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İSTANBUL TECHNICAL UNIVERSITY ★ INSTITUTE OF SCIENCE AND TECHNOLOGY

**ELASTIC WAVE PROPAGATION IN METALS UNDER
HIGH STRAIN RATES**

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**YÜKSEK HIZLI DEFORMASYONLARDA METALLERDE
ELASTİK DALGA YAYINIMI**

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PREFACE

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ABBREVIATIONS

SHPB	: Split Hopkinson Pressure Bar
DU	: Depleted Uranium
EOS	: Equation of State of a Material
HSS	: Tool Steel
JC	: Johnson-Cook Constitutive Equation
MJC	: Modified Johnson-Cook Constitutive Equation
RJC	: Revised Johnson-Cook Equation
ZA	: Zerilli-Armstrong Constitutive Equation
KH	: Khan-Huang Constitutive Equation
BCC	: Body Centered Cubic
FCC	: Face Centered Cubic
HCP	: Hexagonal Compact Packed
BP	: Bodner-Partom Constitutive Equation
MTS	: Mechanical Threshold Stress
LORD	: Laser Occlusive Radius Detector
OFHC	: Oxygen-Free High Conductivity
DSO	: Digital Storage Oscilloscope
CMRR	: Common Mode Rejection Ratio
CMV	: Common Mode Voltage
TSHP	: Torsional Split Hopkinson Pressure Bar

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LIST of SYMBOLS

E_c	: Kinetic Energy
U_p	: Particle Velocity
n	: Strain Hardening Coefficient
m	: Strain Rate Sensitivity
ε	: Strain
σ	: Stress
C_p	: Plastic Wave Velocity
σ^*	: Nominal, Engineering Stress
E	: Young's Modulus of Elasticity
T	: Temperature
T_r	: Reference Temperature
T_m	: Melting Temperature
T_h	: Homologous Temperature
A, B, C, n, m	: Material Constants of Johnson and Cook Constitutive Equation
σ_{yield}	: Yield Stress
ν	: Poisson's Value
ρ	: Mass Density
t	: Time
$V_0, V_{striker}$	: Striker Bar Velocity
C_0	: Elastic Stress Wave Velocity
ε_i	: Incident Strain
ε_R	: Reflected Strain
ε_T	: Transmitted Strain
$\dot{\varepsilon}$	: Strain Rate
L_{bar}	: Length of Pressure Bars
D_{bar}	: Diameter of Pressure Bars
$D_{specimen}$	: Diameter of Specimen
F	: Compressive Force
A_0	: Cross Sectional Area
I	: Acoustic Empedance
λ	: Wavelength
α_{12}	: Transmission Coefficient , from input bar to specimen
α_{23}	: Transmission Coefficient , from specimen to output bar
β	: Reflection Coefficient
v_s	: Particle Velocity
R	: Radius
Z	: Mechanical Empedance
S	: Gage Factor of a Strain Gage
R_g	: Strain Gage Resistance
E_w	: Wheatstone Bridge Output

YÜKSEK HIZLI DEFORMASYONLARDA METALLERDE ELASTİK DALGA YAYINIMI

ÖZET

Bu çalışmada split Hopkinson pressure bar test yönteminin dizaynında dikkate alınması gereken noktalar sunulmaya çalışılmıştır.

Deformasyon hızının yüksek olduğu şekil değişimlerinde, malzemelerin buna karşı olan davranışlarında değişiklikler görülür. Statik deformaşyonda, kuvvet uygulandıktan, denge oluşuncaya kadar geçen sürede katı her bir t anında dengede kabul edilir. Öteyandan, deformasyon hızı yükselirse, tüm cisim uygulanan gerilme etkisinden aynı anda etkilenmez. Bu durumda, dinamik deformasyon cisimde dalgalar halinde ilerlemektedir.

Bu çalışmada, literatürden elde edilen bilgiler ile, elastik, plastik ve şok dalagalarından kısaca bahsedilmiştir.

Elastik dalgalar, SHPB dizaynında önem kazanmaktadır, zira test süresince oluşan elastik dalgalar bir boyutlu olarak SHPB düzeneğinin basma çubuklarında ilerlemektedir (pressure bars, uygulanan kuvvet şiddeti çubukların akma dayanımını geçmeyecek şekilde belirlenir). Düzeneğin basma çubuklarında plastik dalga oluşmaz, ancak numune'nin akma dayanımı uygulanan kuvvet şiddetinden daha düşük olduğu için plastik deformasyon numunede olur.

SHPB testinde elastik dalgalar, çarpan çubuğun basınçlı düzenek (gas-gun) tarafından veya benzer bir yöntem ile alan çubuğun (input bar) yüzeyine eksenel olarak çarptırılmasıyla oluşturulur. Çarpma sonucu, çarpan çubuğun çarpma hızına ve malzeme özelliklerine göre farklı şiddetlerde basma tipi gerilme dalgaları oluşur. Bu basma tipi gerilme dalgaları bir boyutlu olarak dizayn edilen baskı çubuklarında ilerler.

Test yönteminde numune, iki çubuğun, alan ve ileten çubukların, arasına yerleştirilir. Gerilme dalgası alan çubuk-numune arayüzeyine eriştiğinde, numune ve çubuk malzemeleri arasında malzeme özelliği ve kesit alana bağlı olarak ortaya çıkan mekanik empedans farkından dolayı, dalga'nın bir kısmı çekme dalgası olarak geri yansır ve geri kalan kısmı ise ileten çubuğa geçer. Geleneksel yöntemde, çubukların orta noktalarında çubuk yüzeyine yerleştirilmiş gerinim ölçerler, wheatstone köprülerinde dengeleme rezistanslıdır ve voltaj değişimlerini sinyal olarak osiloskop'a gönderirler. Gerinim ölçerlerin boylarındaki uzama ve kısalma wheatstone köprüsü kolları arasında voltaj farkı yaratır. Köprünün kolları arasındaki voltaj farkı osiloskop'a gönderilmeden önce amplifikatör ile güçlendirilir. Osiloskop bu voltaj değişimlerini zamana bağlı olarak kayıtlar eder.

Kaydedilen veriler bilgisayar'a taşınır ve Wheatstone köprüsünün konfigürasyonu da zaten bilindiğine göre, voltaj değerleri baskı çubuklarının maruz kaldığı gerinim değerlerine çevrilirler. İlgili formüller ile, $\epsilon_{yansiyen}$ ve $\epsilon_{iletken}$ değerleri kullanılarak, numunenin maruz kaldığı gerilme ve gerinme değerleri elde edilir. Daha sonra zamana göre çizilmiş iki eğri (gerilme-zaman, gerinme-zaman) birleştirilerek malzemenin dinamik gerilme-gerinme eğrisi oluşturulur.

Belirli bir malzemeye farklı gerinme hızları ve sıcaklıklarda SHPB yöntemi ile testler yaparak çalışmada özetlenmiş olan *bünye denklemlerine* ait, örneğin Johnson-Cook denklemi, malzeme sabitleri belirlenebilir. Bunun için bünye deklemini katsayıları *doğrusal olmayan regresyon analizi* yöntemi ile belirlenebilir. Belirli bir malzeme için katsayılar belirlendikten sonra, malzemenin davranışı ile ilgili bilgilere deney yapmadan bünye denklemleri kullanılarak ulaşılabilir.



ELASTIC WAVE PROPAGATION IN METALS UNDER HIGH STRAIN RATES

SUMMARY

The aim of this study is to present the considerations that should be taken into account in the design of a SHPB.

When deformation occurs at high strain rates, the behaviour of materials change considerably. Quasi-static deformation occurs when the time between the applied load and the equilibration of the solid is very short, in other words the solid is in equilibrium at every time step t during deformation. On the other hand, when the rate of deformation increases the whole structure will not effect from the stress at the same time. Thus, dynamic deformation in solids propagate as waves.

In this study, from literature, three types of waves briefly described, namely elastic, plastic and shock waves.

Elastic Waves are important in the design of a SHPB apparatus, since during the test elastic waves propagate one – dimensionally through the pressure bars of the SHPB. Plastic waves do not occur in the pressure bars (since the applied load pulse is determined such that it does not exceed the yield strength of the pressure bars), however the specimen is deforms plastically because the applied load pulse is determined such that it exceeds the yield strength of the specimen.

Elastic waves in the SHPB test, are generated by impacting the striker bar coaxially in the input bar end face by projectiling it by a one-stage gas gun or similar method. Upon impact, a compressive stress wave is generated depending on the velocity and material properties of the striker bar. This compressive stress wave propagates through the pressure bars which are designed as one-dimensional.

In the test method, the specimen is placed between the two corresponding end faces of the pressure bars, namely the input (incident) and the output (transmitter) bars. When the stress wave reaches the input bar-specimen interface, due to mechanical empedance difference between specimen and pressure bar materials and the cross sectional area, a portion of the stress wave reflects back as a tensile wave and the remaining portion transmits to the output bar. Strain gages which are usually attached in the middle of the pressure bars, in conventinal method, record voltage changes with respect to time (test duration), since they are configured to act as a balancing resistors of a wheatstone bridge. The contraction and elongation of these gages make the wheatstone bridge unbalanced. The wheatstone bridge output, voltage changes, are magnified by an amplifier and then sent to a DSO (Digital Storage Oscilloscope). Then the oscilloscope records these voltage variations with respect to time.

The data is then taken to a computer and since the configuration type of the wheatstone bridge is known, the voltage values are then converted into strain in the pressure bars. By the necessary equations, using $\epsilon_{\text{reflected}}$ and $\epsilon_{\text{transmitted}}$ the strain and stress in the specimen can be calculated. Later, by combining the two curves the dynamic stress-strain curve of the material is formed.

By performing many test to a specific material at different strain rates and temperatures with SHPB, one can determine the material constants of the presented constitutive equations, i.e. Johnson-Cook equation, by fitting the data to the equation by non-linear regression analysis. Once the constants are determined, approximate calculations can be made for that specific material without performing any tests (remaining in the strain rate range at which the test data for the specimen are obtained).



1. INTRODUCTION TO DEFORMATION AND WAVES

1.1 Introduction

The objective of this study is to present the considerations which are taken into account when designing a test method, compressional Split Hopkinson Pressure Bar, which depends on the propagation of elastic waves in one dimensional finite media.

As the conclusion of this study is targeted to investigate the design considerations in a compressional SHPB, to achieve this goal, the following steps are presented;

- i. Presentation of the fundamentals of a compressional SHPB,
- ii. Presentation of the considerations which should be taken into account when dimensioning the pressure bars and specimen,
- iii. Presentation of the selecting criteria for necessary electronic measuring and data acquisition devices such as osilloscope, amplifier and etc.,
- iv. Presentation of the design considerations and understanding the working principles of gas guns used for to projectile the striker bar on the incident bar in the desired velocity range,
- v. Presentation of the empedance matching of materials which are to be investigated with the pressure bar materials,
- vi. Presentation of some of the calibration considerations for the SHPB apparatus,
- vii. Presentation of the mostly used constitutive equations for material characterization at high strain rates.

1.2 Science of Strength of Materials

During operations, structures and machines are acted on by external loads; for instance, the abutments of a railway bridge carry the weight of a moving train and the weight of the bridge itself, the connecting rod of an automobile engine is

subjected to gas pressure in the cylinder. In order that structural elements and machine parts may sustain the loads acting on them without fracture or appreciable deformation, they must be made of a proper material and have the necessary dimensions. These dimensions of structural members are determined by calculation.

The development of the foundations for the design of structural members is the subject matter of a science called *the strength of materials*.

The dimensions of a member are determined with due allowance for the properties of the material of which the member is to be made. To choose the material rationally and to utilize it most efficiently one must have information regarding the most important properties of various structural materials (steel, cast iron, wood, concrete, stone, etc.). These data include in the first place the *strength* of a material, i.e., its ability to resist external loads without fracture.

Experimental studies of the strength of materials receive widespread attention at the present time. The science of strength of materials is, on the one hand, related to the science of materials and materials testing and, on the other hand, closely related to theoretical mechanics. Strength of materials, is based on the laws and theorems of theoretical mechanics and uses its principles as long as they are consistent with the fundamental principles and the problems of strength of materials. For the solution of these problems a number of new concepts are introduced in strength of materials [1].

1.3 Stress – Strain, Deformation and Design of Structures

The most important of these concepts are the *strain* and *stress*. In theoretical mechanics, solids are conventionally considered as absolutely rigid bodies, i.e., as bodies undergoing no change in shape under the action of forces applied to them. Experimental observations show, however, that all solids deform when subjected to forces [1].

Many material properties, such as hardness and strength, are determined from the relationship between the *stress* and *strain* (stress-strain curve) of that material (Fig. 1.1). Stress is the force per unit area applied to a sample, and strain is the relative deformation of the sample as a result of that stress. Various kinds of deformation can occur (elongation, compression, bending, torsion, tension etc.) depending on the

nature of the applied force. If stress to the material is kept below the so-called *yield point*, or proportional limit, the material will spring back to its original undeformed state, in other words, the response is *elastic*. Once this yield strength has been exceeded, however, *plastic flow* occurs, and the material remains permanently deformed. The slope of the initial elastic region, called the *elastic modulus*, is a measure of the material's stiffness; the slope of the later inelastic region is a measure of *work hardening* (since it is the amount of stress needed to achieve a given amount of plastic flow). The values of these material properties depend on the *rate* at which the material is strained, and for instance realistic analyses of armor-penetrator impact require knowing both static and dynamic material properties. Static properties are easily measurable. Moreover they can serve as a starting point for an analysis of the material since dynamic properties often scale in the same direction as the static properties. Nevertheless, it is the dynamic deformation and failure processes that are of paramount interest, and these can only be understood by measuring properties at high strain rates, which will be studied in section 3 [2].

STRESS-STRAIN CURVE

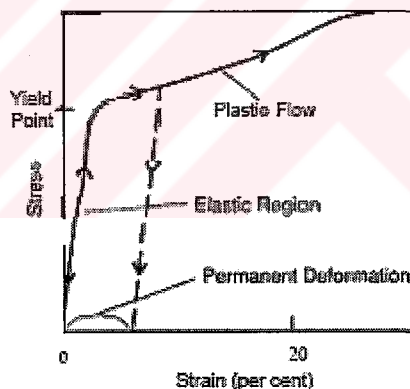


Figure 1.1 : Stress – Strain Curve [2]

Perfectly elastic or absolutely elastic bodies are those in which the deformation produced by external forces disappears completely after they cease to act. Perfectly inelastic bodies are those which completely retain, after the external forces cease to act, the deformation produced in them. There are neither perfectly elastic nor perfectly inelastic bodies in nature. However, some materials such as steel, wood, etc., may be closely approximated by perfectly elastic bodies. But even these materials can be regarded as perfectly elastic only upon to certain limits of loading

established by experiment. Beyond these limits, after the external forces are removed there remains a deformation in the body which cannot be neglected.

External forces acting on a solid produce internal forces which resist the external ones. Thus, for instance, if external forces stretch a solid, the internal forces will resist the stretching. As external forces increase, so do the internal forces. The internal forces in each material can increase only to a certain limit characteristic of this material. The external forces may be so large that the internal forces in a body of given dimensions will not be able to balance them and the body will fracture.

For various types of external forces, strength of materials establishes mathematical relations between external forces, geometric proportions of structural members, the resulting elastic forces and deformations. These relations and the strength characteristics of materials are used to determine the required dimensions of structural members. In establishing these relations certain assumptions and limitations are made. These assumptions and limitations are necessary because it is not possible to cover all the features of the phenomena under study as a whole.

First of all, the material of which the structures are made is considered to be continuous, homogeneous at all points of the body and having the same properties in all directions. The latter property of the material is called isotropy. Indeed, some structural materials, such as cast metals, possess high homogeneity (cast iron is an exception in this case). Other materials, such as wood, possess lower homogeneity as compared with metals. The more homogeneous the material and the more alike its properties in different directions, the closer is the agreement between theoretical and experimental results.

In choosing the material and determining the proportions and dimensions of structural members, the basic factors to be taken into account are: the conditions in which the member will operate, the requirements on its strength, lifetime and economy.

In some cases additional special requirements are imposed on structural parts to be designed; thus, in designing aircraft members and engines, the special requirement is a minimum weight. Of course, different requirements are placed on temporary structures which are built, say, for the duration of an emergency and on structures

which are to serve for many years. Some of the requirements placed on a structure are in contradiction with each other, such as strength, light weight and economy. Thus, increasing the wall thickness of a cylinder of an aircraft piston engine increases the strength and reliability of the cylinder, but its weight becomes heavier; also when the crankshaft of the same engine is drilled, the shaft becomes lighter, but the cost of the machining and hence the total cost is increased. The contradiction of these requirements prompts the development of the science of strength of materials.

In choosing the material and dimensioning the parts of a structure it is necessary to take into account simultaneously all the requirements placed on the structure, both the basic (strength, life-time, economy) and other special ones.

Without knowledge of the fundamentals of strength of materials it is not possible to construct even a simple machine satisfying the technical requirements placed on each construction. In strength of materials experiment and theory are closely interrelated; this science is simultaneously a theoretical and experimental one. All theoretical assumptions and conclusions are verified in practice and only after their validity is confirmed then they are accepted for use. Experiment also comes to the rescue when theory cannot solve a problem because of its extreme complexity.

1.4 Classification Of External Forces

External forces (loads) may act on machines and structural parts in different ways. According to the manner in which they are applied, forces may be divided into *body* and *surface forces*. The *gravitational force* (weight) of earth for example is an example of a body force. Surface forces on the other hand are divided into *distributed and concentrated* ones. Distributed forces are applied over an area or along a length. A snow layer on a roof and gas pressure on the walls of a vessel are examples of distributed loads. Such loads are expressed in units of force per unit area.

A load distributed along a length is expressed in units of force per unit length. A load may be uniformly or non-uniformly distributed along a length or over an area. For instance, the water pressure on a dam is a non-uniformly distributed load because the pressure increases with increasing depth.

Concentrated forces act over a very small area. A concentrated force is considered to be applied at a point for the sake of simplicity. This simplification introduces no serious error in calculations, as a rule. Concentrated forces are measured in units of force.

According to their nature of action, loads are divided into *static (and quasi-static)* and *dynamic ones*. A static load is defined as a load which increases slowly from zero to a certain maximum value and then remains constant or varies only slightly.

On the other hand, an impact load (the action of the ram of a steam hammer on pile) which is a type of a dynamic load, has a time duration only in the fraction of a second. Dynamic loads also include periodic loads varying in time. An example of such loads is the load on the connecting rod of an engine which varies continually in magnitude and direction. Dynamic loads include inertia forces developed during vibration [1].

1.5 Basic Analysis For Dynamic Loads

Many of the loads are dynamic in nature. If an analysis of dynamic loads to be made it can be said that, two more important properties of a structure come into play in addition to the stiffness. These three factors together resist the applied loads. *Stiffness* is proportional to the deformations and is independent of velocities and accelerations. *The mass of the structure* offers resistance proportional to the accelerations. The resistance offered by *damping* is not quite simple. Generally, it is assumed to be proportional to velocities.

The loads can also be classified as *periodic* and *transient*. The loads caused by eccentricities involved in reciprocating or rotating machinery are examples of periodic forces. These have a pattern that repeats itself in time with a known period. Transient loads do not have any regular pattern. The ground accelerations occurring during an earthquake is an example of a transient load. The duration is on the order of few seconds. Blast loading caused by air overpressure due to explosions, is another example of transient loading. Here, the duration is on the order of milliseconds and this type of load is also referred as impulsive load.

Figure 1.2 shows the types of dynamic loads,

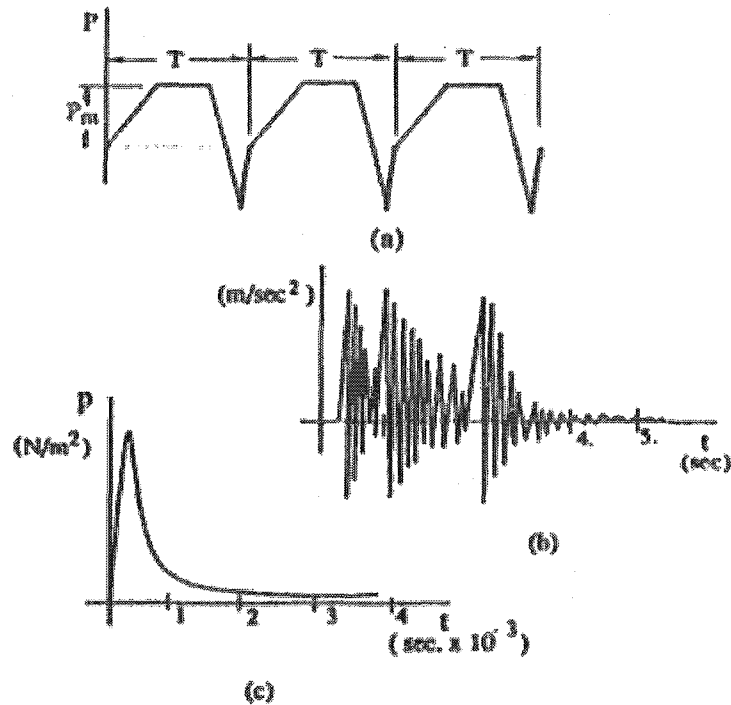


Figure 1.2 : Dynamic Loads. (a) Periodic force, (b) Transient force, (c) Impulsive force

The dynamic analysis of a structure subjected to dynamic loads is expected to provide a timewise history of displacements, stresses, and similar quantities of structure response.

1.6 Dynamic Deformation And Applications

The dynamic behavior of materials is an area of study in many scientific fields. When the materials are subjected to rapidly changing loads, the behavior can differ from those under quasi-static situations. An illustrative example can be the behavior of a sand bag, which was usually used at wars, to stop bullets. Soft, free-flowing sand is effective against impacts of a velocity on the order of 1500 m/s . On the other hand, a simple knife can defeat the sand bag. A solid wood or steel enclosure, however, if not excessively thick, can easily be perforated by a bullet, but a knife would never be able to penetrate it [3].

Dynamic processes refer to high-strain rate events are generally associated with rapid deposition and/or transmission of energy through shock or other modes of wave motion. They can be used to;

- i. Develop fundamental understanding of material response under extreme pressures, temperatures, and deformation and reaction rates,
- ii. Create new materials with unique microstructures and hence properties,
- iii. Monitor dynamic events in materials and probe and characterize the corresponding inelastic flow, fracture and failure modes [4].

In figure 1.3(a) schematic description of the scientific fields that are of importance in the study of dynamic processes can be seen. Though the applications can be divided into two topics such as civilian and military applications.

Figures 1.4 and 1.5 summarize in a schematic manner the common applications where *dynamic deformation* is involved. It is definitely clear that the civilian applications are mostly intended at producing something, whereas the military applications are either intended at defeating or protecting.

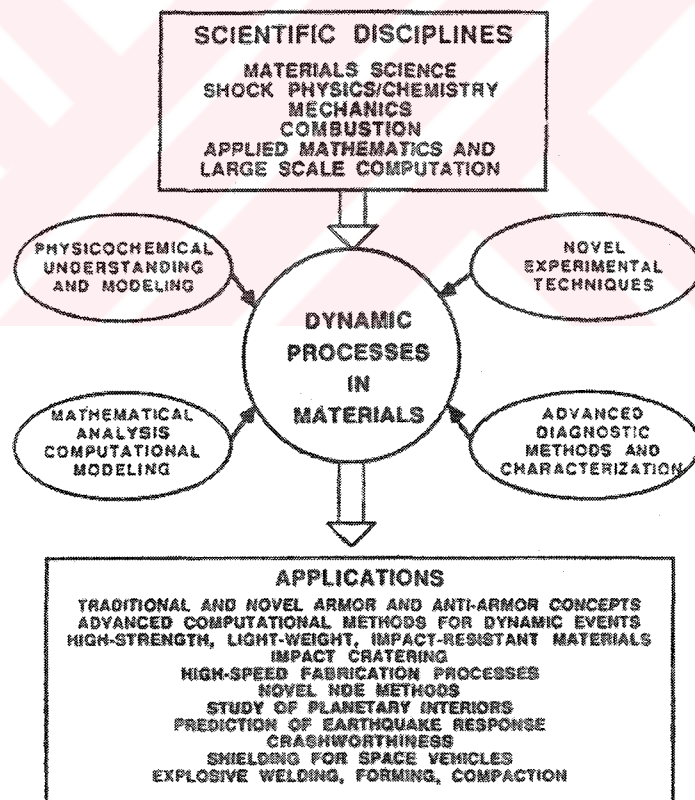


Figure 1.3 : Schematic representation of the contributing sciences (disciplines) and principal applications of dynamic processes in materials. [3]

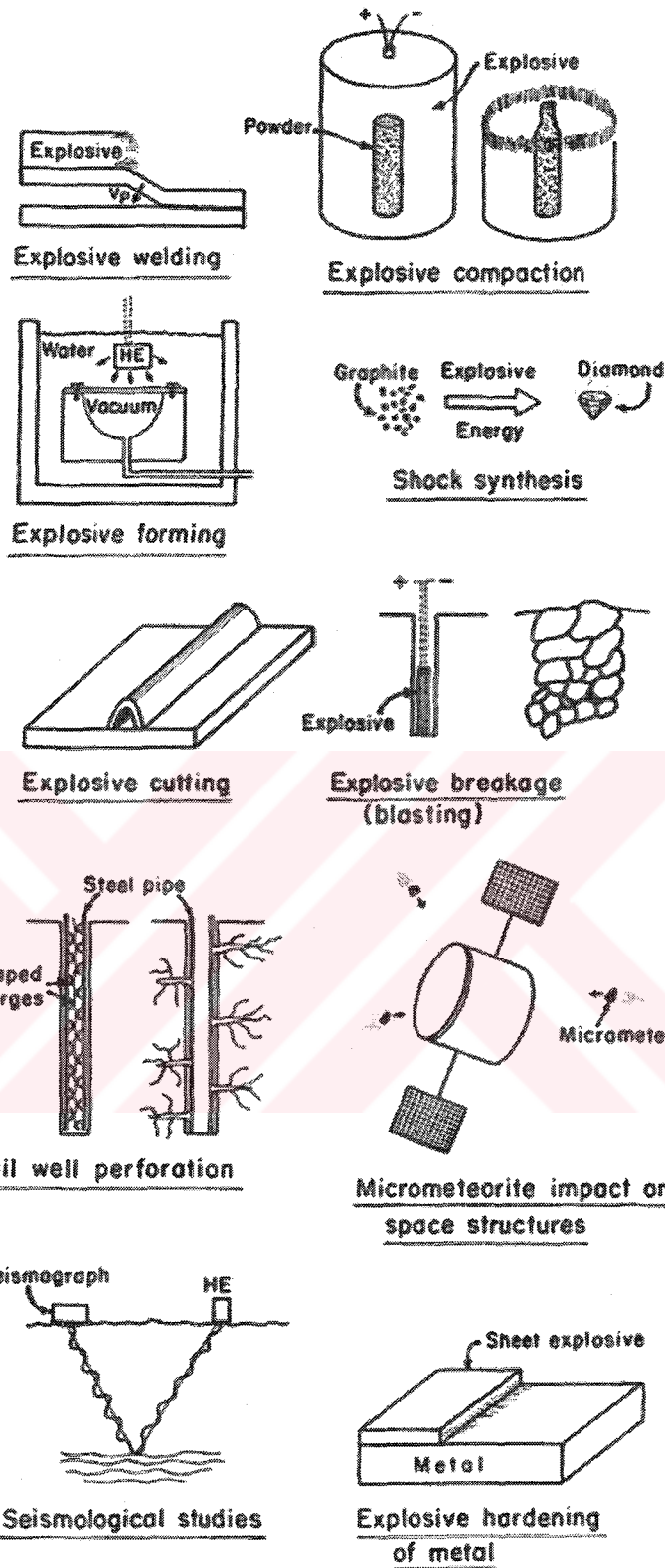


Figure 1.4 : Civilian applications of Dynamic Deformation where high strain rate phenomena is important [3]

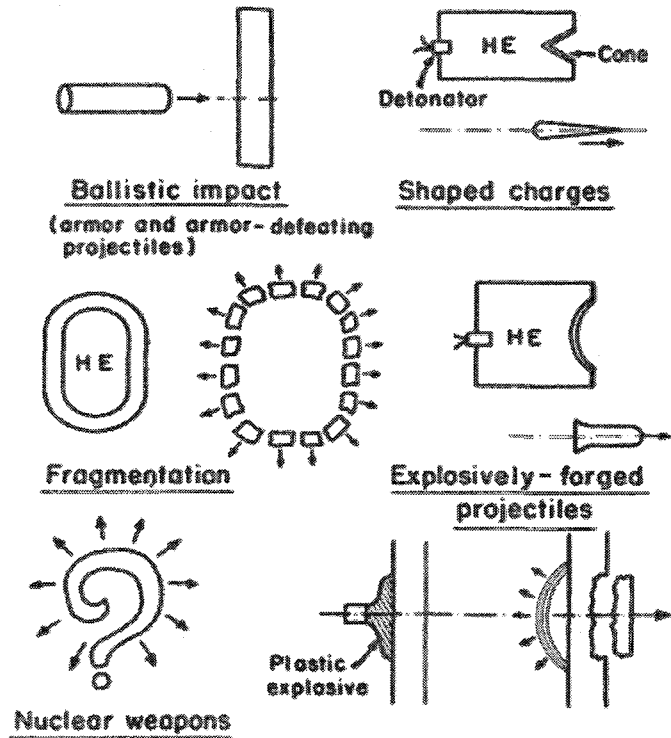


Figure 1.5: Military applications of Dynamic Deformation where high strain rate phenomena is important [3]

Among these applications, the most important one in industry is the explosive welding in which two metal plates are placed on top of each other with a specified spacing and the top plate is propelled at velocity V_p to the bottom one by detonating an explosive over the plates. This velocity produces a very high pressure peak at the impact interface of the plates and as a result the induced jet cleanses surfaces and metalurgically bonds the plates. In this method, metals with very different melting points which can not be welded together conventionally, such as aluminum or titanium to steel, can be welded.

The second important civilian application that has been succesfully implemented industrially is the explosive forming. The detonation of an explosive will produce the energy that drives the metal sheet against an anvil. This can be done in a transmitting medium such as water or through a direct contact operation (explosive in direct contact with metal to be formed).

Shock synthesis which was succesfully industrialized by DuPont for the production of diamond powder (from carbon) uses the energy of shock waves to bond fine metal

powders (in the micrometer range) by transmitting these waves through the powder and densifying it by plastic deformation. The shock wave deposits intense energy at the particle surfaces, often melting them and causing bonding. Metal, polymer and ceramic powders have been consolidated by this method often for the production of hard compacts of boron nitride and diamond for cutting tool applications.

Explosives are also used to cut metals by using a linear shaped charge and this method is named as explosive cutting. Detonation of the explosive produces a metallic jet that severs the metal beneath.

In mining and construction industry rock blasting method, the travelling of compressional and tensional waves through the rocks and fragmenting it, is another important technique.

In oil well drilling applications, the explosively generated shaped charges make everything easier by the help of shaped charges loaded down the hole and detonated in order to produce perforations in the rock for oil recovering.

Dynamic deformation caused by the micrometeorites impacting to space structures and satellites at velocities up to 30 *km/s* produce a great deal on damage and incites scientists to develop shielding mechanisms for space vehicles.

Seismological studies use reflected and refracted *P* and *S* waves from different levels within the earth and rely heavily on a complete understanding of wave propagation in porous media. For instance, underground nuclear detonations can be detected even from very far territories with special instrumentation by means of wave propagating through the earth.

The military applications (figure 1.5) in which impact, explosive detonation, and high-strain rate (dynamic) deformation used are numerous. The defeat of structures through high velocity projectiles and the mechanisms by which the armor can protect these structures are highly important. Among the projectiles used today, the shaped charge is worth describing. An explosive charge placed in contact with a hollow metal cone detonates and deforms the cone into a long rod that is accelerated to very high velocities (up to 10 *km/s*) which has a great penetration capability. Other concepts for projectiles are the kinetic energy penetrator, a long cylindrical rod of a

high-density metal (tungsten or uranium) travelling at a velocity of 1,5–2 km/s. This kinetic energy penetrator has a penetration capability into a steel target equal to approximately one-half of its length. Other forms of projectiles are the explosively forged projectile, which is similar to the shaped-charge one. The cone is replaced by a disk (shaped like a saucer) that is deformed by the explosive into a long rod. Yet another projectile is the one that has a plastic explosive head that deforms upon impact with an armor and then detonates, producing shock waves that propagate through the armor. These waves reflect at the inner surface of the armor and spall it. Thus, fragments are emitted from the inside without total perforation. Fragmentation rounds and projectiles are common and well known. An explosive charge located in the core detonates and transmits the energy to the shell, which fragments and these fragments travel at very high velocities.

In dynamic events, inertia and inner kinetics of materials become an important factor. In all applications, the simple equation relating kinetic energy with the velocity of a mass is of paramount importance;

$$E_C = \frac{1}{2}mv^2 \quad (1.1)$$

The kinetic energy of a mass increases with the square of its velocity. The energy delivered by an object of mass m to a target can be expressed as;

$$dE = Fdl \quad (1.2)$$

where dl is the length over which this force F will act. This kinetic energy is transformed into damage in projectile and target. The blow of a hammer can be presented as an example, such that the faster the blow the more penetration it will impart to the nail. All things being equal, the penetration of the nail quadruples, when its velocity is increased from v to $2v$.

As mentioned earlier, another concept, which is very important, is that the fundamental difference between static (or quasi-static) and dynamic deformation. In quasi-static deformation, at any time, static equilibrium is satisfied, namely, any point in the body has a summation of forces acting on it close to zero. When the deformation is applied at a very high rate by an external force, one portion of the

body is stressed while other portion has not experienced this stress yet. In other words, stress has to travel through the body. Stress and its associated strain or deformation travels in bodies at specified velocities that can be calculated to a good approximation. These are called *waves*, since they have usually well-established velocities. Thus, dynamic deformation often involves wave propagation, whereas quasi-static deformation can be considered as a sequence of states of equilibrium that can be treated by the well-known equations of mechanics of materials (summation of forces equal to zero, summation of moments equal to zero, compatibility of strains, constitutive relations).

As a conclusion, in rigid dynamics it is assumed that, when a force is applied to any one point on a body, the resultant stresses set every other point in motion instantaneously, and the force can be considered as producing linear acceleration of the whole body. In the theory of elasticity, on the other hand, the body is considered as in equilibrium under the action of applied forces, and the elastic deformations are assumed to have reached their static values. These treatments sufficiently accurate for problems in which the time between applied force and the setting up of equilibrium is short compared with the observation time.

However, when dealing with the forces, which are applied for only very short periods of time, or are changing rapidly, the effect must be considered in terms of the propagation of *stress waves*. Mainly, there are three types of stress waves that are described as follows; [3].

- i. Elastic Waves,
- ii. Plastic Waves,
- iii. Shock Waves.

1.7 Dynamic Propagation of Deformation and Lattice Dynamics

The application of an external force to a body can be defined as a dynamic process. However, if the rate of change of the applied force is low, then the process of deformation can be considered as a sequence of steps in which the body can be considered in static equilibrium. Figure 1.6 shows how the distance between atoms changes upon the application of an external force. For each stages of deformation

shown in figure 1.6(b) and (c), the body can be considered under static equilibrium and the methods of mechanics of materials can be applied to determine the internally resisting-stresses. Hence, a section made at AA or BB will yield identical stresses [4].

Also in figure 1.7(a) an array of material points in the undisturbed half-space is shown. 1.7(b) shows the displacements of the material points at a time t resulting from a compressional wave. The points only move in x_1 direction, and their motions depend only on x_1 and t [5].

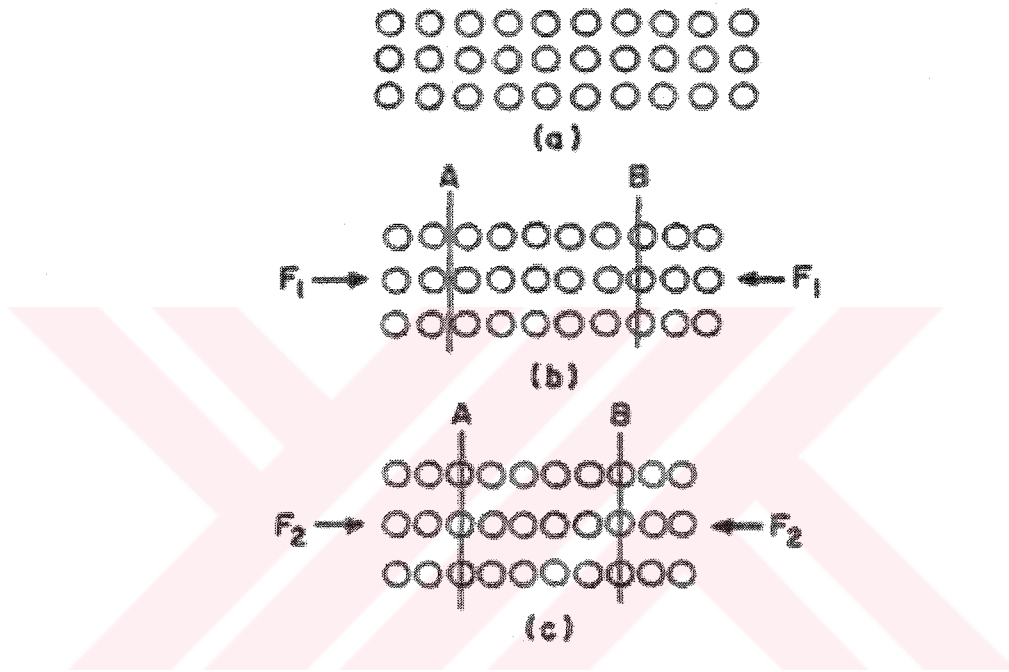


Figure 1.6. : Quasi-Static elastic deformation of a simple 2-D array of atoms ($F_2 > F_1$) [3].

For figure 1.6 however, the internal stresses are not instantaneously transmitted from the force application region to the different regions of the body. The stresses (and strains) are transferred from atom to atom at a certain specific velocity. Figure 1.8 shows the application of a force at a rate dF/dt such that the stresses (and attendant strains) vary from section to section. While section BB has not felt the applied force at time t , section AA has already effected by the external force F .

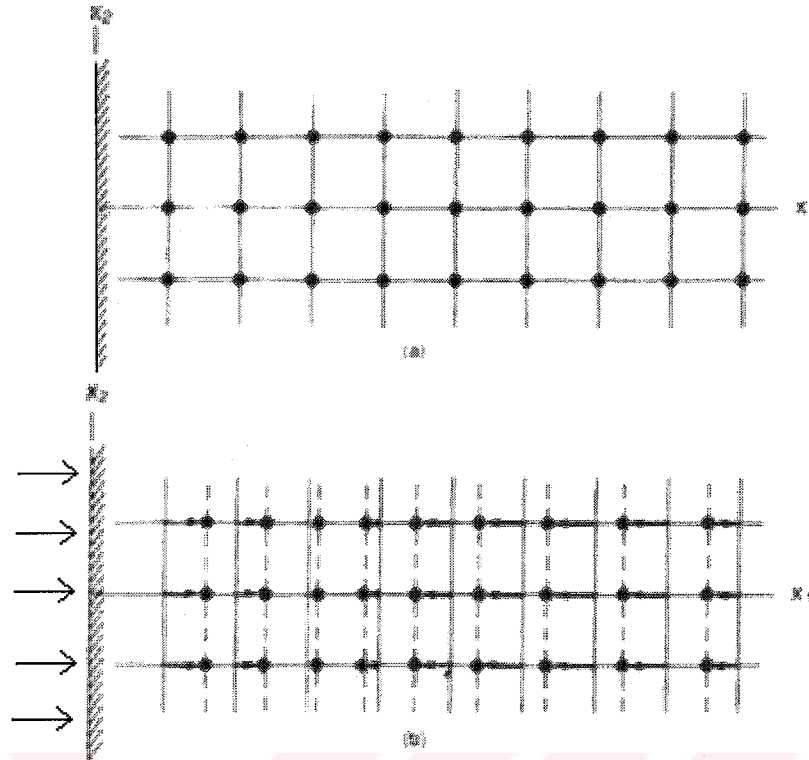


Figure 1.7 : (a) An array of points in the undisturbed half-space, (b) Displacements of the points due to a compressional wave [5].

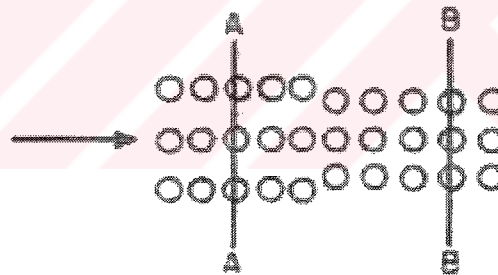


Figure 1.8 : Dynamic elastic deformation of an idealized array of atoms [3].

At an atomistic level, the wave can be defined as the succession of impacts between adjacent atoms. Each atom, upon being accelerated to a certain velocity, transmits all (or part) of its momentum to its neighbor(s). The mass, atom separation, and forces of attraction and repulsion between atoms determine the way in which the stress pulse is carried from one point to the other.

Individual atoms oscillate continuously about their equilibrium position, and the frequency of this oscillation is approximately 10^{13} oscillations per second. These

oscillations, although three dimensional, can be broken down along three dimensions (x_1, x_2, x_3). Figure 1.9 shows an idealized linear array of atoms. The equilibrium position is shown by the solid lines, and the extreme positions along the three axes are shown by dashed lines. This field of study is called *lattice dynamics* and the oscillations are actually not independent, but are coupled in waves called *phonons*. However, to simplify, we can consider that these atoms are only vibrating and thus when the atom is impacted on the left, this atom will transmit this impact to its neighbour on the right.

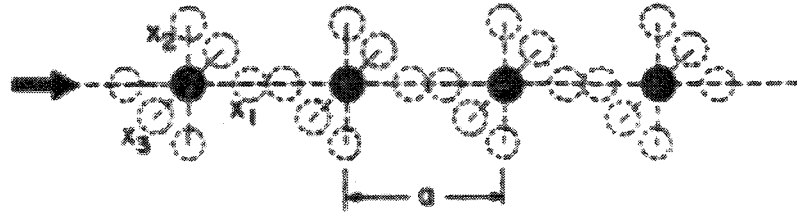


Figure 1.9. : Transmission of disturbances from an atom to an atom.
a = separation (distance between two atoms) [3].

All wave propagation equations can be derived by assuming that the material is a continuum (mass uniformly distributed in space), since they are a direct consequence of Newton's second law. It can be expressed as that these atoms are connected by interatomic forces that can be assumed as minute spring connecting spheres (atoms). Momentum is transferred from sphere to sphere (atom to atom) at a rate that provides the velocity of propagation of disturbances.

When the force on the left is applied, there will be a time lag involved in the transmission of this push equal in average to the period of vibration (10^{-13} s). The atoms in the middle will continue the process, transmitting the *push* (disturbance) to the atom on the right. If the distance between two atoms (separation) is known, the order of magnitude of velocity can be derived. For a solid metal (iron) it can be taken approximately equal to $3\text{\AA}$ (or 3×10^{-10} m). Thus;

$$V = \frac{a}{t} = 3 \times 10^{-10} \times 10^{13} = 3 \times 10^3 \text{ m/s} \quad (1.3)$$

This is very close to the velocity of propagation of an elastic wave in iron (3500 m/s). This simple reasoning explains the physical sequence of momentum transfer at the

atomistic level [3]. The details of elastic wave velocity and the wave motion equation for a cylindrical bar will be mentioned in more details in section 4.

1.8 Waves in Solids

1.8.1 Elastic Waves and Types of Elastic Waves

Different types of elastic waves can propagate in solids, depending on how the motion of the particles of the solid is related to the direction of propagation of the waves themselves and on the boundary conditions. Particle, here represents a small discrete portion of the solid. It is not an atom, which may move in a direction different from the general direction of motion due to crystallographic requirements. The most common types of elastic waves in solids are;

- a. *Longitudinal (irrotational) Waves:* In seismology also known as *push, primary or P waves* and in finite and semi-finite media, they are known as *dilatational waves*. These waves correspond to the motion of the particles back and forth along the direction of wave propagation such that the particle velocity (U_p) is parallel to the wave velocity (U). If the wave is compressive, they have the same sense; if it is tensile, they will have opposite senses. An example of a longitudinal elastic wave is shown in figure 1.10(a). This wave is produced by the impact of a hammer on the left-hand side of the cylinder. Figure 1.10(b) shows, schematically, the impact of a hammer on a semi-finite body. In addition to longitudinal waves, distorsional and Rayleigh waves are also generated. The velocity of a wave in a bounded medium (i.e. cylinder) is different from that in an unbounded medium.
- b. *Distorsional (Shear) Waves:* If the motions of the particles conveying the wave are perpendicular to the direction of the propagation of the wave itself, then this type of wave is called as distorsional wave. All longitudinal strains ($\epsilon_{11}, \epsilon_{22}$ and ϵ_{33}) are zero. An example of distorsional wave is illustrated in figure 1.11. When a bar is subjected to a torsion, the region between the end and the clamp will store elastic energy. Upon release of the clamp, a pulse will travel in the bar, toward the right. This is a distorsional wave, and the

particle displacement is perpendicular to the wave propagation direction (U_p is perpendicular to U).

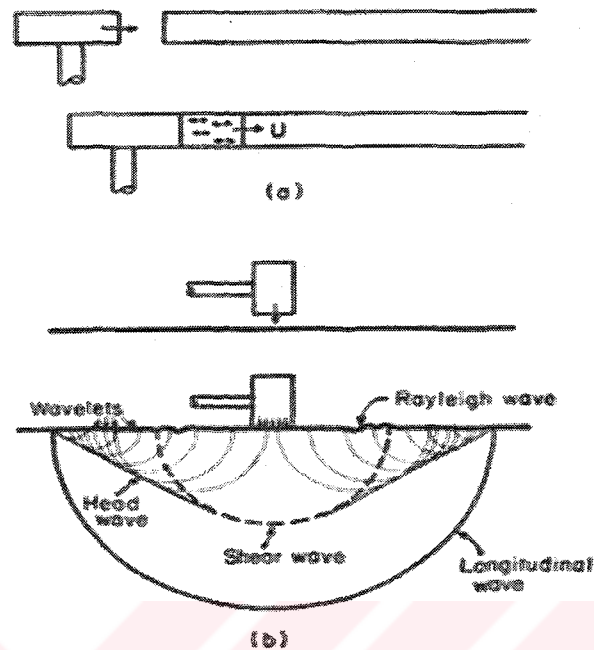


Figure 1.10 : (a) Hammer impacting slender body; (b) Hammer impacting semi-infinite body; notice formation of longitudinal, shear and Rayleigh waves. Interaction of longitudinal wave with free-surface forms wavelets that comprise head wave. [3]

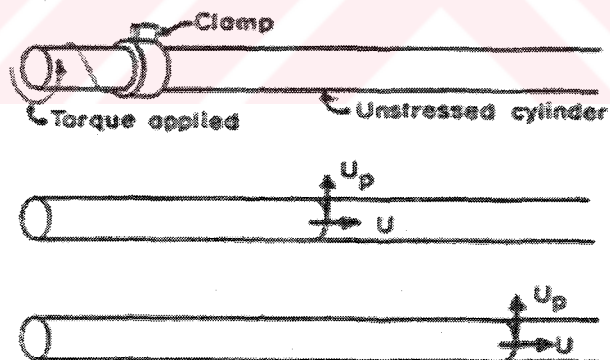


Figure 1.11: Wave and particle motion in release of torsional energy, when clamp is released, particle displacement (on cross-sectional plane) travels along longitudinal axis. [3]

- c. *Surface Waves:* Surface waves are analogous to waves on the surface of water. Objects floating on the water act as markers for the particles of water and move both up and down and back and forth, tracing out elliptical paths as the water moves by. This type of wave is restricted to the region adjacent to the surface, and particle velocity decreases very rapidly (exponentially) as

one moves away from it. The particles describe elliptical trajectories as illustrated in figure 1.12. Surface waves (called Rayleigh waves in solids) are a particular case of interfacial waves when one of the materials has negligible density and elastic wave velocity.

- d. *Interfacial (Stoneley) Waves*: When two semi-finite media with different properties are in contact, special waves form at their interface.
- e. *Waves in Layered Media (LOVE Waves)*: In Seismology, these waves are very important and took its name from Love who was the first person to study these waves. It is known that earthquakes produce waves in which the horizontal component of displacement can be significantly larger than the vertical component, a behavior not consistent with Rayleigh waves. The earth is composed of layers with different properties and special wave patterns are emerged.
- f. *Bending (flexural) Waves (in bars, membranes, plates)*: These waves involve propagation of flexure in a one-dimensional (bar) or in two-dimensional configuration. For the mathematics of wave propagation in plates and shells more information can be found from **Graff** [6].

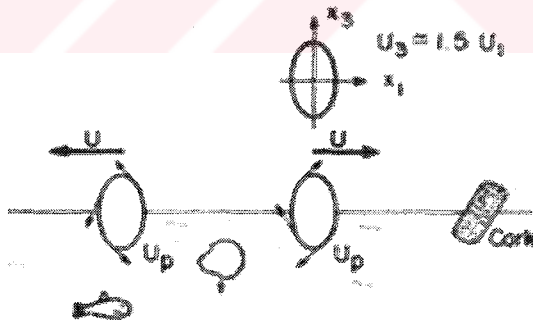


Figure 1.12 : Surface waves; particle and wave velocities shown; cork would describe elliptical trajectory. Rayleigh waves are analogous to surface (gravity) waves in fluids [3].

The surface waves among all are the slowest where longitudinal waves are the fastest ones. Figure 1.13 shows, in a schematic manner, the amplitudes of the longitudinal, shear and Rayleigh waves in half space. The shear wave can undergo a change in particle motion sense ($\pm$); at a point vertically below the load application region of

this particle velocity and, consequently, the shear wave amplitude are zero. The horizontal and vertical components of the particle motions as a function of distance from the surface in the Rayleigh wave are also shown. The exponential decay of the horizontal component, on the right side, is clearly seen. Also the waves decay at different rates. For example, the amplitude of the longitudinal wave decays more readily close to the surface; the width of the band indicates its relative value. The amplitudes of the longitudinal and shear waves decay as r^{-1} in the region away from the free surface. Along the surface however, they decay faster, as r^{-2} . The Rayleigh wave decays much slower at a rate of $r^{-1/2}$ so, it can be picked up at much larger distances.

Figure 1.13 also shows how different waves carry different fractions of energy. Rayleigh waves carry most of the energy, 67%, shear waves come in second place with 26 % and longitudinal (compression) waves come third with only 7% [3].

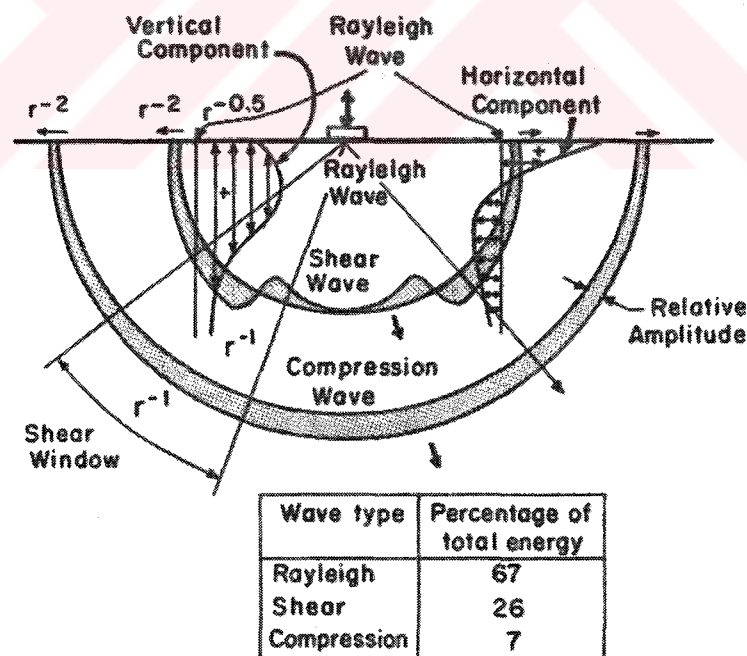


Figure 1.13 : Distribution of displacement and energy in dilatational, shear, and surface waves from a harmonic normal load on a half-space of $\nu = 0.25$ [3].

1.8.2 Plastic Waves and Types of Plastic Waves

When the stress in a ductile material exceeds the elastic limit, plastic deformation sets in. This occurrence is valid for both in quasi-static and dynamic deformation. If a pulse is transmitted to a material that has an amplitude exceeding the elastic limit, this pulse will decompose into an elastic and plastic wave. There are mainly three types of plastic waves [3].

- a. *Plastic Waves in Rods, Wires and Bars:* The classical example of this situation is the impact of a long plastic rod against a rigid target. If the lateral dimensions of the rod (diameter) is small, a state of uniaxial stress is established.
- b. *Plastic Waves in Semi-Infinite Bodies:* When the lateral strains (perpendicular to the direction of propagation of the stress front) are zero, a state of uniaxial strain is established and the resulting plastic wave has a very sharp front; it is therefore called a shock wave.
- c. *Plastic Shear Waves:* Torsional waves in bars or shear waves in semi-finite bodies can, if their amplitude is sufficiently high, generate plastic deformation.

1.8.3 Elastic & Plastic Wave Velocity

Material strength is strain and strain rate dependent, as exemplified by figure 1.14. For metals, the stress-strain curve is often represented by a bilinear function, Figure 1.14 (a), where the first stage is elastic and the second stage is plastic.

The stress-strain curve in many metals approaches more closely a power function of the type;

$$\sigma = \sigma_0 + k\varepsilon^n \quad (1.4)$$

where the strain hardening exponent is $0 < n < 1$ for metals [figure 1.14 (b)]. The strain dependence of the flow stress is schematically illustrated in figure 1.14(c): if the strain rate is changed during the test, the flow stress is accordingly changed.

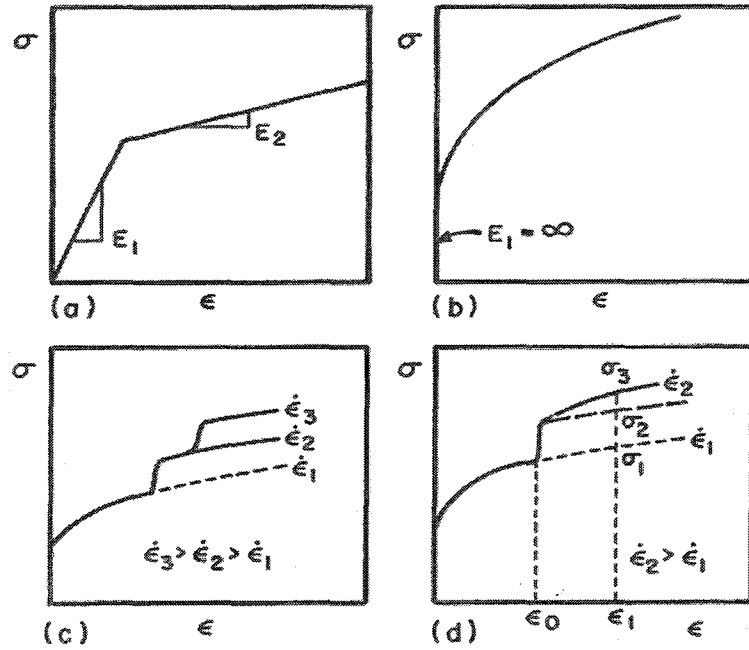


Figure 1.14 : Stress-Strain curves for ductile materials : (a) bilinear elastoplastic; (b) power law work hardening; (c) strain-rate dependent flow stress; (d) strain rate history dependence of flow stress. [3]

This dependence of flow stress on strain rate is often represented by;

$$\sigma = \sigma_0 + k\epsilon^n \dot{\epsilon}^m \quad (1.5)$$

where m is the strain rate sensitivity and usually varies between zero and unity for metals. An additional factor that often has a noticeable effect is the strain rate history. Figure 1.14(d) shows how this effect can change the mechanical response of a material. If the strain rate of a material is changed from $\dot{\epsilon}_1$ to $\dot{\epsilon}_2$ at a strain ϵ_1 , the flow stress changes from σ_1 to σ_2 . However, if the strain rate change occurred earlier, at a strain of ϵ_0 , the work hardening rate of the material could have been modified by the higher strain rate. In that case, the material would exhibit a flow stress σ_3 at a strain ϵ_1 and strain rate $\dot{\epsilon}_2$. Figure 1.14(d) shows how the work hardening at the strain rate $\dot{\epsilon}_2$ is higher than at the strain rate $\dot{\epsilon}_1$. If it were the same, it would follow the dashed line. The treatment of plastic waves increases in complexity as strain rate and strain rate history effects are incorporated [3].

The theory of the propagation of plastic waves in metal wires, when the stress exceeds the yield stress, was developed independently by **Karman - Duwez** [7], **Rakhmatulin**[8] and **Taylor**[2]. In treating the problem, von Karman and

Rakhmatulin used Lagrangian coordinates, whilst Taylor used the coordinates of Euler. When suitable transformations of the notations are made, the two treatments may be shown to give the same result, actually the physical approach is different [3-9]. Even in the simple case of one-dimensional extensional waves, the velocity, C_p or V_p , of a plastic wave of loading is a function of stress or the strain. If σ^* is the “nominal” or “engineering” stress, i.e. the ratio of the total force over a cross section to the original, unstressed area of the section, and if ε is the strain, i.e. the fractional increase in length of an element, C_p is given by the equation;

$$C_p = \sqrt{\left(\frac{1}{\rho} \frac{d\sigma^*}{d\varepsilon} \right)} \quad (1.6)$$

where $d\sigma^*/d\varepsilon$ is the slope of the plastic region of the stress-strain curve and ρ being the density of the material. C_p is the constant only at a fixed strain ε . It can be clearly seen that $d\sigma^*/d\varepsilon$ is equal to E , Young’s Modulus, in the elastic region and so the eqn. (1.16) then reduces to the equation $C = \sqrt{(E/\rho)}$ for the elastic wave velocity in the elastic deformation region. Since this slope, $d\sigma^*/d\varepsilon$, is higher in the elastic region than in the plastic region we can express that;

$$\left(\frac{d\sigma^*}{d\varepsilon} \right)_{el} > \left(\frac{d\sigma^*}{d\varepsilon} \right)_{pl} \quad (1.7)$$

In the case of most metals, $d\sigma^*/d\varepsilon$ decreases as ε increases, when σ^* exceeds the yield stress; plastic waves will usually travel with a lower velocity than elastic waves and in the plastic region, the velocity of propagation will decrease as the stress and the strain increase.

Thus, the velocity of plastic waves is lower than the velocity of elastic waves. A qualitative understanding can be reached from the observation of a stress – strain curve. As the velocity decreases with decreasing work-hardening rate, the tendency is for the front of the wave to “spread out”, or disperse ($V_2 < V_1$) in figure 1.15. [9-11].

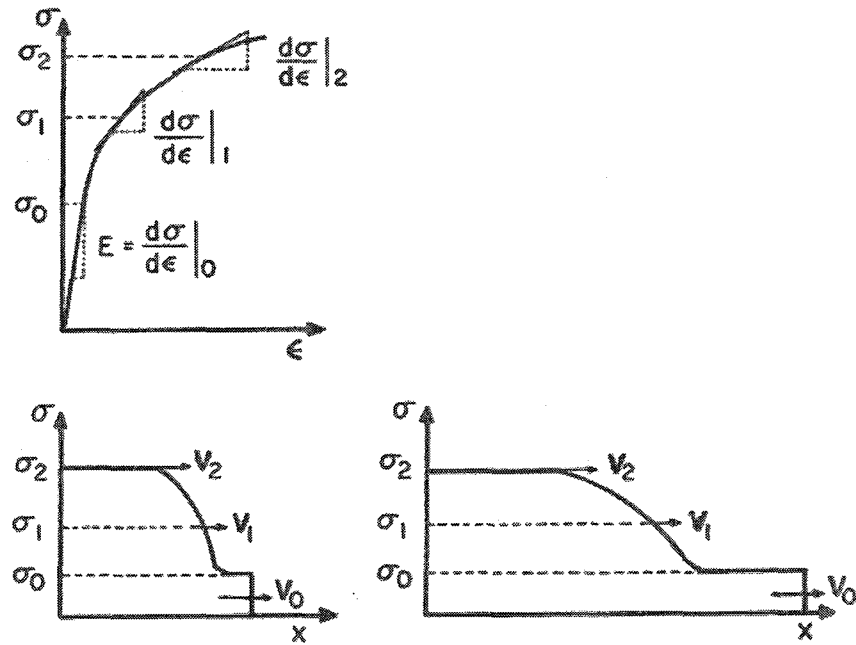


Figure 1.15. : Shape of plastic wave front as a function of time; dispersion of wave is observed due to a decrease in wave velocity with increasing stress. [3]

1.8.4 Shock Waves

Shock waves are a kind of plastic waves, where the strain perpendicular to the applied load can be neglected. To obtain this kind of wave front, very high velocities should be ensured.

Shock waves are characterized by a steep front and require a state of uniaxial strain which allows the buildup of the hydrostatic component of stress to high levels. When this hydrostatic component reaches levels that exceed the dynamic flow stress by several factors, the solid can be assumed as having no resistance to shear. The calculation of shock wave parameters is based, in its simplest form, on the Rankine-Hugoniot conservation equations [3].

Basic assumptions for shock waves can be briefly described as;

- A shock is a discontinuous surface and has no apparent thickness,
- The shear modulus of the material is assumed to be zero, such that it responds to the wave as a fluid; hence the theory is restricted to higher pressures,
- Body forces (such as gravitational) and heat conduction at the shock front are negligible,

- There is no elastoplastic behavior,
- Material does not undergo phase transformations.

The fundamental requirement for the establishment of a shock wave is that the velocity of the pulse, U , increases with increasing pressure;

$$\left(\frac{\partial^2 P}{\partial^2 U} \right) > 0 \text{ as } P \uparrow, U \uparrow, \quad (1.8)$$

The necessary Rankine and Hugoniot equations can be easily developed by considering regions immediately ahead of and behind the shock front. Figure 1.16 illustrates a shock front. Ahead of the front, the pressure is P_o , the density is ρ_o , and the temperature is T_o . Behind this shock front they are named as P , ρ and T , respectively. The velocity of the front is U_s ; the particles (or atoms) are stationary ahead of the front. At the front and behind it, they are moving at a velocity U_p . This displacement of particles is responsible for the pressure build-up.

If the center of reference is considered as the shock front and moves with it at a velocity U_s into a region of particle velocity $U_p = U_o$ and density ρ_o , then the apparent velocity of the fluid moving toward the center of reference is $U_s - U_o$. At the same time, the material leaving the front or receding from the center of reference is $U_s - U_p$.

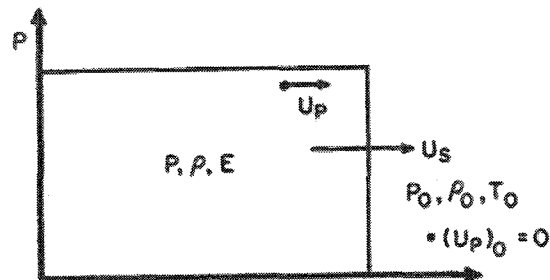


Figure 1.16 : Schematic of a Shock Front [3].

With the help of this information the equations for the conservation of mass, momentum and energy can be developed as;

$$\text{Conservation of Mass} \rightarrow \rho_o U_s = \rho(U_s - U_p) \quad (1.9)$$

$$\text{Conservation of Momentum} \rightarrow (P - P_o) = \rho_o U_s U_p \quad (1.10)$$

$$\text{Conservation of Energy} \rightarrow (E - E_o) = \frac{1}{2}(P + P_o)(V_o - V) \quad (1.11)$$

In these conservation equations there are five variables, (P) is for pressure, (U_p) is for particle velocity, (U_s) is for shock velocity, (V) is for specific volume or density (ρ) and (E) is for energy. Note that the quantity $\rho_o U_s$ is often called as the *shock impedance*.

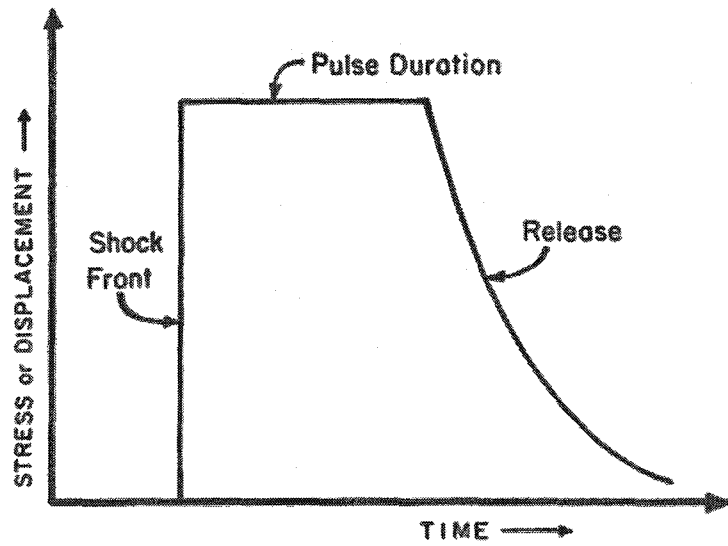
An additional equation is needed if one wants to determine all parameters as a function of one of them. This equation can be conveniently expressed as the relationship between shock and particle velocities by determining it experimentally;

$$U_s = C_o + S_1 U_p \quad (1.12)$$

Equation (1.12) is also known as the equation of state (*EOS*) of a material. Here S_1 is an empirical parameter and C_o is the sound velocity in the material at zero pressure.

The linear relationship between U_s and U_p describes fairly well the shock response of materials not undergoing phase transitions. Values of S_1 and C_o are often tabulated in the literature and by using the necessary values from these tabulations and applying them in equations (1.9) to (1.12), the relationships, $P-U_s$, $P-U_p$, $P-\rho$, $P-V/V_o$, $E-U_s$ and others, can be calculated.

Shock waves are very complicated in real applications, real shock waves exhibit a number of peculiarities that are material and pressure dependent. However, an ideal shock wave profile would predict a discontinuity at the front, a plateau at the top and a gradual return to zero pressure. Figure 1.17 shows a an idealized and a realistic shock wave profile. Since in the application of SHPB bar tests, the shock waves are not included, one can refer to [3, 9,73] for further knowledge.



(a)

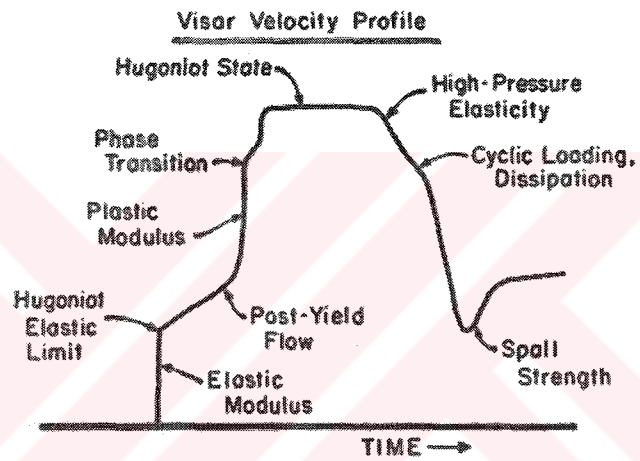


Figure 1.17 : (a) idealized, (b) realistic shock wave profile [3].

2. CONSTITUTIVE EQUATIONS

As it was discussed in section 1, the mechanical behaviors of a material, such as yield stress, ductility and strength, will change under different strain-rate loadings and temperatures. The consideration of rate and temperature dependence of material is very important in the design of structures. To solve mechanical problems with numerical methods and to get high quality results a sufficient description of the material behavior is needed.

This description is needed because the material behaviour cannot be expressed in a general way. Understanding of the micromechanics certainly has made good progress for better solutions however many questions are still open. Therefore scientists have to concentrate on specific deformation or loading conditions, and on certain temperature – strain rate ranges. Hot working, creep, cyclic loading, unloading and history effects are excluded from this concentration. However cold working, monotonic straining transients, as well as temperatures between 50 and 500 K and strain rates between 10^{-5} to 10^5 s^{-1} should be considered in this concentration.

The mathematical formulation describing the macroscopic response of a material to applied force is known as constitutive relation, or equation. In the most general form, a constitutive equation relates the stress of a material to the strain, strain rate, temperature, and loading history experienced by the material. It is not possible, however, for a single relationship to completely describe the deformation behaviour of a material for all combinations of loading conditions and histories. Depending on the intended use, availability of experimental data, and the desired accuracy of the description, a decision must be made as to the amount of effort to be invested. Typically, the effort required to describe the material behaviour is proportional to the degree of accuracy with which the governing physical mechanisms are modelled. An ideal plasticity model in general for metals should be able to describe the materials' properties such as strain-rate dependence, temperature dependence, strain and strain-rate history dependence, work-hardening or strain-hardening behavior (both isotropic and anisotropic hardening); however, a complete description of these phenomena is an extremely difficult issue.

The availability of current high-speed computers allows for the implementation of more complicated, physically based constitutive relations into codes for solving problems involving deformation of materials at high strain rates. The ability to accurately describe material behaviour for a wide range of loading conditions and to predict the mechanical response outside of the measured condition is growing in demand. Despite this, most of the constitutive relations implemented into computer codes describe only the effects of strain hardening, and take some account of thermal softening, but few relations presently account for strain rate effects [12,13].

Among all there are four widely used relations, with approximately the same number of material constants, that do account for strain rate effects, Johnson-Cook, Zerilli-Armstrong, Bodner-Partom, and Khan-Huang constitutive relations. In this chapter, among these most widely used constitutive equations, a brief presentation of the Ludwik Equation, Litonski's model and MTS model will be carried out as well.

2.1 Ludwik Equation

The Johnson-Cook (JC), Zerilli-Armstrong, Bodner-Partom (BP) and Khan-Huan (KH) constitutive relations are relatively simple, one dimensional models that account for the effects of strain, strain rate and thermal softening on flow stress and utilize a Von Mises yield criterion. They both describe the material hardening behaviour based on the well known power-law function introduced by Ludwik (also called parabolic hardening), one of the simplest forms of describing the material behavior:

$$\sigma = \sigma_0 + K\varepsilon^n \quad (2.1)$$

At low and constant strain rates, metals are known to work harden along this well known relationship. Here, σ_0 is the yield stress, n is the work-hardening coefficient, and K is the pre-exponential factor. Using the constants and the work-hardening coefficient n , it is often used to characterize the stress, σ , as a function of the strain, ε , at ambient temperatures and quasi-static loading. Ludwik claims that the strain hardening develops monotonically with a constant to the power n . Indeed, this is often true, at least beyond a transient for the very first plastic deformation, up to a saturation of stress [13].

2.2 Litonski's Model

To include the influence of temperature, T , and strain rate, $\dot{\epsilon}$, the Ludwik equation was extended in several ways by **Klopp, Clifton and Shawki** [14] and expressed as;

$$\sigma = \sigma_0 \epsilon^n T^{-\nu} \dot{\epsilon}_p^m \quad (2.2)$$

where m being the strain rate hardening parameter and n , and ν being the temperature softening parameter.

Litonski split up the strain into a constant, ϵ_0 (approximately representing the elastic part), and plastic strain, ϵ_p , and expressed the temperature influence linearly instead of as a power function;

$$\sigma = \sigma_0 (\epsilon_0 + \epsilon_p)^n (1 - aT)(1 + b\dot{\epsilon}_p)^m \quad (2.3)$$

or,

$$\sigma = \sigma_0 (\epsilon_0 + \epsilon_p)^n [1 - a(T - T_0)](1 + b\dot{\epsilon}_p)^m \quad (2.4)$$

In equation (2.4) the linear thermal softening may seem overly specialized, it is known that the initial rate of softening often dominates the response in the early stages before a shear band is fully formed. Thus, the coefficient $a \equiv -g_T(T_0)$, which determines the initial rate, has great importance, but the remaining shape of the softening function, $g(T)$, has less importance. Later in the process, after the band is fully formed, the shape of the softening function has much greater effect.

In addition to Litonski's model, it is found by other researchers that an exponential expression of temperature provided a better fit to their results for aluminium, copper and mild steel [13];

$$\sigma = F(\epsilon)^n \left(\frac{\dot{\epsilon}}{\dot{\epsilon}_0}\right)^m \exp\left(\frac{W}{T}\right) \quad (2.5)$$

2.3 Johnson-Cook (JC) Constitutive Equation

Johnson and Cook proposed a constitutive model for metals subjected to large strains, high strain rates and high temperatures. The JC constitutive relation has been very successful because of its simplicity and availability of parameters for wide range of materials. It is a constitutive relation that is primarily intended for computations and the basic form of the model is well suited for computations because it uses variables that are readily available in most of the applicable computer codes. It is recognized that more complicated models may certainly give more accurate descriptions of material behavior. Similarly, various models may give better descriptions for various materials. However, in many instances, the computational codes cannot readily incorporate complicated and diverse models [15].

Johnson and Cook proposed the following empirically based, one dimensional relation which describes the variation of the von Mises flow stress, σ , with respect to strain, strain rate and temperature;

$$\sigma = (A + B\varepsilon_p^n) \left(1 + C \ln \dot{\varepsilon}^*\right) \left(1 - T_h^{*m}\right) \quad (2.6)$$

where ε_p is the equivalent plastic strain, $\dot{\varepsilon}^* = \dot{\varepsilon} / \dot{\varepsilon}_0$ is the non-dimensional strain rate, ($\dot{\varepsilon}_0$ is normally to be taken as 1.0 s^{-1}), and;

$$T_h^* = \frac{(T - T_r)}{(T_m - T_r)} \quad (2.7)$$

T_h^* is being the homologous temperature where T is the absolute temperature of the specimen, T_r and T_m are the reference (room) temperature and melting temperature of the material respectively. T_r must be chosen as the lowest temperature of interest or of the lowest temperature of experiments because parameter m is normally a decimal number and T should be larger or equal to T_r to make eqn. (2.7) valid. The five material constants are A , B , n , C and m . It should be noted that all of the temperatures are in units of degrees K [3,15].

In eqn. (2.6), the expression in the first set of brackets gives the dependence on strain, the second one represents instantaneous strain-rate sensitivity and the third

term represents the temperature dependence of stress through thermal softening. This approach does not represent any thermal or strain-rate history effects, but is simple to implement and the parameters are readily obtained from a limited number of experiments.

In general, the JC model represents a set of models that consider that the mechanical behaviors of material are the multiplication effects of strain, strain rate and temperature. This form is simple and has a clear physical interpretation; but the main problem for this form is the characteristic behavior of work hardening it represents. In the JC model, the work hardening rate, $\partial\sigma/\partial\varepsilon$, at certain strain will increase when strain rate increases; the stress-strain curve is upward when strain rate increases. This characteristic implies that the JC model, eqn. (2.6), is appropriate to model the work hardening behavior of metals such as OFHC copper and nickel (figure 2.1). It is certainly not appropriate for modelling any metals where the work hardening rate decreases when strain rate increases, such as tantalum (figure 2.2). It is also not appropriate for materials where the work hardening remains constant with increasing strain rate [15].

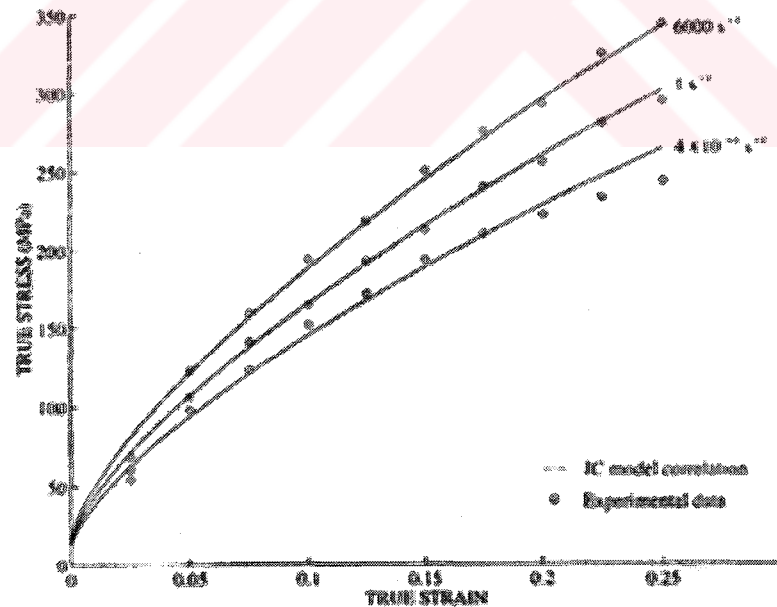


Figure 2.1 : Comparison of experimental data and the Johnson-Cook relation correlations for OFHC Copper [15].

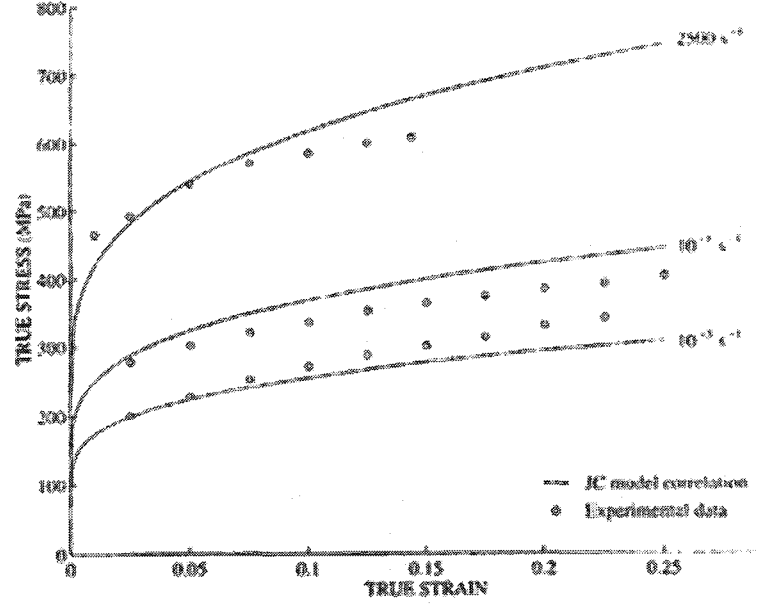


Figure 2.2 : Comparison of experimental data and the Johnson-Cook relation correlations for tantalum [15].

For many ductile materials the yield strength increases more rapidly with strain rate than that described by the form of eqn. (2.6) for strain rates exceeding $10^3/s$. To increase the strain rate sensitivity a modified Johnson-Cook (MJC) strength relation was proposed with the following form;

$$\sigma = (A + B\varepsilon_p^n) \dot{\varepsilon}^{*\alpha} (1 - T_h^{*m}) \quad (2.8)$$

where α is an empirical factor and all the other parameters are as described above. However, the MJC strength relation does not appear to be widely used today perhaps because the strain rate sensitivity is not significantly enhanced over that provided by eqn. (2.8).

Many ductile metals display an enormous increase in yield stress for strain rates in excess of $10^3/s$. This observed behavior provided the motivation for the revised Johnson-Cook (RJC) strength relation. The proposed form of the RJC strength model is;

$$\sigma = (A + B\varepsilon_p^n) \left[1 + C \ln \dot{\varepsilon}^* + D \left(\frac{1}{E - \ln \dot{\varepsilon}^*} - \frac{1}{E} \right) \right] (1 - T_h^m) \quad (2.9)$$

where C , D and E are additional empirical coefficients [16].

Johnson-Cook provided constants for 6 ductile and 6 less ductile materials, including Cu, Fe, Brass, Ni, C-Steel, Tool Steel (HSS), Al alloys and DU (Depleted Uranium). By this broad source of data, the Johnson-Cook equations are often used as constitutive equations for calculations [13]. Table 2.1 shows the constitutive constants for various materials [3].

Table 2.1 : JC Constitutive Constants for Various Materials ($\dot{\epsilon}_0 = 1s^{-1}$) [3].

Material	Description			Constitutive Constants for $\sigma = [\sigma_0 + B\epsilon^n][1 + C \ln \dot{\epsilon}][1 - T^*]$				
	Hardness (Rockwell)	Density (kg/m ³)	Specific Heat (J/kg K)	Melting Temperature (K)	σ_0 (MPa)	B (MPa)	n	C
OFHC copper	F-30	8960	383	1356	90	292	0.31	0.025
Cartridge brass	F-67	8520	385	1189	112	505	0.42	0.009
Nickel 200	F-79	8900	446	1726	163	648	0.33	0.006
Armco iron	F-72	7890	452	1811	175	380	0.32	0.060
Carpenter electrical iron	F-83	7890	452	1811	290	339	0.40	0.055
1006 steel	F-94	7890	452	1811	350	275	0.36	0.022
2024-T351 aluminum	B-75	2770	875	775	265	426	0.34	0.015
7039 aluminum	B-76	2770	875	877	337	343	0.41	0.010
4340 steel	C-30	7830	477	1793	792	510	0.26	0.014
S-7 tool steel	C-50	7750	477	1763	1539	477	0.18	0.012
Tungsten alloy (.07Ni, .03Fe)	C-47	17000	134	1723	1506	177	0.12	0.016
Depleted uranium-0.75% Ti	C-45	18600	117	1473	1079	1120	0.25	0.007

2.4 Zerilli-Armstrong (ZA) Constitutive Equation

Zerilli and Armstrong proposed dislocation mechanics based constitutive relations for material dynamic calculations. The effects of strain hardening, strain-rate hardening and thermal softening based on the thermal activation analysis have been incorporated into a constitutive relation. The relation has a relatively simple expression compared to other dislocation based constitutive models. One of the main points they have proposed was that each material structure type (FCC,BCC,HCP) will have a different constitutive behavior based on the different rate-controlling mechanism for that particular structure. Zerilli and Armstrong based their model on the framework of thermally activated dislocation motion and they analyzed the temperature and strain rate response of typical FCC and BCC metals and observed a significant difference between these materials. Such that, the BCC metals exhibit a much higher temperature and strain rate sensitivity than the FCC metals. Therefore, it has been given two different constitutive relations for BCC and FCC materials [3,15].

For a BCC material, which is modeled similar to Johnson-Cook, the ZA model has the following relation;

$$\sigma = \sigma_a + B e^{-\beta T} + B_o \bar{\epsilon}^n \quad (2.10)$$

and for a FCC material it is;

$$\sigma = \sigma_a + B_o \bar{\epsilon}^{1/2} e^{-\alpha T} \quad (2.11)$$

where σ is the von Misses flow stress, $\bar{\epsilon}$ is the equivalent plastic strain and T is the absolute temperature. The parameter σ_a represents the athermal component of the flow stress and is determined mainly from the effects of solutes and grain boundaries. It is expressed as;

$$\sigma_a = \sigma_G + k d^{-1/2} \quad (2.12)$$

where, σ_G represents the influence of solute and the original density on the effective stress. The second term, known as Hall-Petch term, calculates the stress associated with the plastic flow between grain boundaries at low temperatures. The variable k

represents the microstructural stress density and the variable d is the average grain diameter.

The terms α and β are expressed as;

$$\alpha = \alpha_o - \alpha_1 \ln \dot{\epsilon} \quad (2.13)$$

$$\beta = \beta_o - \beta_1 \ln \dot{\epsilon} \quad (2.14)$$

where, $\dot{\epsilon}$ is the equivalent strain rate and the variables $B, B_o, \beta_o, \beta_1, \alpha, \alpha_o, \sigma_G, k, d$ and n are material constants that are determined from experimental data.

Zerilli-Armstrong stated that their original relation for BCC materials, expressed in eqn. (2.10), was insufficient to account for the effect of twinning, which is important in BCC materials. An improved version of the BCC relation including a simple deformation-twinning model, since it was suggested that plastic flow by slip could be replaced by twinning at high strain rates in the presence of large grains, is as follows;

$$\sigma_a = \sigma_G + kd^{-1/2} \left[(N_T + 1)^{1/2} - 1 \right] \quad (2.15)$$

Note that the Hall-Petch term is modified and N_T represents the average number of twins per grain. Further developments are still made on this constitutive relationship [10,13,17].

2.5 Bodner – Partom (BP) Constitutive Equation

Bodner and Partom [18] formulated a set of constitutive equations to represent elastic-viscoplastic strain hardening material behaviour for large deformations and arbitrary loading histories. An essential feature of the formulation is that the total deformation rate is considered to be separable into elastic and inelastic components, which are functions of state variables at all stages of loading and unloading. The formulation is independent of any yield criteria or loading and unloading conditions. The deformation rate components are determinable from the current state, which permits an incremental formulation of problems. Strain hardening is considered in the equations by introducing plastic work as the representative state variable. The

assumptions of isotropy and isothermal conditions are maintained in this formulation, although strain hardening is known to induce anisotropy.

The general formula of the BP model is based on separation of the total deformation rate $d = \{d_{ij}\}$ into elastic (reversible) and plastic (irreversible) components at all stages of deformation;

$$d = d^e + d^p \quad \text{or} \quad d_{ij} = d_{ij}^e + d_{ij}^p \quad (2.16)$$

where $d = \{d_{ij}\}$ is the symmetric part of the velocity gradient,

$$d = \frac{1}{2}(L + L^T) = \frac{1}{2}(v\nabla + \nabla v) \quad \text{or} \quad d_{ij} = \frac{1}{2}(v_{ij} + v_{ji}) \quad (2.17)$$

The elastic deformation rate $d^e = \{d_{ij}^e\}$ can be related to the Cauchy stress rate by the generalized Hooke's law,

$$d_{ij}^e = \frac{\dot{t}_{ij}}{G} - \frac{\lambda \dot{t}_{kk} \delta_{ij}}{2G(3\lambda + 2G)} \quad (2.18)$$

where G is the elastic shear modulus and λ is an elastic Lamé constant.

When finite deformation and finite rotation are involved, the objective Jaumann rate of the Cauchy stress must be used here,

$$\overset{\circ}{t} = \dot{t} - (w * t - t * w) \quad \text{or} \quad \overset{\circ}{t}_{ij} = \dot{t}_{ij} - (w_{ik} t_{kj} - t_{ik} w_{kj}) \quad (2.19)$$

$w = \{w_{ij}\}$ is the anti-symmetric part of the velocity gradient.

$$w = \frac{1}{2}(L - L^T) = \frac{1}{2}(v\nabla - \nabla v) \quad \text{or} \quad w_{ij} = \frac{1}{2}(v_{ij} - v_{ji}) \quad (2.20)$$

The plastic deformation rate $d^p = \{d_{ij}^p\}$ is related to the deviator of Cauchy stress, as in the classical Prandtl-Ruess equation,

$$d^p = \gamma s \quad \text{or} \quad d_{ij}^p = \gamma s_{ij} \quad (2.21)$$

$s = \left\{ s_{ij} = t_{ij} - \frac{1}{3} t_{kk} \delta_{ij} \right\}$ is the deviator of the Cauchy stress and γ is a proportional factor.

Squaring eqn. (2.21) gives,

$$\gamma^2 = \frac{D_2^p}{J_2} \quad (2.22)$$

D_2^p is the second invariant of the plastic deformation rate. J_2 is the second invariant of s ,

$$D_2^p = \frac{1}{2} d_{ij}^p d_{ij}^p \quad (2.23)$$

$$J_2 = \frac{1}{2} s_{ij} s_{ij} \quad (2.24)$$

γ will be determined by specifying D_2^p as a function of J_2 ,

$$D_2^p = f(J_2) \quad (2.25)$$

This formula can be considered to be a multi-dimensional generalization of the uniaxial result. The particular forms of eqn. (2.25) that seems to be applicable are those used relate dislocation velocity with stress. In those equations, the dislocation velocity is represented as a power function or as an exponential function of the stress.

The particular function used in the BP model is,

$$D_2^p = D_0^p \exp \left(- \left(\frac{n+1}{n} \right) \left(\frac{Z^2}{3J_2} \right)^n \right) \quad (2.26)$$

where D_0 is the limiting or maximum strain rate; Z is a loading history dependent internal variable; n is a material parameter governing the strain rate sensitivity. The preferred functional form of equation (2.25) for a particular material would be essentially empirically based and any monotonically increasing function of D_2^p with J_2 is admissible.

For the constitutive equation of the internal variable, it is assumed that Z is a function of the plastic work,

$$\dot{Z} = \frac{m(Z_1 - Z)\dot{W}^p}{Z_0} \quad (2.27)$$

where m is a material parameter, Z_0 is the initial value of Z , Z_1 is the saturation value of Z and W^p the plastic work,

$$\dot{W}^p = t : d^p = t_{ij} d_{ij}^p \quad (2.28)$$

From eqn. (2.27), Z can be integrated as,

$$Z = Z_1 + (Z_0 - Z_1) \exp\left(-\frac{mW^p}{Z_0}\right) \quad (2.29)$$

For one dimensional case, the BP model reduces to the following equations,

$$\dot{\varepsilon}^e = \frac{\dot{\sigma}}{E} \quad (2.30a)$$

$$\dot{\varepsilon}^e = \frac{2}{\sqrt{3}} \left(\frac{\sigma}{|\sigma|} \right) D_0 \exp\left(-\frac{n+1}{2n} \left(\frac{z}{\sigma} \right)^{2n}\right) \quad (2.30b)$$

$$Z = Z_1 + (Z_0 - Z_1) \exp\left(-\frac{m \int \sigma d\varepsilon^p}{Z_0}\right) \quad (2.30c)$$

where the quantity $\sigma / |\sigma|$ imposes the condition that the plastic strain rate has the same sign as that of the applied stress.

There are five material properties in the BP model. In most of the cases, the experimental data for characterizing the material properties are in the form of stress-strain curves at constant strain rates. For the BP model, two stress – strain curves at different strain rates are sufficient to calculate the material properties. D_0 is the maximum strain rate that the material can experience. It was recommended by

Bodner and Partom that $D_0 = 10^4 s^{-1}$ for quasi-static modeling and $D_0 = 10^6 s^{-1}$ for dynamic modelling.

The predictions of the BP model for commercially pure titanium under uniaxial deformations are shown in figure 2.3.(a) and 2.3.(b) using the material properties determined by Bodner and Partom. The material constants can be determined by fitting computed stress-strain curves to the experimental ones at the slowest and fastest rates. **Huang and Khan [19]** had also proposed a systematic method to determine the material constants for the BP model when modeling the 1100 Aluminium.

For the BP model, the work hardening behavior is not easy to see from equation (2.30) directly. When calculating the stress increment due to strain-rate change, it can be seen from figure 2.3 (c) that $\Delta\sigma$ first increases then decreases beyond certain strain level; but in general, for large deformation, the strain-hardening behavior does not change much at different strain rates.

From figure 2.3 (b), the BP model shows an elastic and almost perfectly plastic behavior at very high strain rate. At very low strain rate, the BP model also reduces to perfectly plastic behavior model after 5% strain. The BP model, using the published constants, does not show a very strong strain-hardening behavior. It is not very sensitive to the parameter m . Therefore, for materials that exhibit strong work-hardening behavior like OFHC copper, m was made a function of W_p by **Bodner and Merzer [20]**. The expression for m was,

$$m = m_0 + m_1 e^{-\alpha W_p} \quad (2.31)$$

which adds two more constants. When m is defined by equation (2.31), the relationship given by equation (2.29) is not valid. The expression for Z when integrated after replacing m by the equation (2.31) yields the following relationship between Z and W_p :

$$Z = Z_1 + (Z_0 - Z_1) \exp(-m_0 W_p) \exp\left(-\frac{m_0 + m_1 - m}{\alpha}\right) \quad (2.32)$$

Bodner and Rubin [21] also presented a modification to the BP model to include a strong strain-rate dependence of strain hardening. The original form of the BP model did not take the temperature effect into consideration. **Bodner and Rajendran [22]** presented a modification to the BP model on temperature effect for copper. All of these modifications introduced more material constants to the model.

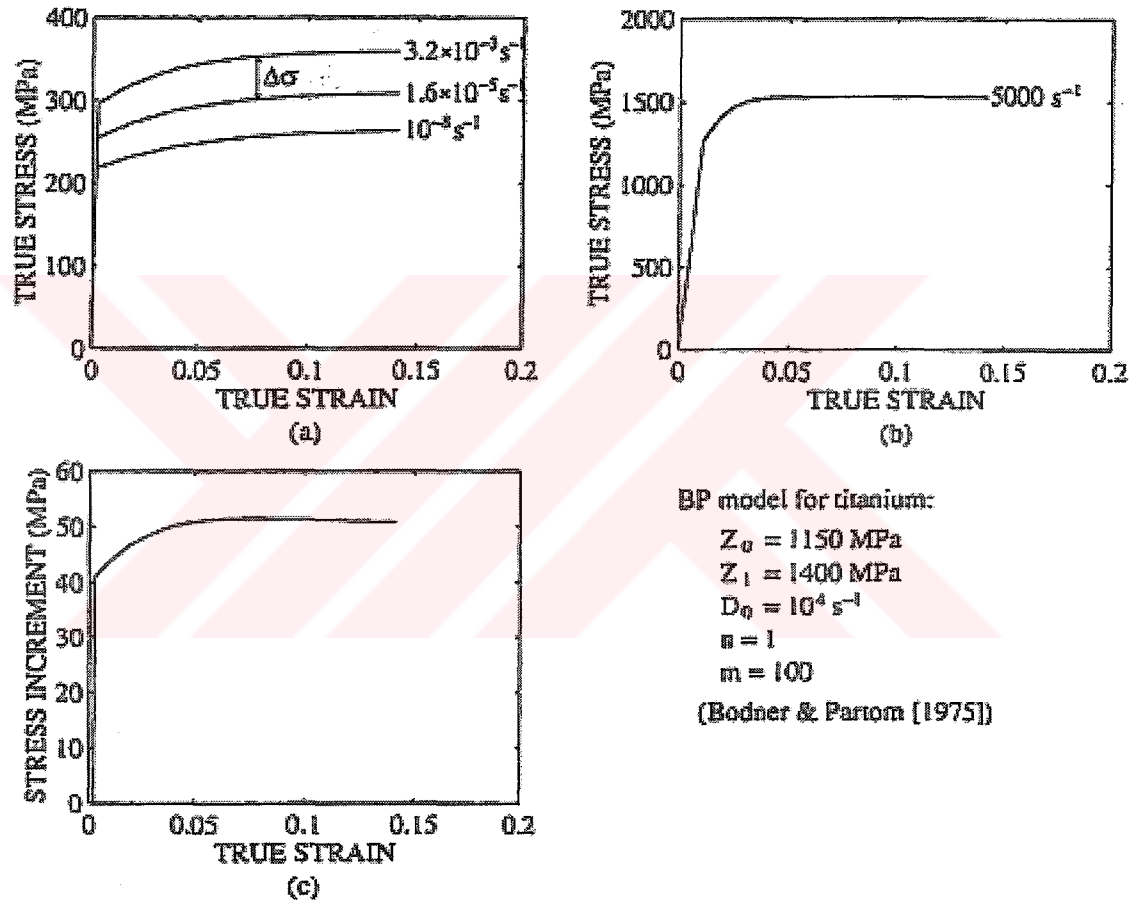


Figure 2.3 : Prediction of the BP model for Ti under uniaxial deformation. [14]. (a) Stress-Strain relations at different strain rates 10^{-8} s^{-1} , $1.6 \times 10^{-5} \text{ s}^{-1}$, and $3.2 \times 10^{-3} \text{ s}^{-1}$; (b) Stress - Strain relation at strain rate 5000 s^{-1} ; (c) Stress increment ($\Delta\sigma$) between strain rate $3.2 \times 10^{-3} \text{ s}^{-1}$ and $1.6 \times 10^{-5} \text{ s}^{-1}$ [20].

2.6 Khan-Huang (KH) Constitutive Equation

In the study of mechanical behaviour of 1100 Aluminum in the strain rate range $10^{-5} - 10^4 s^{-1}$, Khan and Huang proposed a new constitutive model that can predict the experimental results in a very large strain-rate range including the high work-hardening region.

The assumption of the KH model is similar to that of the BP model. The only difference is to determine the proportionality parameter γ also shown earlier in eqn. (2.22);

$$\gamma^2 = \frac{D_2^p}{J_2} \quad (2.32)$$

A constitutive equation is proposed for J_2 as;

$$J_2 = f_1(\varepsilon_2^p) f_2(D_2^p) \quad (2.33)$$

where ε_2^p is the equivalent plastic strain defined by;

$$\varepsilon_2^p = \frac{1}{2} \varepsilon_{ij}^p \varepsilon_{ij}^p \quad (2.34)$$

For one-dimensional case, eqn. (2.33) can be written as;

$$\sigma = g_1(\varepsilon_p) g_2(\dot{\varepsilon}_p) \quad (2.35)$$

where;

$$g_1(\varepsilon_p) = [3f_1(\frac{3}{4}(\varepsilon^p)^2)]^{1/2} \quad (2.36)$$

$$g_2(\varepsilon_p) = [f_2(\frac{3}{4}(\dot{\varepsilon}^p)^2)]^{1/2} \quad (2.37)$$

The particular form of $g_1(\varepsilon^p)$ and $g_2(\dot{\varepsilon}^p)$ are proposed to be;

$$g_1(\varepsilon_p) = \sigma_0 + E_\infty \varepsilon^p - a e^{a \varepsilon^p} \quad (2.38)$$

$$g_2(\dot{\epsilon}^p) = \frac{1}{\left[1 - \frac{\ln(\dot{\epsilon}^p)}{\ln(D_o^p)}\right]^n} \quad (2.39)$$

Five material constants are n, E_∞, σ_o, a and α . D_o^p is arbitrarily chosen to be $10^6 s^{-1}$ in Khan and Huang's work.

The particular form of the KH model describing 1100 Aluminum has the same features of work-hardening behavior as that of the JC model when strain rate changes. The KH model is more capable of predicting the strong work-hardening behavior in a large strain-rate range than that of the BP model. Since the KH model does not consider the temperature effect in the original equations, some modifications will be needed in order to incorporate the temperature effect into the model [19].

2.7 Mechanical Threshold Stress (MTS) Constitutive Model

The mechanical threshold stress (MTS) model uses the basic ingredients of thermally activated dislocation motion and focuses on the determination of the threshold stress (height of barrier in Fig. 2.4; F_o, τ_o , or σ_o). This threshold stress is the flow stress of a certain structure at 0 K. Follansbee and Kocks defined a normalized total activation energy g_o as;

$$\Delta G_o = G(T)b^3 g_o \quad (2.40)$$

where $G(T)$ is the temperature-dependent shear modulus.

Kocks et. al. proposed a general expression for the activation energy dependence on σ of the form;

$$\Delta G = \Delta G_o \left[1 - \left(\frac{\sigma}{\sigma_o} \right)^p \right]^q \quad (2.41)$$

The parameters p and q determine the shape of the activation barrier, ΔG is the free activation energy and ΔG_o is the activation energy at 0 K.

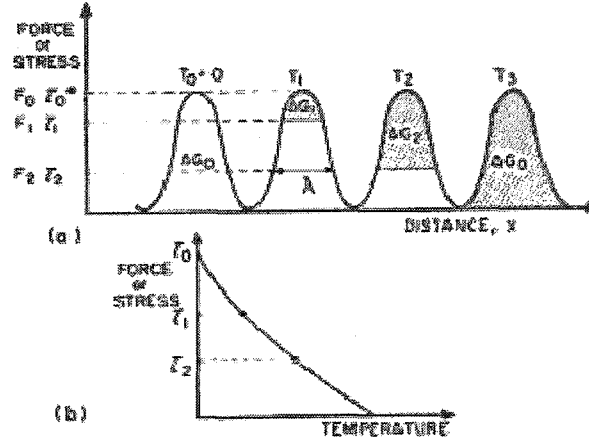


Figure 2.4 : (a) Overcoming of barriers by thermal energy; (b) stress required to overcome obstacle as a function of temperature [4].

Follansbee and Kocks, by considering barriers with $p=1/2$ and $q=3/2$ obtained the following equation from eq. (2.41);

$$\Delta G = \Delta G_o \left[1 - \left(\frac{\sigma}{\sigma_o} \right)^{1/2} \right]^{3/2} \quad (2.42)$$

Substituting eqn. (2.41) into (2.42) yields;

$$\Delta G = G(T)b^3 g_o \left[1 - \left(\frac{\sigma}{\sigma_o} \right)^{1/2} \right]^{3/2} \quad \text{and substituting this into } \Delta G = kT \ln \frac{\dot{\epsilon}_o}{\dot{\epsilon}}, \text{ the}$$

expression now becomes;

$$\left(\frac{\sigma}{G(T)} \right)^{1/2} = \left(\frac{\sigma_o}{G(T)} \right)^{1/2} \left[1 - \left(\frac{kT}{Gb^3 g_o} \ln \frac{\dot{\epsilon}_o}{\dot{\epsilon}} \right)^{2/3} \right] \quad (2.43)$$

By applying eqn. (2.43) and conducting tests at strain rates ranging from 10^{-4} to 10^4 s^{-1} and final strains of 0.05, 0.1, 0.15, 0.2, and 0.25, Follansbee and Kocks obtained the results plotted in figure 2.5(a). It can be seen that the intercepts of these plots with the ordinate axis provide values of σ_o . The tests were conducted by reloading the prestrained specimens (to the different strains of 0.05, 0.1, ..., 0.25) at various temperatures T . In eq. (2.43), when $T = 0$ then;

$$\sigma = \sigma_o \quad (2.44)$$

The intercepts are obtained by extrapolation since there is no possibility to perform tests at 0 K. It can be seen that a straight-line behavior is obeyed, in accordance with eqn. (2.43). It can also be observed that σ_o / G (or σ_o) is dependent on strain rate. The linear behavior indicates that the correct barrier shape was assumed. The values of σ_o increase with strain rate at a constant strain (of 0.10), as indicated in fig. 2.5(b). This lends support to one of the cardinal statements of the MTS model: *Structure evolution is rate sensitive*. Thus, the simple relationship of the type (similar to eqn. 2.1);

$$\sigma = \sigma_o + \kappa \dot{\epsilon}^{1/2} \quad (2.44)$$

which is obtained directly from Taylor's theory of work hardening, is only applicable at a constant strain rate (κ is strain rate dependent). The threshold stress is higher (at a constant strain ϵ) for higher strain rates. This is indicative of the stronger barriers created by high-strain rate deformation. Based on this, Follansbee and Kocks concluded for copper that dislocation drag mechanisms are not operative up to strain rates of 10^4 s^{-1} . The increased threshold stress σ_o is responsible for the increase of strain rate sensitivity (as defined by $m = \delta\sigma / \delta \ln \dot{\epsilon}$) at strain rates of 10^3 s^{-1} and higher. Figure 2.6 shows the experimental results obtained by **Follansbee and Gray [3]** for copper. It is clear that at 10^3 s^{-1} it starts to rise.

The procedure used in establishment of the threshold stress is to first perform the dynamic test to a fixed strain on a number of specimens and then to test these predeformed specimens at a very slow strain rate and decreasing temperatures. The flow stress is then the stress required to overcome the dynamically generated barriers minus the thermal activation. As the testing temperature is decreased, the thermal activation component decreases until it vanishes, at 0 K. Thus, one can find the threshold stress by extrapolation if the correct barrier shape (p and q) is used [3].

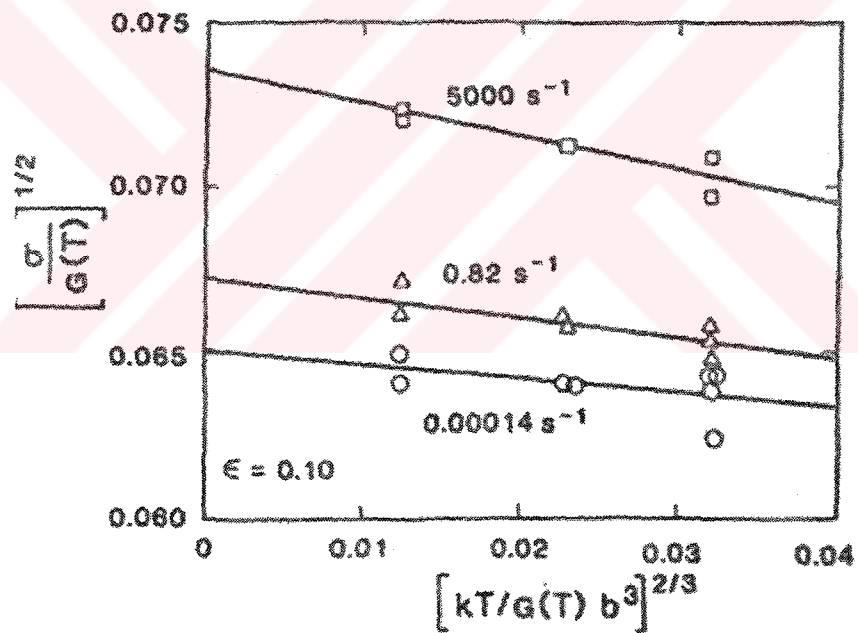
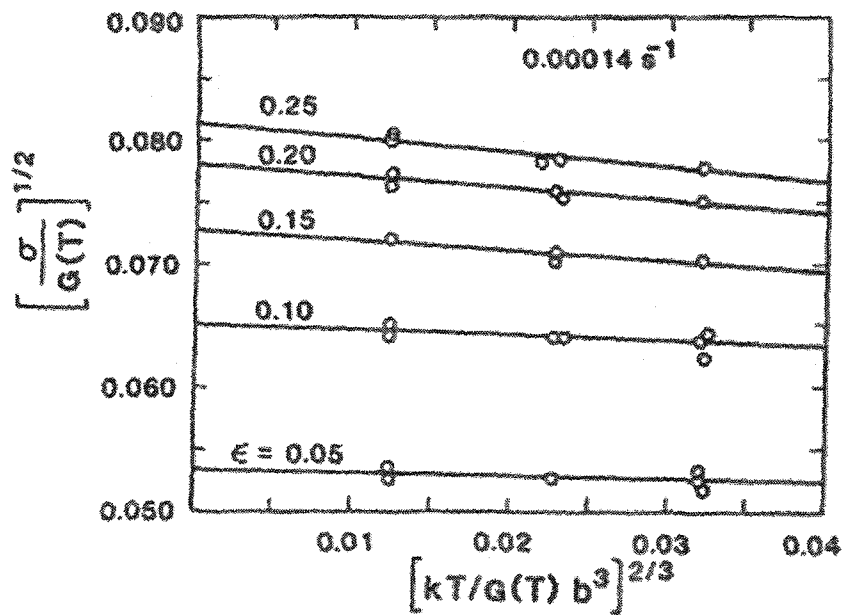


Figure 2.5 : (a) Reload yield stress for copper deformed to different plastic strains (0.05, 0.1, ..., 0.25) and constant strain rate. (b) Reload yield stress rate for copper at a plastic strain of $\epsilon = 0.1$ at different strain rates as a function of temperature according to normalized coordinates provided by eqn. 2.43 [3].

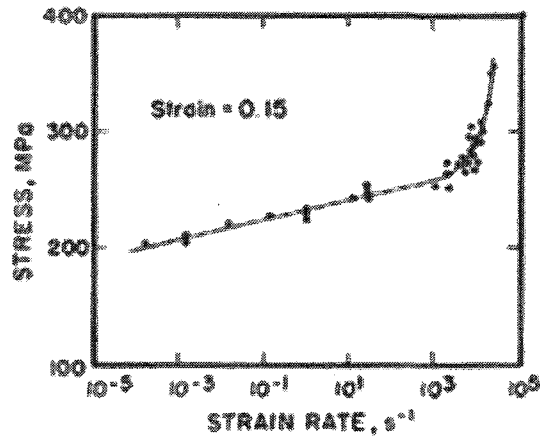


Figure 2.6 : Flow stress for 99.99 Cu at plastic strain of $\epsilon = 0.15$ as a function of strain rate [3].

2.8 Comparison of the Constitutive Equations

Among many constitutive models developed by scientists so far, several of them describing the strain, strain-rate and temperature relations for metals in the large strain and high strain-rate regime, had been explained in this chapter. It should be mentioned that in literature still many useful and important constitutive models which are not described in here exist, such as **Anand's model** [23], **Klopp-Clifton-Shawki** [3,14], **Hoge-Mukherje** [3], **Campbell** [3], **Vinh et. al.** [13] and etc. One can refer to the given references for details of these models.

The Ludwik Equation, perhaps so far the simplest in literature, often used to characterize the stress as a function of the strain at ambient temperatures and quasi-static loading. It is incapable of considering the influence of some parameters such as temperature and strain rate but it was extended by several ways to include the influence of these parameters.

Litonski, included the temperature effect linearly and considered the strain in two separate parts as elastic and plastic and developed the Litonski model.

The Johnson-Cook constitutive relation is an empirical relation that is relatively simple to calibrate for a given material. That is, very few stress-strain curves covering the loading conditions are required to determine the parameters. It is relatively easy to implement into computer codes, inexpensive to use and produces reasonably accurate predictions for a range of materials if loading conditions do not

exceed those used to determine the material parameters. Besides it is a work-hardening rate increasing model, with increasing strain rate, in its original form. On the other hand it has inherent problem of describing the work-hardening behavior of metals, such as tantalum, where the work-hardening rate decreases when strain rate increases [6] .

The Zerilli-Armstrong constitutive relations are semi-empirical models based on the concept of dislocation dynamics. Seperate relations were developed to describe the deformation behaviour of FCC and BCC materials. A subsequent relation was introduced for HCP materials, which generally exhibit behaviour between that of BCC and FCC materials. The ZA model takes the average grain size into account and it presumes that the work-hardening rate is independent of the temperature and strain-rate. For most metals, however, work-hardening behaviors are actually dependent on the temperature and strain rate. Therefore, the ZA model is not a good candidate for modelling the work-hardening behaviors of materials with strong temperature and strain-rate dependences. Overall, the ZA relation tend to give a slightly better correlation, but is still relatively easy to implement into computer codes and inexpensive to use. Similar to the Johnson-Cook relation, however, it is not very accurate when used outside of the range of loading conditions that it was calibrated for, or for large strain calculations.

The Bodner-Partom model, in its original form, is not very sensitive to the strain rate. It can be seen as a constant work-hardening model when the strain rate change is not too much. The BP model also does not take the temperature effect into consideration. Many modification have been made to the model but all these modification increase the number of material constants. *The Khan-Huang model*, based on the same assumption as that of the BP model, proposed a new relation to describe the dependence of work-hardening behavior on strain and strain rate. The KH model is better at predicting the strong work-hardening behavior in a large strain-rate range than the BP model [15].

The Mechanical Threshold Stress Model is a more advanced and physically-based model that relates the macroscopic mechanical response of a material to microscopic properties. It accounts for the evolution of sub-structures as a function of plastic deformation and loading history. It generally is more accurate than both the Johnson-

Cook and Zerilli-Armstrong relations, and once it is calibrated for a particular material it can be used to accurately predict deformation outside the range of calibration loading conditions. This has been attributed to the form used to represent the work hardening behaviour in the MTS relation. Rather than using a power law form, which yields a continuously increasing stress for increases in strain, a function is selected that “saturates” to a maximum stress level for increasing strain. This concept was based on the observation that at large strains most metallic materials tend to approach a finite “saturation stress”, or approach a constant , but small hardening rate at large strains. However, it should be noted that the form of the equation used to describe the work hardening behaviour is not strictly physically-based and therefore, the MTS relation is also semi-empirical. The main disadvantages of the MTS relation are at that it requires an extensive experimental data base to determine the material parameters, it is more difficult to incorporate into computer codes and more computationally expensive. The MTS model can be studied in more details from [3, 23, 13].

3. EXPERIMENTS AT VARIOUS STRAIN RATE LEVELS

The study of material behavior at high strain rates is becoming very important with the desire to fabricate structures that are capable of withstanding high velocity impacts. For computer simulation of such impacts, the need for accurate properties for a variety of materials is required.

In this section several testing methods which achieve strain rates from 10 to $10^8 s^{-1}$ will be underlined. It is hard to achieve the full range of tests by a single test, therefore testing methods are divided into three categories as follows; low strain rate, high strain rate and ultrahigh strain rate tests. Table 3.1 illustrates the different types of tests that exist for various strain rates.

Table 3.1 : Test Techniques at Various Strain Rates [24].

Strain Rate	Testing Methods	Category
10^7 10^6 10^5	-Explosives -Gun Systems -Pulsed Laser -Plate Impact	Ultrahigh Strain Rate
10^4 10^3	-Taylor Anvil Tests -Hopkinson Bar -Expanding Ring	High Strain Rate
10^2 10^1	-High-velocity hydraulic or pneumatic machines; cam plastomer	Low Strain Rate

Under routine operating conditions, many structural materials are subjected, either by accident or design, to high rates of loading, which can cause catastrophic failures or corresponding high rates of straining. In numerous practical engineering situations, from the everyday traffic accident to the impact of runway debris on aircraft composite panels, large strains are applied in milliseconds.

At high-strain rates, the mechanism by which many materials deform is different from that experienced at lower rates, and many material properties are strain-rate sensitive. Consequently, quasi-static stress-strain data may not produce accurate predictions at high-strain rates, and the use of such data in the analysis and design of dynamically loaded structures can lead to designs of parts that are too heavy or prone to premature structural failure. It is essential that the material's deformation characteristics under high-strain conditions are understood so the engineer can make appropriate design choices [25].

Figure 3.1 shows an example of a stress-strain curves for dynamic and quasi-static loading conditions for depleted Uranium, which is an effective armor-piercing material developed at Los Alamos National Laboratory. The dynamic, or high strain-rate, curve shows a higher yield point, and initially, higher work hardening, followed by lower work hardening as the material thermally softens. The curve illustrates the influence of strain rate on the strength and behavior of the material [26].

DYNAMIC VERSUS STATIC

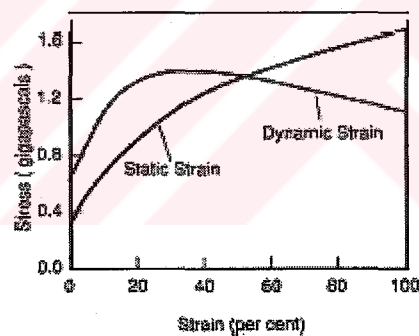


Figure 3.1 : Stress-Strain curves for depleted uranium at strain rates 5000 (Dynamic) and 0.001 (Static) s^{-1} [26].

Among these three strain rates mentioned at the beginning, Low Strain Rates are the easiest to be achieved. These rates range from 10 to $10^2 s^{-1}$ and are marginally higher than quasi-static tests. Simple hydraulic or pneumatic actuators can achieve these rates [24, 27].

On the other hand, high-rate data may be generated using a variety of methods depending on the test-piece size and the energy or velocity demands of the test. Charpy impact pendulum machines employ an instrumented hammer that is swung into a notched square or rectangular section specimen bridged across a set of anvils

positioned in the hammer path. This gives rise to specimen strain rates in the region of $10s^{-1}$ to 10^2s^{-1} . Drop-weight impact testing machines employ a guided free falling or spring-assisted weight and puncture probe. These machines also develop similar strain rates. At the other end of the spectrum are Hopkinson bars and their gas guns that rely on force transmission by a gas pressure assisted rod or by actual projectile firing, using pressurized gases, to achieve strain rates of $3 \times 10^2s^{-1}$ to 10^5s^{-1} .

One method specific to the automotive industry is the catapult rig that has a sledge with attached components, such as steering wheels and seating, attached to the end of a hydraulic ram and accelerated into an obstacle to simulate specific crash conditions. Strain rates of $5 \times 10^3s^{-1}$ may be achieved using such equipment.

Systems that are designed with servohydraulic test machines can combine simultaneous load and velocity performance of 100 kN at 20 m/s. Some frames are capable of generating strain rates in the region of $1s^{-1}$ to 10^3s^{-1} and can be used for conventional static and quasi-static tests. The frames can be configured for high-speed tension, bend and impact tests between -150 C and 600 C. With large oil accumulators driving the actuator piston, these systems are capable of performing constant velocity tests, a capability not available to other high rate testing devices, with the exception of the catapult rig. Test specimens and components may take a range of forms depending on the industry concerned. Suitable candidates for high-strain rate testing include seat belt webbing materials and fasteners, automotive body and dashboard components, forging alloy compression specimens and aircraft panel composite tensile specimens [25].

For high strain rates up to 10^4s^{-1} and among low strain rate test, three main tests are commonly used to determine the mechanical properties of the sample material. Among them, split Hopkinson pressure bar will be studied in details in the next sections.

3.1 Low Strain Rate Testing Methods

3.1.1 Cam Plastomer Testing Machine

Figure 3.2 shows a diagram of a cam plastomer testing machine. This machine uses a cam rotating at a specific velocity to deform the specimen. The compression specimen is placed on an elastic bar. At a certain moment, when the cam is rotating at the desired velocity, the follower is moved into place and the specimen is deformed (shown by the dashed line). Strain rates between 10^{-1} and $10^2 s^{-1}$ have been achieved by this method [3].

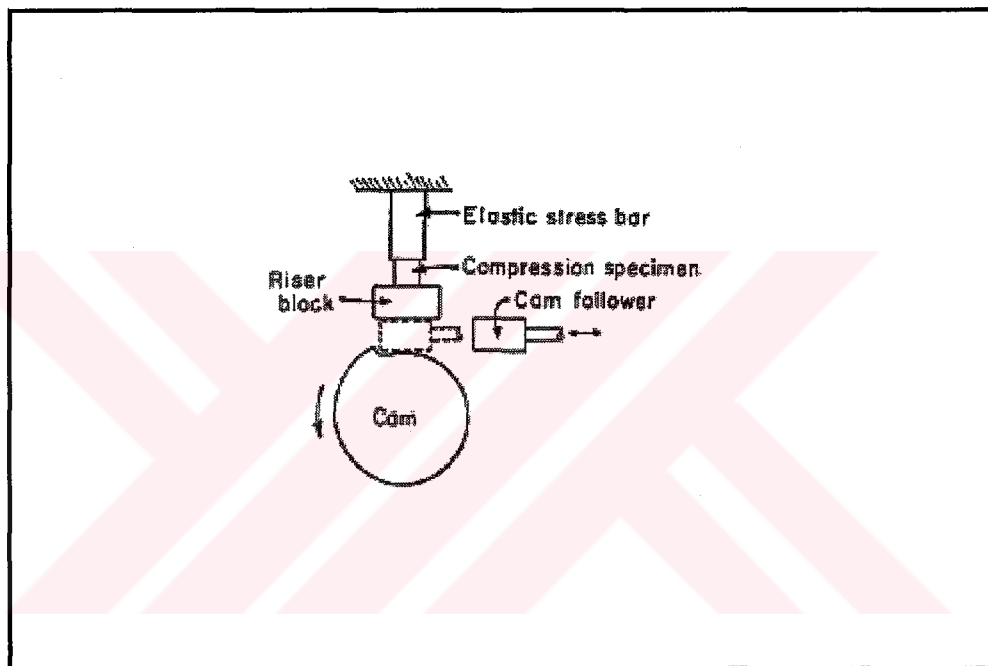


Figure 3.2 : Schematic of Cam Plastomer [24].

3.1.2 High -Velocity Hydraulic & Pneumatic Test Machines

Figure 3.3 shows a schematic of a compressed gas machine. The specimen is placed on the anvil of the machine. A movable piston passes through a reservoir. Support jacks keep the piston upwards (fig. 3a). When the auxiliary reservoir is pressurized after the support jacks are lowered (fig. 3b), the piston accelerates downwards and impacts the anvil. This results in the compression of the specimen (fig. 3c). The machine shown in fig. 3 can be capable of velocities up to $15 m/s$. The diagnostics required to record stress and strain are not shown in the schematic. A simpler version of this machine is known as “drop hammer” and in this method a large mass is being dropped freely on the anvil [3].

There is also another version of this machine which is modified in order to conduct torsional testing to provide transient loadings. High speed hydraulic torsional devices are directly analogous to similar axial tension and compression devices. A torsional actuator is used instead of a standard linear hydraulic actuator.

Hydraulic loading of the actuator is maintained by a servo-valve and servo-control loop for operating at low rotational velocities which provide shear strain rates to about 10 s^{-1} . For higher velocities, a solenoid triggered quick-release valve conveys pressure from an accumulator directly to the actuator. In this mode, strain rates of up to $3 \times 10^2\text{ s}^{-1}$ have been achieved.

More details about these testing devices can be found in [27].

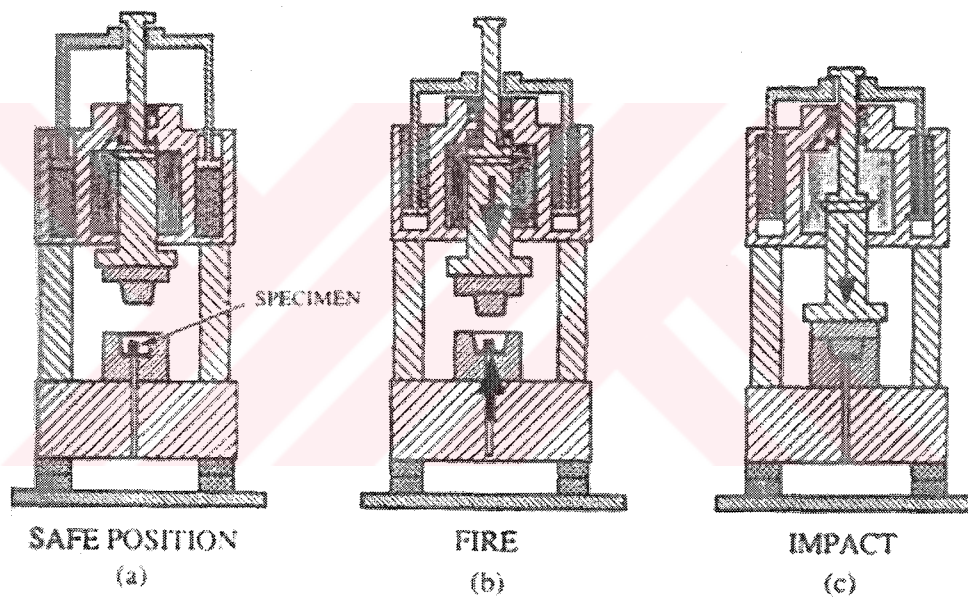


Figure 3.3 : Schematic of a Pneumatic Compressed Gas Machine. (a) Hydraulic jacks keep piston upwards, (b) jacks lowered and reservoir activated, (c) impact [3].

3.2 High Strain Rate Testing Methods

3.2.1 Taylor Anvil (Rod Impact) Tests

In 1947, Taylor conducted an analysis on specimens deformed at high rates of strain and it has become a standard procedure to verify the constitutive behavior of materials. These experiments involved the propagation of plastic deformation as a wave process.

In this method a cylindrical projectile is being propelled into a target which is rigid or symmetric. Figure 3.4(a) shows a specimen deformed quasi-statically under compression. It is clear from the figure that, neglecting frictional end effects, the cross-sectional area is constant at any time during deformation. On the other hand, if deformation is dynamic as shown in figure 3.4(b), the cross-sectional area during deformation and after deformation will not be uniform. The part of the cylinder undergoing impact (the end of the rod impacting on the rigid target in this example) undergoes a higher deformation. The process of deformation is being depicted as a sequence of elastic and plastic wave propagation into the cylinder by **Taylor [3]**.

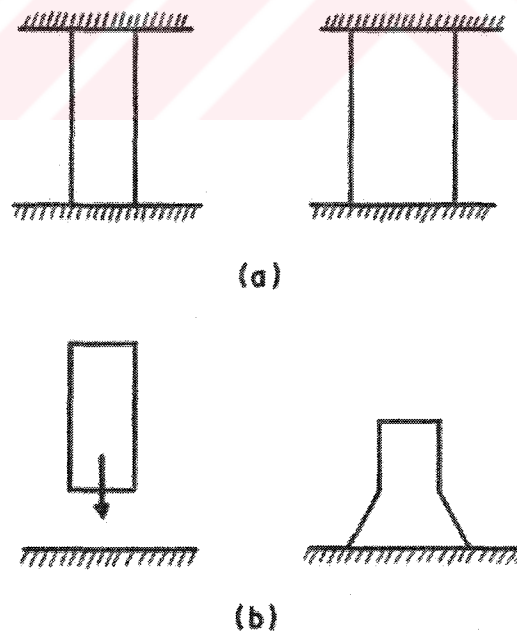


Figure 3.4 : Quasi-static (a) and Dynamic (b) deformation of a cylindrical specimen [3].

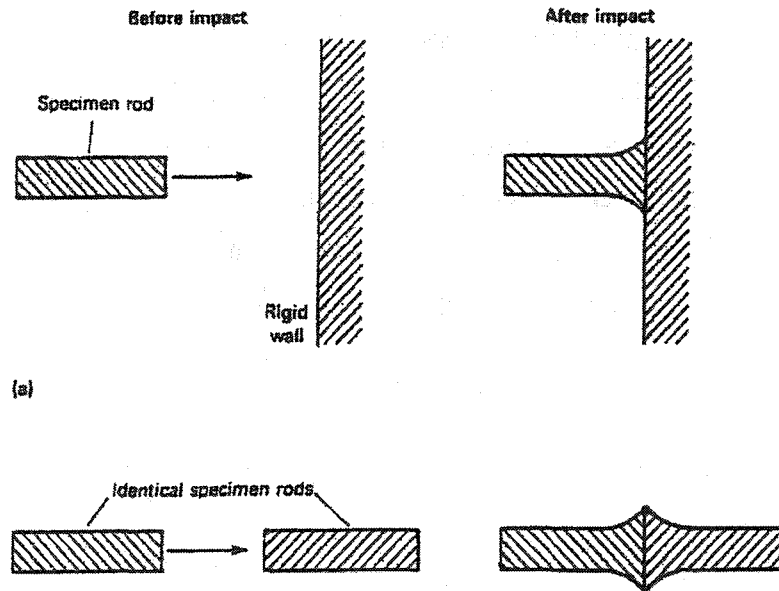


Figure 3.5 : (a) Classic Taylor , (b) Symmetric rod impact tests at ambient temperatures [27].

In figure 3.5, two different configurations for the method is presented. In the Classic Taylor configuration a cylindrical rod is accelerated into a “rigid” plate. The plastic deformation at the impact end shortens the rod, and the fractional change in rod length can be related to the dynamic yield strength by one-dimensional rigid-plastic analysis. This relation was shown to be independent of both the rod aspect ratio and the impact velocity for a wide range of materials such as copper, lead, paraffin wax and various steels. A quarter century later, after the original classic Taylor test, the use of two-dimensional wave propagation code enabled a better understanding of and renewed interest in this technique. Thus, Wilkins and Gunian, using a two-dimensional finite difference code and elastic-plastic model with work-hardening, were able to correctly simulate the final shapes and final lengths of Taylor test specimens of several metallic alloys at ambient temperatures. The simulation results showed good correlation between the dynamic yield strength and the fractional change in rod length for a wide range of impact velocities and rod aspect ratios, confirming the original classic Taylor test conclusions.

On the other hand, in the early 80’s, Symmetric and Asymmetric rod impact tests had been developed. Two major modifications have been made in order to develop the Symmetric Rod Impact Test. The first one was to use a ultrahigh-speed photography to monitor the deformation history of the specimen rod in order to compare the

intermediate and final deformation states with two dimensional computer simulations. This way the reliability of the flow curve determination is improved.

The second modification, as it is being named so, was the replacement of the rigid plate with another rod of the same material and geometry as the impacting rod (fig. 3.5b). This arrangement allowed the impacting ends of the two specimen rods to deform symmetrically so that the boundary condition uncertainties in the analysis that arise from the unknown friction conditions at the rod/plate interface and from the deformation of the rigid plate adjacent to that interface in the classical method were eliminated.

As for the Asymmetric Rod Impact test, it can be concluded that a reverse ballistics variation of the classic Taylor test was used. In this variation, a rigid plate is launched into a stationary specimen rod that is preheated to the desired temperature in order to measure the fractional changes in the length of these metallic specimen rods. The desired temperatures of the metallic rods could be as high as 1000°C. In both symmetric and asymmetric rod impact tests, the heater assembly (for elevated temperature tests) and the recovery pipe (for catching the specimen rods, filled with energy absorbing materials) are placed near the muzzle of the gas gun. In symmetric rod impact tests the alignment of two rods is critical to ensure that impacting ends are parallel and coaxial. It can be concluded that the asymmetric Rod Impact test method is considered to be more suitable for elevated temperature tests, since it would not be very suitable to perform symmetric rod impact tests because of the hard feasibility of heating the moving specimen rod. However there are still investigations processing on this technique and when accomplishments are done the asymmetric rod impact technique may be used to determine dynamic flow curves at ambient or elevated temperatures [27].

Back to the Classic Taylor Impact Test, the process of deformation as a sequence of elastic and plastic wave propagating into the cylinder can be presented by fig. 3.6;

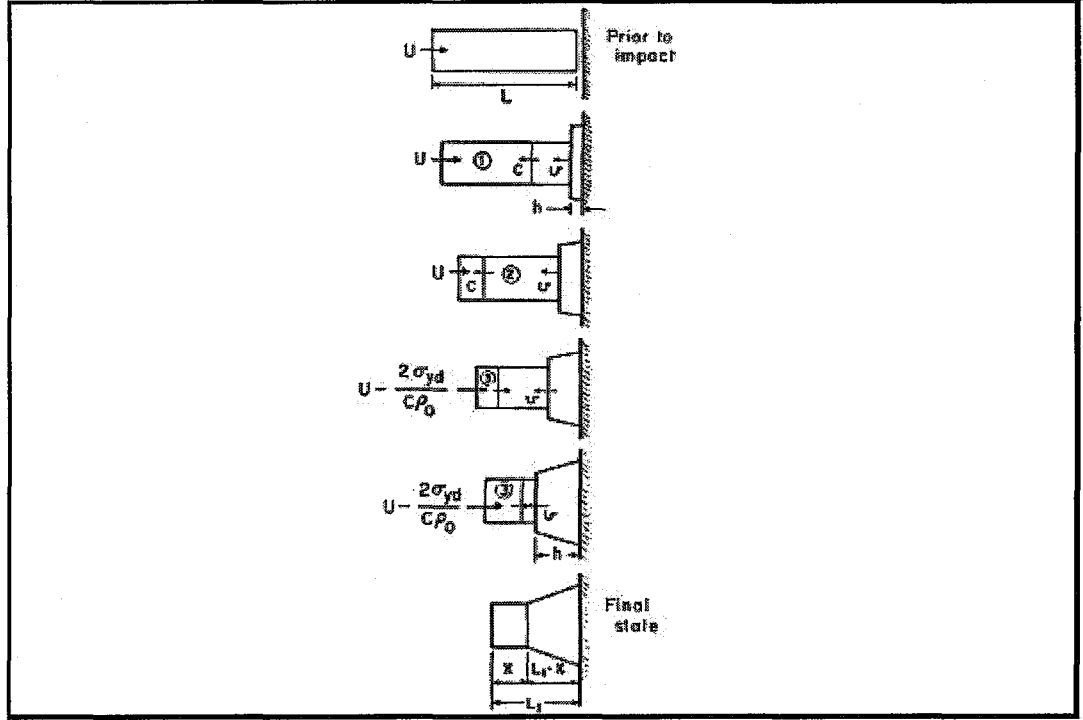


Figure 3.6 : Taylor Impact Specimen, Sequence of deformation after impact of cylindrical projectile against a rigid wall [24].

A cylindrical projectile in length L , impacts a rigid target at a velocity U . At this moment, the elastic wave will be faster than the plastic wave and moves at a velocity C . The elastic wave with velocity C travels until it reaches the back surface of the projectile, reflects from there, and then returns as a release wave. As it returns towards the plastic wave, it interacts with it, and this marks the end of the deformation process since the stress is reduced to zero. The stress within the region that has plastically deformed is assumed to be constant and equal to the yield stress of the material at that strain rate, σ_{dy} (dynamic yield stress) [3].

Briefly, it can be said that the dynamic flow stress of the cylinder material can be estimated by measuring the overall length of the impacted cylinder and the length of the undeformed (rear) section of the projectile by means of the following simple formula:

$$\sigma_{yield} = \frac{\rho U^2 (L - X)}{2(L - L_1) \ln(L/X)} \quad (3.1)$$

where σ is the dynamic yield stress of the material of the projectile, ρ is the density and U being its impact velocity. L , X and L_1 are defined in figure 3.6.

Determination of the dynamic flow curve of a material from a rod impact test is also made by computationally simulating the experiments with a two-dimensional wave propagation computer code. The flow parameters are varied until the computed profiles agree with those determined experimentally at various times during the deformation history.

In simulations, the specimen rods are divided into a series of rectangular zones or computational cells, each of which, in the cylindrical geometry appropriate to the rod impact test, represents an annulus of revolution about the rod axis. The corners of the cells or rods are given an appropriate initial velocity. Then the subsequent node velocities and the resulting rod deformation are determined by solving the Lagrangian equations of motion for a continuous medium. Figure 3.7 shows the original, the intermediate, and the final cell profiles for a typical test simulation.

A rate-independent elastic-plastic model with work hardening is used for describing the plastic deformations in each cell. The simulation results are then can be compared with the experimental results until an agreement is provided.

The computer simulations for rod impact tests, have confirmed that the plastic strain rate, as well as the total strain and the temperature resulting from the local plastic work, varies significantly as a function of time and axial and radial position within the specimen rod. For a particular axial location, strain rates (either peak or average) along the rod axis are often three or four times higher than those near the periphery. Also, there is often an order of magnitude difference in the strain rates among locations near the impact interface and locations one-rod diameter distant.

The experimentally measured rod thickness at a particular axial position is not the result of a region of material undergoing deformation at a specific strain rate, but rather the result of various regions of material deforming at a range of strain rates. Therefore, the rod impact technique should not be viewed as a method for accurately determining the strain rate sensitivity of the flow curve over the range of strain rate observed in the test, namely between about 10^4 and 10^5 s^{-1} , but rather as a means of determining the average flow curve over that range [10].

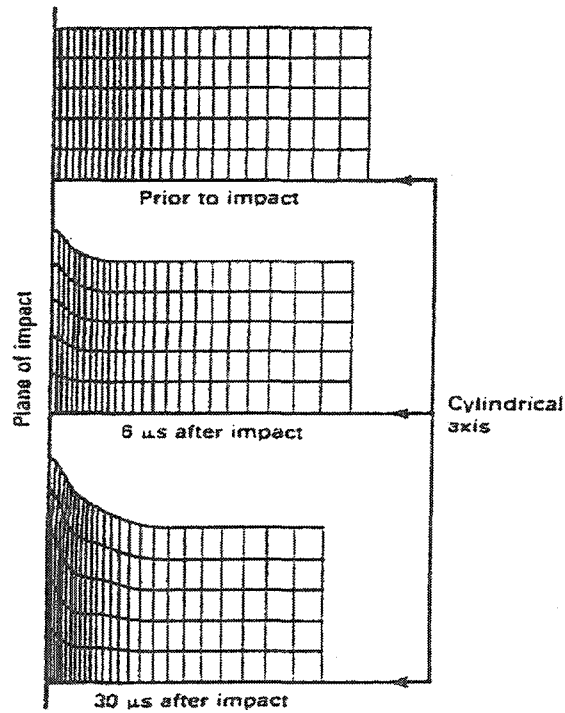


Figure 3.7 : Cell outlines at three stages during simulation of symmetric rod impact [10].

3.2.2 Expanding Ring Test

The other common method in high strain rate testing is the Expanding Ring test. The main difference of this test from others is that, the load type is tension in this technique. The expanding ring test is a highly sophisticated technique for subjecting metals to tensile strain rates over $10^4 s^{-1}$. Although the testing principle is simple, its performance requires specialized equipment available in only a few laboratories. The ring test can determine the high-rate stress-strain relationships, but a simplified, more widely used version can be employed to determine ultimate strain only.

Figure 3.8 shows a possible arrangement of the expanding ring. The material to be tested is placed in a ring around the hollow cylinder. The steel cylinder has a core, in which an explosive is detonated. As the explosive is detonated, a shock wave expands radially, travels outwards and enters the metal ring which in turn causes the cylinder and specimen to expand. In order to determine the stress-strain curve for the material at the imposed strain rate, the velocity history of the ring is recorded by laser interferometry. The velocity history can be related to stress and strain through a

set of simple equations. This technique uses varying amounts of explosives to select the strain rate desired [3].

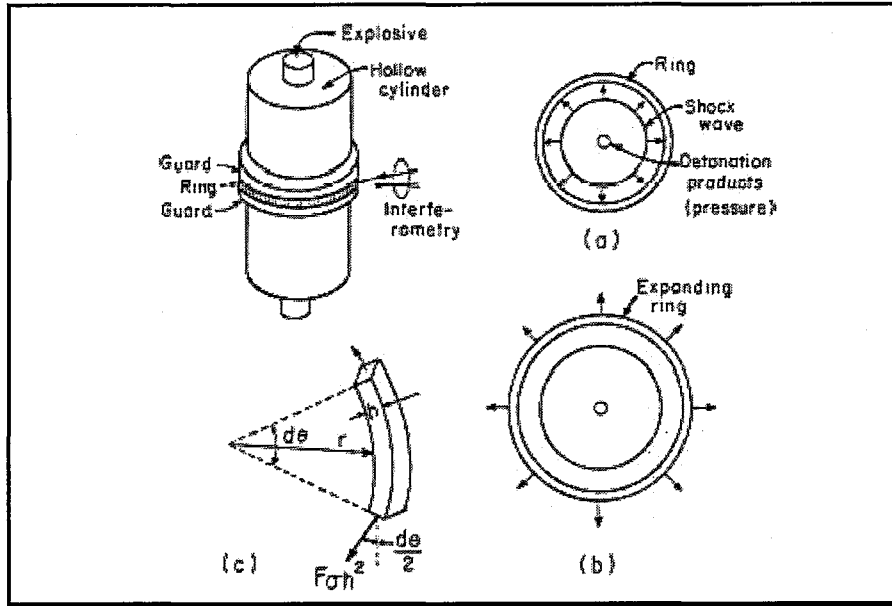


Figure 3.8 : Expanding Ring Technique: (a) Steel block with explosive in core; (b) sections of cylinder just after detonation and during flight of ring; (c) Section of ring [3].

This test involves the sudden radial acceleration of a ring due to detonation of an explosive charge or electromagnetic loading. The ring rapidly becomes a free-flying body, expanding radially, and decelerating due to its own internal circumferential stresses. A thin ring must be used for the analysis to be valid; the wall thickness should be less than one tenth the ring diameter, which is typically 25 mm. If r is the radius of the ring, ρ the density and σ the hoop stress;

$$\sigma = -\rho r \frac{d^2 r}{dt^2} \quad (3.2)$$

To obtain stress-strain data, radial displacement as a function of time must be calculated. Strain is proportional to the change in radius (like engineering strain in tension is $\Delta L / L_0$); thus:

$$\varepsilon = \ln \frac{r}{r_0} \quad (3.3)$$

where r_0 is the initial radius. Stress may be computed from eqn. (3.2) by double differentiation of radial displacement data as a function of time [27].

It should be noticed that since the velocity of the ring is continuously decreasing comparing to its initial value and thus, the strain rate is continuously varying, several number of tests with different explosive charges need to be conducted in order to develop stress-strain curves at constant strain rates. Figure 3.9 shows a charecteristic velocity-time record obtained from laser interferometry measurements;

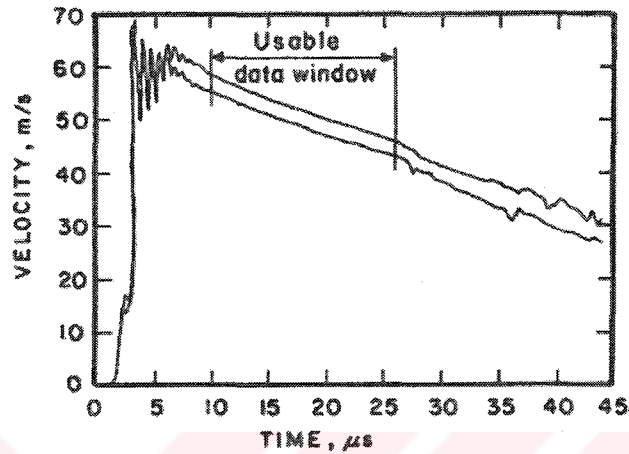


Figure 3.9 : Velocity-time history of copper ring in explosively accelerated device [3].

In fig. 3.9 it can be said that the velocity rapidly rises to 70 m/s. Then, one observes early-time ringing of stress waves, caused by wave reflections. After these wave interactions dampen, the ring slowly and steadily decelerates over the first 45 μ s. If the ring does not rupture, the strain rate will fall to zero at the end of the test. Ring specimens also experience a compressive preload in the radial direction that frequently exceeds the yield stress during the acceleration phase. Because load history is known to affect the subsequent stress-strain behavior of many materials, data obtained from expanding ring tests do not always agree with results from other tests at slightly lower strain rates.

The ring test has two principal advantages. The expanding ring test subjects the material to a state of dynamic uniaxial stress without the wave propagation complications that accompany other high strain rate tests. Also, the maximum strain rate available in the ring test is higher than any other common tension tests involving large plastic strains. In Expanding Ring method, there are alternative ways of accelerating the ring. The acceleration of the ring can be provided by an exploding wire or electromagnetic pulse.

The difficulties, expense and limitations of the expanding ring test preclude its use as a standard test technique for generating high strain rate stress-strain data in tension. However, if subjecting a material to high strain rates in tension without determining stress-strain data is of primary interest, the expanding ring test is much easier to conduct. Because stresses are not calculated in this method, the accurate determination of radial displacement versus time is not as critical. Less precise displacement data provide reasonably accurate determinations of strain rate. The ambiguity arising from possible strain rate history effects still exists when the expanding ring test is used in this simpler manner [3].

3.2.3 Split Hopkinson Pressure Bar Test

The Split-Hopkinson Pressure Bar (SHPB) is the experimental device most applied to characterize materials at high strain rates. The SHPB was firstly proposed by Kolsky and updated by many other scientists. For compression testing with the SHPB technique, a specimen is inserted between the ends of the two bars with colinear axes, as shown in figure 3.10. The device is normally composed by a gas gun and three lined up cylindrical slender bars. The second and third bars, known as input and output bars, are instrumented with strain gages attached on and the specimen is located between them [28].

One important point in the SHPB is that one should not consider the Hopkinson bar as a plastic wave propagation experiment. Nevertheless, it can provide important information (the constitutive relation) needed in the mathematical models on plastic waves [3]. A detailed description of this testing method will be studied in the next section.

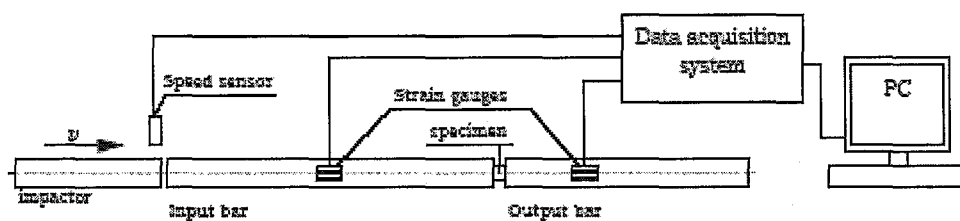


Figure 3.10 : Schematic Representation of the Experimental Set-up [29].

3.3 Ultrahigh Strain Rate Testing Methods

In order to achieve ultrahigh strain rates a very rapid application of energy at the surface of the sample material is required. This can be accomplished by a variety of methods as indicated in Table 3.1. The easiest way to apply a large amount of energy to the sample surface is to detonate explosives that are not in contact with the surface. This would generate the required plastic wave in the sample. Similarly, a pulsed laser of very high power would develop high pressures in the sample through radiation.

Single and dual stage gas guns are capable of launching projectiles towards a sample material at high velocities. Single stage gas guns are capable of propelling a projectile at velocities of up to 1200 *m/s* while two-stage gas guns are capable of velocities up to 8000 *m/s*.

Flyer plates can be used in achieving ultrahigh strain rates as well. A plate is driven by an explosive charge or gas gun to impact against a second stationary plane. Compressed gas guns, propellant guns, magnetic accelerators and explosives have all been used to launch the flyer plate. Extreme precision must be achieved to eliminate relative tilt at the instant of impact.

In order to accelerate a flyer plate by means of an explosive charge for example, plane wave generators are used to simultaneously initiate the entire surface of a main charge. The detonation of this main charge, in turn produces the energy that propels a flyer plate at velocities that can be as high as 3 *km/s* for simple planar systems.

The flyer plate is carried in a gas gun in a plastic sabot. Velocity of the flyer is determined from the transit time between the shorting pin in the gun barrel and time-of-arrival pins in the target. The target is supported by a spall ring that suppresses late time radial tensile waves. Details for this method can be found in references given [3, 27, 30].

4. SPLIT HOPKINSON PRESSURE BAR (KOLSKY BAR)

The present chapter includes the chronological development of the split Hopkinson bar apparatus, its fundamentals and working principle. A time line approach is taken to summarize the major advancements leading to the current arrangement of the split Hopkinson bar apparatus, beginning with its founder. The latter part of the chapter focuses on the other loading techniques of split Hopkinson bar such as, tension and torsion and also modifications that has been done for elevated temperature tests.

4.1 Historical Perspective of SHPB

As the 19th century progressed, there was an increasing awareness that the properties of materials under impact differed from those under static or quasi-static loading.

Experimental investigations were hampered by lack of suitable instrumentation but considerable theoretical progress was made in the understanding of the propagation of stress waves in bounded media such as circular rods. The idea of using a pressure bar to determine material properties was first used by John Hopkinson in 1872. He was able to determine properties of iron wire by transferring the energy of a dropping weight into the wire and measured how much it deformed before breaking. His experimental demonstrations showed that metals are able to withstand a larger impulsive tensile load than they can under static tensile loading. However, it was Dunn in 1897 to be credited with constructing the first true high strain-rate testing machine.

After John Hopkinson's death, one of his son, Bertram Hopkinson was elected to carry on his father's work as Professor of Engineering at Cambridge. He introduced a technique for determining the pressure – time relations due to an impact produced by a bullet or explosive. The key components of Hopkinson's apparatus were a means of developing impact like pressures, a long steel rod, a short steel billet and a ballistic pendulum. A schematic of his device is shown in figure 4.1.

By impacting one end of the rod, a compressive pressure wave of finite length is generated inside the rod, in other words, by using momentum traps of different lengths, he was able to build up a picture of how much momentum passed down the rod as a function of time for different classes of events (assuming that these types of events are repeatable). Because he performed the pioneering work on determining the shapes of impulses travelling down rods, the device, consisting of a long rod to convey a force pulse to a force transducer, became known as the Hopkinson pressure bar. This was rapidly put to use in the 1st. World War, and is still used to this day in its original configuration in blast wave loading studies.

Back to Hopkinson's Apparatus, if it can be summarized shortly, he attached a steel billet at the far end of the rod, held by only a thin layer of grease. His idea was that as the compressive wave traversed down the bar, through the greased joint, and into the billet the wave would be reflected at the far end as a pulse of tension. Since the grease could not withstand any appreciable tensile loads, the billet would fly off with a definite momentum, measured with a ballistic pendulum. The time over which this momentum acts is the round trip time of the longitudinal wave in the billet. By running several tests of identical magnitude but different length of cylindrical billets, a series of pressure – time curves were generated describing the impact event. Hopkinson was always capable of determining the maximum pressure and total duration of these impact events, however, lacking reliable methods of data storage and reduction, he was unsuccessful in generating reliable pressure versus time relationships for the impact experiments. He had, however, unknowingly laid out a method of material testing that would be revisited by future researchers and scientists investigating the dynamic response of materials subjected to high rates of strain.

If figure 4.1 is observed, we can explain as that, Bertram Hopkinson used a bar (B) suspended by two sets of wires that was in line with a box (D) which was also suspended. A short section of rod (C) was placed at the end of the main bar and held in place by a small magnetic force. A bullet was then shot at the end (A) of the longer bar causing a pressure wave to be imparted into the rod. The wave travelled down the long rod and into the short rod causing it to fly off and be caught by the box. The displacement of the box and the rod were measured with a simple

displacement device, allowing the calculation of momentum. The crude measuring devices available at that time limited the accuracy of the results of the experiments.

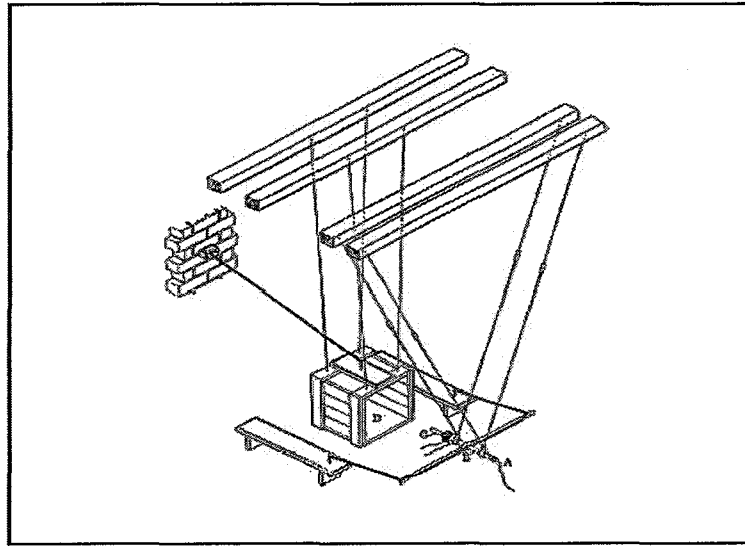


Figure 4.1 : Schematic of First Pressure Bar [24].

It can be said that there was little significant done in this area of research until 1948 when **Davies** [32] performed a critical study on the Hopkinson pressure bar. Davies used a condenser unit in order to measure more accurately the displacement of the bar's end. Davies' idea was that the displacement (strain) in the pressure bar was proportional to the stress developed in the bar. The pressure in the bar was kept well under the elastic limit of the pressure bar material. With this assumption in mind, Davies designed a condenser mechanism to generate an electrical output that was proportional to the displacement of the pressure bar in Hopkinson's original apparatus. One of the primary motivating factors behind Davies' work was the uncertainty associated with the grease applied to the test specimen. He was certain that the grease had effected the dynamics of the experiment but he was unsure how to account for it. Thus, Davies was able to improve the data acquisition process of Hopkinson's experiment by replacing the ballistic pendulum with his condenser strain measurement system while lessening the error and uncertainty associated with the greased test specimen.

The most profound addition to Hopkinson's research came in the late 1940's and early 1950's when Kolsky modified the original design with the addition of a second pressure bar. Kolsky's motivation was that the addition of the second pressure bar

would allow for strain data to be recorded at the input and output interfaces of the test specimen. In addition to specimen strain, this new version of Hopkinson's experiment allowed researchers to calculate specimen stress and strain rate (figure 4.2). Also, a consequence of "sandwiching" the test specimen between the two pressure bars was that homogeneous deformation would be much more easily achieved. Kolsky then further developed this technique by using an electrical condenser system to measure the strains in both pressure bars, known as the input and output pressure bars (figure 4.3). Using the strain data from the two pressure bars, Kolsky was able to derive expressions for calculating specimen stress, strain and strain rate. Due to its improved robustness, versatility and accuracy, Kolsky's version of the Hopkinson bar, which became known as the Kolsky bar or split Hopkinson pressure bar (SHPB), quickly became one of the preferred methods for testing materials at strain rates from 10^2 s^{-1} to 10^4 s^{-1} .

Significant improvements to methods of strain measurement came about in the 1960's with the introduction of strain gage technology. Using the change in resistance of a conductor due to changes in its length and cross sectional area, scientists could then measure the voltage output of a strain gage device and relate it to the displacement of the surface to which the gage was applied. **Krafft, et al [32]**, first implemented strain gage technology in the Hopkinson bar in 1954 when they studied the effects of static and dynamic loading and temperature on the yield stress of iron and mild steel in compression. In 1961, **Hauser, et al [33]**, used strain gages on the SHPB in his studies of static and dynamic compressive loading of iron and mild steel at elevated temperatures. **Lindholm and Yeakly [34]** performed similar research just 3 years later in his studies of the SHPB. Their experiments involved selection and mounting of the strain gages as well as interpretation of the data generated by the experiment. Even in its earliest stages of development, it was found that the strain gages significantly increased the accuracy, resolution and repeatability of the data.

Since the 1970's, the most significant improvements to the Hopkinson bar experiment and its derivatives have come in the form of high-speed computer data acquisition systems (as in figures 3.10 and 4.4). Digital storage oscilloscopes and high-bandwidth signal analyzers have allowed the scientists to obtain much more

highly resolved data with much greater precision. Current researchers routinely use digital storage oscilloscopes with bandwidths as high as 500 MHz. Additional research has been done into pressure bar characteristics, specimen geometry effects and mathematical modeling.

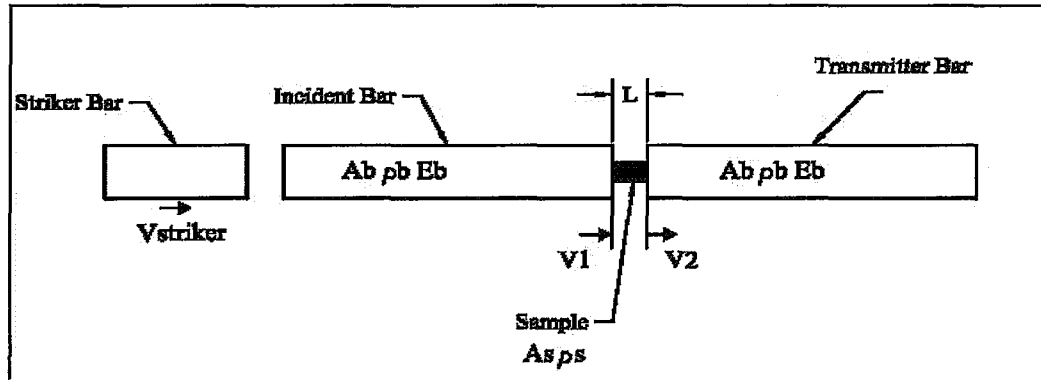


Figure 4.2 : Schematic of Split Hopkinson Pressure Bars and Specimen [39].

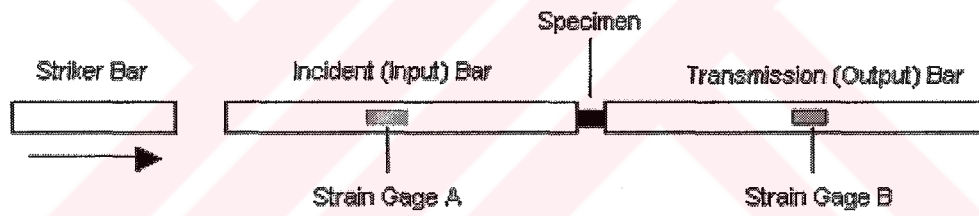


Figure 4.3 : Simplified Schematic of Split Hopkinson Pressure Bar with Strain Gages Attached [39].

Kaiser [35] presented his experimental results in 1998, which included a new dispersion correction technique and a novel numerical method of aligning strain pulses. Other recent advancements have come in strain gage technology, as new grid patterns and sensing element materials have been introduced which have greatly reduced the errors associated with the strain gage. Such errors include lateral strain sensitivity, thermal sensitivity and hysteresis effects.

Ramesh and Narasimhan [36] were the first investigators to publish SHPB data using an optical strain measurement system in 1996. Their optical system, known as the LORD (Laser Occlusive Radius Detector) system, utilized a 670-nanometer (nm)

laser diode capable of generating 3 milliwatts (mW) of output power to measure the change in diameter of the cylindrical specimen typically used in the SHPB. Though their LORD system produced very promising results, little work has been done involving the use of optical measurement methods in the Hopkinson bar since their efforts.

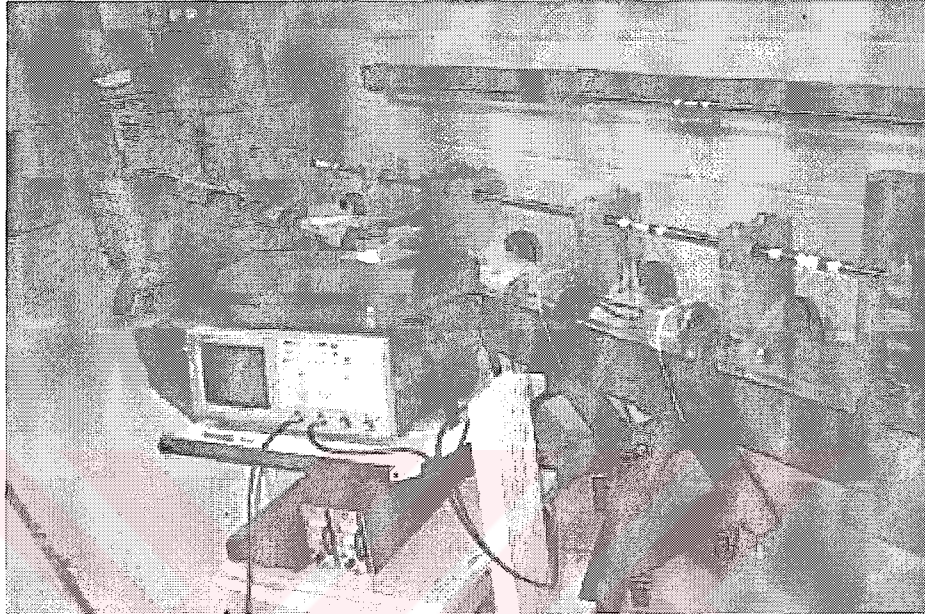


Figure 4.4 : PC Connected SHPB Apparatus with Data Acquisition System [35].

Woldesenbet and Vinson [37] performed tests to determine the effects of specimen geometry on testing results. Their experiments focused on varying the length-to-diameter ratio of the specimen from 0.5 to 2.0, but also included the use of square/rectangular specimens. In comparing the test results for various l/d ratios, "no statistically significant effect of either l/d or geometry could be found." Other investigators have suggested using l/d ratios in the range of 0.5 to 1.0 to minimize the errors due to pressure bar/specimen friction and radial inertia [24, 35, 38-40].

4.2 Fundamentals of Split Hopkinson Pressure Bar Technique

This section introduces and fully develops the theory behind compressive split Hopkinson bar testing. It begins by summarizing the principles of a conventional SHPB and then developing the equation of motion for a rod in axial vibration, followed by the wave interaction principles and cross-sectional and impedance variation investigations. The complete and reduced equations are derived for

calculating the specimen stress, strain, and strain rate. The phenomenon of wave dispersion is discussed. Finally the data acquisition equipments and gas gun systems will be presented. A summary is then included to discuss the equations used to calculate specimen properties.

4.2.1 Principles of Conventional SHPB Technique

As indicated in the previous part, the Hopkinson Bar has been in existence since the early 1900's and today is the most commonly used method for determining material properties at high rates of strain. Since then, many derivatives of Hopkinson's experiment have been developed. One of the most popular derivatives is the Split Hopkinson Pressure Bar (SHPB) or sometimes it is referred as Kolsky bar.

It was identified by Hopkinson that the use of a long elastic bar to study high-rate phenomena is feasible, because the wave propagation behaviour in such a geometry is well understood and mathematically predictable. Thus, the displacements or stresses generated at any point can be deduced by measuring the elastic wave at any point propagates along the bar.

Significant advancements implemented from the areas of testing techniques, numerical methods, and signal processing have improved the accuracy and repeatability of high strain rate testing. Before proceeding with the development of the equations governing the experiment, a basic description of the apparatus will be given as shown in figure 4.5.

In the split Hopkinson bar test, a short cylindrical specimen is "sandwiched" between two long elastic bars, as shown in figure 4.5. Proper alignment of the pressure bars with the specimen ensures a uniaxial state of stress while the thin layer of lubricant applied to the specimen promotes homogeneous deformation in the specimen. The ends of the pressure bars and specimen are machined flat to enforce prescribed boundary conditions. Figure 4.5 presented above, although a very simplified illustration of the SHPB, gives the reader a general idea of the experimental procedure. Note the arrow above the striker bar indicating the axis of motion in the experiment. Not shown in fig. 4.5 are the bearing blocks that support the pressure bars while limiting all motion to the longitudinal axis of the striker, input and output bars. The gas gun system is not shown in this simplified schematic as well. Also the

specimen size has been greatly exaggerated in relation to the test bars to give a clearer picture of the test procedure. The dimensions of the striker bar, pressure bars and test specimen as well as the yield strengths of the pressure bars are important features of the test apparatus.

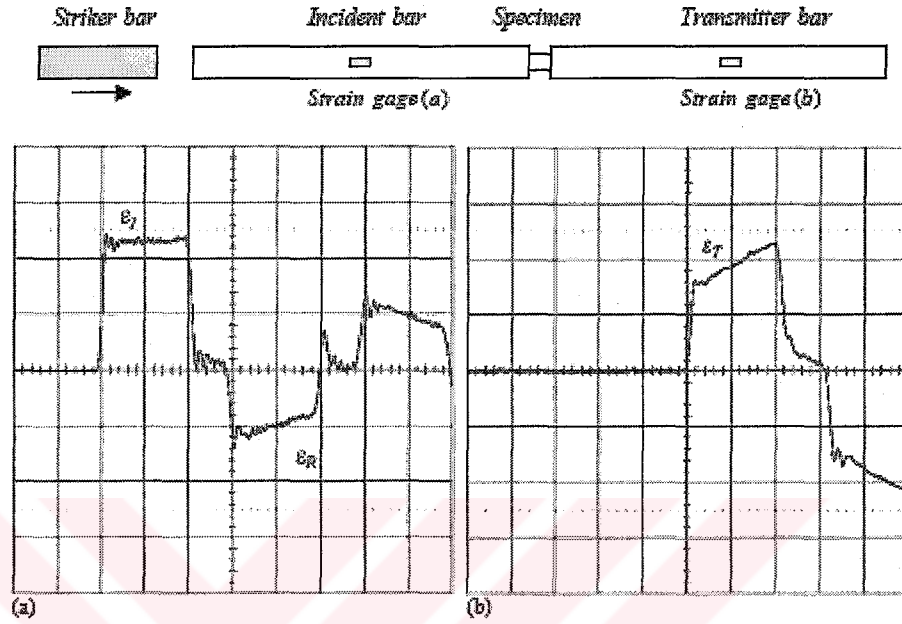


Figure 4.5 : Schematic of Split Hopkinson Pressure Bar ,strain histories in the (a) incident bar, (b) transmitter bar.

To review the experiment, generally a pressure gun fires the striker bar at the input bar, generating an elastic compressive stress wave (incident wave) that travels the length of the input bar to the input bar/specimen interface. A time-dependent strain, $\epsilon_I(t)$, is developed in the input bar and this is measured by as time resolved voltage variations by strain gage A. At the input bar/specimen interface, a portion of the incident wave is transmitted through the specimen and into the output bar. The remainder of the original incident wave reflects back into the input bar. The reflected pulse is reflected as a wave in tension with strain $\epsilon_R(t)$ and recorded by strain gage A again, whereas the transmitted pulse remains in compression and its compressive strain $\epsilon_T(t)$ is recorded in the output bar by strain gage B fig. 4.5 and 4.6. The velocity of the striker bar controls the strain rate achieved, while the length of the striker bar determines the duration of the test. This strain data is recorded on a digital oscilloscope or similar equipment such that the voltage variations versus time signals can be converted to strain in pressure bars vs time and then manipulated to yield

specimen stress and strain as well as strain rate. It is interesting also to mention that wave propagation in a specimen is not considered. Actually, when the wave initially enters the specimen there are reverberations. After approximately three reverberations equilibrium is reached in the specimen.

It should be underlined that, the following conditions should also be carefully observed to obtain meaningful results from SHPB tests;

1. The split bars should remain within elastic regime during operations,
2. The waves propagated in the bars should be one-dimensional longitudinal type,
3. The deformation in the specimen should be uniform through the length of the specimen.

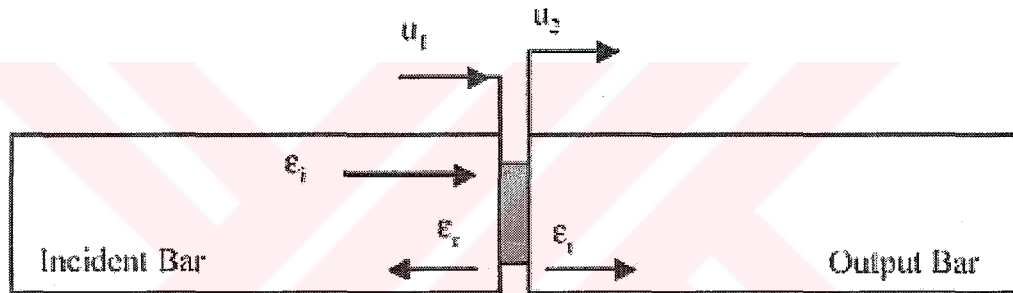


Figure 4.6 : Schematic of Incident and Output Bars, showing $\epsilon_I(t)$, $\epsilon_R(t)$ and $\epsilon_T(t)$.

Typically, the strain rate for the SHPB is limited to $10^4 s^{-1}$. This upper strain level limitation is the consequence of a number of factors including;

1. The effects of mechanical dispersion on signal resolution,
2. Input bar length-to-diameter ratio requirements for one-dimensional wave propagation theory and uniaxial stress, homogeneous deformation,
3. Pressure bar length requirements to ensure separation of the input bar signal.

Longitudinal wave dispersion has adverse effects on all conventional SHPB experiments. According to **Burstow, et al [41]**, the SHPB is limited at medium rates of strain by its time resolution of approximately $1\mu s$ caused by acoustic dispersion (figure 4.7). Dispersion is best defined as the wave propagation velocity dependence

on frequency. In essence, the impact caused by the striker bar impact excites many different frequencies within the input bar. Each of these component frequencies travels at a different velocity within the input bar, causing an oscillation in the time history recorded by the strain gage. This introduces an undesirable amount of uncertainty in the signal in that the peak strain is no longer apparent.

Another important consideration in the SHPB is the geometry of the pressure bars, particularly the input bar. Also known as the incident bar, the input bar “delivers” the mechanical stress wave to the test specimen. A portion of this wave is transmitted into the specimen while the remainder is reflected back into the input bar. The length and diameter of the input bar is critical in enforcing the assumptions of uniaxial stress, homogeneous deformation in the specimen and elastic behavior in the incident pressure bar. In order to ensure that uniaxial, homogeneous deformation is generated in the specimen while the peak stress in the pressure bar is well below its elastic limit, eqn. (4.1) must hold true:

$$\frac{L_{bar}}{D_{bar}} \geq 11 \quad (4.1)$$

In other words, the input bar length-to-diameter must be greater than 10. Also, in order to ensure that the incident and reflected pulses in the input pressure bar can be separated, the bar length must be at least twice the wavelength of the initial compressive wave.

Later in this chapter, it will be shown that specimen strain rate $\dot{\epsilon}$ is inversely proportional to the length of the pressure bar. Thus,

$$\dot{\epsilon} \propto K * \frac{1}{L_{bar}} \quad (4.2)$$

where K is a constant of proportionality and L_{bar} is the pressure bar length. Accordingly, in order to achieve strain rates greater than approximately $10^4 s^{-1}$ pressure bars must be with very small lengths and diameters. However, experimentation becomes impractical with pressure bar diameters much less than 6.5 mm due to the decreased size of the pressure bars and, more importantly, the requirement of a specimen with much decreased diameter.

In addition to the geometry considerations presented above, precise specimen positioning and alignment also play important roles in the deformation event. In order to ensure that homogeneous deformation occurs, the ends of the specimen should be lubricated with a suitable lubricant to reduce the frictional coefficient at the pressure bar-specimen interfaces [42]. Also, some method of ensuring that the specimen is centered between the pressure bars and that it is perfectly aligned longitudinally with the pressure bars is needed. Failure to meet these conditions can result in non-uniaxial stress and non-homogeneous deformation.

Also the locationing of the two strain gages in the two pressure bars are very important, as shown in figure 4.5. The strain Gages are usually placed in a location where no superposition between the reflected wave and incident wave will occur over the duration of the test. Thus, gages must be mounted considering the wavelength of the pulse and the dimension of the striker bar in order to avoid superposition. The strain gage on the incident bar is often located at the bar midpoint so that the incident and reflected waves can be measured independently. If the strain gage was located so close to the incident bar-specimen interface, the leading edge of the reflected wave could interfere with the trailing edge of the incident wave.

Before any experimentation can begin, the researcher must first specify the total strain applied to the specimen. This strain value can then be used, along with the physical dimensions of the specimen and specimen stress constant, to compute the necessary parameter for setting up the experiment to yield the desired strain. These computations will be developed in the following sections. Figure 4.5 and 4.7 shows a typical response of a SHPB when strain signals are recorded by strain gages. These voltage variations will be converted into stress to obtain the stress vs time curve of the material [3, 24, 35, 43 - 45].

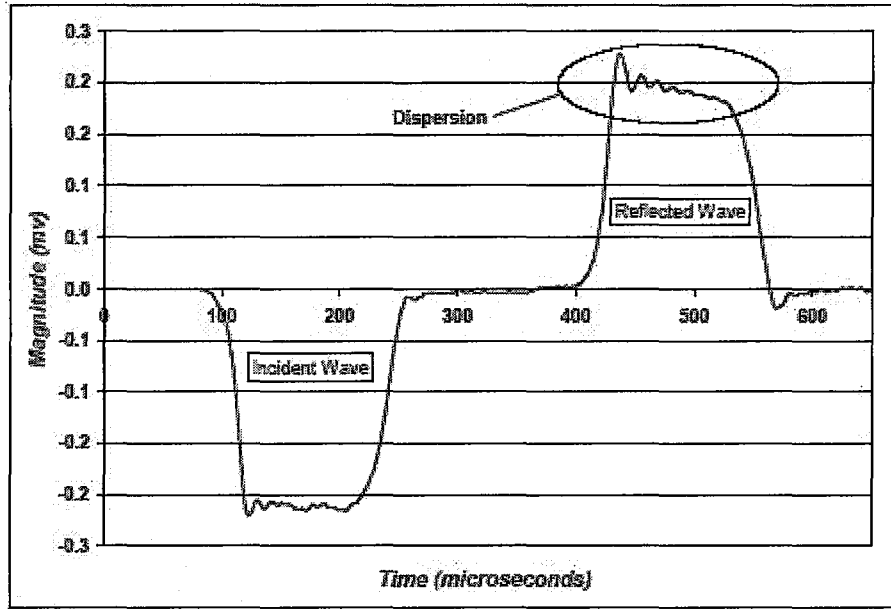


Figure 4.7 : Strain signals recorded by strain gages and converted into stress.

4.2.2 The Equation of Motion of a Bar

In order to understand the dynamics of the SHPB, it is necessary to consider a single, differential element dy of a long rod of cross sectional area A_0 , density ρ and Young's Modulus E . As specified by the experiment, the rod is subjected to an impact along its longitudinal axis. Since the two bars are identical, it is only necessary to consider one of them in developing the equation of motion governing axial vibration. Typically the length-to-diameter ratio of the Hopkinson pressure bar due to some considerations can be kept as much as possible, such as eighty or greater,

$$\frac{L_{bar}}{D_{bar}} \approx 80 \text{ to } 100, \text{ however there are other used ratios in literature;}$$

The derivation of the equation for a uniform bar in axial vibration is studied in literature widely and all of the derivations begin by considering a differential cross section of a bar prior to deformation and then just after the deformation begins. Figure 4.8 depicts a non-deformed pressure bar with differential element.

The differential element has a length dy and cross sectional area A_0 . Prior to impact the bar is in static equilibrium. Just after the impact, particles in the differential element are in compression due to forces F_1 and F_2 , as shown in figure 4.9.

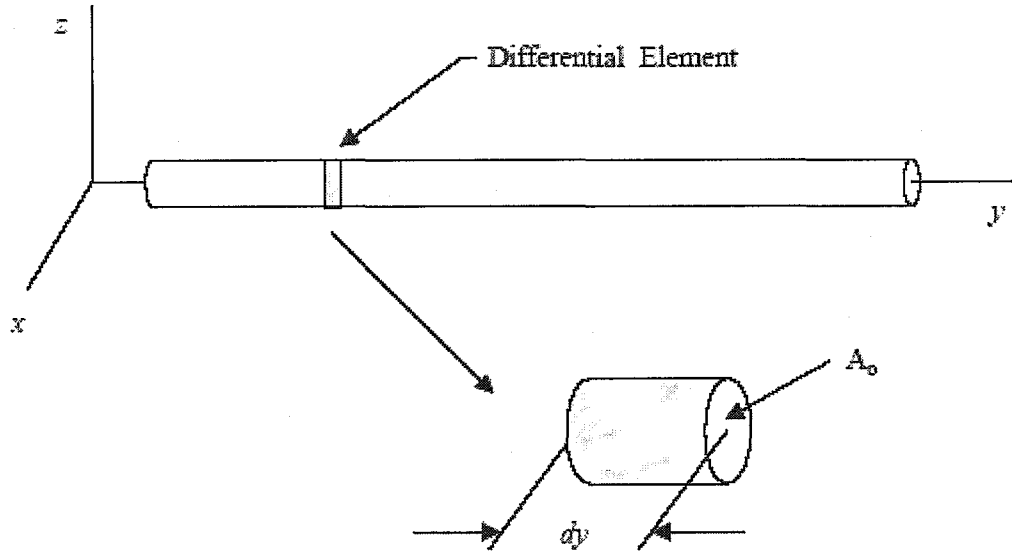


Figure 4.8 : Pressure bar shown with differential element prior to deformation [35].

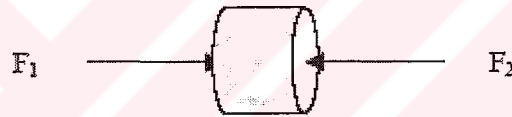


Figure 4.9 : Differential Element in Uniaxial Compression [35].

Assuming perfectly elastic behavior in the pressure bars , the compressive forces F_1 and F_2 can be related to the specimen engineering stress σ by eqn. (4.3):

$$\sigma = \frac{F}{A_o} \quad (4.3)$$

Equation (4.3) simply states that engineering stress is equal to force per initial specimen area. This stress is related to the specimen strain through Hooke's Law, as;

$$E = \frac{\sigma}{\epsilon} \quad (4.4)$$

However note that this relationship is only valid for purely elastic behavior. Continuing with the analysis, the specimen strain ϵ can be redefined in terms of specimen particle displacements u_1 and u_2 , as in figure 4.10, such that;

$$F_1 = A_o E \frac{\partial u_1}{\partial y} \quad (4.5)$$

$$F_2 = A_o E \frac{\partial u_2}{\partial y} \quad (4.6)$$

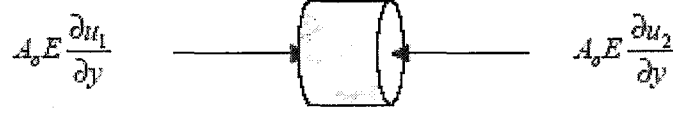


Figure 4.10 : Forces applying compression on differential element [35].

The expressions $\frac{\partial u_1}{\partial y}$ and $\frac{\partial u_2}{\partial y}$ represent the specimen strain at each boundary condition and F_1 and F_2 are uniaxial, $F_1 = -F_2$. By summing up the forces acting on the differential element and by applying Newton's second law, $F = ma$, the following equation describing the motion of the pressure pulses can be expressed as;

$$A_o E \frac{\partial u_1}{\partial y} - A_o E \frac{\partial u_2}{\partial y} = A_o dy \rho \frac{\partial^2 u_1}{\partial t^2} \quad (4.7)$$

Where $\frac{\partial^2 u_1}{\partial t^2}$ is the particle acceleration within the element and it is assumed to be constant across the element. In addition it is also assumed that the bar material is homogeneous across its cross section. Simplifying the equation above yields the bar's equation of motion;

$$C_o \left[\frac{\partial u_1}{\partial y} - \frac{\partial u_2}{\partial y} \right] = \frac{\partial^2 u_1}{\partial t^2} dy \quad (4.8)$$

Here, C_o , is defined as the longitudinal wave velocity (presented in section 1) and it is defined as;

$$C_o = \sqrt{\frac{E}{\rho}} \quad (4.9)$$

The wave velocity can also be identified from the wave pattern based on the following formula;

$$C_o = \frac{2l}{t} \quad (4.10)$$

where $2l$ is the total length of wave propagation and t is the travelling time of the wave within this distance.

On the other hand, further simplification for eqn. (4.8) is present, such that many investigators using the principles of the conservation of momentum, define u_2 in terms of u_1 as follows;

$$u_2 = u_1 + \frac{\partial u_1}{\partial y} dy \quad (4.11)$$

Upon differentiating, this equation becomes;

$$\frac{\partial u_2}{\partial y} = \frac{\partial u_1}{\partial y} + \frac{\partial^2 u_1}{\partial y^2} dy \quad (4.12)$$

By substituting eqn. (4.12) into (4.8), the equation of motion for the bar reduces to;

$$C_o^2 \frac{\partial^2 u_1}{\partial y^2} = \frac{\partial^2 u_1}{\partial t^2} \quad (4.13)$$

The equation of motion for the bar will be used as an aid in simplifying the equations defining the specimen stress, strain, and strain rate in the next sections [3].

4.2.3 Wave Reflection, Refraction and Interaction

In this section the interaction of waves when they encounter a body will be discussed. Figure 4.11 shows the longitudinal waves that are reflected and refracted at the boundary as well as the two transverse waves that are generated at the interface. These reflection and refraction occur when the wave encounters a medium with different acoustic impedance. The acoustic impedance is defined as the product of the mass density by its elastic wave velocity;

$$\text{Acoustic Impedance} = I = \rho \cdot C_0 \quad (4.14)$$

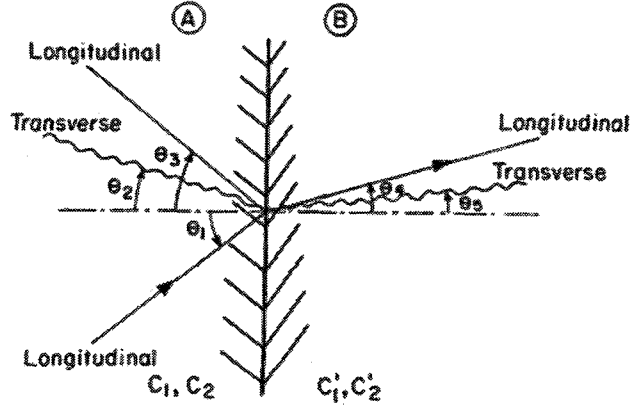


Figure 4.11 : Longitudinal elastic wave incident on boundary between media A and B, generating refracted and reflected waves. C_1 and C_2 are the longitudinal and shear wave velocities in medium A; and vice versa for medium B [3].

The interactions of a wave with an interface are much simpler when the incidence is normal ($\theta = 0^\circ$). When this condition is provided, a longitudinal wave refracts (transmits) and reflects longitudinal waves, and vice versa for shear waves. Figure 4.12(a) shows the front of a wave propagating along a cylinder with cross-sectional area A in a medium in which the wave velocity is C_A . The particle velocity is U_p and the stress is σ . Figure 4.12(b) shows an interface and the forces due to incident, transmitted and reflected waves; figure 4.12(c) shows the particle velocities of the incident, transmitted and reflected waves. The amplitudes of the transmitted and reflected waves can be calculated from the densities and wave velocities of the two media.

The expression relating the uniaxial stress σ to particle and wave velocity is;

$$\sigma = \rho \frac{d_x}{d_t} U_p = \rho C U_p \quad (4.15)$$

If the interface, in figure 4.12, to be considered in equilibrium under three stresses, incident, transmitted and reflected, than it can be expressed as;

$$\sigma_I + \sigma_R = \sigma_T \quad (4.16)$$

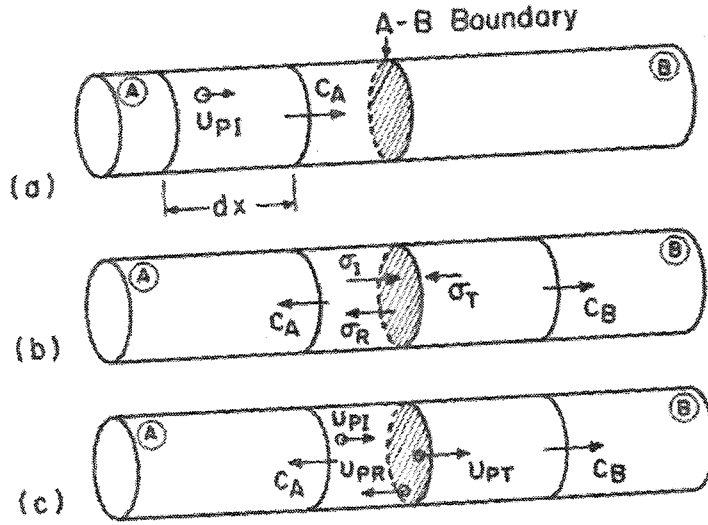


Figure 4.12 : Longitudinal wave incident on boundary between two media A and B in normal trajectory: (a) prior to encounter with boundary, (b) forces exerted on boundary (equilibrium), (c) particle velocities (continuity. Direction of arrows for reflected wave case for Empedance $A >$ Empedance B [3].

For the continuity at the interface we can also express;

$$U_{PI} + U_{PR} = U_{PT} \quad (4.17)$$

The signs of the stresses and particle velocities in figure 4.12 were set in such a manner that medium A has a higher empedance than medium B. The particle velocities are obtained from eqn. (4.15) and defined as;

$$U_{PI} = \frac{\sigma_I}{\rho_A C_A} \quad U_{PT} = \frac{\sigma_T}{\rho_B C_B} \quad U_{PR} = \frac{-\sigma_R}{\rho_A C_A} \quad (4.18)$$

The sign of the reflected particle velocity was inverted because a positive stress causes a negative particle velocity upon reflection. Substituting eqn. (4.18) into (4.17) yields to;

$$\frac{\sigma_I}{\rho_A C_A} - \frac{\sigma_R}{\rho_A C_A} = \frac{\sigma_T}{\rho_B C_B} \quad (4.19)$$

Equations (4.16) and (4.19) provide;

$$\frac{\sigma_T}{\sigma_I} = \frac{2\rho_B C_B}{\rho_B C_B + \rho_A C_A} \quad (4.20)$$

$$\frac{\sigma_R}{\sigma_I} = \frac{\rho_B C_B - \rho_A C_A}{\rho_B C_B + \rho_A C_A} \quad (4.21)$$

From eqn. (4.20) and (4.21), it can be concluded that the impedances of the materials determine the amplitude of transmitted and reflected pulses. When $\rho_B C_B > \rho_A C_A$, a pulse of the same sign as the incident pulse is reflected. In contrast, the sign of the reflected pulse will be opposite of that the incident pulse has.

There are two limiting conditions in this interaction investigation; a wave incident on a free surface ($E=0$) and a wave incident on a rigid boundary ($E=\infty$).

For a free surface (bars with free end), the acoustic impedance is null so;

$$\frac{\sigma_T}{\sigma_I} = 0 \quad \text{and} \quad \frac{\sigma_R}{\sigma_I} = -1$$

and for a rigid boundary, (considering $E=\infty$, leading to $C=\infty$);

$$\frac{\sigma_T}{\sigma_I} = \frac{2}{1 + \frac{\rho_A C_A}{\rho_B C_B}} \cong 2$$

$$\frac{\sigma_R}{\sigma_I} = \frac{1 - \rho_A C_A / \rho_B C_B}{1 + \rho_A C_A / \rho_B C_B} \cong 1$$

Expressions for the particle velocities can also be similarly defined as;

$$\frac{U_{pR}}{U_{pI}} = \frac{\rho_A C_A - \rho_B C_B}{\rho_A C_A + \rho_B C_B} \quad (4.22)$$

$$\frac{U_{pT}}{U_{pI}} = \frac{2\rho_A C_A}{\rho_A C_A + \rho_B C_B} \quad (4.23)$$

For a free surface;

$$\frac{U_{pT}}{U_{pI}} = 2 \quad \text{and} \quad \frac{U_{pR}}{U_{pI}} = 1,$$

For a rigid boundary;

$$\frac{U_{pT}}{U_{pI}} = 0 \quad \text{and} \quad \frac{U_{pR}}{U_{pI}} = \frac{\rho_A C_A / \rho_B C_B - 1}{\rho_A C_A / \rho_B C_B + 1} = -1.$$

Figure 4.13 shows the sequences of stresses and particle velocities for rigid and free boundaries. It is clear in figure that when a compressive wave encounters a free surface, it reflects back as a tensile wave and vice versa; the stress sign is changed, while the particle velocity is maintained. When a rigid boundary is encountered the opposite occurs, the stress sign is maintained and the particle velocity direction is reversed. Additionally it can be reminded that if the particle velocity and wave velocity have the same direction and sense, the stress is compressive, if they have opposite senses, the stress is tensile. More detail about this topic can be found in references given [3, 5].

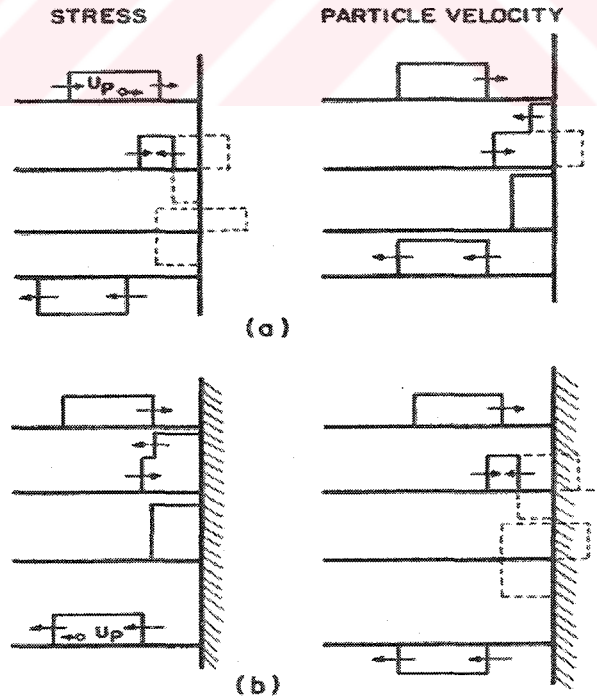


Figure 4.13 : Reflection of a stress pulse (rectangular) at (a) free surface and (b) rigid boundary [3].

4.2.4 Elastic Waves in Cylindrical Bars

In order to analyze the Hopkinson bar experiments it is vital to understand the propagation of elastic waves in cylindrical bars. When a cylindrical bar is impacted with a cylindrical projectile of length L , a rectangular pulse of length $2L$ propagates through the bar if the bar and the projectile are of the same material (It is often the same in SHPB experiments). This is shown schematically in figure 4.14. The impact produces compressive waves propagating at velocities C_0 into projectile and target. As the compressive wave reaches the end of the projectile, it reflects back as shown in figure 4.14. This determines the length of the pulse, $\lambda = 2L$. The velocity of the interface, equal to the particle velocity, can be calculated from the conservation of momentum equation;

Prior to impact : $\rho_0 A_0 L V_0$,

After Impact: $\rho_0 A_0 \lambda U_p = 2\rho_0 A_0 L U_p$.

Thus,

$$U_p = \frac{V_0}{2} \quad (4.24)$$

Substituting eqn. (4.15) into eqn. (4.24) yields;

$$\sigma = \frac{1}{2} \rho C V_0 \quad (4.25)$$

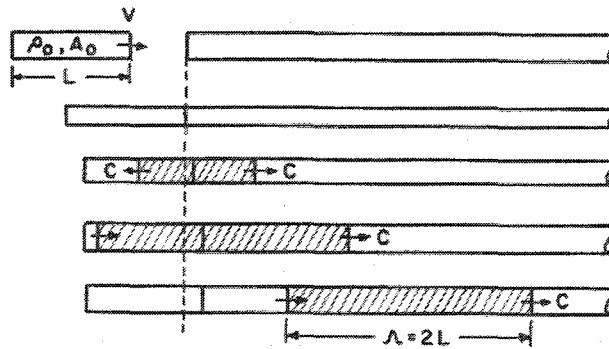


Figure 4.14 : Cylindrical Projectile Impacting Cylindrical Bar [3].

The stress value in eqn. (4.25) is equal to the striker bar's and input bar's stress and dependent on the striker bar velocity for the same bar materials. The expected shape of this propagating pulse is rectangular with a duration of $2L/C$ and this is also dependent on the striker bar length and its material properties. The gages mounted on the Hopkinson Bars record these stress pulses and voltage variations vs time signals from wheatstone bridges are sent to an oscilloscope however the outputs from the oscilloscope will show significant fluctuations (oscillations), called as dispersion effects, as shown in figure 4.15.

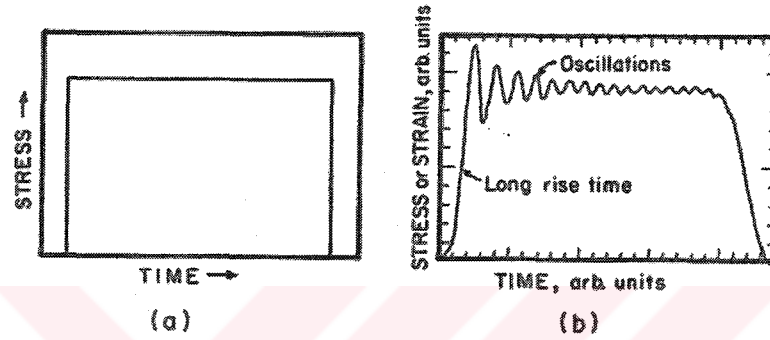


Figure 4.15: (a) Idealized and (b) actual stress pulse recorded in cylindrical bar [3].

This dispersion effect (also shown in figure 4.7) is due to the variation of the longitudinal velocity with the wavelength. If the wave is a pure cosine wave of wavelength λ , the axial displacements and stresses are uniform over the cross section of the bar as long as $R/\lambda \ll 1$, where R is the radius of the bar. In fact, when $R/\lambda < 0.1$, the displacements at the surface of the bar differ from those along the bar axis by less than 5%. The majority of the energy in waves generated in the split Hopkinson bar is contained in wavelengths that exceed $10R$, thus the wave can be assumed to be one-dimensional, and surface displacement measurements are accurate indicators of axial displacements in the pressure bar. However if the wavelength is changed the longitudinal velocity will change. For $R/\lambda \ll 1$, the propagation velocity is the well-known longitudinal sound velocity C (eqn. 4.9). But as λ approaches R , the velocity of these short wavelength components decreases to $0.576C$ (for a bar material with Poisson's ratio of 0.29). The lower velocity of the short wavelength components causes dispersion of the initial wave and leads to the appearance of oscillations, Pochhammer-Chree oscillations that ride waves and build

during propagation down the bar. These oscillations are simply short wavelength frequency components that lag behind the leading edge of the wave.

Note the dispersive effects shown in the input bar strain history in figure 4.15b. Dispersion appears as an oscillation at the peak strain regions and has the effect of distorting the data by altering the peak strain but also by attenuation of the signal. This attenuation is particularly apparent in the reflected pulse. The effects of dispersion can be accounted for in a variety of ways, including using tabulated dispersive corrective values [46] as well as modeling the dispersive effects numerically [35]. Alternatively, these dispersive effects can be lessened or, possibly, completely removed via the use of an instrumentation system that measures strain directly at the specimen [3,39,47].

4.2.5 Variation of Cross Section and Impedance in SHPB

The most commonly encountered discontinuities in split Hopkinson bar testing are step changes in cross sectional area and material properties. Waves encountering discontinuities are usually examined in terms of impedance. Impedance is defined as the ratio of the driving force to the velocity at a point in a structure (pressure bar in this case). Equation (4.26) is an expression for the mechanical impedance of the bars used in Hopkinson bar testing;

$$Z = \frac{F}{V} = s \cdot \rho \cdot C_o \quad (4.26)$$

The variables ρ , s , and C_o are the density, cross sectional area, and longitudinal wave velocity, respectively. The product ρC_o (acoustic impedance, E) has a constant value for a given material; therefore it is often convenient for the two to be grouped together as one term. Notice that all of the impedance variables are physical properties; the impact event does not effect the impedance of the bars. Further notice that for any given bar material an impedance change can only occur by changing the cross sectional area. However, discontinuities occur at the pressure bar – specimen interfaces. Since a wide range of materials may be under investigation, it is important to understand how waves respond to a change in medium. Consider the familiar scenario in which one type of solid is sandwiched between two dissimilar solids of different cross sectional areas, showed in figure 4.16.

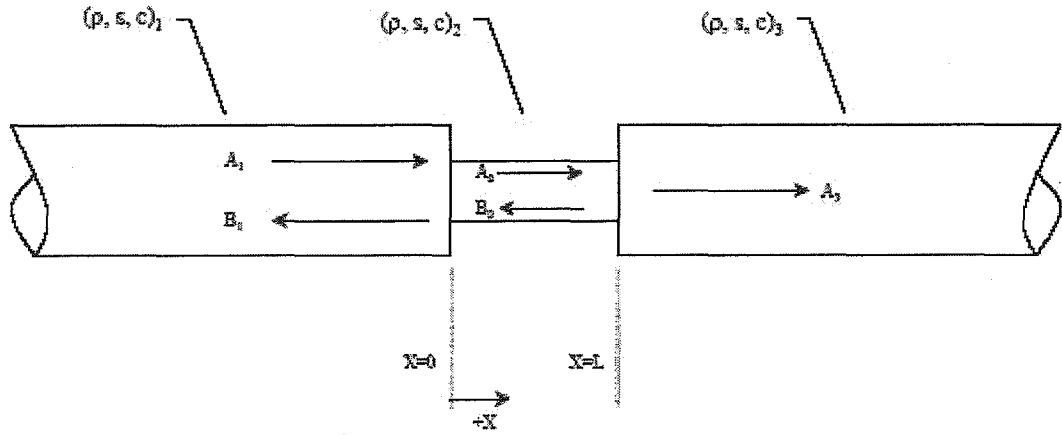


Figure 4.16 : Step Change in Cross-Sectional Area and Empedance in the System [35].

When the cross-sectional area of a cylindrical bar is changed, one has a geometric impedance difference, furthermore due to the different material properties of the specimen and bars there will also be an additional material impedance mismatch. The variables ρ , s , and c are the same variables used in eqn. (4.26). The terms $A1$ - $A3$ and $B1$ - $B2$ denote the stress amplitudes for waves traveling to the right and to the left, respectively. At interface 1 ($X=0$), wave $A1$ is partially reflected and transmitted as waves $B1$ and $A2$, respectively. At interface 2 ($X=L$), wave $A2$ is partially reflected and transmitted as waves $B2$ and $A3$, respectively. The level of reflection or transmission is dependent on the impedance mismatch at each interface.

At each of the two pressure bar - specimen interfaces, the velocity of each material just to the left and right of the interface must be equal, since they are in intimate contact at all times. The forces just to the left and right of each interface must balance one another to satisfy equilibrium. By recognizing that these conditions must be true, equations describing the interface effects on wave propagation may be written;

At interface 1 ($X=0$) :

$$\text{From continuity of velocity} \rightarrow \frac{A_1 - B_1}{(\rho C)_1} = \frac{A_2}{(\rho C)_2} \quad (4.27)$$

$$\text{From force balance} \rightarrow s_1(A_1 + B_1) = s_2(A_2) \quad (4.28)$$

At interface 2 (X=L) :

$$\text{From continuity of velocity} \rightarrow \frac{A_2 - B_2}{(\rho C)_2} = \frac{A_3}{(\rho C)_3} \quad (4.29)$$

$$\text{From force balance} \rightarrow s_2(A_2 + B_2) = s_3(A_3) \quad (4.30)$$

Notice from the four expressions that five stress variables exist, A_1 , A_2 , A_3 , B_1 , and B_2 . To solve explicitly for any of these variables requires an additional expression containing the stress variables. By defining the transmission coefficient, a means of calculating the stress amplitude transmitted through a boundary exists. Subsequently expressed as α , the transmission coefficient is a number ranging in value from zero to one, zero representing a complete reflection and one representing 100% transmission (no reflection). The transmission coefficient is given by;

$$\alpha = \frac{A_t}{A_i} = \frac{\sigma_T}{\sigma_I} \quad (4.31)$$

The subscripts t represents the transmitted and i , the incident stresses, respectively. The transmitted stress wave is in phase with the incident stress wave at the interfaces, therefore only the absolute magnitudes need be considered in eqn. (4.31).

Based on the conservation of energy principal, a conjugate term to the transmission coefficient may be defined as;

$$\beta = 1 - \alpha \quad (4.32)$$

β , the reflection coefficient, is related to α by eqn. (4.32), since the wave either reflects or transmits at a boundary. As the name suggests, the reflection coefficient is a measure of the amount a wave is reflected at a boundary. With knowledge of one coefficient, the other can be calculated simply. The transmission and reflection coefficients can give an investigator valuable insight to properly testing a wide range of materials. With these coefficients defined, expressions describing wave transmission may be solved for at each boundary. The following expressions are for the fraction of a wave entering and leaving the sandwiched solid in figure 4.16, respectively.

$$\alpha_{12} = \frac{2s_1(\rho C)_2}{s_1(\rho C)_1 + s_2(\rho C)_2} \quad (4.33a)$$

$$\alpha_{23} = \frac{2s_2(\rho C)_3}{s_2(\rho C)_2 + s_3(\rho C)_3} \quad (4.33b)$$

By varying the cross sectional areas and ρc values in eqn. (4.33a) and (4.33b), many transmission coefficients can be achieved.

This analysis is very important in Hopkinson bars, where ceramic and/or metal specimens are being sandwiched between high-strength steel bars. The equations 4.33 can also be expressed as by considering in terms of stresses by the help of eqn. (4.20) and (4.21) with adding the cross sectional effect;

$$\sigma_T = \frac{2s_1\rho_2C_2}{s_1\rho_1C_1 + s_2\rho_2C_2} \sigma_i \quad (4.34a)$$

$$\sigma_R = \frac{s_2\rho_2C_2 - s_1\rho_1C_1}{s_1\rho_1C_1 + s_2\rho_2C_2} \sigma_i \quad (4.34b)$$

It can be seen that, when two materials are the same, the parameters ρ and C will have the same values so the above equations will be dependent only on the cross sections [3, 5, 35].

4.2.6 Stress, Strain and Strain Rate Equations for SHPB Technique

For the derivation of stress, strain and strain rate equations within the specimen one should refer to figure 4.8 again. In section 4.2.2, the equation of motion of a bar was

$$\text{expressed in reduced form as } C_o^2 \frac{\partial^2 u_1}{\partial y^2} = \frac{\partial^2 u_1}{\partial t^2}.$$

For harmonic waves (assuming that the stress and strain are harmonic in nature) we can express;

$$E \frac{\partial}{\partial x} \left(\frac{\partial u_1}{\partial y} \right) = \frac{\partial \sigma}{\partial y} \quad (4.35)$$

where σ is the stress across the cross section. So the equation of motion can be rewritten in terms of the pressure and velocity across the bar cross section as;

$$-\frac{\partial \sigma(y,t)}{\partial y} = p \frac{\partial v}{\partial t} \quad (4.36)$$

For a uniaxial state of stress, the pressure is equal to the stress over the pressure bar cross section and that is $p(y,t) = \sigma(y,t)$.

If a positive travelling harmonic wave of the form is assumed;

$$p(y,t) = P e^{i(\omega t - ky)} \quad (4.37)$$

where P is the amplitude of the pressure pulse, ω is the pulse frequency, t is the time (temporal index) and k is the wave number which is defined as $k = \omega / C_o$. Here, y is the spatial location of the wave.

Taking the first derivative of eqn. (4.37) with respect to y the following expression can be made;

$$\frac{\partial \sigma(y,t)}{\partial y} = -ik P e^{i(\omega t - ky)} \quad (4.38)$$

From here, the particle velocity can be expressed as;

$$v(y,t) = \frac{k}{p\omega} P e^{i(\omega t - ky)} = \frac{k}{p\omega} \sigma(y,t) = \frac{1}{pC_o} \sigma(y,t) \quad (4.39)$$

As $\sigma(y,t) = \varepsilon(y,t)E$, the particle velocity in terms of the bar strain can be expressed as;

$$v(y,t) = C_o \varepsilon(y,t) \quad (4.40)$$

However note that for a reflected wave which is propagating in $-y$ direction, the particle velocity is;

$$v(y,t) = -C_o \varepsilon(y,t) \quad (4.41)$$

with these equations above the specimen strain rate can be calculated easily.

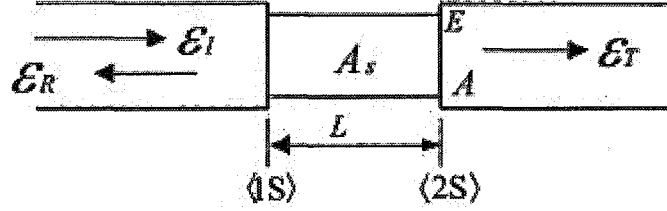


Figure 4.17 : Parameters of Cylindrical Specimen and Bars.

For the specimen shown in figure 4.17, The average strain at any time is given by;

$$\epsilon_s = \frac{u_{1S} - u_{2S}}{L} \quad (4.42)$$

and the average strain rate is;

$$\frac{d\epsilon_s}{dt} = \frac{v_{1S} - v_{2S}}{L} \quad (4.43)$$

The velocity at interface 1S is comprised of the incident (positive travelling wave) and the reflected (negative travelling wave) as;

$$v_{1S} = C_o \epsilon_I - C_o \epsilon_R \quad (4.44)$$

The velocity of interface 2S is a positive quantity as;

$$v_{2S} = C_o \epsilon_T \quad (4.45)$$

So the specimen strain rate in terms of the bar strains from the three equations above will be;

$$\frac{d\epsilon_s}{dt} = -\frac{C_o(\epsilon_T - \epsilon_I + \epsilon_R)}{L} \quad (4.46)$$

It is being assumed that the specimen is deforming uniformly such that;

$$\epsilon_I + \epsilon_R = \epsilon_T \quad (4.47)$$

By this, the equation of strain rate for the specimen can be simplified to;

$$\frac{d\varepsilon_s}{dt} = -2 \frac{C_0}{L} \varepsilon_R \quad (4.48)$$

The negative sign in equation (4.48) is due to the fact that the test specimen is in a state of compression. Integrating equation (4.48) yields to the specimen compressive strain equation below;

$$\varepsilon_s(t) = -2 \frac{C_0}{L} \int_0^t \varepsilon_R(t) dt \quad (4.49)$$

On the other hand for the specimen stress, the average force on the specimen is given by;

$$P_{avg} = \frac{P_{1S} + P_{2S}}{2} \quad (4.50)$$

the forces on the interfaces 1S and 2S (on the end faces of specimen) can be expressed in terms of the incident and reflected bar strains as;

$$P_{1S} = EA(\varepsilon_I + \varepsilon_R) \quad (4.51)$$

$$P_{2S} = EA\varepsilon_T \quad (4.52)$$

So the average force on the specimen in terms of the bar strains can be given as;

$$P_{avg} = \frac{EA}{2} (\varepsilon_I + \varepsilon_R + \varepsilon_T) \quad (4.53)$$

By using the eqn. (4.47) and the equation followed,

$$\sigma_s = \frac{P_{avg}}{A_s} \quad (4.54)$$

The expression for the average specimen stress is found to be;

$$\sigma_s(t) = E \frac{A}{A_s} \varepsilon_T(t) \quad (4.55)$$

Summary of Specimen Strain Rate, Strain and Stress Equations:

For a uniformly deforming specimen, the expressions relating pressure bar strains to specimen properties are greatly reduced. After the striker bar impacts the input bar, as it was mentioned earlier an elastic compressive wave is generated that propagates towards the specimen. Upon reaching the specimen, part of the wave is reflected while the remainder is transmitted through the specimen and into the output bar. By mounting strain gages to the surfaces of the pressure bars, the reflected and transmitted waves can be recorded for use in eqn. (4.56);

$$\frac{d\varepsilon_s(t)}{dt} = -\frac{2C_o}{L} \varepsilon_R(t) \quad (4.56a)$$

$$\varepsilon_s(t) = -\frac{2C_o}{L} \int_0^t \varepsilon_R(t) dt \quad (4.56b)$$

$$\sigma_s(t) = E \frac{A_o}{A} \varepsilon_T(t) \quad (4.56c)$$

The recorded strain histories must first be corrected for dispersion, then examined such that the precise beginning and ending of the reflected and transmitted pulses may be identified. Many testing conditions greatly affect the implementation and effectiveness of eqn. (4.56). For instance the length of the impact pulse determines how much the specimen may be deformed. As mentioned the waves are dispersed, which in effect smears the waveforms. The fundamental assumption that the specimen deforms uniformly must be enforced by lubricating the specimen – pressure bar interfaces and choosing appropriate specimen dimensions. By carefully addressing these issues, dynamic stress-strain relations can be found for a broad range of materials using eqn. (4.56) [43, 44].

4.2.7 Lubrication Effect in SHPB

Friction effect is a very important consideration in SHPB tests. Most studies of friction in compression or extrusion have been conducted at ambient or elevated temperatures. Materials are usually shaped under these conditions because they are softer and easier to work.

The friction effect in a compression test may be estimated by Siebel's analysis:

$$P = \left(1 + \frac{\mu D}{3L}\right) \sigma_{\varepsilon} \quad (4.57)$$

P is the applied mean pressure, μ is the coefficient of friction, D and L are the specimen diameter and length respectively, and σ_{ε} is the material flow stress at strain ε . For fixed specimen geometry, the measured stress value is the only parameter that is a function of μ . Using a specimen aspect ratio, $L/D \approx 0.5$, as proposed by **Davies and Hunter [54]** and their suggested friction coefficient (0.02-0.006), the error in the analysis is found to be between 1.3 and 4 %.

The flow stresses of many pure metallic materials, such as Cu and Al, typically increase at high strain rates by only 25% or less over the quasi-static values. Since alloying and thermal processing tend to reduce rate sensitivity of the metals significantly, the importance of measurement errors involved increases dramatically. Therefore, friction effects must be quantified, particularly for materials having relatively low strain rate sensitivities such as Al alloys. Better estimation of friction effects also allows more precise determination of a material's constitutive equation.

One of the common signs of frictional effects during compression loading of cylindrical specimens is barreling. With good lubrication to prevent friction effects specimens may be deformed even upto 25% without any barreling shape. However the lack of barreling alone is not any indication of the absence of frictional effects.

Two lubricant types can be used for specimen end's lubrication;

(a) *solid lubricants*, (MoS_2 powder, graphite powder and Teflon tape)

(b) *liquid lubricants* (light oil, grease, and silicone-oil).

For elevated temperature tests solid lubrication systems have usually been found to be better than liquid lubricants or greases as;

- i. They are less prone to being squeezed out from between the specimen and the anvils.

- ii. Most liquids and greases burn away under prolonged contact with hot metal. Over the years a number of different solid lubricants have been investigated and recent reviews of high-temperature lubrication may be found in references [48, 49].

Another component of the lubrication system is the surface finish of the specimen. For lubrication to large strains, it is important that the surface is not too smooth or the lubricant will be squeezed out and the friction rise. On the other hand there should be a good surface contact of the bar ends with the specimen ends for the interaction of stress waves. Various methods have been adopted in different studies from simple roughening of the surface, to machining grooves or other profiles. A comprehensive study of this problem for low strain rate compression of metals was conducted by Lovato and Stout. They concluded that boron nitride (BN) powder was the best for high temperature, colloidal graphite for room temperature, and MoS_2 in a resin was the best for cryogenic upsetting. They also found a grooved surface finish gave good results. It should be noted, however, that as the deformation proceeds the area of the cylinder ends increases and that of the cylinder sides decreases. Effectively material from the sides folds over onto the top and bottom of the cylinder. So the friction conditions are bound to vary as the deformation proceeds because unroughened material comes into contact with the anvils [50, 51].

4.2.8 Equipment and Test Method of SHPB

The split Hopkinson pressure bar test apparatus consists of three major components ; a pressure source system, split bar system and data acquisition system. The data acquisition system consists of strain gages, amplifier, wheatstone bridges and an oscilloscope. There are also mounting and alignment hardwares associated with the apparatus. Figure 4.18 shows a schematic of a compressional type SHPB apparatus. Note that there is no standard design for a split Hopkinson pressure bar test apparatus and so the dimensions of the apparatus in Figure 4.18 are not constant and can be varied depending on the desired strain rate, strain and materials of both the bars and specimen. The design parameters of a SHPB depends on the properties of the specimen and loading type which is to be investigated by the scientist however there are certain considerations present.

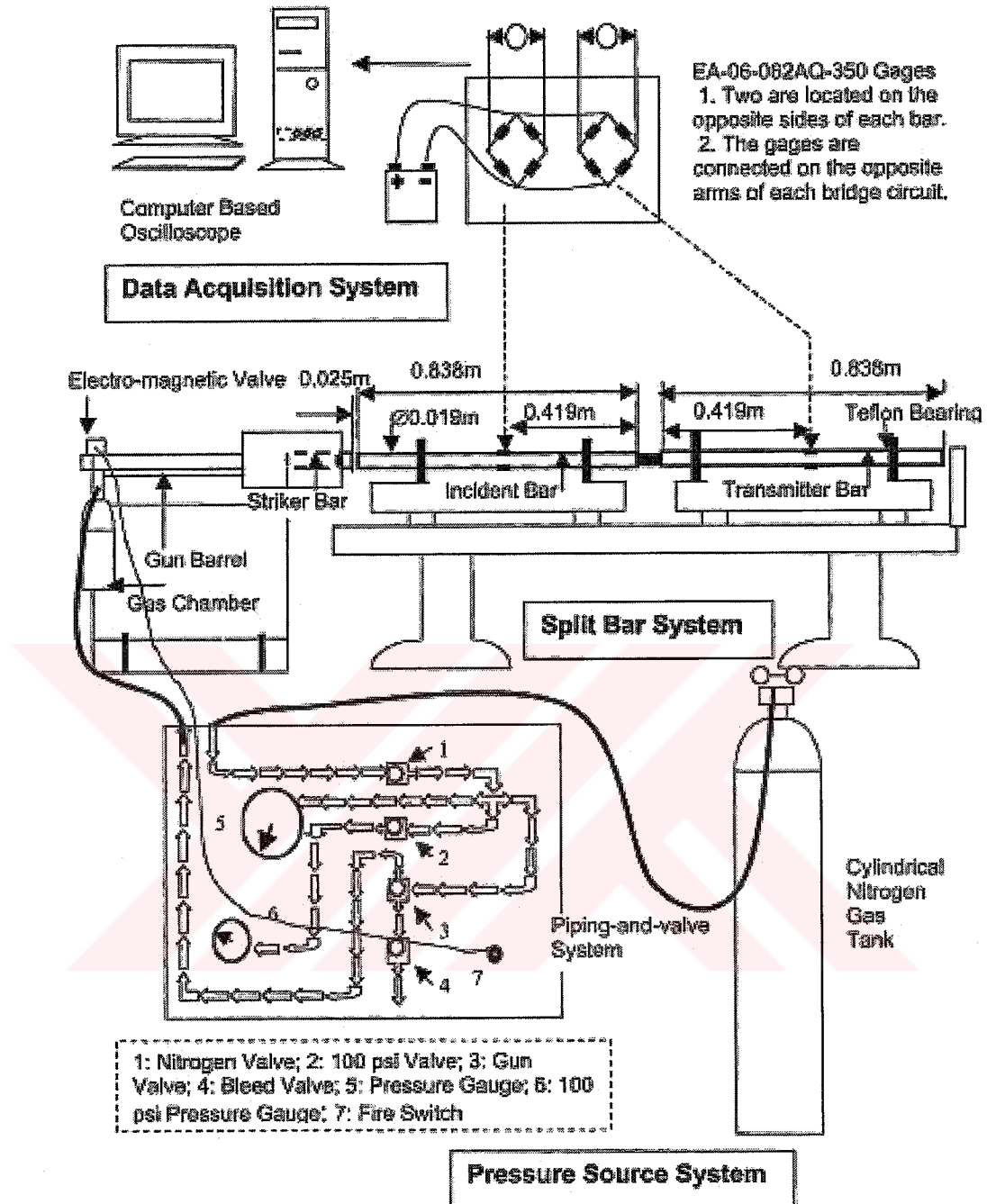


Figure 4.18 : Schematic Diagram of a SHPB System [47].

A. The Pressure Bars:

The pressure bars of the system should be constructed from a high strength material, preferably with a high yield strength so that the bars would remain elastic during the tests. The yield strength of the pressure bar determines the maximum stress attainable within the specimen, because the cross-sectional area of the specimen approaches that of the pressure bar during deformation. The length, L_{bar} , and diameter, D_{bar} , of

the pressure bars are test variables. The diameter chosen depends on the strain rate desired; the highest strain rate tests require the smallest diameter bar, and since SHPB is a one-dimensional wave technique for material characterization, when testing homogenous materials, specimens with small diameters will be adequate. Usually the diameters of bars are less than 20 mm.

The first rule to determine the length of the incident and transmitter bars requires that the bars should be long enough to warrant one-dimensional wave propagations. After determining the suitable pressure bar diameter, an aspect ratio, L_{bar} / D_{bar} , around 10 is usually agreed by researchers as a minimum requirement for a one-dimensional bar.

As the striker bar impacts the incident bar, the wave generated in the latter is highly complex due to the nonuniform contact between the striker bar and the incident bar, and the non-longitudinal waves, such as spherical dilatational wave, generated from the impact. Although Saint Venant's principle states that non-uniform contact force can be quickly dampened out in a distance approximately equal to the bar diameter into the bars if the bars are made of isotropic materials, the non-longitudinal waves may take 10 times the bar diameter to vanish. Moreover, if electrical-resistance strain gages are to be installed in the bars for wave measurements, the reflection waves from each bar end should be considered in determining the bar length. A bar length more than 20 times the bar diameter (10 times from each bar end) may be required to avoid overlap of propagation waves with reflection waves. Thus each bar should also exceed a L_{bar} / D_{bar} ratio of 20.

A second consideration affecting the bar length is the amount of strain desired in the specimen, which is related to the length of the incident wave. Note that the incident wave is also affected by the length of the striker bar because the wave length will be twice the length of the striker bar. To obtain strain as high as 25%, it may be necessary to specify a ratio for L_{bar} / D_{bar} as 80 – 100 or even more.

The length of the pressure bars, input and output, should not have to be equal since the output bar will only be dealing with the transmitted waves. Theoretically, the minimum length of the transmitter bar is $L_{striker} + kD$, $L_{striker}$ is the projectile length,

to ensure that the reflection of the transmitted wave from the free end of the transmitter bar reaches the gage after the tail of the transmitted wave vanishes. Parameter k is determined by the critical distance between the strain gage station and the specimen/bar interface. The minimum value of k is between 1.5 and 5 according to the analysis by *Meng* and *Li* [52].

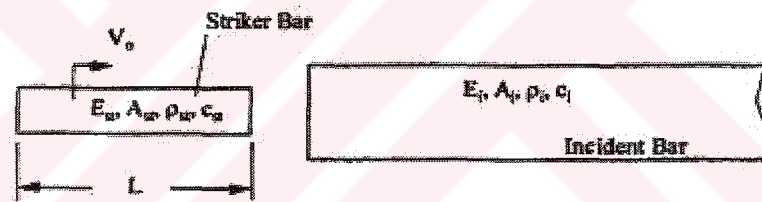
Although longer bars seem to be excellent for one-dimensional wave measurements and can help to reduce the end effect and transverse effect, they also pose serious challenges on the straightness of the bars. As a bar becomes longer, it sags due to its own weight. The curvature of the bar, accompanied by low critical buckling load will have a negative effect on one-dimensional wave propagation.

On the other hand if the material (specimen) to be tested with the method is with a low impedance property, in order to prevent impedance mismatch the bar material should be selected accordingly. If the mechanical impedance difference is too much between the specimen and bar materials the incident wave will be completely reflected or a very little portion of it will be transmitted and this way the results obtained from the test can not be considered as valid results. It was shown in section 4.2.5 that, the impedance matching effects the transmitted and reflected wave ratios. When testing materials are not homogenous, specimens with large diameters are required. Therefore the bars characterizing inhomogeneous materials, such as concrete or wood, usually have a diameter of 75 mm or more.

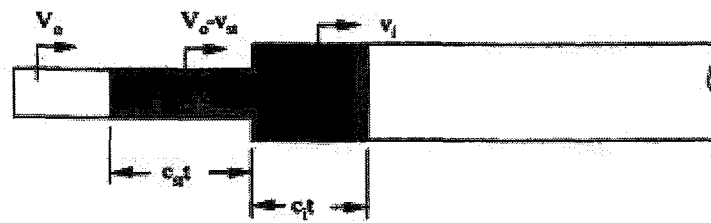
The length of the pressure bars also effect maximum observation time. The measuring duration Δt of a classical SHPB is limited to $\Delta t \leq L_{bar} / C$, C being the wave speed and L_{bar} the length of the bar. Consequently, the total relative displacement Δl between the two bar-specimen interfaces is limited for a given loading speed V , $\Delta l \leq V \cdot \Delta t$. For material behavior testing at a given average strain rate, the maximum measurable strain is limited ($\epsilon_{max} \leq \dot{\epsilon} \Delta t$). For instance, the measurable duration does not exceed 400 μs for a SHPB set-up made of 2m long steel bars ($C \approx 5000$ m/s) and the measurable maximum strain is then limited to 20% for a test performed at an average strain rate of about 500 s^{-1} (and only 1% for 25 s^{-1}) [3, 47, 52, 53].

B. The Striker Bar:

The striker bar should be constructed from the same bar material and in most cases should have a diametrical dimension equal to the bars. Under certain circumstances, specimen strain rate is proportional to the amplitude of the reflected pulse. Since most investigators are interested in materials sensitivity to changing strain rate, testing materials at a constant strain rate is desirable. Thus it would seem to achieve a constant strain rate test would require a constant reflected pulse. The stress pulses in the Hopkinson Bar apparatus are usually generated by firing a shorter bar of similar material into the first pressure bar. In section 4.2.4 the theory of the impact of two collinear rods was described. Before the bars impact, $t < 0$, the bars are stress free and the particle velocity in the striker bar is equal to its rigid body velocity. After the impact, a compressive wave travels into the striker and incident bars as shown in figure 4.19.



a) Striker and incident bars before impact



b) Striker and incident bars after impact

Figure 4.19 : Impact of Two Cylindrical Bars.

Since the materials and the cross-sectional areas will be the same in the SHPB apparatus we can express that;

$$\rho_{st} C_{st} A_{st} = \rho_i C_i A_i = \rho C_o A \quad (4.58)$$

$$\sigma_{st} = \sigma_i = \sigma \quad (4.59)$$

$$v_{st} = v_i = \frac{1}{2} V_o \quad (4.60)$$

$$\sigma_{st} = \sigma_i = \frac{1}{2} \rho C_o V_o \quad (4.61)$$

So the perfect impact of similar bars result in a square shaped pulse of magnitude $\sigma = \frac{V_o \rho C_o}{2}$ and with a duration of $t = \frac{2L_{striker}}{C_o}$. These equations suggest that

for a given striker bar, the stresses generated are proportional to its impact velocity and its duration is only related to physical properties and the length of the striker bar.

Additionally if we can express t in terms of $\epsilon/\dot{\epsilon}$ we can also remind that the length and velocity of the striker bar are chosen to yield the desired total strain and strain rate within the specimen. At a constant strain rate, the maximum strain in the specimen is directly proportional to the length of the striker bar, $L_{striker}$;

$$\epsilon = 2\dot{\epsilon} \frac{L_{striker}}{C_o} \quad (4.62)$$

From inspection of eqn. (4.62) and consideration of momentum conservation between the striker and incident bars, it can be shown that;

$$\dot{\epsilon} \leq \frac{V_o}{L_{striker}} \quad (4.63)$$

Eqn. (4.63) can be approximated for soft metals at high impact velocities. Equation (4.62) can be used to define the striker bar length and velocity required for a desired strain and strain rate. Thus, the definition of the striker bar length will be the first step of determining the pressure bar lengths.

So far in this section, at the very beginning, it was mentioned that the cross-sectional area of the striker bar was all constant through the striker bar. However, one striker shape may not be sufficient in particular cases to achieve the range of strain rate in

dynamic tests using the Split Hopkinson pressure bar. By simply reducing the velocity of the striker will reduce the strain rate but the stress level may not be adequate to fracture a brittle specimen. For this, **Kumar et al. [44]** had worked on the design of an impact striker bar for a split Hopkinson pressure bar and showed how the shape of a striker bar can influence the rise time of the incident waveform, resulting in considerable reduction of stress oscillation. This work was done in order to test non-homogeneous brittle materials such as concrete, brickwork, rock and ceramic to obtain representative dynamic properties. They have designed a tapered striker bar which lengthens the duration of the incident stress wave but which does not increase the magnitude of the incident stress abruptly. In this way, stress oscillation is eliminated and uniform stress on the test specimen is achieved.

They have calculated stress-time histories using a finite difference method for all the striker shapes in figure 4.20 and showed the results in figure 4.21.

The results in figure 4.21 showed that the incident stress jumped abruptly from zero to finite value on impact due to the difference in the cross-sectional area of the striker and the output bar at the contact. The shapes of the striker bars also effected the duration to reach the peak values. Shape 3 had the longest duration to reach the peak value. And the maximum incident stresses for all 5 shapes varied between 150 MPa to 200 MPa with the striking velocities of 10 m/s for each shape. Shape 1 among all was the most similar to the rectangular loading waveform generated by a solid cylindrical bar. Shape 2 had the shortest rise time of less than 100 μ s. Shape 5 was unlikely to provide an oscillation-free stress waveform due to its abrupt change in the diameter along the length. Shape 4 had a very suitable loading waveform but it would not provide a sufficient long rise time to lower the strain rate on a specimen. Shape 3 had the longest rising time duration of more than 300 μ s, caused by the long tapered portion at the impact end of the striker bar. The gradual increase of stress up to a failure value made it ideal to fracture concrete specimens.

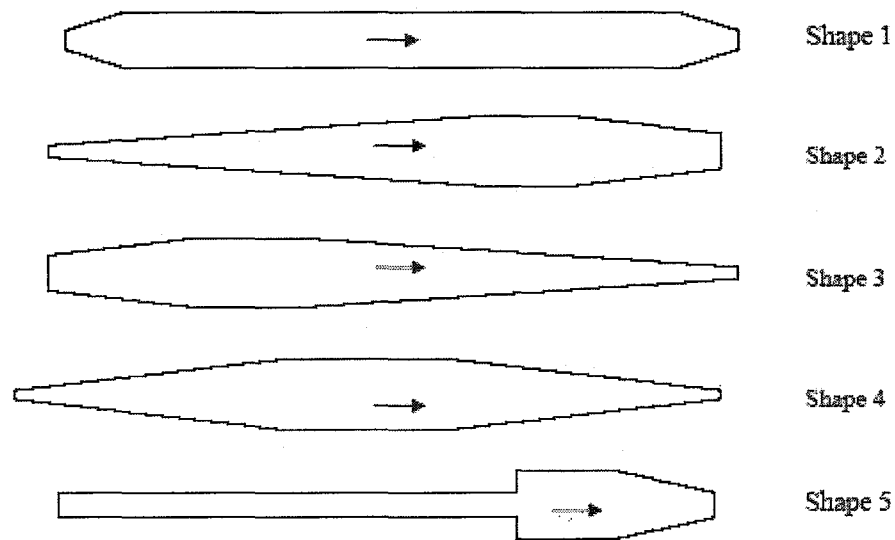


Figure 4.20 : Arbitrarily Shaped Striker Bars [44].

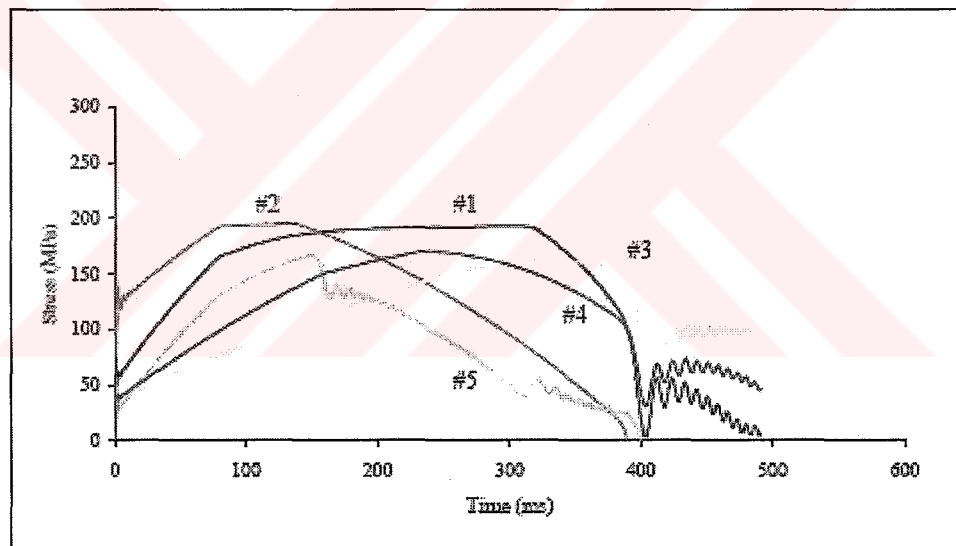


Figure 4.21 : Incident stress waveforms from shaped striker bars [44].

C. Considerations for Specimen and Its Dimensions:

Specimen to be tested is one of the most important parameter effecting in the determination of the pressure bar lengths, diameters and other characters of the SHPB apparatus because they are the materials to be tested and the aparatus should be designed according to the specimen material properties. Most of the test apparatus are designed so that they can be used to determine the high strain rate behavior of a wide range of materials and thus the pressure bars are for that reason being fabricated

out of high strength materials. Properties of specimens such as elastic modulus, yield strength and acoustic impedances and also diameter dimensions should be suitable for the test apparatus.

In metals the flow stress measured in a dynamic test is typically 25% higher than the equivalent static value. An accuracy in measurement of around 2% is needed if strain rate effects are to be reliably characterised. The stress and strain rates in a specimen are affected by several factors, i.e. radial inertia effect and the friction constrain between the specimen and the pressure bars, which may cause the violation of a uniaxial and uniform stress and strain states within the specimen. It is well known that the interface friction and the slenderness ratio (length to diameter ratio) of a specimen are two major factors to influence the stress and strain rates in a specimen, and therefore, to influence the accuracy of a SHPB test.

Davies and Hunter [54] showed that the slenderness ratio plays opposite roles on axial and radial inertia effects and an optimal slenderness ratio exists as;

$$\lambda = \frac{L_{specimen}}{D_{specimen}} = \frac{1}{2} \sqrt{3\nu_{specimen}} \quad (4.64)$$

where, $\nu_{specimen}$ is an effective Poisson's ratio for the specimen under the experimental condition. For metals, $\nu_{specimen}$ varies between 0.33 (elastic deformation) and 0.50 (large plastic deformation) and thus, the optimal slenderness ratio to minimize inertia effects is between 0.5 and 0.61 according to eqn. (4.64). Equation (4.64) is recognized as the optimal slenderness ratio for all kinds of metal specimens in a SHPB test.

Besides Davies and Hunter, many other researchers have investigated the effects of $L_{specimen} / D_{specimen}$ ratio and no ideal ratio has been concluded from their studies. The researchers are also recommending $L_{specimen} / D_{specimen}$ ratios as 0.5, 1.5, 2 and even 3.

The selection of the mentioned ratio can also be based on the following arguments;

- i. One of the fundamental assumptions of the SHPB technique is the uniform deformation of the specimen during the characterization; however, it is more

difficult for a longer specimen to meet the uniformity requirement than a shorter specimen,

- ii. In fact, a longer specimen also suffers lower stability when subjected to compressive loading,
- iii. The stability requirement, however, it can not be satisfied by simply using a very short specimen because of several considerations; (a) one-dimensional assumption, (b) Saint Venant's end effect (impact induced contact), and (c) the frictional effect due to Poisson's effect.

The diameter of specimens, $D_{specimen}$, should always be smaller than the diameter of pressure bars, D_{bar} , to warrant complete wave propagations into and from the specimens. This is especially true for specimen materials with positive Poisson's ratios. They expand transversely when subjected to longitudinal compression. A ratio for $D_{specimen}/D_{bar}$ around 0.75 is commonly accepted by SHPB researchers. As this ratio decreases, the wave patterns seem to be distorted more.

Kaiser [35] also, in his thesis, showed some works how to determine specimen diameter for cylindrical bars and specimens.

The friction is another important consideration in the SHPB tests. The lubrication is also required at the specimen-bar interfaces in order to minimize errors due to friction. It is important to apply a very thin layer of lubricant on the ends because a thick layer of lubricant can affect the timing between the waves recorded on the incident and output pressure bars. For example a thick layer of lubricant on the bar ends will delay the contact time of the incident bar to the specimen and there will be also a similar delay at the specimen – output bar interface. The use of an oil-based molybdenum disulfide, MoS_2 , will be sufficient and succesful for room temperature testing. MoS_2 is composed of sheets and layers and the layers themselves are strong but the bonding between them is weak. Consequently MoS_2 is strong in compression but weak in shear and this is advantageous for producing low friction in compression testings [3, 42, 47, 51, 55].

D. Specimen - Bar Alignment Tool:

Since the specimen diameters will be chosen smaller comparing to the bar diameters due to the expansion of the specimen during the test, it is important to align the specimen and output bars uni-axially. For this alignment purpose a suitable alignment tool can be designed and built for centering the specimen on the output bar, figure 4.22.

The alignment tool can be slid over the end of the pressure bar and the specimen can be placed through the tool. The thin layer of lubricant on the specimen end will be enough to hold the sample in its place until the alignment tool is removed and the input bar is slid against the free end of the specimen.

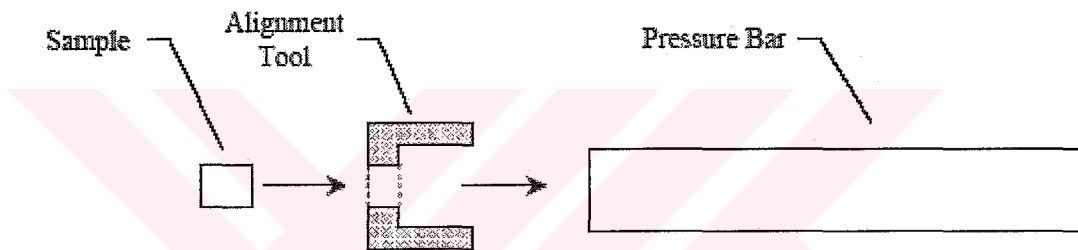


Figure 4.22 : Alignment Tool for aligning the specimen with the pressure bars [35].

E. Data Acquisition System:

A Data Acquisition System of a SHPB mainly consists of strain gages, a wheatstone bridge, an amplifier and a digital oscilloscope or a computer based oscilloscope as also was shown in figure 4.18. Figure 4.23 schematically represents the evolution of the strain signal generated by the SHPB.

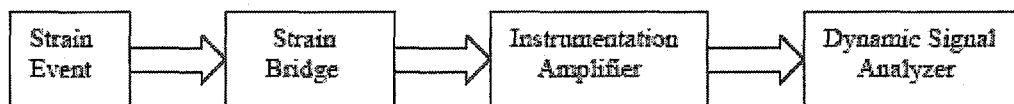


Figure 4.23 : Evolution of SHPB Strain Signal.

i. Strain Gages:

Although strain has been measured by mechanical indicators (extensometers), optical methods, and reluctance transducers, the device used for the vast majority of strain measurements is the resistive strain transducer or strain gage.

The strain gages are the most important equipments of the data acquisition system of a SHPB since they are used for recording the incident, transmitted and reflected strains through the Hopkinson bars.

A strain gage consists essentially of a conductor or semi-conductor of small cross-sectional area which is mounted to the measured surface so that it elongates or contracts with that surface. This deformation of the sensing material causes it to undergo a change in resistance. Hence, a strain gage senses strain by its own deformation and transduces the deformation into a resistance change, figure 4.24. These are extremely versatile due to their small size and ease of installation. For the purposes of pressure bar measurements, a single element strain gage can be used.

The change in resistance of a material due to an applied stress has been termed the piezoresistive effect. The change in resistance of a conductor when subjected to mechanical strain was first reported by Lord Kelvin (W. Thomson) in 1856, further elaborated by H. Tomlinson in 1877, and demonstrated in 1881 by the Russian physicist O.D. Khvol'son.

In design there are four main types of strain gages; i. Metal-wire strain gages, ii. Metal-foil strain gages, iii. Deposited-metal strain gages and iv. Semiconductor strain gages.

Since a strain gage is a resistive elastic unit whose change in resistance is a function of applied strain;

$$\frac{dR_g}{R_g} = S \cdot \epsilon \quad (4.65)$$

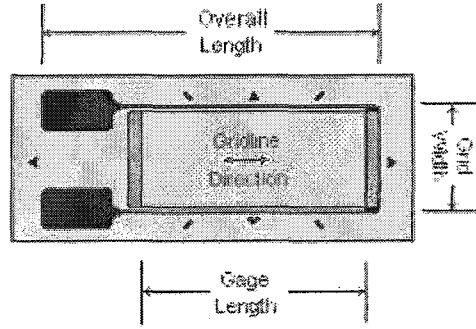


Figure 4.24 : Uniaxial Strain Gage [39].

where R_g is the resistance, ε is the strain, and S is the strain sensitivity factor of the gage material (or gage factor).

Consider a wire strain gage, as illustrated in figure 4.25. The wire is composed of a uniform conductor of electric resistivity r with length l and cross-section area A . Its resistance R_g is a function of the geometry given by;

$$R_g = \rho \frac{l}{A} \quad (4.66)$$

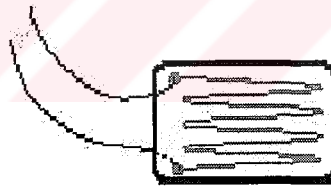


Figure 4.25 : A Wire Strain Gage [56].

The resistance change rate is a combination effect of changes in length, cross-section area, and resistivity:

$$dR_g = \frac{\rho}{A} dl - \frac{\rho l}{A^2} dA + \frac{1}{A} d\rho \Rightarrow \frac{dR_g}{R_g} = \frac{dl}{l} - \frac{dA}{A} + \frac{d\rho}{\rho} \quad (4.67)$$

When the strain gage is attached and bonded well to the surface of an object, the two are considered to deform together. The strain of the strain gage wire along the longitudinal direction is the same as the strain on the surface in the same direction;

$$\varepsilon_l = \frac{dl}{l} \quad (4.68)$$

However, its cross-sectional area will also change due to the Poisson's ratio. Suppose that the wire is cylindrical with initial radius r . The normal strain along the radial direction is then;

$$\varepsilon_r = \frac{dr}{r} = -\nu \frac{dl}{l} \quad (4.69)$$

The change rate of cross-section area is twice as the radial strain, when the strain is small;

$$\frac{dA}{A} = (1 + \varepsilon_r)^2 - 1 = 2\varepsilon_r + \varepsilon_r^2 \approx 2\varepsilon_r = -2\nu \frac{dl}{l} \quad (4.70)$$

Thus, the resistance change rate now becomes ;

$$\frac{dR_g}{R_g} = \frac{dl}{l} - \frac{dA}{A} + \frac{d\rho}{\rho} = (1 + 2\nu) \frac{dl}{l} + \frac{d\rho}{\rho} = (1 + 2\nu)\varepsilon_l - \frac{d\rho}{\rho} \quad (4.71)$$

For a given material, the sensitivity of resistance versus strain can be calibrated by the following equation;

$$S \equiv \frac{dR_g / R_g}{\varepsilon_l} = 1 + 2\nu + \frac{d\rho / \rho}{\varepsilon_l} \quad (4.72)$$

When the sensitivity factor S is given, (usually provided by strain gage vendors) the average strain at the point of attachment of the strain gage can be obtained by measuring the change in electric resistance of the strain gage;

$$\varepsilon_l = \frac{dR_g / R_g}{S} \approx \frac{\Delta R_g}{S} \quad (4.73)$$

There are at least three significant considerations when using strain gages for measuring dynamic signals. The following limitations should be understood before choosing a strain gage, and especially before interpreting dynamic strain data:

- (a) Frequency resolution is lost for wavelengths near that of the gage length,
- (b) Spatial resolution is a concern for pulses on the order of the gage length,
- (c) The ability of a gage to 'track' a signal, or rise time, must be known.

The ability to resolve a wave spatially becomes limited for pulses on the order of the gage length. As the gage length increases, so too does the ability to resolve spatial location. Therefore the total length of the pulse of concern must be much greater than the gage length.

The output of a strain gage tends to give an integrated average of the strains imposed over its gage length. Consider the three different wavelengths and three different gage lengths, shown in figure 4.26:

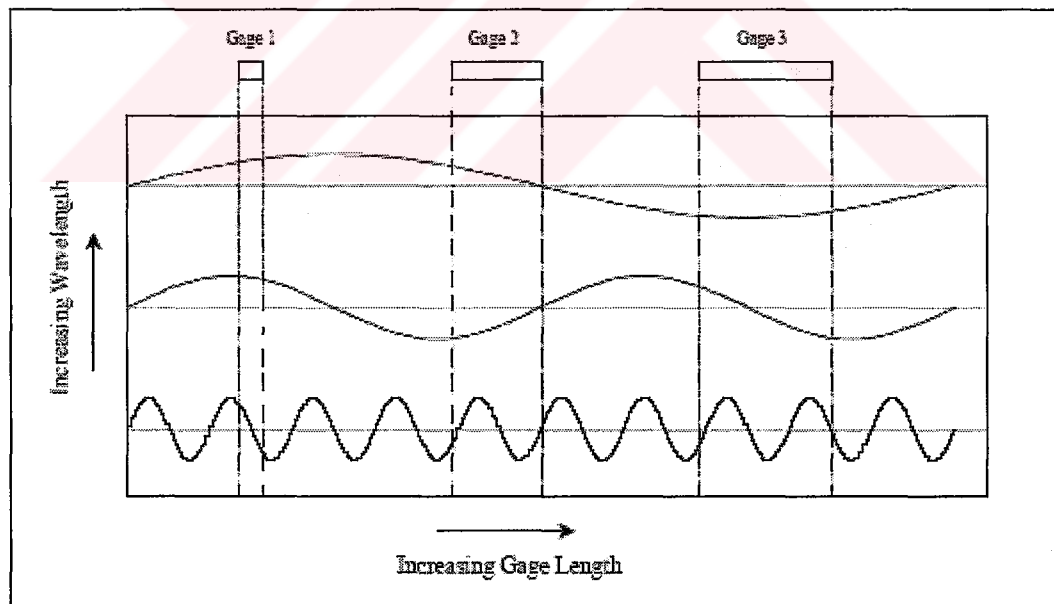


Figure 4.26 : Wavelength vs. Gage length [35].

Imagine what the output of each gage would be (the average strain over the gage length). A number of interesting questions arise for these situations. The smaller the strain gradient across the element length, the closer the output will be to the true

strain, i.e. the special case where the wavelength is equal to the gage length (gage 2 – bottom strain curve). The average output is zero, but the actual strain is not. This will occur for integer multiples of the wavelength. As the wavelength decreases, so too does the ability to resolve the peak strain due to the averaging effect over the gage length. The peak strain is consequentially always estimated low. The following figure illustrates the point. Notice that the steeper the gradient (i.e. short wavelength), the less the peak strain can be resolved.

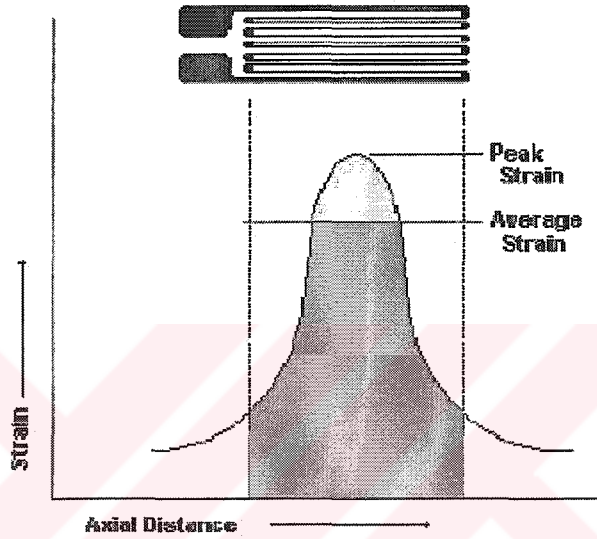


Figure 4.27 : Averaging effect on peak stress [35].

For harmonic waves propagating through a strain gage, the peak output from the gage will always occur when the peak strain is centered in the gage, as illustrated in figure 4.27. To determine the closed form solution for the peak strain reported by a strain gage, the harmonic wave must be integrated over the gage length. By recognizing that the peak of a cosine wave coincides with the center of the gage, the following expression yields the peak strain reported by the strain gage. The peak output of the gage is always less than the true peak strain.

$$\epsilon_{\text{Reported}} = \frac{C_o \sin\left(\frac{\pi \cdot L \cdot f}{C_o}\right)}{\pi \cdot L \cdot f} \quad (4.74)$$

C_o , L , and f are the wave velocity, gage length, and frequency of the cyclic strain signal, respectively.

Upon examination of equation (4.74), it can be concluded that the reported strain follows a sinc function. A plot of the reported peak strain versus frequency for a harmonic input of amplitude one is shown in figure 4.28. Notice that the shorter gages are more accurate at measuring high frequency strains.

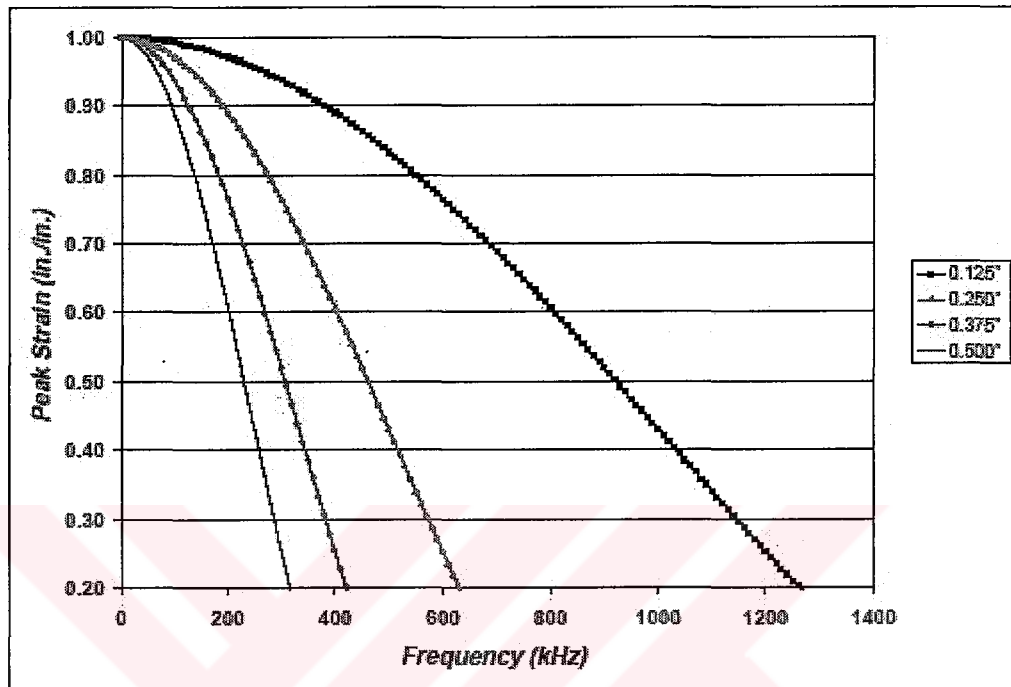


Figure 4.28 : Reported peak strain from various strain gage lengths [35].

Notice that for DC, accurate strain values are reported. As frequency increases, however, the actual strain becomes drastically attenuated. As was expected, shorter gages offer greater frequency response. The effect of the averaging dominates the output of the strain gage for high frequency. The output from the gage is zero when the wavelength is equal to the gage length. To choose a strain gage, the desired accuracy of peak strain and frequency extension must be considered. According to the Measurements Group, Strain gages of less than about 0.125 in (3 mm) gage length tend to exhibit degraded performance - particularly in terms of the maximum allowable elongation, the stability under static strain, and endurance when subjected to alternating cyclic strain. Solving eqn. (4.74) with this gage length and properties of a steel bar, one attains the highest resolved frequency.

For normal applications, both foil and semiconductor strain gages are mounted to the pressure bars by means of an adhesive (cement). The foil and semi-conductor gages

have elastic moduli that are low and so provide no reinforcement but deform to follow the strain in the part.

Foil gages, which are commonly used, are available in a wide variety of configurations, are easy to apply and are relatively inexpensive. Unfortunately they have a low gage factor and are relatively insensitive compared to semiconductor gages. The low gage factor, generally around 2.0, requires the amplification of the bridge output signal. In amplifying the bridge output, the electronic noise is amplified as well.

Semi-conductor gages, on the other hand, also came in a variety of configurations but are relatively difficult to apply and are more expensive than foil gages. Unlike foil gages, semi-conductor gages have no backing and special care must be taken when adhering the gage to material that conducts electricity. The main advantage in using a semi-conductor strain gage is its high gage factor of approximately 150. This higher gage factor allows for recording of the unamplified bridge (wheatstone) output voltage. The high gage factor increases the resolution of the sensor allowing for a much greater signal to noise ratio [35, 56 - 58].

ii. Wheatstone Bridge:

The resistance changes in a strain gage are almost invariably converted into voltage changes. The conversion is performed by passive resistive signal-conditioning networks. The most widely used resistance bridge is the Wheatstone bridge, developed by S.H. Christie in 1833.

The Wheatstone bridge is ideal for making measurements with passive transducers such as thermistors, resistance thermometers and strain gages. A derivative of the Wheatstone bridge is a configuration known as a strain bridge. The strain bridge simply consists of a combination of four resistors and strain gages configured as two parallel circuits. Figure 4.29 shows one particular strain bridge configuration. In the figure, the quarter bridge configuration (strain gage accounts for 1 of 4 resistances) in which a single strain gage is placed in the circuit with three resistors to complete the bridge.

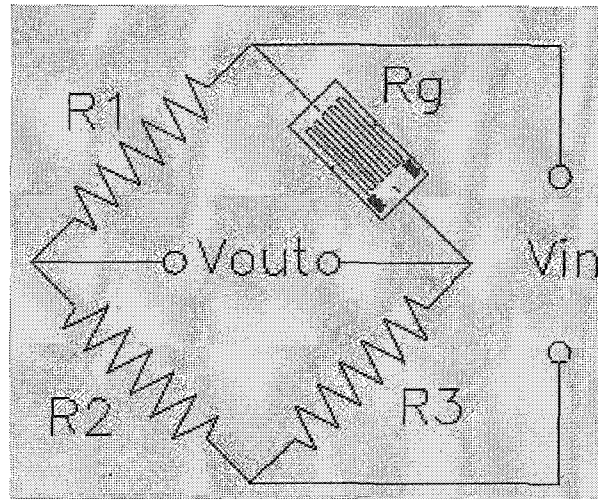


Figure 4.29 : Strain Gage in Quarter Bridge Configuration [36].

Either a DC or AC excitation can be used for the Wheatstone Bridge. The latter is often preferred when the bridge output must be amplified and when interconnecting cables are relatively short. The bridge network is normally balanced (no output) for a condition of zero strain acting on the active gage(s) so that application of strain causes bridge output due to unbalance.

The three resistor values, R_1 , R_2 and R_3 , should be closely matched to the resistance of the strain gage, R_g , so that the bridge will be balanced for the zero input condition. The Quarter Bridge Configuration is useful for metal strain gages at room temperature.

Compensation for temperature effects can be achieved by using an active gage as well as a dummy gage of the same type and material and exposed to the same temperature as the active gage but isolated from mechanical strain. When connected in adjacent bridge arms as shown in figure 4.30, the increase in resistance due to temperature alone is equal for both gages and is not reflected in the bridge output voltage since no bridge unbalance is effected. Only the additional resistance change due to strain in the active gage causes bridge unbalance.

It is important that the dummy gage should be mounted on a plate of material having the same thermal characteristics as the structure under test. The dummy plate must be in good thermal contact with the test structure but this bond must not cause the gage to be strained when load is applied to the structure. The dummy gage is usually not effective in compensating for transient temperature conditions.

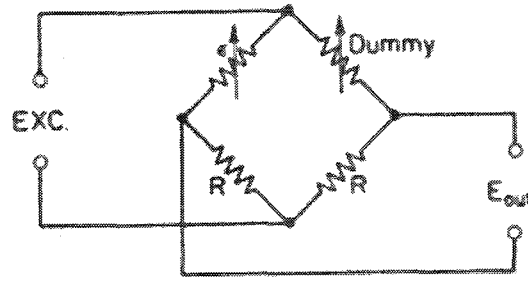


Figure 4.30 : Temperature Compensation by a Dummy Gage [3].

And also in the case of SHPB, any bending strain in the bar should be eliminated. Besides, the use of two active gages has sensitivity twice that of one active gage. The two active gages are located on the opposite side of each bar and on the opposite arm of each circuit for these purposes. For example in figure 4.31, R_1 and R_3 would be active gages (denoted by the boxes) and R_2 and R_4 are resistors of equal resistance to the gages.

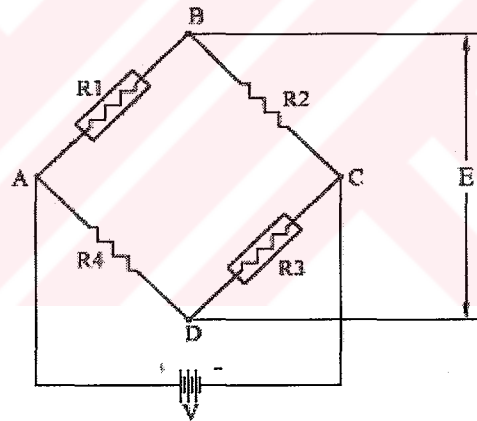


Figure 4.31 : Wheatstone Bridge with Two Active Strain Gages [24].

Voltage (V) is supplied to the bridge at points A and C, while the output voltage (E_w), measured across points B and D, can be related to the strain. The Voltage E_w is then equal to;

$$E_w = V_{BD} = V_{AB} - V_{AD} \quad (4.75)$$

By Ohms law, V_{AB} and V_{AD} can be related to the resistances by;

$$V_{AB} = \frac{R_1}{R_1 + R_2} \cdot V \text{ and } V_{AD} = \frac{R_4}{R_3 + R_4} \cdot V \quad (4.76)$$

This leads to;

$$E_w = \frac{R_1 R_3 - R_2 R_4}{(R_1 + R_2)(R_3 + R_4)} V \quad (4.77)$$

From eqn. (4.73), the resistance can be related to the strain by $R_g = R_o + S\varepsilon$ where R_o is the original resistance, R_g the current resistance and S is the gage factor. Since R_1 and R_3 are the only active gages, the output from the bridge can be related to the strain by;

$$E_w = \frac{(R_{10} + S_1\varepsilon)(R_{30} + S_3\varepsilon) - R_2 R_4}{[(R_{10} + S_1\varepsilon) + R_2][(R_{30} + S_3\varepsilon) + R_4]} V \quad (4.78)$$

Therefore, strain can be derived from the bridge output with the knowledge of the initial resistances and gage factors.

In examining eqn. (4.77), it can be seen that in order for the bridge to be initially in balance ($E_w=0$) at zero strain, $R_1 R_3 = R_2 R_4$. Because of the tolerances on the resistances, it is unlikely that this relation will occur naturally, and therefore, a balancing circuit may be needed. The bridge can also be out of balance due to differential temperatures and the resistance in the lead wires. The balancing circuit must be able to compensate for the difference in resistances due to lead resistance and variability of the gage resistance. There are a variety of circuits that can be used to balance the Wheatstone bridge. An example is given by figure 4.32. By adjusting the R_{trim} , the numerator, can be adjusted to equal zero [24,39,56].

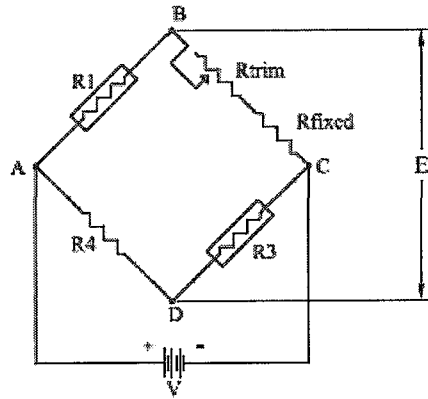


Figure 4.32 : Wheatstone Bridge with Balancing [24].

iii. Differential Amplifier:

Amplifiers receive a signal transmitted by cable (lead cables) from the transducers (strain gages in SHPB). The whole signal is first processed by an amplifier designed to have a wide frequency response. It is usually of a high gain and high stability characteristic. It produces a strong, relatively low impedance, electrical output which precisely reproduces the amplitude-time sequence measured at the transducer. The output of this amplifier is therefore in a suitable form for the application of electronic frequency weighting or for more detailed frequency or other analysis of the signal.

The signals from the Wheatstone bridge circuits are usually very small, typically only a few millivolts. Attempting to process a signal of only a few millivolts can result in a very low signal to noise ratio since electronic noise within the digital storage scope is sure to contribute to the signal. Generally a differential amplifier is used to magnify the signal outputs from each Wheatstone bridge circuit. A typical instrumentation amplifier design is shown below in figure 4.33.

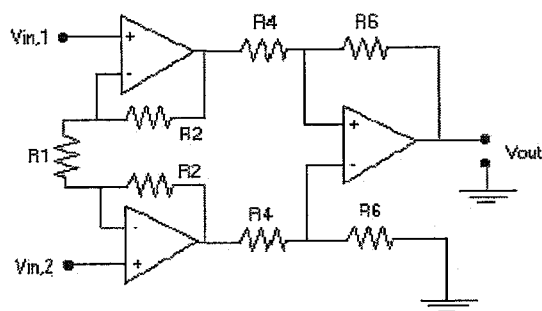


Figure 4.33 : A Typical Amplifier Layout.

The amplifier shown above utilizes three operational amplifiers (op-amps) to provide two stages of amplification (more commonly referred to as gain). In a typical adjustable gain operational amplifier, the gain is set by the selection of R_1 , which may be either a fixed resistor or a potentiometer. As $R_1 \rightarrow 0$ or $R_1 \rightarrow \infty$, the first stage of the instrumentation amplifier tends to behave as a voltage follower, meaning that the first stage gain is 1. The second stage gain is set by the selection of the resistors R_4 and R_6 . With the resistor pairs selected as shown above, the gain of the instrumentation amplifier can be calculated as follows:

$$Gain = \left(1 + \frac{2R_2}{R_1}\right) \left(\frac{R_6}{R_4}\right) \quad (4.79)$$

According to eqn. (4.79), a wide range of gain values is available since resistor values range from a few ohms to hundreds of mega ohms. However, resistor values are normally selected in the range from 1000 *ohms* to 100,000 *ohms*. Resistor values too small result in excessive current draw and resistor values too large allow can cause noise issues. In general, the output of the instrumentation amplifier is limited by the voltage supplied to the op-amps. Without the inclusion of specialized transistor circuitry, the output of the amplifier is limited to the voltage supplied to each individual op-amp, assuming *rail-to-rail* op-amps are used. This voltage is typically 5 *VDC* to 30 *VDC*. Op-amps not classified as *rail-to-rail* are limited to an output voltage typically 2 *VDC* less than the supply voltage. A typical op-amp is shown below in figure 4.34.

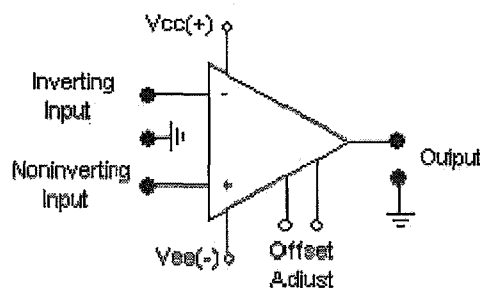


Figure 4.34 : Typical Op-Amp Configuration.

In figure 4.34, V_{cc} is the positive supply voltage and V_{ee} is the negative supply voltage. Typically, $-V_{cc} = V_{ee}$. The offset adjust pins allow the user to make provisions for zeroing the amplifier such that zero input results in zero output. This is

an important feature of the op-amp since noise in the form of bias currents and ground loops can result in a signal output for the zero input condition. Zeroing an amplifier is best achieved by shorting the two inputs together before adjusting the offset. Once the signals are amplified a digital oscilloscope records the strain signals [39, 47, 58, 59].

iv. Oscilloscopes:

The oscilloscope is basically a graph-displaying device. It draws a graph of an electrical signal. In most applications, the graph shows how signals change over time: the vertical (Y) axis represents voltage and the horizontal (X) axis represents time. The intensity or brightness of the display is sometimes called the Z axis, figure 4.35.

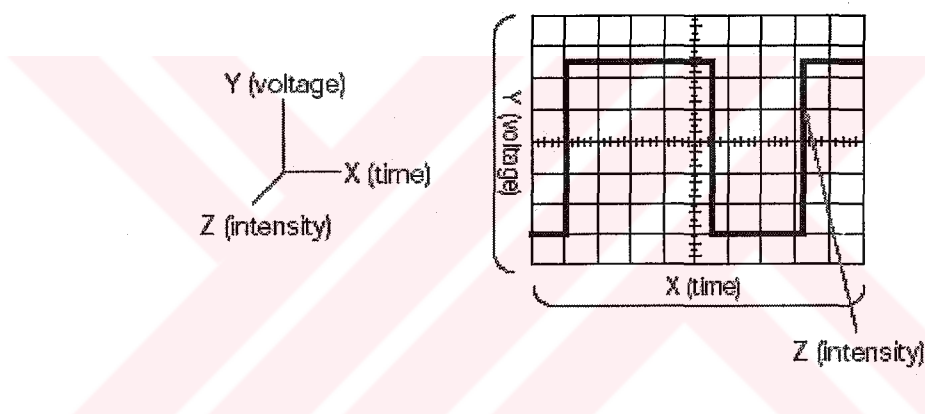


Figure 4.35 : X,Y and Z components of a Displayed Waveform.

The oscilloscope in SHPB test will provide the voltage changes from the wheatstone bridge output signals versus time graph. From these graphs, by using the necessary equations, described under the section of wheatstone bridges, voltage variations can be converted into strain in pressure bars and one can be able to plot the dynamic stress-strain curve of the specimen.

The properties such as bandwidth, sampling rate per second, vertical resolution, input type, dynamic range, memory per channel and rise time are important when choosing the appropriate oscilloscope.

The sampling rate per second needs to be adequate for obtaining a more valid voltage change vs time curve, since it will effect the reliability of the Stress-Strain curve later on. A sampling rate of an oscilloscope is usually given by the vendors. For example

an oscilloscope with two channels (one for the recording of incident strains and the other for transmitted strains) having a total sampling rate of 100 million samples per second (100 MS/s) for a measuring duration of 0.5 ms (500 μs) can acquire 25,000 samples per the duration of the test for each channel which will be subsequent for data manipulation. Because the typical strain (in mV) vs time graph of a SHPB test usually scale in μs as for time and 25,000 samples per the duration of 500 μs means a sample for every 0.02 μs which may be very ideal. Generally in SHPB testings, an oscilloscope with a minimum sampling rate of 2 MS/s is needed in order to capture high frequency details such as Poschammer-Chree oscillations.

The bandwidth determines an oscilloscope's fundamental ability to measure a signal. As signal frequency increases, the capability of the oscilloscope to accurately display the signal decreases. This specification indicates the frequency range that the oscilloscope can accurately measure.

Without adequate bandwidth, the oscilloscope will not be able to resolve high-frequency changes. Amplitude will be distorted, edges will vanish and details will be lost. Thus, this will result as a less reliable data manipulation. An oscilloscope with a bandwidth of 1 or 2 MHz will be fine for SHPB testings but generally it is recommended to use an oscilloscope with 5 MHz or over. In fact, in digital oscilloscopes the maximum sampling rate is at least twice the bandwidth. That means an oscilloscope with a 5 MHz bandwidth is at least capable of providing a sampling rate as 10 MS/s.

To determine a bandwidth of an oscilloscope the following equation (5 times rule) can be used;

$$\text{Oscilloscope Bandwidth Required} = \text{Highest Frequency Component of Measured Signal} * 5$$

An oscilloscope selected using the 5 Times Rule will give less than +/- 2% error in the analysis. However, as signal speeds increase, it may not be possible to achieve this. It should be always kept in mind that higher bandwidth will likely provide more accurate reproduction of the signal coming from the strain gages attached on the pressure bars.

The vertical resolution should also be taken into account in digital oscilloscopes. In SHPB testings, typically the strain gage signals are on the order of $5 - 20 \text{ mV}$. In order to track signals in this range a 12-bit resolution is needed. An 8-bit resolution can also be used but it is recommended to use 12-bit in literature.

Rise time may be a more appropriate performance consideration when it is expected to measure digital signals, such as pulses and steps. The oscilloscope must have sufficient rise time to accurately capture the details of rapid transitions. Rise time describes the useful frequency range of an oscilloscope. To calculate the oscilloscope rise time required for the type of signal type;

$$\text{Oscilloscope Rise Time Required} = \text{Fastest Rise Time of Measured Signal} / 5 ,$$

can be used.

In addition, digital oscilloscopes with a differential input should always be preferred rather than the ones with a single ended input.

It is better to prefer oscilloscopes with a maximum high (preferably 70 dB or more) Common Mode Rejection Ratios (CMRR) so that the noise levels are kept lower in the signals to be recorded. Relevantly, if possible, the Common Mode Voltage (CMV) of the oscilloscope should be in the order of $\pm 10 \text{ V}$ range or more so that an excitation voltage of around 20 V can be used in the wheatstone bridge to increase signal quality [60, 61, 72].

Below is an example of a digital oscilloscope, HP, 100 MHz at the High Voltage Lab, Istanbul Technical University (ITU);

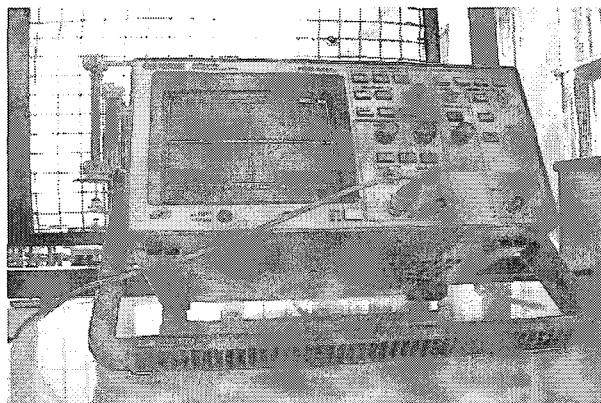


Figure 4.36 : HP Digital Oscilloscope 100 MHz at ITU High Voltage Lab.

v. Incident Wave Generating Techniques:

One of the most common ways of generating a stress wave, as it was mentioned earlier, is done by using a gas gun to impact the striker bar on the incident (input) bar. Guns have been used for many years and will continue to be used as tools to generate impacts. The principal advantage of guns over other techniques are the reproductibility of the results, the excellent planarity and paralellism at impact, and the relative ease to use sophisticated instrumentation and diagnostics. Waves can also be generated by the detonation of explosives at the free end of the incident bar. However, it will be more difficult to ensure a one-dimensional excitation within the incident bar using explosive techniques.

For lower velocity impacts one stage gas guns (max. velocity up to 1100 *m/s*) or propellant guns (max. velocity up to 2500 *m/s*) can be used. For velocities up to 8 km/s, two-stage gas guns are used. Even higher velocities can be reached by an electromagnetic gun known as “rail gun” [3].

Among all, one-stage gas gun is very simple in design. It consists of a breech, a barrel and a target chamber (the input bar in SHPB applications). The high-pressure gas is loaded in the high-pressure chamber, and the projectile is mounted in a sabot which is placed in the barrel. A valve is released and the high pressure gas drives the projectile (striker bar). Light gases such as Hydrogen, Helium or even Air can be used. The maximum velocity can be calculated from the maximum rate of expansion of the gas. Figure 4.37 shows a one-stage gas gun that can accelerate a projectile to a velocity of 1200 *m/s* using helium. It contains a fast-acting valve that is pulled back by a pressure differential and allows the high-pressure chamber to vent into the barrel. The projectile is accelerated in the barrel and its velocity can be measured by the interruption of two laser beams or optical detectors.

Reducing friction in the gas gun is essential to control the repeatability of the test. An example of a different type gas gun is shown in figure 4.38. The gas gun acts like a pneumatic actuator with the exception that the striker is launched from the piston at the end of the stroke. The reservoir is set to a desired pressure through a metering valve. In order to actuate the piston, the fast acting high flow valve is opened allowing the air from reservoir to flow into the cylinder causing the piston and striker

to accelerate forward. As the piston comes to the end of the cylinder it slows down and then stops while launching the striker bar down the barrel.

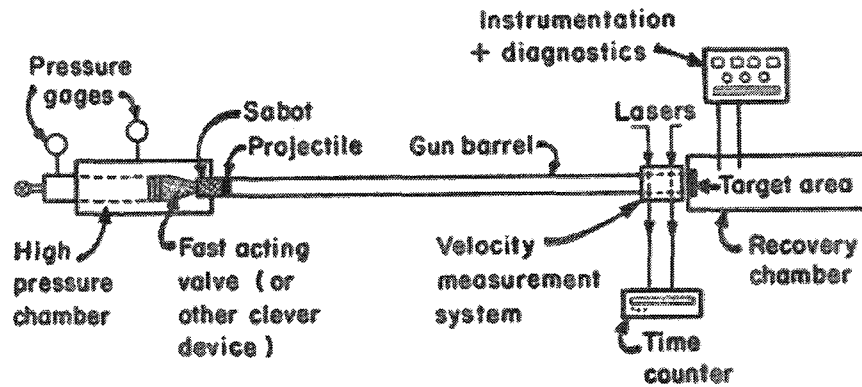


Figure 4.37 : Schematic Representation of one-stage light gas gun.

To predict the acceleration of the piston, a simple force analysis can be applied. Figure 4.39 is a free body diagram of the system. By taking the sum of the forces, the acceleration of the piston is given by;

$$\sum F = m_{system} a_{system} = F_{applied} - F_{friction} \quad (4.80)$$

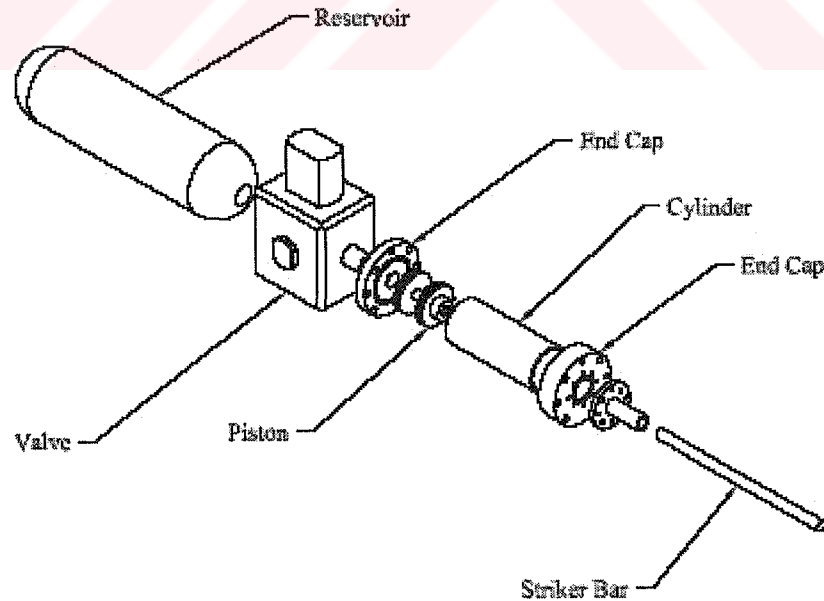


Figure 4.38 : Exploded view of a Gas Gun [24].

where,

$$m_{system} = m_{piston} + m_{striker\ bar} \quad , \quad (4.81)$$

$$F_{applied} = P_{reservoir} \cdot A_{piston} \quad , \quad (4.82)$$

$$a_{system} = a_{striker\ bar} = a_{piston} \quad (4.83)$$

and A_{piston} is the cross-sectional area of the piston. This equation is greatly simplified since it ignores losses through the valve, and as the piston moves, the reduction of pressure inside the reservoir. If linear acceleration is assumed, friction is ignored and the pressure from the reservoir is assumed constant, the velocity of the striker bar when it leaves the piston is given by;

$$V_{striker}^2 = a_{system} d \quad (4.84)$$

in the equation, d is the difference between the cylinder and piston length. This is a very simplified analysis of the event, it does not indicate the critical design parameters.

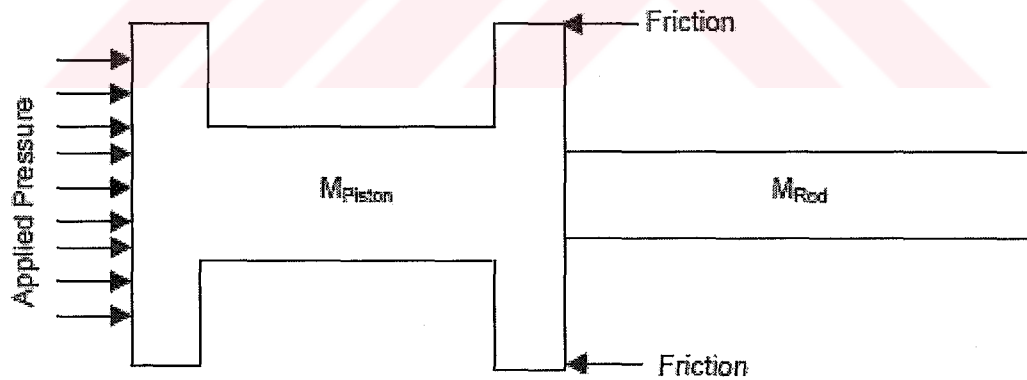


Figure 4.39 : Free Body Diagram of the Piston [24].

For velocities up to 8 km/s or more there are two-stage guns or even three-stage guns are available. Figure 4.40 shows a two stage gas-gun. It uses the first stage to drive a fairly massive piston. This piston is usually accelerated by means of the deflagration of a gunpowder load. The powder is placed in the breech and the first stage has a considerably larger diameter than the second stage. The first stage is filled (between projectile and piston) with a light gas (He or H). The deflagration of the gunpowder

accelerates the piston, compressing the light gas. When the pressure in the first stage reaches a critical level, a diaphragm (rupture disk) opens and starts driving the projectile. The piston continues to move and to compress the gas that drives the projectile. The piston eventually partly penetrates into the launch tube. Very high pressures in the light gas are generated that can drive the projectiles to velocities that can routinely reach 7 km/s . Besides the one-stage and two-stage gas guns there are three-stage and multi-stage gas guns, more information for these systems can be found in references given [64, 65].

Rather than gas guns or explosives there is also another method which is very simple to apply. In this method the incident stress wave can be generated by using a spherical ball with a specific diameter and by impacting it on the input bar. The amplitude of the wave depends on the mass of the impacting sphere and its velocity at impact. The maximum energy associated with this method is relatively small and can only cause very weak materials to fracture. Figure 4.41 shows a schematic of this method and figure 4.42 shows the stress values generated by the impact of spherical mass at different impact velocities [3, 24, 62, 65].

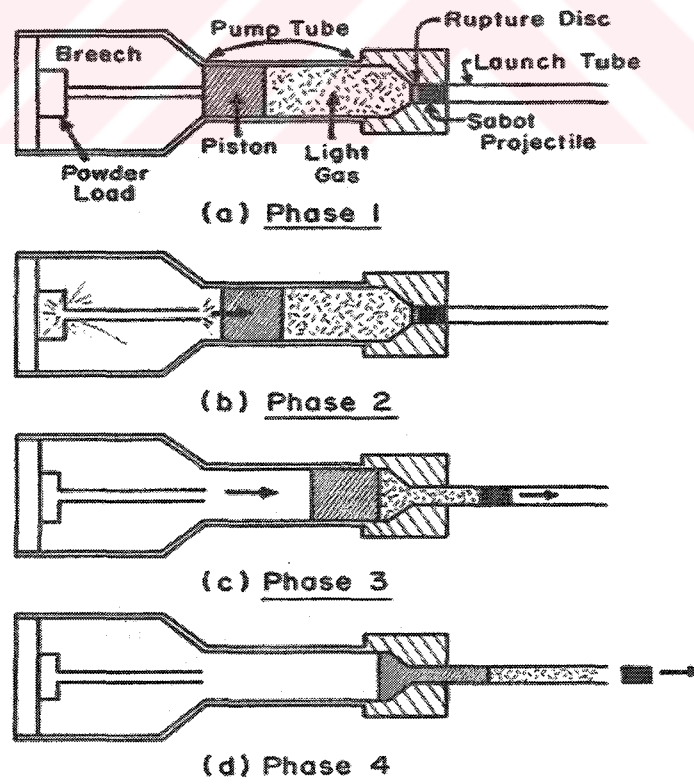


Figure 4.40 : Two-Stage Gas Gun [3].

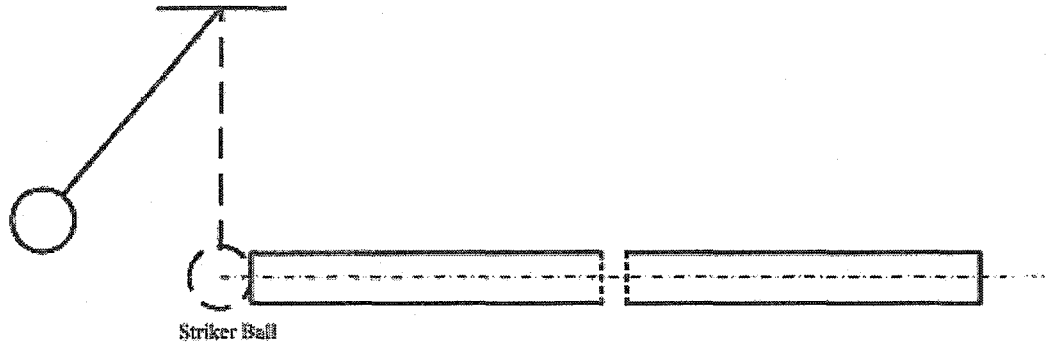


Figure 4.41 : Schematic of a Spherical Ball Hitting a Long Cylinder Bar [65].

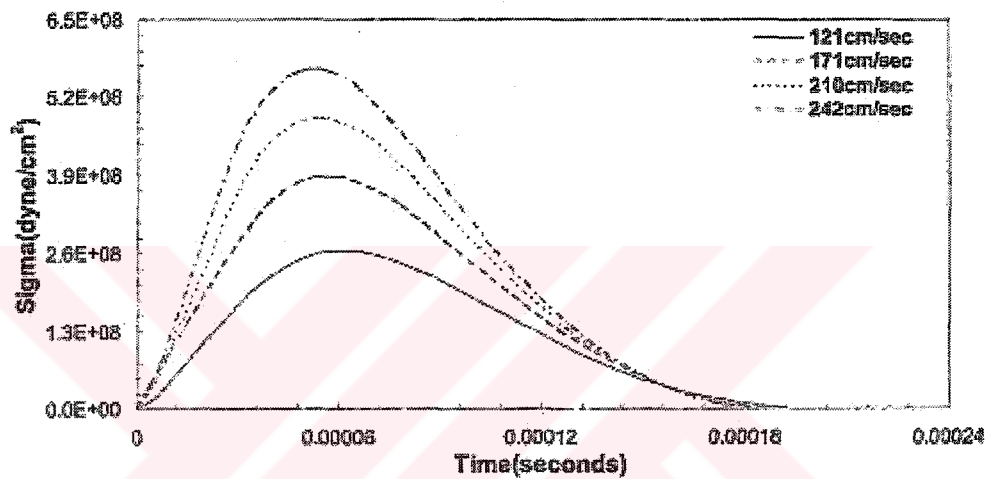


Figure 4.42 : Stress vs. Time at the impact end of a 16 mm dia. bar impacted by a 43 mm dia. Spherical Ball at different impact velocities [65].

vi. Pressure Bar Support System and Bearings:

Accurate bar alignment is required for one-dimensional wave propagation within the pressure bars and for uniaxial compression within the specimen. However, alignment can not be forced by over constraining the pressure bars, because this violates the boundary conditions for one-dimensional wave propagation in an infinite cylindrical solid. This is one reason for a tight specification on curvature in the pressure bars. Generally the pressure bars are mounted to a rigid beam. Mounting brackets with bearings, through which bars pass, are spaced every 100 – 200 mm, depending on the bar diameter, and designed so that each can be adjusted easily for an alignment. A simple check for alignment is to verify that each pressure bar slides and rotates freely through the mounting brackets and that faces of the incident and output bars meet identically when brought together without the specimen in place.

The bearings in the mounting brackets should be designed such that the diameter should be greater than the bar diameter to allow radial expansion. The dimensions of the bearings must be flexible enough to accept bars that vary in size, rigidity and straightness. When a stress pulse is induced in the incident bar by the striker bar, a wave propagates along the length of the bar causing it to radially expand. The amount of radial expansion is a function of the stress, the Elastic Modulus and the poisson's ratio of the material. The apparatus should be designed considering a maximum desired stress to be applied through out the test and its effect for the pressure bar.

Using the following formulas;

$$\epsilon_{longitudinal} = \frac{\sigma}{E} \quad (4.85)$$

$$\epsilon_{radial} = -\epsilon_{longitudinal} \cdot \nu \quad (4.86)$$

with an applied stress, the maximum radial expansion of the pressure bars can be calculated. From the radial expansion, the diametrical expansion of the bars can be determined and this expansion value must be considered when designing the bearing dimensions. By using bushings with different inner diameters, several pressure bars in different diameters can be used in a single SHPB apparatus. Figure 4.43 and 4.44 show two examples of a pressure bar support system.

In figure 4.44, to hold the incident bar and transmitter bar horizontally with minimum friction and deflection and to achieve excellent alignment of the bars, two teflon bearings were framed and fixed on an aluminium rail, which was joined to a solid steel foundation.

Another factor that is important in the design of the support system is the friction. If friction is too great between the bars and the bushing, the stress pulse will be attenuated. Friction has also adverse effect on the repeatability of the experiments since the friction is expected to change with time. These factors all indicate that the design of the bar supports are critical to the operation of the apparatus [24,27,47,65].

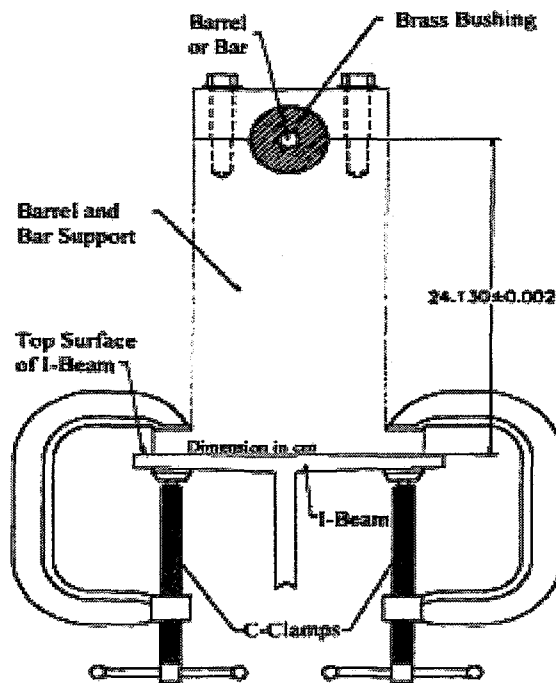


Figure 4.43 : An example of a Bar Support Configuration.

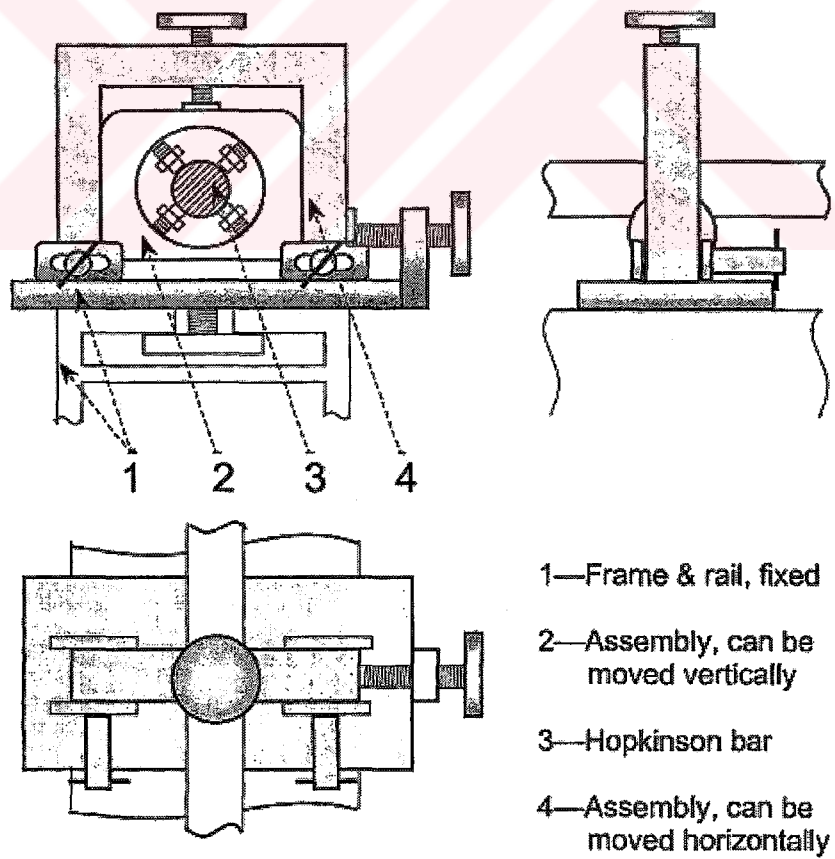


Figure 4.44 : Teflon Bearing and associated frame and rail.

4.2.9 Calibration of The Whole SHPB System

For the calibration of the system three methods will be proposed. In the first one, two strain gages were mounted on the striker bar closer to the impacting end. The installation of the strain gages and the subsequent circuits design should be identical to those used for the incident bar and the transmitter bar. In addition a calibrated load cell, usually used for impact testing, shall be placed in front of the striker bar as shown in figure 4.45. When the striker bar impacts the load cell, the history of the contact force can be detected by the load cell and recorded in a computer. Similarly, the history of the strain waves can be detected by the strain gages attached on the striker bar and recorded in a computer. Based on the recorded strain wave, the wave velocity in the striker bar can be calculated with the use of equation $C_o = 2l/t$ ($2l$ being the total length of wave propagation and t is the travelling time of the wave with in this distance). Subsequently, the dynamic Young's modulus of E of the striker bar can be determined by using eqn. (4.9). The results can be compared to the real Young's modulus of the striker bar after the calculations are done since the material of the striker bar is known.

The stresses in the striker bar can be identified from two independent methods; the load cell and the strain gage on the striker bar. Based on the load cell, the stresses can be obtained by dividing the contact forces with the cross-sectional area of the striker bar. Based on the strain gages, the stresses can be obtained from the multiplication of Young's modulus with the strains. The procedure to determine the strains is as follows;

The change of voltage in a Wheatstone bridge circuit due to the changes in strain gages can be expressed as;

$$\varepsilon = \frac{2 * \Delta E}{E_c S} \quad (4.87)$$

where ΔE is the change of voltage, E_c is the circuit voltage and S is the gage factor. If ΔE is a reading from the amplifier, it should be divided by the gain factor of the amplifier before substituting it to the equation. Note that, the equation is valid for the condition of no strains in the dummy gages if they are used in wheatstone

configuration. After the process a comparison of the stress results from the load cell and strain gages is made. Figure 4.46 shows an example comparison.

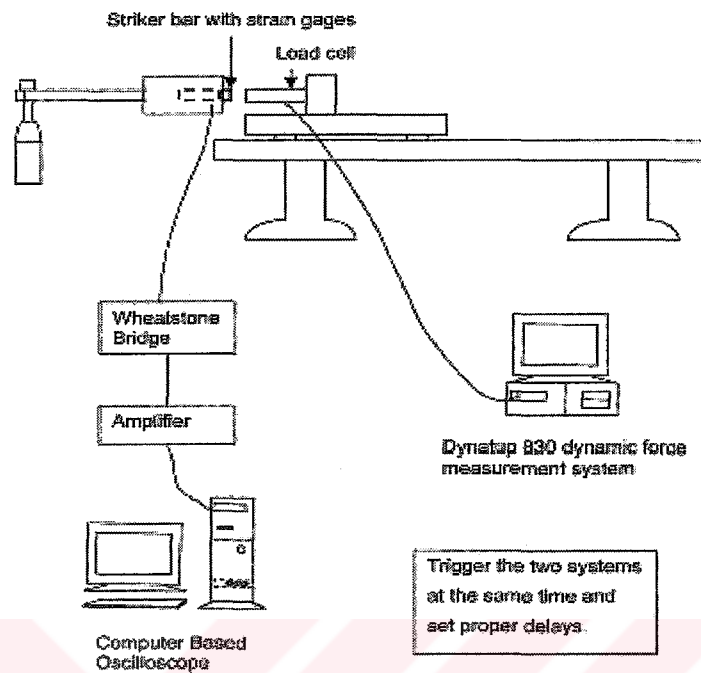


Figure 4.45 : Schematic Diagram of a Striker Bar Impacting on a Load Cell.

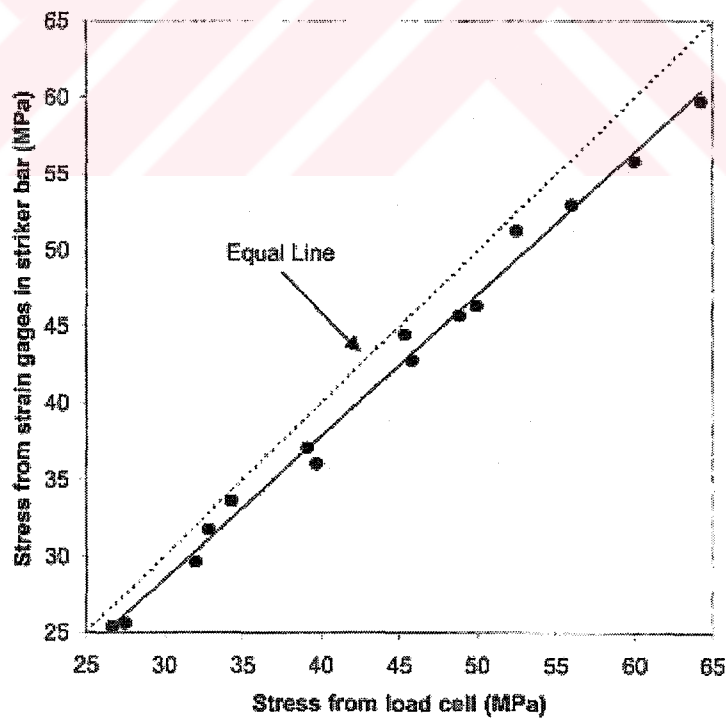


Figure 4.46 : Comparison between the signals from the load cell and those from strain gages on the striker bar.

In the second method, firstly the input bar without output bar and specimen is tested. The maximum strain occurred in the bar can be calculated as follows;

$$\varepsilon_{bar,max} = \frac{V_o}{2C_o} \quad (4.88)$$

where, V_o is the striker bar velocity. Thus, by measuring the strains in the bar the system is checked. Then the output bar is added to the test without the specimen. Both strains can be measured in the same way and the system is then can be calibrated again.

The last method is to compare the calculated result of the deformation occurred in the specimen after the test by measuring the final length of the specimen by a micrometer [43,47,55].

4.2.10 Dispersion Correction Techniques for The SHPB

A basic description of dispersion and attenuation was done at the early beginning of this chapter. The oscillations in the strain – stress history data in SHPB test are very critical. They greatly effect the computer stress-strain curves. One of the reason is the dispersion that occurs due to propagation of of longitudinal waves between the interface and the gage location. In the SHPB test, the pressure bar is excited by a pulse composed of a spectrum of frequencies. The higher frequencies of the pulse are travelling more slowly, thus, they lag behind the lower frequency components. A variation in phase velocity leads to the spreading, or dispersion, of an initially sharp pulse.

Most of the analysis of the SHPB results have been based on the assumption that one dimensional wave theory is adequate in the interpretation of pulses travelling along the bars. In order to be more accurate in the analysis of test results, it is desirable to use a more general theory to take into consideration the dispersion of waves due to the finite size of the bars. The procedure for correcting the recorded waveforms is discussed in details by **Follansbee and Frantz [71]**. According to their description, a program is developed to process the recorded waveforms.

First, the recorded waveforms were transformed from the time domain to the frequency domain using Fourier transformation. The Fourier components that contribute significantly to the magnitude of the whole pulse were determined. The amplitude and phase angle of each Fourier component was determined in the frequency domain.

Second, the mode I roots of the Pochhammer-Chree equation for each frequency component in the waveform using the material and geometrical properties of the bars were obtained (refer to **Follansbee and Frantz [71]** and **Bancroft [46]** for details). The phase angle difference for each Fourier component due to wave propagation over a distance between specimen and strain gage was then determined.

Third, with the preserved amplitudes and the new phase angles in the frequency domain, the corrected waveform was converted back to the time domain with the inverse Fourier transformation (summation of only the significant number of terms) at the incident bar/specimen interface.

After removing the dispersion and reconstructing the waveforms (figure 4.47), the resulting stress-strain curves to most of the experiments were smoother than those obtained with the initial raw data.

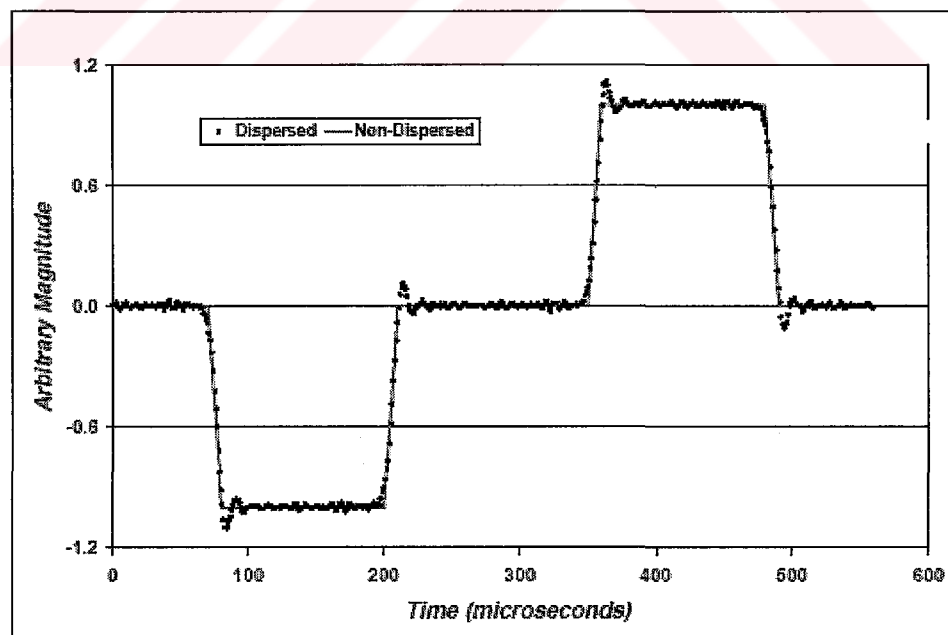


Figure 4.47 : Reconstructed Waveforms, Dispersed and Non-Dispersed Waveforms are overlaid [35].

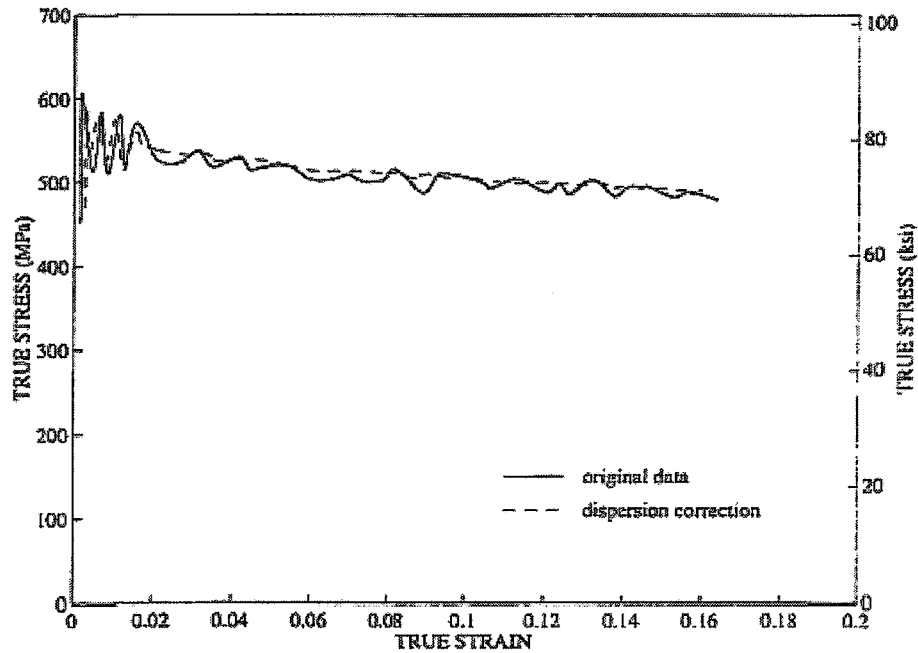


Figure 4.48 : Stress – Strain Relation for Ta at strain rate 2500 s^{-1} ; original data and data after dispersion correction [12].

However, the inherent oscillations in the dynamic stress-strain curves due to the lack of stress equilibrium in the specimens at small strains are not improved very much (figure 4.48) [12].

A detailed description on dispersion correction techniques both experimental and theoretical can be found in references [24,35].

4.2.11 Hopkinson Bar For Testing Low Empedance Materials

One of the requirements for the Hopkinson bar analysis is that a transmitted pulse to be measured. This means that the acoustical empedance of the specimen should be similar to that of the bars. The acoustic empedance was given by eqn. (4.14).

When testing low empedance materials such as foam and rubber, the transmitted pulse is significantly reduced causing the signal to noise ratio to decrease. In order to boost the signal transferred to the transmitter bar, the cross sectional area of the transmitter bar can be reduced, the signal can be measured with a more sensitive measurement technique or the empedance of the bar material can be reduced. Various researchers have made use of these techniques. Some researchers used hollow metallic transmitter bars with an end cap. By reducing the cross sectional area, the transmitted strain is increased for the same force level. This method avoids

any dispersion and attenuation problems that are encountered when viscoelastic bars with lower impedance are used. However the problem with this method is that the wave mechanics at the interface of the hollow tube with the end cap become complex. There can be a variety of reflections off the various interfaces since the end cap is press fitted into the tube and secured with a pin. The transmitted strain seen at the gage location may therefore not be indicative of the true strain being transmitted at the interface [24]. **Song and Chen [80]** and **Chen et al. [81]** used metallic hollow transmitter bars in their works and further information is present in the given references.

Chen et al. [82], suggested another method of increasing the signal in the transmitter bar. This method requires that a small quartz crystal to be embedded into the transmitter bar. The quartz crystal gave a much larger signal than the surface mounted strain gages and had a similar impedance to the aluminium bars used. The quartz crystal is approximately 3 times more sensitive than a conventional strain gage. Problems with this method occur when the wave hits the crystal causing complex reflections and refractions. These reflections and refractions are inevitable due to discontinuities at the crystal interface [24].

Another method used to reduce the impedance of the bar material is to switch from the conventional high strength steel bars to aluminium bars. Aluminium bars have a reduced modulus and density that makes them applicable for testing stiffer polymers and they do not suffer from viscoelastic effects of non-metallic bars. Most strain gage technology is applicable to aluminium bars and few changes need to be made to existing experimental techniques. Magnesium bars can also be used in a similar fashion although this method will only be applicable to stiffer polymers. Since Aluminium and Magnesium bars are linearly elastic, there is little dispersion or attenuation and the conventional Hopkinson equations can be used. On the other hand once the material to be tested enters into the realm of foams and rubbers, the bars must be made out of lower modulus non-metallic bars such as acrylic. **Subhash et al. [83]** advised that the high strain rate loading for the low-density ($< 1 \text{ g/cm}^3$) foams can be implemented using a polymeric split Hopkinson pressure bar and for the high density foams ($> 1 \text{ g/cm}^3$) using a magnesium pressure bar (figure 4.49). In their work, in the case of using a polymeric bar, three strain gages

are used on the incident bar to fully characterize the the dispersion and attenuation phenomenon in the polymeric bar.

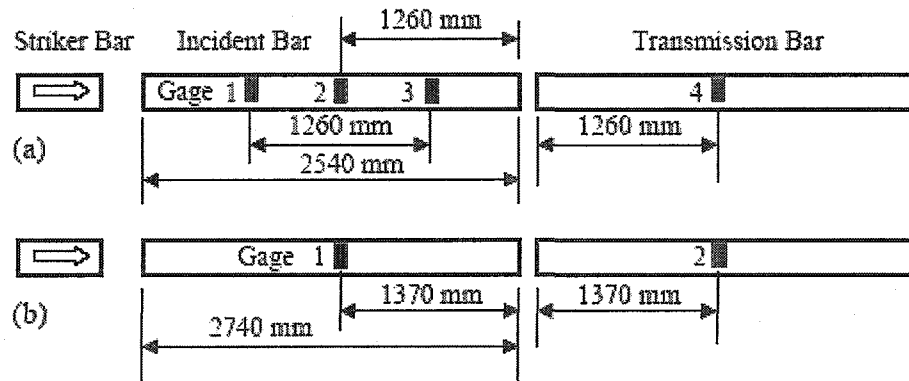


Figure 4.49 : Schematic of the two SHPB systems used for high strain rate testing of polymer foams. (a) polymer SHPB, (b) magnesium SHPB [83].

4.3 Other Applications with Split Hopkinson Pressure Bars

In section 4.2, the fundamentals of compressive SHPB was discussed. The Hopkinson bar is also used in tension, torsion and shear or their combinations. In this section other types of SHPB applications such as Tensional SHPB, Torsional SHPB, Elevated SHPB tests and SHPB test for low impedance materials will be discussed.

4.3.1 SHPB for Tension

The principles of the split-Hopkinson bar in tension are similar to those in compression. The primary differences are the methods of generating a tensile loading pulse, specimen geometry, and the method of attaching the specimen to the two pressure bars.

A number of experimental fixtures were developed for tension and some of them are shown in figure 4.50. In figure 4.50(b) the compressive pulse by-passes the specimen because of a collar. It is allowed to reflect at a free surface and returns as a tension pulse. Another arrangement, using an inertial bar is shown in figure 4.50(c). The striker tube impacts the main tube, causing a compression pulse, and this compression pulse acts on the specimen, pulling it to the right. The inertial bar counteracts this. The result is tension on the specimen.

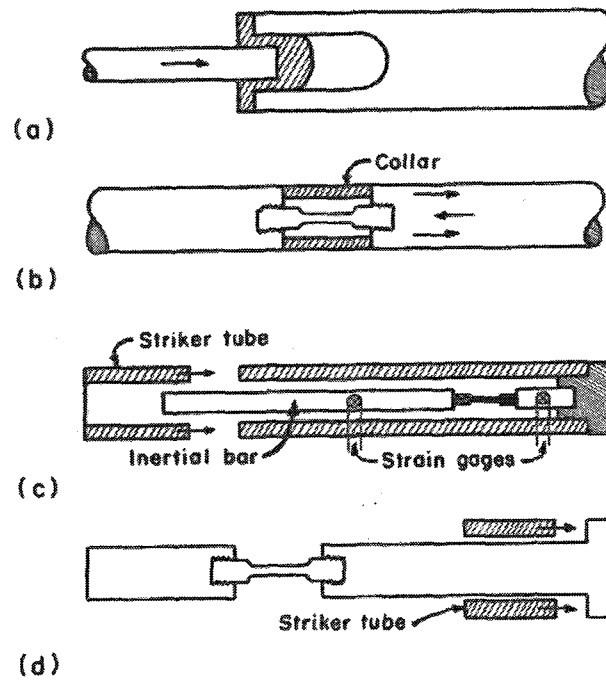


Figure 4.50 : Setups for testing specimens in Hopkinson bar in tension mode: (a) Hat-Shaped Specimen, (b) Collared Specimen, (c) Inertial Bar Configuration, (d) Collar Impacting Bolt Head [3].

Figure 4.51 is a detailed experimental set-up for the collar impacting bolt head arrangement in figure 4.50.

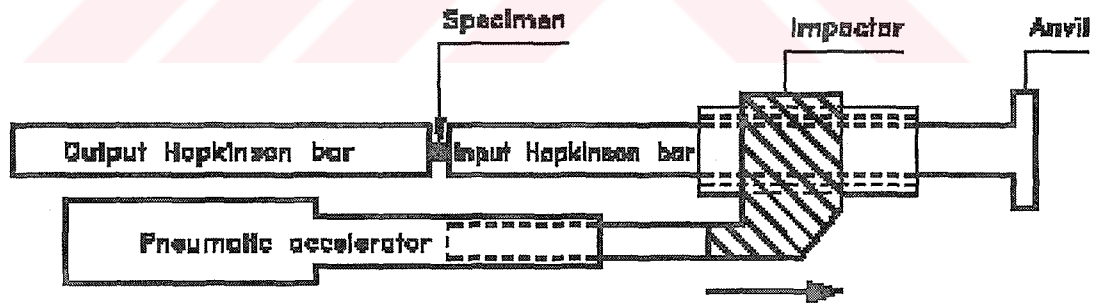


Figure 4.51 : Experimental set-up of a typical split Hopkinson tensile bar set-up.

The events are also same as in the compression tests, a tensile wave, the so called incident wave, is generated and propagates along the input bar towards the specimen. The incident wave interacts with the specimen, generating a reflected wave and a transmitted wave. The strain histories $\varepsilon_I(t)$, $\varepsilon_R(t)$ and $\varepsilon_T(t)$ corresponding with, respectively, the incident, reflected and transmitted wave are usually measured by means of strain gauges at well-chosen points on the Hopkinson bars. From these

measured waves, the history of the stress, the strain and the strain rate can be derived with the same equations given in section 4.2. Note that in order to place the specimen between the pressure bars the specimen can be either glued to the pressure bars or it can be attached to the pressure bars in case of a sheet type specimen with bolt type grips as in figure 4.52 [3, 27, 66- 68].

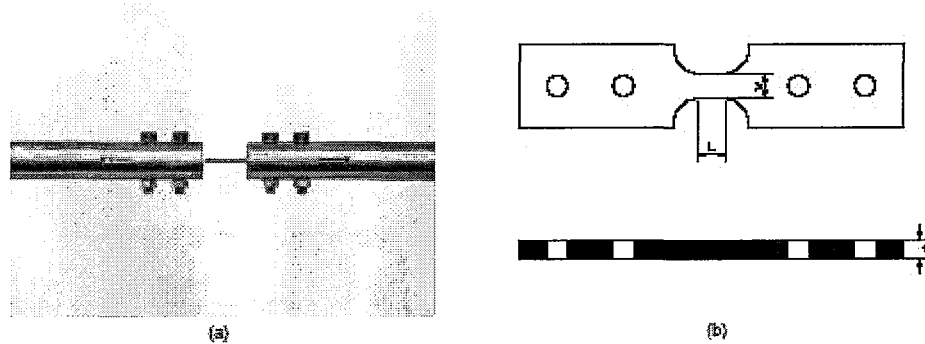


Figure 4.52 : Schematic Description of a Specimen and Grips, (a) tensile specimen, (b) bolt type grips.

4.3.2 SHPB for Torsional Loading

The torsional split Hopkinson (TSHB) technique is widely used to test materials at strain rates that typically range from $5 \times 10^2 s^{-1}$ to $5 \times 10^3 s^{-1}$. In this technique a short material specimen, a thin-walled tube, placed between two bars is loaded beyond the elastic limit by a torsional wave generated in the incident bar. Torsional energy is released by braking the clamp. The stress wave propagations are the same like in the compressional SHPB. Figure 4.53 shows a typical TSHB arrangement.

In the standard procedure for determining the specimen's stress, strain rate, and strain from the recorded elastic waves in the bars it is assumed that the specimen is under a state of pure and uniform shear stress and deformation.

The torsional split Hopkinson bar experiment has not been analyzed as extensively as the compression configuration of the technique since inertia effects in the specimen, friction at the specimen's interfaces, and dispersion of the elastic waves in the bars are not an issue. The specimen's gage (the section that undergoes plastic deformation) is the short thin-walled tube section that is machined in the middle of the thick-walled tube. It is being concluded by investigators that the assumption of uniform state of pure shear stress and strain in the specimen's gage is appropriate for

a specimen that is 2.5 mm long ($L_o = 2.5$ mm). In this the stress concentration near the corners of the specimen's ends does not have a significant effect. Figure 4.54 shows an example geometry for a specimen used in TSHB tests.

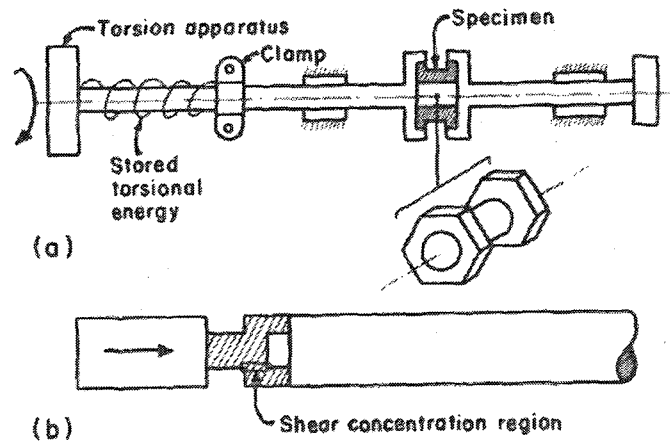


Figure 4.53 : (a) Torsional Hopkinson bar for pure shear configuration, (b) Hat-Shaped specimen for shear stresses in compressional Hopkinson bar.

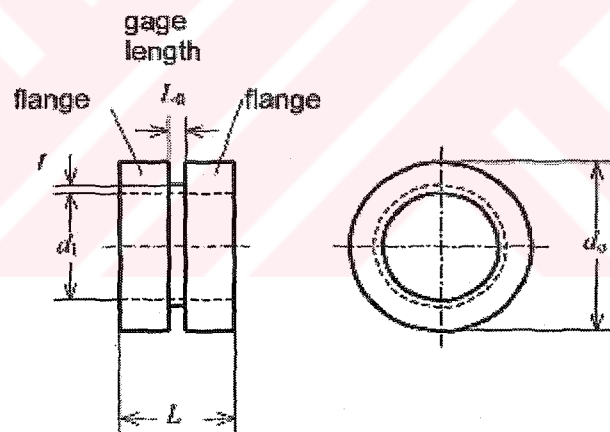


Figure 4.54 : Geometry of a Specimen for Torsion Tests.

With the use of a specimen that has a very short gage length, the torsional split Hopkinson bar technique can be used to study the response of materials at strain rates above $10^4 s^{-1}$.

An important difference exists between an axial pulse (tension or compression) travelling down a cylindrical bar and a torsional pulse. This difference affects both the instrumentation requirements and the shape of the pulse that loads the specimen. When a compressive pulse travels down a bar, it undergoes geometric dispersion,

because the different frequency components in the compressive pulse have different velocities. In particular, the higher frequency components travel at a lower velocity than the main pulse. In contrast, there is no geometric dispersion when a torsional pulse travels along an elastic bar in its primary mode; i.e. all frequency components of the torsional pulse have the same velocity. Hence, a torsional pulse is initiated either by an explosive detonation or by the release of a stored torque does not change its shape as it propagates toward the specimen.

In Hopkinson bar experiments, this absence of geometric dispersion in torsion means that a pulse initiated with a short-rise time will maintain this rise time until it reaches the specimen, independent of the length of the pressure bars. Also, the torsional strain gage stations can be located as near or as far from the specimen as desired and still reveal the correct shape pulse incident on or reflected from the specimen.

There is, however, a disadvantage in a nondispersive pulse. If a torsional pulse is noisy when initiated (i.e., if some high-frequency components are superposed on the main pulse), this characteristic will be maintained, regardless of the length of the pressure bars. As a result, the strain rate imposed on the specimen will not be constant. In an axial Hopkinson bar, such high-frequency components gradually disappear. Hence, an axial pulse tends to become flat, i.e. tends to smooth out, as it travels along an elastic bar. A noisy torsional pulse, such as that associated with explosive initiation of the pulse, will retain this characteristic; it will never become flat and hence will not provide deformation at a constant strain rate. More detail regarding this considerations are fully described in [27].

Besides, the arrangement in figure 4.51(a), shear stresses can also be created in a compressional Hopkinson bar by using a hat-shaped specimen, shown in figure 4.51 (b) [27, 69].

4.3.3 SHPB for Elevated Temperature Tests

Several high-temperature modifications have been proposed and implemented for use with the Hopkinson bar. The most common approach to running high-temperature Hopkinson bar experiment is to simply heat the specimen while it is in contact with the input and output bars (inside some sort of tube furnace), allowing a strong temperature gradient to develop within the bars. If an estimate of the temperature

gradient can be made, its effect can be compensated for by making the proper adjustments to the temperature-dependent elastic modulus, and thus, wave velocities within the bar material. Such a technique, have limitations on the attainable specimen temperature that are imposed by heat conduction in the bars and by the properties of the bar material at very high-temperatures. The thermal cycling associated with repeated testing also has the potential of permanently altering the mechanical properties of the bars, which are typically made of a heat-treated, high-strength steel. A change in the mechanical properties will result as a change in the mechanical impedance of the bars and this will lead to reflections and dispersions of the wave. One solution to this problem can be to provide an insulating layer between the specimen and the bars in the form of a short length of impedance-matched ceramic bar (i.e. aluminium oxide). This method can reduce the temperature gradients in the bar, but it typically requires a large energy input to heat the specimen (since at least small portion of the specimen-ceramic interfaces must be heated as well). One method of providing rapid heating in such an arrangement is induction heating.

A second, more modern approach is to heat the specimen without contacting it with the bars, and then to bring the bars into contact with the specimen just before impact. This set-up has the advantage of reducing temperature gradients in the bars, and the added benefit of requiring relatively small energy inputs to heat the specimen. The major design issue in such a system is the “cold contact time”, the duration of time over which the input and output bars are in contact with the specimen before the arrival of the incident wave (which correspond to the beginning of the recorded specimen deformation history). During this contact time, thermal gradients develop within the specimen, and the overall specimen temperature drops by an amount that is typically not recorded.

Figure 4.55 shows a schematic of a high-temperature Hopkinson bar apparatus. Standard cylindrical specimens are heated using an air-cooled infra-red spot heater, which consists of a high temperature tungsten filament positioned at the focal point of an elliptical reflector.

The elevated high strain tests can also be performed in tension or torsion mode Hopkinson tests by implementing the heating systems accordingly for the desired mode [70, 71].

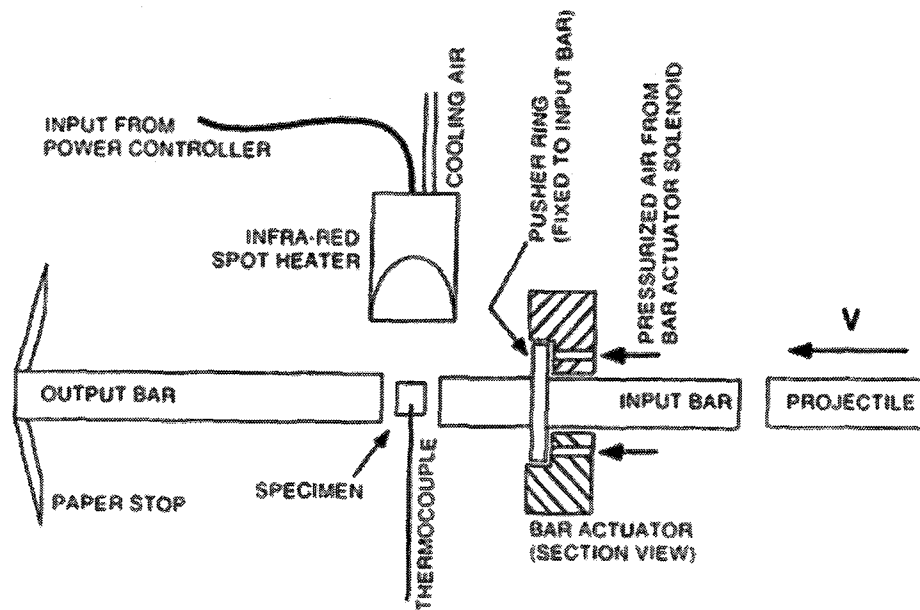


Figure 4.55 : Schematic of the High-Temperature Compressional Hopkinson Bar. Timing of the projectile length and bar actuation is controlled by an electronic delay circuit. Specimen temperature is maintained by a closed loop process controller.

5. CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

Following topics can be considered as conclusions from the present research;

- When strain is dynamic, ($>10^2 s^{-1}$), the strength of a material behaves different than those in the quasi-static or static strain rate, thus strength of a material can be sometimes as much as 25% higher in the dynamic strain range.
- The split Hopkinson pressure bar testing method can be used to test materials in the dynamic deformation range ($500 - 10^4 s^{-1}$) and at various temperatures to obtain regarding stress-strain curves at those strain rates and it can be adapted for compressional, tensional and torsional loadings of materials.
- A split Hopkinson pressure bar apparatus for compressional loading type can be constructed with the related considerations such as one-dimensional elastic wave propagation, impedance matching and wave reflection-refraction and interaction conditions.
- A compressional split Hopkinson pressure bar apparatus design considerations are carried out in this study and it is being realised that, the real difficulties in designing a SHPB apparatus were implementing the data acquisition system, gas gun system and evaluating the data obtained from test results.
- It was understood that there is not any standard SHPB apparatus, since the design parameters change according to the properties of the specimen to be investigated and desired strain rate.
- A one-stage gas gun system design can be done, using pressurised air and capable of impacting the striker bar on the incident bar at different velocities.

- The procedure of converting the voltage variations vs time in the recorded oscilloscope data to strain is the most important issue to evaluate the test data.

5.2 Recommendations for Further Studies

- A split Hopkinson pressure bar apparatus including a one stage gas-gun can be constructed with the given design considerations.
- Several tests with an SHPB for different materials should be performed to investigate their dynamic behavior and perhaps then their Johnsonn-Cook constitutive equation constants should be obtained.
- Pressure bars to for testing Polystrene, Aluminium and Concrete should be designed and constructed to perform test on the mentioned materials.



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