ISTANBUL TECHNICAL UNIVERSITY ★ INSTITUTE OF SCIENCE AND TECHNOLOGY

COSMOLOGICAL CONSTANT RELOADED

M.Sc. Thesis by
Çiğdem ETKER

Department : Physics Engineering

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FOREWORD

First, I want to thank my advisor Assoc. Prof. Dr. Savaş ARAPOĞLU for his intense and advisory supervision throughout all this thesis work. I have learned a lot from him. He supported me at every stage of my work, showed patience and guidence.

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July 2010 Çiğdem ETKER
. Physics Engineer

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ABBREVIATIONS

ACBAR	:	Arcminute Cosmology Bolometer Array Receiver
BAO	:	Baryon Acoustic Oscillations
BB	:	Big Bang
BOOMERanG	:	Balloon Observations Of Millimetric Extragalactic Radiation and Geophysics
CBI	:	Cosmic Background Imager
CDM	:	Cold Dark Matter
CMBR	:	Cosmic Microwave Background Radiation
COBE	:	COsmic Background Explorer
EFE	:	Einstein Field Equations
Fig.	:	Figure
FIRAS	:	Far InfraRed Absolute Spectrophotometer
FRW	:	Friedmann-Robertson-Walker
GR	:	General Relativity
MAXIMA	:	Millimeter Anisotropy eXperiment IMaging Array
MOG	:	MOdified Gravity
SDSS	:	Sloan Digital Sky Survey
SNe Ia	:	Type Ia supernovae
SR	:	Special Relativity
WMAP	:	Wilkinson Microwave Anisotropy Probe

LIST OF FIGURES

LIST OF SYMBOLES

$\mathbf{a}$	:	Scale factor
$\mathbf{A}$	:	Chaplygin gas parameter
$\mathbf{c}$	:	Speed of light
$\mathbf{d_A}$	:	Angular diameter distance
$\mathbf{d_E}$	:	Event horizon distance
$\mathbf{d_H}$	:	Horizon distance
$\mathbf{d_L}$	:	Luminosity distance
$\mathbf{d_p}$	:	Proper distance
ε	:	Energy density
$\mathbf{f}$	:	Flux
$\mathbf{g}_{\mu\nu}$	:	Metric tensor
$\mathbf{G}$	:	Gravitational constant
$\mathbf{G}_{\mu\nu}$	:	Einstein tensor
$\mathbf{H}$	:	Hubble parameter
$\mathbf{K}$	:	Curvature constant
κ	:	Normalized curvature constant
$\mathbf{L}$	:	Luminosity
$\mathbf{\Lambda}$	:	Cosmological constant
$\mathbf{m}$	:	Apparent magnitude
$\mathbf{M}$	:	Absolute magnitude
$\mathbf{\Omega}$	:	Density parameter
$\mathbf{\Omega}_c$	:	Critical density
$\mathbf{p}$	:	Pressure
$\mathbf{q}$	:	Deceleration parameter
ρ	:	Mass density
$\mathbf{R}^{\sigma}_{\mu\rho\nu}$	:	Riemann tensor
$\mathbf{R}$	:	Ricci scalar
$\mathbf{R}_{\mu\nu}$	:	Ricci tensor
$\mathbf{S}$	:	Action
$\mathbf{t_H}$	:	Hubble age
$\mathbf{T}_{\mu\nu}$	:	Energy-momentum tensor
ω	:	Equation of state parameter
$\mathbf{V}$	:	Scalar field potential
$\mathbf{z}$	:	Redshift

COSMOLOGICAL CONSTANT RELOADED

SUMMARY

When Einstein was building his cosmological model he thought that the Universe is static, therefore added Λ to his equations as the cosmological constant to rearrange the geometry of the Universe. However, observations of Hubble proved Einstein was wrong, the Universe was not stationary, it was expanding in time. After almost eighty years Type Ia Supernovae observations showed that the Universe was not only expanding, but also accelerating and caused modern cosmology to readopt the cosmological constant. But this time cosmological constant took its place at the opposite side of the Einstein field equation as a new component in the Universe.

In this thesis work we will start by a short introduction with Einstein's GR, carry on with the Friedmann's expanding Universe model. Later we discuss the by far observational evidences and how they support the idea of the dark energy. We introduce the simplest and yet the best candidate - the cosmological constant - for describing the time evolving Universe. Finally we will discuss other developed theories providing an explanation to the accelerating expansion.

KOZMOLOJIK SABIT YENIDEN

ÖZET

Einstein kendi kozmolojik modelini kurarken evrenin durağan olduğunu düşünmüştü, bu nedenle evrenin geometrisini yeniden düzenlemek için denklemlerine kozmolojik sabit Λ'yı ekledi. Ancak Hubble'ın gözlemleri Einstein'ı haksız çıkardı, evren durağan değildi, zamanla genişliyordu. Bundan yaklaşık seksen yıl sonra Tip Ia Süpernova gözlemleri evrenin yalnızca genişlemediğini, aynı zamanda genişlemenin ivmelenğını ortaya koydu. Bu kanıtla beraber kozmoloji kozmolojik sabiti yeniden sahiplendi, ancak bu kez evrende yeni bir bileşen olarak Einstein alan denklemlerinin sağ tarafına koyuldu, ve gizemli karanlık enerji için bir aday oldu.

Bu tez çalışmasında öncelikle Einstein'ın genel görelilik kuramının kısa bir tanımını veriyoruz ve Friedmann'ın genişleyen evren modeli ile devam ediyoruz. Daha sonra bu güne kadar yapılan deneysel gözlemleri ve sonuçlarının karanlık enerji kavramını nasıl desteklediğini tartışıyoruz, ve hızlanan genişlemeyi açıklamak için bu güne kadarki en basit aday olan kozmolojik sabiti zamanla değişen evrene adapte ediyoruz. Son olarak hızlanan gelişmeyi açıklamaya aday diğer teorileri tartışıyoruz.

1. INTRODUCTION

In the past ten years modern cosmology faced a big change with the evidence of the acceleratingly expansion, provided by the observational results by the luminosity-distance relations of the Type Ia Supernovae. Two groups of scientists, The High Z Supernovae Research Team [1] and then The Supernova Cosmology Project [2], examining Supernovae explosions have published papers on their findings on the accelerating expansion of the Universe in 1998.

The result was unexpected since the gravity should have caused a deceleration. Although the expansion was for certain, the reason for it was indeterminable, therefore the term dark energy is thought to be suitable for this repulsive energy form.

In years particle physicists have discovered that Einstein's cosmological constant can be treated as the vacuum energy density with a negative pressure, playing a dynamical role in the Universe. Therefore the retreated cosmological constant term happens to be a suitable candidate for the dark energy slowing down the effect of gravity. CMB observations showed that the Universe is mainly dominated by the dark energy, therefore dark energy density already overcame the matter density and reversed the direction of the expansion, yielding it to accelerate.

Introducing cosmological constant is not the only way to explain the Universe; it is the simplest one by far, and for that reason we have thought that it is worth studying.

In the construction of this thesis work several text books, lecture notes and articles has been used as source materials. Therefore we want to quote the books [3], [4], [5], the lecture notes [6], [7], [8] and articles [9], [10], [11] as a reference and they can be used as a further reading material.

In Chapter 2 we start our discussion with a brief introduction to Einstein's General Theory of Relativity. Later by introducing the Cosmological Principle we discuss the required conditions to built a cosmological model for an expanding Universe. By assuming the homogeneity and isotropy of the Universe we construct the suitable mathematical model to describe our expanding Universe.

Chapter 3 mainly discusses the dynamics of an expanding Universe. We derive the important equations of the FRW universe, which help us examine the relations between the expansion of the Universe with parameters related to the content of the Universe.

In Chapter 4 we discuss the different ways of measuring the extra galactic distances and how those ways help to determine the scale factor or the density parameters of the Universe.

Chapter 5 is on the late time observations in cosmology as a proof of the expansion, homogeneity and isotropy of the Universe. We start with the supernovae observations showing the accelerated expansion, continue with the CMBR concerning the homogeneity and isotropy and later discuss the BAO results.

Chapter 6 is devoted to the strongest dark energy candidate, the readopted Einstein's Cosmological Constant Λ. We start with the early observations, explain how Hubble's law states the Universe is expanding, retreat Λ as a new component in the Universe. Later we examine the constraints on the cosmological parameters in a dark energy dominated Universe and discuss how the observational data supports the Λ dominated Universe.

Finally, in Chapter 7, we examine other dark energy candidates; mainly quintessence, k-essence, phantom field and modified gravity.

2. THE MATHEMATICAL MODEL OF ISOTROPIC AND HOMOGENEOUS UNIVERSE

2.1 A Brief Review of General Relativity

The mathematical description of our Universe can only be done by using Einstein's theory of general relativity. In GR space and time are handled together as a four dimensional manifold where the line element between two nearby points is given by

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu. \tag{2.1}$$

$g_{\mu\nu}$ on the r.h.s is the metric tensor describing the spacetime. The subscripts μ, ν take values between 0 and 3, 0 corresponding to time coordinate and the remaining corresponding to spatial coordinates. In SR the curvature of spacetime caused by gravity is not taken into consideration. The calculations are done on the flat Minkowski spacetime, which in $4D$ has this line element in inertial coordinates

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2. \tag{2.2}$$

The Minkowski spacetime is flat but also static and therefore only satisfies the conditions in SR. However the gravitational effect of present matter in the Universe causes the spacetime to curve.

Einstein's field equation is the relation between curvature and matter energy density.

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi\, G}{c^4} T_{\mu\nu} \tag{2.3}$$

$T_{\mu\nu}$ on the r.h.s. is called the stress energy tensor and it represents the energy produced by the matter in space. The l.h.s. of the equation only depends on the curvature of spacetime, which is defined by the Riemann tensor $R^\sigma{}_{\mu\rho\nu}$.

By contraction of the indices Ricci tensor $R_{\mu\nu}$ can be derived, which by futher contraction, yields the Ricci scalar R.

(2.3) works in both ways. It tells how matter curves spacetime and how this curvature of spacetime determines matter's motion. Though (2.3) is not false, it is incomplete. Albert Einstein built his cosmological model on the assumption of homogeneity and isotropy, but he also assumed our Universe does not change with time. However by the rules of GR, the Universe should be either contracting or expanding. In orer to reconcile the stationary Universe idea with GR, by introducing the Greek letter Λ as the cosmological constant [12], he modified his field equations as

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \tag{2.4}$$

Einstein used Λ as an independent parameter proportional to the metric effecting the curvature. Hence, the Universe was allowed to be static. Einstein did not predict the expansion of the Universe. After Hubble's redshift observations in 1929 [13] proved Universe's expansion, Einstein erased Λ from his equations and called it the "biggest blunder" of his life.

2.2 The Assumption of Homogeneity and Isotropy - The Cosmological Principle

The ancient astronomy is based on the ideas of the Greek philosophers Plato and his student Aristotle. They introduced a geocentric model, so that the Earth was at the center and everything else the Moon, the Sun, the planets, fixed stars were revolving around the Earth by drawing circles. This idea was adopted by many other philosophers and considered to be true for twenty centuries.

In the 16th century Nicolaus Copernicus's "De Revolutionibus Orbium Coelestium" (On the Revolutions of the Heavenly Bodies) was published. In the book Copernicus formulated the observed motions of celestial objects, putting the Sun, not the Earth, to the center of the Universe. Though Copernicus' heliocentric model is very incomplete to define our Universe, it is considered as the starting point of modern cosmology. Nicolaus Copernicus started a scientific revolution, by suggesting Earth does not occupy a privileged location in the Universe.

"Cosmological Principle" or regarding to his thoughts the "Copernicus Principle" is the assumption stating for any observer, the universe is homogeneous and isotropic on very large spatial scales at all times. Under the influence of Copernicus we may come to a conclusion that no observer is any more special than the other in physics, meaning our Universe shall look exactly the same from one's vantage point on Earth, with someone else's at anywhere else, at the same cosmological epochs.

Saying the the Universe is isotropic is implying that there is no preferred direction in the universe, and saying it is homogeneous is implying there is no preferred place at the universe. These two may appear to be similar, but they are completely different. In a homogeneous Universe to be homogeneous, the average matter density at some point x must be equal to the density at every other point $x^{'}$. However this does not require isotropy. The universe may still seem completely different at point x from point $x^{'}$. However if a universe is isotropic in every direction, then it is also homogeneous at every location.

Cosmological principle allows our Universe to be treated as a perfect cosmic fluid. Galaxies can be treated as dust particles and a volume element of the fluid encloses clusters of galaxies, which is extremely small compared to the whole system. Therefore we consider these assumptions hold true only on large scales, as large as $100 Mpc$ or larger. On smaller scales the universe is neither isotropic nor homogeneous, it is clumpy.

Cosmological principle is being used long before there was any evidence for homogeneity and isotropy of the Universe. Though the Big Bang theory, proposed by Georges Lemaitre, is the most accepted among scientists to explain the existence and the evolution of the Universe, theory directly assumes cosmological principle holds true. Traditional Big Bang theory can not provide any explanation for the horizon and the flatness problem. Suggestions to these problems are made by the inflationary theory [14], proposed by Alan Guth in 1981. Cosmological principle is consistent with evidences provided by late time cosmological tests .

2.3 Metric of Homogeneous and Isotropic Space

Adopting the cosmological principle yields the spatial part of the metric describing the Universe to be a $3-D$ hyperspace with a radius R of constant curvature at any instant time t. Such a spatial metric is maximally symmetric, which implies spherical symmetry, and can be written in the form of

$$ds^2 = \frac{dr^2}{1-Kr^2} + r^2[d\theta^2 + sin^2\theta d\phi^2]. \tag{2.5}$$

Derivation of (2.5) is given in Appendix B in detail. For a homogeneous and isotropic space in more than 2-dimensions, there exists only three possible solutions for the curvature K in the denominator. K can be zero or take positive and negative values, corresponding to flat, closed and open Universes.

$$K = \begin{cases} > 0 & \text{closed} \quad \text{reel R} \\ = 0 & \text{flat} \quad \text{R} \to \infty \\ < 0 & \text{open} \quad \text{imaginary R} \end{cases} \tag{2.6}$$

2.3.1 Comoving Coordinates

Since we have treated the Universe as a perfect cosmic fluid, galaxies follow geodesic worldlines for the fact that their motion is only determined by the self gravity of the Universe. Therefore it is convenient to work with the comoving coordinates, which is the cosmic rest frame.

In this preferred special frame galaxies' coordinates are fixed, only the distance between them alters with time as the Universe expands. The cosmic scale factor $a(t)$ is introduced as a measure of the Universe's expansion. It is a dimensionless function of time defined as the ratio of $R_0(t)$, the current uniform radius of the universe and $R(t)$, the radius at time t as

$$a(t) = \frac{R(t)}{R_0(t)}. \tag{2.7}$$

The scale factor the present moment is adjusted to be $a(t_0) = 1$.

The expansion makes it hard to determine the distance between two objects in the universe. Assume one galaxy is at the comoving coordinate (R_1, θ, ϕ) and the other at (R_2, θ, ϕ), the proper distance between them is calculated by integrating over the radial coordinate

$$d_p(t_0) = a(t) \int_{R_1}^{R_2} dr = a(t)(R_2 - R_1) = a(t)\Delta R. \quad \textbf{(2.8)}$$

Since R_1 and R_2 are fixed, the proper distance is only a function of the scale factor $a(t)$.

A useful quantity used to describe here is the Hubble constant $H(t)$ given by

$$H(t) = \frac{\dot{a}}{a}. \quad \textbf{(2.9)}$$

Hubble constant defines the rate of the expansion and depends on time. However it is called constant, for the fact that at an instant of time it has a constant value all over the Universe. The Hubble constant at current time is denoted by H_0.

The relative velocity of the galaxies, can be obtained by taking a time derivative of $d_p(t_0)$ as

$$\dot{d}_p(t_0) = a\dot{(t)}\Delta R = \frac{\dot{a}}{a} a\Delta R, \quad \textbf{(2.10)}$$

and evaluated as a function of the Hubble constant as

$$v_p(t_0) = H_0 d_p(t_0). \quad \textbf{(2.11)}$$

2.3.2 Friedmann-Robertson-Walker Metric

Unlike Einstein, Russian cosmologist Alexander Friedmann was thinking that our universe is evolving in time. He was in search of a metric which is an exact solution to Einstein's field equations, suitable for the cosmological principle and evolves in time. Friedmann's solutions [15] of negative, positive and zero curvature, expanding spacetime was published in 1922, long before Hubble's redshift observations in 1930's. In modern cosmology FRW metric is used to describe our spatially homogeneous and isotropic Universe. In its most compact form is given by

$$ds^2 = -c^2dt^2 + a(t)^2 \left[dr^2 + S_K^2(r,R)(d\theta^2 + sin^2\theta d\phi^2)\right]. \quad \textbf{(2.12)}$$

The curvature K is of arbitrary magnitude. It is convenient to normalize it with the radius of the curvature as

$$K = \frac{\kappa}{R^2}, \quad \textbf{(2.13)}$$

and allow κ to take only the values $+1$, 0, -1; for closed, flat and open Universes. Depending upon the geometry, $S_K(r,R)$ in (2.12) can be of three different kind.

$$S_K(r,R) = \begin{cases} Rsin(\frac{r}{R}) & \text{closed} \quad \kappa = 1 \\ r & \text{flat} \quad \kappa = 0 \\ Rsinh(\frac{r}{R}) & \text{open} \quad \kappa = -1 \end{cases} \quad \textbf{(2.14)}$$

With a change of variable as $x = S_K(r,R)$; FRW metric can be written in its most general form

$$ds^2 = -c^2dt^2 + a(t)^2 \left[\frac{dx^2}{1 - \frac{\kappa}{R^2}x^2} + x^2(d\theta^2 + sin^2\theta d\phi^2)\right]. \quad \textbf{(2.15)}$$

3. COSMIC DYNAMICS

The assumption of homogeneity and isotropy is very useful for it simplifies the dynamics of the Universe. FRW metric has comoving spatial coordinates and that fixes the curvature side or the Einstein equation in (2.3), and suitably we can treat to matter and energy content of the Universe as a perfect fluid at rest.

Mainly two parameters identify the perfect fluid, its mass density ρ and pressure p, and they are needed to define the stress-energy tensor $T_{\mu\nu}$ of the perfect fluid. The four vector of the perfect fluid in its rest frame is

$$U^{\mu} = c(1,0,0,0), \tag{3.1}$$

then the stress-energy tensor is given by

$$T_{\mu\nu} = (p+\rho c^2)\frac{U_{\mu}U_{\nu}}{c^2} + pg_{\mu\nu}, \tag{3.2}$$

or multiplied by the inverse metric

$$T^{\mu}_{\nu} = diag(-\rho c^2, p, p, p). \tag{3.3}$$

The trace of the stress-energy tensor gives

$$T = -\rho c^2 + 3p. \tag{3.4}$$

By writing (2.3) in it's trace-reversed form

$$R_{\mu\nu} = \frac{8\Pi\, G}{c^4}(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}). \tag{3.5}$$

Calculations on the FRW metric is given in Appendix C, and the related components of the Ricci tensors calculated there.

Choosing $\mu, \nu = 0$ gives

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + \frac{3p}{c^2}). \tag{3.6}$$

And choosing $\mu, \nu = 1$ gives

$$\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + \frac{2\kappa c^2}{a^2 R_0^2} = 4\pi G(\rho - \frac{p}{c^2}). \tag{3.7}$$

Combining equations (3.6) and (3.7) yields

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho - \frac{\kappa c^2}{a(t)^2 R_0^2}. \tag{3.8}$$

The equations in (3.6) and (3.8) are called the Friedmann equations. While (3.8) is commonly referred as the Friedmann equation, (3.6) is referred as the Friedmann's acceleration equation. Friedmann equations describes the universe's dynamical evolution depending on the matter content of the Universe.

Einstein tensor $G_{\mu\nu}$ and $T_{\mu\nu}$ are related with each other with the Einstein field equation

$$G_{\mu\nu} = \frac{8\Pi G}{c^4} T_{\mu\nu}. \tag{3.9}$$

Conracting the differential Bianchi identity

$$R_{\mu\nu[\lambda\sigma;\alpha]} = 0, \tag{3.10}$$

yields

$$\nabla_\mu G^{\mu\nu} = 0. \tag{3.11}$$

Therefore to see how the energy density evolves with time, we shall look at the conservation of the energy-momentum in GR using

$$\nabla_\mu T^{\mu\nu} = 0. \tag{3.12}$$

We set $\nu = 0$, since our perfect fluid is at rest and insert the stress energy tensor of the fluid into (3.12) and evaluate

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + \frac{p}{c^2}), \tag{3.13}$$

Equation (3.13) is called the fluid equation. However only two of these three equations are linearly independent, and there are three unknown parameter in the equation. ρ, p and $a(t)$. Therefore another equation is needed.

The third equation concerning the relation between the mass density ρ and pressure p is the equation of state

$$p = \omega\rho. \tag{3.14}$$

where ω is a constant having values $\frac{1}{3}$, 0, -1 depending on whether radiation, matter or dark energy dominates, respectively.

The continuity equation (3.13) is restated using (3.14)

$$\dot{\rho} = -3\frac{\dot{a}}{a}(1+\omega)\rho. \tag{3.15}$$

The energy density $\varepsilon(t)$ is related to the mass density ρ only by a factor of c^2.

$$\varepsilon(t) = \rho c^2. \tag{3.16}$$

For $\kappa = 0$ and the Friedmann equation in (3.8) becomes

$$H^2 = \frac{8\Pi G}{3}\rho(t). \tag{3.17}$$

$\rho(t)$ derived from here is called the critical density, denoted with $\rho_c(t)$ and is given by

$$\rho_c(t) = \frac{8\Pi G}{3}H(t)^2. \tag{3.18}$$

By using critical density, the dimensionless density parameter $\Omega(t)$ is defined as

$$\Omega(t) = \frac{\rho(t)}{\rho_c(t)}. \tag{3.19}$$

If universe's energy density is less than the critical density, $\Omega(t) < 1$, the gravitational pull of the matter will not be enough to stop the universe's expansion, and the universe will expand forever to end in a 'Big Chill'. Such a universe is negatively curved and has a shape of the surface of a saddle ($\kappa = -1$).

If the density of the universe is greater than the critical density, $\Omega(t) > 1$, the gravitational pull of the matter will prevent the universe from expanding, and eventually pull it back. In the absence of a repulsive force such as dark energy, the universe will collapse back on it self and end in a 'Big Crunch'. Such a universe is positively curved and has a shape of the surface of a sphere ($\kappa = 1$).

For a Universe to be exactly flat, it has to contain equal mass density with ρ_c, where $\Omega(t) = 1$. The curvature can be treated like a content in the Universe, although it is in reality not. However a mass density of

$$\rho_k = -\frac{\kappa c^2}{8\Pi G a^2}, \tag{3.20}$$

can be assigned to curvature. Then if we denote the curvature density by Ω_k, it satisfies the equation

$$1 - \Omega = \Omega_k, \tag{3.21}$$

where Ω is the sum over all other content components of the Universe.

4. MEASUREMENTS IN COSMOLOGY

For a model Universe with several energy components, inserting the precise values for density parameters into the Friedmann equation, the scale factor $a(t)$ for the Universe can be determined. Or we may determine $a(t)$ from observations and then determine the values of Ω. To measure the cosmic expansion, we need to find a way to measure the distance to the astrophysical objects. Measurements to distant astrophysical objects are observed at a younger age of the Universe of a smaller $a(t)$. Astrophysicists use several ways to measure extragalactic distances. Each way works at a different distant scale.

4.1 Redshift, z- Scale Factor, a Relation

The wavelengths of the photons emitted from the astrophysical objects redshift as the Universe expands. To find the relationship between the redshift z and $a(t)$, first consider one galaxy, located at the comoving radius R_e, emitting a photon at time t_e and a second one at time $t_e + \delta t_e$. Then consider another galaxy is located at the origin of the preferred reference system, receiving those photons at times t_0 and $t_0 + \Delta t_0$. To measure the redshift we need to determine how these times are related.

For both emitted photons the current proper distance to the second galaxy are equal,

$$\int_{t_e}^{t_0} \frac{dt}{a(t)} = \int_{t_e+\Delta t_e}^{t_0+\Delta t_0} \frac{dt}{a(t)}. \tag{4.1}$$

Since the comoving coordinate R_e stays unchanged,

$$\int_{t_0}^{t_0+\Delta t_0} \frac{dt}{a(t)} = \int_{t_e}^{t_e+\Delta t_e} \frac{dt}{a(t)}. \tag{4.2}$$

Most objects in the Universe are observed to be redshifting slowly $(t_e - t_0 << 1)$, implying that $a(t)$ of the Universe is oscillating smoothly. Therefore we can assume

$$\frac{\Delta t_0}{a(t_0)} = \frac{\Delta t_e}{a(t_e)}. \tag{4.3}$$

The relation between emitted and observed wavelengths and time are as usual,

$$\lambda_e = c\Delta t_e; \qquad \lambda_0 = c\Delta t_0. \tag{4.4}$$

To define the redshift z,

$$z = \frac{\lambda_0 - \lambda_e}{\lambda_e} \Longrightarrow z = \frac{\Delta t_0}{\Delta t_e} - 1. \tag{4.5}$$

Combining (4.3) and (4.5) reveals the relationship between z and $a(t)$,

$$1 + z = \frac{a(t_0)}{a(t_e)}. \tag{4.6}$$

Our Universe expands, therefore we know $a(t_0) > a(t_e)$, meaning the light emitted from the distant objects redshift.

4.2 Deceleration Parameter, q

Another measure of expansion is the deceleration parameter q_0. To evaluate q_0, $a(t)$ is expanded in a power series around t_0, and the first few terms is enough to define the slow expansion rate.

$$\begin{aligned} a(t) &= a(t_0) + (t-t_0)\dot{a}(t_0) + \frac{1}{2}(t-t_0)^2\ddot{a}(t_0) + \cdots \\ &= a(t_0)[1 + (t-t_0)H_0 + \frac{1}{2}(t-t_0)^2\frac{\ddot{a}(t_0)a(t_0)}{\dot{a}(t_0)^2}{H_0}^2 + \cdots] \\ &= a(t_0)[1 + H_0(t-t_0) - \frac{1}{2}q_0{H_0}^2(t-t_0)^2 + \cdots] \end{aligned} \tag{4.7}$$

The q_0 on the last term is called the deceleration parameter

$$q_0 = -\frac{a(\ddot{t_0})a(t_0)}{\dot{a}^2(t_0)} = -\frac{a(\ddot{t_0})}{a(t_0)H_0^2}. \tag{4.8}$$

Other than the H_0, q_0 is used as a measure for the evolution of the expansion. When $q_0 < 0$, $\ddot{a}(t_0) > 0$ the expansion is accelerating. When $q_0 > 0$, $\ddot{a}(t_0) < 0$ the expansion is decelerating.

q_0 can be predicted for any model Universe by using the acceleration equation (3.6). Writing (3.14) explicitly for each component, combined with (3.6) and dividing with H^2, gives

$$-\frac{\ddot{a}}{aH^2} = \frac{1}{2}\left(\frac{8\Pi G}{3H^2}\right)\sum_i \rho_i(1+3\omega_i). \quad \textbf{(4.9)}$$

Since the term on the l.h.s. is q_0 and the term in the brackets is ρ_c, this can be written as

$$q_0 = \frac{1}{2}\sum_i \Omega_i(1+3\omega_i). \quad \textbf{(4.10)}$$

For a model universe resembling ours which contains mainly Λ and matter q_0 is

$$q_0 = \left[\frac{1}{2}\Omega_{m,0} - \Omega_{\Lambda,0}\right], \quad \textbf{(4.11)}$$

implying for this Universe's expansion to accelerate, the condition

$$\Omega_{\Lambda,0} > \frac{1}{2}\Omega_{m,0} \quad \textbf{(4.12)}$$

needs to be satisfied.

4.3 Luminosity Distance, d_L

The luminosity distance d_L is another way to measure the distance to an astrophysical object. The relationship between an object's flux and it's actual luminosity is given by

$$f = \frac{L}{A}. \quad \textbf{(4.13)}$$

In a static Euclidean Universe $A = 4\pi R^2$, the surface area of the regular 2-sphere where photons were emitted. However in a curved, time evaluating Universe, defined by the FRW metric, $A = a(t_0)^2 4\pi S_K(r,R)^2$. The luminosity distance d_L is given by

$$d_L = \sqrt{\frac{L}{4\pi f}}. \qquad \textbf{(4.14)}$$

The expansion effects the luminosity in two ways. First the photons' energy decrease by the factor of $(1+z)$ due to redshift. And second, the rate of emitted photons decrease by a $(1+z)$ again due to the stretching of space. Therefore the luminosity distance in an expanding, spatially curved Universe is reduced by a factor of $(1+z)^2$ and given by

$$d_L = S_K(r,R)a(t_0)(1+z). \qquad \textbf{(4.15)}$$

4.4 Angular Diameter Distance, d_A

One can measure the angular diameter distance d_A, when an object's length l is known, by one end being at the coordinates (r,θ,ϕ), and the other end being at $(r,\theta+\delta\theta,\phi)$, the angular diameter distance d_A is defined by

$$d_A = \frac{l}{\delta\theta}. \qquad \textbf{(4.16)}$$

Here l is the proper length of the object. In a FRW Universe, the proper length is defined as,

$$l = a(t_e)S_K(r,R)\delta\theta. \qquad \textbf{(4.17)}$$

Using this in (4.16) will give the angular diameter distance in a spatially curved, expanding Universe

$$d_A = \frac{S_K(r,R)a(t_0)}{(1+z)}. \qquad \textbf{(4.18)}$$

In a static Euclidean Universe, where $z << 1$, the angular diameter distance d_A equals to the proper distance $d_p(t_0) = R\delta\theta$. The angular diameter distance and the luminosity distance are related with each other as

$$d_A = \frac{d_L}{(1+z)^2}. \quad \textbf{(4.19)}$$

4.5 Horizon Distance, d_H

The boundary of the Universe is referred as horizon . There are two different horizons to describe, the particle horizon (sometimes called the cosmological horizon) and the event horizon.

The event horizon is the largest distance from which light emitted now can ever reach the observer. It can not be seen for the light from there did not have time to reach us yet. Event horizon distance can be calculated via

$$d_E = c\int_{t_0}^{\infty} \frac{dt}{a(t)}. \quad \textbf{(4.20)}$$

The particle horizon is the maximum distance from which a particle can travel to the observer, since the instant of Big Bang. Particle horizon distance can be calculated via

$$d_H = c\int_{0}^{t_0} \frac{dt}{a(t)}. \quad \textbf{(4.21)}$$

By being smaller than the event horizon, particle horizon is the distance in which particles can stay in causal contact.

The Hubble age of a Universe is given by the inverse of H, when the Universe is expanding with constant velocity

$$t_H = \frac{1}{H}. \quad \textbf{(4.22)}$$

Hubble sphere is defined as a spherical region beyond which a particles velocity exceeds the speed of light and therefore the radius of Hubble sphere is the farthest distance in which particles can stay in causal contact,

$$d_{phs} = ct_H = \frac{c}{H}. \quad \textbf{(4.23)}$$

5. OBSERVATIONS IN COSMOLOGY

5.1 Type Ia Supernovae

One way to specify a cosmological model is to determine the deceleration parameter q_0. By using the magnitude redshift relation q_o can be measured. If the luminosity of an object is known it is identified as a standard candle and redshift distance relation can also be used to determine q_0. The problem is to find an astrophysical object to identify as a standard candle.

The apparent magnitude of an astrophysical object is related to it's flux logarithmically. For two object's having fluxes f_1 and f_2, their apparent magnitude is related as

$$m_1 - m_2 = -2.5log\left(\frac{f_1}{f_2}\right). \quad \textbf{(5.1)}$$

The absolute magnitude (intrinsic brightness) M is defined as the magnitude of a $10pc$ distant source. And $m - M$ is called the distance modulus and can be expressed in terms of the luminosity distance d_L as

$$m - M = 5log(\frac{d_L}{10}). \quad \textbf{(5.2)}$$

The luminosity distance in (4.15)can be expanded and written in terms of H_0 and q_0 and z for a flat Universe as

$$d_L = \frac{c}{H_0}z\left(1 + \frac{1-q_0}{2}z\right). \quad \textbf{(5.3)}$$

For H_0 is a known value and q_0 is related to the density parameters of dark energy and matter with (4.10).

Type Ia Supernovae (SNe Ia) are the result of the evolution of binary star systems with components of one massive star and one smaller star. One way for a SNe

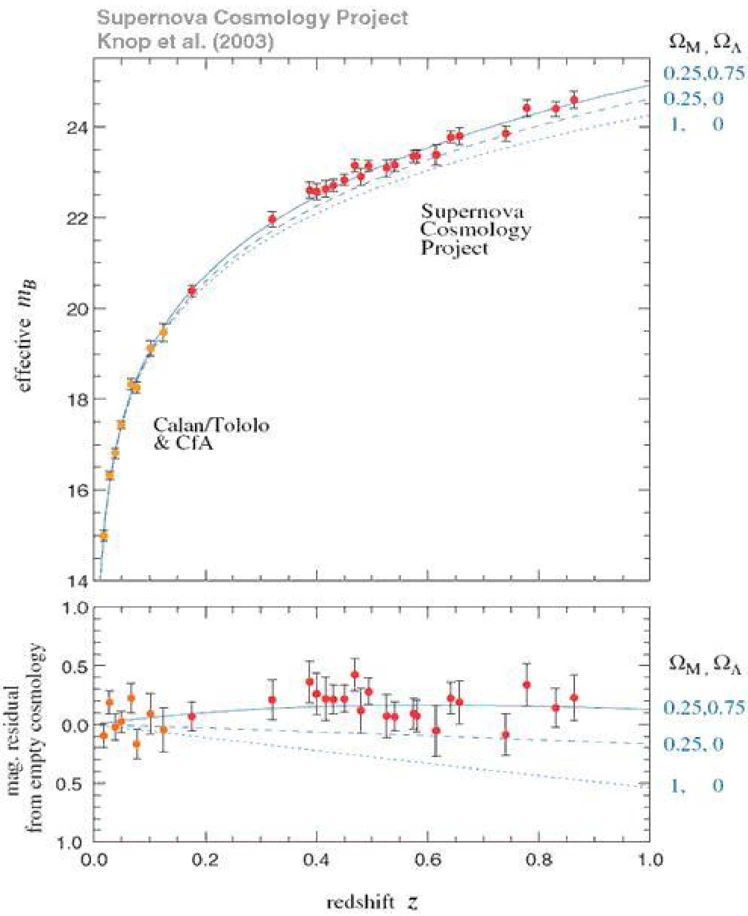

Figure 5.1: Hubble diagram of Type Ia Supernovae /hubbleSNeIa.jpg

Ia to evolve is the small star accreting matter from the massive star causing it to became a carbon-oxygen white dwarf. In time while the massive star becomes a white dwarf, the small star keeps accreting matter. If enough matter is accreted and small star exceeds a mass limit called the Chandrasekhar mass of ≈ 1.4 Solar Masses, it explodes in a supernovae. Since Type Ia Supernovae occur at the Chandrasekhar limit, all have the same uniform luminosity ($M \approx -19.5$) and they are bright standardizable candles that can be observed even at high redshifts ($z \approx 1$).

Type Ia supernovae are mainly characterized by the silicon lines in their spectrum and are distinguished from Type IIa supernovae with the lack of hydrogen lines. Type Ia supernova can also be identified by the second peak in it's near infrared light curve and the correlation between their absolute magnitude M and the entire brightness of the host galaxies they reside.

Fig.5.1 shows the plot of redshift-magnitude relation obtained by the Supernova Cosmology Project. Collected data from supernovae shows that a flat cosmological constant dominated model is much more suitable for the Universe than the matter dominated model.

If matter density is considered as $\Omega_M \approx 0.3$, cosmological constant density is obtained as $\Omega_\Lambda \approx 0.7$.

In May 1998 the first paper on the observations of SNe Ia is published by the High Z-Supernova Team and later in December 1998 another group of scientists Supernova Cosmology Project published a paper on their determination of the universe's expansion at an increasing rate. For that reason Type Ia Supernovae are considered as the first evidence of the existence of dark energy in the Universe and SNe Ia provides constraints on the dark energy equation of the state parameter.

5.2 Cosmic Background Radiation

Cosmic microwave background radiation is the cooled remnant of the early universe, filling the observable sky almost uniformly in all directions. Mostly referred as CMB or CMBR, cosmic background radiation is a form of electromagnetic radiation, shining in the microwave regions of the spectrum, and considered as the most reliable evidence of BB.

CMBR was detected by radio astronomers Arno Penzias and Robert Wilson as a continuous background noise of a radio signal [16]. Later with the help of Robert Dicke, P.J.E. Peebles and D.T. Wilkinson the noise has been interpreted as a remnant of BB [17]. The discovery of CMBR has won Penzias and Wilson the physics Nobel prize in 1978.

In modern cosmology, the "Big Bang" is hot, dense, fluctuating region of space which is a theoretical singularity of GR, referring to the very initial stage of our Universe. Around $10^{-36}s$ after BB a phase transition may have caused that region of the Universe to enter a stage called "false vacuum" and experience a rapid exponential expansion called "Inflation" [14]. During inflation false vacuum's energy density remained constant, however the total energy increased at least by a factor of 10^{75}. This does not violate the conservation of energy since the repulsive gravitational field balances the increasing amount in the matter's energy density. The inflationary era ended around $10^{-33}s$ after BB with the decaying of the false vacuum, the energy it contained is released and transformed into radiation and elementary particles. At the end of Inflation our Universe's dynamics were mainly dominated by radiation. Because the acceleration was exponential, it was slow enough for smoothing out nearly all inhomogeneities and radiation came into a state of thermal equilibrium.

Today we observe them as CMB photons with a nearly uniform $2.7K$ temperature. After recombination the Universe became transparent, and matter became dominant over radiation.

Universe was filled with a hot plasma consisting of electrons, protons, and radiation. The plasma contained also a small amount of heavier elements with neutrons, therefore referred as photon-baryon plasma. Around $10^{13}s$ the Universe's temperature was decreased due to the expansion to $\approx 3000K$, at which radiation did not have enough energy to ionize atoms anymore. This allowed free electrons to bind with a nucleus and neutral atoms, mainly hydrogen atoms were formed. This process is called recombination. With the lack of free electrons, photons could not Thomson scatter from atoms. Photons scattered for the last time directly from the matter and started to wander freely in the space. There happened the decoupling of matter and radiation. The time when decoupling occurred is mostly referred as the "time of last scattering", and the spherical surface where the photons scattered for the last time is called the "surface of last scattering".

The universe keeps expanding acceleratingly. The decoupled photons traveled a very long distance since the early stages of the universe and now filling a greater and a cooler one. Though they are less energetic and fainter, when the sky is observed with a sensitive microwave telescope they can be seen as a faint background glow and this is the CMB, which is the most of the present universe's radiation content. CMBR is important in many ways, mainly because it is a unique source of information, since CMB photons carry information concerning very early universe. The BB theory predicts that CMBR should be in the form

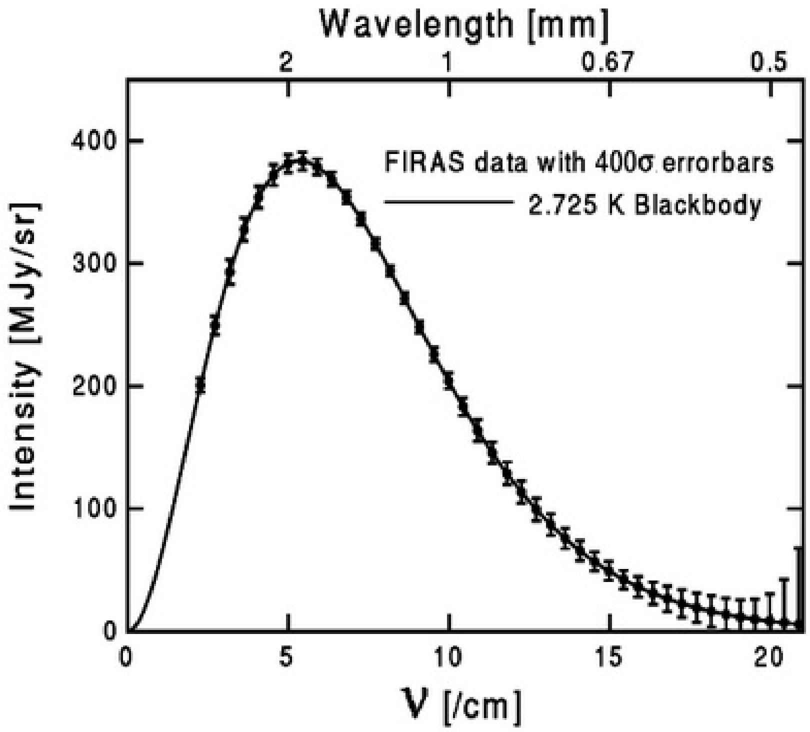

Figure 5.2: The cosmic microwave background spectrum measured by the FIRAS instrument on the COBE satellite

of a blackbody, since the process of multiple scattering should produce one. Fig.5.2 shows that this is in agreement with the experimental data collected with the COBE satellite containing FIRAS instrument, which makes the CMBR spectrum is the most precise thermal black body spectrum ever measured in the nature. The spectrum peaks at $1.9mm$ wavelength, which corresponds to microwave light. Since the time of the last scattering the color temperature of the CMBR spectrum has cooled down to $2.725K$, almost isotropic everywhere in the observable universe. This reduction is obviously the result of the expansion between the time the photons were emitted and now. CMBR temperature is directly proportional to the redshift with the relation

$$T = 2.725(1+z). \tag{5.4}$$

meaning with the universe's further expansion the temperature of CMBR will keep decreasing.

Thus mainly uniform, depending on the size and location of the region examined, CMB has observed to have temperature fluctuations [18,19]. These fluctuations help to examine the origin, evolution and content of the universe. In 1992, COBE satellite measured that these fluctuations are in the order of 10^{-5} [20]. However COBE can only measure from 10^o to 90^o, which are large angular scales and therefore only initial fluctuations could be seen, the structure formation of the universe could not be revealed.

Later observations done with MAXIMA [21], BOOMERanG [22] and finally WMAP revealed that there are small-scale anisotropies, which correspond to the physical scale of today's observed structure. CMBR anisotropies are analyzed by

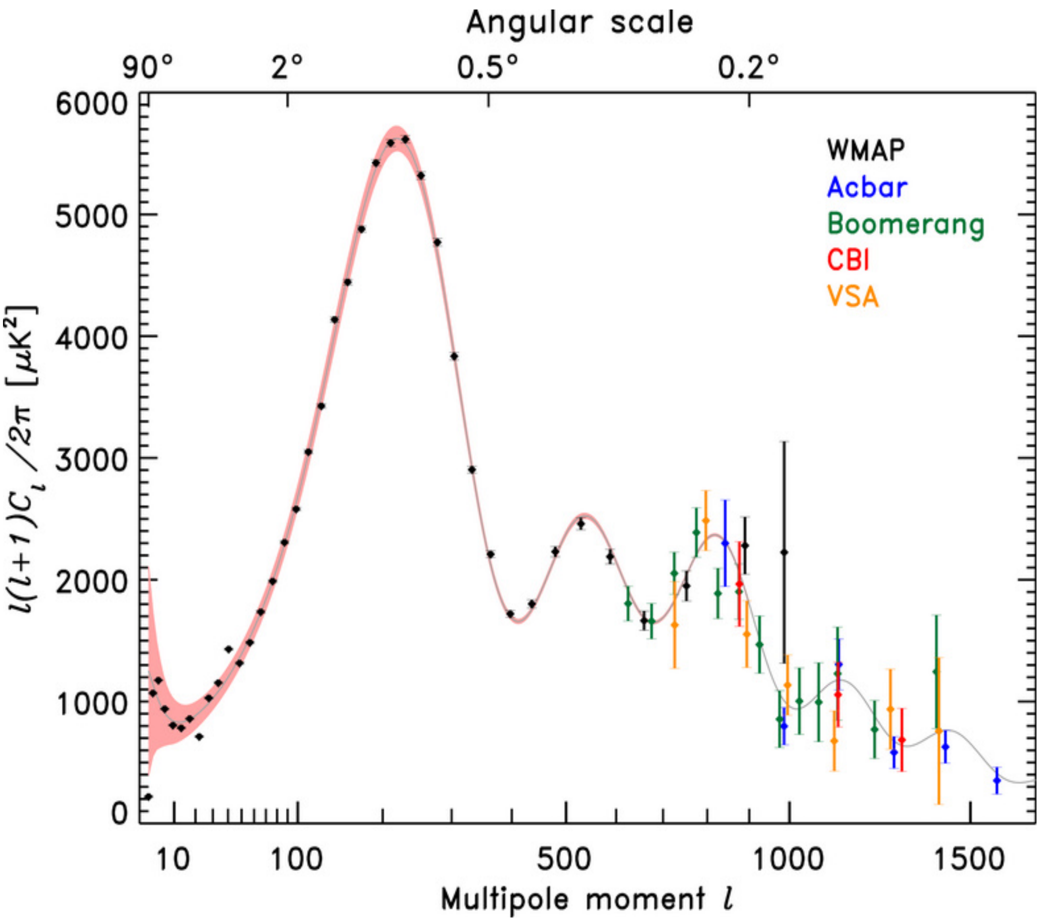

Figure 5.3: The power spectrum ofiMBR temperature anisotropy in terms of the multi pole moment. The data is from the WMAP(2006), Acbar(2004) BOOMERanG(2005), CBI(2004), and VSA(2004) instruments and solid line is a theoretical model.

decomposing the signal into spherical harmonics. The power spectrum is defined by the

$$C_l = \langle |a_{lm}^2 \rangle, \tag{5.5}$$

where a_{lm} are expansion coefficients. Fig.5.3 is the plot $l(l+1)C_l$ against l is usually referred as the CMBR power spectrum.

Primary anisotropies of CMBR spectrum are due to the density fluctuations of matter in the last scattering surface, so they were already present at the time of last scattering and before. These anisotropies may have been caused by the quantum fluctuations in density that existed before inflation and expanded during inflation. These extended fluctuations became the real density perturbations and are the seeds of structure formation of todays Universe.

5.3 Baryon Acoustic Oscillations

The angular sizes of galaxies can be used as a cosmological test using angular diameter distance redshift relation. Baryon acoustic oscillations (BAO) are characteristic sound waves of the over dense areas of baryonic matter at the time of decoupling of photons and matter, which remained as an imprint in the distribution of baryonic matter in the galaxies. As Type Ia Supernovae are used as standard candles , because of their characteristic size BAO can be used as standard rulers of $150Mpc$ for length scales in the Universe to measure distances between the present sound horizon and the sound horizon at the time of last scattering. BAO are discovered by the SDSS whoanalyzed clustering of the galaxies by using two point correlation function. SDSS confirmed the WMAP result that the sound horizon is $150Mpc$ in today's universe [23]. BAO can be used to understand the acceleration of the expansion of Universe when combined with the CMBR observations.

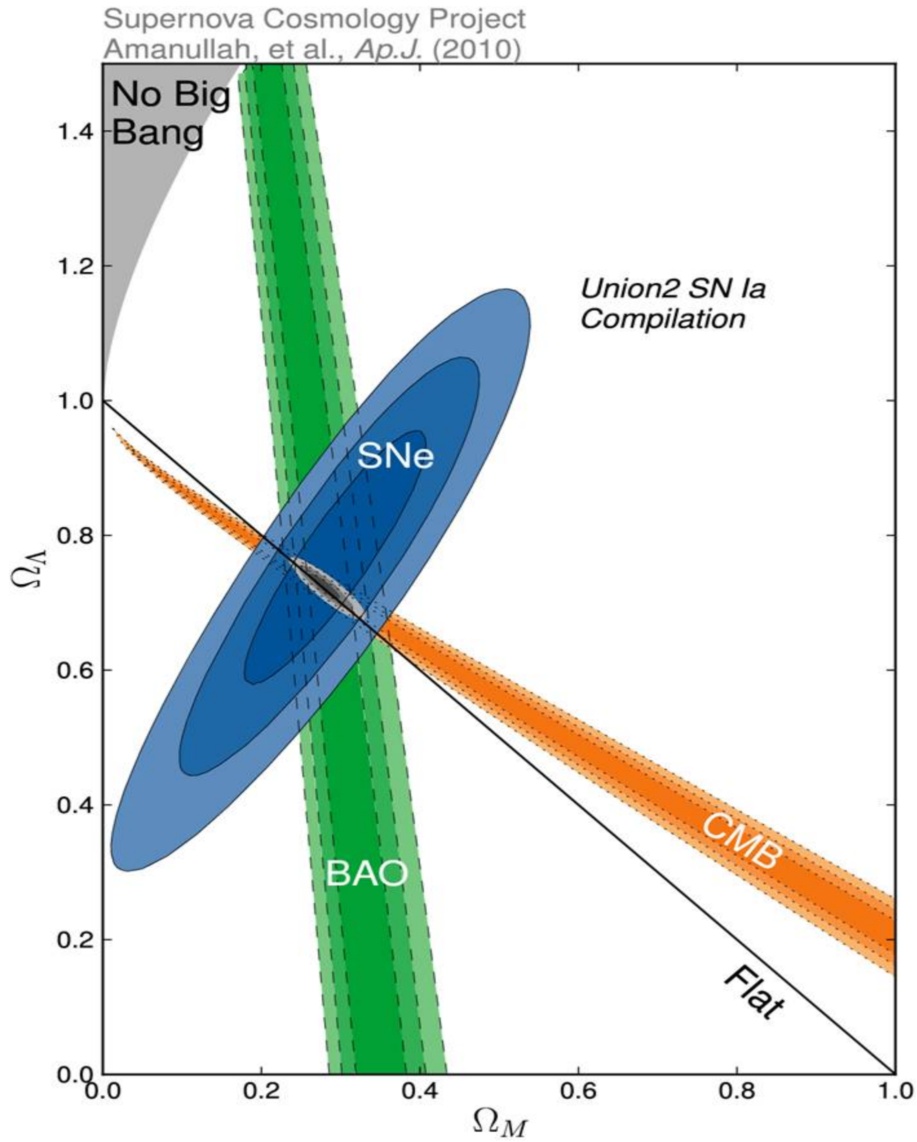

Figure 5.4: Ω_M-Ω_Λ with CMB, BAO, and SCP Union2 SN Constraints

Fig.5.4 shows the constraints on the density parameters of matter and dark energy density from the combined data from the WMAP, SDSS and SNe Ia. The result yields that $\Omega_{matter} = 0.29 \pm 0.02$ and $\Omega_\lambda = 0.7 \pm 0.01$.

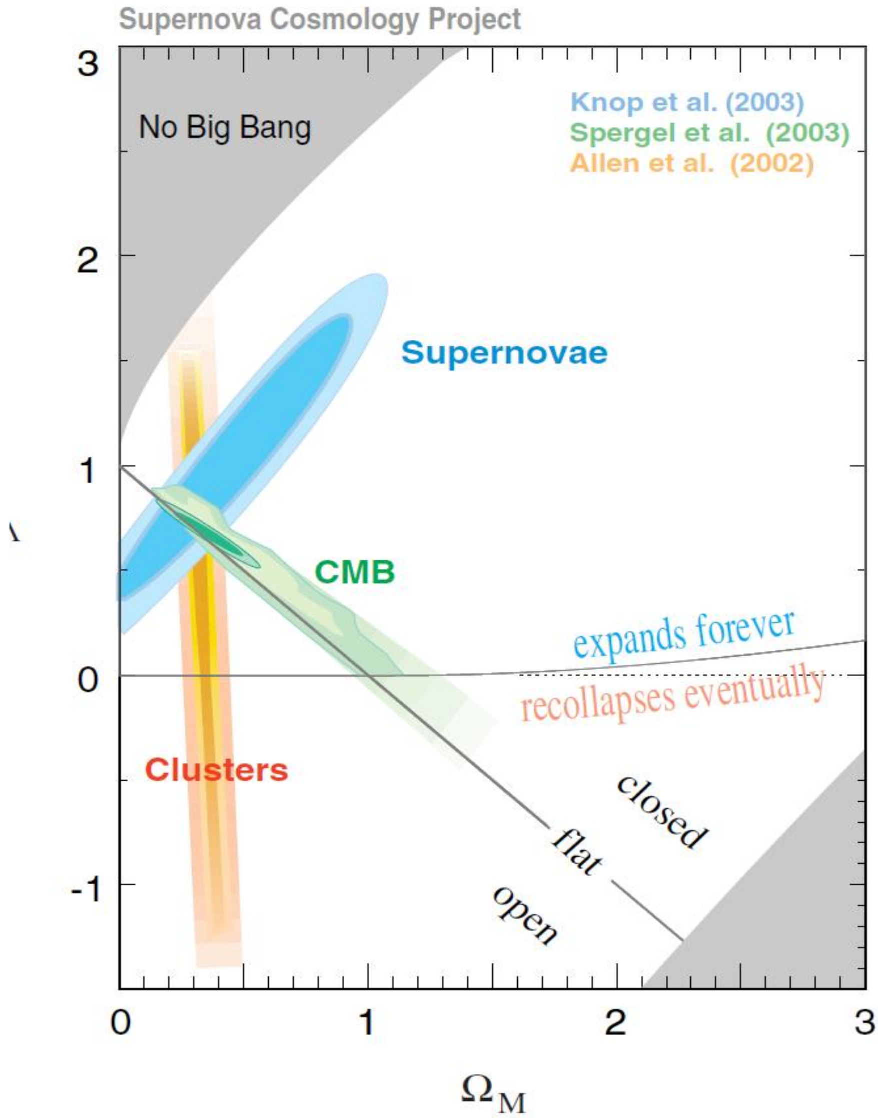

Figure 5.5: BAO data with results from CMB and galaxy cluster data added.

Fig.5.5 shows the constraints on the density parameters of matter and dark energy density from the combined data from the WMAP, SDSS and SNe Ia. The result yields that for the present Universe $\Omega_{matter} = 0.29 \pm 0.02$ and $\Omega_{\lambda} = 0.7 \pm 0.01$ and Fig.5.5 shows the faith of the Universe with these observed values of density parameters.

6. Λ AS A NEW COMPONENT

What made Einstein abandon the cosmological constant was Hubble's observation of redshifting galaxies. Before Hubble, around 1910's there are two remarkable discoveries made by other astronomers, worth mentioning.

The first one was by the American astronomer Henrietta Leavitt in 1912, with the discovery of period-magnitude relationship of Cepheid variable stars [24]. Traditionally for astrophysical objects, magnitude m is used instead of flux f, since they are related as $m \approx -logf$. She noticed stars with greater intrinsic luminosity have longer periods and there is a linear relation between the brightness of the star and it's period. Leavitt's discovery made possible to determine the distances in the Universe by using the observed flux and period.

The second one was by another American astronomer Vesto Slipher [25]. By investigating a spiral nebulae Slipher figured, color patterns of nebulae's light spectrum was changing depending on its components. Slipher also realized, almost all of these light sources were moving away and the lines in their light spectrum were getting reddish with the movement.

Hubble knew about both of these theories. He found and measured 23 other galaxies in a distance of almost 20 million light years. By combining the Doppler shift measurements of radial velocities with distance measurements, Hubble came to a conclusion that all these galaxies are moving away, and the more they are further away they move faster away. Hubble's redshift distance correlation, also referred to as Hubble's law is mathematically expressed as

$$v = H_0 d, \tag{6.1}$$

where d is the proper distance from the galaxy to the observer measured in Mpc, v is the speed of the galaxy, and H_0 is the present Hubble's constant. Though while estimating his constant Hubble made a mistake by a factor of 100, his method of calculation still applies. The estimated correct value of H_0 is

$$H_0 = 70 kms^{-1} Mpc^{-1}. \tag{6.2}$$

After Copernicus's heliocentric model, this was the most revolutionary contribution to cosmology, for this is a solid proof for the expansion of the Universe.

Hubble's discovery also solves a few hundred years of unanswered problem known as Olbers' paradox. An infinite sky in a static universe filled with stars should be extremely hot and bright. Considering an expanding universe makes the problem disappear.

In 1990's, 60 years after Hubble's observation, with the discovery of the universe's accelerating expansion by the observations of Type Ia Supernovae, Einstein's abandoned cosmological constant became a matter of consideration again. As in (2.4) Einstein added the cosmological constant to the l.h.s. of his equation, and treated Λ as a part of the curvature. However Λ can be moved to the r.h.s. of the equation and rewritten it as

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\Pi G}{c^4} T_{\mu\nu} - \Lambda g_{\mu\nu}. \tag{6.3}$$

This makes Λ a part of stress-energy tensor, where it can be treated as a new component of the Universe. Adding the cosmological constant to the matter content of the universe, yields the Friedmann equations take such a form

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4\Pi G}{3}(\rho + 3\frac{p}{c^2}) + \frac{\Lambda c^2}{3}. \tag{6.4}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\Pi G}{3}\rho + \frac{\Lambda c^2}{3} - \frac{\kappa c^2}{a^2 R^2}. \tag{6.5}$$

According to the modern field theory, the cosmological constant behaves almost like the vacuum energy, only multiplied by a proportionality factor 8Π,

$$\Lambda c^2 = 8\Pi G \rho_{vacuum}. \tag{6.6}$$

That implies Λ has constant mass density, which sets $\dot{\rho}$ on the l.h.s of 3.13 to 0 and yields

$$\rho = -p. \tag{6.7}$$

when $c = 1$. A positive mass density ρ associating with a negative pressure p can not be a case neither for matter nor for radiation. Therefor Λ is handled as a new component acting like a repulsion term in the Friedmann equations. From (3.14) we can see Λ is a component with, $\omega = -1$.

For each content component of the Universe corresponds a different equation of state with a different value of ω as listed below.

$$\begin{aligned} \omega_R &= \tfrac{1}{3} \\ \omega_M &= 0 \\ \omega_\Lambda &= -1 \end{aligned} \tag{6.8}$$

R stands for radiation which is the relativistic particles contained in the Universe. M stands for matter and represents both baryonic matter, consisting of protons, neutrons and electrons and the non-baryonic CDM, and Λ is the cosmological constant.

Again each component's mass density shows a different evolution with the scale factor. Integrating the fluid equation in (3.15) gives

$$\rho \propto a^{-3(1+\omega)}. \tag{6.9}$$

As we will use it later, it is convenient to write Friedmann equation (3.8) combined with (6.9)

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\Pi G}{3}\rho_0 a^{-3(1+\omega)} - \frac{\kappa c^2}{a^2 R_0^2}. \tag{6.10}$$

Using the relevant ω values of each component in the Universe, (6.9) would reveal that

$$\begin{aligned} \rho_R &\propto a^{-4} \\ \rho_M &\propto a^{-3} \\ \rho_K &\propto a^{-2} \\ \rho_\lambda &\propto a^{-0}. \end{aligned} \tag{6.11}$$

The $\rho(t)$ in the Friedmann equation is the sum over all mass densities of the different components in a Universe. Friedmann equation in (3.8) can be rearranged by substituting these components separately and expressed in terms of density parameters as

$$\frac{H^2}{H_0^2} = \left(\frac{\Omega_{R0}}{a^4} + \frac{\Omega_{M0}}{a^3} + \frac{\Omega_{K0}}{a^2} + \Omega_{\Lambda 0}\right), \qquad \textbf{(6.12)}$$

where the subscript 0 stands for the values at the present time. It is known that Universe has started with a radiation dominated era followed by a matter dominated era. Later dark energy became more dominant over the matter causing the Universe's expansion to continue and accelerate. By writing Friedmann equation in the form of (6.12) one may obtain how the scale factor $a(t)$ and time t is related in each era, by only taking the related Ω component into account. The relation between $a(t)$ and t is obtained by taking the integral

$$t = \frac{1}{H_0} \int \frac{da}{a\left(\frac{\Omega_{R0}}{a^4} + \frac{\Omega_{M0}}{a^3} + \frac{\Omega_{K0}}{a^2} + \Omega_{\Lambda 0}\right)^{\frac{1}{2}}}. \qquad \textbf{(6.13)}$$

7 year results of WMAP in January 2010 indicate that ordinary matter make up only 4.6 percent of the universe with accuracy to within 0.1 percent, while dark matter make up 23.3 percent to within 1.3 percent accuracy. 72.1 percent of the universe is determined to be dark energy; to within 1.5 percent accuracy which implies that todays Universe is in the dark energy dominated era. The remaining energy density comes from the radiation, which is mainly the cosmic microwave background radiation photons, and neutrinos. The dark energy ω parameter is determined as -1.1 ± 0.14, which makes the cosmological constant ($\omega = -1$) best candidate for the dark energy causing the Universe to accelerate. Assuming that the dark energy is the cosmological constant, with this collected data the curvature parameter is constrained to be within -0.77 percent and $+0.31$ percent, consistent with a flat Universe.

7. OPPONENTS OF COSMOLOGICAL CONSTANT

WMAP observations imply, the value of ω is constrained to be close to -1. Therefore the cosmological constant is in good consistency with observations. Λ has a constant energy density and an equation of state parameter $\omega = -1$, therefore cosmological constant can not provide any information on the time evolution of ω.

As in inflationary cosmology we can consider situations in which ω changes with time, therefore scalar field models as quintessence, k-essence and phantom energy with dynamic equation of state parameters are suggested as candidates for dark energy.

7.1 Quintessence

Quintessence [26] is described by the ordinary scalar field ϕ, minimally coupled to gravity with particular potentials $V(\phi)$ that lead to the inflation [11]. For a flat Universe ($\kappa = 0$) The scalar field ϕ is characterized by the energy density $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$ and pressure $p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$. While the continuity equation in (3.13) becomes

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{dV}{d\phi}, \tag{7.1}$$

the acceleration equation in (3.6) becomes

$$\frac{\ddot{a}}{a} = -\frac{8\Pi G}{3}\left[\dot{\phi}^2 - V(\phi)\right]. \tag{7.2}$$

$\omega = -\frac{1}{3}$ is the limit for an accelerating and decelerating Universe. (7.2) implies universe's acceleration requires $\dot{\phi}^2 < V(\phi)$. Since

$$\omega_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}, \tag{7.3}$$

$w_\phi = -1$ in the down limit $\dot{\phi}^2 << V(\phi)$. Therefore the quintessence field has dynamically evolving equation of state parameter in the range of $-1 < \omega_\phi < -\frac{1}{3}$.

With it's dynamic behavior, quintessence field solves one problem static that Λ can not. Quintessence field's density evolution is similar to the evolution of the radiation density until matter-radiation equality, therefore can reveal valuable information on the physics of the very early Universe.

7.2 k-essence

K-essence (kinetic quintessence) [27] is another a scalar field candidate of dark energy described with a Lagrangian of the form

$$L = p(\phi, X), \tag{7.4}$$

where ϕ is the scalar field and $X = (\frac{1}{2})(\nabla\phi)^2$. Then the energy density of the field ϕ is associated with a negative pressure as

$$\rho = 2X\frac{\partial p}{\partial X} - p = f(\phi)(-X + 3X^2), \tag{7.5}$$

which sets the equation of state parameter for the k-essence field to

$$\omega_\phi = \frac{1-X}{1-3X}, \tag{7.6}$$

meaning for a constant X, ω_ϕ is static. By considering the accelerated Universe requires $-1 < \omega_\phi < -\frac{1}{3}$, as log as X varies in the range of $\frac{1}{2} < X < \frac{2}{3}$ the k-essence scalar field behaves as the dark energy.

7.3 Phantom Field

As mentioned at the end of Ch.6, 7 year WMAP results indicate ω is determined as -1.1 ± 0.14. The scalar field models having an equation of state with $\omega < -1$ are referred as phantom dark energy. Phantom dark energy are seen in models in braneworlds or Brans-Dicke scalar-tensor gravity. However this form of dark energy seems unphysical. If the universe becomes phantom dark energy dominated the increasing negative pressure will result every type of matter in the Universe, even the sub atomic particles to be torn apart and the Universe to end in a so called "Big Rip".

7.4 Several More Scalar Field Candidates For Dark Energy

Another candidate for dark energy are considered as tachyon fields. Tachyon fields are mainly suitable candidates for the inflation at high energy. The equation of state of the tachyon is given by

$$\omega_\phi = \frac{p}{\rho} = \dot{\phi}^2 - 1. \tag{7.7}$$

Tachyon can act as a source of dark energy depending upon the form of it's potential, therefore their dynamics is different. When the Tachyon potentials are close to $V(\phi) \propto \phi^{-2}$, they lead to an accelerated expansion.

There is also a fluid known as a Chaplygin gas which is a special case of a tachyon with a constant potential. Chaplygin gas has the equation of state

$$p = -\frac{A}{\rho}, \tag{7.8}$$

where A is a positive constant. While at the early times Chaplygin gas behaves as a pressureless dust, it leads to the acceleration of the universe at late times can act as the dark energy.

7.5 Modified Gravity Instead of Dark Energy

One way to provide the accelerated expansion for the Universe is to add a component to the stress energy tensor $T_{\mu\nu}$ at the r.h.s of the EFE as we have done previously with the dark energy component as either a cosmological constant or a scalar field. Another way is to modify the geometry of the Universe by rearranging the l.h.s of EFE. Such a need for modified gravity in cosmology arises from the fact that the yet most approved dark energy dominated Universe model is in a big percent invisible and indeterminable.

Since neither Einstein's nor Newton's theorem can provide an information on the orbital velocities for the stars in the outer spiral galaxies there has been a need for a modification in the theory of gravity. To be used instead of dark energy such a modified gravity need to be in agreement with the measurements of the CMB, measurements of the mass power spectrum through the distribution of galaxies, and the luminosity-distance relationship of Type Ia supernovae.

For example a gravity theory is $f(R)$ gravity which is an alternative to the Einstein's theory of gravity. In $f(R)$ gravity the ordinary Lagrangian of the Einstein-Hilbert action

$$S[g] = \int \frac{1}{2(8\Pi G)} R\sqrt{-g}d^4x, \tag{7.9}$$

is generalized as

$$S[g] = \int \frac{1}{2(8\Pi G)} f(R)\sqrt{-g}d^4x, \tag{7.10}$$

yielding to the generalized Friedmann equations to take such a form

$$3FH^2 = \rho_m + \rho_{rad} + \frac{1}{2}(FR - f) - 3H\dot{F}, \tag{7.11}$$

$$-2F\dot{H} = \rho_m + \frac{4}{3}\rho_{rad} + \ddot{F} - H\dot{F}. \tag{7.12}$$

Other main exmamples include the Brans-Dicke Theory proposed as an alternative to the GR and developed by Robert H. Dicke and Carl H. Brans [28], where inverse of the gravitational constant $\frac{1}{G}$ is replaced with the scalar field ϕ which plays the role of a variable gravitational constant and the Lagrangian takes the form

$$S = \int d^4x\sqrt{-g}\left(\frac{\phi R - \omega\frac{\partial_a\phi\partial^a\phi}{\phi}}{16\Pi} + L_M\right). \tag{7.13}$$

The matter term L_M includes the contribution of ordinary matter and electromagnetic fields and the remaining is the gravitational term.

Another example is the Einstein-Gauss-Bonnet gravity [29] where the Gauss-Bonnet term $G = R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\mu\nu\sigma\lambda}R_{\mu\nu\sigma\lambda}$ is included to the Einstein-Hilbert action as

$$S = \int d^D x \sqrt{-g} G, \tag{7.14}$$

which can only apply for $4+1D$ or greater dimensional models.

Last model we will mention is a model of gravity, DGP model [30], where the action consists of two terms, the usual Einstein-Hilbert action in having the $4-D$ spacetime-dimensions and the other term is an equivalent of the Einstein-Hilbert action extended to $5-D$.

8. CONCLUSION

Since 1998 observations of Type Ia Supernovae there has been a need to explain the accelerated expansion of the Universe. The evidence is supported strongly by the observations of the CMBR and baryon acoustic oscillations in the last scattering surface.

Observational evidence indicate the Universe is spatially flat with $\Omega_{M0} = 0.3$, $\Omega_{\Lambda 0} = 0.7$ meaning that most of the Universe is filled with non-baryonic matter and the mysterious dark energy. To explain the cosmic acceleration we have pointed out several canditates such as the dark energy in the form of the cosmological constant or a scalar field and also several modified gravity theories. Amongst them, by far the cosmological constant seems to be the best choice, but there is still a lot to discover. Hopefully future observations will narrow down the candidates by setting limitations, reveal more truth and enlighten us about the unknown Universe.

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APPENDICES

APPENDIX A: GR
APPENDIX B: Derivation of Maximally Symmetric Metric
APPENDIX C: Calculations for the FRW Metric

APPENDIX A

In GR, the relationship between curvature and matter is given by the Einstein equation in (2.3). The l.h.s of this equation is the Einstein tensor $G_{\mu\nu}$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}. \tag{1}$$

Therefore to calculate the Einstein tensor of any spacetime

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu}, \tag{2}$$

we need to obtain the Ricci tensor $R_{\mu\nu}$ and Ricci scalar R. We start by defining the connections on the manifold, the Christoffel symbols

$$\Gamma^{\gamma}_{\mu\nu} = \frac{1}{2} g^{\gamma\beta} (\partial_{\mu} g_{\beta\nu} + \partial_{\mu} g_{\beta\nu} - \partial_{\beta} g_{\mu\nu}). \tag{3}$$

Once the Christoffel symbols are known, Riemann tensor describing the curvature are calculated by using the relation

$$R^{\gamma}{}_{\mu\nu\beta} = \partial_{\nu} \Gamma^{\gamma}_{\beta\mu} - \partial_{\beta} \Gamma^{\gamma}_{\nu\mu} + \Gamma^{\gamma}_{\nu\theta} \Gamma^{\theta}_{\beta\mu} - \Gamma^{\gamma}_{\beta\theta} \Gamma^{\theta}_{\nu\mu}. \tag{4}$$

The Ricci tensor can be obtained by the contraction of the two indices

$$R_{\mu\beta} = R^{\gamma}{}_{\mu\gamma\beta}. \tag{5}$$

With another contraction the Ricci scalar is evaluated

$$R = R^{\gamma}{}_{\gamma}, \tag{6}$$

allowing us to compute $G_{\mu\nu}$ in (1).

APPENDIX B

To achieve 2.5 we first mark that a homogenous and isotropic universe must be maximally symmetric, which also implies spherical symmetry. A maximally symmetric space is the space having maximum possible number of independent Killing vectors which has a number of $\frac{n(n+1)}{2}$ for n dimensional space. In the spherical coordinates the part of such a metric for the three spatial dimensions can be defined as

$$ds^2 = B(r)dr^2 + r^2[d\theta^2 + sin^2\theta d\phi^2]. \tag{7}$$

To obtain (2.5), the spatial metric of the maximally symmetric space, we need to solve B(r).

First we need to evaluate the existing Christoffels for this metric, which appear to be

$$\Gamma^r_{rr} = \frac{B^{'}}{2B}, \qquad \Gamma^r_{\theta\theta} = -\frac{r}{B}$$

$$\Gamma^r_{\phi\phi} = -\frac{rsin^2\theta}{B}, \qquad \Gamma^\theta_{\theta r} = \Gamma^\phi_{\phi r} = \frac{1}{r}$$

$$\Gamma^\theta_{\phi\phi} = -cos\theta sin\theta, \quad \Gamma^\phi_{\theta\phi} = \frac{cos\theta}{sin\theta}.$$

The Ricci tensors R_{rr} and $R_{\theta\theta}$ can be evaluated in two different ways. One way is to use the Christoffels calculated above.

$$R_{rr} = \frac{B^{'}}{rB}, \qquad R_{\theta\theta} = -\frac{1}{B} + \frac{B'r}{2B^2} + 1.$$

This is evaluating the Ricci tensors geometrically, but other than this we can note that the Ricci tensor of a maximally symmetric space can be defined as

$$R_{ij} = K(n-1)g_{ij}. \tag{8}$$

where K stands as the curvature constant and n is the dimension of the space. Combining the metric from Eq.(7) with the differential equation in Eq.(8) yields

$$R_{rr} = 2KB(r), \qquad R_{\theta\theta} = 2Kr^2.$$

The Ricci tensors calculated in two different ways are equal to each other and therefore gives us two equalities

$$\frac{B^{'}}{rB} = 2KB(r), \qquad -\frac{1}{B} + \frac{B'r}{2B^2} + 1 = 2Kr^2.$$

By solving them together, we can evaluate B(r) as;

$$B(r) = \frac{1}{1-Kr^2}, \tag{9}$$

which gives us the spatial part of a maximally symmetric metric as

$$ds^2 = \frac{dr^2}{1-Kr^2} + r^2[d\theta^2 + sin^2\theta d\phi^2]. \tag{10}$$

APPENDIX C

The FRW metric for our Universe is given by;

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2) \right] \quad \textbf{(11)}$$

The existing Christoffel symbols for this metric appear to be;

$$\Gamma^t_{rr} = \frac{a\dot{a}}{(1-Kr^2)}, \qquad \Gamma^r_{rr} = \frac{Kr}{1-Kr^2}$$

$$\Gamma^t_{\theta\theta} = a\dot{a}r^2, \qquad \Gamma^t_{\phi\phi} = a\dot{a}r^2 sin^2\theta$$

$$\Gamma^r_{tr} = \Gamma^\theta_{t\theta} = \Gamma^\phi_{t\phi} = \frac{\dot{a}}{a}, \qquad \Gamma^r_{\phi\phi} = -rsin^2\theta(1-Kr^2)$$

$$\Gamma^r_{\theta\theta} = -r(1-Kr^2), \qquad \Gamma^\phi_{\theta\phi} = \frac{cos\theta}{sin\theta}$$

$$\Gamma^\theta_{r\theta} = \Gamma^\phi_{r\phi} = \frac{1}{r}, \qquad \Gamma^\theta_{\phi\phi} = -cos\theta sin\theta$$

Then we can calculate the Riemann tensors describing the curvature.

$$R^r_{trt} = R^\theta_{t\theta t} = R^{\phi t\phi}_t = -\frac{\ddot{a}}{a}, \qquad R^t_{rtr} = \frac{\ddot{a}a}{1-Kr^2}$$

$$R^t_{\theta t\theta} = a\ddot{a}r^2, \qquad R^t_{\phi t\phi} = a\ddot{a}r^2 sin^2\theta$$

$$R^r_{rr\theta} = \frac{K}{1-Kr^2}, \qquad R^r_{\theta r\theta} = R^\phi_{\theta\phi\theta} = Kr^2 + \dot{a}^2 r^2$$

$$R^r_{\phi r\phi} = Kr^2 sin^2\theta + \dot{a}^2 r^2 sin^2\theta, \qquad R^\theta_{r\theta r} = R^\phi_{r\phi r} = \frac{K+\dot{a}^2}{1-Kr^2}$$

$$R^\theta_{\theta\theta r} = \frac{1}{r^2}, \qquad R^\theta_{\phi\theta\phi} = (K+\dot{a}^2)r^2 sin^2\theta$$

The Ricci tensors are calculated as;

$$R_{00} = -3\frac{\ddot{a}}{a}$$

$$R_{11} = \left(\frac{\ddot{a}a + 2K + 2\dot{a}}{1-Kr^2} \right)$$

$$R_{22} = \left(2\dot{a}^2 + a\ddot{a} + 2K\right) r^2$$

$$R_{33} = \left(2\dot{a}^2 + a\ddot{a} + 2K\right) r^2 sin^2\theta$$

The Ricci scalar of the Friedmann-Robertson-Walker metric appears to be,

$$R = \frac{6}{a^2}(\ddot{a}a + K + \ddot{a}) \quad \textbf{(12)}$$

And finally the Einstein tensors for the FRW metric are obtained as;

$$G_{00} = \frac{3}{a^2}(K+\dot{a}^2)$$

$$G_{11} = -\left(\frac{2a\ddot{a}+K+\dot{a}^2}{1-Kr^2}\right)$$

$$G_{22} = \left(-2a\ddot{a}+K+\dot{a}^2\right)r^2$$

$$G_{33} = \left(-2a\ddot{a}+K+\dot{a}^2\right)r^2 sin^2\theta$$

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