MODELING AND SIMULATION OF ELECTROMAGNETIC PROBLEMS
VIA
THE TRANSMISSION LINE MATRIX METHOD

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İLETİM HATTI MATRİSİ YÖNTEMI İLE ELEKTROMANYETİK PROBLEMLERİN MODELLENMESİ VE SİMÜLASYONU

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<td>1D</td>
<td>One-Dimension</td>
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<tr>
<td>2B</td>
<td>İki Boyutlu</td>
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<tr>
<td>2D or 2-D</td>
<td>Two-Dimensions</td>
</tr>
<tr>
<td>3B</td>
<td>Üç Boyutlu</td>
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<tr>
<td>3D or 3-D</td>
<td>Three-Dimensions</td>
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<td>ABC</td>
<td>Absorbing Boundary Condition</td>
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<td>ASCN</td>
<td>Asymmetrical Condensed Node</td>
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<td>BEM</td>
<td>Bioelectromagnetics</td>
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<td>CISPR</td>
<td>International Special Committee on Radio Interference</td>
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<td>DFFT</td>
<td>Discrete Fast Fourier Transform</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
</tr>
<tr>
<td>DUT</td>
<td>Device Under Test</td>
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<tr>
<td>EE</td>
<td>Ekranlama Etkinliği</td>
</tr>
<tr>
<td>EM</td>
<td>Electromagnetics</td>
</tr>
<tr>
<td>EMC</td>
<td>Electromagnetic Compatibility</td>
</tr>
<tr>
<td>EMI</td>
<td>Electromagnetic Interference</td>
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<td>EN</td>
<td>Expanded Node</td>
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<tr>
<td>EUT</td>
<td>Equipment Under Test</td>
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<td>FCC</td>
<td>Federal Communications Commission</td>
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<td>FD</td>
<td>Frequency Domain</td>
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<td>FDTD</td>
<td>Finite Difference Time Domain</td>
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<td>FFT</td>
<td>Fast Fourier Transform</td>
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<td>GO</td>
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<td>GPR</td>
<td>Ground Penetrating Radar</td>
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<td>HFD</td>
<td>Hızlı Fourier Dönüşümü</td>
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<td>HFSWR</td>
<td>High Frequency Surface Wave Radar</td>
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<td>HSCN</td>
<td>Hybrid Symmetrical Condensed Node</td>
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<td>ICNIRP</td>
<td>International Committee on Non-Ionising Radiation Protection</td>
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<td>IEC</td>
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<td>İHM</td>
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<td>NTFF</td>
<td>Near To Far Field</td>
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<td>OET</td>
<td>Office of Engineering and Technology</td>
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<td>OP</td>
<td>Observation Point</td>
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<td>ÖSO</td>
<td>Özgül Soğurma Oranı (SAR)</td>
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<td>PDE</td>
<td>Partial Differential Equation</td>
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<tr>
<td>PEC</td>
<td>Perfectly Electrical Conductor</td>
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<td>PIFA</td>
<td>Planar Inverted F-Antenna</td>
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<td>PML</td>
<td>Perfectly Matched Layer</td>
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<td>RAM</td>
<td>Random Access Memory</td>
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<tr>
<td>RCS</td>
<td>Radar Cross Section</td>
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<td>RF</td>
<td>Radio Frequency</td>
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<td>SAR</td>
<td>Specific Absorption Rate</td>
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<td>SCN</td>
<td>Symmetrical Condensed Node</td>
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<td>SDA</td>
<td>Spectral Domain Approach</td>
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<td>Sn-RCS</td>
<td>Single RCS</td>
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<td>SSD</td>
<td>Simetrik Sıkıştırılmış Düğüm</td>
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<td>TD</td>
<td>Time Domain</td>
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<td>TLM</td>
<td>Transmission Line Matrix or Transmission Line Modeling</td>
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<td>ZRT</td>
<td>Zero Reflection</td>
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LIST OF SYMBOLS

\[ E \] : Electric Field Vector
\[ H \] : Magnetic Field Vector
\[ D \] : Electric Flux Density Vector
\[ B \] : Magnetic Flux Density Vector
\[ E_k \] : \( k \) (x, y or z) Component of E Field
\[ H_k \] : \( k \) (x, y or z) Component of H Field
\[ E_s \] : Scattered E Field
\[ E_i \] : Incident E Field
\[ |E| \] : Magnitude of E Field
\[ |H| \] : Magnitude of H Field
\[ \mu_0 \] : Free Space Permeability
\[ \mu_r \] : Relative Permeability
\[ \varepsilon_0 \] : Free Space Permittivity
\[ \varepsilon \] : Permittivity Tensor
\[ \varepsilon_r \] : Relative Permittivity
\[ \sigma \] : Conductivity
\[ \sigma^* \] : Magnetic Conductivity
\[ V \] : Voltage
\[ I \] : Current
\[ C \] : Capacitance per Unit Length
\[ L \] : Inductance per Unit Length
\[ R \] : Resistance per Unit Length
\[ G \] : Conductance per Unit Length or Characteristic Admittance of Loss

\[ c, u_o \] : Speed of the Light in Free Space
\[ u_n \] : Velocity of the Waves on the Network
\[ \gamma \] : Propagation Constant
\[ \alpha \] : Attenuation Constant
\[ \rho \] : Tissue Density
\[ \beta \] : Phase Constant
\[ \omega \] : Angular Frequency
\[ f \] : Frequency
\[ f_{\text{max}} \] : Maximum Frequency
\[ f_r \] : Resonant Frequency
\[ k \] : Wave Number
\[ t \] : Time
\[ \Delta \ell \] : Mesh (Unit Cell) Size
\[ \Delta x \] : x-Directed Unit Cell Size
\[ \Delta y \] : y-Directed Unit Cell Size
\( \Delta x \) : z-Directed Unit Cell Size
\( \Delta t \) : Time Step
\( \lambda_o \) : Free Space Wavelength
\( \lambda_{\text{min}} \) : Minimum Wavelength
\( \Gamma \) : Reflection Coefficient
\( k V_n^i \) : Incident Voltage Impulse on Line \( n \) at Time \( t = k \times \Delta \ell / c \)
\( k+1 V_n^r \) : Combined Voltage Impulse Reflected along Line \( n \) at Time \( t = (k + 1) \times \Delta \ell / c \)
\( Y \) : Characteristic Admittance of Permittivity Stub
\( Z \) : Characteristic Impedance of Permeability Stub
\( \hat{G} \) : Normalized Characteristic Admittance of Loss Stub
\( \hat{Y} \) : Normalized Characteristic Admittance of Permittivity Stub
\( \hat{Z} \) : Normalized Characteristic Impedance of Permeability Stub
\( Z_o \) : Intrinsic Impedance of the Free Space
\( Z_{\text{TL}} \) : Characteristic Impedance of the Main Transmission Lines
MODELING AND SIMULATION OF ELECTROMAGNETIC PROBLEMS VIA THE TRANSMISSION LINE MATRIX METHOD

SUMMARY

Today’s electromagnetic (EM) problems are very complex. Analytical solutions are available only for some canonical structures and this has lead to an increased interest in numerical electromagnetics. Today, parallel to the increase in computer’s capacity and speed, numerical approaches have become rather popular.

Improvements in computers have also made it possible to solve EM problems directly in time domain (TD), starting either or from field and network theories. That is why, the transmission line matrix (TLM) and finite difference time domain (FDTD) methods, have enjoyed widespread use in the last decade. TLM is a fast developing technique which was first introduced by P.B. Johns in 1971. At the beginning, three dimensional (3D) problems were simplified and reduced to a generalized 2D nodes (Expanded Node) in the TLM method. Towards the end of the 70’s, 3D TLM began to be applied successfully to a wide variety of EM problems.

This thesis can be considered as an attempt for increasing the realm of complex EM problems which can be satisfactorily addressed by the TLM method. We will consider two such problems concerning Electromagnetic Compatibility (EMC) and Specific Absorption Rate (SAR) calculations under realistic conditions for which hitherto it has not been possible to generate TLM solutions. Our numerical calculations will show clearly that TLM can satisfactorily be applied to these problem areas.

We will also validate our solutions, albeit in a necessarily incomplete manner, by comparing our results with independently generated FDTD solutions of the same problems. It has also to be mentioned that both the TLM and the FDTD algorithms used for this purpose in our work were developed and coded by the author.

TLM is based on network theory and involves TD lumped transmission line modeling of the Maxwell’s equations in discretized spatial domain. This is an entirely different approach from that used in the FDTD, which relies on the direct discretization of the governing differential equations. TLM involves replacing a continuous system by a network or array of lumped elements. Interrelations and analogies between network equations and Maxwell’s equations form the basis of this method, and as such it can be considered as being more physical than strictly mathematical discretization approach. Lumped parameters of the transmission line, such as, inductance and capacitance correspond to the electrical parameters relating to the permeability and permittivity distributions in the corresponding EM problem, respectively. Currents and voltages, on the other hand, correspond to the magnetic
and electrical field components in the system. There are many different TLM versions in the literature. Here, the most powerful of these approaches, namely the symmetrical condensed node (SCN) TLM version is used. The main advantage of this node structure over the others and over the FDTD method is the symmetry it provides and the fact that the calculation of all the 6 field components is accomplished at the same time step. Each SCN-TLM node is represented by a scattering matrix, S, with which the reflected voltage pulses are related to the incident voltage pulses during the simulation time. On the other hand, there are two main drawbacks of the TLM method, which are the requirement of (i) large computer memories, and (ii) high simulation times.

The organization of this work is as follows:

- Chapter 2 is devoted to a fairly complete and detailed treatment of TLM method in 2D and 3D. In this chapter we critically investigate the various TLM algorithms based on different node structures and assess relative merits in relation to their respective computational requirements and the accuracies (numerical dispersion effects) they provide.

- In Chapter 3, two canonical problems, (i) the Green’s function representation in a PEC resonator and, (ii) radiation from an aperture, are considered for the purposes of validation (calibration) of our codes. The calibration is done via comparisons both in TD and frequency domain (FD). Analytical representations of these two canonical problems are derived in the FD, therefore comparisons in this domain are straightforward. The TLM results are transformed to the FD via discrete Fourier transformation (DFT). On the other hand, broad band TD comparisons are difficult and one needs to follow the steps given below:
  - The TLM results are obtained directly by using a broad band pulse as a source and the response is obtained directly in the TD.
  - FD results are calculated separately at chosen sampling frequencies of the broad band pulse used in TLM simulation via analytical representations. Frequency domain results are weighed with the source spectrum and inverse DFT is applied to obtain the TD analytical results. During this process, one must take the frequency resolution criteria into the consideration to get correct results.

- In Chapter 4, after having successfully calibrated our TLM code (and also FDTD which is used to obtain comparison solutions) we proceed to investigate the complex problems which constitute the main original contribution of this work. These are the Shielding Effectiveness (SE) and the Specific Absorption Rate (SAR) simulations for which, where no TLM solutions are available. SE is an effective parameter in EM compatibility (EMC) problems and is used as a criterion for assessing a structure’s susceptibility to EM interference. As a realistic prototype of EMC problems in this thesis we have considered a resonator with an aperture for SE modeling. The second problem we investigated concerns SAR calculations. SAR is the only parameter in bio-EM where device-human tissue interaction is of interest. The determination of SAR is an extremely complex problem and can be addressed either via difficult to
perform laboratory measurements or via numerical methods using simulated tissue prototypes. In this thesis we have considered the nowadays rather actual problem of calculating SAR distributions in human head models.

Extensive calculations for different parameter regimes are done for both problems and the TLM results are compared against the FDTD results and, in the SE case, also with the results given in the literature as obtained via Method of Moments (MoM) and validated experimentally. In all cases our results were in rather good agreement with the comparison solutions used.

- Finally, in Chapter 5 we present some concluding remarks together with suggestions for future work.
İLETİM HATTI MATRİSİ YÖNTEMI İLE ELEKTROMANYETİK PROBLEMLERİN MODELLENMESİ VE SİMÜLASYONU

ÖZET


 Ayrıca IHM yöntemi ile elde edilen sonuçlar bağımsız olarak üretilmiş ZSF yöntemi sonuçları ile karşılaştırılmıştır. Bu amaç için kullanılan hem İHM hem de ZSF yöntemi ile hazırlanan algoritmalar, yazar tarafından oluşturulmuş ve geliştirilmiştir.


Tezin yapısı aşağıdaki gibidir:

- Bölüm 2’de İHM yöntemi hem 2B hem de 3B’da detaylı olarak açıklanmıştır. Bu bölümde çeşitli düğüm yöntemleri kendine has avantajları ve özelliklerinin alt alana ait olarak açıklanmış, hesaplama ihtiyaçları ve hassasiyetleri (sayısal dispersiyon etkileri) verilmiştir.


  - İHM simülasyon sonuçları kaynak olarak geniş bandlı bir darbe kullanarak elde edilir. Sistemin cevabı da aynı şekilde doğrudan zaman domeninde elde edilir.
  - Frekans domeninde elde edilen analitik formüller kullanılarak İHM yönteminde kullanılan kaynağın bandında seçilen örnekleme frekanslarında aynı ayrı sonuçlar elde edilir. Bu frekans domeni sonuçları kaynağın frekans spektrumu ile ağırlanılar ve ters HFD ile zaman domeninde analitik sonuçlar elde edilmiş olur. Bu işlemler sırasında doğru sonuç elde edebilmek için frekans analizi kriterlerine uyulması gerekir.

- Bölüm 4’de, başlangıçta İHM (ve karşılaştırma amaçlı olarak kullanılan ZSF) yöntemi kodlarını kalibrasyon testlerini gerçekleştirdikten sonra tezin asıl ağırlıklı kısmını oluşturan kompleks problemler incelenmiştir. Burada ekranlama etkinliği (EE) ve özgül soğurma oranı (ÖSO) hesaplanır içeren iki karmaşık EM problem ele alınmıştır. ÖSO problemin de henüz İHM yöntemi ile elde edilen sonuçlar mevcut değildir. EE, EM uyumlulukta kullanılan etkin bir parametredir ve bir yapanın EM etkileşime olan bağıskılığını belirleyen bir kriter olarak kullanılır. Bu tezde EE modellemesinde, EM uyumluluk problemlerinin gerçekçi bir prototipi olarak, üzerinde yarık bulunan bir rezonatör ele alınmıştır. İncelediğimiz ikiinci problem ise ÖSO hesaplamalarını içermektedir. ÖSO cihaz-canalı dokular etkileşimini modelleyn biyo-EM problemi parametresidir. ÖSO problemi oldukça karmaşık olup ancak ya yapılmalı çok zor olan laboratuvar
ölçmeleriyle ya da insan doku parametrelerinin simüle edildiği bilgisayar programları yardımcıyla elde edilebilir. Bu tezde insan kafa modelinde ÖSO dağılımını hesaplayan oldukça gerçekçi problemler ele alınmıştır.

Her iki problemde de farklı parametreler için geniş çaplı hesaplamalar yapılmış ve İHM yöntemi sonuçları ZSF yöntemi sonuçlarına ile karşılaştırılmıştır. EE problemi için, literatürde Moment yöntemi ile elde edilmiş ve deneysel olarak doğruluğu test edilmiş sonuçlarla da karşılaştırma yapılmıştır. Tüm durumlarda kullanılan karşılaştırma sonuçları ile İHM yöntemi sonuçlarının çok iyi uyum gösterdiği görülmuştur.

- Son olarak Bölüm 5’de tezde elde edilen sonuçlar değerlendirilmiştir ve bu doğrultuda gelecekte yapılabilecek çalışmalara için öneriler getirilmiştir.
1. INTRODUCTION

Modeling and simulation are the most effective, if not the only way of solving complex electromagnetic (EM) problems whose analytical solutions can not be obtained or are yet unavailable. The method adopted for this work is the transmission-line matrix (TLM) method [1], which can be classified as a differential time domain (TD) method although a frequency domain (FD) formulation of this method has also been proposed [2]. In this thesis, the TD TLM is used in modeling several different EM problems. In each case the numerical results obtained via TLM are validated by comparing them with the results generated by using an alternative method. For the purpose of generating a comparison solution, we have chosen to use the finite difference time domain (FDTD) method [3] in all cases where analytical results are unavailable and no alternative comparison solutions are reported in the literature. Before going further, the terms simulation, simulator, model, validation, verification, etc., are briefly defined, so that no confusion arises when concepts are being discussed.

Simulation is the process of representing the dynamic behavior of one system by the behavior of another system. In engineering, numerical simulation refers to the use of computation to implement a model of some dynamic system or phenomenon. The purpose of simulation is usually to make “numerical experimental measurements” for analyzing/predicting behaviors of physical systems. Thus it can be considered as moving the laboratory into computer environment. Simulation provides a prototyping environment with which one can answer questions of a “what if?” nature about the system being simulated. A model is defined as a physical or mathematical abstraction of a real world process, device, or concept [4]. A simulation model can be defined as the representation of a model in computer code. Simulators are programs developed to implement and execute simulations. Verification is the process of determining that a model operates as intended, which is also referred to as “debugging”. Validation, on the other hand, is the process of assessing an acceptable level of confidence that the inferences drawn from the model correctly represent the real-world system modeled.

Simulation of EM problems is performed both in TD and FD. The FD techniques are especially advantageous in infinite volume, steady state problems, while these
problems can also be addressed with 1D techniques, the latter are more effective in obtaining transient responses in finite volume problems. Recently, the application of TD techniques to infinite volume problems has become increasingly widespread, since (i) the computers have become more powerful, (ii) they have been supported by powerful routines, such as free space simulators (like Perfectly Matched Layer, PML), near-to-far field transformation, etc. The two major advantages of the TD numerical methods are their ability to analyze complex structures with arbitrary geometry, and to yield FD characteristics over a broad band with a single simulation run.

TLM which was first introduced by P.B. Johns in 1971 [1], provides a TD numerical technique for solving network and field problems [5]. EM fields are modeled by filling the computational domain with a network of transmission lines which renders the problem discrete in space and time since pulses launched on the network scatter from point to point in space in a fixed time-step. The theoretical background of some TLM algorithms and some of its early applications to electromagnetic problems are reviewed in a tutorial paper by Hoefer [6].

The progress of the TLM method has been very fast. Today there exists a vast literature on its applications to a variety of problems such as:

- the simulation of propagation in waveguides and eigenvalue analysis [7,8]
- the analysis of planar microstrip structures [9,10]
- the simulation of antenna systems and radiation pattern evaluation [11]
- radar cross-section (RCS) calculations of various objects [12]
- the analysis of electromagnetic compatibility (EMC) problems of the microwave structures [13-15]
- and, ground penetrating radars (GPR) modeling [16].

The most significant steps in the development of TLM will be mentioned in the following in historical order. The first TLM algorithm developed is the expanded node (EN) technique which resembles FDTD because of its unit cell structure. It models electromagnetic phenomenon in 3D by interconnecting two-dimensional shunt and series nodes [17]. Note that in the TLM nomenclature the term node and cell are used interchangeably. There is half a time-step delay between these nodes, and for this reason the network is called as expanded-node network. A numerical
advantage of this TLM node structure over the FDTD method is that all six field components are available at each nodal cell (in contrast to the presence of different nodal components in FDTD [18]), thus making boundary description finer and providing more information at each node. Also, TLM is a one-step iteration method, whereas the FDTD routine is a two-step method. Conceptually, TLM has the advantage that it is a mathematical model with an exact computer solution, which however is closely related to the underlying physics of the problem.

Main disadvantages of both the EN network [17] is that topology of the network graph is quite complicated (it should be noted that FDTD also suffers from this problem). This has made data preparation for modeling of boundaries difficult and liable to error, and the problem is particularly acute when automatic data preparation schemes are implemented. The process of diakoptics for forming substructures [19,20] is also difficult to organize because of half time-steps and spatial separation of different polarizations.

This inconvenience of the EN network has led to the development of an asymmetrical condensed-node, (ASCN) structure by Saguet and Pic [21] (This and other improvements in TLM are described by Saguet and Tedjini [22]). The ASCN algorithm has also been used independently by Amer [23]. The features of this node structure is further explained in [24] and it can be seen that the network topology is simply a 3D Cartesian mesh with two lines, corresponding to two polarizations, in each branch. In this algorithm all scattering processes are referred to one point in space for the node (i.e., cell), all of the field components are also at one point in space, and boundary conditions can be applied at the discretization points or halfway between them. However, it should be noted that in [24] the node assignment is asymmetrical because, depending upon the direction of view, the first connection in the node is either shunt or series. This asymmetry has been carefully investigated by Amer [23], and it has been found that in most problems the errors thereby introduced are generally insignificant. Nevertheless, it does mean that boundaries viewed in one direction have slightly different properties when viewed in the other direction. The scattering at these nodes is evaluated by first calculating the six nodal quantities from the 12 incident pulses and then calculating the 12 reflected pulses. Although this procedure makes the scattering computations efficient, it involves quite lengthy arithmetic. Nevertheless, in spite of these disadvantages, it has been demonstrated [22-24] that the asymmetrical condensed-node (ASCN) technique uses less computer resources than the EN technique.

Probably the most important step in TLM was the introduction of the symmetrical
condensed node (SCN) method. This technique eliminates the disadvantages of asymmetry and cumbersome arithmetic in the ASCN while preserving the advantages of working with condensed nodes. As implied in the word "symmetrical", the most important advantage of this method is its symmetry that permits the 6 field components to be located at the centre of the unit cell and calculated at the same time step. In this study, we utilize SCN version of the TLM method in TD. In the following we will discuss how we build and develop necessary algorithms and supplementary computational codes, perform the canonical calibration tests and subsequently apply the method to obtain solutions of certain complex EM problems.

The main original contribution of this work is that, to the best of our knowledge, it is the first complete and systematic approach in [11,14,25,26]

- Applying TLM to calculating SE and SAR in problems defined by complex, realistic problems,

- Validating the calculated results by comparing them with results generated independently via the FDTD [27].

Another original contribution of the thesis is the work done for

- Assessing the accuracy and determining the ranges of applicability of TLM by comparing the calculated results with those obtained analytically for two canonical problems which are chosen in such a way as to represent some of the basic features of wave phenomena encountered in SE and SAR calculations.

The organization of this study is as follows:

- Chapter 2 is devoted to a fairly complete and detailed treatment of TLM method in 2D and 3D. In this chapter we critically investigate the various TLM algorithms based on different node structures and assess relative merits in relation to their respective computational requirements and the accuracies (numerical dispersion effects) they provide. Implementation of the absorbing boundary condition (PML) to 3D TLM and analysis of numerical dispersion are also given in this chapter.

- In Chapter 3, two canonical problems, (i) the Green’s function representation in a PEC resonator and, (ii) radiation from an aperture, are considered for the purposes of validation (calibration) of our codes. The calibration is done via comparisons both in TD and frequency domain (FD). Analytical representations of these two canonical problems are derived in the
FD, therefore comparisons in this domain are straightforward: The TLM results are transformed to the FD via discrete Fourier transformation (DFT). On the other hand, broad band TD comparisons are difficult and one needs to follow the steps given below:

- The TLM results are obtained directly by using a broad band pulse as a source and the response is obtained directly in the TD.

- FD results are calculated via analytical representations separately at chosen sampling frequencies within the band of the broad band pulse used in TLM simulation. Each frequency results are weighed with the source spectrum and inverse DFT is applied to obtain the TD analytical results. During this process, one must take the frequency resolution criteria into the consideration to get correct results.

- In Chapter 4, after having successfully calibrated our TLM code (and also FDTD which is used to obtain comparison solutions) we proceed to investigate the complex problems which constitute the main original part of this work. These are the Shielding Effectiveness (SE) and the Specific Absorption Rate (SAR) simulations for which, no TLM solutions are currently available in the literature. SE is an effective parameter in EM compatibility (EMC) problems and is used as a criterion for assessing a structure’s susceptibility to EM interference. As a realistic prototype of EMC problems in this thesis we have considered a resonator with an aperture for SE modeling. The second problem we investigated concerns SAR calculations. SAR is the only parameter in bio-EM where device–human tissue interaction is of interest. It is an extremely complex problem and can be obtained either via difficult to perform laboratory measurements or via numerical methods using simulated tissue prototypes. In this thesis we have considered the nowadays rather actual problem of calculating SAR distributions in human head models. Extensive calculations for different parameter regimes are done and the TLM results are compared against the FDTD results and, in the SE case, also with the results given in the literature as obtained via Method of Moments (MoM) and validated experimentally. In all cases our results were in rather good agreement with the comparison solutions used.

- Finally, in Chapter 5 we present some concluding remarks together with suggestions for future work.

Some basic characteristics of the FDTD method are given in Appendix A.
2. THE TRANSMISSION LINE MATRIX METHOD

In this section, transmission line matrix (TLM) method is described. Its formulations are investigated in detail.


The link between field theory and circuit theory has been exploited in developing numerical techniques to solve certain types of partial differential equations arising in field problems with the aid of equivalent electrical networks [28]. In addressing EM problems via numerical techniques three different electrical scales have generally to be considered. In terms of the wavelength $\lambda$ and a typical dimension $\ell$ of the apparatus, these ranges are [29]:

$$\lambda \gg \ell, \quad \lambda \approx \ell, \quad \lambda \ll \ell.$$ 

In general different techniques we used to address EM problems falling into each one of these ranges. When above ranges are considered from left to the right, the preferred techniques are usually within the realm of the circuit theory, microwave theory and geometric optics, respectively. Hence the fundamental laws of circuit theory can be obtained from Maxwell’s equations by applying approximations valid when $\lambda \gg \ell$, although circuit theory was originally not obtained by approximating Maxwell’s equations, but was developed independently from experimentally obtained laws. It should, however, be noted that circuit theory renders an exact representation of the EM phenomena when complemented with distributed parameter elements and that there is an exact correspondence between transmission line circuit theory and Maxwell’s equations. Citing from Silvester and Ferrari, it can be said that, circuits are mathematical abstractions of physically real fields [30].

The idea of replacing a complicated electrical system by a simple equivalent circuit goes back to Kirchhoff and Helmholtz. As a result of Park’s [31], Kron’s [32] and Schwinger’s [33] works, the power and flexibility of equivalent circuits became more obvious to engineers. The recent applications of this idea to scattering problems, originally due to Johns [1], has made the method more popular and attractive.
TLM is a numerical technique for solving field problems using equivalent circuits. It is based on the equivalence between Maxwell’s equations and the equations for voltages and currents on a mesh of continuous two-wire transmission lines. The main feature of this method is the simplicity of formulation and programming for a wide range of applications [6]. As compared with the lumped network model, the transmission-line model is more general and performs better at high frequencies where the transmission and reflection properties of geometrical discontinuities cannot be regarded as lumped [33].

Like other numerical techniques, the TLM method is a discretization process. Unlike other methods such as finite difference and finite element methods, which are mathematical discretization approaches, the TLM is a physical discretization approach. In the TLM, the discretization of a field involves replacing a continuous system by a network or array of lumped elements. For example, consider the discretization of a two-dimensional lossless conductive sheet as shown in Figure 2.1.

![Figure 2.1 Two-dimensional conductive sheet, its partially discretized equivalent and, fully discretized equivalent.](image)

The TLM method involves dividing the solution region into a rectangular mesh of transmission lines. Junctions are formed where the lines cross forming impedance discontinuities. A comparison between the transmission-line equations and Maxwell’s equations allows equivalences to be drawn between voltages and currents on the lines and electromagnetic fields in the solution region. Thus, the TLM method involves two basic steps:

- Replacing the field problem by the equivalent network and deriving analogy between the field and network quantities.
- Solving the equivalent network by TD numerical methods.

Before we apply the method, it is appropriate to briefly review the basic concepts of transmission lines.
2.1.1. Transmission-Line Equations

By using the model in Figure 2.2, the first order, coupled transmission line equations can be given as

\[
- \frac{\partial V(z,t)}{\partial z} = RL(z,t) + L \frac{\partial I(z,t)}{\partial t} \tag{2.1}
\]

and

\[
- \frac{\partial I(z,t)}{\partial z} = GV(z,t) + C \frac{\partial V(z,t)}{\partial t}, \tag{2.2}
\]

or in a second order decoupled form as

\[
\frac{\partial^2 \Phi}{\partial z^2} = LC \frac{\partial^2 \Phi}{\partial t^2} + (RC + GL) \frac{\partial \Phi}{\partial t} + RG \Phi \tag{2.3}
\]

where \(\Phi(z,t)\) stands for either \(V(z,t)\) or \(I(z,t)\). Here, \(R [\Omega/m]\), \(G [S/m]\), \(L [H/m]\), and \(C [F/m]\) are unit length resistance, conductance, inductance and, capacitance, respectively, which are called primary parameters.

Following special cases [34] are of interest:

(a) \(L = C = 0\) yields
\[
\frac{\partial^2 \Phi}{\partial z^2} = k_1 \Phi
\]  
(2.4)

where \( k_1 = RG \). (2.4) is the 1D elliptic partial differential equation called Poisson’s equation.

(b) \( R = C = 0 \) or \( L = G = 0 \) yields

\[
\frac{\partial^2 \Phi}{\partial z^2} = k_2 \frac{\partial \Phi}{\partial t}
\]  
(2.5)

where \( k_2 = GL \) or \( RC \). (2.5) is the 1D parabolic partial differential equation called the diffusion equation.

(c) \( R = G = 0 \) (lossless line) yields

\[
\frac{\partial^2 \Phi}{\partial z^2} = k_3 \frac{\partial^2 \Phi}{\partial t^2}
\]  
(2.6)

where \( k_3 = LC \). This is the 1D hyperbolic partial differential equation called the Helmholtz equation or simply the wave equation. Thus under certain conditions, the 1D transmission line can be used to model problems involving an elliptic, parabolic or hyperbolic partial differential equation (PDE).

Derived from the primary parameters are the secondary ones, for example, for the lossless line, they can be given as:

- the characteristic resistance

\[
R_0 = \sqrt{\frac{L}{C}}
\]  
(2.7a)

- the wave velocity

\[
u = \frac{1}{\sqrt{LC}}
\]  
(2.7b)

- and the reflection coefficient at the load

\[
\Gamma = \frac{R_L - R_o}{R_L + R_o}
\]  
(2.7c)
where $R_L$ is the load resistance. In the following section, the TLM method is applied specifically to wave propagation problem.

2.1.2. Solution of Wave Equations

A 2D TLM network for a 2D propagation problem is pictured in Figure 2.3 [7,35]. At the nodes, the interconnection of pairs of transmission lines forms impedance discontinuities. The complete network of TLM is made up of a large number of such building blocks as depicted in Figure 2.4. Notice that in Figure 2.4 single lines are used to represent a transmission-line pair. Also, a uniform internodal distance of $\Delta l$ is assumed throughout the matrix (i.e., $\Delta l = \Delta x = \Delta z$). We shall first derive equivalences between network and field quantities.

![Figure 2.3 Equivalent network of a two-dimensional TLM shunt node. $L$, $C$ and $\Delta l$ are unit length capacitance and inductance and, mesh size, respectively.](image)

2.1.2.1. Equivalence Between Network and Field Parameters

We refer to Figure 2.3 and use Kirchhoff's current law at node "0" to obtain in the limit as $\Delta l \to 0$

$$\frac{\partial I_x}{\partial z} - \frac{\partial I_x}{\partial x} = 2C \frac{\partial V_y}{\partial t}$$

(2.8a)

Applying Kirchhoff's voltage law in the $x$-$y$ and $y$-$z$ planes gives as $\Delta l \to 0$ gives
Figure 2.4 Transmission-line matrix and boundaries in 2D plane.

\[
\frac{\partial V_y}{\partial x} = -L \frac{\partial A_x}{\partial t} \tag{2.8b}
\]

\[
\frac{\partial V_y}{\partial z} = -L \frac{\partial A_z}{\partial t} \tag{2.8c}
\]

Combining these equations, one obtains the Helmholtz wave equation in two-dimensional space as follows:

\[
\frac{\partial^2 V_y}{\partial x^2} + \frac{\partial^2 V_y}{\partial z^2} = 2LC \frac{\partial^2 V_y}{\partial t^2} \tag{2.9}
\]

In order to show the field theory equivalence of (2.9), Maxwell’s curl equations are considered in scalar form for the 2D situation. Noting that \(\frac{\partial}{\partial y} = 0\), one obtains \(E_x = E_z = H_y = 0\). It is noticed at once that this is a transverse electric (TE) mode with respect to the \(z\)-axis, but a transverse magnetic (TM) mode with respect to the \(y\)-axis. Thus by the principle of analogy, the network in Figure 2.3 can be used for \(E_y, H_x, H_z\) fields as well as \(E_x, E_z, H_y\) fields. For TE to \(z\) waves, Maxwell’s equations reduce to

\[
\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = \varepsilon_0 \frac{\partial E_y}{\partial t}, \tag{2.10a}
\]
\[
\frac{\partial E_y}{\partial x} = -\mu_o \frac{\partial H_z}{\partial t}, \quad (2.10b)
\]

\[
\frac{\partial E_y}{\partial z} = \mu_o \frac{\partial H_x}{\partial t}, \quad (2.10c)
\]

and the Helmholtz equation for \( E_y \) is obtained as

\[
\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} = \mu_o \varepsilon_o \frac{\partial^2 E_y}{\partial t^2}. \quad (2.11)
\]

Comparing (2.10) and (2.11) with (2.8) and (2.9) yields the following equivalence between the various field and network quantities

\[
E_y \equiv V_y, \quad H_x \equiv -I_z, \quad H_z \equiv I_x, \quad \mu_o \equiv L, \quad \varepsilon_o \equiv 2C, \quad (2.12)
\]

Thus in the equivalent circuit:

- the voltage at shunt node corresponds to \( E_y \),
- the current in the \( z \) direction corresponds to \(-H_x\),
- the current in the \( x \) direction corresponds to \( H_z \),
- the inductance per unit length represents the permeability of the medium,
- twice the capacitance per unit length represents the permittivity of the medium.

### 2.1.2.2. Scattering Matrix

Let us define \( k V_i^j \) and \( k V_i^r \) as voltage impulses incident upon and reflected from terminal "\( n \)" of a node at time \( t = k \times \Delta \ell / c \), and that a voltage impulse function of unit magnitude is launched into line 1 of a node, as shown in Figure 2.5a (the characteristic resistance of the line is normalized). Unit-magnitude impulses of voltage and current will then travel towards the junction with unit energy \( (S_i = 1) \). Since three other lines are joined to line 1, its effective terminal resistance is \( 1/3 \). With the knowledge of its reflection coefficient, both the reflected and transmitted voltage impulses can be calculated. The reflection coefficient is
\[
\Gamma = \frac{R_L - R_o}{R_L + R_o} = \frac{1/3 - 1}{1/3 + 1} = -\frac{1}{2},
\]
so that the reflected and transmitted energies are
\[
S_r = S_i \Gamma^2 = \frac{1}{4}, \quad S_t = S_i (1 - \Gamma^2) = \frac{3}{4}
\]
where subscripts \( i, r \) and \( t \) indicate incident, reflected and transmitted quantities, respectively. Thus a voltage impulse of \( V = \sqrt{\frac{3}{4} + 3} \) will be launched into each of the other three terminals as shown in Figure 2.5b.

The more general case of four impulses being incident on four branches of a node can be obtained by applying the superposition principle to the previous case of a single pulse. Hence, if at time \( t = k \times \Delta \ell / c \), voltage impulses \( kV_1^i, kV_2^i, kV_3^i \) and \( kV_4^i \) are incident on lines 1 to 4, respectively, at any junction node as in Figure 2.5c.

In general, the total voltage impulse reflected along line \( n \) at time \( t = (k+1) \times \Delta \ell / c \) will be
\[
k+1V_n^r = \frac{1}{2} \left[ \sum_{m=1}^{4} kV_m^i \right] - kV_n^i, \quad n = 1,2,3,4
\]
This idea is conveniently described by a scattering matrix equation relating the reflected voltages at time \((k + 1) \times \Delta t / c\) to the incident voltages at the previous time step \(k \times \Delta t / c\):

\[
\begin{bmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4
\end{bmatrix}^{k+1} = \frac{1}{2} \begin{bmatrix}
-1 & 1 & 1 & 1 \\
1 & -1 & 1 & 1 \\
1 & 1 & -1 & 1 \\
1 & 1 & 1 & -1
\end{bmatrix} \begin{bmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4
\end{bmatrix}^k
\]

(2.16)

Also an impulse emerging from a node at position \((z, x)\) in the mesh (reflected impulse) becomes automatically an incident impulse at the neighboring node. Hence, incident and reflected voltage pulses are related via

\[
\begin{align*}
{k+1}V_1^i(z, x + \Delta t) &= kV_3^r(z, x) \\
{k+1}V_2^i(z + \Delta t, x) &= kV_4^r(z, x) \\
{k+1}V_3^i(z, x - \Delta t) &= kV_4^r(z, x) \\
{k+1}V_4^i(z - \Delta t, x) &= kV_2^r(z, x).
\end{align*}
\]

(2.17)

Thus by applying (2.16) and (2.17), the magnitudes, positions and directions of all impulses at time \((k + 1) \times \Delta t / c\) can be obtained at each node in the network provided that their corresponding values at time \(k \times \Delta t / c\) are known. The impulse response may, therefore, be found by initially fixing the magnitude, position and direction of travel of impulse voltages at time \(t = 0\) and then calculating the state of the network at successive time intervals.

The propagation of pulses in the TLM model is illustrated in Figure 2.6, where the first two iterations following an initial excitation pulse in a 2D shunt-connected TLM are shown. We have assumed free-space propagation for the sake of simplicity. The scattering process illustrated in Figure 2.6 forms the basic algorithm of the TLM method [6,35].

2.1.2.3. Boundary Representation

Boundaries are usually placed halfway between nodes, in order to ensure synchronization. In practice, this is achieved by making the mesh size \(\Delta l\) an integer fraction of the structure's dimensions.
Any resistive load at boundary C (in Figure 2.4) may be simulated by introducing a reflection coefficient $\Gamma$

$$k V_4(p+1,q) = \Gamma V_4^r(p,q)$$  

where

$$\Gamma = \frac{R_s - 1}{R_s + 1}$$  

and $R_s$ is the surface resistance of the boundary normalized by the line characteristic impedance. If, for example, a perfectly conducting wall ($R_s = 0$) is to be simulated along boundary C, (2.19) gives $\Gamma = -1$, which represents a short circuit and

$$k V_4(p+1,q) = -k V_4^r(p,q)$$

is used in the simulation.

**2.1.2.4. Computation of Fields and Frequency Response**

We continue with the TE mode of (2.10) as our example to calculate $E_y$, $H_x$ and $H_z$. $E_y$ at any point corresponds to the node voltage at the point, $H_z$ corresponds to the net current entering to the node in the $x$ direction (see (2.12)), while $H_x$ is the net current in the negative $z$ direction. For any point $(z=m, x=n)$ on the grid of Figure 2.4, we have for the $k^{th}$ time step
\[ k E_y(m,n) = \frac{1}{2} [k V^i_1(m,n) + k V^i_2(m,n) + k V^i_3(m,n) + k V^i_4(m,n)] \]  
(2.21)

\[ I_z = -k H_x(m,n) = k V^i_2(m,n) - k V^i_4(m,n), \]  
(2.22)

\[ I_x = k H_z(m,n) = k V^i_3(m,n) - k V^i_1(m,n). \]  
(2.23)

Thus, a series of discrete impulses of magnitudes \( E_y, H_x \) and \( H_z \) corresponding to time intervals of \( \Delta t/c \) are obtained via (2.16) and (2.17). (2.21)–(2.23) provide the output-impulse functions for the point representing the response of the system to an impulsive excitation. These output functions may be used to obtain the output waveform. For example, the output waveform corresponding to a pulse input may be obtained by convolving the output-impulse function with the shape of the input pulse.

Sometimes we are interested in the response to a sinusoidal excitation. This is obtained by taking the Fourier transform of the impulse response. Since the response is a series of delta functions, the Fourier transform integral becomes a summation and the real and imaginary parts of the output spectrum are given by [1,6]

\[
\text{Re}\left[ F\left( \frac{\Delta \ell}{\lambda} \right) \right] = \sum_{k=1}^{N} k I \cos \left( \frac{2\pi k \Delta \ell}{\lambda} \right) \tag{2.24a}
\]

\[
\text{Im}\left[ F\left( \frac{\Delta \ell}{\lambda} \right) \right] = \sum_{k=1}^{N} k I \sin \left( \frac{2\pi k \Delta \ell}{\lambda} \right) \tag{2.24b}
\]

where \( F(\Delta \ell/\lambda) \) is the frequency response, \( k I \) is the value of the output impulse response at time \( t = k \times \Delta \ell/c \) and \( N \) is the total number of time intervals for which the calculation is made, i.e., the number of iterations.

2.1.2.5. Output Response and Accuracy of Results

The output impulse function, in terms of voltage or current, may be obtained at any point in the TLM mesh. It consists of a train of impulses of varying magnitude in the TD separated by a time interval \( \Delta \ell/c \). Thus, the frequency response obtained by taking the Fourier transform of the output response consists of series of delta functions in the FD corresponding to the discrete modal frequencies for which a solution exists. For practical reasons, the output response has to be truncated. Due to
finite duration of the impulse response, its Fourier transform is not a line spectrum but rather a superposition of \( \sin(x)/x \) functions.

To investigate the accuracy of the result, let the output response be truncated after \( N \) iterations. Let \( V_{out}(t) \) be the output response calculated for \( 0 < t < N\Delta t/c \). We may regard \( V_{out}(t) \) as an impulse function \( V_{\alpha}(t) \), taken within \( 0 < t < \infty \), multiplied by a unit pulse function \( V_p(t) \) of width \( N\Delta t/c \), i.e.,

\[
V_{out}(t) = V_{\alpha}(t) \times V_p(t)
\]  
(2.25)

where

\[
V_p(t) = \begin{cases} 
1, & 0 \leq t \leq N\Delta t/c \\
0, & \text{elsewhere}
\end{cases}
\]  
(2.26)

Let \( S_{out}(f) \), \( S_{\alpha}(f) \) and \( S_p(f) \) be the Fourier transforms of \( V_{out}(t) \), \( V_{\alpha}(t) \) and \( V_p(t) \), respectively. The Fourier transform of (2.25) is the convolution of \( S_{\alpha}(f) \) and \( S_p(f) \).

Hence

\[
S_{out}(f) = \int_{-\infty}^{\infty} S_{\alpha}(\alpha) S_p(f - \alpha) d\alpha
\]  
(2.27)

where

\[
S_p(f) = \frac{N\Delta t}{c} \sin \left( \frac{\pi Nf\Delta t}{c} \right) e^{-j(\pi Nf\Delta t)/c}
\]  
(2.28)

which is in the form \( \sin(x)/x \). (2.27) and (2.28) show that \( S_p(f) \) is obtained as an approximation to the exact response \( S_{\alpha}(f) \). Clearly, the accuracy of the result depends on \( N \).

### 2.1.3. Inhomogeneous and Lossy Media in TLM

Inhomogeneous and lossy media are introduced by modifications in the node structure. This is done in different manners for shunt and series nodes.

#### 2.1.3.1. General Two-Dimensional Shunt Node

To account for the inhomogeneity of a medium, we introduce additional capacitances at nodes to represent the spatial variation of permittivity [35-37]. We achieve this by
introducing an additional length of line or stub to the node as shown in Figure 2.7a. The stub of length \( \Delta \ell /2 \) is open circuited at the end and its characteristic admittance is \( \hat{Y} \) relative to the unity characteristic admittance assumed for the main transmission line. At low frequencies, the effect of the stub is to add to each node an additional lumped shunt capacitance \( C \hat{Y} \Delta \ell /2 \), where \( C \) is the shunt capacitance per unit length of the main lines that are of unity characteristic admittance. Thus at each node, the total shunt capacitance becomes

\[
C' = 2C\Delta \ell (1 + \hat{Y} / 4)
\]  

(2.29)

To account for the loss in the medium, we introduce a power-absorbing line at each node, which is match-terminated and has characteristic admittance \( \hat{G} \) normalized to the characteristic admittance of the main lines, \( 1/Z_{TL} \), as illustrated in Figure 2.7b.

Due to these additional lines, the equivalent network now becomes that shown in Figure 2.8. Applying Kirchhoff's current law to shunt node "0" in the x-z plane in Figure 2.8 and taking limits as \( \Delta \ell \to 0 \) results in

\[
- \frac{\partial A_z}{\partial z} - \frac{\partial A_x}{\partial x} = \frac{\hat{G} V_y}{Z_{TL} \Delta \ell} + 2C(1 + \hat{Y} / 4) \frac{\partial V_y}{\partial t}
\]  

(2.30)

Figure 2.7 A two-dimensional node with a). Permittivity stub, b). Permittivity and loss stubs

By using scalar Maxwell’s equations with \( \partial / \partial t = 0 \) leads to

18
\[
\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = \sigma E_y + \varepsilon_e \varepsilon _r \frac{\partial E_y}{\partial t} \tag{2.31}
\]

This may be considered as denoting TE_{m0} modes with field components \(H_z, H_x\) and \(E_y\). From (2.30) and (2.31), the following equivalence between the TLM equations and Maxwell’s equations can be drawn:

\[
\begin{align*}
E_y &= V_y, & H_x &= -I_z, & H_z &= I_x, \\
\varepsilon_o &= 2C, & \varepsilon_r &= \frac{4 + \hat{\gamma}}{4}, & \sigma &= \frac{\hat{G}}{Z_{TL} \Delta \ell},
\end{align*} \tag{2.32}
\]

where \(Z_{TL} = \sqrt{L/C}\). From (2.32), the normalized characteristic admittance \(\hat{G}\) of the loss stub is related to the conductivity of the medium by

\[
\hat{G} = \sigma \Delta \ell Z_{TL}. \tag{2.33}
\]

Thus the losses can be varied by altering the value of \(\hat{G}\). Also from (2.32), one observes that the characteristic admittance \(\hat{\gamma}\) of the stubs used in modeling the permittivity variation are related to the relative permittivity of the medium as

\[
\hat{\gamma} = 4(\varepsilon_r - 1). \tag{2.34}
\]
2.1.3.2. Scattering Matrix

We can now write the impulse response of the network comprising of the interconnection of many generalized nodes such as that in Figure 2.8. As in the previous section, if $V_n(z,x)$ is unit voltage impulse reflected from the node at $(z,x)$ into the $n^{th}$ coordinate direction ($n = 1, 2, ..., 5$) at time $k \times \Delta \ell / c$, then at node $(z,x)$ $[6,13],$

$$
\begin{bmatrix}
V_1(z,x) \\
V_2(z,x) \\
V_3(z,x) \\
V_4(z,x) \\
V_5(z,x)
\end{bmatrix}^{t} = [S]
\begin{bmatrix}
V_3(z,x - \Delta \ell) \\
V_4(z - \Delta \ell, x) \\
V_1(z,x + \Delta \ell) \\
V_2(z + \Delta \ell, x) \\
V_5(z,x + \Delta \ell)
\end{bmatrix}^{t}
$$

(2.35)

where $[S]$ is the scattering matrix given by

$$
[S] = \frac{2}{Y_i} \begin{bmatrix}
1 & 1 & 1 & \hat{Y} \\
1 & 1 & 1 & \hat{Y} \\
1 & 1 & 1 & \hat{Y} \\
1 & 1 & 1 & \hat{Y} \\
1 & 1 & 1 & \hat{Y}
\end{bmatrix} - [I]
$$

(2.36)

$[I]$ is a unit matrix and $Y_i = 4 + \hat{Y} + \hat{G}$. The coordinate directions 1, 2, 3 and 4 corresponds to $-x$, $-z$, $+x$ and $+z$, respectively and 5 refers to the “permittivity stub”. Notice that the voltage $V_5$ (see Figure 2.7) scattered into “loss stub” is represented simply by $\hat{G}$. (2.35) can be used in TLM algorithms in the same way as outlined for (2.16).

The output impulse function at a particular node in the mesh can again be obtained by recording the amplitude and the time of the stream of pulses as they pass through the node. By taking the Fourier transform of the output response, the information on the FD characteristic can be extracted.

2.1.3.3. Generalized Series Nodes

Figure 2.9 portrays a basic series-connected TLM node. Since it is the dual of the shunt network, the details of the derivation of the basic TLM equations in the series mesh is quite similar and will not be repeated. The generalized configuration shown in Figure 2.10 is used to represent magnetic media. A short-circuited stub which will
be called the “permeability stub,” of length $\Delta \ell / 2$, supplies an input inductance of value $L \hat{Z} \Delta \ell / 2$ for $\Delta \ell \ll \lambda$.

The transmission line equations for the configuration in Figure 2.10 can now be written down easily as:

\[
\begin{align*}
\frac{\partial I_y}{\partial x} &= -C \frac{\partial V_z}{\partial t}, \\
\frac{\partial I_y}{\partial z} &= C \frac{\partial V_x}{\partial t}, \\
\frac{\partial V_z}{\partial z} - \frac{\partial V_x}{\partial x} &= 2L(1 + \hat{Z} / 4) \frac{\partial I_y}{\partial t}.
\end{align*}
\]

(2.37)
In these equations \( \hat{Z} \) represents the characteristic impedance of the permeability stub which is normalized with impedance of the main transmission lines, \( Z_{TL} \).

Maxwell’s equations describing the corresponding physical problem can be written as,

\[
\frac{\partial H_y}{\partial x} = \varepsilon_0 \frac{\partial E_z}{\partial t}, \quad \frac{\partial H_y}{\partial z} = -\varepsilon_0 \frac{\partial E_x}{\partial t},
\]
\[
-\frac{\partial E_x}{\partial t} + \frac{\partial E_z}{\partial x} = \mu_r \mu_o \frac{\partial H_y}{\partial t}.
\]

(2.38)

Comparing (2.37) with (2.38), we obtain the equivalence relations:

\[
H_y = I_y, \quad E_z = -V_x, \quad E_x = -V_x,
\]
\[
\varepsilon_0 = c, \quad \mu_o = 2L, \quad \mu_r = 1 + \frac{\hat{Z}}{4},
\]

(2.39)

A voltage impulse incident on a series node is scattered in accordance with (2.35), where the scattering matrix is now,

\[
[S] = \frac{2}{Z_t} \begin{bmatrix}
-1 & 1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 & 1 \\
1 & -1 & -1 & 1 & 1 \\
-1 & 1 & 1 & -1 & -1 \\
-\hat{Z} & \hat{Z} & \hat{Z} & -\hat{Z} & -\hat{Z}
\end{bmatrix} + [I]
\]

(2.40)

where \( Z_t = 4 + \hat{Z} \) and \([I]\) is the unit matrix. We note that

(a) The extended transmission line matrix can simulate an environment with twice the relative permeability of the real physical medium. This causes the waves to propagate again with a speed \( 1/\sqrt{2} \) of the actual value. This will be taken into account while processing the output results.

(b) The relative permeability of the medium is simulated by using a short-circuited stub having a normalized characteristic impedance of \( \hat{Z} = 4(\mu_r - 1) \). The length of this short stub has to be \( \Delta \ell / 2 \) to achieve synchronization of the scattering from each node. Analogous to the open-ended stub case, the short-circuited stub stores an additional magnetic energy, thereby, simulating a magnetic material.
2.2. 3D TLM Method: The Symmetrical Condensed Node (SCN)

The topology of SCN shown in Figure 2.11 does not lend itself to a treatment using Thevenin equivalent circuits. Instead, the scattering properties are obtained from general energy and charge conservation principles [38,39]. The node may be viewed as the bringing together of the three structures in Figure 2.12, each representing lines in the three coordinate planes. The scattering matrix, "S" relating reflected "V'" to incident "V" voltages is a 12×12 matrix. Any pulse incident onto a port can, in general, couple to all other ports, so the first task is to ascertain which coupling paths are possible and to establish an analogy between circuit and field quantities. The elements of the scattering matrix will then be calculated. Let us consider a voltage pulse incident on port 1 of the SCN. Since this pulse is x-directed, it is associated with $E_x$ and since it contributes to current $I_z$ as shown in Figure 2.12a, it is also associated with $H_z$. If we examine Maxwell's equations including $E_x$ and $H_z$, we see that $V_j$, in principle, can couple into port 1 (i.e., reflected), ports 2 and 9 (since they are associated with $E_x$ and $H_y$) and port 12 (associated with $E_x$ and $H_z$). Similarly, it must couple to ports 3 and 11, since they are associated with $E_y$ and $H_z$.

![Figure 2.11 The three-dimensional SCN. Node coordinates are (x,y,z). The incident and reflected voltages for port 1 are indicated.](image)

Let the incident pulse on port 1 be equal to 1V (see Figure 2.11). Then, an amount (a) may be reflected, and an amount (b) may be coupled to ports 2 and 9. Symmetry dictates that 2 and 9 couple identically to port 1. Similarly, an amount (c) will propagate directly across the junction to couple into port 12. Using symmetry considerations one can also shows that if the coupling to port 3 is (+d) then it must
be \((-d)\) for port 11. The elements of the first column of the scattering matrix have now been identified. This procedure may be applied to all ports to identify all non-zero elements. The overall matrix is shown below [13]:

\[
S = \begin{bmatrix}
  a & b & d & 0 & 0 & 0 & 0 & 0 & b & 0 & -d & c \\
  b & a & 0 & 0 & 0 & d & 0 & 0 & c & -d & 0 & b \\
  d & 0 & a & b & 0 & 0 & 0 & b & 0 & 0 & c & -d \\
  0 & 0 & b & a & d & 0 & -d & c & 0 & 0 & b & 0 \\
  0 & 0 & 0 & d & a & b & c & -d & 0 & b & 0 & 0 \\
  0 & d & 0 & 0 & b & a & b & 0 & -d & c & 0 & 0 \\
  0 & 0 & 0 & -d & c & b & a & d & 0 & b & 0 & 0 \\
  0 & 0 & b & c & -d & 0 & d & a & 0 & 0 & b & 0 \\
  b & c & 0 & 0 & 0 & -d & 0 & 0 & a & d & 0 & b \\
  0 & -d & 0 & 0 & b & c & b & 0 & d & a & 0 & 0 \\
 -d & 0 & c & b & 0 & 0 & 0 & b & 0 & 0 & a & d \\
 c & b & -d & 0 & 0 & 0 & 0 & b & 0 & d & a
\end{bmatrix}
\]

(2.41)

It now remains to determine the value of each of the unknown coupling parameters \(a\), \(b\), \(c\) and \(d\). These will be determined by demanding that, for a lossless network, the
total incident power must be equal to the total reflected power. This results in the unitary condition for the scattering matrix, namely [40]

\[ S^T S = I \] (2.42)

where the superscript "T" stands for the transpose matrix and "I" is the unit matrix. Imposing this condition together with some circuit/field equivalence relations one obtains [13]:

\[ a = 0, \quad b = 0.5, \quad c = 0, \quad d = 0.5 \] (2.43)

The complete scattering matrix is obtained by substituting (2.43) into (2.41). It should be noted that the voltage scattered into each port is obtained using a simple arithmetic operation involving only four incident voltages.

To achieve synchronization, the block of space represented by the SCN is chosen to be a cube \( \Delta x = \Delta y = \Delta z = \Delta \ell \), thus producing what is described as a regular mesh. This has limitations as regards to modeling nonuniformities and fine features, which will be tackled in Section 2.2.5. Before generalizing the SCN by adding stubs, some practical issues will be explored. These can be most simply understood for the regular mesh just described.

2.2.1. Propagation Properties in a Regular Mesh

Let us consider a \( y \)-polarized plane wave propagating in the \( x \)-direction. This requires the excitation of port 3 on all the nodes on a \( y-z \) plane, i.e., \( kV_j^3 = 1 \). Exciting port 3 only on each of these nodes will produce the following scattered pulses

\[ kV_1^r = kV_4^r = kV_8^r = -kV_{12}^r = 0.5. \]

All other scattered pulses are zero, as can be easily confirmed from the scattering matrix. This means that no energy is reflected back (\( kV_j^3 = 0 \)), and no energy is transmitted through (\( kV_{10}^r = kV_{11}^r = 0 \)). At the next time step, the incident voltages on node \( (x,y,z) \) will be coming from its neighboring nodes. As an example, port 1 of \( (x,y,z) \) will receive the pulse reflected into port 12 of node \( (x,y-\ell,z) \) at the previous time step and other relevant voltage pulses are similarly processed; i.e.,
\[ k_+ V_1^i(x, y, z) = k V_2^r(x, y - 1, z) = -0.5, \quad k_+ V_4^i(x, y, z) = k V_8^r(x, y, z - 1) = 0.5, \]
\[ k_+ V_6^i(x, y, z) = k V_4^r(x, y, z + 1) = 0.5, \quad k_+ V_{12}^i(x, y, z) = k V_1^r(x, y + 1, z) = 0.5. \]

All other incident voltages on node \((x,y,z)\) at the start of time-step \((k + 1)\) are zero. The reflected voltages into each port of node \((x,y,z)\) can then be readily obtained from "S" and are all zero except for \(k_+ V_{11}^r\), which is equal to 1. Hence, the wave propagates without dispersion, and it takes two time-steps to cover distance \(\Delta \ell\). The choice of space and time discretization must then be such that

\[ \frac{\Delta \ell}{\Delta t} = 2u \tag{2.44} \]

where "\(u\)" is the medium propagation velocity. The same result for the propagation velocity is obtained for propagation at 45° which may be studied by exciting ports 3 and 4 simultaneously with all other incident voltages set to zero.

### 2.2.2. Computation in an SCN Mesh

Computation in 3D follows closely the procedures described in connection with 2D meshes. The flow-chart of the TLM algorithm is given in Figure 2.13. Only the main subroutines of the algorithm are given in the figure. The calculation starts by imposing the initial conditions and excitation. The next step is "scattering", whereby the reflected voltages at each node are obtained from

![Flow-chart of the TLM algorithm for limited band excitation.](image-url)

Figure 2.13 Flow-chart of the TLM algorithm for limited band excitation.
The scattering matrix \( S \) has the form given by (2.41) for nodes describing a medium of parameters \((\varepsilon, \mu)\). It also is possible to insert conducting bodies in the simulation space. One way to represent a part of space as a perfect conductor is to describe it by the short-circuited node shown schematically in Figure 2.14a. Many such nodes may be joined together to form conducting wire structures. Clearly, in a short-circuited node, the reflected voltage into each port is simply equal to minus the incident voltage; i.e., \( V_n^r = -V_n^i \). The scattering matrix for the short-circuited node is diagonal with elements equal to \(-1\).

![Figure 2.14](image)

Figure 2.14 a). A short-circuited node and, b). an SCN adjacent to a conducting boundary

Other forms of "\( S \)" are possible to describe nonuniform, lossy materials as will be discussed in subsequent sections. However, scattering in all cases proceeds according to (2.45) with the matrix "\( S \)" determined in such a way as to suit the properties of the medium being modeled.

From the scattered voltages, it is straightforward to obtain the incident voltages at the next time-step on all ports and nodes in the problem. This procedure is similar to that used in 2D simulation and it is described as "connection". Clearly, the incident voltage on port 1 of node \((x,y,z)\) at time-step \((k+1)\), is equal to the voltage reflected into port 12 of node \((x,y-1,z)\) at time-step \(k\); i.e., \( V_{1}^{i}(x,y,z)=k V_{12}^{r}(x,y-1,z) \)

Similar expressions apply for all other ports. An exception must be made for nodes that are adjacent to conducting, matched or open-circuited boundaries. Such a node,
next to a conducting boundary, is shown in Figure 2.14b. The new incident voltages on port 10 and 11 are equal to minus the corresponding reflected voltages; i.e.,\[ k^+ V_{10}^i = -k V_{10}^r \quad \text{and} \quad k^+ V_{11}^i = -k V_{11}^r. \]

The total number of time-steps required depends on the nature of the problem and the frequency resolution required.

At any stage during the simulation, the electric and magnetic field and other quantities are available and may be stored in output files. However, results are obtained directly in the TD. If required, they may be transformed into FD using Fourier transforms.

2.2.3. Output from an SCN Mesh

As already mentioned, all electromagnetic quantities may be obtained at any point in the mesh from the value of the incident voltage pulses. Let us first determine how the electric field component \( E_x \) may be obtained. The total \( x \)-directed voltage is the average of the total voltage on ports 1, 2, 9 and 12; i.e., [13],

\[
V_x = \frac{1}{2} \left[ V_{1}^i + V_{2}^i + V_{9}^i + V_{12}^i \right].
\]

hence

\[
E_x = -\frac{V_{1}^i + V_{2}^i + V_{9}^i + V_{12}^i}{2\Delta \ell}. \tag{2.46a}
\]

Using similar procedures, remaining field components may be calculated as

\[
E_y = -\frac{V_{3}^i + V_{4}^i + V_{8}^i + V_{11}^i}{2\Delta \ell}, \tag{2.46b}
\]

\[
E_z = -\frac{V_{3}^i + V_{6}^i + V_{1}^i + V_{10}^i}{2\Delta \ell}. \tag{2.46c}
\]

The magnetic field component \( H_x \), may be obtained from the current \( I_x \). This current is calculated from the circuit shown in Figure 2.12b, where each line is replaced by its Thevenin equivalent as shown in Figure 2.15, to give

\[
H_x = \frac{V_{4}^i - V_{5}^i + V_{7}^i - V_{8}^i}{2Z_o \Delta \ell}, \tag{2.47a}
\]

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2.2.4. Excitation in an SCN Mesh

Excitation in TD is achieved by injecting voltages into the appropriate ports. In the case of $E_x$, pulses must be injected into ports 1, 2, 9 and 12, as suggested by (2.46a). Choosing $E_o$ as the desired field value and

$$V_1^i = V_2^i = V_9^i = V_{12}^i = -E_o \frac{\Delta \ell}{2}$$

and substituting into (2.47a) gives $E_x = E_o$ as desired.

Magnetic field components are also excited in a similar manner. For example, to excite $H_x$, it is necessary to inject voltages on ports 4, 5, 7 and 8, as suggested by (2.47a). Choosing $H_o$ as the desired value and

$$V_4^i = V_7^i = H_o Z_o \frac{\Delta \ell}{2}, \quad V_5^i = V_8^i = -H_o Z_o \frac{\Delta \ell}{2},$$
and substituting into (2.47a) gives $H_x = H_0$ as desired.

2.2.5. The Variable Mesh

There are two practical requirements that make it necessary to improve and extend the capabilities of the regular SCN described in Section 2.2. First, it should be possible to use a fine mesh ($\Delta \ell$ small) only in areas of rapid field variation and thus save on computer requirements. This calls for a variable $\Delta \ell$ or what sometimes is referred to as a *graded mesh*. Second, it should be possible to model non-homogeneous material properties such as different values of $\varepsilon$ and $\mu$. Modifications must also be made to model electric and magnetic losses.

Both of these requirements must be met while maintaining synchronisation and connectivity (one-to-one correspondence between transmission lines on adjacent ports). It is therefore clear that the block of space modeled by each node is not necessarily a cube and that the material properties in each block may be different. To accommodate these requirements, it is necessary to add stubs to the basic node shown in Figure 2.11.

2.2.5.1. The SCN with Capacitive and Inductive Stubs

To allow for variations in $\varepsilon$, $\mu$, node shape and dimensions, all 12 link transmission lines in the SCN node are chosen to model the same capacitance and inductance ($C$ and $L$), respectively. These are chosen to represent the parameters of the background medium which, in most cases, is air $(\varepsilon_0, \mu_0)$. The impedance $Z_0$ of these lines is chosen the same throughout the mesh and it has a value which in most cases corresponds to free space ($Z_0 = 377 \Omega$). If the time-step of the calculation is then chosen to be $\Delta t$, the propagation time on each line is $\Delta t / 2$ and, hence, each line segment represents

$$C = \frac{\Delta t / 2}{Z_0}, \quad L = \frac{\Delta t}{2} Z_0$$  \hspace{1cm} (2.48)

Clearly, depending on local material properties and the shape of the block modeled by the node, these values are to a greater or lesser extent underestimate of those required. These deficits in capacitance and inductance can be calculated and then added to the SCN as stubs.

The required value for $x$-directed capacitance is

30
\[ C_x = \varepsilon \frac{\Delta y \Delta z}{\Delta x} \]  
\[ (2.49) \]

The capacitance in this direction modeled so far is due to 1, 2, 9 and 12 and is therefore equal to 4C. Hence, the deficiency is

\[ C_x^s = \varepsilon \frac{\Delta y \Delta z}{\Delta x} - 4C \]  
\[ (2.50) \]

This capacitance is added to the node in the form of a stub of characteristic admittance

\[ Y_x = \frac{2C_x^s}{\Delta t} = 2\varepsilon \frac{\Delta y \Delta z}{\Delta x \Delta t} - \frac{8C}{\Delta t} \]

where the round trip on the stub is chosen to be equal to \( \Delta t \) to maintain synchronism. Substituting \( C = \Delta t / 2Z_o \) gives

\[ Y_x = 2\varepsilon \frac{\Delta y \Delta z}{\Delta x \Delta t} - \frac{4}{Z_o} \]
\[ (2.51) \]

Similar expressions may be obtained for the stubs that must be added to account for deficits in the y- and z-directions. It is customary to normalize the stub admittance to that of the background link lines, \( Y_o = 1 / Z_o \); i.e.,

\[ \hat{Y}_x = 2\varepsilon \frac{\Delta y \Delta z}{\Delta x \Delta t Y_o} - 4 \]

Assuming that the background values are those for air \( (Y_o = \sqrt{\varepsilon_o / \mu_o}) \) and substituting in the equation above gives the following expressions for \( \hat{Y}_x \) and, by analogy, for the other capacitance stubs:

\[ \hat{Y}_x = \frac{2\varepsilon_r}{u_0 \Delta t} \frac{\Delta y \Delta z}{\Delta x} - 4, \quad \hat{Y}_y = \frac{2\varepsilon_r}{u_0 \Delta t} \frac{\Delta x \Delta z}{\Delta y} - 4, \quad \hat{Y}_z = \frac{2\varepsilon_r}{u_0 \Delta t} \frac{\Delta x \Delta y}{\Delta z} - 4, \]
\[ (2.52) \]

where \( u_o = 1 / \sqrt{\varepsilon_o \mu_o} \).

Pulses scattered into the capacitive stubs are reflected after time \( \Delta t / 2 \) from their open-circuit terminations and become incident onto the node at the next time-step. Note that if \( \Delta x = \Delta y = \Delta z = \Delta \ell \), then using (2.44), (2.52) becomes,
\[ \hat{Y}_x = \hat{Y}_y = \hat{Y}_z = 4(\varepsilon_r - 1). \]  

(2.53)

A similar approach is adopted to calculate the inductance deficit. Let us consider first the deficit in inductance associated with current on the y-z plane.

The required value

\[ L_x = \mu \frac{\Delta y \Delta z}{\Delta x} \]  

(2.54)

The modeled inductance is associated with lines 4, 5, 7 and 8 and is therefore equal to \(4L\). Hence, the deficit is

\[ L^s_x = \mu \frac{\Delta y \Delta z}{\Delta x} - 4L \]  

(2.55)

This inductance is added to the node in the form of a stub of characteristic impedance

\[ Z_x = \frac{2L^s_x}{\Delta t} = 2\mu \frac{\Delta y \Delta z}{\Delta x \Delta t} - \frac{8L}{\Delta t} = 2\mu \frac{\Delta y \Delta z}{\Delta x \Delta t} - 4Z_0 \]

Normalizing as before to the background value \(Z_0\), the following expressions are obtained for \(\hat{Z}_x\) and, by analogy, for the other inductive stubs:

\[ \hat{Z}_x = \frac{2\mu_r}{u_o \Delta t} \frac{\Delta y \Delta z}{\Delta x} - 4, \quad \hat{Z}_y = \frac{2\mu_r}{u_o \Delta t} \frac{\Delta x \Delta z}{\Delta y} - 4, \quad \hat{Z}_z = \frac{2\mu_r}{u_o \Delta t} \frac{\Delta x \Delta y}{\Delta z} - 4, \]  

(2.56)

Pulses scattered into the inductive stubs are reflected after time \(\Delta t / 2\) from their short-circuit terminations and become incident onto the node at the next time-step.

Note that if \(\Delta x = \Delta y = \Delta z = \Delta \ell\), then (2.56) becomes, by also using (2.44),

\[ \hat{Z}_x = \hat{Z}_y = \hat{Z}_z = 4(\mu_r - 1). \]  

(2.57)

For stability, it is necessary to ensure that all stubs represent component values with positive real parts. Hence, the time-step must be chosen to ensure that all stub impedances and admittances are positive. From the equations given in (2.52) and (2.56) one obtains

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\[ \dot{Z}_x = \frac{2 \mu_r}{u_o \Delta t} \frac{\Delta y \Delta z}{\Delta x} - 4 \geq 0 \quad \text{and} \quad \dot{Y}_x = \frac{2 \varepsilon_r}{u_o \Delta t} \frac{\Delta x \Delta z}{\Delta y} - 4 \geq 0. \]

The worst case corresponds to the propagation in the free space. So by replacing \( \varepsilon_r \) and \( \mu_r \) with 1, we can get the condition for the time step \( \Delta t \) as

\[ \Delta t \leq \frac{1}{2u_o} \frac{\Delta y \Delta z}{\Delta x} \]

(2.58a)

For a regular mesh (\( \Delta x = \Delta y = \Delta z = \Delta \ell \)), the time step \( \Delta t \) becomes

\[ \Delta t \leq \frac{\Delta \ell}{2u_o} \]

(2.58b)

It should be pointed out that, in a large mesh stability must be checked at each node and a suitable value of \( \Delta t \) chosen to prevent negative stub values anywhere in the mesh.

### 2.2.5.2. Scattering in an SCN with Stubs

The three capacitive and three inductive stubs are connected internally to the node and do not interact directly with adjacent nodes. Connection to neighboring nodes is, as for the regular SCN, through the 12 link lines. The scattering process, however, is affected by the presence of the stubs. The capacitive stubs are given port numbers 13 to 15, and the inductive stubs have port numbers 16 to 18. The scattering matrix now has dimensions 18x18. The non-zero elements of the scattering matrix may be identified using the same physical reasoning as outlined in Section 2.2. A pulse incident on port 1 may be partially reflected (amount \( a \)) and will couple to port 2 (amount \( b \)), port 9 (amount \( b \)), port 3 (amount \( d \)), port 11 (amount \( -d \)) and port 12 (amount \( c \)). In addition, since port 1 contributes to \( E_x \) and \( H_z \), it will also couple with the \( x \)-directed capacitive stub (amount \( e \)) and \( z \)-directed inductive stub (amount \( f \)). In a similar manner, the other non-zero elements of "S" may be identified to produce the matrix shown in (2.59). It must now be allowed that elements such as (1,1), which is indicated as \( (a) \), may be different numerically from element (2,2), also indicated as \( (a) \), since symmetry can not be guaranteed in an irregularly shaped node.
The values of these elements may be calculated as before from Kirchhoff’s Laws and from energy conservation. The unitary condition means, in this case, that the scattering matrix must obey the equation

\[ S^T Y S = Y \]

where \( Y \) is an 18×18 diagonal matrix with elements equal to the normalized admittance of the 12 link lines, 3 capacitive and 3 inductive stubs. It turns out, after some algebra, that the coefficients of the scattering matrix are given in (2.60) [38].

\[
\begin{align*}
a &= \frac{\hat{Y}}{2(4 + \hat{Y})} + \frac{\hat{Z}}{2(4 + \hat{Z})} , \\
b &= \frac{4}{2(4 + \hat{Y})} , \\
c &= \frac{\hat{Y}}{2(4 + \hat{Y})} - \frac{\hat{Z}}{2(4 + \hat{Z})} , \\
d &= \frac{4}{2(4 + \hat{Z})} , \\
e &= b , \\
f &= \hat{Z}d , \\
g &= \hat{Y}b , \\
h &= \frac{\hat{Y} - 4}{\hat{Y} + 4} , \\
i &= d , \\
j &= \frac{4 - \hat{Z}}{4 + \hat{Z}} .
\end{align*}
\]

(2.60)

The values of \( \hat{Y} \) and \( \hat{Z} \) used in these formulas are chosen to correspond to the relevant stubs. For example, element \( S_{6,9} \) (\( d \)) represents coupling between ports 6 and 9. Port 6 is associated with \( E_z \) and \( H_y \), while port 9 with \( E_x \) and \( H_y \). Hence, in the formula for \( d \), the value for \( \hat{Z} \) must be that corresponding to \( H_y \); i.e.,

\[
d_{6,9} = \frac{4}{2(4 + \hat{Z}_y)}
\]
Similarly, ports 2 and 9 are associated with $E_x$ and $H_y$; hence, in calculating $S_{2,9} (c)$, $\hat{Y}_x$ and $\hat{Z}_y$ must be used.

### 2.2.5.3. Calculation of EM Fields in an SCN with Stubs

Let us now examine how $E_x$ may be calculated. The procedure adopted in Section 2.2.3 must now be modified to take account of the influence of the $x$-directed capacitive stub (port 13). The capacitance of each link line is $Y_o \Delta t / 2$ (see Section 2.2.5.1), and the capacitance of the stub 13 is $Y_x \Delta t / 2 = \hat{Y}_x Y_o \Delta t / 2$. Hence, the total incident charge on all the ports associated with $E_x$ is

$$Y_o \frac{\Delta t}{2} (V_1^i + V_2^i + V_9^i + V_{12}^i) + Y_o \frac{\Delta t}{2} \hat{Y}_x V_{13}^i$$

From charge conservation, the total charge leaving these lines should be the same as above; hence, in total, the charge is equal to twice the above value. The total capacitance of these lines is

$$4Y_o \frac{\Delta t}{2} + \hat{Y}_x Y_o \frac{\Delta t}{2}$$

Dividing charge by capacitance gives the $x$-directed voltage

$$V_x = \frac{2(V_1^i + V_2^i + V_9^i + V_{12}^i + \hat{Y}_x V_{13}^i)}{4 + \hat{Y}_x}$$

The electric field is then equal to $-V_x / \Delta x$; hence,

$$E_x = -\frac{2(V_1^i + V_2^i + V_9^i + V_{12}^i + \hat{Y}_x V_{13}^i)}{\Delta x (4 + \hat{Y}_x)} \quad (2.61a)$$

Similar expressions apply for the other two electric field components. Hence,

$$E_y = -\frac{2(V_3^i + V_4^i + V_8^i + V_{11}^i + \hat{Y}_y V_{14}^i)}{\Delta y (4 + \hat{Y}_y)} \quad (2.61b)$$

$$E_z = -\frac{2(V_5^i + V_6^i + V_7^i + V_{10}^i + \hat{Y}_z V_{15}^i)}{\Delta z (4 + \hat{Y}_z)} \quad (2.61c)$$
Component $H_x$ may be obtained by considering contributions from ports responsible for currents flowing in the $y$-$z$ plane. These are shown in Figure 2.16, where the inductive stub associated with $H_x$ has been added with positive reference voltage opposing the positive reference current $I_x$.

![Figure 2.16 Circuit for calculating $H_x$ in a node with stubs. Stub shows an inductive effect on the node.](image)

From this circuit, using Thévenin equivalents, the current is obtained, and since $H_x = I_x / \Delta x$,

$$H_x = \frac{2(V_4^i - V_5^i + V_7^i - V_8^i - V_{16}^i)}{\Delta x(4Z_o + Z_oZ_x)}$$  \hspace{1cm} (2.62a)

Similar expressions are obtained for the other two magnetic field components. Hence,

$$H_y = \frac{2(-V_2^i + V_6^i + V_9^i - V_{10}^i - V_{17}^i)}{\Delta y(4Z_o + Z_oZ_y)}$$ \hspace{1cm} (2.62b)

$$H_z = \frac{2(V_1^i - V_3^i + V_{11}^i - V_{12}^i - V_{18}^i)}{\Delta z(4Z_o + Z_oZ_z)}$$ \hspace{1cm} (2.62c)
2.2.5.4. Excitation in an SCN with stubs

Introducing an excitation is similar to that described earlier for a node without any connected stubs. For example, if $E_x$ only is to be excited, the following pulses need to be injected:

$$V_i^1 = V_i^2 = V_i^9 = V_{12}^i = V_{13}^i = -E_o \frac{\Delta x}{2}$$

where $E_o$ is the desired value of $E_x$ in volts per meter.

2.2.5.5. The SCN with capacitive, inductive and lossy stubs

Losses may be incorporated by adding more stubs to the node described in the previous section. These “lossy” stubs may be regarded as infinitely long or, equivalently, as terminated by their own characteristic impedance. In either case, any energy scattered into these stubs is absorbed, and there are no pulses coming from them that are incident on the node. Three lossy stubs associated with electric losses in the $x$-, $y$-, and $z$- directions (ports 19-21) and another three associated with magnetic losses in the $x$-, $y$-, and $z$- directions (ports 22-24) may be introduced to produce the most versatile node capable of dealing with general inhomogeneous anisotropic media. It is, however, rare that such a general node is required. Such a general node requires substantial computational resources, and special algorithms have been developed to increase numerical efficiency [22].

The introduction of electric and magnetic losses is described in [23,24]. Since no incident voltages are coming from the lossy stubs, the scattering matrix has 18 columns, 24 rows. Normally, the pulses scattered into the lossy nodes need not be explicitly calculated, and then $S$ is an $18 \times 18$ matrix. Row 19 of $S$ consists of elements representing ports coupled to losses associated with $E_x$. Hence, coupling is represented by elements 1, 2, 9, 12 (amount $k$), and 13 (amount $p$).

Similarly, row 20 has non-zero elements 3, 4, 8, 11 (amount $k$), and 14 (amount $p$). For row 21, the non-zero elements are 5, 6, 7, 10 (amount $k$), and 15 (amount $p$). Row 22 consists of elements representing losses associated with $H_x$. Non-zero elements are 4, 7 (amount $k'$), 5, 8 (amount $-k'$), and 16 (amount $p'$). Row 23 has non-zero elements 6, 8 (amount $k'$), 2, 10 (amount $-k'$), and 17 (amount $p'$). Finally, Row 24 has elements 1, 11 (amount $k'$), 3, 12 (amount $-k'$), and 18 (amount $p'$).
Stubs 19 through 21 have normalized conductances \( \hat{G}_x \), \( \hat{G}_y \), and \( \hat{G}_z \), respectively. Similarly, stubs 22 to 24 have normalized resistances \( \hat{R}_x \), \( \hat{R}_y \), and \( \hat{R}_z \), respectively. Following a procedure identical to that described in the previous section, the following values for the elements of the 18×18 scattering matrix given in (2.59) are obtained:

\[
\begin{align*}
    a &= \frac{\hat{Y} + \hat{G}}{2(4 + \hat{Y} + \hat{G})} + \frac{\hat{Z} + \hat{R}}{2(4 + \hat{Z} + \hat{R})} & f &= \hat{Z}d \\
    b &= \frac{4}{2(4 + \hat{Y} + \hat{G})} & g &= \hat{Y}b \\
    c &= -\frac{\hat{Y} + \hat{G}}{2(4 + \hat{Y} + \hat{G})} - \frac{\hat{Z} + \hat{R}}{2(4 + \hat{Z} + \hat{R})} & h &= \frac{\hat{Y} - \hat{G} - 4}{\hat{Y} + \hat{G} + 4} \\
    d &= \frac{4}{2(4 + \hat{Z} + \hat{R})} & i &= d \\
    e &= b & j &= \frac{4 - \hat{R} - \hat{Z}}{4 + \hat{R} + \hat{Z}}
\end{align*}
\]

(2.63)

As for the lossless node, the appropriate values of \( \hat{Z} \), \( \hat{Y} \), \( \hat{R} \), and \( \hat{G} \) must be used. As an illustration, \( c_{2,9} \) represents coupling between ports 2 and 9, both of which are associated with \( E_x \) and \( H_y \). Hence,

\[
c_{2,9} = -\frac{\hat{Y}_x + \hat{G}_x}{2(4 + \hat{Y}_x + \hat{G}_x)} - \frac{\hat{Z}_y + \hat{R}_y}{2(4 + \hat{Z}_y + \hat{R}_y)}
\]

The lossy stub parameters are calculated from the following formula:

\[
G_x = \sigma_0 \frac{\Delta y \Delta z}{\Delta x}
\]

Hence the normalized value is

\[
\hat{G}_x = \sigma_0 \frac{\Delta y \Delta z}{\Delta x Y_o}
\]

(2.64)
Parameter $\sigma_{ex}$ is the conductivity associated with electric losses in the $x$-direction. The complex dielectric constant, conductivity, and loss tangent are related by the expressions

$$
\varepsilon^* = \varepsilon_r \varepsilon_0 j \omega = \varepsilon_r \omega - j \frac{\sigma_e}{\omega}
$$

$$
\tan \delta_e = \frac{\sigma_e}{\omega \varepsilon_r \varepsilon_0}
$$

(2.65)

Similar expressions apply for $G_y$ and $G_z$.

To account for magnetic losses with $H_x$, a resistance $R_x$ must be added in series with $L_x$. The power loss is calculated from $P_{mx} = I_x^2 R_x$. Loss tangent may be associated with magnetic losses

$$
\tan \delta_{mx} = \frac{R_x}{\omega L_x}
$$

Therefore,

$$
R_x = \omega L_x \tan \delta_{mx} = \omega \mu \tan \delta_{mx} \frac{\Delta y \Delta z}{\Delta x}
$$

The complex magnetic permeability of the medium is

$$
\mu^* = \mu_r \mu_0 j \mu^* = \mu_r \mu_0 - j \frac{\sigma_m}{\omega}
$$

(2.66)

Hence,

$$
\tan \delta_m = \frac{\sigma_m}{\omega \mu_r \mu_0}
$$

and substituting in the expression for $R_x$ gives

$$
R_x = \sigma_{mx} \frac{\Delta y \Delta z}{\Delta x}
$$

The normalized value is
\[
\hat{R}_x = \sigma_{mx} \frac{\Delta y \Delta z}{\Delta x Z_o}
\] (2.67)

Similar expressions are obtained for \( R_y \) and \( R_z \).

From these expressions, one can easily obtain the formulae of the electromagnetic field components in the lossy media as given below:

\[
E_x = -\frac{2(V^i_1 + V^i_2 + V^i_9 + V^i_{12} + \hat{Y}_x V^i_{13})}{\Delta x (4 + \hat{Y}_x + \hat{G}_x)}
\] (2.68a)

Similar expressions apply for the other two electric field components. Hence,

\[
E_y = -\frac{2(V^i_2 + V^i_4 + V^i_8 + V^i_{11} + \hat{Y}_y V^i_{14})}{\Delta y (4 + \hat{Y}_y + \hat{G}_y)}
\] (2.68b)

\[
E_z = -\frac{2(V^i_3 + V^i_6 + V^i_9 + V^i_{10} + \hat{Y}_z V^i_{15})}{\Delta z (4 + \hat{Y}_z + \hat{G}_z)}
\] (2.68c)

and magnetic field components

\[
H_x = \frac{2(V^i_4 - V^i_5 + V^i_j - V^i_8 - V^i_{16})}{\Delta x Z_o (4 + \hat{Z}_x + \hat{K}_x)}
\] (2.69a)

\[
H_y = \frac{2(-V^i_2 + V^i_6 + V^i_9 - V^i_{10} - V^i_{17})}{\Delta y Z_o (4 + \hat{Z}_y + \hat{K}_y)}
\] (2.69b)

\[
H_z = \frac{2(V^i_1 - V^i_3 + V^i_{11} - V^i_{12} - V^i_{18})}{\Delta z Z_o (4 + \hat{Z}_z + \hat{K}_z)}
\] (2.69c)

2.3. Perfectly Matched Layer (PML) in 3D TLM Method

Perfectly Matched Layers (PML's) are layers especially designed by Berenger to simulate free space at the boundaries of a finite-difference time-domain (FDTD) computational domain. The PML was first implemented in the transmission-line matrix (TLM) [41,42] method using an interface between the FDTD and TLM
network. However, it was shown that the use of a nonuniform TLM-FDTD mesh provides more inaccurate absorbing conditions than obtained by Berenger with the FDTD method. A uniform 2D TLM mesh, which can simulate PML media, was reported in [43] and, recently, Dubard and Pompei have proposed a new SCN for the implementation of the Berenger’s PML’s in a 3D TLM method [44,45]. Two different 3D PML-TLM algorithms were developed at the same time by other authors [46,47]. They differ from the one presented in [44], in that they have many degrees of freedom in building a PML-TLM node. It was shown in [44] that a PML implemented with this modified node provides 30dB more accurate boundary conditions than Higdon’s conditions and matched termination.

2.3.1. Derivation of the PML SCN

PML media are described by splitting the six electromagnetic field components into 12 sub-components and introducing anisotropic electric and magnetic conductivities [48]. For example, in a usual media with isotropic electric conductivity \( \sigma \), relevant Maxwell’s equation can be written as follows:

\[
\varepsilon_0 \varepsilon_r \frac{\partial \mathbf{e}_x}{\partial t} + \sigma \mathbf{e}_x = \frac{\partial \mathbf{h}_z}{\partial y} - \frac{\partial \mathbf{h}_y}{\partial z} \tag{2.70}
\]

In Berenger’s PML media, each electromagnetic field is split into two subcomponents

\[
\mathbf{e}_x = \mathbf{e}_{xy} + \mathbf{e}_{xz} \quad \mathbf{h}_y = \mathbf{h}_{yx} + \mathbf{h}_{yz} \quad \mathbf{h}_z = \mathbf{h}_{zx} + \mathbf{h}_{zy} \tag{2.71}
\]

and (2.70) is replaced by

\[
\varepsilon_0 \varepsilon_r \frac{\partial \mathbf{e}_{xy}}{\partial t} + \sigma_y \mathbf{e}_{xy} = \frac{\partial (\mathbf{h}_{zx} + \mathbf{h}_{zy})}{\partial y}, \quad \varepsilon_0 \varepsilon_r \frac{\partial \mathbf{e}_{xz}}{\partial t} + \sigma_z \mathbf{e}_{xz} = -\frac{\partial (\mathbf{h}_{yx} + \mathbf{h}_{yz})}{\partial z} \tag{2.72}
\]

where \( \sigma_y \) and \( \sigma_z \) are the electric conductivities along the \( y \)- and \( z \)-axes. (2.72) can be rewritten in the following forms:

\[
\varepsilon_0 \varepsilon_r \frac{\partial \mathbf{e}_x - \mathbf{e}_{xy}}{\partial t} + \sigma_y \mathbf{e}_{xy} = \frac{\partial \mathbf{h}_z}{\partial y}, \quad \varepsilon_0 \varepsilon_r \frac{\partial \mathbf{e}_x - \mathbf{e}_{xz}}{\partial t} + \sigma_z \mathbf{e}_{xz} = -\frac{\partial \mathbf{h}_y}{\partial z} \tag{2.73}
\]
or
\[
\frac{\dot{y}_x + 4}{2} \frac{\partial E_x^2}{\partial T} = \frac{\partial H_x}{\partial y} + \frac{\dot{y}_x + 4}{2} \frac{\partial E_{xy}}{\partial T} - \hat{G}_{xy} E_{xy}
\]
(2.74)

\[
\frac{\dot{y}_x + 4}{2} \frac{\partial E_x^2}{\partial T} = \frac{\partial H_y}{\partial z} + \frac{\dot{y}_x + 4}{2} \frac{\partial E_{xz}}{\partial T} - \hat{G}_{xz} E_{xz}
\]

using
\[
X = \frac{x}{u}, \quad Y = \frac{y}{v}, \quad Z = \frac{z}{w}, \quad T = \frac{t}{\Delta t},
\]
\[
E_x = e_x u, \quad E_{xy} = e_{xy} u, \quad E_{xz} = e_{xz} u,
\]
\[
H_y = h_y v Z_o, \quad H_z = h_z w Z_o, \quad \dot{y}_x = \left( \epsilon_r \frac{v w}{u \Delta t} - 1 \right)
\]
(2.75)

\[
\hat{G}_{xy} = \sigma_y \frac{v w}{u} Z_o, \quad \hat{G}_{xz} = \sigma_z \frac{v w}{u} Z_o,
\]

where \(u, v, \) and \(w\) are the sizes of the TLM SCN shown in Figure 2.17. The node is located in the TLM network by indices \((i, j, k)\). \(\Delta \ell\) is the smallest cell size over the network and the time step is set as \(\Delta t \leq \frac{\Delta \ell}{2c}\). \(Z_o\) and \(c\) are the characteristic impedance and free space wave velocity, respectively.

As described in [49], (2.74) is reformulated as
\[
\frac{1}{2} \left[ \frac{\partial (E_x - H_z)}{\partial A_y} - \frac{\partial (E_x + H_z)}{\partial B_y} \right] = \frac{\partial E_{xz}}{\partial T} - \frac{\dot{y}_x + 4}{2} \frac{\partial E_{xy}}{\partial T} - \hat{G}_{xy} E_{xy}
\]
(2.76a)

\[
\frac{1}{2} \left[ \frac{\partial (E_x + H_y)}{\partial A_z} - \frac{\partial (E_x - H_y)}{\partial B_z} \right] = \frac{\partial E_{xy}}{\partial T} - \frac{\dot{y}_x + 4}{2} \frac{\partial E_{xz}}{\partial T} - \hat{G}_{xz} E_{xz}
\]
(2.76b)

using the following new coordinate system where time and space are mixed:

\[
A_x = X + T \quad A_y = Y + T \quad A_z = Z + T
\]
\[
B_x = X - T \quad B_y = Y - T \quad B_z = Z - T
\]
(2.77)
A set of finite-difference equations can be obtained from (2.76) by using central differences at point \((i,j,k)\) and time \(n\Delta t\) in the new coordinate system (2.77). For example, (2.76a) gives

\[
\frac{1}{2} \left[ \frac{E_x(n + 1/2, i, j + 1/2, k) - H_z(n + 1/2, i, j + 1/2, k)}{\Delta t} \right] \\
- \frac{1}{2} \left[ \frac{E_x(n - 1/2, i, j - 1/2, k) - H_z(n - 1/2, i, j - 1/2, k)}{\Delta t} \right] \\
- \frac{1}{2} \left[ \frac{E_x(n - 1/2, i, j + 1/2, k) - H_z(n - 1/2, i, j + 1/2, k)}{\Delta t} \right] \\
+ \frac{1}{2} \left[ \frac{E_x(n + 1/2, i, j - 1/2, k) - H_z(n + 1/2, i, j - 1/2, k)}{\Delta t} \right] \\
+ \frac{\dot{Y}_x + 2}{2} \left[ \frac{E_{xy}(n + 1/2, i, j, k) - E_{xy}(n - 1/2, i, j, k)}{\Delta t} \right] \\
- \left[ \frac{E_{xz}(n + 1/2, i, j, k) - E_{xz}(n - 1/2, i, j, k)}{\Delta t} \right] + \dot{G}_{xy} E_{xy}(n, i, j, k) = 0. \\
(2.78)
\]

In TLM formulation, each elementary plane wave penetrating into the cell along the \(x-, y-,\) and \(z\)-directions of space is associated with a voltage impulse traveling toward the center of the cell through one of the 12 transmission lines linking the node to its six neighbors. For example, an \(x\)-polarized plane wave propagating in the \(y\)-direction is related to the incident voltage impulse \(V_1^i\) on port number 1 at the cell boundary \((i, j-\frac{1}{2}, k)\) and time step \((n-\frac{1}{2})\Delta t\) as follows:

\[
E_x(n - 1/2, i, j - 1/2, k) - H_z(n - 1/2, i, j - 1/2, k) = 2nV_1^i \\
(2.79a)
\]
According to the Huygen’s principle, these incident voltage impulses are scattered at the center of the node which at time step \( n \Delta t \) yield the reflected voltage impulses associated with outgoing plane waves. For example, for an \( x \)-polarized outgoing wave propagating in the \( +y \)-direction, the relation between the \( E_x \) and \( H_z \) field components and the reflected voltage impulse \( n^r V_{12}^r \) on port number 12 at the cell boundary \((i,j+\frac{1}{2},k)\) and time step \((n+\frac{1}{2})\Delta t\) is

\[
E_x(n+\frac{1}{2},i,j+\frac{1}{2},k) - H_z(n+\frac{1}{2},i,j+\frac{1}{2},k) = 2_n V_{12}^r
\]  

(2.79b)

Open and short stubs connected at the center of the node allow to model the permittivity and permeability of various materials. In the case of \( x \)-polarized waves, for example, it is necessary to add two open stubs with normalized characteristic admittance \( \hat{Y}_x \) (numbered 13 and 14), to take into account the two subcomponents \( E_{xy} \) and \( E_{xz} \) as follows:

\[
E_{xy}(n-\frac{1}{2},i,j,k) = 2_n V_{13}^i, \quad E_{xz}(n-\frac{1}{2},i,j,k) = 2_n V_{14}^i
\]  

(2.79c)

and

\[
E_{xy}(n+\frac{1}{2},i,j,k) = 2_n V_{13}^r, \quad E_{xz}(n+\frac{1}{2},i,j,k) = 2_n V_{14}^r
\]  

(2.79d)

Using (2.79a)–(2.79d), (2.78) can be reduced to

\[
n^r V_{12}^r + n V_{12}^i - n V_{12}^i + (\hat{Y}_x + 2)(n V_{13}^i - n V_{13}^i) - 2(n V_{14}^i - n V_{14}^i) + \hat{G}_{xy} E_{xy} (n,i,j,k) = 0
\]  

(2.80)

Another set of finite-difference equations can be obtained from (2.72), which can be rewritten using (2.71) and (2.75) as

\[
\frac{\hat{Y}_x + 4}{2} \frac{\partial E_{xy}}{\partial T} = \frac{\partial H_x}{\partial Y} - \hat{G}_{xy} E_{xy}
\]  

(2.81a)

\[
\frac{\hat{Y}_x + 4}{2} \frac{\partial E_{xz}}{\partial T} = \frac{\partial H_x}{\partial Z} - \hat{G}_{xz} E_{xz}
\]  

(2.81b)

By using central differencing and averaging at point \((i,j,k)\) and time step \((n+\frac{1}{2})\Delta t\) in the coordinate system (2.75), and by using (2.79a)–(2.79d), (2.81a), for example, gives
\[
\frac{\hat{Y}_x + \hat{G}_{xy} + 4}{2} \left[ E_{xy}(n+1, i, j, k) - E_{xy}(n, i, j, k) \right] = \\
\left( n + V_{1}^{i} + n + V_{12}^{i} - n V_{1}^{r} - n V_{12}^{r} - \hat{G}_{xy} E_{xy}(n, i, j, k) \right)
\]

Substituting (2.80) into (2.82), we finally obtain

\[
E_{xy}(n, i, j, k) = \frac{2 \left[ n V_{1}^{i} + n V_{12}^{i} + (\hat{Y}_x + 2) n V_{13}^{i} - 2 n V_{14}^{i} \right] }{\hat{Y}_x + \hat{G}_{xy} + 4} \tag{2.83a}
\]

Performing the same procedure for the other Maxwell’s equations provide the following expressions

\[
E_{zz}(n, i, j, k) = \frac{2 \left[ n V_{2}^{i} + n V_{12}^{i} + (\hat{Y}_x + 2) n V_{14}^{i} - 2 n V_{13}^{i} \right] }{\hat{Y}_x + \hat{G}_{zz} + 4} \tag{2.83b}
\]

\[
E_{yx}(n, i, j, k) = \frac{2 \left[ n V_{3}^{i} + n V_{11}^{i} + (\hat{Y}_y + 2) n V_{15}^{i} - 2 n V_{16}^{i} \right] }{\hat{Y}_y + \hat{G}_{yx} + 4} \tag{2.83c}
\]

\[
E_{yz}(n, i, j, k) = \frac{2 \left[ n V_{3}^{i} + n V_{11}^{i} + (\hat{Y}_y + 2) n V_{16}^{i} - 2 n V_{15}^{i} \right] }{\hat{Y}_y + \hat{G}_{yz} + 4} \tag{2.83d}
\]

\[
E_{zx}(n, i, j, k) = \frac{2 \left[ n V_{6}^{i} + n V_{10}^{i} + (\hat{Y}_z + 2) n V_{17}^{i} - 2 n V_{18}^{i} \right] }{\hat{Y}_z + \hat{G}_{zx} + 4} \tag{2.83e}
\]

\[
E_{zy}(n, i, j, k) = \frac{2 \left[ n V_{5}^{i} + n V_{10}^{i} + (\hat{Y}_z + 2) n V_{18}^{i} - 2 n V_{17}^{i} \right] }{\hat{Y}_z + \hat{G}_{zy} + 4} \tag{2.83f}
\]

\[
H_{xy}(n, i, j, k) = \frac{2 \left[ n V_{5}^{i} - n V_{10}^{i} + \left(1 + \frac{2}{\hat{Z}_x} \right) n V_{19}^{i} - 2 n V_{20}^{i} \right] }{\hat{Z}_x + \hat{R}_{xy} + 4} \tag{2.83g}
\]
\[ H_{x}(n, i, j, k) = \frac{2^{\left( V_{i}^{4} + V_{i}^{2} + V_{i}^{19} \right)}}{Z_{x} + \hat{R}_{xx} + 4} \]  
(2.83h)

\[ H_{y}(n, i, j, k) = \frac{2^{\left( V_{i}^{6} + V_{i}^{21} - \frac{2}{Z_{y}} \right)}}{Z_{y} + \hat{R}_{yx} + 4} \]  
(2.83i)

\[ H_{z}(n, i, j, k) = \frac{2^{\left( V_{i}^{8} + V_{i}^{22} - \frac{2}{Z_{y}} \right)}}{Z_{y} + \hat{R}_{yz} + 4} \]  
(2.83j)

\[ H_{xx}(n, i, j, k) = \frac{2^{\left( V_{i}^{1} + V_{i}^{23} - \frac{2}{Z_{z}} \right)}}{Z_{z} + \hat{R}_{xx} + 4} \]  
(2.83k)

\[ H_{yy}(n, i, j, k) = \frac{2^{\left( V_{i}^{3} + V_{i}^{24} - \frac{2}{Z_{z}} \right)}}{Z_{z} + \hat{R}_{yy} + 4} \]  
(2.83l)

where,

\[ \hat{Y}_{x} = 4 \left( \varepsilon_{r} \frac{vw}{u \Delta t} - 1 \right) \]
\[ \hat{Y}_{y} = 4 \left( \varepsilon_{r} \frac{uv}{v \Delta t} - 1 \right) \]
\[ \hat{Y}_{z} = 4 \left( \varepsilon_{r} \frac{uw}{w \Delta t} - 1 \right) \]

\[ \hat{Z}_{x} = 4 \left( \mu_{r} \frac{vw}{u \Delta t} - 1 \right) \]
\[ \hat{Z}_{y} = 4 \left( \mu_{r} \frac{uv}{v \Delta t} - 1 \right) \]
\[ \hat{Z}_{z} = 4 \left( \mu_{r} \frac{uw}{w \Delta t} - 1 \right) \]

\[ \hat{G}_{xy} = \sigma_{r} \frac{vw}{uZ_{o}} \]
\[ \hat{G}_{xz} = \sigma_{r} \frac{vw}{uZ_{o}} \]
\[ \hat{G}_{yx} = \sigma_{r} \frac{uw}{vZ_{o}} \]
\[ \hat{G}_{yz} = \sigma_{r} \frac{uw}{vZ_{o}} \]
\[ \hat{G}_{zx} = \sigma_{r} \frac{uv}{wZ_{o}} \]
\[ \hat{G}_{zy} = \sigma_{r} \frac{uv}{wZ_{o}} \]

\[ \hat{R}_{xy} = \sigma_{r} \frac{vw}{uZ_{o}} \]
\[ \hat{R}_{xz} = \sigma_{r} \frac{vw}{uZ_{o}} \]
\[ \hat{R}_{yx} = \sigma_{r} \frac{uw}{vZ_{o}} \]
\[ \hat{R}_{yz} = \sigma_{r} \frac{uw}{vZ_{o}} \]
\[ \hat{R}_{zx} = \sigma_{r} \frac{uv}{wZ_{o}} \]
\[ \hat{R}_{zy} = \sigma_{r} \frac{uv}{wZ_{o}} \]  
(2.84)

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In (2.84), \( \sigma_x^*, \sigma_y^*, \) and \( \sigma_z^* \) are the magnetic conductivities along the \( x-, y-, \) and \( z- \)directions, respectively.

Note that, to prevent the term \( 1/\hat{Z} \) to go to infinity in (2.83g)-(2.83l), when \( u = v = w = \Delta \ell \) and \( \mu_r = 1 \), it is necessary to take \( \Delta t \) slightly lower than the maximum time step \( \Delta \ell / (2c) \). The reason for this can be understood when we compare the expression obtained for the normalized admittance of the open stub along \( x \)-direction following the procedure given in Section 2.2.5.1. as,

\[
\hat{Y}_x = 4 \left( \varepsilon_r \frac{vwh}{u\Delta \ell} - 1 \right).
\]

We then deduce that the time step, \( \Delta t \) satisfies \( \Delta t = \frac{\Delta \ell}{2hc} \), where

\[
h = \max \left( \frac{u\Delta \ell}{vw}, \frac{v\Delta \ell}{uw}, \frac{w\Delta \ell}{uv} \right)
\]

over the computation domain. This value of \( h \) ensures that non-negative values for \( \hat{Y} \) and \( \hat{Z} \) are guaranteed for all cells during the computation. In addition, to ensure positive values for \( \hat{Y} \) and \( \hat{Z} \) for all cells, it is sufficient to replace “\( h \)” by “\( h_2 \)”, where \( h_2 = h + \delta \). \( \delta \) is an arbitrary small and positive quantity. In the following we will choose \( \delta = 10^{-7} \) which has proven to be appropriate for all of our numerical calculations. This results in a TLM computation at “almost” maximum time step since \( \Delta t_2 = \frac{\Delta \ell}{2h_2c} \) is slightly less than \( \Delta t = \frac{\Delta \ell}{2hc} \). In the regular mesh \( u = v = w = \Delta \ell \), if the medium is free space \( (\varepsilon_r = \mu_r = 1) \), \( h \) and \( h_2 \) become 1 and \( 1 + 10^{-7} \), respectively. One can readily see that normalized admittance and impedance become \( \hat{Y} = \hat{Z} = 4(h_2 - 1) = 4 \times 10^{-7} \) for this case.

Furthermore, in the case of usual media with isotropic electric and magnetic conductivities \( \sigma \) and \( \sigma^* \), (2.83a) and (2.83b) can be reduced to

\[
E_x = E_{xy} + E_{xz} = \frac{2\left[ nV_1 + nV_2 + nV_3 + nV_4 + \hat{Y}_x(nV_3 + nV_4) \right]}{\hat{Y}_x + \hat{G}_x + 4}
\]

where \( \hat{G}_x = \hat{G}_{xy} = \hat{G}_{xz} \). (2.85) is the \( E_x \) field expression for the standard SCN given in (2.68a) before. It therefore, becomes clear that this PML SCN can be used to simulate not only PML layers but also the physical media filling the problem domain. It can thus be used in the entire computational domain leading to a uniform algorithm.
As described in [49], the scattered voltage waves can now be obtained from field expressions \( E_{a\pm b} \), where \( a \) and \( b \) stand for \( x, y, \) or \( z \), by averaging in the space-time coordinate system defined in (2.77). The 24×24 scattering matrix of this PML SCN has been given in [48]. Here we will use the formulation given below, which can be obtained via reformulation of that given in [48]. However our formulation has some definite computational advantages:

\[
nV'_1 = [E_{xy}(n,i,j,k) + E_{xz}(n,i,j,k)] + [H_{zx}(n,i,j,k) + H_{zy}(n,i,j,k)] - nV'_1
\]

\[
nV'_2 = [E_{xy}(n,i,j,k) + E_{xz}(n,i,j,k)] - [H_{yz}(n,i,j,k) + H_{zy}(n,i,j,k)] - nV'_2
\]

\[
nV'_3 = [E_{yx}(n,i,j,k) + E_{yz}(n,i,j,k)] - [H_{yx}(n,i,j,k) + H_{zx}(n,i,j,k)] - nV'_3
\]

\[
nV'_4 = [E_{yx}(n,i,j,k) + E_{yz}(n,i,j,k)] + [H_{yx}(n,i,j,k) + H_{zx}(n,i,j,k)] - nV'_4
\]

\[
nV'_5 = [E_{zx}(n,i,j,k) + E_{zy}(n,i,j,k)] - [H_{yx}(n,i,j,k) + H_{zx}(n,i,j,k)] - nV'_5
\]

\[
nV'_6 = [E_{zx}(n,i,j,k) + E_{zy}(n,i,j,k)] + [H_{yx}(n,i,j,k) + H_{zx}(n,i,j,k)] - nV'_6
\]

\[
nV'_7 = [E_{xz}(n,i,j,k) + E_{zy}(n,i,j,k)] - [H_{yx}(n,i,j,k) + H_{zx}(n,i,j,k)] - nV'_7
\]

\[
nV'_8 = [E_{xz}(n,i,j,k) + E_{zy}(n,i,j,k)] + [H_{yx}(n,i,j,k) + H_{zx}(n,i,j,k)] - nV'_8
\]

\[
nV'_9 = [E_{xy}(n,i,j,k) + E_{xz}(n,i,j,k)] + [H_{zx}(n,i,j,k) + H_{zy}(n,i,j,k)] - nV'_9
\]

\[
nV'_{10} = [E_{zx}(n,i,j,k) + E_{zy}(n,i,j,k)] - [H_{yx}(n,i,j,k) + H_{zy}(n,i,j,k)] - nV'_{10}
\]

\[
nV'_{11} = [E_{yx}(n,i,j,k) + E_{zy}(n,i,j,k)] + [H_{zx}(n,i,j,k) + H_{zy}(n,i,j,k)] - nV'_{11}
\]

\[
nV'_{12} = [E_{xy}(n,i,j,k) + E_{xz}(n,i,j,k)] - [H_{zx}(n,i,j,k) + H_{zy}(n,i,j,k)] - nV'_{12}
\]
\[ nV_{13}^{r} = E_{xy}(n, i, j, k) - nV_{13}^{i} \]
\[ nV_{14}^{r} = E_{xz}(n, i, j, k) - nV_{14}^{i} \]
\[ nV_{15}^{r} = E_{yx}(n, i, j, k) - nV_{15}^{i} \]
\[ nV_{16}^{r} = E_{yz}(n, i, j, k) - nV_{16}^{i} \]
\[ nV_{17}^{r} = E_{zx}(n, i, j, k) - nV_{17}^{i} \]
\[ nV_{18}^{r} = E_{zy}(n, i, j, k) - nV_{18}^{i} \]
\[ nV_{19}^{r} = nV_{19}^{i} - \hat{Z}_{x}H_{xy}(n, i, j, k) \]
\[ nV_{20}^{r} = nV_{20}^{i} - \hat{Z}_{x}H_{xz}(n, i, j, k) \]
\[ nV_{21}^{r} = nV_{21}^{i} - \hat{Z}_{y}H_{yx}(n, i, j, k) \]
\[ nV_{22}^{r} = nV_{22}^{i} - \hat{Z}_{y}H_{yz}(n, i, j, k) \]
\[ nV_{23}^{r} = nV_{23}^{i} - \hat{Z}_{z}H_{zx}(n, i, j, k) \]
\[ nV_{24}^{r} = nV_{24}^{i} - \hat{Z}_{z}H_{zy}(n, i, j, k) \] (2.86)

Inspection of the above equations reveals that, scattering calculations with PML SCN require 108 additions/subtractions, 42 multiplications, and 12 divisions in place of 54 additions/subtractions and 12 multiplications with the standard SCN. This higher computational cost may be prohibitive, and may be avoided by using the standard SCN in the interior of the problem domain and the PML SCN in the PML boundary layers. This line of thought was followed in this thesis. It has been shown that an appropriate mix of SCN and PML-SCN nodes can be incorporated into the TLM
algorithm making use of the fact that 12-link lines are the same for both types of nodes.

2.3.2. PML SCN Applications

In this section, we aim to test the accuracy of the PML ABC formulation given in the previous section by applying this to some test structures such as an empty resonator and a waveguide. After we ascertain the accuracy of our ABC routine, we will incorporate this into our ABC routine and proceed to validate our main TLM algorithm using the canonical problems given in Chapter 3 before applying TLM to the more complex problems discussed in Chapter 4.

Since a geometric conductivity profile has been extensively used by Berenger [48], the same profile has also been used in the PML applications in this thesis. We have firstly used this profile in the case of radiating dipole located at a corner of a computational domain of $14 \times 14 \times 14$ cells as in [45,48]. The electric dipole $P_e$ was implemented by superposing to the $E_z$ field at node (2,2,2) the following excitation:

$$E_z(n\Delta T) = -\frac{\Delta t}{\varepsilon_0 \Delta \ell^3} \frac{\partial P_e(n\Delta T)}{\partial t}$$ \hspace{1cm} (2.87a)

where

$$P_e(t) = 10^{-10} \exp\left(-\left(\frac{n\Delta t - 3T}{T}\right)^2\right)$$ \hspace{1cm} (2.87b)

Substituting (2.87b) into (2.87a) one obtains

$$E_z(n\Delta T) = -10^{-10} \frac{(n\Delta t - 3T)2\Delta t}{\varepsilon_0 \Delta \ell^3 T^2} \exp\left(-\left(\frac{n\Delta t - 3T}{T}\right)^2\right)$$ \hspace{1cm} (2.87c)

with $T=2$ nsec. Note that, the above expression is the once-differentiated Gaussian pulse.

The cubic cell size $\Delta \ell$ is chosen as 5cm and the time step $\Delta t$ as 83.333 picoseconds. The $E_z$ field is observed at only two cells from the opposite corner at point (2,12,2). The computational domain was surrounded by PML(N-Gg-R). Gg means that a geometric profile of conductivity with a ratio of g was used. N and R stand for number of PML layers and theoretical reflection, respectively. Such a profile is implemented in the TLM using
\[
\sigma(0) = -\frac{\varepsilon_0 c \cdot \text{Ln}(R)}{2} \frac{g-1}{\Delta t} \frac{g^{-1}}{g^N - 1} \\
\sigma(L) = \sigma(0) g^{-L}, \quad \text{for} \quad L=1,2,\ldots,N-1
\]  

(2.88)

while in FDTD simulations, Berenger [48] used

\[
\sigma(0) = -\frac{\varepsilon_0 c \cdot \text{Ln}(R)}{2} \sqrt{g-1} \frac{g^{-1}}{\sqrt{g} - 1} \frac{g^L}{g^{N-1}} \\
\sigma(L) = \sigma(0) \frac{g^{-1}}{\sqrt{g} - 1} g^L, \quad \text{for} \quad L=1,2,\ldots,N-\frac{1}{2}
\]  

(2.89)

The results plotted in Figure 2.18a were computed with the TLM using PML(5-G1.9-6%), PML(6-G1.9-6%), and PML(8-G1.9-6%) and in Figure 2.18b with the FDTD using PML(5-G2.15-1%), PML(6-G2.15-1%), PML(8-G2.15-1%). With such conditions, the numerical value of conductivity \(\sigma(0)\) in the vacuum-PML interface is the same for both simulations (i.e., \(\sigma(0) = 0.0028\)S/m). As seen in Figure 2.18, the results computed with both TLM and FDTD are very close to the corresponding reference solutions calculated in a computational domain 150×150×150 cells. The reference solutions are generated by choosing the computational domain as large as ascertaining that no reflections occur from the boundaries to the excitation and observation points. It can be observed that oscillations appear in both methods. According to Berenger, these oscillations are caused by the evanescent waves, which cannot be correctly absorbed in the first cells of the PML.

![Figure 2.18 E_z field obtained in the time domain with (a) TLM and, (b) FDTD methods using a PML with a geometric profile.](image)

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As the number of PML layers is increased, these spurious oscillations become less. We note that as number of the PML layers is increased beyond 4 our PML-TLM algorithm starts to outperform the FDTD PML simulations. As seen in figures TLM PML with 6 layers works as good as 8 layer FDTD.

In wave-structure interaction problems, it has been found that the domain of validity of the PML technique is bounded by a frequency $f_c$ which depends on the numerical conductivity implemented in the first row of the layer, denoted by $\sigma(0)$. As [48],

$$f_c = \frac{\sigma(0)}{2\pi \varepsilon_o} \quad (2.90)$$

Figure 2.19 shows the radiated field in the FD obtained by Fourier transforming the time responses computed up to 5 microseconds $(60000\Delta t)$. All the results are superimposed on the reference above a frequency, which is approximately 55MHz with the TLM and 60MHz with the FDTD. This is in accordance with the theoretical cutoff frequency (i.e., 50MHz) bounding the domain of validity of the PML computed with (2.90). Note that the vertical axis in Figure 2.19 is the $E_z$ normalized to the maximum value of the FDTD reference value in dB.

![Figure 2.19](image)

Figure 2.19 Normalized $E_z$ field observed in the FD with the TLM and FDTD using 6-layer PML with a geometric profile.

Note that it is important to determine the number of PML layers $N$ in such a way that the following time, $t_c$, is at least 10 times the duration of computation $D_c$ [45]:
\[ t_c = \frac{1}{f_c} \geq 10D_c \] (2.91)

It has been recommended in [45] that it is more appropriate to use a geometric profile of conductivity with a ratio \( g \) of approximately two. The number \( N \) of cells in the PML should be chosen in order to set the numerical value of \( \sigma(0) \) in (2.88) noting that to the FD of interest is bounded by (2.90). Of course, this also depends on the theoretical reflection factor \( R \), which should not be too small in order to ensure the stability of the algorithm. Furthermore, as shown with the numerical results, a value of about 1% for \( R \), seems to be sufficient to obtain good absorption and, to ensure stability.

The second application is the analysis of the unwanted reflections inside a rectangular X-band waveguide (see Figure 2.20). Note that the theoretical cut-off frequency for the dominant TE\(_{10} \) mode in waveguide is 6.5GHz. The waveguide is terminated with six-cell PML layers on both ends and the propagation of the dominant mode is simulated. The time variation of the excitation is a pulse which has \( f_0=10 \)GHz carrier (i.e., a sine wave) frequency and its shape is the first derivative of a

![Figure 2.20 Analyzed X-band waveguide structure. Cross-sectional and axial dimensions are 23mm×10mm, and 105mm (325mm), respectively. A uniform mesh size \( \Delta l=0.5 \)mm is used. Gaussian pulse with a 40GHz bandwidth. The carrier is shown with dashed lines in Figure 2.21. The calculated values for \( E_y \) and for the amplitude of the reflection coefficient for the dominant mode are shown in Figures 2.21 and 2.22, respectively.](image)
Figure 2.21 Time variation of source (dashed line) and wave at the observation point (solid line). Source is chosen as 40GHz bandwidth once-differentiated Gaussian pulse modulated with 10GHz sinusoidal wave.

The following parameters are used in these calculations: The cross-section of the waveguide is chosen 46×20 cells. The length of the waveguide is chosen as 210 and

Figure 2.22 Unwanted reflections computed in an X-band waveguide with the TLM and FDTD using PML terminations. Number of layers is 6.

650 cells for total and incident wave observations, respectively. Calculations for the reflection coefficient are made with both TLM and FDTD and a 6-cell PML layer is used in both methods. 1150 time steps are used in TLM and FDTD algorithms for both 210 and 650 cell lengths. The unit cell size, \( \Delta \ell \) in computation domain is chosen as 0.5mm [50]. The frequency variation of reflection coefficient given in Figure 2.22
shows that the reflection coefficient obtained with TLM is about 10dB worse than that obtained via FDTD. However, since the results obtained with both methods are less than -40dB within the frequency range of interest it is safe to argue that the above mentioned degradation in TLM should be totally insignificant.

As mentioned in [50], inefficiency of PML layers to absorb the evanescent waves cause oscillatory behaviour in TD, which means one should be careful when applying Fourier transforms. Here, windowing followed by zero-padding is applied to the time variations in order to obtain the necessary frequency resolution as well as to get rid of the aliasing effects.

2.4. Dispersion analysis for TLM and FDTD Techniques

Due to the spatial discretization in computer modeling of electromagnetic problems, wave propagation velocity varies with frequency in all numerical schemes. This is true even in cases where the numerical method is used to simulate non-dispersive media. Clearly the errors introduced by discretization will decrease by decreasing the mesh size. The ratio of mesh size to the minimum signal wavelength of interest in a particular problem provides a measure for estimating the level of numerical error associated with any given choice of the mesh size. This phenomenon is usually referred to as numerical dispersion. Although errors introduced due to the numerical dispersion can generally be rendered small via appropriate choice of the mesh size, they nevertheless may become unacceptable in certain applications such as, for example, scattering from objects containing surface irregularities with small radii of curvature, which can not be accurately modeled using even the smallest possible mesh size for given computational resources.

In order to be able to investigate the effects of the numerical dispersion in different modeling techniques, the dispersion relation for a particular discretization scheme must be obtained. In general, this dispersion relation describes the functional relationship between the angular frequency $\omega$ and the propagation vector, $k$,

$$k^2 = \omega^2 \varepsilon \mu = \sqrt{k_x^2 + k_y^2 + k_z^2}$$  \hspace{1cm} (2.92)

where $k_x$, $k_y$ and $k_z$ are the Cartesian components of $k$.

The phase velocity $v_p$ and the group velocity $v_g$ are defined as [51,52]:

55
\[ v_p = \frac{\omega}{k}, \quad v_g = \left( \frac{\partial k}{\partial \omega} \right)^{-1}. \] (2.93)

It follows from (2.92) that the group and phase velocities in non-dispersive media are equal, independent of frequency and are given by:

\[ v_p = v_g = v = \frac{1}{\sqrt{\varepsilon \mu}}. \] (2.94)

The dispersion characteristics \( k = f(\omega) \) for such media are straight lines with a slope of \( \frac{1}{v} = \sqrt{\varepsilon \mu} \) and they are referred as linear dispersion characteristics.

It is shown in [51] that in TLM, as in other discrete numerical schemes, the modeled group and phase velocities follow the physical velocities closely only within a low-frequency band or, equivalently, for sufficiently fine spatial discretization. At higher frequencies, the numerical dispersion increases, thus causing a deviation in modeled wave velocities.

An analysis of the dispersion characteristics of the TLM mesh can be found in [53]. Many other studies related to dispersion analysis are also published in the literature [54,55]. A limitation of the analysis procedure given in [53] is that an equivalent circuit model of TLM node is required. Since a practical equivalent circuit model for the 3D symmetrical condensed node (SCN) does not exist, its dispersion relation cannot be derived by this procedure, except for special cases such as propagation along the axis and along diagonal directions [56].

As noted above, numerical dispersion depends on the electrical size of the spatial mesh size, \( \Delta \ell \). The numerical dispersion effect is inverse proportional to the ratio of minimum wavelength (of interest) to \( \Delta \ell \). Depending on the complexity of the problem, it sometimes may be sufficient to use \( \lambda_{\text{min}} / \Delta \ell \) ratios of 10–20, but for certain problems like SAR and RCS, the values as high as 100–150 may be required. Therefore, before doing a complete simulation analysis the modeler should perform some tests with different \( \lambda_{\text{min}} / \Delta \ell \) values to determine the ratio which optimizes computation time and yields acceptable numerical dispersion.

To show the sensibility of the TLM and FDTD techniques to the numerical dispersion, the two test problems pictured in Figure 2.23, will be considered. The first test problem depicted in Figure 2.23a is the determination of the EM field of a short dipole inside an empty PEC cavity. The formal analytical solution of this
Figure 2.23 EUTs for numerical dispersion analysis. The dimensions for outer and inner cubes are 26cm×26cm×26cm and, 10cm×10cm×10cm, respectively. Source bandwidth is 2.5GHz.

problem can readily be obtained, however its numerical evaluation can conveniently be effected only for special source distributions. The "canonical problems" resulting from such source distributions will be investigated in more detail in the next chapter.

The second test problem shown in Figure 2.23b differs from the first in that it contains an additional scatterer inside the cavity, which is an empty five-faceted cube (i.e., has five PEC walls and the sixth wall is removed).

In choosing these two problems for the purposes of estimating the dependence of numerical error on the problem parameters (geometry, electrical characteristics, mesh size etc.), we used the following two considerations:

i) The geometry is rectangular and termination is PEC, no discretization and termination errors are expected. The only source for error will thus be numerical dispersion,

ii) The second test problem (Figure 2.23b) involves the contributions of edge diffracted fields, a phenomenon for which an accurate computer model can not be constructed even at any discretization level.

For both test problems, the dimensions of the outer cube are taken as 26cm×26cm×26cm, and those of the inner one for the second problem as 10cm×10cm×10cm. Once differentiated Gaussian pulse that has 2.5GHz bandwidth,
is used as a source to excite \( y \)-component of the electric field and the same component is observed at the observation point.

As the first test example, \( E_y \) is calculated in the empty resonator shown in Figure 2.23a using two different mesh sizes. In the first case, the ratio of minimum wavelength (\( \lambda_{\text{min}} = 12 \text{cm} \)) to the mesh size (\( \Delta \ell = 1 \text{cm} \)) is taken as \( \lambda_{\text{min}}/\Delta \ell = 12 \). FDTD and TLM results obtained for this mesh size are shown in Figure 2.24. As seen, the results obtained using both methods agree very well. However, there are some small differences at the peaks indicating presence of errors due to numerical dispersion at high frequencies. Should these errors be unacceptable for a particular application they can be reduced at the expense of longer computation time by decreasing the mesh size. Indeed when the mesh size is halved (\( \Delta \ell = 0.5 \text{cm} \)), and \( \lambda_{\text{min}}/\Delta \ell \) ratio is taken as 24, the agreement between the results obtained via both methods becomes excellent as seen in Figure 2.25. It should be noted that the computation time for this case increases approximately by a factor of 8. The agreement between two sets of calculations made with different numerical codes clearly demonstrates that TLM (and also FDTD) can yield highly accurate results for problems involving PEC structures with rectangular boundaries much larger than the wavelength components of the illuminating signal.

![Figure 2.24 Numerical dispersion analysis of a structure in Figure 2.23a for \( \lambda_{\text{min}}/\Delta \ell = 12 \) (\( \lambda_{\text{min}} = 12 \text{cm} \) and \( \Delta \ell = 1 \text{cm} \)).](image)

The results obtained for the second test problem are also calculated using TLM and FDTD for three different \( \lambda_{\text{min}}/\Delta \ell \) values, 12, 24, and 36, respectively, and plotted in Figures 2.26a and 2.26b.
As seen in the figure, for this specific problem FDTD results converge already for \( \Delta t = \lambda_{\text{min}}/12 \), whereas \( \Delta t = \lambda_{\text{min}}/36 \) has to be used to obtain a similar convergence in TLM. It should be recalled that the structure shown in Figure 2.23b investigated in

Figure 2.25 Numerical dispersion analysis of an EUT in Figure 2.23a for \( \lambda_{\text{min}}/\Delta t = 24 \). (\( \lambda_{\text{min}} = 12\text{cm} \) and \( \Delta t = 0.5\text{cm} \)).

this test involves scattering phenomena (such as edge diffraction) which can not be modeled accurately and hence some residual error (numerical dispersion) may remain in the computed results even when using the finest possible mesh size.

Figure 2.26 Numerical dispersion analysis of an EUT in Figure 2.23b for a). FDTD and, b). TLM. Dashed line: \( \lambda_{\text{min}}/\Delta t = 36 \), Solid line: \( \lambda_{\text{min}}/\Delta t = 24 \) and, Signs \( \lambda_{\text{min}}/\Delta t = 12 \).
Figures 2.26a and 2.26b demonstrate that by decreasing \( \Delta \ell/\lambda_{\text{min}} \) ratio the results obtained via both algorithms converge. But upon closer inspection one observes that in a small region centered at about 9 nsec, quite different results are obtained in TLM and FDTD calculations. To better illustrate this behavior TLM and FDTD results are plotted on top of each other in Figures 2.27a-c, for mesh sizes \( \Delta \ell=\lambda_{\text{min}}/12, \lambda_{\text{min}}/24, \) and \( \lambda_{\text{min}}/36 \), respectively. When we inspect the curves shown in Figure 2.27c corresponding to \( \Delta \ell=\lambda_{\text{min}}/36 \), the above mentioned discrepancy around 9 nsec between the two results is easily seen, despite the fact that both curves exhibit an almost perfect fit at other times. Evidently contributions from some wave phenomena which cannot be modeled accurately with the step size used (\( \Delta \ell=\lambda_{\text{min}}/36 \)) become important around 9 nsec. As noted before, we expected the possibility that the effects of numerical dispersion (error) would persist in this test problem. The fact that we

![Figure 2.27](image)

**Figure 2.27** Numerical dispersion analysis of an EUT in Figure 2.23b for three different discretization. a). \( \lambda_{\text{min}}/\Delta \ell=12 \), b). \( \lambda_{\text{min}}/\Delta \ell=24 \) and, c). \( \lambda_{\text{min}}/\Delta \ell=36 \)

have not observed a persisting discrepancy between TLM and FDTD results in all our calculations made for structures with smooth surfaces, also indicates that numerical dispersion can be attributed to either an undersampling (i.e., large \( \Delta \ell/\lambda_{\text{min}} \) values) or to the presence of surface irregularities with very small radii of curvature.

The lesson to be learned from these two tests is that even though

i) The results calculated for a problem using a numerical code converge appropriately with decreasing \( \Delta \ell \), and
ii) Independently obtained numerical results agree with each other "almost everywhere".

the accuracy of the calculated results can not be assessed with complete confidence, since they still may be contaminated with numerical dispersion. The ultimate performance test for validation of any numerical result can only be made by comparing the calculated results with exact analytical solutions (where available) or with experiment (where possible).

In the next chapter we will investigate some canonical (analytically solvable) problems and show that these can be used to validate our TLM algorithm for problems with smooth (radius of curvature much larger than $\lambda_{min}$) surfaces. However, in this work we will address complex EM problems for which neither analytically nor experimentally obtained solutions are available. We will then resort to a validation in a limited sense, in that we will compare our TLM results with the results obtained via FDTD for the same problem. The level of agreement/disagreement between them will give us a means to estimate with some confidence the accuracy of TLM in addressing these problems.

2.5. Criteria for Parameter Selection

Running simulations via TLM as with many other numerical techniques including FDTD require parameter optimization. In applying TLM to a particular class of problems spatial mesh sizes, $\Delta x$, $\Delta y$, $\Delta z$, time step $\Delta t$, simulation time ($T_{obs} = n \times \Delta t$), source bandwidth $B$, pulse duration $\tau$ are characteristic parameters that should be optimally selected prior to the simulation.

Parameter selection has two main steps:

- **FD Analysis**: The TD simulations are generally used to obtain frequency characteristics of a given EM problem such as the determination of the radiation characteristics or input impedance of an antenna structure, RCS behavior of a chosen target, transmission and/or reflection characteristics of a microstrip network, propagation characteristics of a waveguide, resonance frequencies of a closed enclosure, the shielding effectiveness of an aperture, etc. Whatever the problem at hand, it is desired to obtain the "response" in a given frequency range; from a given minimum frequency $f_{min}$ to a maximum frequency $f_{max}$ with $\Delta f$ incremental frequency steps.
• **TD Discrete Simulation Requirements**: In TD simulations the discretization parameters $\Delta x$, $\Delta y$, $\Delta z$, $\Delta t$, and observation time $T_{obs}=n\times\Delta t$, together with source parameters $B_{eff}$ and $T_{eff}$ need to be specified in keeping with the desired frequency domain information.

The user first specifies minimum and maximum frequencies, $f_{min}$ and $f_{max}$ together with frequency sensitivity $\Delta f$. Suppose, the problem is to find the frequency characteristics of an antenna from DC to 1GHz with 10MHz frequency steps. As shown in Figure 2.28, the signal spectrum extends from DC to a maximum frequency 1GHz. An ideal waveform for broad band TD simulations is one with finite duration in both TD and FD. However, it is not possible to meet both of these requirements.

![Figure 2.28 Source spectrum and sufficient/insufficient sampling effects. Aliasing is inevitable for unsuitable sampling rates.](image)

For example, a TD pulse with rectangular duration is an ideal duration-limited signal. On the other hand, its frequency extent (which is a Sinc function) is infinite (i.e., converging to zero at infinity). Similarly, rectangular spectrum corresponds to an infinite duration pulse in the TD (because of the symmetry property of Fourier Transform). There is an inverse relationship between signal duration and bandwidth; the shorter the pulse to higher the bandwidth, and vice versa. This is expressed using the time-bandwidth product, which states that, for a source signal, the product of bandwidth and duration is constant (i.e., Hartley’s theorem). The best source waveform is the one with minimum time-bandwidth product. A Gaussian pulse has this property and it is usually chosen as the source waveform in these kinds of simulations.
The spectrum of a Gaussian source may be assumed as a band-limited signal as shown in Figure 2.28. As mentioned above, for the above example the maximum frequency of interest (defined either between 50% or 10% amplitude points) is 1GHz. According to the Nyquist sampling criteria, the sampling rate ($f_s$, sampling frequency) must be at least twice the maximum frequency, i.e., $f_s \geq 2 \times f_{\text{max}} = B$. In Figure 2.28, the spectrum of a waveform sampled (discretized) without and with obeying Nyquist criteria are plotted. Discretization in TD makes the signal spectrum periodic in FD (convolution). Without adequate sampling these periodic bands coalesce resulting in loss of information. On the other hand, sampling with adequate rate separates these bands in the FD, which makes extracting one of them possible.

Starting from the frequency analysis requirements and using the sampling criteria, parameter optimization steps can be listed as follows:

- Choose the source waveform with a duration so that its frequency spectrum contains the maximum frequency of interest.

- Maximum frequency determines minimum time step, i.e., $\Delta t_{\text{FFT}} = \frac{1}{2f_{\text{max}}}$ to be used in applying Fast Fourier Transform (FFT). This is the hard limit for frequency analysis. For example for 1GHz as the maximum frequency $\Delta t_{\text{FFT}}$ is obtained as 0.5 nanosecond.

- There are two important factors which have to be considered in choosing maximum simulation (observation) time; (a) the frequency sensitivity $\Delta f$, which determines the observation time as $\Delta f = \frac{1}{T_{\text{obs}}}$ and (b) the requirement that the simulation should continue until all relevant transient effects are observed. Therefore, $T_{\text{obs}}$ is chosen to satisfy both of these conditions. Since $\Delta f$ is given as 10MHz, $T_{\text{obs}}$ is determined as 100 nanoseconds. Number of time steps $n$ will then be 200. If all transients decay after 200 time steps then this choice is appropriate. If on the other hand the structure under investigation is some kind of a resonant structure, which results in ringing effects in TD, then the observation time should be extended so that these delayed arrivals can also be observed.

Up to this point, requirements of discrete Fourier analysis are used in defining time step $\Delta t$ and maximum simulation time $T_{\text{max}}$. There are two important criteria in TD simulations; Courant stability criteria and numerical dispersion.
• Spatial mesh sizes \( \Delta x, \Delta y, \Delta z \) are chosen according to numerical dispersion requirements. This requirement corresponds to satisfying Nyquist sampling criteria in spatial domain. The minimum wavelength (\( \lambda_{\text{min}} \)) component must be sampled with at least two samples, i.e., max of \( \{\Delta x, \Delta y, \Delta z\} \leq \lambda_{\text{min}} / 2 \). However in practice, this condition is far from being adequate and at least \( \lambda_{\text{min}} / 10 \) is required for acceptable results. Depending on the problem at hand it may be required to reduce the step size as much as to \( \lambda_{\text{min}} / 100 \) may be required in order to minimize the numerical dispersion effects explained in Section 2.4.

• Once spatial mesh sizes are determined, time step \( \Delta t \) may directly be chosen from the Courant stability criteria. For example, in the above example since, \( \Delta x = \Delta y = \Delta z = 1 \text{ cm} \), \( \Delta t \) may be chosen as \( 1/(2c) \), where \( c \) is the speed of light. This gives a time-step about 17psec. In general, simulation time step dictated by the stability criteria (17psec) is much less than that required by the FFT analysis (0.5nsec). Therefore in the above example taking \( \Delta t \) as 17psec., the number of time steps (\( n \)) to be used in the simulation run will be about 6000.

When the above requirements are met, we can safely argue that parameter selection is made appropriately and the results obtained via simulations may be accurate and may correspond to the investigated physical system.

2.6. A Comparison of the Computational Resource Requirements for TLM and FDTD Methods

Because of the differences both in the topology and also in the algorithms of TLM and FDTD methods, they naturally have different computational resources. In order to estimate and compare the differences in the computational resource requirements of TLM and FDTD, we have tested the effects of several parameters used in typical computational problems. These tests were performed by using a personal computer, with Pentium-II processor and 192Megabyte SDRAM.

In order to estimate the execution speeds of our TLM and FDTD algorithms, we used as the test problem an empty PEC resonator with the dimensions of 35m×35m×35m. Mesh size is taken as 0.5m which leads to a computation volume of 70×70×70 cells. Total number of cells in the system is 343000. Once-differentiated Gaussian pulse with 10MHz bandwidth is used to excite the system. Iteration is repeated 10000 times using a time step of 0.83nsec. Simulation run times obtained in this problem
for FDTD and TLM methods are 45 minutes and, 7 hour and 30 minutes, respectively. We have performed several other tests with similar results. In general, the ratio of computation times for both methods varied between 1/8 and 1/12, depending on the problem. It should be noted that the number of arithmetic operations per time step in TLM is 3-4 times more than that used in FDTD. This is the main reason for the longer computation time in TLM. Note that all the numbers given above are determined for 343000 cells and for PEC boundaries. During these tests, TLM method needed 22 MB memory, while FDTD used only a memory of 12 MB.

Using our above specified computer platform the maximum computation domain that could be processed was $2.6 \times 10^6$ cells in TLM but $5 \times 10^6$ cells in FDTD. For these maximum domains, TLM needs 163 MB memory (for $2.6 \times 10^6$ cells) but FDTD needs 174 MB memory (for $5 \times 10^6$ cells).

Another test is performed using 5-layer PML-ABC boundaries for a computational domain of $86 \times 86 \times 92 = 680432$ cells excluding the PML boundary layers. In this case, the memory requirements were y requirements 95 MB and 81 MB for TLM and FDTD methods, respectively.

In conclusion, we should point out that depending on the problem investigated TLM’s memory requirement may be up to 2 times more than FDTD, while, as noted above, the execution time of TLM is approximately an order of magnitude larger than that of FDTD. The reason for this increased memory and execution time requirements is that in TLM method 12 different incident and reflected voltage pulses need to be calculated at each time step.

Although it requires more computational resources and more execution time TLM has a definite advantage over FDTD since all 6-field components are located in the center of the cell in TLM whereas they are spatially separated in FDTD Yee’s cell. This property significantly facilitates the modeling of the interfaces and becomes particularly important in modeling EM problems involving several interfaces such as those encountered in SAR calculations to be investigated in Section 4.2.

An additional advantage of TLM over FDTD is that in TLM electric and magnetic field components are computed at the same time step whereas in FDTD they are separated in time by half a time-step.
3. CANONICAL TEST PROBLEMS

Before applying the TLM method to complex EM problems, for which there are yet no existing reference solutions, we would like to calibrate our algorithms against canonical problems, analytical solutions are readily available. The aim of the tests with the canonical problems investigated in this chapter is two fold: First we would like to verify and validate our TLM algorithm and optimize parameter selection using the criteria and considerations discussed in the preceding chapter, secondly we would like to repeat the above exercise for the FDTD algorithm that we have also developed. The reason why we address these canonical problems not only with TLM and also with FDTD is that we are also going to use the FDTD to generate comparison solutions for the complex problems to be investigated in the next chapter. In view of the lack of other reference solutions we believe that these comparisons are the only means for assessing the accuracy of the numerically generated TLM solutions with some confidence level.

In this chapter we will investigate two such canonical problems. The first problem concerns the evaluation of EM fields inside a rectangular PEC cavity excited by an appropriately chosen source distribution, both in TD and in FD. As a second problem we undertake the calculation of radiation from a current distribution in a rectangular aperture on a PEC surface. These problems will be addressed respectively in Sections 3.1 and 3.2.

3.1. Green's Function Representation in a PEC Resonator

The first calibration test for the TLM and FDTD algorithms that we have developed is a rectangular PEC cavity. An exact analytical solution for the Green's function problem is available, and the Green's function problem corresponding to an arbitrary current source distribution can be obtained in dyadic form [57], the components of which may be calculated via triply infinite series involving modal field representations. These series have rather poor convergence properties and hence the calculation of the complete Green's dyad becomes a difficult, if not intractable, numerical task. In the following we will choose a specific source distribution which will excite only the $\text{TE}_{10p}$ type modes of the cavity. This choice will scalarize the
problem and result in a single infinite series representation thereby significantly reducing the complexity of the associated numerical task.

3.1.1. TE_{10p} solution in a PEC resonator

In keeping with the above considerations we assume as the driving source distribution a \( y \)-directed current sheet on a \( z' \in (0,d) \) plane given by

\[
J_y(x',y',z') = a_y J_y(x',y',z') = a_y \sin\left(\frac{\pi x'}{a}\right) \delta(z-z')
\]  

(3.1)

where \( x' \) ranges in \((0,a)\) and the primed coordinates refer to source location. It is straightforward to show that this source distribution will only excite \( y \)-independent fields polarized along \( y \).

In this case the problem of determining the electric field \( E = E_y a_y \) reduces to the solution of the scalar non-homogeneous wave equation

\[
\nabla^2 E_y + k^2 E_y = -j\omega\mu J_y
\]  

(3.2)

with \( \frac{\partial}{\partial y} = 0 \), under the boundary conditions,

\[
E_y(0,0 \leq y \leq b,0 \leq z \leq d) = E_y(a,0 \leq y \leq b,0 \leq z \leq d) = 0
\]

\[
E_y(0 \leq x \leq a,0 \leq y \leq b,0) = E_y(0 \leq x \leq a,0 \leq y \leq b,d) = 0
\]

(3.3)

\( E_y(x) \) at \( z \) due to the current sheet (3.1) at \( z' \) can then be expressed as [58]

\[
E_y(x,z;z')_{10p} = \frac{-4}{d} j\omega\mu \sin\left(\frac{\pi x}{a}\right) \sum_{p=1}^{\infty} \frac{\sin\left(\frac{p\pi z}{d}\right) \sin\left(\frac{p\pi z'}{d}\right)}{k^2 - \left(\frac{\pi}{a}\right)^2 - \left(\frac{p\pi}{d}\right)^2}
\]  

(3.4)

where the subscripts \((1,0,p)\) are used as a reminder to show that the terms of the series can be identified as \( TE_{10p} \) modes of the cavity with associated excitation coefficients.

When wave number, \( k \) is real the cavity is at resonance at the frequencies determined by
\[ f_{1,0,p} = \frac{c}{2 \sqrt{\varepsilon_r}} \sqrt{\left( \frac{1}{a} \right)^2 + \left( \frac{p}{d} \right)^2}. \]  

(3.5)

(3.4) is not valid at the resonance frequencies which render the denominator zero. These singularities are avoided when we assume small losses and define \( k \) as,

\[ k = \frac{\omega}{c} \left[ \varepsilon_r - j \frac{\sigma}{\omega \varepsilon_0} \right]^{1/2} \]

(3.6)

where \( \omega, c, \sigma, \varepsilon_0, \) and \( \varepsilon_r \) are the angular frequency, speed of the light, conductivity, permittivity, and relative dielectric constant, respectively.

It should be noted that calculations at observation points very close to the source plane and for media with very small losses should be avoided. It is clear that the series representation given in (3.4) will converge rather slowly. We have, therefore, run several tests in order to check the dependence of numerical accuracy on the number of terms kept in the calculations. Such a test result is given in Table 3.1 for \( a=100 \text{m}, \ b=100 \text{m}, \ d=100 \text{m}, \) assuming that the resonator is filled with a lossy dielectric (\( \varepsilon_r=1.1 \) and \( \sigma=10^3 \text{ S/m} \)). The values given in Table 3.1 are calculated for \( f=2.6 \text{MHz} \) which is not close to a resonance. As can be seen from the table, with the chosen set of parameters, while only 100 terms are needed for 3-digit accuracy, more than 5000 terms are required for 6-digit accuracy. It should be noted that as losses are reduced in (3.4) or observer approaches to the source plane, the required number of terms increases drastically.

<table>
<thead>
<tr>
<th>Mode Number (( p ))</th>
<th>Field Amplitude</th>
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<td>10</td>
<td>35.7125473</td>
</tr>
<tr>
<td>100</td>
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</tr>
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</tr>
<tr>
<td>10000</td>
<td>35.536554</td>
</tr>
</tbody>
</table>

Table 3.1 The convergence analysis of (3.4) at the frequency of 2.6MHz

3.1.2. Validation via Green's function representation

The TLM algorithm is tested against the Green's function representation for a 100m×100m×100m rectangular cavity, filled with different lossy dielectrics. The
cavity is represented by 100×100×100 cubical cells in TLM and 101×101×101 cells in FDTD (when terminated with a PEC wall, the termination cell is included in TLM, but excluded in FDTD, therefore an extra cell is required in FDTD). The cell sizes are taken as $\Delta x=\Delta y=\Delta z=\Delta \ell=1$ m in both algorithms. The time step is chosen to be 1.8 nanoseconds. The source has the spatial distribution as given in Figure 3.1 (represented by (3.1)). The temporal distribution of the source is represented by a once-differentiated Gaussian pulse. Its time extend between the points where pulse amplitude reduces to $10^6$ of its maximum value, is about 400nsec. This corresponds to a bandwidth of 6MHz in FD (here bandwidth is defined as the range in FD between the points where pulse amplitude reduces to 10% of its maximum value).

![Figure 3.1 Rectangular cavity and spatial source distribution at $z=z'$ plane. Dimensions are 100m×100m×100m](image)

The temporal distribution of the pulse and its frequency spectrum is given in Figure 3.2.

The investigation of this canonical test problem provides a convenient setting for calibrating our TLM algorithm and to assess its accuracy. Moreover, we will use this problem also for comparing TLM results with those obtained via FDTD method. For this problem these comparisons reveal that there is an excellent agreement between results obtained via TLM and FDTD. As noted before the comparison of the numerical results obtained via TLM and FDTD will be used in the next chapter to allow us to gain a certain level of confidence for the validity of the TLM numerical solutions obtained in investigating the complex EM problems which do not have a reference solution.
Figure 3.2 a). Time variation of the pulsed source, once-differentiated Gaussian pulse, b). its frequency spectrum.

We note the following favorable properties of the canonical test problem under consideration:

- Since the PEC cavity has a rectangular shape, it can be discretized without geometrical approximation, therefore there will be no discretization errors.

- The TLM and FDTD techniques can be compared directly with an highly accurate reference solution. Therefore, discrepancies between the reference and computed results will be solely due to the limitations of these techniques (e.g., cell structure and resulting numerical dispersion).

- Choosing the source as in Figure 3.1 reduces number of resonances. In this case there are only three resonances (i.e., at 2.02 MHz, 3.2 MHz, and 4.52 MHz) within the bandwidth of interest (see Figure 3.3).

- The source distribution given in (3.1) does not excite $E_x$ and $E_z$ field components. This gives us another possibility to assess the validity of our calculations in that we can check whether these components are not excited or appear as artifacts in TLM and FDTD algorithms.

The tests and comparisons are carried out both in the FD and TD. In FD, reference solution is directly obtained from (3.4) for each frequency of interest. The TLM and FDTD results in FD are obtained indirectly. First, TD responses are accumulated at the chosen observation point for given source at a given excitation point and frequency responses are then obtained via DFT. On the other hand, when TD results
Figure 3.3 Frequency spectrum of the source (dashed line) and TE_{10p} resonance frequencies computed by (3.5) for the PEC resonator filled with dielectric, \( \varepsilon_r = 1.1 \) (solid line)

are of interest FDTD and TLM yield these directly, whereas reference solution has to be obtained via inverse DFT.

The first comparisons are given in FD and TD in Figures 3.4 and 3.5, respectively, for \( \varepsilon_r = 1.1 \) and \( \sigma = 10^{-4} \) S/m. Here, the source is located at the cell (50,50,25) and the observation is chosen at (50,50,75). As shown in Figure 3.4, no resonances are

Figure 3.4 Frequency spectrum inside the cavity for \( \varepsilon_r = 1.1 \) and \( \sigma = 10^{-4} \) S/m. Dimensions of cavity is 100m×100m×100m, mesh size is 1m and, frequency resolution is 10KHz.

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observed in this case because of the lossy material filling the cavity. The loss effects can be better understood from the TD plots of $E_y$ given in Figure 3.5. The fields emanating from the source decay as they propagate, and noting that a round-trip requires approximately 700nsec we deduce that only waves which undergo one or two bounces on the boundaries of the cavity contribute to the field at the observation point. The wave does not "sense" the reflecting boundaries and consequently no resonances evolve. In Figures 3.4 and 3.5, solid and dashed lines representing the results of TLM calculations and the reference solution, are indistinguishable in the scale of the plots. Although not shown in these figures we have also performed FDTD calculations for the cases corresponding to Figures 3.4 and 3.5. Again we obtained perfect agreement between reference and FDTD solutions, similar to that obtained using TLM.

The second comparisons are given in FD and TD in Figures 3.6 and 3.7, respectively, for $\varepsilon_r=1.1$ and $\sigma=10^5$ S/m, with all other parameters same as in Figures 3.4 and 3.5. As observed in Figure 3.6, the resonance character appears when $\sigma$ is reduced by an order (from $\sigma=10^4$ S/m to $\sigma=10^5$ S/m). This is also observed in TD variations in Figure 3.7.

Here, the field at the observation point diminishes around 8000-10000 nanoseconds, which means that fields have the opportunity to bounce back and forth between the opposite walls of the cavity more than 10 times. In practice, to obtain the resonance signature from the TD simulations in either TLM or FDTD, it is found that 5 or 6 reflections are enough. Again, solid and dashed lines, representing the results of
Figure 3.6 Frequency spectrum inside the cavity for $\varepsilon_r = 1.1$ and $\sigma=10^5$ S/m. Dimensions of cavity and mesh size are as before, frequency resolution is 10KHz.

TLM/FDTD and the Green's function formulations, are indistinguishable in the scale of the plots.

The FD variations in Figures 3.4 (for $\sigma=10^4$ S/m) and 3.6 (for $\sigma=10^5$ S/m) demonstrate that the spectrum inside the cavity is continuous and dominant contributions arise near resonances. It is analytically known that, when $\sigma=0$ pure resonances occur inside the cavity, where the only field contributions come from the resonance frequencies. Unfortunately, the Green's function representation in (3.4) has

Figure 3.7 Time variation of $E_y$ observed at (50,50,75) when the source is at (50.50,25)
singularities at these frequencies, which means we can only approach these frequencies numerically in the limit. This is illustrated in Figure 3.8. Here, field strength versus frequency in FD is shown for $\varepsilon_r=1.1$ and $\sigma=10^6$ S/m, where discrete nature of the resonance contributions are clearly observed. Perfect agreement between the results of TLM/FDTD and the Green's function computations are worthwhile to note. The calculations and comparisons are repeated for $\sigma=10^7$ S/m and $\sigma=10^8$ S/m (when $\varepsilon_r=1.1$) and indistinguishable results are obtained. A comparison in TD for $\varepsilon_r=1.1$ and $\sigma=10^8$ S/m is given in Figure 3.9. The agreement continues to be same even for the late time responses.

![Image](image-url)

Figure 3.8 Frequency spectrum inside the cavity for $\varepsilon_r=1.1$ and $\sigma=10^6$ S/m. Dimensions of cavity and mesh size are as before, frequency resolution is 10KHz.

The simulations are also done for different relative permittivity, and similar results are obtained. A typical example is given in Figure 3.10, where $\varepsilon_r=4.0$ and $\sigma=10^5$ S/m. It should be noted that when $\varepsilon_r$ increases wave velocity decreases and one needs to use finer cell sizes in TLM and FDTD to meet the numerical dispersion requirements. Moreover, one need to take longer simulation time in TD, since the higher the relative permittivity the more energy storage inside the dielectric in the cavity, which dissipates more slowly in time.

It is interesting to show some FD differences between simulation and analytical results for $\varepsilon_r=1.1$ and $\sigma=10^5$ S/m at different observation points. In this case, excitation point is same as before, i.e., (50,50,25), but, two different observation points are chosen, (30,50,80) and (75,50,50), respectively. As seen in Figure 3.11a, FD results are almost perfect for the first observation point, but, a slight difference is seen in the second one (see Figure 3.11b).
Figure 3.9 Time variation of $E_y$ observed at (50,50,75) when the source is at (50,50,25)

It is seen that resonance does not analytically occur at about 3.2 MHz, but does in TLM. It is obvious that this observation point ($z=50$) is the null point for $E_y$ and there must not be any resonance at that frequency computed by (3.5) for $p=2$.

Figure 3.10 Time variation of $E_y$ observed at (50,50,75) when the source is at (50.50,25) ($\varepsilon_r=4.0$ and $\sigma=10^{-4}$ S/m)

It is well known that running simulations with either TLM or FDTD techniques require parameter optimization as explained in Section 2.5. Spatial mesh sizes, $\Delta x$, $\Delta y$, $\Delta z$, time step $\Delta t$, simulation period ($T_{obs}=n\times\Delta t$), source bandwidth $B$, pulse duration $\tau$ are characteristic parameters that should be optimally selected a priori the
Figure 3.11 Frequency domain analysis for the structure ($\varepsilon_r = 1.1$ and $\sigma = 10^{-5}$ S/m) for two different observation points: a). (30,50,80) and, b). (75,50,50). Dimensions of cavity and mesh size are as before, frequency resolution is 10KHz.

simulation. On the other hand, the Green’s function representation is exact, but similar steps should be followed when numerically computed.

Table 3.2 lists the requirements of the Green’s function representation in TD comparisons. As seen from the table, the loss factor ($\sigma$) of the medium determines the frequency resolution ($\Delta f$), which in turn specifies the observation period $T_{obs}$. Theoretically, zero frequency resolution can be obtained in the Fourier transformation, since the integration range in TD is infinite (i.e., from $-\infty$ to $+\infty$). On the other hand, everything is finite and discrete in DFT, which introduces well-known periodicity in both TD and FD, aliasing effects and spectral leakage.

Table 3.2 The requirements of Green’s function formulation (3.4) in TD analysis ($\varepsilon_r = 1.1$)

<table>
<thead>
<tr>
<th>$\sigma$ [S/m]</th>
<th>$\Delta f$ [MHz]</th>
<th>$T_{obs}$ [µs]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-4}$</td>
<td>0.5 MHz</td>
<td>2</td>
</tr>
<tr>
<td>$10^{-5}$</td>
<td>0.1 MHz</td>
<td>10</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>10 kHz</td>
<td>100</td>
</tr>
<tr>
<td>$10^{-7}$</td>
<td>1 kHz</td>
<td>1,000</td>
</tr>
<tr>
<td>$10^{-8}$</td>
<td>100 Hz</td>
<td>10,000</td>
</tr>
</tbody>
</table>
3.2. Radiation from an Aperture

The second canonical test problem that we are going to consider is the calculation of the radiation from an aperture in a PEC surface, assuming a known current distribution on the aperture. In order to be able to insert a current distribution similar to that which will be expected in an EMC problem we have chosen not to specify an arbitrary current distribution in an artificial way. Rather we chose to calculate the aperture current distribution which corresponds to that induced on the aperture on the rectangular enclosure shown in Figure 3.12a by a short dipole located inside the enclosure.

Figure 3.12 The diagram of the problem space for the 3D TLM program. By using the time domain values in the aperture, the time domain fields at points 1, 2 and 3 are calculated directly by the TLM program.

3.2.1. Analytical representation

Once the distribution of the dominant electric field component in the aperture

\[ E_a = E_\alpha^y a_y \]  

(3.16)

is obtained, the corresponding equivalent magnetic surface current can be determined as

\[ M_s = 2E_a \times n = -2E_\alpha^y a_z. \]  

(3.17)

The radiated E field can be calculated via [58-60]
\[ E = -\frac{1}{4\pi} \int_S \nabla_r \times \left[ M_s(r') \frac{e^{-jkR}}{R} \right] dS' = -\frac{1}{4\pi} \int_S \left[ M_s(r') \times \hat{r} \right] \left( jk + \frac{1}{R} \right) \frac{e^{-jkR}}{R} dS', \]

(3.18)

where right hand side \( E \) field is obtained in terms of the magnetic currents \( M \) at the aperture. As shown in Figure 3.13, the cause of the aperture fields is the \( y \)-polarized point source.

![Figure 3.13 Source and observation points together with aperture. Dimension of the aperture is 1.25cm×2.5cm.](image)

The aperture geometry and the coordinate system are shown in Figure 3.13. Substituting (3.17) into (3.18), we can write \( E \) field in terms of the two components:

\[ E_x = -\frac{1}{2\pi \ aper.} \int r_y E_y' \left( jk + \frac{1}{R} \right) \frac{e^{-jkR}}{R} dS', \]

(3.19)

\[ E_y = \frac{1}{2\pi \ aper.} \int r_x E_x' \left( jk + \frac{1}{R} \right) \frac{e^{-jkR}}{R} dS', \]

(3.20)

where \( r_x \) and \( r_y \) represent components of the unit vector \( \hat{r} \) (see Figure 3.13) in the \( x \) and \( y \) directions, respectively.

Next we derive a finite difference algorithm for the evaluation of the integral in (3.20).
Since the $E_y$ is dominant component ($E_x \ll E_y$), only this component is calculated below. Remember that, one can easily duplicate the results for $E_x$ too. First, (3.20) is separated into two parts:

$$
E_y = \frac{1}{2\pi} \int_{\text{aper.}}^{} r_x E_x^y \frac{e^{-jkr}}{R} dS' + \frac{1}{2\pi} \int_{\text{aper.}}^{} r_x E_a^y \frac{e^{-jkr}}{R^2} dS'.
$$

(3.21)

We convert the first term of (3.21) into the TD as;

$$
E_{y1}(x,y,z,t) = \frac{1}{2\pi c} \int_{\text{aper.}}^{} r_x \frac{d}{dt} \frac{E_a^y(y',z';t-R/c)}{R} dy'dz'.
$$

(3.22)

In TLM, the time and distance are discretized as

$$
t = n\Delta t, \quad R = \Delta t R_{\Delta}
$$

(3.23)

where $\Delta t$ is the unit cell size used in the TLM simulation (Note that $\Delta x = \Delta y = \Delta z = \Delta \ell$ that is cubic cell.). Time step $\Delta t$ and cell size $\Delta \ell$ are related to as given in (2.44).

Therefore, the retardation in $E_a^y$ in (3.22) and spatial quantization become

$$
\frac{R}{c} = \frac{R_{\Delta} \Delta \ell}{c} = \frac{R_{\Delta} \Delta t 2c}{c} = 2\Delta t R_{\Delta}, \quad x = i \Delta \ell, \quad y = j \Delta \ell, \quad z = k \Delta \ell
$$

(3.24)

and the aperture field is

$$
E_a^y(y',z';t-R/c) = E_a^y(j',k';n-2R_{\Delta})
$$

(3.25)

The first-order approximation for the time derivative in (3.22) is

$$
\frac{d}{dt} E_a^y(y',z';t-R/c) \approx \frac{E_a^y(j',k';n-2R_{\Delta}) - E_a^y(j',k';n-2R_{\Delta}-1)}{\Delta t}
$$

For convenience, we define the parameter

$$
diff_1 E_a^y(j',k';n-2R_{\Delta}) = E_a^y(j',k';n-2R_{\Delta}) - E_a^y(j',k';n-2R_{\Delta}-1). \quad (3.26)
$$

and finally obtain
\[ E_y(i, j, k; n) = \frac{1}{\pi} \sum_{j', k'} \frac{r_{x}}{R_{\Delta}} \ \text{diff} \ _{\Phi} \ E_y(j', k'; n - 2R_{\Delta}). \] (3.27)

The second term of (3.21) is transformed into the TD by the same discretization procedures and

\[ E_y(i, j, k; n) = \frac{1}{2\pi} \sum_{j', k'} \frac{r_{x}}{R_{\Delta}} \ E_y(j', k'; n - 2R_{\Delta}) \] (3.28)

is obtained. Finally, we write the two components of \( E_y \) together by substituting (3.27) and (3.28) into (3.21);

\[ E_y(i, j, k; n) = \frac{1}{\pi} \sum_{j', k'} \frac{r_{x}}{R_{\Delta}} \left[ \text{diff} \ _{\Phi} \ E_y(j', k'; n - 2R_{\Delta}) + \frac{E_y(j', k'; n - 2R_{\Delta})}{2R_{\Delta}} \right] \] (3.29)

Obviously, One can easily obtain the identical expressions for the calculation of \( E_x \) in (3.19), except with an \( r_y \) term instead of \( r_x \). Note that \( E_x \) and \( E_y \) in TLM method have already expressed in terms of TLM voltage pulses as in (2.61a) and (2.61b).

3.2.2. Validation tests

For validation purposes we have compared the results obtained via on TLM algorithm with those calculated from (3.29). For this purpose we proceeded as follows:

- The TD simulation for the structure given in Figure 3.12 is performed via the TLM simulator.
- During this TD simulation, magnetic surface currents are obtained at each cell of the aperture.
- From these surface current sources, analytical results at the chosen observation points are calculated via (3.29) and are compared with the TD simulation results at the same observation points.

We have compared the time variation of the \( E_y \) component at the 3 observation point locations shown in Figure 3.12b. The problem parameters are outlined below: The once-differentiated Gaussian pulse with 9GHz bandwidth is used to excite the \( y \)-component of \( E \). The structure and chosen observation points are illustrated in Figure
3.12. The TLM computation space is $87 \times 87 \times 87$ cells with a cell size of 0.25 cm. It is terminated by 6-cell PML ABCs at each side [45]. The dimension of the EUT is $20 \times 20 \times 20$, with an aperture of $5 \times 10$ cells in the $y$ and $z$ directions, respectively. The $E_y$ is excited at point (25,38,38) which is the center of the EUT. Point 1 is located at (50,38,38), point 2 is at (65,38,38) and point 3 is at (65,55,38). The calculated results are given in Figure 3.14. These results belong to the observation points 1, 2 and 3, respectively. As seen in Figure 3.14, analytical and simulation results agree very well.

Ringing effects are clearly observed in Figure 3.14 indicating resonant nature of the structure. Note also that the wave arrives at OP2 at a later time than it arrives at OP1 with a delay which is, of course, consistent with the mean path difference between them.

![Figure 3.14 Comparison of analytical results with the TLM solutions. Observation points 1, 2 and 3 show the comparison of TLM data (dashed line) and the time domain data generated via (3.29) (solid line)](image)

Comparisons of the TLM results with the analytical solutions of the investigated canonical problems presented in this section show that the two solutions are in perfect agreement. This is a clear indication that we have implemented the TLM code with all of its functionalities (such as modeling ABC, PEC and calculation of absorbed powers) correctly (verification) and that the calculated results are sufficiently accurate (validation).
4. APPLICATION OF THE TRANSMISSION LINE MATRIX METHOD TO TWO SPECIFIC, COMPLEX PROBLEMS

In this section, the application of TLM to two complex EM wave problems will be discussed. These problems concern the determination of

- Shielding Effectiveness (SE) and,
- Specific Absorption Rate (SAR).

Both in SE and in SAR calculations we have investigated a number of structures which represent typical, albeit somewhat simplified models of real life problems.

SE is the main performance measure for EMC in EM system design. SAR plays a similar role in problems involving the interaction of EM energy with matter such as those related to the nowadays rather popular issue of determining the adverse effects (if any) of mobile phones on users. Due to their wide ranges of application both SE and SAR calculations have been active areas of research in the past two decades, and they continue to be so at an ever-increasing rate. Despite these efforts a flexible (widely if not universally applicable) and dependable (sufficiently accurate, validated) method has not yet evolved.

The difficulties in modeling and simulation of real-life SE or SAR problems stem from two sources: (i) In most, if not all cases, the parameters of the mathematical model can only be crudely or generically determined (as in the case of human head models) and (ii) in general accurate solutions can not be obtained for the resulting mathematical model even with the most advanced computational facilities available today. These difficulties preclude the generation of analytical or measured reference solutions for the class of problems (referred to as “complex EM problems” in this thesis) to be addressed in this section. On the other hand, SAR, one of the TD complex EM problems to be investigated in this section has so far addressed only via the FDTD method [61-63]. Moreover, particularly for SAR calculations, it has not been possible to assess the validity of these FDTD results due to lack of reference and/or independently generated alternative solutions. The main objective of this investigation is to fill in the above mentioned void.
It should be noted that a structure very similar to that used in this thesis for SE calculations has also been considered in the literature [64], where SE calculations obtained via Method of Moments (MoM) were found to be in agreement both with FDTD calculations and also with experimental results. For this reason, we have considered also the results given in [64] as an additional comparison solution in assessing the accuracy of our TLM calculations.

To the best of our knowledge, this thesis constitutes the first systematic treatment of these problems via the TLM method and the first attempt to "validate" the generated results by comparing them with independently generated results obtained via the FDTD method. It has to be noted that the above procedure can not be regarded as validation in the strict sense. However, since these two methods are based on radically different cell structures, and hence, can be regarded as totally independent ways of attacking the problem at hand, we believe that it is safe to assume that the results obtained are accurate whenever there is a close agreement between them. This claim is further substantiated via the close agreement between our SE results with those reported in the literature [64,65]. It will be seen from examples given in this chapter that in most cases the agreement between the results obtained using the two methods ranged from good to excellent as the discretization was progressively refined. On the other hand, we will also give examples to and explanations for some of problematic regions, wherein relatively large discrepancies between the two results continued to exist up to the finest (but evidently not fine enough) discretization levels used. With these remarks we will now proceed to discuss the specific examples of SE and of SAR calculations that we have performed.

4.1. Shielding Effectiveness Calculation

EMC deals with EM interference problems where major design tools are grounding, shielding and filtering. Here we will be concerned with shielding which refers to the control of radiated EM fields. Electrical shielding is the process of preventing radiation from coupling into or out of defined areas or regions. Commonly used shielding materials are metals, metalized plastics (conductive coatings) and conductive composites.

Shielding effectiveness (SE) is a measure of a shield's performance to attenuate electric and/or magnetic fields. It is defined in dB values as

\[
SE_{dB} = 20 \log_{10} \frac{E_{before}}{E_{after}} \tag{4.1}
\]
for electric fields and as,

\[ SE_{dB} = 20 \log_{10} \frac{H_{\text{before}}}{H_{\text{after}}} \]  \hspace{1cm} (4.2)

for magnetic fields. The subscripts "before" and "after" correspond to the field amplitudes measured at an observation point before and after a shield barrier is positioned.

At this point it should be noted that in the literature one encounters conceptually different approaches in SE calculations each of which relies on somewhat different definitions of the SE. The SE is defined by the IEEE simply as the ratio of the fields at a point with and without the shield in place [66]. This is the definition adopted in this work as given by (4.1) and (4.2). Clearly, when the observation point is chosen in the far field of the device there is no need to distinguish between electric and magnetic SE defined in (4.1) and (4.2), respectively. The definitions used here render SE to be a point function which depends on several factors including position and type (plane wave with certain polarization, dipole etc.) of the excitation used, frequency and on the observation point location. Hence, it comes as no surprise that in the literature different authors have calculated SE using different assumptions for these factors. A frequently used approach is to calculate SE of a device under the assumption that the excitation is a (pulsed) plane of wave of given polarization [67,68]. Although this assumption is useful since it allows one to state SE values independent of source position, it is not representative of many EMC problems of practical interest. This is why, some authors [64,69] have used a dipole source at a given position and with a given orientation when performing SE calculations.

On the other hand, in industrial applications the SE of a volume is usually specified by a single number (typically a worst-case figure). This is a fairly simplistic measure as it ignores both the effects of the important factors mentioned above and also relies on the assumption that the introduction of the shield does not change source characteristics which implies that the shield is well removed from the source.

In this thesis we will use the definition given by (4.1) in SE calculations, which is a measure of the ratio of the electric fields at a point with and without the shield in place. It should be noted that the observation point locations considered in our SE calculations are chosen to be within or close to the near field of both the source-excited and the shield diffracted fields. Utilizing near field quantities in (4.1) is in complete compliance with the IEEE definition of the SE as a general point function.
and our calculations may be used to realistically estimate the effects of near field interactions within electrically small devices. On the other hand, the choice of the observation points in the near field allows one to use smaller computational spaces in TLM calculations and helps addressing problems which would otherwise be intractable with the computational resources available to us.

The total shielding effectiveness of a barrier can be approximated by adding the dB values of the reflection and absorption losses of the barrier. When a thick barrier is used, third term may also be used for the additional losses caused by multiple internal reflections inside this barrier.

Shielding effectiveness depends upon a number of factors, such as the EM wave impedance, frequency, polarization and, thickness of the barrier, its composition and the type of the radiating element, among which the factor related to the radiating element is usually dominant. There are two types of radiating elements: sources that behave as electric dipoles (wires) and sources that behave as magnetic dipoles (loops). Electric dipoles have dominant electric type near fields and high impedance, but magnetic dipoles have dominant magnetic type near fields and low impedance. Therefore the latter presents a more difficult shielding problem. Of course radiation fields of both types of sources become similar and can be approximated by plane waves at distances which are a few wavelengths away from these elements.

In shielding practice, depending on the criticality of the problem under consideration, SE values of 30-50dB are considered to be acceptable, whereas, SE values in the range of 60-90dB represent very high quality shields. The shielding performance of a metal box with no holes or seams exceeds 100dB which is usually too high to measure. If the barrier has holes, slots, joints, vents, windows or other discontinuities the shielding effectiveness can only be as good as allowed by such shielding imperfections. One rule of thumb is to find the longest dimension of these discontinuities and determine the frequency at which that length represents a half-wavelength. As a worst-case approximation the barrier can be assumed to yield 0dB shielding effectiveness beyond that frequency.

Shielding boxes are very common in practice since they furnish effective shielding while at the same time providing a structure for supporting circuit boards, easing cooling requirements, and isolating different subsystems.

In this section, shielding effectiveness of boxes with different apertures on its walls are investigated. The structures used in numerical calculations are given in Figure 4.1. These structures are identical in form with those investigated in [64] via MoM
and FDTD calculations and also experimentally. Hence, at the end of this section we will also present the results obtained via our TLM code using the same parameters as was used in this paper in order to be able to compare our results with those reported in [64]. Here, a PEC box of 10cm×10cm×10cm, which may be considered as equipment under test (EUT) of a typical EMC problem, is located inside the TLM computation space of 26cm×26cm×26cm for Figure 4.1a and 40cm×30cm×30cm for Figure 4.1b.

![Diagram](image)

**Figure 4.1** The test structures for SE measurements. EUT is a a). five-facet, b). six-facet cube with an aperture. Dimensions of outer box are 26cm×26cm×26cm and, 40cm×30cm×30cm, respectively. Inner box is 10cm×10cm×10cm for both structure. And mesh size for (a) and (b) is 1cm and, 0.667cm, respectively.

Inside the TLM space, the EUT is placed 8cm above the floor and 4cm away from the x=0 plane. The EUT has rectangular apertures on its sides with different sizes. In Figure 4.1a, the right vertical wall of the EUT is removed (i.e., the whole wall is an aperture). The field radiated from a vertically polarized, 6cm long dipole antenna placed within the EUT is monitored at a distance of 2cm away from the front face of the EUT. The feed point of the dipole and the observation point are located on a line parallel to the floor and 14 cm above it.

This configuration is modeled with a mesh of 26×26×26 nodes in the TLM, each representing a cube of space with sides equal to 1cm. As noted before, in order to yield an identical computational domain the same problem is discretized with 27×27×27 cells when using FDTD. In the simulations, two different boundary conditions were considered at the boundaries of the computational domain: PEC walls simulating a screened room and absorbing boundaries simulating free space.
ABC were modelled by PML [45] condition in TLM and Mur type boundary conditions in FDTD [70]. The dipole inside the EUT is excited by a pulse with the bandwidth of 2.5GHz. As shown in Figure 4.1a, the antenna is placed vertically exciting y-polarised waves and its feed point is at (12,14,13). Same structure and parameters are used in TLM and FDTD calculations.

The TD variation of $E_y$ at the observation point OP1 (20,14,13) calculated with TLM and FDTD methods are given in Figures 4.2a and 4.2b for the five facet PEC cube depicted in Figure 4.1a, respectively, for the two cases wherein the computation space is terminated by ABC and PEC boundaries. The source has a once-differentiated Gaussian function temporal distribution. Although the same source function is used in both cases, it is evident from these figures that there are some discrepancies between the results computed via TLM and FDTD.

![Figure 4.2 Time variation of $E_y$, where EUT in Figure 4.1a is placed a). in free-space, b). in screened room.](image)

As observed in the figure, although the first few peaks of both TLM and FDTD simulations which may be identified as direct wave contributions are in perfect agreement, there are quite high discrepancies in later time responses. The poor agreement in late time responses occur because of the fact that the discretization used (Note that the ratio of minimum wavelength $\lambda_{\text{min}}$ to the mesh size $\Delta\lambda$ is 12 here) fails to model the effects of secondary wave contributions due to scattering from the edges of the aperture. These discrepancies at later times can thus be attributed to the so called “numerical dispersion” phenomenon discussed in Section 2.4. Although some improvement can be obtained in both methods if a finer mesh is used, the errors due to the above mentioned diffracted field contributions will clearly persist. On the other
hand, these differences in TD solutions are suppressed to a large extent in the FD response as shown in Figure 4.3.

![Graph showing resonance frequencies of the screened room (26cm×26cm×26cm) including the EUT (in Figure 4.1a).](image)

**Figure 4.3** Resonance frequencies of the screened room (26cm×26cm×26cm) including the EUT (in Figure 4.1a).

The SE of these structures can now be obtained from FD data using (4.1) or (4.2). Below listed steps have to be followed in SE calculations:

- First, the field produced by a source which is placed inside the EUT is observed at the observation point (chosen outside the EUT) in the TD and electric field \(E_{\text{after}}\) versus time is accumulated.

- Then, the EUT is removed and the same procedure is repeated without changing any parameters (such as excitation and observation points, source amplitude and bandwidth etc.) and electric field \(E_{\text{before}}\) versus time is accumulated.

- Both data \((E_{\text{before}}(t)\) and \(E_{\text{after}}(t)\)) are Fourier transformed (via DFT). The frequency variation of SE at the observation point is then calculated via (4.1).

- It should be noted that in performing simulations the parameters should always be selected according to the criteria explained in Section 2.5.

- It is also the fact that the EUT may not be a closed structure or SE may not be required for only inside/outside separations. It may be a barrier of any shape between two points (the interfering source and the victim).
The computed results for SE corresponding to PEC terminations on the computational domain are given in Figure 4.4. In this figure, bottom and top plots correspond to the direct application of (4.1) (named as raw data analysis) and application of (4.1) after windowing, respectively. Due to the multiple reflections on the PEC boundaries of the computational domain and also at the walls of the five-

![Diagram](image)

Figure 4.4 SE versus frequency obtained with TLM and FDTD methods for EUT in Figure 4.1a; a). windowed data, b). raw data

faceted EUT, the EM field will exhibit a slowly damped ringing character. Long simulation times have to be used in order to obtain frequency behaviours of these kinds of structures, i.e., PEC resonators. Indeed, we must theoretically observe the field for infinite time duration. Running the simulations for a limited period is equivalent to observing fields inside a rectangular time window. If transient response is trapped within this rectangular window, then it does not cause a problem in Fourier transform. Otherwise, using the rectangular window causes an abrupt truncation in the TD data, which causes aliasing and spectral leakage problems and, may yield totally misleading results. To avoid this, different windowing techniques such as Hanning, Hamming etc., are used in truncating the data in the TD simulations without causing severe problems. This is clearly observed in Figure 4.4, where very good agreement is obtained after the application of the Hanning window. However, there are still some discrepancies between TLM and FDTD results at low and high ends of the frequency spectrum, i.e., below about 0.6GHz and above 2.3GHz. This behaviour is to be expected since the source spectrum rapidly decays at both ends (see Figure 3.2) and hence both methods will contain comparatively large errors at these regions.

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The same SE analysis is also performed for the EUT in free space and the results are given in Figure 4.5. As observed from this figure, calculated SE values are around 0dB indicating that there is almost no isolation between the source and the observer, which is to be expected since no barrier exists in between. It is seen that SE becomes negative at around 1.5GHz, which means the presence of the EUT actually increases the field at the observation point. This behaviour can be explained by noting that around this frequency the wave constituents reflected from the walls of the EUT are expected to interfere constructively at the observer.

![Figure 4.5 SE versus frequency obtained with TLM and FDTD methods for EUT in Figure 4.1a.](image)

The second problem to be investigated is the calculation of SE for the structure depicted in Figure 4.1b. In this problem the EUT is taken as a PEC 10cm×10cm×10cm cube with an aperture on one of its faces. The parameters used in the simulation are given below:

- **Source**: 3GHz bandwidth, once-differentiated Gaussian applied to a center fed dipole antenna of length 6.6cm (feed point: (30,22,22)),
- **Mesh size**: 0.667cm (corresponding to $\lambda_{min}/\Delta\lambda=15$),
- **Computation volume**: Free space, 40cm×30cm×30cm (corresponding to $60\times45\times45$ cells),
- **Aperture**: 2.7cm×4cm (corresponding to $4\times6$ cells) located at the center of the front wall of the EUT at x=22cm.
We have calculated the SE of this structure at the following three observation points OP1 (41,22,22), OP2 (30,38,22), and OP3 (10,22,22). The three observation points used in the SE calculations are chosen as follows:

- OP1 is in front of the EUT and directly faces the aperture,
- OP2 is above the EUT, and
- OP3 is behind the EUT.

Locations of different observation points are indicated in the insets of respective figures for convenience. The SE calculated at OP1 is given in Figure 4.6.

![Graph showing SE versus frequency](image)

Figure 4.6 SE versus frequency obtained with TLM and FDTD methods for EUT in Figure 4.1b. Aperture dimensions are 2.7cm×4cm. Observation point is at OP1 (41,22,22).

The variations of SE at the two other observation points OP2 and OP3 are shown in Figures 4.7a and 4.7b, respectively. As seen, OP3 (behind EUT) results in higher SE values when compared to those obtained for OP1 and OP2. SE at OP2 is also higher than that at OP1. We see that both TLM and FDTD methods satisfactorily model this problem over the entire frequency range. We note that SE drops sharply at all observation points around 1.8GHz which approximately corresponds to a resonance of the aperture (aperture size is λ/4 at 1.875GHz).

Following two observations can be made in connection to the variation of SE with frequency at the three observation point locations depicted in Figures 4.6 and 4.7:
Figure 4.7 SE variation of EUT in Figure 4.1b at two different observation points simulated via TLM and FDTD methods: a). OP2 (30,38,22) and, b). OP3 (10,22,22).

i) The general behaviour of the variation of SE exhibits a similar pattern for all observation points indicating that the dominant effect of changing the observation point is just a scaling of the field amplitudes but it does not significantly modify their ratio. This observation is consistent with the experimental and numerical data calculated via FDTD and MoM methods in the literature [64,65,69].

ii) In all cases the calculated SE peaks up at around 1.8GHz which is to be expected since this frequency is near to one of the resonances of the empty enclosure and at the same time to that of the excited aperture. It should however be noted that the peak SE (worst-case) value decreases by about 20dB as the observation point is moved from OP1 where it is in front of the aperture to OP3 where the field radiated by the aperture is shadowed by the EUT. Recalling that in view of the utilized dipole excitation one would have symmetric forward and backward lobes we conclude that the back-scattered field is in this case weaker by about 20dB.

As another example we will give calculated SE results for an EUT which has two apertures. These calculations are performed using exactly the same parameters as those given in the previous single aperture example. The only difference is that EUT now has a second aperture on the top or in the back face in addition to the one in the front face. Apertures are of the same size and are located at the centers of the respective faces. We have considered following three cases:
i) One aperture in the front and one in the back faces placed parallel to each other, i.e., with the longer dimension along the z-axis,

ii) One aperture in front and one on the top faces, placed parallel to each other, i.e., with the longer dimension along the z-axis,

iii) The same as (ii) above but the aperture on the top face is rotated so that its longer dimension is now along the x-axis.

The SE calculations at the observation point OP2 (30,38,22) performed via TLM are given in Figure 4.8a through 4.8c. The positions of the apertures are also indicated in the insets of these figures for convenience. In the same figures the SE variations at OP2 in the case of a single aperture in the front face given in Figure 4.1b, as the TLM result are replicated in dashed lines, for comparison purposes. The results given in Figure 4.8 clearly indicates that the interaction of EM waves emanating from the two apertures may result in decrease or increase of SE at an observation point depending on characteristic parameters, especially the frequency.

Figure 4.8 SE of EUT with multi-apertures located at different faces in free space. Observation point is taken at top (OP2) that is (30,38,22). Dashed lines show the plot corresponding to the single-aperture case given in Figure 4.1b.

In connection with the variations depicted in Figure 4.8 we first note that the overall behaviour of the variation of SE with frequency is very similar in all cases, although a second aperture is now introduced on another face of the EUT. It should again be
noted that this general behaviour is consistent with the data calculated via other numerical methods and reported in the literature [65]. For the cases depicted in Figures 4.8b and 4.8c wherein the second aperture is placed on the top face this can be explained by the fact that the scattered E field from these apertures should be of lower order since they are in the x-z plane, while the exciting dipole used in the calculations is y-polarized. In connection with Figure 4.8a we note that OP2 used in these calculations is along the direction of the exciting y-polarized dipole. Thus the reference field calculated at this location when the EUT is removed represents near field effects only. On the other hand, for the case depicted in Figure 4.8a where there are two slots one in the front and the other on the back-face of the EUT one would expect to obtain an identical variation of SE with that corresponding to the single aperture case depicted in Figure 4.7a and plotted in dashed lines in Figure 4.8a except for an increase of about 6dB, if the observation point location were also symmetric with respect to the two apertures. However, in the calculated example the dipole and hence the observation point is closer to the aperture at the front face of the EUT and this is reflected by the fact that existence of a second aperture on the back face of the EUT introduces a minute effect over a wide portion of the frequency band considered.

As indicated at the beginning of this section we would like to conclude our SE calculations by trying to reproduce the results obtained on identical structures (apart from their dimensions and the excitation), and reported in the literature [64,65]. With reference to Figure 4.1a the parameters used in [64,65] are;

- Dimensions of the EUT : 50cm×50cm×50cm,
- Aperture : 20cm×5cm located at the centre of the front wall of the EUT,
- Source : 1GHz bandwidth, once-differentiated Gaussian pulse excited via a short dipole placed at the centre of the EUT,
- Mesh size : 1.667cm (corresponding to $\lambda_{min}/\Delta\lambda=18$),
- Computation volume : Free space, 150cm×116.7cm×116.7cm (corresponding to 90×70×70 cells).

Similar to the case investigated by us and depicted in Figure 4.1b in [64,65] SE calculations are also performed for the EUT wherein the aperture is increased to cover the front face completely, whereby resulting in a five-faceted cube. The configuration is excited by a short electric dipole placed to the centre of the EUT and the field is observed at a position coinciding with the centre of the aperture and
removed from the aperture by a distance of 25 cm and SE is calculated via (4.1). It should be noted that the results given in [64] are calculated via Method of Moments and FDTD and are also compared with experimentally measured data. It was found in [64] that the results calculated by the two different numerical codes are in agreement with each other and also the experimental data. In Figure 4.9 we present our TLM results obtained using identical parameters as used in [64] and explained above, together with the MoM solutions given in [64].

As seen from Figure 4.9 our TLM results that we have obtained for the two test structures explained above agree very well with those reported in [64,65]. In view of the almost perfect agreement between TLM and FDTD solutions and between TLM and (experimentally validated) MoM solutions presented in this section we believe that it is safe to conclude that our SE calculations via the TLM method are rather accurate.

![Figure 4.9 Variation of the SE with the frequency for the structures shown in Figure 4.1 when the dimensions and excitation are taken to correspond to those utilized in [64] and explained in the text. Solid lines and signs correspond to our TLM results and to the MoM data read from the corresponding figures in [64], respectively. a). SE for EUT with an aperture and, b). for 5-faceted cube.](image)

Moreover, with the aid of the results obtained in the various test problems investigated in this subsection it is possible to state some characteristic features of SE modeling via the TD simulators:

- The TD simulators are very flexible and can be used to obtain accurate estimates of the SE at all observation points for almost all structures (geometry, source characteristics, medium parameters) of practical significance.
• SE of an enclosure, aperture, or a barrier is critically dependent on the problem parameters. In other words, the estimation of the frequency variation of SE for a given realistic structure is a task which can not be addressed using even the best EMC engineering expertise. The test cases presented here illustrate that SE estimations can only be performed either by measurements or via simulations covering all frequencies of interest.

4.2. Specific Absorption Rate (SAR) Modeling

With the ever increasing use of personal electronic and communication devices (such as cell phones) in the last decade EM energy-human tissue interactions have become important issues of public concern. The main parameter used in evaluating the effects of this interaction is the specific absorption rate (SAR). SAR is the time rate at which EM RF energy is imparted to a defined amount of mass of a biological body. There are two alternative ways of obtaining SAR values: laboratory measurements and computer simulations.

A procedure developed for a laboratory measurement by Dr. Niels Kuster of the Swiss Federal Institute of Technology [71] uses a thin-walled fiberglass model (also called phantom) of a human torso filled with a dielectric solution approximating the electrical properties of fluid inside the human skull. A wireless device is operated near the phantom under full RF power. An E-field probe makes localized measurements near the ear area inside the liquid-filled phantom by means of a high-precision positioner. SAR values at different locations are computed using the measured E-field values and presented in form of a color SAR density map.

The procedure for computer simulations is almost same. TLM (and also FDTD) can be used for this purpose. A human head computer image (such as that obtained from an NMR) is used to generate a discrete model (phantom). As many as millions of cells may be used in discrete human head models. Different electrical parameters are assigned to different cell groups. These parameters are chosen in such a way that they are approximately equal to the parameters obtained via laboratory measurements at different parts of the human tissues corresponding to cell groups used in the discretized model in representing these parts. The discrete (simplified) model of the wireless device is placed near the discrete human head phantom and the interaction of the model with EM energy radiated from the device is simulated directly in TD. Since E-fields are calculated at every cell at every time instant, the variation of SAR over the model can be obtained, and if desired, can be presented in form of a colour SAR density map. In some investigations comparisons of measured
and calculated SAR results obtained on simple structures are also given. Here we would like to specifically mention wherein the use of TLM for SAR calculations are given for a uniform spherical region [72]. It is reported that the calculated results are in good agreement with the measured SAR values for this simple test problem.

National and international ruling bodies determine the standards for allowable values of SAR. As noted above SAR specifies the rate at which energy is absorbed in the tissue, and is commonly stated in units of watts per kilogram [W/kg] or milliwatts per gram [mW/g]. SAR is the dosimetric measure with which the basic safety limits have been defined above 10MHz.

In tissues, SAR is proportional to the square of the electrical field induced in the tissue by the external sources (e.g., cellular phone). At any point in the tissue SAR is calculated as

\[ SAR = \frac{\sigma}{\rho} |E|^2 \]  

(4.3a)

where \(|E| [\text{V/m}]\), \(\sigma [\text{S/m}]\) and \(\rho [\text{kg/m}^3]\) are the magnitude of the measured or computed RMS electric field, conductivity and density of the tissue, respectively. More than often SAR values for a group of tissues are of interest, in which case SAR is calculated as the ensemble average. Using its definition, as

\[ SAR = C \frac{\delta T}{\delta t} \]  

(4.3b)

where \(C [\text{joule/kg.}^\circ\text{C}]\) is the specific heat of tissue, \(\delta T [^\circ\text{C}]\) is the temperature rise and \(\delta t [\text{sec}]\) is the exposure duration (see [63,73-76]). However, in this method of determining SAR is not suited to TLM computer simulations and will not be used here.

It should be noted that in order to be able to show the periodicity of the variation of instantaneous power density in this thesis we preferred to use the instantaneous values of the electric field strength, in (4.3a) rather than its RMS value. However, since all calculations involved time harmonic excitations and since in all cases the SAR calculations were performed after the establishment of the steady state conditions wherein the field distribution on all cells considered remained invariant except for the periodic fluctuations dictated by the time harmonic excitation not only our cell-based SAR values but also SAR values obtained by averaging over many cells can be converted into those used in the literature by simply modifying them
with a weighting factor. The reason why we have preferred to calculate SAR via the instantaneous field strengths is that although it is straightforward to convert these into SAR values obtained by using the RMS field values as is applied in most SAR calculations it is generally not possible for SAR values averaged over many cells to estimate the instantaneous peak exposure levels when calculations are based on RMS field values. Since the effects of the instantaneous peak exposure levels are also gaining on interest [77] and since these are obtained in TLM calculations without any additional computational effort we decided to use instantaneous field strength values in (4.3a) for SAR calculations.

The most recognized EM exposure standards, including ICNIRP, adopt the specific absorption rate (SAR), averaged over the whole body (SAR_{WB}), as the basic parameter to establish the safety limit for exposure [25,78-80]. The value of 0.08W/kg is generally accepted as the basic upper safety limit for SAR_{WB}.

Sometimes, although SAR_{WB} becomes may be lower than the basic limit, the local SAR values can exceed the limit in certain parts of the body region, where the power absorption takes place, such as a human head near the antenna of a mobile phone. Limits on local SAR averaged over tissue masses of 1g or 10g have also been introduced in the standards. These limits are 1.6W/kg over 1g and 2W/kg over 10g.

In this section, the calculation of local SAR with TLM method will be investigated. As in the previous section the computed results will be compared with results obtained by applying FDTD to the same problems. Moreover, the SAR values on single computational cells are also calculated which better display differences between results obtained via TLM and FDTD. It will be shown that in regions wherein local SAR limits are met, when viewed at the finer scale of one computational cell size one may encounter peak SAR values which exceed the limits.

At this point it would be appropriate to discuss some features of power calculation in TD TLM simulations. The most important steps in numerical SAR modeling are the calculation of radiated and absorbed power.

- Calculation of absorbed power

As noted in Chapter 2 matched stubs are used TLM nodes to model medium losses. One extra stub per electric field component is required at each cell in order to represent dielectric losses. This results in a cell structure with a total number of 21 ports. Any pulses reflected down the new ports, 19 to 21, are considered lost. Hence no pulses are incident on these lines. This gives a $21 \times 18$ scattering matrix $S$, in the
following form

\[
\begin{bmatrix}
V^r \\
V_{19} \\
V_{20} \\
V_{21}
\end{bmatrix} = SV^i.
\] (4.4)

The total loss (absorbed) power dissipated in the node is then calculated via

\[P_{\text{absorbed}} = G_x V_{19}^2 + G_y V_{20}^2 + G_z V_{21}^2.\] (4.5)

It should be remembered that the same loss power can also be calculated via

\[P_{\text{absorbed}} = \sigma \times \left( E_x^2 + E_y^2 + E_z^2 \right)\] (4.6)

where \(\sigma\) is the conductivity parameter.

- *Calculation of radiated power*

The radiated power is calculated by integrating the poynting vector over a closed virtual surface \(S\) as

\[P = \oint_S (E \times H) \cdot dS\] (4.7)

where

\[dS = dS \cdot n\] (4.8)

and \(n\) is the unit normal vector directed outwards from the surface. Since rectangular coordinate system is used in the TD simulations, virtual surface is also chosen to be rectangular. It is also chosen in such a way that includes the EUT and is not too close (typically at a distance of 8 to 10 cells) either to the EUT or to the boundaries of the computational domain.

Two different types of excitation are used in the simulations. First one is a Gaussian type band-limited pulse and the second one is sinusoidal excitation. But in the latter, Hanning windowing technique is used to avoid excitation instabilities and sinusoidal steady-state error in the system as explained in [81]. Windowing technique is applied to the wave for the duration of two periods in order to effectively suppress the
transients. The time evolution of the resultant sinusoidal wave is shown in Figure 4.10. The mathematical expression of this sine wave is as follows:

$$\text{Source} = \text{AMP} \times \sin(2\pi f) \times 0.5 \times \left(1 - \cos(\pi \times \text{IT} / \text{NP})\right)$$  

(4.10)

where \(\text{IT}\) is the iteration number and \(\text{NP}\) is given by

$$\text{NP} = \frac{2}{f \times \Delta t}$$  

(4.11)

where \(f\) and \(\Delta t\) are frequency and time-step, respectively.

Figure 4.10 Sine wave of which two periods are suppressed by half Hanning window

To test the radiated and absorbed power subroutines used in SAR calculations we will consider the structure depicted in the inset 4.11 and 4.12. This structure is composed of two lossy dielectric cubes one of which is located inside the other. The computation volume of 14cm×14cm×14cm, which is modelled with 70×70×70 cells using the unit cell size, \(\Delta\lambda = 0.2\text{cm}\). Dimensions of the outer dielectric cube used in calculations is 6cm×6cm×6cm (30×30×30 cells), while the dimensions of inner cube are 2.6cm×2.6cm×2.6cm (13×13×13 cells). The electrical parameters of the outer and inner cubes are \(\varepsilon_2 = 5\) and \(\sigma_2 = 0.2\text{S/m}\) and, \(\varepsilon_1 = 2.5\) and \(\sigma_1 = 0.5\text{S/m}\), respectively. The outer cube is placed between \((x;23.52; y:23.52; z:23.52)\), while the inner one is between \((x:31.43; y:31.43; z:31.43)\). The \(y\)-component of electric field is excited at point \((18,18,18)\). Time variation of the radiated power is shown in Figure 4.11 for the case when the computation space is terminated by PEC boundaries which form a resonator. Figure 4.11a corresponds to the response of the resonator. The source is
Figure 4.11 Poynting power for a) an empty resonator, b) EUT in a PEC resonator. The computation volume is 70x70x70 cells and unit cell size, \( \Delta \lambda \) is 0.2cm. Outer dielectric cube has 30x30x30 cells, and inner one has 13x13x13 cells.

Once-differentiated Gaussian pulse having 4GHz bandwidth. The variation in Figure 4.11a corresponds to a radiating source inside a PEC resonator. The positive power values correspond to power radiated outward from the surface (from source towards the PEC walls of the resonator), while the negative values show power flow in the opposite direction, i.e., reflected power travelling inward the virtual surface. The same situation occurs in Figure 4.11b. However, the radiated power decays with time indicating dissipation of the energy of source pulse by the lossy cubes. The radiated and absorbed power calculations are repeated for the sinusoidal excitation with a frequency of 1.8GHz. The results obtained when EUT is enclosed into a PEC resonator (i.e., PEC terminations at the boundaries of the computational domain) are pictured in Figure 4.12. Since the sinusoidal radiating element continuously pumps power into the resonator, the amplitude of the poynting and absorbed powers initially increase with time as observed in the figure until steady-state regime is established at later times. Again, positive and negative radiating powers correspond to outward and inward propagating powers, respectively. Note that for the radiated power the average value is zero since whatever EM energy radiates away from the virtual surface used in calculations, is reflected back by the PEC walls. On the other hand, absorbed power is always positive as expected. It is clearly seen from the figures given in Figures 4.11 and 4.12 that there is an excellent agreement between the results obtained via TLM and FDTD.

We have performed a number of tests with similar configurations, i.e., using different electrical parameters, excitation and observation locations, geometries etc. In all
Figure 4.12 a). Poynting power for sinusoidal excitation for EUT, b). Absorbed power in EUT for sinusoidal excitation. Source frequency is 1.8GHz and period of the power is half of the source period.

cases the results obtained via TLM and FDTD exhibited an almost perfect agreement. We therefore believe that it is safe to assume that

- in the absence of geometrical discretization (cubical structures) errors and,
- in the presence of PEC boundaries

TLM (and also FDTD) can provide very accurate results for absorbed power values in reasonable computer times. However, SAR simulations are usually performed by placing the EUT in free space and not in a PEC resonator. We therefore have critically investigated the various techniques which can be used for simulating free space (absorbing boundary) conditions in the boundaries of the computational domain. A typical output is shown in Figure 4.13 where results obtained via PML ABC for a pulse excitation are given. In this test we have used same geometrical and electrical parameters as for Figures 4.11 and 4.12 except that, now at the boundaries of the computational space we implemented PML type ABC's to simulate free space and used a λ/3 (at 2GHz) thin wire antenna. The solid curve shown in Figure 4.13a shows the variation of the radiated power in the absence of the EUT. As described above, the radiated power is calculated by applying Gauss theorem over a virtual surface which includes the source and is chosen between the EUT and the boundaries of the computational domain. The solid drawn curve in Figure 4.13a, therefore, corresponds to the time variation of the total radiated energy from the source distribution used in the absence of any interaction from a nearby scatterer. Clearly, energy emitted from the antenna may be modified with respect to its distribution
under “free space” conditions when some scatterers (such as the EUT in this example) are placed in its vicinity. The dashed curve in Figure 4.13a is obtained by adding up the power radiated away from the domain in the presence of the EUT with the power absorbed by the EUT. These two constituents are shown separately in Figures 4.13b and 4.13c.

It can be seen from the two curves shown in Figure 4.13a that both the time distribution and that total amount of the energy emitted from the antenna are modified due to the presence of the EUT. By comparing Figure 4.13b with the variation of the power from the source in the absence of the EUT as given by the solid curve in Figure 4.13a it becomes evident that the two curves differ in both amplitude and time extent. Clearly, EM waves intercepted by the EUT slow down on the other hand a substantial part of the energy is absorbed by the EUT which is closer to the source than a major part of the virtual surface used in the calculations. Thus when we compare the two cases, one with and the other without the EUT then we can conclude that when EUT is present the energy corresponding to the absorbed part travels a shorter average distance but at a lower speed. Due to the combined effect of these two factors (e.g., lower speed, shorter distance) it is in general not possible to predict the difference between the time dependences of the pulses with and without the EUT. The remaining discrepancies between the two curves can be attributed to contributions from the following two possible sources:

- Imperfect modeling of ABC,
• The variation of the power radiated by the source as a function of presence/absence of the EUT.

These will be investigated in more detail below:

• As noted before a negative value in the radiated power curve given in Figure 4.13a would mean a net power input through the virtual surface used in the calculations. This would be an indication of reflections from the PML boundary, which are large enough to balance the source power incident on the surface thereby resulting in a negative value. Clearly the effectiveness of the implemented PML algorithm can best be observed at later times where the level of the source pulse diminishes and the reflections gain on importance in determining the direction of the net power flow through the surface. When we concentrate our attention on the time arrivals in Figure 4.13b we note that the sign of the power remains always positive (i.e., net outward flow). This finding proves that there is no appreciable imperfectness in the modeling of ABC. It should be noted that validation tests for PML ABC given in Section 2.3, indicated that our PML algorithm works perfectly for the case of paraxial propagation in waveguides. The above test now shows that a similar conclusion can be drawn also for the most general case involving an isotropic excitation and gives us further confidence that our PML algorithm accurately simulates free space conditions.

• To better show the effects of the presence of EUT on the emission characteristics of the antenna we have calculated the energies emitted from the source both in the absence and presence of the EUT as,

  o Total energy emitted from source in the absence of EUT is 1.21pJ,

  o Total energy emitted from source in the presence of EUT is 1.32pJ.

When the EUT is present about 60% of this energy (0.78pJ) is absorbed by the EUT. This is a clear indication of the fact that the emission characteristics of the antenna are affected from the presence of the EUT, since else, EUT being contained in only one-half of the antenna radiation pattern, the absorbed energy would be expected to be less than 50% of the source energy. On the other hand, it is to be expected that the imaginary part rather than the real part of the dielectric constant of the EUT would be more effective in modifying the antenna characteristics in the above example (Note that for the two media
constituting the EUT $\varepsilon = \varepsilon_0 \left[ \varepsilon_r - j \frac{\sigma}{\omega \varepsilon_0} \right] = \varepsilon_0 [\varepsilon' - j\varepsilon'']$, $\varepsilon' - j\varepsilon''$ assume values 2.5-j10 and 5-j4, respectively). Indeed, we have calculated the total emitted energy for another test wherein the shape of the EUT and the relative dielectric constants of the cubes were kept in tact, but the conductivities of the media were set to zero. In this case we observed that the total energy was within 1% of the total energy emitted in the absence of the EUT (i.e., 1.22pJ).

Similar tests were also performed for FDTD since we intend to further use FDTD calculations as a comparison solutions.

After the above described preliminary tests, we will now proceed to SAR calculations. We will first present the results of SAR calculations for some simpler structures, similar to but more complicated than those investigated above and subsequently address much more complicated structure, which represents a discrete, approximate model of the human head.

Figure 4.14 shows the first structure used in SAR simulations. It is formed by four concentric dielectric lossy cubes with electrical parameters given in Table 4.1. Computation volume is taken as 17.2cm x 17.2cm x 17.2cm. Mesh size, $\Delta \lambda$, is equal to 0.2cm (i.e., corresponding to 86 x 86 x 86 = 636056 cells). The dimensions of the dielectric cubes are 9.2cm x 9.2cm x 9.2cm (46 x 46 x 46 cells) for cube-1 (see Figure 4.14), 6.8cm x 6.8cm x 6.8cm (34 x 34 x 34 cells) for cube-2, 5.2cm x 5.2cm x 5.2cm

![Figure 4.14](image)

Figure 4.14 The structure used in SAR calculations a). perspective view and, b). front view (Dimensions and electrical parameters of the various regions are given in the text)
Table 4.1 Electrical parameters of the cubical EUT pictured in Figure 4.14a at 900MHz

<table>
<thead>
<tr>
<th>Dielectric</th>
<th>$\varepsilon_r$</th>
<th>$\sigma$ [S/m]</th>
<th>$\rho$ [kg/m$^3$]</th>
<th>Number of Cells</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>58.5</td>
<td>1.21</td>
<td>1040</td>
<td>58032</td>
</tr>
<tr>
<td>2</td>
<td>34.5</td>
<td>0.6</td>
<td>1100</td>
<td>21728</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>0.11</td>
<td>1850</td>
<td>11744</td>
</tr>
<tr>
<td>4</td>
<td>55</td>
<td>1.23</td>
<td>1030</td>
<td>5832</td>
</tr>
</tbody>
</table>

(26x26x26 cells) for cube-3, and 3.6cmx3.6cmx3.6cm (18x18x18 cells) for cube-4. Number of cells in each dielectric region is given in the last column of Table 4.1. Note that total number of cells in the computation volume, whose electrical parameters differ from free space, is about 15% of the number of cells on the whole computation volume (i.e., 97336). A $\lambda/3$ dipole driven by a 900MHz sinusoidal wave with averaged power normalized to 600mW, is placed near the structure. At this point it would be appropriate to note that due to differences in the cell structures used in TLM and FDTD special precaution is necessary in average power calculations in order to ensure that both numerical models are driven under identical conditions. In this work we have achieved this by calculating the radiated powers from the antenna in the absence of the structure as outlined in Section 4.2, both with TLM and FDTD codes and normalizing them independently to the same rated average power (600mW here). The $y$-component of electric field is excited at point (15,43,43). 5 layer PML ABC is used to simulate free space in these calculations. The time variation of the total absorbed power in this structure as obtained via TLM and FDTD is plotted in Figure 4.15. It should be noted that the average of the absorbed powers is very close for both methods, while the peak and average power levels differ slightly indicating numerical dispersion effects in two algorithms are somewhat different. It should be noted that the time variation of the total power absorbed in the structure properly converges to a steady state distribution which exhibits a small scale periodic variation at twice the transmitter frequency superimposed on a much larger average value. The averaging effect depicted in Figure 4.15 is to be expected since the signal wavelengths in the various regions of the structure are comparable to the dimensions of the regions. This is in contrast to the variations shown in Figures 4.16 and 4.17 wherein the averaging effect gradually diminishes as the sizes of the regions in which absorbed powers are calculated, are decreased.
At this point it would be appropriate to recall the close relationship between SAR and absorbed power calculations. Indeed, when viewed on a cell-by-cell basis both results differ only by multiplicative factors corresponding to the density of tissue $\rho$, represented by the cell (see 4.3a) and the inverse of the number of cells used in absorbed power calculations (e.g., average value per cell). Hence Figure 4.15 can also be interpreted as the “whole structure SAR” for a homogeneous density distribution (i.e., $\rho=1$ for all cells). Figure 4.15 shows the evolution of the absorbed power in time. Transient response following the onset of the sinusoidal signal will of course die away and steady-state conditions will settle in as time progresses. In SAR calculations this steady-state value is of interest. Moreover, international standards (see Section 4.2) are stated in such a way that SAR is related to the peak absorbed power rather than its average value (Note that this relation involves a spatially varying renormalization using appropriate $\rho$ values). As noted in Section 4.2 more than often local SAR values rather than global SAR values are of interest. The reason for this is that although the SAR value obtained for the whole structure (global SAR) may well be within the limits dictated by the standards, SAR calculated in certain sub-domains (local SAR) may exceed these limits.

An example of local SAR calculations is given in Figure 4.16, where SAR values averaged over a tissue mass of 1g ($\text{SAR}_{1g}$) are calculated both with TLM and FDTD. Clearly there is an excellent agreement between these results. The calculations are performed in the 1g-cell group ($5\times5\times5=125$ cells in this case) located centrally within the innermost dielectric cube and $\text{SAR}_{1g}$ is obtained as the average of $\text{SAR}_{1\text{cell}}$ values over the 1g-cell group. In the figure the variation of SAR is given for the steady-state case. The calculated $\text{SAR}_{1g}$ value for 600mW source is seen to be
approximately 1.8W/kg. It should be noted that variation of the “instantaneous SAR” given in Figure 4.16 differs from the variation of the absorbed power given in Figure 4.15 in that in the former the minima reach up to almost zero power levels. The reason for this can be easily understood by noting that the linear dimensions of domain involving 1g-cell group is about \( \lambda/30 \) whereas that of the whole EUT is about \( \lambda/3 \) and hence the power absorbed in 1g cells can mimic the input signal closely, whereas the power absorbed on the whole structure effectively corresponds to a time-averaged value.

![Graph showing SAR variation](image)

Figure 4.16 SAR\(_{1g}\) variation at the center 1g cell for antenna input power of 600mW.

Another example of our calculations which displays the variation of local SAR at the finest granularity (e.g. on the cell basis) is depicted in Figure 4.17. Here the calculations performed in the central cell of the cell group used in Figure 4.16 are given. One can clearly see that SAR\(_{cell}\) value is rather close to SAR\(_{1g}\) value as expected, but it somewhat differs from it, obviously an indication of the non-uniform illumination in the region containing the 1g-cell group. The rather small differences between TLM and FDTD solutions in Figure 4.17 obtained with a mesh size of \( \Delta\lambda=0.2\)cm, can be attributed to the effects of numerical dispersion discussed in Section 2.4. This point is further illustrated in Figure 4.18 where TLM and FDTD solutions obtained at half resolution (i.e., using a mesh size of \( \Delta\lambda=0.4\)cm) are displayed. It can be seen from these figures that, for this test problem TLM has more favorable convergence characteristics (Note that this conclusion does not apply generally, see discussions in connection with Figure 2.26 in Section 2.4).

In a large structure containing many ten-thousands of cells, the determination of critical sub-domains which may be subject to over-exposures, requires the
Figure 4.17 SAR variation at the center cell for antenna input power of 600mW.

An investigation of the variation of local SAR values over the entire structure. On the other hand, since in TLM or FDTD TD calculations of absorbed powers at each cell are readily available at all time steps they can be conveniently displayed in the form of color SAR density maps. Two such examples are given in Figures 4.19 and 4.20, which display the variation of SAR in each cell in slices taken along horizontal and vertical directions, respectively. These figures show snapshots taken at the same, arbitrary time step after the steady-state conditions are established. Looking at these SAR density maps one can easily identify “critical areas” which, as expected, are

Figure 4.18 SAR variation at the center cell for different mesh sizes obtained via a). FDTD, and b). TLM.
Figure 4.19 SAR values at horizontal slices (defined in the order from left to right in both rows by cell numbers y = 24, 30, 34, 40, 49, 55, 59, and 64, respectively) of the test structure. The rectangular lines on the first windows depict the boundaries of the EUT. The maximum values of SAR cell are indicated on top of each slice.

located in the high loss regions (see Table 4.1) in the vicinity of the antenna. Although, SAR values obtained by taking as the reference domain the “whole EUT”, “1g sub-domain” and “an individual cell”, as depicted in Figures 4.15, 4.16, and 4.16, respectively, are all approximately equal to 2W/kg it is seen from Figures 4.19 and 4.20 that values as much as 6 times larger than the “whole structure” SAR can exist in some cells which are located in the outermost dielectric cube, and close to the antenna. This is further illustrated in Figure 4.21.

The results obtained via TLM for global and local SAR values and the close agreement between these and the FDTD results in the fairly simple test problems considered above, demonstrate that our TLM algorithm is properly implemented and “fine-tuned” for performing accurate SAR calculations. We now proceed to address the complex problem of modeling and simulation of variation of SAR values in the human head. The human head model used in the computations is taken from the literature [82]. This model is derived from actual numerical magnetic resonance (NMR) images. In this model, 7 different tissue types are used and their electrical parameters (\(\varepsilon, \sigma\)) and densities (\(\rho\)) are specified at the frequencies of interest (900MHz) for cellular phone applications. These parameters are given in Table 4.2. It should be noted that, we had to use a scaling of the dimension of the head by a factor of 0.5 (i.e., the head model used in our computations is half the size of a
Figure 4.20 SAR values at vertical slices (defined in the order from left to right in both rows by cell numbers \( z = 24, 30, 34, 40, 49, 55, 59, \) and 64, respectively) of the test structure. The rectangular lines on the first windows depict the boundaries of the EUT.

mature human head and corresponds approximately to the dimensions of the head of a new born baby, in order to meet the memory requirements with the available computational resources. The last column of Table 4.2 shows the number of cells in the head used in calculations.

Figure 4.21 SAR variation in extreme case (Cell location is \((25, 43, 43)\) which is located in the outermost dielectric).
Table 4.2 Electrical parameters of the human head tissues at 900MHz

<table>
<thead>
<tr>
<th>Tissue</th>
<th>( \varepsilon_r )</th>
<th>( \sigma [\text{S/m}] )</th>
<th>( \rho [\text{kg/m}^3] )</th>
<th>Number of Cells</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skin/fat</td>
<td>41.4</td>
<td>0.87</td>
<td>1010</td>
<td>18544</td>
</tr>
<tr>
<td>Muscle</td>
<td>55.0</td>
<td>0.94</td>
<td>1040</td>
<td>27920</td>
</tr>
<tr>
<td>Bone</td>
<td>21.0</td>
<td>0.32</td>
<td>1810</td>
<td>17504</td>
</tr>
<tr>
<td>Brain</td>
<td>35.7</td>
<td>0.61</td>
<td>1040</td>
<td>32528</td>
</tr>
<tr>
<td>Humor</td>
<td>74.1</td>
<td>1.97</td>
<td>1010</td>
<td>128</td>
</tr>
<tr>
<td>Lens</td>
<td>51.3</td>
<td>0.89</td>
<td>1100</td>
<td>64</td>
</tr>
<tr>
<td>Cornea</td>
<td>52.1</td>
<td>1.22</td>
<td>1170</td>
<td>848</td>
</tr>
</tbody>
</table>

Apart from the 7 tissues listed in Table 4.2, the model also includes locations and extends of the air-filled regions (nasal cavity etc.) within the human head. Total number of cells in the computation domain is 688896 (78x96x92 cells). But, number of cells that is tabulated below is only 97536 whereas the number of cells used for representing the human head is about 20% of the total number of cells. A slice of the model is shown in Figure 4.22. The remaining 80% of the cells are used to define the computational domain, which contains the antenna and is terminated by PML ABC.

Figure 4.22 Discretised head model in 2D

The results obtained using both TLM and FDTD codes for \( \text{SAR}_{\text{MAXbrain}} \) calculations are given in Figure 4.23. Before discussing the salient features of the curves in Figure 4.23 it would be appropriate to give the definition of the quantity calculated, i.e., \( \text{SAR}_{\text{MAXbrain}} \), and explain how the calculations are performed.
Figure 4.23 The time variation of maximum SAR in the brain, as averaged over 1g. Period of the variation is half of the period of source that is 1.11 nsec.

SAR values averaged over a tissue mass of 1g (SAR$_{1g}$) were defined in the previous section. The calculated value of SAR$_{1g}$ is a point function (i.e., defined at each cell point) which involves an average effect of absorbed power and density variations in a certain (in this case corresponding to 1g) neighborhood of the point in question. It is therefore clear that SAR$_{1g}$ value obtained for one cell will be a function of the position of the cell and the density distribution (tissue type) in a certain neighborhood of this cell, as well as of the time step used in calculations. In case of sinusoidal excitation the time dependence will exhibit a periodic behavior once the steady-state conditions are reached.

With the aid of the above explanations the definition of SAR$_{1MAXbrain}$ can now be stated as “the maximum of SAR$_{1g}$ values calculated for all brain cells whose entire 1g neighborhoods are contained in brain region.” As noted above SAR$_{1MAXbrain}$ value is also a function of time. SAR$_{1MAXbrain}$ calculations are performed as:

- the number of cells in 1g neighborhood is 125,
- the number of brain cells is 32528,
- the number of internal cells whose 1g neighborhood is contained in the brain region (i.e., excluding boundary cells which are closer than 2 cells to the boundaries of other tissues) is approximately 25000,

At a given time step,

i) Choose an internal cell, and calculate SAR$_{1g}$
ii) Repeat (i) for all internal cells and utilize their maximum value to define SAR_{IMAXbrain} at this time step.

iii) Repeat above calculations for the number of time steps required for obtaining steady-state time variation (e.g., 3000 times here) of SAR_{IMAXbrain} depicted in Figure 4.23.

It is evident from the above that the TLM calculations leading to Figure 4.23 are rather time consuming and impose some severe memory requirements. For the above example we used about 15 hours on a Pentium-3, 850MHz PC with 256MB RAM.

Since the curves depicted in Figure 4.23 correspond to times after the establishment of the steady-state regime we observe that the time variation of the absorbed power in the brain tissue does, of course, exhibit a periodic behavior with a period of 0.55nsec corresponding to signal frequency. We further note that the SAR_{IMAXbrain} as obtained with TLM and FDTD varies in a quasi-sinusoidal fashion, respectively, between 2.3-4W/kg and 2.2-3.8W/kg, with average values of 3.15W/kg for TLM and 3.0W/kg for FDTD. Thus we conclude that the SAR_{IMAXbrain} calculations reveal that

i) Time variation of TLM and FDTD results exhibit essentially identical patterns,

ii) For the discretization level used in these calculations the actual values obtained with TLM and FDTD differ from each other by about 5%. It should be noted that in this problem \( \lambda/\Delta\lambda \) value, which yields a measure for the fineness of discretization ranges from about 160 in free space to about 18 in the humor region. Although the value of 160 can be considered to represent an excellent modeling, the value of 18 yields a resolution which may, and evidently is, too coarse for modeling the problem with sufficient accuracy.

Similar conclusions can be drawn from the curves shown in Figure 4.24. Here TLM and FDTD calculations corresponding to normalized SAR_{tcell} values performed utilizing the same head model and same computational parameters are given for two tissue types: brain in Figure 4.24a and bone in Figure 4.24b. It should be noted that also in this case both algorithms result in almost identical time variations, however with about 8% difference between the calculated amplitudes and averages.

Guided by the above observations we believe that it is safe to conclude that TLM (and also FDTD) is capable of representing the physics of the extremely complex problem of calculating the SAR values in real-to-life situations. Our numerical computations show that utilizing the readily available computational resources of
Figure 4.24 Time variation of normalized SAR in two different head tissues: a). Brain, and b). Bone. Period of the wave is 0.55nsec.

today (Pentium-4, 2GHz, 1GB RAM) these calculations can be performed for solving even more realistic problems (full-size human head models involving more than 7 tissue types used in our model [63]) and will yield solutions with accuracies which should be considered at least as good as “adequate” if not “excellent” for most practical applications (Note that all computer head models are necessarily of a generic nature and the experimental approach which is the only alternative way of obtaining SAR values is prone to quite a number of error sources). On the other hand the numerical errors in TLM calculations can, in principle, be reduced to much lower levels. We note that apart from the problems in realizing PML ABC discussed above, the main difficulty stems from the problems posed by the “sufficiently fine” sampling requirements in the spatial domain. The accumulation of expertise in numerical electromagnetics over the last few decades has thought us that in order to be able to adequately represent a scattering surface we must sample it, as a rule of thumb, along each dimension with at least 10 points or at intervals of $\lambda/10$, whichever is smaller. For the problem of SAR calculations in human head, using the above considerations we can conclude that in order our calculations be acceptable the granularity of the model (i.e., minimum dimensions of regions containing different tissue types) should be not less than 2cm (i.e., ten times the cell size, 0.2cm). This condition was violated in various parts of the numerical head model used in our calculations. We believe that about one order of magnitude smaller cell sizes are required to resolve a realistic human head model in keeping with the above criteria. However, some simple calculations show that in this case the problem will be numerically intractable and remain to be so also in the foreseeable future since this
refinement of the grid size would require the computational work to increase by 2-3 orders of magnitude.

Color density maps can be generated for better visualization of critical areas in the human head which may be subjected to relatively higher SAR values. Such two examples for human head model are given in Figure 4.25 for horizontal slices and in Figure 4.26 for vertical slices. In these figures, the SAR values are given at each cell in different horizontal and vertical slices of head model at the same time step. The maximum SAR values for each slice are indicated in the figures. It can be seen that higher SAR values are obtained in tissues with high conductivity values (see Table 4.2) and also in regions closer to the antenna.

Figure 4.25 SAR values at horizontal slices of the human head. These slices are taken at the same time step for different horizontal slices.

The numerical results given in this chapter demonstrate clearly that TLM method is capable of modeling complex EM problems encountered in SE and SAR calculations. Comparing our results with FDTD calculations we can further conclude that the TLM solutions are also quite accurate.

Although the computer time and memory requirements of FDTD are less than that for TLM (see Section 2.6), when dealing with complex modeling problems such as those encountered in SAR calculations TLM provides better accuracies since it is less vulnerable to discretization errors (i.e., numerical dispersion) (see Section 4.2).
Figure 4.26 SAR values at vertical slices of the human head. These slices are taken at the same time step for different vertical slices.
5. CONCLUSIONS AND DISCUSSIONS

The main objective of the work presented in this thesis can be stated as to demonstrate that TLM method can be applied in obtaining reliable solutions to rather complex problems using fairly modest computational resources. To prove this point we have considered two problem domains: The calculation of Shielding Effectiveness (SE) and Specific Absorption Rate (SAR). We have chosen a specific physical problem from each of these domains; slits on the walls of a rectangular PEC enclosure for SE calculations, and a numerical human head model (generated by other researchers utilizing NMR data [82]) for SAR calculations. It should be noted that both of these problems relate to areas which are of practical interest. Moreover, although the mathematical model which defines the geometries and electrical parameters used in the calculations, can be considered as involving some simplification and/or idealization of the real world problem, the problems investigated in this thesis can nevertheless be regarded as being “realistic”. The implication of dealing such realistic problems is that they define complex EM problems which, usually, can only be addressed via the techniques of numerical EM, or by performing measurements on the actual structures. In either case one needs to compare these results with those obtained independently (by performing other measurements or using other numerical codes) in order to assess their accuracy with some confidence. To the best of our knowledge, the problems of calculating SE and SAR in realistic structures has not yet been addressed using the TLM method, and hence we have not been able to find in the literature reference solutions to be used for comparison purposes (Note that although a somewhat simplified version of the problem that we used in SE calculations was also considered in [83], we have not been able to reproduce these results since the dimensions of the rectangular enclosure and the discretization scheme used in [83] rendered the problem intractable with the computational resources available to us). It should be noted that the structure depicted in Figure 4.1 and considered in our SE calculations has also been investigated in the literature and same results are reported which are obtained using two different numerical techniques (FDTD and MoM) [64,65]. Moreover, the authors did also claim that their numerical solutions agree with the experimental results that they have generated. Using the parameters considered in [64,65] we have been able to accurately reproduce the results reported in the literature. This comparison has
provided an independent validation for our approach. In all other cases, wherein “reference” data that can be used as comparison solutions was not available we have chosen to generate these ourselves and developed apart from the TLM algorithm also an FDTD algorithm for this purpose. After calibrating both algorithms utilizing the canonical test problems discussed in Chapter 3 we have applied them for SE and SAR calculations.

Our numerical results given in Chapter 4 demonstrate that the TLM code we have developed can be used to obtain reliable and quite accurate solutions of these complex EM problems with fairly modest computational resources. We therefore believe that we have successfully fulfilled the above stated objective of this thesis.

Our experience with the TLM (similar remarks also apply for FDTD) indicates that due consideration should be given to the following two areas:

i) Choice of Parameters:

For a given problem this item relates to the appropriate choice of parameters in keeping with

- frequency range and maximum frequency (i.e., minimum wavelength) of interest,
- observation time (frequency resolution/extent of transients)
- spectral content of the source,
- resolution and accuracy requirements in FD and TD,
- mesh size (stability and numerical dispersion),
- data processing requirements (DFT, windowing etc.).

ii) Estimation of Accuracy:

The safest way for estimation of the accuracy of a computed result is to compare these with the results (preferably with known accuracy) obtained via an independent method (such as a different numerical approach or measurement). If should this not be possible one generally resorts to the procedure of repeating a particular numerical experiment by step-wise reducing the mesh size until “convergence” is achieved, i.e., the last obtained result agrees with the former within the specified error bounds.
In this thesis we have given several examples (see Sections 2.4 and 4.2) wherein the numerical results “converge” only in certain temporal or spatial sub-domains while not converging in others. This demonstrates that in order to estimate the accuracy of the calculated results one should “calibrate” the algorithm using representative canonical problems for which exact solutions are available, as done in Chapter 3, or compare the calculated results with those obtained via different numerical approaches, as done in Chapter 4.

The main original contribution of this work is that, to the best of our knowledge, it is the first complete and systematic approach in [11,14,25,26]

- Applying TLM to calculating SE and SAR in problems defined by complex, realistic problems,

- Validating the calculated results by comparing them with results generated independently via the FDTD.

Another original contribution of the thesis is the work done for

- Assessing the accuracy and determining the ranges of applicability of TLM by comparing the calculated results with those obtained analytically for two canonical problems which are chosen in such a way as to represent some of the basic features of wave phenomena encountered in SE and SAR calculations [27].

We believe that TLM will increasingly gain an importance in treating complex EM problems in TD. We list below some of the areas wherein the findings of this work can be applied and/or further developments/extensions in the TLM algorithm are to be expected:

- Developing a sliding window type TLM algorithm for addressing medium and long range propagation problems in a way similar to the SSPE or TDWP approaches [84,85] (see also [86] for some preliminary results obtained by the author using TLM),

- To further investigate the additional advantages which can be gained from using a code which involves a hybrid mix of TLM and FDTD algorithms (some work is done in this field in 2D [86] which needs to be generalized to 3D),

- Developing unit cell structures which reduce the computational burden without degrading accuracy,
• Adaptive mesh sizes (extension of present multi-grid techniques),

• Developing TLM codes suitable to parallel processing techniques,

• More efficient ABC algorithms.
REFERENCES


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Finite Difference Time Domain (FDTD) Method

FDTD Yee’s cell is depicted in Figure A.1. For a lossy and source-free region, iterative FDTD equations are given as

\[ H_x^{n+1}(i,j,k) = H_x^n(i,j,k) + \frac{\Delta t}{\mu_0 \Delta z} [E_y^n(i,j,k) - E_y^n(i,j,k-1)] \]
\[ - \frac{\Delta t}{\mu_0 \Delta y} [E_x^n(i,j,k) - E_x^n(i,j-1,k)] \]  \hspace{1cm} (A.1)

\[ H_y^n(i,j,k) = H_y^{n-1}(i,j,k) + \frac{\Delta t}{\mu_0 \Delta x} [E_x^n(i,j,k) - E_x^n(i-1,j,k)] \]
\[ - \frac{\Delta t}{\mu_0 \Delta z} [E_y^n(i,j,k) - E_y^n(i,j,k-1)] \]  \hspace{1cm} (A.2)
\[ H_{\tilde{n}}(i,j,k) = H_{\tilde{n}-1}(i,j,k) + \frac{\Delta t}{\mu_0 \Delta y} \left[ E_n^x(i,j,k) - E_n^x(i,j-1,k) \right] \]
\[ - \frac{\Delta t}{\mu_0 \Delta x} \left[ E_n^y(i,j,k) - E_n^y(i-1,j,k) \right] \] (A.3)

\[ E_n^x(i,j,k) = \frac{2\varepsilon - \sigma \Delta t}{2\varepsilon + \sigma \Delta t} E_{\tilde{n}-1}^x(i,j,k) \]
\[ - \frac{2\Delta t}{(2\varepsilon + \sigma \Delta t)\Delta z} \left[ H_{\tilde{n}}(i,j,k) - H_{\tilde{n}}(i,j,k-1) \right] \]
\[ + \frac{2\Delta t}{(2\varepsilon + \sigma \Delta t)\Delta z} \left[ H_{\tilde{n}}(i,j,k) - H_{\tilde{n}}(i,j,k-1) \right] \] (A.4)

\[ E_n^y(i,j,k) = \frac{2\varepsilon - \sigma \Delta t}{2\varepsilon + \sigma \Delta t} E_{\tilde{n}-1}^y(i,j,k) \]
\[ - \frac{2\Delta t}{(2\varepsilon + \sigma \Delta t)\Delta x} \left[ H_{\tilde{n}}(i,j,k) - H_{\tilde{n}}(i,j,k-1) \right] \]
\[ + \frac{2\Delta t}{(2\varepsilon + \sigma \Delta t)\Delta x} \left[ H_{\tilde{n}}(i,j,k) - H_{\tilde{n}}(i,j,k-1) \right] \] (A.5)

\[ E_n^z(i,j,k) = \frac{2\varepsilon - \sigma \Delta t}{2\varepsilon + \sigma \Delta t} E_{\tilde{n}-1}^z(i,j,k) \]
\[ - \frac{2\Delta t}{(2\varepsilon + \sigma \Delta t)\Delta y} \left[ H_{\tilde{n}}(i,j,k) - H_{\tilde{n}}(i,j,k-1) \right] \]
\[ + \frac{2\Delta t}{(2\varepsilon + \sigma \Delta t)\Delta y} \left[ H_{\tilde{n}}(i,j,k) - H_{\tilde{n}}(i,j,k-1) \right] \] (A.6)

where \( \Delta x, \Delta y, \Delta z \) and \( \Delta t \) are the spatial steps (cell dimensions) in \( (x,y,z) \) directions and, time step, respectively. \( \tilde{n} \) and \( n \) are time steps for magnetic and electric field components, respectively and they are related to each other by;

\[ \tilde{n} = n + 1/2 \] (A.7)
BIOGRAPHY

M. Orhan ÖZYALÇIN was born in Sivas in 1966. He completed most of his education in Istanbul. ÖZYALÇIN had a B.S. degree from Istanbul Technical University (ITU) in 1987. He became an honor graduate from the Department of Electronics and Communication Engineering. Later on, he was commissioned as an Air Force Lieutenant in 1987 and assigned to 3rd Air Supply and Maintenance Center, Ankara in the same year. He took a 9-month Precision Measurement Equipment Laboratory (PMEL) course in the USA Air Force in 1990. ÖZYALÇIN was awarded with an M.S. degree by Middle East Technical University (METU) in 1996. After the graduation, he was assigned to Turkish Air Force Academy (TUFA) as a teaching assistant. He has been lecturing many courses in the Department of Electronics Engineering in TUFA since 1997. He is mostly interested in time-domain numerical methods such as Transmission Line Matrix and Finite Difference Time Domain. His research interests include EMC, SAR, propagation and scattering problems. ÖZYALÇIN is married with a son.