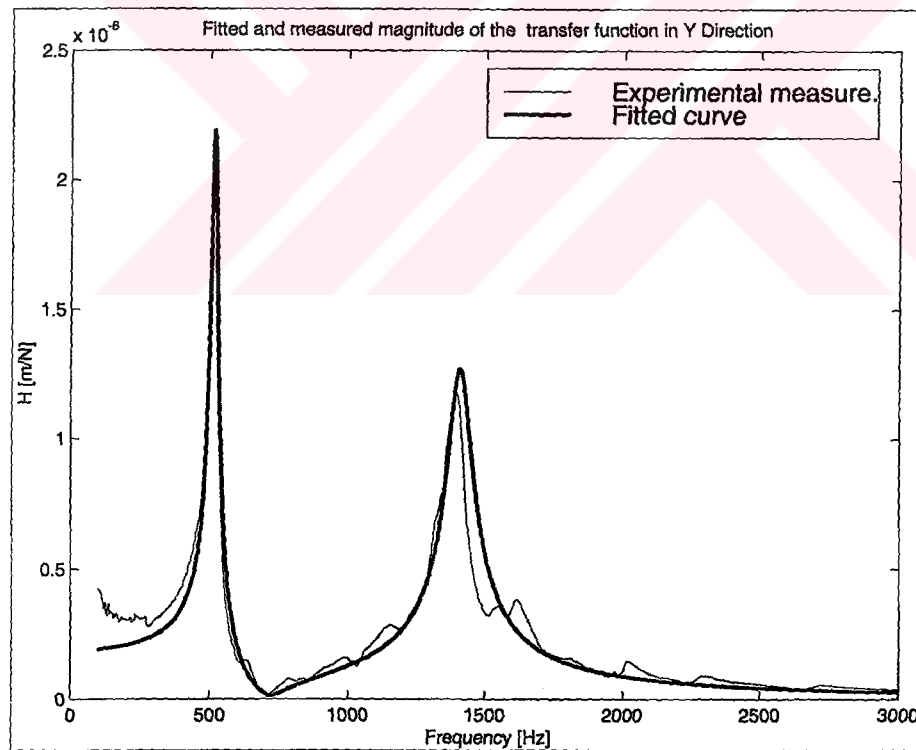
Figure 4.19. Experimental Setup (See Appendix A).

Table 4.6. Measured modal parameters of the bull nose inserted cutter on a machining center with a mechanic chuck.

Direction	Mode	ω_n [Hz]	ζ [%]	Mode Residue [m/N] $(\sigma_k \pm i v_k)$
X	1	452.77	12.37	$(92.02966 - i 186.2195) \cdot 10^{-6}$
	2	1448.53	1.65	$(-41.81562 - i 304.362) \cdot 10^{-6}$
Y	1	516.17	2.43	$(-2.39290 - i 172.1539) \cdot 10^{-6}$
	2	1407.64	3.24	$(40.55052 - i 361.8808) \cdot 10^{-6}$



a) X Direction



b) Y Direction

Figure 4.20. Transfer function of the system from tool tip with Mechanic Chuck, (See Table 4.6 for parameters).

The stability lobes were calculated for Ti₆Al₄V (Figure 4.22). The orthogonal cutting parameters (Table 4.4) were used for time domain stability lobes simulation. Analytical stability lobes were computed using average cutting coefficients (Table 4.3). Time domain and analytical stability lobes were found to be very close to each other. The cutting tests could not be done after 4000 rpm to verify the stability lobes, because the high cutting speed and dry cutting conditions caused the titanium material to burn. Experimental and simulated results are given in Figure 4.22 for the stable and unstable points. It is very difficult to see severe chatter at low spindle speeds, if the stability lobes become narrow for low spindle speeds because of the process damping. In the figure, force, surface finish and FFT results are given at 2000 rpm and 0.5 and 4.5 mm depth of cut. There is no chatter at 0.5 mm depth of cut. This can be seen from the presented results (Figure 4.22). The forces are very stable at 0.5mm, and unstable at 4.5 mm depth of cut. The surface roughness for the unstable point is more than eight times worse than the stable point. The surface roughness is $R_{max}=3\mu\text{m}$ for 0.5 mm and $R_{max}=25\mu\text{m}$ for 4.5 mm at 2000 rpm.

The workpiece transfer function was measured. There was a dominant peak at 1200 Hz in the X direction and 1250 Hz in the Y direction. However, the magnitude of these peaks is very small when compared to the magnitude of the transfer function at the tool tip.

Various cutting tests were conducted with helical ball and cylindrical helical end mills and similar results were obtained. The experimental setup used is shown in Figure 4.19. Detail explanation of experimental setups for transfer function to obtain the dynamic characteristic of machine-tool system, cutting forces to compare with simulated results and chatter detection measurements are given in Appendix A.

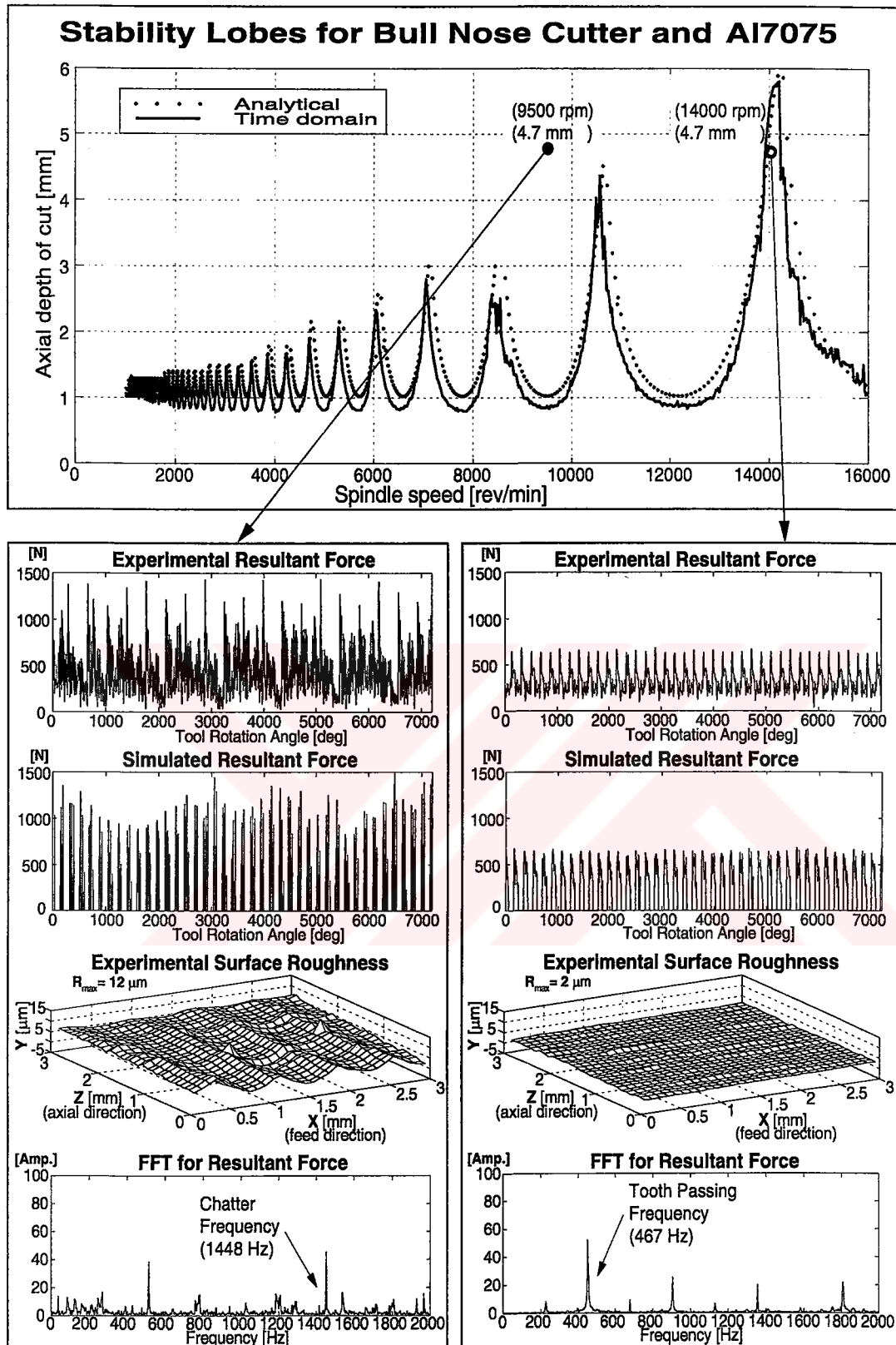


Figure 4.21. Stability lobes for bull nose cutter (see Figure 4.5 for tool geometry). Cutting conditions: half immersion down milling, $N_f = 2$ flutes, feed rate 0.050 mm/tooth. See Table 4.6 for the transfer function parameters. (See Table 4.2 for cutting coefficients).

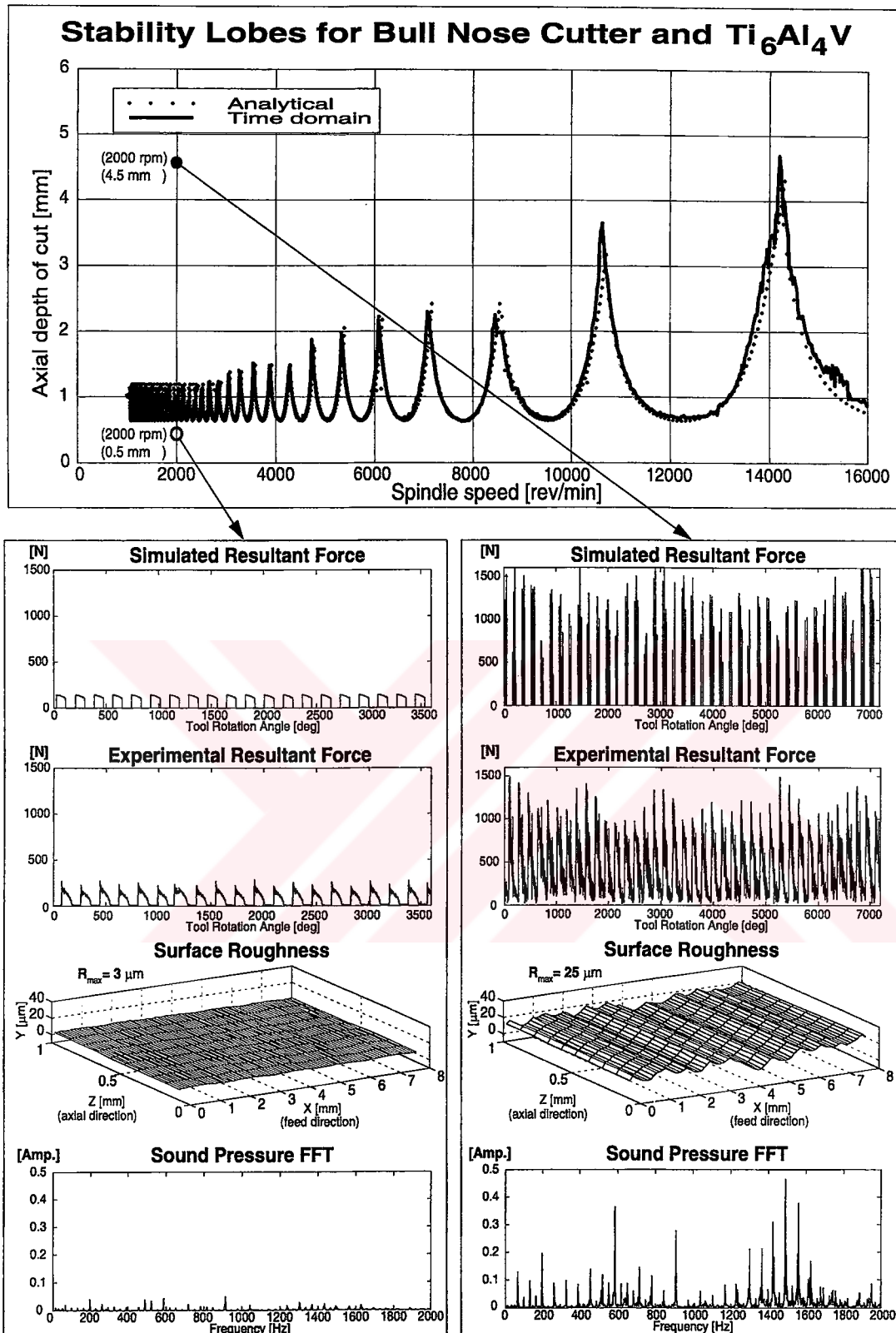


Figure 4.22. Stability lobes for bull nose cutter with two flutes (See Figure 4.5 for tool geometry) and workpiece material Ti_6Al_4V (See Table 4.3 for cutting coefficients). Feed rate 0.050 mm/tooth, half immersion down milling (System transfer function in Table 4.6).

4.6 Summary

A generalized mathematical model for arbitrary end mill cutters has been developed. The model allows parametric design and representation of a variety of end mill shapes and helical flutes. Using a previously developed scheme for solving the exact kinematics of dynamic milling processes, the chip thickness, corresponding cutting forces, vibrations, and dimensional surface finish generated by arbitrarily shaped end mills were predicted in simulations. The mathematical models were then validated by a number of experiments conducted with helical tapered end, ball end, and bull nosed cutters, all of which showed good agreement with the simulation results. The proposed approach allows the planning and analysis of a variety of milling operations used in industry.



5. GENERALIZED GEOMETRIC MODEL OF INSERTED MILLING CUTTERS

5.1 Introduction

In this chapter, a generalized model for inserted cutters is developed. The geometry of a cutting edge is defined in a local coordinate system fixed to the insert. The definition of the cutting edge in a coordinate system fixed to the cutter body, and whose origin is at the cutter body center, is then obtained by coordinate transformation. Assigned lead and axial rake angles for each cutter are included in this transformation.

Inserted cutters are widely used in roughing and finishing of parts. The insert geometry and distribution of inserts on the cutter body vary significantly in industry depending on the application. This section presents a generalized mathematical model of inserted cutters for the purpose of predicting cutting forces, vibrations, dimensional surface finish and stability lobes in milling. The edge geometry is defined in the local coordinate system of each insert, and placed and oriented on the cutter body using the cutter's global coordinate system. The cutting edge locations are defined mathematically, and used in predicting the chip thickness distribution along the cutting zone. Each insert may have a different geometry, such as rectangular, convex triangular or a mathematically definable edge. Each insert can be placed on the cutter body mathematically by providing the coordinates of the insert center with respect to the cutter body center. The inserts can be oriented by rotating them around the cutter body, thus each insert may be assigned to have different lead and axial rake angles. The geometrical definition of a cutting edge for inserted cutters is explained in the next section. By solving the mechanics and dynamics of cutting at each edge point, and integrating them over the contact zone, it is shown that the milling process can be predicted for any inserted cutter. A sample of inserted cutter modeling and analysis examples are provided with experimental verifications.

5.2 Generalized Geometric Model of Inserted Cutters

The purpose of the geometric model is to predict the mechanics and dynamics of cutters with arbitrary insert shapes and distribution. First, the mathematical model of one insert placed on a cutter body is presented. This is followed by the definition of a general cutter with varying inserts along its periphery.

5.2.1 Mathematical Model of Insert

The cutter body is defined in Cartesian coordinate system (XYZ), and the center of the insert is aligned with a second coordinate system (uvw), see Figure 5.1. A vector between the cutter tip and insert centers is,

$$\vec{V}_{IC} = -\sin \varphi I_r \vec{i} + \cos \varphi I_r \vec{j} + I_z \vec{k} \quad (5.1)$$

where I_r is the radial offset in the XY plane, I_z is the axial offset of the insert center from the cutter axis (Figure 5.1). The face of the insert where the cutting edge lies, is aligned with the uw plane of the insert coordinate system (i.e. $v=0$). The insert center is indexed by an angular distance of φ measured counter-clockwise from the Y axis (see Figure 5.1). The insert is placed on the cutter body using the vector defined in equation (5.1).

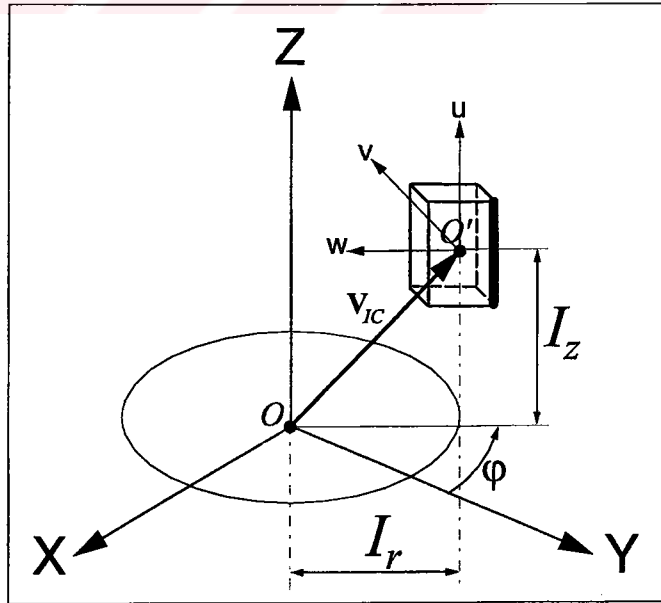


Figure 5.1. Location vector of insert center on the cutter.

The insert shapes can vary depending on the application and manufacturer. The two most commonly used insert shapes are considered to illustrate the generalized modeling approach (Figure 5.2). The rectangular insert is defined by the cutting edge length (b) and insert width (a). The convex triangular insert is defined by the radius (R) of the curved cutting edge, offset (o_f) between the insert center (O) and the cutting edge curvature center (O_c), and by the angular boundaries (θ_s, θ_e) of the cutting edge measured on the (uw) plane clockwise from the local axis (u). The points along the cutting edge of the insert are defined by the vector $\vec{V}_{CE}$ in the coordinate system (uvw),

$$\vec{V}_{CE} = u \vec{i} + v \vec{j} + w \vec{k} \quad (5.2)$$

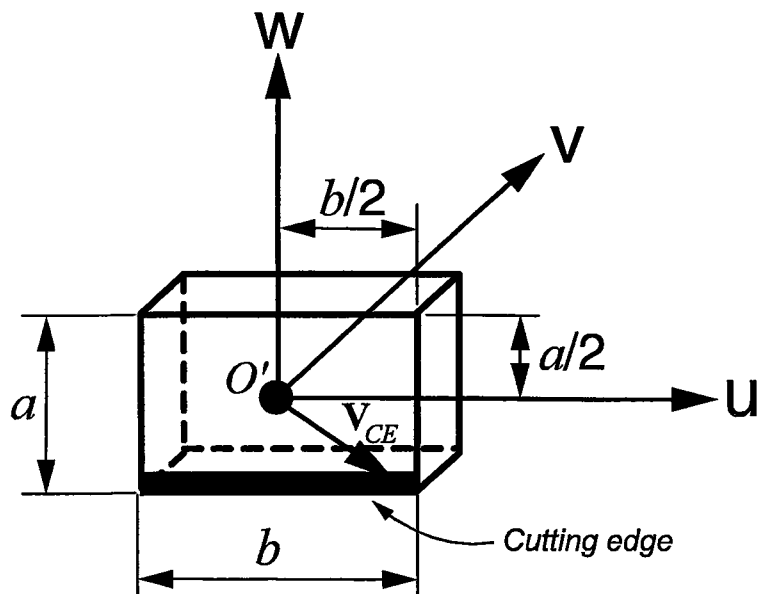
where

$$\begin{aligned} -\frac{b}{2} \leq u \leq \frac{b}{2}; \quad v = 0; \quad w = \frac{a}{2} & \quad \text{for the rectangular insert} \\ \left. \begin{aligned} u &= R \cos \gamma; \quad v = 0; \quad w = -R \cos \gamma + o_f \\ \gamma &= \theta + \sin^{-1} \left(\frac{o_f}{R} \cos \theta \right) \text{ for } \theta_s \leq \theta \leq \theta_e \end{aligned} \right\} & \quad \text{for the convex triangular inserts} \end{aligned}$$

Note that the (uvw) plane of the insert's coordinate system is aligned with the insert face, which contains the cutting edge (i.e. $v=0$). The insert's cutting edge is defined locally in the insert coordinate system (uvw).

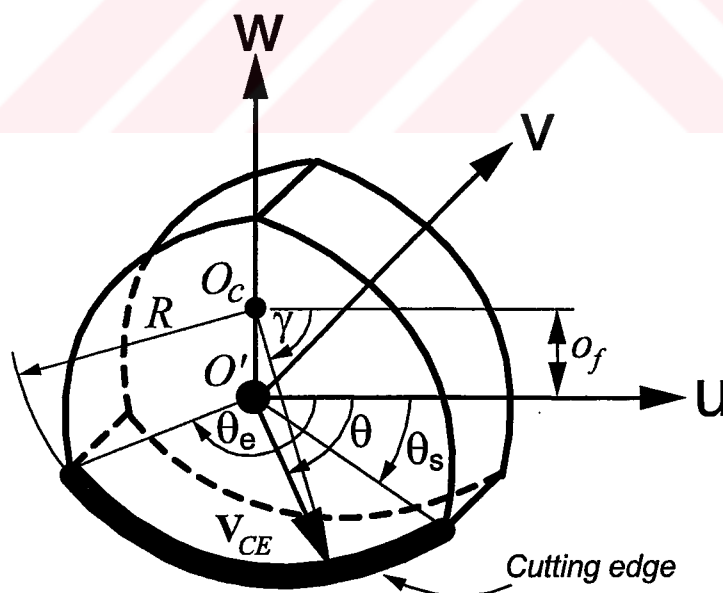
Each insert can be oriented on the cutter body by rotating it around the global cutter body axis X, Y and Z (Figure 5.3). The insert is rotated by a *lead angle* (δ) about axis (Y), *axial rake angle* (β) about axis (X), and *index angle* (φ) about axis (Z). The corresponding rotation matrixes for X, Y and Z are as follows, respectively.

Rectangular Insert



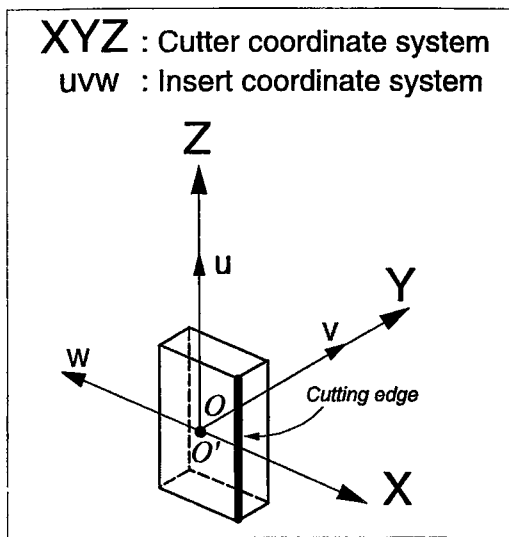
a) Rectangular Insert

Convex Triangular Insert

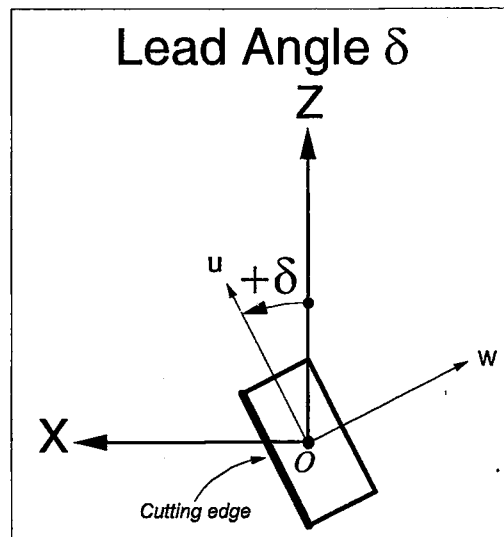


b) Convex triangular insert

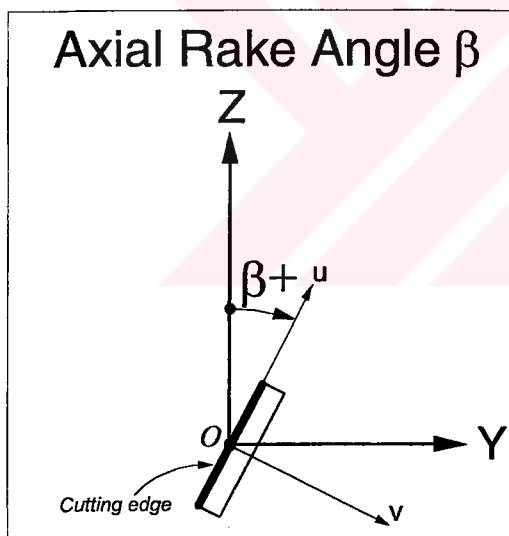
Figure 5.2. Geometry of inserts.



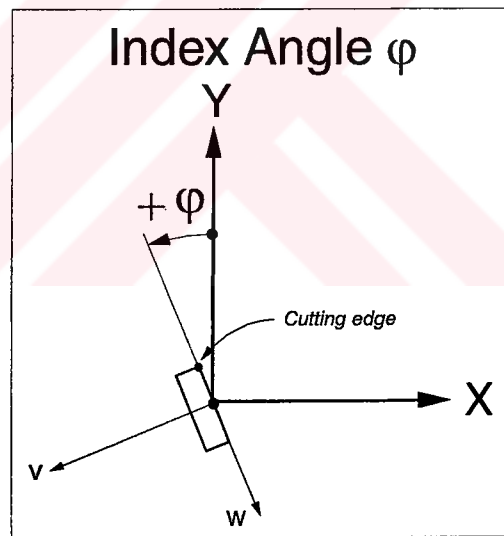
a) Cutter (XYZ) and insert(uvw) coordinates



b) Rotation about Y axis with δ lead angle



c) Rotation about X axis with β axial rake angle



d) Rotation about Z axis with ϕ index angle

Figure 5.3. The insert rotations.

$$\left. \begin{aligned}
\mathbf{R}_x(\beta) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \beta & \sin \beta \\ 0 & -\sin \beta & \cos \beta \end{bmatrix} && \text{for axial rake angle } (\beta) \text{ about axis (X)} \\
\mathbf{R}_y(\delta) &= \begin{bmatrix} \sin \delta & 0 & -\cos \delta \\ 0 & 1 & 0 \\ \cos \delta & 0 & \sin \delta \end{bmatrix} && \text{for lead angle } (\delta) \text{ about axis (Y)} \\
\mathbf{R}_z(\varphi) &= \begin{bmatrix} -\sin \varphi & -\cos \varphi & 0 \\ \cos \varphi & -\sin \varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} && \text{for index angle } (\varphi) \text{ about axis (Z)}
\end{aligned} \right\} \quad (5.3)$$

If the insert is rotated about all three axis, the resulting transformation matrix is,

$$\begin{aligned}
\mathbf{T}_M &= \mathbf{R}_z(\varphi) \cdot \mathbf{R}_x(\beta) \cdot \mathbf{R}_y(\delta) \\
&= \begin{bmatrix} -\sin \varphi \sin \delta - \cos \varphi \sin \beta \cos \delta & -\cos \varphi \cos \beta & \sin \varphi \cos \delta - \cos \varphi \sin \beta \sin \delta \\ \cos \varphi \sin \delta - \sin \varphi \sin \beta \cos \delta & -\sin \varphi \cos \beta & -\cos \varphi \cos \delta - \sin \varphi \sin \beta \sin \delta \\ \cos \beta \cos \delta & -\sin \beta & \cos \beta \sin \delta \end{bmatrix} \quad (5.4)
\end{aligned}$$

The coordinates of the cutting edge in the insert coordinate system become (Figure 5.4):

$$\mathbf{V}_{CER} = \mathbf{T}_M \cdot \mathbf{V}_{CE} = [\mathbf{T}_M] \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad (5.5)$$

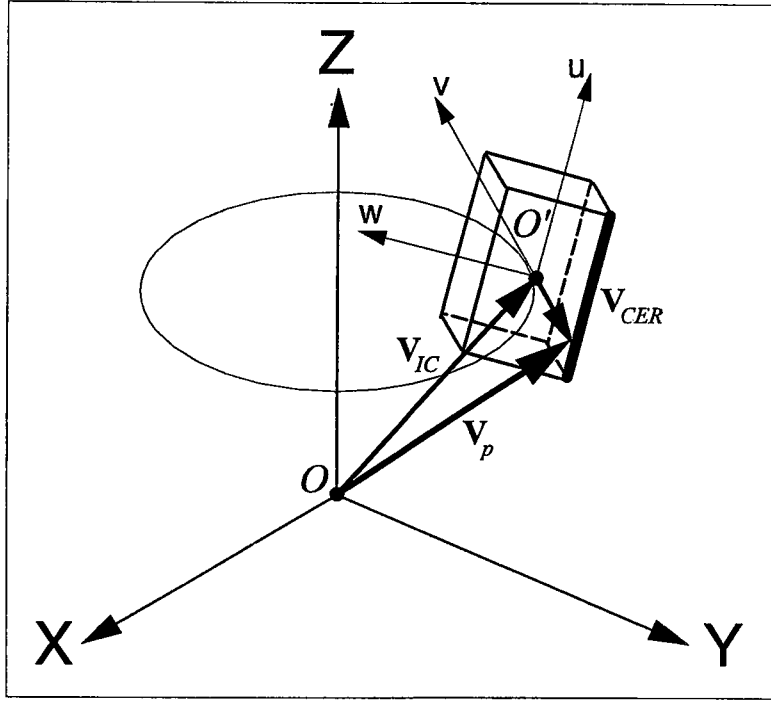


Figure 5.4. The cutting edge position vector.

The final position of the cutting edge with respect to cutter body coordinate system becomes,

$$\begin{aligned} \vec{V}_p = \vec{V}_{IC} + \vec{V}_{CER} = & -\sin \varphi I_r \vec{i} + \cos \varphi I_r \vec{j} + I_z \vec{k} + [T_M] \begin{bmatrix} u \\ v \\ w \end{bmatrix} \\ = & [-\sin \varphi (u \sin \delta - w \cos \delta + I_r) - \cos \varphi \sin \beta (u \cos \delta + w \sin \delta) - v \cos \varphi \cos \beta] \vec{i} \\ & + [\cos \varphi (u \sin \delta - w \cos \delta + I_r) - \sin \varphi \sin \beta (u \cos \delta + w \sin \delta) - v \sin \varphi \cos \beta] \vec{j} \\ & + [\cos \beta (u \cos \delta + w \sin \delta) - v \sin \beta + I_z] \vec{k} \end{aligned} \quad (5.6)$$

Once the insert center location (I_r, I_z) , orientation on the cutter body (δ, β, φ) , and edge dimensions $(a, b; R, \theta_s, \theta_e)$ are specified, a point on the cutting edge can be evaluated for a given elevation or axial depth of cut position (z) from equation (5.6).

5.2.2 Mathematical Model of Multiple Inserts

There are various inserted cutters used in industry depending on machining applications. There may be identical inserts placed on the periphery of the cutter with equal pitch spacing (i.e. face milling cutters), helical flutes where each flute may

contain multiple inserts, or randomly distributed inserts with different shapes and at different locations both in radial and axial directions. A generalized cutter design matrix (D_M ($6 \times N$)) is used for defining a cutter with a general insert distribution,

$$D_M = \begin{bmatrix} t_{11} & t_{12} & t_{13} & \dots; & t_{21} & t_{22} & \dots; & \dots \\ I_{z11} & I_{z12} & I_{z13} & \dots; & I_{z21} & I_{z22} & \dots; & \dots \\ I_{r11} & I_{r12} & I_{r13} & \dots; & I_{r21} & I_{r22} & \dots; & \dots \\ \delta_{11} & \delta_{12} & \delta_{13} & \dots; & \delta_{21} & \delta_{22} & \dots; & \dots \\ \beta_{11} & \beta_{12} & \beta_{13} & \dots; & \beta_{21} & \beta_{22} & \dots; & \dots \\ \underbrace{\varphi_{11} \quad \varphi_{12} \quad \varphi_{13} \quad \dots}_{1st \text{ insert}} & \underbrace{\varphi_{21} \quad \varphi_{22} \quad \dots}_{2nd \text{ insert}} & \dots & \dots \end{bmatrix} \quad (5.7)$$

$\underbrace{\hspace{10em}}_{1st \text{ flute}}$
 $\underbrace{\hspace{10em}}_{2nd \text{ flute}}$
 $\dots$

where N is the total number of inserts on the cutter. The first and the second digits of subindices represent flute number and insert order number from tool tip, respectively. Note that it is not necessary to have equal number of inserts on each flute. Each column represents one insert with its type and position on the cutter body. The first element (t) of each column represents the insert type, i.e. $t = 0$: no insert, $t = 1 \dots 10$: rectangular insert family, $t = 11 \dots 20$ triangular insert family. The remaining five parameters define the position and orientation of the insert on the cutter body. Rectangular inserts have two parameters that are width (a) and length (b) (Figure 5.2a). The convex triangular insert has four parameters that are the starting angle (θ_s) and the end angle (θ_e), radius of arc (R) and the arc radius center offset from the center of insert (o_f) (Figure 5.2b). Each insert shape is separately stored in a database with its associated dimensions.

5.3 Modeling of Cutting Forces

Consider differential cutting forces (dF_t , dF_r , dF_a) that are generated when cutting an infinitesimal chip element which has a local uncut chip thickness of $h(\phi, z)$ normal to the cutting edge, chip height of dz and curved edge contact length of dS (Figure 5.5).

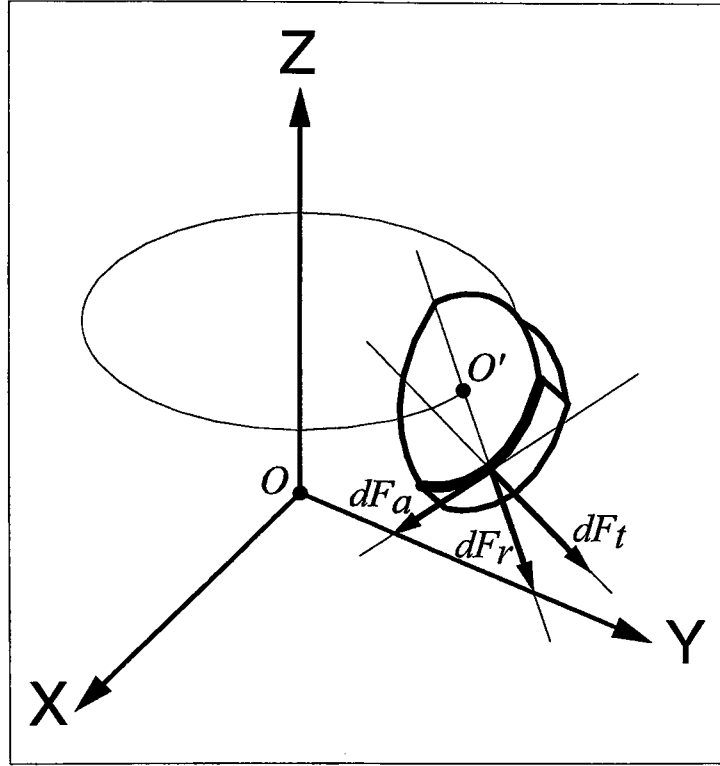


Figure 5.5. Three-dimensional cutting edge profile on the insert and cutting force directions.

$$\left. \begin{aligned} dF_t &= K_{te}(z) dS + K_{tc}(z) h(\phi, z) dz \\ dF_r &= K_{re}(z) dS + K_{rc}(z) h(\phi, z) dz \\ dF_a &= K_{ae}(z) dS + K_{ac}(z) h(\phi, z) dz \end{aligned} \right\} \quad (5.8)$$

The edge force cutting coefficients, K_{te} , K_{re} , K_{ae} represent the non-shearing flank line contact forces, and the force coefficients K_{tc} , K_{rc} , K_{ac} represent shear or rake face contact pressure. The linearized force model used in equation (5.8) is presented in detail by Budak et al. [17]. It should be noted that the inserts used in industry might have varying rake face and curvature along its cutting edge. For example, chip breaking grooves and non-uniform rake face may exist on each insert. The cutting coefficients can be identified from orthogonal cutting parameters (average shear stress, shear angle and friction coefficient) by transforming them into oblique cutting domain [17], if the rake face is uniform like in regular end mills. However, if the insert's cutting edge has a chamfer or radius, and the rake face of the insert is non-uniform, an orthogonal to oblique cutting transformation may not lead to an accurate

evaluation of cutting coefficients [30]. Depending on the insert shape and cutting edge geometry, it may be more appropriate to use mechanistic models in identifying the cutting coefficients as a function of edge position (z), cutting speed, and local chip thickness. The mechanistic approaches for the identification of cutting constants are well presented in literature [13,25,28,30]. The evaluation of chip thickness with and without chatter vibrations has been presented in detail by Altıntaş and Lee [20]. The details of mechanistic identification of cutting constants and milling process modeling can be found in the cited literature [23]. The cutter is divided into small axial disk elements with a differential height of dz . The uncut chip thickness removed by each differential segment along the insert is identified by considering the trochoidal motion of the milling [4] and the regenerative chatter vibration of both cutter and workpiece [10]. If the insert has roundness, the chip thinning is considered using a similar method used in ball end milling [20]. The differential cutting forces are evaluated and transformed into Cartesian coordinates of the cutter similar to the method used for helical end mills. The differential forces are summed along all inserts which are in contact with the workpiece. The resulting cutting forces in the three directions are applied to the structural dynamic models of the cutter and workpiece [10] to evaluate the structural displacements and their influence on the regenerative chip thickness. The dynamic chip thickness ($h(\phi, z)$) is evaluated at small time intervals by rotating the cutter and feeding the workpiece. The time domain simulation model allows the predictions of dynamic chip thickness, cutting forces, vibrations of both cutter and workpiece along the depth of cut, surface finish and chatter stability lobes [20,52]. The chatter stability lobes for the inserted cutters are also identified in frequency domain using the analytical solution proposed by Altıntaş and Budak [67] in 1995 and summarized in Chapter 3. The cited models of milling process mechanics, kinematics, dynamics and process simulations are all used here.

5.4 Simulation and Experimental Results

The proposed generalized inserted cutter model, and its corresponding mechanics and dynamics are integrated into the comprehensive milling process simulation package described in [94]. The simulation program contains all the process models

cited in the previous sections. Two sample cases are presented here to illustrate the proposed model.

5.4.1 Milling of Al 7075 with Cutter I:

A two fluted helical inserted cutter with a 25.4 mm diameter and 4 rectangular inserts on each flute is modeled (Figure 5.6). The cutter body is a Mitsubishi BAP300R 1608WL16, with Mitsubishi APMT1135PDER-H1 HTi20T rectangular inserts ($a=6.35$ mm, $b=9.8$ mm). The design matrix of the cutter is constructed as:

$$D_M = \begin{bmatrix} 2 & 2 & 2 & 2 & ; & 2 & 2 & 2 & 2 \\ 4.6 & 13.1 & 17.7 & 22.3 & ; & 4.7 & 13.2 & 17.8 & 22.4 \\ 9.5 & 9.5 & 9.5 & 9.5 & ; & 9.5 & 9.5 & 9.5 & 9.5 \\ 0.0 & 0.0 & 0.0 & 0.0 & ; & 0.0 & 0.0 & 0.0 & 0.0 \\ 20.0 & 20.0 & 20.0 & 20.0 & ; & 10.0 & 20.0 & 20.0 & 20.0 \\ 0.0 & 40.0 & 80.0 & 120.0 & ; & 190.0 & 270.0 & 310.0 & 350.0 \end{bmatrix} \quad (5.9)$$

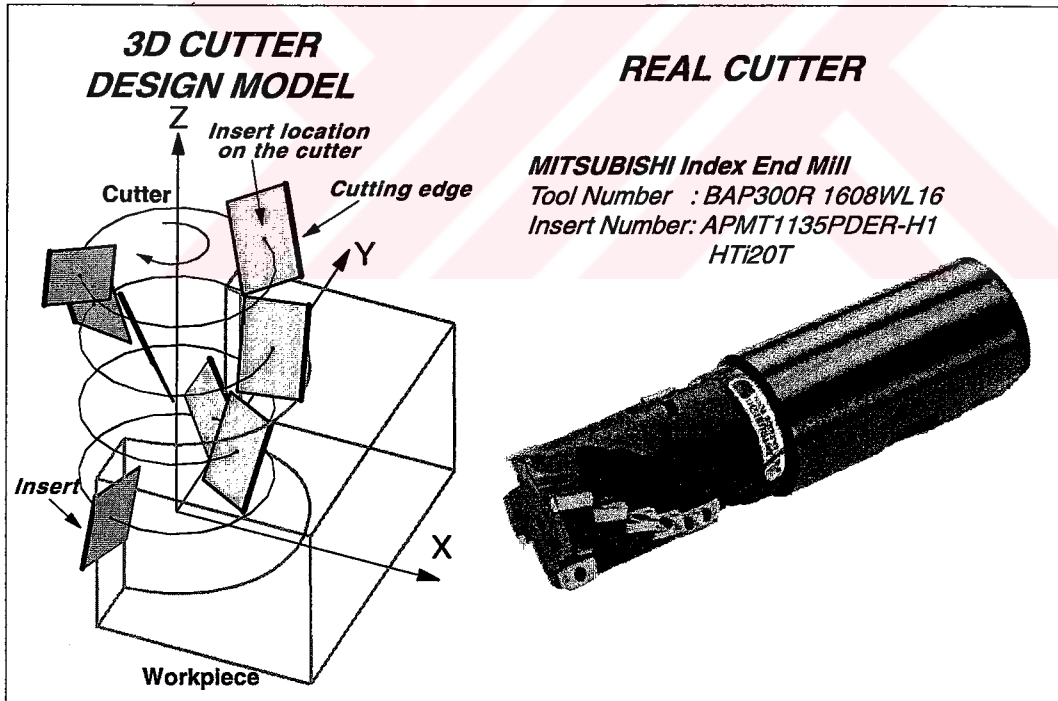


Figure 5.6. Mitsubishi inserted cutter Cutter I (Cutter number: BAP300R 1608WL16; Insert number: APMT1135PDER-H1 “insert dimensions; $a=6.35$ mm, $b=9.8$ mm” HTi20T). See equation (5.9) for insert location parameters. Shank diameter=25.4 mm, Cutter diameter=25.4 mm, 2 flutes, 4 inserts on every flute.

Note that the first insert of the second flute (row 5, column 5) has an axial rake of 10 degrees, where the rest of the inserts have an axial rake of 20 degrees. The lead angles of all inserts are zero, i.e. row 4. The cutter is used for the machining of Al-7075. The cutting coefficients are identified mechanistically from the measured average cutting forces per tooth period. Slot milling tests were conducted with one insert by increasing the axial depth of cut by 1.0 mm increments up to the insert's edge length. Average cutting forces can be seen in Figure 5.7 and Figure 5.8. The insert's rake face geometry varies significantly along the cutting edge. Therefore the cutting coefficients are highly dependent on the axial location of the cutting edge point. The cutting coefficients are identified for each 1.0 mm segment. When the depth of cut is 5.0 mm for example, the measured average forces for 4.0 mm axial depth of cut were subtracted before identifying the cutting coefficients for the 5.0 mm edge zone. The cutting tests were conducted at a feed rate range of 0.025 to 0.200 mm/tooth at 0.025 mm/tooth increments. After identifying the cutting constants for each edge position, a satisfactory polynomial was fitted to represent them along the insert's cutting edge.

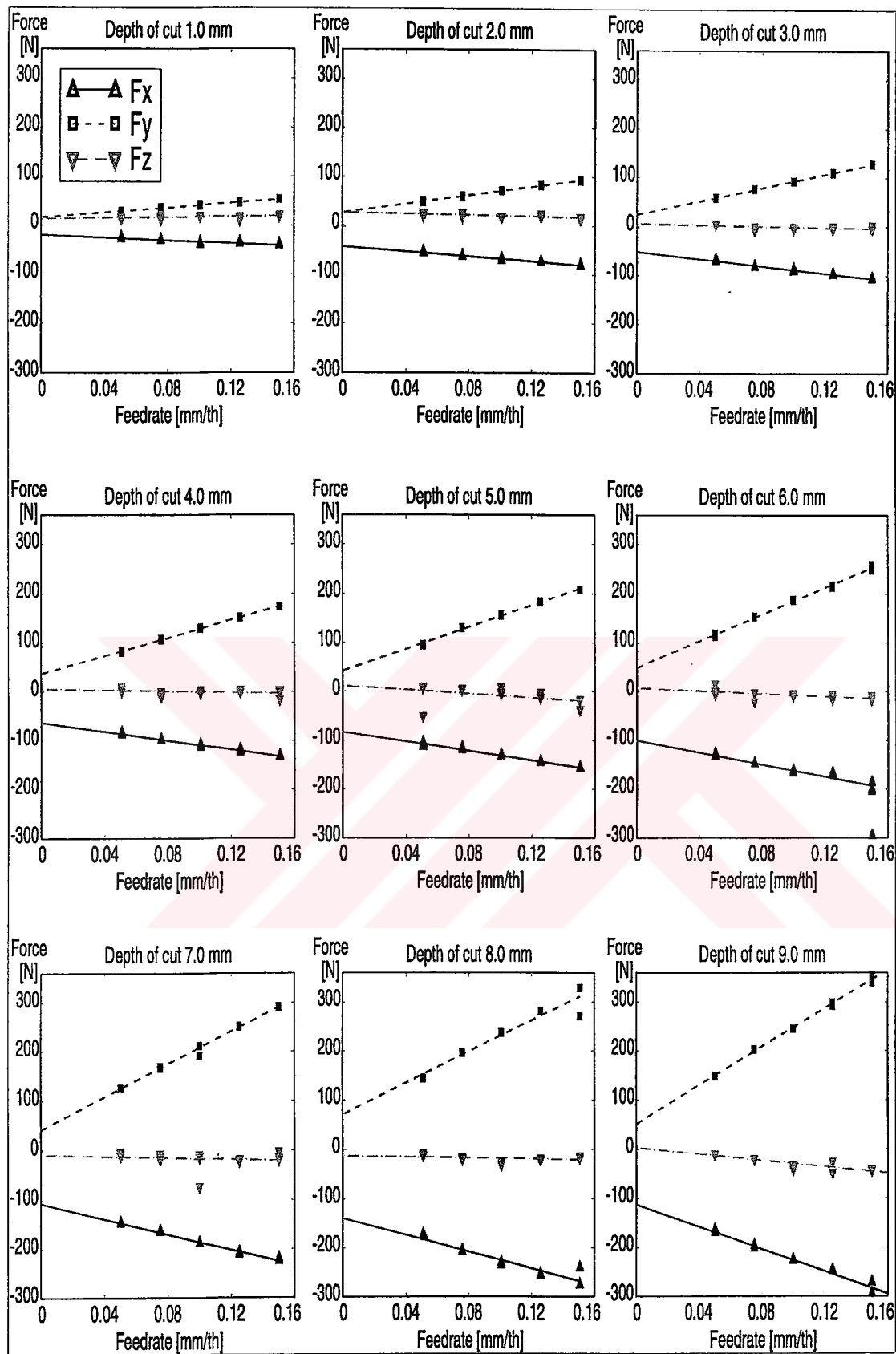


Figure 5.7. The average cutting forces for 10° helix angle, slotting, 3780 rev/min, for Al7075 (Cutter I, see Figure 5.6).

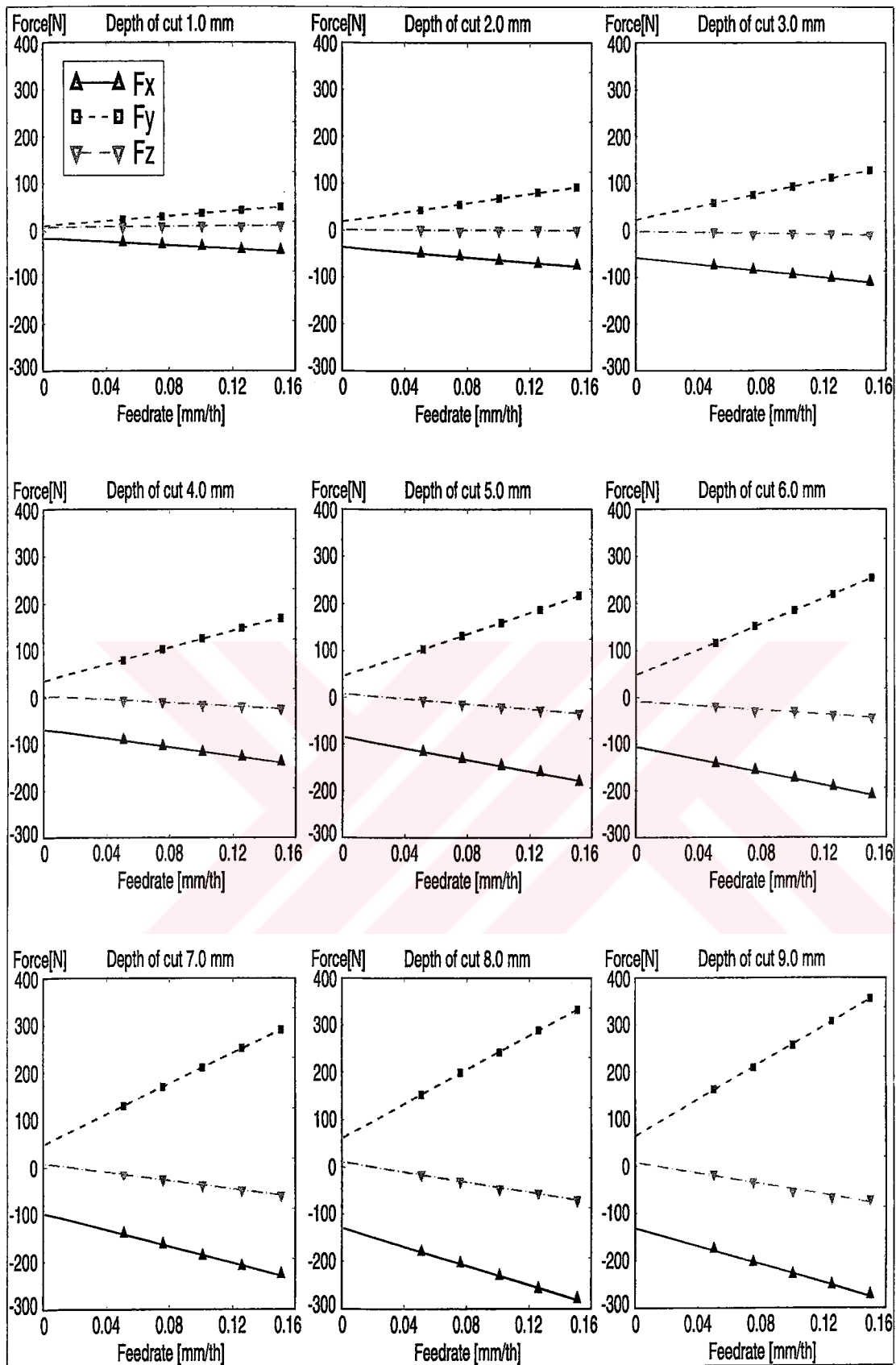


Figure 5.8. The average cutting forces for 20° helix angle, slotting, 3780 rev/min, for Al7075 (Cutter I, see Figure 5.6).

The identified cutting coefficients for the two different inserts are given in Table 5.1 (Figure 5.9 and Figure 5.10). The correlation coefficient for each curve fit is also provided. The closeness of the correlation coefficient to unity represents the closeness of the fit curve to the actual data. Higher order polynomials were also used, but the accuracy improvement was not sufficient to justify the complexity in expressing the cutting coefficients. The correlation coefficients are low, because of the complex geometry on the rake face of the insert. The curve fitting to the cutting coefficients along the z direction was an approximate solution.

Table 5.1. The cutting coefficients for Al7075 with Cutter I ($0 \leq z \leq 9.8\text{mm}$). The edge contact position (z) is expressed in millimeters.

	Cutting Coef.	Fitted curve	Correla.Coeff.
10° helix angle	K_{tc} [N/mm ²]	$7.9z^2 - 90.4z + 1073.5$	0.7984
	K_{rc} [N/mm ²]	$25.98z^2 - 222.3z + 795.1$	0.9198
	K_{ac} [N/mm ²]	$19.55z^2 - 159.4z + 294.55$	0.9164
	K_{te} [N/mm]	$-0.34z^2 - 1.91z + 38.78$	0.7796
	K_{re} [N/mm]	$-3.15z^2 + 22.98z + 23.80$	0.8071
	K_{ae} [N/mm]	$-2.66z^2 + 27.56z - 54.80$	0.9195
20° helix angle	K_{tc} [N/mm ²]	$-5.6z^2 + 22.76z + 925.4$	0.9003
	K_{rc} [N/mm ²]	$-13.0z^2 + 85.6z + 409.1$	0.9242
	K_{ac} [N/mm ²]	$-15.3z^2 + 181.9z - 227.6$	0.8241
	K_{te} [N/mm]	$-0.04z^2 - 2.04z + 33.86$	0.8006
	K_{re} [N/mm]	$-0.46z^2 + 1.38z + 53.86$	0.9672
	K_{ae} [N/mm]	$-0.25z^2 + 2.56z - 6.92$	0.7754

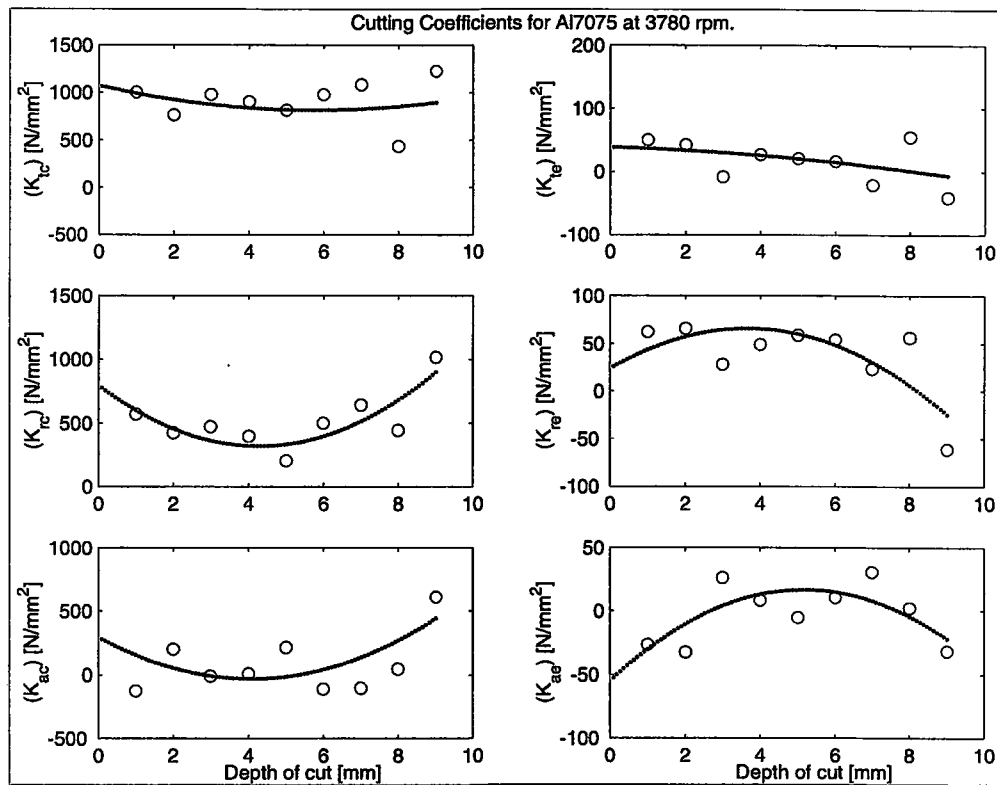


Figure 5.9. The average cutting coefficients for 10° helix angle, slotting, 3780 rev/min, for Al7075 (Cutter I, see Figure 5.6).

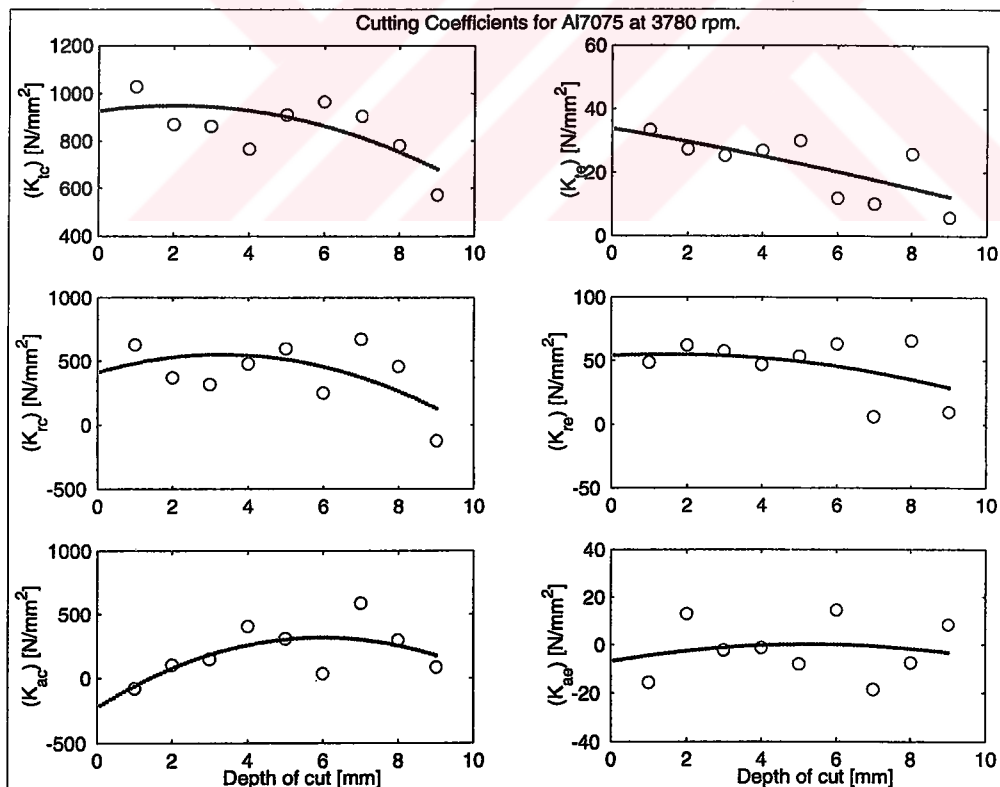


Figure 5.10. The average cutting coefficients for 20° helix angle, slotting, 3780 rev/min, for Al7075 (Cutter I, see Figure 5.6).

The cutter is attached to a taper 40 spindle with a mechanical chuck. The transfer functions ($[\Phi_{xx}(s)] = \{x\}/\{F_x\}$, $[\Phi_{yy}(s)] = \{y\}/\{F_y\}$) of the cutter mounted on the spindle were measured using impact hammer tests. The identified modal parameters are given in Table 5.2 for the machine tool spindle (Figure 5.11), and in Table 5.3 for the workpiece (Figure 5.12). The transfer function model was presented in Section 4.5.3.

Table 5.2. The transfer function parameters from tool tip (Cutter I).

Direction	Mode	ω_n [Hz]	ζ [%]	Mode Residue [m/N] ($\sigma_k \pm i v_k$)
X	1	269.20	4.81	$(10.097 - i 8.760) \cdot 10^{-6}$
	2	447.51	4.66	$(25.835 - i 105.161) \cdot 10^{-6}$
	3	573.79	4.69	$(2.976 - i 57.900) \cdot 10^{-6}$
	4	1837.50	2.94	$(54.251 - i 98.720) \cdot 10^{-6}$
	5	2843.05	3.10	$(108.761 - i 29.515) \cdot 10^{-6}$
	6	3065.05	1.13	$(63.293 - i 0.188) \cdot 10^{-6}$
	7	4007.50	0.81	$(-8.380 - i 31.021) \cdot 10^{-6}$
Y	1	495.47	3.32	$(8.428 - i 169.651) \cdot 10^{-6}$
	2	1850.98	2.68	$(18.614 - i 76.902) \cdot 10^{-6}$
	3	2115.10	1.67	$(16.382 - i 33.327) \cdot 10^{-6}$
	4	3134.97	4.74	$(168.391 - i 191.999) \cdot 10^{-6}$
	5	3991.89	0.91	$(5.962 - i 17.500) \cdot 10^{-6}$

Table 5.3. The transfer function parameters of workpiece.

Direction	Mode	ω_n [Hz]	ζ [%]	Mode Residue [m/N]
X	1	790.56	2.75	$(1.149097 - i 5.934) \cdot 10^{-6}$
	2	1068.29	1.91	$(0.622097 - i 17.641) \cdot 10^{-6}$
Y	1	793.02	2.41	$(4.358 - i 23.716) \cdot 10^{-6}$
	2	1075.30	1.90	$(-0.243 - i 2.185) \cdot 10^{-6}$

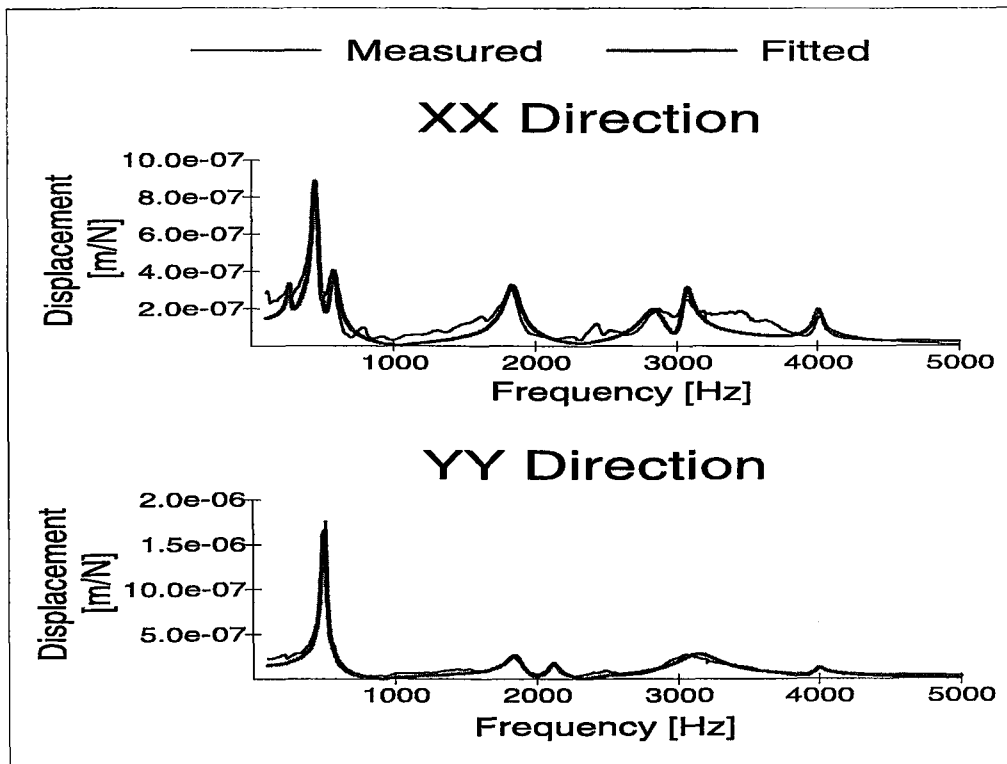


Figure 5.11. The transfer functions for the tool tip (Cutter I, see Figure 5.6).

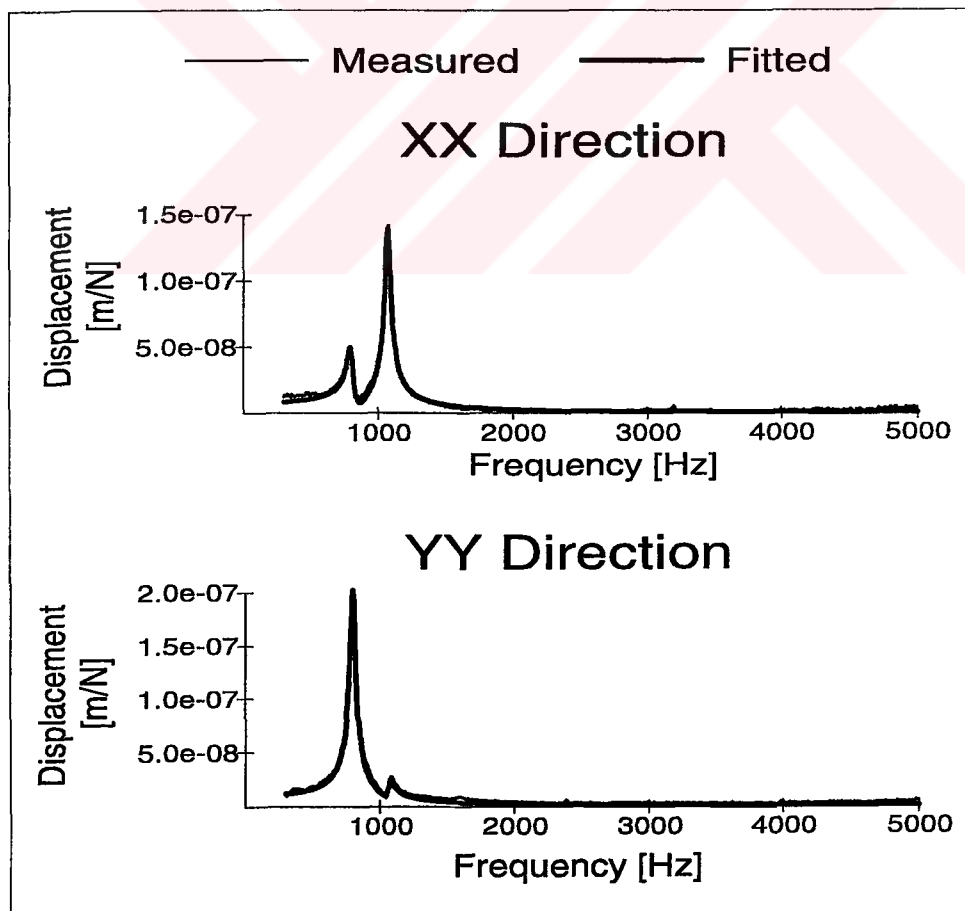


Figure 5.12. The transfer functions for the workpiece.

The use of transfer functions in frequency and time domain for the prediction of chatter vibrations can be found in Altıntaş et al. [20,67]. Using the identified cutting constants, the cutting forces are predicted for slot milling of Al7075 at different depths of cut. The predicted and measured cutting forces agreed well, as can be seen from two sample test results shown in Figure 5.13 and Figure 5.14. The machine tool was unable to handle larger depths of cut due to chatter and its limited torque capacity. The chatter stability lobes of the cutter were predicted using time domain as well as frequency domain analytical methods. The time domain simulation considers the nonlinearities in the process, such as varying cutting constants along the insert, discontinuities along the flute due to inserts, and tool jumping out of cut due to excessive vibrations. The frequency domain analytical solution assumes constant average cutting coefficients and continuous cutting edges like in regular end mills. The stability lobes for half immersion down milling were predicted and experimentally verified with 0.050 mm/tooth feed rate (Figure 5.15). The presence of chatter was monitored by checking the spectrum of sound signals, which were measured using a microphone placed close to the cutting zone. Although the analytical chatter stability assumes a simplified, linear process model, it is still able to predict the chatter stability lobes quite satisfactorily. The stability lobe generation in time domain took approximately two days on a Pentium II machine running at 450 Mhz, whereas the computation time for the frequency domain solution was only a few seconds. The stability lobes predicted the presence of chatter for 6.0 mm of axial depth of cut and 3150 rev/min of spindle speed. The measured and simulated surface finish profile and chatter vibration spectra were found to be in good agreement. The chatter was caused by the first bending mode (473 Hz) of the spindle - cutter assembly. When the speed was increased to 5750 rev/min, the chatter vibrations had disappeared as predicted by the stability lobe diagram and time domain simulations.

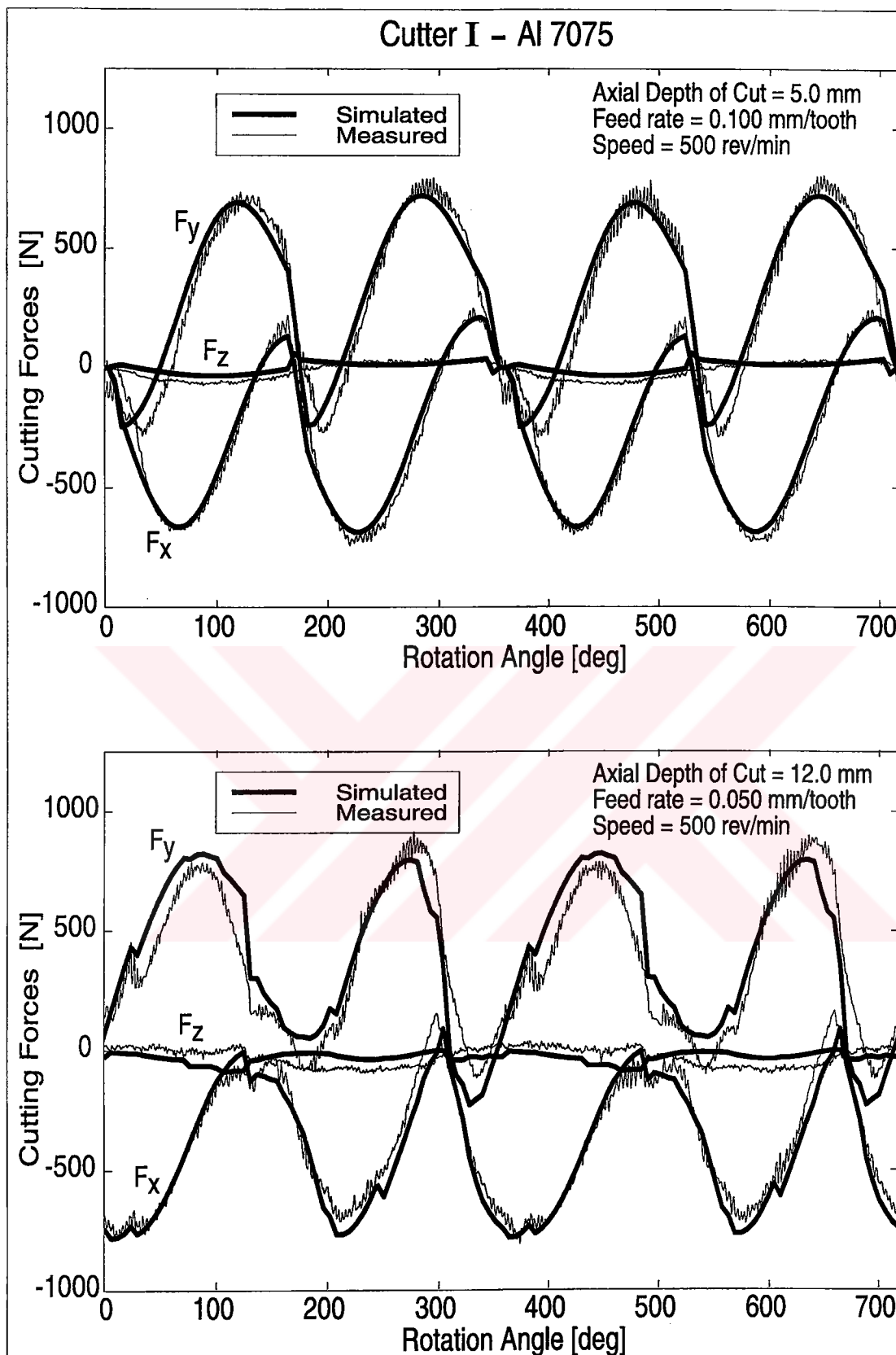


Figure 5.13. Measured and predicted cutting forces for Al-7075 with Cutter I. Cutting type is slotting with two flutes. See Figure 5.6 for tool geometry and Table 5.1 for cutting coefficients.

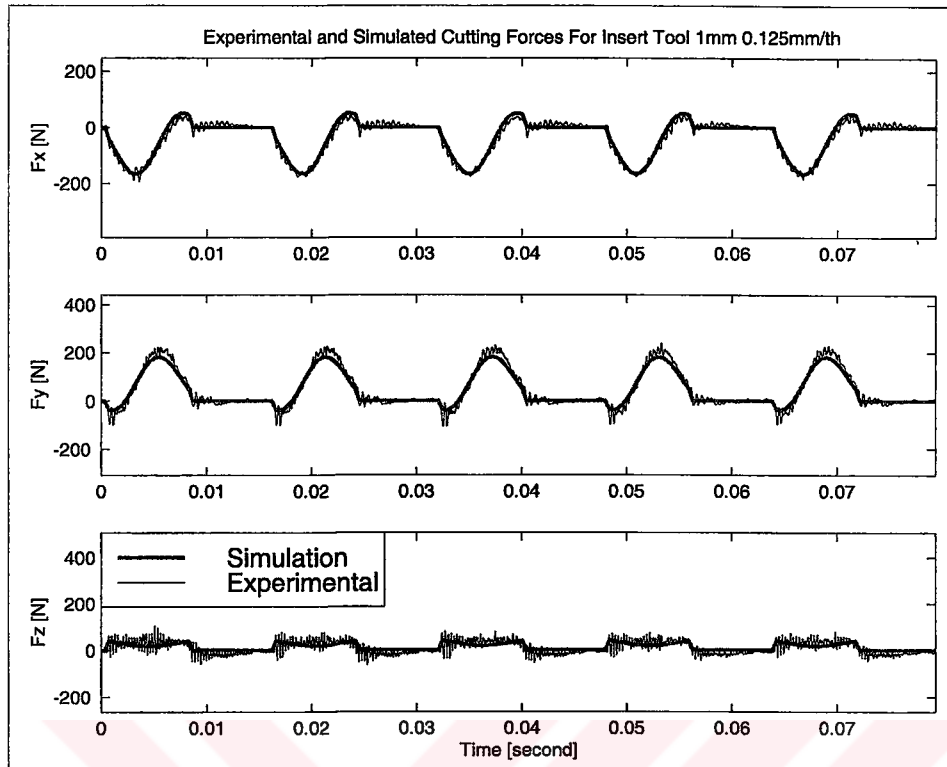


Figure 5.14 a) Al7075, 10 degree helix angle at 3780rpm 1 flute, full immersion

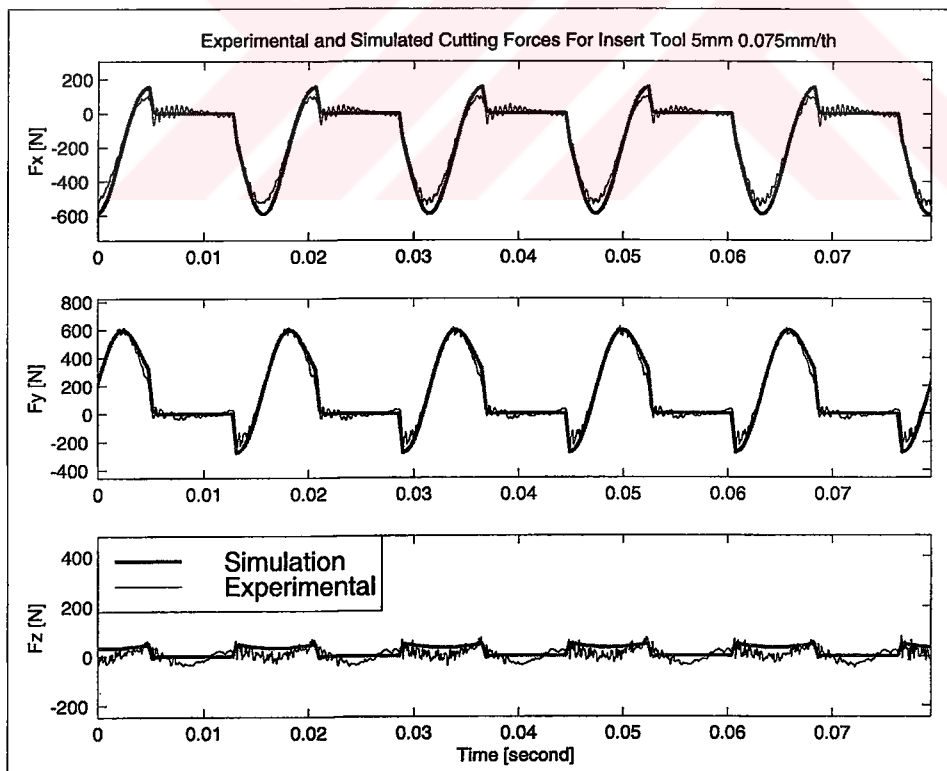


Figure 5.14 b) Al7075, 10 degree helix angle at 3780rpm 1 flute, full immersion

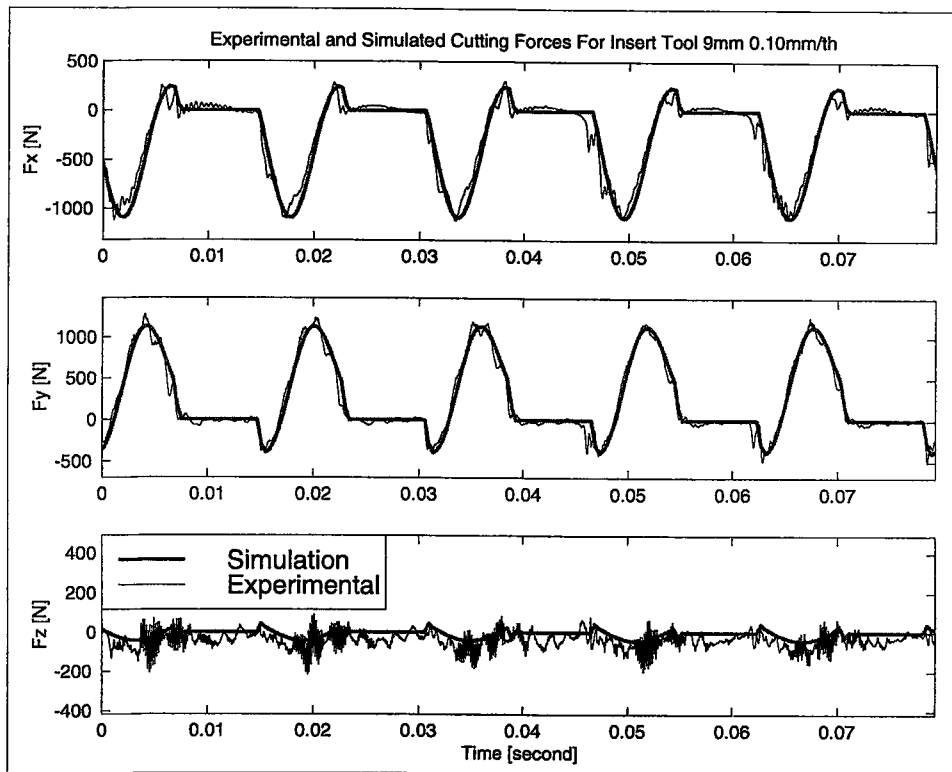


Figure 5.14 c) Al7075, 10 degree helix angle at 3780rpm 1 flute, full immersion

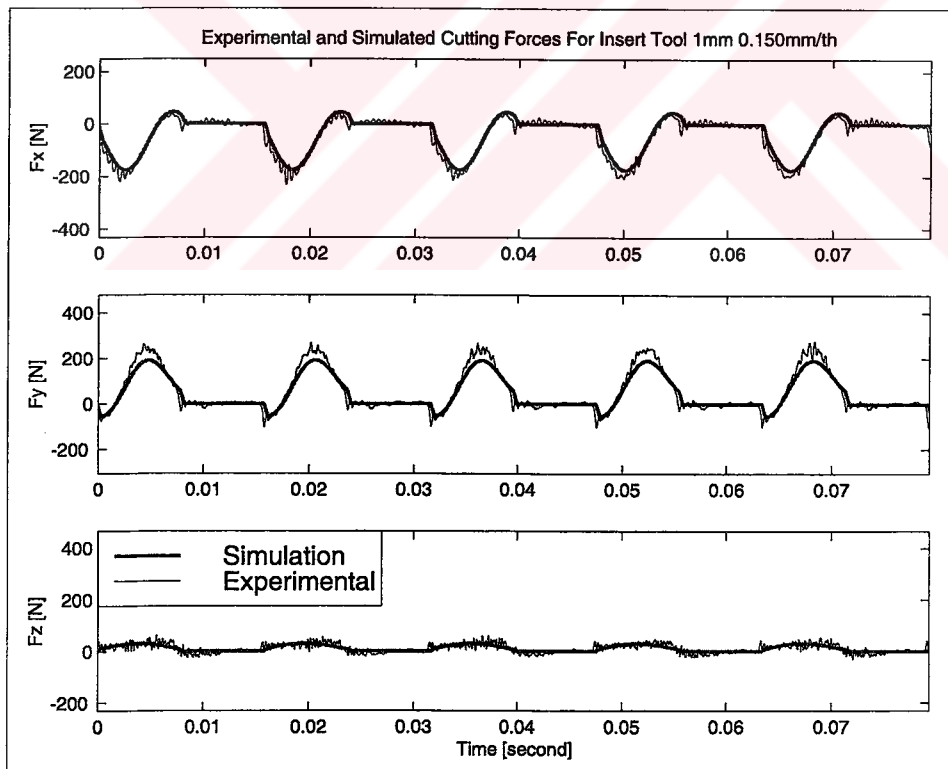


Figure 5.14 d) Al7075, 20 degree helix angle at 3780rpm 1 flute, full immersion

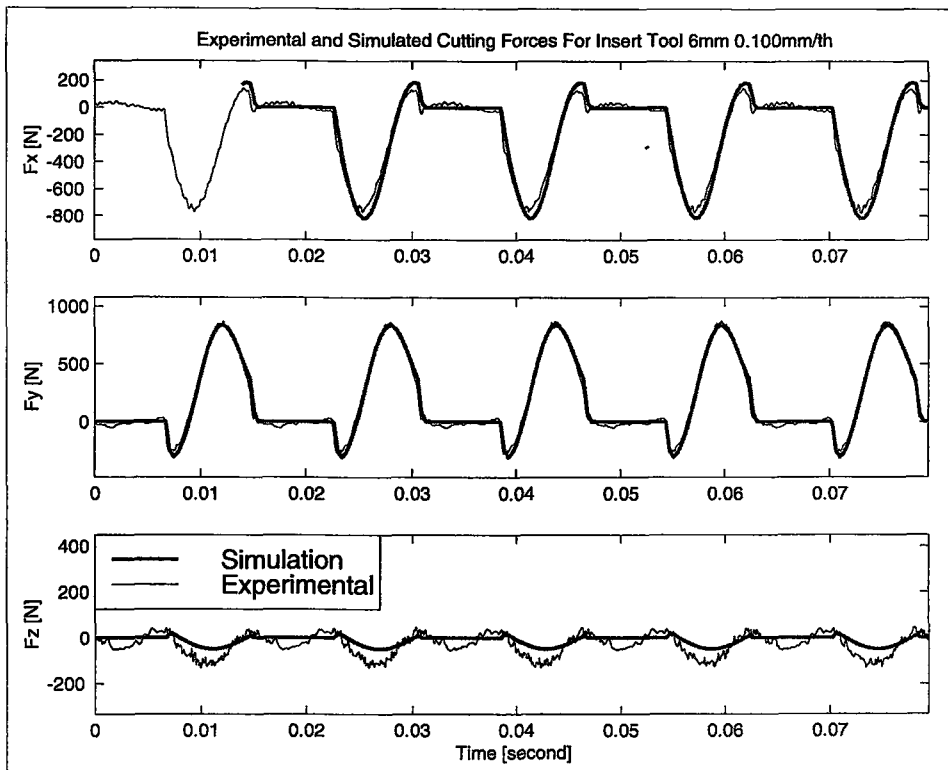


Figure 5.14 e) Al7075, 20 degree helix angle at 3780rpm 1 flute, full immersion

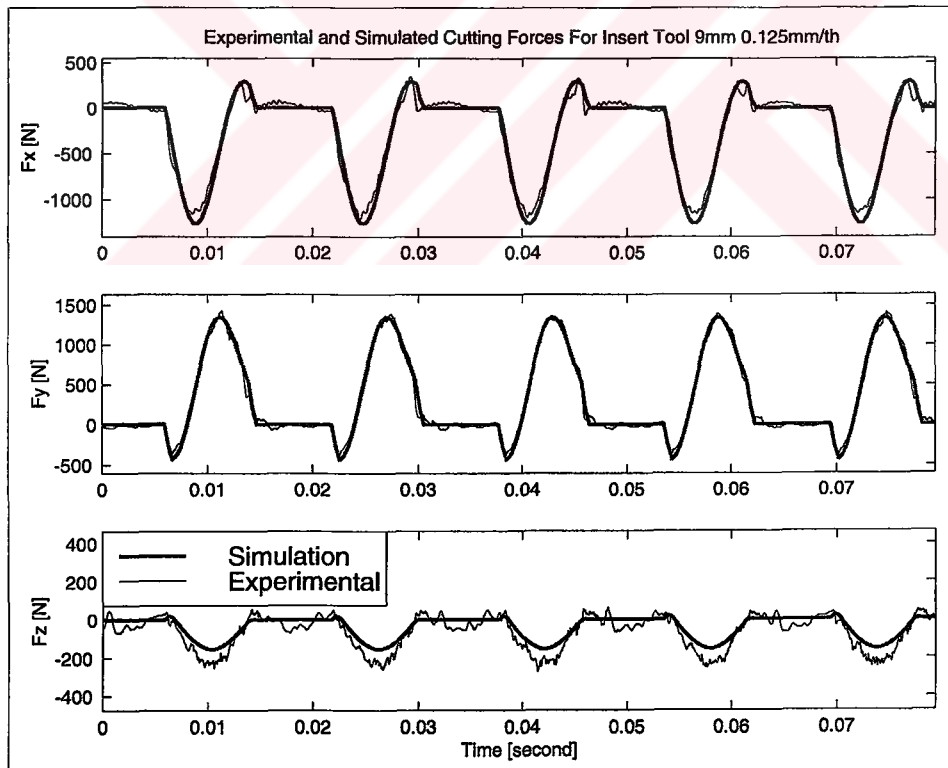


Figure 5.14 f) Al7075, 20 degree helix angle at 3780rpm 1 flute, full immersion

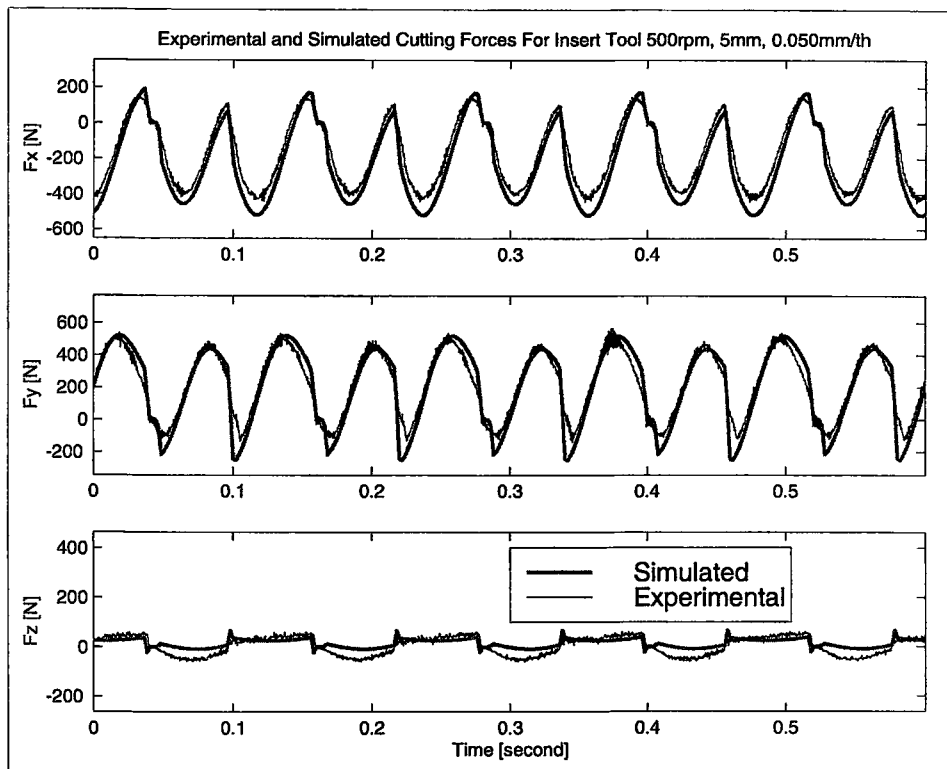
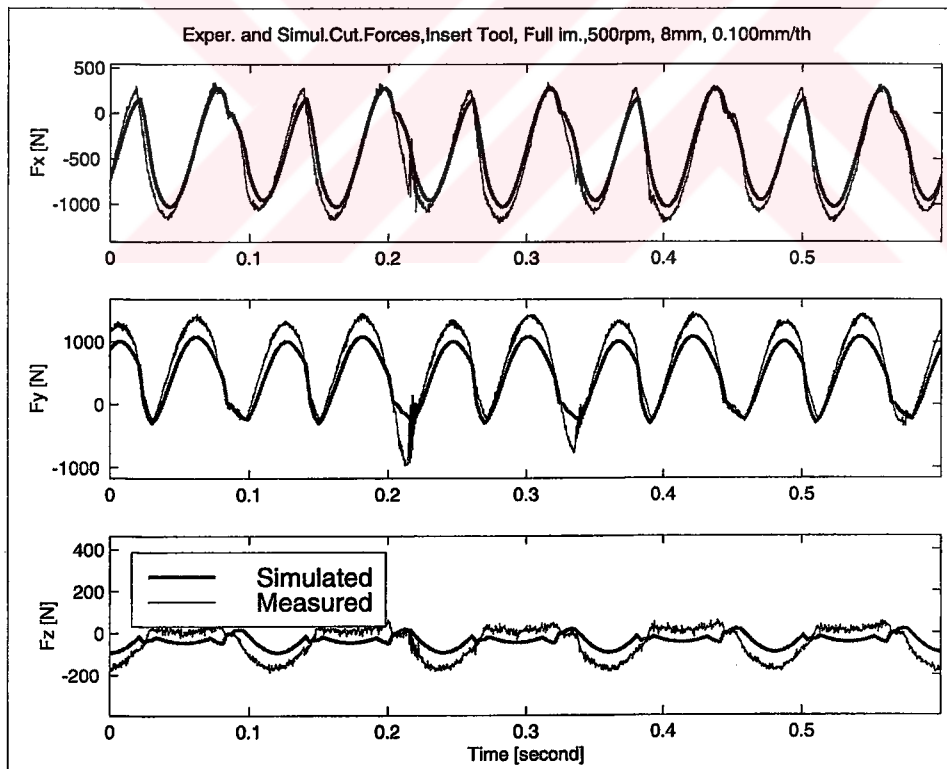


Figure 5.14 g) Al7075, 10 and 20 degree helix angle at 500rpm, 2 flutes, full im.



h) Al7075, 10 and 20 degree helix angle at 500rpm, 2 flutes, full immersion

Figure 5.14. Simulated and measured cutting forces for Al7075 and (Cutter I, Figure 5.6 for tool geometry). See Table 5.1 for cutting coefficients.

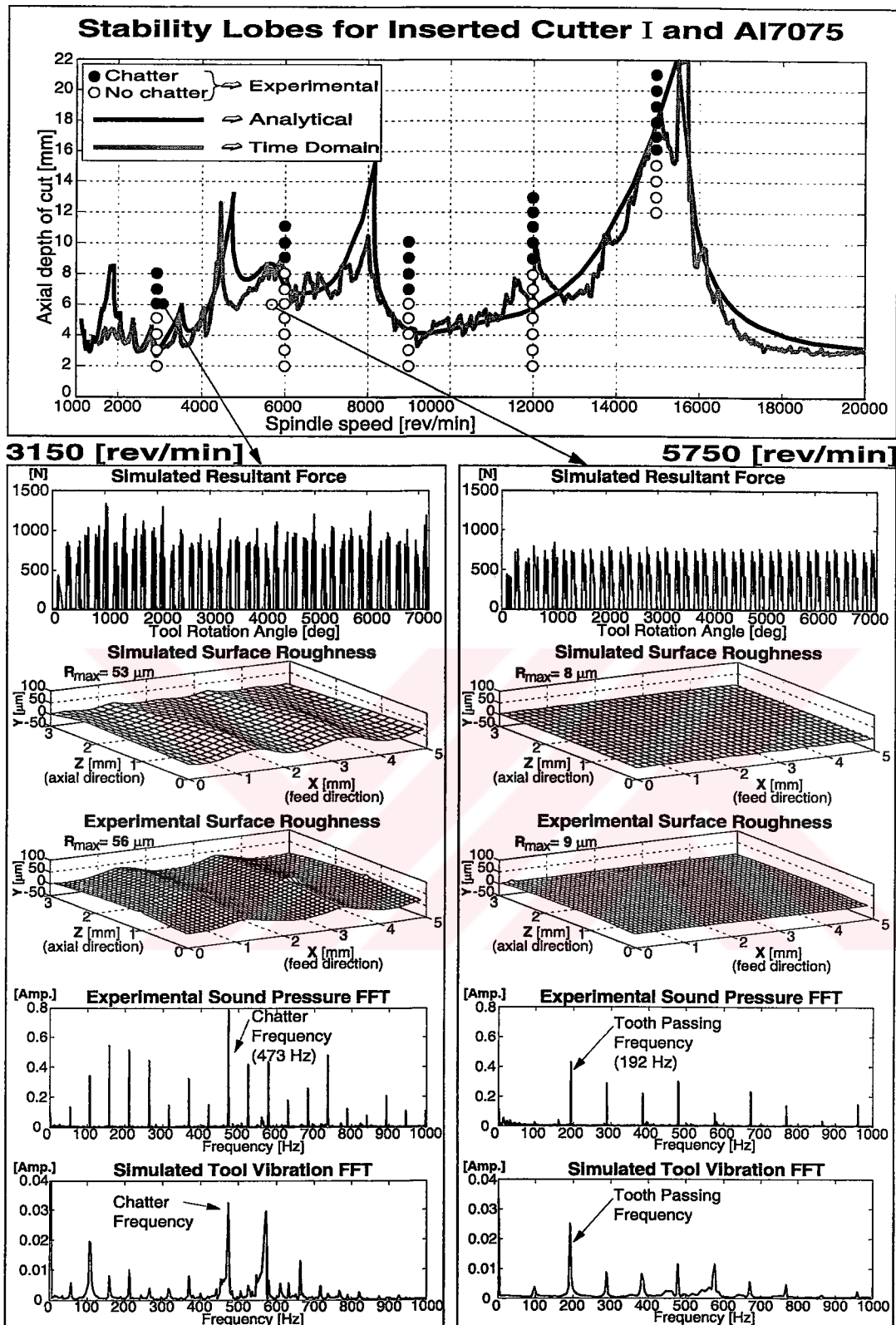


Figure 5.15. Stability lobes for Al-7075 with Cutter I (see Figure 5.6). Cutting conditions: half immersion down milling, $N_f = 2$ flutes, diameter=25.4 mm, feed rate=0.050 mm/tooth. See Table 5.1 for cutting coefficients Al-7075 and Table 5.2 and Table 5.3 for the transfer function parameters.

5.4.2 Milling of Ti₆Al₄V with Cutter II:

In this second case study, a titanium alloy Ti₆Al₄V is milled using a Mitsubishi LER1606W20 R98244 cutter with two flutes and two different rectangular inserts (Mitsubishi CCMX083508ENA and ZCMX083508ERA HTi20T). There are three inserts placed on each flute. The cutter design model is formed as (Figure 5.16):

$$D_M = \begin{bmatrix} 3 & 3 & 3 & ; & 4 & 3 & 3 \\ 4.0 & 12.8 & 21.6 & ; & 5.0 & 16.5 & 25.3 \\ 8.7 & 8.7 & 8.7 & ; & 8.7 & 8.7 & 8.7 \\ 0.0 & 0.0 & 0.0 & ; & 0.0 & 0.0 & 0.0 \\ -10.0 & -10.0 & -10.0 & ; & -10.0 & -10.0 & -10.0 \\ 0.0 & 330.0 & 300.0 & ; & 180.0 & 150.0 & 120.0 \end{bmatrix} \quad (5.10)$$

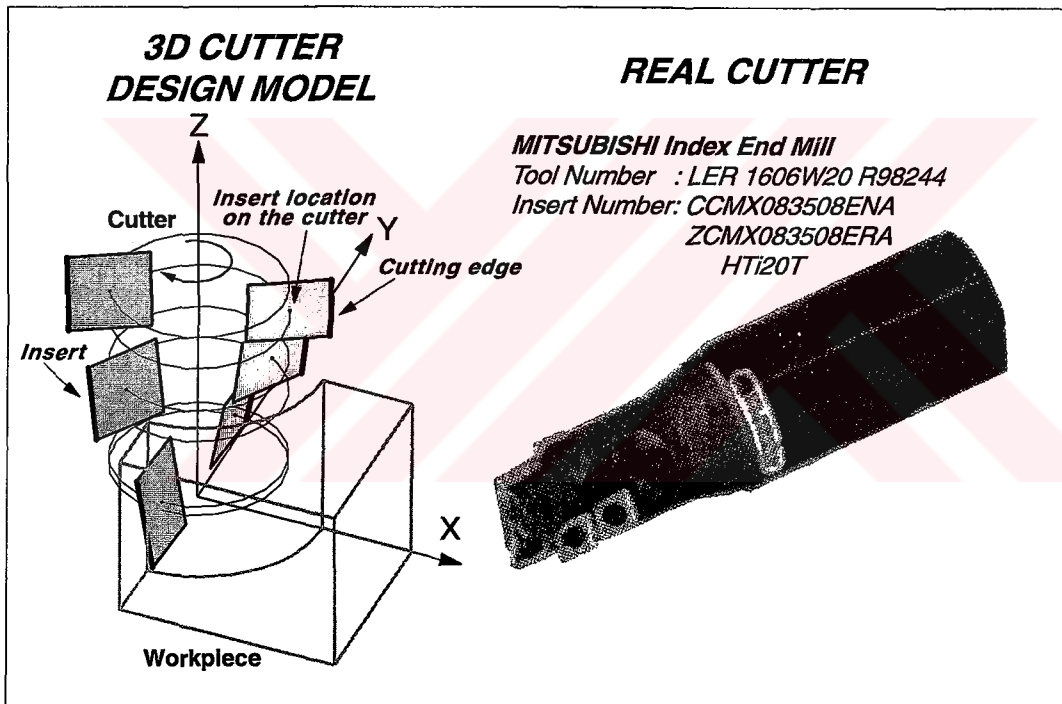


Figure 5.16. Mitsubishi inserted cutter Cutter II (Cutter number: LER1606W20 R98244; Insert numbers: CCMX083508ENA “insert dimensions; $a=8.0$ mm, $b=8.0$ mm” and ZCMX083508ERA “insert dimensions; $a=8.0$ mm, $b=10.0$ mm” HTi20T). See equation (5.10) for insert location parameters. Shank diameter=31.75 mm, Cutter diameter=25.4 mm, 2 flutes, 3 inserts on every flute.

The first row indicates rectangular insert types 3 and 4. Both inserts have identical geometry except the cutting edge lengths. Insert 3 has edge dimensions ($a=8.0$ mm, $b=8.0$ mm), and Insert 4 has ($a=8.0$ mm, $b=10.0$ mm). Insert 4 is used as a wiper blade to smooth the cut surface. The cutting coefficients of the inserts are identified

from slot milling tests conducted at different depths of cut (1.0 mm - 5.0 mm) and feed rates (0.020-0.080 mm/tooth). Average cutting forces can be seen in Figure 5.17 for one insert at the tool tip.

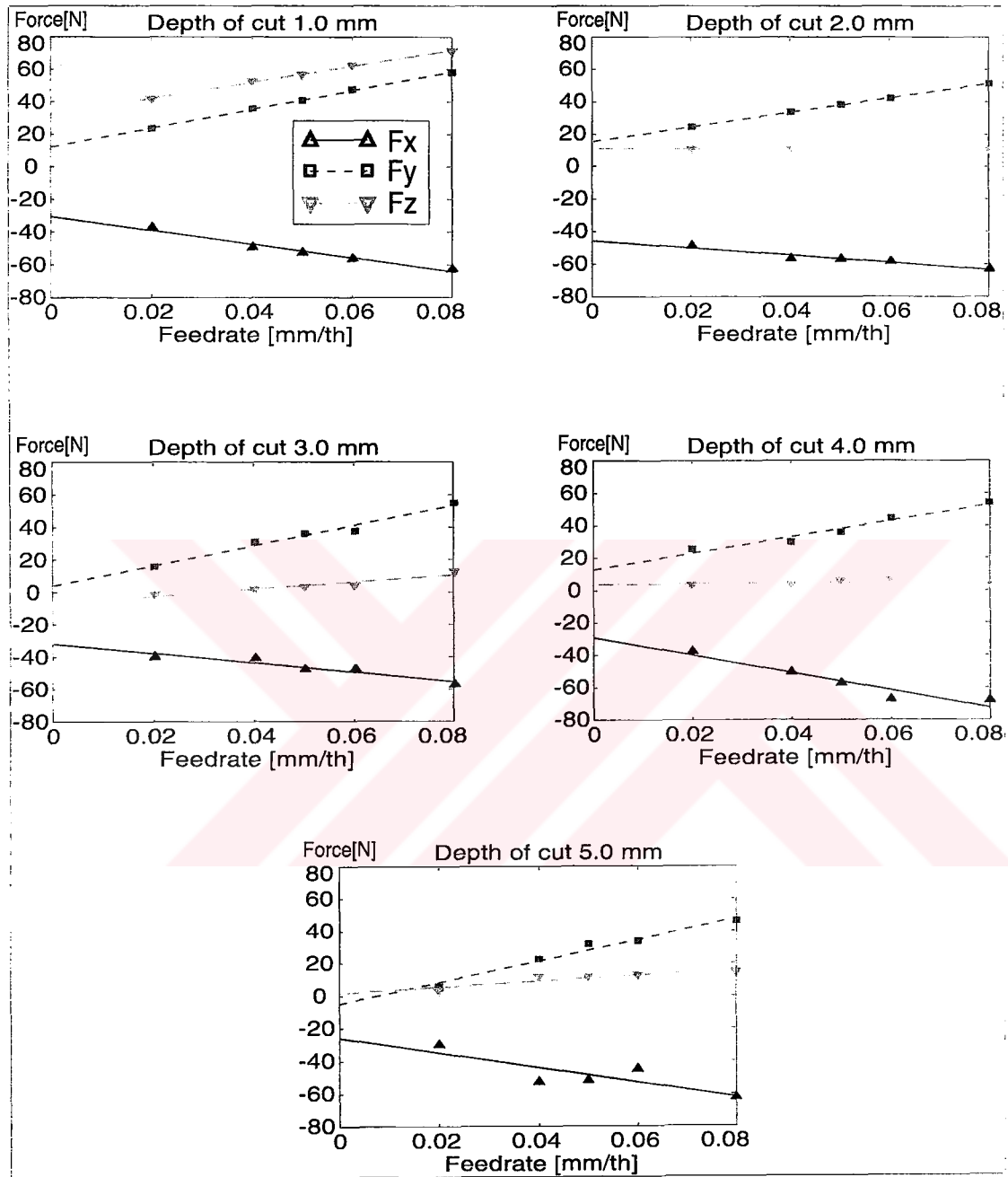


Figure 5.17. The average cutting forces for Ti_6Al_4V (Cutter II, see Figure 5.16), cutting type is slotting, 500 rev/min.

The cutting coefficients are given in Table 5.4 (Figure 5.18). The insert edge geometry does not change beyond the first 5.0 mm length, thus the cutting coefficients remain the same as the values at $z=5.0$ mm. The spindle speed was 500 rev/min. The cutting constants change as a function of the insert's axial contact (z)

with the material. The simulated and experimentally measured cutting forces are given in Figure 5.19 and Figure 5.20. Different feed rate and depth of cut combinations are given to enable comparison between the results. The spindle speed was 500 rev/min. This solution is an approximation, because of the rake face complexity along the z direction on the insert.

Table 5.4. The cutting coefficients for Ti₆Al₄V with Cutter II ($0 \leq z \leq 5.0$ mm). Between 5.0 mm and 10.0 mm, the cutting coefficients are constant and the same as the values used at $z=5.0$ mm. The edge contact position (z) is expressed in millimeter.

Cutting Coef.	Fitted curve	Correlation Coefficient
K_{rc} [N/mm ²]	$81.8z^2 - 405.3z + 2562.1$	0.8232
K_{re} [N/mm ²]	$110.7z^2 - 511.1z + 1862.3$	0.8931
K_{ac} [N/mm ²]	$-135.2z^2 + 935.8z - 1697.9$	0.8803
K_{te} [N/mm]	$-4.83z^2 + 17.50z + 25.80$	0.8260
K_{re} [N/mm]	$-5.84z^2 + 26.74z + 86.4$	0.9563
K_{ae} [N/mm]	$-4.83z^2 + 35.81z - 62.80$	0.9968

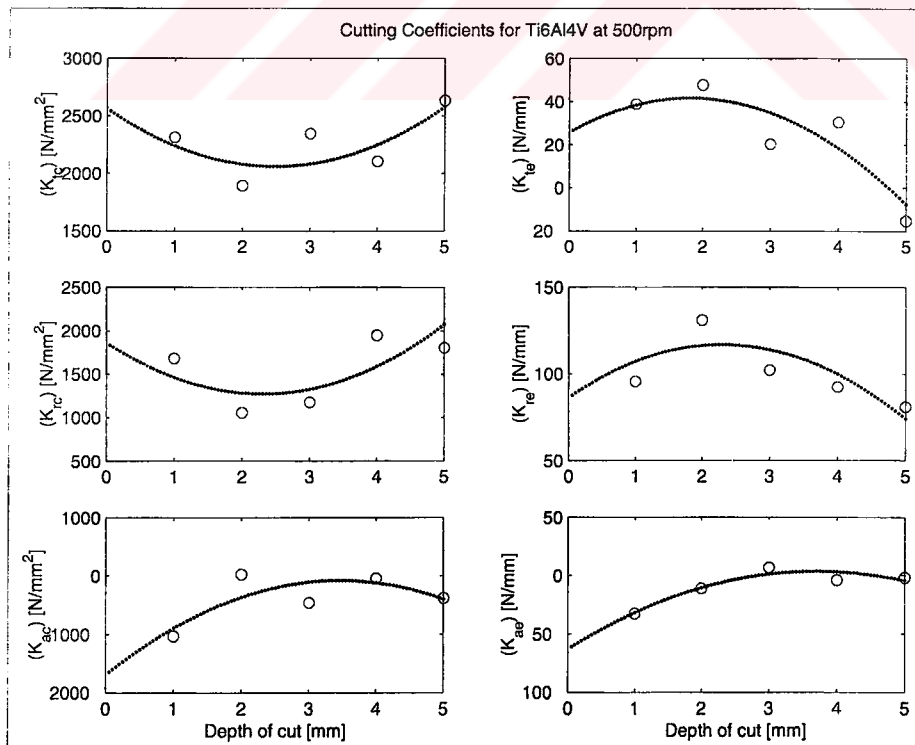


Figure 5.18. The average cutting coefficients for Ti₆Al₄V (Cutter II, see Figure 5.16).

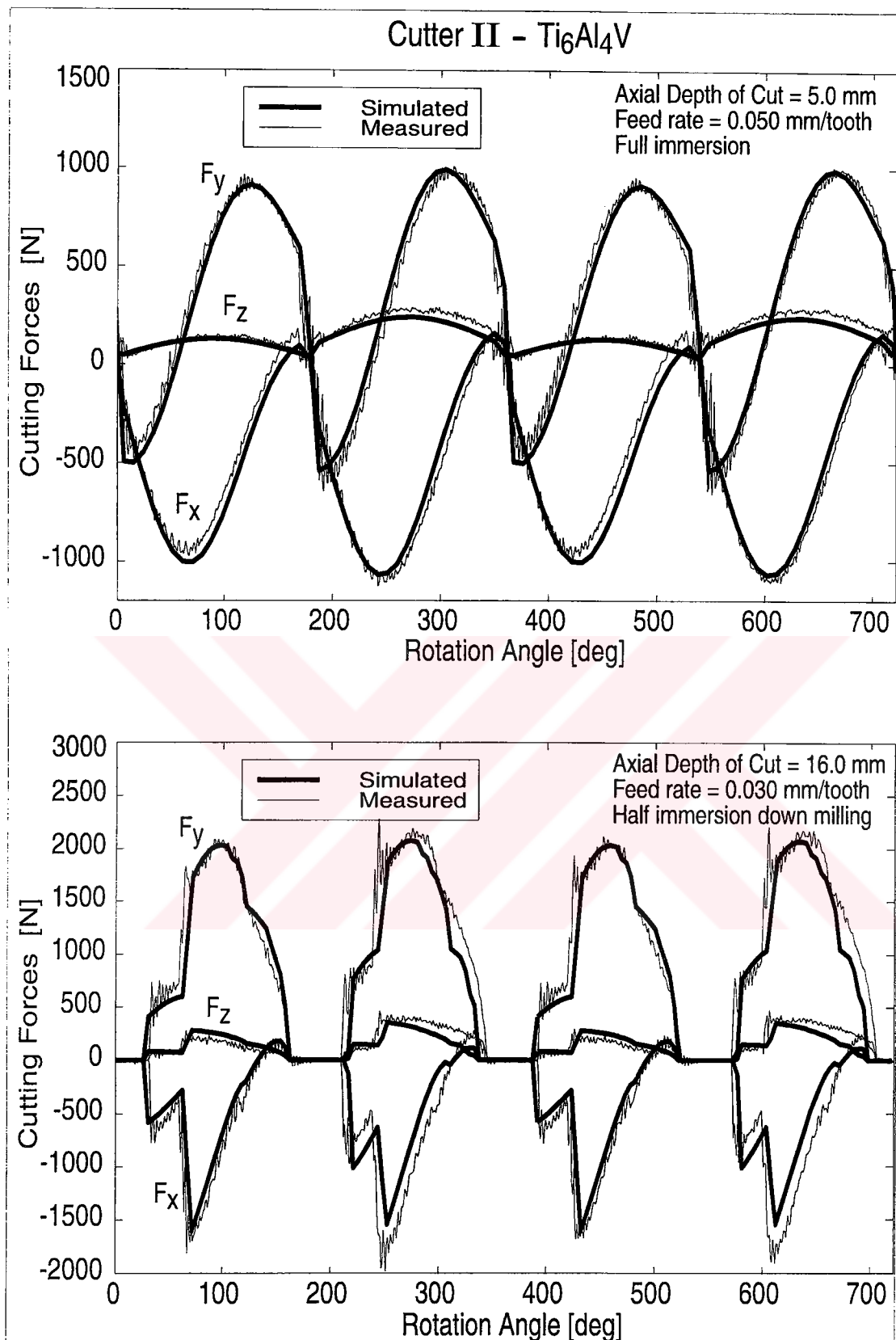
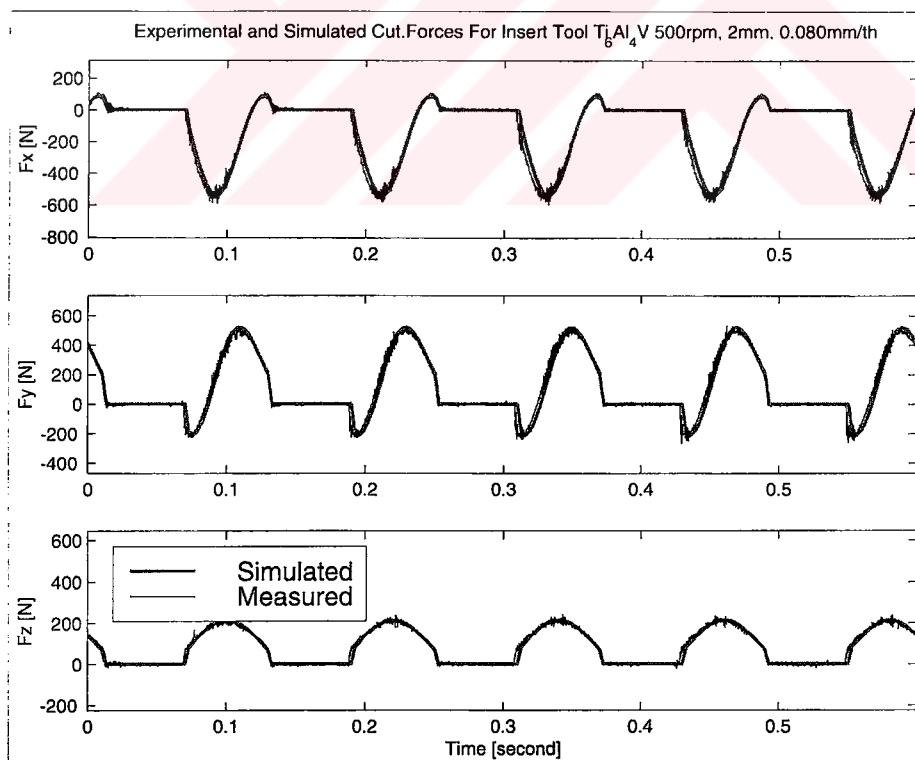
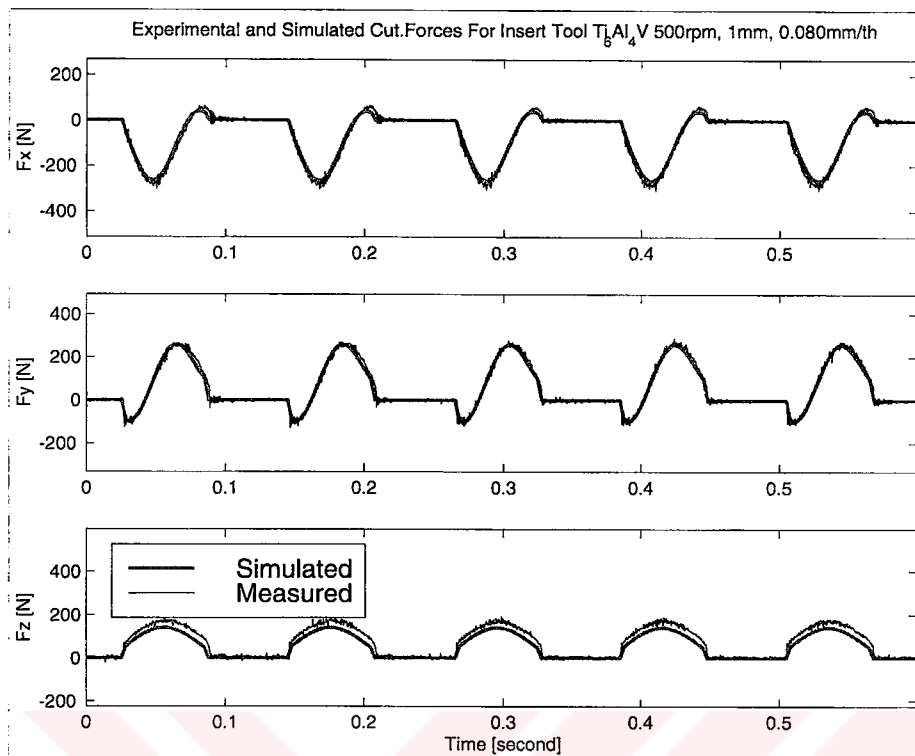


Figure 5.19. Measured and predicted cutting forces for Ti₆Al₄V with Cutter II. Spindle speed is 500 rev/min. See Figure 5.16 for tool geometry and Table 5.4 for cutting coefficients.



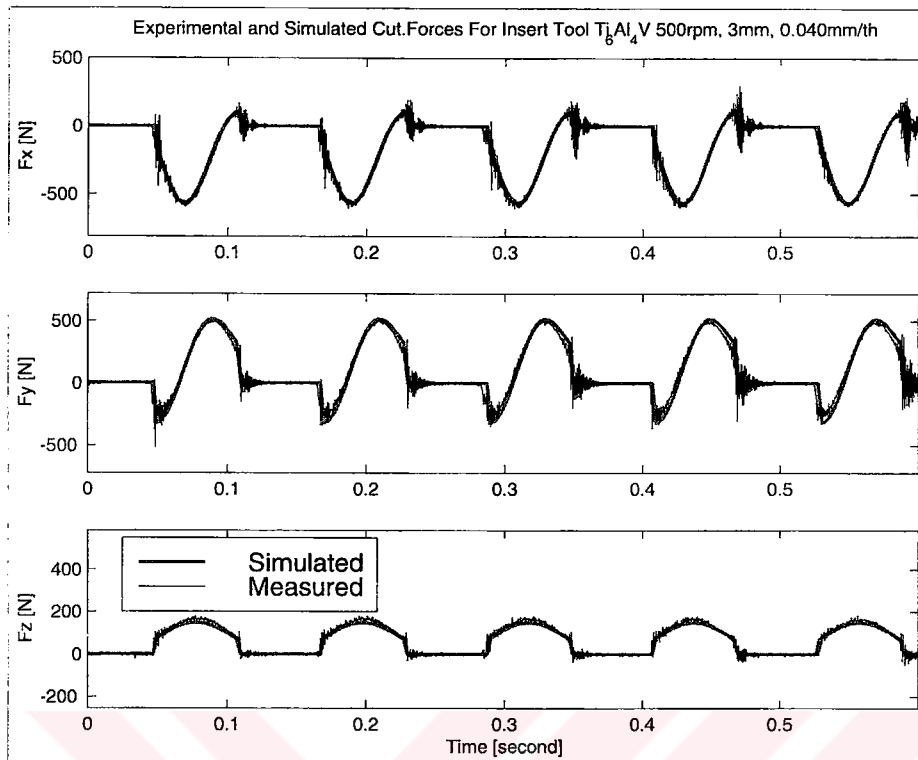


Figure 5.20 c) Ti_6Al_4V at 500rpm, 1 flute, full immersion.

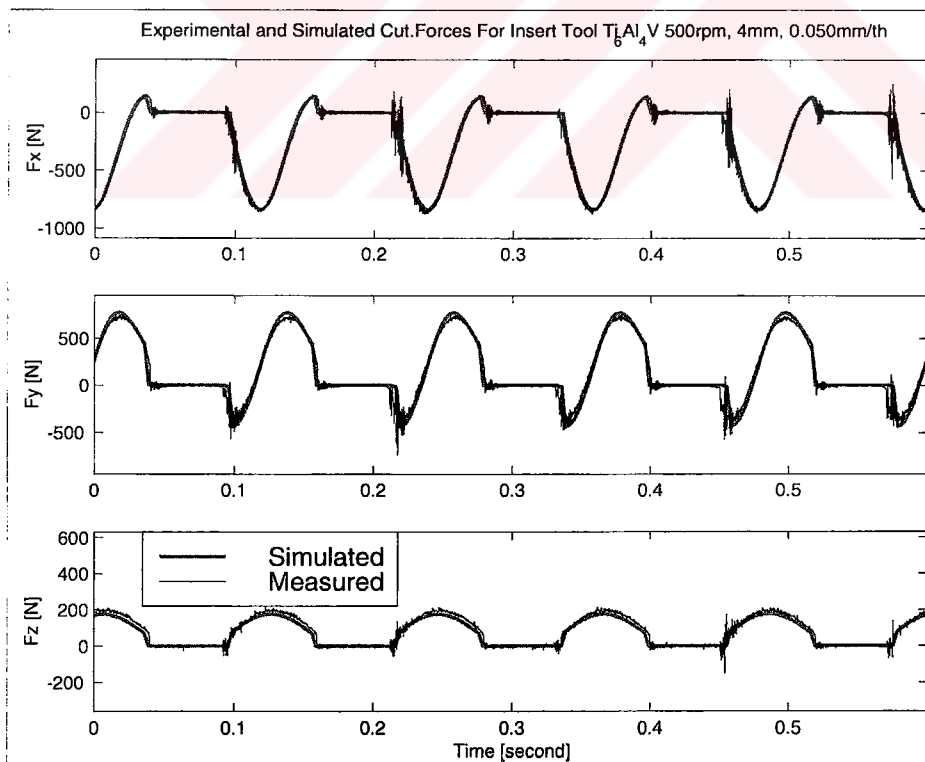


Figure 5.20 d) Ti_6Al_4V at 500rpm, 1 flute, full immersion.

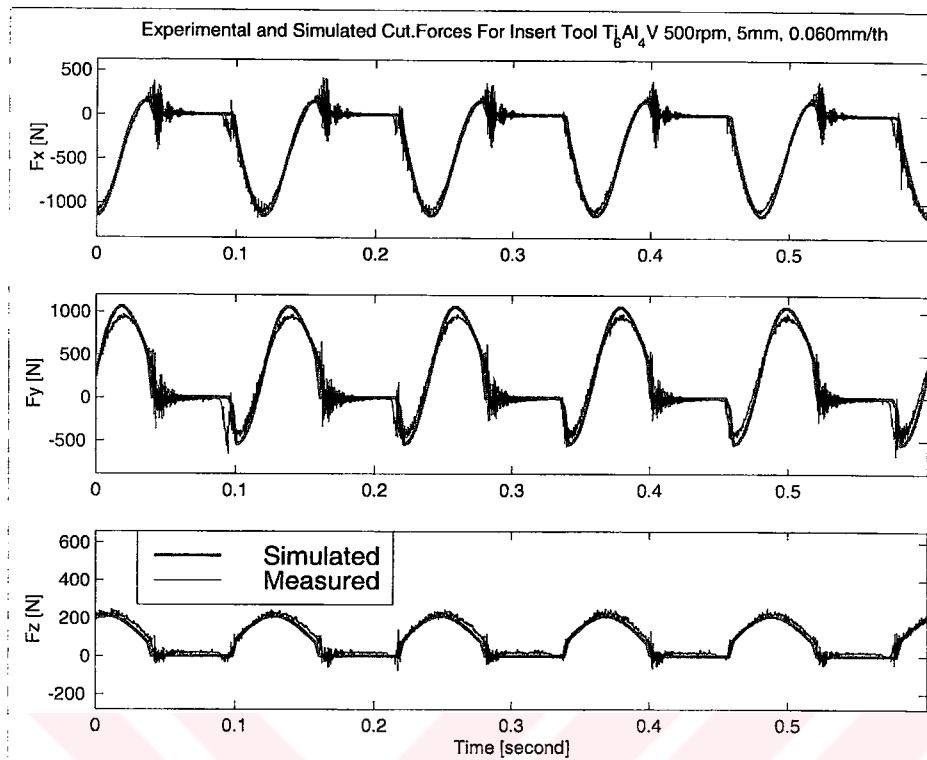


Figure 5.20 e) Ti_6Al_4V at 500rpm, 1 flute, full immersion.

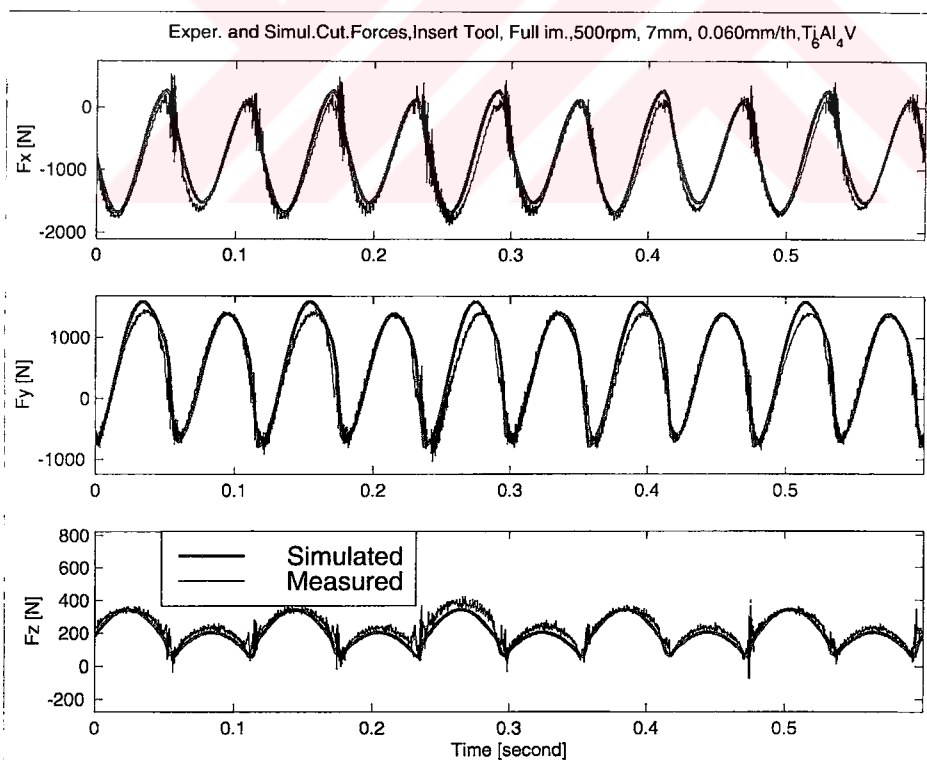


Figure 5.20 f) Ti_6Al_4V at 500rpm, 2 flutes

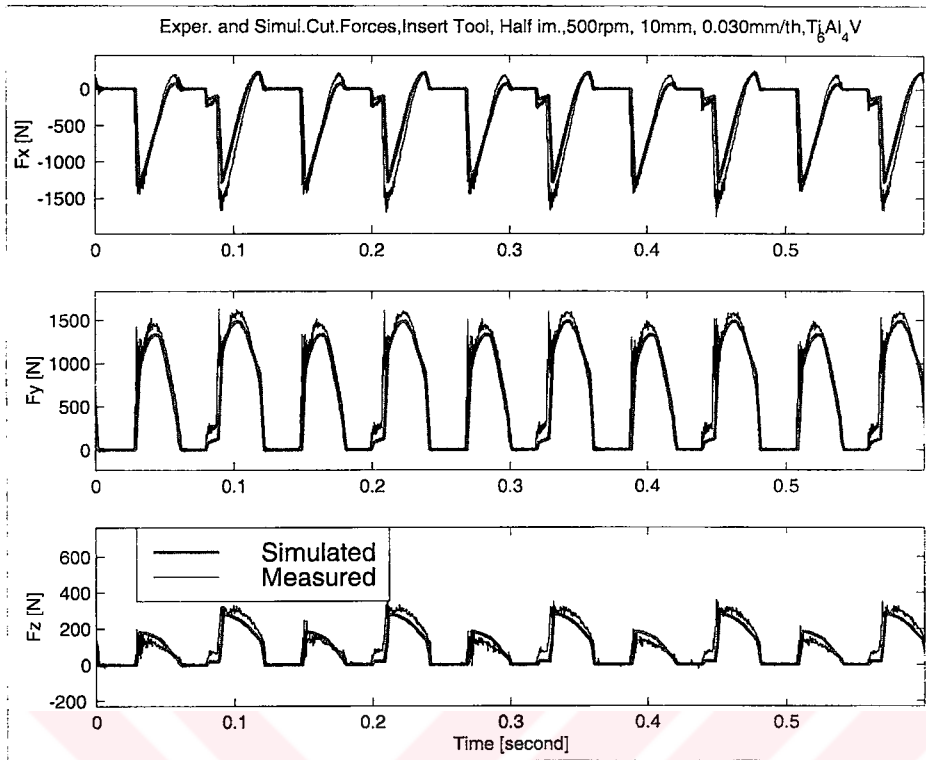


Figure 5.20 g) Ti₆Al₄V at 500rpm, 2 flutes

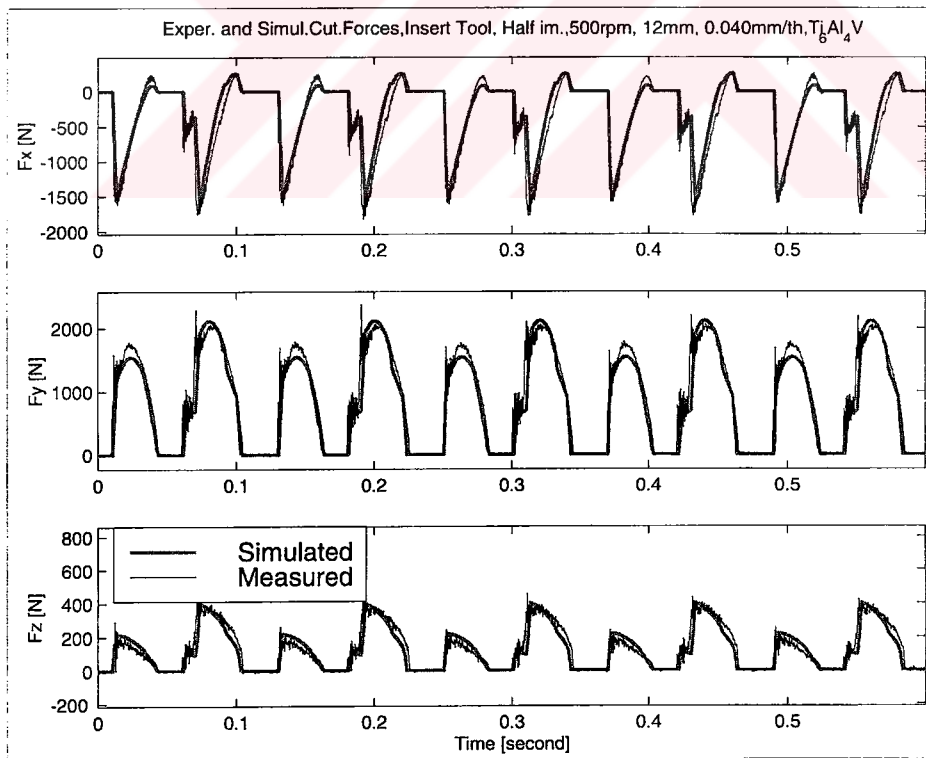
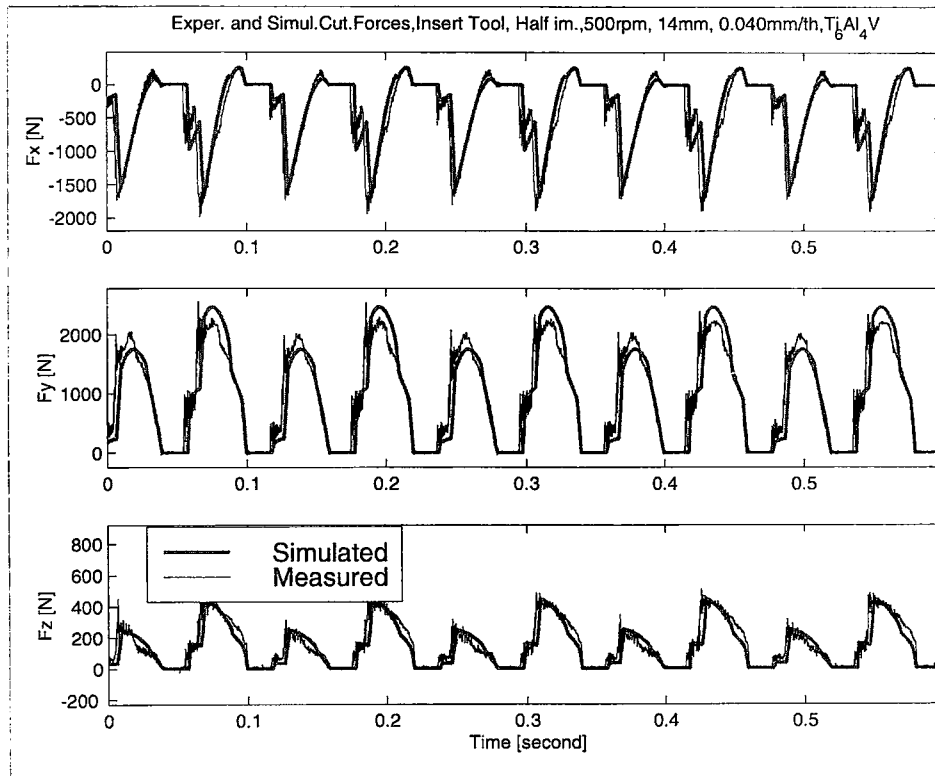


Figure 5.20 h) Ti₆Al₄V at 500rpm, 2 flutes



i) $\text{Ti}_6\text{Al}_4\text{V}$ at 500rpm, 2 flutes

Figure 5.20. Simulated and measured cutting forces for $\text{Ti}_6\text{Al}_4\text{V}$ (Cutter II, Figure 5.16 for the tool geometry). See Table 5.4 for the cutting coefficients.

5.5 Summary

The generalized mathematical models for inserted cutters have been developed. The model allows placing different inserts on the cutter body mathematically. Using a previously developed scheme for solving the exact kinematics of dynamic milling processes, the chip thickness, corresponding cutting forces, vibrations, and dimensional surface finish generated by inserted end mills were predicted in simulations. The mathematical models were verified by a number of experiments conducted with inserted cutters.

6. OPTIMAL DESIGN AND ANALYSIS OF VARIABLE PITCH CUTTERS

In this chapter, an analytical prediction of stability lobes for milling cutters with variable pitch angles is presented. This method requires the cutting constants, number of teeth, and transfer function of the cutter mounted on the machine tool as inputs to a chatter stability expression. The stability is formulated by transforming time varying directional cutting constants into time invariant constants. Constant regenerative time delay in uniform cutters is transformed into non-uniform multiple regenerative time delay for variable pitch cutters. The chatter free axial depth of cut is solved from the eigenvalues of the stability expression, whereas the spindle speed is identified from regenerative phase delays. The proposed technique has been verified with extensive cutting tests and time domain simulations. The practical use of the analytical solution is demonstrated by an optimal tooth spacing design application, which increases the chatter free depths of cut significantly.

6.1 Introduction

The amplitudes of chip thickness, cutting forces and vibrations, which are all interrelated, increase when a milling system experiences chatter vibrations. The cutting forces are periodic at tooth passing frequency when uniform pitch cutters are used. However, when the pitch angles are non-uniform, the process is periodic at spindle frequency or its harmonics as opposed to tooth passing frequency. A general closed form expression for the dynamic end milling process has been modeled for uniform pitch cutters previously by Budak and Altıntaş [67-69,84] and was summarized in Chapter 3.5. This theory is modified here to cover the stability of variable pitch cutters. A detailed static model of cutting forces and chip load generation for variable pitch cutters is given by Shirase and Altıntaş [97].

6.2 Mechanics and Dynamics of Variable Pitch Cutters

When there is a relative vibration between the tool and workpiece structure, the chip thickness varies with time. As a result, cutting forces vary dynamically as well. The dynamic model for end milling is shown in Figure 6.1. The stability formulation given for regular pitch cutters (see Chapter 3.5) is extended to irregular pitch cutters. The coordinate system X and Y can be expressed into the rotating tangential and normal directions at the tip of tooth j as u_j and v_j , respectively,

$$\begin{cases} u_j = -x \cos \phi_j(z) + y \sin \phi_j(z) \\ v_j = -x \sin \phi_j(z) - y \cos \phi_j(z) \end{cases} \quad (6.1)$$

The cutting forces acting on the j -th tooth in the X and Y directions are:

$$\begin{cases} dF_{xj}(\phi, z) = [-dF_{tj}(\phi, z) \cos \phi_j(z) - dF_{rj}(\phi, z) \sin \phi_j(z)] g(\phi_j(z)) \\ dF_{yj}(\phi, z) = [dF_{tj}(\phi, z) \sin \phi_j(z) - dF_{rj}(\phi, z) \cos \phi_j(z)] g(\phi_j(z)) \end{cases} \quad (6.2)$$

where the tangential and radial cutting forces acting on tooth j are expressed as:

$$\begin{aligned} dF_{tj} &= K_t h_j(\phi, z) dz \\ dF_{rj} &= K_r dF_{tj} \end{aligned} \quad (6.3)$$

In the above equation, K_t and K_r are chip load independent and are simplified linear cutting coefficients for radial and tangential directions, dz is the differential axial depth of cut and $h(\phi_j)$ is the chip thickness experienced by the tooth j .

$$h(\phi_j) = g(\phi_j)[s_t \sin \phi_j + (v_{j-1} - v_j)] \quad (6.4)$$

where s_t is the feed rate per tooth, ϕ_j is the immersion angle. v_j , v_{j-1} are the dynamic displacements of the present tooth j and previous tooth $j-1$ in the radial direction. The cutter is assumed to be rigid in the z direction. $g(\phi_j)$ is a unit step function, which determines whether the tooth is in or out of cut, i.e.,

$$g(\phi_j) = \begin{cases} 1 & \text{if } \phi_{st} < \phi_j < \phi_{ex} \\ 0 & \text{if } \phi_j < \phi_{st} \text{ or } \phi_j > \phi_{ex} \end{cases} \quad (6.5)$$

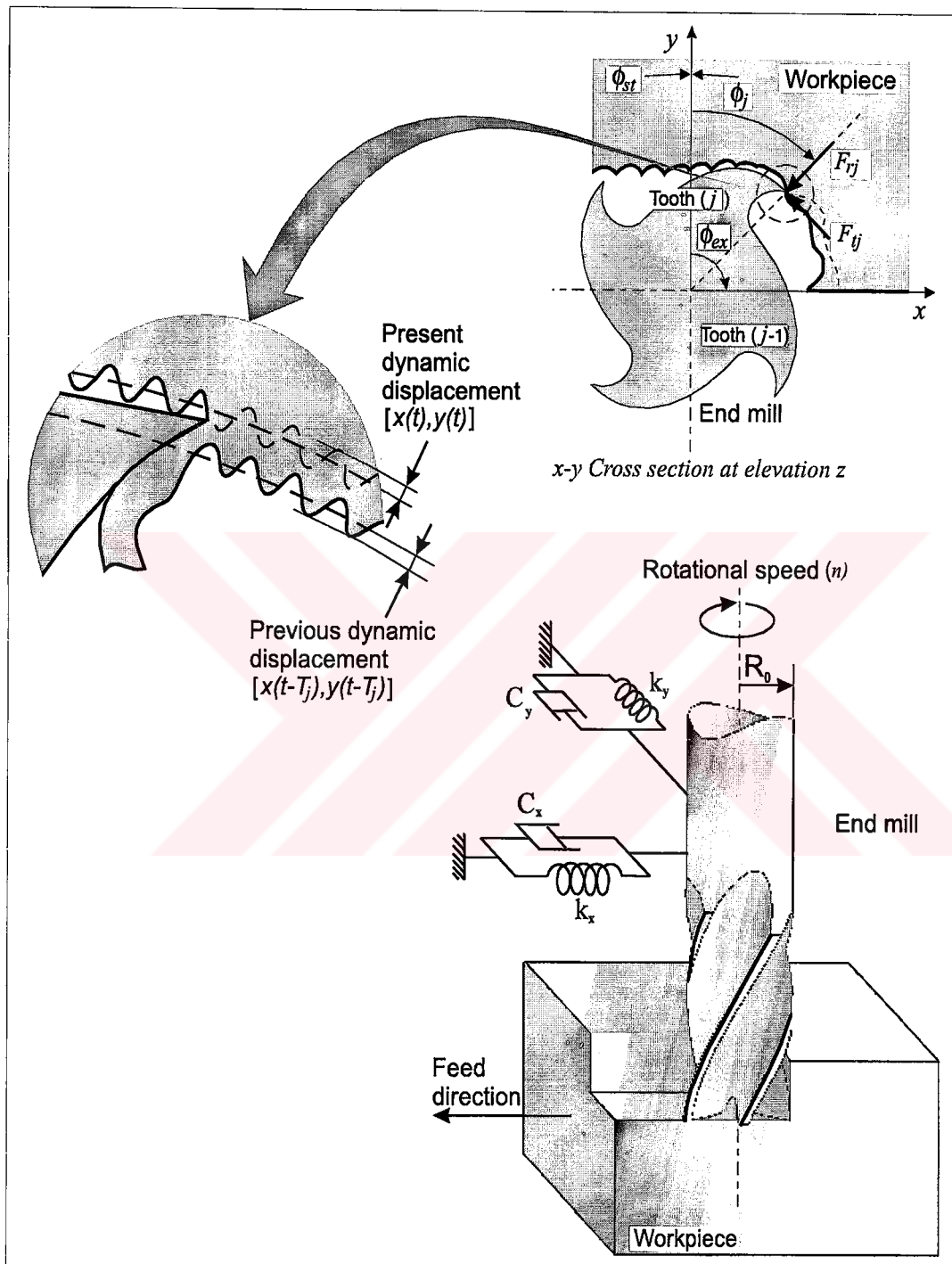


Figure 6.1 Dynamic Model of end milling process

in which ϕ_{st} , ϕ_{ex} are start and exit immersion angles, respectively. The static chip thickness ($s_t \sin \phi_j$) does not contribute to chatter stability, hence it is dropped from equation (6.4). If the cutter displacements are x and y , the radial displacement of tooth j at time t becomes $v_j(t) = -x(t) \sin \phi_j(t) - y(t) \cos \phi_j(t)$. The previous tooth passes from the same radial location one pitch period ago, and leaves a radial displacement $v_{j-1}(t - T_j) = -x(t - T_j) \sin \phi_j(t) - y(t - T_j) \cos \phi_j(t)$. Here T_j represents the time delay between teeth j and $j-1$ due to the rotation of the spindle. Since the pitch angles between the teeth are not constant, the time delay (T_j) between each tooth is also different. Hence, dynamic chip thickness (equation (6.4)) can be rewritten as,

$$h(\phi_j(t)) = g(\phi_j) [\Delta x_j(t) \sin \phi_j(t) + \Delta y_j(t) \cos \phi_j(t)] \quad (6.6)$$

where $\Delta d_j(t) = d_j(t) - d_{j-1}(t - T_j)$, where d denotes for x and y . d_j and d_{j-1} represent the dynamic displacements of the cutter structure at the present and previous tooth periods, respectively. The following differential milling forces can be found by substituting equations (6.6) and (6.3) into (6.2).

$$\left. \begin{aligned} dF_{xj}(\phi, z) &= -K_t [\cos \phi_j(z) + K_r \sin \phi_j(z)] \\ &\quad [\Delta x \sin \phi_j(z) + \Delta y \cos \phi_j(z) + s_t \sin \phi_j(z)] g(\phi_j(z)) dz \\ dF_{yj}(\phi, z) &= K_t [\sin \phi_j(z) - K_r \cos \phi_j(z)] \\ &\quad [\Delta x \sin \phi_j(z) + \Delta y \cos \phi_j(z) + s_t \sin \phi_j(z)] g(\phi_j(z)) dz \end{aligned} \right\} \quad (6.7)$$

where

$$\begin{aligned} \Delta x &= x_j - x_{j-1} \\ \Delta y &= y_j - y_{j-1} \end{aligned} \quad (6.8)$$

The differential dynamic milling forces are integrated within an element i between z_i (lower level) and z_{i+1} (upper level) to determine the elemental milling forces for the flute j . The result of the integration is as follows:

$$\left. \begin{aligned} F_{xj}(i) &= F_{xj}^c - K_t [L_{1j}(i) + K_r L_{2j}(i) + L_{3j}(i) + K_r L_{4j}(i)] \\ F_{yj}(i) &= F_{yj}^c - K_t [L_{2j}(i) - K_r L_{1j}(i) + L_{4j}(i) - K_r L_{3j}(i)] \end{aligned} \right\} \quad (6.9)$$

where F_{xj}^c and F_{yj}^c are cutting forces due to the chip thickness. F_{xj}^c , F_{yj}^c , L_1 , L_2 , L_3 and L_4 are given as follows,

$$\left. \begin{aligned} F_{xj}^c &= -K_t [\cos \phi_j(z) + K_r \sin \phi_j(z)] s_t \sin \phi_j(z) g(\phi_j(z)) dz \\ F_{yj}^c &= K_t [\sin \phi_j(z) - K_r \cos \phi_j(z)] s_t \sin \phi_j(z) g(\phi_j(z)) dz \end{aligned} \right\} \quad (6.10)$$

$$\left. \begin{aligned} L_{1j}(i) &= (1/2) \Delta z \Delta x(i) g(\phi_j, z_i) \sin 2\phi_j(i) \\ L_{2j}(i) &= (1/2) \Delta z \Delta x(i) g(\phi_j, z_i) (1 - \cos 2\phi_j(i)) \\ L_{3j}(i) &= (1/2) \Delta z \Delta y(i) g(\phi_j, z_i) (1 + \cos 2\phi_j(i)) \\ L_{4j}(i) &= (1/2) \Delta z \Delta y(i) g(\phi_j, z_i) \sin 2\phi_j(i) \end{aligned} \right\} \quad (6.11)$$

Finally, the dynamic milling forces for an increment of the rotation are:

$$\begin{Bmatrix} F_{xj}(i) \\ F_{yj}(i) \end{Bmatrix} = \begin{Bmatrix} F_{xj}^c(i) \\ F_{yj}^c(i) \end{Bmatrix} + \frac{1}{2} a K_t \begin{bmatrix} a_{xxj} & a_{xyj} \\ a_{yxj} & a_{yyj} \end{bmatrix} \begin{Bmatrix} \Delta x_j \\ \Delta y_j \end{Bmatrix} \quad (6.12)$$

And the total dynamic parts of these forces are:

$$\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = \frac{1}{2} a K_t \sum_{j=0}^{N-1} \begin{Bmatrix} a_{xxj} \Delta x_j + a_{xyj} \Delta y_j \\ a_{yxj} \Delta x_j + a_{yyj} \Delta y_j \end{Bmatrix} \quad (6.13)$$

where N is the number of teeth on the cutter. The time varying directional dynamic milling force coefficients are given by,

$$\left. \begin{aligned} a_{xxj} &= -g(\phi_j) [\sin 2\phi_j + K_r (1 - \cos 2\phi_j)] \\ a_{xyj} &= -g(\phi_j) [(1 + \cos 2\phi_j) + K_r \sin 2\phi_j] \\ a_{yxj} &= g(\phi_j) [(1 - \cos 2\phi_j) - K_r \sin 2\phi_j] \\ a_{yyj} &= g(\phi_j) [\sin 2\phi_j - K_r (1 + \cos 2\phi_j)] \end{aligned} \right\} \quad (6.14)$$

The total dynamic milling forces can be written in vector form as follows:

$$\{F(t)\} = \frac{1}{2} a K_t \sum_{j=0}^{N-1} [A_j(t)] \{\Delta_j(t)\} \quad (6.15)$$

where

$$\{F(t)\} = \{F_x(t) \ F_y(t)\}^T$$

$[A_j]$ varies with time and is given as follows:

$$[A(t)] = [A_0(t)] \dots [A_{N-1}(t)] = \left[\begin{bmatrix} a_{xx0} & a_{xy0} \\ a_{yx0} & a_{yy0} \end{bmatrix} \dots \begin{bmatrix} a_{xxN-1} & a_{xyN-1} \\ a_{yxN-1} & a_{yyN-1} \end{bmatrix} \right] \quad (6.16)$$

$\{\Delta_j(t)\}$ dynamic displacement of the cutting edge is:

$$\{\Delta_j(t)\} = \left\{ \{\Delta x_0(t) \ \Delta y_0(t)\}^T \dots \{\Delta x_{N-1}(t) \ \Delta y_{N-1}(t)\}^T \right\}$$

Unlike uniform pitch cutters [68], each directional coefficient matrix in equation (6.15) is different due to non-uniform pitch angles between the teeth. However, each matrix is periodic at spindle frequency, hence it can be expressed by its Fourier series expansion as:

$$[A_j(t)] = \sum_{r=-\infty}^{\infty} [A_{rj}] e^{ir\omega t} \quad (6.17)$$

where r represents the frequency spectrum, ω is the spindle frequency and t is the time.

$$[A_{rj}] = \frac{1}{T} \int_0^T [A_j(t)] e^{-ir\omega t} dt \quad (6.18)$$

where T is the tooth period, which varies for each flute.

6.3 Chatter Stability Analysis for Variable Pitch End Mills

The dynamics of a cutter are described by the following linear differential equation:

$$\{r(t)\} = [\Phi(i\omega)] \{F\} \quad (6.19)$$

where the vector $\{r(t)\}$ and $\{F\}$ contain the cutter displacements and milling forces in the X and Y directions, respectively.

$$\{r(t)\} = \begin{Bmatrix} x(t) \\ y(t) \end{Bmatrix}; \quad \{F\} = \begin{Bmatrix} F_x \\ F_y \end{Bmatrix} \quad (6.20)$$

$[\Phi(i\omega)]$ is the transfer function matrix for the cutter-workpiece contact zone,

$$[\Phi(i\omega)] = \begin{bmatrix} \Phi_{xx}(i\omega) & \Phi_{xy}(i\omega) \\ \Phi_{yx}(i\omega) & \Phi_{yy}(i\omega) \end{bmatrix} \quad (6.21)$$

where Φ_{xx} and Φ_{yy} are the relative direct transfer functions between the cutter and workpiece structures in X and Y directions, and Φ_{xy} and Φ_{yx} are the relative cross transfer functions. The dynamic displacement vectors at the present time (t) and pervious tooth period ($t-T_j$) are defined as,

$$\{\Delta_j(t)\} = \begin{Bmatrix} x(t) \\ y(t) \end{Bmatrix} - \begin{Bmatrix} x(t-T_j) \\ y(t-T_j) \end{Bmatrix} = \{r(t)\} - \{r(t-T_j)\} \quad (6.22)$$

where $\{r(t)\} = \{x(t) \ y(t)\}$ represents the relative vibrations between the tool and the workpiece at time t , and $\{r(t-T_j)\}$ denotes the vibration marks left by the previous tooth with a time delay of pitch period T_j . The vibrations at chatter frequency ω_c can be described using harmonic variables as follows:

$$\begin{cases} \{r(i\omega_c t)\} &= [\Phi(i\omega_c)]\{F\}e^{i\omega_c t} \\ \{r(i\omega_c(t-T_j))\} &= e^{-i\omega_c T_j} \{r(t)\} \end{cases} \quad (6.23)$$

The regenerative displacements at a chatter frequency can be expressed as:

$$\begin{aligned} \{\Delta_j(i\omega_c t)\} &= \{r(i\omega_c t)\} - \{r(i\omega_c(t-T_j))\} \\ &= (1 - e^{-i\omega_c T_j})[\Phi(i\omega_c)]\{F\}e^{i\omega_c t} \end{aligned} \quad (6.24)$$

Substituting a regenerative displacement term for a variable pitch cutter into (6.15) leads to the final dynamic milling expression:

$$\{F(t)\}e^{i\omega_c t} = \frac{1}{2} K_t a \sum_{j=0}^{N-1} (1 - e^{-i\omega_c T_j}) [A(t)] [\Phi(i\omega_c)] \{F(t)\}e^{i\omega_c t} \quad (6.25)$$

$[A(t)]$ is periodic in time. The solution of equation (6.25) has the following form:

$$\{F(t)\} = e^{\sigma t} \{S(t)\} \quad (6.26)$$

where $\{S(t)\}$ is the Fourier coefficient and periodic with T (tooth period). Therefore, the system is stable if the real part of all exponents σ 's are negative, and σ represents $-i\omega_c$. The tooth passing frequency is equal to the spindle frequency for a

variable pitch cutter. The characteristic equation of the system is derived using Fourier analysis. By expanding the periodic function ($\{S(t)\}$) into the Fourier series and substituting into equation (6.25), the following equation is obtained:

$$\begin{aligned}
 \{F\} &= \Lambda' \sum_{j=0}^{N-1} (1 - e^{-i\omega_c T_j}) [A(t)] [\Phi(i\omega_c)] \left(e^{\sigma t} \sum_{n=-\infty}^{\infty} \{S_n\} e^{in\omega t} \right) \\
 &= \Lambda' \sum_{j=0}^{N-1} (1 - e^{-i\omega_c T_j}) \sum_{n=-\infty}^{\infty} [A(t)] [\Phi(i\omega_c)] \{S_n\} e^{(in\omega + \sigma)t} \\
 &= \Lambda' \sum_{j=0}^{N-1} (1 - e^{-i\omega_c T_j}) \sum_{n=-\infty}^{\infty} e^{in\omega t} [A(t)] [\Phi(i\omega_c + in\omega)] \{S_n\}
 \end{aligned} \tag{6.27}$$

where Λ' is,

$$\Lambda' = \frac{1}{2} K_t a \tag{6.28}$$

$[A(t)]$ can be expanded into a Fourier series as it is also periodic at the tooth frequency ω , which is the spindle frequency for a variable pitch cutter. When substituted, equation (6.27) will contain a double Fourier series. This is performed by multiplying both sides of equation (6.27) by $1/Te^{-ir\omega t}$ and integrating from 0 to T :

$$\{S_r\} = \Lambda' \sum_{n=-\infty}^{\infty} [D(i\omega_c + in\omega_c)]_{rn} \{S_n\} \quad (r, n = 0, \pm 1, \pm 2, \dots) \tag{6.29}$$

where

$$[D(i\omega_c + in\omega_c)]_{rn} = [B(i\omega_c)] [A]_{rn} [\Phi(i\omega_c + in\omega)] \tag{6.30}$$

where

$$[B(i\omega_c)] = \begin{bmatrix} (1 - e^{-i\omega_c T_0}) [I] \\ (1 - e^{-i\omega_c T_1}) [I] \\ (1 - e^{-i\omega_c T_2}) [I] \\ \vdots \\ (1 - e^{-i\omega_c T_{N-1}}) [I] \end{bmatrix} \tag{6.31}$$

The linear algebraic system given by equation (6.29) can be written in the following matrix form:

$$\begin{Bmatrix} \vdots \\ S_{-1} \\ S_0 \\ S_1 \\ \vdots \end{Bmatrix} = \Lambda' \begin{Bmatrix} \vdots & \vdots & \vdots & \vdots \\ \vdots & [D(i\omega_c - i\omega)]_0 & [D(i\omega_c)]_{-1} & [D(i\omega_c + i\omega)]_{-2} \\ \vdots & [D(i\omega_c - i\omega)]_1 & [D(i\omega_c)]_0 & [D(i\omega_c + i\omega)]_{-1} \\ \vdots & [D(i\omega_c - i\omega)]_2 & [D(i\omega_c)]_1 & [D(i\omega_c + i\omega)]_0 \\ \vdots & \vdots & \vdots & \vdots \end{Bmatrix} \begin{Bmatrix} \vdots \\ S_{-1} \\ S_0 \\ S_1 \\ \vdots \end{Bmatrix} \quad (6.32)$$

The main difference between the expressions for uniform and nonuniform pitch cutters is the summation term with varying pitch periods (T_j), which requires a new solution strategy for chatter stability. The above equation has a non-trivial solution when its determinant is zero:

$$\det[[I] - \Lambda' [D(i\omega_c + in\omega)]_{rn}] = 0 \quad (6.33)$$

6.3.1 Zero Order Approximation

In the zero order approximation, i.e. when only the vibrations at the chatter frequency ω_c are considered, the harmonics of the tooth passing frequency ($n\omega$) have been neglected. The phase difference between the tooth vibrations in two successive periods is ($\omega_c T$). The dynamic milling equation is obtained as follows:

$$\{F\}e^{i\omega_c t} = \Lambda[B(i\omega_c)][A(i\omega_c)]_0[\Phi(i\omega_c)]\{F(t)\}e^{i\omega_c t} \quad (6.34)$$

$[A(i\omega_c)]_0$ is periodic and has the spindle period. Since $[A(i\omega_c)]_0$ is valid only between the entry (ϕ_{st}) and exit (ϕ_{ex}) angles of the cutter, and $T = 1/n$, $dt = d\phi(t)/(2\pi n)$, equation (6.18) can be re-expressed as:

$$[A_0] = \frac{1}{2\pi} \int_{\phi_{st}}^{\phi_{ex}} [A_j(\phi)] d\phi = \frac{1}{2\pi} \begin{bmatrix} \alpha_{xx} & \alpha_{xy} \\ \alpha_{yx} & \alpha_{yy} \end{bmatrix} \quad (6.35)$$

where n is the spindle speed and the integrated functions are,

$$\left. \begin{aligned} \alpha_{xx} &= \frac{1}{2} [\cos 2\phi - 2K_r \phi + K_r \sin 2\phi]_{\phi_{st}}^{\phi_{ex}} \\ \alpha_{xy} &= \frac{1}{2} [-\sin 2\phi - 2\phi + K_r \cos 2\phi]_{\phi_{st}}^{\phi_{ex}} \\ \alpha_{yx} &= \frac{1}{2} [-\sin 2\phi + 2\phi + K_r \cos 2\phi]_{\phi_{st}}^{\phi_{ex}} \\ \alpha_{yy} &= \frac{1}{2} [-\cos 2\phi - 2K_r \phi - K_r \sin 2\phi]_{\phi_{st}}^{\phi_{ex}} \end{aligned} \right\} \quad (6.36)$$

The directional cutting coefficients depend on the immersion angle and K_r coefficient. The variation of directional coefficients with immersion and K_r can be seen in Figure 6.2. They are evaluated over one spindle period and they would be identical both for regular and variable pitch cutters.

The equation (6.34) has a nontrivial solution if its determinant is zero. This can be stated in frequency domain as,

$$\det\left[[I] - \Lambda' [D(i\omega_c)]_0\right] = 0 \quad (6.37)$$

where

$$\begin{aligned} [D(i\omega_c)]_0 &= \begin{bmatrix} [B(i\omega_c)] & [A(i\omega_c)]_0 & [\Phi(i\omega_c)] \end{bmatrix} \\ &= \left(\frac{N}{2\pi} - \frac{1}{2\pi} \sum_{j=0}^{N-1} e^{-i\omega_c T_j} \right) \begin{bmatrix} \alpha_{xx} & \alpha_{xy} \\ \alpha_{yx} & \alpha_{yy} \end{bmatrix} \begin{bmatrix} \Phi_{xx} & \Phi_{xy} \\ \Phi_{yx} & \Phi_{yy} \end{bmatrix} \end{aligned} \quad (6.38)$$

The characteristic equation (6.37) becomes,

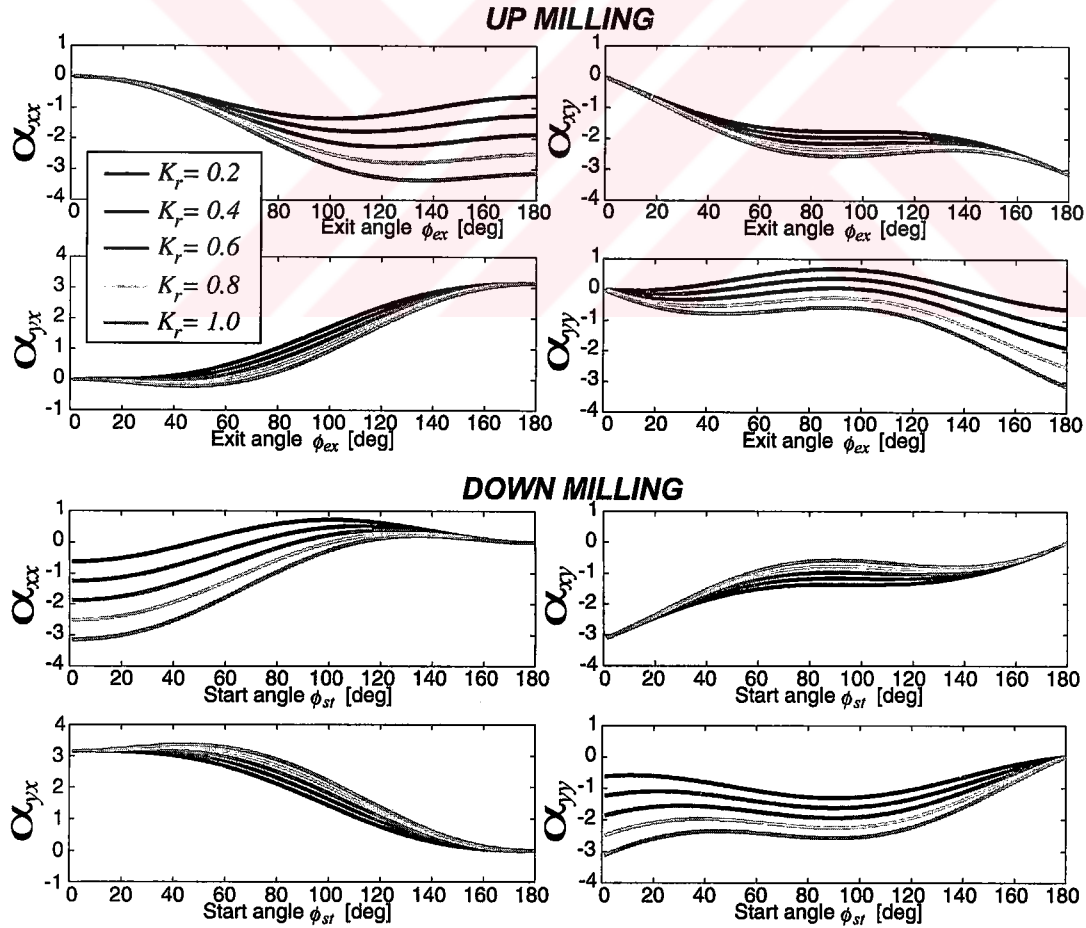


Figure 6.2 Directional cutting coefficients. The start angle in up milling and the exit angle in down milling are 180 degrees.

$$\det \left[[I] + \Lambda E [\Phi_0(i\omega_c)] \right] = 0 \quad (6.39)$$

where the eigenvalue (Λ) and regenerative delay (E) terms are:

$$\left. \begin{aligned} \Lambda &= -\frac{1}{4\pi} K_r a \\ E &= N - \sum_{j=0}^{N-1} e^{-i\omega_c T_j} \end{aligned} \right\} \quad (6.40)$$

and the pitch periods can be found from the pitch angle ($\phi_{p,j}$) between teeth ($j, j-1$) and the spindle speed n [rev/sec], i.e. $T_j = \phi_{p,j} / (2\pi n)$. The oriented transfer function is the product of directional factors and transfer function before orientation:

$$[\Phi_0(i\omega_c)] = \begin{bmatrix} \Phi_{xx}\alpha_{xx} + \Phi_{yx}\alpha_{xy} & \Phi_{xy}\alpha_{xx} + \Phi_{yy}\alpha_{xy} \\ \Phi_{xx}\alpha_{yx} + \Phi_{yx}\alpha_{yy} & \Phi_{xy}\alpha_{yx} + \Phi_{yy}\alpha_{yy} \end{bmatrix} \quad (6.41)$$

There are two unknowns in equation (6.39), the eigenvalue (Λ), and regenerative term (E), which contains known pitch angles and an unknown spindle speed.

The following solution is proposed in evaluating the stability charts. The total chatter vibration wavelength (β_j) left on the surface within the pitch period T_j can be expressed as:

$$\left. \begin{aligned} \beta_j &= \omega_c T_j = 2k_j\pi + \varepsilon_j \\ &= \omega_c \frac{\phi_{pj}}{2\pi n} \end{aligned} \right\} \quad (6.42)$$

where k_j is the number of lobes or vibration marks and ε_j is the fractional wave or phase shift left on the cut surface. The solution starts by scanning dominant modes (i.e. chatter frequency ω_c) in the transfer function, a range of phase shifts (ε_j) and the number of stability lobes (k_j) for tooth number ($j=0$). Hence, β_0 is assigned to the initial wavelength, which leads to the spindle speed for this case.

$$n = \frac{\omega_c \phi_{p0}}{2\pi \beta_0} \quad (6.43)$$

The remaining wavelengths are also evaluated with the known spindle speed (n) from equation (6.42), which leads to the solution of regenerative delay term:

$$E = N - \sum_{j=0}^{N-1} e^{-i\beta_j} \quad (6.44)$$

The resulting characteristic equation (6.39) becomes,

$$\begin{vmatrix} 1 + \Lambda E(\Phi_{xx}\alpha_{xx} + \Phi_{yx}\alpha_{xy}) & \Lambda E(\Phi_{xy}\alpha_{xx} + \Phi_{yy}\alpha_{xy}) \\ \Lambda E(\Phi_{xx}\alpha_{yx} + \Phi_{yx}\alpha_{yy}) & 1 + \Lambda E(\Phi_{xy}\alpha_{yx} + \Phi_{yy}\alpha_{yy}) \end{vmatrix} = 0 \quad (6.45)$$

The eigenvalue for the variable pitch cutter stability can now be solved by substituting E into (Λ) in equation (6.45). If two orthogonal degrees of freedom in the feed (X) and normal (Y) directions are considered (i.e. $\Phi_{xy} = \Phi_{yx} = 0$), the characteristic equation reduces to a quadratic function,

$$a\Lambda^2 + b\Lambda + 1 = 0 \quad (6.46)$$

where

$$\begin{aligned} a &= E^2 \Phi_{xx} \Phi_{yy} (\alpha_{xx}\alpha_{yy} - \alpha_{xx}\alpha_{yx}) \\ b &= E(\Phi_{xx}\alpha_{xx} + \Phi_{yy}\alpha_{yy}) \end{aligned} \quad (6.47)$$

The eigenvalue Λ is obtained as,

$$\Lambda = -\frac{1}{2a} (b \pm \sqrt{b^2 - 4a}) \quad (6.48)$$

Since the stability equation is complex, the resulting eigenvalue has real and imaginary components

$$\Lambda = \Lambda_R + i\Lambda_I \quad (6.49)$$

$$\Lambda = -\frac{1}{4\pi} K_I a_{lim} \quad (6.50)$$

where a_{lim} is the borderline depth of cut for chatter free, stable cutting. However, since the axial depth of cut is a physical value, the eigenvalue solution is acceptable only when the imaginary part (Λ_I) is zero and the real part (Λ_R) is negative. Hence,

$$a_{lim} = -\frac{4\pi\Lambda_R}{K_t} \quad (6.51)$$

Otherwise the solution is rejected. The stability lobes are constructed by scanning wavelength (β_0) and chatter frequency. A typical wavelength starts from $\beta_0 = 10^\circ$ for high speed machining, and increases to $\beta_0 = 3600^\circ$ for low speeds which correspond to $k=10$. Stability lobes are generated at fine increments of $\Delta\beta_0 = 5^\circ$.

6.3.2 Higher Order Approximation

The harmonics (r) have been considered in the solution for regular pitch cutters by Budak and Altıntaş [69,84]. This theory is modified here to cover stability of variable pitch cutters. In this case, the dynamic milling equation is:

$$\sum_{n=-\infty}^{\infty} \{F_r\} e^{in\omega t} = \Lambda[B(i\omega_c)] \sum_{r=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} [A(t)]_r [\Phi(i\omega_c + in\omega)] \{F_n\} e^{i(r+n)\omega t} \quad (6.52)$$

The Fourier coefficients of $[A(t)]$ can be kept under the same summation with the other terms by multiplying both sides of the equation by $1/Te^{-ir\omega t}$ and integrating it from 0 to T . The following is then obtained:

$$\{F_r\} = \Lambda \sum_{n=-\infty}^{\infty} [D(i\omega_c + in\omega)]_{rn} \{F_n\} \quad (r, n = 0, \pm 1, \pm 2, \dots) \quad (6.53)$$

where

$$[D(i\omega_c + in\omega)]_{rn} = [B(i\omega_c)] [A]_{rn} [\Phi(i\omega_c + in\omega)] \quad (6.54)$$

$[A]_{rn}$ is the $(rn)^{th}$ Fourier coefficient of $[A]$,

$$[A]_{rn} = \frac{1}{T} \int_0^T [A(t)] e^{-i(rn)\omega t} dt \quad (6.55)$$

Equation (6.53) has nontrivial solutions if the determinant is zero:

$$\det[\delta_{rn}[I] - \Lambda[B(i\omega_c)] [A]_{rn} [\Phi(i\omega_c + in\omega)]] = 0 \quad (6.56)$$

In the above equation, δ_{rn} is the Kronecker delta and $[I]$ is a (2x2) identity matrix.

The procedure is very similar to the one followed for zero order solution ($[A]_0$).

If $[A]_h$ represents the h^{th} Fourier series coefficient ($h = r, n$),

$$[A]_h = \frac{1}{T} \int_{\phi_{vi}}^{\phi_{ex}} [A(\phi)] e^{-ih\phi} d\phi$$

$$= \frac{1}{2\pi} \begin{bmatrix} \alpha_{xx}^h & \alpha_{xy}^h \\ \alpha_{yx}^h & \alpha_{yy}^h \end{bmatrix} \quad (h = \pm 1, \pm 2, \dots) \quad (6.57)$$

where the following general form for the coefficients α_{xx}^h , α_{xy}^h , α_{yx}^h and α_{yy}^h are obtained:

$$\left. \begin{aligned} \alpha_{xx}^h &= \frac{i}{2} \left[-f_0 K_r e^{-ih\phi} + f_1 e^{-iq_1\phi} - f_2 e^{-iq_2\phi} \right]_{\phi_{vi}}^{\phi_{ex}} \\ \alpha_{xy}^h &= \frac{1}{2} \left[-f_0 K_r e^{-ih\phi} + f_1 e^{-iq_1\phi} + f_2 e^{-iq_2\phi} \right]_{\phi_{vi}}^{\phi_{ex}} \\ \alpha_{yx}^h &= \frac{1}{2} \left[f_0 K_r e^{-ih\phi} + f_1 e^{-iq_1\phi} + f_2 e^{-iq_2\phi} \right]_{\phi_{vi}}^{\phi_{ex}} \\ \alpha_{yy}^h &= \frac{i}{2} \left[-f_0 K_r e^{-ih\phi} - f_1 e^{-iq_1\phi} + f_2 e^{-iq_2\phi} \right]_{\phi_{vi}}^{\phi_{ex}} \end{aligned} \right\} \quad (6.58)$$

where

$$q_1 = 2 + h$$

$$q_2 = 2 - h$$

$$f_0 = \frac{2}{h}$$

$$f_1 = \frac{K_r - i}{q_1}$$

$$f_2 = \frac{K_r + i}{q_2}$$

Equation (6.56) becomes:

$$\det \begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & [I] + \Lambda[D]_{0,-1} & \Lambda[D]_{-1,0} & \Lambda[D]_{-2,1} & \vdots \\ \vdots & \Lambda[D]_{1,-1} & [I] + \Lambda[D]_{0,0} & \Lambda[D]_{-1,1} & \vdots \\ \vdots & \Lambda[D]_{2,-1} & \Lambda[D]_{1,0} & [I] + \Lambda[D]_{0,1} & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} = 0 \quad (6.59)$$

where

$$[D]_{rn} = [B(i\omega_c)] [A]_h [\Phi(i\omega_c + in\omega)] \quad (6.60)$$

The directional dynamic milling force coefficient matrix $[A]_h$ can be expressed as,

$$[A]_h = [\bar{A}]_h [[U]_0 [U]_1 \dots [U]_{N-1}] \quad (6.61)$$

where

$$[\bar{A}]_h = \frac{1}{2\pi} \begin{bmatrix} \alpha_{xx}^h & \alpha_{xy}^h \\ \alpha_{yx}^h & \alpha_{yy}^h \end{bmatrix} \quad (6.62)$$

The dimension of the determinant of equation (6.59) is $[(1+2n) \times (1+2n)] \times 2$, where n is the harmonics of spindle frequency (or tooth passing frequency) for variable pitch cutters. The eigenvalues of equation (6.59) can be solved for a given chatter frequency (ω_c) and spindle frequency (ω). The limiting depth of cut can be calculated according to these eigenvalues.

6.4 Prediction of Chatter Stability lobes

The total wavelength of the vibration mark left in one tooth period can be expressed as (see also Figure 6.3):

$$\beta_j = \omega_c T_j = \varepsilon_j + 2k_j\pi \quad ; \quad j = 0, 1, 2, \dots, N-1 \quad (6.63)$$

where ε_j is the regenerative phase shift and k_j is the number of lobes at the j -th pitch (Figure 6.4). The relationship between the pitch angle $\phi_{p,j}$ and the wavelength β_j can be expressed by,

$$\beta_j = \frac{\omega_c}{\Omega} \phi_{p,j} \quad ; \quad j = 0, 1, 2, \dots, N-1 \quad (6.64)$$

Since $[B(i\omega_c)]$ and $[\Phi(i\omega_c)]$ are complex, the resulting eigenvalues will also be complex, i.e. $\Lambda = \Lambda_R + i\Lambda_I$. However, the critical axial depth of cut a_{lim} should be a real number, thus, any acceptable solution must satisfy that the imaginary part is

equal to zero ($\Lambda_I = 0$). The problem is reduced to find the wavelengths ($\beta_0, \beta_1, \beta_2, \dots, \beta_{N-1}$) which make $\Lambda_I = 0$.

The calculation of stability lobes for variable pitch cutters requires the knowledge of the following process parameters:

- Pitch angles ($\phi_{p,j}$) of the cutter,
- Transfer function of the machine-tool structure ($[\Phi(i\omega_c)]$),
- Cutting force coefficients (K_t, K_r) for the given workpiece-tool material pair,
- Dynamic cutting coefficients ($[A]$) which may be evaluated from equation (6.35) or (6.57),
- And radial immersion of the cut.

The following steps are followed in obtaining the stability lobes:

- Select a chatter frequency (ω_c) from transfer functions of the machine tool system around a dominant mode,

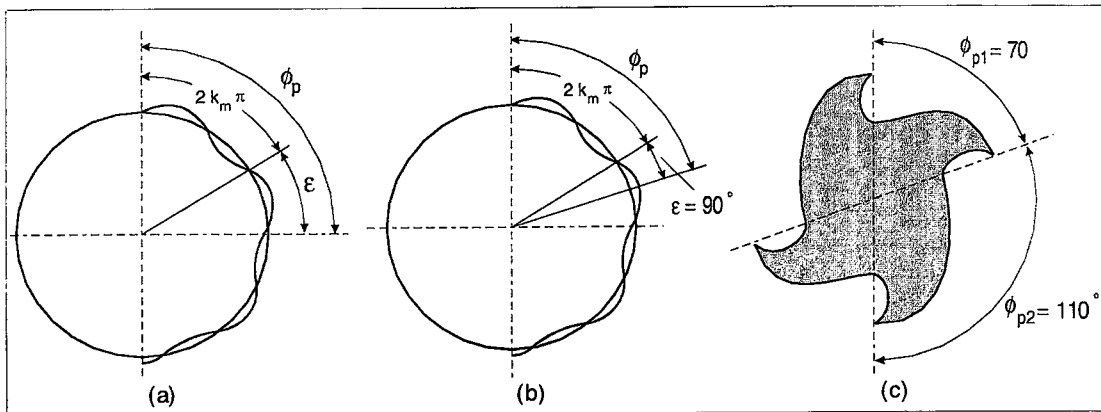


Figure 6.3 Phase shift: a) Wavelength for Regular pitch cutter ($\phi_p = 90^\circ$). b) Wavelength for variable pitch cutter ($\phi_p = 70^\circ$). c) Designed variable pitch cutter angles

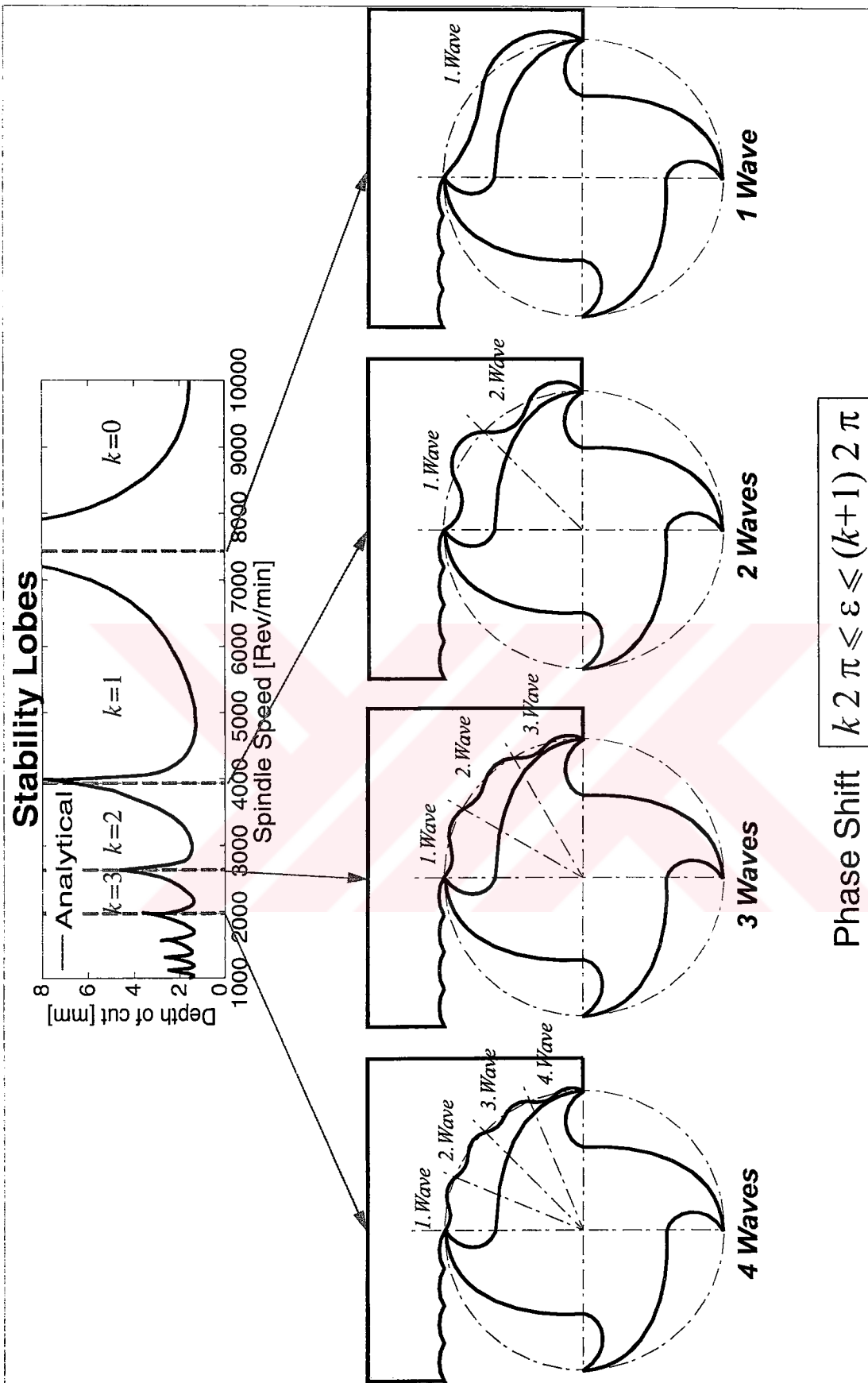


Figure 6.4 Number of waves between two teeth

- Find the wavelength ($\beta_0, \beta_1, \beta_2, \dots, \beta_{N-1}$) which satisfy $\Lambda_l = 0$, then calculate the corresponding a_{lim} by the following procedure,
 - a) Start scanning wavelengths from $\beta_{0\text{min}}$ to $\beta_{0\text{max}}$,
 - b) Calculate the corresponding spindle speed from equation (6.63) as follows:

$$\Omega = \frac{\omega_c \phi_{p0}}{\beta_0} \Rightarrow n = \frac{\Omega 60}{2\pi} \text{ [rev/min]}$$
 - c) Determine the other wavelengths ($\beta_1, \beta_2, \dots, \beta_{N-1}$) by using equation (6.64),
 - d) Calculate the eigenvalues (Λ) from equation (6.39) or (6.59),
 - e) If $\Lambda_l = 0$, calculate a_{lim} using equation (6.51) and store ω_c , Ω or n , β_j and a_{lim} , if not, continue scanning wave length,
- Repeat the procedure by scanning the chatter frequencies around all dominant modes of the structure evident on the transfer functions,
- Plot the results as a_{lim} versus spindle speed n (rev/min).

The flow chart for computing variable pitch end mill cutter stability lobes can be seen in Figure 6.5.

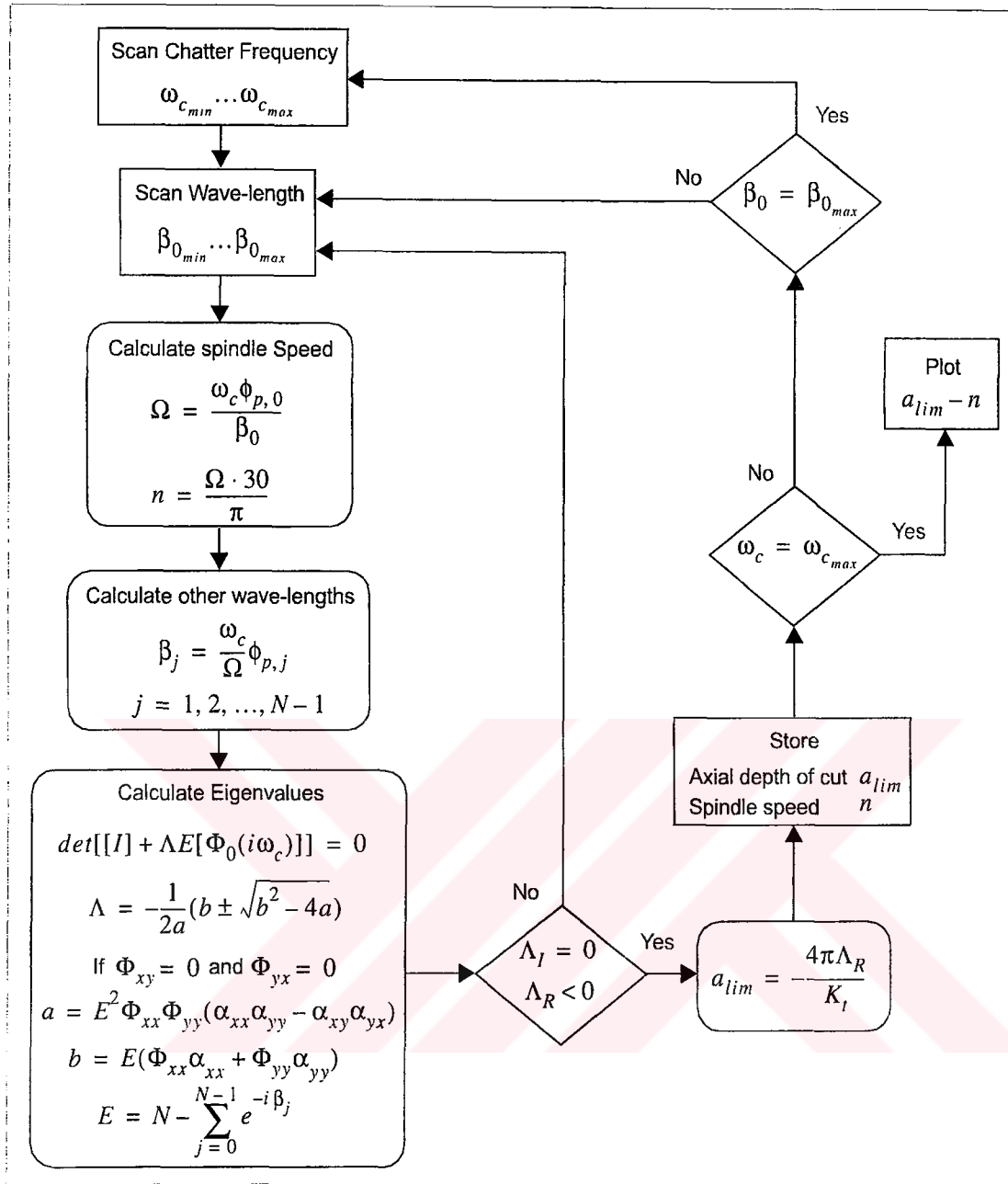


Figure 6.5 Flowchart for variable pitch cutter stability lobes.

6.5 Identification of Optimum Pitch Angles

The main idea in designing a cutter with different pitch angles is to obtain different phase angles between successive surface waves (Figure 6.6) which enable a larger depth of cut to be achieved, without chatter, compared to the uniform pitch case. The pitch angles are chosen in such a manner that the axial depth of cut is maximized for the given spindle speed. Considering equation (6.39), it can be seen from equation (6.63) that N variables need to be solved. In order to obtain a solution to the

eigenvalue equation in (6.39), it is necessary to assume a relation between the pitch angles. In this work, analysis has been made for the cases involving the following assumptions:

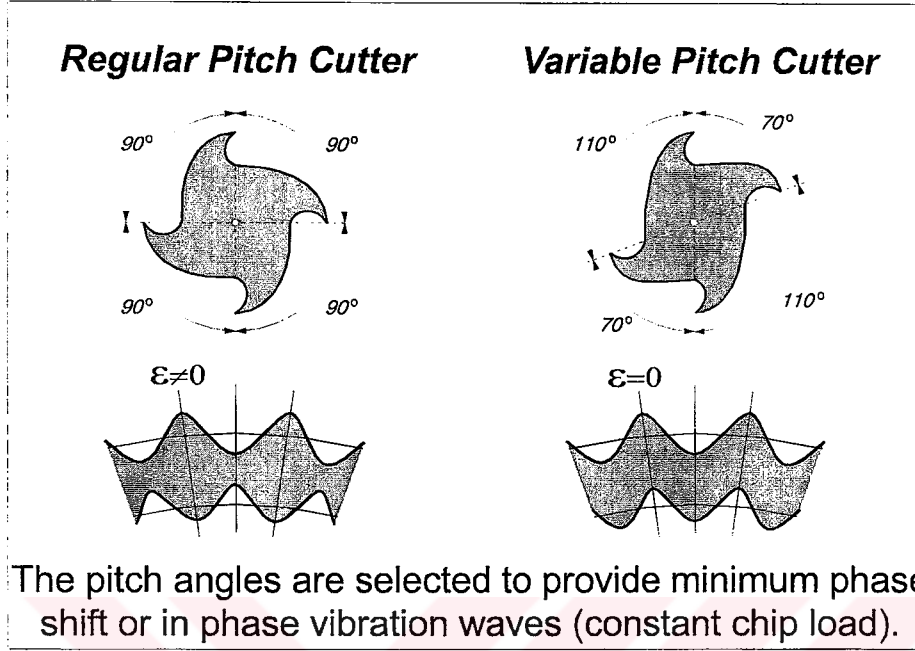


Figure 6.6 Objective of variable pitch cutter design.

I. If the cutter has **two different pitch angles**, then the relationship between its pitch angles is:

$$\phi_{p,0} = \phi_{p,2} = \dots = \phi_{p,2j} ; \phi_{p,1} = \phi_{p,3} = \dots = \phi_{p,2j+1} ; j = 0,1,2,\dots,N-1 \quad (6.65)$$

If the difference between the first and second pitch angle ($P = \phi_{p,1} - \phi_{p,0}$) is known;

$$\phi_{p,j} = \frac{2\pi}{N} - \frac{P}{4N} [2N(-1)^j + (-1)^N - 1] \quad (6.66)$$

If the first pitch angle ($\phi_{p,0}$) is known;

$$\phi_{p,j} = \phi_{p,0} - \frac{((-1)^j - 1)(4\pi - N\phi_{p,0})}{2N + (-1)^N - 1} \quad (6.67)$$

All pitch angles can be found according to these equations.

II. If the variation in the pitch angle is assumed to be **linear**, the pitch angles can be expressed by,

$$\phi_{p,j} = \phi_{p,0} + \Delta\phi_p \quad (6.68)$$

where $\phi_{p,0}$ is the reference pitch angle, j is the pitch number and $\Delta\phi_p$ is the pitch non-uniformity that can be expressed as,

$$\Delta\phi_p = \frac{2(2\pi - N\phi_{p,0})}{N(N-1)} \quad (6.69)$$

III. If the variation in pitch angles is assumed to be **sinusoidal**, the pitch angles can be determined by,

$$\phi_{p,j} = \frac{2\pi}{N} + R \sin\left(j \frac{2\pi}{N}\right) \quad (6.70)$$

where R is a constant, which affects the increment or decrement in the pitch angles.

Two methods are described in the following section for calculating the optimum pitch angles at one spindle speed for given cutting conditions and process dynamics. The first method gives the optimum pitch angles versus the stable axial depth of cut limit by scanning all possible pitch angles. The second method yields the most suitable angle distribution to avoid a certain chatter frequency.

6.5.1 Optimum Pitch Angle Design - Method I

In this method, limit depths of cut are calculated for a range of pitch angles. The following steps are used:

- Select a chatter frequency (ω_c) around one of the dominant modes of the machine tool's transfer function
- Find the wavelengths, $\beta_0, \beta_1, \beta_2, \dots, \beta_{N-1}$ which satisfy $\Lambda_l = 0$, then calculate the corresponding a_{lim} using the following procedure,

a) Start scanning pitch angles from $\phi_{p,0min}$ to $\phi_{p,0max}$ $\left(\phi_{p,0max} \leq \frac{2\pi}{N} \right)$,

b) Calculate the pitch angles $\phi_{p,1}, \phi_{p,2}, \dots, \phi_{p,N-1}$ using one of equations (6.67), (6.68) or (6.70).

- c) Calculate the wavelengths $\beta_0, \beta_1, \beta_2, \dots, \beta_{N-1}$ using equation (6.64),
 - d) Calculate the eigenvalues (Λ) (equation (6.39) or (6.59)),
 - e) If $\Lambda_i = 0$, calculate a_{lim} using equation (6.51) and $\phi_{p,1}, \phi_{p,2}, \dots, \phi_{p,N-1}$, $\beta_0, \beta_1, \beta_2, \dots, \beta_{N-1}$, ω_c and a_{lim} , if not, continue scanning,
- Repeat the above procedure by scanning the chatter frequencies until the end of the considered frequency range for the system transfer function.
 - Plot the results as $\phi_{p,j}$ versus a_{lim} .
 - Select the most suitable pitch angles, which yield the highest depth of cut limit.

The flowchart description of optimum pitch angle calculation can be seen in Figure 6.7.

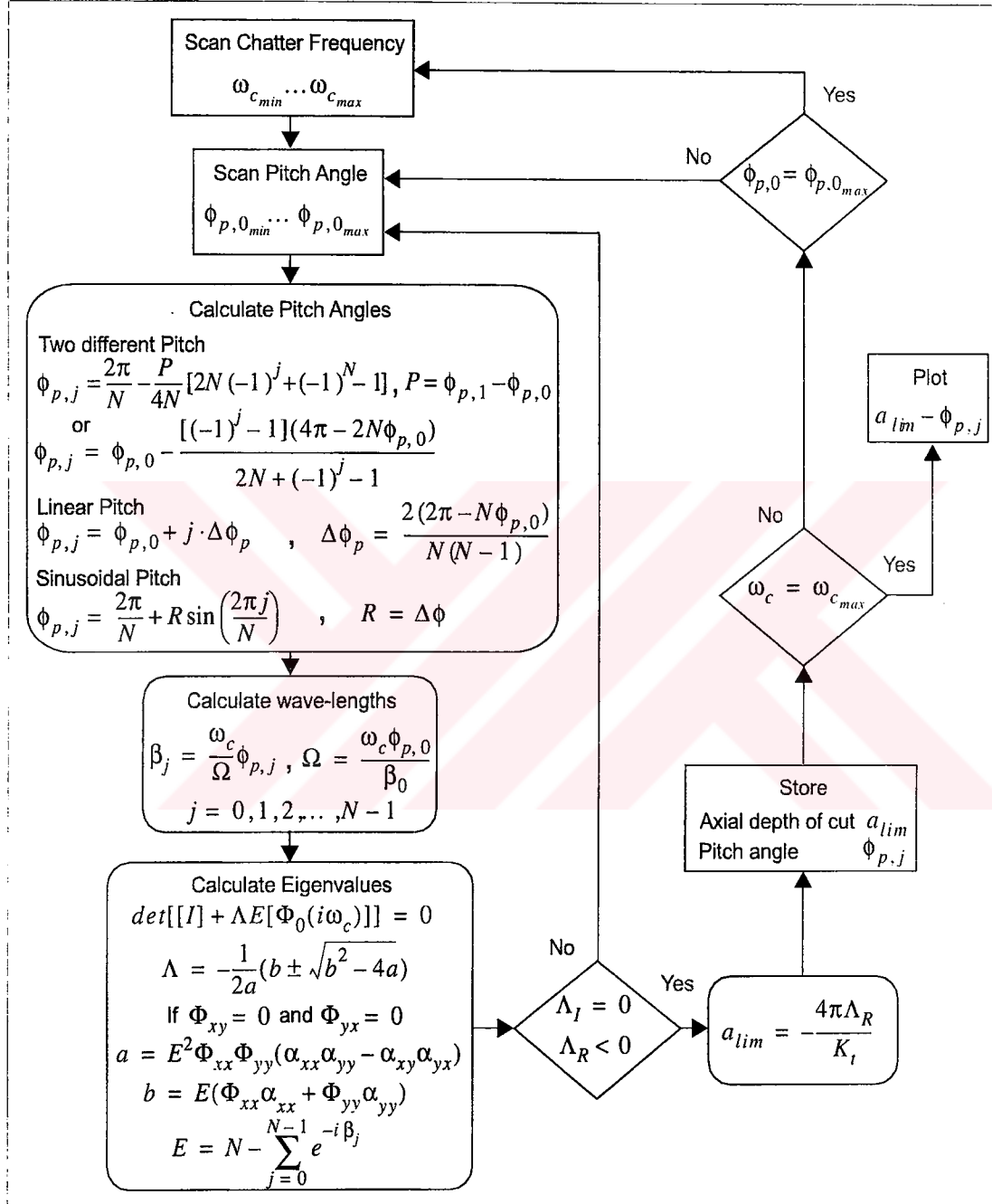


Figure 6.7 Flowchart for optimum pitch angle calculation.

6.5.2 Optimum Pitch Angle Design – Method II

In the second approach, pitch angles are calculated as follows:

- Select the chatter frequency (ω_c),
- Select suitable phase shift for maximum depth of cut,

$$\frac{T}{T_c} = k_m + \frac{\varepsilon}{2\pi} \quad (6.71)$$

where T is the tooth period for a regular pitch cutter, T_c is the chatter vibration period, k_m is the number of complete vibration waves in T and (ε_j) is the phase shift between two adjacent teeth. The phase shift can be found as,

$$\varepsilon_j = \frac{\omega_c \phi_{p,j}}{f_n} - 2\pi k_m \quad (6.72)$$

where $\phi_{p,j}$, ω_c and f_n are the tooth pitch angle (rad), chatter frequency (Hz) and spindle frequency (Hz), respectively. The phase shift for a chatter free large depth of cut is at around 90° for a small value of lobe number (k).

- Find the first pitch angle from the following equation,

$$\phi_{p,j} = \frac{f_n}{\omega_c} (\varepsilon_j + 2\pi k_m) \quad (6.73)$$

- Calculate the other pitch angles, $\phi_{p,1}, \phi_{p,2}, \dots, \phi_{p,N-1}$

6.5.3 A Case Study for Optimum Pitch Angle Cutter Design

Method I: In this case study, cutter design was implemented for a case of cutting Al356 with a 4 fluted cutter, at a spindle speed of 5000 rev/min. In the simulation results shown (Figure 6.8a), it can be seen that the highest depth of cut without chatter is achieved using the two different pitch angle approach. This figure has two peaks. The first peak is around $\phi_{p,0}=55^\circ$ and the second peak is around $\phi_{p,0}=70^\circ$. The larger of the two angle values should be preferred, in order to facilitate easier

chip flow. According to these results, the most suitable pitch angle is 70° for the first pitch angle. Hence, all the pitch angles for 4 flutes are found as 70° - 110° - 70° - 110° (Figure 6.3c).

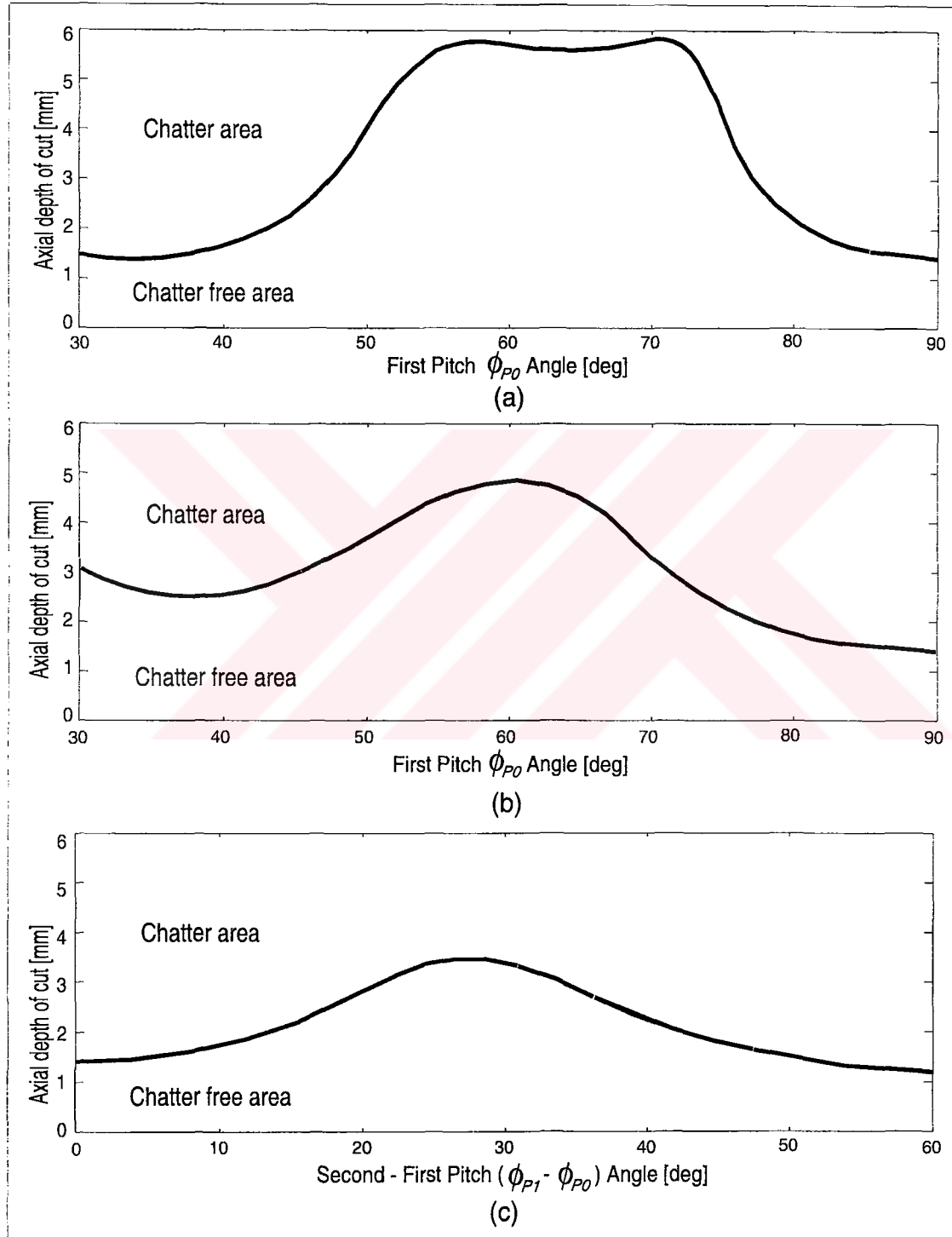


Figure 6.8 Axial depth of cut limit for different pitch angle assumptions at spindle speed of 5000 rev/min for four flutes, material is Al356: a) Two different pitch angles, b) Linearly distributed pitch angles, c) Sinusoidal distributed pitch angles.

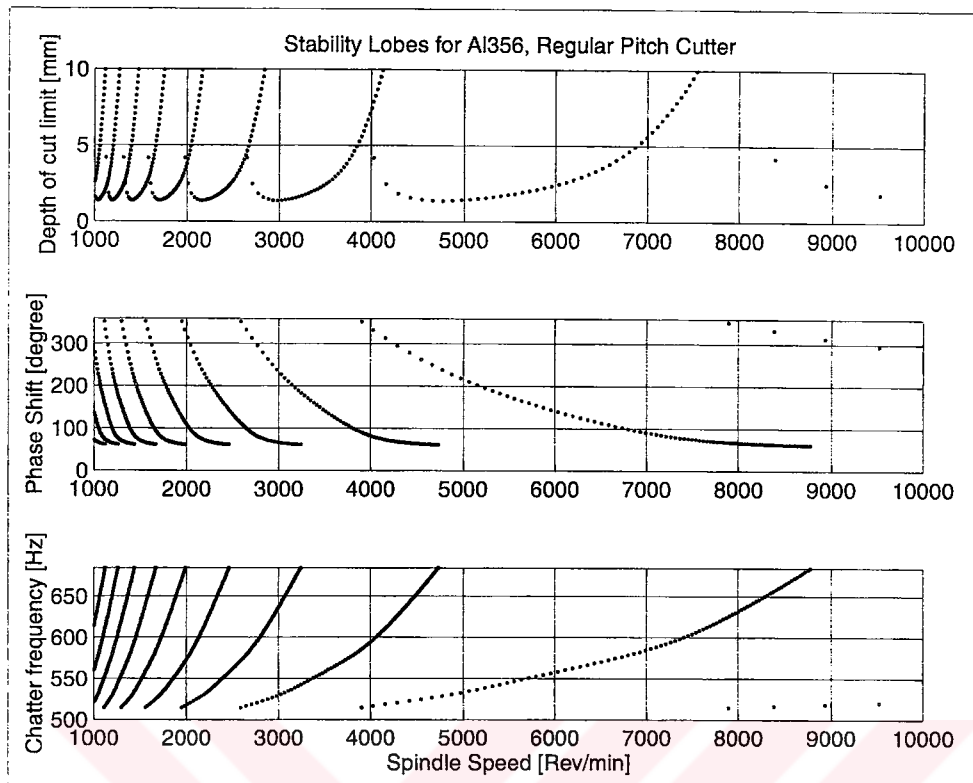


Figure 6.9. Phase shift and chatter frequency for regular pitch cutter and Al356

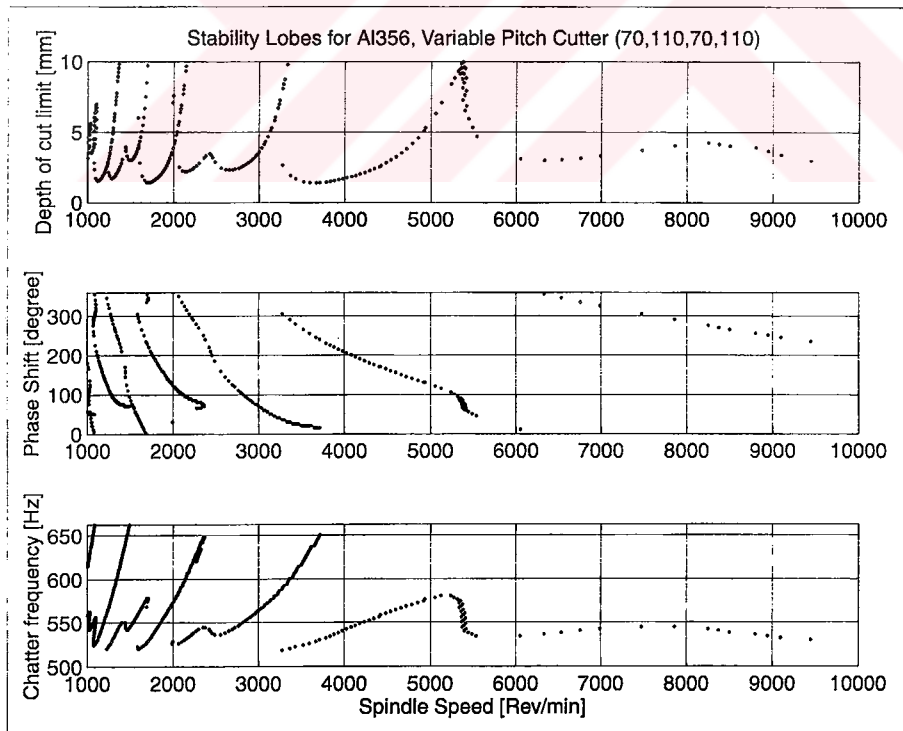


Figure 6.10. Phase shift and chatter frequency for variable pitch cutter and Al356.

Method II: Cutting tests were conducted using a regular pitch cutter and a spindle speed of 5000 rev/min. The results indicated chatter at around 538.75 Hz (Figure 6.20). Phase shift and chatter frequency simulation results can be seen in Figure 6.9 for a regular pitch cutter and Figure 6.10 for a variable pitch cutter. The stability lobe number is $k=1$ around 5000 rev/min (Figure 6.11). Hence, the stability lobe number (k) corresponding to the experimentally detected chatter is taken as 1. The phase shift (ε) is taken as $\pi/2$, because its simulated value is around 90° in stability lobe $k=1$ for the maximum depth of cut (Figure 6.11).

Spindle frequency for 5000 rev/min is,

$$f_n = \frac{5000 \text{ rev/min}}{60} = 83.333 \text{ Hz}$$

If these values are put into equation (6.73), the first tooth pitch angle can be found,

$$\phi_{p,0} = \frac{83.333}{538.75} \left(\frac{\pi}{2} + 2\pi \cdot 1 \right) = 1.2148 \text{ rad} = 69.61 \text{ deg.}$$

The first pitch angle is taken as 70° . The other pitch angles can be found from equation (6.66) or (6.67).

P value is,

$$P = \left(\frac{2\pi}{N} - \phi_{p,0} \right) \cdot 2 = \left(\frac{360}{4} - 70 \right) \cdot 2 = 40 \text{ deg.}$$

Using equation (6.67) for flute number $N=4$:

$$\phi_{p,1} = 70 \text{ deg.}$$

$$\phi_{p,2} = 110 \text{ deg.}$$

$$\phi_{p,3} = 70 \text{ deg}$$

There is a very important point about the phase shift values. The selection of phase shift (ε) should be at the maximum depth of cut area. In fact this value is not fixed. Phase shift values change at the intersection point of every lobe. For example, the value of a phase shift increases at the first intersection point, and decreases at the second intersection point with respect to increasing lobe number (k). Results of these simulations can be seen in Figure 6.11.

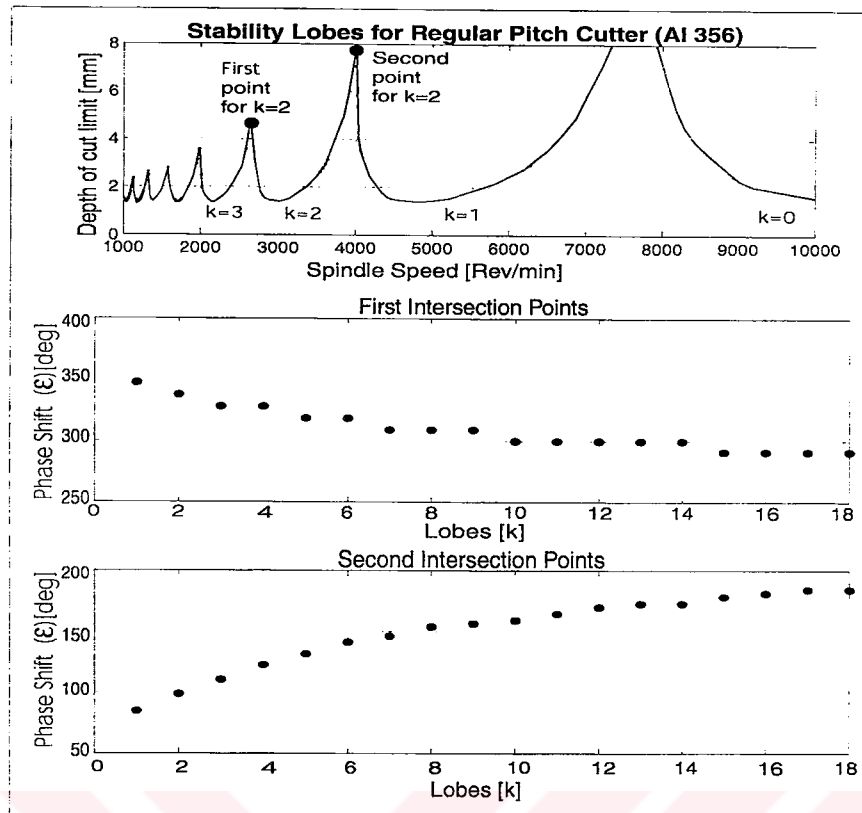


Figure 6.11. The phase shift values at the intersection points of stability lobes (or maximum possible axial depth of cut for every lobe) for Al356.

6.6 Simulation and Experimental Results

The proposed method for analytical stability lobe prediction and a variable pitch cutter design technique have been verified with extensive time domain simulations and cutting tests. In the first set of experiments, a regular pitch four-fluted cutter was used in the milling of Al356 alloy. A carbide cutter with 19.05 mm diameter, 30° helix, and 10° rake angles was used. The transfer function of the cutter and spindle assembly was measured, and the identified modal parameters for this system are presented in Table 6.1 (see also Figure 6.12 and Figure 6.13). Cutting coefficients were calculated using the mechanistic model. The average forces are given in Figure 6.14. The average cutting coefficients can be seen in Table 6.2 for Al7075 and Al356 alloys. Some of the cutting force comparisons between simulations and measurements are given in Figure 6.15, Figure 6.16, Figure 6.17 and Figure 6.18 for a variable and regular pitch cutter with Al7075 and Al356 workpiece materials.

Table 6.1. Transfer function values of the system

Direction	Mode	ω_n [Hz]	ζ [%]	Mode Residue $\sigma_k \pm i v_k$ [m/N]
X	1	443.62	2.86	$(+7.299-i16.34)10^{-6}$
	2	563.55	5.58	$(-6.415-i94.01)10^{-6}$
	3	778.52	5.90	$(-3.545-i7.629)10^{-6}$
Y	1	516.27	2.50	$(-7.710-i128.4)10^{-6}$

Table 6.2. Cutting Coefficients

Coefficients	Al7075	Al356
K_{tc} [N/mm ²]	906.074	696.936
K_{rc} [N/mm ²]	207.784	256.012
K_{ac} [N/mm ²]	258.367	134.854
K_{te} [N/mm]	21.905	24.894
K_{re} [N/mm]	32.416	32.911
K_{ae} [N/mm]	4.742	0.962

The stability lobes are predicted both analytically and using time domain simulations. The stability charts indicate that the milling system would be unstable at the spindle speed of $n=5000$ rev/min, axial depth of cut of $a=5$ mm and feed-rate of $s_f=0.0508$ mm/tooth, which was verified by cutting test results given in Figure 6.20 and Figure 6.21. Cutting forces were measured with a Kistler (Model#9257A) table dynamometer, which had a measured frequency response bandwidth of 1050 Hz with the workpiece clamped. The vibrations were also monitored with an accelerometer attached to the spindle and a microphone. The measurements show the severity of chatter vibrations, which produce large amplitude forces at the chatter frequency $\omega_c=538$ Hz. Although the chatter diminishes when the speed is increased to 7500 rev/min, the poor machinability of special Al356 alloy produces accelerated tool wear. Reduction of the speed and depth of cut is also not desirable due to the decrease in productivity. A variable pitch cutter is designed in order to stabilize the system at the desired cutting speed using the proposed theory. Two alternating pitch

angles were selected. The chatter free axial depths of cut were mapped by scanning the first pitch angle ($\phi_{p,0}$) and the increment $\Delta\phi_{p,0}$ (Figure 6.19).

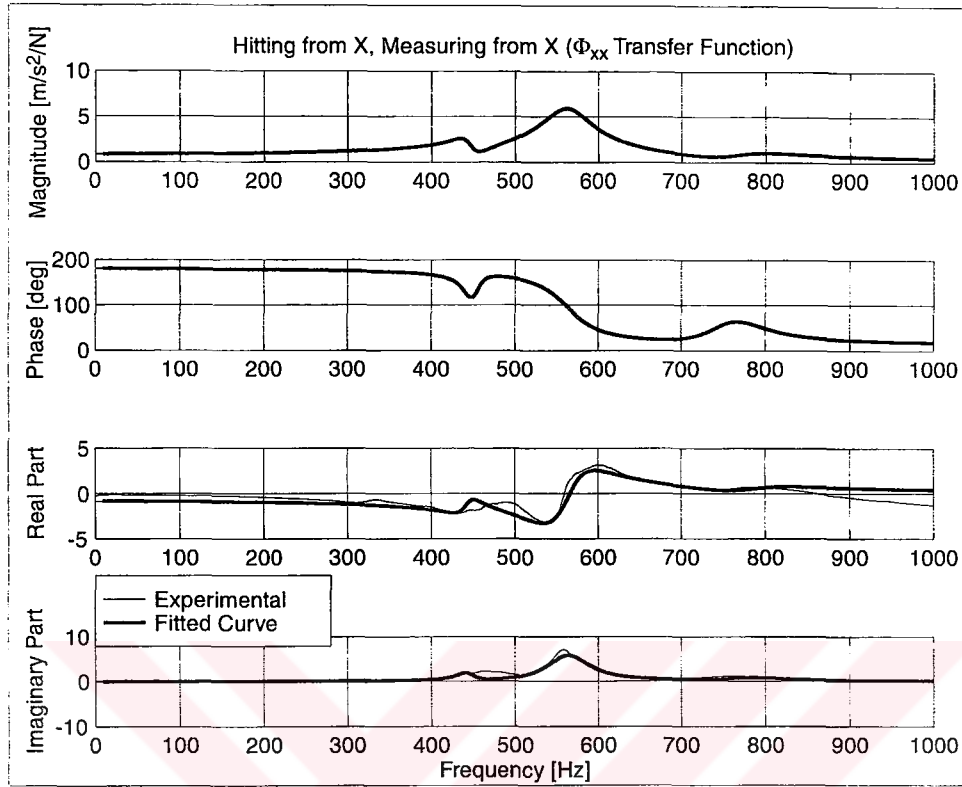


Figure 6.12. Direct transfer function of the system from tool tip in X direction.

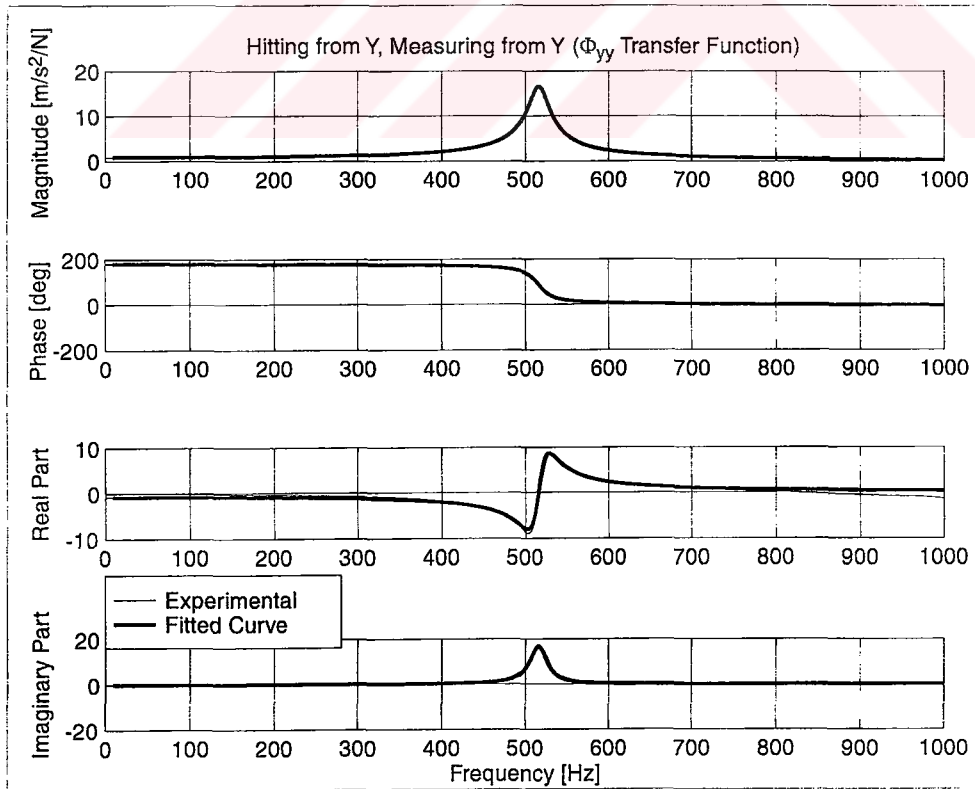


Figure 6.13. Direct transfer function of the system from tool tip in Y direction.

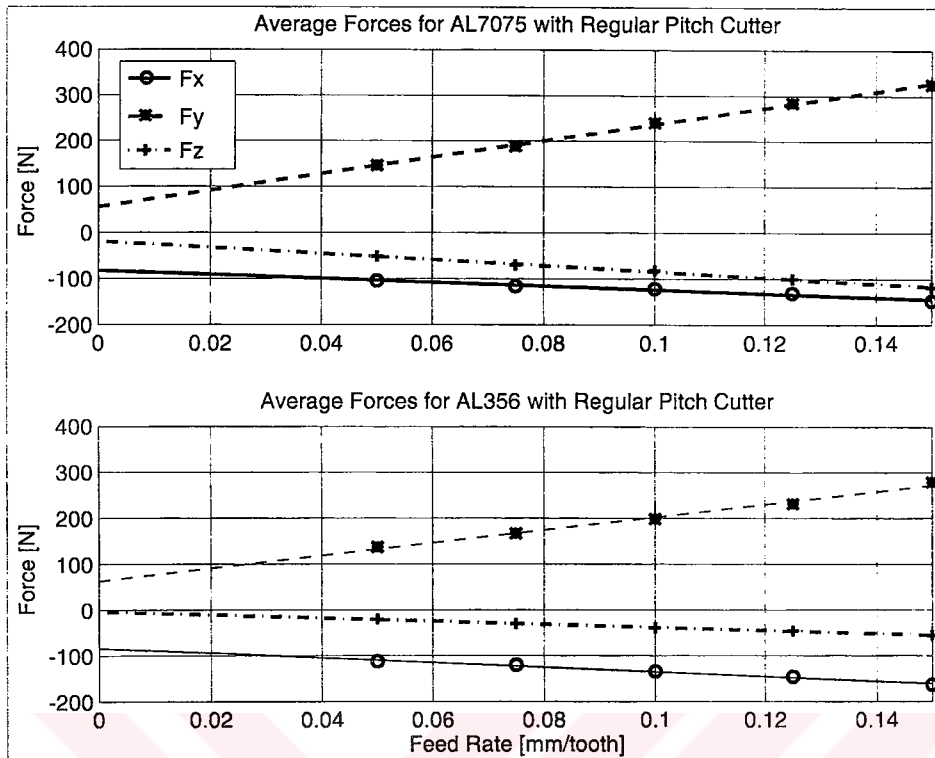


Figure 6.14. Average cutting forces. Spindle speed: 1620 rev/min, depth of cut: 2.0 mm, cutting type is slotting.

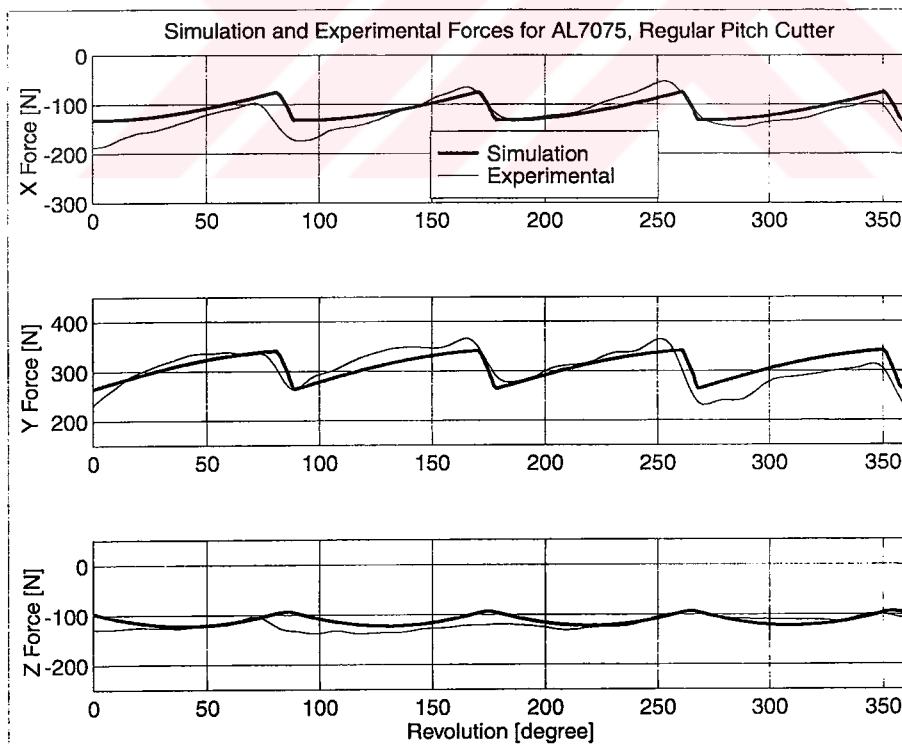


Figure 6.15. Simulated and measured cutting forces. Cutting type is slotting, depth of cut: 2.0 mm, spindle speed: 1620 rev/min, feed rate: 0.150 mm/tooth, Regular pitch cutter, workpiece material: AL7075.

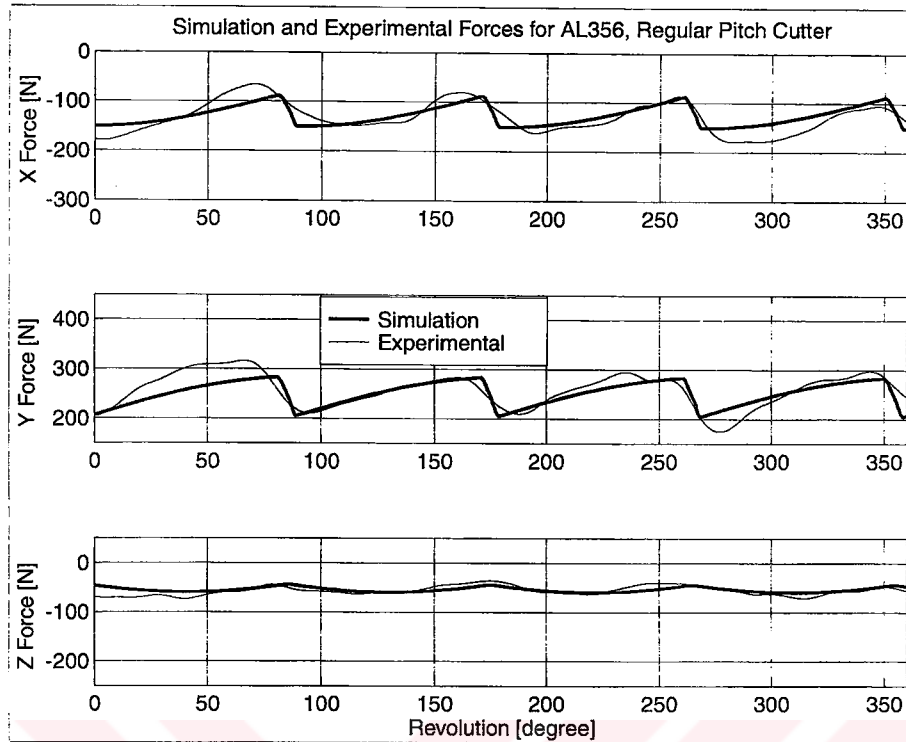


Figure 6.16. Simulated and measured cutting forces. Cutting type is slotting, depth of cut: 2.0 mm, spindle speed: 1620 rev/min, feed rate: 0.150 mm/tooth, Regular pitch cutter, workpiece material: Al356.

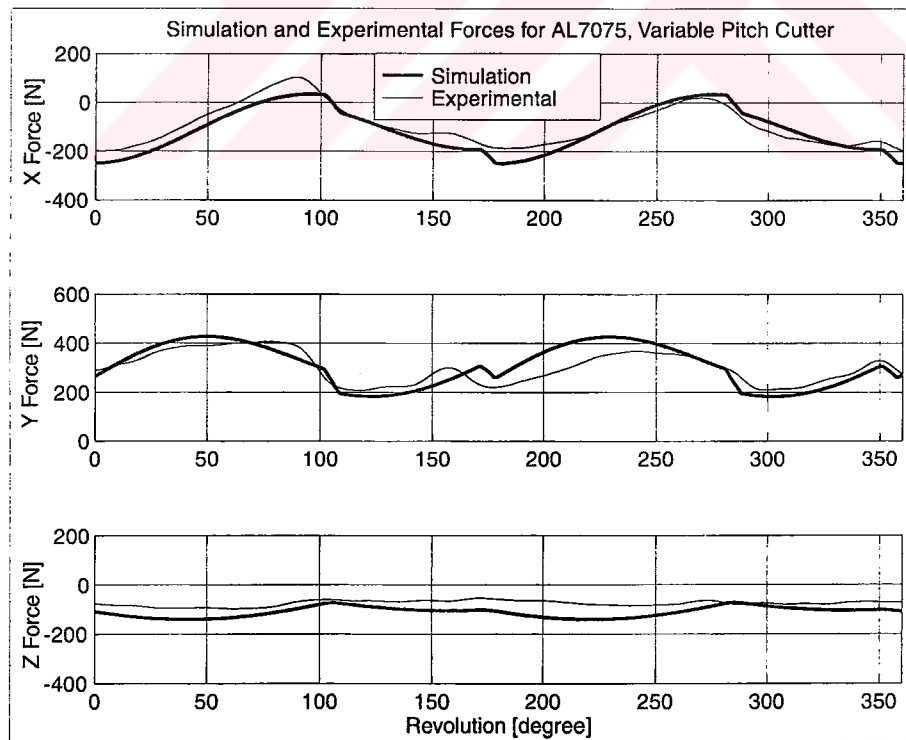


Figure 6.17. Simulated and measured cutting forces. Cutting type is slotting, depth of cut: 2.0 mm, spindle speed: 1620 rev/min, feed rate: 0.150 mm/tooth, Variable pitch cutter, workpiece material: Al7075.

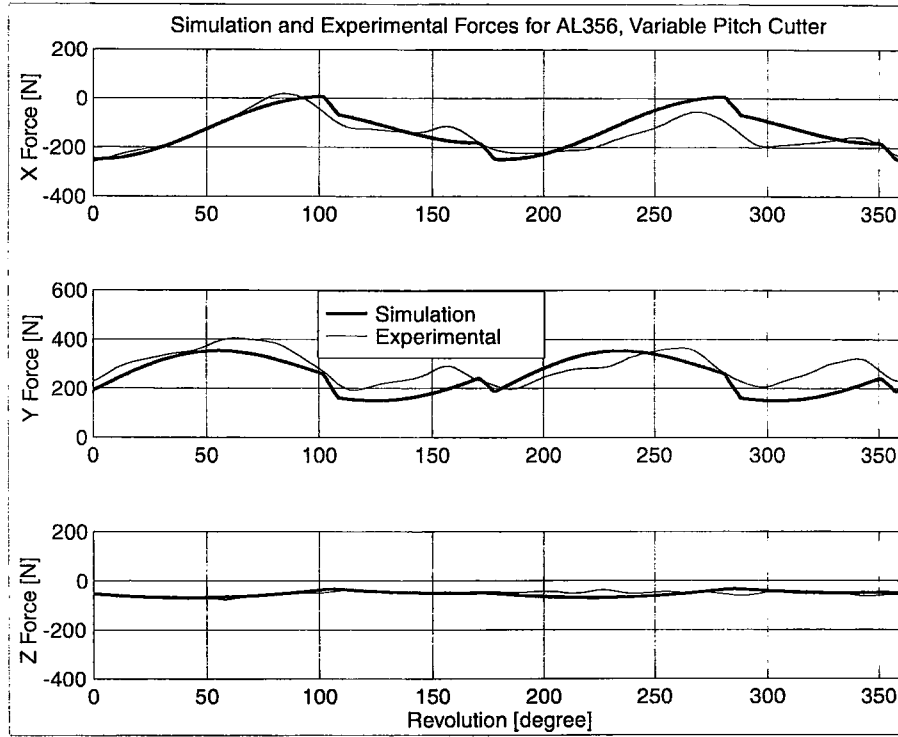


Figure 6.18. Simulated and measured cutting forces. Cutting type is slotting, depth of cut: 2.0 mm, spindle speed: 1620 rev/min, feed rate: 0.150 mm/tooth, Variable pitch cutter, workpiece material: AL356.

The analytical solutions indicate that the first optimal pitch angle is $\phi_{p,0}=70^\circ$ and the increment is 40° , which results in a cutter with varying pitch angles of 70° - 110° - 70° - 110° (Figure 6.19). Note that smaller pitch angles (e.g. $\phi_{p,0}=55^\circ$) would not provide a desirable chip evacuation, hence should not be adopted. If there were 5 teeth on the cutter, than the pitch angles would have been 70° - 75° - 70° - 75° - 70° , or 70° - 70° - 70° - 70° - 80° . Although the most optimal pitch angle is 70° , the fractional angular space has to be distributed to remaining tooth spacing angles. The cutter could also be designed using the vibration wave expression (6.42). The analytical stability method indicates that the maximum depth of cut for one lobe is achieved when the regenerative phase shift is $\epsilon = 90^\circ$ for this particular uniform pitch cutter, see Figure 6.20. However, it must be noted that this value changes as a function of lobe number, spindle speed, chatter frequency and depth of cut.

The cutter pitch angles can be approximately tuned for a spindle speed of $n=5000$ rev/min (83.33 Hz) and a chatter frequency of $\omega_c = 538$ Hz as :

$$\phi_{p,0} = \frac{83.33}{538} \left(\frac{\pi}{2} + 2\pi \cdot 1 \right) = 1.215 \text{ rad} = 70^\circ$$

The remaining pitch angle has to be $\phi_{p,1} = (360 - 2 \times 70) / 2 = 110^\circ$. A cutter, which was manufactured by Robb-Jack tool manufacturer using an eight-axis CNC cutter grinder, was tested at the same cutting conditions. As can be seen from Figure 6.20, the chatter vibrations diminished at the desired spindle speed of $n=5000 \text{ rev/min}$. The cutting force and vibration spectrum amplitudes are reduced significantly, and the system is dominated by the second harmonics of the spindle (i.e. pitch passing) frequency (166 Hz, 332 Hz, ...).

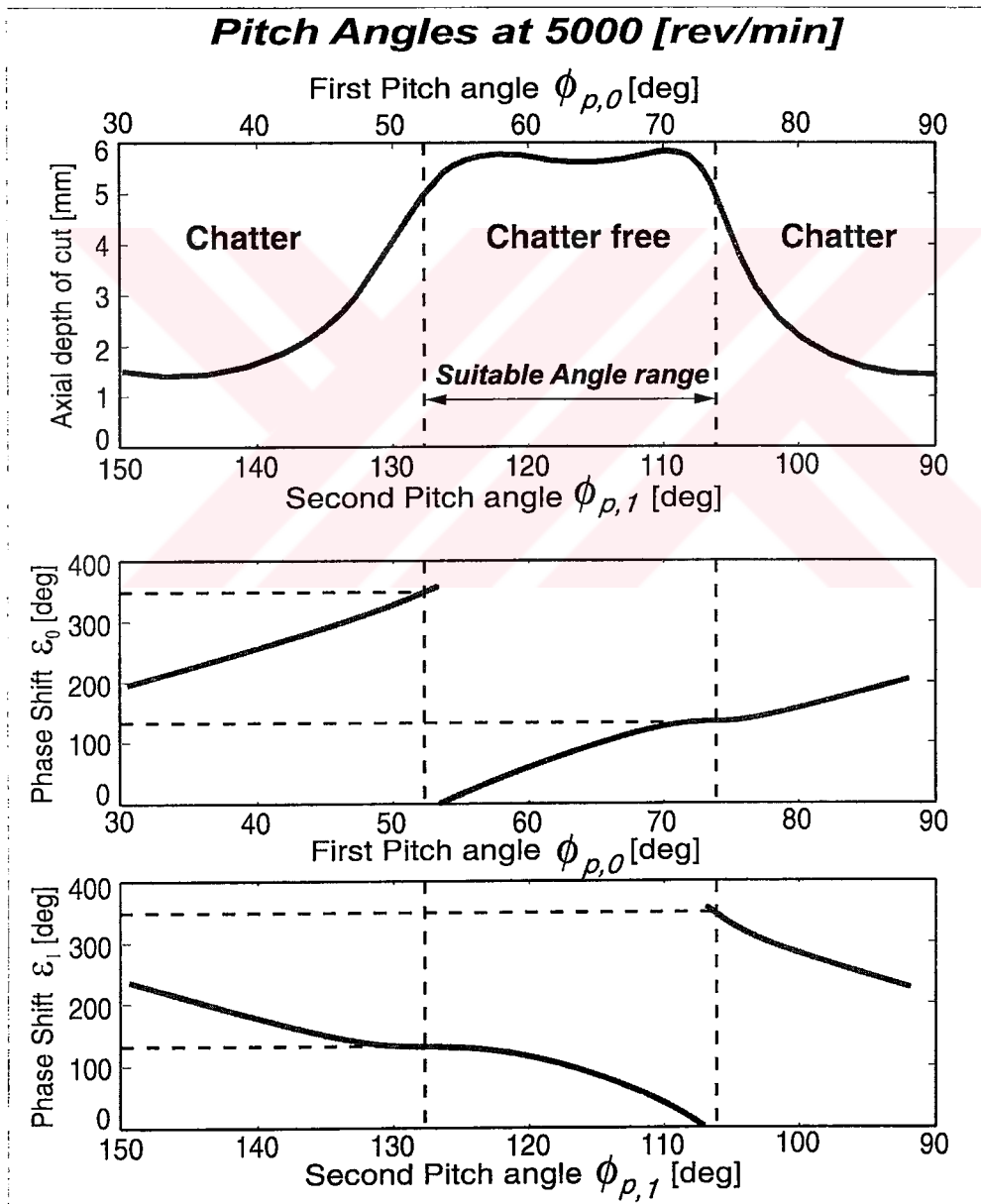


Figure 6.19. Analytical solution of pitch angles and limit axial depth of cuts for machining Al356 at 5000 rev/min with a four fluted cutter.

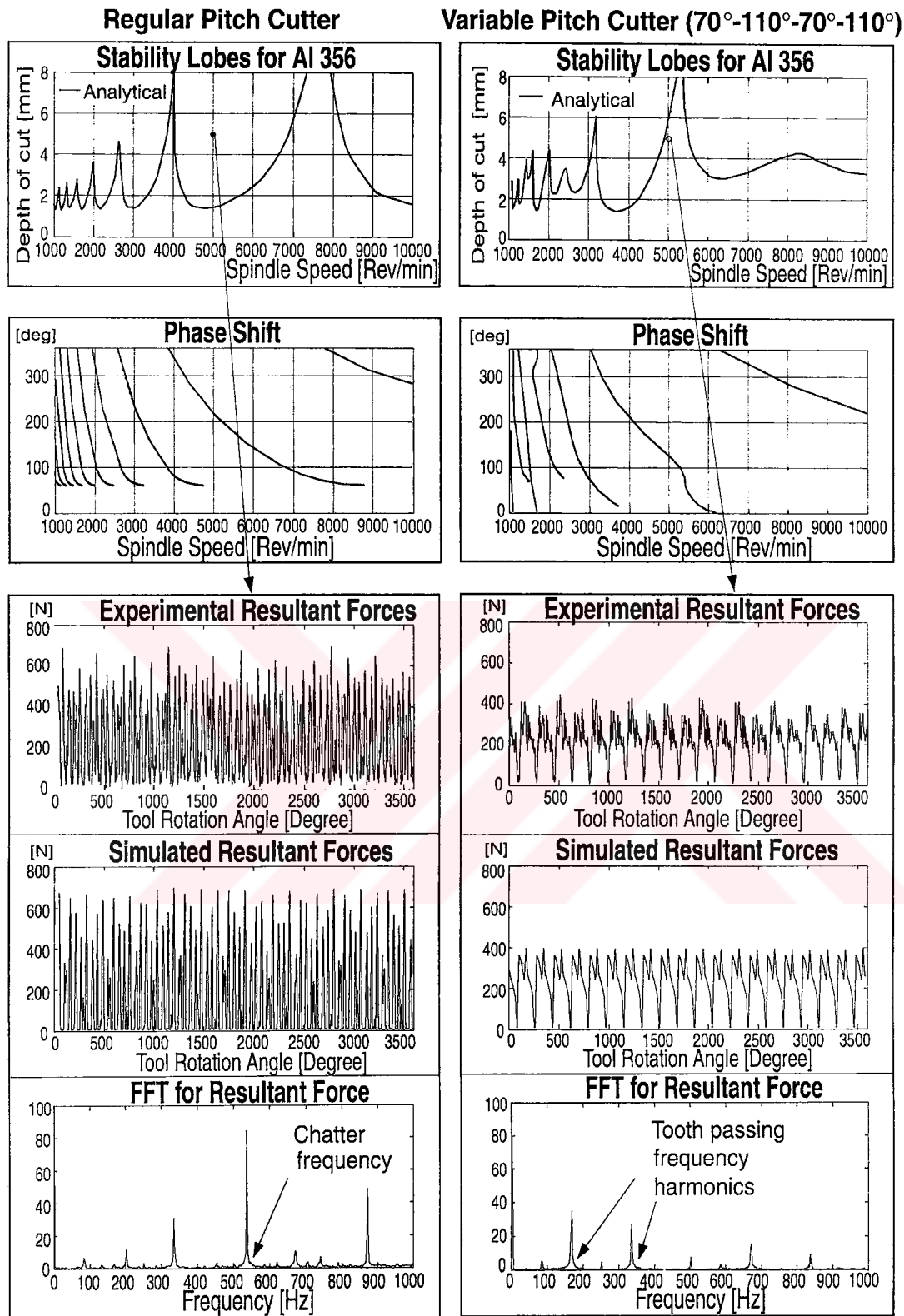


Figure 6.20. Stability lobes for regular and variable pitch cutter for half immersion down milling of work material Al356. Rob Jack carbide cutter with 0.75 inch diameter and four flutes. Cutting constants are given in Table 6.2. Special case: axial depth of cut $a=5.0$ mm, feed rate $s_f=0.0508$ mm/tooth, spindle speed $n=5000$ rev/min.

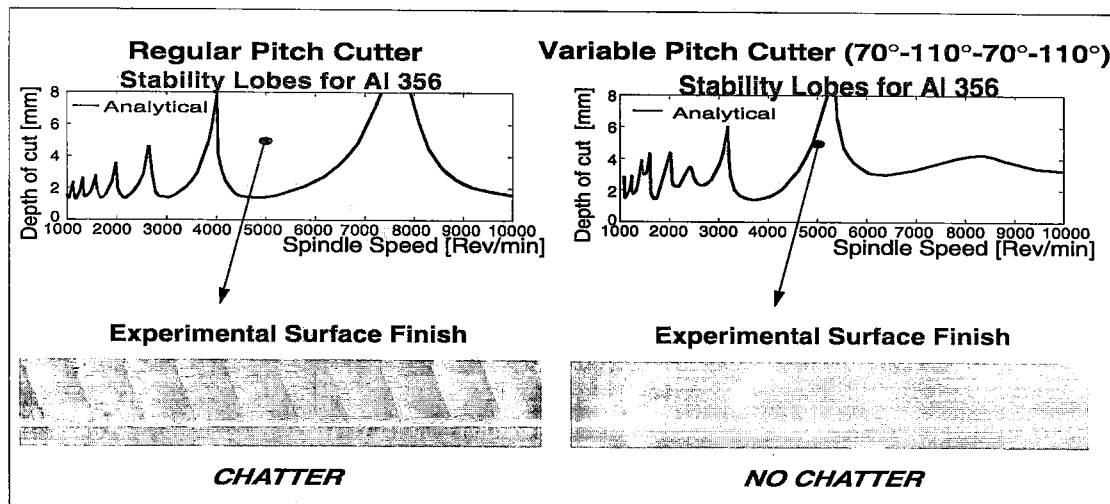


Figure 6.21. Experimental surface finish for regular and variable pitch cutter at the same spindle speed.

A second set of tests was conducted on Al7075. Analytically predicted stability lobes agreed well with the time domain simulations and with a matrix of cutting tests conducted at different speeds and depths of cut (Figure 6.22, Figure 6.23 and Figure 6.24). Note that the time domain simulations include nonlinear cutting constants, tools jumping out of cut, feed marks, and chip load formation using exact kinematics of dynamic milling [10]. The cutting mode was half immersion down milling. The presence of chatter was detected by comparing the increases in the amplitudes of cutting forces and the cutting force spectra as suggested by Smith and Tlustý [51]. When chatter occurs, force and vibration amplitudes increase exponentially with the axial depth of cut. For chatter-free machining, which experiences only forced vibrations, forces and vibration, amplitudes vary linearly with the axial depth of cut. The mechanics of variable pitch cutters [97] and the generalized time domain model of mechanics and dynamics of milling [70] are presented by Altıntaş et. al. When the system was unstable at $n=4000$ rev/min, $a=5$ mm, the surface finish was quite poor with a measured average peak to valley roughness of 0.042 mm. The simulations had predicted an average roughness of 0.045 mm. The surface waves have a length of 1.516 mm. When the spindle speed was increased to its desired value, which is $n=5300$ rev/min, the chatter vibrations diminished completely and an acceptable surface finish with an average roughness of 0.004 mm was achieved. Note that the stability peak is not exactly at 5000 rev/min but at 5300 rev/min. This is due to a hang in the chatter frequency at 5000 rev/min when the pitch angles are modified.

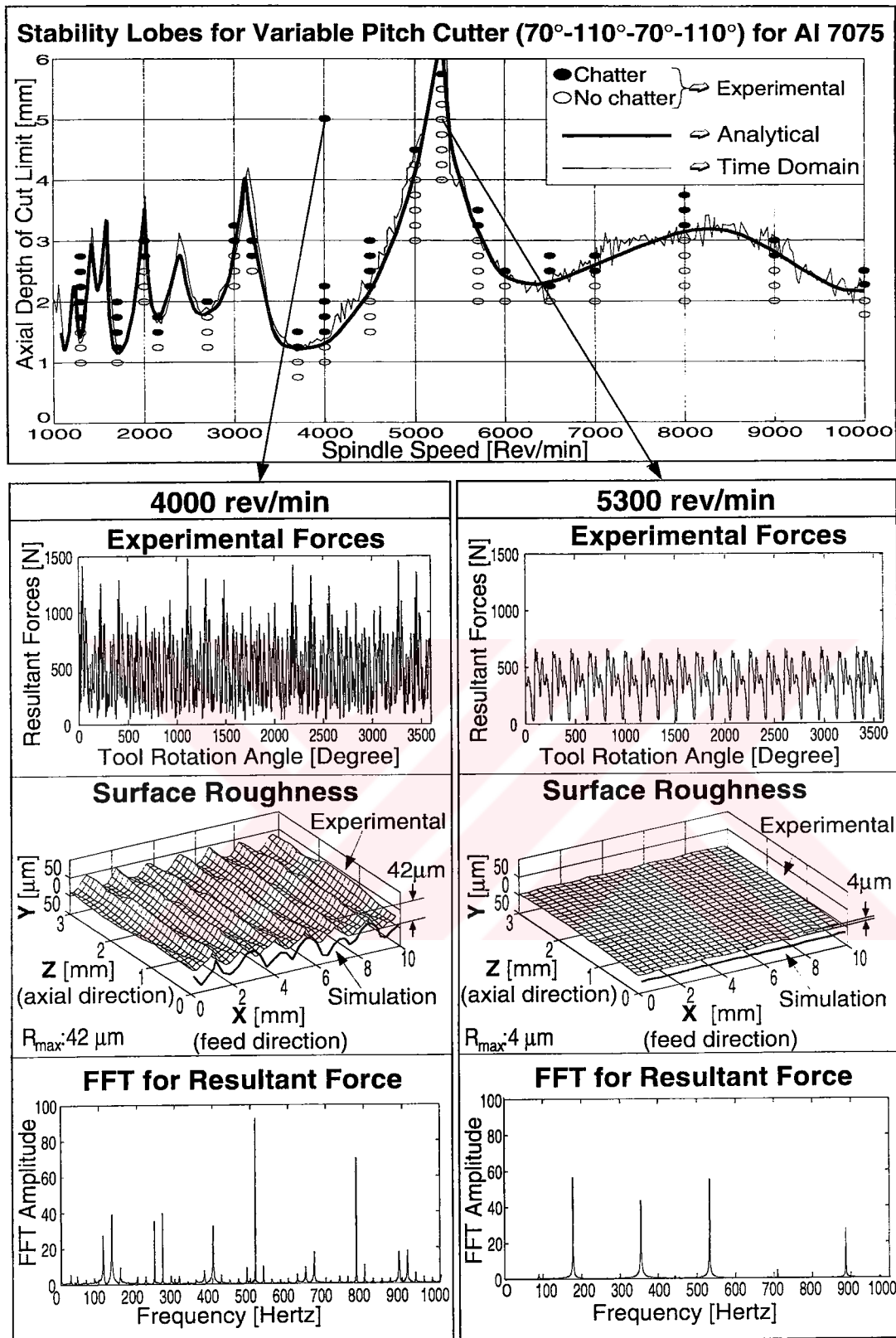


Figure 6.22. Stability lobes, forces and surface roughness obtained by experiments and simulations. Cutter is Rob Jack carbide with 0.75 inch diameter, 30° helix angle end mill. Depth of cut $a=5.0$ [mm], feed rate 0.12 [mm/tooth]. Half immersion down milling of Al7075. Cutting constants are in Table 6.2.

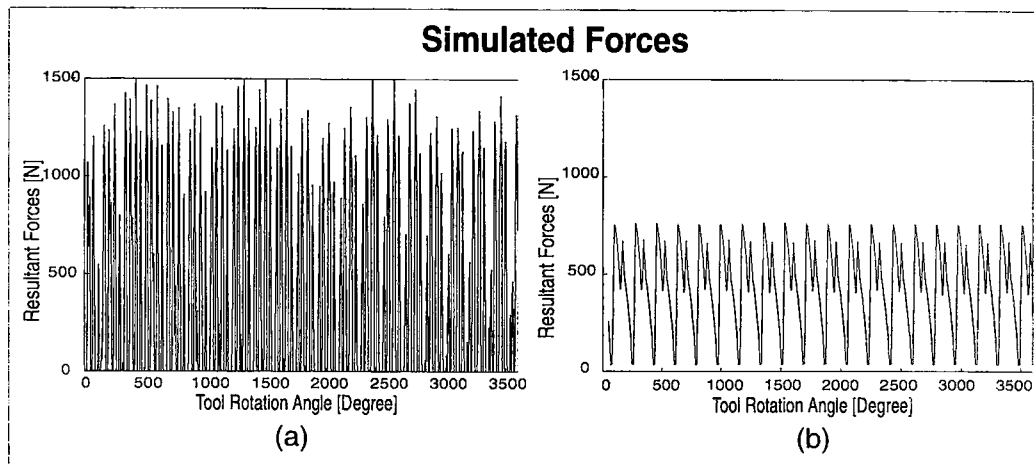


Figure 6.23. Simulated forces for variable pitch cutter, 4 flutes, 5.0 mm depth of cut, workpiece material is Al7075. Cutting coefficients are in Table 6.2. a) Spindle speed is 4000 rev/min, b) Spindle speed is 5300 rev/min

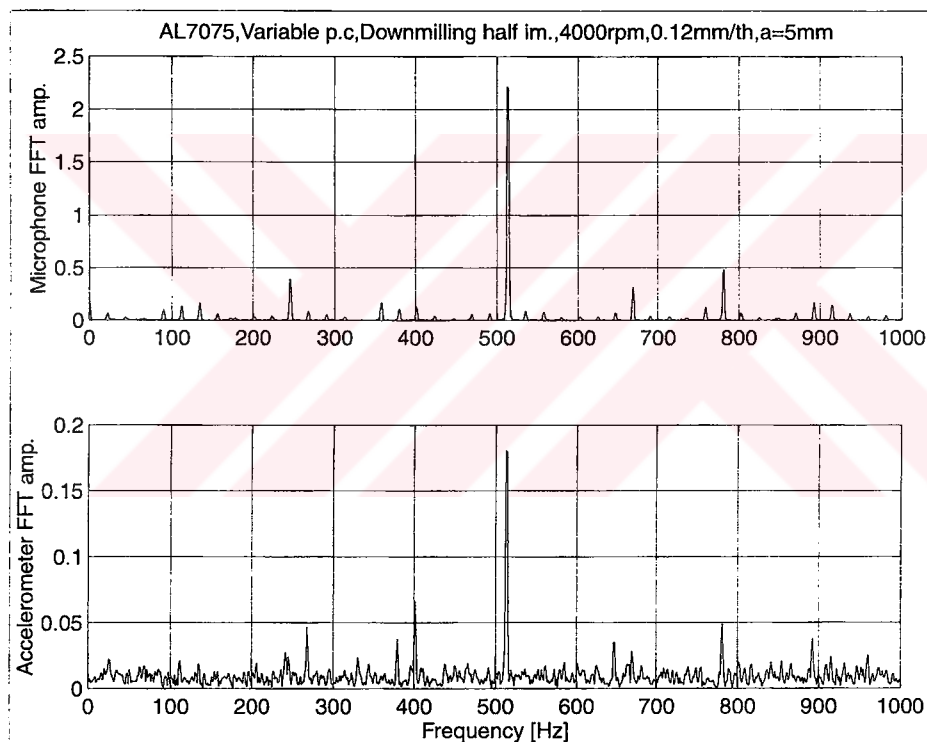


Figure 6.24. Sound pressure and acceleration FFT results for variable pitch cutter.

6.6.1 Stability Lobes for Higher Order Approximations

Stability lobes were drawn according to a zero order approximation. The first, the second and the third order approximations were calculated as an example for a variable pitch cutter. In this example, transfer functions have two modes in the X and Y directions. The first mode is at 500 Hz, the second mode is placed at 560 Hz one spindle frequency apart for investigating the difference between zero order and

higher order solutions of the stability lobes. Stability lobes for higher order approximations can be seen in Figure 6.25. Stability lobe values of different approximations are very close to each other. Axial depth of cut limits were calculated at 3600 rev/min. for the zero, the first, the second and the third order solutions which are 0.7806 mm, 0.8216 mm, 0.8164 mm and 0.8152 mm, respectively (Figure 6.26). The difference is only 0.041 mm between the zero order and the first order solutions. The difference between the first and higher order solutions is negligible. Hence, it can be concluded that zero order solution has enough accuracy. Both time domain and analytical solutions (Figure 6.27) proved that the higher harmonics of tooth or spindle passing frequencies do not improve the accuracy of the stability solution, because the vibration energy is centered at the chatter frequency [71]. Even when there was a structural mode coinciding with the harmonics of the spindle speed and the chatter frequency, the stability results did not change significantly.

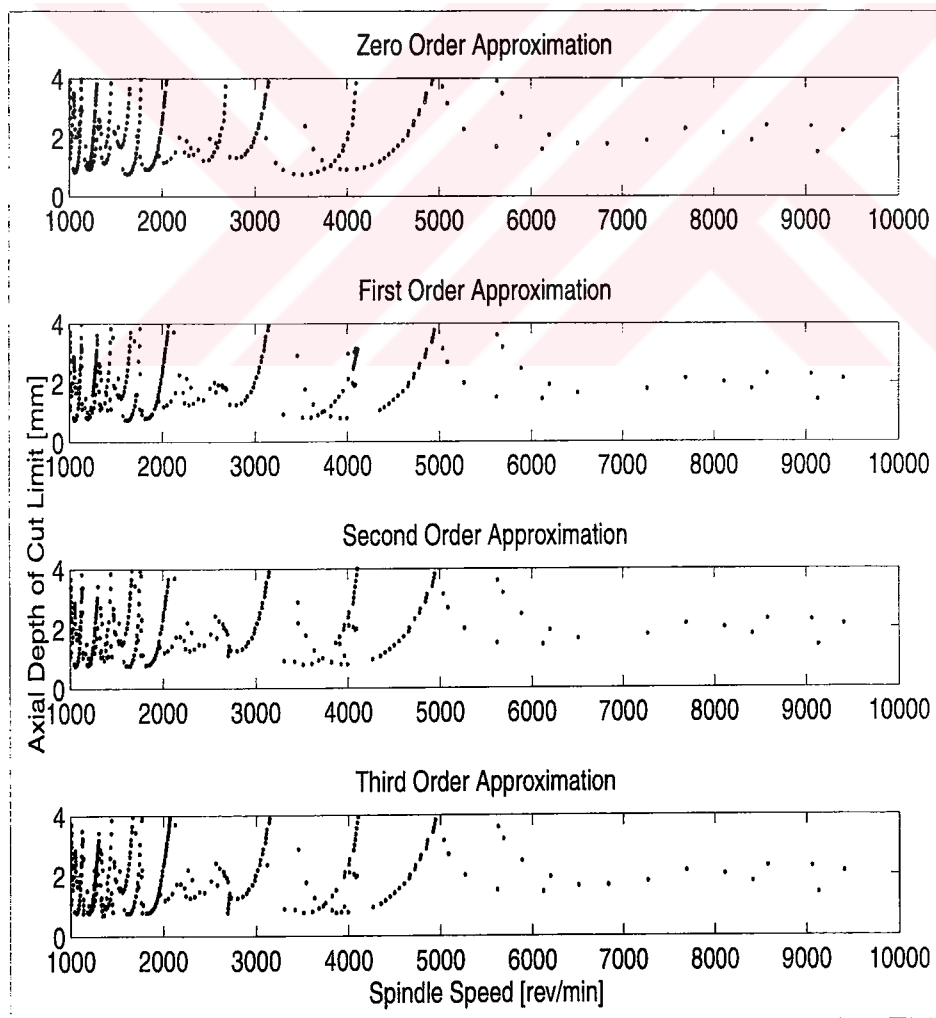


Figure 6.25. Stability lobes for higher order approximations as an analytical example.

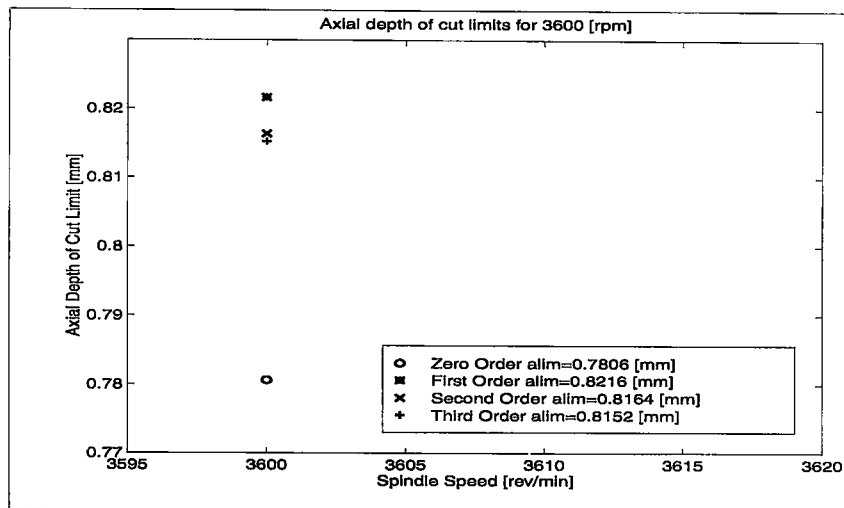


Figure 6.26. Axial depth of cut limit values at a spindle speed for higher order approximations as an analytical example.

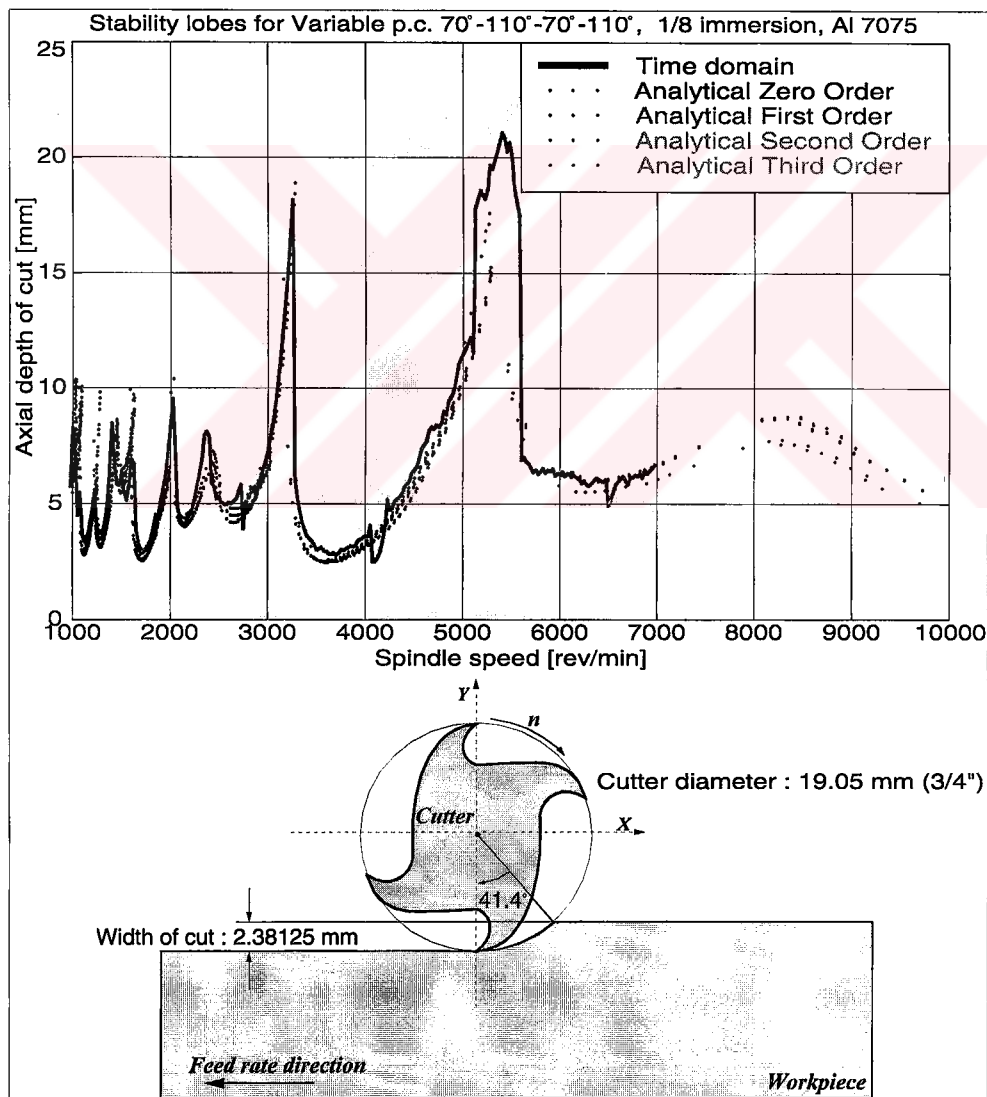


Figure 6.27. Comparison of higher order and time domain solution for small width of cut.

The effect of the cutting coefficient K_r can be seen in Figure 6.28a. Because of the flank wear, the tool wear increases the radial cutting coefficient. In Figure 6.28 an extreme point is shown for which radial and axial cutting coefficients are equal to each other. In practice, K_r is around 0.2-0.5 for metal cutting. The stability lobes are almost identical especially at the low spindle speeds. The change for high spindle speeds is very little. Up and down milling stability lobes are the same, if the transfer functions and all characteristics of the structure are the same in both X and Y directions (see Figure 6.2 for directional coefficients). If the dynamic behavior of the system is different in X and Y directions, the stability lobes can be seen in Figure 6.28.

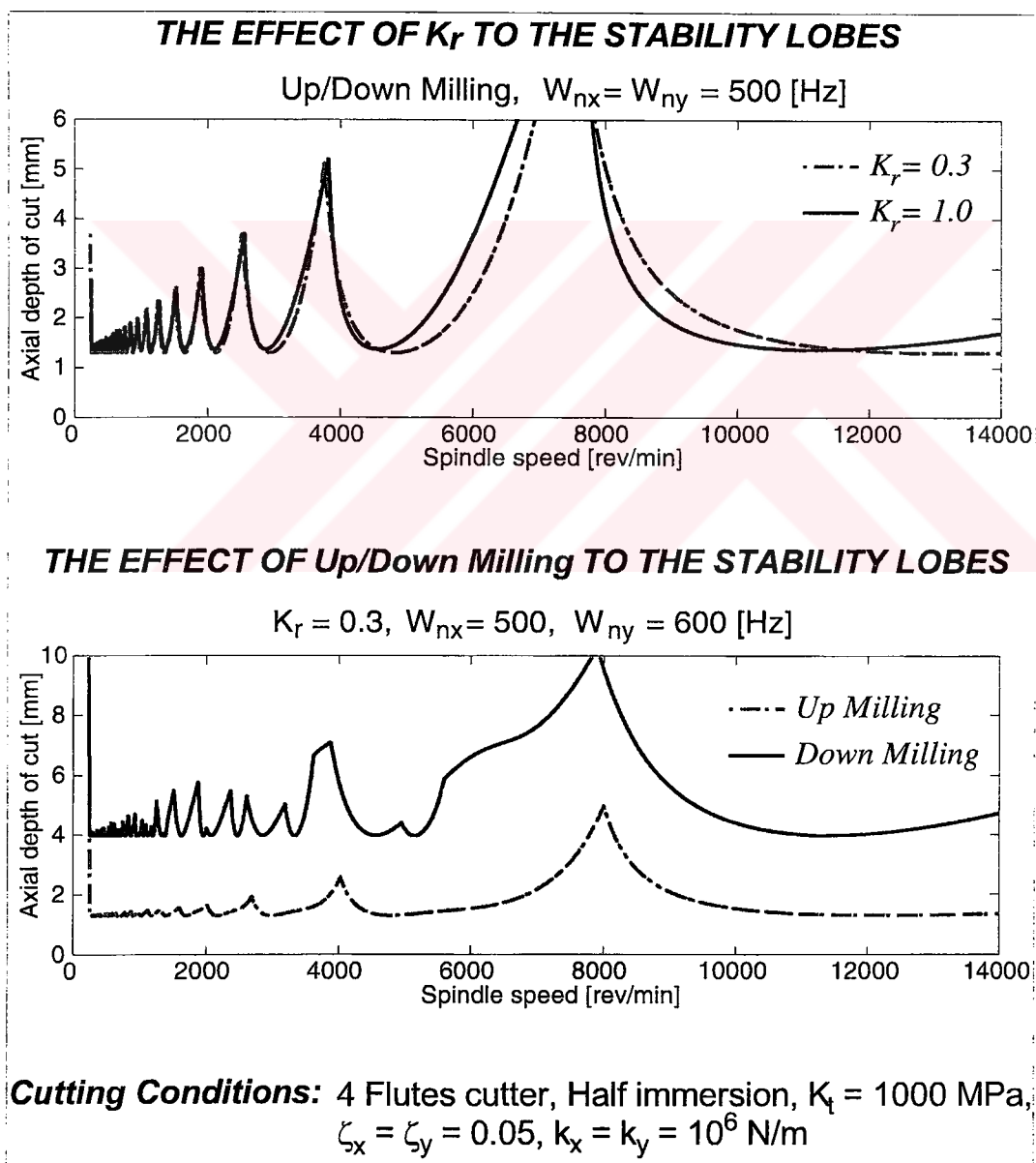


Figure 6.28. The effect of cutting parameters to the stability lobes. a) The effect of tool wear, b) The effect of up and down milling.

6.6.2 Surface Finish Results

Surface finish results were obtained at 4000 and 5300 rev/min for a 5.0 mm depth of cutting case on Al7075 alloy material, and a 0.12 mm/tooth feed rate. Surface roughness simulation and experimental results can be seen in Figure 6.29 and Figure 6.30. The simulated and the experimental surface finish results agree well with each other. At 5300 rev/min spindle speed, no chatter was observed. When the system was unstable at $n=4000$ rev/min, the surface finish was quite poor with a measured average peak to valley roughness of 0.042 mm (Figure 6.29). The simulation predicted an average roughness of 0.045 mm. When the spindle speed was increased to the desired $n=5300$ rev/min, the chatter vibrations diminished completely and an acceptable surface finish with an average roughness of 0.004 mm was achieved (Figure 6.30).

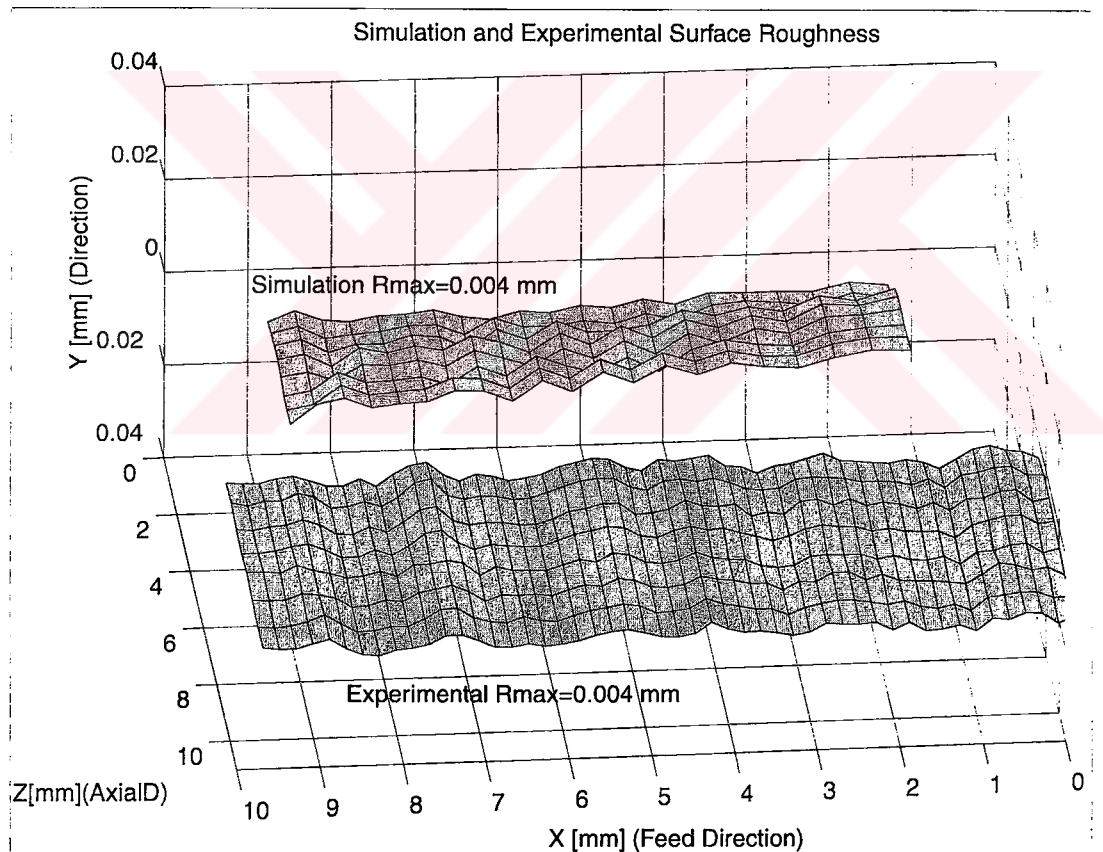


Figure 6.29. Simulation and experimental surface roughness for Al7075, spindle speed: 5300 rev/min, feed rate: 0.12 mm/tooth, axial depth of cut: 5.0 mm, variable pitch cutter is used.

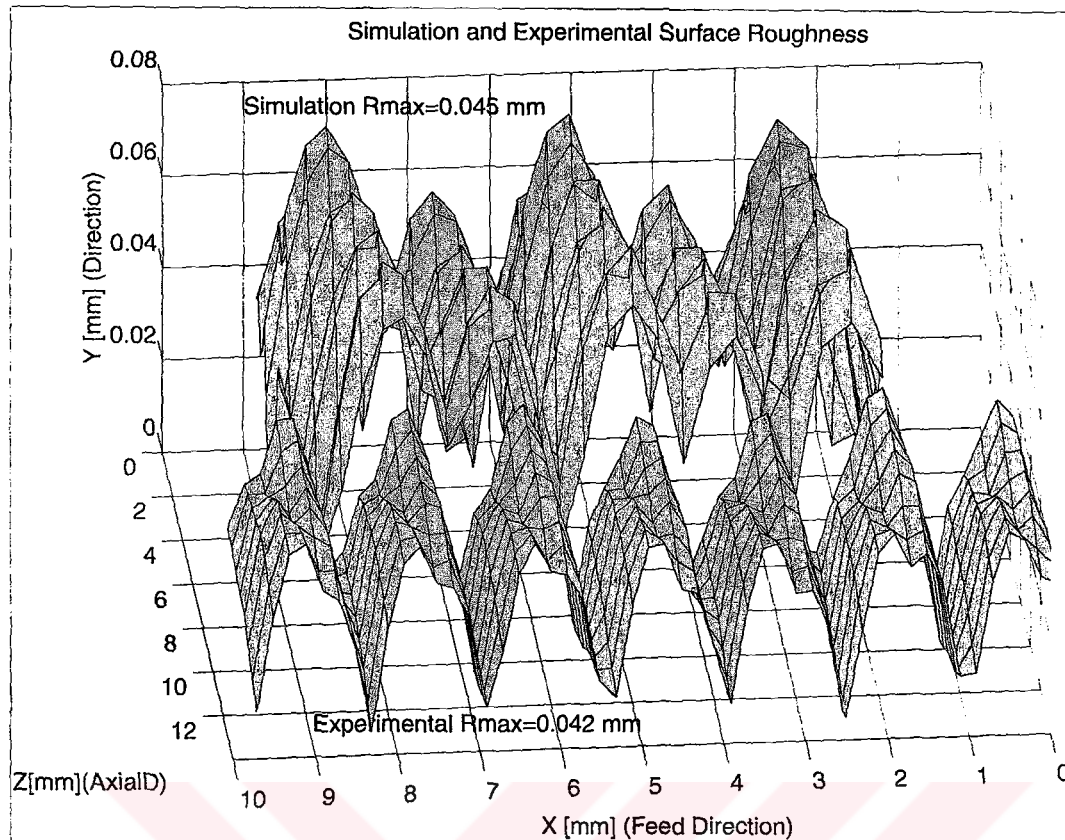


Figure 6.30. Simulation and experimental surface roughness for Al7075, spindle speed: 4000 rev/min, feed rate: 0.12 mm/tooth, axial depth of cut: 5.0 mm variable pitch cutter is used.

The results of surface finish obtained from the simulation and experiments are compared. The amplitudes of the surface roughness obtained in the simulations and the experiments were very close to each other.

The average wavelength left on the surface at 4000 rev/min was $\lambda = 1.516$ mm. Paths of a variable pitch cutter with four teeth are shown in Figure 6.31a. In Figure 6.31b the Y axis vibration of the tool and Y coordinate of the cutting edge are shown. As was noted by Smith and Tlustý [51], the surface finish is generated only when the teeth are at the point P_j . This is a variable pitch cutter, therefore, all degrees between the teeth can be different. Hence tooth-passing frequency is equal to the spindle frequency for every tooth. This results in a flat surface finish (by simulation and experiment) when the rotational and vibration frequencies are equal [51], or in fact when they may be divided without any fractional remainder, with the vibration frequency being higher. In the case of Figure 6.31, the frequencies do not divide evenly and the resultant surface generated by time domain simulation is given in Figure 6.31c. The physical surface finish is shown in Figure 6.31d.

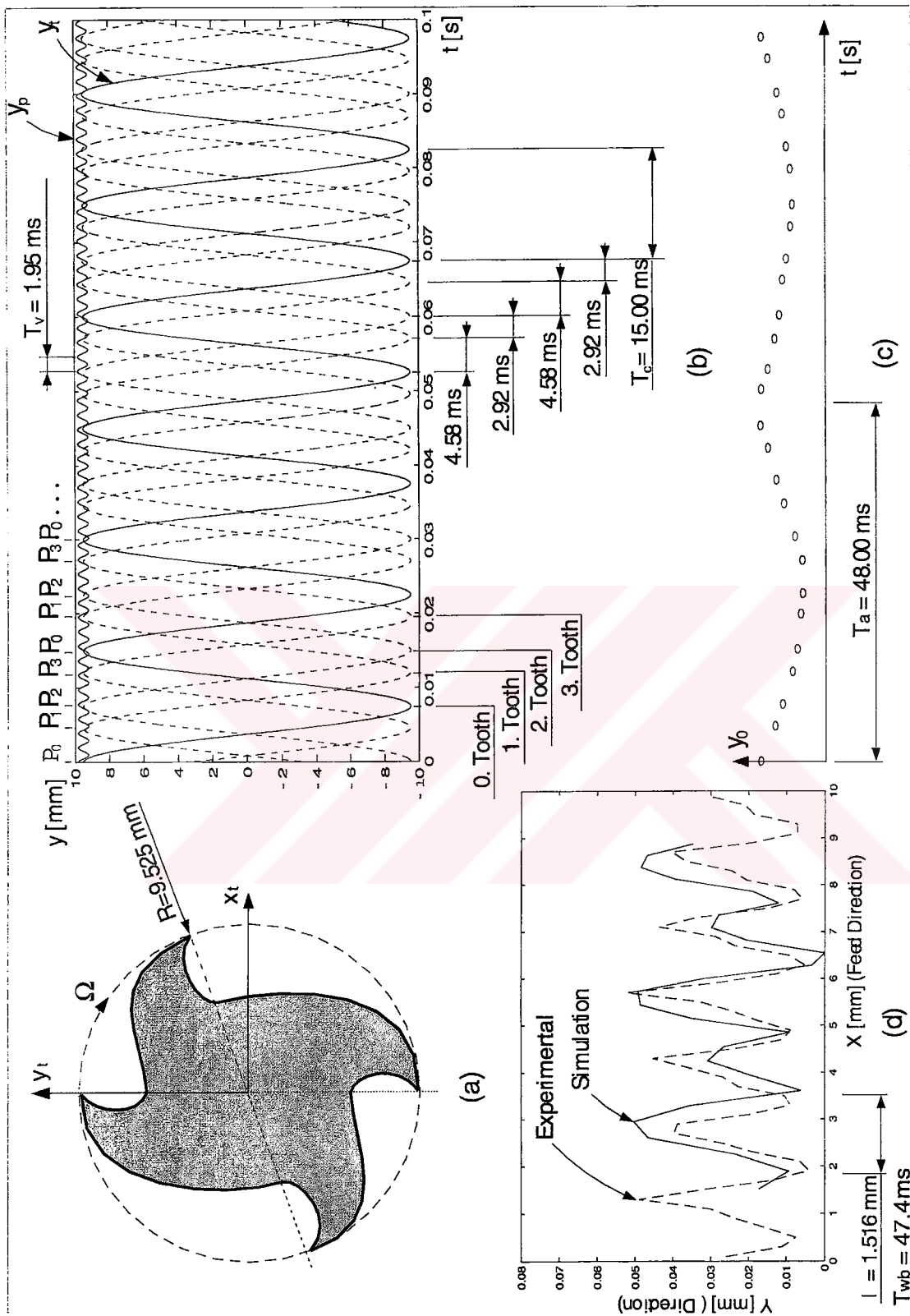


Figure 6.31. Illustrations of surface finish generation in milling.

The spindle (or tooth) period was $T_c=0.015$ sec. for 4000 rev/min, and the vibration period of the tool-workpiece was $T_v=0.00195$ sec ($T_v=1/\omega_c$) for $\omega_c=512.5$ Hz (Figure 6.24). The workpiece was moving at a feed rate of $f=32$ mm/sec = 0.48 mm/rev. Since in Figure 6.31d the average wavelength (λ) is 1.516 mm, the period is $T_{wb}=\lambda/f=1.516[\text{mm}]/32[\text{mm/sec}]=0.0474$ sec which is greater than both T_c and T_v . The ratio of process periods is $r=T_c/T_v=7.6875$. This means there are 7.6875 vibration waves in a single tooth period. The aliasing period is $T_a=N T_c$ where:

$$\begin{aligned} N &= [\text{frac}(r)]^{-1} & \text{if } \text{frac}(r) \leq 0.5 \\ N &= [1 - \text{frac}(r)]^{-1} & \text{if } \text{frac}(r) > 0.5 \end{aligned} \quad (6.74)$$

where $\text{frac}(r)=0.6875$ is the fractional component of r . In our case, $N=3.2$ and the aliasing period is $T_a=N T_c=3.2 \cdot 0.015=0.0480$ sec. This is in good agreement with the washboarding period obtained in simulation [4,9,10,59].

6.7 Summary

An analytical model is presented for the prediction of stability lobes for milling with variable pitch cutters. In addition, two methods of selecting optimal pitch angles are presented. The proposed methods have been verified both experimentally and using time domain simulations which consider various non-linearities in the process. The proposed methods are used for eliminating chatter by tuning the cutter pitch angles to cut a specific part material with a specific machine tool that has known structural dynamics.

7. CONCLUSIONS

The mechanics and dynamics of general tool geometry have been studied in this thesis. General tool geometry has been developed for solid end mills, as well as indexable cutters such as inserted end mills and face mills. Variable pitch cutters, which increase productivity at pre-optimized cutting speeds, have also been studied.

The contributions of this thesis can be summarized in the following main categories with the related results:

- A generalized mathematical model of arbitrary end mills is presented. The model allows parametric design and representation of a variety of end mill shapes and helical flutes. Sample design examples include cylindrical, ball, tapered helical end mills as well as inserted bull nosed cutters. The model allows the evaluation of local cutting edge geometry along the flute. Using previously developed exact kinematics of dynamic milling, the chip thickness and the corresponding cutting forces, vibrations, and the dimensional surface finish generated by end mills with arbitrary geometry can be predicted. The mathematical models are verified by a number of experiments conducted with helical tapered end, ball end and bull nosed cutters. The proposed approach allows the design and analysis of a variety of milling cutters and operations used in industry.
- A generalized mathematical model for inserted cutters has been developed. The model allows the placing of different inserts on the cutter body. Each insert is placed on the cutter body by defining its center from the cutter body coordinate system. The inserts can be oriented by rotating them about the three axes of the cutter body coordinate frame. The insert geometry is defined individually in a local coordinate system by its edge dimensions and shapes. By combining the vectorial representation of the insert center and edges, the cutting edge of each insert is determined. The chip thickness, cutting force,

chatter vibrations, dimensional surface finish and stability lobes of the milling process generated by arbitrarily inserted cutters are evaluated. The proposed model allows the analysis of general indexed cutters, and it is experimentally verified using two industrial indexed cutters in milling of Al-7075 and Ti₆Al₄V alloys.

- An analytical model is presented for the prediction of stability lobes when milling with variable pitch cutters. The method provides chatter-free depths of cut and spindle speeds for a given set of relative transfer functions between the cutter and workpiece, part material, cutting edge dependent cutting constants and cutter geometry. In addition, a method of selecting optimal pitch angles is presented. The method creates a large stability zone around a desired speed range, which is otherwise unstable when regular pitch cutters are used. The proposed methods have been verified both experimentally and using time domain simulations which consider various non-linearities in the process. The proposed method is used for eliminating chatter vibrations by tuning the cutter pitch angles to cut a specific part material machined on a specific machine tool with known structural dynamics. Hence, variable pitch cutter design is most suitable in eliminating chatter vibrations when the chatter is caused by time invariant structural dynamics of spindles or parts.

Results such as cutting forces, workpiece and spindle vibrations, stability lobes, chatter frequencies and surface finish profiles were obtained through frequency domain analyses and time domain simulations, which were in close agreement. A laser displacement sensor, accelerometers, a force dynamometer, an impact hammer with force sensor, an acoustic microphone, and a laser surface scanner were used to measure input and output variables of the milling simulations. The experimental and simulated results were very close to each other under these carefully controlled conditions. Throughout all this work, more than two thousands cutting tests were done to verify the theory, which is developed in this thesis. Some of the cutting tests were repeated several times to obtain accurate results. Some of the tests results, which gave wrong outcomes, were not given in the thesis. The simulations incorporating the theoretical work developed in this thesis, can be an quite useful tool to identifying the optimum cutting conditions for improved productivity.

The three main contribution areas of this thesis are;

- Arbitrary end mills: a generalized geometric model for arbitrary end mills has been developed. The mechanics and the dynamics of the system have been investigated.
- Inserted (indexable) cutters: a generalized geometric model for inserted (indexable) cutters has been developed. The mechanics and dynamics of this type of cutters have been investigated.
- Variable pitch cutters: a method to predict the chatter stability lobes analytically, and analytical design technique to prevent chatter for variable pitch cutters have been developed.

In the generalized geometric model for arbitrary end mills, the envelope is defined in a similar manner to the parametric representation used by APT and CAD/CAM softwares. Mathematical models of the envelope geometry and cutting edge positions were developed. These models were presented for each zone of cutter body in general cases. Mechanics and dynamics of this geometry were investigated using a technique similar to the one given by Altıntaş and Lee [20] for ball end milling.

In the generalized geometric model for inserted cutters, a model for the definition of cutting edge positions and design models of the multiple inserted cutters in a matrix form were developed. The cutting edge positions were defined by a vector, which extends from cutter body center to the cutting edge at any given level z . In this model, two different inserts were considered. A new variable cutting coefficient model based on the average cutting coefficients method (see Budak et. al [17] for details), along the z direction was used for the inserted cutters. The average cutting coefficients are very accurate at each level, however, they vary along the z direction. A second order polynomial was fitted to the average cutting coefficients along the z direction. If the cutting edge rake face geometry is not very complex, the curve fitting is more accurate. In this study, two different applications were performed with two different insert geometries and two materials, which are AL7075 and Ti_6Al_4V . The correlation coefficients were low, because of the complex geometry on the rake face of the insert. The dynamic investigations of inserted cutter were carried out using time domain and analytical solutions proposed by Altıntaş and Budak [67].

A solution method based on the analytical stability lobes for variable pitch cutters was developed by modifying uniform pitch cutters stability lobe solution, which is previously developed by Altıntaş and Budak [67]. The static model of cutting forces and chip load generation model for variable pitch cutters are given by Shirase and Altıntaş [97]. The stability formulation developed previously for regular pitch cutters is extended to irregular pitch cutters in this study. In addition, an analytical method of selecting optimum pitch angles was developed. This method gives all possible pitch angles at a specified speed range, where it is tuned to provide minimum regeneration for a given set of machine tool dynamics, i.e. spindle-cutter assembly.

Additional research to follow this work is recommended in several areas. In this thesis, only vibrations in the X and Y directions were considered. The dynamics in the Z direction should also be considered for machines with more than three axes. The cutting process assumed that the feed direction is along the X-axis. In actual cutting the feed may be in any direction. Therefore the oriented transfer functions can be included in the calculations. The force model used in this thesis can be improved by including tool wear and thermal effects, which alter cutting coefficients with time, changing cutting speed, wear and immersion. The inter-relation of thermal effects and tool wear can also be studied. The static and dynamic simulation models can be added to CAD/CAM software for optimizing the cutter path in the planning stage. The real time chatter detection methods and also the adaptive control algorithms can be tested using time domain simulations. The cutting conditions are constant in the simulation. Conditions such as width of cut, engagement of tool and workpiece, and dynamic behavior of the workpiece structure can vary throughout the cutting process.

BIBLIOGRAPHY

- [1] **Taylor, F. W.**, 1907. On the Art of Cutting Metals, *Transactions of the ASME*, Vol. 28.
- [2] **Airey, J., and Oxford, C. J.**, 1921. On the Art of Milling, *Transactions of the ASME*, Vol. 43, pp. 549.
- [3] **Sawin, N. N.**, 1926. Theory of Milling Cutters, *Mechanical Engineering*, Vol. 48, pp. 1203-1209.
- [4] **Salomon, C.**, 1926. Die Frasarheit, *Werkstattstechnik*, Vol. 20, pp. 469-474.
- [5] **Parsons, F.**, 1923. Power Required for Cutting Metal, *Transaction of the ASME*, Vol. 49, pp. 193-227.
- [6] **Boston, O.W., and Kraus, C.E.**, 1932. Elements of Milling, *Transaction of the ASME*, Vol. 54, pp. 71-104.
- [7] **Boston, O.W., and Kraus, C.E.**, 1932. Elements of Milling – Part II, *Transaction of the ASME*, Vol. 56, pp. 355-371.
- [8] **Martellotti, M.E.**, 1945. An Analysis of the Milling Process. Part II: Down Milling, *Transactions of the ASME*, Vol. 67, pp. 233-251.
- [9] **Martellotti, M.E.**, 1941. An Analysis of the Milling Process, *Transactions of the ASME*, Vol. 63, pp. 677-700
- [10] **Montgomery, D. and Altıntaş, Y.**, 1991, Mechanism of Cutting Force and Surface Generation in Dynamic Milling, *Transactions of ASME Journal of Engineering for Industry*, Vol. 113, pp. 160-168.
- [11] **Lee, P.**, 1995. Mechanics and Dynamics of Ball End Milling, Master's thesis, *University of British Columbia*.
- [12] **Campomanes, M. L.**, 1998. Dynamics of Milling Flexible Structures, Master's thesis, *University of British Columbia*.
- [13] **Fu, H. J., Devor, R. E., and Kapoor, S. G.**, 1984. A Mechanistic Model for the Prediction of the Force System in Face Milling Operation, *ASME Journal of Engineering for Industry*, Vol. 106(1), pp 81-88.
- [14] **Sutherland, J.W. and DeVor, R.E.**, 1986. An Improved Method for Cutting Force and Surface Prediction in Flexible End Milling System,

Transactions of the ASME Journal of Engineering for Industry, Vol. 108, pp. 269-279.

- [15] **Bayoumi A. E., Yucesan, G., and Kendall, L. A.**, 1994. An Analytic Mechanistic Cutting Force Model for Milling Operations: A Theory and Methodology, *Transaction of the ASME*, Vol. 116, pp. 324-330.
- [16] **Bayoumi A. E., Yucesan, G., and Kendall, L. A.**, 1994. An Analytic Mechanistic Cutting Force Model for Milling Operations: A Case Study of Helical Milling Operation, *Transaction of the ASME*, Vol. 116, pp. 331-339.
- [17] **Budak, E., Altıntaş, Y. and Armarego, E.J.A.**, 1996. Prediction of Milling Force Coefficients From Orthogonal Cutting Data, *Transactions of ASME*, Vol. 118, pp. 216-224.
- [18] **Spiewak, S. A.**, 1994. Analytical Modeling of Cutting Point Trajectories in Milling, *Transactions of the ASME, Journal of Engineering for Industry*, Vol. 116, pp. 440-448.
- [19] **Lazoglu, I., and Liang, S. Y.**, 1997. Analytical Modeling of Force System in Ball End Milling, *Journal of Machining Science and Technology*, Vol. 1 (No.2), pp. 219-234.
- [20] **Altıntaş, Y. and Lee, P.**, 1998. Mechanics and Dynamics of Ball End Milling, *Transaction of ASME, Journal of Manufacturing Science and Engineering*, Vol. 120, pp. 684-692.
- [21] **Yucesan, G. and Altıntaş, Y.**, 1996. Prediction of Ball End Milling Forces, *Transaction of ASME, Journal of Engineering for Industry*, Vol. 1(No. 1), pp. 95-103.
- [22] **Ramaraj, T. C. and Eleftheriou, E.**, 1994. Analysis of the Mechanics of Machining with Tapered End Milling Cutters, *Transactions of the ASME*, Vol. 116, pp. 398-404.
- [23] **Ehmann, K. F., Kapoor, S. G., DeVor, R. E., and Lazoglu, I.**, 1997. Machining Process Modeling: A Review, *Journal of Manufacturing Science and Engineering Transaction of the ASME*, Vol. 119(No.4-B), pp. 655-663.
- [24] **Leu, M. C., Wang, L. and Blackmore, D.**, 1997. A Verification Program for 5-axis NC Machining with General APT Tools, *Annals of the CIRP*, Vol. 46 (No. 1), pp. 419-424.
- [25] **Gu, F., Melkote, S. N., Kapoor, S. G., and Devor, R. E.**, 1997. A Model for the Prediction of Surface Flatness in Face Milling, *Journal of Manufacturing Science and Engineering Transaction of the ASME*, Vol. 119 (No. 4), pp 476-484.

- [26] **Spence, A., Altıntaş, Y.,** 1994. A Solid Modeler Based Milling Process Simulation and Planning System, *Transactions of the ASME, Journal of Engineering for Industry*, Vol. 116, pp. 61-69.
- [27] **Wertheim, R., Satran, A., and Ber, A.,** 1994. Modifications of the Cutting Edge Geometry and Chip Formation in Milling, *Annals of the CIRP*, Vol. 43(No. 1), pp. 63-68.
- [28] **Kim, H. S., and Ehmann, K. F.,** 1993. A Cutting Force Model for Face Milling Operations, *International al Journal of Machine Tools and Manufacture*, Vol 33(5), pp. 651-673.
- [29] **Weck, M., Altıntaş, Y. and Beer, C.,** 1994. Cad Assisted Chatter Free NC Tool Path Generation in Milling, *International Journal of Machine Tools and Manufacture*, Vol. 34(6), pp. 879-891.
- [30] **Endres, W. J., DeVor, R. E., and Kapoor, S. G.,** 1995. A Dual-Mechanism Approach to the Prediction of Machining Forces: Part I - Model Development: Part II - Calibration and Validation, *Transactions of ASME Journal of Engineering for Industry*, Vol. 117, pp. 526-541.
- [31] **Choudhury, S. K., and Mathew, J.,** 1995. Investigations of the Effect of Non-Uniform Insert Pitch on Vibration During Face Milling, *International al Journal of Machine Tools and Manufacture*, Vol 35(No.10), pp. 1435-1444.
- [32] **Thusty, J. and Ismail, F.,** 1981. Basic Nonlinearity in Machining Chatter, *Annals of the CIRP*, Vol. 30, pp. 21-25.
- [33] **Kline, W.A., DeVor, R.E. and Zdeblick, W.J.,** 1980. A Mechanistic Model for the Force System in End Milling with Application to Machining Airframe Structures, *In North American Manufacturing Research Conference Proceedings, Society of Manufacturing Engineers*, Vol. XVIII, pp. 297, Dearborn, MI.
- [34] **Altıntaş, Y. and Spence, A.,** 1991. End Milling Force Algorithms For Cad Systems, *Annals of CIRP*, Vol. 50(1), pp. 31-34.
- [35] **Armarego, E. J. A. and Brown, R. H.,** 1969. The Machining of Metals, *Prentice-Hall*.
- [36] **Smith, S. and Thusty, J.,** 1991. An Overview of Modeling and Simulation of the Milling Process. *Transactions of the ASME, Journal of Engineering for Industry*, Vol. 113(2), pp. 169-175.
- [37] **Wang, W. P.,** 1988. Solid Modeling for Optimizing Metal Removal of Three-dimensional NC End Milling, *Journal of Manufacturing Systems*, Vol. 7(1), pp. 57-65.
- [38] **Sabberwal, A.J.P.,** 1961. Chip Section and Cutting Force During the Milling Operation, *Annals of the CIRP*, Vol. 10, pp. 197-203.

- [39] **Koenigsberger, F. and Sabberwal, A.J.P.**, 1961. An Investigation Into the Cutting Force Pulsations During Milling Operations, *International Journal of Machine Tool Design and Research*, Vol. 1, pp. 15-33.
- [40] **Armarego, E.J.A. and Epp, C.J.**, 1970. An Investigation of Zero Helix Peripheral Up-Milling, *International Journal of Machine Tool Design and Research*, Vol. 10, pp. 273-291.
- [41] **Thusty, J. and McNeil, P.**, 1970. Dynamics of Cutting Forces in End Milling, *Annals of the CIRP*, Vol. 24, pp. 21-25.
- [42] **Zorev, N.N.**, 1966. Metal Cutting Mechanics, *Pergamon Press*.
- [43] **Radulescu, R. and Kapoor, S.G. and DeVor, R.E.**, 1993. Investigation of Variable Spindle Speed Face Milling for Tool-work Structures with Complex Dynamics: Part I-Simulation Results, Part II- Physical Explanation, *ASME Publication PED*, Vol. 64, pp. 603-627.
- [44] **Altıntaş, Y. and Chan, P.K.**, 1992. In Process Detection and Suppression of Chatter in Milling, *International Journal Machine Tools and Manufacture*, Vol. 32, No. 3, pp. 329-347.
- [45] **Thusty, J.**, 1965. A Method of Analysis of Machine Tool Stability, *Proceeding MTDR*, pp. 5-14.
- [46] **Koenigsberger, F. and Thusty J.**, 1967. Machine Tool structures-Vol.I: Stability Against Chatter, *Pergamon Press*.
- [47] **Tobias, S.A.**, 1965. Machine Tool Vibration, *Blackie and Sons Ltd.*.
- [48] **Thusty, J.**, 1985. Machine Dynamics, *Handbook of High Speed Machining Technology*, pp. 49-153.
- [49] **Thusty, J.**, 1986. Dynamics of High-Speed Milling, *Transactions of the ASME Journal of Engineering for Industry*, Vol. 108(2), pp. 59-67.
- [50] **Smith, S. and Thusty, J.**, 1987. Update in High Speed Milling Dynamics, *ASME 1987 Winter Annual Meeting PED.*, Vol. 25, pp. 153-165.
- [51] **Smith, S. and Thusty, J.**, 1993. Efficient simulation programs for chatter in milling, *Annals of the CIRP*, Vol. 42(1), pp. 463-466.
- [52] **Altıntaş, Y., Montgomery, D. and Budak, E.**, 1992. Dynamic Peripheral Milling of Flexible Structures, *Transactions of ASME Journal of Engineering for Industry*, Vol. 114, pp. 137-145.
- [53] **Lee, A.C. and Liu, C.S.**, 1991. Analysis of Chatter Vibration in the End Milling Process, *International Journal of Machine Tool Design and Research*, Vol. 31(4), pp. 471-479.
- [54] **Varterasian, J.H.**, 1971. White Noise: A Deterrent to Milling Cutter Chatter, *Manufacturing Engineering and Management*, Vol. 67 (4), pp. 26-28.

- [55] **Hohn, R.E., Sridhar, R. and Long, G.W.**, 1968. A Stability Algorithm for a Special Case of the Milling Process, *Transactions of the ASME Journal of Engineering for Industry*, pp. 325-329.
- [56] **Merchant, M.E.**, 1944. Basic Mechanistic of the Metal Cutting Process, *Transactions of the ASME journal of Applied Mechanics*, Vol. A, pp. 168-175.
- [57] **Opitz, H.**, 1968. Chatter Behavior of Heavy Machine Tools, *Quarterly technical report no.2-61(052)-916 Research and Technology Division*, Wright-Patterson Air Force Base, Ohio.
- [58] **Opitz, H. and Bernardi, F.**, 1970. Investigation and Calculation of the Chatter Behavior of Lathes and Milling Machines, *Annals of the CIRP*, Vol. 18, pp. 335-343.
- [59] **Montgomery, D.**, 1990. Milling of Flexible Structures, Master's thesis, *University of British Columbia*.
- [60] **Sridhar, R., Hohn, R.E. and Long, G.W.**, 1968. A Stability Algorithm for the General Milling Process, *Transactions of the ASME Journal of Engineering for Industry*, pp. 330-334.
- [61] **Tsao, T.C., McCarthy, M.W. and Kapoor, S.G.**, 1993. New Approach to Stability Analysis of Variable Speed Machining System, *International Journal of Machine Tools and Manufacture*, Vol. 33(No. 6), pp. 791-808.
- [62] **Minis, I. and Yanushevsky, R.**, 1993. A New Theoretical Approach for Prediction of Machine Tool Chatter in Milling, *Transactions of the ASME Journal of Engineering for Industry*, Vol. 115, pp. 1-8.
- [63] **Minis, I., Yanushevsky, R., Tembo, R. and Hocken, R.**, 1990. Analysis of Linear and Nonlinear Chatter in Milling, *Annals of the CIRP*, Vol. 39, pp. 459-462.
- [64] **Lee, P. and Altıntaş, Y.**, 1996. Prediction of Ball End Milling Forces from Orthogonal Cutting Data, *International Journal of Machine Tools and Manufacturing*, Vol. 36, pp. 1059-1072.
- [65] **Lee, A.C., Liu, C.S. and Chiang, S.T.**, 1991. Analysis of Chatter Vibration in a Cutter-Workpiece System, *International Journal of Machine Tool Design and Research*, Vol. 31(2), pp. 221-234.
- [66] **Lee, Y.Y. and Lu, W.F.**, 1992. An Improved Method for Dynamic Behaviors in End Milling Process, *ASME Winter Annual Meeting Anaheim USA*.
- [67] **Altıntaş, Y. and Budak, E.**, 1995. Analytical Prediction of Stability Lobes in Milling, *Annals of CIRP*, Vol. 44(1), pp. 357-362.

- [68] **Budak, E. and Altıntaş, Y.**, 1995. Analytical Prediction of Chatter Stability in Milling. Part I: General Formulation, *Proceedings of the ASME Dynamic Systems and Control Division*, Vol. 57-1, pp. 545-556.
- [69] **Budak, E. and Altıntaş, Y.**, 1995. Analytical Prediction of Chatter Stability in Milling. Part II: Application of the General Formulation to Common Milling System, *Proceedings of the ASME Dynamic Systems and Control Division*, Vol. 57-1, pp. 557-565.
- [70] **Altıntaş, Y. and Lee, P.**, 1996. A General Mechanics and Dynamics Model for Helical End Mills, *Annals of the CIRP*, Vol. 45, pp. 59-64.
- [71] **Altıntaş, Y., Shamoto, E., Lee, P. and Budak, E.**, 1999. Analytical Prediction of Stability Lobes in Ball End Milling, *Transactions of ASME Journal of Engineering for Industry*, (in press).
- [72] **Slaveicek, J.**, 1965. The Effect of Irregular Tooth Pitch on Stability of Milling, *In Proceeding of the 6th MTDR Conference*, Pergamon Press, London, pp. 15-22.
- [73] **Vanherck, P.**, 1967. Increasing Milling Machine Productivity by Use of Cutter with Non-constant Edge Pitch, *In Proceeding of the 6th MTDR Conference*, Vol. 8, pp. 947-960.
- [74] **Opitz, H., Dregger, U., and Roese, H.**, 1966. Improvement of the Dynamic Stability of the Milling Process by Irregular Tooth Pitch, *In Proceeding of the 7th MTDR Conference*, Vol. 9, pp. 213-228.
- [75] **Stabler, G.V.**, 1964. The Chip Flow Law and Its Consequences, *Advances in Machine Tool Design and Research*, pp. 243-251.
- [76] **Stone, B.J.**, 1970. The Effect on the Chatter Behavior of Cutters With Different Helix Angles on Adjacent Teeth, *Advances in Machine Tool Design and Research, Proceedings of the 11th International MTDR Conference*, University of Birmingham, Vol. A, pp. 169-180.
- [77] **Doolan, P., Phadke, M.S. and Wu, S.**, 1975. Computer Design of Vibration Free Face Milling Cutters, *ASME Journal of Engineering for Industry*, pp. 925-930.
- [78] **Thusty, J., Ismail, F. and Zaton, W.**, 1983. Use of Special Milling Cutters Against Chatter, *SME, NAMRC 11*, University of Wisconsin, Madison, pp. 408-415.
- [79] **Altıntaş, Y.**, 2000. Manufacturing Automation: Principles of Metal Cutting and Machine Tool Control, *Cambridge University Press*. ISBN 0521650291, (in press).
- [80] **Armarego, E.J.A. and Deshpande, N.P.**, 1991. Computerized End-Milling Force Predictions with Cutting Models Allowing Eccentricity and Cutter Deflections, *Annals of the CIRP*, Vol. 40 (No. 1), pp. 25-29.

- [81] **Armarego, E.J.A. and Deshpande, N.P.**, 1989. Computerized Predictive Cutting Models for Forces in End-Milling Including Eccentricity Effects, *Annals of the CIRP*, Vol. 38 (No. 1), pp. 45-49.
- [82] **Armarego, E.J.A. and Deshpande, N.P.**, 1993. Force Prediction Models and Cad/Cam Software for Helical Tooth Milling Processes. I. Basic Approach and Cutting Analyses, *International Journal of Production Research*, Vol. 31 (No. 8), pp. 1991-2009.
- [83] **Armarego, E.J.A. and Whitfield, R.C.**, 1985. Computer Based Modelling of Popular Machining Operations for Force and Power Predictions, *Annals of the CIRP*, Vol. 34, pp. 65-69.
- [84] **Budak, E.**, 1994. Mechanics and Dynamics of Milling Thin Walled Structure, PhD's thesis, *University of British Columbia*.
- [85] **Kline, W.A. and DeVor, R.E.**, 1983. The Effect of Runout on Cutting Geometry and Forces in End Milling, *International Journal of Machine Tool Design and Research*, Vol. 23, pp. 123-140.
- [86] **Smith, S. and Thusty, J.**, 1988. Modeling and Simulation of the Milling Process, *ASME 1988 Winter Annual Meeting Publication*, Vol. 33, pp. 17-26.
- [87] **Yucesan, G. and Altıntaş, Y.**, 1994. Improved Modeling of Cutting Force Coefficients in Peripheral Milling, *International Journal of Machine Tools and Manufacture*, Vol. 34(4), pp. 473-487.
- [88] **Masuko, M.**, 1956. Fundamental Researches on the Metal Cutting (12nd report)- Force Acting on a Cutting Edge and its Experimental Discussion, *Trans. Japanese Society of Mechanical Engineers*, Vol. 22, pp. 371-377.
- [89] **Albrecht, P.**, 1960. New Developments in the Theory of the Metal Cutting Process: Part I - The Ploughing Process in Metal Cutting, *Transactions of the ASME Journal of Engineering for Industry*, pp. 348-358.
- [90] **Sisson T.R. and Kegg, R.L.**, 1969. An Explanation of Low Speed Chatter Effects *Transaction of ASME Journal of Engineering for Industry*, Vol. 91, pp. 951-955.
- [91] **Thusty, J.**, 1978. Analysis of the State of Research in Cutting Dynamics, *Annals of the CIRP*, Vol. 27, pp. 583-589.
- [92] **Thusty, J.**, 1993. High-Speed Machining, *Annals of the CIRP*, Vol. 42, pp. 733-738.
- [93] **Childs, J. J.**, Numerical Control Part Programming, *Industrial Press*.
- [94] **Krystof, J.**, 1939. Berichte uber Betriebswissenschaftliche Arbeiten, Bd., 12. VDI Verlag.

- [95] **Merchant, M.E.**, 1945. Mechanics of the Cutting Process, *Journal of Applied Physics*, Vol. 16, pp. 267-318.
- [96] **MillPro**, 1998. Milling Simulation Program, University of British Columbia - Manufacturing Automation Laboratory.
- [97] **Shirase, K. and Altıntaş, Y.**, 1995. Cutting Force and Dimensional Surface Error Generation in Peripheral Milling with Variable Pitch Helical End Mills, *International Journal of Machine Tools and Manufacture*, Vol. 36(5), pp. 567-584.



APPENDIX A

Throughout this study, more than two thousands cutting tests were done to verify the developed mathematical models. There were mainly three different types of experimental setups, namely, transfer function measurement (Figure A. 1 and Figure A. 2), cutting force measurement (Figure A. 3 and Figure A. 4) and chatter detection (Figure A. 5).



Figure A. 1. Experimental setup to measure transfer functions. 1- FFT Signal Analyzer, 2- Charge Amplifiers, 3- Hammer with force transducer, 4- Accelerometer or Laser sensor.

Experimental setup for the measurement of transfer functions is shown in Figure A. 1. The transfer function was measured in X and Y directions at the tool tip, when the cutter was stationary. The dynamic vibrations during the machining were modeled based on the measured transfer functions. In the experimental setup, an impact hammer with a piezoelectric force transducer was used. The force transducer delivers an impact force, which is a narrow pulse and has a flat Fourier. The force sensor and the mass of the hammer must be selected according to the structural properties of the cutting tool, which are material, rigidity and mass. The vibrations in the structure were measured with accelerometer. Alternatively, a laser displacement sensor was used in the measurement of the amplitudes of the vibrations. The input and output values were collected with a Fourier analyzer, which displays the measured data in time or frequency domain. The equipment used in the measurement of transfer functions is given in Table A. 1.

Table A. 1. Equipment used in the measurement of transfer functions.

No	Name	Specifications and Parameters									
1	FFT Signal Analyzer	HP 3569A Hand-held Real-time Dual-Channel Dynamic Signal Analyzer The standard HP 3569A provides one and two channel octave measurements (both 1/3 and 1/1 resolutions) with the following center frequency bands: <table> <tr> <th>Resolution</th><th>1Channel</th><th>2 Channel</th></tr> <tr> <td>1/3 Octave</td><td>1.6 Hz to 20 kHz</td><td>1.6 Hz to 10 kHz</td></tr> <tr> <td>1/1 Octave</td><td>2 Hz to 16 kHz</td><td>2 Hz to 8 kHz</td></tr> </table>	Resolution	1Channel	2 Channel	1/3 Octave	1.6 Hz to 20 kHz	1.6 Hz to 10 kHz	1/1 Octave	2 Hz to 16 kHz	2 Hz to 8 kHz
Resolution	1Channel	2 Channel									
1/3 Octave	1.6 Hz to 20 kHz	1.6 Hz to 10 kHz									
1/1 Octave	2 Hz to 16 kHz	2 Hz to 8 kHz									
2	Charge Amplifiers	Charge Amplifier-Kistler Model5004 Output voltage $\pm 10V$ Output current $\leq \pm 5mA$ Output impedance 100Ω Frequency response, with standard 5311 filter (-3dB) ≈ 0 to 180 kHz Power 100 to 130 VAC PCB Charge Amplifiers Model: 480E09 (sensor power unit) INTERTECHNOLOGY Inc. Ontario-Canada									

3	Hammer With force transducer	Impulse Force Hammer Model 086C05 Serial 7902 - PCB Range 0-5000 lb [22000N] Frequency Range 5kHz Linearity Error <1.0% Discharge Time Constant 2000 s Output Impedance 100 ohms Output Bias 10.1 Volts Sensitivity 9.67 mV/g Head Diameter 1 inch [25.4 mm] Hammer Sensitivity (S_f) 0.22 mV/N
		Impulse Force Hammer Model 086C80 Serial 7953 - PCB Range 0-50 lb [220N] Frequency Range 15 kHz Linearity Error <2.0% Discharge Time Constant 100 s Output Impedance 100 ohms Output Bias 10.2 Volts Sensitivity 5.35 mV/g Head Diameter 0.25 inch [6.3 mm] Hammer Sensitivity (S_f) 22.2 mV/N
		Impulse Force Hammer Model 9722A2000 Serial C129289 - Kistler Range 0-500 lb [2000N] Frequency Range 9.3kHz Discharge Time Constant 500 s Output Impedance 100 ohms Output Bias 11 Volts Sensitivity 10.07 mV/g Head Diameter 0.69 inch [17.5 mm] Hammer Sensitivity (S_f) 2.13 mV/N

4	Accelerometer or Laser sensor	Accelerometer Model 353B11 Serial 9690 - PCB Voltage Sensitivity 5.42 mV/g Transverse Sensitivity 1.3 % Resonant Frequency 95 kHz Time Constant 0.6 s Output Bias Level 8.2 V Range 1000 $\pm$ g Temp. Range -65/+250 °C
		Accelerometer Model 353B31 Serial 8757 - PCB Voltage Sensitivity 51.6 mV/g Transverse Sensitivity 1.0 % Resonant Frequency 38.5 kHz Time Constant 0.7 s Output Bias Level 9.1 V Range 100 $\pm$ g Temp. Range -65/+250 °C
		Accelerometer Model 8702B500 Serial C128797 - Kistler Voltage Sensitivity 10.09 mV/g Transverse Sensitivity 0.2 % Resonant Frequency 54.0 kHz Time Constant 0.8 s Output Bias Level 10.3 V Range 500 $\pm$ g Temp. Range -54/+120 °C
		Accelerometer Model 8628B5 Serial C121339 - Kistler Voltage Sensitivity 1010 mV/g Transverse Sensitivity 1.0 % Resonant Frequency 9.4 kHz Time Constant 1.0 s Output Bias Level 11.4 V Range 5 $\pm$ g Temp. Range 0/+65 °C

4	Accelerometer or Laser sensor	DynoVision Laser Sensor LTS 15/2.9 Measuring Distance 15mm Effective Range 2.9mm Accuracy $\pm 2\mu\text{m}$ Resolution $0.2\mu\text{m}$ Measuring Frequency 100kHz Temperature Range 0 – 50 °C Humidity Max 90% Distance Output (X) $-8.0/+6.5$ VDC Intensity Output (I) 0 – 0.5 VDC Enable Output (E) -0.7 VDC in range $+1.7$ VDC out of range Load (X,I,E) $R > 10\text{k}\Omega$ Sensitivity $5\text{mV}/\mu\text{m}$ Weight 750 (± 50) grams Power Supply ± 15 VDC $\pm 1\%$ Power Consumption Max. 5W Laser 30mW (780nm) When the sensor is not in range the X (distance) output is $+13\text{VCD}$
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After the measurement of the transfer function of the system at the tool tip, the measured data, which has frequency, real and imaginary part of the magnitude, were analyzed by using Model Analysis Software developed in Manufacturing Automation Laboratory. All of the modes in the measurement data were identified with this software, which gives all of the mode parameters such as mode frequency, damping ratio and residues in displacements. The output is displacement when the laser displacement sensor is used. The output is accelerations, if the accelerometer is used. The output accelerations need to be integrated twice to obtain displacements. The equations to calculate the transfer function values are given in the thesis. In the milling simulation, these dynamic parameters were considered to predict the vibrations, which may originate from the cutter, spindle or tool holder (Figure A. 2).

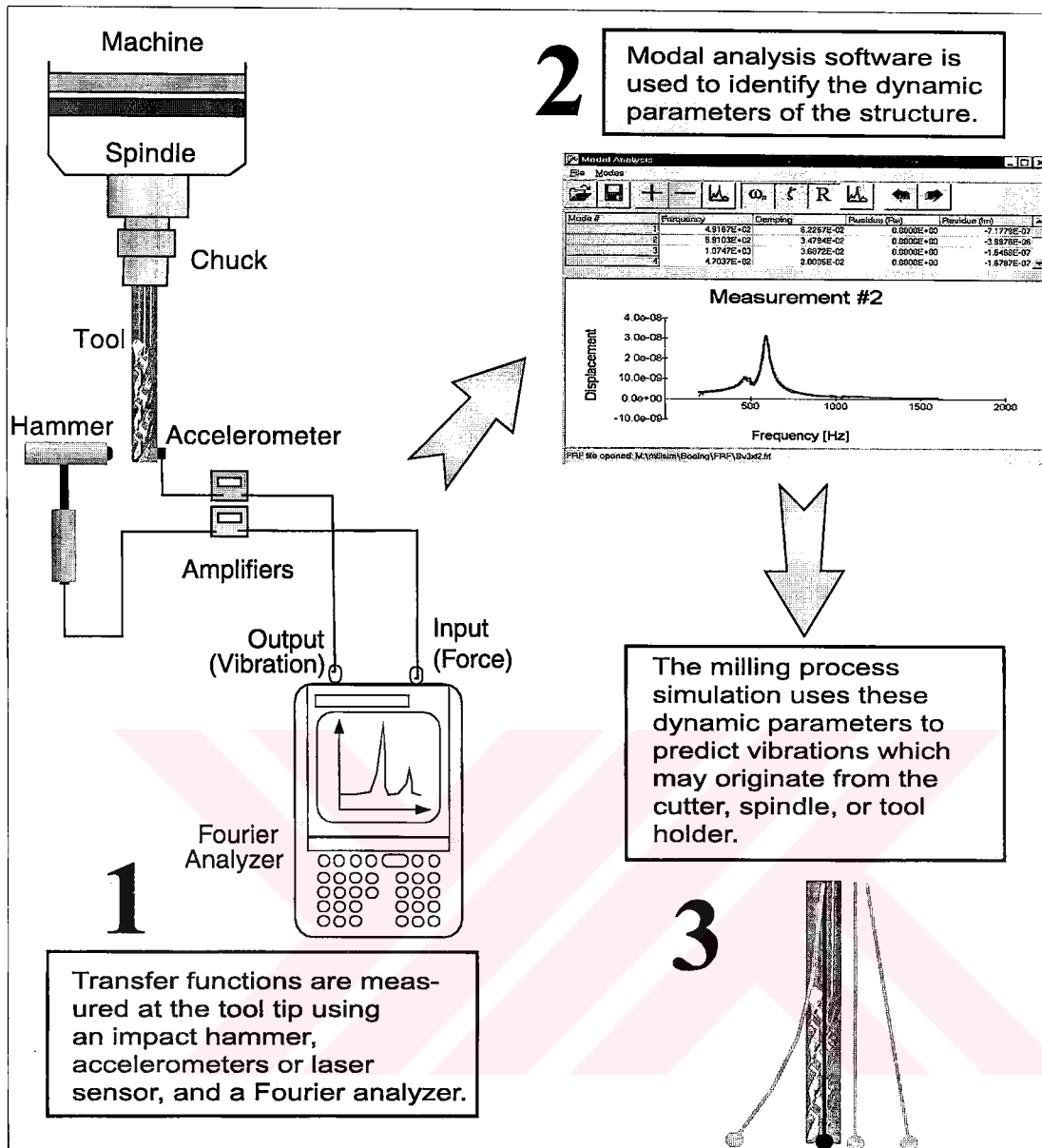


Figure A. 2. Experimental setup, illustration of transfer function and calculation steps.

Experimental setup for the cutting force measurement is shown in Figure A. 3 The cutting forces were measured using the three component Kistler dynamometer with piezoelectric force transducers. The dynamometer outputs dynamic forces in X (feed), Y (normal) and Z (axial) directions. The data were collected using IMM (Intelligent Machining Module) and Labview programs. In Labview, MalTf 2.1 data acquisition program was used. IMM and MalTf 2.1 for Labview are developed in Manufacturing Automation Laboratory. The equipment used in the measurement of cutting forces is shown in Table A. 2.

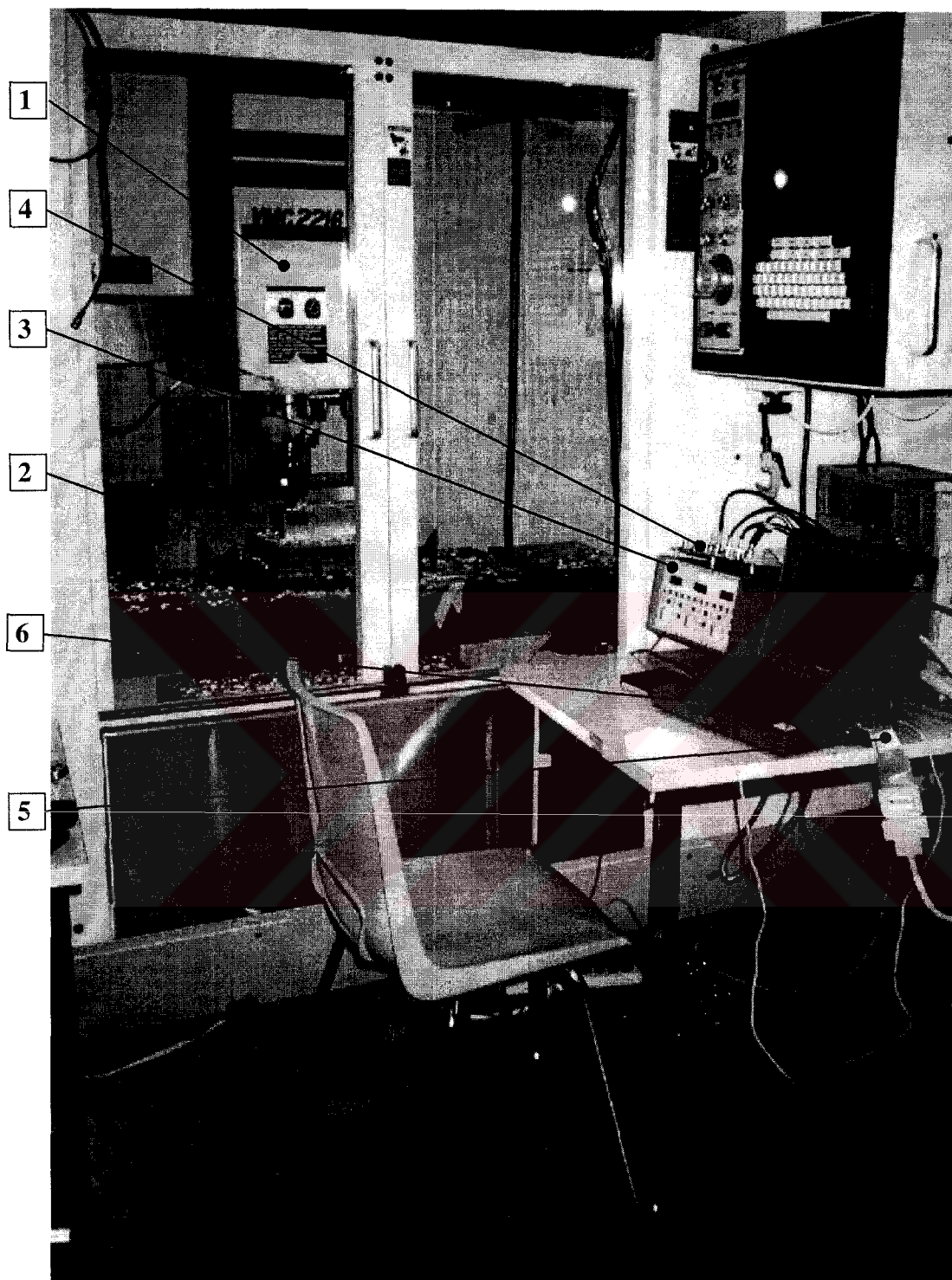


Figure A. 3. Experimental setup for the cutting forces measurements. 1- Machining center, 2- Dynamometer, 3- Charge amplifier, 4- BNC adapter board, 5- Data acquisition card, 6- Data acquisition program.

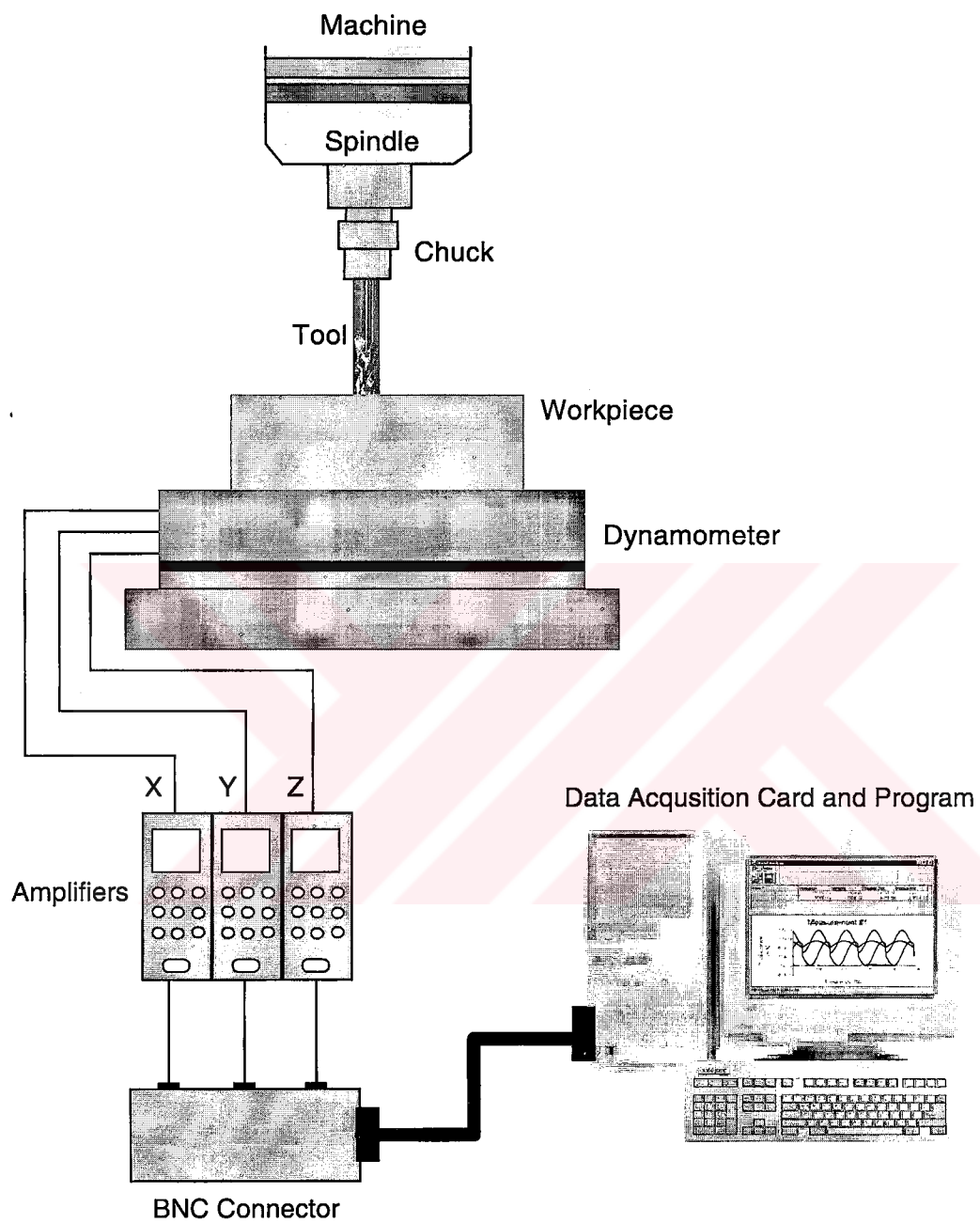


Figure A. 4. Experimental setup for the illustration of cutting forces measurements.

Table A. 2. Equipment used in measurement of cutting forces.

No	Name	Specifications and Parameters
1	Machining Center	FADAL Vertical Machining Center VMC 2216 Table Size 991X406mm Cutting Feed Rate (X/Y/Z) 2.54-6350mm/min (9525mm/min) Longitudinal (X Axis) 558.8mm Cross (Y Axis) 406.4mm Vertical (Z Axis) 508mm Main Motor 15 HP (11.2 KW) Accuracy, Axis Positioning $\pm 0.00508\text{mm}$ Accuracy, Axis Repetability $\pm 0.00254\text{mm}$ Spindle Speed 75-15,000 RPM Spindle Taper No.40 ATC, Number of Tools 21 Max.Tool Diameter 76.2mm Max. Tool Length 381mm Max. Tool Weight 6.8kg
		Superieor Electronic Vertical Milling Machine Table Size 1320X330mm Cutting Feed Rate (X/Y/Z) 0-457mm/min Longitudinal (X Axis) 711.2mm Cross (Y Axis) 304.8mm Vertical (Z Axis) 101.6mm Main Motor 7.5 HP (5.52 KW) Spindle Speed 39-1500 RPM Number of Speed 16 Spindle Taper No.50

2	Dynamometer	Dynamometer Kistler Model 9257A Serial 285906 Measuring range [N] $F_x=0\ldots 5000$, $F_y=0\ldots 5000$, $F_z=0\ldots 10000$ Sensitivity [pC/N] $F_x=-7.83$, $F_y=-7.95$, $F_z=-3.74$ Linearity [$\leq \pm$ %FSO] $F_x=0.3$, $F_y=0.3$, $F_z=0.3$ Weight 7.3 kg The top late area of 170 x 100 mm accommodates the workpiece.	
		Three Component Dynamometer 9255B Measuring range [N] $F_x=0\ldots 20000$, $F_y=0\ldots 20000$, $F_z=0\ldots 40000$ Weight 52 kg The work-holding top plate area measures 260 x 260 mm.	
3	Charge Amplifier	Charge Amplifier-Kistler Model 5010A Output voltage $\pm 10V$ Output current $\leq \pm 5mA$ Output impedance 100 Ω Bandwidth 0...100 kHz	
		Charge Amplifier-Kistler Model 5017B Multi channel Charge Amplifier 3 channels	
4	BNC Adapter Board	BNC connector For DSP Board	BNC Adapter Boards 2080 8 differential, 16 single-ended channel +5 VDC, 12mA, 750 mA max Dimensions: 20 by 12.4 by 4.1 cm

5	Data Acquisition Card	Spectrum DSP Digital Signal Process Board C31 and C32 4 channels input	E Series Multifunction I/O 250 kS/s, 12-Bit, 16 Analog Inputs DAQCard-AI-16E-4 16 single ended, 8 differential channels 500 kS/s single channel scanning 250 kS/s sampling rate 250 kS/s stream-to-disk rate 12 bit resolution
6	Data Acquisition Program	IMM- Intelligent Machining Module Developed in Manufacturing Automation Laboratory	Labview with MalTf 2.1 MalTf 2.1 developed in Manufacturing Automation Laboratory

The experimental setup of chatter detection is shown in Figure A. 5. The equipment used in this setup is given in Table A. 3. A number of different cutting tests were done to verify the stability lobes for the cutters. The results of these tests were given in the thesis. The analytical or time domain solutions of the stability lobes were verified with experimental cutting tests at different cutting conditions, which are axial depth of cut and spindle speed. In the cutting tests, the cutting forces, sound pressure and accelerations of the spindle and the workpiece were measured using a three component dynamometer, an acoustic microphone and accelerometers, respectively. In order to find the dominant frequency, FFT of the time domain measurements is used. If the dominant frequency at spindle or tooth passing period, there is no chatter vibrations in the system, though, there may be forced vibrations. If the dominant frequency at around one of the system modes, there is chatter vibrations in the system.

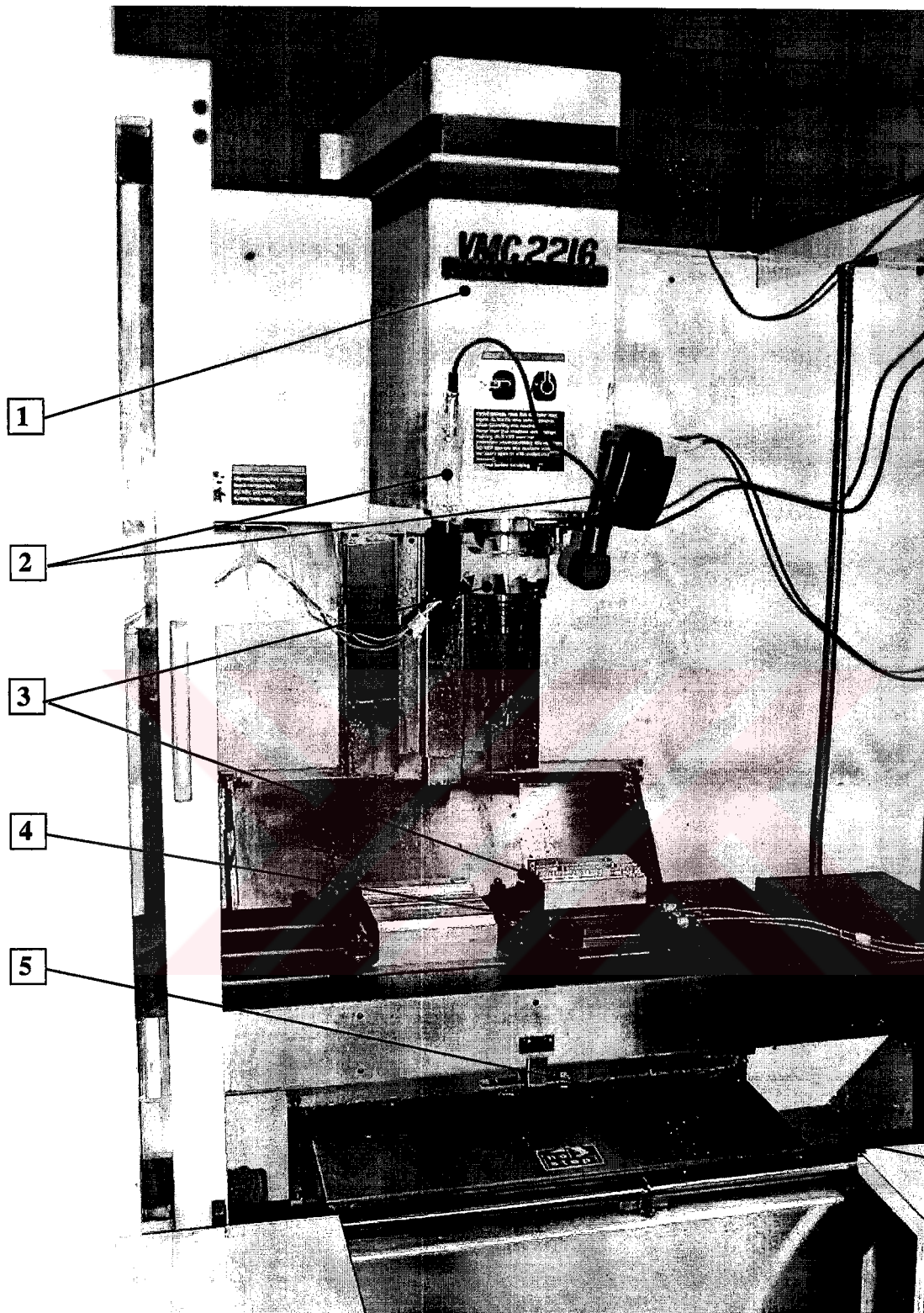


Figure A. 5. Experimental setup for chatter detection. 1- Vertical Machining Center, 2- Acoustic Microphones, 3- Accelerometers, 4- Dynamometer, 5- Data Acquisition System.

Table A. 3. Equipment used in chatter detection.

No	Name	Specifications and Parameters
1	Machining Center	See Table A. 2.
2	Acoustic Microphone	Optimus Unidirectional Condenser Microphone Microphone type Electret Condenser Frequency Response 15,000 Hz Directivity Unidirectional Impedance 600 Ohms Sensitivity -66 ± 3 dB Signal to Noise Ratio 40 dB (at 94 dB SPL) Battery type 1.5 Volt AA Type Current Drain Less Than 1 mA
3	Accelerometer	See Table A. 1
4	Dynamometer	See Table A. 2
5	Data Acquisition System	See Table A. 2

A laser surface scanner was used to scan the surface roughness. In the experiments, a number of different surface finish qualities were obtained in stable and unstable cutting conditions. In this thesis, some surface samples with different surface roughness were given for comparison between the stable and unstable cutting conditions in terms of surface quality. Small parts of the finished surfaces were cut out of workpieces in 5x1x0.5cm sizes for the laser surface scanner, which has 0.1 μm accuracy.

AUTOBIOGRAPHY

Şerafettin ENGİN was born in 1969 in Trabzon. He obtained his B.Sc. degree in Mechanical Engineering from İstanbul Technical University (İTÜ) in 1991. He then continued his graduate studies at the same university and finished his M.Sc. degree in 1994. In the same year, he joined the Ph.D. program offered at İTÜ. Two years into his Ph.D. degree, he went to The University of British Columbia in Canada to conduct the research presented in his Ph.D. thesis study. Currently, he is working as a research assistant at İTÜ, in the Materials and Manufacturing Technology Division.

