

**İSTANBUL TECHNICAL UNIVERSITY ★ INSTITUTE OF SOCIAL SCIENCES**

**EFFECTS OF A QUALITY DIFFERENTIATION AMONG DOWNSTREAM  
FIRMS DUE TO AN UPSTREAM QUALITY INVESTMENT**

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**TEDARİKÇİNİN KALİTE YATIRIMI SONUCU ÜRETİCİLERİN  
KALİTELERİNİN FARKLILAŞMASININ ETKİLERİ**

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## **FOREWORD**

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Buğra SEÇGİN

Engineer

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# **EFFECTS OF A QUALITY DIFFERENTIATION AMONG DOWNSTREAM FIRMS DUE TO AN UPSTREAM QUALITY INVESTMENT**

## **SUMMARY**

We analyze the effects of a quality differentiation of downstream firms due to an upstream quality investment. There is an upstream firm that supplies the intermediate goods to two downstream manufacturers. Manufacturers have different quality efficiencies and a quality investment of upstream supplier creates a quality differentiation among the manufacturers. The degree of quality differentiation is important in determining the effects of low quality firm's quality, on its rival. The effects are positive when there is significant differentiation. When quality efficiencies are close, downstream firms behave almost like identical firms and the quality increase of low quality firm affects rival firm negatively. Also when there is significant quality differentiation, increase in investment costs favors the firm with low quality where as the firm with high quality is always negatively affected. As an extension, we investigate the effects of a different timing. Moving first gives the firm with high quality an opportunity to leave its rival out of the market.

## **TEDARİKÇİNİN KALİTE YATIRIMI SONUCU, ÜRETİCİLERİN KALİTE FARKLILAŞMASININ ETKİLERİ**

### **ÖZET**

Tedarikçi firmanın yaptığı kalite yatırımı sonucu, üretici firmaların kalite farklılaşmasının etkilerini inceledik. Bir tedarikçi firma, iki üreticiye yarı mamül sağlamaktadır. Üreticilerin kalite verimlilikleri farklıdır, ve tedarikçinin yapacağı bir kalite yatırımı, üreticiler arasında bir kalite farklılaşması yaratır. Bu kalite farklılaşmasının derecesi, düşük kaliteli firmanın kalitesinin rakibi üzerindeki etkilerini belirlemekte önemlidir. Bu etkiler, eğer kaydadeğer bir farklılaşma varsa pozitifdir. Kalite verimlilikleri yakın ise, firmalar neredeyse aynı şekilde davranır ve bu durumda düşük kaliteli firmanın kalitesindeki bir artış rakip firmayı negatif etkiler. Ayrıca, yatırım maliyetlerindeki artış, kaydadeğer bir kalite farklılaşması olduğunda, düşük kaliteli üreticinin yararına olduğu halde, yüksek kaliteye sahip üreticiyi her zaman olumsuz etkiler. Ek olarak, farklı zamanlamaların etkilerini araştırdık. İlk hareket etmek, yüksek kaliteli şirkete rakibini piyasanın dışında bırakma şansı verir.

## 1. INTRODUCTION

The purpose of this paper is to examine the effects of a quality differentiation of downstream firms due to an upstream quality investment. There are two downstream manufacturer firms in the model, that produce the same product, and an upstream firm that supply the manufacturers with the intermediate goods. Upstream firm makes a quality investment, to increase the quality level of its intermediate goods. Downstream firms have different abilities to gain advantage from this quality investment. We may say that one of the firms has a better infrastructure, has a higher knowledge stock or more qualified employees which are not put into use when semi products have basic quality. Like a better sword favors the better swordsman, a quality increase favors the firm that has more tools to benefit from quality.

For example think of two machine parts manufacturer. Let one of them have a better thermal processing facility, which can heat up to higher temperatures compared to that of the rival. If the supplier of steel makes a quality investment, and start producing a new alloy that gives better strength after tempered at higher temperatures, the firm with a better thermal processing unit becomes advantageous. The parts it produces are more rigid compared to that of rival. For another example, consider two software companies, one of which has more talented programmers. If they are supplied with faster computers, the company with talented programmers make a difference, where as a faster computer would mean less to the other firm's programmers. Or think of a case where there are two plastic toys manufacturers. In the case where the supplier of plastic makes a quality investment and starts producing a kind of plastic that is easier to give form, the manufacturer that has better designers is favored. Many more examples can be given about the case.

Of course in real life, the firm that has a better infrastructure, a higher knowledge stock or more qualified employees might have a better quality compared to that of rival for any given intermediate good quality level. Yet assuming that both firms have same quality without an upstream investment is not totally wrong and it

provides a base point to compare changes due to upstream investment. Also it makes the model more simple and solvable.

Within the model, four decisions are made. Upstream firm makes a quality investment decision, and sets a wholesales price. Downstream firms make cost reducing investment decisions and set their output levels. All of these decisions affect the whole supply chain. A quality investment of supplier eventually yields higher quality final goods, which increases both the demand for downstream final goods and upstream intermediate goods. Downstream output level decision affects both its profits and rival's profits. It also affects the demand for upstream firm. Wholesale price decision affects both manufacturers' output decisions, and their demand for intermediate goods. Cost reducing investment of a downstream firm provides lower price and an increased output, which increases the demand supplier faces.

The main target of this paper is to analyze the effects of a quality differentiation of manufacturers due to quality investment which is made by the supplier. Within the model, we use a coefficient to represent this differentiation, and we call it "quality efficiency". It is a value between 0 and 1, and it determines the percentage of quality investment being used effectively. If we go back to our first example, the machine parts manufacturer may have a thermal processing unit that fulfills the needs of the new alloy. In this case, quality efficiency is 1. Or, thermal processing unit can heat up to temperatures higher than what is needed for basic alloy, but not as much as what is needed for new alloy. In this case, quality efficiency value is in somewhere between 0 and 1. Both downstream firms have a different quality efficiency values, which causes the differentiation between the firms.

We also examine the effects of investment costs. Within the model, as in real world, every investment decision has a cost. An investment might be cheap or expensive depending on the cost structure. To investigate this, we use separate coefficients for downstream investment and upstream investment.

Analyzing the model, we have come to following main results. If firms' quality differentiation is not significant, firms behave almost like identical firms, yet, still the firm with higher quality efficiency serves to a larger portion of the market. In this setting, either firms quality efficiency has a negative effect on the other firm.

An increase in quality differentiation favors the firm with high quality efficiency, and a decrease in quality differentiation favors the firm with low quality efficiency. Think of the toys manufacturers. The firm with better designers wants the other firm's designers to have very low talent. By this way, former firm can have a lot better toys compared to its rival and make more sales. On the other hand, firm with less talented designers wants the other firm's designers to have as low talent as possible, so that quality of toys are very similar. Any situation that widens the quality differentiation gap favors the first firm, and any situation that narrows it favors the latter.

If the downstream firms' quality differentiation is quite significant, firm with the high quality efficiency dominates the market, leaving the other firm a little portion. In this setting, firm with low quality efficiency still wants the quality differentiation gap to be as narrow as possible. However the firm with high quality efficiency is better off when rival has as high quality efficiency as possible. This may sound weird, yet there is a simple explanation. Upstream firm has incentive to invest more when downstream firms together can make a better use of it. Thus, dominant firm can use every bit of quality increase to its advantage since its rival has a very low ability to gain advantage from quality compared to dominant firm.

Our last findings are about investment costs. As expected, expensive investments cause losses for all three firms, unless quality differentiation between the manufacturers is very big. In that case, the firm with less quality is favored by expensive investments. That is understandable since both upstream and downstream investments reinforce other firm's superiority. Consider the machine parts manufacturers. If one of the manufacturers has a thermal processing unit that can not reach the temperatures new alloy needs, and its rival has the sufficient equipment, the former would not want upstream to make any quality investment. This is because, new alloy makes the firm with outdated technology to serve only to a little portion of the market. In the case of no quality investment, both manufacturers share the market equally.

## 1.1 Literature Review

Foros (2004) investigates a similar case where there are two retailers one of which is vertically integrated and invests in quality. Both firms have different levels of ability to offer value-added services. His main goal is to find effects of a price regulation on consumer surplus and welfare. Also, he investigates the possibilities for integrated firm to use over investment as a foreclosure tool.

Wickelgren (2004) pays attention to investment incentives when two firms have differentiated products and they sell their goods through bargaining. He uses a linear city model, and adds up holdup problem. By this paper, Wickelgren finds that competition can improve incentive for investment.

In their paper, Buehler and Shmutzler (2008) investigates a case where there are two upstream firms and two downstream firms. They seek to find the effects both vertical integration and cost reducing downstream investment. Their main findings are as follows. Vertical integration decreases rivals innovation efforts, thus not integrating vertically is a less likely outcome. Also, they find out that asymmetric integration is an equilibrium outcome.

Erkal (2007) investigates the effects of demand for specific investments on a downstream duopoly's product variety. She uses an extension of standard hoteling model. She finds that for fully specialized suppliers to emerge, downstream firms should require specific inputs and they should be willing to make contracts. Also, if upstream firms semi products are not specialized, downstream firms produce more similar outputs to increase investment incentives for the upstream firm.

In their paper, Banerjee and Lin (2003) examine the incentives to make cost reducing investments in a duopoly. They find that a downstream firm has an incentive to invest in cost reduction since it increases intermediate goods' price and eventually raises rival's cost. Therefore, there is more investment incentive for a downstream duopoly compared to a downstream monopoly. They also find that a contract that stabilizes the intermediate goods' price eliminates the opportunistic behavior of upstream supplier and increases welfare.

Milliou (2004) analyzes the effects of research and development spillovers on innovation incentives of firms, and on welfare. He creates a model where there are

two upstream and two downstream firms two of which are vertically integrated. There is information flow from downstream firm to its rival through upstream firms. He finds that in a case where both downstream firms make cost reducing investments, the information flow favors the integrated firm and harms the non integrated firm. He also finds that if information flow is not much, product differentiation is high and investment is not costly, then use of firewalls decreases welfare.

In his paper, Ishii (2004) investigates the effects of cooperative research and development in a market with two upstream suppliers and two downstream manufacturers. His findings are as follows. Firstly, if the spillover between the upstream firms is not too high, vertical research and development cartel causes a higher social welfare than a horizontal research and development cartel. Secondly, vertical research and development cartel yields a higher welfare than a non-cooperative research and development in every case. He also finds that a vertical research joint venture accelerates tech improvement, thus a vertical research joint venture yields largest social welfare if related firms can coordinate.

In their work, Banerjee and Lin (2001) investigates the incentives for upstream and downstream firms to form a research joint venture. They make a model which consists of an upstream firm and  $n$  number of downstream firms, where upstream firm make investments and downstream firms share the investment costs thus, firms that don't participate in cost sharing can't benefit from the investment. Their research shows that upstream firm wants a larger number of firms to be in research joint venture compared to the downstream firms.

Symeonidis (2003) compares Cournot and Bertrand equilibria of a duopoly which produce differentiated products, and make product research and development. He finds that firms have higher sales price they achieve higher profits when they compete for outputs. He also finds that, if there is only little spillover and significant product differentiation, total output, consumer surplus and welfare are higher for Bertrand. For the opposite case, total output, consumer surplus and welfare are higher for Cournot equilibrium. One of the main findings of his paper is that in certain circumstances, Cournot competition is better for both firms and consumers than Bertrand competition.

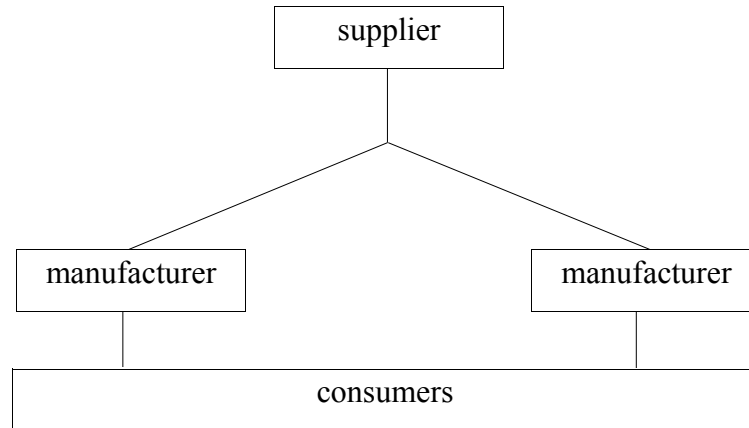
In his paper, Toshimitsu (2003) examines a quality differentiated duopoly where firms determine their quality before production. He finds that a research and development subsidy to the firm with high quality increases social welfare and a subsidy to the firm with low quality increases welfare only if firms are in a Bertrand competition.

In their paper, Lin and Saggi (2002) studies the relationship between cost reducing investments and product developing investments under Cournot and Bertrand competitions. They find that product and process research and development affect each other positively. Thus, cooperation in product development has a positive effect on both type of investments, where as, cooperation in cost reducing investment has a negative effect on both.

In his work, Aoki (2003) investigates the effects of commitment to quality choice on equilibrium qualities and welfare under Bertrand and Cournot competition. He finds that the firm that has the advantage of moving first picks a higher quality thus achieves higher profits.

## 2. THE MODEL

As we state in the introduction, our main goal is to explore the consequences of an upstream quality investment on downstream firms, when quality investment leads to a quality differentiation among the downstream firms. In our model, there is a single upstream supplier that supplies the downstream firms with intermediate goods. Two downstream manufacturers purchase intermediate goods from upstream supplier at a price the supplier determines. Manufacturers process the semi products and offer final goods to the market. Each manufacturer determines its own output, and sales price is determined in the market. Manufacturers also make cost reducing investments to become more competitive. Without an upstream quality investment, the final products are of basic quality and each downstream firm shares the market equally. Figure (2.1) shows the supply chain.



**Figure 2.1:** Supply chain of the model

The amount of quality upstream firm achieves after an investment, is not directly transferred to customers. Each downstream firm has its own knowledge stock and infrastructure which determine the percent of upstream quality transferred to the customers. Therefore, after the investment, a quality differentiation among the downstream manufacturers occurs.

In order to model this situation, we use a simplified version of the utility function of Toshimitsu (2003). In his model,  $\theta$  is used as a multiplier of quality (  $U = \theta(\text{quality})$  ) and it represents consumer choices of price over quality. In our

model, we assume that there is a single consumer choice of price over quality and it is same for every individual in the market. Consumers are instead defined by their maximum willingness to pay, which is represented by  $\theta$ . In this respect, the utility of a consumer is as follows:

$$U = \theta \quad \{ 1 > \theta > 0 \} \quad (2.1)$$

where  $\theta$  represents consumer type, and it is homogeneously distributed between 0 and 1.

The surplus of  $n^{\text{th}}$  consumer who buys a single unit of product is:

$$S = \theta_n + q - p \quad (2.2)$$

where  $q$  represents initial quality, and  $p$  represents initial price of the product. Since the downstream firms are identical, there is only a single sales price. If this consumer buys one unit of the product, he/she benefits from the quality and suffers from the price of the product. If the difference between price and quality is bigger than this consumer's willingness to pay ( $\theta_n$ ), then he/she does not make a purchase.

We use the model Foros (2004) uses to define downstream firms' ability to offer value added services to create a quality differentiation. After an upstream investment, supplier increases its intermediate goods quality by  $y$  amount. However, downstream firms can transfer only a certain percent ( $\beta$ ) of this quality to the market. Hence, consumer  $n$  who buys a single unit of product from firm  $i$  has the following surplus:

$$S_n = \theta_n + q + y * \beta_i - P_i \quad (2.3)$$

where  $y$ ,  $\beta_i$  and  $P_i$  indicate upstream investment, downstream firm  $i$ 's quality efficiency and firm  $i$ 's sales price respectively. If this consumer buys from firm  $j$ , then he has the following surplus:

$$S_n = \theta_n + q + y * \beta_j - P_j \quad (2.4)$$

where  $\beta_j$  and  $P_j$  indicate downstream firm  $j$ 's quality efficiency and sales price respectively.

Here we make the assumption in Foros (2004). Since consumers are homogenous in their evaluation of quality, prices of downstream firms should be equal once the effect of quality is removed. Otherwise firms can not be active in the market at the same time.

$$P = P_i - y * \beta_i = P_j - y * \beta_j \quad (2.5)$$

It is obvious that surplus should be non negative for any consumer to buy a unit of product. So we can say that the last consumer that buys one unit of product from firm i has zero surplus after the transaction.

$$S_{min} = \theta_{min} + q - P = 0 \quad (2.6)$$

Rearranging equation (2.6):

$$\theta_{min} = P - q \quad (2.7)$$

Since we know from equation (2.1) that consumers are distributed between 1 and 0, we can say that the total number of goods sold (total demand) is:

$$X_{total} = 1 - \theta_{min} \quad (2.8)$$

If we substitute equation (2.7) in (2.8), then we have:

$$X_{total} = 1 + q - P \quad (2.9)$$

Then we substitute quality adjusted prices of firm i and firm j in equation (2.5) for  $p$  in equation (2.9) and find demand functions both downstream firm faces:

$$X_{total} = 1 + q + y * \beta_i - P_i \quad (2.10)$$

$$X_{total} = 1 + q + y * \beta_j - P_j \quad (2.11)$$

Since  $X_i + X_j = X_{total}$ , rearranging equations (2.10) and (2.11) we find inverse demand functions as follows:

$$P_i = 1 + q - X_i - X_j + y * \beta_i \quad (2.12)$$

$$P_j = 1 + q - X_i - X_j + y * \beta_j \quad (2.13)$$

As it can easily be seen, both initial quality and quality due to an investment has a positive impact on sales prices, where as quantities of downstream firms have a negative effect.

Upstream firm supplies downstream firms with intermediate product. Downstream firms process intermediate product and manufacture final good. The price of final good is determined within the market through the inverse demand functions (2.12) and (2.13). We assume that for both upstream and downstream firms, marginal costs of production are zero ( $c=0$ ). Upstream firm charges downstream firms with a wholesale price  $w$ . Both supplier and manufacturer have quadratic investment costs. The profit function of supplier is:

$$\Pi_{up} = w(X_i + X_j) - k y^2 \quad (2.14)$$

First term of the equation is total revenue of the upstream firm, second term is the total cost of investment.  $y$  represents the amount of quality investment upstream firm makes and coefficient  $k$  represents the marginal cost of investment. If  $k$  is high, total cost of investment is high. (and vice versa). We assume  $k > 1$  so that the second order conditions hold.

Downstream firms have the following profit functions:

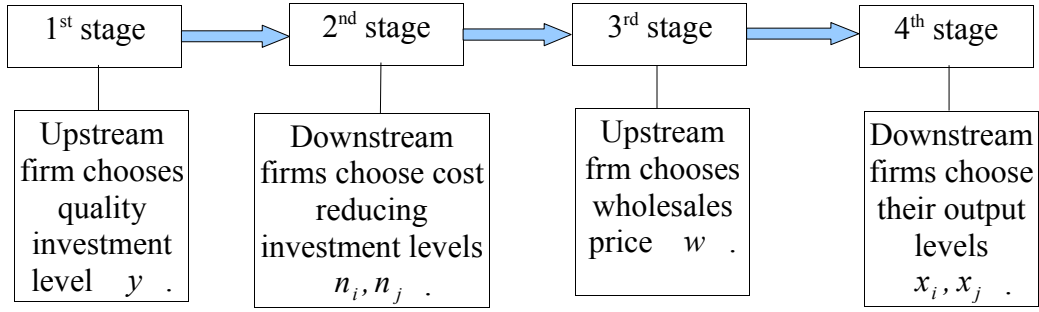
$$\Pi_i = X_i(P_i - w + n_i) - m n_i^2 \quad (2.15)$$

$$\Pi_j = X_j(P_j - w + n_j) - m n_j^2 \quad (2.16)$$

In both equations, first term is total revenue whereas second term is the total cost of cost reducing investment.  $n_i$  and  $n_j$  are cost reducing investments of firm  $i$  and firm  $j$  respectively. Coefficient  $m$  represents the marginal cost of investment. If  $m$  is high, total cost of investment is high (and vice versa). We assume that  $m > 1$  so that the second order conditions hold.

Our model consists of a four stage game. In the first stage, upstream firm decides quality investment level, in the second stage, downstream firms decide their cost reducing investment levels simultaneously, in the third stage, upstream firm decides wholesale price and in the forth stage, downstream firms compete in quantities (firms choose their quantities simultaneously). We assume that there is perfect

information. We solve the model with backwards induction method. Figure (2.2) shows the timing of the model.



**Figure 2.2:** Timing of the game

#### *4<sup>th</sup> stage*

In this stage of the game, both downstream firms determine the quantities they produce. Profit functions of the firms are given in equations (2.15) and (2.16). If we set the first order conditions of these functions equal to zero, we obtain the following reaction functions:

$$\frac{\partial \Pi_i}{\partial X_i} = 1 + q - w + n_i - 2X_i - x_j + y\beta_i = 0 \quad X_i = \frac{1 + q - w + n_i - x_j + y\beta_i}{2} \quad (2.17)$$

$$\frac{\partial \Pi_j}{\partial X_j} = 1 + q - w + n_j - X_i - 2x_j + y\beta_j = 0 \quad X_j = \frac{1 + q - w + n_j - x_i + y\beta_j}{2} \quad (2.18)$$

If we solve equations (2.17) and (2.18) for the Cournot-Nash Equilibrium, we obtain:

$$X_i^c = \frac{1}{3}(1 - c + q - w + 2n_i - n_j + 2y\beta_i - y\beta_j) \quad (2.19)$$

$$X_j^c = \frac{1}{3}(1 - c + q - w + 2n_j - n_i + 2y\beta_j - y\beta_i) \quad (2.20)$$

(Superscript *c* indicates the equilibrium values of the fourth stage)

We have to check for second order conditions to make sure profit function is concave with respect to quantities. Second order conditions should be negative for a maximum:

$$\frac{\partial^2 \Pi_i}{\partial X_i^2} = -2 < 0$$

$$\frac{\partial^2 \Pi_j}{\partial X_j^2} = -2 < 0 \quad (2.21)$$

Equation (2.21) ensures that profit function is concave with respect to quantities (X). We also have to check the stability of the equilibrium. For the equilibrium to be stable and unique, the following condition should hold:

$$\left(\frac{\partial^2 \Pi_i}{\partial X_i^2}\right) * \left(\frac{\partial^2 \Pi_j}{\partial X_j^2}\right) > \left(\frac{\partial^2 \Pi_i}{\partial X_i \partial X_j}\right) * \left(\frac{\partial^2 \Pi_j}{\partial X_i \partial X_j}\right) \quad (2.22)$$

Left part of the inequality yields 4, and right part of the inequality yields 1 so the stability condition holds.

### 3rd stage

In this stage of the game, upstream firm determines its wholesale price. Profit function of the upstream firm is given in equation (2.14). Substituting the equilibrium outputs in equations (2.19) and (2.20) into profit function of the upstream firm, we obtain:

$$\Pi_u^c = \frac{1}{3}(2w + 2qw - 2w^2 - 3ky^2 + wn_i + wn_j + wy\beta_i + wy\beta_j) \quad (2.23)$$

Setting first order conditions equal to zero and solving for wholesale price, gives us the wholesale price which maximizes the upstream profit.

$$\frac{\partial \Pi_u^c}{\partial w} = \frac{1}{3}(2 + 2q - 4w + n_i + n_j + y\beta_i + y\beta_j) = 0$$

$$w^w = \frac{1}{4}(2 + 2q + n_i + n_j + y\beta_i + y\beta_j) \quad (2.24)$$

(Superscript  $w$  indicates the equilibrium values of the third stage)

Second order conditions should be negative for a maximum:

$$\frac{\partial^2 \Pi_u^c}{\partial w^2} = -\left(\frac{4}{3}\right) < 0 \quad (2.25)$$

Equation (2.25) shows us that second order condition is satisfied.

2nd stage

In the second stage of the game, firms simultaneously choose their cost reducing investment levels to maximize their profit. We substitute the equilibrium wholesale price and equilibrium quantities into the profit functions and obtain:

$$\Pi_i^w = \frac{1}{144}((49 - 144m)n_i^2 + 14n_i A + A^2)$$

$$A = (2 + 2q - 5n_j + 7y\beta_i - 5y\beta_j) \quad (2.26)$$

$$\Pi_j^w = \frac{1}{144}((49 - 144m)n_j^2 + 14n_j T + T^2)$$

$$T = (2 + 2q - 5n_i + 7y\beta_j - 5y\beta_i) \quad (2.27)$$

Differentiating  $\Pi_i^w$  with respect to  $n_i$ , and  $\Pi_j^w$  with respect to  $n_j$ , setting them equal to zero and solving for  $n_i$  and  $n_j$ , we obtain the following investment reaction functions:

$$n_i = \frac{7(2 + 2q - 5n_j + 7y\beta_i - 5y\beta_j)}{-(49 - 144m)} \quad (2.28)$$

$$n_j = \frac{7(2 + 2q - 5n_i - 5y\beta_i + 7y\beta_j)}{-(49 - 144m)} \quad (2.29)$$

Solving equations (2.28) and (2.29) simultaneously, equilibrium investment levels can be found as follows:

$$n_i^d = \frac{7(-7 + 12m - 7q + 12mq - 7y\beta_i + 42my\beta_i - 30my\beta_j)}{49 - 588m + 864m^2} \quad (2.30)$$

$$n_j^d = \frac{7(-7 + 12m - 7q + 12mq - 30my\beta_i - 7y\beta_j + 42my\beta_j)}{49 - 588m + 864m^2} \quad (2.31)$$

(Superscript  $d$  indicates the equilibrium values of the second stage)

We have to check for second order conditions to make sure profit function is concave with respect to cost reducing investment levels. Second order conditions should be negative for a maximum.

$$\frac{\partial^2 \Pi_i^w}{\partial n_i^2} = \frac{49}{72} - 2m < 0 \quad (2.32)$$

$$\frac{\partial^2 \Pi_j^w}{\partial n_j^2} = \frac{49}{72} - 2m < 0 \quad (2.33)$$

Since  $m > 1$ , second order conditions are satisfied. Equations (2.32) and (2.33) ensure that profit function is concave with respect to cost reducing investment levels ( $n$ ). We also have to check the stability of the equilibrium. For the equilibrium to be stable and unique, the following condition should hold:

$$\left(\frac{\partial^2 \Pi_i^w}{\partial n_i^2}\right) * \left(\frac{\partial^2 \Pi_j^w}{\partial n_j^2}\right) > \left(\frac{\partial^2 \Pi_i^w}{\partial n_i \partial n_j}\right) * \left(\frac{\partial^2 \Pi_j^w}{\partial n_i \partial n_j}\right) \quad (2.34)$$

The left part of inequality (2.33) yields  $\left(\frac{49}{72} - 2m\right)^2$ , right part of the inequality yields  $\frac{1225}{5184}$ . Since  $m > 1$ , stability condition is satisfied.

### *1st Stage*

In this stage, the upstream firm chooses its quality investment level. Substituting the equilibrium outputs, equilibrium wholesale price and equilibrium downstream investment levels into the profit function of the upstream firm, we obtain:

$$\Pi_u^d = \frac{864m^2(1+q)^2 - k(7-72m)^2 y^2 + 216m^2 y(\beta_i + \beta_j)(4+4q+y\beta_i+y\beta_j)}{(7-72m)^2} \quad (2.35)$$

Upstream firm maximizes its profit with respect to quality investment level ( $y$ ), so:

$$\frac{\partial \Pi_u^d}{\partial y} = \frac{k(7-72m)^2 y - 216m^2(\beta_i + \beta_j)(2+2q+y\beta_i+y\beta_j)}{-7+72m} = 0 \quad (2.36)$$

If we solve for y:

$$y^u = \frac{432m^2(1+q)(\beta_i + \beta_j)}{-k(7-72m)^2 + 216m^2(\beta_i + \beta_j)^2} \quad (2.37)$$

(Superscript  $u$  indicates the equilibrium values of the third stage)

Second order conditions should be negative for a maximum:

$$\frac{\partial^2 \Pi_u^d}{\partial y^2} = \frac{-2k(7-72m)^2 + 432m^2(\beta_i + \beta_j)^2}{(7-72m)^2} < 0 \quad (2.38)$$

Equation (2.38) is always smaller than zero when  $m > 1$ ,  $k > 1$ , and  $1 > \beta_i > \beta_j$ , so second order condition is satisfied.

All equilibrium values for endogenous variables can be found in the appendix.



### 3. COMPARATIVE STATIC ANALYSIS

Our goal is to investigate the effects of quality differentiation due to an upstream quality investment, among the downstream firms. We construct a model that serves our intentions and solve it in section 2. In this section, we make a comparative static analysis of the model to see the effects of exogenous variables ( $k$ ,  $m$ ,  $\beta_i$  and  $\beta_j$ ) on endogenous variables ( $X_i, X_j$ ,  $n_i, n_j$ ,  $P_i, P_j$ ,  $w$ ,  $y$ ).

#### 3.1 The Effects of Firm i's Quality Efficiency ( $\beta_i$ )

It is best to explore the effect of firm i's quality efficiency on firm i's sales price since  $\beta_i$  affects it directly. As given in equation (2.11), quality has a positive effect on firm i's inverse demand function. The quality firm i offers to its customers is  $y\beta_i$  which is only  $\beta_i$  percent of the upstream quality investment. An increase in own quality efficiency causes an upward shift of the demand firm i faces. An upward shift increases the price firm i can sell its products in the market. Although, bigger profit margin, which is caused by high prices, gives an incentive to produce more outputs, the decrease in prices due to output increase is less than the increase in prices due to the shift of the demand curve. So we can say that an increase in firm i's quality efficiency increases its sales price. (See Appendix A.4 ln[87])

Increased sales price, due to increased own quality efficiency directly affects the output of firm i. It is easy to see in firm i's profit function in equation (2.14) that the profit margin of the firm is related to sales price, upstream wholesale price and own cost reducing investment level. Once sales price increases, firm i's profit margin grows. An increased profit margin, gives firm i an incentive to increase its production. To sum up, an increase in firm i's quality efficiency increases its own output level (See Appendix A.4ln[88]).

As firm i's quality efficiency increases, the price it can supply its goods to the market increases. This gives firm i an incentive to produce more. Even though there

exists a negative effect on market price due to increased output, the positive effect caused by increase in quality is still higher. Therefore, firm i's profit is higher for higher levels of quality it can offer (See Appendix A.4ln[89]).

A higher  $\beta_i$  means, firm i can offer more quality to its customers. Quality increase yields and upward shift on the demand curve firm i faces, which enables firm i to sell its products at higher prices, thus gives firm i a higher profit margin and profit. As its profit increases, firm i has more resources to make cost reducing investments. Firm i's investment level increases as own quality efficiency increases (See Appendix A.4ln[90]). This also affects firm i's output level. Firm i's profit margin increases due to cost reduction. Since it is more profitable, firm i chooses a higher output level.

Following proposition helps us explain firm i's quality efficiency's effect on the rival firm's sales price:

*Proposition 1.1:* If  $\beta_i$  and  $\beta_j$  are sufficiently close, inefficient firm's sales price increases with the quality efficiency level of the rival.

*Proof:* Derivative of  $P_j^u$  with respect to  $\beta_i$  is:

$$\frac{\partial P_j^u}{\partial \beta_i} = \frac{-432m^2(1+q)(-216m^2\beta_i^2\beta_j + 24\beta_i(k(-7+72m)-18m\beta_j^2) - \beta_j * A)}{(k(7-72m)^2 - 216m^2(\beta_i + \beta_j)^2)^2}$$

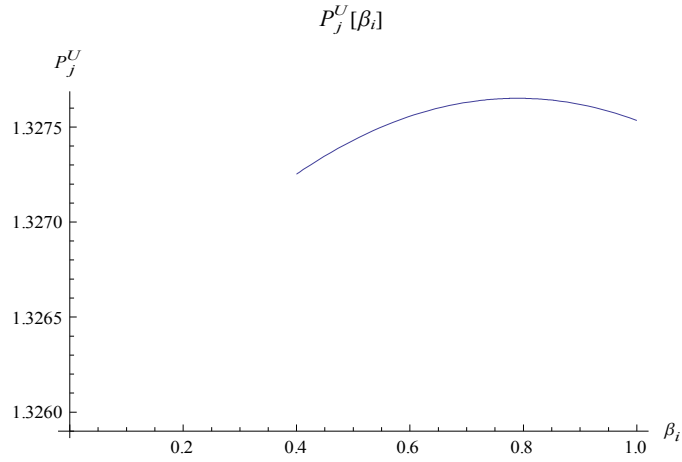
$$A = (k(-7+48m)(-7+72m) + 216m^2\beta_j^2) \quad (3.1)$$

The sign of equation (3.42) is determined by the following equation (See Appendix A.4ln[91]):

$$\beta_j(k(-7+48m)(-7+72m) + 216m^2(\beta_i^2 + \beta_j^2)) - 24m\beta_i(k(-7+72m) - 18m\beta_j^2) \quad (3.2)$$

The result of equation (3.43) is positive when  $(\beta_i < 1.7\beta_j)$  and it is negative when  $(\beta_i \geq 2.3\beta_j)$  ( $k > 1$ ,  $m > 1$ ,  $q > 0$  and  $1 > \beta_i > \beta_j$ ), (See Appendix A.4 ln[92] ln[93]).

Figure (3.1) shows that  $P_j^U(\beta_i)$  function is concave. Firm j's sales price increases for an increase of  $\beta_i$  at lower values of  $\beta_i$  and decreases for an increase of  $\beta_i$  at higher values of  $\beta_i$ .



**Figure 3.1:** Change of firm j's sales price by  $\beta_i$   
 $k = 11, m = 7, q = 1, \beta_j = 0.5$

Although it sounds odd, there is a simple explanation for this relation between firm j's price and rival's quality efficiency. Increase in  $\beta_i$  gives upstream firm more incentive to invest in quality. As upstream firm invests more, firm j's quality level and prices increase. Although, high  $\beta_i$  yields high total output, and increase in total output makes a negative effect on price, the positive effect due to quality increase is higher as long as there isn't a significant difference in  $\beta_i$  and  $\beta_j$ . If the gap between two downstream firms quality efficiencies widen, negative effect due to increase in total output is too much because of high  $\beta_i$  and the positive effect due to quality increase is too small due to low quality efficiency of firm j, thus firm j's price level declines.

A high quality efficiency, enables firm i to produce more outputs and as a result, supply a bigger portion of the consumers. As  $\beta_i$  increases, the market portion of firm i gets even bigger, leaving less consumers to firm j. Also, a high  $\beta_i$  yields high profits for firm i, enabling it to invest more on cost reduction compared to firm j. Lower production costs also help firm i to dominate the market, causing firm j to stay on low outputs. Although an increase in  $\beta_i$  causes upstream firm to make more quality investment, this positive effect makes a little difference for firm j. As a result, firm j's output level decreases as rivals quality efficiency increases (See Appendix A.4 In[94]). Even if the quality efficiencies of both firms have close

values, meaning price of firm j is affected positively by rivals quality efficiency increase, the result does not change.

An increase in firm i's quality efficiency yields an upward shift of the demand for firm i. This shift enables firm i to sell at a higher sales price, which results in having firm i to have an incentive to produce more. Firm i supplies a bigger portion of the demand and firm j's output decreases. Even though firm j benefits higher price for higher levels of  $\beta_i$  when  $\beta_i$  and  $\beta_j$  are not significantly different, the greater negative effect due to low outputs causes a reduction in firm j's profit. Firm j's profit decreases as rivals quality efficiency increases (See Appendix A.4 ln[95]).

High quality efficiency of firm i enables firm i to offer its customers higher quality, which shifts the demand it faces upwards. Elevated demand yields high market price, thus high output, making firm j to produce less output. As the gap between two firms' quality efficiencies widen, firm j's portion of the market gets smaller and its profit decreases. As a result, firm j doesn't have much sources to spare for cost reducing investment. So, we can say that an increase in firm i's quality efficiency decreases rivals investment level (See Appendix A.4 ln[96]).

Before we talk about firm i's quality efficiency's effect on upstream firm, we have to make a point on how  $\beta_i$  affects total output. As we discuss earlier, an increase in  $\beta_i$  increases firm i's output, but decreases firm j's output. However the changes in downstream firms' output levels are not the same. The effect of  $\beta_i$  on firm i's output is a direct effect, where as the effect on firm j's output is an indirect effect. Therefore, the change in firm i's output exceeds the change in firm j's output. As a result, total output increases as  $\beta_i$  increases (See Appendix A.4 ln[97]).

Increase of  $\beta_i$  affects upstream wholesales price in two ways. First of all, a higher  $\beta_i$  increases the demand for the product, which causes downstream firm do increase its output. More output means, more input, so the downstream firm demands more intermediate product from the upstream firm. As the demand for the intermediate product increases, upstream firm chooses a higher wholesale price. Second effect of  $\beta_i$  on wholesale price is indirect. Since a higher  $\beta_i$  increases downstream profits through increased demand and increased prices, the downstream firm has more incentive to make cost reducing investment. Cost reducing investment enables the firm to increase its output further more, which as a result

increases the demand for the intermediate product further more and makes the upstream firm choose a higher wholesale price . We can say that an increase in  $\beta_i$  increases upstream wholesales price (See Appendix A.4 ln[98]).

An increase in firm i's quality efficiency, increases its own output level and it reduces the rivals output level. Increase in its own output is greater than decrease in rivals output, as a result, total output increases. An increase in total output yields a greater demand for intermediate goods, enabling upstream supplier to choose a higher wholesale price. Increased output level and high wholesale price increases upstream firm's profit. So the effect of firm i's quality efficiency on upstream profit is positive (See Appendix A.4 ln[99]).

$\beta_i$  represents the percentage of upstream quality investment transmitted to consumers. A higher  $\beta_i$  means, upstream quality investment is used more efficiently. An increase in  $\beta_i$  affects upstream investment level in three ways. First effect is caused by the increase in output of the downstream firms. Second effect is a result of upstream firm charging higher wholesales prices due to increased effect of of quality investment. Third, as downstream firms are more quality efficient, they have higher profits due to increased prices and outputs. Higher profits enable them to do more cost reducing investments, which further increases their outputs. An increased output level and an increased wholesale price enables the upstream firm to make more profit thus have more resources to invest more in quality. We can easily say that an increase in firm i's quality efficiency increases upstream investment level (See Appendix A.4 ln[100]).

### 3.2 The Effects of Firm j's Quality Efficiency ( $\beta_j$ )

It is best to start exploring the effects of firm j's quality efficiency on firm j's sales price. Firm j faces an inverse demand function, that  $y\beta_j$  affects directly. A high quality efficiency enables firm j to offer more quality to its customers. As a result firm j faces an elevated demand curve. Shifted demand enables firm j to sell its products at higher prices. Although, higher prices gives firm j an incentive to produce more, which causes a negative effect on price, the positive effect due to quality increase is greater. So, firm j's sales price increases as own quality efficiency increases (See Appendix A.4 ln[112]).

A quality investment shifts the demand curve upwards. Since  $\beta_j$  determines firm j's ability to gain advantage from an upstream investment, it is the main factor that identifies the level of shift together with the upstream investment level. A higher  $\beta_j$  yields a bigger shift. A bigger shift enables firm j to sell at a higher sales price, which gives the firm an incentive to increase its output level (See Appendix A.4 ln[113]).

If firm j's quality efficiency increases, the firm faces a shifted demand curve, which enables it to sell at a higher sales price. Increased demand yields increased output level for firm i thus an increased total output. Increased output makes the upstream firm to charge a higher wholesale price, yet it also makes the upstream firm invest more on quality. The negative effect of increased wholesale price is exceeded by the positive effects of increased output, higher price and increased upstream investment, so firm j's profit increases as  $\beta_j$  increases (See Appendix A.4 ln[114]).

As we state above, as firm j's quality efficiency increases, its profit increases due to two factors. First of all, firm j's sales price increases due to a quality increase. Also, firm j can supply more output to the market due to shifted demand caused by high quality. If firm j achieves to gain high profit, it can spare more resources on cost reducing investments. So we can say that investment level of firm j is positively affected by an increase in own quality efficiency (See Appendix A.4 ln[115]).

Unlike the effect of  $\beta_i$  on firm j's sales price, an increase in  $\beta_j$  always affects firm i's sales price positively (See Appendix A.4 ln[102]). As we discuss earlier, an increase in quality efficiency of one downstream firm affects rivals sale price in two ways. First one is a negative effect due to an increased output, and the second one is a positive effect due to an increased upstream quality investment. Since firm i has higher quality efficiency, an increased quality investment always favors firm i. Therefore, the positive effect always exceeds the negative effect, and firm i's sales price increases as rival's quality efficiency increases.

Following proposition helps us explain firm j's quality efficiency's effect on the rival firm's output level:

*Proposition 1.2:* If  $\beta_i$  and  $\beta_j$  are sufficiently close, efficient firm's output level decreases as the quality efficiency level of the rival firm increases.

*Proof:* Derivative of  $X_i^u$  with respect to  $\beta_j$  is

$$\frac{\partial X_i^u}{\partial \beta_j} = \frac{(5184m^3(1+q)(-H\beta_i^3 + 60k(7-72m)m\beta_j - 432m^2\beta_i^2\beta_j + \beta_i A - H\beta_j^2))}{(-7+12m)(k(7-72m)^2 - 216m^2(\beta_i + \beta_j)^2)}$$

$$A = k(-7+12m)(-7+72m)$$

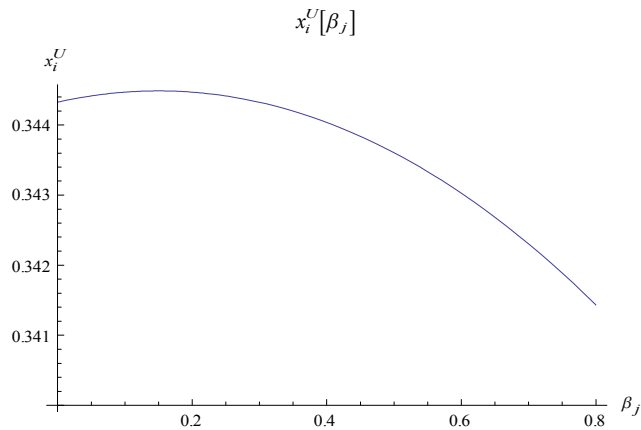
$$H = 216m^2 \quad (3.3)$$

And the sign of equation (3.3) depends on the sign of following equation: (See Appendix A.4 ln[103])

$$216m^2\beta_i^3 + 60k(7-72m)m\beta_j + 432m^2\beta_i^2\beta_j + \beta_i(k(-7+12m)(-7+72m) + 216m^2\beta_j^2) \quad (3.4)$$

Sign of equation (3.4) is positive when  $\beta_i > 12\beta_j$  and it is negative when  $\beta_i \leq 3.5\beta_j$  for  $k > 1$ ,  $m > 1$ ,  $q > 0$  and  $1 > \beta_i > \beta_j$  (See Appendix A.4 ln[104] and [105]).

Figure (3.2) shows that  $X_i^u(\beta_j)$  function has a maximum at  $\beta_j = 0.15$ .



**Figure 3.2:** Change of firm i's output level by  $\beta_j$   
 $k = 11$ ,  $m = 7$ ,  $q = 1$ ,  $\beta_i = 0.8$

Increase in quality efficiency of firm j affects firm i's output level in a couple of ways. First, there is a direct effect due to increase in firm j's output level. This causes firm i to choose a lower output level. Secondly, there is an indirect effect due

to upstream quality investment increase. When there isn't a considerable quality differentiation, the indirect effect is so slight, it does not exceed the direct effect. However, in the case where quality differentiation is great, the negative effect is slight and the positive effect exceeds it. In this scenario, firm j shares a very tiny portion of the market, and an increase in its output barely affects firm i. Although, the increase in upstream quality investment is proportionally small as well, firm i makes a good use of it due to its superior quality efficiency.

Following proposition helps us explain firm j's quality efficiency's effect on the rival firm's profit:

*Proposition 1.3:* If  $\beta_i$  and  $\beta_j$  are sufficiently close, efficient firm's profit decreases as the quality efficiency level of the rival firm increases.

*Proof:* Derivative of  $\Pi_i^u$  with respect to  $\beta_j$  is

$$\frac{\partial \Pi_i^u}{\partial \beta_j} = \frac{(864m^3(-49+144m)(1+q)^2(A+H(\beta_i^2-\beta_j^2))T)}{(-7+12m)(k(7-72m)^2-216m^2(\beta_i+\beta_j)^2)}$$

$$A = k(-7+12m)(-7+72m)$$

$$H = 216m^2$$

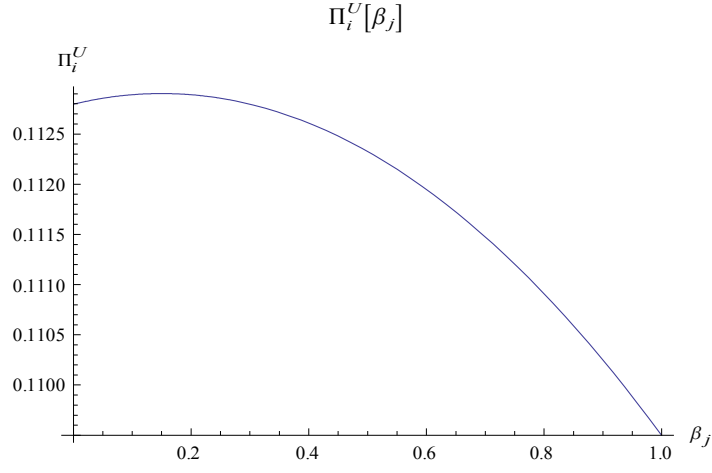
$$T = (-H\beta_i^3 + 60k(7-72m)m\beta_j - 432m^2\beta_i^2\beta_j + \beta_i A - H\beta_j^2) \quad (3.5)$$

And the sign of equation (3.5) depends on the sign of following equation: (See Appendix A.4 ln[106])

$$216m^2\beta_i^3 + 60k(7-72m)m\beta_j + 432m^2\beta_i^2\beta_j + \beta_i(k(-7+12m)(-7+72m) + 216m^2\beta_j^2) \quad (3.6)$$

Sign of equation (3.6) is positive when  $\beta_i > 12\beta_j$  and it is negative when  $\beta_i \leq 3.5\beta_j$  for  $k > 1$ ,  $m > 1$ ,  $q > 0$  and  $1 > \beta_i > \beta_j$  (See Appendix A.4 ln[107] and [108]).

Figure (3.3) shows that  $\Pi_i^u(\beta_j)$  function has a maximum at  $\beta_j = 0.15$ .



**Figure 3.3:** Change of firm i's profit by  $\beta_j$   
 $k = 11, m = 7, q = 1, \beta_i = 0.8$

This effect of  $\beta_j$  on firm i's profit is directly proportional to the effect of  $\beta_j$  on firm i's output. As it can be seen in the profit function of firm i in equation (2.14) the profit of firm is directly affected by firm i's output level. Therefore, the conditions we discuss earlier for firm i's output level applies directly to firm i's profit. In the cases where firm i's output shows a decline due to an increase in rivals quality efficiency, firm i's profit also shows a decline (and vice versa).

The conditions that we come up as we explore  $\beta_j$  effects on firm i's output level and profit show themselves as we investigate the effects of firm j's quality efficiency on firm i's investment level, thus we make the following proposition:

*Proposition 1.4:* If  $\beta_i$  and  $\beta_j$  are sufficiently close, efficient firm's investment level decreases as the quality efficiency level of the rival firm increases.

*Proof:* Derivative of  $n_i^u$  with respect to  $\beta_j$  is

$$\frac{\partial n_i^u}{\partial \beta_j} = \frac{(3024m^2(1+q)(-H\beta_i^3 + 60k(7-72m)m\beta_j - 432m^2\beta_i^2\beta_j + \beta_i A - H\beta_j^2))}{(-7+12m)(k(7-72m)^2 - 216m^2(\beta_i + \beta_j)^2)^2}$$

$$A = k(-7+12m)(-7+72m)$$

$$H = 216m^2$$

(3.7)

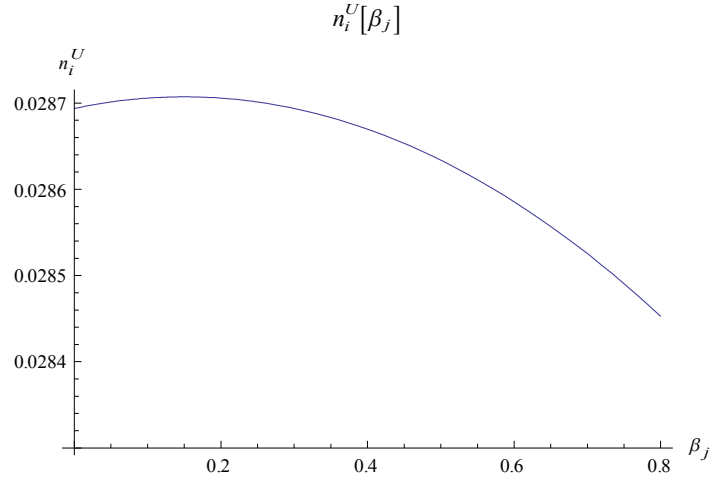
And the sign of equation (3.7) depends on the sign of following equation (See Appendix A.4 ln[109]):

$$216m^2\beta_i^3 + 60k(7-72m)m\beta_j + 432m^2\beta_i^2\beta_j + \beta_i(k(-7+12m)(-7+72m) + 216m^2\beta_j^2)$$

(3.8)

Sign of equation (3.8) is positive when  $\beta_i > 12\beta_j$  and it is negative when  $\beta_i \leq 3.5\beta_j$  for  $k > 1$ ,  $m > 1$ ,  $q > 0$  and  $1 > \beta_i > \beta_j$  (See Appendix A.4 in [110] and [111]).

Figure (3.4) shows that  $n_i^u(\beta_j)$  function has a maximum at  $\beta_j = 0.15$ .



**Figure 3.4:** Change of firm i's investment level by  $\beta_j$   
 $k = 11$ ,  $m = 7$ ,  $q = 1$ ,  $\beta_i = 0.8$

As we discuss earlier output and profit of firm i are affected positively from an increase in rival's quality efficiency if quality differentiation among the firms is very large, and affected negatively when quality differentiation is not significant. The same conditions apply when we examine the effects of  $\beta_j$  on firm i's investment level. This is predictable since firms spare resources for investment proportional to their profits. If profit of a firm has tendency to decrease, the investment level of the firm has the same tendency (and vice versa).

### 3.3 The Effects of Upstream Investment Cost ( $k$ )

The effects of  $k$  are more direct for upstream firm compared to the downstream firms since it is upstream firm's investment cost. Therefore, it is best to start by examining the effects of  $k$  on the upstream firm.  $k$  represents the efficiency of upstream quality investment. If  $k$  increases, upstream investment becomes inefficient and expensive. Therefore, an increase in  $k$  decreases upstream quality

investment level (See Appendix A.4 ln[136]). If investment cost of upstream investment is high, upstream firm decreases its quality investment level. A decrease in quality shifts the demand curve which downstream firms face, down, causing downstream firms to produce less output. Decreased output means decreased intermediate product demand, and decreased intermediate product demand causes upstream firm to charge a lower wholesale price. So, upstream wholesale price decreases as upstream investment cost increases (See Appendix A.4 ln[134]).

A decrease in upstream firm's quality investment due to increased cost of investment, decreases the total output and makes upstream firm choose a lower wholesales price. Also, a bigger portion of upstream firm's profit disappears through investment costs. The indirect effect of decreased output plus decreased wholesale price, and the direct effect of high investment cost causes upstream profit to decrease as  $k$  increases (See Appendix A.4 ln[135]).

The effects of upstream investment cost is carried to downstream firms through changes in quality investment level. A high investment cost yields low quality and a low investment cost yields high quality. Firm  $i$  is negatively affected from an upstream quality investment cost increase since quality investment is what creates a quality differentiation among the firms. Quality differentiation favors firm  $i$  since it has a higher quality efficiency and as a result, higher quality. A decrease in upstream investment level caused by increased upstream investment cost reduces the quality firm  $i$  offers its customers. Reduced quality decreases the demand for firm  $i$ 's product, causing the sales price to decrease. A low sales price yields lower profit margins, causing firm  $i$  to cut itself on production, and choose a lower output level. So, an increase in  $k$  decreases firm  $i$ 's output level (See Appendix A.4 ln[122]).

Increase in upstream investment cost causes a decrease in upstream investment level. This reduction affects firm  $i$ 's sale price in two ways. First effect is the direct effect of quality on sales price which is negative due to quality reduction. The second effect is positive and it is a result of decreased overall output. The direct effect of quality exceeds the indirect effect of quantity, so sales price is negatively affected from an upstream investment cost increase (See Appendix A.4 ln[121]).

As it can be seen in profit function of firm i in equation (2.14), profit of the firm i is affected positively by firm i's output and sales price and it is affected negatively by upstream wholesale price. We state that firm i's sale price and output level are negatively affected by an increase in upstream investment cost. We also indicate that an increase in k decreases the wholesale price of upstream firm. Although, reduction in wholesale price favors firm i's profit a little, the negative effects of low price and low output level are far greater. As a result, firm i's output level decreases as upstream investment cost increases (See Appendix A.4 ln[123]).

Firm with high  $\beta$  achieves high price and output level as a result of increased quality due to an upstream quality investment. High price and output level yields high profits, thus more opportunity to invest on cost reduction. If upstream investment cost increases, quality level decreases lowering price, output, profit and investment level of firm i (See Appendix A.4 ln[124]).

When we explore the effects of upstream investment cost on firm j, we see that the effects can be negative or positive depending on the quality efficiency of the firm. A general conclusion for the following four analysis is that an increase in upstream investment cost makes firm j better off if the quality differentiation among the firms is significantly large. With this in mind, we can move on to our next proposition:

*Proposition 1.5:* If  $\beta_i$  and  $\beta_j$  are sufficiently close, inefficient firm's sales price decreases as the investment cost of upstream firm increases.

*Proof:* Derivative of  $P_j^u$  with respect to  $k$  is:

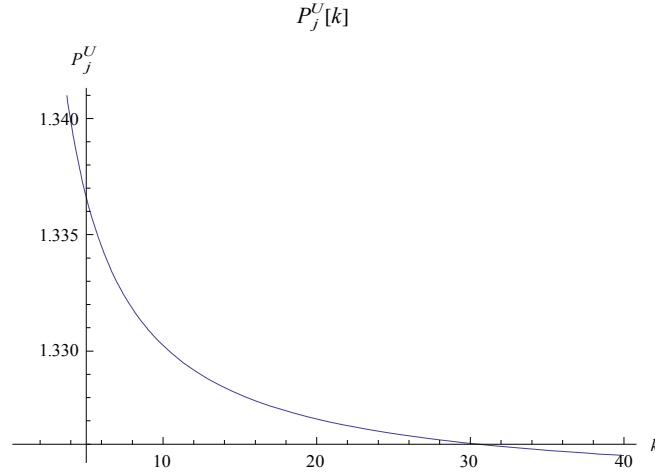
$$\frac{\partial P_j^u}{\partial k} = \frac{432m^2(-7+72m)(1+q)(\beta_i+\beta_j)(12m\beta_i+(7-60m)\beta_j)}{(k(7-72m)^2-216m^2(\beta_i+\beta_j)^2)^2} \quad (3.9)$$

this equation is negative or positive depending on the sign of (See Appendix A.4 ln[125]):

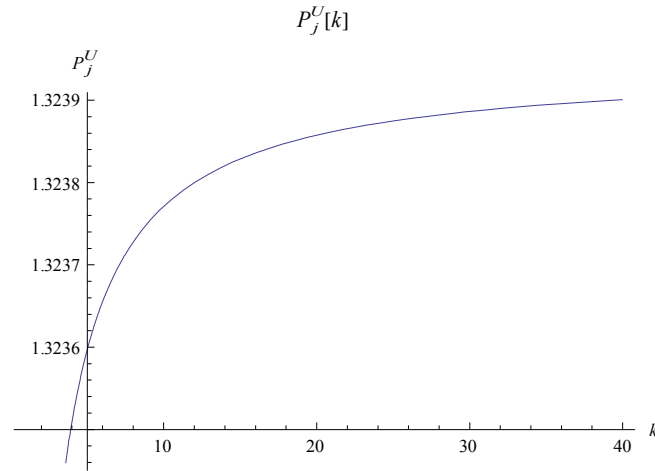
$$12m\beta_i+(7-60m)\beta_j \quad (3.10)$$

It is positive when  $\beta_i \geq 5\beta_j$  and negative when  $\beta_i \leq 4.4\beta_j$  while  $k > 1$ ,  $q > 0$  and  $1 > \beta_i > \beta_j$  (See Appendix A.4 in [126] and [127]). The values of  $\beta_i$  and  $\beta_j$  that is outside the given conditions tend inconclusive results.

Figures (3.5) and (3.6) show  $P_j^u(k)$  functions for two different situations.



**Figure 3.5:** Change of firm j's sales price by  $k$   
 $m = 7, q = 1, \beta_j = 0.5 \quad \beta_i = 0.8$



**Figure 3.6:** Change of firm j's sales price by  $k$   
 $m = 7, q = 1, \beta_j = 0.15 \quad \beta_i = 0.8$

A reduction in upstream investment level because of an increase in investment cost decreases downstream sales prices thus output levels. If downstream firms' quality efficiencies are not significantly different, the direct negative effect of quality decrease on firm j's price exceeds the indirect positive effect of reduced output on price and firm j's sales price falls. However, in the case where  $\beta_j$  is very low compared to  $\beta_i$ , indirect positive effect of output decrease on firm j's sales price exceeds the direct negative effect quality decrease on sales price. As a result, this

time firm j's sales price is affected positively from an increase in upstream firm's investment cost.

Following proposition helps us explain upstream investment cost's effect on the firm j's output level.

*Proposition 1.6:* If  $\beta_i$  and  $\beta_j$  are sufficiently close, inefficient firm's output level decreases as the investment cost of upstream firm increases.

Proof: Derivative of  $X_j^u$  with respect to  $k$  is:

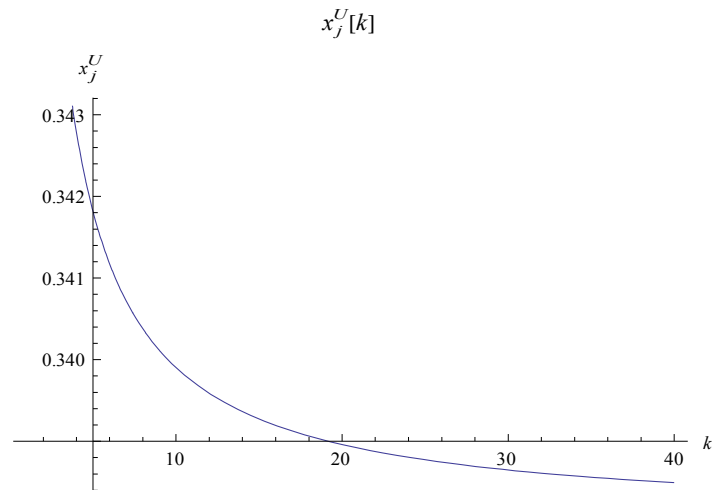
$$\frac{\partial X_j^u}{\partial k} = \frac{(5184m^3)(-7+72m)(1+q)(\beta_i+\beta_j)(30m\beta_i+7(1-6m)\beta_j)}{(-7+12m)(k(7-72m)^2-216m^2(\beta_i+\beta_j)^2)^2} \quad (3.11)$$

and this equation is negative or positive depending on the sign of (See Appendix A.4 ln[128]):

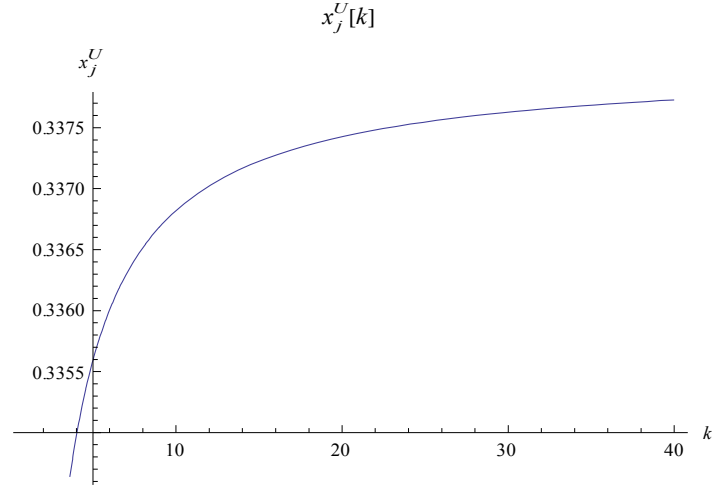
$$30m\beta_i+7(1-6m)\beta_j. \quad (3.12)$$

It is positive when  $\beta_i \geq 1.4\beta_j$  and negative when  $\beta_i \leq 1.1\beta_j$  while  $k > 1$ ,  $q > 0$  and  $1 > \beta_i > \beta_j$  (See Appendix A.4 ln[129] and [130]).

Figure (3.7) and (3.8) show  $X_j^u(k)$  functions for two different situations.



**Figure 3.7:** Change of firm j's output level by  $k$   
 $m = 7, q = 1, \beta_j = 0.7, \beta_i = 0.8$



**Figure 3.8:** Change of firm j's output level by  $k$   
 $m = 7, q = 1, \beta_j = 0.5 \quad \beta_i = 0.8$

If two downstream firms' quality efficiencies are not significantly different, firm j suffers from a reduction of upstream quality due to an increase in upstream investment cost. Although firm j serves to a lower portion of the market due to its low quality efficiency, it still benefits from a quality increase. However when  $\beta_j$  and  $\beta_i$  are significantly different, firm j serves only to a limited portion of the market. Rival firm benefits a lot more from a quality investment compared to firm j. As a result, a decrease in quality investment level reduces the negative effects of low  $\beta_j$  for the firm j. Its output level increases even though total output is less.

As we discuss earlier, output of a firm is directly proportional to its profit level and investment of a firm is directly proportional to its profit.

The conditions that explain how upstream investment cost affects firm j's output level are exactly the same as the conditions that explain upstream investment cost's effects on firm j's profit and investment level.

Signs of derivatives of  $X_j^u$ ,  $\Pi_j^u$ ,  $n_j^u$  with respect to  $k$  depends on a condition which is same for all three derivatives.

$$\frac{\partial X_j^u}{\partial k} = \frac{(5184m^3)(-7+72m)(1+q)(\beta_i+\beta_j)(30m\beta_i+7(1-6m)\beta_j)}{(-7+12m)(k(7-72m)^2-216m^2(\beta_i+\beta_j)^2)^2} \quad (3.11)$$

$$\frac{\partial \Pi_j^u}{\partial k} = \frac{864m^3(-7+72m)(1+q)^2(-49+144m)(\beta_i+\beta_j)T(A+H(\beta_i^2+\beta_j^2))}{(7-12m)^2(k(7-72m)^2-216m^2(\beta_i+\beta_j)^2)^3}$$

$$A = k(-7 + 12m)(-7 + 72m)$$

$$H = 216m^2$$

$$T = (30m\beta_i + 7(1 - 6m)\beta_j) \quad (3.13)$$

$$\frac{\partial n_j^u}{\partial k} = \frac{3024m^2(-7 + 72m)(1 + q)(\beta_i + \beta_j)(30m\beta_i + 7(1 - 6m)\beta_j)}{(-7 + 12m)(k(7 - 72m)^2 - 216m^2(\beta_i + \beta_j)^2)} \quad (3.14)$$

All parts of the three equations are clearly positive ( while  $k > 1$ ,  $q > 0$  and  $1 > \beta_i > \beta_j$  ) except the following part: (See Appendix A.4 In[128], [131] and [132])

$$30m\beta_i + 7(1 - 6m)\beta_j \quad (3.15)$$

Equation (3.15) yields a positive result when  $\beta_i \geq 1.4\beta_j$  and a negative result when  $\beta_i \leq 1.1\beta_j$ . And since it is present in all three derivatives, we can say that upstream investment cost affects firm j's output, firm j's profit and firm j's investment level in parallel courses.

If upstream investment cost increases, upstream firm's quality investment level decreases. A decrease in quality causes a downward shift of the demand curve that downstream firms face. This downward shift yields lower sales price, which reduces downstream firms' output productions. When quality efficiencies of downstream firms are close, reduced output and low prices lower both firm i's and firm j's profits and when firm j's profit decreases, firm j has less resources to spend on cost reducing investment. However, when downstream firm's  $\beta$ 's are significantly different, firm i gains a significant advantage over firm j through quality investment and dominates the market using its superior quality. In this case, any decrease in upstream quality investment suppresses this advantage firm i has and favors firm j. Firm j's profits increase thus it has more resources to invest in cost reduction due to increased profit.

### 3.4 The Effects of Downstream Investment Cost ( $m$ )

A firm makes an investment only if it is efficient and profitable. If the cost of downstream investment increases, it makes the investment inefficient. For a given

level of investment, firm has to pay more research cost. In this sense we can say that, firm i's investment level decreases as cost of investment increases. (See Appendix A.4 ln[144]).

Investment reduces production costs and causes an increase in firm i's output level. Firm i, which has a better quality in every case, uses investment to enhance its superiority over firm j. Firm i has better quality which leads to higher sales price, higher output and as a result higher profit compared to its rival. Since firm i has more resources to invest in cost reduction, it uses investment as a tool to dominate the market even more. If cost reducing investments are not efficient, meaning they have a high cost, firm i's investment level decreases thus less investment yields less production (See Appendix A.4 ln[138]).

Downstream firms use cost reducing investments to enhance their output levels. If investments are costly, firms can't increase their output through lowering their production costs. This affects sales price of firm i in two ways. First, reduced output makes a positive effect on price. Second, as output is reduced, upstream profit decreases and upstream firm can't spare as much resources to quality investment, which shifts the demand curve downward and makes a negative impact on firm i's sales price. The positive effect through decreased output exceeds the negative effect through decreased quality. As a result,  $p$  is higher for higher downstream investment costs (See Appendix A.4 ln[137]).

As we explore the effects of downstream investment cost on firm i's profit, we see there exists several different relations. Firm i's profit increases as cost of downstream investment  $m$  increases when upstream investment cost is high  $k \geq 7$  or there isn't a significant difference in between downstream firm's quality efficiencies (  $\beta_i \leq 1.05 \beta_j$  ). If there is a significant difference in between downstream firms' quality efficiencies and upstream investment cost is low two things happen. When  $\beta_i$  is high, and  $k$  is very low and the difference between downstream firms' quality efficiencies is big enough, firm i's profit decreases when the cost of investment  $m$  increases. For other cases, firm i's profit first decreases, then it increases as cost of downstream investment  $m$  increases. In the following part, we explain the situation mathematically.

Derivative of  $\Pi_i''$  with respect to  $m$  is

$$\frac{\partial \Pi_i''}{\partial m} = \frac{(7(1+q)^2(A)(L-432m^2(\beta_i+\beta_j)(F+2\beta_i(7kH+54m^2R\beta_j^2)+S)))}{((-7+12m)^3(k(7-72m)^2-216m^2(\beta_i+\beta_j)^2)^3)}$$

$$A = k(-7+12m)(-7+72m)+216m^2(\beta_i^2-\beta_j^2)$$

$$H = (-686+9m(1057+12m(-371+552m)))$$

$$T = (2401-18m(3871+12m(-1883+3336m)))$$

$$L = k^2(7-72m)^2(7-12m)^2(49-216m)$$

$$F = 108(49-204m)m^2\beta_i^3+108(49-204m)m^2\beta_i^2\beta_j$$

$$R = (-49+204m)$$

$$S = \beta_j(kT+108m^2R\beta_j) \quad (3.16)$$

and this equation is negative or positive depending on the sign of (See Appendix A.4 ln[139]).

$$L-432m^2(\beta_i+\beta_j)(F+2\beta_i(7kH+54m^2R\beta_j^2)) \quad (3.17)$$

1- The result of this equation is always negative when  $k \geq 7$  or  $\beta_i \leq 1.05\beta_j$  (See Appendix A.4 ln[140] and ln[141]).

2- For different values of  $\frac{\beta_i}{\beta_j}$ , there are specific  $k'$  ( $k' < 3$ ) and  $k''$  ( $k'' < 7$ )

values. For any  $k'$ , result of equation is negative when  $k < k'$ . If

$k'' > k > k'$ , the result of equation is negative for lower values of  $m$ , and positive for higher values of  $m$ . If  $k > k''$ , the result of the equation is always positive.

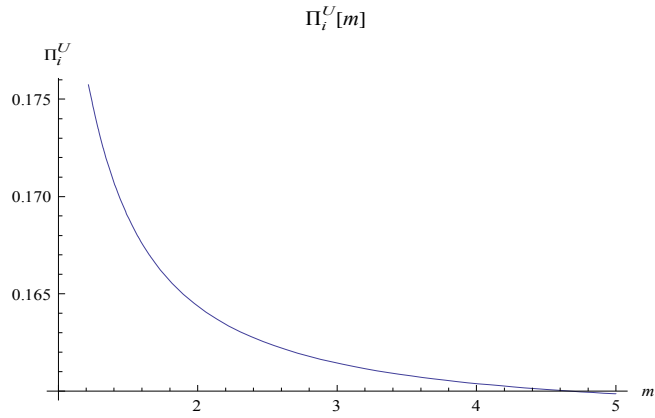
$k'$  exists only if  $\beta_i > 0.67$  ( $k > 1$ ,  $m > 1$ ,  $q > 0$  and  $1 > \beta_i > \beta_j$ ) Table (3.1)

shows  $k'$  and  $k''$  values for different sets of  $\frac{\beta_i}{\beta_j}$  ratio.

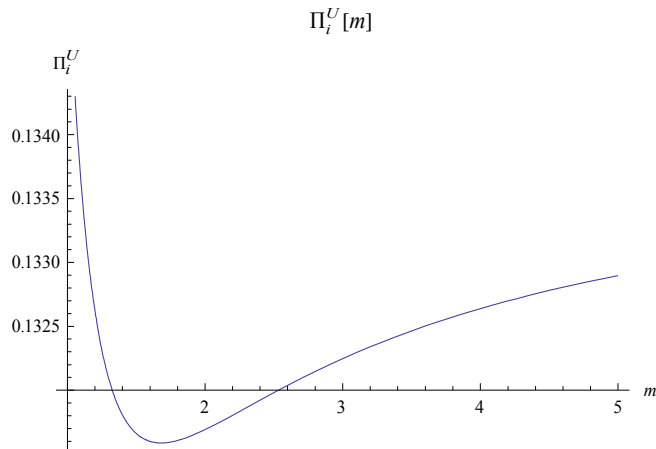
**Table 3.1:** Table shows  $k^l$  and  $k^u$  values for various  $\beta_i$  and  $\beta_j$  combinations.

| $\beta_i$ | $\beta_j$ | $k^l$ | $k^u$ |
|-----------|-----------|-------|-------|
| 0.8       | 0.1       | 1.42  | 3.85  |
| 0.8       | 0.2       | 1.38  | 3.68  |
| 0.99      | 0.6       | 1.62  | 3.93  |
| 0.9       | 0.3       | 1.68  | 4.44  |
| 0.79      | 0.2       | 1.34  | 3.57  |
| 0.99      | 0.01      | 2.16  | 5.96  |

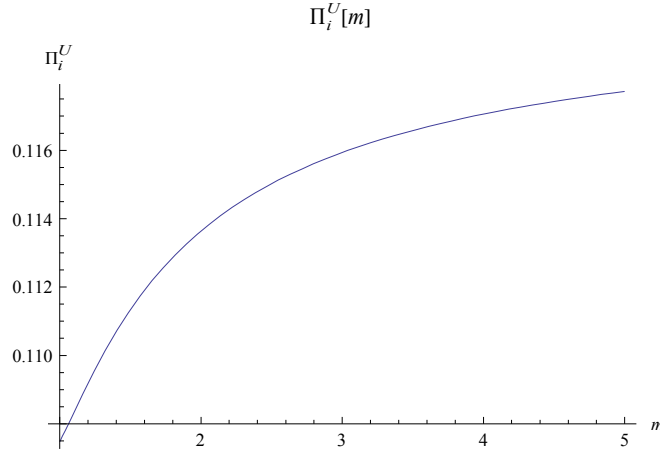
Figures (3.9), (3.10) and (3.11) show  $\Pi_j^u(m)$  functions for different situations:



**Figure 3.9:** Change of firm i's profit by  $m$   
 $k = 1.30, q = 1, \beta_i = 0.9 \quad \beta_j = 0.3 \quad k < k^l$



**Figure 3.10:** Change of firm i's profit by  $m$   
 $k = 2.5, q = 1, \beta_i = 0.9 \quad \beta_j = 0.3 \quad k^u > k > k^l$



**Figure 3.11:** Change of firm i's profit by  $m$   
 $k = 6, q = 1, \beta_i = 0.9 \quad \beta_j = 0.3 \quad k > k''$

Firm i is dominant firm in the market as a result of its high product quality. Firm i is in best situation when its quality efficiency is very high compared to its rival, and upstream quality investment is very high. In this case, firm i's investment level is very high and firm j's investment level is very low, thus any increase in downstream investment cost makes firm i worse off.

When upstream quality investment is low or downstream firms quality efficiencies are close, both downstream firms' sales prices, output levels thus profits have close values. In this case, both firms make similar levels of investment. After the investment, both firms' output level increases which reduces the total price. The increase in own output is less than the decrease in price since price is affected by both firms' output. As a result, firm i's profit decreases. It is not wrong to say that making investment is not profitable in this scenario. Yet, making investment is best response for firm i because in the case where firm i does not invest and firm j invests, firm i becomes worse off. To sum up, if both firms have similar output levels, an increase in investment cost increases firm i's profit because it reduces investment level, which is not profitable in this setting.

For moderate levels of upstream investment and  $\beta_i$  and  $\beta_j$  difference, firm i suffers a slight profit loss due to a slight increase in investment cost. Yet for higher values of  $m$ , firm i's profit increases as investment cost increases. The decrease in profit is caused by loss of advantage against firm j, and the increase is caused by reduced inefficient investment.

When we explore  $m$  effects on firm  $j$ , it is best to start explaining the effect of downstream investment cost on firm  $j$ 's investment level since investment level is directly affected by investment cost.

Firm  $j$ 's investment level decreases as cost of downstream investment  $m$  increases when upstream investment level is  $k \geq 2$  or downstream firms' quality efficiencies are similar ( $\beta_i \leq 1.5 \beta_j$ ). When  $k < 2$  and  $\beta_i > 1.5 \beta_j$ , firm  $j$ 's investment level increases slightly and then decreases as cost of downstream investment level increases.

Derivative of  $n_j^u$  with respect to  $m$  is

$$\frac{\partial n_j^u}{\partial m} = \frac{-(T(A + L(-H - 648m^3\beta_j^3 + \beta_j V - 9m\beta_i(7k(-7 + 8m(-7 + 36m)) + 72m^2\beta_j^2))))}{((7 - 12m)^2(k(7 - 72m)^2 - 216m^2(\beta_i + \beta_j)^2))}$$

$$A = k^2(7 - 12m)^2(7 - 72m)^2$$

$$H = -648m^3\beta_i^3 - 648m^3\beta_i^2\beta_j$$

$$T = 504(1 + q)$$

$$L = 12m(\beta_i + \beta_j)$$

$$V = k(343 + 3m(-1127 + 24m(49 + 180))) \quad (3.18)$$

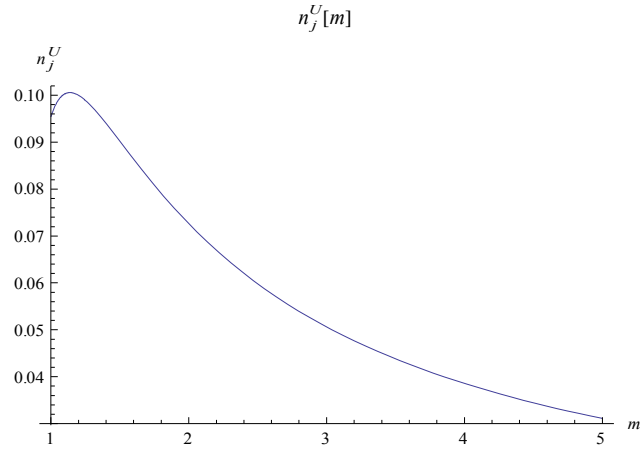
and this equation is negative or positive depending on the sign of (See Appendix A.4 ln[152]):

$$A + L(-H - 648m^3\beta_j^3 + \beta_j V - 9m\beta_i(7k(-7 + 8m(-7 + 36m)) + 72m^2\beta_j^2)) \quad (3.19)$$

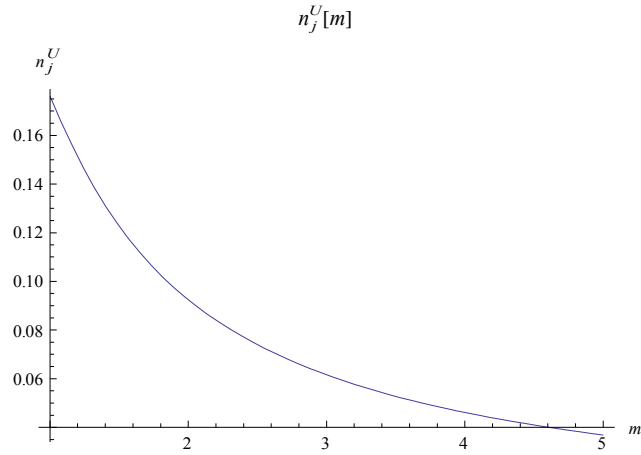
1- The result of this equation is always negative when  $k \geq 2$  or  $\beta_i \leq 1.5 \beta_j$  (See Appendix A.4 ln[153] and [154]).

2- When  $k < 2$  and  $\beta_i > 1.5 \beta_j$ , the result of equation is positive for smaller values of  $m$  and negative for greater values of  $m$ . When  $m \geq 2$  the result of equation is always negative ( $k > 1, m > 1, q > 0$  and  $1 > \beta_i > \beta_j$ ).

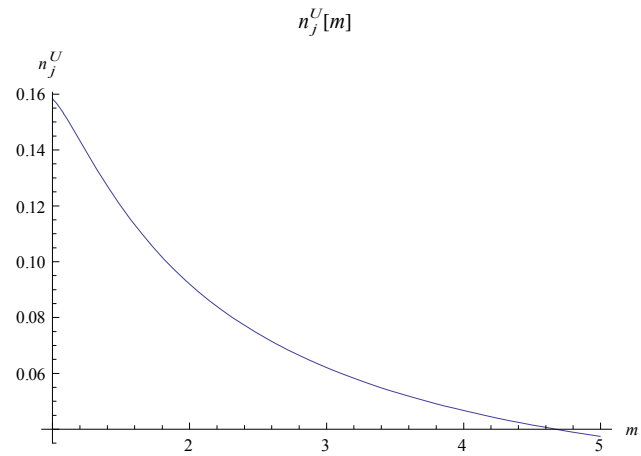
Figures (3.12), (3.13) and (3.14) show how  $n_j^u(m)$  function behaves in different situations.



**Figure 3.12:** Change of firm j's investment level by  $m$   
 $k = 1.02, q = 1, \beta_i = 0.95 \quad \beta_i = 0.1$



**Figure 3.13:** Change of firm j's investment level by  $m$   
 $k = 3, q = 1, \beta_i = 0.95 \quad \beta_i = 0.1$



**Figure 3.14:** Change of firm j's investment level by  $m$   
 $k = 1.02, q = 1, \beta_i = 0.95 \quad \beta_i = 0.6$

Both downstream firms suffer from an increase in downstream investment cost since it makes R&D more expensive and reduces firms' investment level. However, in the case where upstream investment is very high, there is a significant difference in downstream firms' quality efficiencies and downstream investment cost is low, firm j benefits from an increase in investment cost. This is because, high quality investment combined with great difference between  $\beta_i$  and  $\beta_j$ , causes firm j to serve only a small portion of the market. Thus, firm i makes more investment in cost reduction compared to firm j, to improve its advantage further. Therefore, an increase in investment cost cancels this advantage of firm i. However, if investment becomes more expensive, firm j's investment level again decreases due to lack of own cost reducing investment.

When we examine the effect of downstream investment cost on firm j's output, we encounter a set of relations that are given below.

Firm j's output level decreases as cost of downstream investment  $m$  increases when upstream investment cost is high ( $k \geq 10$ ) or there isn't a significant difference in between downstream firm's quality efficiencies ( $\beta_i \leq 1.07\beta_j$ ). If there is a significant difference in between downstream firms' quality efficiencies ( $\beta_i > 1.07\beta_j$ ) and upstream investment cost is low ( $k < 10$ ), following two things happen. When  $\beta_i$  is high, and  $k$  is very low and the difference between downstream firms' quality efficiencies is big enough, firm j's output level increases when the cost of investment  $m$  increases. For other cases, firm j's output level first increases, then it decreases as cost of downstream investment  $m$  increases. The mathematical explanation is given below.

Derivative of  $X_i''$  with respect to  $m$  is

$$\frac{\partial X_i''}{\partial m} = \frac{-(T(A+L(H-98k(1-6m)^2\beta_j+108m^2\beta_j^3+\beta_i(k(49+72m(-14+47m))))))}{((7-12m)^2(k(7-72m)^2-216m^2(\beta_i+\beta_j)^2))}$$

$$A=k^2(7-12m)^2(7-72m)^2$$

$$H=-108m^2\beta_i^3-108m^2\beta_i^2\beta_j+108m^2\beta_j^2$$

$$T=84(1+q)$$

$$L=432m^2(\beta_i+\beta_j) \quad (3.20)$$

and this equation is negative or positive depending on the sign of (See Appendix A.4 ln[146]):

$$k^2(7-72m)^2-(7-12m)^2-L(H-98k(1-6m)^2\beta_j+108m^2\beta_j^3+\beta_i(k(49+72m(-14+47m)))) \quad (3.21)$$

1- The result of this equation is always negative when  $k \geq 10$  or  $\beta_i \leq 1.07\beta_j$  (See Appendix A.4 ln[147] and [148]).

2- For different values of  $\frac{\beta_i}{\beta_j}$ , there are specific  $k'$  ( $k' < 2$ ) and  $k''$  ( $k'' < 10$ )

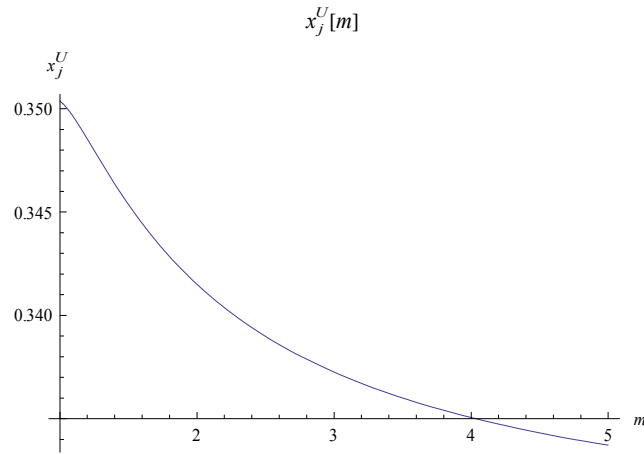
values. For any  $k'$ , result of equation is positive when  $k < k'$ . If  $k'' > k > k'$ , the result of equation is positive for lower values of m, and negative for higher values of m. If  $k > k''$ , the result of the equation is always negative.  $k'$  only exists if  $\beta_i > 0.73$  ( $k > 1$ ,  $m > 1$ ,  $q > 0$  and  $1 > \beta_i > \beta_j$ ). Table (3.2) shows  $k'$

and  $k''$  values for different sets of  $\frac{\beta_i}{\beta_j}$  ratio.

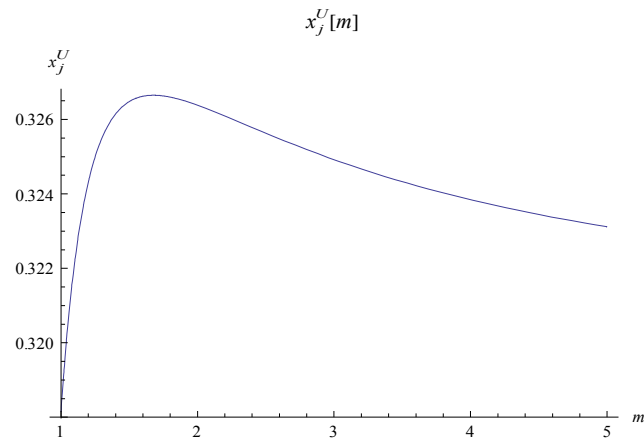
**Table 3.2:** Table shows  $k'$  and  $k''$  values for various  $\beta_i$  and  $\beta_j$  combinations.

| $\beta_i$ | $\beta_j$ | $k'$ | $k''$ |
|-----------|-----------|------|-------|
| 0.8       | 0.1       | 1.2  | 6.2   |
| 0.8       | 0.2       | 1.12 | 5.87  |
| 0.99      | 0.6       | 1.07 | 5.93  |
| 0.9       | 0.3       | 1.33 | 7.03  |
| 0.79      | 0.2       | 1.09 | 5.72  |
| 0.99      | 0.01      | 1.88 | 9.67  |

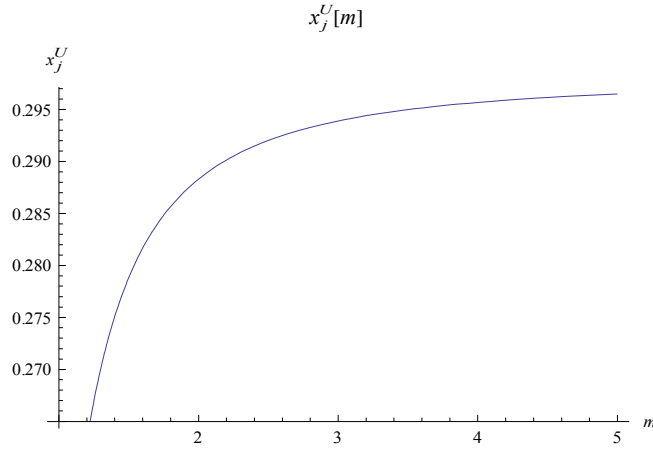
Figures (3.15), (3.16) and (3.17) show how  $X_j^u(m)$  function behaves in different situations:



**Figure 3.15:** Change of firm j's output level by  $m$   
 $k = 8, q = 1, \beta_i = 0.9 \quad \beta_i = 0.3 \quad k > k''$



**Figure 3.16:** Change of firm j's output level by  $m$   
 $k = 3, q = 1, \beta_i = 0.9 \quad \beta_i = 0.3 \quad k'' > k > k'$



**Figure 3.17:** Change of firm j's output level by  $m$   
 $k = 1.2, q = 1, \beta_i = 0.9 \quad \beta_j = 0.3 \quad k'' > k'$

An increase in downstream investment cost affects firm j's output level in three ways, depending on the upstream investment cost and the difference between downstream quality efficiencies. If upstream investment is very low, due to a high investment cost, the difference in downstream firm's quality efficiencies is neglected regardless of how big it is. Firm i's output level is still more than that of firm j, yet there isn't much difference between them. In this case, increased downstream investment cost decreases investment level of firm j, causing higher production cost thus a negative effect on its output level. This also happens in the case where downstream firm's quality efficiencies have very close values because downstream firms' output levels are again similar.

If upstream investment cost is low, which means upstream firms makes more investment in quality, the separation in downstream firms' output level increases. If there is significant difference in quality efficiency values of downstream firms, firm j serves to a very low portion of the market. In this case, firm i uses cost reducing investments as a tool to enhance its superiority, where as firm j's investment level is very low because of low profit. If the cost of investment is high, firm i's tool becomes expensive, which makes firm j better off and its output level is positively affected.

In cases where neither upstream investment is too high, nor the difference between downstream firms quality efficiencies is too much, firm j is negatively affected by rivals cost reducing investment, but also, it has resources to invest in its own cost reduction. In this situation, a slight increase in investment cost is beneficial for firm j, as it affects firm i more than firm j. But if the cost of investment increases too

much, firm j's output level decreases since it becomes more expensive to make any investment and reduce production cost.

Downstream firms use cost reducing investments to enhance their output levels. If investments are costly, firms can't increase their output through lowering their production costs. This affects sales price of firm j in two ways. First, reduced output makes a positive effect on price. Second, as output is reduced, upstream profit decreases and upstream firm can't spare as much resources to quality investment, which shifts the demand curve downward and makes a negative impact on firm j's sales price. The positive effect through decreased output exceeds the negative effect through decreased quality. As a result, firm j's sales price is higher for higher downstream investment costs (See Appendix A.4 ln[145]).

Firm i can make more investment compared to firm j in every possible case. When firm j makes a cost reducing investment and increases its output, firm i makes more cost reducing investment and increases its own output even more. This increases total output and decreases sales price. Any advantage firm j gains from an investment is canceled because of reduced sales price. We can say that cost reducing investment is not profitable for firm j, yet if firm j does not invest and firm i does, firm j is worse off, so firm i has to invest. Therefore, an increase in downstream investment cost decreases inefficient downstream investment level thus increases profit for firm j (See Appendix A.4 ln[151]).

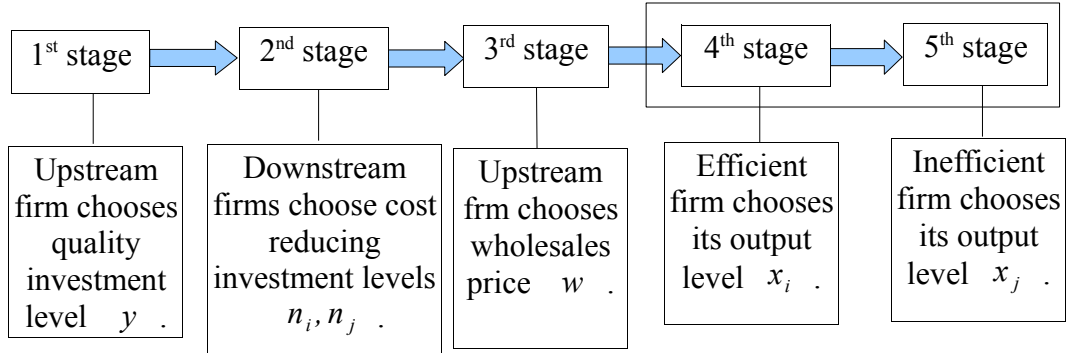
Lastly, we explore the effects of downstream investment cost on upstream firm. As we state earlier, increased investment cost reduces firm i's output level in every case. But we can't say the same thing for firm j's output level, there are some cases in which firm j's output level increases as downstream investment cost increases. We can speak of a total output reduction even for these cases (See Appendix A.4 ln[155]) since firm i has the advantage of having higher quality efficiency and is always one step ahead of firm j. Therefore, in a situation where firm j's output increases and firm i's output level decreases, the decrease of firm i's output is always higher than the increase in firm j's output. Once it is clear that total output level decreases as downstream investment cost increases, we can investigate the effects of this on the upstream firm. First of all, obviously, upstream firm faces a lower demand, due to an increase in  $m$ . This prevents the supplier to choose a higher wholesale price. So we can say that if downstream investment cost increases,

upstream wholesales price decreases (See Appendix A.4 ln[156]). Lower wholesale price combined with a lower output, upstream firm's profit decreases as well (See Appendix A.4 ln[157]). As upstream firm's profit decreases, the resources it can spare for quality investment decreases. As a result, upstream quality investment is negatively effected from a downstream investment cost increase (See Appendix A.4 ln[158]).

#### 4.EXTENSION

*Firm i moves first*

In this section, we want to analyze the impact of a new timing of the model. Despite the first model, firms choose their output levels sequentially. Our purpose is to see the effect of a quality differentiation when firms' outputs are not identical. Inverse demand functions the firms faces, and the profit functions of upstream and downstream firms are identical to the previous model. The timing of the game is as follows; in the first stage, upstream firm chooses its quality investment level, in the second stage downstream firms determine their cost reducing investment level, in the third stage upstream firm chooses its wholesale price, in the fourth stage downstream firm with higher quality efficiency chooses its output level, in the fifth stage firm with lower quality efficiency chooses its output level. We assume that there is perfect information. Figure (4.1) shows timing of the game.



**Figure 4.1:** Timing of the game

Again we solve this model through backwards induction. Since first three stages are identical with the previous model we solve in this paper, we show the solution of those steps in the appendix.

*Solution for 4<sup>th</sup> and 5<sup>th</sup> stages*

In this stage of the game, both downstream firms determine the quantities they produce. Profit functions of the firms are given in equations (2.14) and (2.15). If we set the first order condition of firm j's profit function equal to zero, we obtain the following reaction function for firm j:

$$\frac{\partial \Pi_j}{\partial X_j} = 1 + q - w + n_j - X_i - 2x_j + y\beta_j = 0 \quad X_j^R = \frac{1 + q - w + n_j - x_i + y\beta_j}{2} \quad (4.1)$$

We substitute  $X_j^R$  into profit function of firm i and obtain:

$$\Pi_i^R = -m n^2 - \frac{x_i(-1 - q + w - 2n_i + n_j + x_i - 2y\beta_i + y\beta_j)}{2} \quad (4.2)$$

We set first order condition of equation (4.2) equal to zero, solve for  $x_i$  and obtain the equilibrium output level of firm i:

$$\frac{\partial \Pi_i^R}{\partial X_i} = 1 + q + 2n_i + 2y\beta_i - w - n_j - 2x_i - y\beta_j = 0$$

$$X_i^{SI} = \frac{1 + q + 2n_i + 2y\beta_i - w - n_j - y\beta_j}{2} \quad (4.3)$$

Substituting the equilibrium output of firm i into equation (4.1), we obtain equilibrium output level of firm j:

$$X_j^{SI} = \frac{1 + q - 2n_i - 2y\beta_i - w + 3n_j + 3y\beta_j}{4} \quad (4.4)$$

(Superscript *SI* indicates the equilibrium values of the fourth and fifth stage)

Second order conditions should be negative for a maximum.

$$\frac{\partial^2 \Pi_i}{\partial X_i^2} = -2 < 0 \quad (4.5)$$

$$\frac{\partial^2 \Pi_j}{\partial X_j^2} = -2 < 0 \quad (4.6)$$

Equations (4.5) and (4.6) ensure that profit function is concave with respect to quantities (X). We also have to check the stability of the equilibrium. For the equilibrium to be stable and unique, the following condition should hold:

$$\left(\frac{\partial^2 \Pi_i}{\partial X_i^2}\right) * \left(\frac{\partial^2 \Pi_j}{\partial X_j^2}\right) > \left(\frac{\partial^2 \Pi_i}{\partial X_i \partial X_j}\right) * \left(\frac{\partial^2 \Pi_j}{\partial X_i \partial X_j}\right) \quad (4.7)$$

Left part of the inequality is equal to 4, right part of it is equal to 1, so the condition holds.

As we stated earlier, the rest of the solution is in the appendix together with the equilibrium values of endogenous variables. One thing that reveals itself at this point is that in order for firm j's output level to be non negative, there should not be significant difference among the downstream firms' quality efficiencies.

$\beta_i \leq 1.16 \beta_j$  is the necessary condition for firm j to have a non negative output. Firm i has upper hand for quality. Being a first mover gives firm i an opportunity to leave firm j out of the market if there is a significant quality differentiation between the firms. Therefore we take  $\beta_i \leq 1.16 \beta_j$  as one of the conditions as we used comparative static analysis to see the effects of endogenous variables (quality efficiencies and investment costs) on the endogenous variables (outputs, sales prices, profits, downstream investment, upstream investment and wholesale price). Our goal in this section is to explore any changes caused by different timing of this model. The ones we find are as follows.

#### *Effect of $\beta_i$*

In the previous model, we see that firm i's quality efficiency has a conditional effect on firm j's sales price. We don't observe the same effect in this model, since we have an upper limit for quality differentiation. There is only a positive effect of firm i's quality efficiency on rivals sales price (See Appendix A.5 ln[99]). This positive effect occurs through three steps. As  $\beta_i$  increases, firm i's output increase, firm j's output decrease. Increase in firm i's output is greater than the decrease in firm j's output so total output increases. Increase of total output creates a negative effect on sales price of firm j, however it also causes upstream firm to make more quality investment which eventually affects sales price of firm j positively. The positive

effect exceeds the negative effect so firm j's sales price increases as rivals quality efficiency increases.

### *Effect of $\beta_j$*

In the previous model, we observe that the effects of firm j's quality efficiency on rival firm are conditional. Within this model, since the quality differentiation has an upper limit, firm i's quality efficiency has only negative effect on firm i's output level, investment level and profit (See Appendix A.5 in [109], [110] and [111]). An increase in firm j's quality efficiency increases firm j's output level. As firm j's output increases, market share of firm j increases thus market share of firm i decreases. This reduction in firm i's output level causes firm i's profit to decrease. Finally, firm i has less resources to spare for cost reducing investment, as a result its investment level decreases.

In this section, we analyze a situation in which firm i has the advantage to move first when downstream firms decide their output levels. Remember that firm i already has an advantage of having higher quality. If it also has an advantage of having the first movement, then firm i has the opportunity to leave firm j outside of market if quality differentiation is significant. In the cases where quality differentiation is not significant, we don't see conditional situations that we observe in our first model.

### *Firm j moves first*

In this section, we analyze a situation where firm j which has a lower quality, has an advantage of having the first move in output competition. Our goal is to see the effects of this timing on the model. Inverse demand functions the firms faces, and the profit functions of upstream and downstream firms are identical to the previous models. The timing of the game is as follows; in the first stage, upstream firm chooses its quality investment level, in the second stage downstream firms determine their cost reducing investment level, in the third stage upstream firm chooses its wholesale price, in the fourth stage downstream firm with lower quality efficiency chooses its output level, in the fifth stage firm with higher quality efficiency chooses its output level. We assume that there is perfect information. Figure (4.2) shows timing of the game.

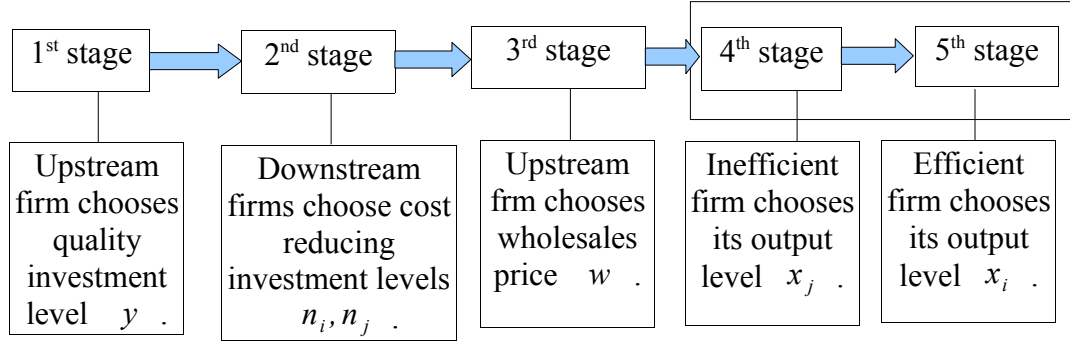


Figure 4.2: Timing of the game

Again we solve this model through backwards induction. Since first three stages are identical with the previous model we solve in this paper, we show the solution of those steps in the appendix.

#### *Solution for 4<sup>th</sup> and 5<sup>th</sup> stages*

In this stage of the game, both downstream firms determine the quantities they produce. Profit functions of the firms are given in equations (2.15) and (2.16). If we set the first order condition of firm i's profit function equal to zero, we obtain the following reaction function for firm i:

$$\frac{\partial \Pi_i}{\partial X_i} = 1 + q - w + n_i - x_j - 2x_i + y\beta_i = 0 \quad X_i^R = \frac{1 + q - w + n_i - x_j + y\beta_i}{2} \quad (4.8)$$

We substitute  $X_j^R$  into profit function of firm i and obtain:

$$\Pi_j^R = -mn^2 - \frac{x_j(-1 - q + w - 2n_j + n_i + x_j - 2y\beta_j + y\beta_i)}{2} \quad (4.9)$$

We set first order condition of equation (4.9) equal to zero, solve for  $x_j$  and obtain the equilibrium output level of firm i:

$$\frac{\partial \Pi_j^R}{\partial X_j} = 1 + q + 2n_j + 2y\beta_j - w - n_i - 2x_j - y\beta_i = 0$$

$$X_j^{s2} = \frac{1 + q + 2n_j + 2y\beta_j - w - n_i - y\beta_i}{2} \quad (4.10)$$

Substituting the equilibrium output of firm i into equation (4.8), we obtain equilibrium output level of firm i:

$$X_i^{S2} = \frac{1+q-2n_j-2y\beta_j-w+3n_i+3y\beta_i}{4} \quad (4.11)$$

(Superscript  $S2$  indicates the equilibrium values of the fourth and fifth stage)

We have to check for second order conditions to make sure profit function is concave with respect to quantities. Second order conditions should be negative for a maximum.

$$\frac{\partial^2 \Pi_i}{\partial X_i^2} = -2 < 0 \quad (4.12)$$

$$\frac{\partial^2 \Pi_j}{\partial X_j^2} = -2 < 0 \quad (4.13)$$

Equations (4.12) and (4.13) ensure that profit function is concave with respect to quantities ( $X$ ). We also have to check the stability of the equilibrium. For the equilibrium to be stable and unique, the following condition should hold:

$$\left(\frac{\partial^2 \Pi_i}{\partial X_i^2}\right) * \left(\frac{\partial^2 \Pi_j}{\partial X_j^2}\right) > \left(\frac{\partial^2 \Pi_i}{\partial X_i \partial X_j}\right) * \left(\frac{\partial^2 \Pi_j}{\partial X_i \partial X_j}\right) \quad (4.14)$$

Left part of the inequality is equal to 4, right part of it is equal to 1, so the condition holds. As we stated earlier, the rest of the solution is in the appendix together with the equilibrium values of endogenous variables. After solving the model, we make a comparative static analysis to see the effects of exogenous variables on endogenous variables. Our goal is to see the consequences of a different timing. Our findings are as follows

#### *Effects of $\beta_i$*

On the original model, we observe that the effects of firm i's quality efficiency on firm j's sales price are conditional. We observe a similar case here, yet with different conditions. Here we make the following preposition:

*Proposition 1.7:* If  $\beta_i$  and  $\beta_j$  are sufficiently close, inefficient firm's sales price increases as the quality efficiency level of rival firm increases.

*Proof:* Derivative of  $P_j^u$  with respect to  $\beta_i$  is:

$$\frac{\partial P_j^u}{\partial \beta_i} = \frac{-(A(-768(5-6m)^2 m^2 \beta_i^2 \beta_j - 48m(-5+6m)\beta_i(H) - \beta_j(C + 192(17-24m)^2 m^2 \beta_j^2)))}{(k(85+6m(163+192m))^2 + 192m^2((-17+24m)\beta_j + 2(-5+6m)\beta_i)^2)^2}$$

$$A = 3456m^2(-3+4m)(-5+6m)(1+q)$$

$$H = k(85+6m(-163+192m)) + 16(17-24m)m\beta_j^2$$

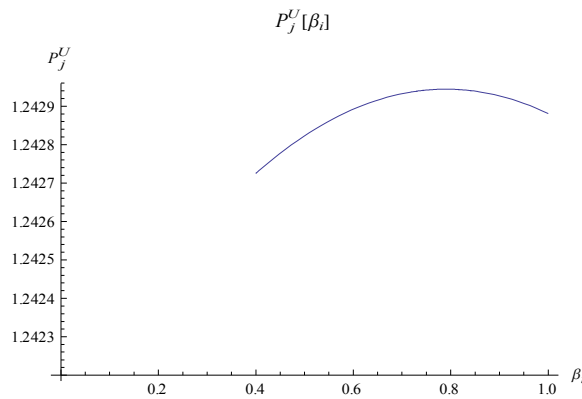
$$C = k(85+6m(-95+96m))(85+6m(-163+192m)) \quad (4.15)$$

The sign of equation (5.8) is determined by the following equation (See Appendix A.6 ln[96]):

$$\beta_j(C + 192m^2(4(5-6m)^2 \beta_i^2 + (17-24m)^2 \beta_j^2)) - 48m(-5+6m)\beta_i(H) \quad (4.16)$$

The result of equation (4.16) is positive when  $(\beta_i \leq 1.8\beta_j)$  and it is negative when  $(\beta_i \geq 2.4\beta_j)$  while  $k > 1$ ,  $m > 1$ ,  $q > 0$  and  $1 > \beta_i > \beta_j$  (See Appendix A.6 ln[97] and [98]).

Figure (4.3) shows that  $P_j^u(\beta_i)$  function is concave. Firm j's sales price increases for an increase in lower values of  $\beta_i$  and decreases for an increase in higher values of  $\beta_i$ .



**Figure 4.3:** Change of firm j's sales price by  $\beta_i$   
 $k = 11$ ,  $m = 7$ ,  $q = 1$ ,  $\beta_j = 0.4$

Our reasoning for the relation between firm i's quality efficiency and firm j's sales price in the original model applies this model as well. An increase in firm i's quality efficiency increases total output, and eventually causes an increase in upstream firm's quality investment. The positive effect of quality increase exceeds the

negative effect of output increase there isn't a significant quality differentiation. This model allows a larger quality differentiation compared to the original model, since firm j having the advantage of being first mover, produces more output, and an increase in  $\beta_i$  has less effect on total output. If the quality differentiation among the firms is significant, than the negative effect of output increase exceeds the positive effect of quality increase.

#### *Effects of $\beta_j$*

As we examine the effects of firm j's quality efficiency, we see that it has a conditional effect on rival firm, just like it has in the original model. However, the conditions are a little different due to the change in production levels of downstream firms. Following proposition helps us explain firm j's quality efficiency's effect on the rival firm's output level:

*Proposition 1.8:* If  $\beta_i$  and  $\beta_j$  are sufficiently close, efficient firm's output level decreases as the quality efficiency level of rival firm increases.

*Proof:* Derivative of  $X_i''$  with respect to  $\beta_j$  is:

$$\frac{\partial X_j''}{\partial \beta_j} = \frac{(41472m^3(-3+4m)(1+q)(768(5-6m)^2m^2\beta_i^3 - A + H + \beta_i(C + 192(17-24m)^2m^2\beta_j^2)))}{(k(85+6m(163+192m))^2 + 192m^2((-17+24m)\beta_j + 2(-5+6m)\beta_i)^2)^2}$$

$$A = 56km(-17+24m)(85+6m(-163+192m))\beta_j$$

$$H = 768m^2(-5+6m)(-17+24m)\beta_i^2\beta_j$$

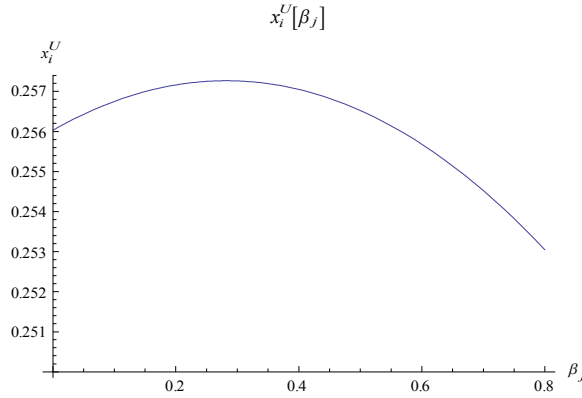
$$C = k(85-418m+480m^2)(85+6m(-163+192m)) \quad (4.17)$$

The sign of equation (4.17) is determined by the following equation (See Appendix A.6 ln[108]):

$$\beta_j(C + 192m^2(2(5-6m)\beta_i + (-17+24m)\beta_j)^2) - A \quad (4.18)$$

The result of equation (4.18) is negative when and it is  $(\beta_i < 2.3\beta_j)$  positive when  $(\beta_i > 3\beta_j)$  while  $k > 1$ ,  $m > 1$ ,  $q > 0$  and  $1 > \beta_i > \beta_j$  (See Appendix A.6 ln[109] and [110]).

Figure (4.4) shows that  $X_i''(\beta_j)$  function is concave. Firm i's output level increases for an increase in lower values of  $\beta_j$  and decreases for a increase in higher values of  $\beta_j$ .



**Figure 4.4:** Change of firm i's output level by  $\beta_j$   
 $k = 11, m = 7, q = 1, \beta_i = 0.8$

Our explanations of relations in the original model applies here. Firm j's quality efficiency affects firm i's output level through a direct and an indirect effect. If firm j's quality efficiency increases, its output and market share increases which automatically causes a reduction in firm i's output. However as we discuss earlier, an increase in quality efficiency causes an increase in total output, which allows upstream firm to make more quality investment. Increased quality favors firm i and enables it to increase its output level. When there is a significant quality differentiation, the positive effect exceeds the negative effect. If quality differentiation is not much, then the negative effect exceeds positive effect and as a result firm i's output level decreases.

We discuss in the original model that the effects of  $k$  on output level, profit and investment level are the same. Here, we can say the same thing about the effects of  $\beta_j$  on firm i's output level, profit and investment level.

The conditions that explain how firm j's quality efficiency affects firm i's output level are exactly the same as the conditions that explain firm j's quality efficiency's effects on firm i's profit and investment level.

Signs of derivatives of  $X_i''$ ,  $\Pi_i''$ ,  $n_i''$  with respect to  $\beta_j$  depends on a condition which is same for all three derivatives.

$$\frac{\partial X_j^u}{\partial \beta_j} = \frac{(41472m^3(-3+4m)(1+q)(768(5-6m)^2 m^2 \beta_i^3 - A + H + \beta_i(C + 192(17-24m)^2 m^2 \beta_j^2)))}{(k(85+6m(163+192m))^2 + 192m^2((-17+24m)\beta_j + 2(-5+6m)\beta_i)^2)^2}$$

$$A = 56k m(-17+24m)(85+6m(-163+192m))\beta_j$$

$$H = 768m^2(-5+6m)(-17+24m)\beta_i^2\beta_j$$

$$C = k(85-418m+480m^2)(85+6m(-163+192m)) \quad (4.17)$$

$$\frac{\partial \Pi_j^u}{\partial \beta_j} = \frac{(3456m^3 S T(768(5-6m)^2 m^2 \beta_i^3 - A + H + \beta_i(C + 192(17-24m)^2 m^2 \beta_j^2)))}{(k(85+6m(163+192m))^2 + 192m^2((-17+24m)\beta_j + 2(-5+6m)\beta_i)^2)^3}$$

$$A = 56k m(-17+24m)(85+6m(-163+192m))\beta_j$$

$$H = 768m^2(-5+6m)(-17+24m)\beta_i^2\beta_j$$

$$C = k(85-418m+480m^2)(85+6m(-163+192m))$$

$$T = (k(-5+6m)(85+6m(-163+192m)) + 192m^2(\beta_i - \beta_j)(2(-5+6m)\beta_i + (-17+24m)\beta_j))$$

$$S = (-3+4m)(-289+576m)(1+q)^2 \quad (4.19)$$

$$\frac{\partial n_j^u}{\partial \beta_j} = \frac{(S(768(5-6m)^2 m^2 \beta_i^3 - A + H + \beta_i(C + 192(17-24m)^2 m^2 \beta_j^2)))}{(k(85+6m(163+192m))^2 + 192m^2((-17+24m)\beta_j + 2(-5+6m)\beta_i)^2)^2}$$

$$A = 56k m(-17+24m)(85+6m(-163+192m))\beta_j$$

$$H = 768m^2(-5+6m)(-17+24m)\beta_i^2\beta_j$$

$$C = k(85-418m+480m^2)(85+6m(-163+192m))$$

$$S = 29376m^2(-3+4m)(1+q) \quad (4.20)$$

The sign of equations (4.17), (4.19) and (4.20) are determined by the following equation (while  $k>1$ ,  $q>0$  and  $1>\beta_i>\beta_j$ ), (See Appendix A.6 ln[108], [114] and [111]):

$$\beta_j(C + 192m^2(2(5-6m)\beta_i + (-17+24m)\beta_j)^2) - A \quad (4.21)$$

Equation (4.21) yields a positive result when  $\beta_i \geq 3\beta_j$  and a negative result when  $\beta_i \leq 2.3\beta_j$ . And since it determines the signs of all three derivatives, we can say that quality efficiency of firm j affects firm i's output, firm i's profit and firm i's investment level in parallel courses. This is not contrary to intuition. If we look at firm i's profit function (2.14) we see that output level has a direct effect on firm i's profit. Also, investment levels are affected the same way since profit creates a resource to spare on investments. Any change in profit affects investment level proportionally.

In this version of the model, we see that firm j's quality efficiency affects firm j in quite an interesting way, thus we explain this in the following proposition:

*Proposition 1.09:* If  $\beta_i$  and  $\beta_j$  are not sufficiently close, inefficient firm's output level decreases as own quality efficiency level increases.

*Proof:* Derivative of  $X_j''$  with respect to  $\beta_j$  is:

$$\frac{\partial X_j''}{\partial \beta_j} = \frac{-(S(768(5-6m)^2 m^2 \beta_i^3 - A + H + \beta_i(C + 192(17-24m)^2 m^2 \beta_j^2)))}{(\partial \beta_j (k(85+6m(163+192m))^2 - 192m^2((-17+24m)\beta_j + 2(-5+6m)\beta_i)^2)^2}$$

$$A = k(-17+24m)(85+6m(-163+192m))\beta_j$$

$$H = 768m^2(-5+6m)(-17+24m)\beta_i^2\beta_j$$

$$C = k(-85+2m(13+96m))(85+6m(-163+192m))$$

$$S = 41472m^3(-3+4m)(1+q) \quad (4.22)$$

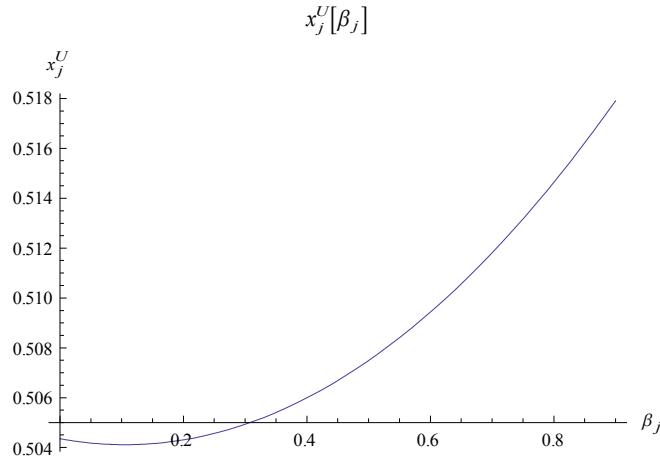
The sign of equation (4.22) is determined by the following equation (See Appendix A.6 ln[118]):

$$768m^2(-5+6m)\beta_i^2((-5+6m)\beta_i + (-17+24m)\beta_j) - A(-17+80m) - \beta_i(-C - 192(17-24m)^2 m^2 \beta_j^2)$$

$$(4.23)$$

The result of equation (4.23) is positive when and it  $(\beta_i < 3\beta_j)$  is negative when  $(\beta_i > 10\beta_j)$ , while  $k > 1$ ,  $m > 1$ ,  $q > 0$  and  $1 > \beta_i > \beta_j$  (See Appendix A.6 ln[119] and [120]).

Figure (4.5) shows  $X_j^u(\beta_j)$  function. Function has a minimum at  $\beta_j=0.10$  for given values. Slope is downward for lower values of  $\beta_j$  and upward for higher values of  $\beta_j$ .



**Figure 4.5:** Change of firm j's output level by  $\beta_j$   
 $k = 11, m = 7, q = 1, \beta_i = 0.9$

It is odd to see any negative effect of firm j's quality efficiency on firm j's output level. However there is a simple explanation. As firm j's quality efficiency increases, total output increases which causes upstream firm to increase its investment in quality. If quality efficiency of firm i is a lot greater than that of firm j, then, increase in firm i's output level can be bigger than the increase in firm j's output level. Then, firm i's market share increases hence firm j's output level decreases. If quality differentiation is not as significant, the increase in firm i's output level due to increased quality is less than the increase in firm j's output due to higher quality efficiency. In this case, firm j's output level increases as  $\beta_j$  increases.

As we know that firm j's output level, profit and investment level are connected, we can say that the effects of firm j's quality efficiency shows parallel courses in firm j's output level, profit and investment level.

The conditions that explain how firm j's quality efficiency affects firm j's output level are exactly the same as the conditions that explain firm j's quality efficiency's effects on firm j's profit and investment level.

Signs of derivatives of  $X_j^u$ ,  $\Pi_j^u$ ,  $n_j^u$  with respect to  $\beta_j$  depends on a condition which is same for all three derivatives.

$$\frac{\partial X_j^u}{\partial \beta_j} = \frac{-(S(768(5-6m)^2 m^2 \beta_i^3 - A + H + \beta_i(C + 192(17-24m)^2 m^2 \beta_j^2)))}{(k(85+6m(163+192m))^2 - 192m^2((-17+24m)\beta_j + 2(-5+6m)\beta_i)^2)^2}$$

$$A = k(-17+24m)(85+6m(-163+192m))\beta_j$$

$$H = 768m^2(-5+6m)(-17+24m)\beta_i^2\beta_j$$

$$C = k(-85+2m(13+96m))(85+6m(-163+192m))$$

$$S = 41472m^3(-3+4m)(1+q) \quad (4.22)$$

$$\frac{\partial n_j^u}{\partial \beta_j} = \frac{-(T(768(5-6m)^2 m^2 \beta_i^3 - A + H + \beta_i(C + 192(17-24m)^2 m^2 \beta_j^2)))}{(k(85+6m(163+192m))^2 - 192m^2((-17+24m)\beta_j + 2(-5+6m)\beta_i)^2)^2}$$

$$T = 17280m^3(-3+4m)(1+q) \quad (4.24)$$

$$\frac{\partial \Pi_j^u}{\partial \beta_j} = \frac{-(RT(768(5-6m)^2 m^2 \beta_i^3 - A + H + \beta_i(C + 192(17-24m)^2 m^2 \beta_j^2)))}{(k(85+6m(163+192m))^2 - 192m^2((-17+24m)\beta_j + 2(-5+6m)\beta_i)^2)^3}$$

$$T = (k(-17+24m)(85+6m(-163+192m)) - 384m^2(\beta_i - \beta_j)(2(-5+6m)\beta_i + (-17+24m)\beta_j))$$

$$R = 6912m^3(-3+4m)(-25+72m)(1+q)^2 \quad (4.25)$$

The sign of equations (4.22), (4.24) and (4.25) are determined by the following equation (while  $k > 1$ ,  $q > 0$  and  $1 > \beta_i > \beta_j$ ), (See Appendix A.6 ln[118], [121] and [124]):

$$768m^2(-5+6m)\beta_i^2((-5+6m)\beta_i + (-17+24m)\beta_j) - A(-17+80m) - \beta_i(-C - 192(17-24m)^2 m^2 \beta_j^2)$$

$$A = k(-17+24m)(85+6m(-163+192m))\beta_j$$

$$C = k(-85+2m(13+96m))(85+6m(-163+192m)) \quad (4.26)$$

Equation (4.26) yields a positive result when  $\beta_i \leq 3\beta_j$  and a negative result when  $\beta_i \geq 10\beta_j$ . And since it determines the signs of all three derivatives, we can say that quality efficiency of firm j affects firm j's output, firm j's profit and firm j's investment level in parallel courses. This means that if quality differentiation between the downstream firms is too great, an increase in firm j's quality efficiency decreases firm j's output level as we state earlier. This loss in output is directly transferred to profit of firm j since output is a key factor in profit function. Also, as the profits decrease, firm j's has less resources to invest in cost reduction.

### Effects of $k$

As we investigate the effects of upstream investment cost in this new model, we see that the only effects of  $k$  that differ from the original model are  $k$ 's effect on firm  $j$ . As in the original model, upstream investment cost affects firm  $j$  in different ways depending on the conditions. The conditions are different in this model, which are stated in the following proposition:

*Proposition 1.10:* Firm  $j$ 's output level increases if as upstream investment cost  $k$  increases when downstream firms' quality efficiencies are not sufficiently close ( $\beta_i > 1.5\beta_j$ ). Firm  $j$ 's output level decreases with an upstream investment cost increase when downstream firms' quality efficiencies are sufficiently close ( $\beta_i < 1.125\beta_j$ ).

*Proof:* Derivative of  $X_j^u$  with respect to  $k$  is:

$$\frac{\partial X_j^u}{\partial \beta_j} = \frac{20736m^3(-3+4m)(85+6m(-163-192m))(1+q)A}{(k(85+6m(163+192m))^2 - 192m^2((-17+24m)\beta_j + 2(-5+6m)\beta_i)^2)^2}$$

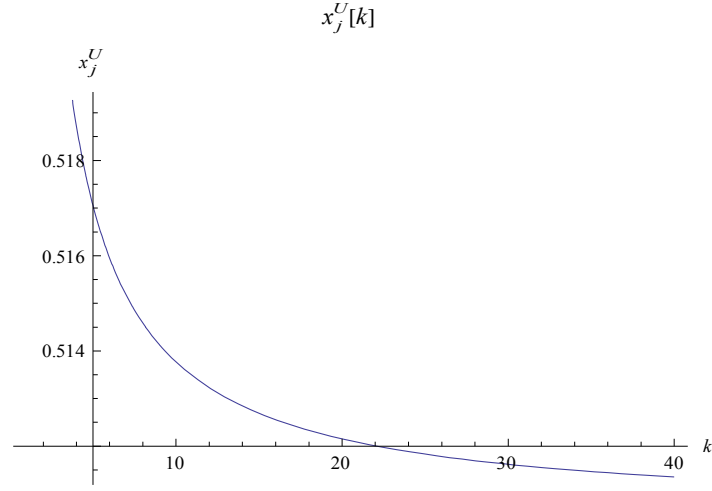
$$A = (56m\beta_i + (17-80m)\beta_j)(2(-5+6m)\beta_i + (-17+24m)\beta_j) \quad (4.27)$$

and this equation is negative or positive depending on the sign of (See Appendix A.6 ln[137]):

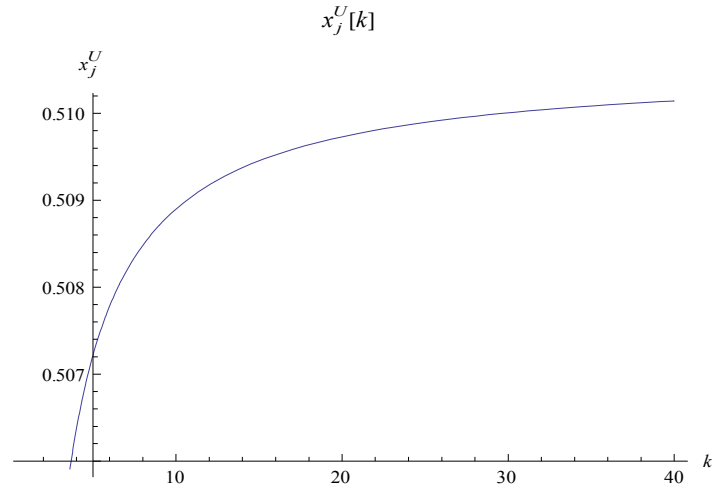
$$56m\beta_i + (17-80m)\beta_j \quad (4.28)$$

It is positive when  $\beta_i > 1.5\beta_j$  and negative when  $\beta_i < 1.25\beta_j$  while  $k > 1$ ,  $q > 0$  and  $1 > \beta_i > \beta_j$  (See Appendix A.6 ln[138] and [139]).

Figures (4.6) and (4.7) show  $X_j^u(k)$  functions for two different situations.



**Figure 4.6:** Change of firm j's output level by  $k$   
 $m = 7, q = 1, \beta_j = 0.7 \quad \beta_i = 0.8$



**Figure 4.7:** Change of firm j's output level by  $k$   
 $m = 7, q = 1, \beta_j = 0.5 \quad \beta_i = 0.8$

The same reasoning applies here, as it is in the original model. If quality differentiation among the downstream firms is not too big, quality increase affects them almost equally. In this case, an increase in upstream investment cost naturally causes a reduction in firm j's output level. However, if quality differentiation is significant, then quality increase favors firm i, and harms firm j. In this case, an increase in upstream investment cost increases firm j's output level. Also, just like in the original model, the effect of upstream investment cost on firm j's production is parallel to the effects on firm j's profit and investment levels.

The impact of upstream investment cost on firm j's sales price is conditional as it is in the original model. The conditions for this model are different than that of the original model, thus they are stated in the following proposition.

*Proposition 1.11:* Firm j's sales price increases if as upstream investment cost  $k$  increases when downstream firms' quality efficiencies are significantly different ( $\beta_i > 8\beta_j$ ). Firm j's sales price decreases with an upstream investment cost increase when downstream firms' quality efficiencies are not significantly different ( $\beta_i \leq 5.5\beta_j$ ).

Proof: Derivative of  $P_j^u$  with respect to  $k$  is:

$$\frac{\partial P_j^u}{\partial k} = \frac{A(2(-5+6m)\beta_i + (-17+24m)\beta_j)(24m(-5+6m)\beta_i + (-85+18(43-48m)m)\beta_j)}{(k(85+6m(163+192m))^2 - 192m^2((-17+24m)\beta_j + 2(-5+6m)\beta_i)^2}$$

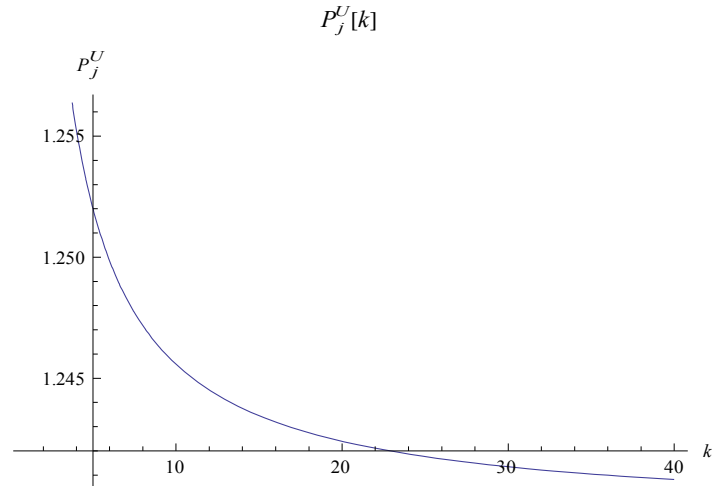
$$A = 1728m^2(-3+4m)(85+6m(-163+192m))(1+q) \quad (4.29)$$

and this equation is negative or positive depending on the sign of (See Appendix A.6 ln[146]):

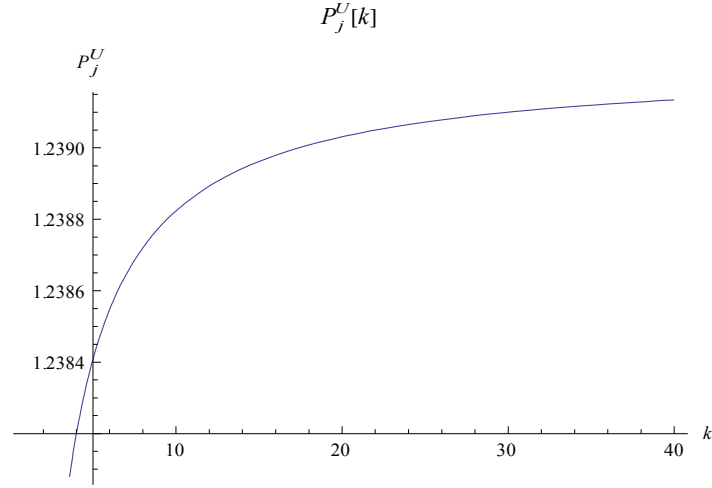
$$30m\beta_i + 7(1-6m)\beta_j \quad (4.30)$$

It is positive when  $\beta_i \geq 8\beta_j$  and negative when  $\beta_i \leq 5.5\beta_j$  while  $k > 1$ ,  $q > 0$  and  $1 > \beta_i > \beta_j$  (See Appendix A.6 ln[147] and [148]).

Figures (4.8) and (4.9) show  $P_j^u(k)$  functions for two different situations.



**Figure 4.8:** Change of firm j's sales price by  $k$   
 $m = 7, q = 1, \beta_j = 0.5 \quad \beta_i = 0.8$



**Figure 4.9:** Change of firm j's sales price by  $k$   
 $m = 7, q = 1, \beta_j = 0.09 \quad \beta_i = 0.8$

Upstream firm's investment decreases when it faces a high investment cost. This reduction on quality investment affects downstream firms sales price in two ways. First, as quality decreases, downstream firms' demands curves shift downwards, which directly affects the sales price negatively. Secondly, reduced demand causes downstream manufacturers to produce less output, which indirectly affects sales price positively. For firm i, the negative effect always exceeds the positive effect, since firm i has a high quality. For firm j, however, the magnitude of these effects are conditional. If the quality differentiation among the firms is not significant, then the effect on firm j is similar to the effect on firm i. However, if the quality differentiation is quite big, then, the positive effect is higher for firm j. It can be explained as follows. In order for quality differentiation to be big, firm i has to have a high quality efficiency, and firm j has to have a low quality efficiency. An decrease in upstream investment level due to high investment cost, affects firm i a lot and increases the loss of total output. However, the negative effect on firm j's price is very little since firm j's has a low quality efficiency.



## 6. CONCLUSION

Our purpose in this paper is to investigate the effects of a quality differentiation of two manufacturers due to an upstream supplier quality investment. To achieve our goal, we construct a model of a supply chain that includes an upstream firm and two downstream firms. We use a coefficient we call quality efficiency of downstream firms to create a quality differentiation. This coefficient is a multiplier of upstream quality investment and it determines the percentage of upstream quality level transferred to the consumers. The model is a four stage game where upstream firm chooses quality investment level and wholesale price, and downstream firms choose their output levels and cost reducing investment levels. We solve the model through backwards induction method. Once we find the equilibrium values of endogenous variables, we make a comparative static analysis to see the effects of exogenous variables on endogenous variables. As we explore our model, we see that the degree of quality differentiation plays an important role in determining firm  $j$ 's (firm having lower quality) quality's effect on firm  $i$  (firm having high quality). We also see that, investment costs' effect on the firm with low quality also depends on the degree of quality differentiation.

As an extension, we investigate the effects of a different timing on our model. We let one of the downstream firms to make the first move when choosing output levels. Here, we see that if the firm that makes the first move is the firm with high quality, it has the opportunity to leave the rival firm out of market when quality differentiation is significant.



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## **APPENDICES**

**APPENDIX A.1 :** Equilibrium outcomes of first model

**APPENDIX A.2 :** Solution and equilibrium outcomes of second model

**APPENDIX A.3 :** Solution and equilibrium outcomes of third model

**APPENDIX A.4 :** Mathematica printout of first model

**APPENDIX A.5 :** Mathematica printout of second model

**APPENDIX A.6 :** Mathematica printout of third model



## APPENDIX A.1

Equilibrium outcomes of first model

Upstream investment level:

$$y^u = \frac{-432m^2(1+q)(\beta_i + \beta_j)}{-k(7-72m)^2 + 216m^2(\beta_i + \beta_j)^2} \quad (\text{A.1.1})$$

Upstream wholesale price:

$$w^u = \frac{36km(-7+72m)(1+q)}{k(7-72m)^2 - 216m^2(\beta_i + \beta_j)^2} \quad (\text{A.1.2})$$

Upstream profit:

$$\Pi_u^u = \frac{864km^2(1+q)^2}{k(7-72m)^2 - 216m^2(\beta_i + \beta_j)^2} \quad (\text{A.1.3})$$

Firm i's output level:

$$X_i^u = \frac{12m(1+q)(k(-7+12m)(-7+72m) + 216m^2(\beta_i^2 - \beta_j^2))}{(-7+12m)(k(7-72m)^2 - 216m^2(\beta_i + \beta_j)^2)} \quad (\text{A.1.4})$$

Firm j's output level:

$$X_j^u = \frac{12m(1+q)(k(-7+12m)(-7+72m) + 216m^2(-\beta_i^2 + \beta_j^2))}{(-7+12m)(k(7-72m)^2 - 216m^2(\beta_i + \beta_j)^2)} \quad (\text{A.1.5})$$

Firm i's sales price:

$$P_i^u = \frac{(1+q)(k(-7+48m)(-7+72m)+216m^2(\beta_i^2-\beta_j^2))}{k(7-72m)^2-216m^2(\beta_i+\beta_j)^2} \quad (\text{A.1.6})$$

Firm j's sales price:

$$P_j^u = \frac{(1+q)(k(-7+48m)(-7+72m)+216m^2(-\beta_i^2+\beta_j^2))}{k(7-72m)^2-216m^2(\beta_i+\beta_j)^2} \quad (\text{A.1.7})$$

Firm i's profit:

$$\Pi_i^u = \frac{m(-49+144m)(1+q)^2(k(49-588m+864m^2)+216m^2\beta_i^2-216m^2\beta_j^2)^2}{(7-12m)^2(-k(7-72m)^2+216m^2\beta_i^2+432m^2\beta_i\beta_j+216m^2\beta_j^2)^2} \quad (\text{A.1.8})$$

Firm j's profit:

$$\Pi_j^u = \frac{m(-49+144m)(1+q)^2(k(49-588m+864m^2)-216m^2\beta_i^2+216m^2\beta_j^2)^2}{(7-12m)^2(-k(7-72m)^2+216m^2\beta_i^2+432m^2\beta_i\beta_j+216m^2\beta_j^2)^2} \quad (\text{A.1.9})$$

Firm i's investment level:

$$n_i^u = \frac{7(1+q)(k(49-588m+864m^2)+216m^2\beta_i^2-216m^2\beta_j^2)}{(-7+12m)(k(7-72m)^2-216m^2\beta_i^2-432m^2\beta_i\beta_j-216m^2\beta_j^2)} \quad (\text{A.1.10})$$

Firm j's investment level:

$$n_j^u = \frac{7(1+q)(k(49-588m+864m^2)-216m^2\beta_i^2+216m^2\beta_j^2)}{(-7+12m)(k(7-72m)^2-216m^2\beta_i^2-432m^2\beta_i\beta_j-216m^2\beta_j^2)} \quad (\text{A.1.11})$$

## APPENDIX A.2

Solution and equilibrium outcomes of the second model:

### *3<sup>rd</sup> Stage*

In this stage of the game, upstream firm determines its wholesale price. Profit function of the upstream firm is given in equation (2.13). Substituting the equilibrium outputs in equations (4.3) and (4.4) into profit function of the upstream firm, we obtain:

$$\Pi_u^{SI} = \frac{1}{4}(3w + 3qw - 3w^2 - 4ky^2 + 2w n_i + w n_j + 2w y \beta_i + w y \beta_j) \quad (\text{A.2.1})$$

Setting first order conditions equal to zero and solving for wholesale price, gives us the wholesale price which maximizes the upstream profit.

$$\frac{\partial \Pi_u^c}{\partial w} = \frac{1}{4}(3 + 3q - 6w + 2n_i + n_j + 2y \beta_i + y \beta_j) = 0$$
$$w^w = \frac{1}{6}(3 + 3q + 2n_i + n_j + 2y \beta_i + y \beta_j) \quad (\text{A.2.2})$$

Second order condition should be negative for a maximum.

$$\frac{\partial^2 \Pi_u^c}{\partial w^2} = -\left(\frac{3}{2}\right) < 0 \quad (\text{A.2.3})$$

Equation (A.2.3) shows us that second order condition is satisfied.

### *2<sup>nd</sup> Stage*

In the second stage of the game, firms simultaneously choose their cost reducing investment levels to maximize their profit. We substitute the equilibrium

wholesale price and equilibrium quantities into the profit functions and obtain:

$$\Pi_i^w = \frac{1}{288} (4(25-72m)n_i^2 + 20n_i(3+3q-7n_j+10y\beta_i-7y\beta_j) + (3+3q-7n_j^2+10y\beta_i-7y\beta_j)^2) \quad (\text{A.2.4})$$

$$\Pi_j^w = \frac{(196n_i^2(289-576m)n_j^2 + (34n_j-28n_i+A)A - 476n_in_j)}{576} \quad (\text{A.2.5})$$

$$A = 3+3q-14y\beta_i+17y\beta_j$$

Differentiating  $\Pi_i^w$  with respect to  $n_i$ , and  $\Pi_j^w$  with respect to  $n_j$ , setting them equal to zero and solving for  $n_i$  and  $n_j$ , we obtain the following investment reaction functions:

$$n_i = \frac{5(3+3q-7n_j+10y\beta_i-7y\beta_j)}{2(-25+72m)} \quad (\text{A.2.6})$$

$$n_j = \frac{17(3+3q-14n_i-14y\beta_i+17y\beta_j)}{(-289+576m)} \quad (\text{A.2.7})$$

Solving equations (A.2.6) and (A.2.7) simultaneously, equilibrium investment levels can be found as follows:

$$n_i^d = \frac{5(-17+24m-17q+24mq-17y\beta_i+80my\beta_i-56my\beta_j)}{85-978m+1152m^2} \quad (\text{A.2.8})$$

$$n_j^d = \frac{17(-5+6m-5q+6mq-28my\beta_i-5y\beta_j+34my\beta_j)}{85-978m+1152m^2} \quad (\text{A.2.9})$$

We have to check for second order conditions to make sure profit function is concave with respect to cost reducing investment levels. Second order conditions should be negative for a maximum

$$\frac{\partial^2 \Pi_i^w}{\partial n_i^2} = \frac{25}{36} - 2m < 0 \quad (\text{A.2.10})$$

$$\frac{\partial^2 \Pi_j^w}{\partial n_j^2} = \frac{289}{288} - 2m < 0 \quad (\text{A.2.11})$$

Since  $m > 1$ , second order conditions are satisfied. Equations (A.2.10) and (A.2.11) ensure that profit function is concave with respect to cost reducing investment levels ( $n$ ). We also have to check the stability of the equilibrium. For the equilibrium to be stable and unique, the following condition should hold:

$$\left(\frac{\partial^2 \Pi_i^w}{\partial n_i^2}\right) * \left(\frac{\partial^2 \Pi_j^w}{\partial n_j^2}\right) > \left(\frac{\partial^2 \Pi_i^w}{\partial n_i \partial n_j}\right) * \left(\frac{\partial^2 \Pi_j^w}{\partial n_i \partial n_j}\right) \quad (\text{A.2.12})$$

First part of the inequality yields a result that is bigger than 1 given the condition that  $m > 1$ . Right part of the inequality is equal to  $\frac{4165}{10368}$ . The stability condition is satisfied.

### *1<sup>st</sup> Stage*

In this stage, the upstream firm chooses its quality investment level. Substituting the equilibrium outputs, equilibrium wholesale price and equilibrium downstream investment levels into the profit function of the upstream firm, we obtain:

$$\Pi_u^d = \frac{192m^2(9(-3+4m)(1+q) + (-17+24m)y\beta_i + 2(-5+6m)y\beta_j)^2}{(84+6m(-163+192m))^2} \quad (\text{A.2.13})$$

Upstream firm maximizes its profit with respect to quality investment level ( $y$ ), so:

$$\frac{\partial \Pi_u^d}{\partial y} = \frac{-2(k(85+6m(-163+192m))^2 y - 192m^2((-17+24m)\beta_i + 2(-5+6m)\beta_j)A)}{(84+6m(-163+192m))^2} = 0$$

$$A = (9(-3+4m)(1+q) + (-17+24m)y\beta_i + 2(-5+6m)y\beta_j) \quad (\text{A.2.14})$$

If we solve for y we obtain:

$$y^u = \frac{-1728m^2(-3+4m)(1+q)((-17+24m)\beta_i + 2(-5+6m)\beta_j)}{-k(85+6m(163+192m))^2 + 192m^2((-17+24m)\beta_i + 2(-5+6m)\beta_j)^2}$$

**(A.2.15)**

Second order conditions should be negative for a maximum.

$$\frac{\partial^2 \Pi_u^d}{\partial y^2} = \frac{(-k(85+6m(163+192m))^2 + 192m^2((-17+24m)\beta_i + 2(-5+6m)\beta_j)^2)}{(84+6m(-163+192m))^2} < 0$$

**(A.2.16)**

Equation (A.2.16) is always smaller than zero when  $m > 1$ ,  $k > 1$ , and  $1 > \beta_i > \beta_j$ , so second order condition is satisfied.

Equilibrium Values

Upstream investment level:

$$y^u = \frac{-1728m^2(-3+4m)(1+q)((-17+24m)\beta_i + 2(-5+6m)\beta_j)}{-k(85+6m(163+192m))^2 + 192m^2((-17+24m)\beta_i + 2(-5+6m)\beta_j)^2}$$

**(A.2.17)**

Upstream wholesale price:

$$w^u = \frac{144k m(-3+4m)(85+6m(-163-192m))(1+q)}{k(85+6m(163+192m))^2 + 192m^2((-17+24m)\beta_i + 2(-5+6m)\beta_j)^2}$$

**(A.2.18)**

Upstream profit:

$$\Pi_u^u = \frac{15552k(3-4m)^2 m^2 (1+q)^2}{k(85+6m(163+192m))^2 + 192m^2((-17+24m)\beta_i + 2(-5+6m)\beta_j)^2}$$

**(A.2.19)**

Firm i's output level:

$$X_i^u = \frac{12(1+q)(k m(-17+24m)(85+6m(-163+192m))+384m^3 A)}{k(85+6m(163+192m))^2+192m^2((-17+24m)\beta_i+2(-5+6m)\beta_j)^2}$$

$$A=(\beta_i-\beta_j)((-17+24m)\beta_i+2(-5+6m)\beta_j)$$

**(A.2.20)**

Firm j's output level:

$$X_j^u = \frac{24m(1+q)(k(-5+6m)(85+6m-(163+192m))-192m^2 A)}{k(85+6m(163+192m))^2+192m^2((-17+24m)\beta_i+2(-5+6m)\beta_j)^2}$$

$$A=(\beta_i-\beta_j)((-17+24m)\beta_i+2(-5+6m)\beta_j)$$

**(A.2.21)**

Firm i's sales price:

$$P_i^u = \frac{(1+q)(k(85-654m+720m^2)(85+6m(-163+192m))+384m^2(-5+6m) A)}{k(85+6m(163+192m))^2+192m^2((-17+24m)\beta_i+2(-5+6m)\beta_j)^2}$$

$$A=(\beta_i-\beta_j)((-17+24m)\beta_i+2(-5+6m)\beta_j)$$

**(A.2.22)**

Firm j's sales price:

$$P_j^u = \frac{(1+q)(k(85-654m+720m^2)(85+6m(-163+192m))-192m^2(-17+24m) A)}{k(85+6m(163+192m))^2+192m^2((-17+24m)\beta_i+2(-5+6m)\beta_j)^2}$$

$$A=(\beta_i-\beta_j)((-17+24m)\beta_i+2(-5+6m)\beta_j)$$

**(A.2.23)**

Firm i's profit:

$$\Pi_i^u = \frac{m(-25+72m)(1+q)^2(k(-17+24m)(85+6m(-163+192m))+384m^2 A)}{(k(85+6m(163+192m))^2+192m^2((-17+24m)\beta_i+2(-5+6m)\beta_j)^2)^2}$$

$$A=(\beta_i-\beta_j)((-17+24m)\beta_i+2(-5+6m)\beta_j)$$

**(A.2.24)**

Firm j's profit:

$$\Pi_j^u = \frac{m(-289+576m)(1+q)^2(k(-5+6m)(85+6m(-163+192m))-192m^2A)}{(k(85+6m(163+192m)))^2+192m^2((-17+24m)\beta_i+2(-5+6m)\beta_j)^2}$$

$$A=(\beta_i-\beta_j)((-17+24m)\beta_i+2(-5+6m)\beta_j) \quad (\text{A.2.25})$$

Firm i's investment level:

$$n_i^u = \frac{5(1+q)(k(-17+24m)(85+6m(-163+192m))+384m^2A)}{k(85+6m(163+192m))^2+192m^2((-17+24m)\beta_i+2(-5+6m)\beta_j)^2}$$

$$A=(\beta_i-\beta_j)((-17+24m)\beta_i+2(-5+6m)\beta_j) \quad (\text{A.2.26})$$

Firm j's investment level:

$$n_j^u = \frac{17(1+q)(k(-5+6m)(85+6m(-163+192m))-192m^2A)}{k(85+6m(163+192m))^2+192m^2((-17+24m)\beta_i+2(-5+6m)\beta_j)^2}$$

$$A=(\beta_i-\beta_j)((-17+24m)\beta_i+2(-5+6m)\beta_j) \quad (\text{A.2.27})$$

### APPENDIX A. 3

Solution and equilibrium outcomes of the third model

#### *3<sup>rd</sup> Stage*

In this stage of the game, upstream firm determines its wholesale price. Profit function of the upstream firm is given in equation (2.13). Substituting the equilibrium outputs in equations (4.10) and (4.11) into profit function of the upstream firm, we obtain:

$$\Pi_u^{s2} = \frac{1}{4}(3w + 3qw - 3w^2 - 4ky^2 + 2wn_j + wn_i + 2wy\beta_j + wy\beta_i) \quad (\text{A.3.1})$$

Setting first order conditions equal to zero and solving for wholesale price, gives us the wholesale price which maximizes the upstream profit.

$$\begin{aligned} \frac{\partial \Pi_u^{s2}}{\partial w} &= \frac{1}{4}(3 + 3q - 6w + 2n_j + n_i + 2y\beta_j + y\beta_i) = 0 \\ w^w &= \frac{1}{6}(3 + 3q + 2n_j + n_i + 2y\beta_j + y\beta_i) \end{aligned} \quad (\text{A.3.2})$$

Second order conditions should be negative for a maximum:

$$\frac{\partial^2 \Pi_u^{s2}}{\partial w^2} = -\left(\frac{3}{2}\right) < 0 \quad (\text{A.3.3})$$

Equation (A.3.3) shows us that second order condition is satisfied.

#### *2nd Stage*

In the second stage of the game, firms simultaneously choose their cost reducing investment levels to maximize their profit. We substitute the equilibrium wholesale price and equilibrium quantities into the profit functions and obtain:

$$\Pi_i^w = \frac{((289 - 576m)n_i^2 + 34n_i(-14n_j + A) + (A - 14n_j)^2)}{576}$$

$$A = 3 + 3q + 17y\beta_i - 14y\beta_j \quad (\text{A.3.4})$$

$$\Pi_j^w = \frac{(49n_i^2 + 4(25 - 72m)n_j^2 + 20n_i(2(B) + B^2 - 14n_i(B + 10n_j)))}{288}$$

$$B = 3 + 3q - 7y\beta_i + 10y\beta_j \quad (\text{A.3.5})$$

Differentiating  $\Pi_i^w$  with respect to  $n_i$ , and  $\Pi_j^w$  with respect to  $n_j$ , setting them equal to zero and solving for  $n_i$  and  $n_j$ , we obtain the following investment reaction functions:

$$n_i = \frac{17(3 + 3q - 14n_j - 14y\beta_j + 17y\beta_i)}{(-289 + 576m)} \quad (\text{A.3.6})$$

$$n_j = \frac{5(3 + 3q - 7n_i + 10y\beta_j - 7y\beta_i)}{2(-25 + 72m)} \quad (\text{A.3.7})$$

Solving equations (A.3.6) and (A.3.7) simultaneously, equilibrium investment levels can be found as follows:

$$n_i^d = \frac{17(-5 + 6m - 5q + 6mq - 28my\beta_j - 5y\beta_i + 34my\beta_i)}{85 - 978m + 1152m^2} \quad (\text{A.3.8})$$

$$n_j^d = \frac{5(-17 + 24m - 17q + 24mq - 17y\beta_j + 80my\beta_j - 56my\beta_i)}{85 - 978m + 1152m^2} \quad (\text{A.3.9})$$

We have to check for second order conditions to make sure profit function is concave with respect to cost reducing investment levels. Second order conditions

$$\text{should be negative for a maximum: } \frac{\partial^2 \Pi_i^w}{\partial n_i^2} = \frac{289}{288} - 2m < 0 \quad (\text{A.3.10})$$

$$\frac{\partial^2 \Pi_j^w}{\partial n_j^2} = \frac{25}{36} - 2m < 0 \quad (\text{A.3.11})$$

Since  $m > 1$ , second order conditions are satisfied. Equations (A.3.10) and (A.3.11) ensure that profit function is concave with respect to cost reducing investment levels ( $n$ ). We also have to check the stability of the equilibrium. For the equilibrium to be stable and unique, the following condition should hold:

$$\left(\frac{\partial^2 \Pi_i^w}{\partial n_i^2}\right) * \left(\frac{\partial^2 \Pi_j^w}{\partial n_j^2}\right) > \left(\frac{\partial^2 \Pi_i^w}{\partial n_i \partial n_j}\right) * \left(\frac{\partial^2 \Pi_j^w}{\partial n_i \partial n_j}\right) \quad (\text{A.3.12})$$

First part of the inequality yields a result that is bigger than 1 given the condition that  $m > 1$ . Right part of the inequality is equal to  $\frac{4165}{10368}$ . The stability condition is satisfied.

### 1st Stage

In this stage, the upstream firm chooses its quality investment level. Substituting the equilibrium outputs, equilibrium wholesale price and equilibrium downstream investment levels into the profit function of the upstream firm, we obtain:

$$\Pi_u^d = \frac{192m^2(9(-3+4m)(1+q) + (-17+24m)y\beta_j + 2(-5+6m)y\beta_i)^2}{(84+6m(-163+192m))^2} \quad (\text{A.3.13})$$

Upstream firm maximizes its profit with respect to quality investment level ( $y$ ), so:

$$\frac{\partial \Pi_u^d}{\partial y} = \frac{k(85+6m(-163+192m))^2 - 192m^2(2(-5+6m)\beta_i + (-17+24m)\beta_j)A}{(84+6m(-163+192m))^2} = 0$$

$$A = (9(-3+4m)(1+q) + 2(-5+6m)y\beta_i + (-17+24m)y\beta_j) \quad (\text{A.3.14})$$

If we solve for  $y$  we obtain:

$$y^u = \frac{-1728m^2(-3+4m)(1+q)((-17+24m)\beta_j + 2(-5+6m)\beta_i)}{-k(85+6m(163+192m))^2 + 192m^2((-17+24m)\beta_j + 2(-5+6m)\beta_i)^2} \quad (\text{A.3.15})$$

Second order conditions should be negative for a maximum:

$$\frac{\partial^2 \Pi_u^d}{\partial y^2} = \frac{(-k(85+6m(163+192m))^2 + 192m^2((-17+24m)\beta_j + 2(-5+6m)\beta_i)^2)}{(84+6m(-163+192m))^2} < 0 \quad (\text{A.3.16})$$

Equation (A.3.16) is always smaller than zero when  $m>1$ ,  $k>1$ , and  $1 > \beta_i > \beta_j$ , so second order condition is satisfied.

Equilibrium Outcomes

Upstream investment level:

$$y^u = \frac{-1728m^2(-3+4m)(1+q)((-17+24m)\beta_j + 2(-5+6m)\beta_i)}{-k(85+6m(163+192m))^2 + 192m^2((-17+24m)\beta_j + 2(-5+6m)\beta_i)^2} \quad (\text{A.3.17})$$

Upstream wholesale price:

$$w^u = \frac{144km(-3+4m)(85+6m(-163-192m))(1+q)}{k(85+6m(163+192m))^2 - 192m^2((-17+24m)\beta_j + 2(-5+6m)\beta_i)^2} \quad (\text{A.3.18})$$

Upstream profit:

$$\Pi_u^u = \frac{15552k(3-4m)^2 m^2 (1+q)^2}{k(85+6m(163+192m))^2 - 192m^2((-17+24m)\beta_j + 2(-5+6m)\beta_i)^2} \quad (\text{A.3.19})$$

Firm i's output level:

$$X_i^u = \frac{24m(1+q)(k(-5+6m)(85+6m-(163+192m))+192m^2 A)}{k(85+6m(163+192m))^2 - 192m^2((-17+24m)\beta_j + 2(-5+6m)\beta_i)^2}$$

$$A = (\beta_i - \beta_j)((-17+24m)\beta_j + 2(-5+6m)\beta_i) \quad (\text{A.3.20})$$

Firm j's output level:

$$X_j^u = \frac{12(1+q)(k m(-17+24m)(85+6m(-163+192m))-384m^3 A)}{k(85+6m(163+192m))^2-192m^2((-17+24m)\beta_j+2(-5+6m)\beta_i)^2}$$

$$A = (\beta_i - \beta_j)((-17+24m)\beta_j+2(-5+6m)\beta_i) \quad (\text{A.3.21})$$

Firm i's sales price:

$$P_i^u = \frac{(1+q)(k(85-654m+720m^2)(85+6m(-163+192m))-192m^2(-17+24m)A)}{k(85+6m(163+192m))^2+192m^2((-17+24m)\beta_j+2(-5+6m)\beta_i)^2}$$

$$A = (\beta_i - \beta_j)((-17+24m)\beta_j+2(-5+6m)\beta_i) \quad (\text{A.3.22})$$

Firm j's sales price:

$$P_j^u = \frac{(1+q)(k(85-654m+720m^2)(85+6m(-163+192m))-384m^2(-5+6m)A)}{k(85+6m(163+192m))^2-192m^2((-17+24m)\beta_j+2(-5+6m)\beta_i)^2}$$

$$A = (\beta_i - \beta_j)((-17+24m)\beta_j+2(-5+6m)\beta_i) \quad (\text{A.3.23})$$

Firm i's profit:

$$\Pi_i^u = \frac{m(-289+576m)(1+q)^2(k(-5+6m)(85+6m(-163+192m))+192m^2 A)}{(k(85+6m(163+192m))^2-192m^2((-17+24m)\beta_j+2(-5+6m)\beta_i)^2)^2}$$

$$A = (\beta_i - \beta_j)((-17+24m)\beta_j+2(-5+6m)\beta_i) \quad (\text{A.3.24})$$

Firm j's profit:

$$\Pi_j^u = \frac{m(-25+72m)(1+q)^2(k(-17+24m)(85+6m(-163+192m))-384m^2 A)}{(k(85+6m(163+192m))^2-192m^2((-17+24m)\beta_j+2(-5+6m)\beta_i)^2)^2}$$

$$A = (\beta_i - \beta_j)((-17+24m)\beta_j+2(-5+6m)\beta_i) \quad (\text{A.3.25})$$

Firm i's investment level:

$$n_i^u = \frac{17(1+q)(k(-5+6m)(85+6m)(-163+192m)+192m^2 A)}{k(85+6m(163+192m))^2-192m^2((-17+24m)\beta_j+2(-5+6m)\beta_i)^2}$$

$$A=(\beta_i-\beta_j)((-17+24m)\beta_j+2(-5+6m)\beta_i) \quad (\text{A.3.26})$$

Firm j's investment level:

$$n_j^u = \frac{5(1+q)(k(-17+24m)(85+6m(-163+192m))-384m^2 A)}{k(85+6m(163+192m))^2-192m^2((-17+24m)\beta_j+2(-5+6m)\beta_i)^2}$$

$$A=(\beta_i-\beta_j)((-17+24m)\beta_j+2(-5+6m)\beta_i) \quad (\text{A.3.27})$$

## APPENDIX A.4

```

In[1]:= Remove["Global' *"]

Remove::rmnsm : There are no symbols matching "Global' *". >>

In[2]:= Text[inverse demand for quality differentiated products];
Solve[x1 + x2 == 1 - (p1 - y β1) + q, p1];

In[4]:= Solve[x1 + x2 == 1 - (p2 - y β2) + q, p2];

In[5]:= p1 = 1 + q - x1 - x2 + y β1;
p2 = 1 + q - x1 - x2 + y β2;

In[7]:= Π1 = x1 * (p1 - w + n1) - m n1^2;

In[8]:= Π2 = x2 * (p2 - w + n2) - m n2^2;

In[9]:= Πup = (x1 + x2) * w - k (y)^2;

In[10]:= Text[STAGE 4];

In[11]:= Solve[{D[Π1, x1] == 0, D[Π2, x2] == 0}, {x1, x2}]

Out[11]= {{x1 →  $\frac{1}{3} (1 + q - w + 2 n_1 - n_2 + 2 y \beta_1 - y \beta_2)$ ,
x2 →  $\frac{1}{3} (1 + q - w - n_1 + 2 n_2 - y \beta_1 + 2 y \beta_2)$ }}

In[12]:= Text[second order condition];

In[13]:= FullSimplify[D[D[Π1, x1], x1] ≤ 0, {1 > β1 > β2 > 0, m > 1, k > 1, q > 0}]

Out[13]= True

In[14]:= FullSimplify[D[D[Π2, x2], x2] ≤ 0, {1 > β1 > β2 > 0, m > 1, k > 1, q > 0}]

Out[14]= True

```

```
In[15]:= Text[stability condition];
FullSimplify[D[D[Π1, x1], x1] * D[D[Π2, x2], x2] >
D[D[Π1, x1], x2] * D[D[Π2, x1], x2], {1 > β1 > β2 > 0, m > 1, k > 1, q > 0}]
```

```
Out[16]= True
```

```
In[17]:=
Text[STAGE 4 RESULTS];
```

```
In[18]:= X1c =  $\frac{1}{3} (1 + q - w + 2 n_1 - n_2 + 2 y \beta_1 - y \beta_2)$ ;
X2c =  $\frac{1}{3} (1 + q - w - n_1 + 2 n_2 - y \beta_1 + 2 y \beta_2)$ ;
```

```
In[20]:= Simplify[P1c = p1 /. x1 → X1c /. x2 → X2c];
Simplify[P2c = p2 /. x1 → X1c /. x2 → X2c];
```

```
In[22]:= Simplify[Π1c = Π1 /. p1 → P1c /. x1 → X1c /. x2 → X2c];
Simplify[Π2c = Π2 /. p2 → P2c /. x1 → X1c /. x2 → X2c];
Simplify[Πuc = Πup /. x1 → X1c /. x2 → X2c];
```

```
In[25]:=
Text[STAGE 3];
```

```
In[26]:= Simplify[Solve[D[Πuc, w] == 0, w]]
```

```
Out[26]= {{w →  $\frac{1}{4} (2 + 2 q + n_1 + n_2 + y \beta_1 + y \beta_2)$ }}
```

```
In[27]:= Text[second order condition];
FullSimplify[D[D[Πuc, w], w] ≤ 0, {1 > β1 > β2 > 0, m > 1, k > 1, q > 0}]
```

```
Out[28]= True
```

```
In[29]:=
Text[STAGE 3 RESULTS];
```

```
In[30]:= ww =  $\frac{1}{4} (2 + 2 q + n_1 + n_2 + y \beta_1 + y \beta_2)$ ;
```

```
In[31]:= Simplify[X1w = X1c /. w → ww];
Simplify[X2w = X2c /. w → ww];
```

```
In[33]:= Simplify[P1w = P1c /. x1c → X1w /. x2c → X2w /. w → ww];
Simplify[P2w = P2c /. x1c → X1w /. x2c → X2w /. w → ww];
```

```
In[35]:= Simplify[Π1w = Π1c /. P1c → P1w /. x1c → X1w /. x2c → X2w /. w → ww];
Simplify[Π2w = Π2c /. P2c → P2w /. x1c → X1w /. x2c → X2w /. w → ww];
FullSimplify[Πuw = Πuc /. x1c → X1w /. x2c → X2w /. w → ww];
```

```
In[38]:=
Text[STAGE 2];
```

In[39]:= **Solve**[{**D**[ $\Pi_{1w}$ ,  $n_1$ ] == 0, **D**[ $\Pi_{2w}$ ,  $n_2$ ] == 0}, { $n_1$ ,  $n_2$ }]

$$\text{Out[39]} = \left\{ \left\{ n_1 \rightarrow \frac{7(-7 + 12m - 7q + 12mq - 7y\beta_1 + 42my\beta_1 - 30my\beta_2)}{49 - 588m + 864m^2}, \right. \right. \\ \left. \left. n_2 \rightarrow \frac{7(-7 + 12m - 7q + 12mq - 30my\beta_1 - 7y\beta_2 + 42my\beta_2)}{49 - 588m + 864m^2} \right\} \right\}$$

In[40]:= **Text**[second order condition];

In[41]:= **FullSimplify**[**D**[**D**[ $\Pi_{1w}$ ,  $n_1$ ],  $n_1$ ] ≤ 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]

Out[41]= True

In[42]:= **FullSimplify**[**D**[**D**[ $\Pi_{2w}$ ,  $n_2$ ],  $n_2$ ] ≤ 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]

Out[42]= True

In[43]:= **Text**[stability condition];

**FullSimplify**[**D**[**D**[ $\Pi_{1w}$ ,  $n_1$ ],  $n_1$ ] \* **D**[**D**[ $\Pi_{2w}$ ,  $n_2$ ],  $n_2$ ] > **D**[**D**[ $\Pi_{1w}$ ,  $n_1$ ],  $n_2$ ] \* **D**[**D**[ $\Pi_{2w}$ ,  $n_1$ ],  $n_2$ ], {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]

Out[44]= True

In[45]:=

**Text**[STAGE 2 RESULTS];

$$\text{In[46]} = n_{1d} = \frac{7(-7 + 12m - 7q + 12mq - 7y\beta_1 + 42my\beta_1 - 30my\beta_2)}{49 - 588m + 864m^2}; \\ n_{2d} = \frac{7(-7 + 12m - 7q + 12mq - 30my\beta_1 - 7y\beta_2 + 42my\beta_2)}{49 - 588m + 864m^2};$$

In[48]:= **Simplify**[ $X_{1d} = X_{1w} / . n_1 \rightarrow n_{1d} / . n_2 \rightarrow n_{2d}$ ];  
**Simplify**[ $X_{2d} = X_{2w} / . n_1 \rightarrow n_{1d} / . n_2 \rightarrow n_{2d}$ ];  
**Simplify**[ $w_d = w_w / . n_1 \rightarrow n_{1d} / . n_2 \rightarrow n_{2d}$ ];

In[51]:= **Simplify**[ $P_{1d} = P_{1w} / . X_{1w} \rightarrow X_{1d} / . X_{2w} \rightarrow X_{2d} / . n_1 \rightarrow n_{1d} / . n_2 \rightarrow n_{2d}$ ];  
**Simplify**[ $P_{2d} = P_{2w} / . X_{1w} \rightarrow X_{1d} / . X_{2w} \rightarrow X_{2d} / . n_1 \rightarrow n_{1d} / . n_2 \rightarrow n_{2d}$ ];

In[53]:= **FullSimplify**[  
 $\Pi_{1d} = \Pi_{1w} / . P_{1w} \rightarrow P_{1d} / . X_{1w} \rightarrow X_{1d} / . X_{2w} \rightarrow X_{2d} / . n_1 \rightarrow n_{1d} / . n_2 \rightarrow n_{2d}$ ];  
**FullSimplify**[ $\Pi_{2d} =$   
 $\Pi_{2w} / . P_{2w} \rightarrow P_{2d} / . X_{1w} \rightarrow X_{1d} / . X_{2w} \rightarrow X_{2d} / . n_1 \rightarrow n_{1d} / . n_2 \rightarrow n_{2d}$ ];  
**FullSimplify**[ $\Pi_{ud} = \Pi_{uw} / . X_{1w} \rightarrow X_{1d} / . X_{2w} \rightarrow X_{2d} / . n_1 \rightarrow n_{1d} / . n_2 \rightarrow n_{2d}$ ];

In[56]:=

**Text**[STAGE 1];

In[57]:= **FullSimplify**[**Solve**[**D**[ $\Pi_{ud}$ , y] == 0, y]]

$$\text{Out[57]} = \left\{ \left\{ y \rightarrow -\frac{432m^2(1+q)(\beta_1+\beta_2)}{-k(7-72m)^2+216m^2(\beta_1+\beta_2)^2} \right\} \right\}$$

In[58]:= **Text**[second order condition];

In[59]:= **FullSimplify**[**D**[**D**[ $\Pi_{ud}$ ,  $y$ ],  $y$ ]  $\leq 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[59]= True

In[60]:=

**Text**[**STAGE 1 RESULTS**];

In[61]:= 
$$y_u = -\frac{432 \text{ m}^2 (1 + q) (\beta_1 + \beta_2)}{-k (7 - 72 \text{ m})^2 + 216 \text{ m}^2 (\beta_1 + \beta_2)^2};$$

In[62]:= **Simplify**[ $w_u = w_d /. y \rightarrow y_u$ ];  
**Simplify**[ $x_{1u} = x_{1d} /. y \rightarrow y_u$ ];  
**Simplify**[ $x_{2u} = x_{2d} /. y \rightarrow y_u$ ];  
**Simplify**[ $n_{1u} = n_{1d} /. y \rightarrow y_u$ ];  
**Simplify**[ $n_{2u} = n_{2d} /. y \rightarrow y_u$ ];  
**Simplify**[ $p_{1u} = p_{1d} /. x_{1d} \rightarrow x_{1u} /. x_{2d} \rightarrow x_{2u} /. y \rightarrow y_u$ ];  
**Simplify**[ $p_{2u} = p_{2d} /. x_{1d} \rightarrow x_{1u} /. x_{2d} \rightarrow x_{2u} /. y \rightarrow y_u$ ];  
**Simplify**[ $\Pi_{1u} =$   
 $\Pi_{1d} /. p_{1d} \rightarrow p_{1u} /. x_{1d} \rightarrow x_{1u} /. x_{2d} \rightarrow x_{2u} /. n_{1d} \rightarrow n_{1u} /. n_{2d} \rightarrow n_{2u} /. y \rightarrow y_u$ ];  
**Simplify**[ $\Pi_{2u} = \Pi_{2d} /. p_{2d} \rightarrow p_{2u} /. x_{1d} \rightarrow x_{1u} /. x_{2d} \rightarrow x_{2u} /. n_{1d} \rightarrow n_{1u} /. n_{2d} \rightarrow n_{2u} /.$   
 $y \rightarrow y_u$ ];  
**Simplify**[ $\Pi_{uu} = \Pi_{ud} /. x_{1d} \rightarrow x_{1u} /. x_{2d} \rightarrow x_{2u} /. y \rightarrow y_u$ ];  
**Simplify**[ $x_{\text{Total}} = x_{1u} + x_{2u}$ ];

In[73]:=

In[74]:=

**Text**[**equilibrium**];

In[75]:= **FullSimplify**[ $y_u > 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[75]= True

In[76]:= **FullSimplify**[ $w_u > 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[76]= True

In[77]:= **FullSimplify**[ $x_{1u} > 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[77]= True

In[78]:= **FullSimplify**[ $x_{2u} > 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[78]= True

In[79]:= **FullSimplify**[ $p_{1u} > 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[79]= True

In[80]:= **FullSimplify**[ $p_{2u} > 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[80]= True

In[81]:= **FullSimplify**[ $n_{1u} > 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[81]= True

In[82]:= **FullSimplify**[ $n_{2u} > 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[82]= True

```

In[83]:= FullSimplify[ $\Pi_{uu} > 0, \{1 > \beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0\}$ ]
Out[83]= True

In[84]:= FullSimplify[ $\Pi_{1u} > 0, \{1 > \beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0\}$ ]
Out[84]= True

In[85]:= FullSimplify[ $\Pi_{2u} > 0, \{1 > \beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0\}$ ]
Out[85]= True

In[86]:=
Text[ $\beta_1$  effects];

In[87]:= FullSimplify[D[P1u,  $\beta_1$ ] > 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0$ }]
Out[87]= True

In[88]:= FullSimplify[D[X1u,  $\beta_1$ ] > 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0$ }]
Out[88]= True

In[89]:= FullSimplify[D[ $\Pi_{1u}$ ,  $\beta_1$ ] > 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0$ }]
Out[89]= True

In[90]:= FullSimplify[D[n1u,  $\beta_1$ ] > 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0$ }]
Out[90]= True

In[91]:= FullSimplify[D[P2u,  $\beta_1$ ] < 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0$ }]
Out[91]=  $\beta_2 \left( k (-7 + 48 m) (-7 + 72 m) + 216 m^2 (\beta_1^2 + \beta_2^2) \right) < 24 m \beta_1 \left( k (-7 + 72 m) - 18 m \beta_2^2 \right)$ 

In[92]:= FullSimplify[D[P2u,  $\beta_1$ ] > 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0, 10 \beta_1 \leq 17 \beta_2$ }]
Out[92]= True

In[93]:= FullSimplify[D[P2u,  $\beta_1$ ] < 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0, 10 \beta_1 \geq 23 \beta_2$ }]
Out[93]= True

In[94]:= FullSimplify[D[X2u,  $\beta_1$ ] < 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0$ }]
Out[94]= True

In[95]:= FullSimplify[D[ $\Pi_{2u}$ ,  $\beta_1$ ] < 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0$ }]
Out[95]= True

In[96]:= FullSimplify[D[n2u,  $\beta_1$ ] < 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0$ }]
Out[96]= True

In[97]:= FullSimplify[D[XTotal,  $\beta_1$ ] > 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0$ }]
Out[97]= True

In[98]:= FullSimplify[D[wu,  $\beta_1$ ] > 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0$ }]
Out[98]= True

In[99]:= FullSimplify[D[ $\Pi_{uu}$ ,  $\beta_1$ ] > 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0$ }]
Out[99]= True

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In[100]:= FullSimplify[D[yu, β1] > 0, {1 > β1 > β2 > 0, m > 1, k > 1, q > 0}]
Out[100]= True

In[101]:=
Text[β2 effects];

In[102]:= FullSimplify[D[P1u, β2] > 0, {1 > β1 > β2 > 0, m > 1, k > 1, q > 0}]
Out[102]= True

In[103]:= FullSimplify[D[X1u, β2] < 0, {1 > β1 > β2 > 0, m > 1, k > 1, q > 0}]
Out[103]= 216 m2 β13 + 60 k (7 - 72 m) m β2 +
          432 m2 β12 β2 + β1 (k (-7 + 12 m) (-7 + 72 m) + 216 m2 β22) < 0

In[104]:= FullSimplify[D[X1u, β2] > 0, {1 > β1 > β2 > 0, m > 1, k > 1, q > 0, β1 ≥ 12 β2}]
Out[104]= True

In[105]:= FullSimplify[D[X1u, β2] < 0, {1 > β1 > β2 > 0, m > 1, k > 1, q > 0, 10 β1 ≤ 35 β2}]
Out[105]= True

In[106]:= FullSimplify[D[Π1u, β2] < 0, {1 > β1 > β2 > 0, m > 1, k > 1, q > 0}]
Out[106]= 216 m2 β13 + 60 k (7 - 72 m) m β2 +
          432 m2 β12 β2 + β1 (k (-7 + 12 m) (-7 + 72 m) + 216 m2 β22) < 0

In[107]:= FullSimplify[D[Π1u, β2] < 0, {1 > β1 > β2 > 0, m > 1, k > 1, q > 0, 10 β1 ≤ 35 β2}]
Out[107]= True

In[108]:= FullSimplify[D[Π1u, β2] > 0, {1 > β1 > β2 > 0, m > 1, k > 1, q > 0, β1 ≥ 12 β2}]
Out[108]= True

In[109]:= FullSimplify[D[n1u, β2] < 0, {1 > β1 > β2 > 0, m > 1, k > 1, q > 0}]
Out[109]= 216 m2 β13 + 60 k (7 - 72 m) m β2 +
          432 m2 β12 β2 + β1 (k (-7 + 12 m) (-7 + 72 m) + 216 m2 β22) < 0

In[110]:= FullSimplify[D[n1u, β2] < 0, {1 > β1 > β2 > 0, m > 1, k > 1, q > 0, 10 β1 ≤ 35 β2}]
Out[110]= True

In[111]:= FullSimplify[D[n1u, β2] > 0, {1 > β1 > β2 > 0, m > 1, k > 1, q > 0, β1 ≥ 12 β2}]
Out[111]= True

In[112]:= FullSimplify[D[P2u, β2] > 0, {1 > β1 > β2 > 0, m > 1, k > 1, q > 0}]
Out[112]= True

In[113]:= FullSimplify[D[X2u, β2] > 0, {1 > β1 > β2 > 0, m > 1, k > 1, q > 0}]
Out[113]= True

In[114]:= FullSimplify[D[Π2u, β2] > 0, {1 > β1 > β2 > 0, m > 1, k > 1, q > 0}]
Out[114]= True

In[115]:= FullSimplify[D[n2u, β2] > 0, {1 > β1 > β2 > 0, m > 1, k > 1, q > 0}]
Out[115]= True

```

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In[116]:= FullSimplify[D[XTotal,  $\beta_2$ ] > 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]
Out[116]= True

In[117]:= FullSimplify[D[wu,  $\beta_2$ ] > 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]
Out[117]= True

In[118]:= FullSimplify[D[ $\Pi_{uu}$ ,  $\beta_2$ ] > 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]
Out[118]= True

In[119]:= FullSimplify[D[yu,  $\beta_2$ ] > 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]
Out[119]= True

In[120]:=
Text[k effects];

In[121]:= FullSimplify[D[P1u, k] < 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]
Out[121]= True

In[122]:= FullSimplify[D[X1u, k] < 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]
Out[122]= True

In[123]:= FullSimplify[D[ $\Pi_{1u}$ , k] < 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]
Out[123]= True

In[124]:= FullSimplify[D[n1u, k] < 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]
Out[124]= True

In[125]:= FullSimplify[D[P2u, k] < 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]
Out[125]= 12 m  $\beta_1$  + (7 - 60 m)  $\beta_2$  < 0

In[126]:= FullSimplify[D[P2u, k] > 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0,  $\beta_1 \geq 5 \beta_2$ }]
Out[126]= True

In[127]:= FullSimplify[D[P2u, k] < 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0, 10  $\beta_1 \leq 44 \beta_2$ }]
Out[127]= True

In[128]:= FullSimplify[D[X2u, k] < 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]
Out[128]= 30 m  $\beta_1$  + 7 (1 - 6 m)  $\beta_2$  < 0

In[129]:= FullSimplify[D[X2u, k] < 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0, 10  $\beta_1 \leq 11 \beta_2$ }]
Out[129]= True

In[130]:= FullSimplify[D[X2u, k] > 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0, 5  $\beta_1 \geq 7 \beta_2$ }]
Out[130]= True

In[131]:= FullSimplify[D[ $\Pi_{2u}$ , k] < 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]
Out[131]= 30 m  $\beta_1$  + 7 (1 - 6 m)  $\beta_2$  < 0

In[132]:= FullSimplify[D[n2u, k] < 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]
Out[132]= 30 m  $\beta_1$  + 7 (1 - 6 m)  $\beta_2$  < 0

```

In[133]:= **FullSimplify**[D[X<sub>Total</sub>, k] < 0, {1 > β<sub>1</sub> > β<sub>2</sub> > 0, m > 1, k > 1, q > 0}]

Out[133]= True

In[134]:= **FullSimplify**[D[w<sub>u</sub>, k] < 0, {1 > β<sub>1</sub> > β<sub>2</sub> > 0, m > 1, k > 1, q > 0}]

Out[134]= True

In[135]:= **FullSimplify**[D[π<sub>uu</sub>, k] < 0, {1 > β<sub>1</sub> > β<sub>2</sub> > 0, m > 1, k > 1, q > 0}]

Out[135]= True

In[136]:= **FullSimplify**[D[y<sub>u</sub>, k] < 0, {1 > β<sub>1</sub> > β<sub>2</sub> > 0, m > 1, k > 1, q > 0}]

Out[136]= True

**Text**[m effects];

In[137]:= **FullSimplify**[D[P<sub>1u</sub>, m] > 0, {1 > β<sub>1</sub> > β<sub>2</sub> > 0, m > 1, k > 1, q > 0}]

Out[137]= True

In[138]:= **FullSimplify**[D[X<sub>1u</sub>, m] < 0, {1 > β<sub>1</sub> > β<sub>2</sub> > 0, m > 1, k > 1, q > 0}]

Out[138]= True

In[139]:= **FullSimplify**[D[π<sub>1u</sub>, m] < 0, {1 > β<sub>1</sub> > β<sub>2</sub> > 0, m > 1, k > 1, q > 0}]

Out[139]=  $k^2 (7 - 72 m)^2 (7 - 12 m)^2 (49 + 216 m) <$   
 $432 m^2 (\beta_1 + \beta_2) \left( 108 (49 - 204 m) m^2 \beta_1^3 + 108 (49 - 204 m) m^2 \beta_1^2 \beta_2 + \right.$   
 $2 \beta_1 \left( 7 k (-686 + 9 m (1057 + 12 m (-371 + 552 m))) + 54 m^2 (-49 + 204 m) \beta_2^2 \right) +$   
 $\left. \beta_2 \left( k (2401 - 18 m (3871 + 12 m (-1883 + 3336 m))) + 108 m^2 (-49 + 204 m) \beta_2^2 \right) \right)$

In[140]:= **FullSimplify**[D[π<sub>1u</sub>, m] > 0, {1 > β<sub>1</sub> > β<sub>2</sub> > 0, m > 1, k ≥ 7, q > 0}]

Out[140]= True

In[141]:= **FullSimplify**[D[π<sub>1u</sub>, m] > 0, {1 > β<sub>1</sub> > β<sub>2</sub> > 0, m > 1, k > 1, q > 0, 18 β<sub>1</sub> ≤ 19 β<sub>2</sub>}]

Out[141]= True

In[142]:= **FullSimplify**[D[π<sub>1u</sub>, m] < 0, {β<sub>1</sub> == 0.8, β<sub>2</sub> == 0.1, m > 1, k == 1.41, q > 0}]

Out[142]= True

In[143]:= **FullSimplify**[D[π<sub>1u</sub>, m] > 0, {β<sub>1</sub> == 0.8, β<sub>2</sub> == 0.1, m > 1, k == 3.86, q > 0}]

Out[143]= True

In[144]:= **FullSimplify**[D[n<sub>1u</sub>, m] < 0, {1 > β<sub>1</sub> > β<sub>2</sub> > 0, m > 1, k > 1, q > 0}]

Out[144]= True

In[145]:= **FullSimplify**[D[P<sub>2u</sub>, m] > 0, {1 > β<sub>1</sub> > β<sub>2</sub> > 0, m > 1, k > 1, q > 0}]

Out[145]= True

In[146]:= **FullSimplify**[D[X<sub>2u</sub>, m] < 0, {1 > β<sub>1</sub> > β<sub>2</sub> > 0, m > 1, k > 1, q > 0}]

Out[146]=  $432 m^2 (\beta_1 + \beta_2) \left( -108 m^2 \beta_1^3 - 98 k (1 - 6 m)^2 \beta_2 - 108 m^2 \beta_1^2 \beta_2 + 108 m^2 \beta_2^3 + \right.$   
 $\left. \beta_1 \left( k (49 + 72 m (-14 + 47 m)) + 108 m^2 \beta_2^2 \right) \right) < k^2 (7 - 72 m)^2 (7 - 12 m)^2$

In[147]:= **FullSimplify**[D[X<sub>2u</sub>, m] < 0, {1 > β<sub>1</sub> > β<sub>2</sub> > 0, m > 1, k ≥ 10, q > 0}]

Out[147]= True

In[148]:= **FullSimplify**[**D**[**X**<sub>2u</sub>, **m**] < 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, **m** > 1, **k** > 1, **q** > 0, 13  $\beta_1 \leq 14 \beta_2$ }]

Out[148]= True

In[149]:= **FullSimplify**[**D**[**X**<sub>2u</sub>, **m**] > 0, { $\beta_1 == 0.8$ ,  $\beta_2 == 0.1$ , **m** > 1, **k** == 1.19, **q** > 0}]

Out[149]= True

In[150]:= **FullSimplify**[**D**[**X**<sub>2u</sub>, **m**] < 0, { $\beta_1 == 0.8$ ,  $\beta_2 == 0.1$ , **m** > 1, **k** == 6.21, **q** > 0}]

Out[150]= True

In[151]:= **FullSimplify**[**D**[ $\Pi_{2u}$ , **m**] > 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, **m** > 1, **k** > 1, **q** > 0}]

Out[151]= True

In[152]:= **FullSimplify**[**D**[**n**<sub>2u</sub>, **m**] < 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, **m** > 1, **k** > 1, **q** > 0}]

Out[152]=  $k^2 (7 - 72 m)^2 (7 - 12 m)^2 + 12 m (\beta_1 + \beta_2) (648 m^3 \beta_1^3 + k (343 + 3 m (-1127 + 24 m (49 + 180 m))) \beta_2 + 648 m^3 \beta_1^2 \beta_2 - 648 m^3 \beta_2^3 - 9 m \beta_1 (7 k (-7 + 8 m (-7 + 36 m)) + 72 m^2 \beta_2^2)) > 0$

In[153]:= **FullSimplify**[**D**[**n**<sub>2u</sub>, **m**] < 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, **m** > 1, **k** ≥ 2, **q** > 0}]

Out[153]= True

In[154]:= **FullSimplify**[**D**[**n**<sub>2u</sub>, **m**] < 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, **m** > 1, **k** > 1, **q** > 0, 2  $\beta_1 \leq 3 \beta_2$ }]

Out[154]= True

In[155]:= **FullSimplify**[**D**[**X**<sub>Total</sub>, **m**] < 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, **m** > 1, **k** > 1, **q** > 0}]

Out[155]= True

In[156]:= **FullSimplify**[**D**[**w**<sub>u</sub>, **m**] < 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, **m** > 1, **k** > 1, **q** > 0}]

Out[156]= True

In[157]:= **FullSimplify**[**D**[ $\Pi_{uu}$ , **m**] < 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, **m** > 1, **k** > 1, **q** > 0}]

Out[157]= True

In[158]:= **FullSimplify**[**D**[**y**<sub>u</sub>, **m**] < 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, **m** > 1, **k** > 1, **q** > 0}]

Out[158]= True



## APPENDIX A.5

```

In[1]:= Remove["Global' *"]

Remove::rmnsm : There are no symbols matching "Global' *". >>

In[2]:= Text[inverse demand for quality differentiated products];
Solve[x1 + x2 == 1 - (p1 - y β1) + q, p1]

Out[3]= {{p1 → 1 + q - x1 - x2 + y β1}}

In[4]:= Solve[x1 + x2 == 1 - (p2 - y β2) + q, p2]

Out[4]= {{p2 → 1 + q - x1 - x2 + y β2}}

In[5]:= p1 = 1 + q - x1 - x2 + y β1;
p2 = 1 + q - x1 - x2 + y β2;

In[7]:= Π1 = x1 * (p1 - w + n1) - m n1^2;

In[8]:= Π2 = x2 * (p2 - w + n2) - m n2^2;

In[9]:= Πup = (x1 + x2) * w - k (y)^2;

In[10]:= Text[STAGE 4];

In[11]:= Solve[{D[Π2, x2] == 0}, {x2}]

Out[11]= {{x2 →  $\frac{1}{2} (1 + q - w + n_2 - x_1 + y \beta_2)$ }}

In[12]:= Text[second order condition];

In[13]:= FullSimplify[D[D[Π2, x2], x2] ≤ 0, {1 > β1 > β2 > 0, m > 1, k > 1, q > 0}]

Out[13]= True

In[14]:= x2a =  $\frac{1}{2} (1 + q - w + n_2 - x_1 + y \beta_2)$ ;

In[15]:= Simplify[p1a = p1 /. x2 → x2a];
Simplify[Π1a = Π1 /. p1 → p1a];

In[17]:= Solve[{D[Π1a, x1] == 0}, {x1}]

Out[17]= {{x1 →  $\frac{1}{2} (1 + q - w + 2 n_1 - n_2 + 2 y \beta_1 - y \beta_2)$ }}

In[18]:= FullSimplify[D[D[Π1, x1], x1] ≤ 0, {1 > β1 > β2 > 0, m > 1, k > 1, q > 0}]

Out[18]= True

```

In[19]:= **Text[stability condition];**  
**FullSimplify[D[D[ $\Pi_1$ ,  $x_1$ ],  $x_1$ ] \* D[D[ $\Pi_2$ ,  $x_2$ ],  $x_2$ ] > D[D[ $\Pi_1$ ,  $x_1$ ],  $x_2$ ] \* D[D[ $\Pi_2$ ,  $x_1$ ],  $x_2$ ],**  
**{1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]**

Out[20]= True

In[21]:= **Text[STAGE 4 RESULTS];**

In[22]:=  $x_{1s} = \frac{1}{2} (1 + q - w + 2 n_1 - n_2 + 2 y \beta_1 - y \beta_2);$   
**Simplify[ $x_{2s} = x_{2a} /. x_1 \rightarrow x_{1s}$ ];**

In[24]:= **Simplify[ $P_{1s} = p_1 /. x_1 \rightarrow x_{1s} /. x_2 \rightarrow x_{2s}$ ];**  
**Simplify[ $P_{2s} = p_2 /. x_1 \rightarrow x_{1s} /. x_2 \rightarrow x_{2s}$ ];**

In[26]:= **Simplify[ $\Pi_{1s} = \Pi_1 /. p_1 \rightarrow P_{1s} /. x_1 \rightarrow x_{1s} /. x_2 \rightarrow x_{2s}$ ];**  
**Simplify[ $\Pi_{2s} = \Pi_2 /. p_2 \rightarrow P_{2s} /. x_1 \rightarrow x_{1s} /. x_2 \rightarrow x_{2s}$ ];**  
**Simplify[ $\Pi_{us} = \Pi_{up} /. x_1 \rightarrow x_{1s} /. x_2 \rightarrow x_{2s}$ ];**

In[29]:= **Text[STAGE 3];**

In[30]:= **Simplify[Solve[D[ $\Pi_{us}$ , w] == 0, w]]**

Out[30]=  $\left\{ \left\{ w \rightarrow \frac{1}{6} (3 + 3 q + 2 n_1 + n_2 + 2 y \beta_1 + y \beta_2) \right\} \right\}$

In[31]:= **Text[second order condition];**  
**FullSimplify[D[D[ $\Pi_{us}$ , w], w] ≤ 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]**

Out[32]= True

In[33]:= **Text[STAGE 3 RESULTS];**

In[34]:=  $w = \frac{1}{6} (3 + 3 q + 2 n_1 + n_2 + 2 y \beta_1 + y \beta_2);$

In[35]:= **Simplify[ $x_{1w} = x_{1s} /. w \rightarrow w_w$ ];**  
**Simplify[ $x_{2w} = x_{2s} /. w \rightarrow w_w$ ];**

In[37]:= **Simplify[ $P_{1w} = P_{1s} /. x_{1s} \rightarrow x_{1w} /. x_{2s} \rightarrow x_{2w} /. w \rightarrow w_w$ ];**  
**Simplify[ $P_{2w} = P_{2s} /. x_{1s} \rightarrow x_{1w} /. x_{2s} \rightarrow x_{2w} /. w \rightarrow w_w$ ];**

In[39]:= **FullSimplify[ $\Pi_{1w} = \Pi_{1s} /. P_{1s} \rightarrow P_{1w} /. x_{1s} \rightarrow x_{1w} /. x_{2s} \rightarrow x_{2w} /. w \rightarrow w_w$ ];**  
**FullSimplify[ $\Pi_{2w} = \Pi_{2s} /. P_{2s} \rightarrow P_{2w} /. x_{1s} \rightarrow x_{1w} /. x_{2s} \rightarrow x_{2w} /. w \rightarrow w_w$ ];**  
**FullSimplify[ $\Pi_{uw} = \Pi_{us} /. x_{1s} \rightarrow x_{1w} /. x_{2s} \rightarrow x_{2w} /. w \rightarrow w_w$ ];**

In[42]:= **Text[STAGE 2];**

In[43]:= **Solve[{D[ $\Pi_{1w}$ ,  $n_1$ ] == 0, D[ $\Pi_{2w}$ ,  $n_2$ ] == 0}, { $n_1$ ,  $n_2$ }]**

Out[43]=  $\left\{ \left\{ n_1 \rightarrow \frac{5 (-17 + 24 m - 17 q + 24 m q - 17 y \beta_1 + 80 m y \beta_1 - 56 m y \beta_2)}{85 - 978 m + 1152 m^2}, \right. \right.$   
 $\left. n_2 \rightarrow \frac{17 (-5 + 6 m - 5 q + 6 m q - 28 m y \beta_1 - 5 y \beta_2 + 34 m y \beta_2)}{85 - 978 m + 1152 m^2} \right\}$

In[44]:= **Text[second order condition];**

In[45]:= **FullSimplify**[**D**[**D**[ $\Pi_{1w}$ ,  $n_1$ ],  $n_1$ ]  $\leq 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[45]= True

In[46]:= **FullSimplify**[**D**[**D**[ $\Pi_{2w}$ ,  $n_2$ ],  $n_2$ ]  $\leq 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[46]= True

In[47]:= **Text**[**stability condition**];  
**FullSimplify**[**D**[**D**[ $\Pi_{1w}$ ,  $n_1$ ],  $n_1$ ] \* **D**[**D**[ $\Pi_{2w}$ ,  $n_2$ ],  $n_2$ ]  $> \mathbf{D}[\mathbf{D}[\Pi_{1w}, n_1], n_2] * \mathbf{D}[\mathbf{D}[\Pi_{2w}, n_1], n_2]$ ,  
{ $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[48]= True

In[49]:= **Text**[**STAGE 2 RESULTS**];

$$\text{In[50]:= } n_{1d} = \frac{5(-17 + 24m - 17q + 24mq - 17y\beta_1 + 80my\beta_1 - 56my\beta_2)}{85 - 978m + 1152m^2};$$

$$n_{2d} = \frac{17(-5 + 6m - 5q + 6mq - 28my\beta_1 - 5y\beta_2 + 34my\beta_2)}{85 - 978m + 1152m^2};$$

In[52]:= **Simplify**[ $X_{1d} = X_{1w} /. n_1 \rightarrow n_{1d} /. n_2 \rightarrow n_{2d}$ ];  
**Simplify**[ $X_{2d} = X_{2w} /. n_1 \rightarrow n_{1d} /. n_2 \rightarrow n_{2d}$ ];  
**Simplify**[ $w_d = w_w /. n_1 \rightarrow n_{1d} /. n_2 \rightarrow n_{2d}$ ];

In[55]:= **Simplify**[ $P_{1d} = P_{1w} /. X_{1w} \rightarrow X_{1d} /. X_{2w} \rightarrow X_{2d} /. n_1 \rightarrow n_{1d} /. n_2 \rightarrow n_{2d}$ ];  
**Simplify**[ $P_{2d} = P_{2w} /. X_{1w} \rightarrow X_{1d} /. X_{2w} \rightarrow X_{2d} /. n_1 \rightarrow n_{1d} /. n_2 \rightarrow n_{2d}$ ];

In[57]:= **Simplify**[ $\Pi_{1d} = \Pi_{1w} /. P_{1w} \rightarrow P_{1d} /. X_{1w} \rightarrow X_{1d} /. X_{2w} \rightarrow X_{2d} /. n_1 \rightarrow n_{1d} /. n_2 \rightarrow n_{2d}$ ];  
**Simplify**[ $\Pi_{2d} = \Pi_{2w} /. P_{2w} \rightarrow P_{2d} /. X_{1w} \rightarrow X_{1d} /. X_{2w} \rightarrow X_{2d} /. n_1 \rightarrow n_{1d} /. n_2 \rightarrow n_{2d}$ ];  
**FullSimplify**[ $\Pi_{ud} = \Pi_{uw} /. X_{1w} \rightarrow X_{1d} /. X_{2w} \rightarrow X_{2d} /. w_w \rightarrow w_d /. n_1 \rightarrow n_{1d} /. n_2 \rightarrow n_{2d}$ ];

In[60]:= **Text**[**STAGE 1**];

In[61]:= **FullSimplify**[**Solve**[**D**[ $\Pi_{ud}$ ,  $y$ ] == 0,  $y$ ]]

$$\text{Out[61]= } \left\{ \left\{ y \rightarrow - \frac{1728m^2(-3 + 4m)(1 + q)((-17 + 24m)\beta_1 + 2(-5 + 6m)\beta_2)}{-k(85 + 6m(-163 + 192m))^2 + 192m^2(((-17 + 24m)\beta_1 + 2(-5 + 6m)\beta_2)^2)} \right\} \right\}$$

In[62]:= **Text**[**second order condition**];

In[63]:= **FullSimplify**[**D**[**D**[ $\Pi_{ud}$ ,  $y$ ],  $y$ ]  $\leq 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[63]= True

In[64]:= **Text**[**STAGE 1 RESULTS**];

$$\text{In[65]:= } y_u = - \frac{1728m^2(-3 + 4m)(1 + q)((-17 + 24m)\beta_1 + 2(-5 + 6m)\beta_2)}{-k(85 + 6m(-163 + 192m))^2 + 192m^2(((-17 + 24m)\beta_1 + 2(-5 + 6m)\beta_2)^2)};$$

In[66]:= **FullSimplify**[ $y_u > 0$ , { $q > 0$ ,  $w > 0$ ,  $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ }]

Out[66]= True

In[67]:= **Simplify**[ $w_u = w_d /. y \rightarrow y_u$ ];  
**Simplify**[ $X_{1u} = X_{1d} /. y \rightarrow y_u$ ];  
**Simplify**[ $X_{2u} = X_{2d} /. y \rightarrow y_u$ ];  
**Simplify**[ $n_{1u} = n_{1d} /. y \rightarrow y_u$ ];  
**Simplify**[ $n_{2u} = n_{2d} /. y \rightarrow y_u$ ];  
**Simplify**[ $P_{1u} = P_{1d} /. X_{1d} \rightarrow X_{1u} /. X_{2d} \rightarrow X_{2u} /. y \rightarrow y_u$ ];  
**Simplify**[ $P_{2u} = P_{2d} /. X_{1d} \rightarrow X_{1u} /. X_{2d} \rightarrow X_{2u} /. y \rightarrow y_u$ ];  
**Simplify**[ $\Pi_{1u} = \Pi_{1d} /. P_{1d} \rightarrow P_{1u} /. X_{1d} \rightarrow X_{1u} /. X_{2d} \rightarrow X_{2u} /. n_{1d} \rightarrow n_{1u} /. n_{2d} \rightarrow n_{2u} /. y \rightarrow y_u$ ];  
**Simplify**[ $\Pi_{2u} = \Pi_{2d} /. P_{2d} \rightarrow P_{2u} /. X_{1d} \rightarrow X_{1u} /. X_{2d} \rightarrow X_{2u} /. n_{1d} \rightarrow n_{1u} /. n_{2d} \rightarrow n_{2u} /. y \rightarrow y_u$ ];  
**Simplify**[ $\Pi_{uu} = \Pi_{ud} /. X_{1d} \rightarrow X_{1u} /. X_{2d} \rightarrow X_{2u} /. y \rightarrow y_u$ ];  
**Simplify**[ $X_{Total} = X_{1u} + X_{2u}$ ];

In[78]:=

In[79]:= **Text**[equilibrium];

In[80]:= **FullSimplify**[ $y_u > 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[80]= True

In[81]:= **FullSimplify**[ $w_u > 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[81]= True

In[82]:= **FullSimplify**[ $X_{1u} > 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[82]= True

In[83]:= **FullSimplify**[ $X_{2u} > 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[83]=  $k (-5 + 6 m) (85 + 6 m (-163 + 192 m)) > 192 m^2 (\beta_1 - \beta_2) ((-17 + 24 m) \beta_1 + 2 (-5 + 6 m) \beta_2)$

In[84]:= **FullSimplify**[ $X_{2u} > 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ ,  $6 \beta_1 \leq 7 \beta_2$ }]

Out[84]= True

In[85]:= **FullSimplify**[ $P_{1u} > 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[85]= True

In[86]:= **FullSimplify**[ $P_{2u} > 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[86]= True

In[87]:= **FullSimplify**[ $n_{1u} > 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[87]= True

In[88]:= **FullSimplify**[ $n_{2u} > 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[88]=  $k (-5 + 6 m) (85 + 6 m (-163 + 192 m)) > 192 m^2 (\beta_1 - \beta_2) ((-17 + 24 m) \beta_1 + 2 (-5 + 6 m) \beta_2)$

In[89]:= **FullSimplify**[ $n_{2u} > 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ ,  $6 \beta_1 \leq 7 \beta_2$ }]

Out[89]= True

In[90]:= **FullSimplify**[ $\Pi_{uu} > 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[90]= True

```

In[91]:= FullSimplify[ $\Pi_{1u} > 0, \{1 > \beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0\}$ ]
Out[91]= True

In[92]:= FullSimplify[ $\Pi_{2u} > 0, \{1 > \beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0\}$ ]
Out[92]=  $\left( k (-5 + 6 m) (85 + 6 m (-163 + 192 m)) - 192 m^2 (\beta_1 - \beta_2) ((-17 + 24 m) \beta_1 + 2 (-5 + 6 m) \beta_2) \right)^2 > 0$ 

In[93]:= FullSimplify[ $\Pi_{2u} > 0, \{1 > \beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0, 6 \beta_1 \leq 7 \beta_2\}$ ]
Out[93]= True

In[94]:=
Text[ $\beta_1$  effects];

In[95]:= FullSimplify[D[P1u,  $\beta_1$ ] > 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0, 6 \beta_1 \leq 7 \beta_2$ }]
Out[95]= True

In[96]:= FullSimplify[D[X1u,  $\beta_1$ ] > 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0, 6 \beta_1 \leq 7 \beta_2$ }]
Out[96]= True

In[97]:= FullSimplify[D[ $\Pi_{1u}$ ,  $\beta_1$ ] > 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0, 6 \beta_1 \leq 7 \beta_2$ }]
Out[97]= True

In[98]:= FullSimplify[D[n1u,  $\beta_1$ ] > 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0, 6 \beta_1 \leq 7 \beta_2$ }]
Out[98]= True

In[99]:= FullSimplify[D[P2u,  $\beta_1$ ] > 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0, 6 \beta_1 \leq 7 \beta_2$ }]
Out[99]= True

In[100]:= FullSimplify[D[X2u,  $\beta_1$ ] < 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0, 6 \beta_1 \leq 7 \beta_2$ }]
Out[100]= True

In[101]:= FullSimplify[D[ $\Pi_{2u}$ ,  $\beta_1$ ] < 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0, 6 \beta_1 \leq 7 \beta_2$ }]
Out[101]= True

In[102]:= FullSimplify[D[n2u,  $\beta_1$ ] < 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0, 6 \beta_1 \leq 7 \beta_2$ }]
Out[102]= True

In[103]:= FullSimplify[D[XTotal,  $\beta_1$ ] > 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0, 6 \beta_1 \leq 7 \beta_2$ }]
Out[103]= True

In[104]:= FullSimplify[D[wu,  $\beta_1$ ] > 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0, 6 \beta_1 \leq 7 \beta_2$ }]
Out[104]= True

In[105]:= FullSimplify[D[ $\Pi_{uu}$ ,  $\beta_1$ ] > 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0, 6 \beta_1 \leq 7 \beta_2$ }]
Out[105]= True

In[106]:= FullSimplify[D[yu,  $\beta_1$ ] > 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0, 6 \beta_1 \leq 7 \beta_2$ }]
Out[106]= True

In[107]:=
Text[ $\beta_2$  effects];

```

```

In[108]:= FullSimplify[D[P1u,  $\beta_2$ ] > 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0, 6  $\beta_1 \leq 7 \beta_2$ }]
Out[108]= True

In[109]:= FullSimplify[D[X1u,  $\beta_2$ ] < 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0, 6  $\beta_1 \leq 7 \beta_2$ }]
Out[109]= True

In[110]:= FullSimplify[D[ $\Pi_{1u}$ ,  $\beta_2$ ] < 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0, 6  $\beta_1 \leq 7 \beta_2$ }]
Out[110]= True

In[111]:= FullSimplify[D[n1u,  $\beta_2$ ] < 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0, 6  $\beta_1 \leq 7 \beta_2$ }]
Out[111]= True

In[112]:= FullSimplify[D[P2u,  $\beta_2$ ] > 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0, 6  $\beta_1 \leq 7 \beta_2$ }]
Out[112]= True

In[113]:= FullSimplify[D[X2u,  $\beta_2$ ] > 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0, 6  $\beta_1 \leq 7 \beta_2$ }]
Out[113]= True

In[114]:= FullSimplify[D[ $\Pi_{2u}$ ,  $\beta_2$ ] > 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0, 6  $\beta_1 \leq 7 \beta_2$ }]
Out[114]= True

In[115]:= FullSimplify[D[n2u,  $\beta_2$ ] > 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0, 6  $\beta_1 \leq 7 \beta_2$ }]
Out[115]= True

In[116]:= FullSimplify[D[XTotal,  $\beta_2$ ] > 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0, 6  $\beta_1 \leq 7 \beta_2$ }]
Out[116]= True

In[117]:= FullSimplify[D[wu,  $\beta_2$ ] > 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0, 6  $\beta_1 \leq 7 \beta_2$ }]
Out[117]= True

In[118]:= FullSimplify[D[ $\Pi_{uu}$ ,  $\beta_2$ ] > 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0, 6  $\beta_1 \leq 7 \beta_2$ }]
Out[118]= True

In[119]:= FullSimplify[D[yu,  $\beta_2$ ] > 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0, 6  $\beta_1 \leq 7 \beta_2$ }]
Out[119]= True

```

## APPENDIX A.6

```

In[1]:= Remove["Global' *"]

Remove::rmnsm : There are no symbols matching "Global' *". >>

In[2]:= Text[inverse demand for quality differentiated products];
Solve[x1 + x2 == 1 - (p1 - y β1) + q, p1];

In[4]:= Solve[x1 + x2 == 1 - (p2 - y β2) + q, p2];

In[5]:= p1 = 1 + q - x1 - x2 + y β1;
p2 = 1 + q - x1 - x2 + y β2;

In[7]:= Π1 = x1 * (p1 - w + n1) - m n1^2;

In[8]:= Π2 = x2 * (p2 - w + n2) - m n2^2;

In[9]:= Πup = (x1 + x2) * w - k (y)^2;

In[10]:=

Text[STAGE 4];

In[11]:= Solve[{D[Π1, x1] == 0}, {x1}]

Out[11]= {{x1 -> 1/2 (1 + q - w + n1 - x2 + y β1)}}

In[12]:= Text[second order condition];

In[13]:= FullSimplify[D[D[Π2, x2], x2] ≤ 0, {q > 0, w > 0, β1 > β2 > 0, m > 0, k > 0}]

Out[13]= True

In[14]:= x1a = 1/2 (1 + q - w + n1 - x2 + y β1);

In[15]:= Simplify[p2a = p2 /. x1 -> x1a];
Simplify[Π2a = Π2 /. p2 -> p2a /. x1 -> x1a];

In[17]:= Solve[{D[Π2a, x2] == 0}, {x2}]

Out[17]= {{x2 -> 1/2 (1 + q - w - n1 + 2 n2 - y β1 + 2 y β2)}}

In[18]:= FullSimplify[D[D[Π1, x1], x1] ≤ 0, {q > 0, w > 0, β1 > β2 > 0, m > 0, k > 0}]

Out[18]= True

```

```
In[19]:= Text[stability condition];
FullSimplify[D[D[Pi1, x1], x1] * D[D[Pi2, x2], x2] > D[D[Pi1, x1], x2] * D[D[Pi2, x1], x2],
{q > 0, w > 0, beta1 > beta2 > 0, m > 0, k > 0}]
```

```
Out[20]= True
```

```
In[21]:=
```

```
Text[STAGE 4 RESULTS];
```

```
In[22]:= X2s = 1/2 (1 + q - w - n1 + 2 n2 - y beta1 + 2 y beta2);
Simplify[X1s = X1a /. x2 -> X2s];
```

```
In[24]:= Simplify[P1s = p1 /. x1 -> X1s /. x2 -> X2s];
Simplify[P2s = p2 /. x1 -> X1s /. x2 -> X2s];
```

```
In[26]:= Simplify[Pi1s = Pi1 /. p1 -> P1s /. x1 -> X1s /. x2 -> X2s];
Simplify[Pi2s = Pi2 /. p2 -> P2s /. x1 -> X1s /. x2 -> X2s];
Simplify[Pius = Piup /. x1 -> X1s /. x2 -> X2s];
```

```
In[29]:=
```

```
Text[STAGE 3];
```

```
In[30]:= Simplify[Solve[D[Pius, w] == 0, w]]
```

```
Out[30]= {{w -> 1/6 (3 + 3 q + n1 + 2 n2 + y beta1 + 2 y beta2)}}
```

```
In[31]:= Text[second order condition];
FullSimplify[D[D[Pius, w], w] <= 0, {1 > beta1 > beta2 > 0, m > 1, k > 1, q > 0}]
```

```
Out[32]= True
```

```
In[33]:=
```

```
Text[STAGE 3 RESULTS];
```

```
In[34]:= w = 1/6 (3 + 3 q + n1 + 2 n2 + y beta1 + 2 y beta2);
```

```
In[35]:= Simplify[X1w = X1s /. w -> w];
Simplify[X2w = X2s /. w -> w];
```

```
In[37]:= Simplify[P1w = P1s /. X1s -> X1w /. X2s -> X2w /. w -> w];
Simplify[P2w = P2s /. X1s -> X1w /. X2s -> X2w /. w -> w];
```

```
In[39]:= FullSimplify[Pi1w = Pi1s /. P1s -> P1w /. X1s -> X1w /. X2s -> X2w /. w -> w];
FullSimplify[Pi2w = Pi2s /. P2s -> P2w /. X1s -> X1w /. X2s -> X2w /. w -> w];
FullSimplify[Piuw = Pius /. X1s -> X1w /. X2s -> X2w /. w -> w];
```

```
In[42]:=
```

```
Text[STAGE 2];
```

In[43]:= **Solve**[{**D**[ $\Pi_{1w}$ ,  $n_1$ ] == 0, **D**[ $\Pi_{2w}$ ,  $n_2$ ] == 0}, { $n_1$ ,  $n_2$ }]

$$\text{Out[43]} = \left\{ \left\{ n_1 \rightarrow \frac{17(-5 + 6m - 5q + 6mq - 5y\beta_1 + 34my\beta_1 - 28my\beta_2)}{85 - 978m + 1152m^2}, \right. \right. \\ \left. \left. n_2 \rightarrow \frac{5(-17 + 24m - 17q + 24mq - 56my\beta_1 - 17y\beta_2 + 80my\beta_2)}{85 - 978m + 1152m^2} \right\} \right\}$$

In[44]:= **Text**[second order condition];

In[45]:= **FullSimplify**[**D**[**D**[ $\Pi_{1w}$ ,  $n_1$ ],  $n_1$ ] ≤ 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]

Out[45]= True

In[46]:= **FullSimplify**[**D**[**D**[ $\Pi_{2w}$ ,  $n_2$ ],  $n_2$ ] ≤ 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]

Out[46]= True

In[47]:= **Text**[stability condition];

**FullSimplify**[**D**[**D**[ $\Pi_{1w}$ ,  $n_1$ ],  $n_1$ ] \* **D**[**D**[ $\Pi_{2w}$ ,  $n_2$ ],  $n_2$ ] > **D**[**D**[ $\Pi_{1w}$ ,  $n_1$ ],  $n_2$ ] \* **D**[**D**[ $\Pi_{2w}$ ,  $n_1$ ],  $n_2$ ], {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]

Out[48]= True

In[49]:=

**Text**[STAGE 2 RESULTS];

$$\text{In[50]} = n_{1d} = \frac{17(-5 + 6m - 5q + 6mq - 5y\beta_1 + 34my\beta_1 - 28my\beta_2)}{85 - 978m + 1152m^2}; \\ n_{2d} = \frac{5(-17 + 24m - 17q + 24mq - 56my\beta_1 - 17y\beta_2 + 80my\beta_2)}{85 - 978m + 1152m^2};$$

In[52]:= **Simplify**[ $X_{1d} = X_{1w} /. n_1 \rightarrow n_{1d} /. n_2 \rightarrow n_{2d}$ ];  
**Simplify**[ $X_{2d} = X_{2w} /. n_1 \rightarrow n_{1d} /. n_2 \rightarrow n_{2d}$ ];  
**Simplify**[ $w_d = w_w /. n_1 \rightarrow n_{1d} /. n_2 \rightarrow n_{2d}$ ];

In[55]:= **Simplify**[ $P_{1d} = P_{1w} /. X_{1w} \rightarrow X_{1d} /. X_{2w} \rightarrow X_{2d} /. n_1 \rightarrow n_{1d} /. n_2 \rightarrow n_{2d}$ ];  
**Simplify**[ $P_{2d} = P_{2w} /. X_{1w} \rightarrow X_{1d} /. X_{2w} \rightarrow X_{2d} /. n_1 \rightarrow n_{1d} /. n_2 \rightarrow n_{2d}$ ];

In[57]:= **Simplify**[ $\Pi_{1d} = \Pi_{1w} /. P_{1w} \rightarrow P_{1d} /. X_{1w} \rightarrow X_{1d} /. X_{2w} \rightarrow X_{2d} /. n_1 \rightarrow n_{1d} /. n_2 \rightarrow n_{2d}$ ];  
**Simplify**[ $\Pi_{2d} = \Pi_{2w} /. P_{2w} \rightarrow P_{2d} /. X_{1w} \rightarrow X_{1d} /. X_{2w} \rightarrow X_{2d} /. n_1 \rightarrow n_{1d} /. n_2 \rightarrow n_{2d}$ ];  
**FullSimplify**[ $\Pi_{ud} = \Pi_{uw} /. X_{1w} \rightarrow X_{1d} /. X_{2w} \rightarrow X_{2d} /. w_w \rightarrow w_d /. n_1 \rightarrow n_{1d} /. n_2 \rightarrow n_{2d}$ ];

In[60]:=

**Text**[STAGE 1];

In[61]:= **FullSimplify**[**Solve**[**D**[ $\Pi_{ud}$ , y] == 0, y]]

$$\text{Out[61]} = \left\{ \left\{ y \rightarrow - \frac{1728m^2(-3 + 4m)(1 + q)(2(-5 + 6m)\beta_1 + (-17 + 24m)\beta_2)}{-k(85 + 6m(-163 + 192m))^2 + 192m^2(2(-5 + 6m)\beta_1 + (-17 + 24m)\beta_2)^2} \right\} \right\}$$

In[62]:= **Text**[second order condition];

In[63]:= **FullSimplify**[**D**[**D**[ $\Pi_{ud}$ ,  $y$ ],  $y$ ]  $\leq 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[63]= True

In[64]:=

**Text**[**STAGE 1 RESULTS**];

In[65]:= 
$$y_u = - \frac{1728 m^2 (-3 + 4 m) (1 + q) (2 (-5 + 6 m) \beta_1 + (-17 + 24 m) \beta_2)}{-k (85 + 6 m (-163 + 192 m))^2 + 192 m^2 (2 (-5 + 6 m) \beta_1 + (-17 + 24 m) \beta_2)^2};$$

In[66]:= **FullSimplify**[ $y_u > 0$ , { $q > 0$ ,  $w > 0$ ,  $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ }]

Out[66]= True

In[67]:= **Simplify**[ $w_u = w_d /. y \rightarrow y_u$ ];  
**Simplify**[ $X_{1u} = X_{1d} /. y \rightarrow y_u$ ];  
**Simplify**[ $X_{2u} = X_{2d} /. y \rightarrow y_u$ ];  
**Simplify**[ $n_{1u} = n_{1d} /. y \rightarrow y_u$ ];  
**Simplify**[ $n_{2u} = n_{2d} /. y \rightarrow y_u$ ];  
**Simplify**[ $P_{1u} = P_{1d} /. X_{1d} \rightarrow X_{1u} /. X_{2d} \rightarrow X_{2u} /. y \rightarrow y_u$ ];  
**Simplify**[ $P_{2u} = P_{2d} /. X_{1d} \rightarrow X_{1u} /. X_{2d} \rightarrow X_{2u} /. y \rightarrow y_u$ ];  
**Simplify**[ $\Pi_{1u} = \Pi_{1d} /. P_{1d} \rightarrow P_{1u} /. X_{1d} \rightarrow X_{1u} /. X_{2d} \rightarrow X_{2u} /. n_{1d} \rightarrow n_{1u} /. n_{2d} \rightarrow n_{2u} /. y \rightarrow y_u$ ];  
**Simplify**[ $\Pi_{2u} = \Pi_{2d} /. P_{2d} \rightarrow P_{2u} /. X_{1d} \rightarrow X_{1u} /. X_{2d} \rightarrow X_{2u} /. n_{1d} \rightarrow n_{1u} /. n_{2d} \rightarrow n_{2u} /. y \rightarrow y_u$ ];  
**Simplify**[ $\Pi_{uu} = \Pi_{ud} /. X_{1d} \rightarrow X_{1u} /. X_{2d} \rightarrow X_{2u} /. y \rightarrow y_u$ ];  
**Simplify**[ $X_{Total} = X_{1u} + X_{2u}$ ];

In[78]:=

In[79]:=

**Text**[**equilibrium**];

In[80]:= **FullSimplify**[ $y_u > 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[80]= True

In[81]:= **FullSimplify**[ $w_u > 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[81]= True

In[82]:= **FullSimplify**[ $X_{1u} > 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[82]= True

In[83]:= **FullSimplify**[ $X_{2u} > 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[83]= True

In[84]:= **FullSimplify**[ $P_{1u} > 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[84]= True

In[85]:= **FullSimplify**[ $P_{2u} > 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[85]= True

In[86]:= **FullSimplify**[ $n_{1u} > 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[86]= True

In[87]:= **FullSimplify**[ $n_{2u} > 0$ , { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]

Out[87]= True

```

In[88]:= FullSimplify[ $\Pi_{uu} > 0, \{1 > \beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0\}$ ]
Out[88]= True

In[89]:= FullSimplify[ $\Pi_{1u} > 0, \{1 > \beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0\}$ ]
Out[89]= True

In[90]:= FullSimplify[ $\Pi_{2u} > 0, \{1 > \beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0\}$ ]
Out[90]= True

In[91]:=
Text[ $\beta_1$  effects];

In[92]:= FullSimplify[D[P1u,  $\beta_1$ ] > 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0$ }]
Out[92]= True

In[93]:= FullSimplify[D[X1u,  $\beta_1$ ] > 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0$ }]
Out[93]= True

In[94]:= FullSimplify[D[ $\Pi_{1u}$ ,  $\beta_1$ ] > 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0$ }]
Out[94]= True

In[95]:= FullSimplify[D[n1u,  $\beta_1$ ] > 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0$ }]
Out[95]= True

In[96]:= FullSimplify[D[P2u,  $\beta_1$ ] > 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0$ }]
Out[96]=  $\beta_2 \left( k (85 + 6 m (-95 + 96 m)) (85 + 6 m (-163 + 192 m)) + 192 m^2 (4 (5 - 6 m)^2 \beta_1^2 + (17 - 24 m)^2 \beta_2^2) \right) > 48 m (-5 + 6 m) \beta_1 \left( k (85 + 6 m (-163 + 192 m)) + 16 (17 - 24 m) m \beta_2^2 \right)$ 

In[97]:= FullSimplify[D[P2u,  $\beta_1$ ] > 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0, 10 \beta_1 \leq 18 \beta_2$ }]
Out[97]= True

In[98]:= FullSimplify[D[P2u,  $\beta_1$ ] < 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0, 10 \beta_1 \geq 23 \beta_2$ }]
Out[98]= True

In[99]:= FullSimplify[D[X2u,  $\beta_1$ ] < 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0$ }]
Out[99]= True

In[100]:= FullSimplify[D[ $\Pi_{2u}$ ,  $\beta_1$ ] < 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0$ }]
Out[100]= True

In[101]:= FullSimplify[D[n2u,  $\beta_1$ ] < 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0$ }]
Out[101]= True

In[102]:= FullSimplify[D[XTotal,  $\beta_1$ ] > 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0$ }]
Out[102]= True

In[103]:= FullSimplify[D[wu,  $\beta_1$ ] > 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0$ }]
Out[103]= True

In[104]:= FullSimplify[D[ $\Pi_{uu}$ ,  $\beta_1$ ] > 0, {1 >  $\beta_1 > \beta_2 > 0, m > 1, k > 1, q > 0$ }]
Out[104]= True

```

In[105]:= **FullSimplify**[**D**[**y<sub>u</sub>**,  **$\beta_1$** ] > 0, {1 >  **$\beta_1$**  >  **$\beta_2$**  > 0, m > 1, k > 1, q > 0}]

Out[105]= True

In[106]:=

**Text**[ **$\beta_2$**  effects];

In[107]:= **FullSimplify**[**D**[**P<sub>1u</sub>**,  **$\beta_2$** ] > 0, {1 >  **$\beta_1$**  >  **$\beta_2$**  > 0, m > 1, k > 1, q > 0}]

Out[107]= True

In[108]:= **FullSimplify**[**D**[**X<sub>1u</sub>**,  **$\beta_2$** ] > 0, {1 >  **$\beta_1$**  >  **$\beta_2$**  > 0, m > 1, k > 1, q > 0}]

Out[108]=  $\beta_1 \left( k \left( 85 - 418 m + 480 m^2 \right) \left( 85 + 6 m \left( -163 + 192 m \right) \right) + 192 m^2 \left( 2 \left( -5 + 6 m \right) \beta_1 + \left( -17 + 24 m \right) \beta_2 \right)^2 \right) > 56 k m \left( -17 + 24 m \right) \left( 85 + 6 m \left( -163 + 192 m \right) \right) \beta_2$

In[109]:= **FullSimplify**[**D**[**X<sub>1u</sub>**,  **$\beta_2$** ] < 0, {1 >  **$\beta_1$**  >  **$\beta_2$**  > 0, m > 1, k > 1, q > 0, 10  **$\beta_1$**  ≤ 23  **$\beta_2$** }]

Out[109]= True

In[110]:= **FullSimplify**[**D**[**X<sub>1u</sub>**,  **$\beta_2$** ] > 0, {1 >  **$\beta_1$**  >  **$\beta_2$**  > 0, m > 1, k > 1, q > 0,  **$\beta_1$**  ≥ 3  **$\beta_2$** }]

Out[110]= True

In[111]:= **FullSimplify**[**D**[ **$\Pi_{1u}$** ,  **$\beta_2$** ] > 0, {1 >  **$\beta_1$**  >  **$\beta_2$**  > 0, m > 1, k > 1, q > 0}]

Out[111]=  $\beta_1 \left( k \left( 85 - 418 m + 480 m^2 \right) \left( 85 + 6 m \left( -163 + 192 m \right) \right) + 192 m^2 \left( 2 \left( -5 + 6 m \right) \beta_1 + \left( -17 + 24 m \right) \beta_2 \right)^2 \right) > 56 k m \left( -17 + 24 m \right) \left( 85 + 6 m \left( -163 + 192 m \right) \right) \beta_2$

In[112]:= **FullSimplify**[**D**[ **$\Pi_{1u}$** ,  **$\beta_2$** ] < 0, {1 >  **$\beta_1$**  >  **$\beta_2$**  > 0, m > 1, k > 1, q > 0, 10  **$\beta_1$**  ≤ 23  **$\beta_2$** }]

Out[112]= True

In[113]:= **FullSimplify**[**D**[ **$\Pi_{1u}$** ,  **$\beta_2$** ] > 0, {1 >  **$\beta_1$**  >  **$\beta_2$**  > 0, m > 1, k > 1, q > 0,  **$\beta_1$**  ≥ 3  **$\beta_2$** }]

Out[113]= True

In[114]:= **FullSimplify**[**D**[**n<sub>1u</sub>**,  **$\beta_2$** ] > 0, {1 >  **$\beta_1$**  >  **$\beta_2$**  > 0, m > 1, k > 1, q > 0}]

Out[114]=  $\beta_1 \left( k \left( 85 - 418 m + 480 m^2 \right) \left( 85 + 6 m \left( -163 + 192 m \right) \right) + 192 m^2 \left( 2 \left( -5 + 6 m \right) \beta_1 + \left( -17 + 24 m \right) \beta_2 \right)^2 \right) > 56 k m \left( -17 + 24 m \right) \left( 85 + 6 m \left( -163 + 192 m \right) \right) \beta_2$

In[115]:= **FullSimplify**[**D**[**n<sub>1u</sub>**,  **$\beta_2$** ] < 0, {1 >  **$\beta_1$**  >  **$\beta_2$**  > 0, m > 1, k > 1, q > 0, 10  **$\beta_1$**  ≤ 23  **$\beta_2$** }]

Out[115]= True

In[116]:= **FullSimplify**[**D**[**n<sub>1u</sub>**,  **$\beta_2$** ] > 0, {1 >  **$\beta_1$**  >  **$\beta_2$**  > 0, m > 1, k > 1, q > 0,  **$\beta_1$**  ≥ 3  **$\beta_2$** }]

Out[116]= True

In[117]:= **FullSimplify**[**D**[**P<sub>2u</sub>**,  **$\beta_2$** ] > 0, {1 >  **$\beta_1$**  >  **$\beta_2$**  > 0, m > 1, k > 1, q > 0}]

Out[117]= True

In[118]:= **FullSimplify**[**D**[**X<sub>2u</sub>**,  **$\beta_2$** ] > 0, {1 >  **$\beta_1$**  >  **$\beta_2$**  > 0, m > 1, k > 1, q > 0}]

Out[118]=  $k \left( -17 + 24 m \right) \left( -17 + 80 m \right) \left( 85 + 6 m \left( -163 + 192 m \right) \right) \beta_2 + \beta_1 \left( -k \left( -85 + 2 m \left( 13 + 96 m \right) \right) \left( 85 + 6 m \left( -163 + 192 m \right) \right) - 192 \left( 17 - 24 m \right)^2 m^2 \beta_2^2 \right) > 768 m^2 \left( -5 + 6 m \right) \beta_1^2 \left( \left( -5 + 6 m \right) \beta_1 + \left( -17 + 24 m \right) \beta_2 \right)$

In[119]:= **FullSimplify**[**D**[**X<sub>2u</sub>**,  **$\beta_2$** ] > 0, {1 >  **$\beta_1$**  >  **$\beta_2$**  > 0, m > 1, k > 1, q > 0,  **$\beta_1$**  ≤ 3  **$\beta_2$** }]

Out[119]= True

In[120]:= **FullSimplify**[**D**[**X**<sub>2u</sub>,  $\beta_2$ ] < 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0,  $\beta_1 \geq 10 \beta_2$ }]

Out[120]= True

In[121]:= **FullSimplify**[**D**[ $\Pi_{2u}$ ,  $\beta_2$ ] > 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]

Out[121]=  $k (-17 + 24 m) (-17 + 80 m) (85 + 6 m (-163 + 192 m)) \beta_2 +$   
 $\beta_1 (-k (-85 + 2 m (13 + 96 m)) (85 + 6 m (-163 + 192 m)) - 192 (17 - 24 m)^2 m^2 \beta_2^2) >$   
 $768 m^2 (-5 + 6 m) \beta_1^2 ((-5 + 6 m) \beta_1 + (-17 + 24 m) \beta_2)$

In[122]:= **FullSimplify**[**D**[ $\Pi_{2u}$ ,  $\beta_2$ ] > 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0,  $\beta_1 \leq 3 \beta_2$ }]

Out[122]= True

In[123]:= **FullSimplify**[**D**[ $\Pi_{2u}$ ,  $\beta_2$ ] < 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0,  $\beta_1 \geq 10 \beta_2$ }]

Out[123]= True

In[124]:= **FullSimplify**[**D**[ $n_{2u}$ ,  $\beta_2$ ] > 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]

Out[124]=  $k (-17 + 24 m) (-17 + 80 m) (85 + 6 m (-163 + 192 m)) \beta_2 +$   
 $\beta_1 (-k (-85 + 2 m (13 + 96 m)) (85 + 6 m (-163 + 192 m)) - 192 (17 - 24 m)^2 m^2 \beta_2^2) >$   
 $768 m^2 (-5 + 6 m) \beta_1^2 ((-5 + 6 m) \beta_1 + (-17 + 24 m) \beta_2)$

In[125]:= **FullSimplify**[**D**[ $n_{2u}$ ,  $\beta_2$ ] > 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0,  $\beta_1 \leq 3 \beta_2$ }]

Out[125]= True

In[126]:= **FullSimplify**[**D**[ $n_{2u}$ ,  $\beta_2$ ] < 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0,  $\beta_1 \geq 10 \beta_2$ }]

Out[126]= True

In[127]:= **FullSimplify**[**D**[**X**<sub>Total</sub>,  $\beta_2$ ] > 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]

Out[127]= True

In[128]:= **FullSimplify**[**D**[**w**<sub>u</sub>,  $\beta_2$ ] > 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]

Out[128]= True

In[129]:= **FullSimplify**[**D**[ $\Pi_{uu}$ ,  $\beta_2$ ] > 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]

Out[129]= True

In[130]:= **FullSimplify**[**D**[**y**<sub>u</sub>,  $\beta_2$ ] > 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]

Out[130]= True

In[131]:= **Text**[k effects];

In[132]:= **FullSimplify**[**D**[**P**<sub>1u</sub>, k] < 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]

Out[132]= True

In[133]:= **FullSimplify**[**D**[**X**<sub>1u</sub>, k] < 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]

Out[133]= True

In[134]:= **FullSimplify**[**D**[ $\Pi_{1u}$ , k] < 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]

Out[134]= True

In[135]:= **FullSimplify**[**D**[ $n_{1u}$ , k] < 0, {1 >  $\beta_1$  >  $\beta_2$  > 0, m > 1, k > 1, q > 0}]

Out[135]= True

In[136]:= **FullSimplify**[**D**[ $X_{2u}$ ,  $k$ ] < 0, { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]  
 Out[136]=  $56 m \beta_1 + (17 - 80 m) \beta_2 < 0$

In[137]:= **FullSimplify**[**D**[ $X_{2u}$ ,  $k$ ] > 0, { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ ,  $2 \beta_1 \geq 3 \beta_2$ }]  
 Out[137]= True

In[138]:= **FullSimplify**[**D**[ $X_{2u}$ ,  $k$ ] < 0, { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ ,  $8 \beta_1 \leq 9 \beta_2$ }]  
 Out[138]= True

In[139]:= **FullSimplify**[**D**[ $\Pi_{2u}$ ,  $k$ ] < 0, { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]  
 Out[139]=  $56 m \beta_1 + (17 - 80 m) \beta_2 < 0$

In[140]:= **FullSimplify**[**D**[ $\Pi_{2u}$ ,  $k$ ] > 0, { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ ,  $2 \beta_1 \geq 3 \beta_2$ }]  
 Out[140]= True

In[141]:= **FullSimplify**[**D**[ $\Pi_{2u}$ ,  $k$ ] < 0, { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ ,  $8 \beta_1 \leq 9 \beta_2$ }]  
 Out[141]= True

In[142]:= **FullSimplify**[**D**[ $n_{2u}$ ,  $k$ ] < 0, { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]  
 Out[142]=  $56 m \beta_1 + (17 - 80 m) \beta_2 < 0$

In[143]:= **FullSimplify**[**D**[ $n_{2u}$ ,  $k$ ] > 0, { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ ,  $2 \beta_1 \geq 3 \beta_2$ }]  
 Out[143]= True

In[144]:= **FullSimplify**[**D**[ $n_{2u}$ ,  $k$ ] < 0, { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ ,  $8 \beta_1 \leq 9 \beta_2$ }]  
 Out[144]= True

In[145]:= **FullSimplify**[**D**[ $P_{2u}$ ,  $k$ ] < 0, { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]  
 Out[145]=  $24 m (-5 + 6 m) \beta_1 + (-85 + 18 (43 - 48 m) m) \beta_2 < 0$

In[146]:= **FullSimplify**[**D**[ $P_{2u}$ ,  $k$ ] > 0, { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ ,  $\beta_1 \geq 8 \beta_2$ }]  
 Out[146]= True

In[147]:= **FullSimplify**[**D**[ $P_{2u}$ ,  $k$ ] < 0, { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ ,  $2 \beta_1 \leq 11 \beta_2$ }]  
 Out[147]= True

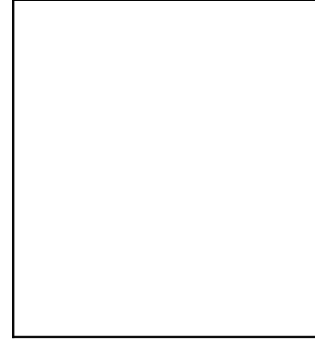
In[148]:= **FullSimplify**[**D**[ $X_{Total}$ ,  $k$ ] < 0, { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]  
 Out[148]= True

In[149]:= **FullSimplify**[**D**[ $w_u$ ,  $k$ ] < 0, { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]  
 Out[149]= True

In[150]:= **FullSimplify**[**D**[ $\Pi_{uu}$ ,  $k$ ] < 0, { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]  
 Out[150]= True

In[151]:= **FullSimplify**[**D**[ $y_u$ ,  $k$ ] < 0, { $1 > \beta_1 > \beta_2 > 0$ ,  $m > 1$ ,  $k > 1$ ,  $q > 0$ }]  
 Out[151]= True

## **CURRICULUM VITA**



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