ISTANBUL TECHNICAL UNIVERSITY ★ INSTITUTE OF SCIENCE AND TECHNOLOGY

INFLATION AND NON-GAUSSIANITY PARAMETER

M.Sc. THESIS

Gülay KARAKAYA

Department of Phyiscs Engineering

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ENFLASYON VE GAUSS OLMAYAN PARAMETRE

YÜKSEK LİSANS TEZİ

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Fizik Mühendisliği

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Gülay KARAKAYA, a M.Sc. student of ITU Institute of Science and Technology 509101126 successfully defended the thesis entitled **"INFLATION AND NON-GAUSSIANITY PARAMETER"**, which he/she prepared after fulfilling the requirements specified in the associated legislations, before the jury whose signatures are below.

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To my dear parents,

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January 2014

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ABBREVIATIONS

NG	**:** Non-Gaussianity
NL	**:** Non-Linear
BD	**:** Bunch-Davies
FRW	**:** Friedmann Robertson Walker
CMB	**:** Cosmic Microwave Background
GUT	**:** Grand Unified Theory

LIST OF SYMBOLS

$(-,+,+,+)$	:	The signature of the metric
∂_μ	:	Partial Derivative
$\eta_{\mu\nu}$	:	Minkowskian Metric Tensor
∇_μ	:	Covariant Derivative
$T_{\mu\nu}$	:	Energy-Momentum Tensor
$R^{\sigma}_{\mu\nu\gamma}$	:	Riemann Tensor
$R_{\mu\nu}$	:	Ricci Tensor
R	:	Ricci Scalar
ϕ	:	Scalar Field
$\Gamma^{\gamma}_{\mu\nu}$	:	Christoffel Symbol

INFLATION AND NON-GAUSSIANITY PARAMETER

SUMMARY

Inflation is the accelerated expanding phase of the universe. For this phase the geometry is nearly de-Sitter where Hubble parameter is almost constant. Theoretically there are many mechanisms which can produce inflation. The simplest among these mechanisms are single field models of inflation, which we will concrete on.

In inflation the spectrum of primordial curvature perturbations, ζ generated during inflation. The spectrum of primordial curvature perturbations generated during inflation. Inflation paradigm means accelerating expansion driven by a scalar field(inflaton) in early universe. On the theoretical side, in constructing the realistic inflation model based on the particle physics, a lot of models have been predicted:

First of models for the standard formulation of the power spectrum of primordial curvature perturbation. It is presented a new formulation of the power spectrum of curvature perturbations generated inflation, using the fact that the Wronskian of the scalar field perturbation equation is constant.

Second of models are analysed by the non-Gaussianity of the primordial curvature perturbations generated during inflation. The non-Gaussianity is produced by Gaussian field. At early time we assume that the universe is homogeneous and isotropy. Inflation predicts gaussian statistics, the bispectrum of the perturbation, which is related three-point correlation functions, is exactly zero. Otherwise, the bispectrum has non-zero value. Recently, this non-Gaussianity(non-Linearity) of the primordial perturbations also has been a focus of constant attention all over the world.

For the case of slow roll inflation the kinetic energy should be much than the potential energy for at least 50 e-folds. There are many potentials $V(\phi)$ that satisfy this property. In a sense there is a big ambiguity of the correct theory of inflation . On the other hands CMB experiment offers us to check the predictions of different models of inflation. One important quantity is the primordial curvature perturbations,ζ generated during inflation. The two point correlation function of ζ is a measurable quantity.

Another important quantity is called the non-gaussianity parameter. Simple models of inflation predict gaussian perturbations. İt is therefore important to check this prediction with CMB data. The non-gaussianity parameter is a measure of the deviation from this prediction. Recent(WMAP,Planck) experiments put upper limits to this parameter.

Agullo and Parker (also Ganc using different techniques) showed that for slow-roll inflation with non-vacuum initial state, one can get enhanced non-gaussianity. They calculated three point correlation function using tree order mode function, u, of scaler field inflaton. Kahya, Onemli, Woodard computed the two loop correction to the mode function by interaction between spectator and ζ. In this work, we extended the calculation of Ganc using this corrected mode function. We calculated bispectrum, powerspectrum in the non-vacuum initial state. In other words we studied the quantum effects on the bispectrum in the non-vacuum initial state to put further constraints on f_{NL} parameter.

ENFLASYON VE GAUSS OLMAYAN PARAMETRE

ÖZET

Enflasyon evrenin ivmelendirilmiş genişleyen aşamasıdır. Bu aşama için geometri hubble parametresinin hemen hemen sabit olduğu yerde yaklaşık olarak de-Sitter'dır. Teorik olarak enflasyonu üretebilen çok mekanizma vardır. Bu mekanizmalar arasında en basiti bizim ilgileneceğimiz enflasyonun tek alan modelleridir.

İlkel eğrilik perturbasyonları, ζ enflasyon boyunca oluşmaktadır. Enflasyon paradigmasının anlamı evrenin ilk zamanlarında, skaler bir alanın yol açtığı ivmeli genişlemedir. Teorik açıdan, parçacık fiziğine dayalı gerçekçi bir enflasyon modeli oluşturmak için çok sayıda model öne sürülmüştür:

İlk model, ilk evren eğriliğindeki dalgalanlanmaların güç spektrumunun standart formulasyonunu içermektedir. Bu model, skaler alan perturbasyon denkleminin Wronskian'ının sabit olmasından faydalanarak enflasyon sırasında oluşan eğrilik dalgalanmalarının güç spektrumunun yeni bir formülasyonunu ortaya koymuştur.

İkinci model enflasyon sırasında oluşan ilk evren eğriliğindeki dalgalanmaların Gausyen olmayan dağılımıyla analiz edilebilir. Gauss olmayan bir gauss alanıyla üretilir.Biz ilk zamanlarda evrenin homojen ve izotropik olduğunu varsayıyoruz. Enflasyon tarafından öngörülen Gausyen dağılıma göre dalgalanmaların üçlü korelasyon fonksiyonlarına bağlı ikiz-spektrumu tam olarak sıfırdır. Aksi takdirde ikiz-spektrum sıfırdan farklı değerlere sahiptir. Son yıllarda, ilk evren dalgalanmalarının Gausyen olmayan (lineer olmayan) bu özelliği dünyanın her yerinde sürekli bir araştırma konusu olmuştur.

Yavaş yuvarlanma enflasyon durumunda evren en az e^{50} olana kadar kinetik enerji potansiyel enerjiden çok daha büyük olmalıdır. Bu özelliği sağlayan $V(\phi)$ çok potansiyelleri vardır. Bir anlamda enflasyonun doğru teorisinin büyük bir belirsizliği vardır. Diğer bir deyişle CMB deneyi enflasyonun farklı modellerinin tahminlerinin kontrol etmemiz için bize sunuyor. Bir önemli nicelikte enflasyon boyunca gelişen ilkel eğrilik pertürbasyonları, zeta'dır. Zeta'nın iki nokta korelasyon fonksiyonu ölçülebilir bir niceliktir.

Diğer önemli nicelik gauss olmayan parametre olarak adlandırılır. Enflasyonun basit modelleri gauss pertürbasyonlarını tahmin eder. Bu yüzden CMB verisiyle bu tahmini kontrol etmek önemlidir. Gauss olmayan parametre bu tahminden sapmanın bir ölçümüdür. Son günlerdeki deneyler bu parametre için üst sınır koyarlar.

Agullo ve Parker(Ganc'da farklı teknikler kullanarak) yavaş yuvarlanma koşulu durumunda, vakum olmayan başlangıç durumuyla gauss olmayan durumu arttırılabileceğini gösterdiler. Onlar skaler alan inflatonun ağaç mertebesinde mod fonksiyonu, u'yu kullanarak üç nokta korelasyon fonksiyonunu hesapladılar. Kahya, Onemli, Woodard izleyici ve zeta arasındaki etkileşimle mod fonksiyonu için iki ilmik düzeltmesini hesapladılar. Bu işte biz düzeltilmiş mod fonksiyonu kullanarak Ganc'ın hesaplamalarını devam ettirdik. Biz vakum olmayan başlangıç durumunda ikiz spektrum ve güç spektrumunu hesapladık. Diğer bir deyişle biz f_{NL} parametresi üzerinde başka

kısıtlamalar daha koymak için vakum olmayan başlangıç durumunda bispektrum üzerinde kuantum etkilerini çalıştık.

1. INTRODUCTION

According to modern cosmology the origin of the inhomogeneity of our universe is quantum fluctuations. Then these fluctuations become classical the evolution of our space and time. These fluctuations are the seeds of throught the CMB radiation and complex structures; such as galaxies and stars [18].

A theory of early universe, the cosmic inflation gives a mechanism to produce inhomogeneity in the universe. It makes specific testable predictions for the global structure and the inhomogeneity in the universe. The predictions may be summarized as follows [18]

(a)The universe which is observable, is spatially flat.

(b)On large angular scales, the observable universe is homogeneous and isotropic, apart from tiny fluctuations.

(c)The primordial inhomogeneity is the scale-invariant.

(d)Statistics of the primordial homogeneity is Gaussian statistics.

The most robust evidence of inflation comes from the measured angular distribution ($\frac{\Delta T}{T}$) of CMB.

Since non-gaussianity has infinite degrees of freedom, testing a Gaussian hypothesis is difficult; One statistical method showing CMB, which is consistent with Gaussian does not mean CMB being really Gaussian. In this sense, we see that different statistical methods can bring different results.

Inflation also produces fluctuations in the energy density of the early universe. We observe adiabatic fluctuations which look the same in the different components of the universe(e.g. dark matter, photons). We can parametrize them with a single quantity ζ. Fluctuations are from random field, which we characterize statistically. The measured quantity is consistent with a Gaussian field. The value of two point correlation function of $\zeta - \zeta$ can be obtained from observations of [8]

A Gaussian field is characterized completely by its power spectrum. The power spectrum is also proportional the Fourier components of the two point function [6].

A non-Gaussian field has statistical information beyond the power spectrum. Measuring the non-Gaussianity(e.g. the bispectrum is a powerful probe of inflation). Non-gaussianity is related to the three point(bispectrum) or higher-point correlation functions in the primordial curvature perturbation [19].

Experimental limits on non-Gaussianity are generically given in terms of a scalar variable f_{NL}. In the limit of small deviation, from de-Sitter the consistency relation predicts that the value of f_{NL} should be of order of the slow-roll parameter $\mathscr{O}(1/100)$ in the certain limit. At the same time, the consistency relation relates a particular geometrical limit of the 3-point function of density perturbations to the spectrum and tilt of the 2-point function. In order to rule out the suggested consistency relation, data should be able to characterize the dependence of the 3-point function on the shape of the triangle in momentum space to isolate the limit $k_1 << k_2, k_3$. The three point function can be expressed in terms of two point correlation functions. In the limit $k_1 \to 0$ the sub-leading corrections are dominated by $(k_3/k_1)^2$ for $k_3 << k_1, k_2$ [18].

This thesis has the following structure: chapter2 gives an introduction to standard cosmology. In the next chapter, we explained inflation, the inflationary scenarios and the problem of initial conditions. At chapter 4, we explained the quantization of the scalar field and power spectrum. At chapter 5, we computed f_{NL} parameter for slow-roll inflation with BD-vacuum state, bispectrum and power spectrum with non-BD vacuum state, and our conclusions comprise chapter 6.

2. INTRODUCTION TO COSMOLOGY

General Relativity is Einstein's theory of space, time and gravitation. Thanks to General Relativity, we can understand the relationship between curvature, gravity and space and time [1].

2.1 General Relativity

2.1.1 The Equivalence Principle and The Metric

General relativity and the Metric is based on the Principle of the Equivalence, Gravitation and Inertia. The Equivalence Principle tells us that for any space-time geometry in an arbitrary gravitational field, there exist a locally inertial coordinate system in which the effects of gravitation can be neglected in a small space-time neighbourhood of that point. If we know the equations in the absence of gravitation, we can write the equations for any small physical system in a gravitational field [3].

To define a space in the form of linear vector spaces, it is necessary to define the distance between any two points. It is called the metric.

This infinitesimal space and time interval can be written as

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu \tag{2.1}$$

where $\eta_{\mu\nu}$ is called metric tensor and x^μ stands for space and time coordinates(μ=0,1,2,3).

The metric tensor is a symmetric tensor which describes the geometry of space and time($\eta_{\mu\nu} = \eta_{\nu\mu}$). The inverse of the metric tensor, $\eta^{\mu\nu}$ is defined as

$$\eta^{\mu\nu}\eta_{\nu\sigma} = \eta_{\lambda\sigma}\eta^{\lambda\mu} = \delta^\mu_\sigma \tag{2.2}$$

When we make a coordinate transformation from x^μ to $x^{\mu\prime}$, the metric tensor takes the following form,

$$\eta'_{\rho\sigma}(x') = \eta_{\mu\nu}(x)\frac{\partial x^\mu}{\partial x'^\rho}\frac{\partial x^\nu}{\partial x'^\sigma} \tag{2.3}$$

Similarly the coordinate differentials have the obvious transformation property,

$$dx'^{\rho} = \frac{\partial x'^{\rho}}{\partial x^{\mu}} dx^{\mu} \tag{2.4}$$

so that

$$\eta'_{\rho\sigma}(x')dx'^{\rho}dx'^{\sigma} = \eta^{\mu\nu}(x)\frac{\partial x^{\mu}}{\partial x'^{\rho}}\frac{\partial x^{\nu}}{\partial x'^{\sigma}}\frac{\partial x'^{\rho}}{\partial x^{k}}dx^{k}.\frac{\partial x'^{\sigma}}{\partial x'^{\lambda}}dx^{\lambda} \tag{2.5}$$

$$\eta'_{\rho\sigma}(x')dx'^{\rho}dx'^{\sigma} = \eta_{\mu\nu}(x)dx^{\mu}dx^{\nu} \tag{2.6}$$

$$ds^2 = ds'^2 \tag{2.7}$$

This is called the equivalence of infinitesimal space-time interval.

2.1.2 Tensors, vectors, scalars

There are two kinds of vectors; Contravariant and covariant vectors. They differ from each-other based on the transformation rules. Contravariant vectors are denoted with upper indices and are transformed according to following equation,

$$A'^{\mu}(x') = \frac{\partial x'^{\mu}}{\partial x^{\nu}} A^{\nu}(x) \tag{2.8}$$

and covariant vectors are denoted with lower indices and are transformed according to following equation,

$$A'_{\mu}(x') = \frac{\partial x^{\nu}}{\partial x'^{\mu}} A_{\nu}(x) \tag{2.9}$$

The covariant derivative of a vector field V^{ν} is given by

$$\nabla_{\mu}V^{\nu} = \partial_{\mu}V^{\nu} + \Gamma^{\nu}_{\mu\sigma}V^{\sigma} \tag{2.10}$$

A scalar S(x) is a quantity whose value at a physical space time point is unchanged by a coordinate transformation.

So we can write as

$$S'(x') = S(x) \tag{2.11}$$

The derivative $V'_{\mu} = \partial S/\partial x^{\mu}$ of a scalar $S(x)$ is a covariant vector:

$$V_{\rho'} = \frac{\partial S}{\partial x^{\rho'}} = \frac{\partial S}{\partial x^{\mu}}\frac{\partial x^{\mu}}{\partial x'^{\rho}} = V_{\mu}\frac{\partial x^{\mu}}{\partial x'^{\rho}} \tag{2.12}$$

There are other physical quantities which have more complicated transformation properties. These are called tensors. Tensors can have both contravariant and covariant indices. To give an example a third rank tensor under Lorentz transformations can be written as

$$T^{\gamma}_{\alpha\beta} \rightarrow T'^{\gamma}_{\alpha\beta} = \Lambda^{\gamma}_{\delta}\Lambda^{\varepsilon}_{\alpha}\Lambda^{\zeta}_{\beta}T^{\delta}_{\varepsilon\zeta} \tag{2.13}$$

where Λ is Lorentz transformation matrix.

A contravariant or a covariant vector can be thought as a tensor with one index, and a scalar with no indices. There is a unique connection which gives us a way of relating vectors in the tangent spaces of nearby points that we can construct from the metric and it is called the Christoffel symbol [3],

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2}g^{\lambda\rho}(\partial_{\mu}g_{\rho\nu} + \partial_{\nu}g_{\mu\rho} - \partial_{\rho}g_{\mu\nu}) \tag{2.14}$$

The covariant derivative of a tensor of arbitrary rank is

$$\begin{aligned}\nabla_{\sigma}T^{\mu_1\mu_2\ldots\mu_k}_{\nu_1\nu_2\ldots\nu_\ell} &= \partial_{\sigma}T^{\mu_1\mu_2\ldots\mu_k}_{\nu_1\nu_2\ldots\nu_\ell} + \Gamma^{\mu_1}_{\sigma\lambda}T^{\lambda\mu_2\ldots\mu_k}_{\nu_1\nu_2\ldots\nu_\ell} + \Gamma^{\mu_2}_{\sigma\lambda}T^{\mu_1\lambda\ldots\mu_k}_{\nu_1\nu_2\ldots\nu_\ell} + \ldots \\ &\quad -\Gamma^{\lambda}_{\sigma\nu_1}T^{\mu_1\mu_2\ldots\mu_k}_{\lambda\nu_2\ldots\nu_\ell} - \Gamma^{\lambda}_{\sigma\nu_2}T^{\mu_1\mu_2\ldots\mu_k}_{\nu_1\lambda\ldots\nu_\ell} - \ldots\end{aligned} \tag{2.15}$$

The response of matter to spacetime curvature is easy to understand: Free particles move along paths of geodesics which are the shortest distances in curved geometries. The geodesics equation is

$$\frac{d^2X^{\mu}}{d\lambda^2} + \Gamma^{\mu}_{\rho\sigma}\frac{dX^{\rho}}{d\lambda}\frac{dX^{\sigma}}{d\lambda} = 0 \tag{2.16}$$

where $X^{\mu}(\lambda)$ is a parametrized curve, λ is an affine parameter.

2.1.2.1 The Energy-Momentum Tensor

We define the energy-momentum tensor (sometimes called the stress-energy tensor), $T^{\mu\nu}$ to describe the energy and momentum of a fluid. This tensor is a symmetric(2,0) tensor. $T^{\mu\nu}$ is the flux of P^{μ} across a surface of constant [1] X^{ν} (μ , ν = 0 ,1 ,2 ,3) where P^{μ} is the momentum four-vector:

$$P^{\mu} = \frac{dX^{\mu}}{d\lambda} \tag{2.17}$$

and X^{ν} is any global inertial coordinate system.

Let's look the components of the energy-momentum tensor to understand what they mean. T^{00} is the flux of P^0(energy) in the X^0(time) direction or simply the energy density, ρ.

Similarly, $T^{0i} = T^{i0}$ is the momentum density and T^{ij} are the momentum flux or the stress (i,j components represent the spatial components); they stand for the forces between neighbouring infinitesimal elements of the fluid [1].

In the absence of gravitation any set of particles and/or fields will have energy-momentum tensor $T^{\mu\nu}$, which has property of being conserved,

$$\frac{\partial T^{\mu\nu}}{\partial X^{\nu}} = 0 \tag{2.18}$$

In the presence of a gravitational field, the conservation law is,

$$T^{\mu\nu}_{;\nu} = \frac{\partial T^{\mu\nu}}{\partial X^{\nu}} + \Gamma^{\mu}_{\kappa\nu} T^{\kappa\nu} + \Gamma^{\nu}_{\kappa\nu} T^{\mu\kappa} \tag{2.19}$$

Let's look the energy-momentum tensor for the perfect fluid. If a comoving observer sees the fluid around him as isotropic in all directions, this fluid is called a perfect fluid. There is no heat conduction ($T^{0i} = T_{i0}$) and no viscosity($T^{ij} = p$) [3].

$$T^{ij} = \delta_{ij} p, T^{i0} = T^{0i} = 0, T^{00} = \rho \tag{2.20}$$

where i and j take the values 1, 2, 3. The coefficients p and ρ are the pressure and energy density. The general form of a perfect fluid on a curved background have the following form,

$$T^{\mu\nu} = (p+\rho) U^{\mu} U^{\nu} + p g^{\mu\nu} \tag{2.21}$$

2.1.3 Classical Field Theory

General Relativity is a particular example of a classical field theory, because the metric $g_{\mu\nu}(x)$ is a dynamical field.

In field theory, we use space-time dependent fields $\phi(x^{\mu})$ and the action S becomes a functional of these fields. A functional is simply function of an infinite number of variables [1].

In classical mechanics the action S is the time integral of the Lagrangian, L. But in a field theory, the Lagrangian can be expressed as the spatial integral of a Lagrangian density which is denoted by $\mathscr{L}$. A Lagrangian density is a function of one or more fields $\phi(x)$ and their derivatives $\partial_{\mu}\phi$ [4]. So the action takes the form,

$$S = \int L dt = \int \mathscr{L}(\phi, \partial_{\mu}\phi) d^4x \tag{2.22}$$

We determine the equations of motion by the principle of least action i.e $\delta S = 0$, keeping the end points fixed,

$$\begin{aligned}
\delta S &= \int \left[\frac{\partial \mathscr{L}}{\partial \phi} \delta\phi + \frac{\partial \mathscr{L}}{\partial(\partial_\mu \phi)} \delta(\partial_\mu \phi) \right] d^4x \\
&= \int \left[\frac{\partial \mathscr{L}}{\partial \phi} - \partial\mu \Big(\frac{\partial \mathscr{L}}{\partial(\partial_\mu \phi)} \Big) \delta\phi + \partial\mu \Big(\frac{\partial \mathscr{L}}{\partial(\partial_\mu \phi)} \Big) \delta\phi \right] d^4x \\
&= 0. \qquad \textbf{(2.23)}
\end{aligned}$$

The boundary term is zero for any variation $\delta\phi(x,t)$ that decay suitably fast at $x \to \infty$, and obeys

$$\delta\phi(x,t_i) = \delta\phi(x,t_f) = 0. \qquad \textbf{(2.24)}$$

Then $\delta S = 0$ gives us the following field equation,

$$\partial_\mu \Big(\frac{\partial \mathscr{L}}{\partial(\partial_\mu \phi))} \Big) - \frac{\partial \mathscr{L}}{\partial \phi} = 0. \qquad \textbf{(2.25)}$$

These are called Euler-Lagrange Equations.

2.1.4 The gravitational field equations

Einstein equation governs the response of space-time curvature to the presence of matter and energy. Einstein equation can be written mathematically as

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}, \qquad \textbf{(2.26)}$$

where $T_{\mu\nu}$ is the energy-momentum tensor, $R_{\mu\nu}$ is the Ricci tensor, G is the Newton's constant, R is the Ricci scalar and $g_{\mu\nu}$ is the metric tensor. Here we have set $c = 1$. While the expression on the left hand side of the Einstein equation is a measure of the curvature of space time, the right hand side measures the energy and momentum of matter [1].

The Ricci tensor is defined as the part of the curvature of space-time that determines the degree to which matter will tend to converge or diverge in time. This is a symmetric tensor and we can define it mathematically as

$$R_{\mu\nu} = \frac{\partial \Gamma^\lambda_{\mu\lambda}}{\partial x^\nu} - \frac{\partial \Gamma^\lambda_{\mu\nu}}{\partial x^\lambda} + \Gamma^\kappa_{\mu\lambda}\Gamma^\lambda_{\nu\kappa} - \Gamma^\kappa_{\mu\nu}\Gamma^\lambda_{\lambda\kappa}. \qquad \textbf{(2.27)}$$

$$R_{\mu\nu} = R^\lambda_{\mu\lambda\nu} \qquad \textbf{(2.28)}$$

Here $R^{\lambda}_{\mu\lambda\nu}$ is called the Riemann tensor and it is defined as

$$R^{\rho}_{\sigma\mu\nu} = \partial_{\mu}\Gamma^{\rho}_{\nu\sigma} - \partial_{\nu}\Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}. \tag{2.29}$$

All we need to know about the curvature of a manifold is given by the Riemann tensor; it is zero for flat geometry($R^{\rho}_{\sigma\mu\nu} = 0$).

The Ricci scalar(the scalar curvature) is defined as the trace of the Ricci tensor.It is expressed as

$$R = g^{\mu\nu}R_{\mu\nu} = R^{\mu}_{\mu} = R^{\nu}_{\nu}. \tag{2.30}$$

The Newtonian limit of the gravitational field equation is Poisson equation. Poisson equation is written as

$$\nabla^2\Phi = 4\pi G T^{00} = 4\pi G\rho \tag{2.31}$$

where G is Newton's constant, Φ is the Newtonian potential and ρ is the energy density.

Solving Poisson equation for potential is necessary to know the charge density distribution. If the charge density is zero, the Poisson equation transforms the Laplace equation. If the charge density corresponds to Boltzmann distribution, the equation becomes Poisson -Boltzmann equation.

2.1.5 Friedman Robertson Walker Metric and Friedman Equations

The universe in which we live obeys to the certain principles which are related to the actual values of the cosmological parameters. The universe is defined as homogeneous and isotropic around us on scales of larger than 100 million light years. According to modern cosmology, the universe looks the same at all direction in space at any point which is called Cosmological Principle [5].

Our space-time can be represent as $Rx\Sigma$, where R represents the time direction and Σ is a maximally symmetric three manifold. The space-time metric can be written as the following form

$$ds^2 = -dt^2 + a(t)^2 d\sigma^2 \tag{2.32}$$

where t is time coordinate, a(t) is the scale factor, spatial part of the metric is

$$d\sigma^2 = \gamma_{ij}(u)du^i du^j \tag{2.33}$$

where u^i are spatial coordinates (i=1,2,3) and γ_{ij} is a maximally symmetric three dimensional metric.

The metric of a maximally symmetric space can be written as,

$$d\sigma^2 = \gamma_{ij}(u)du^i du^j = e^{2\beta r}dr^2 + r^2 d\Omega^2. \quad \textbf{(2.34)}$$

Here r is the radial coordinate and the metric on the two-sphere is

$$d\Omega^2 = d\theta^2 + sin^2\theta d\phi^2, \quad \textbf{(2.35)}$$

where

$$\beta = -\frac{1}{2}In(1-kr^2) \quad \textbf{(2.36)}$$

With this coordinate choice, the metric takes the following form,

$$ds^2 = -dt^2 + a^2(t)\Big[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\Big] \quad \textbf{(2.37)}$$

and is called FRW metric.

The k parameter takes three different values:-1, 0 or 1. The universe with $k=-1$ is called an open universe. $k=0$ corresponds to flat universe and $k=1$ corresponds to closed universe.

We will use perfect fluid model to describe matter and energy. The energy-momentum tensor of a perfect fluid is

$$T^{\mu\nu} = (\rho + p)U^\mu U^\nu + pg_{\mu\nu} \quad \textbf{(2.38)}$$

where U^μ is the four velocity. In comoving frame, the four velocity takes the following form,

$$U^\mu = (1,0,0,0) \quad \textbf{(2.39)}$$

Let's consider the zero component of the conservation of energy equation:

$$0 = \nabla_\mu T^\mu_0 = -\partial_0\rho - 3\frac{\dot{a}}{a}(\rho + p) \quad \textbf{(2.40)}$$

The equation of state is defined as

$$p = w\rho \quad \textbf{(2.41)}$$

where w is a generally constant for a single fluid.

Using the equation(2.40) and (2.41), the conservation of energy equation takes the following form,

$$\frac{\dot{\rho}}{\rho} = -3(1+w)\frac{\dot{a}}{a}. \quad \textbf{(2.42)}$$

Solution of this equation

$$\rho \propto a^{-3(1+w)}. \quad \textbf{(2.43)}$$

The energy density of a universe which is dominated by matter, is proportional to a^{-3} .Similarly, universes whose energy density is dominated by radiation, has the energy density is proportional to a^{-4}. For a universe dominated by cosmological constant the energy density becomes constant [1].

The Einstein equation is for $\mu = 0$ and $\nu = 0$ takes the following form,

$$-3\frac{\ddot{a}}{a} = 4\pi G(\rho + 3p). \quad \textbf{(2.44)}$$

And for $\mu = i$ and $\nu = j$, the equation becomes,

$$\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{k}{a^2} = 4\pi G(\rho - p). \quad \textbf{(2.45)}$$

Using the last two equations, one can get

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \quad \textbf{(2.46)}$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p). \quad \textbf{(2.47)}$$

These are called the Friedman equations.

The rate expansion of our universe is characterized by the Hubble parameter,

$$H = \frac{\dot{a}}{a} \quad \textbf{(2.48)}$$

At the present epoch, the value of the Hubble parameter has the following value, H_0. We parametrize the Hubble constant as [22]

$$H_0 = 67.11 km/sec/Mpc. \quad \textbf{(2.49)}$$

3. INFLATION

3.1 Inflation

Inflation is known as the accelerated expansion of our universe. We know that matter is distributed homogeneously and isotropically on scales larger than a few hundred Mpc. The universe evolves according to the Hubble law. The fractional amplitude of density perturbations $\delta\varepsilon/\varepsilon$ is nearly equal to 10^{-5}. We think that inflation begins at energy levels scales closed to GUT scale and explains all the observations we have just mentioned above.

3.1.1 Problem of initial conditions

Let's look at the initial conditions characterizing matter. There are two things that we have to know about initial conditions. First, how the matter is distributed in space, i.e. its location, and how they are moving, i.e. its initial field of velocities. Inflation is proposed to solve initial condition problems. The first of these problems is called the horizon problem.

Homogeneity, isotropy (horizon) problem: The present homogeneous,isotropic domain of the universe is at least large as the present horizon scale, $ct_0 \sim 10^{28}$cm. We assume that the inhomogeneity can not be solved by expansion, so we may reach this result that the size of the homogeneous, isotropic region from which our universe originated at $t = t_i$ was larger than

$$l_i \sim ct_0 \frac{a_i}{a_0}. \tag{3.1}$$

when we compare this scale to the size of causal region $l_c \sim ct_i$, it gives

$$\frac{l_i}{l_c} \sim \frac{t_0}{t_i}\frac{a_i}{a_0}. \tag{3.2}$$

If the primordial radiation dominates at $t_i{\sim}t_{Pl}$, its temperature is at the order of $10^{32}K$. Therefore

$$\frac{a_i}{a_0}{\sim}\frac{T_0}{T_{pl}} \sim 10^{-32} \tag{3.3}$$

and we find

$$\frac{l_i}{l_c} \sim \frac{10^{17}}{10^{-43}} 10^{-32} \sim 10^{28}. \tag{3.4}$$

At the initial planckian time, with a fractional variation not exceeding $\delta\varepsilon/\varepsilon{\sim}10^{-5}$ there are 10^{84} causally disconnected regions which energy density was smoothly disturbed. Because no signals can propagate faster than light, causal physical processes can't be responsible for such a not natural fine-tuned matter distribution [6].

Supposing that the scale factor grows as some power of time, we can guess $\frac{a}{t}{\sim}\dot{a}$, then we can write as,

$$\frac{l_i}{l_c}{\sim}\frac{t_0}{t_i}\frac{a_i}{a_0}{\sim}\frac{\dot{a}_i}{\dot{a}_0} \tag{3.5}$$

where l_c is the size of a causal region, l_i is the size of the early universe. Assuming that gravity was always attractive, we can write as the following form,

$$\frac{l_i}{l_c}{\sim}\frac{\dot{a}_i}{\dot{a}_0} > 1. \tag{3.6}$$

So the universe was decelerating expansion. We can always say that the homogeneity scale was larger than the scale of causality. For this reason we can also use the horizon problem instead of the homogeneity problem.

Flatness problem: We observed that the universe is almost flat. We described this flatness by the cosmological parameter($\Omega(t)$)which tells us if our universe has energy density closed to the critical energy.Its definition is ,

$$\Omega(t) \equiv \frac{\varepsilon(t)}{\varepsilon^{cr}(t)} \tag{3.7}$$

where

$$\Omega^{cr} = \frac{3H^2}{8\pi G}. \tag{3.8}$$

Using the definition of $\omega(t)$ friedmann equation can be written as

$$\Omega(t) - 1 = \frac{k}{(Ha)^2}. \tag{3.9}$$

The initial value of this parameter can be written in terms of its present value is

$$\Omega_i - 1 = (\Omega_0 - 1)\frac{(Ha)_0^2}{(Ha)_i^2} \leq 10^{-56}. \tag{3.10}$$

One can see from this equation, Ω should be extremely closed to one, corresponding to a flat universe [6].

The origin of all the problems that we discussed is related to the difference between initial velocity and present velocity of our universe. If the initial velocity has a value bigger than its present value, we can never solve these problems. Since gravity is attractive for non-particles, it is natural to expected the value of the velocity to decrease. One obvious of this problem is to make gravity repulsive.

3.1.2 How can gravity become "repulsive"?

To look how gravity becomes repulsive, we should analyse the Friedmann equation which is given by

$$\ddot{a} = -\frac{4\pi G}{3}(\varepsilon + 3p)a. \tag{3.11}$$

If $\varepsilon + 3p$ is bigger than zero, $\ddot{a}$ becomes negative. It means that gravity decelerates the expansion. When $\varepsilon + 3p$ is smaller than zero, the universe is accelerated.

Inflation can solve the initial conditions problem only if $t_f > 75H_i^{-1}$, since

$$\frac{a_f}{a_i} \sim exp(H_i t_f) \sim exp(\frac{H_i^2}{\dot{H}_i}) > 10^{33}. \tag{3.12}$$

Therefore during inflation the universe must grow up at least 75 e-folds.

3.1.3 How to realize the equation of state $p \approx -\varepsilon$

Inflaton is a scalar field which drives inflation. For a scalar field we show that it can be described as an ideal fluid. So for a scalar field, the energy-momentum tensor is given by the expression

$$T^{\alpha}_{\beta} = \varphi^{,\alpha}\varphi_{,\beta} - \left[\frac{1}{2}\varphi^{,\gamma}\varphi_{,\gamma} - V(\varphi)\right]\delta^{\alpha}_{\beta} \tag{3.13}$$

where

$$\varphi_{,\beta} \equiv \frac{\partial\varphi}{\partial X^{\beta}}, \varphi^{,\alpha} \equiv g^{\alpha\gamma}\varphi_{,\gamma} \tag{3.14}$$

We know that for a perfect fluid the energy-momentum tensor is

$$T^{\alpha}_{\beta} = (\varepsilon + p)U^{\alpha}U_{\beta} - p\delta^{\alpha}_{\beta} \tag{3.15}$$

where the equation of state $p = p(\varepsilon)$, U^{α} is the four-velocity. For the homogeneous classical field the energy density is

$$\varepsilon = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) \tag{3.16}$$

and the pressure is

$$p = \frac{1}{2}\dot{\varphi}^2 - V(\varphi). \quad \textbf{(3.17)}$$

For this scalar field obeys the Klein-Gordon equation,

$$\ddot{\varphi} + 3H\dot{\varphi} + V_{,\varphi} = 0 \quad \textbf{(3.18)}$$

where

$$V_{,\varphi} \equiv \frac{\partial V}{\partial \varphi}. \quad \textbf{(3.19)}$$

This equation has the same form of the equation for a harmonic oscillator with a friction term proportional to the Hubble parameter H. In order to get the equation of state $p \approx -\varepsilon$ the potential term in (3.18) and (3.19) should be much bigger than the kinetic energy term. Therefore the velocity of the scaler field should stay small for a long time(at least for 75 e-folds). This means that the friction term in (3.18) is much bigger than the acceleration term; i.e $\ddot{\varphi} << 3H\dot{\varphi}$. This is known as slow-roll approximation [6]. The slow-roll conditions are

$$|\dot{\varphi}^2| << V, \qquad |\ddot{\varphi}| << 3H\dot{\varphi} \sim |V_{,\varphi}|. \quad \textbf{(3.20)}$$

The above slow-roll conditions are recast in terms of requirements on the derivatives of the potential. These are

$$\left(\frac{V_{,\varphi}}{V}\right)^2 << 1, \qquad \left(\frac{V_{,\varphi\varphi}}{V}\right) << 1. \quad \textbf{(3.21)}$$

3.1.4 "Menu" of scenaros

We require a scalar condensate satisfying the slow-roll conditions for successful inflation. We can involve two or more scalar condensates, we are assumed to be equally relevant during inflation. It increases the number of possibilities, at the same time decreases the predictive power of inflation. [6].

Although many of these models are successfully the initial condition problem, they predict very close numbers for some observations, e.g. CMB power spectrum. Because of this cosmologists have to perform very careful experiments which should be sensitive enough to tell us the correct theory of inflation.

Let us give an recent example [24];

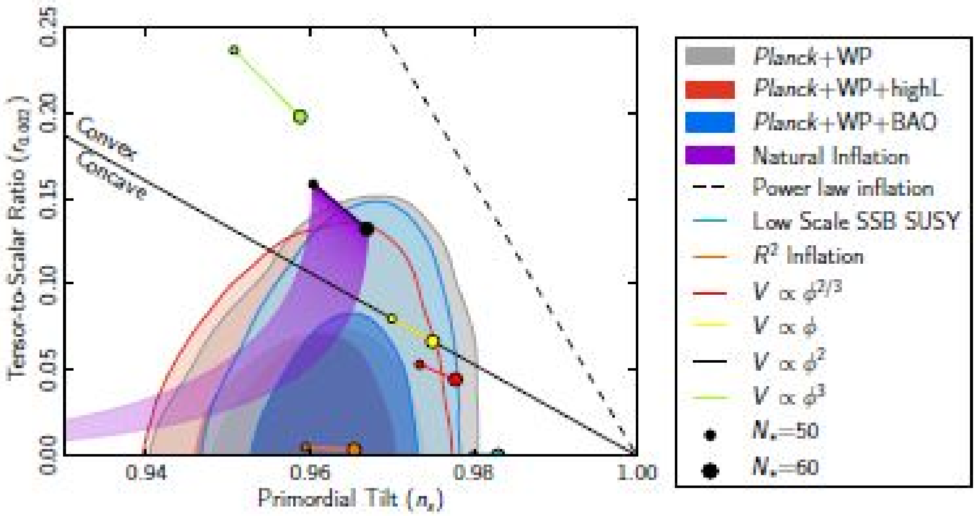

Figure 3.1: Marginalized joint %68 and %95 CL regions for n_s and r_{002} from Planck in combination with other data sets compared to the theoretical predictions of selected inflationary models.

We also need a graceful exit from inflation. After that with the help of reheating mechanism the inflaton field should decay to radiation. The reheating mechanism still not understood in all details. The main reason of this is the need for a correct theory beyond standard model. One can see from the figure of the planck paper that this intolerance coming from reheating is represented with lines on this figure.

4. QUANTIZITION OF THE SCALAR FIELD

4.1 Quantization of the scalar field

One of the central issues of contemporary cosmology is to explain the origin of primordial inhomogeneities and to understand their observed spectrum. Inflation predicts the inflationary cosmology proposed the origin of primordial inhomogeneities is quantum fluctuations and inflation also predict the shape of the spectrum of density perturbations [6].

During the period of inflation there are quantum fields which oscillate. The question is how do this small quantum fields evolve during inflation so that they become stars and galaxies. To answer this question we must investigate the quantum fluctuations. These fluctuations have significant amplitudes only on scales close to the Planckian length. They are extended to galactic scales with nearly unchanged amplitudes. To investigate quantum fluctuations, we need to have quantization on the fields.

4.1.1 Characterizing perturbations

Small inhomogeneities can be characterized by the energy density fluctuations $\delta\varepsilon/\varepsilon_0$ or by the spatial distribution of the gravitational potential. We can describe these fluctuations with random fields. The particular configuration a random field effects, subdivides an infinite universe into a set of large spatial regions. If a given configuration $f(x)$ exists, it can be described by a probability distribution function [6].

We can use Fourier methods to describe the random process. In a given region of volume V, the Fourier expansion of the function $f(x)$ is expressed as

$$f(x) = \frac{1}{\sqrt{V}} \sum_k f_k e^{ikx}. \quad \textbf{(4.1)}$$

The complex coefficients f_k are $f_k = a_k + ib_k$ and have dimension $cm^{3/2}$. Since $f(x)$ is a real function, it is required to have $f_{-k} = f_k^*$. In this case the real and imaginary parts of f_k must satisfy the constraints $a_{-k} = a_k$ and $b_{-k} = -b_k$. In different spatial regions, the values of coefficients a_k and b_k are changing.

Given a very large number N of such regions with the definition of the probability distribution functions $p(a_k,b_k)$,

$$dN = Np(a'_k,b'_k)da_k db_k \quad \textbf{(4.2)}$$

where the value of a_k lie between a'_k and $a'_k + da_k$ and the value of b_k lie between b'_k and $b'_k + db_k$. Inflation predicts only homogeneous and isotropic Gaussian processes. So the probability distribution function is

$$p(a_k,b_k) = \frac{1}{\pi\sigma_k^2} e^{-\frac{a_k^2}{\sigma_k^2}} e^{-\frac{b_k^2}{\sigma_k^2}} \quad \textbf{(4.3)}$$

where the variance depends only on $k = |k|$; The independent variables a_k and b_k are equal to $\sigma_k^2/2$.

One can calculate the expectation value of the product of continuous Fourier coefficients

$$< f_k f_{k'} >= \sigma_k^2 \delta(k+k') \quad \textbf{(4.4)}$$

where

$$f(x) = \int f_k e^{ikx} \frac{d^3k}{(2\pi)^{\frac{3}{2}}}. \quad \textbf{(4.5)}$$

A gaussian field can be defined by the spatial two point correlation function

$$\xi_f(x-y) \equiv < f(x) f(y) > . \quad \textbf{(4.6)}$$

This function is major of the magnitude of the field fluctuations. In an homogeneous and isotropic universe the two point correlation function only depends on the distance between the points x and y ($\xi_f = \xi_f(|x-y|)$).

Substituting (4.5) into (4.6) and averaging over the ensembles, we obtain that

$$\xi_f(|x-y|) = \int \frac{\sigma_k^2 k^3}{2\pi^2} \frac{sin(kr)}{kr} \frac{dk}{k} \quad \textbf{(4.7)}$$

where $r \equiv |x-y|$. In this equation we define the dimensionless variance as

$$\delta_f^2(k) \equiv \frac{\sigma_k^2 k^3}{2\pi^2}. \quad \textbf{(4.8)}$$

This variance expresses as the typical squared amplitude of the fluctuation on scales $\lambda \sim 1/k$.

Fourier modes evolve independently for small perturbations. When the non-linear regime starts to occur, different Fourier modes start to interact which is very complicated to analyse.

We look at the initial perturbation spectrum which is generated during inflation. When we take $\sigma_k^2 \equiv |\phi_k|^2$,the dimensionless variance has the following form,

$$\delta_\Phi^2(k) \equiv \frac{\Phi_k^2 k^3}{2\pi^2} \tag{4.9}$$

which is called $\delta_\Phi^2(k)$ as the power spectrum.

4.1.2 Free quantum fields and vacuum

A classical field is expressed as a function of space-time $\phi(x,t)$ where t is the time and x is a three dimensional coordinate in space.

Fourier transformation for the scalar field is

$$\phi(x,t) = \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}} e^{ikx} \phi_k(t). \tag{4.10}$$

The functions $\phi_k(t)$ are called the modes of the field ϕ.

To quantize the field, each mode $\phi_k(t)$ is threaded as a separate harmonic oscillator. We can define as π [7],

$$\pi(x) = \frac{\partial L[\phi, \dot{\phi}]}{\partial \dot{\phi}(x)}. \tag{4.11}$$

The field ϕ and its canonical momenta π should satisfy the standard commutation relations:

$$[\phi(x,t), \phi(y,t)] = [\pi(x,t), \pi(y,t)] = 0, \tag{4.12}$$

and

$$[\phi(x,t), \pi(y,t)] = [\pi(x,t), \phi(y,t)] = i\delta(x-y) \tag{4.13}$$

where we have set $\hbar = 1$.

The dynamical variable can be expanded in terms of the raising and lowering operators. This is called the free field expansion.

$$\phi(t,x) = \frac{1}{\sqrt{2}} \int [u_k^*(t) e^{ikx} a_k^- + u_k(t) e^{-ikx} a_k^+] \frac{d^3k}{(2\pi)^{\frac{3}{2}}} \tag{4.14}$$

where a_k^-, a_k^+ are annihilation, creation operators and $u_k(t)$ are the mode functions for $\phi(t)$. Between creation, annihilation operators, the commutation relations are

$$[a_k^-, a_{k'}^-] = [a_k^+, a_{k'}^+] = 0, [a_k^-, a_{k'}^+] = \delta(k-k'). \quad \textbf{(4.15)}$$

So the mode functions $u_k(t)$ require the normalization condition

$$u_k' u_k^* - u_k u_k^{*\prime} = 2i \quad \textbf{(4.16)}$$

where $u_k' \equiv du_k/d\eta$, $\eta = \int dt/a$ and η is the conformal time. $u_k(t)$ mode functions are also certain complex-valued solutions of the equation,

$$\ddot{u}_k + \omega_k^2 u_k = 0 \quad \textbf{(4.17)}$$

where ω_k is the frequency.

The next step in quantization is to describe the vacuum state $|0>$ as the state annihilated by operators a_k^-;

$$a_k^- |0> = 0 \quad \textbf{(4.18)}$$

where $|0>$ is the ground state which is empty, it is called, the vacuum. If the $\omega_k^{'}$s are time-independent, the vector $|0>$ corresponds to the familiar Minkowski vacuum.

4.1.3 Spectrum,Spectral Index

In this chapter finally we want to calculate the correlation function, or equivalently, the power spectrum of the gravitational potential.

For the initial vacuum state, $|0>$ the correlation function at $\eta > \eta_i$,

$$\begin{aligned} <0|\phi(\eta,x)\phi(\eta,y)|0> \;&=\; <0|\int [u_k^*(\eta)e^{ikx}a_k^- + u_k(\eta)e^{-ikx}a_k^+]\frac{d^3k}{(2\pi)^{\frac{3}{2}}} \\ &\times \int [u_k^*(\eta)e^{iky}a_k^- + u_k(\eta)e^{-iky}a_k^+]\frac{d^3k}{(2\pi)^{\frac{3}{2}}}|0> \quad \textbf{(4.19)} \end{aligned}$$

and then

$$<0|\phi(\eta,x)\phi(\eta,y)|0> = \int \frac{|u_k|^2 k^2 dk}{4\pi^2}\int_0^{\pi} e^{ikrcos\theta} sin\theta d\theta \quad \textbf{(4.20)}$$

$$<0|\phi(\eta,x)\phi(\eta,y)|0> = \int \frac{|u_k|^2 k^3}{2\pi^2}\frac{sin(kr)}{kr}\frac{dk}{k} \quad \textbf{(4.21)}$$

where $r \equiv |x-y|$, $u \equiv cos\theta$ and θ is the angle between the $\vec{k}$ vector and r.

According to the definition of the power spectrum in (4.6) and (4.7), we can write as

$$\delta_{\Phi}^{2}(k,\eta)=\frac{|u_{k}|^{2}k^{3}}{2\pi^{2}}. \quad \textbf{(4.22)}$$

The expansion for the spectral index is

$$n_{s}-1 \equiv \frac{dIn\delta_{\Phi}^{2}}{dInk}, \quad \textbf{(4.23)}$$

where it is calculated at the time of horizon-crossing. Spectral index (Spectral tilt), is of order 10^{-10} and $1+\omega$ can be predicted as $\sim 10^{-2}$ on galactic scales where ω is equal to p/ε. Thus the final energy density is of order 10^{-12} times the Planckian density. Within a limited range of scales, one can always approximate the spectrum by the power law, $\delta_{\Phi}^{2} \propto k^{n_{s}-1}$ where n_{s} is the spectral index and it is defined as [6]. The concrete value of n_{s} is related to particular scenario.

5. INFLATION AND NON-GAUSSIANITY PARAMETER

Inflationary cosmology explains the observed inhomogeneities in the distribution of galaxies and the anisotropies of the cosmic microwave background [16]. In this stage, the fluctuations in the scalar curvature perturbations are produced by quantum fluctuations in the scalar field ϕ.

The origin of the observed inhomogeneities in our universe is the perturbation of background metric. One can express the metric of a flat Friedmann universe with small perturbations as

$$ds^2 = [{}^{(0)}g_{\alpha\beta} + \delta g_{\alpha\beta}(x^\gamma)]dx^\alpha dx^\beta \tag{5.1}$$

where $|\delta g_{\alpha\beta}| << |{}^{(0)}g_{\alpha\beta}|$. One can categorized $\delta g_{\alpha\beta}$ into different types: scalars, vectors and tensors. We would be interested only in scalar perturbations during this work. Although four scalar functions to categorize scalar perturbations, we will reduce that to only two functions with choosing a particular gauge. To give an example the metric in Longitudinal(Conformal-Newtonian gauge) takes the form of

$$ds^2 = a^2[(1+2\phi)d\eta^2 - (1-2\psi)\delta_{ij}dx^i dx^j] \tag{5.2}$$

It turns out that ϕ is equal to ψ if one uses ij-component of the Einstein equation for this metric.

The quantity that is used to describe scalar curvature perturbation is denoted by ζ and ζ is defined in terms of the metric perturbations ϕ [20], [21] as

$$\zeta = \frac{2}{3}\frac{\mathscr{H}\Phi' + \Phi}{1+\omega} + \Phi \tag{5.3}$$

where $\mathscr{H}$ is defined as $\frac{a'}{a}$, prime is the conformal time derivative.

We rewrite ζ in Fourier components quantizing ζ using Quantum Field Theory in curved space-time [15]

$$\zeta(\eta, x) = \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}}[u_k^*(\eta)a_k e^{ikx} + u_k(\eta)e^{-ikx}a_k^+]. \tag{5.4}$$

All the k modes in the integral evolve independent of each other for small perturbations. Let us define the random variables S_N as finite sum over N of k modes

$$S_N = \sum_{i=1}^{N} k_i. \tag{5.5}$$

Here we assume all the k_i to obey the same probability distribution. Central limit theorem tells us that for large enough N, the probability density of the random variables S_N is given by the Gaussian distribution. Equality ζ and its two point correlation function will obey Gaussian statistics [2].

5.1 Non-Gaussianity

The most recent CMB experiment, planck satellite, find that statistics of the CMB anisotropies is gaussian [23]. But there are still ongoing efforts to see if there is any deviation from gaussianity [24]. In this work we try to understand the quantum origin of a possible non-gaussianity. The three point function is the lowest order non-Gaussianity and is expressed in the term of bispectrum as

$$< \zeta_{k_1}\zeta_{k_2}\zeta_{k_3} >= (2\pi)^3\delta^{(3)}\Big(\sum_i k_i\Big)B_\zeta(k_1,k_2,k_3) \tag{5.6}$$

where ζ_k is the Fourier transform of the primordial curvature perturbation and the δ-function is required by translation invariance. The bispectrum $B_\zeta(k_1,k_2,k_3)$ is a real-valued function of three variables and is expressed in terms of f_{NL} parameters. We define primordial NG as the one generated either during inflation or after it when the comoving curvature perturbation becomes constant on super horizon scales.The non-Gaussianity is produced from a background Gaussian field ζ_G by a purely local process [10],

$$\zeta(x) = \zeta_G(x) + \frac{3}{5} f_{NL}\zeta_G^2(x) + ... \tag{5.7}$$

On the right hand side of this equation the first term is the contribution from the Gaussian field and the second term from the non-Gaussian field.

Using (5.7) it is straightforward to see that the bispectrum is

$$B_\zeta^{loc}(k_1,k_2,k_3) = \frac{6}{5} f_{NL}[P_\zeta(k_1)P_\zeta(k_2) + P_\zeta(k_2)P_\zeta(k_3) + P_\zeta(k_3)P_\zeta(k_1)]. \tag{5.8}$$

Here the measured power spectrum $P_\zeta(k) \propto k^{-3}$ and the wavenumbers can have various values which must satisfy $k_1 + k_2 + k_3 = 0$. One can choose different values of k. We see that the bispectrum peaks at the squeezed limit(local limit) where one of the wave numbers is much smaller than the others. In this case, the bispectrum is [12]

$$B_{\zeta}^{loc}(k_1,k_2,k_3)_{k_3<<k_1\approx k_2}\approx\frac{12}{5}f_{NL}P(k_1)P(k_3). \tag{5.9}$$

Cremenilli and Zaldarriaga showed a consistency relation for all single-field inflation models in the squeezed limit,

$$B_{\zeta}(k_3<<k_1)=(1-n_s)P(k_1)P(k_3) \tag{5.10}$$

Equation (5.9) and (5.10) tells us that

$$f_{NL}\approx\frac{5}{12}(1-n_s)\approx 0.01 \tag{5.11}$$

Despite of this consistency relation, Agullo and Parker [17] demonstrated that there can be enhancement in the local limit bispectrum for slow-roll inflation with non-vacuum initial state.

5.1.1 Non-Gaussianity from a Non-Bunch Davies initial state

For slow-roll inflation with a Non-Bunch-Davies initial state we must compute the bispectrum. To compute this we will take the action

$$S=\frac{1}{2}\int d^4x\sqrt{-g}[R-(\nabla\phi)^2-2V(\phi)] \tag{5.12}$$

where R is the Ricci scalar, ϕ is a scalar field and $V(\phi)$ is the potential.Here the slow-roll parameters are defined as

$$\varepsilon\equiv\frac{1}{2}\Big(\frac{M_pV'}{V}\Big)^2\sim\frac{1}{2}\frac{\dot\phi^2}{H^2}<<1. \tag{5.13}$$

$$\eta=\frac{M_{pl}^2V''}{V}\sim\frac{-\ddot\phi}{H\dot\phi}+\frac{1}{2}\frac{\dot\phi^2}{H^2}. \tag{5.14}$$

Maldecana show that this action at second and third order(in term of ζ)takes the following form [10]. So this actions are obtained using different gauges. The actions are

$$S_2=\frac{1}{2}\int d^4x\frac{\dot\phi^2}{H^2}[a^3\dot\zeta^2-a(\partial\zeta)^2]. \tag{5.15}$$

$$S_3=\int d^4x\frac{\dot\phi^4}{H^4}a^5H\dot\zeta^2\partial^{-2}\dot\zeta\equiv\int L_3(t)dt. \tag{5.16}$$

To lowest order in slow-roll, the equation of motion for ζ is given by

$$-\partial_t(a^3\frac{\dot{\phi_0}^2}{H^2}\dot{\zeta})+a\frac{\dot{\phi_0}^2}{H^2}\partial^2\zeta=0. \quad \textbf{(5.17)}$$

The three point correlation function for ζ, using in-in formalism can be written in the following form [13],

$$<\zeta^3(t^*)>=-i\int_{t_0}^{t^*}dt' <0|[\zeta^3(t^*),H_I(t')]|0> \quad \textbf{(5.18)}$$

where H_I is the interaction Hamiltonian and H_I is equal to $-L_3$. The quantum field ζ can be written in terms of annihilation, creation operators(a_k, a_k^+)[16]

$$\zeta_k(t)=u_k(t)a_k+u_k^*(t)a_{-k}^* \quad \textbf{(5.19)}$$

where $u_k(t)$ are the mode functions. Bunch-Davies(BD) vacuum is a preferred vacuum state in de-Sitter space. In the early time limit ($\eta\rightarrow-\infty$) of each mode BD vacuum goes to Minkowski vacuum and contains only positive frequency mode [7]. For slow-roll inflation, the BD vacuum state is

$$u_k(\eta)=\frac{H^2}{\dot{\phi}}\frac{1}{\sqrt{2k^3}}(1+ik\eta)e^{-ik\eta}. \quad \textbf{(5.20)}$$

To generalized BD vacuum state let us assume that there are some particles at the initial state. These can be done by applying a Bogoliubov transformation on the BD u:

$$\begin{aligned}\tilde{u}_k(t) &= \alpha_k u_k(t)+\beta_k(t)u_k^*(t)\\ &= \alpha_k\frac{H^2}{\dot{\phi}}\frac{1}{\sqrt{2k^3}}(1+ik\eta)e^{-ik\eta}+\beta_k\frac{H^2}{\dot{\phi}}\frac{1}{\sqrt{2k^3}}(1-ik\eta)e^{ik\eta}\end{aligned} \quad \textbf{(5.21)}$$

for slow-roll inflation.

The coefficients α_k and β_k satisfy $|\alpha|^2-|\beta|^2=1$(the normalization condition) [7]. To determine the Bogoliubov coefficients α_k and β_k, it is necessary to know the mode functions $v_k(\eta)$ and $u_k(\eta)$ and their derivatives at only one value of η. N_k is the number of particles in mode k. We define θ_k as the relative phase between α_k and β_k and is an additional unconstrained parameter [9]. Therefore we can express all quantities in terms of N_k and θ_k. A particular choice of θ_k can enhance f_{NL}. α_k and β_k can be written as

$$\alpha_k=\sqrt{N_k+1}e^{ik\theta_k},\beta_k=\sqrt{N_k}. \quad \textbf{(5.22)}$$

We compute the power spectrum by replacing u with $\tilde{u}$ in the quantization of ζ, i.e

$$\begin{aligned} P(k) &= |u_k|^2_{\eta\to 0} \\ &= \frac{H^4}{\dot{\phi}^2}\frac{1}{2k^3}|\alpha_k+\beta_k|^2 \\ &= \frac{H^4}{\dot{\phi}^2}\frac{1}{2k^3}\Big(1+2N_k+2\sqrt{N_k(N_k+1)}cos(\theta_k\Big) \end{aligned}$$

Here N_k is continuous and must be compatible with the observed spectral tilt,

$$1-n_s \equiv \frac{-dlog(k^3P(k))}{d\log k} = -2\eta_{SR}+6\varepsilon-\frac{dlog(1+2N_k)}{d\log k} = 0.032 \quad \textbf{(5.23)}$$

where ε and η_{SR} are taken small values and N_k must change slowly([17]).

To calculate the bispectrum, we plug (5.19) and (5.16) into (5.18) and we find that

$$B_\zeta(k_1,k_2,k_3) = 2i\frac{\dot{\phi}^4}{H^6}\sum_i\Big(\frac{1}{k_i^2}\Big)\tilde{u}_{k_1}(\tilde{\eta})\tilde{u}_{k_2}(\tilde{\eta})\tilde{u}_{k_3}(\tilde{\eta})\int_{\eta_0}^{\tilde{\eta}}d\eta\frac{1}{\eta^3}u'_{k_1}u'_{k_2}u'_{k_3}+c.c \quad \textbf{(5.24)}$$

We replace u by $\tilde{u}$ to account for our initial state, so the integral takes the following form,

$$\begin{aligned} \int_{\eta_0}^{\tilde{\eta}}d\eta\frac{1}{\eta^3}u'_{k_1}u'_{k_2}u'_{k_3} &= \frac{H^6}{\dot{\phi}^3}\sqrt{\frac{k_1k_2k_3}{8}}\int_{\eta_0}^{\tilde{\eta}}d\eta(\alpha^*_{k_1}e^{ik_1\eta}+\beta^*_{k_1}e^{-ik_1\eta}) \\ &\times(\alpha^*_{k_2}e^{ik_2\eta}+\beta^*_{k_2}e^{-ik_2\eta})(\alpha^*_{k_3}e^{ik_3\eta}+\beta^*_{k_3}e^{-ik_3\eta}) \quad \textbf{(5.25)} \end{aligned}$$

For the BD case($\alpha=1$, $\beta=0$), The integral gives $[1/i(k_1+k_2+k_3)]e^{i(k_1+k_2+k_3)\eta}|^{\tilde{\eta}}_{\eta_0}$. In the squeezed limit, it becomes $(1/2ik_1)e^{2ik_1\eta}|^{\tilde{\eta}}_{\eta_0}$. For the left endpoint, as usual , $\tilde{\eta}\to 0$. In this way we find an expression for the bispectrum,

$$\begin{aligned} B_\zeta(k_1,k_2,k_3) &= \frac{i}{4}\frac{H^6}{\dot{\phi}^4}\frac{1}{k_1k_2k_3}\sum_i\frac{1}{k_i^2}(\alpha_{k_1}+\beta_{k_1})(\alpha_{k_2}+\beta_{k_2})(\alpha_{k_3}+\beta_{k_3}) \\ &\times\int_{\eta_0}^{\tilde{\eta}}d\eta(\alpha^*_{k_1}e^{ik_1\eta}+\beta^*_{k_1}e^{-ik_1\eta}) \\ &\times(\alpha^*_{k_2}e^{ik_2\eta}+\beta^*_{k_2}e^{-ik_2\eta})(\alpha^*_{k_3}e^{ik_3\eta}+\beta^*_{k_3}e^{-ik_3\eta})+c.c. \quad \textbf{(5.26)} \end{aligned}$$

For $k_3 << k_1$;

$$B_\zeta(k_1,k_2,k_3) = \frac{H^6}{\dot{\phi}^2}\frac{1}{4k_1k_2k_3^4}\mathcal{N} \quad \textbf{(5.27)}$$

where

$$\mathcal{N} \equiv \prod_i (\alpha_{k_i} + \beta_{k_i})(\alpha^*_{k_1}\beta^*_{k_1}\alpha^*_{k_3} + \beta^*_{k_1}\alpha^*_{k_2}\alpha^*_{k_3}) - (\alpha\leftrightarrow\beta) + c.c$$

$$= 2N_1 + 2N_2 + 4N_1N_2 + 4N_1\sqrt{N_2(N_2+1)}\cos\theta_2 + 4N_2\sqrt{N_1(N_1+1)}\cos\theta_1$$

$$+2\sqrt{N_2(N_2+1)}\cos\theta_2 + 2\sqrt{N_1(N_1+1)}\cos\theta_1$$

$$+4\sqrt{N_1N_2(N_1+1)(N_2+1)}\cos\theta_1\cos\theta_2$$

$$+4\sqrt{N_1N_2(N_1+1)(N_2+1)}\sin\theta_1\sin\theta_2$$

$$+4\sqrt{N_1N_3(N_1+1)(N_3+1)}\sin\theta_1\sin\theta_3$$

$$+4\sqrt{N_2N_3}\sqrt{(N_2+1)(N_3+1)}\sin\theta_2\sin\theta_3 \quad \textbf{(5.28)}$$

where $N_i \equiv N_{k_i}$, $\theta_i \equiv \theta_{k_i}$(i takes the values 1, 2, 3). N_{k_i} are particle numbers in k mode and θ_{k_i} are the phase angles between the Bogoliubov parameters.

We calculate f_{NL} for various values of the cutoff k_{cut}. When k_{cut} is very large, $N_k \approx N_{k_0} \approx constant$ [9]. We suppose either that $\theta_k \approx constant$ or $\theta \approx k\eta_0$.

Even though $k_3 << k_1 \approx k_2$, η_0 is sufficiently large. Thus these terms will oscillate greatly during the integral over k_2 and average out [9]. So we only take the first three terms on the above equation:

$$N_{\theta_k \approx k\eta_0} = 4(N_{k,0} + N^2_{k,0}) \quad \textbf{(5.29)}$$

For either form of θ_k, the numerical integral is unchanged. We find

$$f_{NL} = 76\frac{\dot{\phi}^2}{H^2}\frac{\mathcal{N}}{\tilde{P}^2} = 1,52\varepsilon\frac{\mathcal{N}}{\tilde{P}^2} \quad \textbf{(5.30)}$$

where we have defined $P(k) = H^4/(2k^3\dot{\phi}^2)\tilde{P}$ and

$$\tilde{P} = 1 + 2N_k + 2\sqrt{N_k(N_k+1)}cos\theta_k$$

For the case that $\theta_k \approx k\eta_0$, we find the following form,

$$f_{NL} = 1.52\frac{\varepsilon}{0.01}\left(1 - \frac{1}{1 + 4N_{k,0} + 4N^2_{k,0}}\right) \quad \textbf{(5.31)}$$

When we take $\theta_k \approx constant$, we find larger values and even negative f_{NL} for $N_{k,0} > 0.017$, A maximum value of f_{NL} is

$$f_{NL} = 1.52\frac{\varepsilon}{0.01}(\frac{1}{4} + 9N_{k,0} + 9N_{k,0}{}^2). \quad \textbf{(5.32)}$$

and a minimum value of f_{NL} also is

$$f_{NL} = -1.52\frac{\varepsilon}{0.01}[2N_{k,0}(1+N_{k,0}) + \sqrt{N_{k,0}(N_{k,0}+1)}(1+2N_{k,0})]. \quad \textbf{(5.33)}$$

5.1.2 The Bunch Davies Limit of the Non-linearity parameter

Let's calculate the bispectrum in the limit of $\beta = 0$, using equation (5.21), where we have,

$$\tilde{u} = \frac{H^2}{\dot{\phi}}(1 + ik\eta)e^{-ik\eta}. \quad \textbf{(5.34)}$$

For the power spectrum,we get

$$P(k) = |\tilde{u}_k|^2_{\tilde{\eta}\to 0} = |\alpha_k|^2\frac{H^4}{\phi^2}\frac{1}{2k^3} = \frac{H^4}{\dot{\phi}^2}\frac{1}{2k^3}, \quad \textbf{(5.35)}$$

where $\alpha^2 = 1$. For the bispectrum we need to evaulate the integral (5.27) which is

$$\begin{aligned}\int_{\eta_0}^{\tilde{\eta}} d\eta \frac{1}{\eta^3} u'_{k_1} u'_{k_2} u'_{k_3} &= \frac{H^6}{\dot{\phi}^3}\sqrt{\frac{k_1,k_2,k_3}{8}} \int_{\eta_0}^{\tilde{\eta}} d\eta \, \alpha^*_{k_1} e^{ik_1\eta} \alpha_{k_2} e^{ik_2\eta} \alpha^*_{k_3} e^{ik_3\eta} \\ &= \frac{H^6}{\dot{\phi}^3}\sqrt{\frac{k_1,k_2,k_3}{8}} \frac{1}{i(k_1+k_2+k_3)} e^{i(k_1+k_2+k_3)\eta}|^{\tilde{\eta}}_{\eta_0}. \quad \textbf{(5.36)}\end{aligned}$$

therefore Bispectrum becomes

$$B_\zeta(k_1,k_2,k_3) = \frac{i}{4}\frac{H^6}{\dot{\phi}^4}\frac{1}{k_1k_2k_3}\sum_i \frac{1}{k_i{}^2}\frac{1}{k_1+k_2+k_3}(e^{i(k_1+k_2+k_3)\tilde{\eta}} - e^{i(k_1+k_2+k_3)\eta_0}), \quad \textbf{(5.37)}$$

where $\tilde{\eta} \to 0$.

In the local limit where $k_3 << k_1$ the bispectrum will have the following form

$$B_\zeta(k_1,k_2,k_3) = \frac{i}{4}\frac{H^6}{\dot{\phi}^4}\frac{1}{k_1k_2k_3}\frac{1}{k_3{}^2}\frac{1}{2k_1}(1 - e^{i(2k_1)\eta_0}). \quad \textbf{(5.38)}$$

5.2 Two Loop Correction to the Non-Linearity Parameter

Let us investigate a model where there is an extra spectator scalar field σ,

$$\mathscr{L} = \left[\frac{R}{16\pi G} - \frac{1}{2}\varphi_{,\mu}\varphi_{,\nu}g^{\mu\nu} - V(\varphi) - \frac{1}{2}\sigma_{,\mu}\sigma_{,\nu}g^{\mu\nu} - U(\sigma)\right]\sqrt{-g} \tag{5.39}$$

where R is the D-dimensional Ricci scalar and a comma denotes ordinary differentiation.

This is a generalized case of the equation (5.12). After using ADM formalism one can show that the lowest order effects of the spectator scalar potential on the inflaton field can be represented with the following lagrangian [8],

$$\mathscr{L} = \frac{\varepsilon}{D-1}a^{D-1}e^{(D-1)\zeta}U(\sigma) \tag{5.40}$$

where $U(\sigma)$ is the spectator scalar potential and D is the number of the dimension.

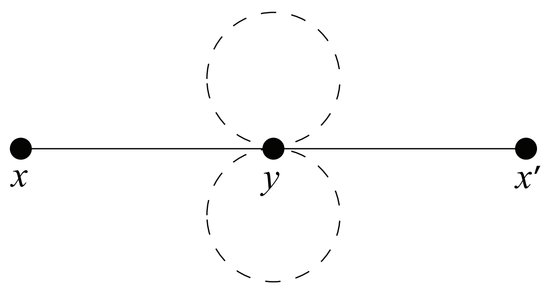

Figure 5.1: Two loop correction

In this figure, Solid lines represent ζ and dashed lines represent σ.

We assuming $U(\sigma) = \lambda\sigma^4/4!$, the diagram in Fig 5.1 derives from the $\zeta^2\sigma^4$ term of the interaction(5.40),

$$\Delta\mathscr{L} = \frac{(D-1)}{48}\lambda\varepsilon a^{D-1}\zeta^2\sigma^4. \tag{5.41}$$

Kahya, Onemli and Woodard [8] calculated the two loop correction(Fig 5.1) to the mode function of the inflaton using the above interaction and found the following expression(5.40),

$$\tilde{u} \approx \frac{H}{\sqrt{2k^3}}\left\{1 + \frac{\lambda GH^2}{96\pi^3}\log^3(a)\right\} \tag{5.42}$$

We will study the effects of modified mode function (5.42) on the non-gaussianity parameter.

Let us rewrite the loop corrected mode function for the non-BD vacuum:

$$\begin{aligned}
\tilde{u}_k(t) &= \Big(\alpha_k(t)u_k(t)+\beta_k(t)u_k^*(t)\Big)\left\{1+\frac{\lambda GH^2}{96\pi^3}\log^3(a)\right\} \\
&= \Big(\alpha_k\frac{H^2}{\dot\phi}\frac{1}{\sqrt{2k^3}}(1+ik\eta)e^{-ik\eta}+\beta_k\frac{H^2}{\dot\phi}\frac{1}{\sqrt{2k^3}}(1-ik\eta)e^{ik\eta}\Big) \\
&\quad\times\left\{1+\frac{\lambda GH^2}{96\pi^3}\log^3(a)\right\}. \qquad \textbf{(5.43)}
\end{aligned}$$

using the mode function given by the above equation, the power spectrum becomes

$$P(k)=|u_k|^2_{\eta\to0}=\frac{H^4}{\dot\phi^2}\frac{1}{2k^3}|\alpha_k+\beta_k|^2\left\{1+\frac{\lambda GH^2}{96\pi^3}\log^3(a)\right\}^2$$

and in terms of N_k and θ_k

$$P(k) = \frac{H^4}{\dot\phi^2}\frac{1}{2k^3}\Big(1+2N_k+2\sqrt{N_k(N_k+1)}cos(\theta_k)\Big)\left\{1+\frac{\lambda GH^2}{48\pi^3}\log^3(a)\right\}. \qquad \textbf{(5.44)}$$

Using equation (5.23), the spectral index is obtained as

$$1-n_s=-2\eta_{SR}+6\varepsilon+\frac{\lambda G}{96\pi^3}\Big(2\eta-8\varepsilon-\frac{3H}{\log a}\Big)-\left\{1+\frac{\lambda G}{96\pi^3}\right\}\frac{dlog(1+2N_k)}{dlogk}. \qquad \textbf{(5.45)}$$

To calculate the bispectrum, we need to evaluate the integral in equation(5.24) with the modified mode function

$$\begin{aligned}
\int_{\eta_0}^{\tilde\eta} d\eta\frac{1}{\eta^3}u'_{k_1}u'_{k_2}u'_{k_3} &= \int_{\eta_0}^{\tilde\eta}\frac{d\eta}{\eta^3}\prod_i\Big(\frac{H^2}{\dot\phi}k_i^2\eta(\alpha_{k_i}^*e^{ik_i\eta}+\beta_{k_i}^*e^{-ik_i\eta})\left\{1+\frac{\lambda GH^3}{32\pi^3}\log^3(a)\right\} \\
&\quad+\frac{H^2}{\dot\phi}\frac{1}{\sqrt{2k_i^3}}\frac{\lambda GH^3}{32\pi^3}\log^2 a(\alpha_{k_i}^*(1-ik_i\eta)e^{ik_i\eta} \\
&\quad+\beta_{k_i}^*(1+ik_i\eta)e^{-ik_i\eta})\Big) \qquad \textbf{(5.46)}
\end{aligned}$$

where i takes 1, 2, 3 values.

We can divide (5.46) into three parts,

$$\int_{\eta_0}^{\tilde\eta} d\eta\frac{1}{\eta^3}{u'_{k_1}}^*{u'_{k_2}}^*{u'_{k_3}}^*=I_1+I_2+I_3 \qquad \textbf{(5.47)}$$

Let's look at first part of this integral which is defined as ,

$$I_1 \equiv \int d\eta \frac{H^6}{\dot{\phi}^2}(k_1^2k_2^2k_3^2)\Big(\alpha^*_{k_1}\alpha^*_{k_2}\alpha^*_{k_3}e^{i(k_1+k_2+k_3)\eta} + \alpha^*_{k_1}\alpha^*_{k_2}\beta^*_{k_3}e^{i(k_1+k_2-k_3)\eta}$$

$$+\alpha^*_{k_1}\beta^*_{k_2}\alpha^*_{k_3}e^{i(k_1-k_2+k_3)\eta} + \alpha^*_{k_1}\beta^*_{k_2}\beta^*_{k_3}e^{i(k_1-k_2-k_3)\eta} + \beta^*_{k_1}\alpha^*_{k_2}\alpha^*_{k_3}e^{i(k_2-k_1+k_3)\eta}$$

$$+\beta^*_{k_1}\alpha^*_{k_2}\beta^*_{k_3}e^{i(k_2-k_1-k_3)\eta} + \beta^*_{k_1}\beta^*_{k_2}\alpha^*_{k_3}e^{i(-k_2-k_1+k_3)\eta} - \beta^*_{k_1}\beta^*_{k_2}\beta^*_{k_3}e^{i(k_2+k_1+k_3)\eta}\Big)$$

$$\times\left\{1+\frac{\lambda GH^3}{96\pi^3}\log^3(a)\right\}^2 \qquad \textbf{(5.48)}$$

expressing η in terms a using $\eta = -\frac{1}{Ha}$, we can calculate the above integral

$$I_1 = \frac{H^6}{\dot{\phi}^2}(k_1k_2k_3)^2\Big[\frac{\alpha^*_{k_1}\alpha^*_{k_2}\alpha^*_{k_3}}{i(k_1+k_2+k_3)}e^{-i(k_1+k_2+k_3)/Ha} + \frac{\alpha^*_{k_1}\alpha^*_{k_2}\beta^*_{k_3}}{i(k_1+k_2-k_3)}e^{-i(k_1+k_2-k_3)/Ha}$$

$$+\frac{\alpha^*_{k_1}\beta^*_{k_2}\alpha^*_{k_3}}{i(k_1-k_2+k_3)}e^{-i(k_1-k_2+k_3)/Ha} + \frac{\alpha^*_{k_1}\beta^*_{k_2}\beta^*_{k_3}}{i(k_1-k_2-k_3)}e^{-i(k_1-k_2-k_3)/Ha}$$

$$+\frac{\beta^*_{k_1}\alpha^*_{k_2}\alpha^*_{k_3}}{i(k_2-k_1+k_3)}e^{-i(k_2-k_1+k_3)/Ha} + \frac{\beta^*_{k_1}\alpha^*_{k_2}\beta^*_{k_3}}{i(k_2-k_1-k_3)}e^{-i(k_1+k_2-k_3)/Ha}$$

$$+\frac{\beta^*_{k_1}\beta^*_{k_2}\alpha^*_{k_3}}{i(-k_1-k_2+k_3)}e^{-i(-k_1-k_2+k_3)/Ha} - \frac{\beta^*_{k_1}\beta^*_{k_2}\beta^*_{k_3}}{i(k_1+k_2+k_3)}e^{i(k_1+k_2+k_3)/Ha}\Big]$$

$$+\sum_k C_k\frac{H^8}{\dot{\phi}^2}(k_1^2k_2^2k_3)^2\frac{\lambda GH^2}{48\pi^3}\Big[\frac{1}{k}\Big(-\frac{6k}{Ha}\log aF_3(1,1,1;2,2,2;-\frac{ik}{Ha})$$

$$-\frac{6k}{Ha}F_4(1,1,1,1;2,2,2,2;-\frac{ik}{Ha}) - i(3E_i(-\frac{ik}{Ha})(\log(-\frac{1}{Ha})+\log a)^2 + e^{-\frac{ik}{Ha}}\log^3 a$$

$$+6\log a\log(-\frac{1}{Ha})log(\frac{ik}{Ha}) + 3\log(-\frac{1}{Ha})(2\log a + \log(-\frac{1}{Ha}))\Gamma(0,\frac{ik}{Ha})$$

$$-3\log^2(-\frac{1}{Ha})\log a + 6\gamma\log a\log(-\frac{1}{Ha}) + 3\log^2(-\frac{1}{Ha})\log(\frac{ik}{Ha})$$

$$-2log^3(-\frac{1}{Ha}) + 3\gamma log^2(-\frac{1}{Ha})\Big)\Big], \qquad \textbf{(5.49)}$$

where C_k are

$$\begin{aligned}
&\alpha^*_{k_1}\alpha^*_{k_2}\alpha^*_{k_3}, \quad k=k_1+k_2+k_3; \quad \alpha^*_{k_1}\alpha^*_{k_2}\beta^*_{k_3}, \quad k=k_1+k_2-k_3;\\
&\alpha^*_{k_1}\beta^*_{k_2}\alpha^*_{k_3}, \quad k=k_1-k_2+k_3; \quad \alpha^*_{k_1}\beta^*_{k_2}\beta^*_{k_3}, \quad k=k_1-k_2-k_3;\\
&\beta^*_{k_1}\alpha^*_{k_2}\alpha^*_{k_3}, \quad k=k_3+k_2-k_1; \quad \beta^*_{k_1}\alpha^*_{k_2}\beta^*_{k_3}, \quad k=k_2-k_1-k_3;\\
&\alpha^*_{k_1}\beta^*_{k_2}\alpha^*_{k_3}, \quad k=k_3-k_1-k_2; \quad \beta^*_{k_1}\beta^*_{k_2}\beta^*_{k_3}, \quad k=-k_1-k_2-k_3
\end{aligned} \tag{5.50}$$

F is the hypergeometric function, γ is the Euler's constant, Ei is the exponential integral function.For equation (5.49) the summation is done for different values of k which is give in C_k. These for different values come from for distinct terms appear in equation (5.51).

Similarly I_2 is defined as

$$\begin{aligned}
I_2 \;\equiv\; & \frac{H^9}{\dot{\phi}^3}\frac{\lambda G}{32\pi^3}\Big(\frac{{k_1}^2{k_3}^2}{\sqrt{2{k_2}^3}}+\frac{{k_2}^2{k_3}^2}{\sqrt{2{k_1}^3}}+\frac{{k_1}^2{k_2}^2}{\sqrt{2{k_3}^3}}\Big)\int\frac{d\eta}{\eta}\\
& \times a\log^2 a\Big(\alpha^*_{k_1}\alpha^*_{k_2}\alpha^*_{k_3}e^{i(k_1+k_2+k_3)\eta}+\alpha^*_{k_1}\alpha^*_{k_2}\beta^*_{k_3}e^{i(k_1+k_2-k_3)\eta}\\
& +\alpha^*_{k_1}\beta^*_{k_2}\alpha^*_{k_3}e^{i(k_1-k_2+k_3)\eta}+\alpha^*_{k_1}\beta^*_{k_2}\beta^*_{k_3}e^{i(k_1-k_2-k_3)\eta}\\
& +\beta^*_{k_1}\alpha^*_{k_2}\alpha^*_{k_3}e^{i(k_2-k_1+k_3)\eta}+\beta^*_{k_1}\alpha^*_{k_2}\beta^*_{k_3}e^{i(k_2-k_1-k_3)\eta}\\
& +\beta^*_{k_1}\beta^*_{k_2}\alpha^*_{k_3}e^{i(-k_2-k_1+k_3)\eta}-\beta^*_{k_1}\beta^*_{k_2}\beta^*_{k_3}e^{i(k_2+k_1+k_3)\eta}\Big)
\end{aligned} \tag{5.51}$$

The above integral gives

$$
\begin{aligned}
I_2 &= -\frac{H^9}{\dot{\phi}^3}\frac{\lambda G}{32\pi^3}\Big(\frac{{k_1}^2{k_3}^2}{\sqrt{2{k_2}^3}}+\frac{{k_2}^2{k_3}^2}{\sqrt{2{k_1}^3}}+\frac{{k_1}^2{k_2}^2}{\sqrt{2{k_3}^3}}\Big) \\
&\times\sum_k D_k\Big\{\frac{1}{3H^2a}\Big(6k^2F_4(1,1,1,1;2,2,2,2;-\frac{ik}{Ha}) \\
&+6k^2(\log a-1)F_3(1,1,1;2,2,2;-\frac{ik}{Ha})+Ha(-6ik(\log a-1)Ei(-\frac{ik}{Ha}) \\
&+6Hae^{-ik/Ha}+3Ha\log^2 ae^{-ik/Ha}-3ik\log^2 a\log(ik/Ha) \\
&-6Ha\log ae^{-ik/Ha}+3ik\log a\log(-ik/Ha)-3ik\log a\log(iHa/k) \\
&-3ik\log(-ik/Ha)+3ik\log(iHa/k)-3ik\log^2 a\Gamma(0,ik/Ha)-ik\log^3 a \\
&-3i\gamma k\log^2 a+3ik\log^2 a+6i\gamma k\log a-6ik\log a-6i\gamma k+6ik)\Big)\Big\} \qquad \textbf{(5.52)}
\end{aligned}
$$

where D_k are

$$
\begin{aligned}
&\alpha^*_{k_1}\alpha^*_{k_2}\alpha^*_{k_3}, \quad k=k_1+k_2+k_3; \quad \alpha^*_{k_1}\alpha^*_{k_2}\beta^*_{k_3}, \quad k=k_1+k_2-k_3; \\
&\alpha^*_{k_1}\beta^*_{k_2}\alpha^*_{k_3}, \quad k=k_1-k_2+k_3; \quad \alpha^*_{k_1}\beta^*_{k_2}\beta^*_{k_3}, \quad k=k_1-k_2-k_3; \\
&\beta^*_{k_1}\alpha^*_{k_2}\alpha^*_{k_3} \quad k=k_3+k_2-k_1; \quad \beta^*_{k_1}\alpha^*_{k_2}\beta^*_{k_3}, \quad k=k_2-k_1-k_3; \\
&\alpha^*_{k_1}\beta^*_{k_2}\alpha^*_{k_3}, \quad k=k_3-k_1-k_2; \quad \beta^*_{k_1}\beta^*_{k_2}\beta^*_{k_3}, \quad k=-k_1-k_2-k_3 \qquad \textbf{(5.53)}
\end{aligned}
$$

The last part of the integral is defined as

$$
\begin{aligned}
I_3 &\equiv \int d\eta\frac{H^9}{\dot{\phi}^3}\frac{\lambda G}{32\pi^3}ia\log^2 a(k_1+k_2+k_3)\Big(-\alpha^*_{k_1}\alpha^*_{k_2}\alpha^*_{k_3}e^{i(k_1+k_2+k_3)} \\
&-\alpha^*_{k_1}\alpha^*_{k_2}\beta^*_{k_3}e^{i(k_1+k_2-k_3)\eta}+\alpha^*_{k_1}\beta^*_{k_2}\alpha^*_{k_3}e^{i(k_1-k_2+k_3)\eta} \\
&+\alpha^*_{k_1}\beta^*_{k_2}\beta^*_{k_3}e^{i(k_1-k_2-k_3)\eta}-c.c\Big). \qquad \textbf{(5.54)}
\end{aligned}
$$

The integral above becomes

$$
\begin{aligned}
I_3 &= \frac{iH^8}{\dot{\phi}^3}\frac{\lambda G}{32\pi^3}(k_1+k_2+k_3)\sum_k E_k\Big[\frac{1}{3Ha}\Big(6ikF_4(1,1,1,1;2,2,,2,2;-\frac{ik}{Ha})\\
&\quad +\log a(6ikF_3(1,1,1;2,2,2;-\frac{ik}{Ha})+Ha\log a(3\log(\frac{ik}{Ha})\\
&\quad +3\Gamma(0,ik/Ha)+\log a+3\gamma))\Big)\Big] \qquad \textbf{(5.55)}
\end{aligned}
$$

where time independent coefficients E_k are defined as

$$
\begin{aligned}
&\alpha^*_{k_1}\alpha^*_{k_2}\alpha^*_{k_3}, \quad k=k_1+k_2+k_3; \quad \alpha^*_{k_1}\alpha^*_{k_2}\beta^*_{k_3}, \quad k=k_1+k_2-k_3;\\
&\alpha^*_{k_1}\beta^*_{k_2}\alpha^*_{k_3}, \quad k=k_1-k_2+k_3; \quad \alpha^*_{k_1}\beta^*_{k_2}\beta^*_{k_3}, \quad k=k_1-k_2-k_3;\\
&\beta^*_{k_1}\alpha^*_{k_2}\alpha^*_{k_3} \quad k=k_3+k_2-k_1; \quad \beta^*_{k_1}\alpha^*_{k_2}\beta^*_{k_3}, \quad k=k_2-k_1-k_3;\\
&\alpha^*_{k_1}\beta^*_{k_2}\alpha^*_{k_3}, \quad k=k_3-k_1-k_2; \quad \beta^*_{k_1}\beta^*_{k_2}\beta^*_{k_3}, \quad k=-k_1-k_2-k_3 \qquad \textbf{(5.56)}
\end{aligned}
$$

The bispectrum has the following final form using equation(5.24),

$$
\begin{aligned}
B_\zeta(k_1,k_2,k_3) &= 2i\frac{\dot{\phi}^4}{H^6}\sum_i\Big(\frac{1}{k_i{}^2}\Big)(\alpha_{k_1}+\beta_{k_1})(\alpha_{k_2}+\beta_{k_2})(\alpha_{k_3}+\beta_{k_3})\\
&\quad \times\Big\{1+\frac{\lambda GH^2}{32\pi^3}\log^3 a\Big\}\times I+c.c \qquad \textbf{(5.57)}
\end{aligned}
$$

where I is defined as $I_1+I_2+I_3$.

6. CONCLUSIONS AND RECOMMENDATIONS

Our universe is homogeneous and isotropic for scales larger than hundred Mpc and it is expanding according to Hubble's law. The metric that describes this geometry is called FRW(Friedmann Robertson Walker). Einstein equations tell us the dynamical behaviour for this geometry via Friedmann equations. Standard big bang cosmology successfully explains many observables such as nuclear abundance light elements. But standard cosmology could not explain major unsolved puzzles such as flatness, horizon and initial perturbations.

Inflation, an accelerated expanding phase of the universe offers an explanations for the problems of standard big bang cosmology. There are various mechanisms to produce inflation; e.g. scaler field(inflaton) driven inflation with appropriate potential, lambda driven inflation etc. We believe that quantum fluctuations, during this inflationary phase, are the origin of structures in the universe.

Universe filled with Cosmic Microwave Background Radiation(CMB)that we observe WMAP and Planck experiments are the main reason why physicist threat cosmology seriously. The error bars are much smaller than ordinary astrophysical experiments. Statistical analysis of CMB experiments is done using a tool called the two point correlation function(power spectrum).

Each model of inflation has a specific prediction for power spectrum. This gives us a chance to test sum of these models. In our thesis we were interested in the non-gaussianity parameter, f_{NL}. It is an important parameter that we can use to rule out sum inflationary models.

It was shown that spectator scaler fields interact with inflaton and modify the mode function. A two loop calculation was done by Kahya, Onemli, Woodard and it was shown that $\zeta - \zeta$ correlator becomes time-dependent. It was also shown by Ganc that one can get enhance $f_N L$ parameter for non-vacuum initial state(non-BD). To do that Ganc calculated the bispectrum(three point correlation function) with ordinary mode function. In this work we calculated f_{NL} parameter for the non-Bunch Davies vacuum state with the modified mode function.

We found that the bispectrum has the following final form

$$\begin{aligned} B_\zeta(k_1,k_2,k_3) &= 2i\frac{\dot{\phi}^4}{H^6}\sum_i\left(\frac{1}{{k_i}^2}\right)(\alpha_{k_1}+\beta_{k_1})(\alpha_{k_2}+\beta_{k_2})(\alpha_{k_3}+\beta_{k_3}) \\ &\quad \times\left\{1+\frac{\lambda GH^2}{32\pi^3}\log^3 a\right\}I+c.c. \end{aligned} \quad \mathbf{(6.1)}$$

where I is defined as $I_1+I_2+I_3$. I_1,I_2,I_3 is given by (5.53),(5.56) and (5.59). In future work we would like to calculate f_{NL} numerically to put constraint on coupling constant between spectator field and inflaton.

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