ISTANBUL TECHNICAL UNIVERSITY ★ INSTITUTE OF SOCIAL SCIENCES

VERTICAL MERGER IN SUPPLY CHAIN: COORDINATION UNDER NASH BARGAINING SOLUTIOIN

M.A. Thesis by

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İSTANBUL TEKNİK ÜNİVERSİTESİ★ SOSYAL BİLİMLER ENSTİTÜSÜ

TEDARİK ZİNCİRİNDE DİKEY BİRLEŞME: NASH PAZARLIK ÇÖZÜMÜ ALTINDA KOORDİNASYON

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Abdulmecit YILDIRIM, a **M.A.** student of ITU **Institute of Social Sciences** student ID 412091012, successfully defended the **thesis/dissertation** entitled “**VERTICAL MERGER IN SUPPLY CHAIN: COORDINATION UNDER NASH BARGAINING SOLUTION”** which he prepared after fulfilling the requirements specified in the associated legislations, before the jury whose signatures are below.

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FOREWORD

I would like to thank my advisors Benan Zeki Orbay and M. Özgür Kayalıca for their help and patience while directing me to scientific study. I am glad to have opportunity to study with my advisors, and teachers at Istanbul Technical University. During my education at ITU I have gained the ability of scientific thinking, and a critical perspective to the events. I have learned how to identify a research problem, making presentation during regular Haydar Kazgan seminars.

I am grateful to my parents for their endless support and love. Without their support I could not successfully accomplished my education.

Istanbul Abdulmecit YILDIRIM

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ABBREVIATIONS

SCM	**:** Supply Chain Management
M	**:** Manufacturer
R	**:** Retailer
C1	**:** First Chain
C2	**:** Second Chain

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VERTICAL MERGER IN SUPPLY CHAIN: COORDINATION UNDER NASH BARGAINING SOLUTIOIN

SUMMARY

In this study, we discuss cooperation decision in two echelon supply chain consisting of two competitive manufacturers and two competitive retailers. We assume that, each manufacturer produces a homogenous price dependent product, and sells the goods through an exclusive retailer to end consumers. The manufacturers also utilize advertising campaign to improve product image, and to get consumer attention. We consider a sequential move game structure where manufacturers decide advertising level in the first stage and wholesale price in the second stage of the game. The competing retailers decide retail price in the final stage of the game. We determine the behaviors of agents in supply chain for a vertical merger. The vertical merger occurred between one manufacturer and one retailer, and we obtain two competing channel after the merger. We assume that the manufacturer in the first chain is more productive than the manufacturer in the second chain. When the firms merged, each manufacturer provides her exclusive retailer with the product at a cost equal to marginal production cost. In order to ensure the coordination among vertically related supply chain members, we use Nash arbitration scheme as a revenue sharing contract. We compare Nash bargaining solution with equal division rule. We find that under Nash arbitration scheme the total profit of chain, and individual profit of each member increase. Moreover, we determine the effect of exogenous variables on the equilibrium values through numerical examples.

TEDARİK ZİNCİRİNDE DİKEY BİRLEŞME: NASH PAZARLIK ÇÖZÜMÜ ALTINDA KOORDİNASYON

ÖZET

Bu çalışmada, iki rakip üretici ve iki rakip perakendeciden oluşan iki aşamalı bir tedarik zincirinde firmaların işbirliği kararlarını inceledik. Her bir üretici homojen bir ürün üretmekte ve bu ürünü kendi perakendecisi vasıtasıyla tüketicilere satmaktadır. Ayrıca, üretici reklam kampanyası yoluyla ürün imajını artırmayı ve tüketicilerin dikkatini çekmeyi amaçlamaktadır. Üreticinin birinci aşamada reklam kararını, kinci aşamada toptan fiyat kararını aldığı ardışık hareketli bir oyun yapısını inceledik. Ardışık karar alma oyunun son aşamasında perakendeci firma ürünün perakende fiyatını belirliyor. Tedarik zincirindeki firmaların dikey birleşme kararlarını belirlemeye çalıştık. Bir üretici ve bir perakendeci arasında oluşan dikey birleşme kararı sonrasında rekabetçi iki tedarik zinciri oluşmaktadır. Birinci zincirdeki üretici ikinci zincirdekinden daha üretkendir. Firmalar dikey olarak birleştiklerinde, her bir üretici ürünü perakendeciye üretim maliyetinden satmaktadır. Dikey olarak ilişkili olan firmaların işbirliğini sağlamak için Nash paylaşım kuralını bir kâr paylaşım kontratı olarak inceledik. Ayrıca, Nash bölüşüm kuralını eşit kâr paylaşım kuralıyla karşılaştırdık. Nash bölüşüm kuralı altında zincirin toplam kârı ve her bir firmanın bireysel kârının arığını bulduk. Ek olarak, dışsal parametrelerin denge değerleri üzerine olan etkilerini nümerik olarak inceledik.

1. INTRODUCTION

Recently, the coordination problem among supply chain members has been studied in many researches. Constant wholesale price, revenue sharing, buyback, price discount sharing, cost sharing are some specific contracts types that has been applied to make the firms in supply chain to work in collaboration. Cachon and Lariviere [2005] and Zhou and Yang [2008] studied a revenue sharing contract where the manufacturer sells the product at a price less than marginal production cost, and the retailer share a percentage of revenue with manufacturer. Cachon and Laiviere [2000] study revenue sharing contract in multi echelon supply chain, and they compare with other coordination contracts. Giannoccaro and Pontrandolfo [2002] study a three stage supply chain consisting of a manufacturer, a distributer, and a retailer. The contract model is characterized by two different contracts: the first contract is offered between distributer and retailer, the second contract is offered between manufacturer and retailer. Leng and Zhu [2009] study a two echelon supply chain with one manufacturer and one retailer. They study revenue sharing contract satisfying Nash arbitration scheme and Shapley value. Li et al. [2008] study a supply chain system with one manufacturer and one retailer. The manufacturer decides output and retail price, and the retailer determines revenue sharing contract under Nash bargaining solution.

Most of the studies about revenue sharing contracts in supply chain management focus on optimum sharing coefficient. In other words, these studies focus on a proportion of sales revenue paid from one player to the other. However, in this study we concern about Nash bargaining scheme as a revenue sharing contract.

In this study our aim is to analyze the behavior of supply chain members under vertical merger. The profit of the agents in the supply chain depends on both own behavior and other member behaviors. We analyze the possible action of channel members under three scenarios. In the first scenario, we investigate the case when all members of the supply chain under decentralized channel system. In the second scenario, we analyze whether the firms in the chain have incentives for merger. Also we investigate the conditions for a profitable merger. In addition to that, we analyze the behavior of rival chain when one of them has incentive for merger. In other words, we investigate the possible action of a chain when rival chain gets more profit with merger. In the third scenario, we analyze the Nash equilibrium of the chains, and share the profit of the channel system according to Nash bargaining solution.

2. LITERATURE REVIEW

2.1 Contracts and Coordination in SCM

Bernstein and Federgruen [2003] consider a two-echelon supply chain consists of one supplier and n-competing retailers. The supplier distributes a single product or closely substitutable products to n-competing retailers who face deterministic price dependent demand function. The price of each retailer that he can charge for his product depends on the all retailers' order quantity. The author first characterizes the solution of centralized system in which all parameters are determined by a single agent. Then, they proceed with the decentralized channel system where the supplier chooses a whole sale pricing scheme, and the retailers determine their policy variables dependent on the supplier strategy. They compare the equilibrium under Cournot and Bertrand competition. Finally, Bernstein and Federgruen [2003] derived a perfect coordination mechanism. Under Cournot competition, for example, the mechanism applies a discount from whole sale price. Under this discount scheme the centralized solution arises as Nash equilibrium. The authors assess the value of coordination within a decentralized system. They consider the Stackelberg game structure in which supplier acts as a leader and retailers as followers. They show that Stackelberg solution often result in major loses in the supply-chain-wide profit.

Lal [1990] construct a model consist of a manufacturer and a retailer. In this model Lal [1990] study the role of franchising arrangement in improving supply chain coordination. He focuses on the use of royalties and monitoring the franchise activities to improve coordination between chain members.

Wanhang [2004] introduces the notion of a rational sharing rule in a cooperative game structure. According to Wanhang [2004] not every coordinating sharing rules are stable. Sometimes, it is possible to find a subset of the players who can do better when not participating in any contracts. However, under the rational contract that is offered by Wanhang [2004] every subsets of the players receive no less than the amount when collaborating on their own.

Horn and Wolinsky [1988] consider an industry consist of two firms that produce either substitutable or complement products, and suppliers of the two firms. They consider two cases regarding the structure of the upstream industry. In one case the supplier are independent of one another and in the other case suppliers merged and acts as a monopoly input supplier. The whole sale price is determined in bargaining between firms and suppliers.

Ziss [1995] construct a model that allows either upstream or downstream merger, and leave the remaining stage intact as a duopoly. The game structure between manufacturer and retailer is as follows; in the first stage the manufacturer simultaneously choose a two part tariff consisting of a wholesale price and a fixed fee, in the second stage the retailers simultaneously choose retail price.

Milliou and Petrakis [2007] consider a model with two upstream and two downstream firms. Their model explores the role of bargaining and contract type in horizontal merger that take place in the upstream sectors of vertically related industries. The game structure is as follows; in the stage one the upstream firms take merger decision. In other words, they decide whether to merge or not. In the stage two, they choose contract types. If the upstream firms decide to merge horizontally, in stage three, monopolist upstream firm bargains with downstream firms separately and simultaneously for the contract terms. They show that if upstream firms prefer to remain separated, two part tariff contracts are always chosen in equilibrium.

Fontenay and Gons [2005] construct a model to analyze vertical integration under upstream competition and monopoly. In their model there are two upstream and two downstream firms. They demonstrate that vertical integration of upstream monopolist creates a higher industry profit then upstream competition.

Klastorion et al. [2002] consider a decentralized two-echelon distribution system consisting of a manufacturer and multiple retailers. In this model a price discount scheme is offered to retailer who places his order which coincides with the beginning of manufacturer production cycle.

Trivedi [1998] constructs a differentiated product model to analyze channel competition at the both manufacturer and retailer levels. In this model Trivedi [1998] investigates the effect of various power structures on equilibrium price and profits. In other words, the model analyzes the market structure when the manufacturer and retailer play the Stackelberg leader respectively.

Xiao and Yang [2008] investigate the competition between two independent supply chains each of which consist of a risk-neutral supplier acting as Stackelberg leader and a risk averse retailer acting as Stackelberg follower. In this game structure the retailers compete in service investment and retail price, and the suppliers compete in wholesale price under demand uncertainty. They found that risk sensitivity/demand uncertainty of one retailer influence the optimal decision of agents in the rival channel. The degree of the service investment of one retailer influences the optimal retail price and service level of his rival.

Not only price, but also some important non-price factors influence the consumers' preferences. Iyer [2000] construct a model to analyze the effort of manufacturer coordinating channel when retailer compete in price, and non-price factors such as the provision of product information, free repair, faster check-out etc.

Esmaeili [2009] construct a model to analyze the interaction among seller and buyer by using non-cooperative and cooperative game theory. In non-cooperative situation seller-Stackelberg and buyer-Stackelberg scenarios are analyzed separately. For the cooperative situation a Pareto efficient solution is provided for the model. They show that selling and marketing expenditure are smaller in cooperative than non-cooperative games.

Dumrangsiri et al. [2008] consider a supply chain model in which the single product is sold to the retailer as well as to the consumer directly by the manufacturer in a single selling period. Consumers are free to choose either the retailer channel or the direct channel, but their decision depends on the price and service quality. In this model the manufacturer decides the price of the direct channel, and the retailer decides the retail price and order quantity. The result show that if the retailer's marginal cost is high , and the wholesale price, consumer valuation and the demand variability is low the centralized dual channel outperforms the centralized single channel.

McGuire and Staelin [1983] study a channel structure with two upstream manufacturers and two downstream retailers. In this supply chain system each of two manufacturers sells its product through a single retailer. The two different manufacturers are either vertically integrates with their retailer or not. A static linear demand and cost function are used. Different game structures are analyzed and the results are compared and contrast. The game structures used in this model are as follow; firstly, the manufacturer sells its product through retailer store to end consumes. Secondly, the manufacturer directly sells its product to end consumers. Thirdly, the manufacturer sells its product through retailer company and directly to end consumers. The results show that consumers are best off when the manufacturer sells its product directly to end consumers. If the manufacturers act in collision profits are greater and retail prices are lower with a pure company store than with privately owned dealers

2.2 Advertisement Affects in SCM

He et al. [2009] investigate a theoretical analysis of co-op advertising plans in a dynamic stochastic supply chain consisting of one manufacturer and one retailer. In their model the manufacturer produces a product and wholesale it to the retailer who then sales the product to end consumers. They use a Stackelberg game structure in which the manufacturer acts as a Stackelberg leader and the retailer acts as a Stackelberg follower. In particular, before the selling season the manufacturer announces a whole sale price and the percentage of retail advertising that it will contribute, then the retailer orders retail quantity. They show that in the absence of coordinated co-op advertising and pricing strategies, the optimal price is always higher in decentralized channel structure, and the advertising amount is below the optimal advertising level.

Li et al. [2002] investigate cooperative and non-cooperative game structure to analyze the local and brand advertising effects on pricing, production and profit of agents. They analyze the system equilibriums under three models. They found that the channel profit is maximized in the cooperative game structure in which the cost of local advertising is shared between the agents. In order to find the best sharing policy, a cooperative bargaining model is constructed by the authors. However, the analysis conducted by Li et al. [2002] depends on the assumption that the manufacturer's marginal profit is large enough. Xie and Ai [2006] extended the result found by Li et al. [2002] to the case where the manufacturer's marginal profit is not large enough.

Huang et al. [2002] consider a single manufacturer and single retailer supply chain in which a single product is produced and sold. They try to determine optimal national advertising and local advertising efforts for the supply chain members. Two different strategies namely partnership and leader-follower strategies are analyzed in terms of national advertising investment, local advertising expenditure, and sharing rules for advertising expenses. They show that the system total profit, national advertising investment, and local advertising spending are higher under partnership strategy than leader follower strategy.

Szmerekovsky and Zhang [2009] consider a model to analyze the pricing and two-tier advertising decisions between a manufacturer and a retailer. In their model the non-linear demand depends on the retail price and advertisement expenses by the manufacturer and the retailer. They consider a game theoretical environment where the manufacturer is more powerful and acting as a Stackelberg leader, and the retailer acting as follower. They show that, sharing the cost of local advertising between manufacturer and retailer is not a good strategy to improve system profit. However, it is better for the system profit if the manufacturer advertises nationally and offers a lower wholesale price to the retailer.

As emphasized in many papers consumers' purchasing decision is influenced not only by product selling price but also by service quality, advertisement etc. Tsay and Agrawal [2000] consider a distribution system where two competing retailers order a common product from a single manufacturer. Then the retailers use service quality and selling price to compete for end users. Tsay and Agrawal [2000] focus on the impact of competition on selling price, total sales, service quality and profitability. They also characterize a wholesale pricing scheme to coordinate the whole system.

One of the main objectives of the firms is to gain profit or promote product as much as possible. In order to reach this aim retailer always offer discount to the consumers. Ping et al. [2010] study a vertical cooperative advertising in a two echelon supply chain system when retailer offer discount to consumers. Firstly they investigate Stackelberg game structure where the manufacturer act as a leader and the retailer acts as follower in a price sensitive market environment. Then, they consider a Nash co-op game structure, and compared and contrast the result found in the two scenarios. They show that cooperative strategy is better than non-cooperative strategy in terms of advertising level, discount, order quantity, and system profit. For the coordination of the supply chain Ping et al. [2010] share the profit between manufacturer and retailer with Nash bargaining model.

2.3 Revenue Sharing Contracts in SCM

Zhou and Yang [2008] construct a multi-echelon supply chain model with a deterministic price-sensitive demand. In their study they propose a supply chain contract model to coordinate pricing decision through revenue sharing mechanism.

Krishnan and Winter [2010] studied a supply chain model consisting of one manufacturer and two competing retailers. The manufacturer distributes the product through retailers in a infinite horizon discrete time period. Their study reexamines revenue-sharing contracts in a vertical supply chain incorporating downstream competition. The retailer ability to carry over inventory from one period to the next extends the traditional static framework to incorporate dynamics one.

Li et al. [2008] construct a model with two-echelon supply chain consisting of one upstream manufacturer and one downstream retailer. They consider the model under the consignment contract with revenue sharing. The market demand is price sensitive and uncertain. In this supply chain the manufacturer produces a single-period product, and sells through retailer to end consumers. The manufacturer decides the delivery quantity and the retail price, and the retailer decides the revenue shares. They show that the optimal retail price is the same as the optimal retail price the centralized supply chain, and is always less than the optimal retail price in a non-cooperative situation. Numerically they found that, in a cooperative situation the supply chain channel supplies more product quantity with a lower retail price. Therefore, the supply chain members and the end consumers obtain more benefits from the cooperation.

Giannoccaro and Pontrondolfo [2004] focus on the supply chain contracts addressing buyer-supplier coordination. Their model based on revenue sharing contract. They propose the model to coordinate a three-level supply chain. Two different contracts are used in the model. The first contract is offered by the distributer to the retailer, and the second contract is offered by the manufacturer to the distributer. They show that the integrated design of two contracts lets the retailer and the distributer select order quantities that are optimal for the whole supply chain.

Wang [2004] construct a model consisting of one supplier and n-retailers in a non-cooperative game structure. The contract in this model based on profit sharing scheme which is decided with the participation of all members rather than one powerful agent.

Cachon and Lariviere [2005] study a single supplier, single retailer supply chain model which includes both deterministic and stochastic demand function. They aim to investigate how revenue sharing contract alters the performance of supply chain. Then, they compare and contrast revenue sharing mechanism to several other contracts, such as, buy-back, price discount, quantity discount, franchise. The revenue of supply chain is determined by retailers' purchase quantity and retail price. They found that the model assures the supply chain coordination, and arbitrarily allocates the chain profit between the agents. Finally the authors extend their model by including competing retailers.

3. THE BASIC MODEL

As we mentioned in the introduction section, we investigate the incentive behaviors of the firms for vertical merger. Our main objective is to analyze the behaviors of the agents for a profitable merger. In our model there are two upstream and two downstream firms. The upstream firms (here after manufacturers, index as M1 and M2) produce a homogenous product, and sell the product to downstream firms (here after retailers index as, R1 and R2) with a whole sale price determined by the manufacturers. The retailers then sell the product to end consumers. The retailers compete with each other by pricing to maximize their profit. The manufacturers also compete by whole sale pricing, and advertising to gain maximum profit depending on the competitive retailers' response. The following figure represents the nature of the supply chain model.

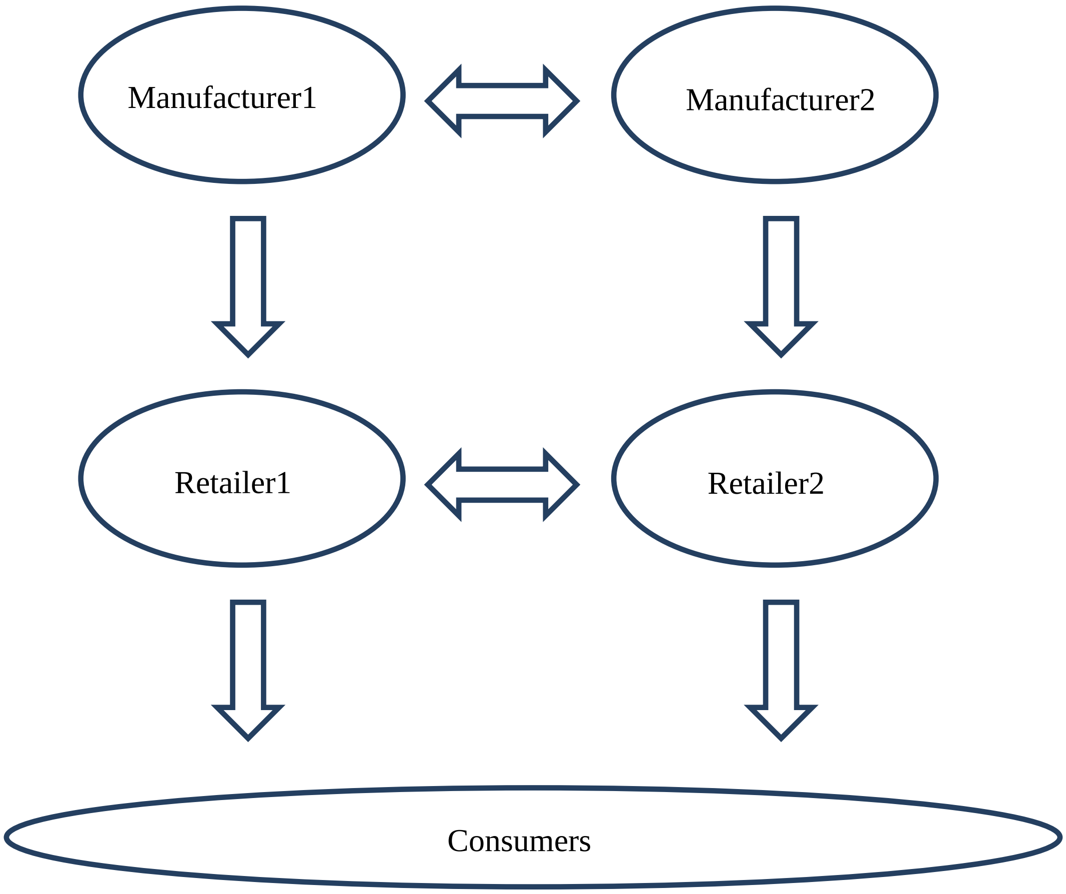

Figure 3.1: Supply chain model structure

Following Leng and Zhu [2009] who proposes $p = a - (q_1 + q_2)$, we use the following deterministic linear price sensitive inverse demand function. Moreover, to analyze the effect of advertising on equilibrium values we add an advertisement term to our inverse demand function, and then we got our inverse demand function: $p = \alpha - \beta(q_1 + q_2) + \theta(A_1 + A_2)$

In our model we use a homogenous product model in which the final product is sold to end consumers with same retail price p. Table 3.1 summarizes the description of parameters.

Table 3.1: Definition of parameters

$\boldsymbol{p}$	Retail price
$\boldsymbol{\alpha}$	Demand intercept
$\boldsymbol{q_i}$	The order quantity of retailer *i, i=1,2*
$\boldsymbol{A_i}$	Advertising expenditure of manufacturer *i, i=1,2*
$\boldsymbol{\beta}$	The demand coefficient
$\boldsymbol{\theta}$	Advertising coefficient
$\boldsymbol{w_i}$	Wholesale price of manufacturer *i* for retailer *i, i=1,2*
$\boldsymbol{\rho}$	The cost of advertising
$\boldsymbol{c_i}$	The marginal production cost of manufacturer *i, i=1,2*

The coefficients β and θ are assumed to be positive constants, and the parameters are assumed to satisfy the following condition $\alpha > c_2 > c_1 > 0$

3.1 Model Analysis: Benchmark Case (No Advertisement A_i = 0)

3.1.1 The Competitive Model

In this section, we will investigate a non-cooperative situation between supply chain agents. In this model both manufacturers and retailers remain separate, and maximize their own profit. We assume that each manufacturer sells its product to only one specific retailer. In other words, the supply chain system that we consider in this part is an exclusive dealer channel [Choi, 1996]. In stage 1, the manufacturers decide their advertisement level. In stage 2, the manufacturers determine a whole sale price for their product. The retailers decide the order quantity in the final stage. The agents of the supply chain compete *a la Cournot*. Figure 3.2 summarizes the game structure for the competitive model. We use backwards induction to find the sub-game perfect Nash Equilibriums.

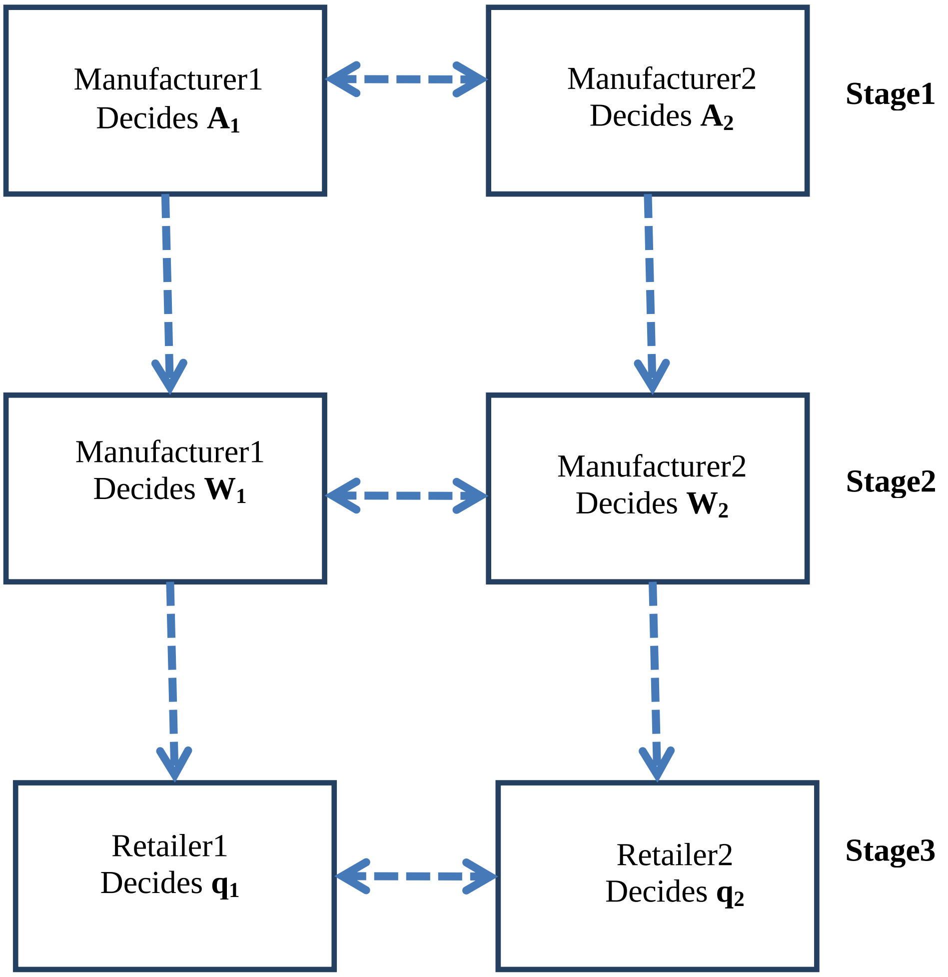

Figure 3.2: The stages of the game

Due to the drop of advertisement term in general inverse demand function, the inverse demand function for this section reduced to:

$$p = \alpha - \beta(q_1 + q_2) \tag{3.1}$$

The profit function of retailers (R1 and R2) is:

$$\pi\text{R1} = (p - w_1)q_1 \tag{3.2}$$

$$\pi\text{R2} = (p - w_2)q_2 \tag{3.3}$$

The retailer1 pays w_1 for the product ordered from manufacturer1, and retailer2 pays w_2 for the product ordered from manufacturer2. Retailers will take into account the behavior of other agents, and make the optimal decision to maximize their own profits.

Stage2

At the third stage we discus retailers how to decide the optimal order quantity. At this stage the retailers decide their order quantity levels that maximize their profit taking the other players actions into consideration. In other words, a retailer can find the optimal decisions by reacting the other retailer's response. The equilibrium quantities found at this stage are a function of wholesale price, and other exogenous parameters. The Cournot equilibrium quantities are found by maximizing the profit function of each retailer. Taking the first order derivative of **(3.2)** and **(3.3)** as shown below,

$$\frac{\partial \pi R1}{\partial q_1} = 0$$

$$\frac{\partial \pi R2}{\partial q_2} = 0$$

We obtain the following reaction functions;

$$q_1 = -\frac{-\alpha + \beta q_2 + w_1}{2\beta} \qquad (\mathbf{3.4})$$

$$q_2 = -\frac{-\alpha + \beta q_1 + w_2}{2\beta} \qquad (\mathbf{3.5})$$

Solving $(\mathbf{3.4})$ and $(\mathbf{3.5})$ simultaneously we get the following Cournot-Nash equilibriums

$$q_1^* = \frac{\alpha - 2w_1 + w_2}{3\beta} \qquad (\mathbf{3.6})$$

$$q_2^* = \frac{\alpha + w_1 - 2w_2}{3\beta} \qquad (\mathbf{3.7})$$

We have to check second order conditions to be sure that the equilibrium quantities that we found in $(\mathbf{3.6})$ and $(\mathbf{3.7})$ maximize the retailers' profit.

$$\frac{\partial^2 \pi R1}{\partial q_1^2} = -2\beta < 0$$

$$\frac{\partial^2 \pi R2}{\partial q_2^2} = -2\beta < 0 \qquad (\mathbf{3.8})$$

Since β is positive the conditions in $(\mathbf{3.8})$ is satisfied. Hence we showed that profit functions of retailers are concave with respect to quantities. Moreover, we must show that the equilibriums are stable and unique. To show the conditions for the uniqueness and stability the following determinant should be hold.

$$\begin{vmatrix} \frac{\partial^2 \pi R1}{\partial q_1^2} & \frac{\partial^2 \pi R1}{\partial q_1 q_2} \\ \frac{\partial^2 \pi R2}{\partial q_2 q_1} & \frac{\partial^2 \pi R2}{\partial q_2^2} \end{vmatrix} > 0 \qquad (\mathbf{3.9})$$

The inequality $(\mathbf{3.9})$ is already satisfied, since the computation of the inequality $(\mathbf{3.9})$ gives $3\beta^2$ which is always greater than zero.

Stage1

In the second stage of the game, the manufacturers choose their wholesale price levels. The profit functions of the manufacturers are given bellow.

$$\pi \mathrm{M1} = (w_1 - c_1)q_1 \qquad (\mathbf{3.10})$$

$$\pi \mathrm{M2} = (w_2 - c_2)q_2 \qquad (\mathbf{3.11})$$

After the substitution of equilibrium quantities q_1^* and q_2^* that we found in stage2 into the equations $(\mathbf{3.10})$ and $(\mathbf{3.11})$ we obtain;

$$\pi \mathrm{M1} = \frac{(-c_1 + w_1)(\alpha - 2w_1 + w_2)}{3\beta} \qquad (\mathbf{3.12})$$

$$\pi \mathrm{M2} = \frac{(-c_2 + w_2)(\alpha + w_1 - 2w_2)}{3\beta} \qquad (\mathbf{3.13})$$

If we set the derivative of $(\mathbf{3.12})$ and $(\mathbf{3.13})$ with respect w_1 and w_2 equal to zero, and solve the equations for the wholesale prices simultaneously, we obtain the optimal wholesale prices that maximize the profits of manufacturers.

$$\frac{\partial \pi M1}{\partial w_1} = 0$$

$$\frac{\partial \pi M2}{\partial w_2} = 0 \qquad (\mathbf{3.14})$$

$$w_1 = \frac{1}{4}(\alpha + 2c_1 + w_2) \qquad (\mathbf{3.15})$$

$$w_2 = \frac{1}{4}(\alpha + 2c_2 + w_1) \qquad (\mathbf{3.16})$$

Solving $(3.\mathbf{14})$ and $(3.\mathbf{15})$ simultaneously we get the following Cournot-Nash equilibriums;

$$w_1^* = \frac{1}{15}(5\alpha + 8c_1 + 2c_2) \qquad (\mathbf{3.17})$$

$$w_2^* = \frac{1}{15}(5\alpha + 2c_1 + 8c_2) \qquad (\mathbf{3.18})$$

For a maximum profit, second order condition should be negative.

$$\frac{\partial^2 \pi M1}{\partial w_1^2} = -\frac{4}{3\beta} < 0 \qquad (\mathbf{3.19})$$

$$\frac{\partial^2 \pi M2}{\partial w_2^2} = -\frac{4}{3\beta} < 0 \qquad (\mathbf{3.20})$$

Equations $(\mathbf{3.19})$ and $(\mathbf{3.20})$ verify that the second order conditions.

After substitution of Nash-Counot equilibrium quantities and wholesale prices into the profit functions **(3.2), (3.3), (3.12),** and **(3.13)** we obtain the following optimal equilibrium profits for manufacturers and retailers.

Table 3.2: Nash-Cournot optimal profits (A=0)

	Profits
π^*M1	$\frac{2(5\alpha - 7c_1 + 2c_2)^2}{675\beta}$
π^*R1	$\frac{4(5\alpha - 7c_1 + 2c_2)^2}{2025\beta}$
π^*M2	$\frac{2(5\alpha + 2c_1 - 7c_2)^2}{675\beta}$
π^*R2	$\frac{4(5\alpha + 2c_1 - 7c_2)^2}{2025\beta}$

3.1.2 The Centralized Model

In the centralized channel structure each manufacturer and her exclusive retailer act together to maximize the channel total profit. In this model, each manufacturer produce a homogenous product with a marginal cost c_i *(i=1, 2)*, and sell the product to the retailer with wholesale price equal to marginal production cost. Then the retailer sells the product to end consumers with a retail price ***p*.** The two chains compete *a la Cournot*. The total system profit is shared between manufacturer and retailer according to Nash bargaining rule.

We use backwards induction to find the sub-game perfect Nash Equilibriums. The profit functions of chain 1 and chain 2 are given in equations $(\mathbf{3.21})$ and $(\mathbf{3.22})$.

$$\pi \mathrm{C1} = (p - c_1)q_1 \tag{3.21}$$

$$\pi \mathrm{C2} = (p - c_2)q_2 \tag{3.22}$$

p is the inverse demand function in $(\mathbf{3.1})$. (Ci indicate: Chain i, i=1,2)

In this stage of the game the retailers compete for quantity level. Differentiating $\pi\mathrm{C1}$ and $\pi\mathrm{C2}$ with respect to q_1 and q_2 respectively, setting the derivatives equal to zero and solving for q_1 and q_2 , we get the following reaction functions for quantity.

$$\mathrm{q}_1 = -\frac{-\alpha + c_1 + \beta q_2}{2\beta} \tag{3.23}$$

$$\mathrm{q}_2 = -\frac{-\alpha + c_2 + \beta q_1}{2\beta} \tag{3.24}$$

If we simultaneously solve the equations $(\mathbf{3.23})$ and $(\mathbf{3.24})$, we get the optimal quantity levels as follows.

$$q_1^* = \frac{\alpha - 2c_1 + c_2}{3\beta} \tag{3.25}$$

$$q_2^* = \frac{\alpha + c_1 - 2c_2}{3\beta} \tag{3.26}$$

In order to be sure about the concavity of profit functions with respect to quantities, we have to check the second order conditions.

$$\frac{\partial^2 \pi C1}{\partial q_1^2} = -2\beta < 0 \tag{3.27}$$

$$\frac{\partial^2 \pi C1}{\partial q_1^2} = -2\beta < 0 \tag{3.28}$$

Since $\beta > 0$ the second order conditions are satisfied. Since the profit functions of the channel 1 and channel 2 are concave in q_1 and q_2, we are sure about profit maximizing level of quantity. In the centralized model, there is no wholesale price, since we assume the members of the each chain collaborate to maximize the system profit. The manufacturers provide retailer with product at a cost equal to marginal production cost. Therefore, in centralized channel structure we have only one decision variable which is quantity sold in the retail market. Clearly the stability conditions are satisfied since the following determinant is hold.

$$\begin{vmatrix} \frac{\partial^2 \pi R1}{\partial q_1^2} & \frac{\partial^2 \pi R1}{\partial q_1 q_2} \\ \frac{\partial^2 \pi R2}{\partial q_2 q_1} & \frac{\partial^2 \pi R2}{\partial q_2^2} \end{vmatrix} = 3\beta^2 > 0$$

Hence the equilibrium Nash-Cournot quantity decision makes the system total profit maximal. After substitution of equilibrium quantities that we found in equation $(\mathbf{3.25})$ and $(\mathbf{3.26})$ into the profit function of chain 1 and chain 2, we obtain the Nash-Cournot equilibrium profits as in $(\mathbf{3.29})$ and $(\mathbf{3.30})$**.**

$$\pi^*\mathrm{C1} = \frac{(\alpha - 2c_1 + c_2)^2}{9\beta} \qquad (\mathbf{3.29})$$

$$\pi^*\mathrm{C2} = \frac{(\alpha + c_1 - 2c_2)^2}{9\beta} \qquad (\mathbf{3.30})$$

3.1.3 The Model Comparison

In this section we will compare the different behaviors of supply chain members. The competitive model and centralized model will be analyzed and compared according to profits. The analysis will be conducted for each chain separately.

Chain1 Analysis

Is merger profitable for supply chain members? For a merger to be profitable, the total profit after the merger should be greater than the total profit of manufacturer and retailer when they make decision independently. Therefore the following inequality must be verified.

$$\pi^*\mathrm{C1} > \pi^*\mathrm{M1} + \pi^*\mathrm{R1} \qquad (\mathbf{3.31})$$

The left hand side of inequality in $(\mathbf{3.31})$ indicates merger profit of the supply chain. The right hand side, on the other hand, indicates the total profit of manufacturer and retailer under the decentralized channel structure. When we replace the corresponding terms into the inequality $(\mathbf{3.31})$, we obtain the inequality in the equation $(\mathbf{3.32})$**.**

$$\frac{(\alpha - 2c_1 + c_2)^2}{9\beta} > \left(\frac{2(5\alpha - 7c_1 + 2c_2)^2}{675\beta} + \frac{4(5\alpha - 7c_1 + 2c_2)^2}{2025\beta}\right) \qquad (\mathbf{3.32})$$

After simplifying the equation $(\mathbf{3.32})$ for α, we get the condition stated in $(\mathbf{3.33})$**.**

$$2c_2 - c_1 < \alpha < 10.7c_2 - 9.7c_1 \qquad (\mathbf{3.33})$$

Chain 2 Analysis

If we conduct the similar analysis for the second chain, we realized that the condition stated in equation ($\mathbf{3.34}$) is not satisfied. That is to say, merger is not profitable for the second chain of the model.

$$\pi^*\mathrm{C2} > \pi^*\mathrm{M2} + \pi^*\mathrm{R2} \qquad (\mathbf{3.34})$$

We reach the conclusion that the total profit of the second chain under competitive model is always greater than the profit under centralized channel structure.

$$\frac{(\alpha + c_1 - 2c_2)^2}{9\beta} < \left(\frac{4(5\alpha - 7c_1 + 2c_2)^2}{2025\beta} + \frac{2(5\alpha + 2c_1 - 7c_2)^2}{675\beta}\right) \qquad (\mathbf{3.35})$$

Proposition 1: If the market size is small enough merger is profitable for chain1, given that the first chain manufacturer has a smaller marginal production cost than the second chain manufacturer.

Proposition 2**:** Merger, acting in collusion, is not profitable for the second chain given that second channel manufacturer has a larger production cost.

Due to the higher production cost chain 2 is not as efficient as chain 1. It is clear that, if the model include only chain 2, then merger would be profitable for the second chain. However, the presence of competition between two chains leads to the condition stated in the proposition 2.

3.1.4 Numerical Analysis

To make the statements clear, we conduct some numerical values to exogenous variables satisfying stability and second order conditions. In our analysis we make the following assumptions.

$$\alpha > c_2 > c_1 > 0, \beta > 0$$

The following numerical values of parameters verify all necessary conditions.

$$c_1 = 6, c_2 = 10, \beta = 1$$

As illustrated in the following figure, as far as $c_1 < c_2$ merger is profitable for the first chain.

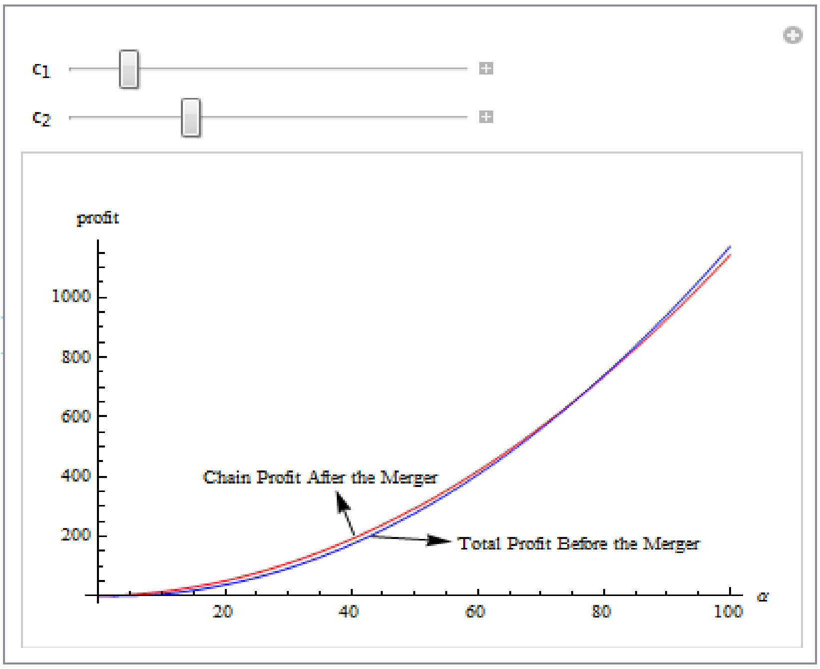

Figure 3.3: Chain 1 profit analysis

On the other hand, chain 2 does not gain any advantage from merger. Figure 3.4 shows that, for any value c_1 less than c_2 the total profit of the manufacturer2 and retailer2 before the merger is greater than the chain profit after the merger.

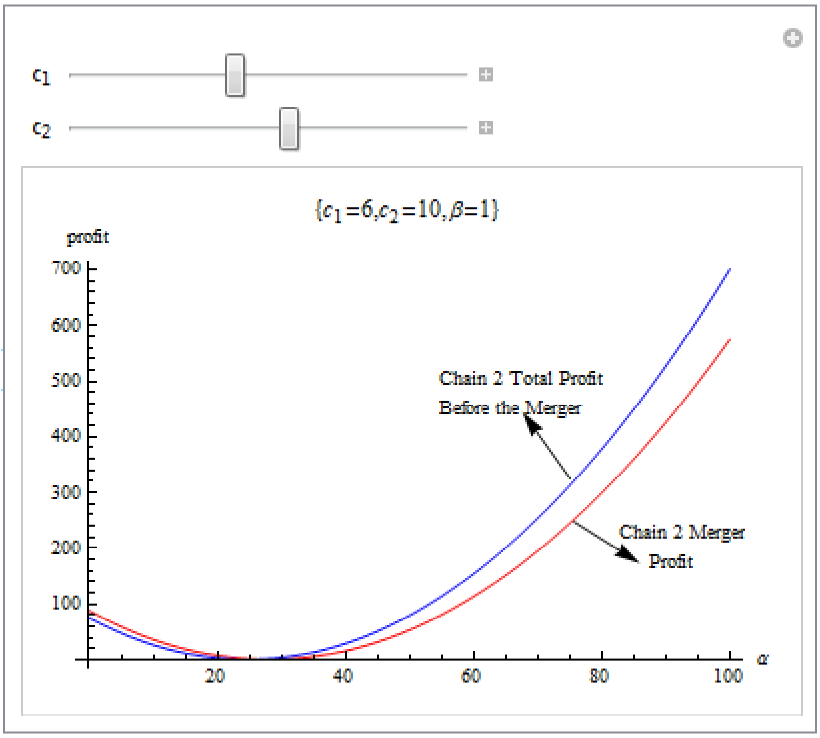

Figure 3.4: Chain 2 profit analysis

At least for some α chain 1 has an incentive for merger, but chain 2 not. Under that condition chain1 members will act together. Put differently, the manufacturer and the retailer in chain 1 will merge for small market sizes. We have shown that merger is not profitable for chain 2. However, if chain 1 merges, then chain 2 will either merge or remain separated. We have already shown the merger decision of the second chain above. Now, we are going to analyze the condition when the chain 2 remains separated.

The merger profit function of chain1 and the independent profits of chain 2 agents are the same as the equation in $(\mathbf{3.21})$, $(\mathbf{3.3})$ and **(3.11)** respectively. Again we use backward induction to find Nash-Cournot equilibriums. In stage 2, chain 1 and retailer 2 decide their profit maximizing quantity level. Taking the derivative of profit functions with respect to quantities, setting equal to zero, and solving the obtained equations simultaneously, we obtain Nash-Cournot equilibrium quantities that make the profit of chain 1 and retailer2 maximal.

$$eq_1 = \frac{\alpha - 2c_1 + w_2}{3\beta} \qquad (\mathbf{3.36})$$

$$eq_2 = \frac{\alpha + c_1 - 2w_2}{3\beta} \qquad (\mathbf{3.37})$$

In stage 2, second order conditions is hold, since $-2\beta < 0$

In stage 1 manufacturer 2 decides wholesale price for retailer 2. After we substitute the Cournot equilibrium quantities stated in the equation $(\mathbf{3.36})$ and $(\mathbf{3.37})$**,** into the profit function $(\mathbf{3.11})$ we obtain the new profit function as shown by the equation $(\mathbf{3.38})$ below.

$$\pi\mathrm{M2} = (w_2 - c_2)eq_2 \qquad (\mathbf{3.38})$$

In order to maximize the profit function in $(\mathbf{3.38})$**,** one take the derivative of the equation with respect to wholesale price, and then solve for w_2. We obtain the following reaction function:

$$\frac{\partial \pi M2}{\partial w_2} = 0$$

$$ew_2 = \frac{1}{4}(\alpha + c_1 + 2c_2) \qquad (\mathbf{3.39})$$

The second order condition for wholesale price level of the manufacturer 2 is equal to:

$$\frac{\partial^2 \pi M1}{\partial w_2^2} = -\frac{4}{3\beta} < 0 \qquad (\mathbf{3.40})$$

Since the second order derivative with respect wholesale price is negative, we can state that the profit function of manufacturer two is concave in wholesale price, and the first order condition provide the profit maximizing level of wholesale price.

After we substitute the equilibrium wholesale price we have found in $(\mathbf{3.39})$ into the equilibrium quantities that stated by the expression in $(\mathbf{3.36})$ and $(\mathbf{3.37})$**,** we obtain the following Nash-Cournot equilibrium quantities.

$$eq_1^* = \frac{5\alpha - 7c_1 + 2c_2}{12\beta} \qquad (\mathbf{3.41})$$

$$eq_2^* = \frac{\alpha + c_1 - 2c_2}{6\beta} \qquad (\mathbf{3.42})$$

After we substitute Nash-Cournot equilibrium quantities and wholesale price into the profit function of chain 1, manufacturer 2 and retailer 2 we obtain the following Nash-Cournot equilibrium profits.

$$\bar{\pi}\mathrm{C1} = \frac{(5\alpha - 7c_1 + 2c_2)^2}{144\beta} \qquad (\mathbf{3.43})$$

$$\bar{\pi}\mathrm{M2} = \frac{(\alpha + c_1 - 2c_2)^2}{24\beta} \qquad (\mathbf{3.44})$$

$$\bar{\pi}\mathrm{R2} = \frac{(\alpha + c_1 - 2c_2)^2}{36\beta} \qquad (\mathbf{3.45})$$

Comparison of Second Chain Agents Decision

The possible reactions of the second chain members for the merger decision of chain 1 are either merger or remain separated. In this section, we will compare the profit of second chain members under two different conditions. In order to show which reaction of chain 2 is a best response to merger decision of chain 1, we will compare the total profit of second chain under two cases (merger and remain separated).

$$\pi^*\mathrm{C2} > \bar{\pi}\mathrm{M2} + \bar{\pi}\mathrm{R2} \qquad (\mathbf{3.46})$$

Lefts hand side of equation $(\mathbf{3.46})$ is the profit of chain 2 when the agents of the chain take merger decision as a response to the merger decision of chain 1. Right hand side of equation $(\mathbf{3.46})$ is the total profit of the second chain, when the members of the chain remain separated. We show that if chain 1 takes merger decision, then the best response reaction for chain 2 is to take merger decision. If we replace the profit functions $(\mathbf{3.30})$, $(\mathbf{3.44})$**,** $(\mathbf{3.45})$ in $(\mathbf{3.46})$ and simplifies the inequality we obtain the following equation which is always true**.**

$$\frac{(\alpha + c_1 - 2c_2)^2}{\beta} > 0 \qquad (\mathbf{3.47})$$

Hence, we prove that at least for some small market sizes chain 1 has incentive for merger. In other words, acting in collusion is a rational behavior for the first chain agents. Therefore, manufacturer 1 and retailer 1 will cooperate. Although the second chain merger profit is smaller than the total profit of the second chain agents in competitive model, it is greater than the total profit of second chain agents when chain 1 merge, but chain 2 remain separated. Therefore, the game will result in merger decision of both chains.

3.2 Inclusion of Advertisement in the Model

In benchmark case we conduct the core analysis of the model without advertisement. In this section we include the advertisement in the model, and we perform a similar analysis. All the assumptions in section 3.1 are also viable in this section. We have examined the behavior of agents in each chain without advertisement. Now, we will investigate the effect of advertisement on members' decision in each chain. In this section we also share the profit between the agents according to Nash bargaining scheme.

3.2.1 The Competitive Model

In this section we investigate the pure decentralized model in which all the agents aim to maximize their own profit. The follow of the game is represented in the figure 3.2 above. As we emphasized in benchmark case the nature of the game is sequential move game. In the first stage of the game, the manufacturers compete to decide optimal advertisement level. In the second stage of the game, the manufacturers determine the wholesale price. In the final stage of the game, the retailers compete to decide quantities which they sold to end consumers. We apply backward induction method in order to find optimal quantities, optimal wholesale price, and optimal advertisement level. Backward induction is a process of reasoning backward in time, from the end of a problem or a situation, to determine a sequence of optimal actions. It proceeds by first considering the last time a decision might be made and choosing what to do in any situation at that time (Gang, 1998). The inverse demand function is;

$$p = \alpha - \beta(q_1 + q_2) + \theta(A_1 + A_2) \quad (\mathbf{3.48})$$

Stage 3: Retailer Determine Output Levels

Taking the rival firm quantity as given, each retailer determines their profit maximizing output level. The equilibrium quantities that we have found at this stage are a function of wholesale prices and advertisements. The profit functions of retailer1 and retailer2 are given in $(\mathbf{3.49})$ and $(\mathbf{3.50})$ respectively.

$$\pi R1 = q_1(\alpha + \theta(A_1 + A_2) - \beta(q_1 + q_2) - w_1) \quad (\mathbf{3.49})$$

$$\pi R2 = q_2(\alpha + \theta(A_1 + A_2) - \beta(q_1 + q_2) - w_2) \quad (\mathbf{3.50})$$

After we simultaneously solve the reaction function that we have found by setting the first order condition equal to zero, we obtain the following profit maximizing quantities for retailer1 and retailer2 respectively.

$$\frac{\partial \pi R1}{\partial q_1} = 0$$

$$\bar{q}_1 = \frac{\alpha + \theta A_1 + \theta A_2 - 2w_1 + w_2}{3\beta} \quad (\mathbf{3.51})$$

$$\frac{\partial \pi R2}{\partial q_2} = 0$$

$$\bar{q}_2 = \frac{\alpha + \theta A_1 + \theta A_2 + w_1 - 2w_2}{3\beta} \quad (\mathbf{3.52})$$

Second order conditions of the profit functions in the equation $(\mathbf{3.49})$and $(\mathbf{3.50})$ equal to -2β which is always negative, since we assume that β is positive. Hence, we can conclude that the profit functions of retailers are concave in q_1 and q_2. The stability condition is satisfied, since the determinant in $(\mathbf{3.53})$ is always satisfied.

$$\begin{vmatrix} \frac{\partial^2 \pi R1}{\partial q_1^2} & \frac{\partial^2 \pi R1}{\partial q_1 q_2} \\ \frac{\partial^2 \pi R2}{\partial q_2 q_1} & \frac{\partial^2 \pi R2}{\partial q_2^2} \end{vmatrix} = 3\beta^2 > 0 \qquad (\mathbf{3.53})$$

Since the stability is a necessary condition for concavity, the equilibrium quantities in ($\mathbf{3.51}$) and ($\mathbf{3.52}$) are unique.

Stage 2: Manufacturers Determine Wholesale Price Level

In this stage of the game, each manufacturer choose her wholesale price level taking the rival wholesale price decision as given. After we substitute the equilibrium quantities we have found in equation ($\mathbf{3.51}$)and ($\mathbf{3.52}$) into the manufacturers' profit functions, we obtain the new profit functions as shown below;

$$\pi \mathrm{M1} = (w_1 - c_1)\bar{q}_1 - \frac{\rho A_1^2}{2} \qquad (\mathbf{3.54})$$

$$\pi \mathrm{M2} = (w_2 - c_2)\bar{q}_2 - \frac{\rho A_2^2}{2} \qquad (\mathbf{3.55})$$

In order to find the optimal wholesale price level, we take the first order derivatives of the equation ($\mathbf{3.54}$) and ($\mathbf{3.55}$) with respect to wholesale prices w_1 and w_2 respectively. Setting the derivatives equal to zero, and solving for w_1 and w_2 we get the following reaction functions;

$$\frac{\partial \pi M1}{\partial w_1} = 0$$

$$w_1 = \frac{1}{4}(\alpha + \theta A_1 + \theta A_2 + 2c_1 + w_2) \qquad (\mathbf{3.56})$$

$$\frac{\partial \pi M2}{\partial w_2} = 0$$

$$w_2 = \frac{1}{4}(\alpha + \theta A_1 + \theta A_2 + 2c_2 + w_1) \qquad (\mathbf{3.57})$$

When we solve the equations ($\mathbf{3.56}$)and ($\mathbf{3.57}$) simultaneously, we get the profit maximizing advertisement level of manufacturers as follows;

$$\overline{\mathrm{w}}_1 = \frac{1}{15}(5\alpha + 5\theta A_1 + 5\theta A_2 + 8c_1 + 2c_2) \qquad (\mathbf{3.58})$$

$$\overline{\mathrm{w}}_2 = \frac{1}{15}(5\alpha + 5\theta A_1 + 5\theta A_2 + 2c_1 + 8c_2) \qquad (\mathbf{3.59})$$

Second order conditions of $(\mathbf{3.54})$and $(\mathbf{3.55})$ are equal to $(-\frac{4}{3\beta})$ which always negative. Therefore, the profit functions of manufacturers are concave in wholesale prices. The stability conditions are tested by the following determinant.

$$\begin{vmatrix} \frac{\partial^2 \pi M1}{\partial w_1^2} & \frac{\partial^2 \pi M1}{\partial w_1 w_2} \\ \frac{\partial^2 \pi M2}{\partial w_2 w_1} & \frac{\partial^2 \pi M2}{\partial w_2^2} \end{vmatrix} = \frac{5}{3\beta^2} > 0 \qquad (\mathbf{3.60})$$

Since the stability and second order conditions are satisfied, we can assure that the optimal wholesale prices can make the manufacturers' profit maximal.

Stage 1: Manufacturers Determine Advertisement Level

In this stage of the game the manufacturers set their advertisement level for their product. The advertisement is very important for a product's retail price, product image, and product competitiveness. Therefore, manufacturers give importance to advertisement. After substituting the optimal wholesale prices that we found in stage 2 into the manufacturers' profit functions, we restructure the profit functions as shown in $(\mathbf{3.61})$ and $(\mathbf{3.62})$

$$\pi\mathrm{M1} = (\overline{w}_1 - c_1)\overline{q}_1 - \frac{\rho A_1^2}{2} \qquad (\mathbf{3.61})$$

$$\pi\mathrm{M2} = (\overline{w}_2 - c_2)\overline{q}_2 - \frac{\rho A_2^2}{2} \qquad (\mathbf{3.62})$$

We find the optimal advertisement level which makes the manufacturers' profit maximal by simultaneously solving the reaction functions that we found by taking the first order derivatives of $(\mathbf{3.61})$ and $(\mathbf{3.62})$ and setting equal to zero.

$$\frac{\partial \pi M1}{\partial A_1} = 0$$

$$\overline{\mathrm{A}}_1 = \frac{4\theta(15\alpha\beta\rho + (4\theta^2 - 21\beta\rho)c_1 + (-4\theta^2 + 6\beta\rho)c_2)}{15\beta\rho(-8\theta^2 + 27\beta\rho)} \qquad (\mathbf{3.63})$$

$$\frac{\partial \pi M2}{\partial A_2} = 0$$

$$\overline{\mathrm{A}}_2 = \frac{4\theta(15\alpha\beta\rho + (-4\theta^2 + 6\beta\rho)c_1 + (4\theta^2 - 21\beta\rho)c_2)}{15\beta\rho(-8\theta^2 + 27\beta\rho)} \qquad (\mathbf{3.64})$$

For concavity of advertisement level the denominator term in parenthesis should be different than zero, that is;

$(-8\theta^2 + 27\beta\rho) \neq 0$

The second order conditions for advertisement level of manufacturers are equal to;

$$\frac{\partial^2 \pi M1}{\partial A_1^2} = \frac{\partial^2 \pi M2}{\partial A_2^2} = \frac{4\theta^2 - 27\beta\rho}{27\beta} \qquad (\mathbf{3.65})$$

We assume that the equation ($\mathbf{3.65}$) is less than zero, in order to provide the profit functions to be concave in advertisement level. Under that condition we can state that the equation in **(3.63)** and **(3.64)** are profit maximizing advertisement levels. To check the stability conditions we calculate the following determinant.

$$\begin{vmatrix} \dfrac{\partial^2 \pi M1}{\partial A_1^2} & \dfrac{\partial^2 \pi M1}{\partial A_1 A_2} \\ \dfrac{\partial^2 \pi M2}{\partial A_2 A_1} & \dfrac{\partial^2 \pi M2}{\partial A_2^2} \end{vmatrix} = \begin{vmatrix} \dfrac{4\theta^2}{27\beta} - \rho & \dfrac{4\theta^2}{27\beta} \\ \dfrac{4\theta^2}{27\beta} & \dfrac{4\theta^2}{27\beta} - \rho \end{vmatrix} = -\frac{8\theta^2\rho}{27\beta} + \rho^2 > 0 \qquad (\mathbf{3.66})$$

In order to ensure the stability condition is hold, $\frac{\beta\rho}{\theta^2} > 0.296$ should be verified.

At the end of this sequential move game, we obtain the following optimal profits illustrated in table 3.3.

Table 3.3: Nash-Cournot optimal profits

	Profits of Agents
π^*M1	$\frac{2(-4\theta^2 + 27\beta\rho)(15\alpha\beta\rho + (4\theta^2 - 21\beta\rho)c_1 + (-4\theta^2 + 6\beta\rho)c_2)^2}{225\beta^2\rho(8\theta^2 - 27\beta\rho)^2}$
π^*R1	$\frac{4(15\alpha\beta\rho + (4\theta^2 - 21\beta\rho)c_1 + (-4\theta^2 + 6\beta\rho)c_2)^2}{25\beta(8\theta^2 - 27\beta\rho)^2}$
π^*M2	$\frac{2(-4\theta^2 + 27\beta\rho)(15\alpha\beta\rho + (-4\theta^2 + 6\beta\rho)c_1 + (4\theta^2 - 21\beta\rho)c_2)^2}{225\beta^2\rho(8\theta^2 - 27\beta\rho)^2}$
π^*R2	$\frac{4(5\alpha + 5\theta A_1 + 5\theta A_2 + 2c_1 - 7c_2)^2}{2025\beta}$

3.2.2 The Centralized Model

This model is a vertically integrated model in which each manufacturer merges with his exclusive retailer. Therefore, we have two competing chains after the merger occurred. In this model C1 indicates first chain, and C2 indicates second chain. The duopolistic vertical chains compete *a la Cournot* in distribution market. The purpose of each chain is to maximize the total channel profit while competing with the second chain for order quantity and advertisement level. Sharing the profit according to Nash bargaining scheme is the main incentive for mergers. That is, the system total profit will be divided between the manufacturer and retailer for each chain. Backward induction is implemented in order to find the Nash-Cournot equilibrium levels. The nature of the game is illustrated in the figure 3.5 below.

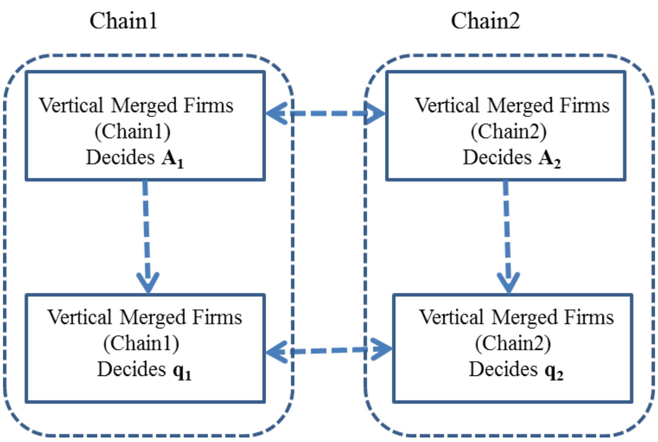

Figure 3.5: Vertical merger

In the vertical merger model the manufacturer sells the product with a price equal to marginal production cost. In other words, there is no wholesale price in this model. The manufacturer and retailer of each chain act together to maximize the channel total profit. The firms in each chain will share the chain's profit according to two rules: Nash Bargaining scheme, and equal division rule. Also, we will show which sharing rule encourages the firms for merger. We will compare the profit gain of each agents of the chain under both division rules with the profit of the agents in competitive model.

We have the following profit function for each chain;

$$\pi C1 = (p - c_1)q_1 - \frac{\rho A_1^2}{2} \tag{3.66}$$

$$\pi C2 = (p - c_2)q_2 - \frac{\rho A_2^2}{2} \quad (\mathbf{3.67})$$

As we see in the profit functions **(3.66)** and $(\mathbf{3.67})$ the vertically merged firms have production and advertising costs. We use a quadratic cost function for advertisement, to ensure the diminishing return of investment.

The game will be solved in two stages by the help of backward induction. In the first stage the chains decide their advertisement level, and in the second stage they choose their output levels.

Stage 2: Each Chain Determine Quantity Level

At the second stage of the game, each chain determines the quantity level taking the rival chain quantity level as given. The equilibriums of this stage are a function of advertisement level.

To find the Nash-Cournot equilibrium quantity decision of each chain, we maximize the profit functions of each chain stated in **(3.66)** and $(\mathbf{3.67})$**.** The first order conditions that we found by taking the first order derivatives of the profit functions are solved for each quantity level. Then we obtain the following Nash-Cournot equilibrium quantities;

$$\frac{\partial \pi C1}{\partial q_1} = 0$$

$$\bar{q}_1^{vm} = \frac{\alpha + \theta A_1 + \theta A_2 - 2c_1 + c_2}{3\beta} \quad (\mathbf{3.68})$$

$$\frac{\partial \pi C2}{\partial q_2} = 0$$

$$\bar{q}_2^{vm} = \frac{\alpha + \theta A_1 + \theta A_2 + c_1 - 2c_2}{3\beta} \quad (\mathbf{3.69})$$

The second order conditions are satisfied since we assume that β is positive.

$$\frac{\partial^2 \pi C1}{\partial q_1^2} = -2\beta$$

$$\frac{\partial^2 \pi C2}{\partial q_2^2} = -2\beta$$

Hence, we can say that the profit functions of vertically merged firms are concave in q_1 and q_2, and the equilibrium quantities makes the chain's profit maximal. The stability conditions of stage 2 is satisfied, since the determinant in $(\mathbf{3.70})$ is always positive. So, the quantities that we have been found by solving first order conditions are unique Nash-Counot equilibriums.

$$\begin{vmatrix} \frac{\partial^2 \pi C1}{\partial q_1^2} & \frac{\partial^2 \pi C1}{\partial q_1 q_2} \\ \frac{\partial^2 \pi C2}{\partial q_2 q_1} & \frac{\partial^2 \pi C2}{\partial q_2^2} \end{vmatrix} = 3\beta^2 > 0 \qquad (\mathbf{3.70})$$

Stage 1: Each Chain Determine Advertisement Level

At the first stage of the game, the manufacturer of each chain determine the equilibrium advertising level taking the rival advertising decision as given. For this stage of the game, we reconstruct the profit functions by substituting the equilibrium quantities in the equations $(\mathbf{3.68})$ and $(\mathbf{3.69})$ into the vertically merged chains' profit functions. Then we obtain the new profit function of chain1 and chain2 respectively as;

$$\pi\mathrm{C1} = (p - c_1)\bar{q}_1^{vm} - \frac{\rho A_1^2}{2} \qquad (\mathbf{3.71})$$

$$\pi\mathrm{C2} = (p - c_2)\bar{q}_2^{vm} - \frac{\rho A_2^2}{2} \qquad (\mathbf{3.72})$$

When we maximize the profit functions of chains by solving the first order conditions with respect to advertising levels, we get the following reaction functions;

$$\frac{\partial \pi C1}{\partial A_1} = 0$$

$$\mathrm{A}_1 = -\frac{2\theta(\alpha + \theta A_2 - 2c_1 + c_2)}{2\theta^2 - 9\beta\rho} \qquad (\mathbf{3.73})$$

$$\frac{\partial \pi C2}{\partial A_2} = 0$$

$$\mathrm{A}_2 = \frac{2\theta(\alpha + \theta A_1 + c_1 - 2c_2)}{-2\theta^2 + 9\beta\rho} \qquad (\mathbf{3.74})$$

When we solve these equations simultaneously, we obtain the Nash-Cournot equilibrium advertising levels that maximize the profit of vertically merged chains as in $(\mathbf{3.75})$ and $(\mathbf{3.76})$;

$$\overline{\mathrm{A}}_1=\frac{2\theta(3\alpha\beta\rho+2(\theta^2-3\beta\rho)c_1+(-2\theta^2+3\beta\rho)c_2)}{3\beta\rho(-4\theta^2+9\beta\rho)} \qquad (\mathbf{3.75})$$

$$\overline{\mathrm{A}}_2=\frac{2\theta(3\alpha\beta\rho+(-2\theta^2+3\beta\rho)c_1+2(\theta^2-3\beta\rho)c_2)}{3\beta\rho(-4\theta^2+9\beta\rho)} \qquad (\mathbf{3.76})$$

We require the denominator terms in parenthesis to be different than zero to assure the concavity of advertising level.

In order to ensure the profit functions of the chains are concave in equilibrium advertising levels, the second order conditions should be verified.

$$\frac{\partial^2\pi C1}{\partial A_1^2}=\frac{2\theta^2}{9\beta}-\rho \qquad (\mathbf{3.77})$$

$$\frac{\partial^2\pi C2}{\partial A_2^2}=\frac{2\theta^2}{9\beta}-\rho \qquad (\mathbf{3.78})$$

The equations in $(\mathbf{3.77})$ and $(\mathbf{3.78})$ are satisfied for some specific values of exogenous variables. Therefore, the profit functions are concave in advertising level, and the first order conditions can make chains' profit maximal.

The stability conditions of advertising level are checked by the following determinant;

$$\begin{vmatrix}\frac{\partial^2\pi C1}{\partial A_1^2} & \frac{\partial^2\pi C1}{\partial A_1A_2}\\ \frac{\partial^2\pi C2}{\partial A_2A_1} & \frac{\partial^2\pi C2}{\partial A_2^2}\end{vmatrix}=-\frac{4\theta^2\rho}{9\beta}+\rho^2>0 \qquad (\mathbf{3.79})$$

The expression in $(\mathbf{3.79})$ is verified if $\frac{\beta\rho}{\theta^2}>\frac{4}{9}$, then we assure the uniqueness of Nash-Cournot equilibriums for advertising levels.

After we replace the equilibrium advertising level into the profit functions $(\mathbf{3.71})$ and $(\mathbf{3.72})$**,** we obtain the optimal profits of both chains in terms of exogenous variables as illustrated in table 3.4 below.

Table 3.4: Nash-Cournot optimal profits under vertical merger

	Vertical Merger Profits
$\pi^*\text{C1}$	$\dfrac{(-2\theta^2+9\beta\rho)(3\alpha\beta\rho+2(\theta^2-3\beta\rho)c_1+(-2\theta^2+3\beta\rho)c_2)^2}{9\beta^2\rho(4\theta^2-9\beta\rho)^2}$
$\pi^*\text{C2}$	$\dfrac{(-2\theta^2+9\beta\rho)(3\alpha\beta\rho+(-2\theta^2+3\beta\rho)c_1+2(\theta^2-3\beta\rho)c_2)^2}{9\beta^2\rho(4\theta^2-9\beta\rho)^2}$

3.2.3 Comparison of the Profits

In this part we will compare the competitive and centralized model profits. As we stated in benchmark case, if a merger is profitable, the merger profit should be greater than the total profit of the agents under competitive case. Therefore, we will investigate whether the profit increase or not when firms vertically merged. For a profitable merger the following condition should be hold for each chain;

$$\pi^*\text{C1} > \pi^*\text{M1} + \pi^*\text{R1} \tag{3.80}$$

$$\pi^*\text{C2} > \pi^*\text{M2} + \pi^*\text{R2} \tag{3.81}$$

The left hand sides of the expression $(\mathbf{3.80})$ and $(\mathbf{3.81})$ is the total merger profit of chain1 and chain 2 respectively. After we replace the corresponding profits into the expression $(\mathbf{3.80})$ and $(\mathbf{3.81})$ we obtain the following inequalities;

$$\frac{(-2\theta^2+9\beta\rho)(3\alpha\beta\rho+2(\theta^2-3\beta\rho)c_1+(-2\theta^2+3\beta\rho)c_2)^2}{9\beta^2\rho(4\theta^2-9\beta\rho)^2} > \frac{2(-4\theta^2+27\beta\rho)(15\alpha\beta\rho+(4\theta^2-21\beta\rho)c_1+(-4\theta^2+6\beta\rho)c_2)^2}{225\beta^2\rho(8\theta^2-27\beta\rho)^2} + \frac{4(15\alpha\beta\rho+(4\theta^2-21\beta\rho)c_1+(-4\theta^2+6\beta\rho)c_2)^2}{25\beta(8\theta^2-27\beta\rho)^2} \tag{3.82}$$

$$\frac{(-2\theta^2+9\beta\rho)(3\alpha\beta\rho+(-2\theta^2+3\beta\rho)c_1+2(\theta^2-3\beta\rho)c_2)^2}{9\beta^2\rho(4\theta^2-9\beta\rho)^2} > \frac{2(-4\theta^2+27\beta\rho)(15\alpha\beta\rho+(-4\theta^2+6\beta\rho)c_1+(4\theta^2-21\beta\rho)c_2)^2}{225\beta^2\rho(8\theta^2-27\beta\rho)^2} + \frac{4(15\alpha\beta\rho+(-4\theta^2+6\beta\rho)c_1+(4\theta^2-21\beta\rho)c_2)^2}{25\beta(8\theta^2-27\beta\rho)^2} \tag{3.83}$$

Due to the complexity of the expressions in $(\mathbf{3.82})$ and $(\mathbf{3.83})$ we will proceeds the analysis by assigning numerical values to the parameters satisfying second order and stability conditions for each stage. Throughout our numerical analysis the following conditions are valid.

$\alpha > c_2 > c_1 > 0, \beta > 0, \theta > 0, \rho > 0$

As in benchmark case, we assigned the following numerical values to the exogenous variables. The second order conditions and stability conditions are verified

$c_1 = 6, c_2 = 10, \beta = 1, \theta = 0.5, \rho = 0.5$

The figure 3.6 shows that, the merger profit of chain 1 is greater than the total profit of manufacturer 1 and retailer 1 under competitive model.

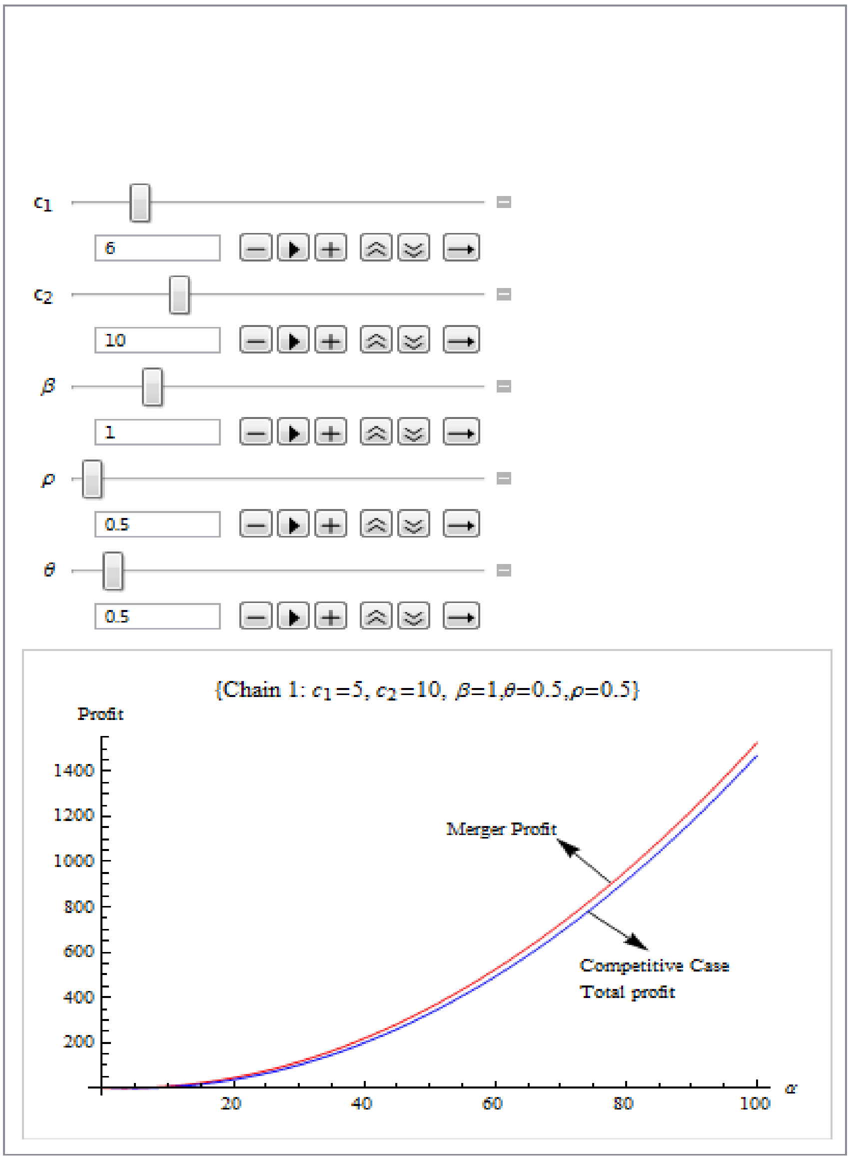

Figure 3.6: Chain 1 profits before and after merger

Observation 3.1: Taking the other variables' numerical values fixed, as far as $c_1 < c_2$ the merger profit of the chain 1 is greater than the total profit of manufacturer 1 and retailer 1 when they act in a competitive environment.

Observation 3.2: Taking the numerical values of other variables as fixed, as θ increases the gap between the profits before and after the merger is increase.

Observation 3.3: Taking the numerical values of other variables as fixed, for large values of ρ the merger is profitable only in small market sizes.

Observation 3.4: Taking the numerical values of other variables as fixed, for large values of β the merger is profitable only in small market sizes.

When we apply a similar analysis for the second chain we obtain the figure 3.7

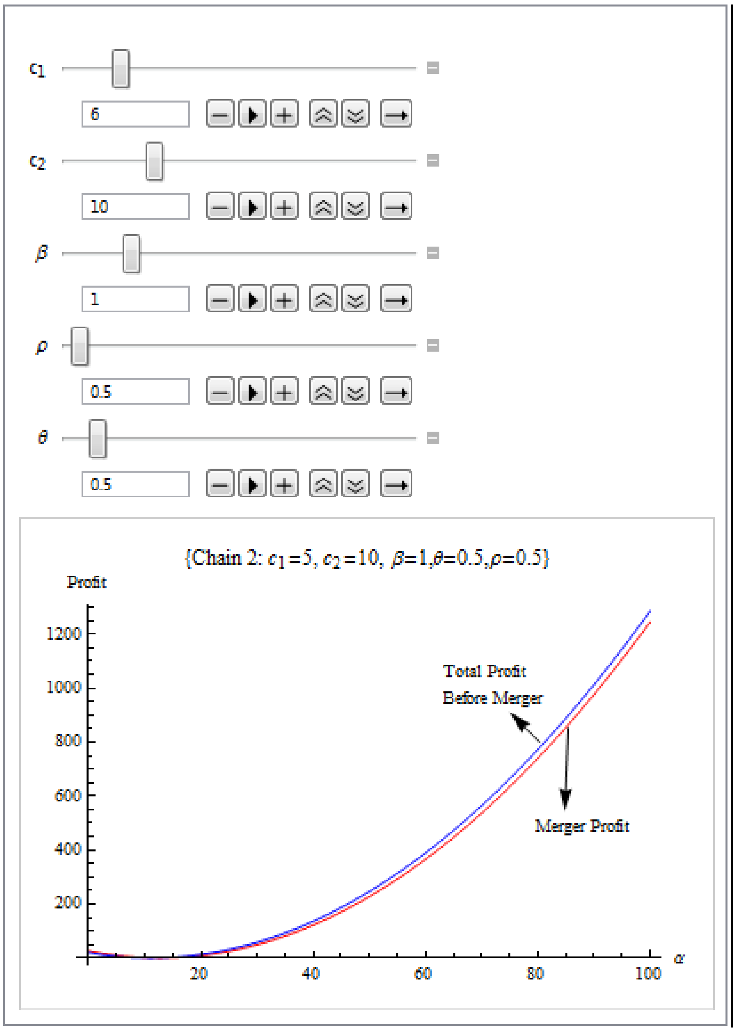

Figure 3.7: Chain 2 profits before and after merger

As we show in the figure 3.7 merger is not profitable for the second chain. That is to say, the total profit of manufacturer 2 and retailer 2 under the competitive model is greater than the merger profit of the channel.

Observation 3.5: Taking the numerical values of other variables as fixed, as far as $c_2 > c_1$ the merger profit of the second chain is smaller than the total profit of manufacturer 2 and retailer 2 under the competitive environment.

Observation 3.6: Taking the numerical values of other variables as fixed, as θ increases the merger will be more profitable.

Observation 3.7: Taking the numerical values of other variables as fixed, as ρ increases the merger and non-merger profits decreases, and the gap between profits before and after the merger increases.

Observation 3.8: Taking the numerical values of other variables as fixed, if $\beta < 1$ merger is profitable, but if $\beta > 1$ the merger profit is less than total competitive profit.

Hence, we observe that at least for some specific values of parameters merger is profitable for the first chain which has a smaller marginal production cost, but it is not profitable for the second chain. So, merger is a rational behavior for first chain's agents. In the following section, we will analyze the possible reaction of the second chain's agents when first chain members take merger decision.

3.2.4 Semi Centralized Model (Chain 1 Merge, Chain 2 Separated)

When chain 1 takes merger decision, chain 2 agents have two possible reactions; merge or remain separated. We have been showed that merger is not profitable for the second chain. In other words, the second chain merger profit does not exceed the total profit of members of the second chain when they independently maximize their own profits. We have analyzed the merger decision of chain 2 agents when chain 1 takes merger decision. Now, we will investigate the non-merger decision of chain 2 when chain 1 merges. The figure 3.8 illustrates the nature of the game when first chain merge, but second chain remains separated.

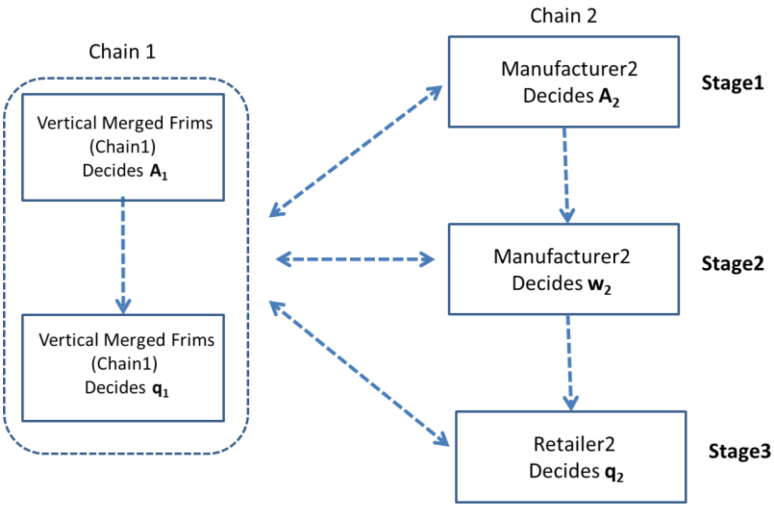

Figure 3.8: Chain 1 merge, chain 2 remains separated

The main difference of this case is that, in the second chain the manufacturer sells the product with a wholesale price larger than marginal production cost, but the first chain manufacturer provides product for her exclusive retailer at a price equals to marginal production cost. We will analyze this model in three stages by using backward induction. At the first stage of the game, the vertically merged chain and manufacturer 2 choose their advertising level. At the second stage of the game, manufacturer two determine wholesale price for her product. At the last stage of the game retailer 2 compete with vertically merged firms for output level. We will analyze the model from last stage of the game to the first by using backward induction method.

Stage 3: Vertically Merged Chain and Retailer 2 Determine Quantity Level

At the third stage of the game, vertically merged firms and second retailer determines their quantity level, taking the rival decision as given. The equilibrium quantities that we have been found at this stage are the function of advertising levels and second manufacturer wholesale price level. The profit functions are shown below.

$$\pi\text{C1} = (p - c_1)q_1 - \frac{\rho A_1^2}{2} \qquad (\mathbf{3.84})$$

$$\pi\text{R2} = (p - w_2)q_2 \qquad (\mathbf{3.85})$$

In order to find the profit maximal quantities at this stage, we differentiate $(\mathbf{3.84})$ and **(3.85)** with respect to q_1 and q_2, and then we simultaneously solve the first order conditions for q_1 and q_2 by setting the derivatives equal to zero. Then, we obtain the following Nash-Cournot quantities;

$$\frac{\partial \pi C1}{\partial q_1} = 0$$

$$\bar{q}_1 = \frac{\alpha + \theta A_1 + \theta A_2 - 2c_1 + w_2}{3\beta} \tag{3.86}$$

$$\frac{\partial \pi R2}{\partial q_2} = 0$$

$$\bar{q}_2 = \frac{\alpha + \theta A_1 + \theta A_2 + c_1 - 2w_2}{3\beta} \tag{3.87}$$

The second order conditions are satisfied since;

$$\frac{\partial^2 \pi C1}{\partial q_1^2} = \frac{\partial^2 \pi R2}{\partial q_2^2} = -2\beta < 0$$

The stability condition is also satisfied since the following determinant is always positive.

$$\begin{vmatrix} \frac{\partial^2 \pi C1}{\partial q_1^2} & \frac{\partial^2 \pi C1}{\partial q_1 q_2} \\ \frac{\partial^2 \pi R2}{\partial q_2 q_1} & \frac{\partial^2 \pi R2}{\partial q_2^2} \end{vmatrix} = 3\beta^2 > 0 \tag{3.88}$$

Stage 2: manufacturer 2 Determine Wholesale Price

At the second stage of the game the second manufacturer choose wholesale price level. After we substitute the equilibrium quantities in the expression $(\mathbf{3.87})$ into the second manufacturer's profit function we obtain the following profit function for this stage of the game.

$$\pi\text{M2} = (w_2 - c_2)\bar{q}_2 - \frac{\rho A_2^2}{2} \tag{3.89}$$

When we maximize the profit of manufacturer 2 by solving the first order condition for w_2, we obtain the profit maximizing wholesale price level in terms of advertising level of chain 1 and manufacturer 2.

$$\frac{\partial \pi M2}{\partial w_2} = 0$$

$$\bar{w}_2 = \frac{1}{4}(\alpha + \theta A_1 + \theta A_2 + c_1 + 2c_2) \qquad \mathbf{(3.90)}$$

Second order condition is verified since we assume that β is positive.

$$\frac{\partial^2 \pi R2}{\partial q_2^2} = -\frac{4}{3\beta} < 0$$

Stage 1: Vertically Merged Chain 1 and Manufacturer 2 Determine Advertising Level

At the first stage of the game, this is last stage of backward induction method, manufacturer 2 and vertically merged firms simultaneously determine the profit maximizing advertising level. We obtain the profit functions for this stage, after we substitute the equilibrium output and wholesale price level into expression in ($\mathbf{3.84}$) and ($\mathbf{3.89}$).

$$\pi C1 = (p - c_1)\bar{q}_1 - \frac{\rho A_1^2}{2} \qquad (\mathbf{3.91})$$

$$\pi M2 = (\bar{w}_2 - c_2)\bar{q}_2 - \frac{\rho A_2^2}{2} \qquad (\mathbf{3.92})$$

Differentiating $\pi C1$ and $\pi M2$ with respect to advertising levels A_1 and A_2 respectively, setting the derivatives equal to zero, and solving for A_1 and A_2 , we obtain the following reaction function for advertising levels.

$$\frac{\partial \pi C1}{\partial A_1} = 0$$

$$A_1 = -\frac{5\theta(5\alpha + 5\theta A_2 - 7c_1 + 2c_2)}{25\theta^2 - 72\beta\rho} \qquad (\mathbf{3.93})$$

$$\frac{\partial \pi M2}{\partial A_2} = 0$$

$$A_2 = -\frac{\theta(\alpha + \theta A_1 + c_1 - 2c_2)}{\theta^2 - 12\beta\rho} \qquad (\mathbf{3.94})$$

By solving the equations ($\mathbf{3.93}$) and ($\mathbf{3.94}$) simultaneously, we obtain the Nash-Cournot equilibrium advertising levels as follows;

$$\bar{A}_1 = \frac{5\theta(5\alpha\beta\rho + (\theta^2 - 7\beta\rho)c_1 - (\theta^2 - 2\beta\rho)c_2)}{\beta\rho(-31\theta^2 + 72\beta\rho)} \qquad (\mathbf{3.95})$$

$$\bar{A}_2 = \frac{\theta(6\alpha\beta\rho + (-5\theta^2 + 6\beta\rho)c_1 + (5\theta^2 - 12\beta\rho)c_2)}{\beta\rho(-31\theta^2 + 72\beta\rho)} \quad (\mathbf{3.96})$$

In order to show the profit functions are concave in advertising level and the equilibriums we have found can make the profits maximal, we have to show that the second order conditions are verified.

$$\frac{\partial^2 \pi C1}{\partial A_1^2} = \frac{25\theta^2}{72\beta} - \rho < 0 \quad (\mathbf{3.97})$$

$$\frac{\partial^2 \pi M2}{\partial A_2^2} = \frac{\theta^2}{12\beta} - \rho < 0 \quad (\mathbf{3.98})$$

If the expression in ($\mathbf{3.97}$) and ($\mathbf{3.98}$) are satisfied, then we can say second order conditions provide sufficient evidence for equilibriums to be exact point that maximize the profits. After we simplify the expression in ($\mathbf{3.97}$) and ($\mathbf{3.98}$) we obtain the following conditions;

$$\frac{\beta\rho}{\theta^2} > \frac{25}{72} \quad (\mathbf{3.99})$$

$$\frac{\beta\rho}{\theta^2} > \frac{1}{12} \quad (\mathbf{3.100})$$

Since the expression in ($\mathbf{3.99}$) is binding, it is necessary condition for the second order conditions to be satisfied. We also have to check the stability of the equilibrium advertising levels. For a stable and unique equilibrium of advertising level the following determinant should be verified.

$$\begin{vmatrix} \frac{\partial^2 \pi C1}{\partial A_1^2} & \frac{\partial^2 \pi C1}{\partial A_1 A_2} \\ \frac{\partial^2 \pi M2}{\partial A_2 A_1} & \frac{\partial^2 \pi M2}{\partial A_2^2} \end{vmatrix} = -\frac{31\theta^2\rho}{72\beta} + \rho^2 > 0 \quad (\mathbf{3.101})$$

For the expression in ($\mathbf{3.101}$) to be hold**,** $\frac{\beta\rho}{\theta^2} > 0.43$

Since we choose the parameters regarding second order and stability condition, the profit functions are concave in advertising level. The stability condition in ($\mathbf{3.101}$) is a binding condition for the second order conditions in ($\mathbf{3.97}$) and ($\mathbf{3.98}$)**.** Hence, all of the necessary conditions are satisfied for an equilibrium to be unique and stable.

The Nash-Cournot optimal profit functions are obtained by substituting the equilibrium output, advertising, and wholesale price levels into the profit functions stated in $(\mathbf{3.84})$**,** $(\mathbf{3.85})$ and $(\mathbf{3.89})$**.** The table 3.5 shows the equilibrium profits.

Table 3.5: Profits of agents when chain 1 merge, chain 2 remain separated

Profits of Agents When Chain 1 Merge, Chain 2 Remain Separated	
$\ddot{\pi}\mathrm{C1}$	$\dfrac{(-25\theta^2+72\beta\rho)(5\alpha\beta\rho+(\theta^2-7\beta\rho)c_1-(\theta^2-2\beta\rho)c_2)^2}{2\beta^2\rho(31\theta^2-72\beta\rho)^2}$
$\ddot{\pi}\mathrm{M2}$	$\dfrac{(-\theta^2+12\beta\rho)(6\alpha\beta\rho+(-5\theta^2+6\beta\rho)c_1+(5\theta^2-12\beta\rho)c_2)^2}{2\beta^2\rho(31\theta^2-72\beta\rho)^2}$
$\ddot{\pi}\mathrm{R2}$	$\dfrac{4(6\alpha\beta\rho+(-5\theta^2+6\beta\rho)c_1+(5\theta^2-12\beta\rho)c_2)^2}{\beta(31\theta^2-72\beta\rho)^2}$

Comparison of Profits for Chain 2

As we mentioned above, when chain 1 takes merger decision chain 2 will either merge or remain separated. Now, we will compare the profits of chain 2 when they collaborate and when they act independently.

$$\pi^*\mathrm{C2} > \ddot{\pi}\mathrm{M2} + \ddot{\pi}\mathrm{R2} \qquad (\mathbf{3.102})$$

The left hand side of $(\mathbf{3.102})$ is the merger profit of chain 2, and right hand side is the total profit when the agents of the second chain act independently. After we replace the corresponding profits into the expression $(\mathbf{3.102})$ we obtain the following inequality.

$$\frac{(-2\theta^2+9\beta\rho)(3\alpha\beta\rho+(-2\theta^2+3\beta\rho)c_1+2(\theta^2-3\beta\rho)c_2)^2}{9\beta^2\rho(4\theta^2-9\beta\rho)^2}$$
$$> \frac{(-\theta^2+12\beta\rho)(6\alpha\beta\rho+(-5\theta^2+6\beta\rho)c_1+(5\theta^2-12\beta\rho)c_2)^2}{2\beta^2\rho(31\theta^2-72\beta\rho)^2}$$
$$+ \frac{4(6\alpha\beta\rho+(-5\theta^2+6\beta\rho)c_1+(5\theta^2-12\beta\rho)c_2)^2}{\beta(31\theta^2-72\beta\rho)^2} \qquad (\mathbf{3.103})$$

Due to the complexity of the expression in $(\mathbf{3.103})$ we will proceeds the analysis by assigning numerical values to the parameters satisfying second order and stability conditions for each stage.

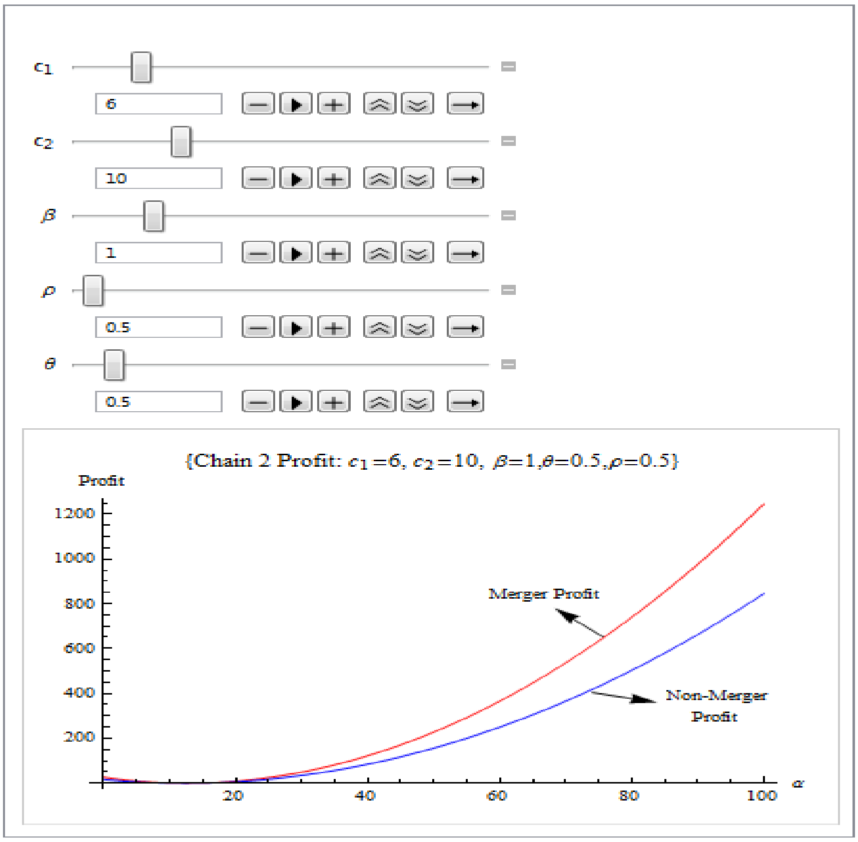

Figure 3.9: Comparison of profits for chain 2

Observation 3.9: When chain 1 takes merger decision, the second chain merger profit is greater than non-merger profit independent of marginal production costs of manufacturers.

As a result, at least for some numerical values of parameter satisfying second order and stability conditions at each stage, merger is profitable for chain 1. When chain 1 takes merger decision, the best response reaction of chain 2 is merger decision, too. So, the profits of the chain will be divided between the agents of each chain according to Nash bargaining scheme when both of chains take merger decision.

4. PROFIT DIVISION

4.1 Nash Arbitration Scheme

The profit division for each chain will be done according to Nash bargaining rule. Since chain 1 is dominant in the process of taking merger decision, in case of disagreement the outside options for chain 1 agents are the profits under competitive model. Chain 2 will choose her best reaction according to chain 1 behavior. We showed that, the merger profit of chain 2 is smaller than the total profit of manufacturer 2 and retailer 2 when they act in competitive model. However, when chain 1 take merger decision, taking merger decision is more profitable than remaining separated. Since chain 2 chooses her action after chain 1 makes decision, in case of disagreement chain 2 agents' outside options are the profits when they operate in the market independently. The profits that will be divided and disagreement options are illustrated in the table 4.1.

Table 4.1: Division profits and disagreement options

	Profit for Allocation		Disagreement Option
Chain1	$\frac{(-2\theta^2+9\beta\rho)(3\alpha\beta\rho+2(\theta^2-3\beta\rho)c_1+(-2\theta^2+3\beta\rho)c_2)^2}{9\beta^2\rho(4\theta^2-9\beta\rho)^2}$	**M1**	$\frac{2(-4\theta^2+27\beta\rho)(15\alpha\beta\rho+(4\theta^2-21\beta\rho)c_1+(-4\theta^2+6\beta\rho)c_2)^2}{225\beta^2\rho(8\theta^2-27\beta\rho)^2}$
		R1	$\frac{4(15\alpha\beta\rho+(4\theta^2-21\beta\rho)c_1+(-4\theta^2+6\beta\rho)c_2)^2}{25\beta(8\theta^2-27\beta\rho)^2}$
Chain2	$\frac{(-2\theta^2+9\beta\rho)(3\alpha\beta\rho+(-2\theta^2+3\beta\rho)c_1+2(\theta^2-3\beta\rho)c_2)^2}{9\beta^2\rho(4\theta^2-9\beta\rho)^2}$	**M2**	$\frac{(-\theta^2+12\beta\rho)(6\alpha\beta\rho+(-5\theta^2+6\beta\rho)c_1+(5\theta^2-12\beta\rho)c_2)^2}{2\beta^2\rho(31\theta^2-72\beta\rho)^2}$
		R2	$\frac{4(6\alpha\beta\rho+(-5\theta^2+6\beta\rho)c_1+(5\theta^2-12\beta\rho)c_2)^2}{\beta(31\theta^2-72\beta\rho)^2}$

Nash arbitration scheme is concerned with allocating the profit on the negotiation set. Any allocation on the set is Pareto optimal, and equal or greater than the disagreement point. As we mentioned above, the disagreement point is the profit incurred by the players in a non-cooperate game. The Nash arbitration scheme can be obtained by solving;

$$max(u_1 - d_1)(u_2 - d_2) \qquad \mathbf{(4.1)}$$

$$\begin{aligned} s.t. \quad & u_1 \geq d_1 \\ & u_2 \geq d_2 \\ & (u_1, u_2) \in P \end{aligned}$$

u_i denotes the player i's allocated profit, d_i denotes the disagreement option, *i=1,2;* and *P* is the set of Pareto optimal solutions.

In our study, the manufacturer and retailer of each chain bargain for the allocation of the chain merger profit. So, the Pareto optimal set is the total of u_1 and u_2 which is equal to chain merger profit. After we replace the corresponding terms into the expression $(\mathbf{4.1})$**,** and solve the maximization problem for u_1 and u_2 we obtain the following allocations for chain 1 and chain 2 members.

Table 4.2: Profits under Nash arbitration scheme

	Profits Under Nash Arbitration Scheme
M1	$\frac{1}{450\beta^2}\left(\frac{25(-2\theta^2+9\beta\rho)(3\alpha\beta\rho+2(\theta^2-3\beta\rho)c_1+(-2\theta^2+3\beta\rho)c_2)^2}{\rho(4\theta^2-9\beta\rho)^2}-\frac{36\beta(15\alpha\beta\rho+(4\theta^2-21\beta\rho)c_1+(-4\theta^2+6\beta\rho)c_2)^2}{(8\theta^2-27\beta\rho)^2}+\frac{2(-4\theta^2+27\beta\rho)(15\alpha\beta\rho+(4\theta^2-21\beta\rho)c_1+(-4\theta^2+6\beta\rho)c_2)^2}{\rho(8\theta^2-27\beta\rho)^2}\right)$
R1	$\frac{1}{450\beta^2}\left(\frac{25(-2\theta^2+9\beta\rho)(3\alpha\beta\rho+2(\theta^2-3\beta\rho)c_1+(-2\theta^2+3\beta\rho)c_2)^2}{\rho(4\theta^2-9\beta\rho)^2}+\frac{36\beta(15\alpha\beta\rho+(4\theta^2-21\beta\rho)c_1+(-4\theta^2+6\beta\rho)c_2)^2}{(8\theta^2-27\beta\rho)^2}+\frac{2(4\theta^2-27\beta\rho)(15\alpha\beta\rho+(4\theta^2-21\beta\rho)c_1+(-4\theta^2+6\beta\rho)c_2)^2}{\rho(8\theta^2-27\beta\rho)^2}\right)$
M2	$\frac{1}{36\beta^2}\left(-\frac{72\beta(6\alpha\beta\rho+(-5\theta^2+6\beta\rho)c_1+(5\theta^2-12\beta\rho)c_2)^2}{(31\theta^2-72\beta\rho)^2}+\frac{9(-\theta^2+12\beta\rho)(6\alpha\beta\rho+(-5\theta^2+6\beta\rho)c_1+(5\theta^2-12\beta\rho)c_2)^2}{\rho(31\theta^2-72\beta\rho)^2}+\frac{2(-2\theta^2+9\beta\rho)(3\alpha\beta\rho+(-2\theta^2+3\beta\rho)c_1+2(\theta^2-3\beta\rho)c_2)^2}{\rho(4\theta^2-9\beta\rho)^2}\right)$
R2	$\frac{1}{36\beta^2}\left(\frac{72\beta(6\alpha\beta\rho+(-5\theta^2+6\beta\rho)c_1+(5\theta^2-12\beta\rho)c_2)^2}{(31\theta^2-72\beta\rho)^2}+\frac{2(-2\theta^2+9\beta\rho)(3\alpha\beta\rho+(-2\theta^2+3\beta\rho)c_1+2(\theta^2-3\beta\rho)c_2)^2}{\rho(4\theta^2-9\beta\rho)^2}+\frac{9(\theta^2-12\beta\rho)(-6\alpha\beta\rho+(5\theta^2-6\beta\rho)c_1+(-5\theta^2+12\beta\rho)c_2)^2}{\rho(31\theta^2-72\beta\rho)^2}\right)$

4.2 Equal Division Rule

Instead of Nash arbitration scheme if we equally allocate the profit of each chain between the agents of the channel, we obtain the results illustrated in table 4.3

Table 4.3: Profits under equal division rule

	Equal Allocation of Profits
M1	$\dfrac{(-2\theta^2+9\beta\rho)(3\alpha\beta\rho+2(\theta^2-3\beta\rho)c_1+(-2\theta^2+3\beta\rho)c_2)^2}{18\beta^2\rho(4\theta^2-9\beta\rho)^2}$
R1	$\dfrac{(-2\theta^2+9\beta\rho)(3\alpha\beta\rho+2(\theta^2-3\beta\rho)c_1+(-2\theta^2+3\beta\rho)c_2)^2}{18\beta^2\rho(4\theta^2-9\beta\rho)^2}$
M2	$\dfrac{(-2\theta^2+9\beta\rho)(3\alpha\beta\rho+(-2\theta^2+3\beta\rho)c_1+2(\theta^2-3\beta\rho)c_2)^2}{18\beta^2\rho(4\theta^2-9\beta\rho)^2}$
R2	$\dfrac{(-2\theta^2+9\beta\rho)(3\alpha\beta\rho+(-2\theta^2+3\beta\rho)c_1+2(\theta^2-3\beta\rho)c_2)^2}{18\beta^2\rho(4\theta^2-9\beta\rho)^2}$

4.3 Numerical Analysis of Profit Allocation

Due to the complexity of algebraic analysis, we will conduct a numerical analysis by assigning specific values to the parameters satisfying second order, and stability condition at each stage of the game. As we have emphasized above, throughout the numerical analysis the following assumptions are valid.

$\alpha > c_2 > c_1 > 0, \beta > 0, \theta > 0, \rho > 0$

The following numerical values of parameters verify all necessary conditions.

$\alpha = 50,\ c_1 = 6,\ c_2 = 10,\ \beta = 0.5,\ \rho = 0.5$

Table 4.4: Nash allocation scheme

	Non-Merger Profits	Total Chain Profits	Merger Profits	Total Chain Profits
M1	192	330.2	204.7	355.6
R1	138.2		150.9	
M2	92	156	127.8	227.6
R2	64		99.8	

By this numerical example, as illustrated in table 4.4, we can find that the agents of both chain gain more profit when they act in collusion. In other words, merger is profitable for both chains. The profit of each agent of chain 1 increases by more than 12 units. The non-merger profits of chain 2 are the profits when chain 1 merges. The profit of each agent of the chain 2 increases by more than 35 units. We can conclude that, Nash arbitration scheme can be used as a revenue sharing contract to coordinate the supply chain agents.

Table 4.5: Equal allocation of profits

	Non-Merger Profits	**Total Chain Profits**	**Merger Profits**	**Total Chain Profits**
M1	192	330.2	177.8	355.6
R1	138.2		177.8	
M2	92	156	113.8	227.6
R2	64		113.8	

As shown in the table 4.5, equal division is less profitable for manufacturers, but opposite for the retailers. For example, under equal division rule retailer 2 gains more than 49 units, but the manufacturer 2 gains about 21 units after the merger. More importantly, the profit of the manufacturer 1 after the merger is less than non-merger case. Therefore, manufacturer 1 does not merge under equal division rule. So, equal division rule cannot be used as a revenue sharing rule to coordinate the supply chain agents.

Table 4.6: Model comparison

	Non-Merger Profits (Competitive Model)	Total Profit	Chain1 Merge, Chain2 Separated (Semi Centralized Model)	Total Profit	Both Chain Merge (Centralized Model)	Total Profit
M1	192	330.2	M1+R1	476	M1+R1	355.5
R1	138.2					
M2	143.3	246.5	92	156	M2+R2	227.5
R2	103.2		64			

The table 4.6 compares the profits of three models. As we see in the table, independent of chain 2 behaviors, merger is profitable for chain 1. In any case, merger is not profitable for chain 2. However, when chain 1 agents cooperate, then merger is more profitable than remain separated for chain 2 agents.

5. THE EFFECT OF EXOGENOUS VARIABLES ON EQUILIBRIUM VALUES

In figure 5.1 and 5.2 we let advertising coefficient changes from 0 to 1 given other exogenous variables fixed, the optimal profits of manufacturers and retailers will increase before and after merger. (The superscripts *a* and *b* indicate the profits after and before merger, respectively). The effect of an increase in θ has lager positive effect on merger profits. For large value of θ the gap between the profits of manufacturers under Nash allocation scheme will decrease.

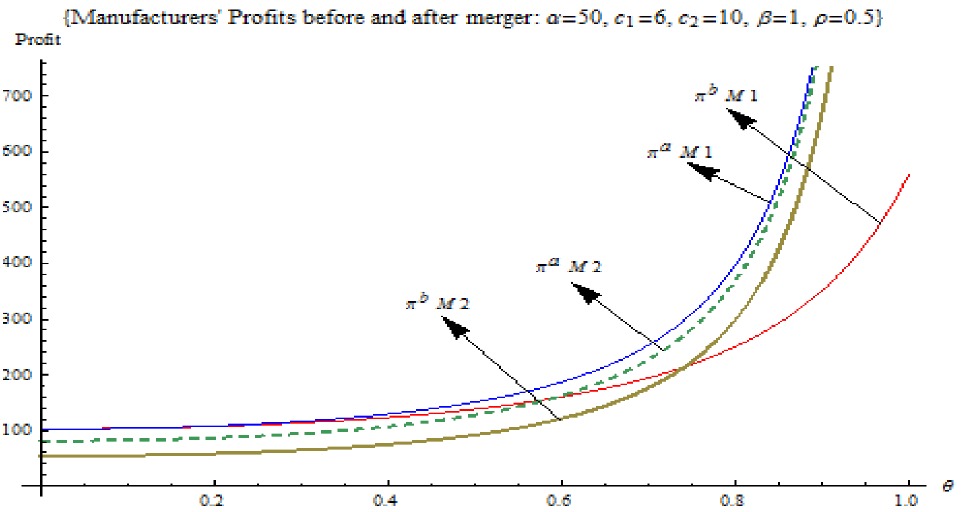

Figure 5.1: The effect of θ on manufacturers' profits

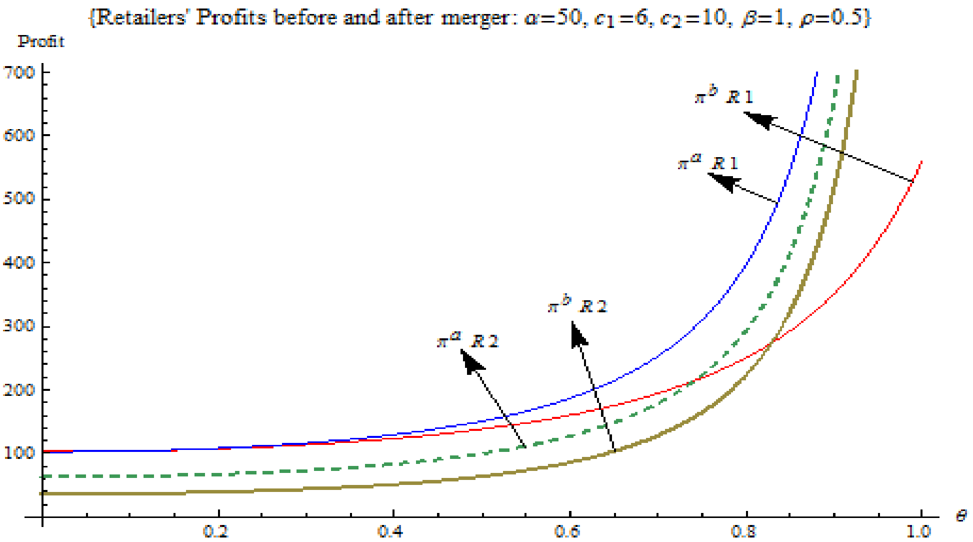

Figure 5.2: The effect of θ on retailers' profits

In figure 5.3 and 5.4 we let advertising cost, ρ, changes from 0.2 to 1 given other exogenous variables fixed, the optimal profits of manufacturers and retailers decreases. If advertisement is highly costly, then merger may loss its attraction for manufacturer 1. Even though the cost of advertising is high, the difference between profits of manufacturer 2 before and after merger is considerable high.

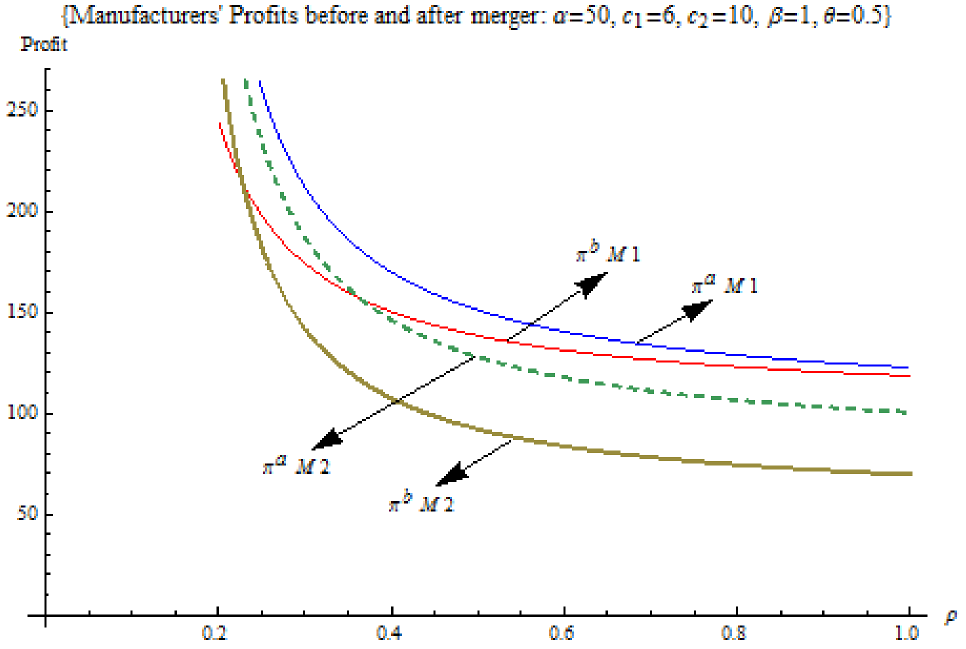

Figure 5.3: The effect of ρ on manufacturers' profits

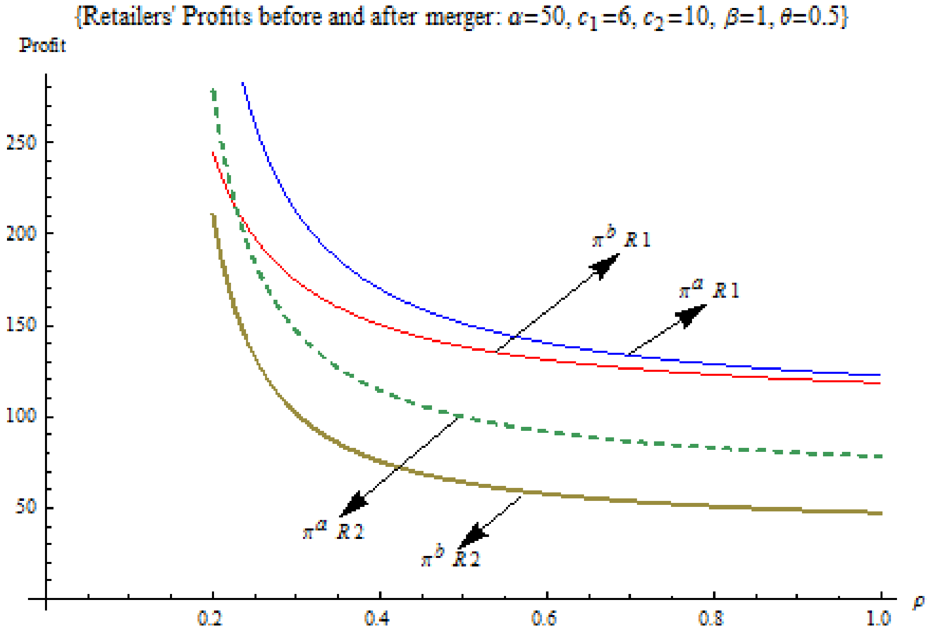

Figure 5.4: The effect of ρ on retailers' profits

In table 5.1 we will analyze the effect of θ on equilibrium advertising level, output, and price for both chains. In this numerical example, we let θ increasing from 0.1 to 1. Given other exogenous variables fixed, the optimal advertising levels increase from 2.2 to 170.6, and from 1.6 to 165.3 for chain 1 and chain 2 respectively. We can find that, the optimal advertising levels are positively affected by θ.

Observation 5.1: The advertising expenses of chain 1 is greater than advertising expenses of chain 2 regardless of the degree of θ.

It is reasonable, because chain 1 has larger market share than chain 2. That is to say, chain 2 has more costumer than chain 1 which means chain 1 agents spends more expense in advertising while competing with chain 2.

Another finding is that, the retailer price is rising by an increase in θ. It is meaningful because increases in θ positively affects the brand image which means the costumers will likely demand the product.

Table 5.1: The effect of $\boldsymbol{\theta}$ on equilibrium values

$\boldsymbol{\theta}$	$\boldsymbol{A_1^*}$	$\boldsymbol{A_2^*}$	$\boldsymbol{q_1^*}$	$\boldsymbol{q_2^*}$	$\boldsymbol{p^*}$
0.1	2.2	1.6	16.1	12.1	22.1
0.4	9.8	7.6	18.3	14.3	24.3
0.7	25	21.3	26.8	22.8	32.8
0.9	62.4	57.6	52	48	58
1	170.6	165.3	128	124	134

Observation 5.2: When $\boldsymbol{\rho}$ increases; advertising expenses, output levels, and retail price decreases (see table 5.2). It is reasonable, because when $\boldsymbol{\rho}$ rises advertising becomes more expensive. Therefore, firms will reduce their advertising expenses which lead to a reduction output level and retail price.

Table 5.2: The effect of ρ on equilibrium values

$\boldsymbol{\rho}$	$\boldsymbol{A_1^*}$	$\boldsymbol{A_2^*}$	$\boldsymbol{q_1^*}$	$\boldsymbol{q_2^*}$	$\boldsymbol{p^*}$
0.2	55.8	49.2	33.5	29.5	39.5
0.3	26.9	22.5	24.2	20.2	30.2
0.5	13.3	10.7	20	16	26
0.7	8.9	6.9	18.6	14.6	24.6
0.9	6.7	5.2	17.9	13.9	23.9
1	5.9	4.6	17.8	13.8	23.8

6. CONCLUSION

In this paper, we study a two echelon supply chain system consisting of two competing manufacturer and two competing retailers. Each manufacturer produces a price dependent product and sells through an exclusive retailer to end consumers. We have studied the manufacturer-retailer coordination behaviors in vertically merged supply chain by applying Nash arbitration scheme as a revenue sharing contract. The optimal pricing, advertising, and quantity decision are provided for duopolistic manufacturers and retailer under a vertically merged supply chain. We perform the analysis of mergers that are assumed to be legally possible. We have showed that regardless of chain 2 behaviors, merger is profitable for chain 1 agents. We investigate the possible behaviors of chain 2 agents when chain 1 takes merger decision. We have showed that the best response behavior of chain 2 to the merger decision of chain 1 is merger decision. Hence, we perform Nash arbitration scheme in order to allocate the profit of each channel between manufacturer and retailer. We also compare the Nash arbitration scheme with the equal division rule in which the profit of merger is equally divided between the agents of the chains. We can conclude that, Nash arbitration scheme can be used as a revenue sharing contract to coordinate the supply chains.

We use some numerical examples to show how exogenous variables affect the optimal advertising level, pricing, and quantity level of each chain. We figure out that when a chain market share becomes large, advertising expenses raises. In addition, the optimal advertising expenses, output levels, and retail price decreases.

A potential topic for the future research could be to analyze a common retailer supply chain with competitive retail market. The study of vertical merger in this paper could be extended by including horizontal merger. In general one side of supply chain is more powerful than the other side of the chain. So, analyzing the model under Stackelberg game structure may help to determine other dimensions of supply chain studies. In addition to Nash arbitration scheme, Talmudic division, shapley value could be used as a revenue sharing contract to coordinate supply chain agents.

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