ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL OF SCIENCE ENGINEERING AND TECHNOLOGY

ISOGEOMETRIC STRUCTURAL ANALYSIS OF BEAMS AND PLATES

M.Sc. THESIS

M. Erden YILDIZDAĞ

Department of Naval Architecture and Ocean Engineering

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İSTANBUL TEKNİK ÜNİVERSİTESİ ★ FEN BİLİMLERİ ENSTİTÜSÜ

KİRİŞ VE LEVHALARIN İZOGEOMETRİK ANALİZ YAKLAŞIMIYLA STATİK VE DİNAMİK DAVRANIŞLARININ İNCELENMESİ

YÜKSEK LİSANS TEZİ

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M. Erden Yıldızdağ, a **M.Sc.** student of ITU **Graduate School of Science, Engineering and Technology** student ID **508121010**, successfully defended the **thesis** entitled "**ISOGEOMETRIC STRUCTURAL ANALSIS OF BEAMS AND PLATES**", which he prepared after fulfilling the requirements specified in the associated legislations, before the jury whose signatures are below.

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Date of Submission : 02 May 2014
Date of Defense : 30 May 2014

To P.S.; "The Woman" of my "Age of Reason",

FOREWORD

This is not the end of an era, just the moment that I feel something reprieved. There are still lots of choices and acts around me to keep myself exist.
I would like to express my deep appreciation and thanks to my supervisor Prof. Dr. A. Ergin for his endless support throughout this work. I would also thank to Prof. Dr. Ö. Gören who has couraged me about pursuing my capacity since bachelor years and of course Dr. M. C. Dökmeci who always shares his impressive knowledge with me. I must thank to M. Demirtaş, I think I could not have started this thesis without his valuable sharings. I greatfully acknowledge the TUBITAK for supporting my education with domestic scholarship programme. I also acknowledge Türk Loydu Foundation for supporting my future educational objectives.
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May 2014 M. Erden YILDIZDAĞ

TABLE OF CONTENTS

Page

ABBREVIATIONS

CAD **:** Computer Aided Design
FEA **:** Finite Element Analysis
NURBS **:** Non-Uniform Rational B-Splines
IGA **:** Isogeometric Analysis
BEM **:** Boundary Element Method

LIST OF TABLES

Page

LIST OF FIGURES

Page

ISOGEOMETRIC STRUCTURAL ANALYSIS OF BEAMS AND PLATES

SUMMARY

Isogeometric analysis (IGA) is introduced and applied structural problems related with beams and plates in this thesis framework. The main idea of isogeometric analysis is to combine Computer Aided Design (CAD) and Finite Element Analysis (FEA) tools and join them in a single tool because design (CAD) and analysis (FEA) stages are crucial for engineering processes and both of them have used different basis for many years. While classic FEA tries to approximate actual geometry using low-order and simply defined basis functions, IGA uses geometric definition defined exactly by CAD tools. Thus, geometry creates analysis in IGA and basis functions employed for both CAD and FEA are the same.

In this thesis, non-uniform rational B-spline (NURBS) is chosen for analysis basis because NURBS is quite common geometric tool in CAD technology due to its distinguishable flexibility and precision about creating curves and surfaces. Before getting into details of IGA, theory of NURBS and its antecedents; Bézier curves and B-splines is introduced and some examples are illustrated. Then, the theory of IGA is explained using Galerkin's method as numerical method. Actually, isoparametric finite element method which is very well known approach in FEA literature is invoked also in IGA. Actual geometry has its place in pyhsical domain and integration over elements should be performed in this domain. However, basis functions and their derivatives can be calculated in parametric domain and this makes necessary to define mapping from parametric domain to physical domain. On the other hand, another mapping should be defined for numerical integration on parent domain.

As numerical method, Galerkin's approach is chosen and its steps and outputs are showed. In order to implement Galerkin's method, weak (variational) form of the problem is defined and then algebraic equations are obtained defining weight and test functions. IGA formulations are explained for four different problems; bending of beams, free vibration analysis of beams, plate bending and free vibration analysis of beam. And these problems are also used for numerical examples, IGA is performed with different NURBS structures and compared with analytic and FEA solutions.

KİRİŞ VE LEVHALARIN İZOGEOMETRİK ANALİZ YAKLAŞIMIYLA STATİK VE DİNAMİK DAVRANIŞLARININ İNCELENMESİ

ÖZET

Bu tez çalışmasında, izogeometrik analiz teorisinin kiriş ve levhalarla ilgili yapısal problemler üzerine uygulamaları incelenmiştir. İzogeometrik teorinin temel amacı, geometrik tanım için kullanılan fonksiyonların, aynı zamanda analizin şekil (dağılım, yaklaşım, test, deneme) fonksiyonları olarak kullanarak, bilgisayar destekli tasarım (BDT) ile sonlu elemanlar analizini (SEA) tek bir çatı altında toplamaktır.

Bir mühendislik sürecine bakıldığında, tasarım ve analizin iki önemli aşamayı oluşturduğu açıktır. Tasarım, oluşturulmak istenen sistemin geometrik bilgisinin oluşturulduğu ve geometrinin detaylı bir şekilde temsil edilip sunulduğu aşamayı oluştururken, analiz kısmı yapının iç ve dış yükler altında göstereceği ilgili kritik tepkilerin olayın doğasına giderek matematiksel modelleme ile ön görülmesini sağlayan aşamadır. Fakat, tipik bir mühendislik sürecinde bu kadar önem arz eden iki aşama uzun yıllardır birbirinden ayrı durmaktaydı. Tasarım aşamasından sonra analize geçildiğinde geometri tekrar tanımlanıp, ağ yapısı oluşturma işlemi yapılması gereklidir. Sonlu elemanlar analizi mevcut geometriye, Lagrange polinomları gibi tanımlanması ve hesaplanması basit, düşük mertebeli fonksiyonlar ile yakınsamaya çalışırken, bilgisayar destekli dizayn, gelişen teknoloji ile beraber arzu edilen geometriyi yüksek mertebeli (Bézier eğrileri, B-spline veya uniform olmayan B-spline eğrileri (NURBS)) eğriler ile tanımlar. Dolayısıyla tasarım aşamasından sonra analize geçildiğinde geometri tekrar tanımlanıp, ağ yapısı oluşturma işlemleri klasik SEA yaklaşımının muhtemel olarak ortaya çıkarabileceği bir takım yakınsama, hassasiyet, vb. hataları azaltmak için son yıllarda yüksek doğruluklu geometrik tanımla ile ilgili yapılan çalışmalar hızla artmış ve çeşitli yeni yaklaşımlar araştırmacılar tarafından ortaya konmuştur. Bu çalışmada da, BDT ve SEA aşamalarını tek bir çatı altında toplayarak, NURBS tabanlı izogeometrik teori tanıtılmış ve bu iki farklı yapının (BDT ve SEA) nasıl bir araya getirildiği, geliştirilen izogeometrik kodların bir bilgisayar destekli dizayn programı ile ilişkilendirerek gösterilmiştir.

Bu kapsamda, izogeometrik analizin temellerinden bahsetmeden önce, ilk olarak Bézier, B-spline ve NURBS eğrilerinin matematiksel temelleri, birbirleriyle olan ilişkileri ve sahip oldukları özellikler ayrıntılı bir şekilde tanıtılmış ve böylelikle klasik sonlu eleman şekil fonksiyonları ve ağ yapısı oluşturma konularında farklı ve benzer olan özellikleri ortaya konmuştur. Daha sonra, izogeometrik analiz için fazlasıyla önem taşıyan ağ yapısı modifikasyonlarından bahsedilmiş, klasik SEA ağ modifikasyonlarından farkları ve benzerlikleri ortaya konmuştur. Sonrasında, bu eğrilerin analizin şekil fonksiyonları olarak nasıl kullanıldığından bahsedilmiştir.

İzogeometrik analiz ile BDT yardımıyla oluşturulan geometrinin direk sistemin analizi için de kullanılması sağlanır ve bu durumda klasik SEA yaklaşımında yer alan geometriyi düşük mertebeli fonksiyonlarla tanımlamaya (yakınsamaya) çalışma ve sonrasında gerekli olan ağ yapısı oluşturma işlemleri de ortadan kalkmış olur. Bu durumda mevcut zaman kayıplarının önüne geçilmesini sağlar. Tez kapsamında analizlerin şekil fonksiyonları olarak NURBS eğrileri kullanılmıştır.

İzogeometrik teori, sonlu elemanlar literatüründe fazlasıyla popüler olan izoparametrik sonlu elemanlar yaklaşımını temel alır. Nümerik yöntem olarak Galerkin yöntemi seçilmiştir. Galerkin yöntemi sonlu elemanlar analizi literatüründe de sıklıkla karşılaşılabilen bir nümerik çözüm yöntemidir (Galerkin Finite Element). Bu yöntem de ilk olarak incelenen problemin matematik modeli diferansiyel formda (strong form) elde edilir ve daha sonrasında problemin varyasyonel formu (variational or weak form) tanımlanır. Sonrasında, bu form içerisinde ağırlık (weight) ve deneme (test) fonksiyonları tanımlanarak lineer denklem sistemi ve bu sistemin matris formu (katılık matrisi, yük ve yer değiştirme vektörleri) elde edilir. Ağırlık ve deneme fonksiyonları NURBS eğrileri ile tanımlanır. SEA yaklaşımında yer alan, ağ yapısı tanımlayan düğüm noktalarının (nodes) yerini, izogemetrik analizde, eğrilerin tanımlanmasını sağlayan kontrol noktaları (control points) alır.

Analizde dikkate alınması gereken fiziksel, parametrik ve doğal koordinatları barındıran üç farklı tanım bölgesi vardır. NURBS şekil fonksiyonları parametrik uzayda tanımlanıp ve değerleri hesaplanırken, mevcut fiziksel uzaydaki tanım kontrol noktaları ile sağlanır. Parametrik uzaydaki değerler elde edilebildiği için integrasyon bu uzayda gerçekleştirilir ve dönüşüm matrisi (Jacobian) yardımıyla fiziksel uzaydaki değerler elde edilir. Sayısal integrasyon ise doğal koordinatları barındıran eşlenik uzayda gerçekleştirilir, bunun için de yine ayrı bir dönüşüm matrisi daha tanımlanır.

Sayısal örnek kısmında ise dört farklı yapısal probleme; kiriş basit eğilmesi, kiriş modal analizi, levha eğilmesi ve levha modal analizi problemlerine yer verilmiştir. İlgili sayısal örneklerin izogeometrik analiz formülasyonları ve çözüm için izlenen yol, katılık ve kütle matrislerinin, yük vektörlerinin nasıl edildiği anlatılmıştır. Kiriş problemleri için Euler-Bernoulli kiriş modeli seçilirken levha problemleri için klasik plak teorisi (Kircchoff plak teorisi) göz önünde bulundurularak problemler çözülmüştür.

Problemlerde ele alınan geometriler ticari bir BDT programı olan Rhinoceros 5 ile oluşturulmuştur, programın kendi bünyesinde barındırdığı komutlarla ağ modifikasyonunu yapma olanağına sahiptir. İzogeometrik analiz kodları ise Matlab R2012b programı yardımıyla oluşturulmuştur. Gerekli bütün temel kodların algoritmaları Piegl ve Tiller (1997) ve Rogers (2001) kaynaklarında bulunmaktadır. Ayrıca, yine Rhinoceros 5 içerisinde oluşturulan altprogramlarla, oluşturulan geometrilerin bilgileri dış ortama alınıp, Matlab yardımıyla oluşturulan izogeometrik analiz kodları ile ilişkilendirilmiştir. Böylece, bir BDT programıyla hazırlanan kodların birlikte aynı ortamda çalışmaları sağlanmıştır.

Analizler farklı NURBS yapıları için tekrar edilmiş ve sonuçlar hem kendi aralarında hem de analitik ve SEA sonuçlarıyla karşılaştırılmıştır. Verilen sonuçlarla bu yöntemin geçerliliği, standart SEA yaklaşımına göre güçlü yanları ortaya konmaya çalışılmıştır. Herşeyden önce yöntem BDT ve SEA çalışma ortamlarını bir araya getirerek daha integre bir yapı sunmaktadır.

Nihai sonuçlar göstermektedir ki izogeometrik analiz yakınsama ve hassasiyet konusunda üstün özelliklere sahiptir. Bunun da en önemli sebebi yüksek mertebeden eğrilerin analizin şekil fonksiyonları olarak atanmasıdır. Seyrek olarak oluşturulmuş ağ yapılarında bile doğru sunuca çok yakın sonuçlar verdiği karşılaştırmalarla ortaya konmuştur. Fonksiyonların mertebelerini arttırarak düşük serbestlik dereceleri ve dolayısıyla düşük yoğunluklu ağ yapısı elde ederek, işlem hacminin küçültülmesine de olanak sağlamaktadır.

1. INTRODUCTION

A typical engineering process has two crucial stages; design and analysis. While engineering design allows obtaining exact geometric representation of desired systems that will be manufactured, analysis stage gives information about critical reactions come from the nature of the problem that the system and its surroundings may encounter.

Finite element analysis (FEA) which is the most powerful and popular analysis tool for analysis stages had its origin in the 1950s and 1960s whereas computer aided design (CAD) technology had its origins later, in the 1970s and 1980s and each one is working with different geometric representations (Rypl and Patzak, 2011).

In classic FEA approach, generated mesh approximates the actual geometry using low-order elements which may lead accuracy problems of solution and each loop of design cycle may need a new and probably much expensive meshing process (Rypl and Patzak, 2011). On the other hand, CAD allows creating desired exact geometries using high-order and flexible curves and splines like Bézier curves, B-splines, non-uniform rational B-splines or T-splines.

Thus, the importance of the accurate geometrical model have shown up in the last decades and numerous authors have been focused on implementation of accurate or exact geometric representations for finite element simulations (Demirtas, 2011).

The gap between geometric definitions of FEA and CAD is obviated with new proposed method; isogeometic analysis (IGA) which was firstly introduced by Hughes, Cottrell and Bazilevs (2005). IGA allows unifying the fields of FEA and CAD using same geometric definitions for both design and analysis stages.

1.1 Objectives and Overview of Thesis

One of the most important projects about IGA is “Geopdes” and it provides a common and flexible framework to implement and test IGA in different fields like linear elasticity, fluid mechanics and electromagnetism (“Geopdes”, n.d.). It is open source and can be used with Matlab or Octave. However, there is an important issue about CAD geometry which should be noticed in the FAQ page of website.

Although this project has very useful IGA examples, it does not give any support for any CAD software as mentioned in FAQ page of website (“Geopdes”, n.d.). It shows that there is not still exact integration between CAD and FEA as stated by Cottrell et al. (2009).

Regarding this case, main motivation of this thesis is to solve structural problems with NURBS-based IGA creating communication with CAD tools. For this purpose, commercial CAD program Rhinoceros 5 has been used as CAD tool to link geometry with IGA codes via Rhinoscript.

In the second chapter, definitions and properties of NURBS and its pregenitors namely Bézier curves and B-splines are explained in detail. In the third chapter, theory of isogeometric analysis, differences and similarities with classic finite element method are stated. Also, steps for Galerkin’s method which should be followed to obtain algebraic equations are introduced showing four different structural problems which are related with beams and plates. Then. in the fourth chapter, results of numerical examples are presented. Also, results have been compared with analytic and FEA solutions.

1.2 Literature Review

The history of finite element method may be originated to Johann Bernoulli’s “The Brachistochrone Problem” at the end of the 17^{th} century. This initiated the fundamental contributions to mathematics and computational methods.

The name “Finite Element Method” started to become popular after Clough (1960) used this name firstly in his work. Then, lots of engineers and mathematicians have intrigued with this new method to apply into different fields. On the other hand, they have also focused on mathematical point of view of this new method improving solution approximations and geometrical representation to reduce error, convergence and stability problems.

Isoparametric element approach is one of the most important improvement to define curved elements using the same interpolation functions to approximate the solution and geometry. Ergatoudis et al. (1968) described firstly the theory of a new family of isoparametric elements to improve the accuracy of solution and used in two-dimensional problems. This new theory had a significant effect on the finite element researches.

On the other hand, the foundation of CAD technology is originated to studies of Bézier (1966, 1967 & 1972) which Bernstein polynomials were used to create curves and surfaces. Also, the term “spline” was firstly used by Schoenberg (1946). B-splines and non-uniform rational B-splines were introduced in 1970s; in studies of Riesenfield (1972) and Versprille (1975), respectively.

After 1980s, CAD technology has begun to appear in FEA. Gontier and Vollmer (1995) solved large displacement analysis of beams using Bézier curves for analysis basis. Kagan et al. (1998) formulated B-spline finite element method and used this method on linear rod and plate problems. Beside the FEA, in several studies (Okan and Umpleby, 1985a, 1985b; Maniar, 1995; Kashiwagi, 1998), this idea has been also used for boundary element method (BEM).

In 2005, new NURBS-based finite element method which uses CAD geometric definition as also analysis basis was stated by Hughes et al. and they named this method as isogeometric analysis. Lots of researchers from different disciplines have been interested in this novel method and it has been applied to a wide range of problems to underline the better approximation and accuracy properties compared with classic FEA.

Isogeometric approach has been used in many solid mechanics problems. Reali (2006) and Cottrell et al. (2005) performed isogeometric structural vibration problems and compared the results with classic finite element method. Benson et al. implemented Reissner-Mindlin shell formulation for isogeometric analysis and examined the method with benchmark examples (2010). Kiendl et al. developed a Kirchhoff-Love shell element on the basis of the isogeometric approach (2009). Benson et al. presented NURBS-based isogeometric shell for large deformations formulated without rotational degree of freedom (2011). Guo et al. employed isogeometric analysis for laminated composites using a displacement-based isogeometric layerwise theory (2014). Shojaee et al. studied isogeometric free vibration analysis of Kirchhoff plates implementing standard Galerkin method (2012). Kapoor and Kapania proposed nonlinear NURBS-based isogeometric analysis of laminated composite plates using first-order, shear-deformable laminate composite plate theory (2012). Nguyen-Xuan et al. proposed a formulation based on a fifth-order shear deformation theory in combination with isogeometric analysis for composite sandwich plates (2013). Weeger et al. analyzed nonlinear Euler-Bernoulli beam vibrations (2013). Auricchio et al. (2007) studied plane incompressible elastic problems using “stream-function” formulation. Casanova and Gallego (2013) analyzed composite shells introducing third-order shear deformation theory. Lee and Park (2013) studied free vibrations of Timoshenko beams. Nguyen and Nguyen-Xuan analyzed delamination process for composites using high-order Bézier elements (2013). Lu (2009) introduced family of isogeometric elements of smooth, curved geometries. Lu and Zhou analyzed large deformations of rod-like structures (2010). Shojaee et al. (2012) presented natural frequencies and buckling analysis of thin symmetrically laminated composite plates regarding the classical plate theory. Bouclier et al. (2012) proposed isogeometric formulations for curved Timoshenko beams.

IGA has been also applied to different fields like shape optimization (Wall et al., 2008; Nagy et al., 2010, 2013; Qian, 2010; Koo et al., 2013; Yoon et al., 2013; Nortoft and Gravesen, 2013; Park et al., 2013), fluid mechanics (Belibassakis et al., 2012; Bazilevs et al., 2007a, 2007b, 2013; Bazilevs and Hughes, 2005, 2008), magnetics (Vazquez and Buffa, 2010; Buffa et al., 2009), fluid-structure interaction problems (Bazilevs et al., 2006, 2008; Heinrich et al., 2012), functionally graded materials (Valizadeh et al., 2012; Tran et al., 2013), contact problems (Temizer et al., 2010).

On the other hand, new IGA developments have been proposed like using T-splines (Bazilevs et al, 2009; Uhm and Youn, 2009; Scott et al., 2012), adaptive mesh refinement (Dörfel et al., 2008), trimmed geometries (Schmidt et al., 2012; Kim et al., 2009). And also Cottrell et al. (2007) and Bazilevs et al. (2006) treated better approximation and accuracy properties of IGA compared with classic FEA in their studies.

2. NON-UNIFORM RATIONAL B-SPLINES (NURBS)

Before getting into details of IGA, non-uniform rational B-splines (NURBS) which is basis for both CAD and IGA solutions are defined in this chapter. Also, its antecedents namely, Bézier curves and B-splines are presented before NURBS to understand their properties and characteristics. At the end of the chapter, refinement processes which have analogously equal to mesh refinement in FEA are presented.

2.1 Antecedents of NURBS

2.1.1 Bézier curves

Parametrically defined Bézier curves were firstly introduced by the French engineer, Pierre Bezier who had been working for Renault automobile company for 42 years until his retirement in 1975; he used his definition to design different products like car bodies, aircraft wings and yatch hull (Rogers, 2001). Although this convenient technique for shape definition of free-form curves and surfaces was developed originally by Bézier regarding geometrical point of view, it was proved that the mathematical basis is exactly equal to Bernstein basis or polynomial approximation function (Rogers, 2001). Gordon and Riesenfeld (1974) named this method as "Bernstein-Bézier Methods" in their work.

nth-degree Bézier curve is defined by

$$C(u) = \sum_{i=0}^{n} J_{i,n}(u)\boldsymbol{B_i} \qquad 0 \le u \le 1. \tag{2.1}$$

$J_{i,n}(u)$ shows nth-degree Bernstein polynomial and defined as follows,

$$J_{i,n}(u) = \frac{n!}{i!\,(n-i)!} u^i (n-i)^{n-i}. \tag{2.2}$$

And $\mathbf{B_i}$ represents the control points (geometric coefficients) of the form. Also, u represents the parametric locations.

Some properties of Bézier curve are given as follows (Piegl and Tiller, 1997; Rogers, 2001);

- Nonnegativity: $J_{i,n}(u) \geq 0$ for all i, n and $0 \leq u \leq 1$.
- Partition of unity: $\sum_{i=0}^{n} J_{i,n}(u) = 1$ for all $0 \leq u \leq 1$.
- $J_{0,n}(0) = J_{n,n}(1) = 1$.
- $J_{i,n}(u)$ attains exactly one maximum on the interval [0, 1], that is, at $u = i/n$.
- Symmetry: for any n, the set of polynomials $\{J_{i,n}(u)\}$ is symmetric with respect to u=1/2.
- Recursive definition:

$$J_{i,n}(u) = (1-u)J_{i,n-1}(u) + uJ_{i-1,n-1}(u),$$

$$J_{i,n}(u) \equiv 0 \text{ if } i < 0 \text{ or } i > n.$$

- Derivatives:

$$J'_{i,n}(u) = \frac{dJ_{i,n}(u)}{du} = n(J_{i-1,n-1}(u) - J_{i,n-1}(u)),$$

$$J_{-1,n-1}(u) = J_{n,n-1}(u) = 0.$$

- Convex hull property: the curves are contained in the convex hulls of their defining control points.
- The basis functions are real.
- The degree of the polynomial defining the curve segment is one less than the number of control polygon points.
- The curve generally follows the shape of the control polygon.
- The first and last points on the curve are coincident with the first and last points of the control polygon.

Moreover, the curve does not oscillate about any straight line more often than the control polygon (variation-diminishing property) and remain invariant under an affine transformation. Examples of Bézier basis functions and Bézier curve are given in Figure 2.1, 2.2 and 2.3.

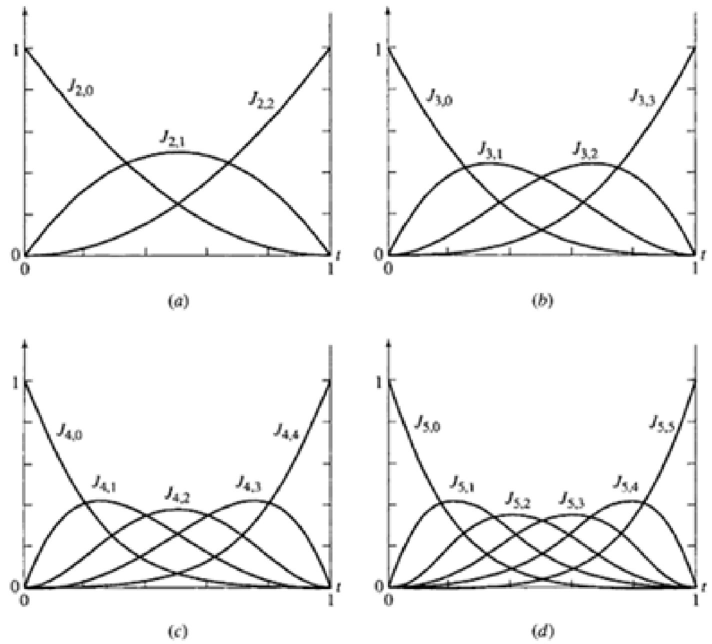


Figure 2.1 : Bezier (Bernstein) basis (blending) functions (Rogers, 2001).

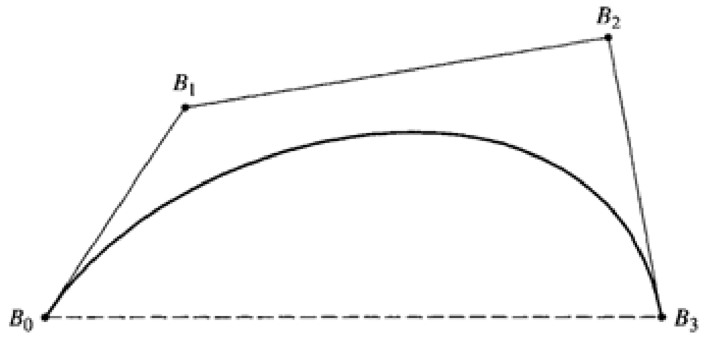


Figure 2.2 : A Bézier curve and its control polygon (Rogers, 2001).

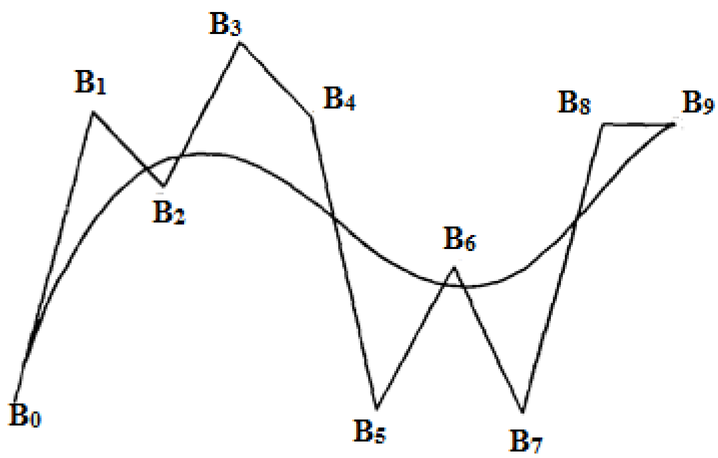


Figure 2.3 : Example of a Bézier curve for 9-sided polygon (Gordon and Riesenfeld, 1974)

2.1.2 B-splines

Although Bézier method is one of the most suitable method for interactive approximation of curves and it can be extended to surface description providing easy way to control the surface parameters as Forrest (1971) stated (1971), this method has two main drawbacks that affect the flexibility of resulting curve. First, order of a curve depends on the number of vertices of control polygon and the second comes from the nature of the basis functions; non-zero character of basis functions does not allow making local changes on the curve and any local change affects the entire curve (Rogers, 2001).

The problems come with Bézier method can be solved with using piecewise polynomial or piecewise rational (Peigl and Tiller, 1997). Thus, the parametric domain should be divided into pieces to define functions. The theory of B-splines was firstly introduced by Schoenberg in 1946 and recursive formula for practical computational use was stated by Cox and de Boer independently in 1971 and 1972 respectively and later, Riesenfeld and Gordon used B-spline basis to define curve (Rogers, 2001). B-spline curves and surfaces were introduced into CAD/CAM technology with Riesenfield's work (Piegl, 1991).

2.1.2.1 Knot vectors

In order to define B-splines, partitions of the domain should be stated with knot vectors which have non-decreasing ($\xi_i \leq \xi_{i+1}$) set of parametric coordinates. They are written as $\Xi = \{\xi_1, \xi_2, \dots, \xi_{n+p+1}\}$ where $\xi_i \in \mathbb{R}$ is the ith knot, i is the knot index, $i = 1,2, \dots, n+p+1$, p is the polynomial order, and n is the number of basis functions (Cotrell et al., 2009).

Mainly, there are two kinds of knot vector namely, periodic and open are used and both types may be either uniform or non-uniform. A knot vector is said to be open if its first and last knots are repeated p+1 times and basis functions generated from open knot vectors are interpolatory at ends of the parametric domain interval (Cotrell et al., 2009).

2.1.2.2 B-Spline basis functions

Recursive formula for B-spline basis functions for p=0 is given by

$$N_{i,0}(\xi) = \begin{cases} 1, & if\ \xi_i \le \xi \le \xi_{i+1} \\ 0, & otherwise. \end{cases} \tag{2.3}$$

For p= 1, 2, 3… they are defined by

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi). \tag{2.4}$$

(2.3) and **(2.4)** are called Cox-de Boor recursion formula and some B-spline functions are shown in Figure 2.4. Clearly, for p=0 and p=1, B-splines are the same as standard finite element functions. However, B-spline functions with higher degree differ from classic FEA functions.

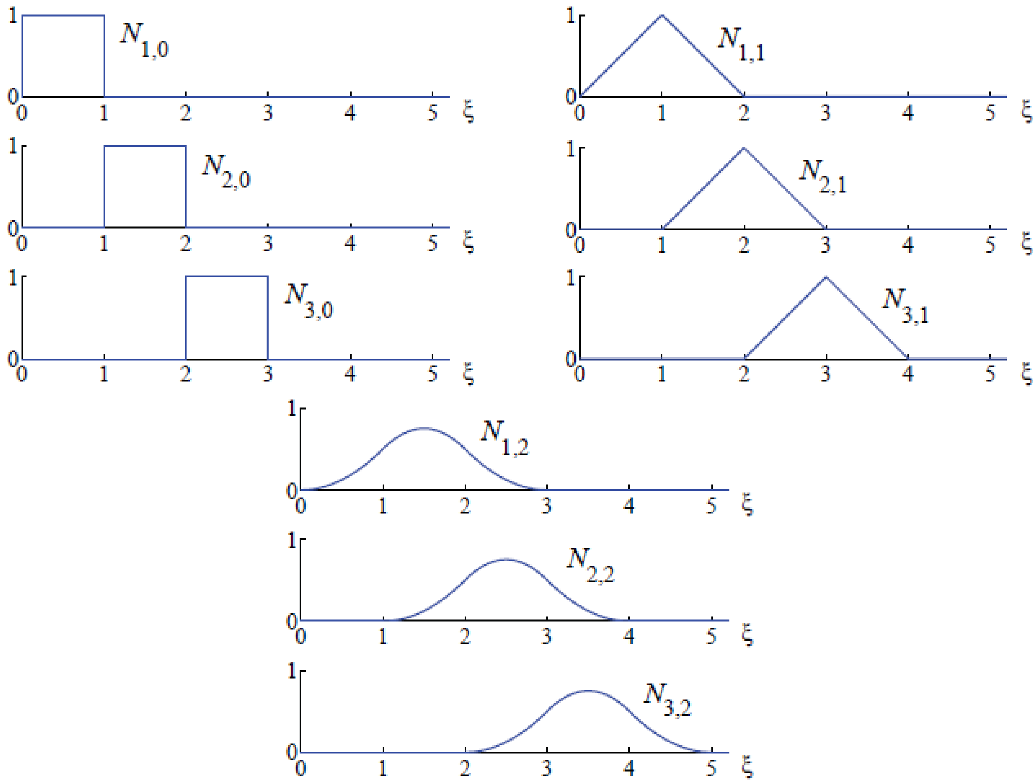


Figure 2.4 : Basis functions of order 0, 1, and 2 for uniform knot vector Ξ= {0, 1,2,3,4 …} (Cotrell et al., 2009).

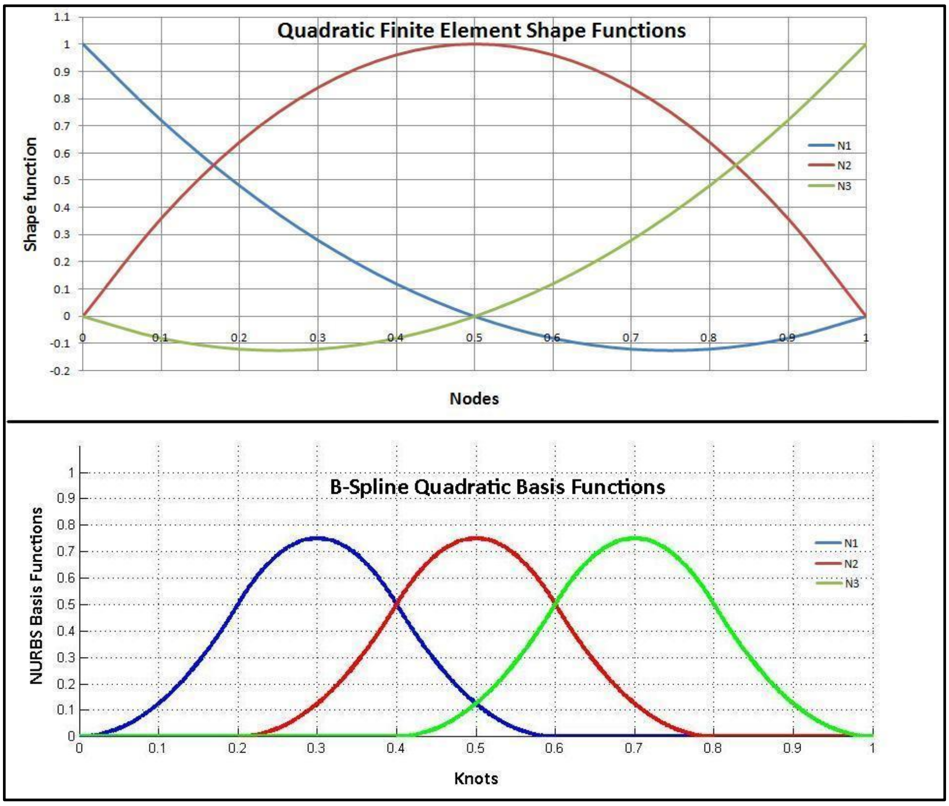


Figure 2.5 : Comparison of quadratic finite element shape functions and B-spline basis functions (Demirtas, 2011).

In order to understand the difference between classic FEA basis functions and B-spline basis functions, a comparison between B-spline quadratic basis functions and quadratic finite element shape functions is shown in Figure 2.5.

2.1.2.3 B-spline curves

Piecewise-polynomial B-spline curve is given by

$$C(\xi) = \sum_{i=1}^{n} N_{i,p}(\xi)\boldsymbol{B_i}. \tag{2.5}$$

Here, $\boldsymbol{B_i}$ represents the control points (control net), and the $N_{i,p}(\xi)$ are the pth-degree B-spline basis functions as defined with **(2.3)** and **(2.4)**.

Some properties of B-splines are given below (Peigl and Tiller, 1997; Rogers, 2001);

- $N_{i,0}(\xi)$ is a step function, equal to one only on the half-open interval $\xi \in [\xi_i, \xi_{i+1})$.
- For p>0, $N_{i,p}(\xi)$ is a linear combination of two (p-1)-degree basis functions.

- Computation of a set of basis functions requires specification of a knot vector, Ξ, and the degree, p.
- The $N_{i,p}(\xi)$ are piecewise polynomials, defined on the entire real line; generally only the interval $[\xi_0,\xi_{n+p+1}]$ is of interest.
- The half-open interval, $[\xi_i, \xi_{i+1})$, is called the ith knot span; it can have zero length, since knots need not be distinct.
- The computation of the pth-degree functions generates a truncated triangular table.

$$
\begin{array}{cccc}
N_{1,0} & & & \\
& N_{1,1} & & \\
N_{2,0} & & N_{1,2} & \\
& N_{2,1} & & N_{1,3} \\
N_{3,0} & & N_{2,2} & \vdots \\
& N_{3,1} & \vdots & \\
N_{4,0} & \vdots & & \\
\vdots & & &
\end{array}
$$

- The maximum order of the curve equals the number of control polygon vertices and the maximum degree is one less.
- The curve exhibits the variation-diminishing property. Thus, the curve does not oscillate about any straight line more often than its control polygon oscillates about the line.
- Any affine transformation is applied to the curve by applying it to the control polygon vertices.

Figure 2.6 and 2.7 illustrate examples of B-spline curves, their basis functions are given with their knot vectors. Clearly, knot vectors show parts of the domain and each part has limited number of functions which have non-zero values and control points give physical data (coordinates) to create desired curve.

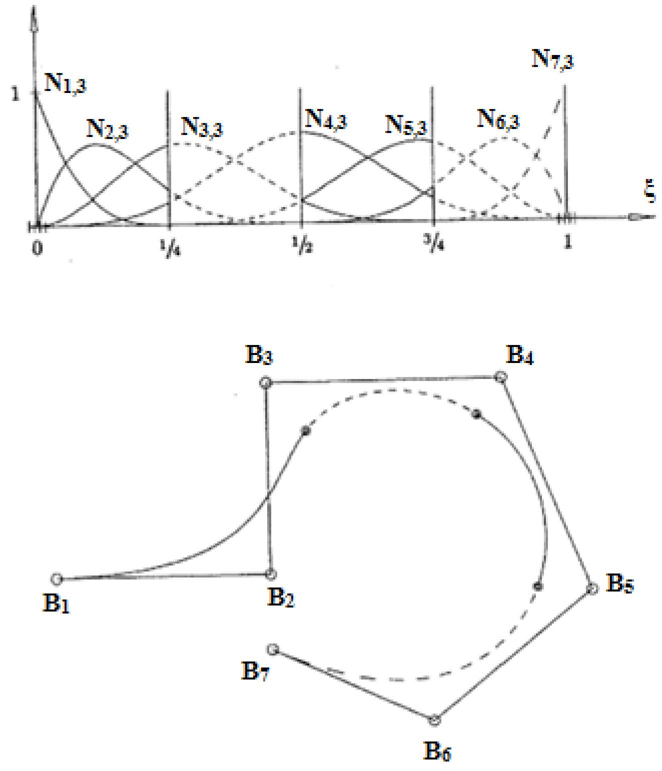


Figure 2.6 : Cubic basis functions Ξ=[0 0 0 0 0.25 0.5 0.75 1 1 1 1] and a cubic curve using the basis functions (Peigl and Tiller, 1997).

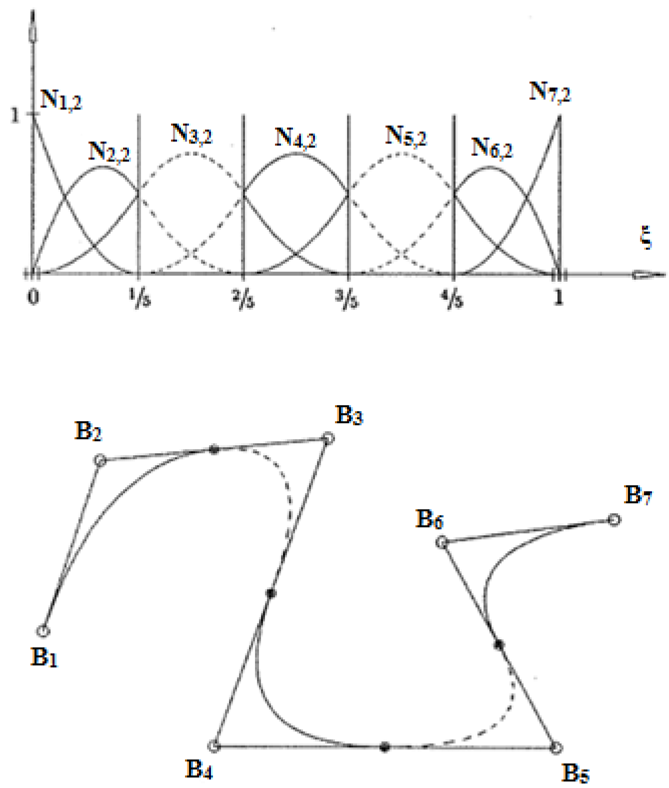


Figure 2.7 : Quadratic basis functions on Ξ=[0 0 0 0.2 0.4 0.6 0.8 1 1 1] and quadratic curve using the basis functions (Peigl and Tiller, 1997).

2.1.2.4 B-spline surfaces

B-spline surface is obtained by taking a bidirectional net of control points, two knot vectors, and the products of the univariate B-spline functions. Tensor product B-spline surface is defined by

$$S(\xi,\eta) = \sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,p}(\xi)M_{j,q}(\eta)\,\boldsymbol{B_{i,j}} \qquad \textbf{(2.6)}$$

Local support of basis functions are shown as $\widehat{N}_{i,j;p,q}(\xi,\eta) = N_{i,p}(\xi)M_{j,q}(\eta)$ and it is exactly $[\xi_i, \xi_{i+p+1}]$ x $[\eta_j, \eta_{j+p+1}]$ (Cotrell et al., 2009). An example of biquadratic B-spline surface is given in Figure 2.8.

i	j	$\mathbf{B}_{i,j}$
1	1	(0, 0)
1	2	(−1, 0)
1	3	(−2, 0)
2	1	(0, 1)
2	2	(−1, 2)
2	3	(−2, 2)
3	1	(1, 1.5)
3	2	(1, 4)
3	3	(1, 5)
4	1	(3, 1.5)
4	2	(3, 4)
4	3	(3, 5)

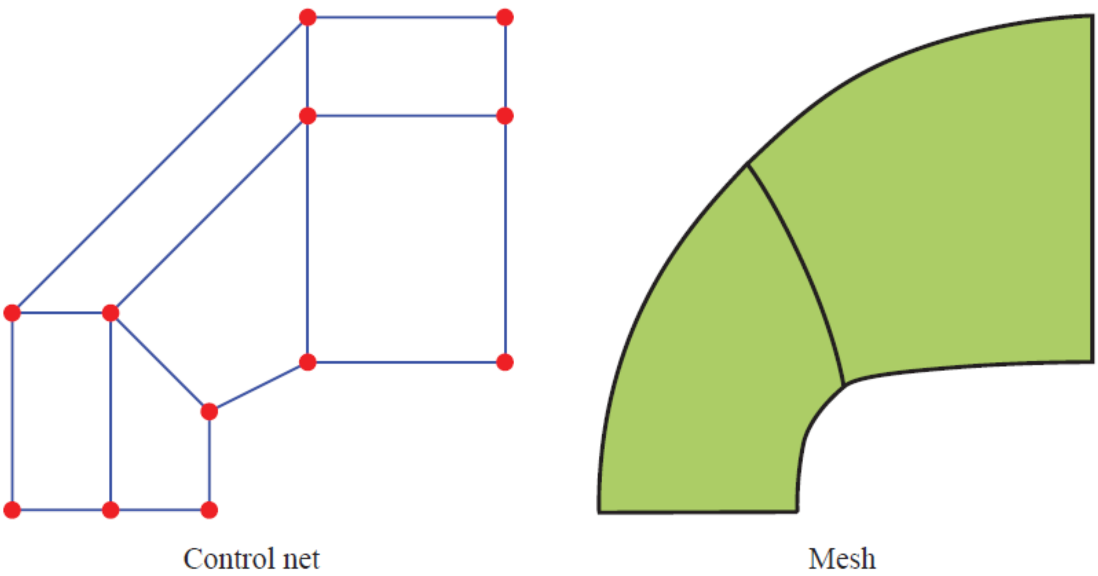


Figure 2.8 : Control points, control net and mesh for biquadratic surface with Ξ= {0, 0, 0, 0.5, 1, 1, 1} and $\boldsymbol{\mathcal{H}}$ = {0, 0, 0, 1, 1, 1} (Cotrell et al., 2009).

2.2 NURBS

Implicit and the parametric polynomials are the two common nonlinear mathematical forms used to define curves and surfaces (Tiller, 1983). While implicit form has certain advantages to design analytic shapes (conics, quadrics, circles, etc.), parametric polynomials such as B-spline that is mentioned in previous chapter can represent free-form curves and surfaces very well.

In order to define both types of geometries using same mathematical form, rational B-splines are used. The idea of rational B-splines was firstly stated with Versprille's work in 1975 (Piegl, 1991).

After that, three main groups namely Boeing, Structural Dynamics Research Corporation (SDRC) and University of Utah made certain contributions developing new modelers which work with NURBS (Piegl, 1991).

2.2.1 Description of NURBS

NURBS basis function is given by

$$R_i^p(\xi) = \frac{N_{i,p}(\xi) w_i}{W(\xi)} \tag{2.7}$$

where $W(\xi)$ is defined as weighting function

$$W(\xi) = \sum_{i=1}^{n} N_{i,p}(\xi) w_i \tag{2.8}$$

where the w_i are weights of control points, and the $N_{i,p}(\xi)$ are the pth-degree classical B-spline basis functions. Then, a NURBS curve is defined by

$$\boldsymbol{C}(\xi) = \sum_{i=1}^{n} R_i^p(\xi) \boldsymbol{B}_i \tag{2.9}$$

where the components of $\boldsymbol{B}_i$ are the control points. Also, rational surfaces are defined analogously in terms of basis functions by

$$R_{i,j}^{p,q} = \frac{N_{i,p}(\xi) M_{j,q}(\eta) w_{i,j}}{\sum_{\tilde{i}=1}^{n} \sum_{\tilde{j}=1}^{n} N_{\tilde{i},p}(\xi) M_{\tilde{j},q}(\eta) w_{\tilde{i},\tilde{j}}} \tag{2.7}$$

Here, each control point has an additional "weight" and it shows how the control point affects the curve. Additional weights give another important feature to represent geometry exactly compared with B-splines. If all weights are taken equal to 1, it can be easily seen that rational B-spline turns into B-spline. Thus, B-splines are a special case of NURBS.

Moreover, most of the properties which are mentioned in previous sections are also valid for NURBS and Figure 2.9 illustrates an example of NURBS curve.

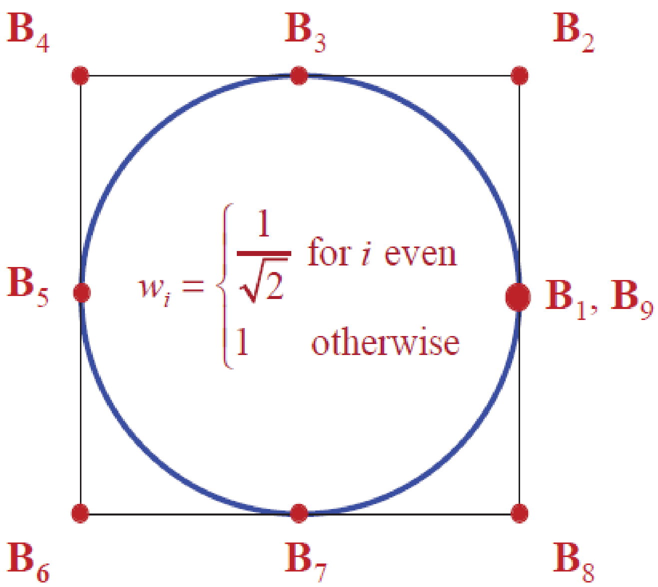


Figure 2.9 : NURBS representation of a circle (Cottrell et al., 2009).

2.3 Refinements

Mesh refinement processes consist of knot insertion, order elevation and combined refinement (both knot insertion and order elevation). They have the same role as the mesh refinement strategies in FEA have.

Knot insertion process is simply based on adding new knots into the solution space without changing actual geometry and order (Figure 2.10). This process is similar to classic h-refinement strategy in finite element analysis.

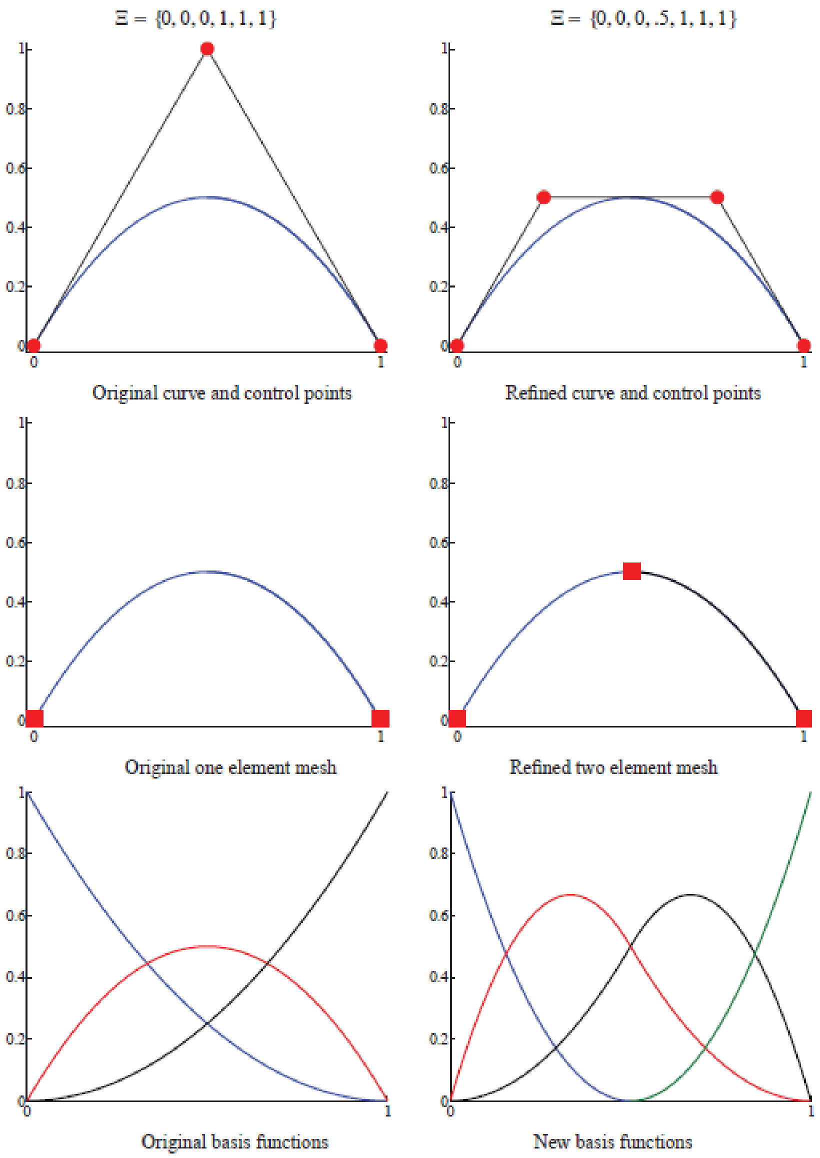


Figure 2.10 : Knot insertion process (Cotrell et al., 2009).

Order elevation process enriches the basis increasing the order of polynomial without changing the original curve and parametrization of the domain (Figure 2.11). Piegl and Tiller (1994) proposed order elevation steps as follows:

- decomposition of B-spline curve into piecewise Bézier curves,
- degree elevation operation on the Bézier pieces,
- removing unnecessary knots.

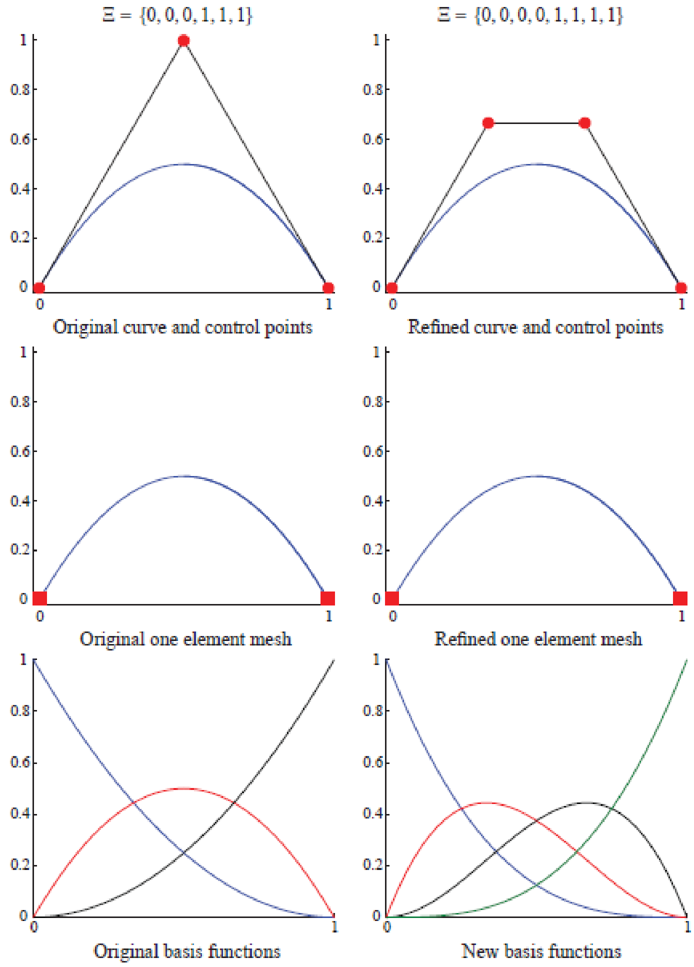


Figure 2.11: Order elevation process (Cottrell et al., 2009).

On the other hand, combined refinement which is more flexible higher-order refinement consists of both knot insertion and order elevation and there is no known anologous practice in standard FEA.

3. FUNDAMENTALS OF ISOGEOMETRIC ANALYSIS

NURBS-based isogeometric analysis employs same basis functions, namely non-uniform rational B-splines (NURBS) to describe both exact geometry and the approximate solution. In this chapter, it is presented how NURBS are used as analysis basis and how the numerical methods are used to assemble the system.

3.1 Using NURBS as Analysis Basis

The main difference between isogeometric analysis and classic finite element analysis is that, in classic FEA, basis functions are used to approximate known geometry and then the unknown solution fields (nodes) while IGA uses exact geometric representation to approximate the solution fields (knots). Figure 3.1 shows this comparing simply.

Classical FEA:	Geometry	$\Longleftarrow$	Fields
		imposed on	
Isogeometric Analysis:	Geometry	$\Longrightarrow$	Fields

Figure 3.1 : Main difference between classic FEA and new concept of IGA (Cottrell et al., 2009).

Isoparametric approach is invoked to implement NURBS as analysis basis and sufficient conditions for a basic convergence proof are also satisfied (see Hughes, 2000). Each point on physical domain can be represented in parametric domain with such geometrical mapping,

$$\boldsymbol{x}(\xi) \equiv \begin{Bmatrix} x(\xi) \\ y(\xi) \\ z(\xi) \end{Bmatrix} = \textstyle\sum_{a=1} N_a(\xi) \begin{Bmatrix} x_a^e \\ y_a^e \\ z_a^e \end{Bmatrix}. \qquad \textbf{(3.1)}$$

$\boldsymbol{x}(\xi)$ represents geometrical mapping from parametric domain to physical domain (inverse of $\boldsymbol{x}$ also exists). Then, approximated solution field in parametric domain can be given as,

$$\widetilde{u^h} = \sum_{A=1} N_A(\xi) d_A. \qquad \textbf{(3.2)}$$

Using inverse of $\boldsymbol{x}(\xi)$, solution field in physical domain is obtained as,

$$u^h = \widetilde{u^h} * \boldsymbol{x}^{-1}. \qquad \textbf{(3.3)}$$

$\boldsymbol{d_A}$ represents the control variables (degrees of freemdom per knot), nodes from standard FEA become knots in IGA. In order to find solution field, different numerical methods are applied ($\boldsymbol{u^h} \approx \boldsymbol{u}$).

For this purpose, Galerkin's method is chosen for all numerical examples in this thesis. Other techniques such as collocation, least-square or meshless methods can also be implemented.

3.2 Galerkin's Method

In order to use Galerkin' method, strong form of the problem should be stated firstly. Then, weak (variational) form of the problem is defined and at the last step, coupled system of linear algebraic equations are obtained with Galerkin's method. For further information, see Cottrell et al. (2009, chapter 3).

System of linear algebraic equations are written in matrix form to get stiffness matrix (**K)**, force vector (**F)** and displacement vector (**d)**,

$$\boldsymbol{Kd} = \boldsymbol{F}. \qquad \textbf{(3.4)}$$

In Figure 3.2, calculation for an element is lucidly explained, there are three spaces named; physical, parametric and parent. Also, there are two mapping; from physical domain to parametric domain and parametric domain to parent domain where the Gaussian quadrature is performed. Here, standard change of variables rules is valid.

In next sections of this chapter, it is shown how the process is achieved to get stiffness matrix, force and displacement vectors in problems related with beams and plates.

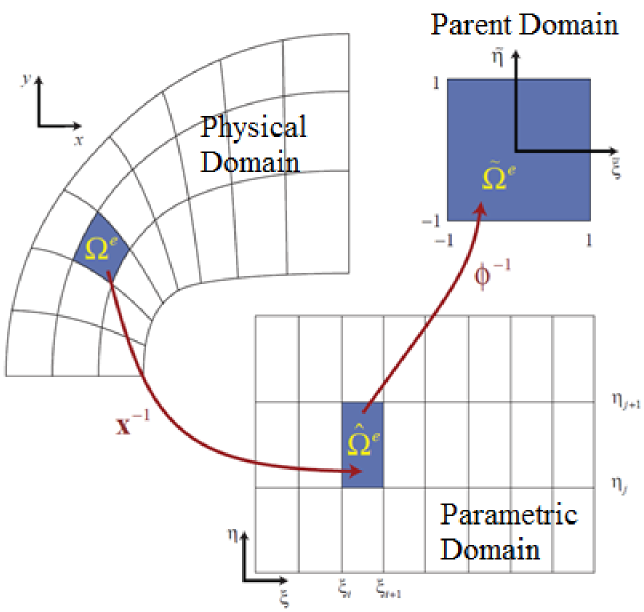


Figure 3.2 : Integration performed on one element (Cottrell et al., 2009).

3.3 Formulations for 1-D and 2-D Structural Problems

In this section, IGA formulations are defined for some problems that are used for numerical examples in the next chapter. Most importantly, these formulations have “rotation-free” characteristics. This means that only unknowns in questions are displacements (x or y direction) and rotations can be computed as displacement derivatives. In order to implement boundary conditions on rotations, there are two different strategies; weak boundary condition imposition and Lagrangian multiplier technique, please see Reali (2004) for further information.

3.3.1 Beam bending problem

In this section, 1-D beam bending formulation regarding Euler-Bernoulli Beam Theory is presented. Strong form of the problem is governed by,

$$\frac{d^2}{dx^2}\left(EI\frac{d^2u}{dx^2}\right) - q = 0, \qquad 0 < x < L. \tag{3.5}$$

Then, weak form of the solution is given as,

$$0 = \int_0^L \left(EI\frac{d^2v}{dx^2}\frac{d^2u}{dx^2} - vq\right)dx - \left[\frac{d}{dx}\left(EI\frac{d^2u}{dx^2}\right)v\right]_{x=0} - \left[\left(EI\frac{d^2u}{dx^2}\right)\left(-\frac{dv}{dx}\right)\right]_{x=0} + \left[\frac{d}{dx}\left(EI\frac{d^2u}{dx^2}\right)v\right]_{x=L} + \left[\left(EI\frac{d^2u}{dx^2}\right)\left(-\frac{dv}{dx}\right)\right]_{x=L}. \tag{3.6}$$

Using Galerkin's approximation, weight functions (u) and test functions (v) are defined as,

$$u(x) = \sum_{i=1}^{n_{en}} \boldsymbol{d_i} R_i(x) \tag{3.7}$$

and

$$v(x) = \sum_{j=1}^{n_{en}} \boldsymbol{b_j} R_j(x). \tag{3.8}$$

$\boldsymbol{d_i}$ shows the unknown field of the problem while $\boldsymbol{b_j}$ is choosen arbitrarily. Also, $\boldsymbol{R}(\boldsymbol{x})$ represents the NURBS basis functions. **(3.7)** and **(3.8)** are put into **(3.5)** and matrix form of the solution is obtained ($\boldsymbol{Kd} = \boldsymbol{F}$) and stiffness matrix and force vector are defined as

$$\boldsymbol{K_{ij}} = \int_\Omega EI\frac{d^2R_i}{dx^2}\frac{d^2R_j}{dx^2}d\Omega \tag{3.9}$$

and

$$\boldsymbol{F_i} = \int_\Omega qR_i d\Omega. \tag{3.10}$$

3.3.2 Beam free vibration problem

Free vibrations of Euler-Bernoulli beam are governed by the differential equation,

$$\frac{d^4u}{dx^4} - w^2u = 0, \quad 0 < x < L. \tag{3.11}$$

Using the same procedures as previous section, equation is written in the form of,

$$\boldsymbol{M}\ddot{\boldsymbol{w}} + \boldsymbol{K}\boldsymbol{w} = 0. \tag{3.12}$$

$\boldsymbol{K}$ is stiffness matrix defined as **(3.9)** and additional $\boldsymbol{M}$ is called mass matrix,

$$\boldsymbol{M_{ij}} = \int_{\Omega} \rho R_i R_j d\Omega. \tag{3.13}$$

The general solution of the free vibration equation can be written as,

$$\boldsymbol{w} = \overline{w}\, exp(i\omega t). \tag{3.14}$$

Substituting **(3.14)** into **(3.12)**, natural frequencies of beams can be calculating by solving eigenvalue equation,

$$(\boldsymbol{K} - \omega^2\boldsymbol{M})\overline{w} = 0. \tag{3.15}$$

3.3.3 Plate bending problem

The strong form of plate bending equation for homogenous and isotropic plates regarding Classical Plate Theory in rectangular coordinates can be expressed as follows,

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \tag{3.16}$$

where q is the lateral loading and D represents the bending stiffness of the plate,

$$D = \frac{Eh^3}{12(1-\nu^2)}. \tag{3.17}$$

$\boldsymbol{E}$, $\boldsymbol{h}$ and $\boldsymbol{\nu}$ are Young's modulus, Poisson's ratio and the thickness of the plate, respectively. Generalized strains and stresses (pseudo-strains and pseudo-stresses) are defined as,

$$\boldsymbol{\varepsilon_p} = \left\{-\frac{\partial^2}{\partial x^2} \quad -\frac{\partial^2}{\partial y^2} \quad -2\frac{\partial^2}{\partial x \partial y}\right\}^T w \tag{3.18}$$

and

$$\boldsymbol{\sigma_p} = \{M_x \quad M_y \quad M_{xy}\}^T \tag{3.19}$$

$\boldsymbol{M_x}$, $\boldsymbol{M_y}$ and $\boldsymbol{M_{xy}}$ are components of bending and twisting moments. Then, generalized Hooke's law for thin plates is defined as

$$\boldsymbol{\sigma_p} = \boldsymbol{D\varepsilon_p} \tag{3.20}$$

where $\boldsymbol{D}$ is a constant matrix of material property and the thickness of the plate,

$$\boldsymbol{D} = D\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \tag{3.21}$$

Galerkin's method gives stiffness matrix as,

$$\boldsymbol{K_{ij}} = \int_{\Omega} \boldsymbol{B_i^T D B_j} d\Omega \tag{3.22}$$

where

$$\boldsymbol{B_i} = \left\{-\frac{\partial^2 R_i}{\partial x^2} \quad -\frac{\partial^2 R_i}{\partial y^2} \quad -2\frac{\partial^2 R_i}{\partial x \partial y}\right\}^T \tag{3.23}$$

and force vector as,

$$\boldsymbol{F_i} = \int_{\Omega} q R_i d\Omega. \tag{3.24}$$

3.3.4 Plate free vibration problem

The strong form of the problem regarding classical plate theory (Kircchoff theory of plates) for homogeneous and isotropic plates can be defined as (Shojaee et al., 2012),

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + \rho h \ddot{w} = q. \tag{3.25}$$

Repeating the same procedure, final dynamical discrete equation is obtained. Additional mass matrix is given as,

$$\boldsymbol{M_{ij}} = \int\limits_{\Omega} \rho\left(R_i R_j h + \frac{\partial R_i}{\partial x}\frac{\partial R_j}{\partial x}\frac{h^3}{12} + \frac{\partial R_i}{\partial y}\frac{\partial R_j}{\partial y}\frac{h^3}{12}\right) d\Omega. \tag{3.26}$$

The general solution of equation is written as **(3.14)** and substituting this equation into **(3.12)** gives eigenvalue equation **(3.15)**.

4. NUMERICAL EXAMPLES

In this chapter, 1-D and 2-D numerical examples of structural problems stated in the Chapter-3 are performed with IGA approach and compared with analytical and FEA solutions. For this purpose, NURBS curves and surfaces designed with Rhinoceros 5 are used. Subroutines (Appendix A and B) generated with Rhinoscript transfer NURBS data (knot vectors, control points and weights) into IGA codes generated with Matlab. Moreover, different types of NURBS have been also used and compared to show how IGA approach is superior to standard FEA and has better approximation properties.

4.1 Application 1: Bending of Beam

Bending of cantilever beam under uniform lateral load is considered as first example. Material properties; Young's modulus (E), density (ρ) and geometric properties; length of beam (L), cross section (h,b) and uniform lateral load (q) are given in Table 4.1 and Figure 4.1. Stiffness matrix and force vector are calculated with **(3.9)** and **(3.10),** respectively.

Table 4.1 : Material properties for application 1.

E	200 GPa
ρ	7850 kg/m^3
L	1m
h	0.1m
b	0.1m
q	50 kN/m

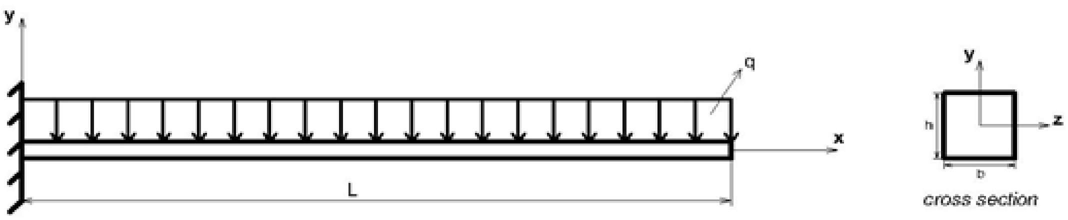


Figure 4.1 : Description of application 1.

IGA solution has been performed with four different NURBS structures and results (maximum deflection and stress distribution) have been compared with analytical solution in Table 4.2 and 4.3. Analytic results are calculated using **(4.1)** and **(4.2)**.

$$\delta = \frac{qx^2}{24EI}(6L^2 - 4Lx + x^2) \quad \textbf{(4.1)}$$

$$M = \frac{q}{2}(L^2 - 2Lx + x^2) \quad \textbf{(4.2)}$$

Table 4.2 : Comparison of maximum deflections for application 1.

	10 points (p=3)	20 points (p=3)	30 points (p=3)	40 points (p=3)	Analytical solution
Max. Deflection (mm)	3.750242	3.749605	3.749682	3.750116	3.750000

Table 4.3 : Comparison of stress distributions according to parametric locations for application 1.

Parametric Location	Max. stresses (Mpa)				
	10 points (p=3)	20 points (p=3)	30 points (p=3)	40 points (p=3)	Analytical solution
0	149.6030	149.8800	149.7065	149.5657	150.0000
0.1	121.5920	121.5945	121.4143	121.3044	121.5000
0.2	96.2720	96.0992	96.1564	95.8436	96.0000
0.3	73.2382	73.4341	73.3442	73.3341	73.5000
0.4	53.9705	54.0077	53.9348	53.9132	54.0000
0.5	37.7692	37.5149	37.5731	37.4460	37.5000
0.6	23.9693	24.0004	23.9624	23.9556	24.0000
0.7	13.2670	13.4517	13.4610	13.4651	13.5000
0.8	6.2271	6.0392	6.0228	5.9986	6.0000
0.9	1.6312	1.5238	1.5058	1.5017	1.5000
1	0.5096	0.0859	0.0348	0.0185	0.0000

4.2 Application 2: Free Vibration Analysis of Beam

In second application, isogeometric free vibration analysis of cantilever beam is considered. Material and geometric properties are the same as first application (Table 4.1 and Figure 4.1). Stiffness matrix, force vector and mass matrix are calculated with using **(3.9)**, **(3.10)** and **(3.13)**, respectively. Table 4.4 shows results of four different NURBS structures and analytic solution to make a comparison between IGA and analytic results. In addition, Figure 4.2 represents first five vibration shapes.

Table 4.4 : Comparison of first five vibration frequencies.

Mode numbers	First five vibration frequencies (Hertz)				
	10 points (p=3)	20 points (p=3)	30 points (p=3)	40 points (p=3)	Analytic solution
1	81.5349	81.5432	81.5421	81.5366	81.5389
2	511.0611	511.0379	511.044	510.9906	510.9952
3	1432.0173	1430.993	1431.021	1430.615	1430.7994
4	2814.152	2804.644	2803.544	2803.888	2803.8007
5	4682.5167	4633.564	4635.768	4635.562	4634.8786

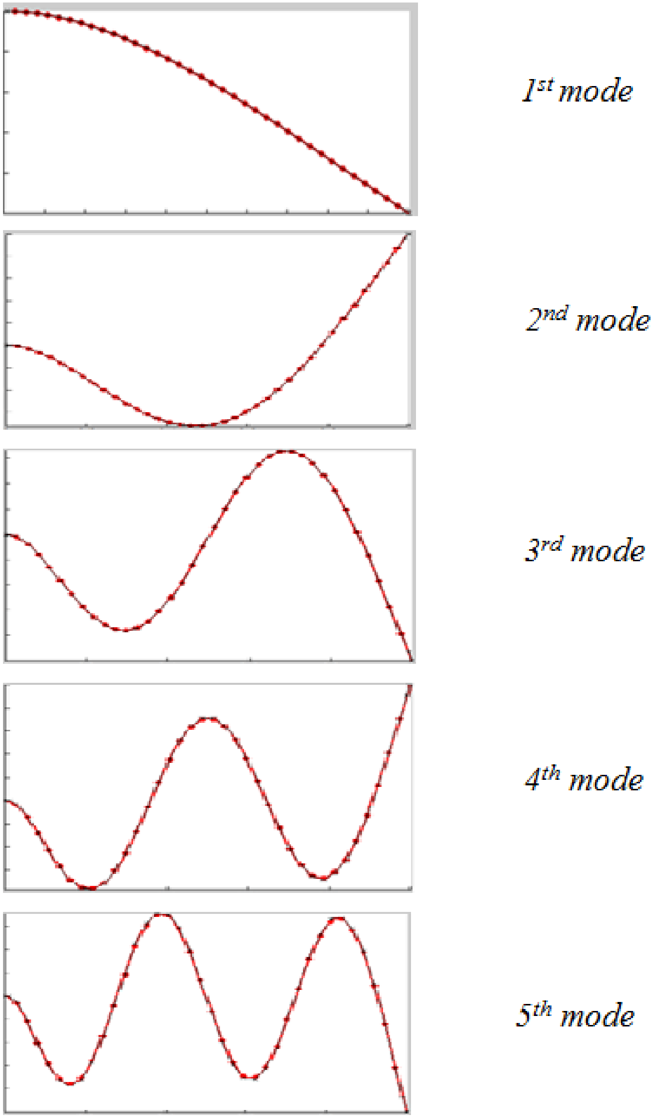


Figure 4.2 : First five vibration shapes of cantilever beam.

4.3 Application 3: Plate Bending

Bending of rectangular simply-supported plate under uniform lateral load regarding classic plate theory (Kirchhoff plate theory) is considered in this application (Figure 4.3). Material properties; Young's modulus (E), Poisson's ratio (ν), density (ρ) and geometric properties; lengths through x and y directions (a and b), thickness of plate (t), uniform lateral load (q) are given in Table 4.5. In order to calculate stiffness matrix and force vector, **(3.22)**, **(3.23)** and **(3.24)** are used.

Table 4.5 : Material properties for application 3.

E	200 GPa
ν	0.3
ρ	7850 kg/m^3
a	1 m
b	1 m
t	0.01 m
q	10 kN/m

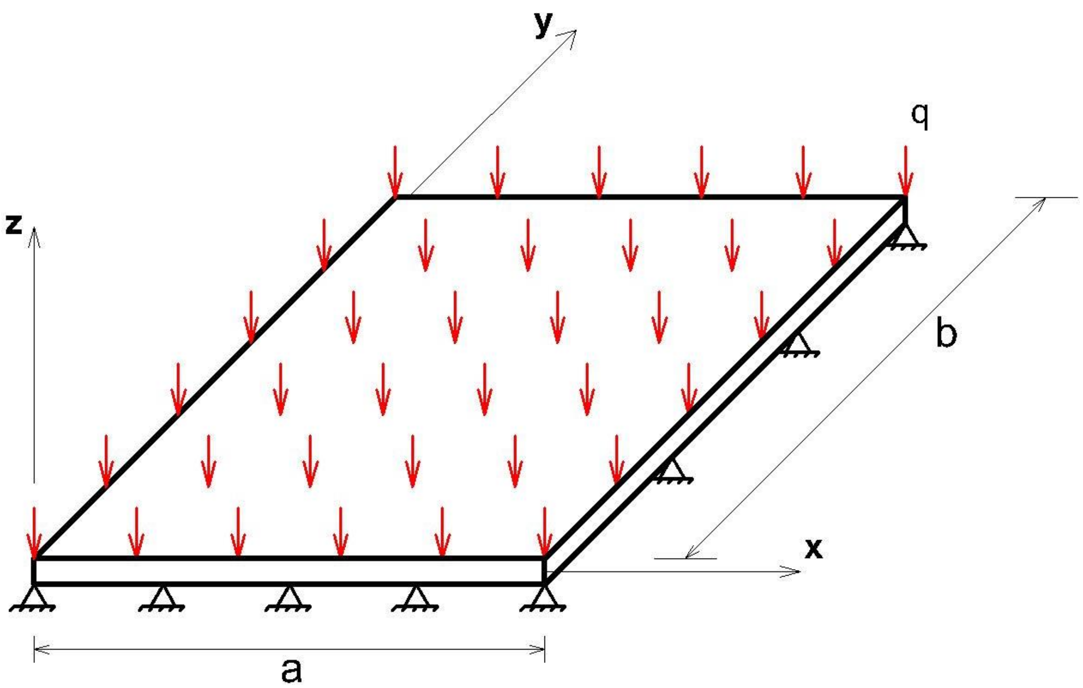


Figure 4.3 : Description of application 3.

Isogeometric analysis of plate has been performed with four different 3rd degree and four different 4th degree NURBS surfaces. Table 4.6, 4.7, 4.8 and 4.9 gives information about results of IGA and also FEA solutions, deflection and stress components calculated at midpoint of the plate have been compared.

Table 4.6 : Comparison of maximum deflections calculated with IGA (3rd degree elements) and FEA solutions.

Maximum deflection (at midpoint of the plate)				
10x10 points p=3	20x20 points p=3	30x30 points p=3	40x40 points p=3	ANSYS solution
2.2523 mm	2.2238 mm	2.2204 mm	2.2192 mm	2.2283 mm

Table 4.7 : Comparison of stress values calculated with IGA (3rd degree elements) and FEA solutions.

	Stresses at the midpoint of plate				
	10x10 points p=3	20x20 points p=3	30x30 points p=3	40x40 points p=3	ANSYS solution
σ_x (Mpa)	28.6558	28.6901	28.7657	28.6731	28.7980
σ_y (Mpa)	28.6558	28.6901	28.7657	28.6731	28.7980

Table 4.8 : Comparison of maximum deflections calculated with IGA (4th degree elements) and FEA solutions.

Maximum deflection (at midpoint of the plate)				
10x10 points p=4	20x20 points p=4	30x30 points p=4	40x40 points p=4	ANSYS solution
2.3120 mm	2.2311 mm	2.2231	2.2207 mm	2.2283 mm

Table 4.9 : Comparison of stress values calculated with IGA (4th degree elements) and FEA solutions.

	Stresses at the midpoint of plate				
	10x10 points p=4	20x20 points p=4	30x30 points p=4	40x40 points p=4	ANSYS solution
σ_x (Mpa)	28.7437	28.7153	28.7141	28.7732	28.7980
σ_y (Mpa)	28.7437	28.7274	28.7141	28.7732	28.7980

4.4 Application 4: Free Vibration Analysis of Plate

In this example, isogeometric free vibration analysis of rectangular simply-supported plate is studied. Material and geometric properties of the plate are the same as previous application (Table 4.5 and Figure 4.3). In order to calculate stiffness matrix, force vector and mass matrix, **(3.22)**, **(3.23)**, **(3.24)** and **(3.26)** are used.

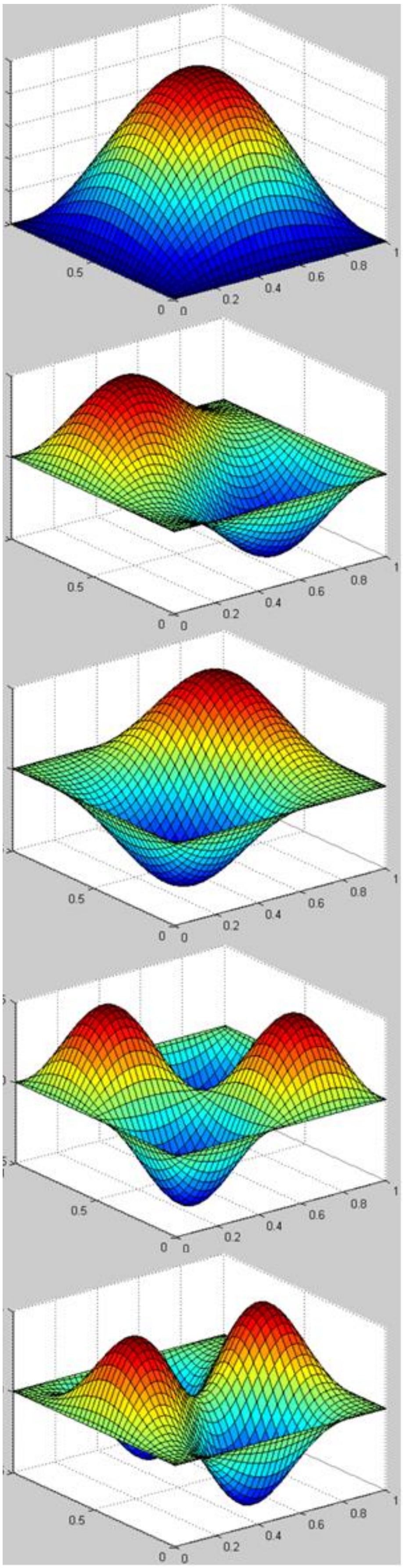
Table 4.10 and 4.11 compare the vibration frequencies calculated with different NURBS structures and analytic solution while Figure 4.4 illustrates first five vibration shapes calculated with IGA.

Table 4.10 : Comparison of free vibration frequencies calculated with IGA (3rd degree elements) and analytic solution.

10x10 points p=3	20x20 points p=3	30x30 points p=3	40x40 points p=3	Analytic solution
47.9813	47.9824	47.9824	47.9825	47.9865
119.9472	119.9407	119.9407	119.9415	119.9662
119.9472	119.9407	119.9407	119.9415	119.9662
191.8306	191.8800	191.8814	191.8827	191.9458
240.1133	239.8318	239.8262	239.8372	239.9323

Table 4.11 : Comparison of free vibration frequencies calculated with IGA (4th degree elements) and analytic solution.

Free vibration frequencies (Hertz)				
10x10 points p=4	20x20 points p=4	30x30 points p=4	40x40 points p=4	Analytic solution
47.9844	47.9822	47.9824	47.9823	47.9865
119.9428	119.9414	119.9549	119.9397	119.9662
119.9428	119.9436	119.9549	119.9397	119.9662
191.8824	191.8864	191.9031	191.8801	191.9458
239.9510	239.8419	239.8252	239.8244	239.9323



1st mode

2nd mode

3rd mode

4th mode

5th mode

Figure 4.4 : First five vibration shapes of rectangular simply-supported plate.

5 CONCLUSION

In this thesis framework, a novel numerical method; isogeometric analysis, its main idea and advantages has been introduced. It has been also validated with numerical examples relate with beam and plate problems. Beam problems have been modeled regarding Euler-Bernoulli beam theory where as classic plate theory (Kircchoff plate theory) is taken into account for plate problems. It can be clearly pointed out that isogeometric analysis is powerful alternative for classic finite element analysis.

Firstly, isogeometric analysis uses CAD geometry directly to analyze the system. Thus, design and analysis stages are not seperated from each other and exact geometric representation is used in analysis. This obviates FEA geometry approximation and time-consuming mesh processes and also possible convergence and accuracy problems. IGA uses basis functions which have higher continuity instead of well-known C0 continous Lagragian polynomials. On the other hand, control points of CAD geometry become degrees of freedom with IGA.

Secondly, accurate results can be obtained even with coarse mesh structure. This feature can be seen in numerical examples. NURBS structures which have lower element number (lower degrees of freedom) can give accurate results comparing with FEA or analytic solution and decrease analysis time.

The main objective of this thesis has been also achieved with create communication between a commercial CAD program (Rhinoceros 5) and generated IGA code in Matlab 2012b. This unifying feature provides “exact” integration between CAD and FEA as stated by Cottrell et al. (2009).

IGA can be implemented into different fields and some suggestions can be made considering naval architecture studies for future works,

- Linear and non-linear buckling analysis of beams and plates
- Linear and non-linear buckling analysis of stiffened plates
- Fluid-structure interaction problems
- Hydrodynamics problems (BEM applications)

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APPENDICES

APPENDIX A : Rhinoscript code for curve elements
APPENDIX B : Rhinoscript code for surface elements

APPENDIX A

Rhinoceros has useful tool to write subroutines (macros) to use in program. This subroutine takes the information (knot vector, control point coordinates and their weights) of NURBS curves and write them into a Excel datasheet.

```
Sub NURBSCURVEtoEXCEL()
Const rhCurve = 4
Dim arrCurves
arrCurves = Rhino.GetObjects("Select curves to export", rhCurve, True, True)
Dim objXL
Set objXL = CreateObject("Excel.Application")
objXL.Visible = True
objXL.WorkBooks.Add
objXL.Sheets("Sayfa1").Select
Dim intIndex1, intIndex2, intIndex3, intIndex4
intIndex1 = 1
intIndex2 = 1
intIndex3 = 1
intIndex4 = 1
intIndex5 = 4
Dim arrKnots, arrPoints, arrWeights, intCount
For Each arrCurve In arrCurves
        arrKnots = Rhino.CurveKnots(arrCurve)
        For Each dblKnot In arrKnots
                objXL.Cells(intIndex1, intIndex2).Value = Round(dblKnot, 4)
                intIndex1 = intIndex1 + 1
        Next
```

```
        intIndex2 = intIndex2 + 1
        intIndex1 = 1
        Next
objXL.Sheets("Sayfa2").Select
For Each arrCurve In arrCurves
        arrPoints = Rhino.CurvePoints(arrCurve)
        For Each arrPoint In arrPoints
                objXL.Cells(intIndex1, intIndex3).Value = Round(arrPoint(0), 4)
                objXL.Cells(intIndex1, intIndex3 + 1).Value = Round(arrPoint(1), 4)
                objXL.Cells(intIndex1, intIndex3 + 2).Value = Round(arrPoint(2), 4)
                intIndex1 = intIndex1 + 1
        Next
        intIndex3 = intIndex3 + 4
        intIndex1 = 1
Next
For Each arrCurve In arrCurves
        arrWeights = Rhino.CurveWeights(arrCurve)
        For Each arrWeight In arrWeights
                objXL.Cells(intIndex1, intIndex5).Value = Round(arrWeight, 4)
                intIndex1 = intIndex1 + 1
        Next
        intIndex5 = intIndex5 + 4
        intIndex1 = 1
Next
objXL.UserControl = True
End Sub
```

APPENDIX B

This subroutine takes the information (knot vectors, control point coordinates and their weights) of NURBS surfaces and write them into a Excel datasheet.

```
Sub NURBSSURFACEtoEXCEL()

Const rhSurface = 8

Dim arrSurfaces

arrSurfaces = Rhino.GetObjects("Select surfaces to export", rhSurface, True, True)

Dim objXL

Set objXL = CreateObject("Excel.Application")

objXL.Visible = True

objXL.WorkBooks.Add

objXL.Sheets("Sayfa1").Select

Dim intIndex1, intIndex2, intIndex3, intIndex4

intIndex1 = 1

intIndex2 = 1

intIndex3 = 1

intIndex4 = 2

intIndex5 = 4

Dim arrKnots, arrPoints, arrWeights, intCount

For Each arrSurface In arrSurfaces

        arrKnots = Rhino.SurfaceKnots(arrSurface)

        For Each dblKnot In arrKnots(0)

                objXL.Cells(intIndex1, intIndex2).Value = Round(dblKnot, 4)

                intIndex1 = intIndex1 + 1

        Next

        For Each dblKnot In arrKnots(1)
```

```
            objXL.Cells(intIndex3, intIndex4).Value = Round(dblKnot, 4)
            intIndex3 = intIndex3 + 1
        Next
        intIndex2 = intIndex2 + 2
        intIndex4 = intIndex4 + 2
        intIndex1 = 1
        intIndex3 = 1
Next
objXL.Sheets("Sayfa2").Select
For Each arrSurface In arrSurfaces
        arrPoints = Rhino.SurfacePoints(arrSurface)
        For Each arrPoint In arrPoints
            objXL.Cells(intIndex1, intIndex3).Value = Round(arrPoint(0), 4)
            objXL.Cells(intIndex1, intIndex3 + 1).Value = Round(arrPoint(1), 4)
            objXL.Cells(intIndex1, intIndex3 + 2).Value = Round(arrPoint(2), 4)
            intIndex1 = intIndex1 + 1
        Next
        intIndex3 = intIndex3 + 4
        intIndex1 = 1
Next
For Each arrSurface In arrSurfaces
        arrWeights = Rhino.SurfaceWeights(arrSurface)
        For Each arrWeight In arrWeights
            objXL.Cells(intIndex1, intIndex5).Value = Round(arrWeight, 4)
            intIndex1 = intIndex1 + 1
        Next
```

```
        intIndex5 = intIndex5 + 4
        intIndex1 = 1
Next
objXL.UserControl = True
End Sub
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CURRICULUM VITAE

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