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ISTANBUL TECHNICAL UNIVERSITY ★ INSTITUTE OF SCIENCE AND TECHNOLOGY

**EFFECTS OF INCLINED CUT-OFFS AND FOUNDATION SOIL
ON SEEPAGE FLOW BENEATH A HYDRAULIC STRUCTURE**

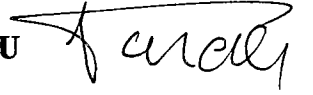
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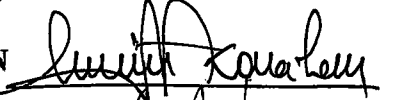


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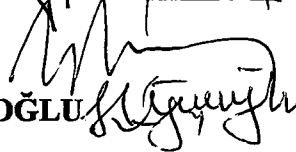
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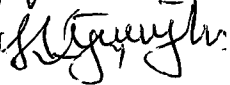
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NOTATION

b = horizontal dam base length (L).

C'_L, C_L = The calculated and the standard weighted creep ratios.

d = dam base depth in case of a depressed base (L).

D = sheet pile depth in Terzaghi's solution (L).

$\Delta x, \Delta z$ = horizontal and vertical unit distances (L).

FD = foundation soil depth (L).

F_s = safety factor against piping.

H = head difference between upstream and downstream levels (L).

$h_{i,j}$ = piezometric head value obtained from the finite difference equation for the node at the location i,j (L).

$(h_{i,j}^n)_{corr}$ = corrected value of $h_{i,j}$ at the n^{th} iteration level (L).

$(h_{i,j}^{n-1})_{corr}$ = corrected value of $h_{i,j}$ at the previous iteration level (L).

i_{cr} = critical hydraulic gradient.

i_e = exit hydraulic gradient.

k_x/k_z = horizontal to vertical permeability ratio.

L_h = creep distance along horizontal surfaces (L).

L_v = creep distance along vertical surfaces (L).

L_θ = creep distance along sloping surfaces (L).

L_w = weighted creep distance (L).

Q = seepage discharge ($L^3/T/L$).

$\bar{Q}$ = non-dimensional seepage discharge.

s = sheet pile length (L).

s_h, s_v = horizontal and vertical components of the sheet pile length, s (L).

x, z = distances along the coordinates in the (x, z) Cartesian system (L).

USL, DSL = upstream and downstream lengths (L).

USH, DSH = upstream and downstream water levels (L).

v = velocity of flow (L/T).

$v_{i,j}$ = velocity of flow at the node i,j (L/T).

ω (or RF) = relaxation factor, $1 \leq \omega \leq 2$.

θ = slope angle of the side walls of the depressed dam base or the sheet pile (deg.).

ε (or TOL) = residual or head difference between two successive iteration levels (L).

λ = correction factor for sloping surfaces.

EFFECTS OF INCLINED CUT-OFFS AND FOUNDATION SOIL ON SEEPAGE FLOW BENEATH HYDRAULIC STRUCTURES

ABSTRACT

If a dam is founded on a pervious foundation, the head formed by the dam acts on the foundation and generates under seepage. The seeping flow generates erosive forces, which tend to pull soil particles with the flow. If these forces exceed the resisting forces (friction and cohesion), then the soil particles will be washed out. The result is an increased velocity and greater erosion, ultimately causing the formation of a channel or “pipe”, which moves upstream beneath the dam. Flow increases tremendously if the pipe reaches the impounded water and causes the failure of such a structure. This process is known as piping phenomenon. Piping occurs if the exit hydraulic gradient at the downstream point approaches the critical hydraulic gradient. In order to prevent piping, it is necessary to reduce the velocity of the seeping water to a safe value. This can be accomplished by lengthening the seepage path. One of the methods of such a lengthening is to introduce cut-off walls (may be a depressed base of concrete or impervious soils or steel sheet piles) within the dam foundation.

Using different solution methods, such as the conformal analysis, empirical formulas, electrical analog models, experimental works using physical as well as numerical models and the flow net technique many researchers had examined the confined seepage through pervious soils beneath hydraulic structures and studied their safety against piping phenomenon. Estimation of the exit gradient, uplift pressure, and flow rate were provided for various problems including the cases of a depressed dam base with vertical side walls, embedded vertical sheet piles, and a flat bottom dam with one or more embedded vertical sheet pile(s). However, limited literature is available concerning the use of inclined walls or sheet piles. Moreover, most of these solutions were based on the assumption of a homogeneous and isotropic foundation soil having an infinite depth. Such an assumption is usually not met depending on various in situ conditions that possess anisotropy, heterogeneity, limited soil depth, .. etc. It appears that no solution is available in the ready accessible literature to determine the effect of using a depressed dam base with inclined sidewalls and a flat dam base with either an inclined toe or heel and toe sheet pile(s) on seepage flow through anisotropic soils of limited depths. Numerical methods such as finite difference or finite element can solve these problems easily and accurately on a case-by-case basis.

For these reasons, a numerical model using finite difference approximation is prepared to solve the governing equations of groundwater seepage under a dam base for the dependence on:

- the horizontal to vertical permeability ratio,
- the geometry of the dam base, and
- the foundation soil characteristics.

In this study, three dam sections with different base configurations are selected and analysed. The first configuration uses a depressed base with different sloping sidewalls, whereas the others use a flat dam base with vertical or inclined steel sheet pile(s) at the toe only or at the heel and toe, respectively. In the first two configurations variation of the horizontal to vertical permeability ratio and base to foundation soil depths ratio are considered. In the third configuration the governing equations are solved for the case of a homogenous and isotropic soil.

The model computes the piezometric head distribution within the pervious foundation soil beneath the dam and shows the effects of the base sidewalls or sheet pile inclination, anisotropy ratio, and sheet pile to foundation soil depths ratio on exit gradient, flow rate, and uplift pressure. The values of the exit gradient, flow rate, and uplift pressure are shown to be affected by changing the slope angle of the base sidewalls or the sheet pile and varying the above mentioned ratios. The model results indicate that the finite difference technique gives comparable values for similar cases solved with other solution methods and offer a more general approach that permits the usage of the desired anisotropy ratio and foundation soil depth.

Application of Lane's method to check the safety of the depressed dam base against failure due to piping showed that different weights should be considered in the calculation of the total length of the sloping surfaces depending on a number of factors. From the analysis of the model results of such a dam base, a new criterion for calculating the creep length, based on the weighted creep concept first suggested by Lane, is derived.

For the case of a flat dam base with an inclined toe sheet pile, the developed design charts could be used efficiently to approximate the exit gradient and the uplift pressure values under such a dam base. Unlike the available charts, which are based on a homogeneous and isotropic soil of infinite depth, the design charts presented herein can account for variable anisotropy ratios and depths of strata.

For the case of a flat dam base with symmetrical heel and toe sheetings (vertical or inclined), the overall trend indicates a small reduction in exit gradient and uplift pressure values as compared with the case of a toe sheet pile alone. Since the difference in results between these two configurations is not great, it could be argued that the analysis of a dam base with only an inclined toe sheet pile is more simpler, computationally less time consuming, and provides a reasonable design from the practical and economical points of view.

The thesis consists of eight chapters. Control measures and the available methods for securing safety against piping phenomenon are presented in Chapter Two after an introduction. In Chapter Three, the fundamental principles of groundwater flow including mathematical description of flow through porous media are given. The finite difference technique along with the boundary conditions and iterative methods are given in Chapter Four. Detailed steps involved in the development of the depressed dam base model altogether with evaluation of its results are then presented. In Chapter Six, investigation and evaluation of the flat dam base model with a sheet pile at its toe (vertical or incline) are given. Chapter Seven includes the investigation and evaluation of the flat dam base model with vertical or inclined heel and toe sheetings. Finally, a summary and conclusions of the study are presented.

Key words Dam; inclined cut-off, numerical solution; piping; seepage

EĞİMLİ PALPLANŞ VE TEMEL ZEMİNİ ÖZELLİKLERİNİN HİDROLİK YAPILAR ALTINDAKİ SIZMAYA ETKİSİ

ÖZET

Bir baraj geçirimli bir temel üzerinde yapılırsa, haznedeki su baraja etki eder ve baraj temel zemininde sızmalar meydana gelir. Sızan akım aşındırıcı kuvvet oluşturur ve kuvvet ise zemin parçacıklarını su ile birlikte aşağıya çeker. Bu aşındırıcı kuvvetler sürtünme ve kohezyondan oluşan direnç kuvvetlerini aşarsa, zemin parçacıkları aşağıya sürüklenir. Bunun sonucunda sızma hızı ve erozyon artar, nihayet bazı borular veya kanalcıklar meydana getirir ve borular giderek mansaptan menbaya doğru ilerler. Bu borucuk menba yüzeyindeki suya ulaştıncaya yapının yıkılmasına sebep olur. Bu olay borulanma olarak bilinir. Borulanma, çıkış noktasındaki hidrolik eğim, kritik hidrolik eğime yaklaşıncaya başlar. Borulanmayı önlemek için sızan suyun hızını belli bir güvenli değere düşürmek gerekir. Bu ise sızma yolu uzunluğunu uzatarak başarılabilir. Bu uzatma metotlarından biri baraj temeli içinde kat-of (katof) duvarları yapmaktır. Bu katoflar geçirimsiz malzemeden, betondan veya çelik palplanştan yapılır. Pek çok araştırmacı, konform tasvir, amprik formüller, elektrik analogi modelleri, deneysel çalışmalar, nümerik metotlar ve akım ağı teknikleri kullanarak, hidrolik yapıların altındaki geçirimli zeminden sızacak basınçlı akımı araştırmışlar ve borulanma olayına karşı yapıların güvenliği üzerinde çalışmışlardır.

Çıkış hidrolik eğimi, kaldırma basıncı, ve akım miktarı hususlarının tahmini, temele gömülmüş baraj temeli için düşey yan duvarlı, düşey çelik palplanş ve düz tabanlı yapılar için yapılmıştır. Bununla birlikte, eğimli katof duvarları ve palplanşlar konusunda pek az yayına rastlanmıştır. Üstelik bu bahsedilen çözümlerin pek çoğu temelin sonsuz derinlikte, homojen ve izotrop bir zemin durumu için gerçekleştirilmiştir. Böyle kabuller izotrop ve homojen olmayan ve sınırlı temel derinliği olan zeminler için uygun değildir. Baraj temellerinde eğimli palplanş veya katof durumları izotrop ve homojen olmayan zemin için sızma probleminin çözümüne literatürde rastlanmamıştır. Sonlu farklar veya sonlu elemanlar metotları gibi nümerik metotlar bu tür problemleri kolaylıkla ve hassaslıkla çözebilir.

Bu sebeplerden dolayı, sonlu fark yaklaşımını kullanan bir nümerik model, bir baraj temeli altındaki zeminde sızma probleminin denklemlerini aşağıdaki şartlara bağlı olarak çözmek için hazırlanmıştır:

1. Yatay permabilitenin düşey permabiliteye oranı,
2. Baraj tabanının geometrisi,
3. Temel zemininin özellikleri.

Bu çalışmada, farklı baraj taban şekilleri için üç baraj enkesiti seçilmiştir. Birincisinde, yere gömülmüş farklı eğimde taban duvarları kullanılmıştır. Diğerlerinde düz bir taban alınmış, fakat menba uca, ortaya veya mansap uca eğimli

veya düşey çelik palplanşlar yerleştirilmiştir. İlk iki durumda, yatay permabilitenin düşey permabiliteye oranının değişimi ile taban genişliğinin temel zemini derinliklerine oranının değişimi irdelenmiştir. Üçüncü baraj kesitinde ise, akımı yöneten denklemler, homojen ve izotrop zeminler için çözülmüştür.

Modelde, baraj altındaki geçirimli zemin içinde piyezometrik yük dağılımları hesaplanmış ve temel yan duvarların veya palplanşların eğiminin, anizotropi oranının, palplanş boyunun temel zemin derinliğine oranının çıkış basınç eğimine, akım miktarına ve kaldırma basıncına etkileri belirlenmiştir.

Çıkış eğimi, akım miktarı ve kaldırma basıncı değerlerinin taban duvarları eğim açıları ile veya palplanş eğim açılarından etkilendiği ve yukarıda bahsedilen oranlarla değiştiği gösterilmiştir. Sonlu farklar metodu ile bulunan sonuçlar diğer metotlarla bulunan sonuçlarla iyi bir uyum sağlamıştır. Ayrıca bu sonuçlar farklı zemin derinlikleri için ve istenen permabilite oranlarının kullanımında daha genel bir yaklaşım sağlamıştır.

Borulanma sebebiyle yıkılmaya karşı düz tabanlı bir baraj tabanının güvenliğini kontrol için Lane metodunun uygulanışında, eğimli yüzeylerin toplam uzunluğunun hesabında bazı faktörlere bağlı olarak farklı ağırlıkların dikkate alınması gerektiği gösterilmiştir. Böyle bir baraj temeli için model analizi çalışmalarından anlaşıldığına göre, ağırlıklı krep kavramına dayanan ve ilk defa Lane tarafından önerilen krep uzunluğu hesabında yeni bir kriter türetilmiştir.

Tabanı düz olan ve eğimli palplanş yerleştirilen durum için geliştirilen tasarım grafikleri, bu tür barajların altında oluşacak çıkış eğimi ve kaldırma basınçlarının belirlenmesi için etkin kullanılabileceklerdir. Homojen ve izotrop olmayan zeminler ve derinliği sınırlı temeller için, şu ana kadar mevcut olmayan grafikler de pratikte kullanılmak üzere hazırlanmış ve sunulmuştur.

Tabanı düz olan baraj için, hem topuk hem de baş uca konan simetrik palplanşlar, çıkış eğimlerini ve kaldırma basınçlarını, sadece topuğa konan palplanş durumuna göre oldukça az azaltmıştır. Bu iki durum arasındaki fark büyük olmadığından, sadece topuk tarafına eğimli palplanş koymak daha kolay, daha az zaman alıcı, tasarımı daha kolay ve daha ekonomiktir.

Tez 8 bölümden oluşmaktadır. Borulanma olayına karşı güvenlik artırıcı metotlar ve kontrol önlemleri İkinci Bölüm'de sunulmuştur. Üçüncü Bölüm'de, boşluklu ortamlarda akımın matematik tanımlanması dahil, yeraltı suyu akımının temel ilkeleri verilmiştir. Sınır şartları ile birlikte sonlu farklar tekniği ve iterasyon metotları Dördüncü Bölüm'de açıklanmıştır. Beşinci Bölüm'de, oluşturulan model değerlendirmeleri ve sonuçlar takdim edilmiştir. Altıncı Bölüm'de, topukta palplanş bulunan düz taban baraj modelinin inceleme ve değerlendirmeleri verilmiştir. Yedinci Bölüm'de hem topukta hem de baş uca palplanş durumu için inceleme ve değerlendirme yapılmıştır. Son bölümde, çalışmanın özeti ve sonuçları sunulmuştur.

Anahtar kelimeler: Baraj, eğimli katof, nümerik çözüm, borulanma, sızma.

1. INTRODUCTION

1.1 General

Seepage is the continuous movement of water from the upstream side of a dam toward its downstream side. Internal erosion of the foundation or embankment caused by seepage is known as piping. Generally, erosion starts at the downstream toe and works back toward the reservoir, forming channels or pipes under the dam, which may cause its failure (Fig. 1.1). The failure usually occurs through the collapse of the pipe roof and overtopping in the channel thus formed. The channels or pipes follow paths of maximum permeability and may not develop until many years after construction.

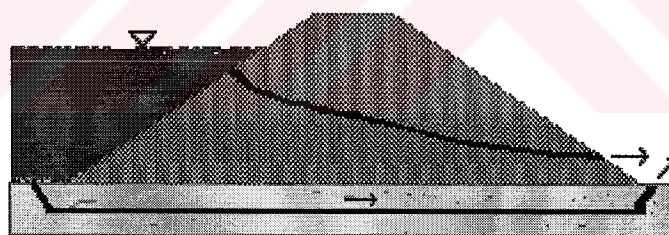


Fig. 1.1 Piping formation in a dam body or its foundation

Resistance of the embankment or foundation to piping depends on the plasticity of the soil, the gradation, and the degree of compactness. Plastic clays with a plasticity index >15 , for both well and poorly compacted are the materials which are most resistant to piping. Minimum piping resistance is found in poorly compacted, through to well-graded cohesionless soils with practically no binder. It is also found in uniform, fine, cohesionless sand, even when well compacted. Settlement cracks in resistant materials may also produce piping. Piping occurs if the exit gradient at the downstream point, i_e , approaches its critical value, i_{cr} .

In order to prevent piping, it is necessary to decrease the hydraulic gradient of the water flow and reduce the velocity of the seeping water to a safe value as it emerges on the downstream side. This can be accomplished by the following measures,

- (1) for any dam, a minimal foundation treatment is the stripping of about 1.0 meter of the original ground upper layer. In the case of alluvial soils, this is usually a layer with a high content of organic matter, whilst in the case of rocky soils this is generally a highly weathered layer, and
- (2) a full or partial watertight barrier (cut-off) may be constructed to eliminate or reduce the underseepage in case of shallow or deep pervious foundations, respectively. From Darcy's law, the amount of seepage can be calculated from the soil permeability and the hydraulic gradient. Since the soil permeability and the acting head are given parameters, the only factor that can be adjusted is the length of the seepage path. Therefore, piping can be avoided by lengthening such a path within the dam and its foundations. The seepage paths can be increased by introducing,
 - cut-off walls of concrete or impervious soil (may be a depressed base),
 - steel sheet piles,
 - grouted sediment formations, and
 - horizontal upstream blankets of impervious material.

1.2 Literature Review

Within the scope of designing new hydraulic structures, safeguards against the occurrence of piping must be verified. Various calculation rules are available to verify such safeguards, varying from simple empirical rules for the first design, to advanced calculation models to design or test more precisely. Many investigators have studied the problem of water seepage under dams or weirs as a factor in estimating their structural safety against piping failure. One of the earliest methods in this field was developed by Bligh (1910) who explained the well-known theory, line of creep, that the safety of masonry dams on earth foundations depends on the length of the percolation path, which is a long the line of contact of the structure and its foundation. In this method, once the total head difference, H , is known, the safe creep length, L_w , is determined by multiplying H by a proper creep ratio, which is

the reverse of a safe uniform gradient. This method is overconservative and later modified by Lane (1935) on the basis of about 280 existing dams study. Terzaghi (1922) conducted some model tests on a single row of vertical sheet piles and devised a method for determining the upward pressure and path of percolation. He found that the failure due to piping takes place within a distance of $D/2$ from the dam toe, where D is the sheet pile depth. The stability of this type of structure can be determined by considering a soil prism on the downstream side of unit thickness with section equal to $D^2/2$. Knowing the average hydraulic head at the base of such a soil prism, the uplifting pressure can be calculated and used along with the prism submerged weight in order to determine the safety factor against piping. In the same year, Pavlovsky (1922) made use of the conformal transformation and obtained a large number of individual solutions as applied to weir foundation design. His investigations comprised also model tank tests and the electric analogy method. It was first mentioned in English by Leliavsky (1929, 1965, and 1979). Harza (1935) investigated the safety of hydraulic structures against piping using an analytical method which is applicable to some familiar types of foundations, and graphical and electrical analogy methods to some more complicated situations. The safety factor against piping is considered as the ratio of the critical hydraulic gradient (i_{cr}) to the exit gradient (i_e). The critical hydraulic gradient is taken as the ratio of the submerged unit weight of soil to unit weight of water whereas the exit gradient is calculated from the flow net by dividing the head lost between the last two equipotential lines by the length of the flow element immediately at the toe. He also presented charts for the maximum exit gradient of dams constructed over deep homogeneous deposits. On the other hand, Lane (1935) investigated the safety of dams against piping and suggested an empirical approach to the problem. His approach, termed as the weighted creep, is based on the statistical analysis of more than 280 dams on pervious foundations. The weighted creep distance is considered as the vertical creep plus one-third the horizontal creep, which when divided by the total head, gives the safety factor against piping, depending on the soil type. He assumed that any surfaces sloping more than 45° to be considered as vertical and those at a flatter slope as horizontal creeps.

In the same years, Khosla *et al.* (1936) threw much light on the problem of seepage of water under hydraulic structures by giving the results of the measurements of

upward pressure on the Panjnad Weir. Their problems, based on the conformal transformation and approximate diagrams computed from standard model tests, respectively, aimed at precise results for the simpler foundation types accompanied by clear design charts facilitating their applications. They also suggested an empirical solution for the case of a rectangular depressed apron on a homogeneous and isotropic soil having an infinite depth.

As mentioned by Harr (1962), Verigin (1940) has analysed seepage flow around an inclined sheet pile embedded in a homogeneous and isotropic porous medium of infinite depth using conformal mapping.

For structures other than a single row of sheet piles, such as a flat dam base with embedded vertical sheet piles, Terzaghi (1943) recommended that the stability of several soil prisms of size $D'D/2$ at the dam toe be investigated to find the minimum factor of safety, where $0 < D' \leq D$. Harr (1962) solved various problems by mathematical analysis using complex variable method. Polubarinova-kochina (1962) derived a mathematical approximation for the seepage under flat bottom dams with a single embedded vertical sheet pile where the soil is homogeneous, isotropic, and has an infinite depth. Estimates of the exit gradient are provided for the case of embedded sheet piles. They also gave the hydrodynamical flownet for an inclined sheet pile in a porous medium of infinite depth. Moreover, they gave a solution for seepage under flat bottom dams resting on a heterogeneous isotropic porous medium consisting of two layers. The thickness of each layer is equal to half the total thickness of the underlying stratum. However, Scott (1963) had also solved the problem by graphical methods. Furthermore, Terzaghi and Peck (1967) suggested analysing of the soil prism at the dam toe. They assumed that piping takes place within a distance of $D/2$ from the dam toe, where D is the sheet pile depth. They take the safety factor against piping as the ratio of the critical hydraulic gradient (i_{cr}) to the calculated exit gradient (i_e). The average critical hydraulic gradient for most of soils is found through experimentation by Terzaghi (1960) to be unity. The exit gradient is calculated from the average vertical hydraulic gradient over the soil prism at the dam toe and its depth. Cedergren (1977) studied the problem and gave numerous examples of flow-net technique. By making use of the finite element method, Christian (1980) outlined the steps involved in solving for the potential and

stream functions that are needed in flow-net construction. Using electrical analog and digital computer methods, Rushton and Redshaw (1978) studied the confined seepage of water beneath hydraulic structures with and without vertical sheet piles and gave solutions to several problems. Van and Harr (1981) while agreeing that the mechanics of piping are almost impossible to analyse, suggested that attempts at analysis could be made using the global gradient approach. In this method, the global seepage gradient (horizontal hydraulic gradient) for a given structure is compared to gradients of structures that have failed. Gradients lower than those for failed structures are considered to be safe. The seepage through the permeable foundation of an earth dam with an impervious core and an impervious vertical sheet pile placed at the centre was studied by Kalkani and Michali (1984). This problem is usually solved in practice by neglecting the effect of the dam. For this reason, they considered different permeability ratios of the foundation and depths of the sheet pile to evaluate the total seepage through for both the dam and its foundation and the simplified seepage only through the foundation. The results indicated that there is a limited range of permeability ratios and cut-off depths, over which simplification of the model will give an error of calculated flows (10% error), which is acceptable in engineering applications. Based on experimental works, Koenders and Sellmeijer (1992) reported the results of a mathematical model of piping phenomenon. The model uses a computer routine and takes into account most of the observed features of erosion under dams. The model consists of a dam with a flat bottom placed on a granular material which is eroded from the piping channel and deposited in a sand boil. Using a curve fitting-technique, a formula has been found for calculating the ratio H/L (the total head/the creep length). Vertical seepage line components, such as at cut-off walls, cannot be included in this model however. The model results are compared against Lane (1935) weighted creep method and showed a shorter seepage length.

Griffiths and Fenton (1993) used the random field theory and the finite element technique with Monte Carlo simulations to study the effect of the stochastic permeability on confined seepage beneath water retaining structures. Analyses have been performed for a flat dam base with two vertical sheet piles. They studied the effect of the standard deviation and correlation structure of the permeability on the output statistics relating to seepage quantities, exit gradients and uplift pressures.

Flow rates and other quantities are shown to be affected significantly by the standard deviation and the correlation structure of soil permeability. Meyer *et al* (1994) examined the failure potential of the debris-avalanch dam at Castle Lake. A three-dimensional groundwater model of the dam was constructed to examine the dam's stability against failure by heave, piping, and internal erosion. They suggested that the weighted creep ratio would be more precisely defined if the horizontal creep is divided by the ratio $\sqrt{k_x/k_z}$ than by dividing by 3 as in the weighted creep method. Abbas (1994) obtained a solution, using conformal mapping, to determine the effect of introducing an inclined sheet pile at the downstream end of a hydraulic structure on exit gradient and uplift pressure. It is found that the use of a sheet pile at the dam toe inclined downstream with an angle not more than approximately 30° increases the factor of safety against uplift and piping. His solution is applicable in the case of a homogeneous and isotropic soil having an infinite depth. Skempton and Brogan (1994) performed tests on internally unstable sandy gravel. Their experiments show that a significant proportion of the sand content is washed out by piping at hydraulic gradients far lower than the critical gradients given by a classical theory. Furthermore, Griffiths and Fenton (1998) use the random field theory and the finite element technique with Monte Carlo simulations to study the statistics of the exit gradient predictions as a function of soil permeability variance and spatial correlation. Both two- and three-dimensional boundary-value problems are considered. Their approach enables conclusions to be drawn about the probability of critical conditions and hence failure at a given site. Tomlinson and Vaid (2000) presented an experimental study of piping erosion. Various artificial granular filter and base soil combinations are tested in a permeameter under variable confining pressures to determine the critical gradient where soil erodes through the filter. By monitoring head, settlement, confining pressure, amount of eroded soil, and water outflow rate, the onset of piping can be determined. This study showed that the grain-size ratio is the most important parameter affecting piping erosion. Recently, Badv and Sargordi (2001) used an analytical model based on a piping code based on Koenders and Sellmeijer (1992) design formula and a probabilistic method. Six diversion dams were selected to investigate their safety against piping. The partial safety factors of the previously mentioned design formula were calculated and used in designing water-retaining structures for piping in Urmia region (Iran). The

comparison of Koenders and Sellmeijer (1992) method with the empirical methods of Bligh (1910) and Lane (1935) showed that this method requires a shorter seepage length and produces a more economical design for water retaining structures. Ojha, *et al.* (2001) studied the influence of porosity on piping models of levee failure. They showed that the choice of the permeability function is critical for the piping model. In a recent study, a method is suggested for the approximate estimation of the time for progression of internal erosion and piping, and development of a breach leading to failure in embankment dams and their foundations (Fell *et al.*, 2003). This method accounts for the nature of the soils in the dam core, the foundation, and the materials in the downstream zone of the dam. They also discussed the factors that influence the likelihood of breaching for different modes of failure. It has been assumed that dams which have factors making it more likely for a breach to form are likely to breach more quickly than those with neutral or less likely factors. The main factors considered are the ability to support a roof, the likelihood of pipe enlargement, limitation of flows by upstream zones, and the materials in downstream zone of embankment or foundation. This method is based on a detailed study of piping failure conducted by Foster *et al.* (2000a,b).

From the above literature review it is seen that several researchers had examined the confined seepage through pervious soils beneath hydraulic structures and studied their safety against piping phenomenon. Estimation of the exit gradient, uplift pressure, and flow rate were provided for various problems including the cases of a depressed dam base with vertical sidewalls, embedded sheet piles (vertical or inclined), and a flat bottom dam with one or more embedded sheet piles. Most of these solutions were based on the assumption of a homogeneous and isotropic foundation soil having an infinite depth. However, this assumption is usually not met depending on various in situ conditions that possess anisotropy, heterogeneity, finite soil depth, etc. Numerical methods such as finite difference can solve these problems easily and accurately on a case-by-case basis.

It appears that no solution is available in the ready accessible literature to determine the effect of using a depressed base with inclined sidewalls and a flat dam base with either an inclined toe or heel and toe sheet pile(s) on seepage flow through anisotropic soils of limited depths.

1.3 Statement of the Problem

Due to the importance of the above mentioned reasons, it is proposed to study by means of numerical techniques the two-dimensional steady-state seepage through the pervious foundation soils under hydraulic structures for the dependence on,

- the horizontal to vertical permeability ratio,
- the geometry of the cut-off (three dam base configurations are considered; (1) a depressed dam base with different sloping sidewalls, (2) a flat bottom dam with an inclined toe sheet pile, and (3) a flat bottom dam with symmetrical inclined heel and toe sheet piles), and finally,
- the foundation soil characteristics (such as its depth, areal extent, and boundary type).

1.4 Method of Solution

In this study, a two-dimensional numerical model using the finite difference approximation is used in order to investigate the flow within the pervious soil beneath the above mentioned dam base configurations. The model is based on steady-state seepage, which exists when the reservoir level is at the same elevation (the maximum water elevation) long enough for the highest pore pressure to develop everywhere in the seepage zone. The pressures computed from the analysis of such steady-state seepage are used to evaluate the safety of the downstream portion against piping or heaving during full reservoir. In reality, the reservoir is never continuously full; however, the pressures indicated by the steady-state seepage analysis are taken into consideration in order to represent the maximum values required in a safe design.

The pressures in the seepage zones considered in this study are computed by application of the finite difference technique. The basic idea of this technique is to replace derivatives at a point by ratios of the changes in appropriate variables (say the piezometric heads) over a small but finite interval. Thus,

$$\frac{dh}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\Delta h}{\Delta x} \approx \frac{\Delta h}{\Delta x} \quad (1.1)$$

The smallness of Δx in Eq. (1.1) depends on the particular problem. Discussion of the sufficiently small nodal spacings selection is available in Rushton and Tomlinson (1977) whose results are also summarized by Rushton and Redshaw (1978). This type of approximation is made at a finite number of points within the study area and reduces a continuous boundary-value problem into a set of algebraic equations. The reasons of choosing the finite differences are,

- (1) finite difference techniques are fundamentally simple,
- (2) they are suitable for problems having regular boundary conditions,
- (3) they lead directly to simultaneous equations without introduction of further mathematical manipulations, and
- (4) digital computer solutions to the finite difference equations can be easily obtained by iterative methods, which are suited for machine computations, without the use of matrices.

The procedure is to overlay the region of interest (the study area) with a rectangular grid network and to develop the difference equation at each grid intersection, (nodal points). A digital computer program, using FORTRAN 77, is prepared to solve the resulting set of simultaneous equations for selected boundary conditions. The program is compiled using the Microsoft FORTRAN PowerStation 4.0 code. The developed program is broadly similar to that given by Rushton and Redshaw (1978) for confined seepage. However, it has been modified to allow for suitable representation of the boundary conditions for each dam base configuration (see the Appendix). Darcy's law is used to derive the boundary equations valid at the various boundary nodes. These conditions must necessarily be altered as the configuration of the dam base itself alters. The program is divided into four parts, namely,

- (1) initialization of the piezometric head, $h_{i,j}$, at all nodal points,
- (2) definition of the boundary conditions,
- (3) computation of the new piezometric heads based on the old values, and finally,
- (4) printout of the results.

The program uses iterations and tests for convergence. In the iteration methods, the unknown heads are initially assigned arbitrary values. However, since the first piezometric heads are assumed values, the finite difference equation governing the

flow within the study area will not be satisfied at all the nodes. For the correction of $h_{i,j}$ value, the successive over relaxation process is used where the value of $h_{i,j}$ obtained from the finite difference expression is over corrected as follows,

$$(h_{i,j}^{m+1})_{corr} = \omega h_{i,j}^{m+1} + (1 - \omega)(h_{i,j}^m)_{corr} \quad (1.2)$$

where m is the iteration level, and ω is the relaxation factor, $1 \leq \omega \leq 2$. With the new heads, the process is repeated and the iteration is carried out until the values of $(h_{i,j}^{m+1})_{corr}$ are equal to $(h_{i,j}^m)_{corr}$. That is, until the difference between the answers generated at two successive iteration levels is less than the preset error tolerance (Wang and Anderson, 1982). The more accurate the first trial, the fewer steps will be needed in the relaxation process. If the convergence criterion is satisfied, the program terminates and prints out the results. Assembled quantities, which will be derived from the numerical solution, are,

- (1) the piezometric head, $h_{i,j}$, at all interior nodal points,
- (2) the quantity of seepage, Q , which will be evaluated with a Darcy relationship,
- (3) the exit gradient at the dam toe, i_e , calculated from the piezometric heads, and finally,
- (4) the uplift pressure assumed as a percent from the total head difference, h/H .

For the case of a dam with a depressed base, at first its thickness is assumed to be zero. Thus representing the case of a dam base flush with the streambed. All other situations including the depressed base are assumed to be symmetric and have a constant depth as $d = 6$ m, but with different sidewall angles such as $\theta = 26.5^\circ, 45^\circ, 63.5^\circ$, and 90° .

For the case of a flat bottom dam with an inclined toe sheet pile, the inclination is towards the downstream, however, any desired angle can easily be incorporated into the model ($0^\circ \leq \theta \leq 90^\circ$).

Moreover, for the case of a flat bottom dam with inclined heel and toe sheet piles, the inclination of those piles is toward the upstream and downstream sides, respectively. The numerical computations are performed for each dam base configuration.

The dam rests on a pervious homogeneous and anisotropic soil, where its foundation depth, FD , is finite ranging from 12 m to 200 m and consists of sand with specific permeability ratios. During the years 1922-35, Terzaghi and others have shown that the average permeability of a natural deposit of soil in a horizontal direction (k_x) is always larger than that in a vertical direction (k_z); and that $k_x = \mu k_z$ in which μ varies between 2 and 10. For distinctly stratified soils, the value of μ may become much larger than 10 (Casagrande, 1935). For this reason, the permeability ratio is taken as $k_x/k_z = 1, 5, 10, 15, 20$, and 25. For the homogeneous and isotropic case, the permeability value at all nodal points is assumed to be 10^{-5} m/s. This value is typical for very fine sands and silt in which piping is likely to occur (Griffiths and Fenton, 1998).

The principal directions of anisotropy in the study area are considered to be in the directions of x and z coordinates in a Cartesian coordinate system. The foundation soil extends to impermeable boundaries forward and backward from the dam heel and toe, respectively. Although the flow field extends an infinite distance upstream and downstream, it is assumed here that it is bounded by artificial limits, which are impervious boundaries.

The required input data are the values of all the parameters that describe the geometry and properties of the study area, which are,

- the upstream and downstream lengths, (USL, DSL),
- dam base length, (b),
- foundation soil depth, (FD),
- water height at upstream and downstream sides, (USH, DSH),
- saturated unit weight of foundation soil, (SATWT),
- unit weight of water, (WATWT),
- maximum number of iterations, (MAXIT),
- relaxation factor, (RF),
- error tolerance, (TOL),
- horizontal and vertical grid distances and hydraulic conductivities, (DX, DZ, XK, ZK),
- sheet pile length, (s), and
- the depressed base depth, (d).

The output is in the form of tables including the head values at the pre-specified number of nodes in the problem solution domain. These tables include the calculated seepage magnitude under the dam base, the exit gradient at the downstream side, and the uplift pressure at some key points expressed as a percent of the total head.

1.5 Objectives of the Study

The main goal is to secure a picture of the water flow beneath hydraulic structures having inclined cut-offs and improve some of the basic formulas or charts used in design of masonry dams on sand. Such approaches based upon the assumption of homogeneous isotropic material may be impractical. If based upon the best available data representing the variations in material, it will be difficult and costly in obtaining knowledge by pumping tests, borings, and other available methods. Nevertheless, numerical solutions are valuable in revealing basic formulas and design charts for the desired dam base configuration and foundation soil characteristics. Additionally, development of a numerical model is sought in order to compute the piezometric head distribution in porous medium under some hydraulic structures by implementing the finite difference method. In order to make an efficient use of the model, the results are compiled and plotted to form different non-dimensional sets of design charts which can be used in the estimation of the exit gradient, uplift pressure, and flow rate under the studied dam base configurations, in both homogeneous isotropic and anisotropic media of limited depth. Among the objectives is also the improvement of the internal consistency of Lane (1935) method for dam safety against piping. This includes,

- (1) taking the influence of the permeability ratio and foundation soil characteristics into consideration, and
- (2) correcting the limitation of Lane's recommendation to treat slopes differently according as whether they are steeper or flatter than 45 degrees. From the analysis of the model results, it is aimed to reach at a new criterion for calculating the weighted creep length necessary for assuring safety against piping.

These are helpful for a comparative study of the seepage effects on the stability of hydraulic structures for various cutoff designs and foundation material characteristics.

2. UNDERSEEPAGE CONTROL AND SAFETY AGAINST PIPING

2.1 General

Unless properly treated, seepage of reservoir water through a dam foundation can present serious problems regardless of the type of foundation--rock, soil, or piling. The primary purpose of controlling the seepage of water through a dam foundation is to prevent the foundation material from piping and washing away, which could result in structural failure due to loss of support. A secondary purpose of controlling seepage is to reduce foundation uplift pressures. In order to attenuate, control, collect, and/or direct the seepage discharge, careful and thorough geotechnical and hydraulic studies and evaluations must be made, and proper cut-off (control) features must be designed.

2.1.1 Earth Foundations

All dams or weirs on earth foundations are normally subject to seepage through their foundation soil. Seepage control is necessary to prevent excessive uplift pressures, piping and erosion of material by migration into open joints in the foundation soil. The purpose of the project, i.e., long-term storage, flood control, etc., may impose limitations on the allowable quantity of seepage. Generally, siltation of the reservoir with time will tend to diminish underseepage. Conversely, the use of some underseepage control methods, such as relief wells and toe drains, may increase the quantity of underseepage.

Some of the methods of underseepage control in dam foundations are cut-offs (compacted backfill trenches, slurry trench, concrete bases or walls, and steel sheet piles), upstream impervious blankets, relief wells, and trench drains. In order to select an underseepage control method for a particular foundation, the relative merits and efficiency of different methods should be evaluated by means of flow nets or approximate methods using numerical techniques (Visvanathan, N., 1999). The

changes in the quantity of underseepage, factor of safety against uplift and piping, and uplift pressures at various locations should be determined for each particular foundation varying the anisotropy ratio of the permeability of the foundation to cover the possible range of expected field conditions.

Cut-offs: (1) Complete versus partial cut-off. When the dam foundation consists of a relatively thick deposit of pervious alluvium, the designer must decide whether to make a complete cut-off or allow a certain amount of underseepage to occur under controlled conditions. It is necessary for a cut-off to penetrate a homogeneous isotropic foundation at least 95 percent of the full depth before there is any appreciable reduction in seepage beneath a dam (Muskat, 1937). The effectiveness of the partial cut-off in reducing the quantity of seepage decreases as the ratio of the dam width to the cut-off penetration depth increases. Partial cut-offs are effective only when they extend down into an intermediate stratum of lower permeability. (2) Compacted backfill trench. This method for control of underseepage consists of excavating a trench beneath the impervious dam floor through pervious foundation strata and backfilling it with compacted impervious material. Because of the improvement in earth-moving equipment, they have been increasingly more favorable economically, and are now installed under the great majority of earth dams (Fig. 2.1). The cost increases rapidly below a certain depth, and the construction tends to be expensive and troublesome. The compacted backfill trench is the only method for underseepage control, which provides a full-scale exploration trench that allows the designer to see the actual natural conditions and to adjust the design accordingly, permits treatment of exposed bedrock as necessary, and provides access for installation of filters to control seepage and prevent piping.

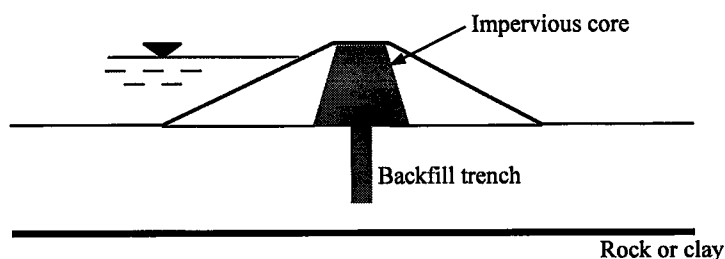


Fig. 2.1 Schematic sketch for a dam with a partial backfill trench

When constructing a complete cut-off, the trench must fully penetrate the pervious foundation and be carried a short distance into unweathered and relatively impermeable foundation soil or rock. If the gradation of the impervious backfill is such that the pervious foundation material does not provide protection against piping, an intervening filter layer between the impervious backfill and the foundation material is required on the downstream side of the cut-off trench. The cut-off trench excavation must be kept dry to permit proper placement and compaction of the impervious backfill. Dewatering systems of wellpoints or deep wells are generally required during excavation and backfill operations when below groundwater level (Sherard *et al*, 1967). Because construction of an open cut-off trench with dewatering is a costly procedure, the trend has been toward use of the slurry trench cut-off. (3) Slurry trench. When the cost of dewatering and/or the depth of the pervious foundation render the compacted backfill trench too costly and/or impractical, the slurry trench cutoff may be a viable method for control of underseepage (Fig. 2.2). Using this method, a trench is excavated through the pervious foundation using sodium bentonite clay and water slurry to support the sides. The slurry-filled trench is backfilled by displacing the slurry with a backfill material that contains enough fines (material passing the No. 200 sieve) to make the cut-off relatively impervious but sufficient coarse particles to minimize settlement of the trench forming the soil-bentonite cut-off.

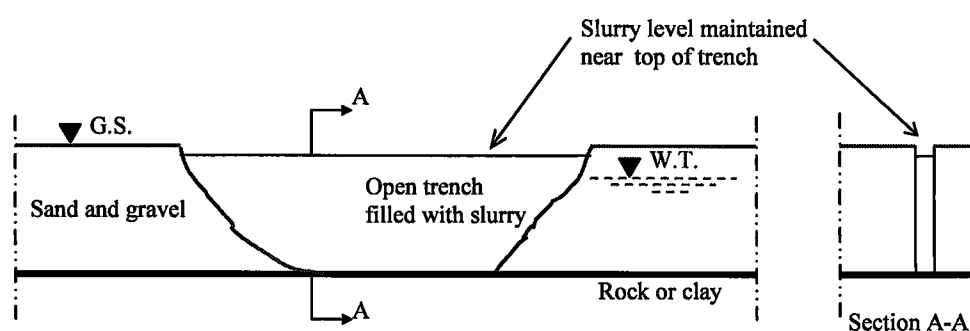


Fig. 2.2 Schematic sketch for a slurry trench

Alternatively, a cement may be introduced into the slurry-filled trench which is left to set or harden forming a cementbentonite cut-off. The slurry trench cut-off is not recommended when boulders, talus blocks on buried slopes, or open jointed rock exist in the foundation due to difficulties in excavating through the rock and slurry

loss through the open joints (Sherard *et al*, 1967). When a slurry trench is relied upon for seepage control, the initial filling of the reservoir must be controlled and piezometers located both upstream and downstream of the cut-off must be read to determine if the slurry trench is performing as planned. If the cut-off is ineffective, remedial seepage control measures must be installed prior to further raising of the reservoir pool. Normally, the slurry trench should be located under or near the upstream toe of the dam. (4) Concrete bases or walls: Shallow depth concrete bases may be constructed where shallow wide trenches can be excavated by conventional equipment. If the excavation reaches the design depth, then the wide trench is backfilled with concrete under controlled conditions in order to establish the depressed dam base (Fig. 2.3).

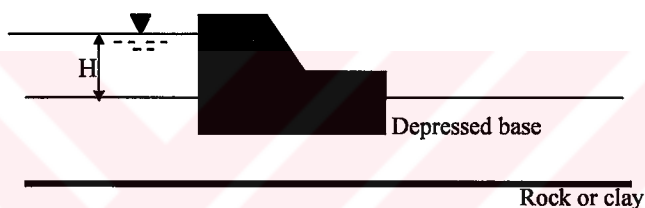


Fig. 2.3 Schematic sketch for a dam with a concrete depressed base

However, when the depth of the pervious foundation is excessive and/or the foundation contains cobbles, boulders, or cavernous limestone, the concrete cut-off wall may be an effective method for control of underseepage. Using this method, a cast-in-place continuous concrete wall is constructed by tremie placement of concrete in a bentonite-slurry-supported trench. Since the wall in its simpler structural form is a rigid diaphragm, earthquakes could cause its rupture; therefore, concrete cutoff walls should not be used at a site where earthquake shocks are likely. (5) Steel sheet piling. This type of underseepage barrier is still preferred under certain circumstances where the alternative cut-off methods are very expensive or time consuming. The sheet pile wall is more effective in a homogeneous than in a stratified foundation. For the most satisfactory use of sheet pile walls, the underlying bed-rock should be sufficiently weathered and soft to allow the pile to penetrate. Hard rock with an irregular upper surface may cause the piles to deflect and break. The piling should be used upstream and usually also downstream and should be embedded 1 to 2 ft into the base of the concrete at the top (Fig 2.4). The piling will

be driven to a depth that satisfies the seepage cut-off requirements indicated by analysis (Sherard *et al*, 1967).

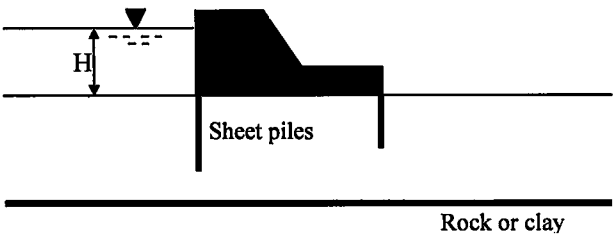


Fig. 2.4 Schematic sketch for a dam with upstream and downstream sheet piling

Upstream impervious blanket: When a complete cut-off is not required or is too costly, an upstream impervious blanket tied into the impervious body of the dam may be used to minimize underseepage. An example is shown in Fig. 2.5. Upstream impervious blankets should not be used when the reservoir head exceeds 200 ft because the hydraulic gradient acting across the blanket may result in piping and serious leakage. Downstream underseepage control measures (relief wells or toe trench drains) are generally required for use with upstream blankets to control underseepage and/or prevent excessive uplift pressures and piping through the foundation. Effectiveness of upstream impervious blankets depends upon their length, thickness, and vertical permeability and on the stratification and permeability of soils on which they are placed. Moreover, Effectiveness of upstream impervious blankets may be defined in terms of criteria as head efficiency and flow efficiency (Novak, *et al* (1990) and Ağralıoğlu *et al*, 1999).

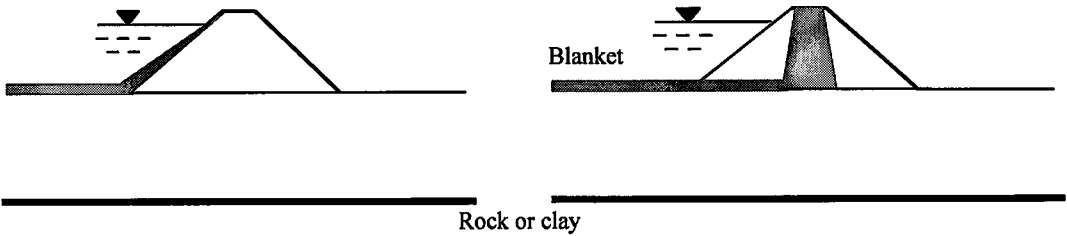


Fig. 2.5 Schematic sketch for a dam with an impervious upstream blanket

Relief wells: When a complete cut-off is not required or is too costly, relief wells installed along the downstream toe of the dam may be used to prevent excessive uplift pressures and piping through the foundation (Fig 2.6). Relief wells increase the quantity of underseepage from 20 to 40 percent, depending upon the foundation conditions.

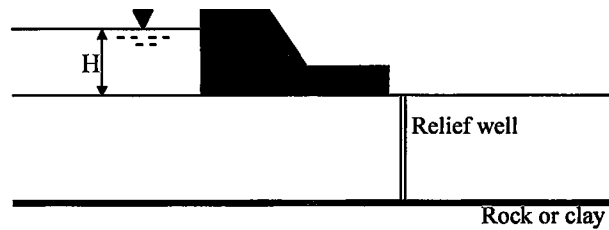


Fig. 2.6 Schematic sketch for a dam with a water pressure relief well

Relief wells may be used in combination with other underseepage control measures (upstream impervious blanket or downstream seepage berm) to prevent excessive uplift pressures and piping through the foundation (Gan and McDonald, 1999). Relief wells are applicable where the pervious foundation has a natural impervious cover. The well screen section, surrounded by a filter if necessary, should penetrate into the principal pervious stratum to obtain pressure relief, especially where the foundation is stratified. The wells, including screen and riser pipe, should have a diameter which will permit the maximum design flow without excessive head losses but in no instance should the inside diameter be less than 6 in. Relief wells should be located so that their tops are accessible for cleaning, sounding for sand, and pumping to determine discharge capacity. Relief wells should discharge into open ditches or into collector systems outside of the dam base which are independent of toe drains or surface drainage systems. Experience with relief wells indicates that with the passage of time the discharge of the wells will gradually decrease due to clogging of the well screen and/or reservoir siltation. Therefore, the amount of well screen area should be designed oversized and a piezometer system installed between the wells to measure the seepage pressure, and if necessary additional relief wells should be installed.

Trench drain: When a complete cut-off is not required or is too costly, a trench drain may be used in conjunction with other underseepage control measures (upstream impervious blanket and/or relief wells) to control underseepage. A trench drain is a trench generally containing a perforated collector pipe and backfilled with filter

material. Trench drains are applicable where the top stratum is thin and the pervious foundation is shallow so that the trench can penetrate into the aquifer. The existence of moderately impervious strata or even stratified fine sands between the bottom of the trench drain and the underlying main sand aquifer will render the trench drain ineffective. Where the pervious foundation is deep, a trench drain of practical depth would only attract a small portion of underseepage, and detrimental underseepage would bypass the drain and emerge downstream of the drain, thereby defeating its purpose. Trench drains may be used in conjunction with relief well systems to collect seepage in the upper pervious foundation that the deeper relief wells do not drain. If the volume of seepage is sufficiently large, the trench drain is provided with a perforated pipe. A trench drain with a collector pipe also provides a means of measuring seepage quantities and of detecting the location of any excessive seepage.

2.1.2 Rock Foundations

General considerations: Seepage should be cut off or controlled by drainage whenever economically feasible. Safety must be the governing factor for selection of a seepage control method.

Cut-off trenches: Cut-off trenches are normally employed when the character of the foundation is such that construction of a satisfactory grout curtain is not practical. Cut-off trenches are normally backfilled with compacted impervious material, bentonite slurry, or neat cement. Construction of trenches in rock foundations normally involves blasting using the presplit method with primary holes deck-loaded according to actual foundation conditions. After blasting, excavation is normally accomplished with a backhoe. Cut-off of seepage within the foundation is obtained by connecting an impervious portion of the foundation to the impervious portion of the structure by backfilling the trench with an impervious material. In rock foundations, as in earth foundations, the impervious layer of the foundation may be sandwiched between an upper and a lower pervious layer, and a cut-off to such an impervious layer would reduce seepage only through the upper pervious layer. However, when the thicknesses of the impervious and upper pervious layers are sufficient, the layers may be able to resist the upward seepage pressures existing in the lower pervious layer and thus remain stable.

Upstream impervious blankets: Impervious blankets may sometimes give adequate control of seepage water for low head structures, but for high head structures it is usually necessary to incorporate a downstream drainage system as a part of the overall seepage control design. The benefits derived from the impervious blanket are due to the dissipation of a part of the reservoir head through the blanket. The proportion of head dissipated is dependent upon the thickness, length, and effective permeability of the blanket in relation to the permeability of the foundation rock. A filter material is normally required between the blanket and foundation.

Grouting: Grouting of rock foundations is used to control seepage. Seepage in rock foundations occurs through cracks and joints, and effectiveness of grouting depends on the nature of the jointing (crack width, spacing, filling, etc.) as well as on the grout mixtures, equipment, and procedures. (1) A grout curtain is constructed beneath the impervious zone of a dam by drilling grout holes and injecting a grout mix. A grout curtain consisting of a single line of holes cannot be depended upon to form a reliable seepage barrier; therefore, a minimum of three lines of grout holes should be used in a rock foundation. Through a study of foundation conditions revealed by geologic investigations, the engineer and geologist can establish the location of the grout curtain in plan, the depths of the grout holes, and grouting procedures. Once grouting has been initiated, the extent and details of the program should be adjusted, as drilling yields additional geological information and as observations of grout take and other data become available. (2) Careful study of grouting requirements is necessary when the foundation is crossed by faults, particularly when the shear zone of a fault consists of badly crushed and fractured rock. It is desirable to seal off such zones by area (consolidation) grouting. When such a fault crosses the proposed dam axis, it may be advisable to excavate along the fault and pour a wedge-shaped concrete cap in which grout pipes are placed so that the fault zone can be grouted at depth between the upstream and downstream toes of the dam. The direction of grout holes should be oriented to optimize the intersection of joints and other defects. (3) Many limestone deposits contain solution cavities. When these are suspected to exist in the foundation, one line (or more) of closely spaced exploration holes is appropriate, since piping may develop or the roofs of undetected cavities may collapse, resulting in dam failure. All solution cavities below the base of the dam should be grouted with multiple lines of grout holes. (4) The

effectiveness of a grouting operation may be evaluated by pre- and post-grouting pressure injection tests for evaluating the water take and the foundation permeability. (5) Development of grouting specifications is a difficult task, and it is even more difficult to find experienced and reliable organizations to execute a grouting program so as to achieve satisfactory results. Grouting operations must be supervised by engineers and geologists with specialized experience (Sherard *et al*, 1967).

2.2 Previous Studies on Safety against Piping

2.2.1 General

Preventing failure from under seepage or piping had been studied by Bligh (1910) and Lane (1935), depending on the distribution of the seepage force below the structure. Furthermore, Terzaghi (1922, 1943) and Harza (1935) studied the same phenomenon depending on the weight of the first element of the foundation material at the dam toe and the seepage force (upward) acting on it.

2.2.2 Terzaghi's Method

Terzaghi (1922) conducted some model tests with a single row of sheet piles as shown in Fig. 2.7 and found that the failure due to piping takes place within a distance of $D/2$ from the sheet piles, where D is the depth of penetration of the sheet piles. Thus, the stability of this type of structure can be determined by considering a soil prism on the downstream side of unit thickness and of section $D^2/2$.

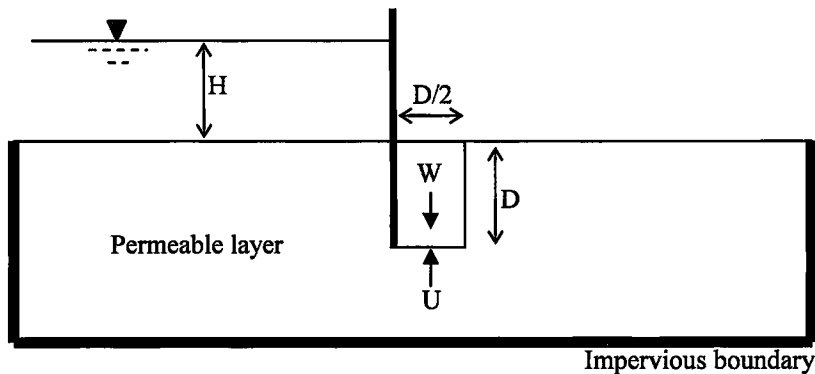


Fig. 2.7 Safety against piping failure for a single-row sheet pile structure

The submerged weight, W , of the soil prism acting vertically downward is given by,

$$W = 0.5 D^2 \gamma_s \quad (2.1)$$

where γ_s is the soil submerged unit weight. The hydraulic uplifting pressure, U , is expressed as,

$$U = 0.5 \gamma_w D h_a \quad (2.2)$$

where h_a is the average head acting on the base of the element, and γ_w is the water unit weight. In the case of equilibrium, $W = U$ and therefore,

$$0.5 D^2 \gamma_s = 0.5 \gamma_w D h_a \quad (2.3)$$

or

$$D \gamma_s = \gamma_w h_a \quad (2.4)$$

On the other hand, the safety factor, F_s , against piping can be written as,

$$F_s = \frac{D \gamma_s}{h_a \gamma_w} = \frac{i_{cr}}{i_m} \quad (2.5)$$

where i_{cr} is the critical hydraulic gradient and is equal to γ_s / γ_w , and i_m is the average hydraulic gradient and is equal to h_a / D . It is clear that this method is not applicable in cases of dams without cut-off walls. For structures other than a single row of sheet piles, such as that in Fig. 2.8, Terzaghi (1943) recommended that the stability of several soil prisms of size $D'D/2$ be investigated to find the minimum factor of safety, where $0 < D' \leq D$.

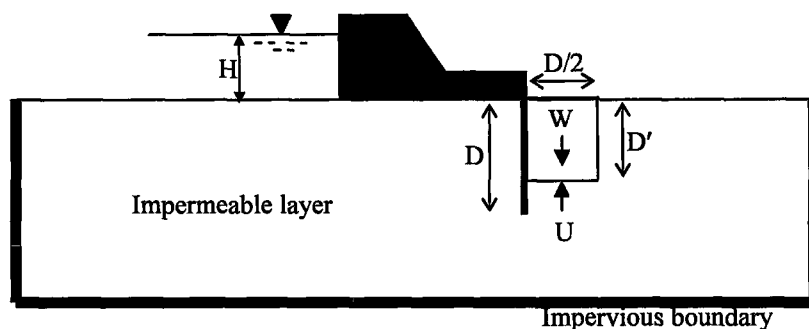


Fig. 2.8 Terzaghi's prism for safety against piping failure under a dam

2.2.3 Harza's Method

Harza (1935) investigated the safety of hydraulic structures against piping and stated that the safety factor, F_s , can be written as,

$$F_s = \frac{i_{cr}}{i_e} \quad (2.6)$$

where i_e is the maximum exit gradient, which can be found from the flow net as shown in Fig. 2.9. The submerged weight of the soil prism, W , acting vertically downward is expressed as,

$$W = \gamma_s \Delta z^2 \quad (2.7)$$

where Δz is the length of the first element of the flow net directly at the dam toe (see Fig. 2.9). The hydraulic uplifting pressure is,

$$U = h_a \gamma_w \Delta z \quad (2.8)$$

where U is the uplifting pressure. Additionally, $h_a = \Delta h = H / N_d$, H is the total hydraulic head, and N_d is the number of potential drops. Hence, Eq. (2.8) can be rewritten as,

$$U = \Delta z \Delta h \gamma_w \quad (2.9)$$

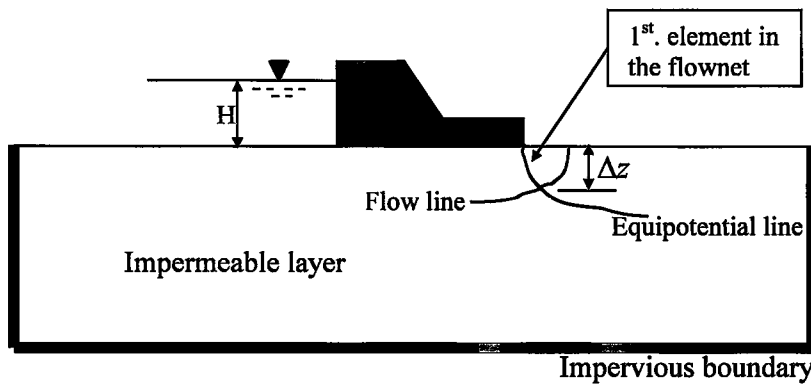


Fig. 2.9 Harza's element for calculation of safety factor against piping failure

However, for equilibrium, $W = U$, one can see that,

$$\gamma_s \Delta z^2 = \gamma_w \Delta z \Delta h \quad (2.10)$$

where the factor of safety is given as,

$$F_s = \frac{\gamma_s \Delta z^2}{\gamma_w \Delta z \Delta h} = \frac{\gamma_s \Delta z}{\gamma_w \Delta h} \quad (2.11)$$

Consideration of $i_{cr} = \frac{\gamma_s}{\gamma_w}$, and $i_e = \frac{\Delta h}{\Delta z}$ leads to,

$$F_s = \frac{i_{cr}}{i_e} \quad (2.12)$$

It is clear that this method gives a general equation, which is applicable in all cases.

2.2.4 Bligh's Method

Bligh (1910) studied the safety of dams against piping taking into consideration that the creep length L_p is equal to $\overline{AB} + 2\overline{BD} + \overline{BC}$ (Fig. 2.10).

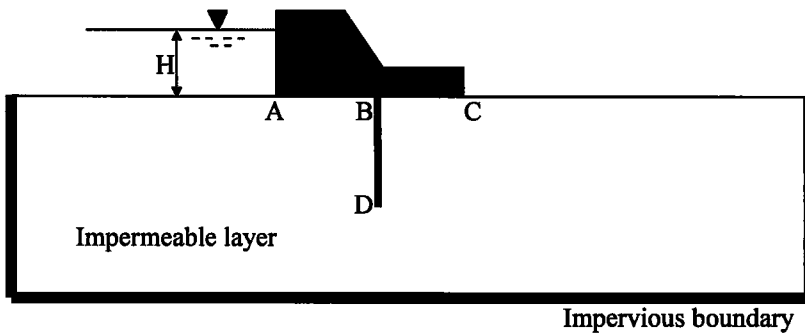


Fig. 2.10 Schematic sketch for calculation of creep distance

It is assumed that the head, H , is dissipated at a constant rate through the length, L_p . This constant rate can be calculated as follows,

$$C'_B = \frac{L_p}{H} \quad (2.13)$$

In order that the structure be safe against piping, the calculated C'_B must be greater than the standard value C_B given in Table 2.1. i.e, $C'_B > C_B$.

Table 2.1 Safe values for Bligh’s creep ratio (C_B)

Class	Description of material	Coeff. C_B
1	River beds of light silt and mud, as that of the Nile	18
2	Fine micaceous sand, as that in the Himalayan rivers and in such rivers as the Colorado in the US	15
3	Coarse grained sand, as in central and south India	12
4	Boulders or shingle and gravel and sand mixed	5 to 9

2.2.5 Lane’s Method

Lane (1935) used a different approach and developed a new method, called the weighted creep method, depending on the fact that in a sedimentary deposit the resistance to flow in the horizontal direction is much less than that in the vertical direction. In this method, Lane compared distances of flow along the soil-concrete contact surface for more than 280 dams and found that as this distance increased, the piping failure occurred less frequently. Therefore, he suggested an empirical approach for the weighted creep, L_w , as,

$$L_w = L_v + \frac{L_h}{3} \tag{2.14}$$

where L_v is the total length of flow lines along the vertical and steeply sloping surfaces ($> 45^\circ$), and L_h is the total length of flow lines along the horizontal and slightly sloping surfaces ($< 45^\circ$), as in Fig. 2.10. It is, therefore, possible to write

$$L_h = \frac{\overline{AB} + \overline{BC}}{3} \tag{2.15}$$

and

$$L_v = 2\overline{BD} \tag{2.16}$$

In this method it is assumed that the head H along the line L_w will be dissipated at a ratio of $1/C'_L$ along the vertical length and at a ratio of $1/3 C'_L$ along the horizontal length. Therefore, Lane's creep coefficient can be written as,

$$C'_L = \frac{k}{v_{\max}} = \frac{L_w}{H} = \frac{\frac{L_h}{3} + L_v}{H} \tag{2.17}$$

The ratio $k/v_{\max}$ represents the maximum exit gradient at the dam toe. It consists of the permeability coefficient, k , and the maximum velocity at which the seeping water can emerge below the dam without carrying away the foundation soil particles, $v_{\max}$. Lane arrived at standard values for his coefficient, C_L , as given in Table 2.2 depending on the soil type.

Table 2.2 Safe values for Lane's weighted creep ratio (C_L)

Material	Safe weighted creep ratio, C_L
Very fine sand and silt	8.5
Fine sand	7.0
Medium sand	6.0
Coarse sand	5.0
Fine gravel	4.0
Coarse gravel	3.0
Soft to medium clay	2.0 - 3.0
Hard clay	1.8
Hard pan	1.6

The calculated value of the coefficient C'_L should be greater than the standard coefficient value for a specific soil type in order that the structure be safe against piping, which requires that $C'_L > C_L$.

3. MATHEMATICAL DESCRIPTION OF GROUNDWATER FLOW

3.1 Introduction

The first great advance in modern saturated-flow theory was made by Henry Darcy, who in 1856 discovered through experimentation that the saturated flow of water through a column of soil is directly proportional to the head difference and inversely proportional to the length of the column. Later, Dupuit (1863), Forchheimer (1886, 1898), and Slichter (1899) did theoretical work independently that led to the development of an equation identical in form with the Laplace equation which describes the movement of groundwater under steady-state conditions.

A rigorous development of the groundwater flow equation generally begins with a mass and momentum balance. For the momentum balance, Darcy's equation, which is based on empirical evidence, is used whereas continuity or conservation is considered for the mass balance.

3.1.1 Darcy's Law

The flow of water in a porous medium is governed by Darcy's law, which may be written as follows

$$v = k i \quad (3.1)$$

where v is the seepage velocity, k is the hydraulic conductivity of the medium, and i is the hydraulic gradient.

Since Darcy's velocity considers the total cross-sectional area normal to the direction of flow, the average Darcy's velocity is computed by dividing Darcy's velocity by the porosity, α (Bouwer and Rice, 1968) as,

$$v_{ave} = \frac{v}{\alpha} \quad (3.2)$$

The hydraulic gradient results from a difference in groundwater potential, dh , across an element, dL , of the medium. Hence,

$$i = -\frac{dh}{dL} \quad (3.3)$$

The negative sign indicates that the flow takes place in the direction of decreasing head. The groundwater potential, h , equals the sum of the kinetic, pressure and potential energy heads, as follows,

$$h = \frac{v^2}{2g} + \frac{p}{\rho g} + z \quad (3.4)$$

where v is the seepage velocity, p is the water pressure, g is the gravitational acceleration, ρ is the water density, and finally z is the elevation.

The kinetic energy is in effect usually negligible in groundwater seepage studies (Cedergren, 1977, pp. 87-9). Therefore Eq. (3.4) reduces to,

$$h = \frac{p}{\rho g} + z \quad (3.5)$$

The three-dimensional generalization of Darcy's law requires that the one-dimensional form in Eq. (3.1) be true for each of the x , y , and z directions of the flow, as

$$v_x(x, y, z) = -k_x(x, y, z) \frac{\partial h(x, y, z)}{\partial x} \quad (3.6.1)$$

$$v_y(x, y, z) = -k_y(x, y, z) \frac{\partial h(x, y, z)}{\partial y} \quad (3.6.2)$$

and

$$v_z(x, y, z) = -k_z(x, y, z) \frac{\partial h(x, y, z)}{\partial z} \quad (3.6.3)$$

respectively.

If the principal directions of anisotropy coincide with the coordinate axes x , y , and z , the components of the seepage velocity, v , will take the following form (Bear, 1972, pp. 143-8)

$$v_x = -k_x \frac{\partial h}{\partial x} \quad (3.7.1)$$

$$v_y = -k_y \frac{\partial h}{\partial y} \quad (3.7.2)$$

and

$$v_z = -k_z \frac{\partial h}{\partial z} \quad (3.7.3)$$

Note that the space derivatives are partial derivatives because the head is now a function of all the three-space coordinates. For solving problems of groundwater flow, Darcy's law alone is not sufficient. It gives only three equations between four unknowns, the three components of the velocity vector and the groundwater potential. An additional equation may be obtained by realizing that the flow has to satisfy the principle of the conservation of mass. Whatever the pattern of flow, no mass can be gained or lost. This leads to an equation of continuity.

3.1.2 Continuity Equation for Steady-State Flow

The equation of continuity is a statement of the law of conservation of matter. When applicable, it states that mass can be neither created nor destroyed. To derive the equation of continuity of flow, consider an elementary soil prism as shown in Fig. 3.1. The flows entering the soil prism in the x , y , and z directions can be given from Darcy's law as follows,

$$q_x = v_x A_x = v_x dy dz \quad (3.8.1)$$

$$q_y = v_y A_y = v_y dx dz \quad (3.8.2)$$

and

$$q_z = v_z A_z = v_z dx dy \quad (3.8.3)$$

where q_x , q_y , and q_z are the flows entering the prism in the x, y, and z directions, respectively, k_x , k_y , and k_z are the coefficients of permeability in the x, y, and z directions, respectively, and h is the hydraulic head.

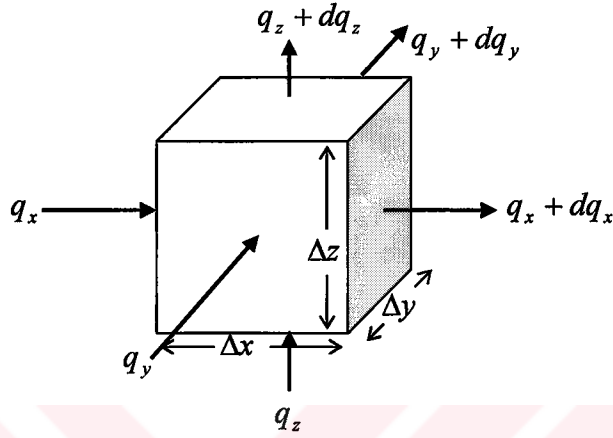


Fig. 3.1 Continuity in a volume element

On the other hand, the respective flows leaving the prism in the x, y, and z directions are,

$$q_x + dq_x = (v_x + \frac{\partial v_x}{\partial x} dx) dy dz \quad (3.9.1)$$

$$q_y + dq_y = (v_y + \frac{\partial v_y}{\partial y} dy) dx dz \quad (3.9.2)$$

and

$$q_z + dq_z = (v_z + \frac{\partial v_z}{\partial z} dz) dx dy \quad (3.9.3)$$

For steady-state flow, continuity requires that $flow_{IN} = flow_{OUT}$. Which implies that,

$$q_x + q_y + q_z = (q_x + dq_x) + (q_y + dq_y) + (q_z + dq_z) \quad (3.10)$$

Combination of Eqs. (3.8) and (3.9) leads to the steady state continuity expression as follows,

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0 \quad (3.11)$$

3.2 General Equation for Steady-State Flow

Substituting Darcy's law component by component from Eq. (3.7) into Eq. (3.11) gives,

$$\frac{\partial}{\partial x} \left(k_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(k_y \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left(k_z \frac{\partial h}{\partial z} \right) = 0 \quad (3.12)$$

This is the full three-dimensional steady-state equation used in most groundwater or seepage problems. For two-dimensional flow in the x-z plane, this expression reduces to,

$$\frac{\partial}{\partial x} \left(k_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial z} \left(k_z \frac{\partial h}{\partial z} \right) = 0 \quad (3.13)$$

3.2.1 Homogeneous and Isotropic Soil

If the soil is homogeneous and isotropic with respect to permeabilities, $k_x = k_z = k$, then Eq. (3.13) simplifies to Laplace equation as,

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \quad (3.14)$$

3.2.2 Homogeneous and Anisotropic Soil

Sedimentary deposits allow the flow to proceed more easily along the planes of deposition than across them. A soil that exhibits this characteristic is defined as being anisotropic, although if the condition of anisotropy is the same everywhere in the stratum, it is still homogeneous. In such a case, the horizontal coefficient of permeability, k_x , is greater than the vertical, k_z . The applicable equation in anisotropic soil, assuming that the directional permeability to be invariant, is given as, (Rushton and Redshaw, 1978)

$$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \quad (3.15)$$

This may be reduced to the Laplace equation by means of a simple transformation as $x' = nx$, from which it is possible to obtain $\frac{\partial x'}{\partial x} = n$, or $\frac{\partial x}{\partial x'} = \frac{1}{n}$, where n is the transformation factor. Using the algebraic chain rule it is possible to obtain,

$$\frac{\partial h}{\partial x'} = \frac{\partial h}{\partial x} \frac{\partial x}{\partial x'} + \frac{\partial h}{\partial z} \frac{\partial z}{\partial x'} \quad (3.16)$$

however, since $\frac{\partial z}{\partial x'} = 0$, it reduces to,

$$\frac{\partial h}{\partial x'} = \frac{\partial h}{\partial x} \frac{1}{n} \quad (3.17)$$

and

$$\frac{\partial^2 h}{\partial x'^2} = \frac{\partial(\frac{\partial h}{\partial x'})}{\partial x} \frac{\partial x}{\partial x'} = \frac{\partial(\frac{\partial h}{\partial x} \frac{1}{n})}{\partial x} \frac{1}{n} = \frac{\partial^2 h}{\partial x^2} \frac{1}{n^2} \quad (3.18)$$

substitution of $\frac{\partial^2 h}{\partial x'^2}$ from Eq. (3.18) into Eq. (3.15), gives finally,

$$k_x n^2 \frac{\partial^2 h}{\partial x'^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \quad (3.19)$$

which becomes Laplace equation in the variables x' and z if $n^2 = \frac{k_z}{k_x}$ or

$n = \sqrt{k_z / k_x}$. Thus,

$$\frac{\partial^2 h}{\partial x'^2} + \frac{\partial^2 h}{\partial z^2} = 0 \quad (3.20)$$

Once the transformation has been effected, a problem involving an anisotropic medium reduces to that of an isotropic medium. Finally, the results can be re-plotted to the true dimensional scale. Harr (1962) has considered a number of problems involving stratified soils.

3.2.3 Heterogeneous and Isotropic Soil

In this case, the three principal hydraulic conductivities at a point in the aquifer are equal, but their values vary from point to point. Therefore, Eq. (3.13) takes the form,

$$\frac{\partial}{\partial x}(k_x(x, z)\frac{\partial h}{\partial x}) + \frac{\partial}{\partial z}(k_z(x, z)\frac{\partial h}{\partial z}) = 0 \quad (3.21)$$

3.2.4 Heterogeneous and Anisotropic Soil

This is the most general case in which the three principal hydraulic conductivities at a point in the aquifer are not equal and their values vary from point to point. Therefore, Eq. (3.13) becomes,

$$\frac{\partial}{\partial x}(k_x(x, z)\frac{\partial h}{\partial x}) + \frac{\partial}{\partial z}(k_z(x, z)\frac{\partial h}{\partial z}) = 0 \quad (3.22)$$

Conditions for Eqs. (3.21) and (3.22) are that the hydraulic conductivity functions must be continuous, and have continuous first derivatives everywhere in the flow domain. Since the hydraulic conductivities k_x , k_y , and k_z are functions of the spatial variables x , y , and z , it is impossible to reduce this equation to any form of Laplace equation. Therefore, they can only be solved numerically using spatial discretization schemes (e.g. finite difference, finite element, integrated finite difference, boundary element, etc.).

4. NUMERICAL METHOD USED

4.1 General

Using techniques of calculus, it is sometimes possible to write an expression relating different variables as a function of the space coordinates. Thus, an analytical solution is obtained. From such a solution, it is possible to calculate the values of the unknown variable at all points in the system. However, in order to obtain an analytical solution, several simplifying assumptions have to be made (e.g., homogenization and idealization of the system). For many problems, these assumptions are not realistic. Hence, in these cases, it is advisable to resort to approximate methods using numerical techniques.

A numerical solution yields values for only predetermined finite number of points in the problem solution domain. By limiting the need to know the piezometric head to a reasonable number of points N , a partial differential equation can be converted into a set of N algebraic equations involving N unknown heads. Using a digital computer, solution of these equations can be obtained. The numerical technique that is considered in the present study is the finite difference approximation.

4.2 Finite Difference Solution

Consider a finite set of nodal points on a grid network as shown in Fig. 4.1 where the points are spaced horizontally by a distance Δx and vertically by Δz . For locating any nodal point in the grid, an integer ordered pair (i,j) is specified. Relative to the ordered pair $(1,1)$ located in the upper left-hand corner, (i,j) is a distance $(i-1)\Delta x$ in the positive x direction and $(j-1)\Delta z$ in the negative z direction.

In the finite difference approximation, derivatives are replaced by differences taken between nodal points (Wang and Anderson, 1982). With non-uniform spacing as shown in Fig. 4.2, the first part in Eq. (3.13) is obtained by taking a difference

between the first derivatives calculated midway between $(i+1,j)$, (i,j) and $(i-1,j)$, (i,j) , respectively.

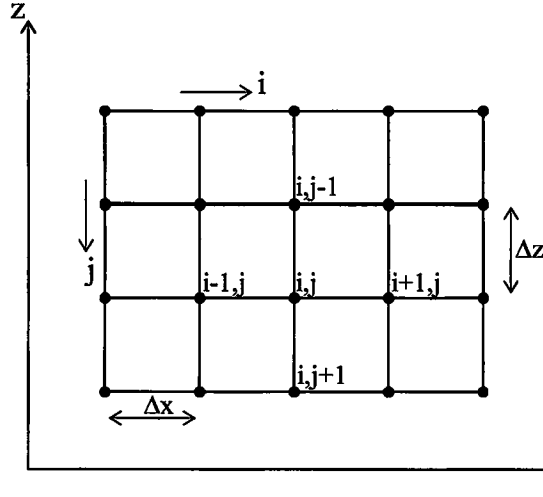


Fig. 4.1 Finite difference grid with uniform spacing

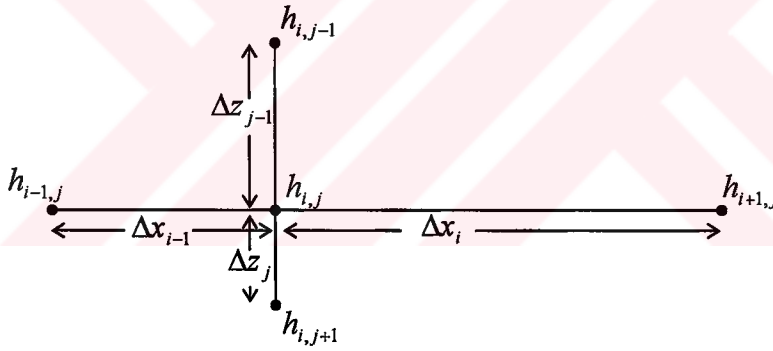


Fig. 4.2 Nodal points with non-uniform spacing

Therefore, the first derivative midway between the nodes at $(i+1,j)$ and (i,j) in the x -direction is taken as,

$$\frac{\partial h}{\partial x_1} = \frac{h_{i+1,j} - h_{i,j}}{\Delta x_i} \quad (4.1)$$

and the flow per unit area between $(i+1,j)$ and (i,j) can be expressed as,

$$k_{x_1} \frac{h_{i+1,j} - h_{i,j}}{\Delta x_i} \quad (4.2)$$

Similarly, the first derivative midway between the nodes at $(i-1,j)$ and (i,j) is,

$$\frac{\partial h}{\partial x_2} = \frac{h_{i,j} - h_{i-1,j}}{\Delta x_{i-1}} \quad (4.3)$$

and the flow per unit area between (i-1,j) and (i,j) becomes,

$$k_{x_2} \frac{h_{i,j} - h_{i-1,j}}{\Delta x_{i-1}} \quad (4.4)$$

The second derivative is then obtained by taking a difference between the first derivatives calculated midway between (i+1,j), (i,j) and (i-1,j), (i,j), respectively, which leads to,

$$\begin{aligned} \frac{\partial}{\partial x} (k_x \frac{\partial h}{\partial x}) &= \frac{k_{x_1} \frac{h_{i+1,j} - h_{i,j}}{\Delta x_i} - k_{x_2} \frac{h_{i,j} - h_{i-1,j}}{\Delta x_{i-1}}}{\frac{\Delta x_i + \Delta x_{i-1}}{2}} \\ &= \frac{2}{\Delta x_i + \Delta x_{i-1}} \left[k_{x_1} \frac{h_{i+1,j} - h_{i,j}}{\Delta x_i} - k_{x_2} \frac{h_{i,j} - h_{i-1,j}}{\Delta x_{i-1}} \right] \end{aligned} \quad (4.5)$$

where

$$k_{x_1} = \frac{(k_x)_{i+1,j} + (k_x)_{i,j}}{2}, \text{ and } k_{x_2} = \frac{(k_x)_{i-1,j} + (k_x)_{i,j}}{2}$$

Similarly in the z-direction, it is possible to write,

$$\frac{\partial}{\partial z} (k_z \frac{\partial h}{\partial z}) = \frac{2}{\Delta z_j + \Delta z_{j-1}} \left[k_{z_1} \frac{h_{i,j-1} - h_{i,j}}{\Delta z_{j-1}} - k_{z_2} \frac{h_{i,j} - h_{i,j+1}}{\Delta z_j} \right] \quad (4.6)$$

where

$$k_{z_1} = \frac{(k_z)_{i,j-1} + (k_z)_{i,j}}{2}, \text{ and } k_{z_2} = \frac{(k_z)_{i,j+1} + (k_z)_{i,j}}{2}$$

By considering Eqs. (4.5) and (4.6), the finite difference form of Eq. (3.13) will be,

$$\begin{aligned} &\frac{2}{\Delta x_i + \Delta x_{i-1}} \left[k_{x_1} \frac{h_{i+1,j} - h_{i,j}}{\Delta x_i} - k_{x_2} \frac{h_{i,j} - h_{i-1,j}}{\Delta x_{i-1}} \right] + \\ &\frac{2}{\Delta z_j + \Delta z_{j-1}} \left[k_{z_1} \frac{h_{i,j-1} - h_{i,j}}{\Delta z_{j-1}} - k_{z_2} \frac{h_{i,j} - h_{i,j+1}}{\Delta z_j} \right] = 0 \end{aligned} \quad (4.7)$$

If $\Delta x_i = \Delta x_{i-1} = \Delta x$, and $\Delta z_j = \Delta z_{j-1} = \Delta z$, then Eq. (4.7) can be written as,

$$\begin{aligned} \frac{1}{\Delta x} \left[k_{x_1} \frac{h_{i+1,j} - h_{i,j}}{\Delta x} - k_{x_2} \frac{h_{i,j} - h_{i-1,j}}{\Delta x} \right] + \\ \frac{1}{\Delta z} \left[k_{z_1} \frac{h_{i,j-1} - h_{i,j}}{\Delta z} - k_{z_2} \frac{h_{i,j} - h_{i,j+1}}{\Delta z} \right] = 0 \end{aligned} \quad (4.8)$$

Furthermore, multiplication by $\Delta x \Delta z$ gives,

$$\begin{aligned} \Delta z \left[k_{x_1} \frac{h_{i+1,j} - h_{i,j}}{\Delta x} - k_{x_2} \frac{h_{i,j} - h_{i-1,j}}{\Delta x} \right] + \\ \Delta x \left[k_{z_1} \frac{h_{i,j-1} - h_{i,j}}{\Delta z} - k_{z_2} \frac{h_{i,j} - h_{i,j+1}}{\Delta z} \right] = 0 \end{aligned} \quad (4.9)$$

Each term in Eq. (4.9) represents the flow between the indicated nodes for unit width. Considering that $B = \Delta x / \Delta z$, one can write that,

$$\begin{aligned} \frac{1}{B} \left[k_{x_1} (h_{i+1,j} - h_{i,j}) - k_{x_2} (h_{i,j} - h_{i-1,j}) \right] + \\ B \left[k_{z_1} (h_{i,j-1} - h_{i,j}) - k_{z_2} (h_{i,j} - h_{i,j+1}) \right] = 0 \end{aligned} \quad (4.10)$$

Multiplication of this expression by B , yields,

$$\begin{aligned} \left[k_{x_1} (h_{i+1,j} - h_{i,j}) - k_{x_2} (h_{i,j} - h_{i-1,j}) \right] + \\ B^2 \left[k_{z_1} (h_{i,j-1} - h_{i,j}) - k_{z_2} (h_{i,j} - h_{i,j+1}) \right] = 0 \end{aligned} \quad (4.11)$$

Finally, the solution for $h_{i,j}$ gives,

$$h_{i,j} = \frac{k_{x_1} h_{i+1,j} + k_{x_2} h_{i-1,j} + B^2 [k_{z_1} h_{i,j-1} + k_{z_2} h_{i,j+1}]}{k_{x_1} + k_{x_2} + B^2 (k_{z_1} + k_{z_2})} \quad (4.12)$$

In the derivation of this expression, the forward finite difference approximation is used as described by Rushton and Redshaw (1978). In this equation, the unknown piezometric head, $h_{i,j}$, at the time step $m+1$ is written explicitly in terms of the heads at the time step m . Hence, Eq. (4.12) may be rewritten as,

$$h_{i,j}^{m+1} = \frac{k_{x_1} h_{i+1,j}^m + k_{x_2} h_{i-1,j}^m + B^2 [k_{z_1} h_{i,j-1}^m + k_{z_2} h_{i,j+1}^m]}{k_{x_1} + k_{x_2} + B^2 (k_{z_1} + k_{z_2})} \quad (4.13)$$

For homogeneous and isotropic soils (equal permeabilities) and a square finite difference grid ($B = 1$), this expression will reduce to,

$$h_{i,j}^{m+1} = \frac{1}{4}(h_{i-1,j}^m + h_{i+1,j}^m + h_{i,j-1}^m + h_{i,j+1}^m) \quad (4.14)$$

Eq. (4.13) forms the heart of the developed computer program. There will be one expression in the form of Eq. (4.13) for each interior node (i,j) of the problem domain. By applying such an equation to each interior node, it is possible to obtain a set of algebraic equations to be solved for selected boundary conditions.

4.3 Boundary Conditions

The solution of a differential equation requires specifications of boundary conditions, which constrain the problem and make the solution unique. The different types of boundary conditions are:

- 1- head is known for surfaces bounding the flow region (**Dirichlet condition**). This is a valid condition if the groundwater potential remains at a constant value, because the aquifer is in a hydraulic continuity with the sea or a large lake or river. The condition is that $h = \text{constant}$. It should also be noted that in order to obtain a unique distribution of heads in a steady-state problem, there must be at least one node on a Dirichlet condition boundary type. This condition is a mathematical requirement (Hildebrand, 1976). In unsteady-state problems, initial conditions are sufficient to ensure uniqueness.
- 2- Flow is known across surfaces bounding the flow region (**Neumann condition**). Usually, it is difficult to determine the quantity of water that is transferred in or out the flow region. Often, estimation can be made by considering water balance of the adjacent strata. The simplest form of a known flow boundary is that there is no flow across the boundary. Examples are line of symmetry, groundwater divide, and impermeable boundary. For these situations the condition is $\partial h / \partial s = 0$, where h is the hydraulic head, and s is the direction normal to the boundary.
- 3- Some combination of Dirichlet and Neumann conditions are known for surfaces bounding the flow region (**mixed condition**).

If inconsistent or incomplete boundary conditions are specified, the problem itself is regarded as ill defined.

4.4 Finite Difference Expressions for Boundary Nodes

The finite difference expressions for nodes on external boundaries must satisfy both the general equations governing the flow within the study area, and the boundary conditions at those nodes. To derive the finite difference equations necessary on the boundary, Darcy's law is used to express the flow within the region of influence of point 0 as shown in Fig. 4.3 (Remson, *et al*, 1971). The region of influence of this point is bounded by lines parallel to the grid axes and midway between adjacent grid points. The sides of the influence region are either Δx or $(\Delta x / 2)$ in length and Δz or $(\Delta z / 2)$ in height.

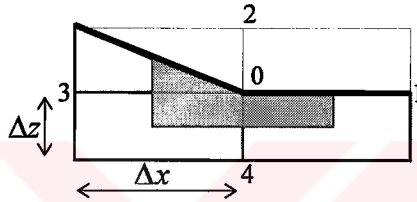


Fig. 4.3 The region of influence of the nodal point 0

4.4.1 Corner Node Adjacent to an Inclined Boundary and the Flow Region is Below the Boundary

This situation is schematized in Fig. 4.4 with the notations at hand.

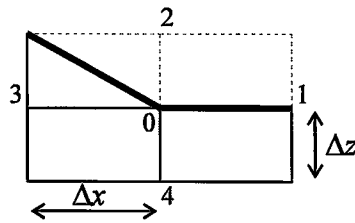


Fig. 4.4 Corner node adjacent to an inclined boundary, the flow region is below the boundary

It is possible from Darcy's law that, the flow between the points 0 & 1 is

$$Q_{0-1} = kiA = -k_{x1} \frac{h_1 - h_0}{\Delta x_i} \frac{\Delta z_j}{2}; \text{ flow between 0 \& 2 is } Q_{0-2} = 0; \text{ flow between 0 \& 3}$$

$$Q_{0-3} = kiA = k_{x2} \frac{h_0 - h_3}{\Delta x_{i-1}} \frac{\Delta z_j + \Delta z_{j-1}}{2}, \text{ and finally the flow between 0 and 4 is}$$

$$\text{expressed as } Q_{0-4} = kiA = k_{z2} \frac{h_0 - h_4}{\Delta z_j} \frac{\Delta x_i + \Delta x_{i-1}}{2}.$$

Since the flow is steady, continuity condition asserts that $\sum Q = 0$. Therefore, one can write,

$$-k_{x1} \frac{h_1 - h_0}{\Delta x_i} \frac{\Delta z_j}{2} + k_{x2} \frac{h_0 - h_3}{\Delta x_{i-1}} \frac{\Delta z_j + \Delta z_{j-1}}{2} + k_{z2} \frac{h_0 - h_4}{\Delta z_j} \frac{\Delta x_i + \Delta x_{i-1}}{2} = 0 \quad (4.15)$$

If $\Delta x_i = \Delta x_{i-1} = \Delta x$, $\Delta z_j = \Delta z_{j-1} = \Delta z$, and $B = \Delta x / \Delta z$, then,

$$-\frac{k_{x1}}{2B}(h_1 - h_0) + \frac{k_{x2}}{B}(h_0 - h_3) + Bk_{z2}(h_0 - h_4) = 0 \quad (4.16)$$

Solution for h_0 leads to,

$$h_0 = \frac{k_{x1}h_1 + 2k_{x2}h_3 + 2B^2k_{z2}h_4}{k_{x1} + 2k_{x2} + 2B^2k_{z2}} \quad (4.17)$$

For homogeneous and isotropic soil where $\Delta x \neq \Delta z$ one can see that,

$$h_0 = \frac{h_1 + 2h_3 + 2B^2h_4}{3 + 2B^2} \quad (4.18)$$

However, for a square grid where $\Delta x = \Delta z$ it becomes as,

$$h_0 = \frac{1}{2.5}(h_3 + h_4 + \frac{1}{2}h_1) \quad (4.19)$$

4.4.2 Corner Node Adjacent to an Inclined Boundary and the Flow Region is Above the Boundary

This boundary type is exemplified as shown in Fig. 4.5.

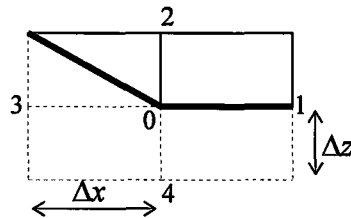


Fig. 4.5 Corner node adjacent to an inclined boundary, the flow region is above the boundary

For steady-state flow, continuity asserts that $\sum Q = 0$ and accordingly,

$$-k_{x1} \frac{h_1 - h_0}{\Delta x_i} \frac{\Delta z_{j-1}}{2} - k_{z1} \frac{h_2 - h_0}{\Delta z_{j-1}} \frac{\Delta x_i + \Delta x_{i-1}}{2} = 0 \quad (4.20)$$

If $\Delta x_i = \Delta x_{i-1} = \Delta x$, $\Delta z_j = \Delta z_{j-1} = \Delta z$, and $B = \Delta x / \Delta z$, then,

$$-\frac{k_{x1}}{2B}(h_1 - h_0) - Bk_{z1}(h_2 - h_0) = 0 \quad (4.21)$$

Solution for h_0 gives,

$$h_0 = \frac{k_{x1}h_1 + 2B^2k_{z1}h_2}{k_{x1} + 2B^2k_{z1}} \quad (4.22)$$

For homogeneous and isotropic soil where $\Delta x \neq \Delta z$ one can obtain,

$$h_0 = \frac{h_1 + 2B^2h_2}{1 + 2B^2} \quad (4.23)$$

Finally in the case of a square grid where $\Delta x = \Delta z$ it becomes,

$$h_0 = \frac{1}{1.5} \left(h_2 + \frac{1}{2} h_1 \right) \quad (4.24)$$

4.4.3 90° - Boundary Corner Node, flow is above the boundary

The corresponding configuration of the points in such a case is given as shown in Fig. 4.6.

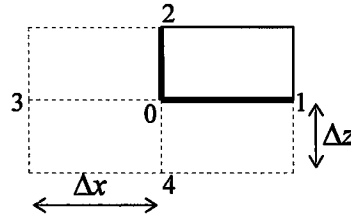


Fig. 4.6 90°-Boundary corner node, flow is above the boundary

Since the flow is steady, continuity asserts that $\sum Q = 0$. Therefore, one can write,

$$-k_{x1} \frac{h_1 - h_0}{\Delta x_i} \frac{\Delta z_{j-1}}{2} - k_{z1} \frac{h_2 - h_0}{\Delta z_{j-1}} \frac{\Delta x_i}{2} = 0 \quad (4.25)$$

If $\Delta x_i = \Delta x_{i-1} = \Delta x$, $\Delta z_j = \Delta z_{j-1} = \Delta z$, and $B = \Delta x / \Delta z$, then,

$$-\frac{k_{x1}}{2B}(h_1 - h_0) - \frac{Bk_{z1}}{2}(h_2 - h_0) = 0 \quad (4.26)$$

Solution for h_0 gives,

$$h_0 = \frac{k_{x1}h_1 + B^2k_{z1}h_2}{k_{x1} + B^2k_{z1}} \quad (4.27)$$

For homogeneous and isotropic soil with $\Delta x \neq \Delta z$ it becomes,

$$h_0 = \frac{h_1 + B^2h_2}{1 + B^2} \quad (4.28)$$

Finally, for a square grid ($\Delta x = \Delta z$) one can write that,

$$h_0 = \frac{1}{2}(h_1 + h_2) \quad (4.29)$$

4.4.4 90° - Boundary Corner Node, flow is under the boundary

The configuration of the five points (0, 1, 2, 3, and 4) is prepared as shown in Fig.4.7 to locate the corner node on a 90°-boundary.

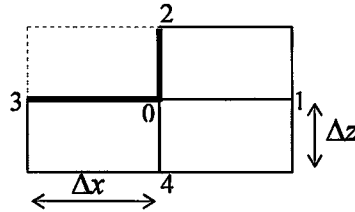


Fig. 4.7 90°- Boundary corner node, flow is under the boundary

Likewise, for steady-state flow, continuity implies that $\sum Q = 0$ and one can write,

$$\begin{aligned} & -k_{x1} \frac{h_1 - h_0}{\Delta x_i} \frac{\Delta z_j + \Delta z_{j-1}}{2} + k_{x2} \frac{h_0 - h_3}{\Delta x_{i-1}} \frac{\Delta z_j}{2} - k_{z1} \frac{h_2 - h_0}{\Delta z_{j-1}} \frac{\Delta x_i}{2} + \\ & + k_{z2} \frac{h_0 - h_4}{\Delta z_j} \frac{\Delta x_i + \Delta x_{i-1}}{2} = 0 \end{aligned} \quad (4.30)$$

If $\Delta x_i = \Delta x_{i-1} = \Delta x$, $\Delta z_j = \Delta z_{j-1} = \Delta z$, and $B = \Delta x / \Delta z$, it is possible to obtain the following expression,

$$-\frac{k_{x1}}{B}(h_1 - h_0) + \frac{k_{x2}}{2B}(h_0 - h_3) - \frac{Bk_{z1}}{2}(h_2 - h_0) + Bk_{z2}(h_0 - h_4) = 0 \quad (4.31)$$

Solution for h_0 yields,

$$h_0 = \frac{2k_{x1}h_1 + k_{x2}h_3 + B^2k_{z1}h_2 + 2B^2k_{z2}h_4}{2k_{x1} + k_{x2} + B^2k_{z1} + 2B^2k_{z2}} \quad (4.32)$$

For homogeneous and isotropic soil where $\Delta x \neq \Delta z$, it becomes

$$h_0 = \frac{2h_1 + h_3 + B^2h_2 + 2B^2h_4}{3 + 3B^2} \quad (4.33)$$

Finally, for a square grid where $\Delta x = \Delta z$, it simplifies to,

$$h_0 = \frac{1}{3}(h_1 + h_4 + \frac{1}{2}h_2 + \frac{1}{2}h_3) \quad (4.34)$$

4.4.5 Boundary Node on an Inclined Surface

Figure 4.8 includes the configuration of nodal points, which lie on a straight inclined boundary.

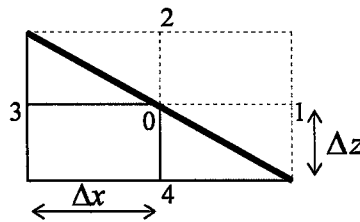


Fig. 4.8 Boundary node on an inclined surface

In a steady-state flow, continuity condition is $\sum Q = 0$, which leads to,

$$k_{x2} \frac{h_0 - h_3}{\Delta x_{i-1}} \frac{\Delta z_j + \Delta z_{j-1}}{2} + k_{z2} \frac{h_0 - h_4}{\Delta z_j} \frac{\Delta x_i + \Delta x_{i-1}}{2} = 0 \quad (4.35)$$

If $\Delta x_i = \Delta x_{i-1} = \Delta x$, $\Delta z_j = \Delta z_{j-1} = \Delta z$, and $B = \Delta x / \Delta z$, then one can obtain

$$\frac{k_{x2}}{B}(h_0 - h_3) + Bk_{z2}(h_0 - h_4) = 0 \quad (4.36)$$

Solution for h_0 gives,

$$h_0 = \frac{k_{x2}h_3 + B^2k_{z2}h_4}{k_{x2} + B^2k_{z2}} \quad (4.37)$$

For homogeneous and isotropic soil where $\Delta x \neq \Delta z$ it reduces to,

$$h_0 = \frac{h_3 + B^2h_4}{1 + B^2} \quad (4.38)$$

However, for a square grid where $\Delta x = \Delta z$ it can be written as

$$h_0 = \frac{1}{2}(h_3 + h_4) \quad (4.39)$$

4.4.6 Boundary Node on a Horizontal or Vertical Surface

This boundary condition is exemplified with the nodal configuration shown in Fig. 4.9.

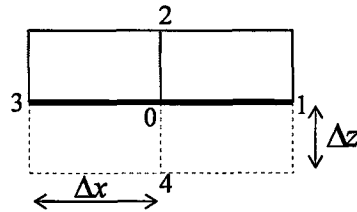


Fig. 4.9 Boundary node on a horizontal or vertical surface, the flow region is above the boundary

Similar to the previous boundary conditions, continuity in a steady state flow asserts that $\sum Q = 0$, and hence,

$$-k_{x1} \frac{h_1 - h_0}{\Delta x_i} \frac{\Delta z_{j-1}}{2} - k_{z1} \frac{h_2 - h_0}{\Delta z_{j-1}} \frac{\Delta x_i + \Delta x_{i-1}}{2} + k_{x2} \frac{h_0 - h_3}{\Delta x_{i-1}} \frac{\Delta z_{j-1}}{2} = 0 \quad (4.40)$$

If $\Delta x_i = \Delta x_{i-1} = \Delta x$, $\Delta z_j = \Delta z_{j-1} = \Delta z$, and $B = \Delta x / \Delta z$, then,

$$-\frac{k_{x1}}{2B}(h_1 - h_0) - Bk_{z1}(h_2 - h_0) + \frac{k_{x2}}{2B}(h_0 - h_3) = 0, \quad (4.41)$$

Solution for h_0 leads to,

$$h_0 = \frac{k_{x1}h_1 + 2B^2k_{z1}h_2 + k_{x2}h_3}{k_{x1} + 2B^2k_{z1} + k_{x2}} \quad (4.42)$$

For homogeneous and isotropic soil where $\Delta x \neq \Delta z$ it simplifies to,

$$h_0 = \frac{h_1 + 2B^2h_2 + h_3}{2(1 + B^2)} \quad (4.43)$$

However, for a square grid where $\Delta x = \Delta z$ it becomes,

$$h_0 = \frac{1}{4}(h_1 + h_3 + 2h_2) \quad (4.44)$$

The relationships for the piezometric head at similar grid nodal points but for different directions of flow are derived from the above equations with suitable indices representing the location of the respective nodal points. A summary of the previously mentioned boundary conditions is given in Fig. 4.10.

4.5 Iterative Methods

4.5.1 General

The earliest iterative methods were based on finite-difference approximations and solutions to seepage problems were obtained by the hand calculations of the relaxation method (Allen, 1954). However, in recent years a wide range of finite-difference techniques suited to the digital computer have been devised. Remson et al. (1971) and Rushton and Redshaw (1978) describe the mathematical basis of many of these techniques.

In problems having a large number of nodes, simultaneous solutions by hand are impractical. Instead, iterative methods, which consist of guessing and adjusting, may be used where the unknown heads are initially assigned by arbitrary values.

	Heterog. & Aniso: $h_0 = \frac{k_{x1}h_1 + 2k_{x2}h_3 + 2B^2k_{z2}h_4}{k_{x1} + 2k_{x2} + 2B^2k_{z2}}$
	Homog. & Iso : $h_0 = \frac{h_1 + 2h_3 + 2B^2h_4}{3 + 2B^2}$
	Heterog. & Aniso: $h_0 = \frac{k_{x1}h_1 + 2B^2k_{z1}h_2}{k_{x1} + 2B^2k_{z1}}$
	Homog. & Iso : $h_0 = \frac{h_1 + 2B^2h_2}{1 + 2B^2}$
	Heterog. & Aniso: $h_0 = \frac{k_{x1}h_1 + B^2k_{z1}h_2}{k_{x1} + B^2k_{z1}}$
	Homog. & Iso : $h_0 = \frac{h_1 + B^2h_2}{1 + B^2}$
	Heter. & Aniso: $h_0 = \frac{2k_{x1}h_1 + k_{x2}h_3 + 2B^2k_{z1}h_2 + B^2k_{z2}h_4}{2k_{x1} + k_{x2} + 2B^2k_{z1} + B^2k_{z2}}$
	Homog. & Iso : $h_0 = \frac{2h_1 + h_3 + 2B^2h_2 + B^2h_4}{3 + 3B^2}$
	Heterog. & Aniso: $h_0 = \frac{k_{x2}h_3 + B^2k_{z2}h_4}{k_{x2} + B^2k_{z2}}$
	Homog. & Iso : $h_0 = \frac{h_3 + B^2h_4}{1 + B^2}$
	Heterog. & Aniso: $h_0 = \frac{k_{x1}h_1 + 2B^2k_{z1}h_2 + k_{x2}h_3}{k_{x1} + 2B^2k_{z1} + k_{x2}}$
	Homog. & Iso : $h_0 = \frac{h_1 + 2B^2h_2 + h_3}{2(1 + B^2)}$

Fig. 4.10 Summary of boundary conditions

4.5.2 Gauss-Seidel Iteration

In this method, the process is to work through the grid in an orderly way; that is, to start at the first interior node at the top left corner and sweep across from left to right and down line by line as if a page is read, as shown in Fig 4.11. In this way, it is always possible to use two newly computed values in the iteration formula. That is, the piezometric heads at the nodes to the left and above node (i,j) will be computed before the head at node (i,j). Thus the Gauss-Seidel iteration formula is:

$$h_{i,j}^{m+1} = \frac{k_{x_1} h_{i+1,j}^m + k_{x_2} h_{i-1,j}^{m+1} + B^2 [k_{z_1} h_{i,j-1}^{m+1} + k_{z_2} h_{i,j+1}^m]}{k_{x_1} + k_{x_2} + B^2 (k_{z_1} + k_{z_2})} \quad (4.45)$$

In other words, $h_{i-1,j}$ and $h_{i,j-1}$ will be known at the $m+1$ iteration level when it is time to evaluate $h_{i,j}$ at the $m+1$ iteration level. Since the first piezometric heads are assumed values, Eq. (4.45) will not be satisfied. The change between two successive Gauss-Seidel iterations is called the residual ε , which is defined as,

$$\varepsilon = h_{i,j}^{m+1} - h_{i,j}^m \quad (4.46)$$

Finally, replacement of $h_{i,j}^m$ with $h_{i,j}^{m+1}$ after each calculation, the Gauss-Seidel procedure relaxes the residual at every node and the solution will converge to the true values.

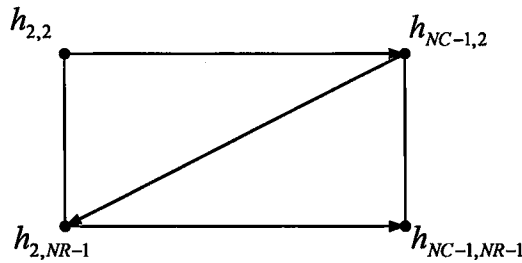


Fig. 4.11 Gauss-Seidel iteration, the arrows indicate the order of iteration

4.5.3 Successive Over-Relaxation Method

For the correction of $h_{i,j}$ value, the successive over relaxation (SOR) process is used.

In SOR, the residual is multiplied by a relaxation factor ω , and the new value $h_{i,j}^{m+1}$ is given by,

$$h_{i,j}^{m+1} = h_{i,j}^m + \omega \varepsilon \quad (4.47)$$

Substitution of $h_{i,j}^{m+1}$ from Eq. (4.45) into Eq. (4.46) and the result subsequently into Eq. (4.47), one can obtain the following expression as,

$$h_{i,j}^{m+1} = (1-\omega)h_{i,j}^m + \omega \frac{k_{x_1} h_{i+1,j}^m + k_{x_2} h_{i-1,j}^m + B^2 [k_{z_1} h_{i,j-1}^{m+1} + k_{z_2} h_{i,j+1}^m]}{k_{x_1} + k_{x_2} + B^2 (k_{z_1} + k_{z_2})} \quad (4.48)$$

where m is the iteration level, and ω is the relaxation factor, ($1 \leq \omega \leq 2$). With the new heads, the process is repeated and the iteration is carried out until the values of $(h_{i,j}^{m+1})_{corr}$ equal $(h_{i,j}^m)_{corr}$. That is, until the difference between the answers generated at two successive iteration levels is less than the preset error tolerance (Wang and Anderson, 1982). If the convergence criterion is satisfied, then the program terminates and prints out the results.

4.6 Approximation Using Taylor's Series Expansion

The approximation introduced by the finite-difference approach can also be identified by considering Taylor's series. Therefore, with reference to Fig. 4.1, it is possible to have the following expression,

$$h_{i+1,j} = h_{i,j} + \frac{\Delta x}{1!} \left(\frac{dh}{dx} \right)_{i,j} + \frac{\Delta x^2}{2!} \left(\frac{d^2 h}{dx^2} \right)_{i,j} + \frac{\Delta x^3}{3!} \left(\frac{d^3 h}{dx^3} \right)_{i,j} + \dots \quad (4.49)$$

Similarly,

$$h_{i-1,j} = h_{i,j} - \frac{\Delta x}{1!} \left(\frac{dh}{dx} \right)_{i,j} + \frac{\Delta x^2}{2!} \left(\frac{d^2 h}{dx^2} \right)_{i,j} - \frac{\Delta x^3}{3!} \left(\frac{d^3 h}{dx^3} \right)_{i,j} + \dots \quad (4.50)$$

Summation of Eqs. (4.49) and (4.50) leads to,

$$h_{i+1,j} - 2h_{i,j} + h_{i-1,j} = \Delta x^2 \left(\frac{d^2 h}{dx^2} \right)_{i,j} + \frac{\Delta x^4}{12} \left(\frac{d^4 h}{dx^4} \right)_{i,j} + \dots \quad (4.51)$$

Solving for $(d^2 h / dx^2)_{i,j}$, one can obtain,

$$\left(\frac{d^2 h}{dx^2} \right)_{i,j} = \frac{h_{i+1,j} - 2h_{i,j} + h_{i-1,j}}{\Delta x^2} - \frac{\Delta x^2}{12} \left(\frac{d^4 h}{dx^4} \right)_{i,j} + \dots \quad (4.52)$$

The terms

$$\frac{\Delta x^2}{12} \left(\frac{d^4 h}{dx^4} \right)_{i,j} + \dots \quad (4.53)$$

are called the truncation error between the exact and the approximate differentials and are proportional to Δx^2 . If in a certain problem the truncation error is zero (i.e., Δx tends toward zero), then the finite-difference solution is identical to the theoretical solution. Moreover, If two solutions are obtained, the second having a mesh interval equal to half of that for the first solution, then the truncation error for the second solution is only one quarter of that for the first. As a result, the second solution will be more accurate. Neglecting the truncation error, the second derivative at the node (i,j) in the x-direction can be obtained from Eq. (4.52) as,

$$\left(\frac{d^2 h}{dx^2} \right)_{i,j} = \frac{h_{i+1,j} - 2h_{i,j} + h_{i-1,j}}{\Delta x^2} \quad (4.54)$$

In a similar manner, another expression can be obtained for the second derivative in the z-direction, which when substituted in Laplace's equation with that in the x-direction gives an identical form to Eq. (4.14).

5. INVESTIGATION OF A DEPRESSED DAM BASE

5.1 General

The provision of a concrete depressed dam base in an excavated trench is sometimes preferred in order to avoid the uncertainty of sheeting (possible opening of interlocking joints in deriving sheet piles); or to distribute the load better over the undisturbed foundation material than a piled foundation (thereby, increasing its resistance to percolation instead of decreasing it); or to form an admirable tie for the entire structure. On the contrary to the previous advantages, this type of a dam base is usually more expensive, and causes still greater delay in initiate work on the superstructures and has some (although less) element of uncertainty due to possible cracking apart from the main structure.

Due to the importance of such a cut-off, it is decided to investigate the effects of the sloping surfaces of the dam base and variation of the permeability ratios and foundation soil depths on exit gradient, uplift pressure, and seepage quantity. It is aimed that further research about seepage under such a base configuration will be made to clear up some of the design problems. In safe designs, it is significant to insure that the exit gradient is within the safe limits. Therefore, it is required to determine such gradients for different soil conditions under different dam base profiles. This study assumes:

- 1- a dam with an impervious depressed base including sloping walls with different angles, such as $\theta = 26.5^\circ, 45^\circ, 63.5^\circ$, and 90° ,
- 2- several permeability ratios in the pervious foundation soil beneath the dam, as $k_x/k_z = 1, 5, 10, 15, 20$, and 25 , and
- 3- several foundation soil depths, such as $FD = 20, 40, 80, 100, 120$, and 200 m.

In all cases the base depth of the dam, d , and the creep length, L_c , are considered constants at 6 m and 52 m, respectively. The creep length is the total length of

contact between the dam base and the foundation soil. The dam section, as illustrated in Fig. 5.1, is selected to describe the procedure of the numerical solution of the depressed base. Five different dam base configurations are considered. The first one has a dam base flush with the streambed while the others have depressed bases with different sidewall angles as shown in Fig. 5.1. Using the developed computer program (in the Appendix), the numerical computations for the piezometric head at all nodal points in the study area are performed for each dam base configuration. The dam rests on a pervious homogeneous and anisotropic soil. The foundation soil extends to impermeable boundaries forward and backward from the dam heel and toe, respectively. Its depth is finite and consists of sand with specific permeability ratios. The upstream water height is a constant at 5 m, whereas the downstream water level is zero. The flow is steady, two-dimensional in the x-z plane, and Eq. (3.13) is applicable. Although the flow field extends an infinite distance upstream and downstream, it is assumed here that it is bounded by the artificial limits AF, DE and FE, which are impervious boundaries.

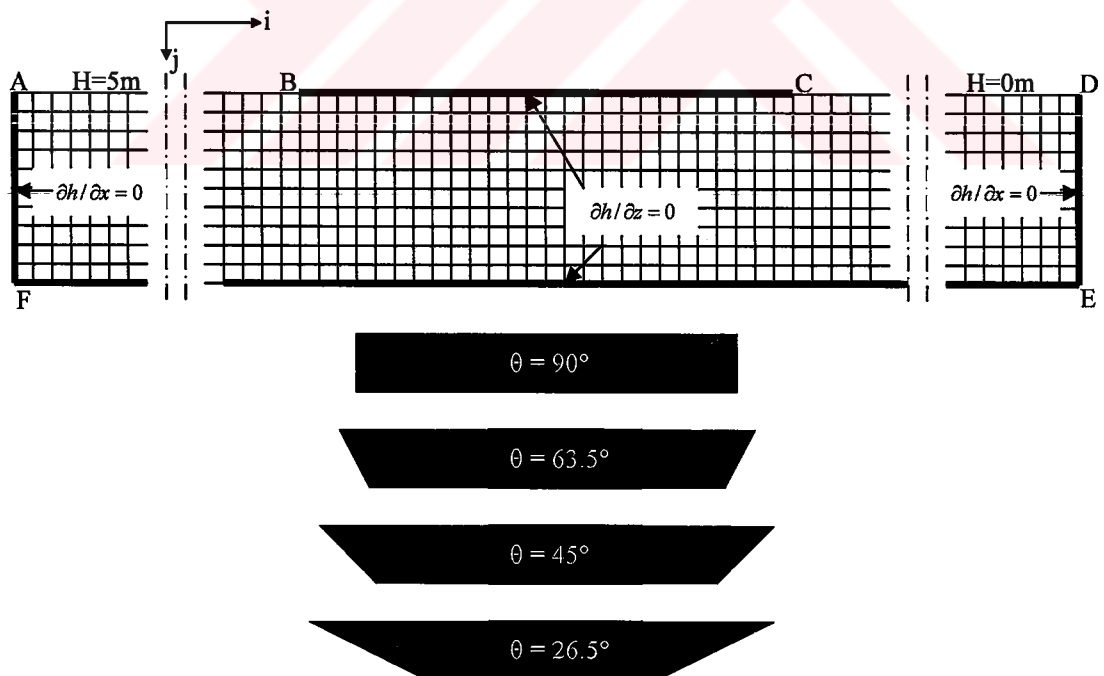


Fig. 5.1 Different dam base configurations for the 2D finite difference model

It should be noted that θ is the angle between the upper horizontal line and the sidewalls of the dam base.

5.2 Input Data

The input data representing the geometry and properties of the study area in the present investigation are as follows:

Upstream Length, $USL = 100$ m.
Length of Dam Floor, $b = 52$ m.
Downstream Length, $DSL = 100$ m.
Depth of Foundation Soil, $FD = 20$ m.
Height of Water at Upstream, $USH = 5$ m.
Height of Water at Downstream, $DSH = 0$ m.
Hydraulic Conductivity, $ZK = 0.00001$ m/s.
Hydraulic Conductivity, $XK = 0.00001$ m/s.
Maximum No of Iterations, $MAXIT = 1000$
Relaxation Factor, $RF = 1.8$
Error Tolerance, $TOL = 0.0001$
Horizontal Grid Distance, $DX = 2$ m.
Vertical Grid Distance, $DZ = 2$ m.
Depth of Dam Base, $d = 6$ m.

The values of all the above mentioned parameters are kept constant for each run with each angle of inclination for the sidewalls, except the following changes,

- 1- the foundation soil depth, FD , takes different values such as 20, 40, 80, 100, 120, and 200 m,
- 2- the horizontal hydraulic conductivity, k_x , is considered to be 1, 5, 10, 15, 20, and 25 times the vertical hydraulic conductivity, k_z , and
- 3- each of the horizontal and vertical grid distances is taken as one unit in the cases where θ equals 63.4° and 26.5° , respectively.

5.3 Application of a Dam Base Flush with the Streambed, $\theta = 0^\circ$

5.3.1 Steps of Numerical Computation

The important steps in the developed computer program using the forward-difference solution are summarized in the flow chart shown in Fig. 5.2. The computer program is run for each foundation soil depth with each of the permeability ratios stated in the previous Section. A total number of thirty-six computer runs are executed for each dam base configuration.

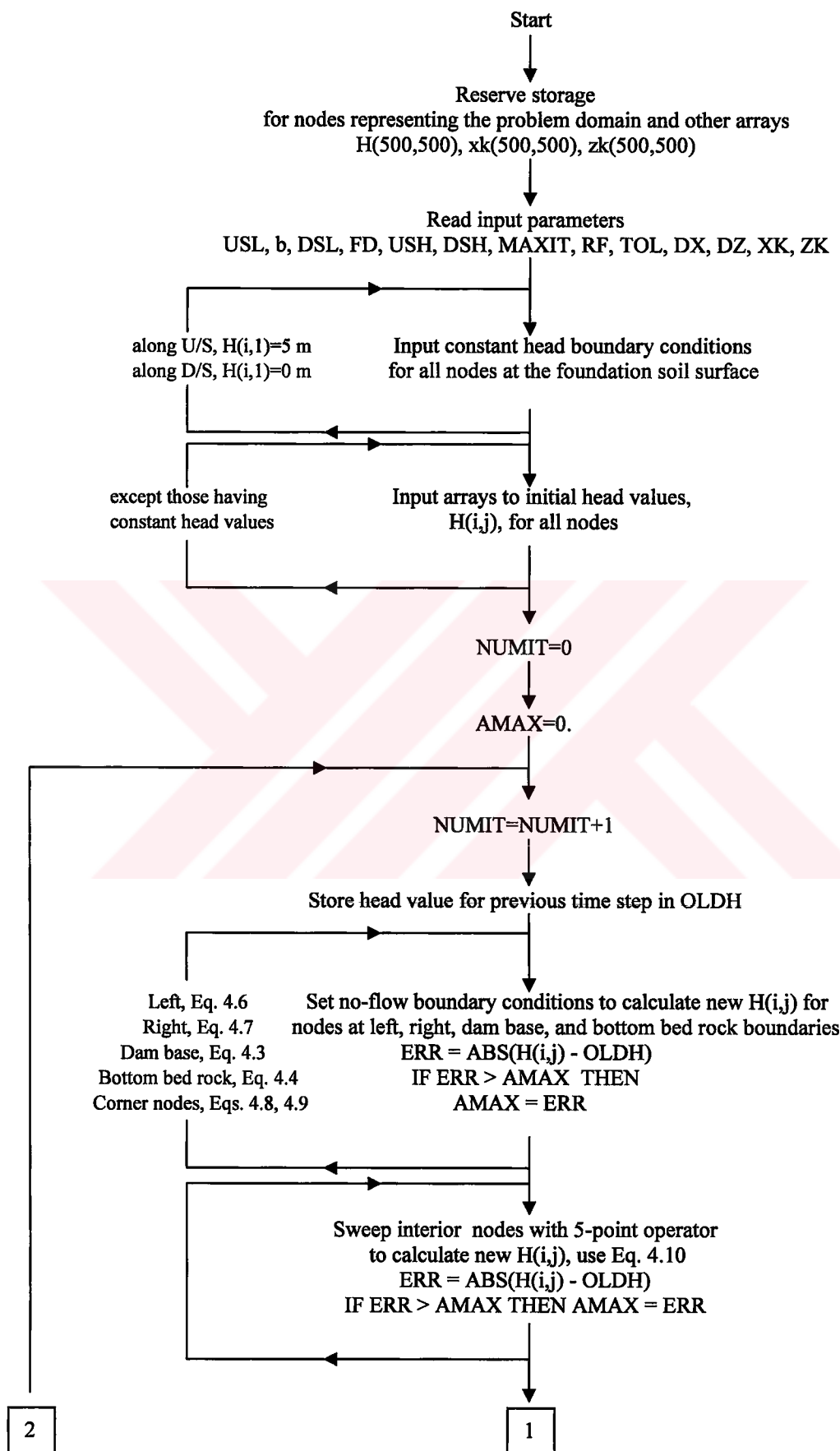


Fig. 5.2 (Cont.)

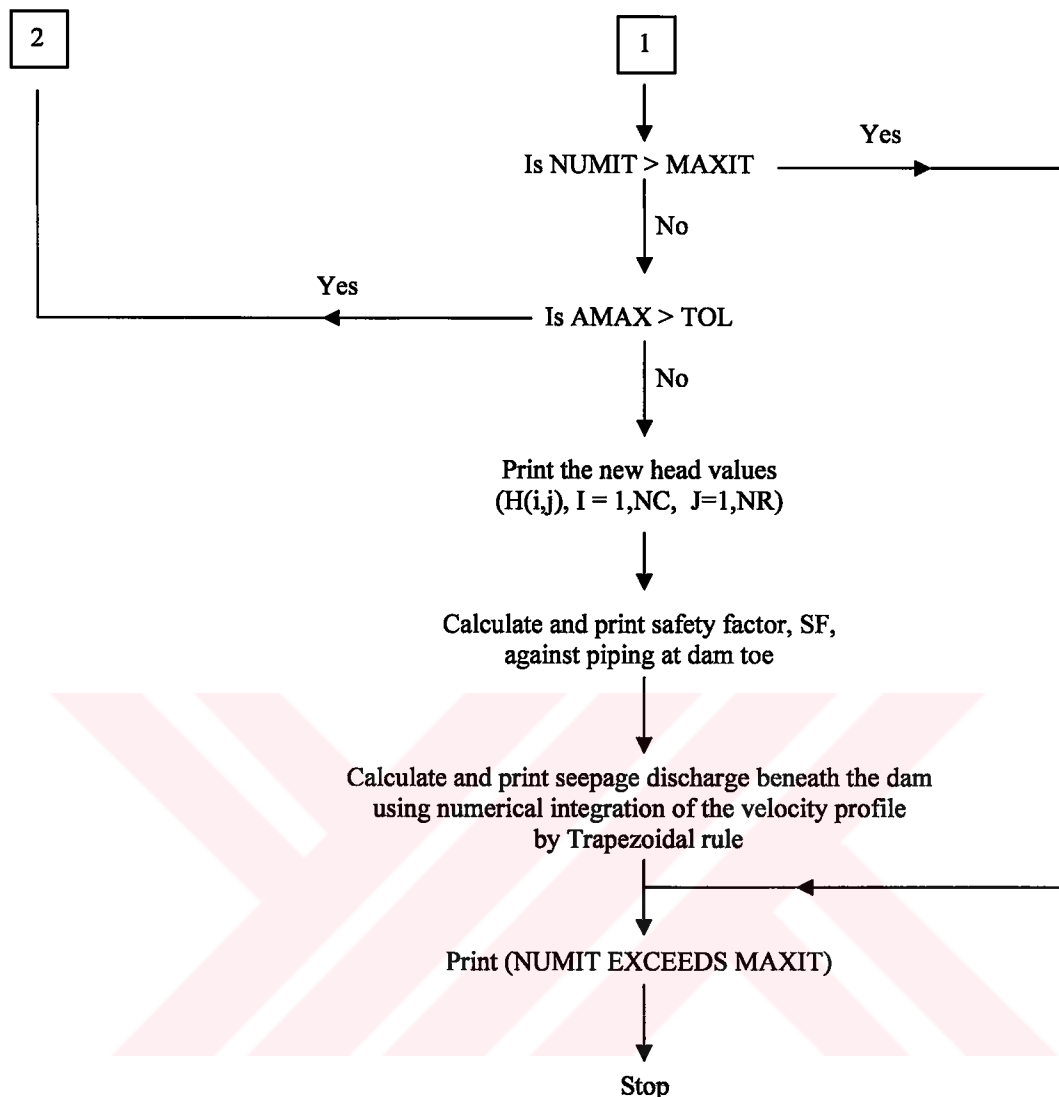


Fig. 5.2 Flow chart for a dam base flush with the streambed

The index convention is used in this program, where I represents the column number from left to right, and J the row number from top to bottom. The I and J of the FORTRAN program refer to the i and j of Fig. 5.1. The z-coordinate is taken as positive downwards to conform with the printing of the results on a computer. In the computer program, comment cards are included to make it comparatively easy to understand and use. Steps of determining the values of $h_{i,j}$ at the various nodal points within the saturated flow region ABCDEF as shown in Fig. 5.1 are as follows,

- 1- assign initial values for the piezometric heads, $h_{i,j}$, at the various grid points.

These values are arbitrary and affect the number of iterations. If the initial heads are closer to those of the final results, the number of iterations is reduced. For

simplicity, the initial heads, $h_{i,j}$ initial, are set equal to 5 m, except for those nodes along the upstream and downstream soil surfaces, which are constant head boundaries.

2- Consider the heads for row 1 (i.e., $j = 1$), (Fig. 5.1),

- I- The $h_{i,j}$ for $j = 1$ and $i = 1, 2, \dots, 51$ are equal to 5 m, and the $h_{i,j}$ for $j = 1$ and $i = 77, 78, \dots, 127$ are equal to 0 m. These are correct values based on the boundary condition that these heads are constants, and
- II- The $h_{i,j}$ for $j = 1$, and $i = 52, 53, \dots, 76$ and the $h_{i,j}$ for $j = 11$, and $i = 2, 3, \dots, 126$ are estimated values. The flow condition for such grid points is similar to that shown in Fig. 4.9, which specify no-flow boundary conditions along horizontal straight boundaries. The applicable finite difference equations are as follows,

$$h_{i,j} = \frac{k_{x1}h_{i+1,j} + 2B^2k_{z2}h_{i,j+1} + k_{x2}h_{i-1,j}}{k_{x1} + k_{x2} + 2B^2k_{z2}} \quad (\text{for } j = 1 \text{ and } i = 52, 53, \dots, 76) \quad (5.1)$$

and

$$h_{i,j} = \frac{k_{x1}h_{i+1,j} + 2B^2k_{z1}h_{i,j-1} + k_{x2}h_{i-1,j}}{k_{x1} + k_{x2} + 2B^2k_{z1}} \quad (\text{for } j = 11 \text{ and } i = 2, 3, \dots, 126) \quad (5.2)$$

where

$$k_{x1} = \frac{(k_x)_{i+1,j} + (k_x)_{i,j}}{2}, \quad k_{x2} = \frac{(k_x)_{i-1,j} + (k_x)_{i,j}}{2}, \quad k_{z1} = \frac{(k_z)_{i,j-1} + (k_z)_{i,j}}{2}, \text{ and}$$

$$k_{z2} = \frac{(k_z)_{i,j+1} + (k_z)_{i,j}}{2}$$

Since the first piezometric heads are assumed values, Eqs. (5.1) and (5.2) will not be satisfied at all the nodes. For example, from the initial guess (iteration level $m = 1$), the head value for the grid point $j = 1$ and $i = 76$ can be calculated from, $h_{i-1,j} = 5$, $h_{i,j} = 5$, $h_{i+1,j} = 0$, and $h_{i,j+1} = 5$. If these values are substituted into Eq. (5.1), it is possible to obtain $h_{i,j}^2 = 3.75$, instead of 5 which is the assumed value of $h_{i,j}^1$. The residual, ε , is given by the relation $\varepsilon = h_{i,j}^{m+1} - h_{i,j}^m$. Therefore, $\varepsilon = 3.75 - 5 = -1.25$,

instead of zero. By replacing $h_{i,j}^m$ by $h_{i,j}^{m+1}$, after each iteration, the Gauss-Seidel procedure relaxes the residuals to zero at every node, and leads to a solution for each algebraic equation. The computer program automatically replaces old values in the head array with new values as soon as they are computed. For the correction of $h_{i,j}^{m+1}$ value, the successive over relaxation process is used. In this method, the Gauss-Seidel residual is multiplied by a relaxation factor ω , and the corrected value $(h_{i,j}^{m+1})_{corr}$ is given by,

$$(h_{i,j}^{m+1})_{corr} = h_{i,j}^m + \omega \varepsilon \quad (5.3)$$

where ω is the relaxation factor, $1 \leq \omega \leq 2$, ε is the residual, which is the head difference between two successive iteration levels, $(h_{i,j}^{m+1})_{corr}$ is the corrected value of $h_{i,j}$ at the $m+1$ iteration level, and $h_{i,j}^m$ is the value of $h_{i,j}$ obtained from the finite difference equation such as Eq. (4.13) or Eq. (4.14) at the m iteration level. Therefore, $(h_{i,j}^2)_{corr} = 5 + (1.8*(-1.25)) = 2.75$. The corrected piezometric heads for the grid points $j = 1$ and $i = 52, 53, \dots, 75$ can be determined in a similar manner.

3- The $h_{i,j}$ for $i = 1$, and $j = 2, 3, \dots, 10$ and $h_{i,j}$ for $i = 127$, and $j = 2, 3, \dots, 10$ are estimated values. The flow condition for these grid points is similar to that shown in Fig 4.9, which specify no-flow boundary conditions along vertical straight boundaries. The applicable finite difference equations are as follows,

$$h_{i,j} = \frac{2k_{x1}h_{i+1,j} + B^2k_{z1}h_{i,j-1} + B^2k_{z2}h_{i,j+1}}{2k_{x1} + B^2k_{z1} + B^2k_{z2}} \quad (\text{for } i = 1 \text{ and } j = 2, 3, \dots, 10) \quad (5.4)$$

and

$$h_{i,j} = \frac{2k_{x2}h_{i-1,j} + B^2k_{z1}h_{i,j-1} + B^2k_{z2}h_{i,j+1}}{2k_{x2} + B^2k_{z1} + B^2k_{z2}} \quad (\text{for } i = 127 \text{ and } j = 2, 3, \dots, 10) \quad (5.5)$$

With proper values of the piezometric head, the residual and the corrected heads are found as in step 2.

4- The $h_{i,j}$ for the corner nodes $i = 1, j = 11$ and $i = 127, j = 11$ are estimated values.

The flow condition for these grid points is similar to that shown in Fig. 4.6, where the applicable finite difference equations are as follows,

$$h_{i,j} = \frac{k_{x1}h_{i+1,j} + B^2k_{z1}h_{i,j-1}}{k_{x1} + B^2k_{z1}} \quad (\text{for } i = 1, j = 11) \quad (5.6)$$

and

$$h_{i,j} = \frac{k_{x2}h_{i-1,j} + B^2k_{z1}h_{i,j-1}}{k_{x2} + B^2k_{z1}} \quad (\text{for } i = 127, j = 11) \quad (5.7)$$

With proper values of the piezometric head, the residual and the corrected heads are found as in step 2.

5- Consider the nodes $i = 2, 3, \dots, 126$ and $j = 2, 3, \dots, 10$, where the DO loops move the five-point operator systematically through these interior points. The applicable finite difference equation is Eq. (4.13) as,

$$h_{i,j} = \frac{k_{x1}h_{i+1,j} + k_{x2}h_{i-1,j} + B^2[k_{z1}h_{i,j-1} + k_{z2}h_{i,j+1}]}{k_{x1} + k_{x2} + B^2(k_{z1} + k_{z2})} \quad (5.8)$$

The heads $h_{i,j}$ are corrected in a similar manner to step 2.

6- With the new heads, repeat steps 2 through 5. This iteration must be carried out until the residuals become less than a pre-specified error tolerance. That is, until the difference between the head values generated at two successive iteration levels is negligible. In this investigation, the solution converges when all the unknown head values change by less than 0.0001 m from one iteration to the next.

5.3.2 Examples of the Model Output

The output is in the form of tables including the piezometric head at the pre-specified number of nodes in the problem solution domain. An example of the numerical computations for the case of $FD = 20$ m with $k_x/k_z = 1$ and 10 is given in Figs. 5.3 and 5.4, respectively, where only the $h_{i,j}$ values within the limits of the dam length

are quoted. The flow rate under the dam base, Q , and its non-dimensional form, $\bar{Q}$, and the exit gradient values at the dam toe are also given in these figures. Since, the dam base length and the foundation soil depth in the first run are taken as 52 m and 20 m, respectively, the number of columns and rows within the limits of the dam floor are 26 and 11, respectively (e.g Fig. 5.3). These numbers, depend on the horizontal and vertical grid distances. In the present case, these distances are taken as 2 m.

5.3.3 Exit Gradient

From the calculated piezometric head values, the hydraulic gradient is determined along the upper most flow line forming the dam base, (Fig 5.3). The hydraulic gradient is found to be large at the entrance then drops gradually to reach its minimum at the middle of the dam base, and it rises again until it reaches a large value at the exit. Particularly, the vertical escape gradient, dh/dz , increases toward the surface of the foundation soil especially near the dam toe. From this observation, the exit gradient, i_e , is taken to be the rate of head change at the exit point closest to the soil surface at the dam toe (Griffiths and Fenton, 1993; 1998). It is considered to be the maximum value of the exit gradient, which may cause piping or heaving. The exit gradient, i_e , is computed using a two-point numerical differentiation as shown in Fig. 5.5.

From the second line of the piezometric head values in Fig. 5.3, the head below the dam toe is computed to be 0.28 m (rounded to two decimals). The piezometric head at the immediate toe is shown from the same figure to be 0.00 (see the last number of the first line of the piezometric heads). This value corresponds to the constant head along the downstream side, which is specified as a boundary condition before starting the computations. From these head values, and for a vertical grid distance of 2 m, the exit gradient is calculated as, $i_e = (0.28 - 0.00)/2 = 0.14$.

This method of determination of i_e gives almost the same value as that computed by Harza (1935) method, which considers the first element of the flownet closest to the dam toe.

$\theta = 0^\circ$, FD = 20 m and $kx/kz = 1$
piezometric head values (m)

$\theta = 0^\circ$, $FD = 20$ m and $k_x/k_z = 10$
piezometric head values (m)

5.00	4.62	4.32	4.08	3.87	3.68	3.51	3.35	3.20	3.06	2.91	2.78	2.64	2.51	2.37	2.23	2.10	1.96	1.81	1.66	1.50	1.33	1.14	.93	.68	.38	.00
4.34	4.21	4.06	3.90	3.75	3.60	3.45	3.31	3.17	3.03	2.90	2.77	2.64	2.51	2.38	2.24	2.11	1.98	1.84	1.70	1.56	1.41	1.26	1.11	.95	.80	.66
4.04	3.94	3.83	3.72	3.60	3.48	3.36	3.23	3.11	2.99	2.87	2.75	2.63	2.51	2.39	2.26	2.14	2.02	1.90	1.78	1.65	1.53	1.41	1.29	1.18	1.07	.97
3.84	3.76	3.67	3.57	3.47	3.37	3.26	3.16	3.05	2.94	2.83	2.72	2.62	2.51	2.40	2.29	2.18	2.07	1.96	1.86	1.75	1.64	1.54	1.44	1.35	1.25	1.17
3.70	3.62	3.54	3.46	3.37	3.28	3.19	3.09	3.00	2.90	2.80	2.70	2.61	2.51	2.41	2.31	2.21	2.11	2.02	1.92	1.83	1.74	1.64	1.56	1.47	1.39	1.31
3.60	3.52	3.45	3.37	3.29	3.21	3.12	3.04	2.95	2.86	2.78	2.69	2.60	2.51	2.42	2.33	2.24	2.15	2.06	1.98	1.89	1.81	1.72	1.64	1.56	1.49	1.42
3.52	3.45	3.38	3.31	3.23	3.16	3.08	3.00	2.92	2.84	2.76	2.67	2.59	2.51	2.43	2.34	2.26	2.18	2.10	2.02	1.94	1.86	1.78	1.71	1.63	1.56	1.49
3.47	3.40	3.33	3.26	3.19	3.12	3.05	2.97	2.90	2.82	2.74	2.66	2.59	2.51	2.43	2.35	2.28	2.20	2.12	2.05	1.97	1.90	1.83	1.75	1.68	1.62	1.55
3.43	3.37	3.30	3.23	3.16	3.09	3.02	2.95	2.88	2.81	2.73	2.66	2.58	2.51	2.43	2.36	2.29	2.21	2.14	2.07	1.99	1.92	1.85	1.79	1.72	1.65	1.59
3.41	3.35	3.28	3.22	3.15	3.08	3.01	2.94	2.87	2.80	2.73	2.66	2.58	2.51	2.44	2.36	2.29	2.22	2.15	2.08	2.01	1.94	1.87	1.80	1.74	1.67	1.61
3.40	3.34	3.28	3.21	3.14	3.08	3.01	2.94	2.87	2.80	2.73	2.65	2.58	2.51	2.44	2.37	2.30	2.22	2.15	2.08	2.01	1.94	1.88	1.81	1.75	1.68	1.62

EXITI and SF using nodes at D/S side & hyd. grad.
.33 3.02

Horizontal hyd. gradients along the dam base

.1902	.1490	.1225	.1051	.0933	.0852	.0794	.0753	.0724	.0703	.0689	.0680	.0676	.0676	.0680	.0690	.0705	.0726	.0756	.0798	.0856	.0938	.1057	.1233	.1500	.1915
-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------

Vertical hyd. gradients along the DSL

.3308	.2802	.2418	.2120	.1884	.1692	.1533	.1400	.1286	.1188	.1102	.1026	.0958	.0897	.0842	.0793	.0747	.0706	.0668	.0632	.0600	.0570	.0542	.0516	.0492	.0470
.0449	.0430	.0411	.0395	.0379	.0365	.0351	.0339	.0327	.0316	.0306	.0297	.0289	.0282	.0275	.0269	.0264	.0259	.0255	.0252	.0249	.0247	.0245	.0244	.0244	.0244

Q= 9.610257E-05
Qbar= 1.922051E-01
NUMIT= 631

Fig. 5.4 An example of the model output, for $\theta = 0^\circ$, $FD=20$ m, and $k_x/k_z=10$

However, the exit gradient value at the toe is mathematically infinite in the present case and in cases of $\theta < 90^\circ$, the apparent toe exit gradient is taken into consideration in order to show its maximum value.

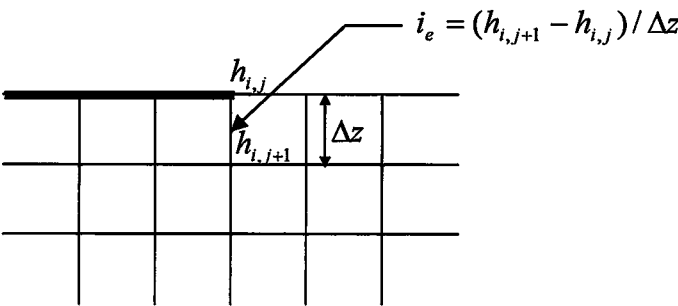


Fig. 5.5 Schematic sketch for numerical calculation of i_e at the dam toe where $\theta = 0^\circ$

The exit gradient values calculated for different foundation soil depths and permeability ratios are presented in Table 5.1.

Table 5.1 Exit gradients for a dam base flush with the streambed, $\theta = 0^\circ$

FD(m)	Exit gradient, i_e					
	k_x / k_z					
	1	5	10	15	20	25
20	0.140	0.265	0.330	0.373	0.405	0.430
40	0.165	0.283	0.345	0.386	0.417	0.440
80	0.176	0.290	0.350	0.390	0.421	0.447
100	0.178	0.292	0.352	0.391	0.422	0.448
120	0.180	0.292	0.353	0.393	0.424	0.450

5.3.4 Flow Rate

The flow rate (seepage discharge) is computed by the numerical integration using trapezoidal rule for the velocity values, $v_{i,j}$, over the distance between the impermeable bed and the bottom floor of the dam base. The computation is performed along the centre line of the dam (Rushton and Redshaw, 1978). Centre of

the dam nodes are used herein, since the nodal velocity has only a horizontal component. In this case, the vertical nodal velocity is zero. Similar techniques for calculating the net flow by the finite difference and the finite element methods are described by Naouss and Najjar (1995) and Desai (1977), respectively.

Consider Δz and Δx to be the distances between the pair of adjacent streamlines and equipotential lines, respectively. Hence the head drop Δh across the two equipotential lines is the difference between the head values along the two lines. The discharge, Q , flowing through the cross-sectional area per unit width of the cross-section may be calculated from the discrete values of the piezometric head without drawing the flownet. Therefore,

$$Q = k \frac{\Delta h}{\Delta x} \Delta z \quad (5.9)$$

As the quantity of flow between any two streamlines is constant, the total flow for homogeneous and anisotropic soil between the two fixed boundaries is obtained by summing the result for each streamline as follows,

$$\begin{aligned} Q &= \frac{\Delta z}{2} v_{m4,1} + \Delta z \sum_{j=2}^{NR-1} v_{m4,j} + \frac{\Delta z}{2} v_{m4,NR} \\ &= \frac{\Delta z}{2} \left[k_{x1} \frac{h_{m4+1,1} - h_{m4,1}}{\Delta x} \right] + k_{x1} \sum_{j=2}^{NR-1} \left(\frac{h_{i,j} - h_{m4+1,j}}{\Delta x} \Delta z \right) + \frac{\Delta z}{2} \left[k_{x1} \frac{h_{m4+1,NR} - h_{m4,NR}}{\Delta x} \right] \quad (5.10) \end{aligned}$$

The non-dimensional form of the flow rate, $\bar{Q}$, is computed using the following ratio,

$$\bar{Q} = \frac{Q}{kH} \quad (5.11)$$

From Eq. (5.10), the computed discharge is shown in Fig. 5.3 to be $Q = 1.465 \text{ m}^3/\text{s}/\text{m}$. Its non-dimensional value is given in the same figure using Eq. (5.11). For a permeability coefficient of unity, it is calculated as $\bar{Q} = 1.465/5 = 0.293$.

The seepage discharge values calculated for different foundation soil depths and permeability ratios are presented in Table 5.2.

Table 5.2 Non-dimensional discharges under a flat dam base, $\theta = 0^\circ$

FD (m)	Non-dimensional seepage discharge, $\bar{Q}$					
	k_x / k_z					
	1	5	10	15	20	25
20	0.290	0.225	0.192	0.172	0.158	0.147
40	0.460	0.310	0.243	0.206	0.182	0.164
80	0.660	0.357	0.258	0.212	0.185	0.166
100	0.713	0.361	0.259	0.212	0.185	0.166
120	0.748	0.363	0.259	0.212	0.185	0.167

5.3.5 Comparison of the Model Results with the Previous Studies

In order to check the accuracy of the computer results for this dam base, they have been compared to that of Weaver (1932) equation for calculating the uplift pressure under a dam with a flat base built directly on the ground surface. Using conformal transformation, Weaver (1932) gave the following equation,

$$h = \frac{H}{\pi} \cos^{-1} \left(2 \frac{x}{b} - 1 \right) \quad (5.12)$$

where h is the piezometric head value, x/b is the ratio of distance from upstream end to total length of dam floor, H is the total head difference between upstream and downstream water levels = 5 m, and b is the dam length = 52 m.

In the derivation of Weaver (1932) equation, the foundation soil depth, FD , is considered to be infinite. For this reason, the computer results used in the comparison are that for $FD = 100$ m, (Table 5.3). These results are found to be unchanged for depths greater than the specified depth (i.e., for $FD \geq 2b$), and can be used to simulate an infinite depth in an analytical solution with acceptable accuracy. The comparison is done for the uplift pressures expressed as the ratio of the computed piezometric head over the total head difference.

Examples for the calculation of the entries of Table 5.3 are presented here. In the first row, for $x/b = 0$ (i.e. at the dam heel, where $x = 0$), Eq. (5.12) will be

$h = \frac{H}{\pi} \cos^{-1}(-1)$, from which $h/H = 1$, as given in the second column. On the other

hand, from the computer results of Fig. 5.6, the piezometric head, h ; at the dam heel is 5 m, this value corresponds to the water level at the upstream side, and represents the specified constant head boundary condition at that node. However, since the total head difference, H , between the upstream and downstream water levels is 5 m, the ratio $h/H = 1$ as given in the third column. Moreover, for the second line in the same table, for $x = 4$ m, and $x/b = 4/52 = 0.0769 \approx 0.077$, Eq. (5.12) will be

$h = \frac{H}{\pi} \cos^{-1}(-0.846)$, from which $h/H = 0.821$, as shown in the second column.

Similarly, the piezometric head, h , along the dam floor at a distance $x = 4$ m from the dam heel, is found from the computer results to be 4.19 m (Fig. 5.6). Hence, the ratio $h/H = 4.19/5.0 = 0.83$ which is given in the third column. In the second line, the relative error between the results of the present numerical model and Weaver

(1932) solution is calculated as, $\frac{0.83 - 0.821}{0.821} * 100 = 1.08$ %. Other entries in Table

5.3 are computed in a similar manner.

Table 5.3 Head ratios from Weaver (1932) equation and the model results

h/H			
x/b	Weaver's equation	Numerical solution	Relative error (%)
0	1	1	0
0.077	0.821	0.830	1.08
0.154	0.743	0.754	1.45
0.230	0.681	0.690	1.30
0.308	0.625	0.630	0.80
0.461	0.524	0.528	0.75
0.577	0.450	0.452	0.40
0.769	0.319	0.316	-0.90
0.884	0.221	0.210	-5.0
1	0	0	0

From Table 5.3, it is observed that the maximum relative error is ≤ 5 %, which is acceptable in practical studies. In a graphical form, the previous results can be represented as in Fig. 5.7.

$\theta = 0^\circ$, $FD = 100$ m and $k_x/k_z=1$
piezometric head values (m)

5.00	4.49	4.19	3.96	3.77	3.60	3.45	3.30	3.16	3.03	2.89	2.77	2.64	2.51	2.38	2.26	2.13	2.00	1.86	1.72	1.57	1.41	1.24	1.05	.82	.51	.00
4.65	4.39	4.15	3.95	3.76	3.60	3.44	3.30	3.16	3.02	2.89	2.76	2.64	2.51	2.38	2.26	2.13	2.00	1.86	1.72	1.58	1.42	1.25	1.07	.86	.62	.36
4.45	4.27	4.08	3.90	3.74	3.58	3.43	3.29	3.15	3.02	2.89	2.76	2.64	2.51	2.39	2.26	2.13	2.00	1.87	1.73	1.59	1.44	1.28	1.11	.93	.74	.56
4.30	4.16	4.01	3.85	3.70	3.55	3.41	3.27	3.14	3.01	2.88	2.76	2.63	2.51	2.39	2.26	2.14	2.01	1.88	1.75	1.61	1.47	1.32	1.17	1.01	.85	.71
4.19	4.07	3.93	3.79	3.65	3.52	3.38	3.25	3.13	3.00	2.88	2.75	2.63	2.51	2.39	2.27	2.15	2.02	1.90	1.77	1.64	1.50	1.36	1.22	1.08	.95	.83
4.09	3.98	3.86	3.74	3.61	3.48	3.36	3.23	3.11	2.99	2.87	2.75	2.63	2.51	2.39	2.28	2.16	2.04	1.91	1.79	1.67	1.54	1.41	1.28	1.16	1.04	.92
4.01	3.90	3.80	3.68	3.56	3.44	3.33	3.21	3.09	2.97	2.86	2.74	2.63	2.51	2.40	2.28	2.17	2.05	1.93	1.81	1.70	1.58	1.46	1.34	1.22	1.11	1.01
3.93	3.84	3.73	3.63	3.52	3.41	3.30	3.18	3.07	2.96	2.85	2.74	2.62	2.51	2.40	2.29	2.18	2.07	1.95	1.84	1.73	1.61	1.50	1.39	1.28	1.18	1.09
3.86	3.77	3.68	3.58	3.48	3.37	3.27	3.16	3.05	2.94	2.84	2.73	2.62	2.51	2.41	2.30	2.19	2.08	1.97	1.87	1.76	1.65	1.55	1.44	1.34	1.25	1.16
3.80	3.72	3.63	3.53	3.44	3.34	3.24	3.13	3.03	2.93	2.82	2.72	2.62	2.51	2.41	2.31	2.20	2.10	1.99	1.89	1.79	1.69	1.59	1.49	1.39	1.30	1.22
3.75	3.67	3.58	3.49	3.40	3.30	3.21	3.11	3.01	2.91	2.81	2.71	2.61	2.51	2.41	2.31	2.21	2.11	2.01	1.92	1.82	1.72	1.63	1.53	1.44	1.36	1.28
3.70	3.62	3.54	3.45	3.36	3.27	3.18	3.09	2.99	2.90	2.80	2.71	2.61	2.51	2.42	2.32	2.22	2.13	2.03	1.94	1.84	1.75	1.66	1.57	1.49	1.41	1.33
3.65	3.57	3.50	3.41	3.33	3.24	3.16	3.07	2.98	2.89	2.79	2.70	2.61	2.51	2.42	2.33	2.24	2.14	2.05	1.96	1.87	1.78	1.70	1.61	1.53	1.45	1.38
3.61	3.53	3.46	3.38	3.30	3.22	3.13	3.05	2.96	2.87	2.78	2.69	2.60	2.52	2.43	2.34	2.25	2.16	2.07	1.98	1.90	1.81	1.73	1.65	1.57	1.49	1.42
3.57	3.50	3.42	3.35	3.27	3.19	3.11	3.03	2.94	2.86	2.77	2.69	2.60	2.52	2.43	2.34	2.26	2.17	2.09	2.00	1.92	1.84	1.76	1.68	1.61	1.53	1.46
3.53	3.46	3.39	3.32	3.24	3.17	3.09	3.01	2.93	2.85	2.77	2.68	2.60	2.52	2.43	2.35	2.27	2.18	2.10	2.02	1.94	1.86	1.79	1.71	1.64	1.57	1.50
3.50	3.43	3.36	3.29	3.22	3.14	3.07	2.99	2.91	2.84	2.76	2.68	2.60	2.52	2.44	2.36	2.28	2.20	2.12	2.04	1.96	1.89	1.81	1.74	1.67	1.60	1.54
3.46	3.40	3.33	3.27	3.20	3.12	3.05	2.98	2.90	2.83	2.75	2.67	2.59	2.52	2.44	2.36	2.28	2.21	2.13	2.06	1.98	1.91	1.84	1.77	1.70	1.63	1.57
3.43	3.37	3.31	3.24	3.17	3.10	3.03	2.96	2.89	2.82	2.74	2.67	2.59	2.52	2.44	2.37	2.29	2.22	2.15	2.07	2.00	1.93	1.86	1.79	1.73	1.66	1.60
3.41	3.35	3.28	3.22	3.15	3.09	3.02	2.95	2.88	2.81	2.74	2.66	2.59	2.52	2.45	2.37	2.30	2.23	2.16	2.09	2.02	1.95	1.88	1.82	1.75	1.69	1.63
3.38	3.32	3.26	3.20	3.13	3.07	3.00	2.94	2.87	2.80	2.73	2.66	2.59	2.52	2.45	2.38	2.31	2.24	2.17	2.10	2.03	1.97	1.90	1.84	1.78	1.71	1.66
3.36	3.30	3.24	3.18	3.12	3.05	2.99	2.92	2.86	2.79	2.72	2.66	2.59	2.52	2.45	2.38	2.31	2.25	2.18	2.11	2.05	1.98	1.92	1.86	1.80	1.74	1.68
3.33	3.28	3.22	3.16	3.10	3.04	2.98	2.91	2.85	2.78	2.72	2.65	2.59	2.52	2.45	2.39	2.32	2.26	2.19	2.13	2.06	2.00	1.94	1.88	1.82	1.76	1.70
3.31	3.26	3.20	3.14	3.08	3.02	2.96	2.90	2.84	2.78	2.71	2.65	2.58	2.52	2.46	2.39	2.33	2.26	2.20	2.14	2.08	2.02	1.96	1.90	1.84	1.78	1.73
3.29	3.24	3.18	3.13	3.07	3.01	2.95	2.89	2.83	2.77	2.71	2.65	2.58	2.52	2.46	2.40	2.33	2.27	2.21	2.15	2.09	2.03	1.97	1.91	1.86	1.80	1.75
3.27	3.22	3.17	3.11	3.06	3.00	2.94	2.88	2.82	2.76	2.70	2.64	2.58	2.52	2.46	2.40	2.34	2.28	2.22	2.16	2.10	2.04	1.99	1.93	1.87	1.82	1.77
3.26	3.21	3.15	3.10	3.04	2.99	2.93	2.87	2.82	2.76	2.70	2.64	2.58	2.52	2.46	2.40	2.34	2.28	2.23	2.17	2.11	2.05	2.00	1.94	1.89	1.84	1.78
3.24	3.19	3.14	3.09	3.03	2.98	2.92	2.87	2.81	2.75	2.70	2.64	2.58	2.52	2.46	2.41	2.35	2.29	2.23	2.18	2.12	2.07	2.01	1.96	1.90	1.85	1.80
3.23	3.18	3.13	3.07	3.02	2.97	2.91	2.86	2.80	2.75	2.69	2.64	2.58	2.52	2.47	2.41	2.35	2.30	2.24	2.19	2.13	2.08	2.02	1.97	1.92	1.87	1.82
3.21	3.16	3.11	3.06	3.01	2.96	2.91	2.85	2.80	2.74	2.69	2.63	2.58	2.52	2.47	2.41	2.36	2.30	2.25	2.19	2.14	2.09	2.03	1.98	1.93	1.88	1.83
3.20	3.15	3.10	3.05	3.00	2.95	2.90	2.85	2.79	2.74	2.69	2.63	2.58	2.52	2.47	2.41	2.36	2.31	2.25	2.20	2.15	2.10	2.04	1.99	1.94	1.89	1.85
3.19	3.14	3.09	3.04	2.99	2.94	2.89	2.84	2.79	2.74	2.68	2.63	2.58	2.52	2.47	2.42	2.36	2.31	2.26	2.21	2.16	2.10	2.05	2.00	1.95	1.91	1.86
3.18	3.13	3.08	3.03	2.98	2.93	2.88	2.83	2.78	2.73	2.68	2.63	2.58	2.52	2.47	2.42	2.37	2.32	2.26	2.21	2.16	2.11	2.06	2.01	1.97	1.92	1.87
3.17	3.12	3.07	3.03	2.98	2.93	2.88	2.83	2.78	2.73	2.68	2.63	2.58	2.52	2.47	2.42	2.37	2.32	2.27	2.22	2.17	2.12	2.07	2.02	1.98	1.93	1.88

Fig. 5.6 (Cont.)

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3.16 3.11 3.06 3.02 2.97 2.92 2.87 2.82 2.77 2.73 2.68 2.63 2.57 2.52 2.47 2.42 2.37 2.32 2.27 2.22 2.18 2.13 2.08 2.03 1.98 1.94 1.89
3.15 3.10 3.06 3.01 2.96 2.92 2.87 2.82 2.77 2.72 2.67 2.62 2.57 2.52 2.48 2.43 2.38 2.33 2.28 2.23 2.18 2.13 2.09 2.04 1.99 1.95 1.90
3.14 3.09 3.05 3.00 2.96 2.91 2.86 2.82 2.77 2.72 2.67 2.62 2.57 2.53 2.48 2.43 2.38 2.33 2.28 2.23 2.19 2.14 2.09 2.05 2.00 1.96 1.91
3.13 3.09 3.04 3.00 2.95 2.91 2.86 2.81 2.77 2.72 2.67 2.62 2.57 2.53 2.48 2.43 2.38 2.33 2.29 2.24 2.19 2.14 2.10 2.05 2.01 1.96 1.92
3.12 3.08 3.04 2.99 2.95 2.90 2.86 2.81 2.76 2.72 2.67 2.62 2.57 2.53 2.48 2.43 2.38 2.34 2.29 2.24 2.20 2.15 2.10 2.06 2.01 1.97 1.93
3.12 3.08 3.03 2.99 2.94 2.90 2.85 2.81 2.76 2.71 2.67 2.62 2.57 2.53 2.48 2.43 2.38 2.34 2.29 2.25 2.20 2.15 2.11 2.06 2.02 1.98 1.93
3.11 3.07 3.03 2.98 2.94 2.89 2.85 2.80 2.76 2.71 2.67 2.62 2.57 2.53 2.48 2.43 2.39 2.34 2.29 2.25 2.20 2.16 2.11 2.07 2.03 1.98 1.94
3.11 3.07 3.02 2.98 2.94 2.89 2.85 2.80 2.76 2.71 2.66 2.62 2.57 2.53 2.48 2.43 2.39 2.34 2.30 2.25 2.21 2.16 2.12 2.07 2.03 1.99 1.95
3.10 3.06 3.02 2.98 2.93 2.89 2.84 2.80 2.75 2.71 2.66 2.62 2.57 2.53 2.48 2.44 2.39 2.34 2.30 2.25 2.21 2.17 2.12 2.08 2.04 2.00 1.96
3.10 3.06 3.02 2.97 2.93 2.89 2.84 2.80 2.75 2.71 2.66 2.62 2.57 2.53 2.48 2.44 2.39 2.35 2.30 2.26 2.21 2.17 2.13 2.08 2.04 2.00 1.96
3.10 3.05 3.01 2.97 2.93 2.88 2.84 2.80 2.75 2.71 2.66 2.62 2.57 2.53 2.48 2.44 2.39 2.35 2.30 2.26 2.22 2.17 2.13 2.09 2.05 2.01 1.97
3.09 3.05 3.01 2.97 2.92 2.88 2.84 2.79 2.75 2.71 2.66 2.62 2.57 2.53 2.48 2.44 2.39 2.35 2.30 2.26 2.22 2.17 2.13 2.09 2.05 2.01 1.97
3.09 3.05 3.01 2.96 2.92 2.88 2.84 2.79 2.75 2.70 2.66 2.62 2.57 2.53 2.48 2.44 2.39 2.35 2.31 2.26 2.22 2.18 2.13 2.09 2.05 2.01 1.97
3.09 3.05 3.00 2.96 2.92 2.88 2.83 2.79 2.75 2.70 2.66 2.62 2.57 2.53 2.48 2.44 2.39 2.35 2.31 2.26 2.22 2.18 2.13 2.09 2.05 2.01 1.97
3.09 3.05 3.00 2.96 2.92 2.88 2.83 2.79 2.75 2.70 2.66 2.62 2.57 2.53 2.48 2.44 2.39 2.35 2.31 2.26 2.22 2.18 2.13 2.09 2.05 2.01 1.97
3.09 3.05 3.00 2.96 2.92 2.88 2.83 2.79 2.75 2.70 2.66 2.62 2.57 2.53 2.48 2.44 2.39 2.35 2.31 2.26 2.22 2.18 2.13 2.09 2.05 2.01 1.97

```

EXITI and SF using nodes at D/S side & hyd. grad.
.17 5.61

horizontal hyd. gradients along the dam base
.2532 .1512 .1134 .0953 .0849 .0781 .0734 .0700 .0675 .0657 .0645 .0637 .0634 .0634 .0639 .0647 .0660 .0679 .0704 .0739 .0788 .0857 .0963 .1147 .1532 .2568

vertical hyd. gradients along the DSL
.1783 .1239 .0964 .0801 .0693 .0614 .0554 .0506 .0467 .0434 .0406 .0382 .0361 .0342 .0325 .0310 .0296 .0283 .0272 .0262 .0252 .0243 .0235 .0227 .0220 .0214
.0208 .0202 .0197 .0192 .0188 .0184 .0180 .0176 .0173 .0170 .0167 .0164 .0162 .0160 .0158 .0156 .0155 .0153 .0152 .0151 .0150 .0149 .0149 .0149

Q= 3.568260E-01
Qbar= 7.136520E-01
NUMIT= 1406

Fig. 5.6 An example of the model output, for $\theta = 0^\circ$, $FD = 100$ m and $k_x/k_z=1$

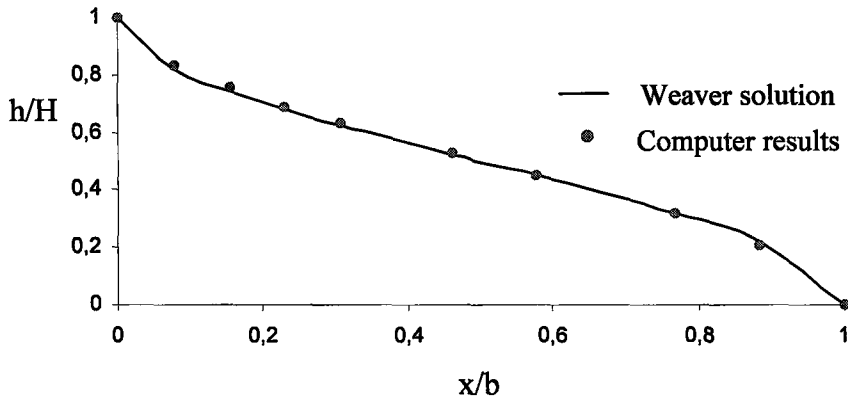


Fig. 5.7 Comparison of uplift pressures under a dam base flush with the streambed using Weaver (1932) equation and the present model results

From Fig. 5.7, it is seen that the head values that were accepted after the last run with a foundation soil depth greater than or equal to twice the dam base length (i.e, where $FD \geq 2b$) are in rather good conformance with the calculated values using Weaver (1932) equation.

5.4 Application of a Dam with a Depressed Base, $\theta = 90^\circ$

5.4.1 Steps of Numerical Computation

After making the necessary changes of representing the no-flow boundary conditions for the nodes along the sidewalls and the floor of the dam base (Fig. 5.8), the flow chart in Fig. 5.2 applies for the cases of a dam with a depressed base, where $\theta = 26.5^\circ$, 45° , 63.4° , and 90° . The new modified parts can be seen from the computer programs presented in the Appendix. The related program is run for each foundation soil depth with each of the permeability ratios stated previously in Section 5.2.

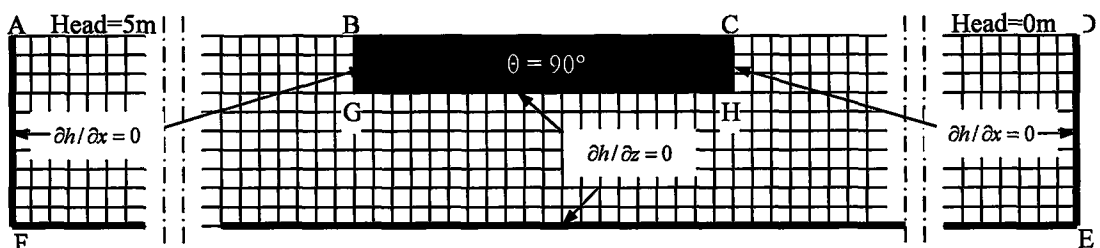


Fig. 5.8 Schematic sketch for a dam with a depressed base, $\theta = 90^\circ$

The steps of determining the values of $h_{i,j}$ at the various nodal points within the saturated flow region ABGHCDEF shown in Fig. 5.8 are almost similar as those presented in Section 5.3.1. However, the following changes are to be noted. In step 1, the $h_{i,j}$ for $j = 1$ and $i = 1, 2, \dots, 51$ are equal to 5 m and the $h_{i,j}$ for $j = 1$ and $i = 71, 72, \dots, 121$ are equal to 0 m. Eq. (5.1) is applicable for the nodes at $j = 4$ and $i = 52, 53, \dots, 70$. Similarly, Eq. (5.2) is applicable for the nodes at $j = 11$ and $i = 2, 3, \dots, 120$. In the same way, Eq. (5.4) is used to determine $h_{i,j}$ for $i = 1$ and $j = 2, 3, \dots, 10$ and for $i = 71$ and $j = 2, 3$, whereas Eq. (5.5) is applicable in the evaluation of $h_{i,j}$ for $i = 121$ and $j = 2, 3, \dots, 10$ and for $i = 51$ and $j = 2, 3$. Moreover, the $h_{i,j}$ for the corner nodes at $i = 1$ and $j = 11$, and at $i = 127$ and $j = 11$ are estimated using Eqs. (5.6) and (5.7), respectively. However, the $h_{i,j}$ value for the corner nodes at $i = 51$ and $j = 4$, and at $i = 71$ and $j = 4$ is similar to that derived from Fig. 4.7. These are the boundary corner nodes between the sidewalls of the depressed base and its lower horizontal floor. The applicable finite difference equations at those nodes are as follows,

$$h_{i,j} = \frac{2k_{x2}h_1 + k_{x1}h_3 + B^2k_{z1}h_2 + 2B^2k_{z2}h_4}{2k_{x2} + k_{x1} + B^2k_{z1} + 2B^2k_{z2}} \quad (\text{for } i = 51, j = 4) \quad (5.13)$$

and

$$h_{i,j} = \frac{2k_{x1}h_1 + k_{x2}h_3 + B^2k_{z1}h_2 + 2B^2k_{z2}h_4}{2k_{x1} + k_{x2} + B^2k_{z1} + 2B^2k_{z2}} \quad (\text{for } i = 71, j = 4) \quad (5.14)$$

Finally, the values of $h_{i,j}$ for all the interior nodes where $i = 2, 3, \dots, 120$ and $j = 2, 3, \dots, 10$, with the exception of the nodes at $j = 1, 2, \dots, 4$ and $i = 51, 52, \dots, 71$, are calculated using Eq. (5.8).

5.4.2 Exit Gradient

Applying the same principles as described in Section 5.3.3, the computation of the exit gradient, i_e , is made using a two-point numerical differentiation as shown in Fig. 5.9.

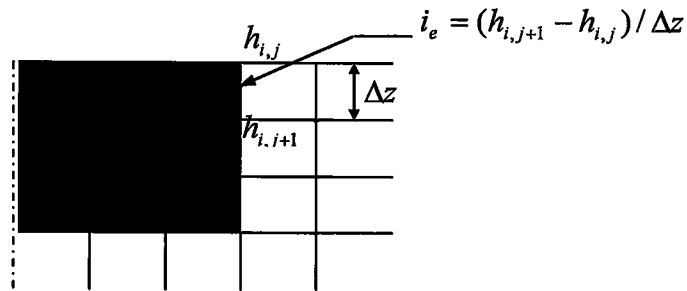


Fig. 5.9 Schematic sketch for numerical calculation of i_e at the dam toe where $\theta = 90^\circ$

The calculated exit gradient values are collected from the different computer runs and presented as shown in Table 5.4. The results are tabulated for various permeability ratios and foundation soil depths. From this table, as well as all similar tables, it is seen that the exit gradient increases as the permeability ratio or the foundation soil depth increases.

Table 5.4 Exit gradients for a depressed dam base, $\theta = 90^\circ$

$FD(m)$	Exit gradient, i_e					
	k_x / k_z					
	1	5	10	15	20	25
20	0.089	0.140	0.160	0.181	0.187	0.201
40	0.110	0.156	0.180	0.196	0.205	0.214
80	0.119	0.167	0.190	0.200	0.215	0.218
100	0.121	0.168	0.191	0.201	0.220	0.220
120	0.123	0.168	0.193	0.202	0.222	0.221

5.4.3 Flow Rate

The computation of the seepage discharge for the present configuration follows the same principles as described in Section 5.3.4. The discharge values calculated for different foundation soil depths and permeability ratios are collected and presented in Table 5.5. From this table, it is seen that the seepage discharge decreases as the permeability ratio increases, and increases as the foundation soil depth increases.

Table 5.5 Non-dimensional discharges under a depressed dam base, $\theta = 90^\circ$

$FD(m)$	Non-dimensional seepage discharge, $\bar{Q}$					
	k_x / k_z					
	1	5	10	15	20	25
20	0.225	0.156	0.126	0.109	0.098	0.088
40	0.410	0.244	0.178	0.143	0.121	0.105
80	0.605	0.288	0.192	0.149	0.124	0.106
100	0.657	0.293	0.192	0.149	0.124	0.107
120	0.690	0.297	0.192	0.149	0.124	0.107

5.4.4 Comparison of the Model Results with a Previous Study

The validity of the model results, for the case of a depressed base where $\theta = 90^\circ$, is examined and compared to Khosla *et al* (1936) solution for calculating the pressure under a dam with a depressed base having vertical sidewalls as follows: Consider that $\alpha = b / d$, where b is the dam floor length and d is the dam base depth. In this case, b is equal to 40 m and d takes the values of 4, 8, 12, 16, 20, 24, 28, 32, 36, and 40 m, respectively. The ratio h_{D_1} / H is interpolated from Khosla *et al* (1936) curve (as presented in Leliavsky, 1965) for such a dam base, where h_{D_1} is the piezometric head at point D_1 as shown in Fig. 5.10, and H is the total head difference acting on the foundation soil (see the second column of Table 5.6).

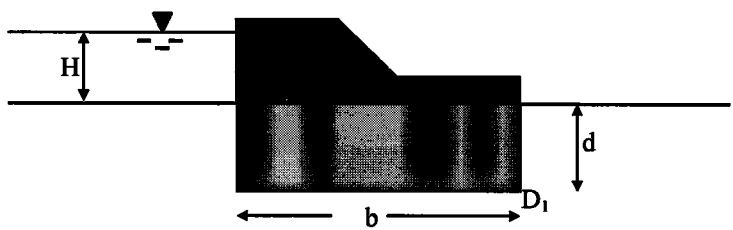


Fig. 5.10 Schematic sketch for a depressed dam base with vertical sidewalls

Examples from the present numerical computations of the piezometric head values, for the cases where the depressed base depth $d = 8, 16$, and 24 m are given in Fig. 5.11 (i.e, for $1/\alpha = 0.2, 0.4$, and 0.6 , respectively).

piezometric head values along the periphery of the depressed dam base (m)

(a)

(b)

©

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The model results are given as the piezometric head values along the periphery of the dam base. For $1/\alpha = 0.2$, the piezometric head value at point D_1 is 0.931, as shown in Fig. 5.11. Hence, the ratio $h_{D_1}/H = 0.931/5 = 0.186$ as presented in the third column of Table (5.6). Other entries in the same column are calculated in a similar manner.

Table 5.6 Head ratios from Khosla *et al* (1936) solution and Computer results

h_{D_1} / H			
$1/\alpha$	Khosla's solution	Computer results	Relative error (%)
0	0	0	0
0.1	0.138	0.138	0
0.2	0.186	0.186	0
0.3	0.220	0.218	0.92
0.4	0.238	0.240	0.83
0.5	0.248	0.250	0.80
0.6	0.265	0.270	1.85
0.7	0.275	0.280	1.78
0.8	0.285	0.290	1.72
0.9	0.295	0.304	2.96
1	0.305	0.310	1.61

The foundation soil depth, FD , in Khosla *et al* (1936) solution, is considered to be infinite. For this reason, the computer results used in the comparison are for $FD = 120$ m. The computer results are found to be unchanged for greater depths. In a non-dimensional graphical form, these results can be represented as shown in Fig. 5.12.

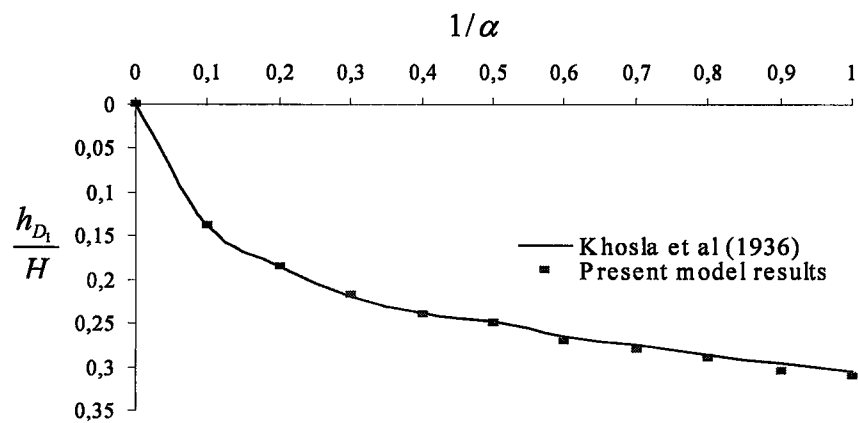


Fig. 5.12 Comparison of piezometric heads at point D_1 in a depressed base, $\theta = 90^\circ$

It is seen that the head values that were accepted after the last run with a foundation soil depth greater than twice the base length are in rather good conformance with the calculated values using Khosla *et al* (1936) solution.

5.4.5 Correction of an empirical equation for uplift under depressed dam floors

Khosla *et al* (1936) derived an empirically equation, from laboratory tests, for calculating the uplift pressure under depressed dam floors as follows: let ϕ_D and ϕ_E represent, respectively, the ratios h_D/H and h_E/H calculated from the theoretical case of a dam floor with a pile-line at end (Fig. 5.13a). Here, h_D and h_E are the piezometric head values at points D and E , respectively, and H is the total head difference between the upstream and downstream water levels. Based on such ratios, the pressure at the lower right corner (point D1 of Fig. 5.13b) in a depressed dam base with vertical sidewalls, is given by the following empirical formula, assuming the same ratio of dam width to depth, $b/d = \alpha$, (as presented in Leliavsky, 1965):

$$\phi_{D1} = \frac{h_{D1}}{H} = \phi_D - \frac{2}{3}(\phi_E - \phi_D) + \frac{3}{\alpha^2} \quad (5.15)$$

However, for a dam floor with pile-line at end, Khosla *et al* (1936) obtained the pressure equations at points D and E using conformal transformation (Fig. 5.13a) as:

$$h_D = \frac{H}{\pi} \cos^{-1}\left(\frac{\lambda-1}{\lambda}\right) \quad (5.16)$$

and

$$h_E = \frac{H}{\pi} \cos^{-1}\left(\frac{\lambda-2}{\lambda}\right) \quad (5.17)$$

where $\lambda = \frac{\sqrt{1+\alpha^2}+1}{2}$, and $\alpha = \frac{b}{d}$.

Application of Eq. (5.15) for the calculation of the pressure at point D1 with higher values of α , say for $\alpha = 1$, gives a value of $\phi_{D1} > 3$, which is far beyond the limits of Khosla *et al* (1936) curve as well as the results of the present numerical model. It is to be noted that the maximum ordinate value, which represents the correct pressure

in Khosla *et al* (1936) curve for all possible values of α , as well as the results of the present numerical model, does not exceed 0.31 (Fig. 5.12).

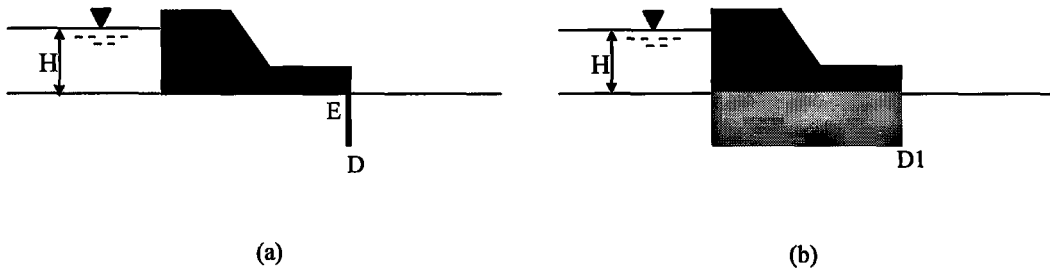


Fig. 5.13 Schematic sketch showing the characteristic points E, D, and D1 for (a) dam with a toe sheet pile, and (b) dam with a depressed base

Correction of Eq. (5.15) is performed for only its third part (i.e., $3/\alpha^2$). The correction method is illustrated from the tabulated values shown in Table 5.7. The procedure starts with the determination of the summation of the 1st and 2nd parts of this equation for different α values, then, the differences between this summation and the numerical results of the present model are obtained. The relation between the resulting differences (as the dependent variable) and α (as the independent variable) is then drawn in Fig. 5.14. From regression analysis, it is aimed to obtain an equation, which fits the curve of Fig. 5.14. The equation of the regression curve, which will be used in the estimation of the dependent variable for a given value of the independent variable, is determined with Microsoft Excel program using a power function (Fig. 5.15). The R-value, which describes the correlation coefficient between the two data sets, is found to be $R = 0.998$. This value shows that the choice of the power function used in the regression is quite reasonable.

From the previous analysis, it is found that the correct expression for the calculation of the pressure under a depressed dam base (i.e. ϕ_{D1}) is:

$$\phi_{D1} = \frac{h_{D1}}{H} = \phi_D - \frac{2}{3}(\phi_E - \phi_D) + \frac{0.0517}{\alpha^{2.016}} \quad (5.18)$$

The relative error in the calculation of ϕ_{D1} using the results of the present numerical model and Eq. (5.18) is determined in Col. 13, and is found to have a maximum value of approximately 1%. The close agreement between these results proves the accuracy of Eq. (5.18) in comparison to Eq. (5.15).

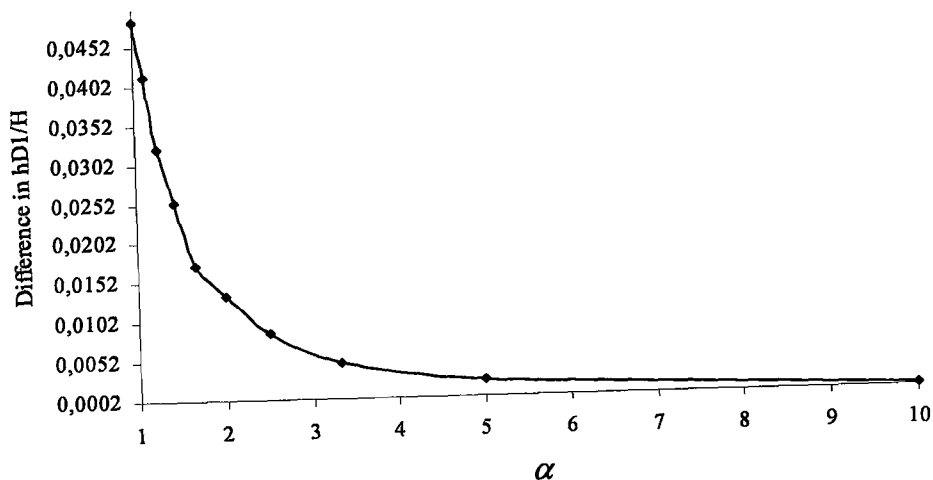


Fig. 5.14 The relation of the pressure difference (between the computer results and the summation of the 1st and 2nd parts of Khosla equation) and the ratio of dam width to depth

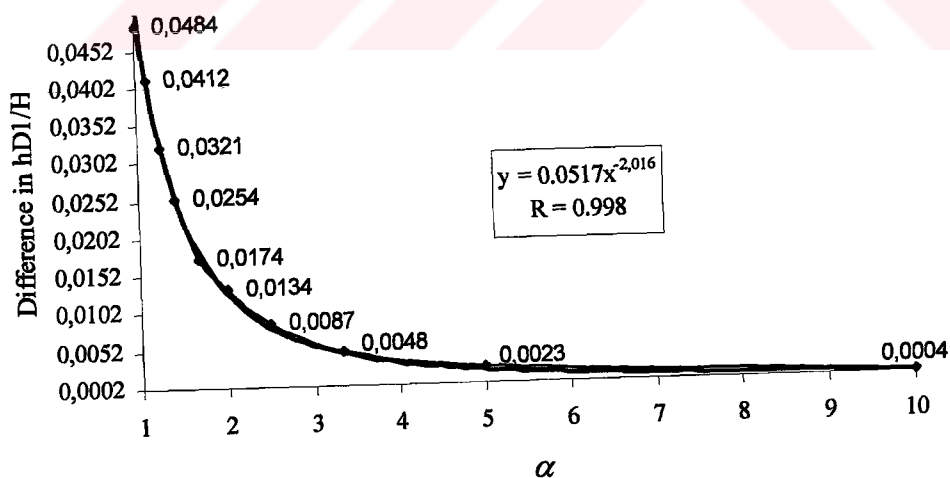


Fig. 5.15 Regression analysis using a power function fitted on the curve representing the relation of the pressure difference (between the computer results and the summation of the 1st and 2nd parts of Khosla equation) and the ratio of dam width to depth

Table 5.7 Data used in the correction of Khosla *et al* (1936) equation for the uplift pressure under a depressed dam base with vertical sidewalls

b	d	α	$1/\alpha$	λ	ϕ_E Eq. (5.17)	ϕ_D Eq. (5.16)	ϕ_{D1}			Difference using the equation on Fig. 5.15	From Col. 8+Col. 11	Relative error %
							From 1 st and 2 nd parts of Eq. (5.15) [$\phi_D - \frac{2}{3}(\phi_E - \phi_D)$]	From present model	Difference using Col. 9-Col. 8			
Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6	Col. 7	Col. 8	Col. 9	Col. 10	Col. 11	Col. 12	Col. 13
40	4	10	0.1	5.5249	0.2799	0.1946	0.1378	0.1382	0.0004	0.0005	0.1383	0.07
40	8	5	0.2	3.0495	0.3884	0.2655	0.1837	0.186	0.0023	0.0020	0.1857	-0.16
40	12	3.333	0.3	2.2401	0.4661	0.3134	0.2117	0.2165	0.0048	0.0046	0.2162	-0.14
40	16	2.5	0.4	1.8463	0.5268	0.3486	0.2298	0.2385	0.0087	0.0082	0.2380	-0.21
40	20	2	0.5	1.6180	0.5762	0.3754	0.2416	0.255	0.0134	0.0128	0.2544	-0.24
40	24	1.667	0.6	1.4718	0.6171	0.3963	0.2491	0.2665	0.0174	0.0185	0.2676	0.41
40	28	1.429	0.7	1.3719	0.6517	0.4128	0.2536	0.279	0.0254	0.0252	0.2788	-0.07
40	32	1.25	0.8	1.3004	0.6812	0.4260	0.2559	0.288	0.0321	0.0330	0.2889	0.31
40	36	1.111	0.9	1.2474	0.7065	0.4367	0.2568	0.298	0.0412	0.0418	0.2986	0.20
40	40	1	1.0	1.2071	0.7285	0.4453	0.2566	0.305	0.0484	0.0517	0.3083	1.07

5.5 Application of a Dam with a Depressed Base, $\theta = 63.4^\circ$

5.5.1 Steps of Numerical Computation

The considered saturated flow region, ABGHCDEF, is shown in Fig. 5.16. The figure shows the depressed base with its sloping sidewalls making an angle of 63.4° with its horizontal top line.

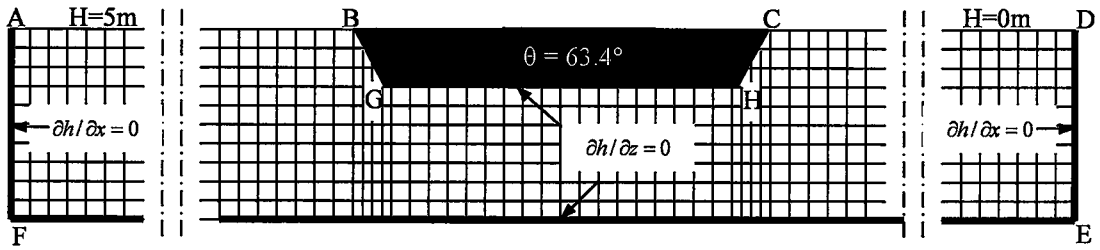


Fig. 5.16 Schematic sketch for a dam with a depressed base, $\theta = 63.4^\circ$

The numerical steps are similar as those used in the case of a depressed base with vertical sidewalls. Slight modifications are performed on the computer program in order to take account of the boundary conditions along the sloping sidewalls (see the Appendix). The following changes are of concern. The $h_{i,j}$ for $j = 1$ and $i = 1, 2, \dots, 101$ are equal to 5 m and the $h_{i,j}$ for $j = 1$ and $i = 145, 146, \dots, 245$ are equal to 0 m. The $h_{i,j}$ for $j = 4$ and $i = 105, 106, \dots, 141$ and the $h_{i,j}$ for $j = 11$ and $i = 2, 3, \dots, 244$ are estimated using Eqs. (5.1) and (5.2), respectively. Furthermore, The $h_{i,j}$ for the left and right no-flow boundaries at $i = 1$ and $j = 2, 3, \dots, 10$ and $i = 245$ and $j = 2, 3, \dots, 10$ are estimated from Eqs. (5.4) and (5.5), respectively. The $h_{i,j}$ values for the nodes along the upstream and downstream sloping sidewalls are estimated using a no-flow boundary condition similar to that shown in Fig. 4.8. In this situation, the applicable finite difference equations are as follows,

$$h_{i,j} = \frac{k_{x2}h_{i-1,j} + B^2k_{z2}h_{i,j+1}}{k_{x2} + B^2k_{z2}} \quad (\text{for } i = 102, j = 2 \text{ and } i = 103, j = 3) \quad (5.19)$$

and

$$h_{i,j} = \frac{k_{x1}h_{i+1,j} + B^2k_{z2}h_{i,j+1}}{k_{x1} + B^2k_{z2}} \quad (\text{for } i = 144, j = 2 \text{ and } i = 143, j = 3) \quad (5.20)$$

In the case where the foundation soil depth is 20 m, the $h_{i,j}$ for the corner nodes at $i = 1$ and $j = 11$, $i = 245$ and $j = 11$ are found from Eqs. (5.6) and (5.7), respectively. However, for the corner nodes at $i = 104$ and $j = 4$ and at $i = 142$ and $j = 4$, the applicable finite difference equations are as follows,

$$h_{i,j} = \frac{k_{x1}h_{i+1,j} + 2k_{x2}h_{i-1,j} + 2B^2k_{z2}h_{i,j+1}}{k_{x1} + 2k_{x2} + 2B^2k_{z2}} \quad (\text{for } i = 104 \text{ and } j = 4) \quad (5.21)$$

and

$$h_{i,j} = \frac{2k_{x1}h_{i+1,j} + k_{x2}h_{i-1,j} + 2B^2k_{z2}h_{i,j+1}}{2k_{x1} + k_{x2} + 2B^2k_{z2}} \quad (\text{for } i = 142 \text{ and } j = 4) \quad (5.22)$$

5.5.2 Exit Gradient

The schematic sketch shown in Fig. 5.17 is used for the numerical calculation of the exit gradient at the dam toe.

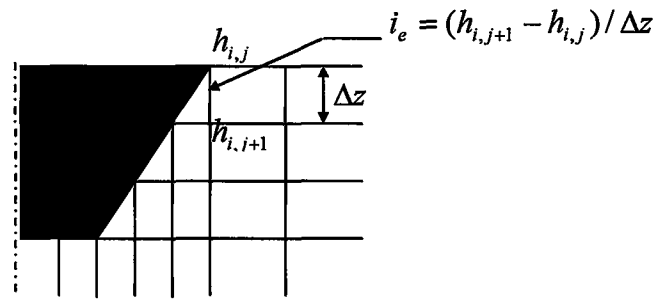


Fig. 5.17 Schematic sketch for numerical calculation of i_e at the dam toe where $\theta = 63.4^\circ$

The exit gradient values are collected from the computer runs and presented in Table 5.8. From this table, for the same permeability ratio, it is observed that the exit gradients increase as the foundation soil depth increases. However, it is clear from

the data presented that the exit gradient value has a minimal increment as the foundation soil depth becomes greater than the dam base width.

Table 5.8 Exit gradients for a depressed dam base, $\theta = 63.4^\circ$

FD(m)	Exit gradient, i_e					
	k_x / k_z					
	1	5	10	15	20	25
20	0.094	0.153	0.174	0.187	0.198	0.206
40	0.117	0.173	0.194	0.207	0.217	0.225
80	0.127	0.182	0.203	0.216	0.227	0.238
100	0.129	0.184	0.206	0.220	0.230	0.250
120	0.130	0.186	0.209	0.223	0.237	0.257

5.5.3 Flow Rate

The seepage discharges and its non-dimensional values under such a dam base are collected from the results of the corresponding computer runs and presented in Table 5.9.

Table 5.9 Non-dimensional discharges under a depressed dam base, $\theta = 63.4^\circ$

FD(m)	Non-dimensional seepage discharge, $\bar{Q}$					
	k_x / k_z					
	1	5	10	15	20	25
20	0.224	0.157	0.127	0.110	0.980	0.890
40	0.408	0.244	0.179	0.144	0.122	0.106
80	0.605	0.290	0.193	0.150	0.125	0.108
100	0.657	0.294	0.194	0.150	0.125	0.108
120	0.690	0.295	0.194	0.151	0.125	0.109

5.6 Application of a Dam with a Depressed Base for $\theta = 45^\circ$ or 26.5°

5.6.1 Exit Gradients

Following similar steps with the same computer program used in the case of a depressed dam base where $\theta = 63.4^\circ$, the values of $h_{i,j}$ at the various nodal points in the saturated flow region in cases of $\theta = 45^\circ$ and 26.5° are evaluated. From these piezometric head values, the exit gradients at the immediate toe for the depressed base where $\theta = 45^\circ$ or 26.5° are computed and presented in Tables (5.10) and (5.11), respectively.

Table 5.10 Exit gradients for a depressed dam base, $\theta = 45^\circ$

FD (m)	Exit gradient, i_e					
	k_x / k_z					
	1	5	10	15	20	25
20	0.101	0.160	0.181	0.194	0.203	0.210
40	0.126	0.180	0.200	0.212	0.220	0.226
80	0.138	0.188	0.207	0.217	0.225	0.231
100	0.140	0.189	0.208	0.218	0.226	0.233
120	0.158	0.190	0.208	0.219	0.227	0.234

Table 5.11 Exit gradients for a depressed dam base, $\theta = 26.5^\circ$

FD (m)	Exit gradient, i_e					
	k_x / k_z					
	1	5	10	15	20	25
20	0.113	0.187	0.213	0.226	0.235	0.241
40	0.140	0.210	0.234	0.246	0.253	0.259
80	0.154	0.220	0.241	0.252	0.259	0.264
100	0.156	0.221	0.243	0.253	0.260	0.266
120	0.158	0.222	0.244	0.255	0.262	0.268

5.6.2 Flow Rates

From the results of the previous Section, the non-dimensional discharge values for the cases where $\theta = 45^\circ$ or 26.5° are calculated for different foundation soil depths and permeability ratios and presented in Tables (5.12) and (5.13), respectively. It should be noted that these discharges are computed from the seepage velocities at the centre line of the dam base.

Table 5.12 Non-dimensional discharges under a depressed dam base, $\theta = 45^\circ$

<i>FD</i> (m)	Non-dimensional seepage discharge, $\bar{Q}$					
	k_x / k_z					
	1	5	10	15	20	25
20	0.221	0.156	0.127	0.110	0.981	0.891
40	0.402	0.244	0.179	0.145	0.123	0.107
80	0.599	0.291	0.194	0.151	0.126	0.109
100	0.652	0.294	0.195	0.151	0.126	0.109
120	0.686	0.296	0.195	0.151	0.126	0.109

Table 5.13 Non-dimensional discharges under a depressed dam base, $\theta = 26.5^\circ$

<i>FD</i> (m)	Non-dimensional seepage discharge, $\bar{Q}$					
	k_x / k_z					
	1	5	10	15	20	25
20	0.228	0.163	0.132	0.114	0.102	0.924
40	0.409	0.251	0.185	0.149	0.127	0.111
80	0.605	0.298	0.200	0.156	0.129	0.112
100	0.659	0.302	0.201	0.156	0.130	0.112
120	0.694	0.304	0.201	0.156	0.130	0.113

5.7 Summary of the Results for the Depressed Dam Base

A summary of the exit gradient and the seepage discharge values from the previously studied cases is given in Tables 5.14 and 5.15, respectively.

Table 5.14 Summary of exit gradients for the depressed dam base

FD (m)	k_x / k_z	Exit gradient, i_e				
		$\theta = 0^\circ$	$\theta = 26.5^\circ$	$\theta = 45^\circ$	$\theta = 63.4^\circ$	$\theta = 90^\circ$
20	1	0.140	0.117	0.11	0.109	0.093
	5	0.265	0.19	0.168	0.155	0.143
	10	0.330	0.217	0.188	0.175	0.167
	15	0.373	0.23	0.199	0.187	0.181
	20	0.405	0.238	0.207	0.197	0.191
	25	0.430	0.244	0.213	0.205	0.20
40	1	0.165	0.146	0.137	0.136	0.114
	5	0.283	0.216	0.19	0.177	0.16
	10	0.345	0.239	0.208	0.196	0.182
	15	0.386	0.25	0.218	0.207	0.196
	20	0.417	0.257	0.225	0.216	0.206
	25	0.440	0.262	0.23	0.225	0.213
80	1	0.176	0.16	0.149	0.148	0.122
	5	0.290	0.226	0.199	0.187	0.166
	10	0.350	0.247	0.215	0.205	0.187
	15	0.390	0.256	0.223	0.216	0.20
	20	0.421	0.263	0.23	0.227	0.209
	25	0.447	0.268	0.235	0.238	0.217
100	1	0.178	0.162	0.152	0.15	0.124
	5	0.292	0.228	0.20	0.189	0.167
	10	0.352	0.248	0.216	0.208	0.188
	15	0.392	0.258	0.225	0.221	0.201
	20	0.422	0.264	0.231	0.23	0.211
	25	0.448	0.27	0.237	0.25	0.218
120	1	0.180	0.164	0.153	0.15	0.125
	5	0.292	0.229	0.20	0.19	0.168
	10	0.353	0.248	0.216	0.211	0.189
	15	0.393	0.258	0.226	0.224	0.202
	20	0.424	0.265	0.231	0.237	0.212
	25	0.450	0.272	0.238	0.257	0.22

Table 5.15 Summary of non-dimensional discharges for the depressed dam base

FD (m)	k_x / k_z	Non-dimensional seepage discharge, $\bar{Q}$				
		$\theta = 0^\circ$	$\theta = 26.5^\circ$	$\theta = 45^\circ$	$\theta = 63.4^\circ$	$\theta = 90^\circ$
20	1	0.290	0.228	0.221	0.222	0.222
	5	0.225	0.163	0.156	0.156	0.155
	10	0.192	0.132	0.127	0.126	0.126
	15	0.172	0.142	0.110	0.109	0.109
	20	0.158	0.102	0.098	0.0978	0.0975
	25	0.147	0.092	0.089	0.0888	0.0884
40	1	0.460	0.409	0.402	0.402	0.402
	5	0.310	0.251	0.244	0.244	0.244
	10	0.243	0.185	0.179	0.178	0.177
	15	0.206	0.149	0.145	0.144	0.143
	20	0.182	0.127	0.122	0.121	0.120
	25	0.164	0.111	0.107	0.106	0.105
80	1	0.660	0.605	0.599	0.605	0.605
	5	0.357	0.298	0.291	0.290	0.288
	10	0.258	0.200	0.194	0.193	0.191
	15	0.212	0.156	0.151	0.150	0.148
	20	0.185	0.130	0.126	0.125	0.123
	25	0.166	0.112	0.108	0.108	0.107
100	1	0.713	0.659	0.652	0.652	0.652
	5	0.361	0.302	0.294	0.294	0.292
	10	0.259	0.200	0.195	0.194	0.192
	15	0.212	0.156	0.151	0.150	0.149
	20	0.185	0.130	0.126	0.125	0.123
	25	0.166	0.112	0.109	0.108	0.107
120	1	0.748	0.694	0.690	0.690	0.690
	5	0.363	0.304	0.296	0.294	0.292
	10	0.259	0.201	0.195	0.194	0.192
	15	0.212	0.156	0.151	0.150	0.149
	20	0.185	0.130	0.126	0.125	0.123
	25	0.167	0.113	0.109	0.108	0.107

5.8 Evaluation of the Results of the Depressed Dam Base Model

5.8.1 Exit Gradient

From the computer results of the piezometric head distribution and the exit gradients, it is found that the exit gradient, i_e , decreases with increasing the sidewall angle, θ , of the dam base, (table 5.14). The result is an increase in the safety factor against piping. The safety factor is considered as $F_s = 1/i_e$, where the value of 1 corresponds to average in situ conditions and represents the critical exit gradient at which piping occurs (Terzaghi, 1960).

Figure 5.18 shows the relative effectiveness of the various sloping planes of contact in increasing such a safety factor. It is seen that the lowest safety factor occurs in the case of using a dam base without any depression. That is for a horizontal base built directly on the foundation soil surface, with $\theta = 0^\circ$. On the other hand, the maximum safety factor occurs in the case of a depressed base, where $\theta = 90^\circ$. For other θ values, i.e. for $0^\circ < \theta < 90^\circ$, the safety factors lie between the previous ranges. From the same figure, it is seen that the safety factors reduce rapidly for k_x/k_z ranging from 1 to about 10 and reduce at a slower rate for higher values. The same characteristic trends for the safety factors can be observed in case of varying the foundation soil depth while keeping the other parameters unchanged. These safety factors will have greater values as the foundation soil depth decreases. Still, the main conclusions derived from Fig. 5.18 are also applicable.

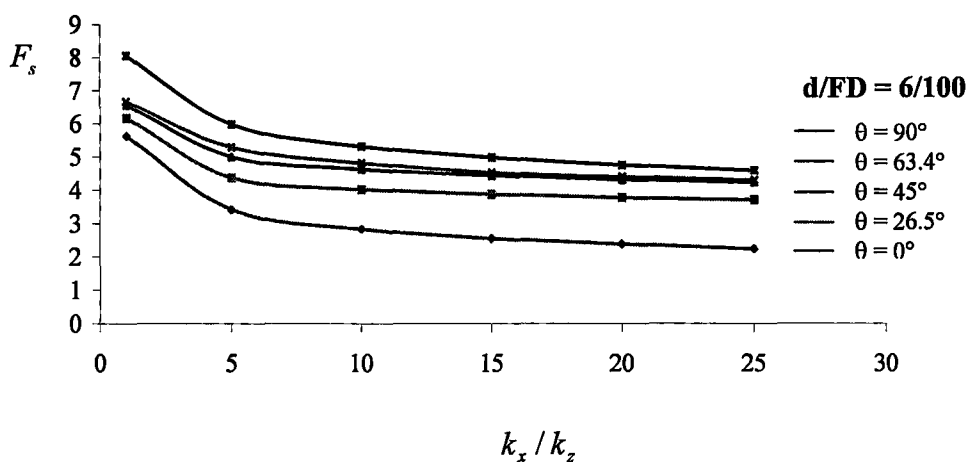


Fig. 5.18 Safety factor for different dam base configurations

From this observation and for practical application of such a dam base, it is required to find an expression to fit the computer results of the present numerical model for the studied configurations. Such an expression can be obtained starting from Darcy's law as,

$$v = ki = k \frac{H}{L_c} \quad (5.23)$$

where v is the seepage discharge or Darcy's velocity, k is the permeability coefficient, i is the hydraulic gradient, H is the total head difference between the water levels at the upstream and downstream sides, and L_c is the total creep length along the contact between the dam floor and the foundation soil. For a given size distribution of material, there appears to be a definite maximum velocity at which water can emerge below the dam without carrying away the foundation material. Combining this value of v with k which also depends on the material, Eq. (5.23) can be written as,

$$\frac{k}{v} = \frac{L_c}{H} \quad (5.24)$$

or

$$\frac{1}{\frac{v}{k}} = \frac{L_c}{H} \quad (5.25)$$

The ratio, v/k , can be evaluated at the dam toe, and it is equal to the exit gradient, i_e , calculated in a similar manner as previously stated in this chapter as given by Griffiths and Fenton, 1998. This is considered to be the maximum exit gradient value, which may cause piping or heaving. It is true in a constant or random permeability field, (Griffiths and Fenton, 1998).

The reverse of this ratio, $1/(v/k)$, represents the safety factor against piping. It seems similar to i_{cr}/i_e with i_{cr} equal to unity, which is the average critical gradient for most of soils, (Terzaghi, 1960). With reference to Eq. (5.25), the left-hand side can be found from the calculated exit gradient for the selected dam base configuration. The other variables, on the right-hand side are given data. By equating

both sides, using the known data in the right hand-side and the computed results in the left-hand side, it is found that equality is not achieved. In order to achieve equality, it is seen that the only parameter that can be adjusted is the creep length, L_c .

In order to adjust the creep length, Lane (1935) method of representing such a length is considered. On the basis of a study of about 280 existing dams, Lane (1935) observed that a vertical cut-off is more effective in reducing the velocity of flow through foundation materials than an equivalent horizontal distance. Therefore, a greater resistance to flow along vertical contacts should be considered as compared with horizontal ones. Lane (1935) recommended that a unit horizontal length of contact be considered to be one third as effective as a unit length of vertical contact. Hence, in computing the necessary seepage length to prevent piping or heaving, Lane (1935) introduced the following empirical relation with the weighted creep length, L_w , as

$$L_w = \frac{\sum L_h}{3} + \sum L_v \quad (5.26)$$

where $\sum L_h$ and $\sum L_v$ are the sum of all horizontal and vertical distances along the contact between the dam base and the foundation soil beneath it, respectively. For a dam with horizontal floor of length b and upstream and downstream vertical sheet piles of depths s_1 and s_2 , the creep length will be $L_w = (b/3) + 2s_1 + 2s_2 = C_L H$.

Eq. (5.26) is based on the following assumptions:

- 1- any surfaces with in the dam base sloping more than 45° with the horizontal are considered as vertical and those at a flatter slope as horizontal creeps,
- 2- the horizontal to vertical permeability ratio, k_x/k_z , is equal to 3, and
- 3- $\frac{\text{Loss of head per unit length of vertical contact}}{\text{Loss of head per unit length of horizontal contact}} = 6$, and
- 4- the hydraulic gradient, H/L_w , is considered uniform along the creep distance, L_w , which is the upper most flow line.

Many researchers who studied the safety of dams against piping have compared their results to that of Lane and suggested some modifications on Lane's method, as follows,

- Khosla (1935) stated that assignment of equal weight to all vertical contact planes and to all planes at angles greater than 45° is essentially erroneous.
- Similarly, Davis (1935) stated that it is not necessary for the inclined surfaces to be as steep as 45° to be as effective as vertical surfaces as suggested by Lane. Sloping surfaces at angles of 43° or 44° should be considered, according to Lane's method, as horizontal creep. However, it seems reasonable to consider them to offer more security against piping than purely horizontal surfaces.
- On the other hand, Meyer *et al.*, (1994) suggested that because in most soils k_z is less than k_x , the weighted creep length would be more precisely defined by dividing the horizontal length of the flow path by the ratio $\sqrt{k_x/k_z}$ than by dividing by 3 as suggested by Lane.
- As it has already been mentioned in Chapter 1, Koenders and Sellmeijer (1992) reported the results of a mathematical model of piping phenomenon. Using a curve fitting-technique, a formula has been found to calculate the ratio H/L_w (the total head/the creep length). The model results are compared against the weighted creep method of Lane and is found to give shorter creep lengths.
- Similarly, Badv and Sargordi (2001) used an analytical model based on Koenders and Sellmeijer (1992) design formula and a probabilistic method in designing water-retaining structures for piping in Urmia region (Iran). The comparison of the results of this formula with the empirical methods of Bligh (1910) and Lane (1935) showed that it requires a shorter seepage length and produces a more economical design for water retaining structures.

Despite of these drawbacks, Lane's method is widely used in practical applications for preliminary dimensioning of dam floors. Therefore, Eq. (5.25) can be rewritten as,

$$\frac{k}{v} = \frac{1}{\frac{v}{k}} = \frac{\frac{L_h}{3} + L_v}{H} \quad (5.27)$$

Since it is recommended to divide the horizontal creep length by $\sqrt{k_x/k_z}$ (Meyer *et al*, 1994), Eq. (5.27) becomes,

$$\frac{1}{i_e} = \frac{\frac{L_h}{\sqrt{k_x/k_z}} + L_v}{H} \quad (5.28)$$

Application of the known data and the computed results to Eq. (5.28) shows that equality is achieved if the creep length along the vertical and sloping surfaces is multiplied by a correction factor (say λ). For this reason, Eq. (5.28) can be rewritten as,

$$\frac{1}{i_e} = \frac{\frac{L_h}{\sqrt{k_x/k_z}} + \lambda L_\theta}{H} \quad (5.29)$$

where λ is the correction factor for the creep along the sloping surfaces, with the assumption that $\lambda = f(\theta, d/FD, k_x/k_z)$, and L_θ is the length of the sloping part of the dam base. Examples for explanation of the previous discussion are given here. The exit gradients from Table 5.14 for $\theta = 45^\circ$ and 90° , where $k_x/k_z = 10$, and $FD = 20$ m are found to be 0.188 and 0.167, respectively. Insertion of these values in the left-hand sides of Eqs. (5.25), (5.27), and (5.28) gives $1/0.188 = 5.32$, and $1/0.167 = 5.98$, when $\theta = 45^\circ$ and 90° , respectively. However, the right-hand sides of the same equations will be 10.4, 5.72, and 5.6 for $\theta = 45^\circ$, and 10.4, 5.06, and 4.93, for $\theta = 90^\circ$, respectively. This proves that equality is not achieved by using any of these equations. On the other hand, equality is achieved by application of Eq. (5.29) if the correction factor, λ , for the sloping surfaces, is taken as 0.9 and 1.44 when $\theta = 45^\circ$ and 90° , respectively. Hence, in computing dam safety against piping, the creep length along the sloping surfaces should be given different weights in accordance with the slope angle, θ , the ratio of base to foundation soil depths, d/FD , and the

permeability ratio, k_x/k_z . In other words, the weighted creep length first suggested by Lane is found to vary depending on the above mentioned parameters. Solving for λ it is possible to obtain from Eq. (5.29) that

$$\lambda = \frac{\frac{H}{i_e} - \frac{L_h}{\sqrt{k_x/k_z}}}{L_\theta} \quad (5.30)$$

Knowing all the parameters on the right hand side of this equation, the values of the correction factor, λ , for the studied cases, are computed and collected as shown in Table 5.16.

Table 5.16 Values of the correction factor, λ , for sloping surfaces of the depressed dam base

FD (m)	k_x/k_z	λ			
		$\theta = 26.5^\circ$	$\theta = 45^\circ$	$\theta = 63.4^\circ$	$\theta = 90^\circ$
20	5	0.547	0.81	1.13	1.40
	10	0.552	0.90	1.23	1.44
	15	0.56	0.93	1.26	1.44
	20	0.566	0.95	1.26	1.45
	25	0.57	0.96	1.25	1.41
40	5	0.43	0.602	0.84	1.11
	10	0.473	0.746	1.0	1.22
	15	0.50	0.804	1.07	1.26
	20	0.51	0.835	1.08	1.27
	25	0.517	0.857	1.09	1.28
80	5	0.39	0.532	0.73	1.00
	10	0.448	0.70	0.92	1.16
	15	0.477	0.77	0.99	1.20
	20	0.492	0.81	1.00	1.23
	25	0.501	0.83	1.00	1.24
100	5	0.384	0.524	0.71	0.98
	10	0.445	0.70	0.896	1.14
	15	0.472	0.76	0.97	1.20
	20	0.489	0.81	0.98	1.21
	25	0.50	0.82	0.92	1.23
120	5	0.38	0.524	0.70	0.989
	10	0.445	0.70	0.87	1.15
	15	0.472	0.758	0.93	1.20
	20	0.487	0.81	0.94	1.22
	25	0.491	0.815	0.883	1.21

5.8.1.1 Effects of Base Sidewall Inclination

Various angles for the sidewalls are imposed on the depressed dam base, including θ equal to 26.5°, 45°, 63.4°, and 90°, keeping the ratios of the permeability (horizontal to vertical) and the dam base to foundation soil depths constant at 10 and 0.3, respectively. The computed values of the correction factor, λ , for the selected cases, are shown in Table 5.16. A plot of the coefficient λ variation with angle θ shows that λ increases as θ increases, (Fig. 5.19). The increment is in the form of a sine curve, for $0^\circ \leq \theta \leq 90^\circ$.

The relation between the coefficient λ and the angle θ for other values of d/FD and k_x/k_z is seen to have the same trend as that shown in Fig. 5.19, but with different values.

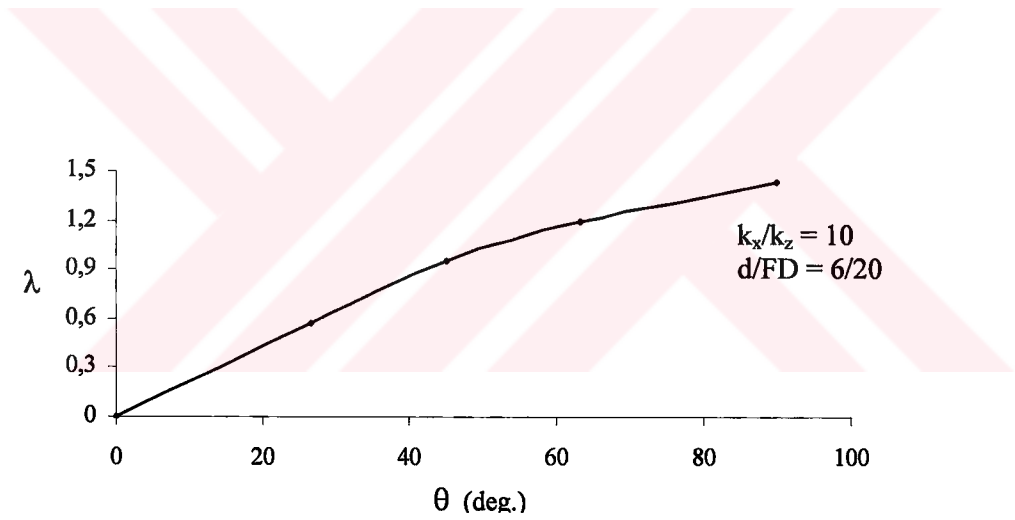


Fig. 5.19 Correction factor against sidewall angle

5.8.1.2 Effects of Permeability Ratio

A second set of analysis is conducted by varying the permeability ratio keeping the sidewall angle, and the ratio of the dam base to foundation soil depths constant at 26.5° and 0.3, respectively. It is found that the permeability ratio does not have a significant effect on the parameter λ compared to the angle of the sidewalls. A slight increase in the value of λ is seen with increasing the permeability ratio, as shown in Fig. 5.20.

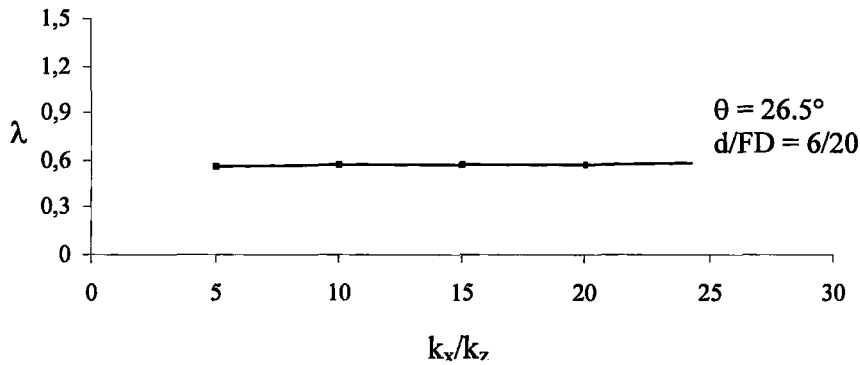


Fig. 5.20 Correction factor against permeability ratio

5.8.1.3 Effects of the Ratio of Base to Foundation Soil Depths

A third set of analysis is conducted in which the ratio of the base to foundation soil depths is varied but the sidewall angle and the permeability ratio were kept constant at 26.5° and 10, respectively. The value of the correction factor, λ , is found to have a slight increase for $d / FD < 0.1$ and a gradual increase for $d / FD > 0.1$, as shown in Fig. 5.21.

The relation between the coefficient λ and d / FD for other θ and k_x / k_z values is seen to have the same trend as that shown in Fig. 5.21, but with different values.

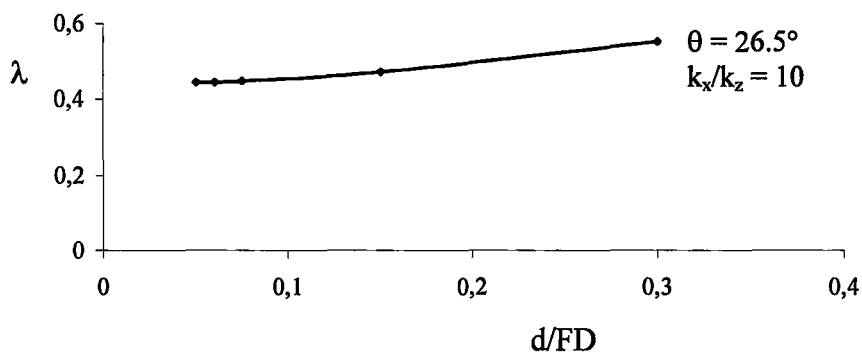


Fig. 5.21 Correction factor against base to foundation soil depths ratio

5.8.1.4 Combined Effects

Referring to Table 5.16, four sets are drawn for the correction factors against the side wall angles of the depressed dam base, using d/FD ratio equal to $6/20$, $6/40$, $6/80$, and $6/120$, respectively, (Fig. 5.22). In each set, two values for k_x/k_z are selected as equal to 10 and 20 and a curve is drawn for each value. Figure 5.22 shows that the curve of λ for $k_x/k_z = 20$ is higher than that for $k_x/k_z = 10$. Moreover, these two curves are observed to have the same trend as that for a sine curve. For convenience of presentation, the sine curve for $0^\circ \leq \theta \leq 90^\circ$ is included in the same figure.

It is also seen that as the ratio d/FD increases, the values of the correction factor, λ , decrease, and tend to fall on the sine curve, particularly for the situation at which $k_x/k_z = 10$ and FD is greater than twice the base width. Therefore, it is possible to write the following expression,

$$\lambda = \sin\theta \quad (\text{for } k_x/k_z = 10 \text{ and } FD > 100 \text{ m}) \quad (5.31)$$

Consideration of the above mentioned situation, requires a safety factor against failure due to piping in the following form:

$$C = \frac{L_w}{H} = \frac{\frac{L_h}{\sqrt{k_x/k_z}} + L_\theta \sin\theta}{H} \quad (5.32)$$

This expression is applicable in case of a depressed dam base without sheet piles. For other values of d/FD and k_x/k_z , the correction factor, λ , can be found by devising special figures as that shown in Fig. 5.22 after introducing the desired data of d/FD and k_x/k_z into the computer program.

It is also observed that increasing the k_x/k_z ratio gives rise to more rapid head loss. This means that the safety against piping decreases in stratified than in homogeneous and isotropic soil.

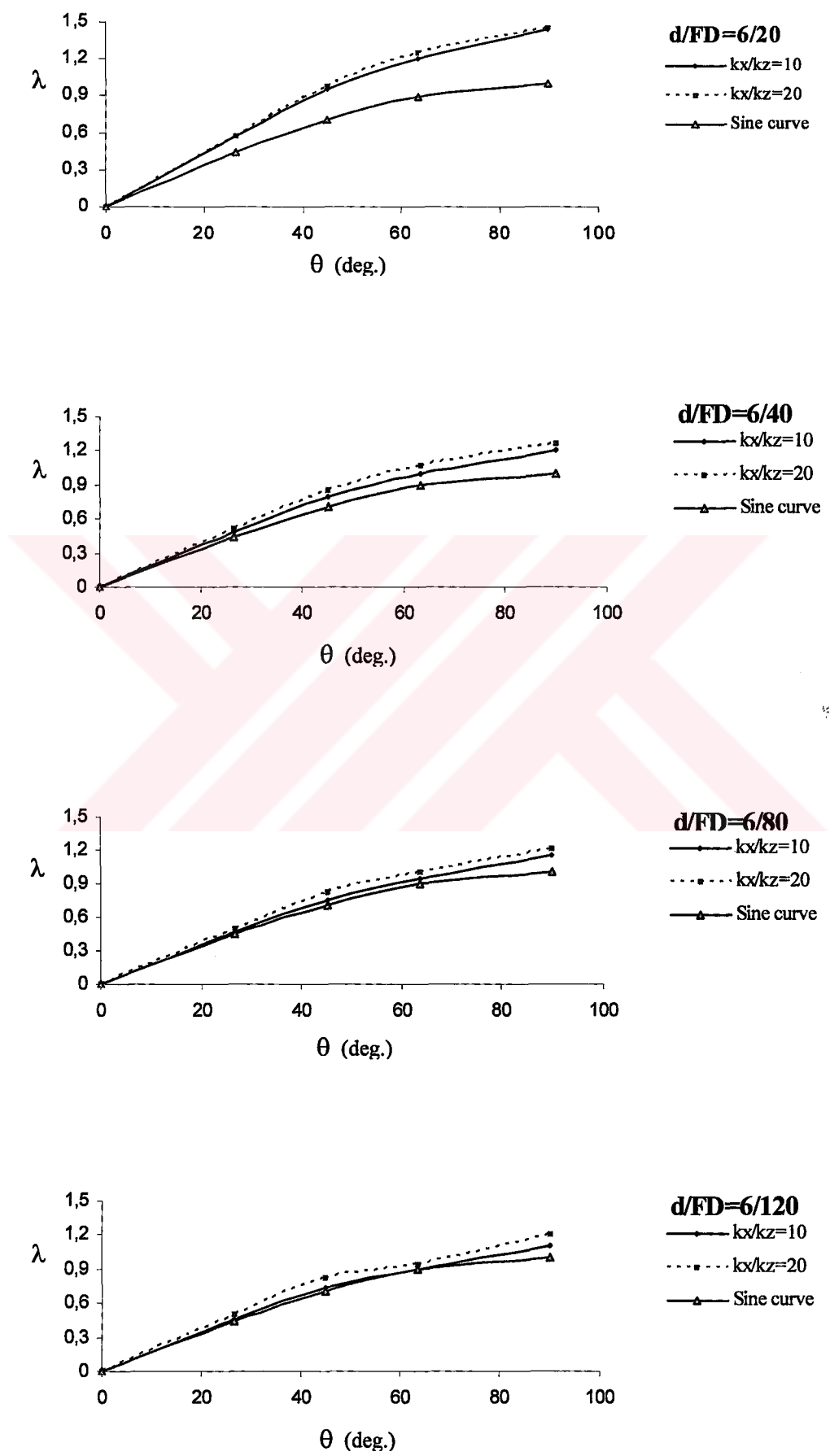


Fig. 5.22 Correction factors for different θ , d/FD , and k_x/k_z

5.8.2 Flow Rate

The non-dimensional form of the flow rate, $\bar{Q}$, for the studied configurations, is computed in this chapter and a summary of its values is presented in Table 5.15. The computation of such a flow rate is useful in estimating the loss of water from the reservoir. The variation of $\bar{Q}$ with the slope angle of the base sidewall, θ , for $k_x/k_z = 1, 10, \text{ and } 25$ and for $d/FD = 6/20$ and $6/120$ is drawn in Figs. 5.23 and 5.24, respectively. From these figures it is observed that as expected, the seepage discharge increases with increasing the foundation soil depth, FD . Moreover, the stratification, in general, reduces the seepage discharges as compared with that of homogeneous and isotropic soil.

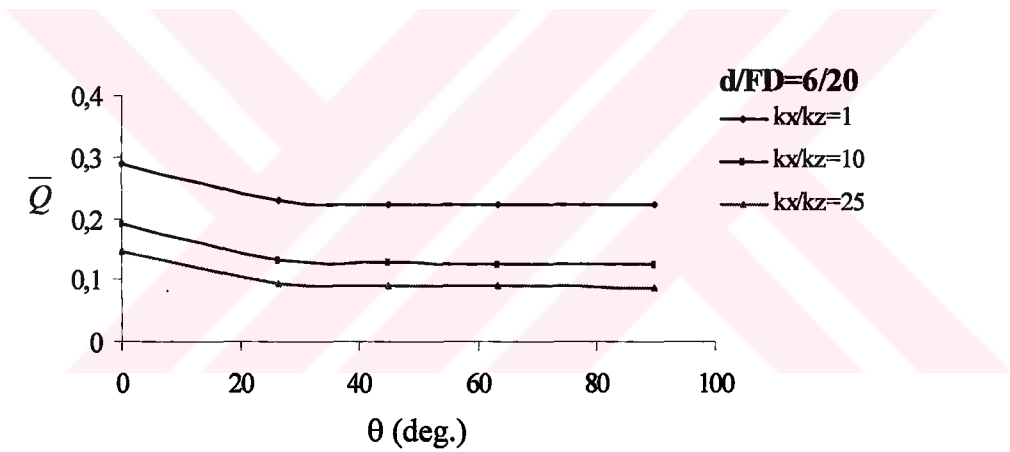


Fig. 5.23 Non-dimensional discharge versus sidewall angle for $FD = 20$ m

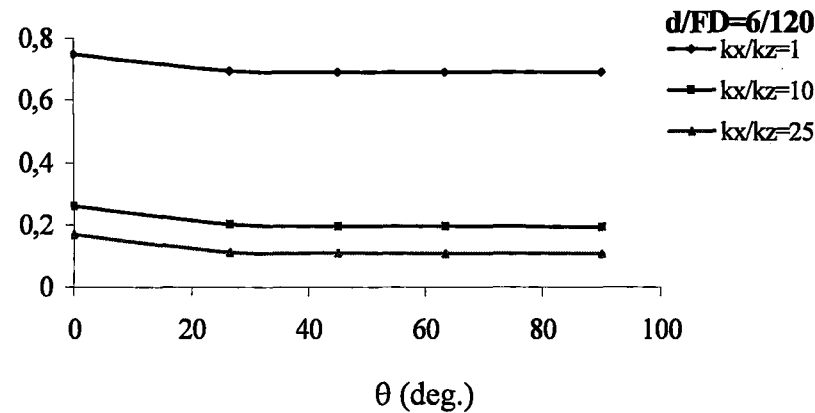


Fig. 5.24 Non-dimensional discharge versus sidewall angle for $FD = 120$ m

Variation of the foundation soil depth, FD , from 20 m to 200 m indicates that the calculated seepage discharge changes by only a small amount for FD values greater than $2b$, as seen from Fig. 5.25. This agrees well with Harza (1935) statement, that the water flowing under the dam tends to concentrate as much as possible along the shortest path with the depressed base. In other words, it follows a strip along the dam base. Therefore, it is possible to assume that a relatively deeper depth can replace the infinite soil depth. It is believed that the error will not be great as most of the seepage flow will concentrate close to the foundation profile of the dam base. In practical cases, most foundation soils have finite depths.

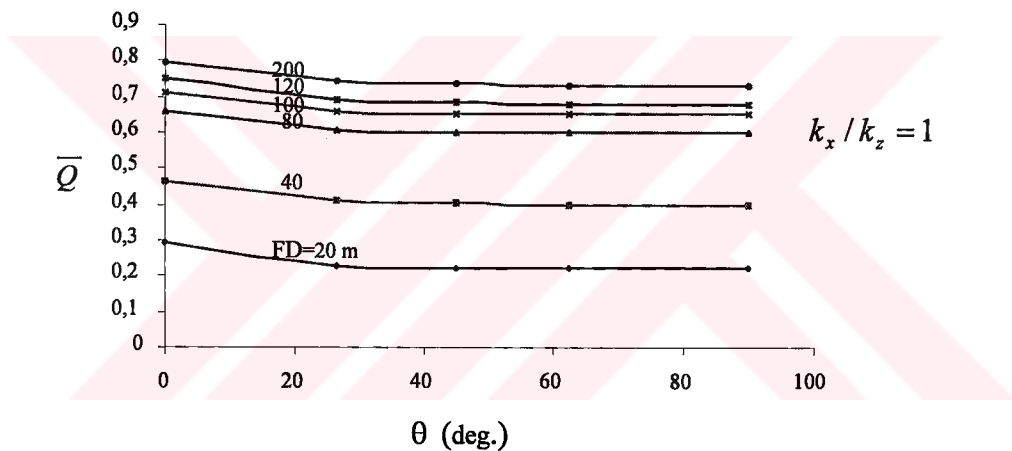


Fig. 5.25 Non-dimensional discharge versus sidewall angle for different FD

5.8.3 Uplift Pressure

5.8.3.1 Effect of Impermeable Sub-Soil Location

The effect of impermeable subsoil location on uplift pressure is investigated in connection with a dam foundation that is supported by a depressed concrete cut-off (where θ equals 90°). This is the dam base configuration that gives the highest safety factor against piping and lowest seepage discharge among the studied dam base configurations, (Fig. 5.26).

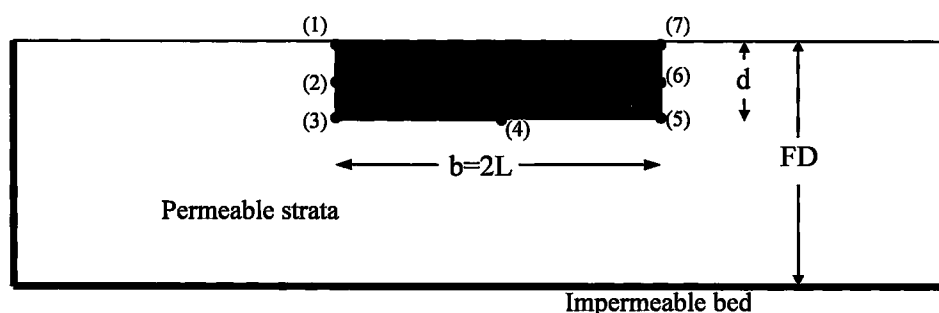


Fig. 5.26 Dam section selected to study the effect of impermeable sub-soil location on uplift pressure where θ equals 90°

In dealing with a flat foundation, only two variables are of concern, namely, the width of the dam floor, b , and the depth of the permeable strata, FD . However, for the case considered, the situation is more involved because in addition to these two factors, its solution depends on another characteristic dimension, viz., the depth, d , of the depressed dam base. Hence, it is possible to define two dimensionless ratios for an effective study as FD/b and d/b .

For practical applications, it is intended to know the limit of the latter ratio in the calculation of the true upward pressure. The principle danger from the upward pressure on a dam base on earth foundation is that, it will be sufficient to lift some part of the dam, breaking a new outlet opening for the percolation which will give such a short percolation path that the dam will fail. For this purpose, a numerical solution (using the computer program given in the Appendix) is applied to the hypothetical dam foundation cross-section shown in Fig. 5.26. The results of the numerical solution supply the required information on this subject. Table 5.17 contains the calculated uplift pressures, expressed in percents of the total head difference, at different points on the line of creep which is the stream line hugging the profile of the solid dam floor. These pressures are shown against the various d/b ratios, for which they are calculated.

The calculations are performed for different depths of the dam base and the foundation soil, i.e. $d = 0, 4, 8$, and 16 m; and $FD = 20, 40, 80$, and 120 m. The foundation soil is considered to be homogeneous and isotropic ($k_x/k_z = 1$). For a dam floor of 40 m in width, the values of the above mentioned ratios are considered as follows: $FD/b = 0.5, 1, 2$, and 3 , and $d/b = 0, 0.1, 0.2$, and 0.4 .

Table 5.17 Uplift pressures under the depressed dam base for $k_x / k_z = 1$

$FD = 20 \text{ m}$

$\begin{array}{c} \text{Abscissa} \\ d/b \end{array}$	$-L$	$-\frac{3}{4}L$	$-\frac{1}{2}L$	$-\frac{1}{4}L$	0	$\frac{1}{4}L$	$\frac{1}{2}L$	$\frac{3}{4}L$	b
0	1.0	0.798	0.69	0.594	0.502	0.410	0.314	0.204	0.0
0.1	0.888	0.757	0.668	0.584	0.502	0.420	0.336	0.246	0.114
0.2	0.866	0.751	0.665	0.584	0.502	0.421	0.339	0.253	0.136
0.4	0.892	0.785	0.691	0.597	0.502	0.408	0.313	0.218	0.110

$FD = 40 \text{ m}$

$\begin{array}{c} \text{Abscissa} \\ d/b \end{array}$	$-L$	$-\frac{3}{4}L$	$-\frac{1}{2}L$	$-\frac{1}{4}L$	0	$\frac{1}{4}L$	$\frac{1}{2}L$	$\frac{3}{4}L$	b
0	1.0	0.786	0.678	0.588	0.504	0.418	0.326	0.218	0.0
0.1	0.870	0.734	0.650	0.575	0.504	0.432	0.357	0.271	0.133
0.2	0.834	0.716	0.640	0.571	0.504	0.438	0.368	0.291	0.17
0.4	0.803	0.701	0.632	0.567	0.505	0.442	0.377	0.307	0.203

$FD = 80 \text{ m}$

$\begin{array}{c} \text{Abscissa} \\ d/b \end{array}$	$-L$	$-\frac{3}{4}L$	$-\frac{1}{2}L$	$-\frac{1}{4}L$	0	$\frac{1}{4}L$	$\frac{1}{2}L$	$\frac{3}{4}L$	b
0	1.0	0.782	0.676	0.588	0.506	0.422	0.334	0.226	0.0
0.1	0.864	0.727	0.645	0.574	0.507	0.439	0.367	0.284	0.142
0.2	0.823	0.705	0.633	0.569	0.508	0.447	0.382	0.308	0.186
0.4	0.780	0.682	0.620	0.564	0.511	0.457	0.401	0.337	0.234

$FD = 120 \text{ m}$

$\begin{array}{c} \text{Abscissa} \\ d/b \end{array}$	$-L$	$-\frac{3}{4}L$	$-\frac{1}{2}L$	$-\frac{1}{4}L$	0	$\frac{1}{4}L$	$\frac{1}{2}L$	$\frac{3}{4}L$	b
0	1.0	0.782	0.676	0.590	0.508	0.426	0.336	0.228	0.0
0.1	0.863	0.727	0.646	0.577	0.510	0.444	0.372	0.289	0.145
0.2	0.822	0.705	0.634	0.572	0.513	0.453	0.389	0.316	0.192
0.4	0.777	0.680	0.621	0.567	0.515	0.465	0.411	0.348	0.245

The difference in uplift pressure values calculated from the model (Table 5.17), is seen to be negligible for soil depths greater than, or equal to, twice the width of the dam floor. From this observation the numerical results in Table 5.17, for the case of $FD = 120$ m, is considered to simulate Pavlovsky (1922) solution which used the conformal transformation and based on infinite soil depth. This is done in order to verify the accuracy of the computer results

With reference to Fig. 5.26, the computer results of the uplift pressure, for the case of homogeneous and isotropic soil with a depth greater than twice the dam width (i.e., $k_x/k_z = 1$ and $FD = 120$ m), are compared to Pavlovsky's solution (for $d/b = 0.1$), and the agreement is good as shown in Fig. 5.27.

Bligh (1910) simplified assumption considering a linear uplift pressure distribution is also added in the same figure. This assumption tends to overestimate the uplift pressure on the upstream and underestimate it on the downstream sides of the dam base. When calculating the required weight of the solid dam or apron, the designer must bear this point in mind.

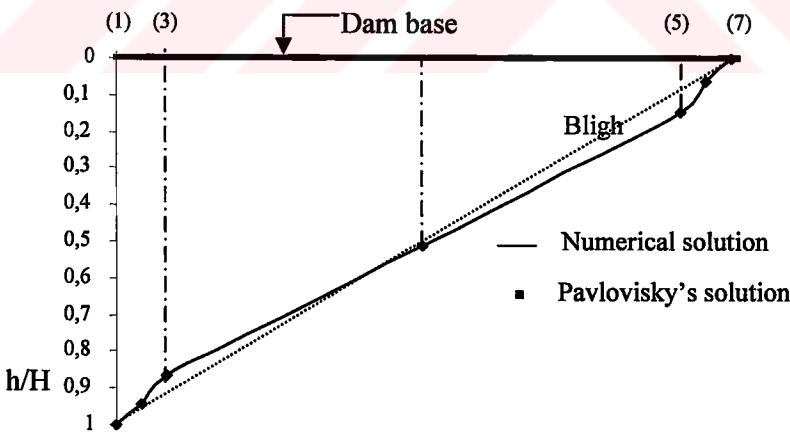
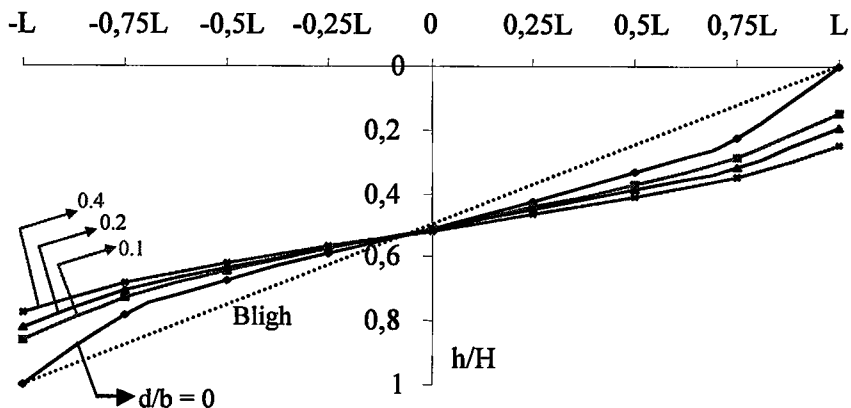
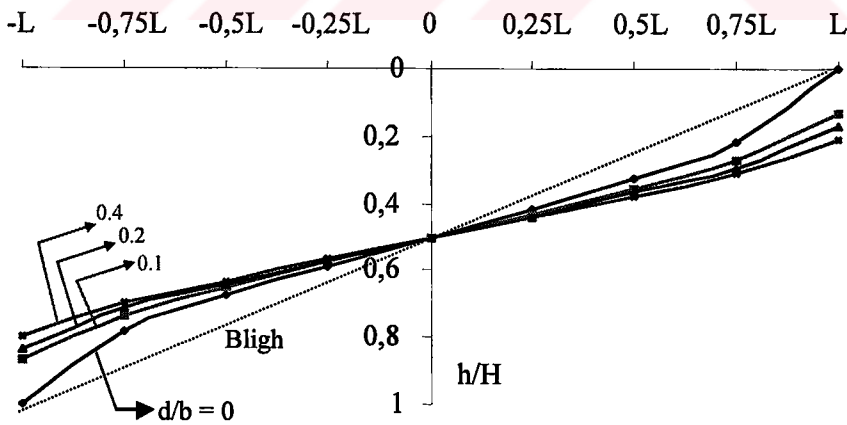


Fig. 5.27 Comparison of uplift pressure curves for $d/b = 0.1$ and $k_x/k_z = 1$

Using the uplift pressure values shown in Table 5.16, the effect of varying the ratio d/b can be seen from Fig. 5.28. It is to be noted that since the chosen dam base is symmetric, the uplift pressure at the centre line is approximately equal to $0.5 H$, where H is the head difference between the upstream and downstream water levels.



(a)



(b)

Fig. 5.28 Uplift pressure curves along the dam base where $k_x/k_z = 1$,
(a) for $FD/b = 3$ and (b) for $FD/b = 1$

It is possible to substantiate the following remarks from Table 5.17 and Fig. 5.28:

- (a) owing to the effect of the impermeable subsoil, approximately 10 % accuracy is adopted as a criterion of admissible error,
- (b) the effect of the impermeable subsoil, in the calculation of the uplift pressure, could be disregarded for values of $FD/b \geq 2$, and
- (c) so long as the thickness of the solid dam base does not exceed one tenth of the dam floor width, the uplift pressure will be almost the same as for zero thickness (less than 10 percent difference). In the majority of weir and dam designs, an ideal straight line can, therefore, be substituted for the uplift pressure under a real apron. This conclusion refers only to the upward pressure of percolating water, which is not intended for applications to downward force resistance to it. This is tantamount to saying that the weight of the apron cannot be disregarded under any circumstances.

5.8.3.2 Effect of Anisotropy Ratio

Another set of analysis is conducted by varying the permeability ratio (horizontal to vertical) as $k_x/k_z = 10, 20$, and 50 , respectively, keeping the soil depth, FD , equal to 120 m. The uplift pressure values corresponding to these permeability ratios are found from the numerical solution and summarized as shown in Table 5.18.

Table 5.18 Uplift pressures at points along the depressed dam base expressed as a percent of the total head for different anisotropy ratios, for $d/b = 0.1$

	h/H						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$k_x/k_z=1$	1.0	0.943	0.863	0.510	0.145	0.061	0.0
10	1.0	0.917	0.796	0.524	0.236	0.096	0.0
20	1.0	0.912	0.780	0.538	0.273	0.112	0.0
50	1.0	0.909	0.771	0.570	0.336	0.138	0.0

Complete uplift diagrams based upon the values shown in Table 5.18, are given in Fig. 5.29. From this figure, it is seen that increasing the permeability ratio causes a decrease in the uplift pressure at the upstream and an increase at the downstream sides of the dam base.

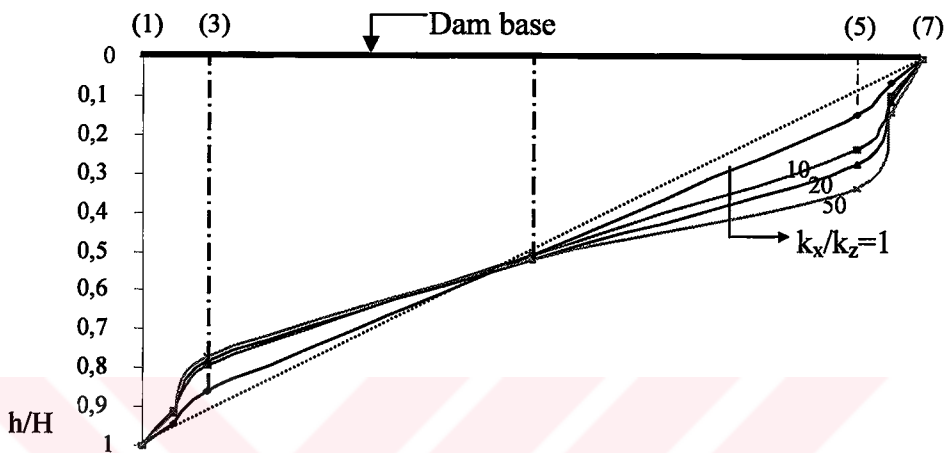


Fig. 5.29 Uplift pressure curves for different permeability ratios, for $d/b = 0.1$

A comparison between the percentage increment in the uplift pressures at the upstream and downstream bottom corners of the dam base against the increment in the permeability ratio is given in Figs. 5.30 and 5.31, respectively.

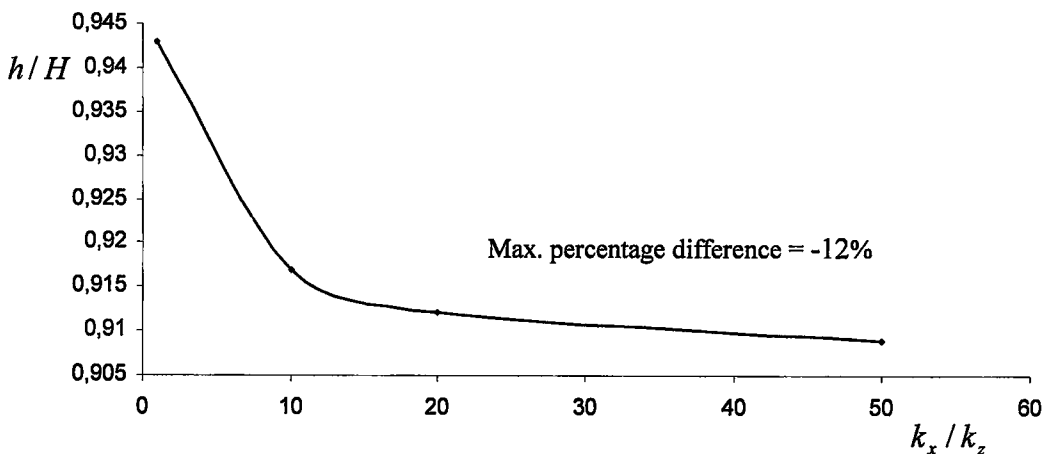


Fig. 5.30 Uplift pressure variation for different permeability ratios at the upstream bottom corner of the depressed dam base (point 3)

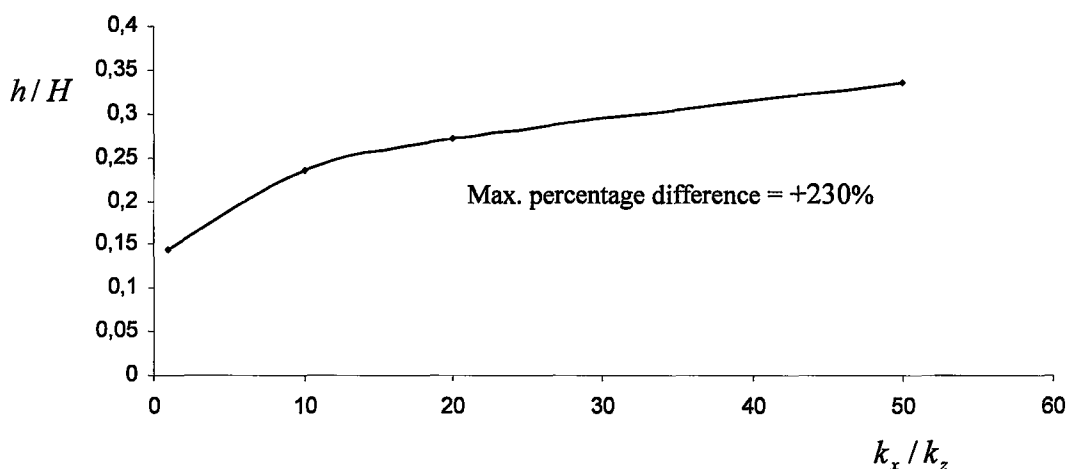


Fig. 5.31 Uplift pressure variation for different permeability ratios at the downstream bottom corner of the depressed dam base (point 5)

5.9 Comparison of the Depressed Dam Base Results with Lane's Method

An example of comparing the results for the calculation of the weighted creep length, L_w , using Lane (1935) method and the approach of the present study, to the same two-dimensional problem, is given in Fig. 5.32.

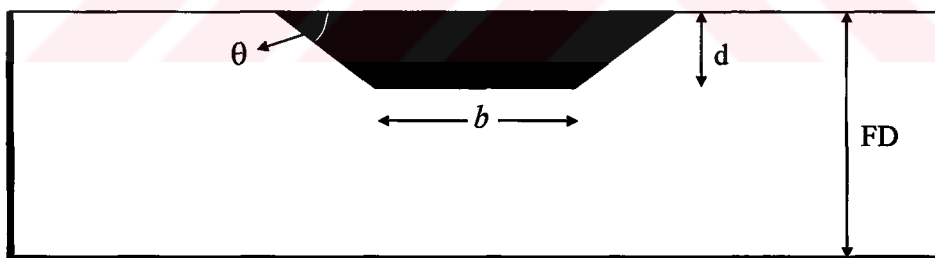


Fig. 5.32 Dam section selected for comparison with Lane's method

This is a hypothetical cross-section across the dam base, and the pervious foundation soil beneath it. The numerical solution that is used in the comparison is that for $k_x/k_z = 10$ and $FD = 100$ m, for this reason, the correction factor for calculating the length of the sloping parts of the dam base will be: $\lambda = \sin\theta$.

The percentage difference in calculating L_w using the two above-mentioned methods is presented for different d and b values as shown in Table 5.19. In order to evaluate the percentage difference, the following expression is used,

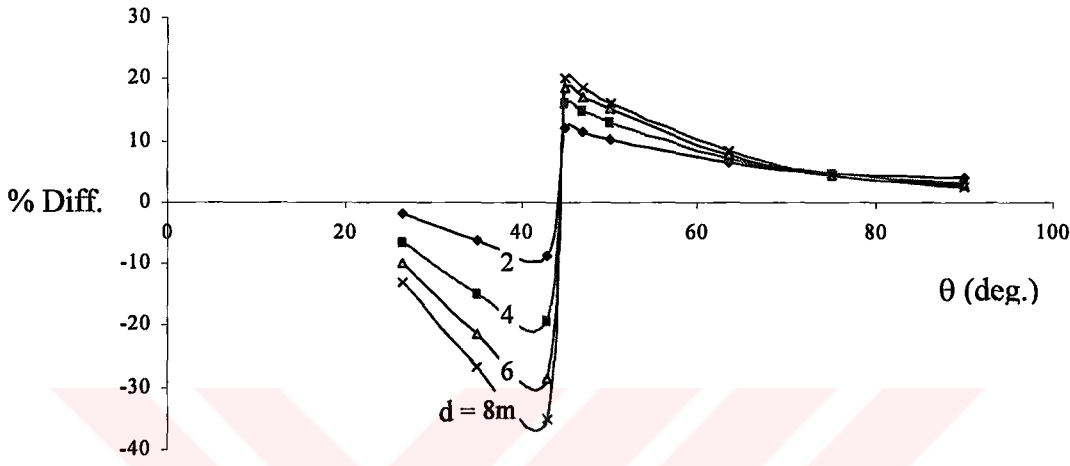
$$\% \text{ difference} = \frac{(L_w)_{Lane} - (L_w)_{thesis}}{(L_w)_{Lane}} * 100 \tag{5.33}$$

where $(L_w)_{Lane}$ is the weighted creep length calculated using Lane’s method, and $(L_w)_{thesis}$ is the weighted creep length calculated using the approach of the present study.

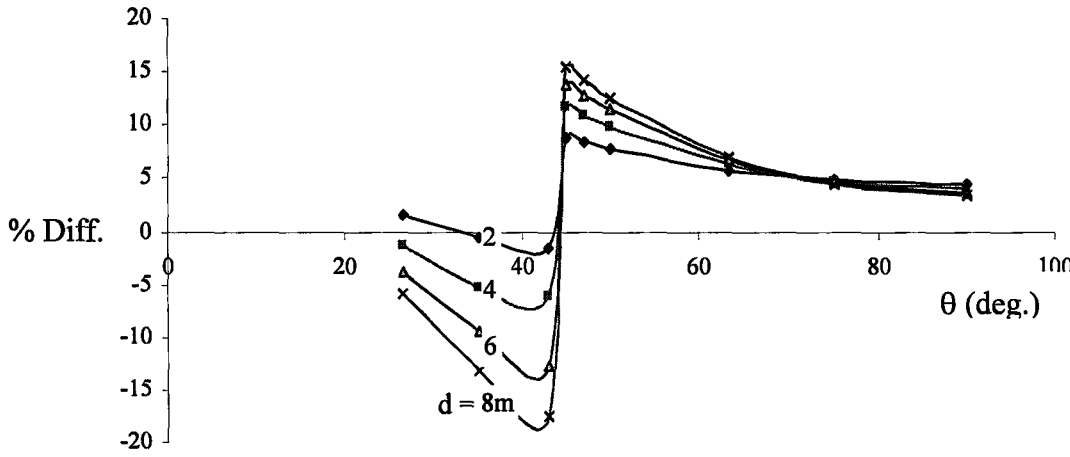
Table 5.19 Percentage differences in the calculation of the weighted creep length, using Lane’s method and the approach of the present study

		% Differences								
		θ								
<i>b</i> (m)	<i>d</i> (m)	26.5°	35°	43°	45°	47°	50°	63.4°	75°	90°
40	2	-1.8	-6.3	-8.8	12.0	11.4	10.3	6.5	4.7	4.0
	3	-4.5	-10.8	-14.6	14.0	13.4	11.9	6.8	4.6	3.3
	4	-6.7	-15.0	-19.4	16.0	14.9	13.1	7.0	4.5	3.2
	5	-8.8	-18.0	-24.2	17.5	16.1	14.1	7.6	4.4	2.79
	6	-10.0	-21.4	-28.4	18.6	17.1	15.0	7.8	4.3	2.76
	8	-13.0	-26.6	-35.0	20.0	18.6	16.2	8.2	4.2	2.38
70	2	0.6	-1.87	-3.36	9.8	9.3	8.4	6.0	4.9	4.4
	3	1.0	-4.9	-7.1	11.5	10.7	9.7	6.2	4.77	3.97
	4	-2.8	-7.7	-10.6	13.0	12.0	10.7	6.5	4.68	3.81
	5	-4.2	-10.2	-13.8	14.2	13.1	11.7	6.8	4.6	3.49
	6	-5.6	-12.6	-16.9	15.0	14.1	12.4	7.1	4.53	3.39
	8	-8.5	-16.8	-22.4	17.0	15.6	13.7	7.4	4.4	3.08
90	2	1.5	-0.5	-1.6	8.9	8.4	7.8	5.8	4.9	4.55
	3	0.0	-2.9	-4.6	10.4	9.8	8.9	6.1	4.83	4.2
	4	-1.3	-5.2	-6.1	11.7	10.9	9.8	6.4	4.75	4.0
	5	-2.4	-7.4	-10.2	12.8	11.9	10.6	6.6	4.7	3.84
	6	-3.8	-9.4	-12.8	13.8	12.8	11.4	6.78	4.62	3.66
	8	-6.0	-13.1	-17.6	15.5	14.3	12.6	7.0	4.5	3.34

A plot of the percent differences of the creep length versus the slope angles of the dam base sidewalls for different d and b values is given in Fig. 5.33. The comparison is presented for two b values as 40 m and 90 m, respectively.



(a)



(b)

Fig. 5.33 % Differences in the calculation of L_w using Lane's method and the approach of the present study, (a) $b = 40$ m and (b) $b = 90$ m

From Fig. 5.33, the following remarks can be deduced.

- 1- the differences in L_w calculated using Lane's method and the approach of the present study is negative for $\theta < 45^\circ$ and positive for $\theta \geq 45^\circ$. In other words, the creep length calculated using the approach of the present study is longer than that using Lane's method in case of $\theta < 45^\circ$. That is because the sloping sidewalls of the dam base are considered to offer more resistance to flow in the present study. It seems obvious that the application of the dam weight and its load to the soil foundation will result in a decrease in the voids and a tighter contact against the dam base. Thus, if the dam base is properly designed and constructed, directly supported by the foundation material, and bear heavily upon it, the horizontal creep should be given a higher value than that suggested by Lane.
- 2- the % differences decrease as b increases while keeping d constant and increase as d increases while keeping b constant.
- 3- for higher values of d other than that mentioned in step 2 and for the same b , the % differences show the same trend and are greater.
- 4- for θ values around 45° , the % differences become greater:
 - for $d = 2$ m to 8 m and $b \approx 40$ m, the % differences = -10 % to -35 %.
 - for $d = 2$ m to 8 m and $b \approx 100$ m, the % differences = -2 % to -18 %.
- 5- as θ approaches 0° or 90° , the % differences decrease and reach its minimum:
 - for $d = 2$ m to 8 m and $b \approx 40$ m, the % differences ≈ 4 % to 2 %.
 - for $d = 2$ m to 8 m and $b \approx 100$ m, the % differences ≈ 4.6 % to 3.3 %.

An example for the calculation of the entries in Table 5.18 is given below for its 1st entry (i.e., for $b = 40$ m, $d = 2$ m, and $\theta = 26.5^\circ$).

(1) using Lane (1935) method

$$L_w = \frac{L_h}{3} + L_v$$

Since $\theta < 45^\circ$, the sloping parts of the dam base will be considered horizontal, and as a consequence,

$$L_w = \frac{40 + 2 * 4.472}{3} = 16.3 \text{ m}$$

(2) using the approach of the present study one can obtain.

$$L_w = \frac{L_h}{\sqrt{k_x / k_z}} + \lambda L_\theta = \frac{40}{\sqrt{10}} + 2 \sin 26.5 * (4.472) = 16.6 \text{ m}$$

Hence, the percentage difference is calculated as,

$$\% \text{ difference} = \frac{16.3 - 16.6}{16.3} * 100 = -1.8 \%$$



6. INVESTIGATION OF A FLAT DAM BASE WITH AN INCLINED TOE SHEET PILE

6.1 Introduction

Hydraulic structures are often built on permeable soil foundations. The water under pressure percolates through the soil and exerts uplift pressures and may carry soil particles with it leading to piping. Therefore, a hydraulic structure founded on permeable soil has to be designed against uplift and piping. Cut-offs like vertical or inclined steel sheet piles can be provided to reduce uplift pressure and exit velocity, resulting in an appreciable saving in the dimensions of the structure.

Due to relatively expensive sheet piling and long experience it is known that leakage through the interlocks between the individual piling is considerable. Therefore, it is chosen less frequently. However, in some circumstances where the alternative cut-off methods are expensive or time consuming, steel sheet piling can provide the best under-seepage barrier, (Lane and Wohlt, 1961). In order to prevent the occurrence of slits between the elements, they are given a special form as that shown in Fig. 6.1.

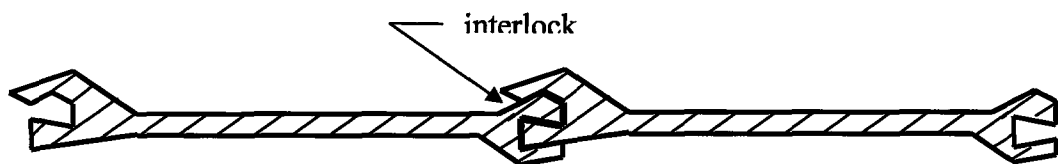


Fig. 6.1 Profile of two steel sheet piles

Another disadvantage of steel sheet piling is the possibility that cobbles or boulders may damage the wall during the driving. Often, with no indication or trouble at the surface, the interlocks tear open or the piling curls. Therefore, driving and pulling tests should be made at sites, where there is some doubt. In order to reduce leakage in the interlocks and prevent breaking or curling, a special design for the sheet piling

was used in the lower 85 ft of the foundation material at Swift Creek Dam in Washington (Burke, 1958). This design consists of a double row of straight section sheet piling welded to 18-in wide-flange steel beams. After the driving, the spaces between the beams are excavated and the cells are filled with concrete. The pressure head loss that offered by a sheet pile wall might be considerable under low dams with relatively short seepage paths.

Using different approaches, Khosla *et al* (1936), Harr (1962), Leliavsky (1965), Terzaghi and Peck (1967), Rushton and Redshaw (1978), Griffiths and Fenton (1993, 1998), and others gave solutions for various problems of embedded vertical sheet piles and a flat bottom dam with one or more embedded vertical sheet pile(s). However limited literature is available for confined seepage through pervious soils beneath hydraulic structures with inclined sheet piles.

Applying conformal mapping, Verigin (1940) has analysed seepage flow around an inclined sheet pile embedded in a porous medium of infinite depth. Polubarinova-Kochina (1962) gave the hydrodynamical flownet for an inclined cut-off in a porous medium of infinite depth. However, Abbas (1994) used conformal transformation and gave a solution for seepage flow below a flat dam base with an inclined sheet pile at its toe on a homogeneous and isotropic soil having an infinite depth. The design charts based on his solution are clear and useful in the case of a homogeneous and isotropic foundation soil having an infinite depth. However, this assumption is usually not met depending on various in situ conditions that possess anisotropy, heterogeneity, limited soil depth, etc. Numerical methods such as finite difference or finite element can solve these problems easily and accurately on a case-by-case basis.

In the present study, a steady-state 2D finite difference model is developed to calculate the piezometric head distribution within the pervious foundation soil beneath a dam (or weir) with an inclined sheet pile at its toe. The goal is to investigate the effects of the slope angle of the sheet pile, θ , variation of horizontal to vertical permeability ratio, k_x/k_z , and sheet pile to foundation soil depths ratio, s/FD , on exit gradient and uplift pressure, (Fig. 6.2). Analytical solutions or approximations for the exit gradient and uplift pressure in the case of a flat bottom dam with an inclined toe sheet pile on anisotropic soil of limited depth are non-existent.

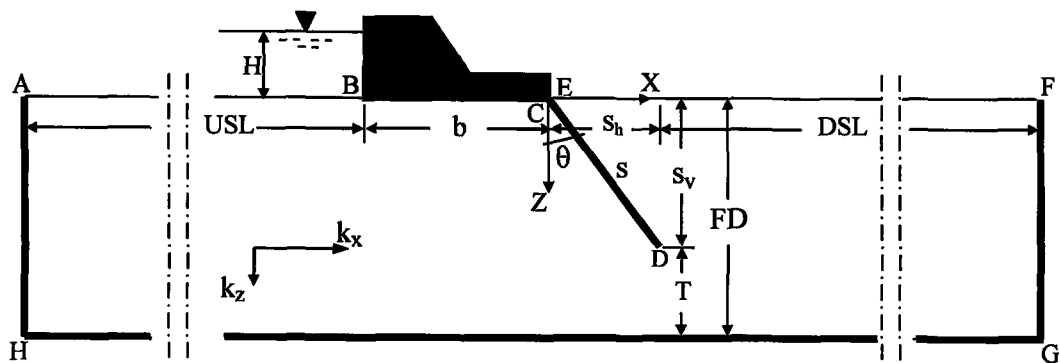


Fig. 6.2 Schematic sketch describing the physical characteristics of the seepage problem

6.2 Governing Equation

The general equation governing the steady-state two-dimensional flow (Eq. 3.13) is described in detail in Chapter 3. This equation is used to represent the flow under the dam base configuration considered in the present case.

6.3 Finite Difference Solution

The finite difference method is described in Chapter 4, yet, for clear presentation, it may be beneficial to reproduce the following short paragraphs. The finite difference method is a numerical approach for solving partial differential equations such as those governing the two-dimensional steady-state flow of a fluid through a porous medium. In the case of confined problems with simple geometry and boundary conditions, the finite difference method can easily be implemented. In this method, the problem domain is discretized with a grid of points, where the coordinates are denoted by the symbols i and j , (Fig. 6.3).

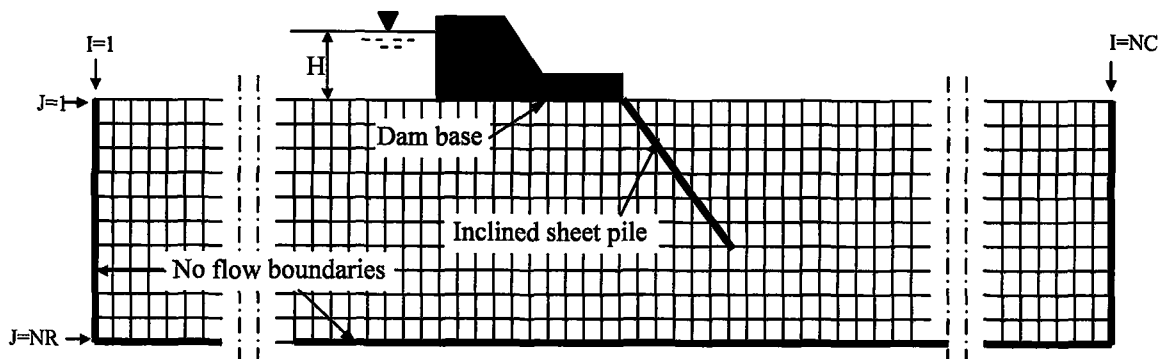


Fig. 6.3 Schematic sketch of finite difference grid

The goal is to predict the value of the unknown variable (for example, the piezometric head, $h_{i,j}$) at all nodal points in the problem domain. This can be done with non-uniform spacings, (as presented in Section 4.2). The first part in Eq. (3.13) is obtained by taking a difference between the first derivatives calculated midway between $(i+1,j)$, (i,j) and $(i-1,j)$, (i,j) as in Fig. 4.2. In general,

$$\frac{\partial}{\partial x}(k_x \frac{\partial h}{\partial x}) = \frac{2}{\Delta x_i + \Delta x_{i-1}} \left[k_{x_1} \frac{h_{i+1,j} - h_{i,j}}{\Delta x_i} - k_{x_2} \frac{h_{i,j} - h_{i-1,j}}{\Delta x_{i-1}} \right] \quad (6.1)$$

and similarly,

$$\frac{\partial}{\partial z}(k_z \frac{\partial h}{\partial z}) = \frac{2}{\Delta z_j + \Delta z_{j-1}} \left[k_{z_1} \frac{h_{i,j-1} - h_{i,j}}{\Delta z_{j-1}} - k_{z_2} \frac{h_{i,j} - h_{i,j+1}}{\Delta z_j} \right] \quad (6.2)$$

Substitution of Eqs. (6.1) and (6.2) into Eq. (3.13) and solving for the piezometric head, $h_{i,j}$, leads to,

$$h_{i,j} = \frac{Ah_{i+1,j} + Bh_{i-1,j} + Ch_{i,j-1} + Dh_{i,j+1}}{E} \quad (6.3)$$

where $h_{i,j}$ is the piezometric head at the nodal point i,j with parameters

$$A = \frac{k_{x_1}}{\Delta x_i(\Delta x_i + \Delta x_{i-1})}, B = \frac{k_{x_2}}{\Delta x_{i-1}(\Delta x_i + \Delta x_{i-1})}, C = \frac{k_{z_1}}{\Delta z_{j-1}(\Delta z_j + \Delta z_{j-1})},$$

$$D = \frac{k_{z_2}}{\Delta z_j(\Delta z_j + \Delta z_{j-1})}, \text{ with } E = A + B + C + D$$

$$k_{x_1} = \frac{(k_x)_{i+1,j} + (k_x)_{i,j}}{2}, k_{x_2} = \frac{(k_x)_{i-1,j} + (k_x)_{i,j}}{2},$$

$$k_{z_1} = \frac{(k_z)_{i,j-1} + (k_z)_{i,j}}{2}, \text{ and } k_{z_2} = \frac{(k_z)_{i,j+1} + (k_z)_{i,j}}{2}$$

Eq. (6.3) forms the heart of the computer program, which is a modified form of the main program used in the case of a depressed dam base in Chapter 5. There will be one expression as the form of Eq. (6.3) for each interior node (i,j) of the problem domain. By applying such an equation to each interior node, it is possible to obtain a

set of algebraic equations to be solved for selected boundary conditions to produce the piezometric head at the desired nodes.

6.4 Boundary Conditions

The finite difference expressions for nodes on external boundaries are described in detail in Chapter 4 (Section 4.4). An example of deriving such finite difference expressions for nodes along the bottom impervious bedrock with non-uniform spacings (as presented in Section 4.4.6) is given herein. Using Eq. (4.40), it is possible to have,

$$-k_{x1} \frac{h_{i+1,j} - h_{i,j}}{\Delta x_i} \frac{\Delta z_{j-1}}{2} - k_{z1} \frac{h_{i,j-1} - h_{i,j}}{\Delta z_{j-1}} \frac{\Delta x_i + \Delta x_{i-1}}{2} + k_{x2} \frac{h_{i,j} - h_{i-1,j}}{\Delta x_{i-1}} \frac{\Delta z_{j-1}}{2} = 0 \quad (6.4)$$

Solving for the piezometric head, $h_{i,j}$, one can obtain

$$h_{i,j} = \frac{A_1 h_{i+1,j} + B_1 h_{i-1,j} + C_1 h_{i,j-1}}{A_1 + B_1 + C_1} \quad (6.5)$$

where

$$A_1 = \frac{k_{x1} \Delta z_{j-1}}{\Delta x_i (\Delta x_i + \Delta x_{i-1})}, B_1 = \frac{k_{z1} \Delta z_{j-1}}{\Delta x_{i-1} (\Delta x_i + \Delta x_{i-1})}, \text{ and } C_1 = \frac{k_{x2}}{\Delta z_{j-1}}$$

Similar expressions can be derived for other boundary nodes. A summary of finite difference forms for interior and boundary nodes is shown in Fig. 6.4.

6.5 Applied Numerical Model

The digital computer program used in the present case is similar to that used in the case of a depressed dam base. However, it has been modified to allow for suitable representation of the boundary conditions for each slope angle, θ , of the sheet pile.

The dam section, as illustrated in Fig. 6.3, is selected to describe the procedure of the numerical solution. Different dam base configurations are considered. The first one uses a flat dam base with a vertical sheet pile at its toe ($\theta = 0^\circ$) while the others have an inclined sheet pile with different slope angles (any desired angle, $0^\circ < \theta < 90^\circ$, can be inserted in the model).

$M1=USL/X$, $M2=b/X$, $M3=N_h+1$, ($M3=1$ if $\theta=0^\circ$), $M4=DSL/X$, $M5=M1+M2$ and $M6=M5+M3+M4$ $NR=N_v+NT+1$, $NC=M6+1$, $X=Z=$ one unit distance N_v, N_h = required No. of subdivisions of vertical & horizontal components of the pile length (S). If $\theta > 0^\circ$ ($S_v=S*\cos(\theta)$, $S_h=S*\sin(\theta)$, $Z1=S_v/N_v$, $X1=S_h/N_h$, $T=FD - S_v$, $NT=0.6FD$, and $Z2=T/NT$) If $\theta = 0^\circ$ ($S_{nodes} = S/Z$)	Interior nodes ($I=2$, $NC-1$ & $J=2$, $NR-1$), except nodes below the tip and along U/S & D/S sides of pile $H(I,J)=(AH(I+1,J)+BH(I-1,J)+CH(I,J-1)+DH(I,J+1))/E$ $A=XK1/(DX(I)*(DX(I-1)+DX(I)))$ $B=XK2/(DX(I-1)*(DX(I-1)+DX(I)))$ $C=ZK1/(DZ(J-1)*(DZ(J-1)+DZ(J)))$ $D=ZK2/(DZ(J)*(DZ(J-1)+DZ(J)))$ $E=A+B+C+D$
Nodes below U/S tip of pile ($J=2$, NR), $IN=I+2$ IF ($J.GT.N_v+1$.AND.I.EQ.$M5+M3$) $H(I,J)=(A*H(IN,J)+B*H(I-1,J)+C*H(I,J-1)+D*H(I,J+1))/E$ $A=XK1/(DX(I+1)*(DX(I-1)+DX(I+1)))$ $B=XK2/(DX(I-1)*(DX(I-1)+DX(I+1)))$ $C=ZK1/(DZ(J-1)*(DZ(J-1)+DZ(J)))$ $D=ZK2/(DZ(J)*(DZ(J-1)+DZ(J)))$ $E=A+B+C+D$	Nodes below D/S tip of pile ($J=2$, NR) IF ($J.GT.N_v+1$.AND.I.EQ.$M5+M3+1$) $H(I,J)=H(I-1,J)$
Top no flow boundary ($J=1$, ($I=M1+2$, $M5$)) $H(I,J)=(A*H(I+1,J)+B*H(I-1,J)+C*H(I,J+1))/(A+B+C)$ $A=XK1*DZ(J)/(DX(I)*(DX(I-1)+DX(I)))$ $B=XK2*DZ(J)/(DX(I-1)*(DX(I-1)+DX(I)))$ $C=ZK2/DZ(J)$	Bottom no flow boundary ($J=NR$, ($I=2$, $NC-1$)) $H(I,J)=(A*H(I+1,J)+B*H(I-1,J)+C*H(I,J-1))/(A+B+C)$ $A=XK1*DZ(J-1)/(DX(I)*(DX(I-1)+DX(I)))$ $B=XK2*DZ(J-1)/(DX(I-1)*(DX(I-1)+DX(I)))$ $C=ZK1/DZ(J-1)$
Corner node between the base and the pile ($J=1$, $I=M5+1$, and $\theta = 0^\circ$) $H(I,J)=(A*H(I-1,J)+B*H(I,J+1))/(A+B)$ $A=XK2*DZ(J)/DX(I-1)$ $B=ZK2*DX(I-1)/DZ(J)$	Corner node between the base and the pile ($J=1$, $I=M5+1$, and $\theta > 0^\circ$) $H(I,J)=(A*H(I-1,J)+B*H(I,J+1))/(A+B)$ $A=XK2*DZ(J)/(DX(I-1)*(DX(I-1)+DX(I)))$ $B=ZK2/DZ(J)$
Left no flow boundary ($I=1$ and $J=2$, $NR-1$) D/S of pile if $\theta=0^\circ$ ($J=2$, $S_{nodes}+1$ and $I=M5+2$) $H(I,J)=(A*H(I+1,J)+B*H(I,J-1)+C*H(I,J+1))/(A+B+C)$ $A=XK1/DX(I)$ $B=ZK1*DX(I)/(DZ(J-1)*(DZ(J-1)+DZ(J)))$ $C=ZK2*DX(I)/(DZ(J)*(DZ(J-1)+DZ(J)))$	Right no flow boundary ($I=NC$ and $J=2$, $NR-1$) U/S of pile if $\theta=0^\circ$ ($J=2$, $S_{nodes}+1$ and $I=M5+1$) $H(I,J)=(A*H(I-1,J)+B*H(I,J-1)+C*H(I,J+1))/(A+B+C)$ $A=XK2/DX(I-1)$ $B=ZK1*DX(I-1)/(DZ(J-1)*(DZ(J-1)+DZ(J)))$ $C=ZK2*DX(I-1)/(DZ(J)*(DZ(J-1)+DZ(J)))$
Left bottom corner node ($I=1$ and $J=NR$) $H(I,J)=(A*H(I+1,J)+B*H(I,J-1))/(A+B)$ $A=XK1*DZ(J-1)/DX(I)$ $B=ZK1*DX(I)/DZ(J-1)$	Right bottom corner node ($I=NC$ and $J=NR$) $H(I,J)=(A*H(I-1,J)+B*H(I,J-1))/(A+B)$ $A=XK2*DZ(J-1)/DX(I-1)$ $B=ZK1*DX(I-1)/DZ(J-1)$
Nodes on U/S side of inclined pile ($J=2$, $NR-1$) IF ($J.LE.N_v+1$.AND.I.EQ.$M5+J$) $H(I,J)=(A*H(I-1,J)+B*H(I,J+1))/(A+B)$ $A=XK2/(DX(I-1)*(DX(I-1)+DX(I)))$ $B=ZK2/(DZ(J)*(DZ(J-1)+DZ(J)))$	Nodes on D/S side of inclined pile ($J=2$, $NR-1$) IF ($J.LE.N_v+1$.AND.I.EQ.$M5+J+1$) $H(I,J)=(A*H(I+1,J)+B*H(I,J-1))/(A+B)$ $A=XK1/(DX(I)*(DX(I-1)+DX(I)))$ $B=ZK1/(DZ(J-1)*(DZ(J-1)+DZ(J)))$

Fig. 6.4 Finite difference equations for interior and boundary nodes, a grid with non-uniform spacings and different k_x/k_z ratios

In the case of using a thin sheet pile, as in this study, extra nodes have to be inserted at the location of the sheet pile. Those additional nodes are required to have different piezometric heads on the upstream and downstream sides of the sheet pile. Beneath the sheet pile the nodes are set to have the same piezometric head. The dam rests on a pervious homogeneous and anisotropic soil, which extends to impermeable boundaries forward and backward from the dam heel and toe, respectively. Its depth is finite and consists of sand with specific permeability ratios.

As discussed in Section 1.3, the permeability ratios are taken as $k_x/k_z = 1, 5, 10$, and 20. For the homogeneous and isotropic case, the permeability value at all nodal points is assumed to be 10^{-5} m/s. This value is typical for very fine sands and silt in which piping phenomenon is likely to occur, (Griffiths and Fenton, 1998). The principal directions of anisotropy in the study area are considered to be in the directions of the x and z coordinates. The upstream water height is a constant at 5 m, whereas the downstream level is zero. The flow is steady, two-dimensional in the x - z plane, and Eq. (3.13) is applicable. Although the flow field extends an infinite distance upstream and downstream, it is assumed here that it is bounded by the artificial limits AH and FG, which are assumed to be impervious boundaries. The required input data are the values of all the parameters that describe the geometry and properties of the study area. For example, upstream and downstream lengths (USL and DSL), dam length (b), foundation soil depth (FD), water height at upstream and downstream sides (USH and DSH), maximum number of iterations ($MAXIT$), relaxation factor (ω), error tolerance (TOL), horizontal and vertical unit distances and hydraulic conductivities (x , z and k_x , k_z), sheet pile length (s), and desired number of subdivisions of the horizontal and vertical components of s in order to have a reasonable grid distance along these lengths (N_h and N_v).

The output is in the form of tables including the piezometric head values at the pre-specified number of nodes in the problem solution domain. The exit gradients at all nodes along the downstream side are also given in these tables. The exit gradient, i_e , is taken to be the rate of head change at the exit points closest to the soil surface along the downstream side and computed using a two-point numerical differentiation as shown in Fig. 6.5.

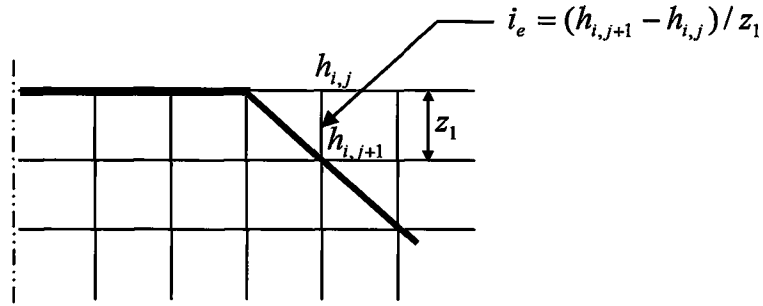


Fig. 6.5 Schematic sketch showing the nodal locations used in the numerical calculation of the exit gradient, i_e

6.6 Results and Discussion

6.6.1 Comparison of the Model Results with a Previous Study

In order to check the accuracy of the developed model results for the considered dam base configuration, the computation of the exit gradient and uplift pressure in a homogeneous and isotropic medium having an infinite depth is selected and compared with the available analytical solution obtained by Abbas (1994) using conformal mapping. Worthy to mention that a close agreement between the two solutions is reached for values of $FD/b > 2$ in the numerical solution. For the sake of simplicity and convenience of presentation, only two curves for the exit gradients, corresponding to two sheet pile inclinations are presented at a time. That is for $\theta = 0^\circ$ and 30° when $b/s = 1$ (Table 6.1), and for $\theta = 0^\circ$ and 45° when $b/s = 5$ (Table 6.2). Similarly, four curves for the uplift pressure are selected and compared. That is for $b/s = 1, 3, 5$, and 10 (Table 6.3). It should be noted that the pressure is computed at the inner corner of the boundary node between the sheet pile and the dam base, expressed as the ratio of the piezometric head at that node to the total head difference between the upstream and downstream water levels.

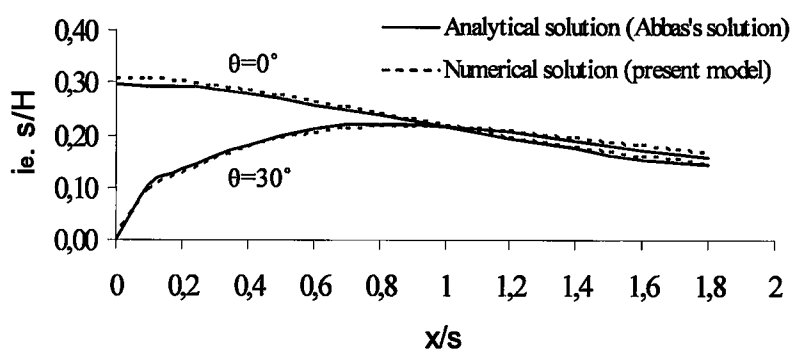
The results using the above-mentioned parameters, can best be presented in non-dimensional graphical forms. Therefore, the comparison is done for the exit gradient values along the downstream side (Fig. 6.6), and the uplift pressure at the inner corner of the boundary node between the dam base and the toe sheet pile, expressed as a percent of the total head difference (Fig. 6.7). The exit gradient and the uplift pressure values predicted by the numerical model are in remarkable agreement with the values obtained by the analytical solution of Abbas (1994).

Table 6.1 Analytical and numerical results of exit gradients at the downstream bed for a dam base with a toe sheet pile (where $k_x/k_z = 1$ and $b/s = 1$)

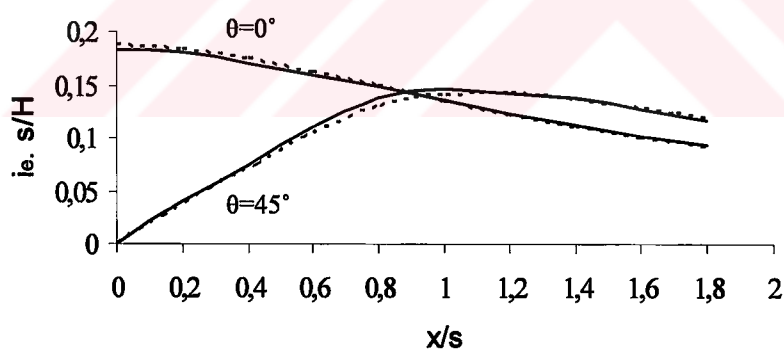
Conformal analysis results				Computer results			
$\theta = 0^\circ, b/s=1$		$\theta = 30^\circ, b/s=1$		$\theta = 0^\circ, b/s=1$		$\theta = 30^\circ, b/s=1$	
x/s	$i_e.s/H$	x/s	$i_e.s/H$	x/s	$i_e.s/H$	x/s	$i_e.s/H$
0	0.294	0	0	0	0.3056	0	0
0.2	0.290	0.2	0.137	0.1	0.3041	0.1	0.0956
0.4	0.278	0.4	0.182	0.2	0.2997	0.2	0.1275
0.6	0.258	0.6	0.21	0.3	0.2929	0.3	0.1534
0.8	0.236	0.8	0.22	0.4	0.284	0.4	0.1745
1	0.215	1	0.215	0.5	0.2737	0.5	0.1913
1.2	0.193	1.2	0.205	0.6	0.2625	0.6	0.2037
1.4	0.175	1.4	0.191	0.7	0.2507	0.7	0.2119
1.6	0.155	1.6	0.173	0.8	0.2389	0.8	0.2162
1.8	0.143	1.8	0.159	0.9	0.2272	0.9	0.2171
				1	0.2159	1	0.2153
				1.1	0.205	1.1	0.2114
				1.2	0.1947	1.2	0.2061
				1.3	0.1849	1.3	0.1998
				1.4	0.1757	1.4	0.1929
				1.5	0.1671	1.5	0.1857
				1.6	0.159	1.6	0.1785
				1.7	0.1515	1.7	0.1714
				1.8	0.1444	1.8	0.1644

Table 6.2 Analytical and numerical results of exit gradients at the downstream bed for a dam base with a toe sheet pile (where $k_x/k_z = 1$ and $b/s = 5$)

Conformal analysis results				Computer results			
$\theta = 0^\circ, b/s=5$		$\theta = 45^\circ, b/s=5$		$\theta = 0^\circ, b/s=5$		$\theta = 45^\circ, b/s=5$	
x/s	$i_e.s/H$	x/s	$i_e.s/H$	x/s	$i_e.s/H$	x/s	$i_e.s/H$
0	0.183	0	0	0	0.1869	0	0
0.2	0.18	0.2	0.04	0.2	0.1834	0.1414	0.0251
0.4	0.171	0.4	0.075	0.4	0.1744	0.2828	0.0501
0.6	0.16	0.6	0.115	0.6	0.162	0.4242	0.0748
0.8	0.148	0.8	0.14	0.8	0.1484	0.5656	0.0982
1	0.136	1	0.148	1	0.1348	0.707	0.1183
1.2	0.123	1.2	0.143	1.2	0.122	0.807	0.1291
1.4	0.113	1.4	0.138	1.4	0.1104	0.907	0.1367
1.6	0.102	1.6	0.128	1.6	0.1	1.007	0.1411
1.8	0.094	1.8	0.118	1.8	0.0906	1.107	0.1426
						1.207	0.142
						1.307	0.1396
						1.407	0.1362
						1.507	0.1321
						1.607	0.1277
						1.707	0.1231
						1.807	0.1185



(a)



(b)

Fig. 6.6 Comparison of numerical and analytical results of $i_e. s/H$ versus x/s for different values of θ (where $k_x/k_z=1$): (a) for $b/s=1$ and (b) for $b/s=5$

Table 6.3 Uplift pressures at inner corner node between the toe sheet pile and the dam base, expressed as a percent of the total head difference

θ	h_c/H from Conformal analysis				h_c/H from Computer results			
	$b/s=1$	$b/s=3$	$b/s=5$	$b/s=10$	$b/s=1$	$b/s=3$	$b/s=5$	$b/s=10$
0°	0.74	0.494	0.39	0.285	0.77	0.52	0.42	0.29
30°	0.67	0.46	0.374	0.27	0.71	0.48	0.38	0.27
45°	0.64	0.44	0.354	0.25	0.66	0.46	0.36	0.25

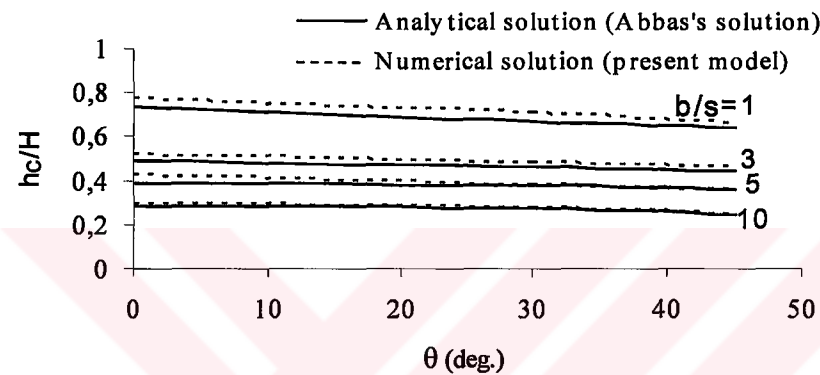


Fig. 6.7 Comparison of numerical and analytical results of h_c/H versus θ for different values of b/s and for $k_x/k_z=1$

The comparison, for all the combinations of θ and b/s , revealed a maximum difference in the exit gradients of about 9.6%, and in the uplift pressure of about 7.9%. These differences can further be reduced by choosing a smaller error tolerance between the head values generated at two successive iterations (in the present model $TOL = 0.0001$), by testing for the optimum FD that simulates the infinite foundation soil depth, and by reducing the size of the mesh-grid.

The grid size and boundary mesh are optimized to provide correct estimation of the exit gradients and uplift pressures as well as to reduce the model computation time. The selected grid size corresponds to one unit length in the horizontal and vertical directions for the case where $\theta = 0^\circ$. For other θ values, the grid size in the horizontal direction, along the USL , b , and DSL , corresponds to one unit length. However, it is $\Delta x = (s \sin \theta) / N_h$, for the nodes along the length s_h . The vertical grid size is $\Delta z = (s \cos \theta) / N_v$ and $\Delta z = T / NT$ along the lengths s_v and T , respectively, ($T = FD - s_v$ and NT is an arbitrary length calculated from FD to give a

reasonable grid size). Since, the foundation soil depth is assumed to have different values, NT is calculated as a percent of each of those values for each case considered. The relation of $NT = 0.6 FD$ is found to give reasonable vertical grid distances. The boundaries of the mesh are set at distances using various upstream and downstream lengths. These lengths are calculated in the model depending on the dam base length, (i.e. $USL = N_1 b$ and $DSL = N_2 b$, where N_1 and N_2 are arbitrary integers representing how long are these lengths compared to the base length). The upstream and downstream lengths are optimized so that the solution would not vary significantly by choosing a smaller length. For each dam base configuration, the computation starts with $N_1 = N_2 = 1$. Other runs are executed with $N_1 = N_2 = 2, 3, 4$, etc. until the previous condition is satisfied. From the model results, it is seen that the effect of the mesh boundaries is minimized, if these boundaries are set at a distance equal to or greater than four times the base width. An example for the model computations showing the effect of the upstream and downstream lengths on exit gradients is given hereafter. This example is selected and presented for $\theta = 30^\circ$, $b/s = 1$, $k_x/k_z = 1$, and $FD > 2b$ ($FD = 50$ m, and $b = 10$ m). The results from the computer runs with different $N_1 = N_2$ values are collected and tabulated as shown in Table 6.4. In order to summarize these data, they are drawn into a non-dimensional graph, altogether with the analytical solution of Abbas (1994) for the same parameters (Fig. 6.8). The effect of the upstream and downstream lengths can be found by comparing the model results for a particular choice of $N_1 = N_2$ with that of Abbas (1994). From Fig. 6.8, it is clear that the computer solution converges to the analytical one as these lengths are set at distances greater than four times the dam base width as mentioned before. It should be noted that the computer results of the exit gradients along the downstream side presented in Table 6.4 are only quoted for a distance $x/s = 1.8$, in order to be able to do the comparison with Abbas (1994) solution. The same method of the previous computation is followed throughout all the other model runs for different values of the above parameters.

Alternative conditions on the left and right boundaries (AH and FG) are applied to investigate the effect of the boundary type on the model results. Therefore, instead of representing them as no flow boundaries, an alternative approach is adopted by considering a fixed piezometric head at all nodes along those boundaries, i.e.

$h_{1,j} = USH$ and $h_{NC,j} = DSH$, ($j = 2, NR$). Execution of the computer program with each of these boundary types, keeping all the other input data unchanged, showed that the model results are exactly similar, provided that these boundaries are set at the suitable locations that their effects are minimized.

Table 6.4 Exit gradients along the downstream side for different choices of $N1 = N2$

x/s	i _e , s/H			
	N1=N2=1	N1=N2=2	N1=N2=4	N1=N2=6
0	0	0	0	0
0.1	0.1283	0.10970	0.10040	0.09740
0.2	0.1711	0.1463	0.1339	0.1299
0.3	0.2061	0.1761	0.1611	0.1563
0.4	0.2352	0.2007	0.1834	0.1778
0.5	0.2593	0.2205	0.2011	0.1949
0.6	0.2786	0.2358	0.2145	0.2076
0.7	0.2935	0.2468	0.2235	0.2161
0.8	0.3044	0.2538	0.2286	0.2207
0.9	0.3119	0.2573	0.2302	0.2218
1	0.3169	0.2581	0.2291	0.2202
1.1	0.3198	0.2569	0.2259	0.2165
1.2	0.3215	0.2543	0.2213	0.2114
1.3	0.3224	0.2507	0.2156	0.2053
1.4	0.3227	0.2467	0.2094	0.1985
1.5	0.3229	0.2425	0.2029	0.1916
1.6	-	0.2383	0.1964	0.1845
1.7	-	0.2344	0.1899	0.1775
1.8	-	0.2307	0.1837	0.1708

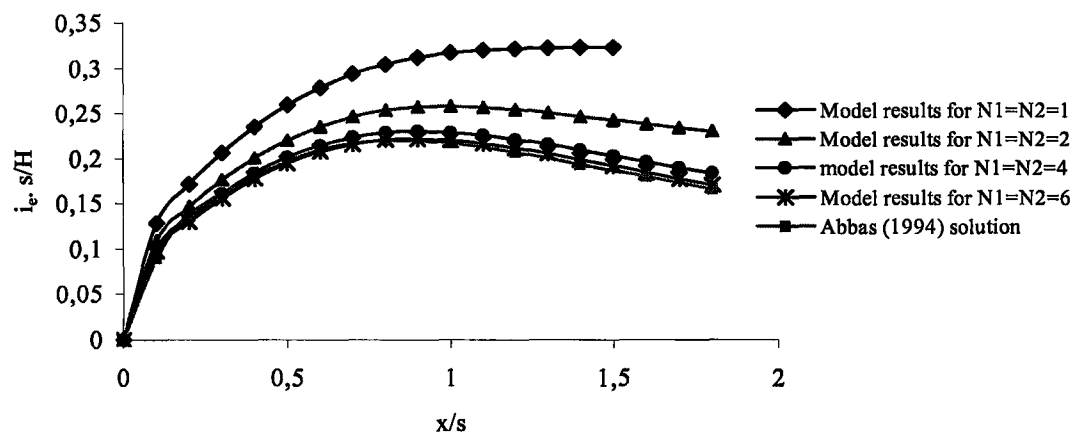


Fig. 6.8 Effect of upstream and downstream lengths on exit gradient values

6.6.2 Effect of θ on Exit Gradient and Uplift Pressure

From the computer results of the piezometric head distribution and the explanations of Abbas (1994), it is observed that the exit gradient at point E (in Fig. 6.2), except for $\theta = 0^\circ$, is zero. Beyond point E the exit gradient increases to a maximum value and then decreases with distance away from the dam toe, (see for example Column 6, Table 6.5). The results in that column show that $i_e.s/H$ is zero at $x/s = 0$, which represents the exit gradient exactly at the dam toe, the exit gradient increases to 0.229 at $x/s = 0.9$, then decreases with distance along the downstream and reaches a value of 0.182 at $x/s = 1.8$. However, For the case of using a vertical sheet pile ($\theta = 0^\circ$), the exit gradient have a maximum value of 0.3085 at the immediate dam toe and then decreases continuously with distance away from the toe. It reaches a value of 0.1524 at $x/s = 1.8$ (Column 2 in the same Table). For the case when $\theta = 0^\circ$, the horizontal grid distance is selected to be $\Delta x = 1$ m everywhere within the study area. Therefore, for a sheet pile of 10 m in length, the grid points along the downstream side are located every $x/s = 0.1$ (Column 1, Table 6.5).

Table 6.5 The computer results of $i_e.s/H$ for different θ values where $k_x/k_z=1$

$\theta = 0^\circ$		$\theta = 20^\circ$		$\theta = 30^\circ$		$\theta = 45^\circ$		$\theta = 60^\circ$	
x/s (1)	$i_e.s/H$ (2)	x/s (3)	$i_e.s/H$ (4)	x/s (5)	$i_e.s/H$ (6)	x/s (7)	$i_e.s/H$ (8)	x/s (9)	$i_e.s/H$ (10)
0	0.3085	0	0	0	0	0	0	0	0
0.1	0.307	0.0684	0.1452	0.1	0.09740	0.1414	0.0400	0.1732	0.0086
0.2	0.3027	0.1368	0.1644	0.2	0.1299	0.2828	0.0800	0.3464	0.0346
0.3	0.296	0.2052	0.18	0.3	0.1563	0.4242	0.1195	0.5196	0.0775
0.4	0.2874	0.2736	0.1926	0.4	0.1778	0.5656	0.1567	0.6928	0.1353
0.5	0.2773	0.342	0.2027	0.5	0.1949	0.707	0.1886	0.866	0.1979
0.6	0.2663	0.442	0.2137	0.6	0.2076	0.807	0.2057	0.966	0.225
0.7	0.2549	0.542	0.2207	0.7	0.2161	0.907	0.2175	1.066	0.2421
0.8	0.2434	0.642	0.2243	0.8	0.2207	1.007	0.2241	1.166	0.2491
0.9	0.232	0.742	0.2251	0.9	0.2218	1.107	0.226	1.266	0.2482
1	0.2211	0.842	0.2236	1	0.2202	1.207	0.2244	1.366	0.2418
1.1	0.2105	0.942	0.2204	1.1	0.2165	1.307	0.2202	1.466	0.2324
1.2	0.2006	1.042	0.2159	1.2	0.2114	1.407	0.2143	1.566	0.2214
1.3	0.1912	1.142	0.2106	1.3	0.2053	1.507	0.2074	1.666	0.21
1.4	0.1823	1.242	0.2049	1.4	0.1985	1.607	0.1999	1.766	0.1987
1.5	0.1741	1.342	0.1989	1.5	0.1916	1.707	0.1924	1.866	0.1879
1.6	0.1663	1.442	0.1929	1.6	0.1845	1.807	0.185		
1.7	0.1591	1.542	0.1869	1.7	0.1775				
1.8	0.1524	1.642	0.1812	1.8	0.1708				
		1.742	0.1756						
		1.842	0.1704						

For the cases when the sheet pile is inclined towards the downstream side, $\Delta x = 1$ m everywhere except within the limits of the horizontal component, s_h , of the sheet pile. The grid points along the length s_h are located at distances depending on the required number, N_h , of subdivisions of that length. If $\theta = 20^\circ$, and $s = 10$ m, then $s_h = s \sin \theta = 3.42$ m. If $N_h = 5$, $\Delta x = 3.42/5 = 0.684$ m, and the grid points will be located every $x/s = 0.0684$ upto the end of the horizontal component length, the grid points are then located every $x/s = 0.1$ (Column 3, Table 6.5). The values in Columns 5, 7, and 9 are determined in a similar manner. The collected results are drawn into a graphical form as shown in Fig. 6.9. This figure is selected and drawn for $b/s = 1$. The purpose is to show the relative effectiveness of the various sloping angles in decreasing such an exit gradient. Moreover, it is observed that the exit gradient, after a distance $x/s \approx 1.0$ beyond point E, decreases with increasing the sheet pile inclination. It is also seen that when the exit gradient reaches its maximum, it starts to decrease for angles up to approximately 30° and increase for higher angles.

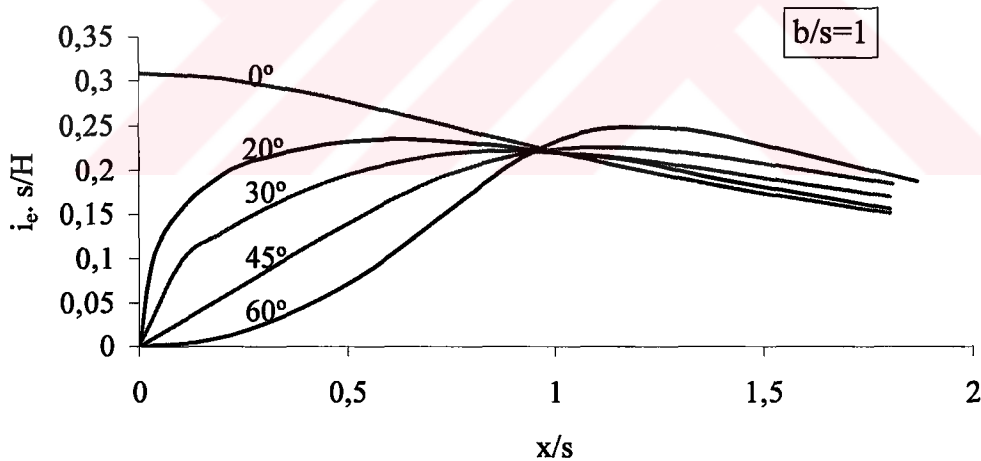


Fig. 6.9 Variation of exit gradient with x/s for different values of θ where $k_x/k_z = 1$ and $b/s = 1$

From the same figure, it is seen that the reduction in exit gradient increases as θ increases. The result is an increase in the safety factor against piping phenomenon. Therefore, it can be concluded that using an inclined toe sheet pile, with θ less than approximately 30° towards the downstream side, is beneficial in increasing such a safety factor. On the other hand, the danger from undermining will be shifted further downstream from the toe of the structure.

The same characteristic trends are observed when the model is run with other b/s values such as 3, 5, and 10 (Fig. 6.10).

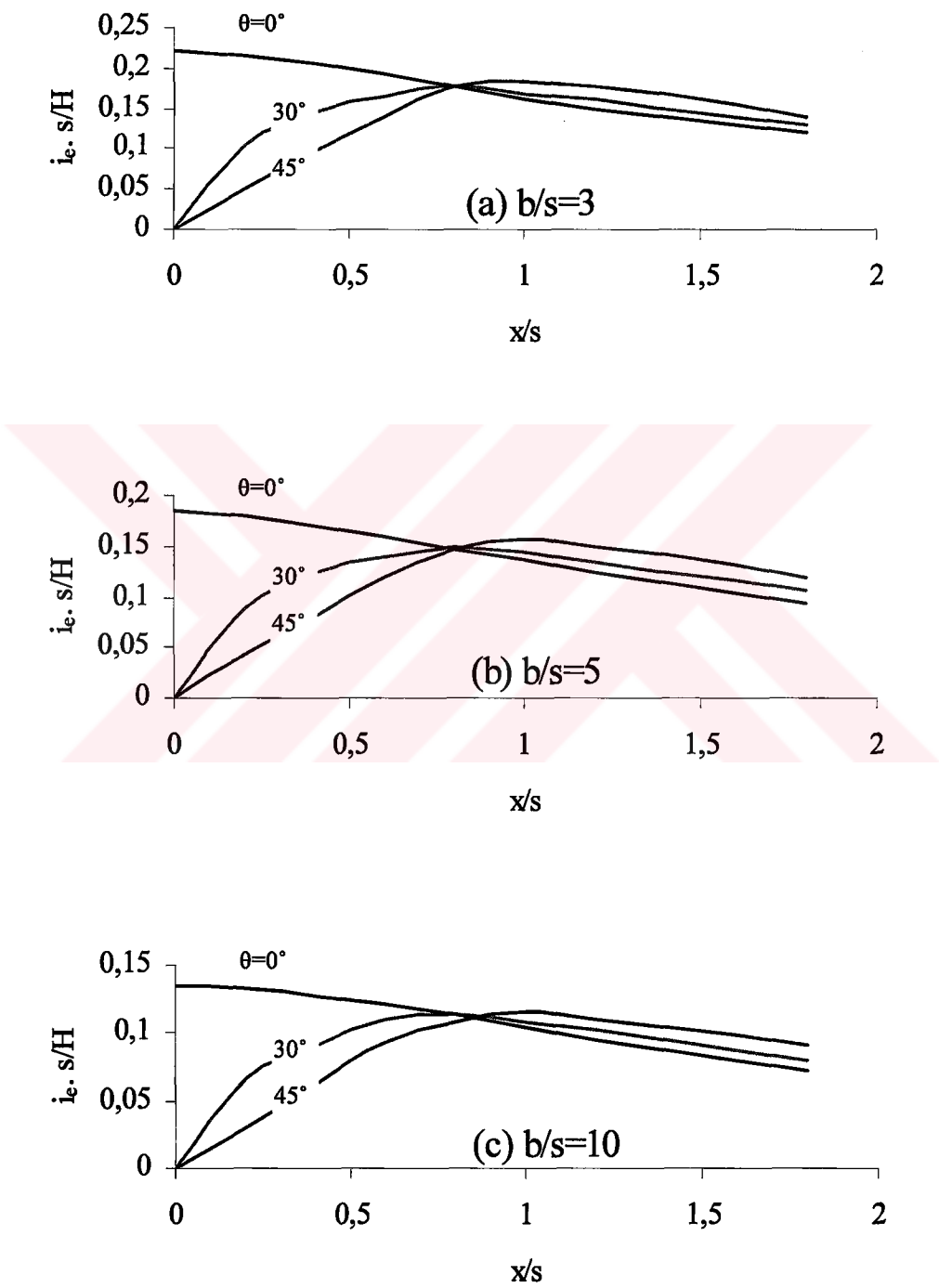


Fig. 6.10 Variation of exit gradient with x/s for different θ values where $k_x/k_z = 1$
(a) for $b/s = 3$, (b) for $b/s = 5$, and (c) for $b/s = 10$

In Fig. 6.10, the relation between $i_e.s/H$ and x/s is drawn for only three angles of inclination for the sheet pile as $\theta = 0^\circ, 30^\circ$, and 45° . The exit gradient, as is expected, decreases as b/s increases. Still, the main conclusions derived from the case with $b/s = 1$ are also applicable to the cases with other values.

Placing the sheet pile at the dam heel is not recommended under any angle of inclination. The same conclusion is also valid for the case of toe sheeting inclined towards the upstream side. Such a solution produces a singularity at the end of the dam, with a mathematically infinite velocity. Thus, the exit gradient curve will be tangent to the vertical at the immediate toe indicating an infinite gradient, and unstable soil condition in close proximity to the toe. An example to such a problem is selected and presented in Fig. 6.11.

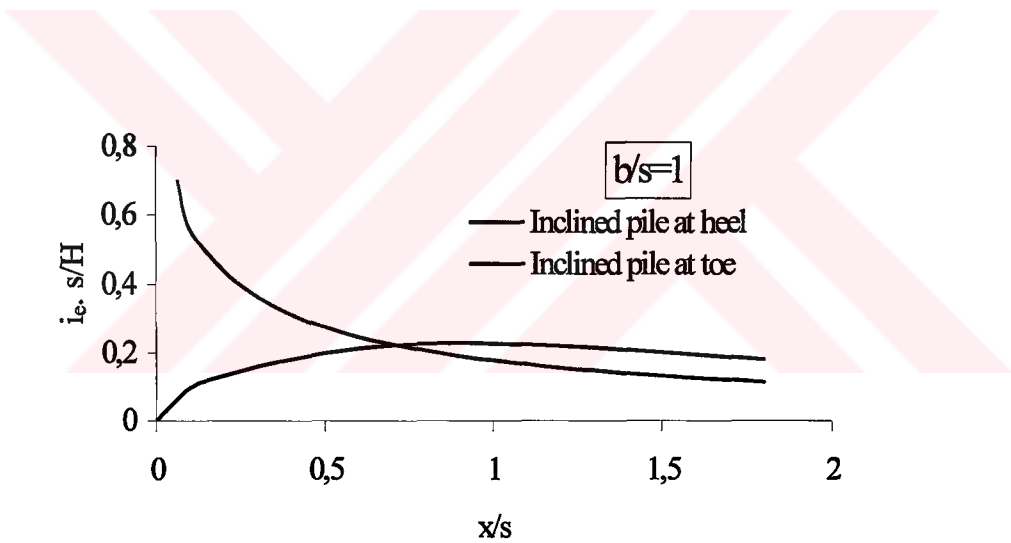


Fig. 6.11 Variation of exit gradient with x/s for two locations of the sheet pile where $\theta = 30^\circ$, $k_x/k_z = 1$, and $b/s = 1$

Figure 6.11 shows a comparison of the exit gradients for two locations of the inclined sheet pile placed at the dam toe or heel, respectively. The sheet pile inclined towards the downstream and upstream sides, when it is placed at the toe or heel, respectively. The figure proves the efficiency of the toe sheeting in reducing the risk of piping near the toe. Because of the excessive escape gradient at the immediate toe as well as for obvious practical reasons, such as rain erosion, the base of a dam on soft material should never be flush with the foundation surface at the toe. Therefore, in order to

render the structure safe, the base level of the dam floor must always be depressed or provided with sheet piling at its toe.

From Fig. 6.7 it is seen that the uplift pressure at the key point C, expressed as a percent of the total head, decreases as the sheet pile inclination increases. However, this reduction in pressure is small. The maximum reduction occurs in the case where $b/s = 1$ and it is about 14 % for θ changing from 0° to 45° .

6.3.3 Effect of k_x/k_z on Exit Gradient and Uplift Pressure

6.3.3.1 Effect of k_x/k_z on Exit Gradient

The effect of anisotropy on the exit gradient is clearly indicated in Fig. 6.12. Even though the model is capable of calculating the exit gradient for any angle of inclination for the sheet pile and the desired ratio of dam base width to sheet pile depth, the results herein are presented only for three sheet pile inclinations as $\theta = 0^\circ$, 30° , and 45° , and for base width to sheet pile depth as $b/s = 1, 3$, and 5 , respectively. The values of the anisotropy ratio considered at this stage are taken as $k_x/k_z = 1, 5, 10$ and 20 respectively. In order to show the effect of the anisotropy ratio on exit gradients, the model results are collected from the different studied cases using the above-mentioned parameters, and summarized as presented in Tables 6.6, 6.7 and 6.8, respectively. These tables are used in the preparation of Figs. 6.12a, 6.12b and 6.12c that show the relation between the exit gradients on the ordinate expressed as $i_e \cdot s/h$ and the distance along the downstream side on the abscissa expressed as x/s for different sheet pile inclination angles. The increase in the anisotropy ratio causes an increase in the exit gradient values. Differences are observed in the exit gradient values when the results of the proposed anisotropy ratios in this study are compared with the available analytical solution obtained by conformal mapping, Abbas (1994), for the case of a homogeneous and isotropic soil. These differences at the toe are, for the case where the sheet pile inclination angle $\theta = 0^\circ$, comparatively small as shown in Fig. 6.12a, whereas they are almost doubled at a distance equals $x/s = 1.8$ for an anisotropy ratio of $k_x/k_z = 10$. It is to be noted that the differences between the exit gradients calculated for different permeability ratios reach a minimum value as the anisotropy ratio increases.

Table 6.6 Summary of the exit gradients along the downstream side for different k_x/k_z and b/s values where $\theta = 0^\circ$

x/s	$i_e.s/H$ for $\theta = 0^\circ$ and $b/s = 1$					$i_e.s/H$ for $\theta = 0^\circ$ and $b/s = 3$					$i_e.s/H$ for $\theta = 0^\circ$ and $b/s = 5$				
	k_x/k_z					k_x/k_z					k_x/k_z				
	1	5	10	20		1	5	10	20		1	5	10	20	
0	0.3056	0.3154	0.3238	0.339		0.2339	0.279	0.2985	0.3205		.1870	.2401	.2617	.2824	
0.1	0.3041	0.3151	0.3237	0.3389		0.2328	0.2787	0.2983	0.3204		.1861	.2399	.2616	.2824	
0.2	0.2997	0.3143	0.3233	0.3387		0.2296	0.278	0.2979	0.3202		.1836	.2392	.2612	.2822	
0.3	0.2929	0.3129	0.3226	0.3384		0.2247	0.2768	0.2973	0.3199		.1797	.2382	.2607	.2819	
0.4	0.284	0.311	0.3217	0.338		0.2183	0.2751	0.2964	0.3195		.1746	.2368	.2599	.2815	
0.5	0.2737	0.3086	0.3206	0.3375		0.2108	0.273	0.2953	0.3189		.1687	.2350	.2589	.2810	
0.6	0.2625	0.3058	0.3192	0.3368		0.2026	0.2705	0.294	0.3183		.1622	.2328	.2577	.2803	
0.7	0.2507	0.3025	0.3176	0.3361		0.194	0.2676	0.2924	0.3175		.1555	.2303	.2563	.2796	
0.8	0.2389	0.2989	0.3157	0.3352		0.1854	0.2644	0.2907	0.3166		.1486	.2276	.2547	.2787	
0.9	0.2272	0.295	0.3137	0.3342		0.1768	0.2609	0.2887	0.3156		.1418	.2246	.2529	.2777	
1	0.2159	0.2908	0.3115	0.3332		0.1685	0.2571	0.2866	0.3144		.1352	.2214	.2510	.2767	
1.1	0.205	0.2864	0.3091	0.332		0.1604	0.2532	0.2843	0.3132		.1288	.2180	.2489	.2755	
1.2	0.1947	0.2817	0.3066	0.3307		0.1528	0.2491	0.2818	0.3119		.1227	.2144	.2467	.2743	
1.3	0.1849	0.277	0.3039	0.3294		0.1455	0.2448	0.2792	0.3105		.1169	.2107	.2443	.2729	
1.4	0.1757	0.2721	0.3011	0.328		0.1387	0.2404	0.2765	0.309		.1115	.2070	.2419	.2715	
1.5	0.1671	0.2672	0.2982	0.3265		0.1322	0.236	0.2737	0.3074		.1063	.2032	.2393	.2700	
1.6	0.159	0.2622	0.2953	0.3249		0.1261	0.2315	0.2708	0.3058		.1015	.1993	.2367	.2684	
1.7	0.1515	0.2572	0.2922	0.3233		0.1204	0.227	0.2678	0.3041		.0969	.1954	.2340	.2668	
1.8	0.1444	0.2522	0.289	0.3216		0.1151	0.2225	0.2647	0.3023		.0926	.1915	.2312	.2650	

Moreover, the difference in exit gradient starts increasing slightly with increasing the ratio of the base width to the sheet pile depth.

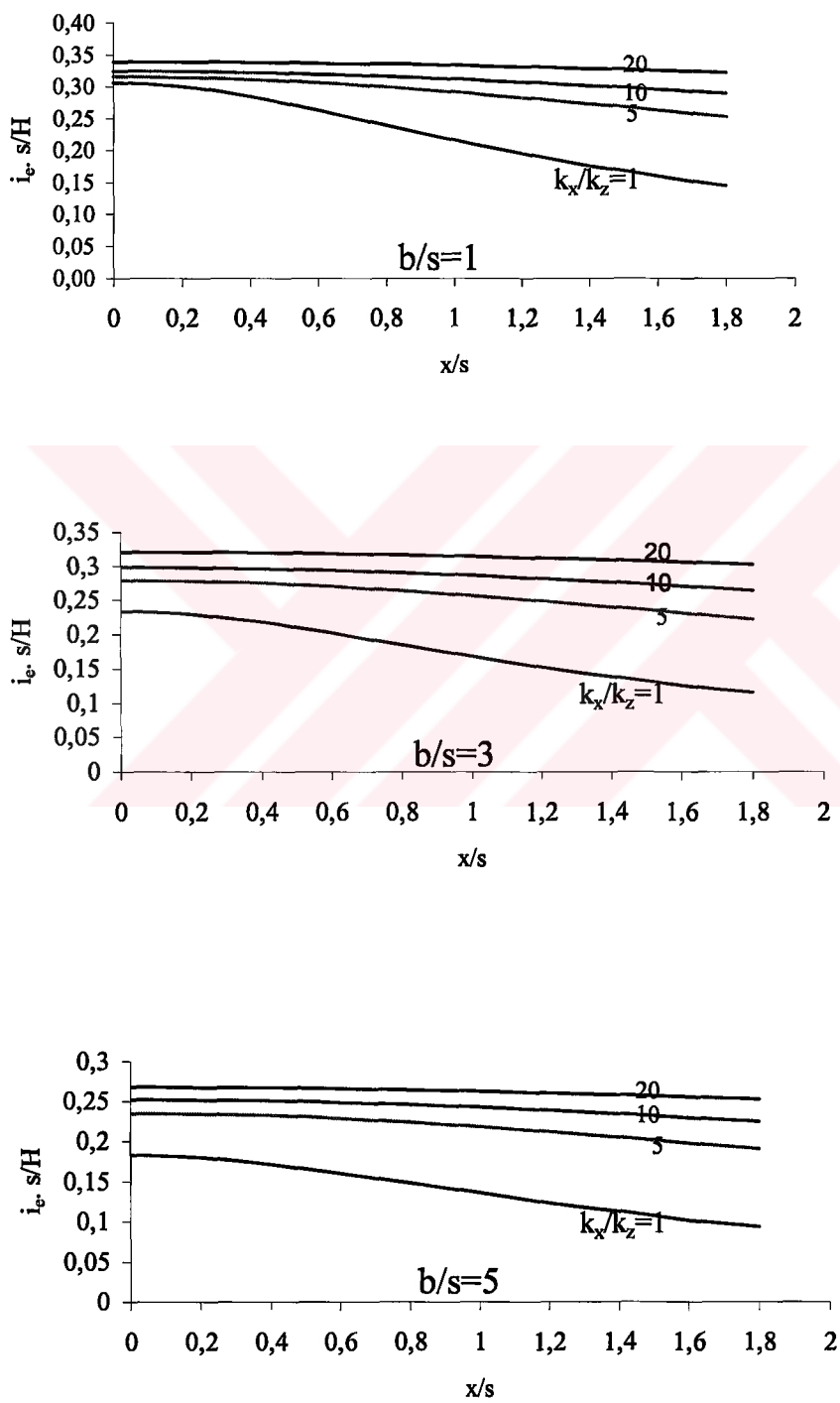


Fig. 6.12a Effect of anisotropy ratio (k_x/k_z) on exit gradient for $\theta = 0^\circ$ and for different values of b/s

Table 6.7 Summary of the exit gradients along the downstream side for different k_x/k_z and b/s values where $\theta = 30^\circ$

$i_e.s/H$ for $\theta = 30^\circ$ and $b/s = 1$													$i_e.s/H$ for $\theta = 30^\circ$ and $b/s = 5$				
x/s	k_x/k_z				k_x/k_z				k_x/k_z				k_x/k_z				
	1	5	10	20	1	5	10	20	1	5	10	20	1	5	10	20	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1	0.0956	0.2082	0.2521	0.2929	0.0699	0.1801	0.2286	0.2749	0.0597	0.1572	0.2052	0.2538	0.0597	0.1572	0.2052	0.2538	
0.2	0.1275	0.2221	0.2605	0.2978	0.0931	0.1921	0.2362	0.2795	0.0796	0.1677	0.212	0.258	0.0796	0.1677	0.212	0.258	
0.3	0.1534	0.2336	0.2676	0.3021	0.1121	0.202	0.2427	0.2835	0.0959	0.1764	0.2178	0.2618	0.0959	0.1764	0.2178	0.2618	
0.4	0.1745	0.243	0.2738	0.3059	0.1276	0.2102	0.2483	0.2871	0.1092	0.1835	0.2228	0.265	0.1092	0.1835	0.2228	0.265	
0.5	0.1913	0.2507	0.2789	0.3092	0.1398	0.2169	0.253	0.2902	0.1198	0.1893	0.227	0.2679	0.1198	0.1893	0.227	0.2679	
0.6	0.2037	0.2569	0.2832	0.3121	0.149	0.2222	0.2568	0.2928	0.1279	0.194	0.2305	0.2703	0.1279	0.194	0.2305	0.2703	
0.7	0.2119	0.2617	0.2868	0.3146	0.1551	0.2264	0.26	0.2951	0.1334	0.1977	0.2334	0.2724	0.1334	0.1977	0.2334	0.2724	
0.8	0.2162	0.2652	0.2896	0.3167	0.1583	0.2294	0.2626	0.2971	0.1366	0.2004	0.2356	0.2742	0.1366	0.2004	0.2356	0.2742	
0.9	0.2171	0.2676	0.2918	0.3185	0.1591	0.2315	0.2645	0.2987	0.1378	0.2022	0.2374	0.2757	0.1378	0.2022	0.2374	0.2757	
1	0.2153	0.269	0.2934	0.3199	0.1579	0.2327	0.2659	0.3	0.1373	0.2034	0.2387	0.2768	0.1373	0.2034	0.2387	0.2768	
1.1	0.2114	0.2694	0.2944	0.3211	0.1551	0.2332	0.2668	0.301	0.1356	0.2038	0.2395	0.2777	0.1356	0.2038	0.2395	0.2777	
1.2	0.2061	0.2691	0.295	0.3219	0.1513	0.2329	0.2672	0.3017	0.133	0.2036	0.2399	0.2784	0.133	0.2036	0.2399	0.2784	
1.3	0.1998	0.268	0.2951	0.3225	0.1467	0.232	0.2673	0.3021	0.1297	0.2029	0.2399	0.2788	0.1297	0.2029	0.2399	0.2788	
1.4	0.1929	0.2663	0.2948	0.3229	0.1417	0.2306	0.2669	0.3024	0.126	0.2017	0.2395	0.2789	0.126	0.2017	0.2395	0.2789	
1.5	0.1857	0.2641	0.2941	0.323	0.1364	0.2286	0.2662	0.3024	0.1221	0.2001	0.2389	0.2789	0.1221	0.2001	0.2389	0.2789	
1.6	0.1785	0.2614	0.293	0.3229	0.1311	0.2263	0.2651	0.3022	0.1181	0.1981	0.238	0.2787	0.1181	0.1981	0.238	0.2787	
1.7	0.1714	0.2584	0.2917	0.3226	0.1258	0.2237	0.2638	0.3018	0.1141	0.1959	0.2368	0.2783	0.1141	0.1959	0.2368	0.2783	
1.8	0.1644	0.255	0.29	0.3221	0.1207	0.2207	0.2622	0.3012	0.1102	0.1934	0.2353	0.2777	0.1102	0.1934	0.2353	0.2777	

For other θ values, the exit gradient at the immediate toe is zero and increases rapidly, almost doubled, at a distance $x/s = 0.1$ downstream from the toe for anisotropy ratio of $k_x/k_z = 10$ (Figs. 6.12b and 6.12c).

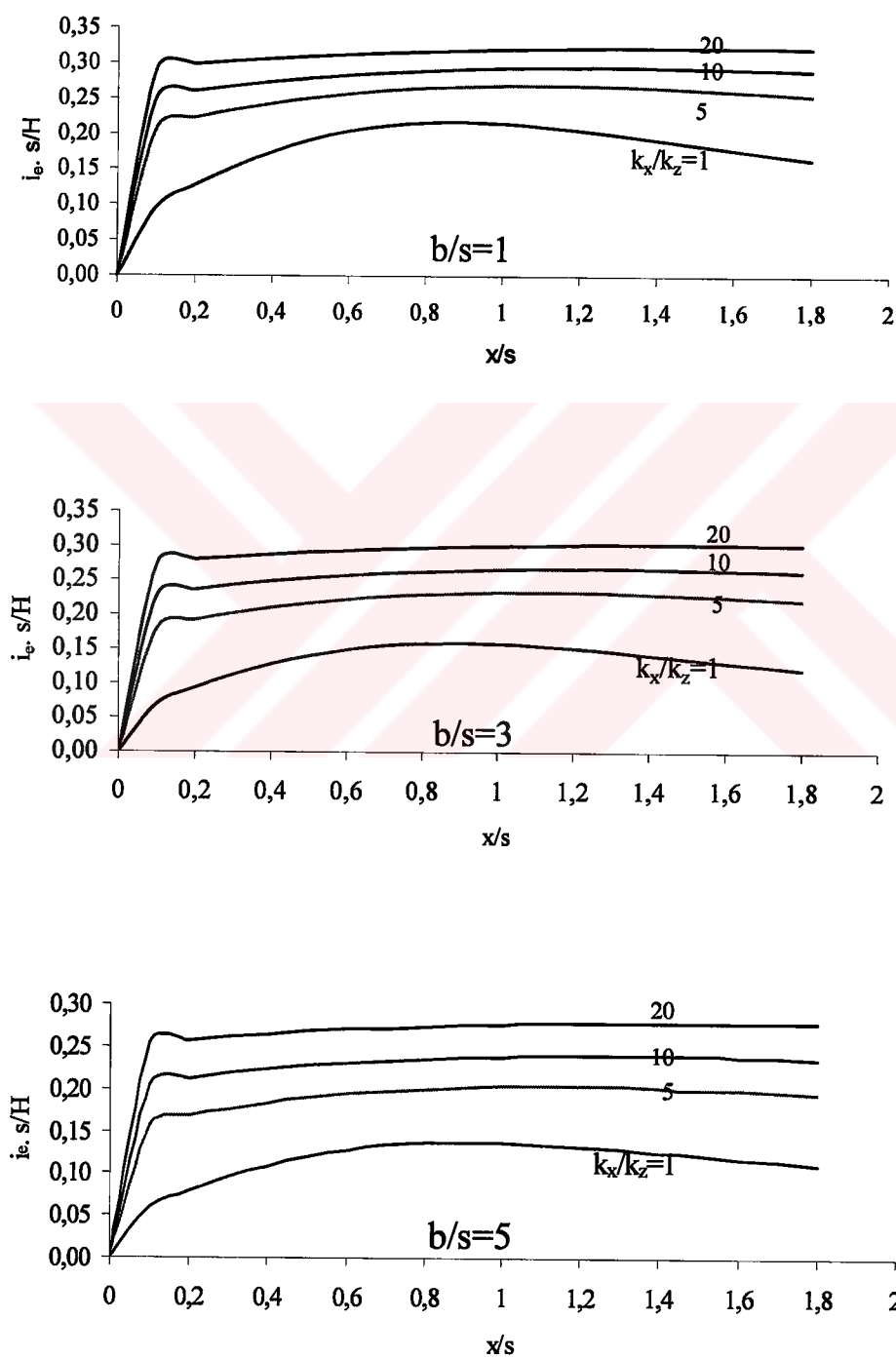


Fig. 6.12b Effect of anisotropy ratio (k_x/k_z) on exit gradient for $\theta = 30^\circ$ and for different values of b/s

Table 6.8 Summary of the exit gradients along the downstream side for different k_x/k_z and b/s values where $\theta = 45^\circ$

x/s	$i_e.s/H$ for $\theta = 45^\circ$ and $b/s = 1$					$i_e.s/H$ for $\theta = 45^\circ$ and $b/s = 3$					$i_e.s/H$ for $\theta = 45^\circ$ and $b/s = 5$				
	k_x/k_z					k_x/k_z					k_x/k_z				
	1	5	10	20		1	5	10	20		1	5	10	20	
0	0	0	0	0		0	0	0	0		0	0	0	0	
0.1414	0.0393	0.1644	0.2272	0.2865		0.0293	0.1387	0.2017	0.2653		0.0251	0.1288	0.1949	0.2703	
0.2828	0.0787	0.1973	0.2499	0.3008		0.0585	0.1664	0.2219	0.2785		0.0501	0.1545	0.2144	0.2838	
0.4242	0.1175	0.2236	0.2684	0.3128		0.0874	0.1886	0.2383	0.2897		0.0748	0.1752	0.2303	0.2952	
0.5656	0.154	0.2448	0.2835	0.3229		0.1146	0.2065	0.2517	0.299		0.0982	0.1919	0.2433	0.3049	
0.707	0.1852	0.2614	0.2956	0.3314		0.1379	0.2205	0.2624	0.3068		0.1183	0.2053	0.2539	0.3129	
0.807	0.2017	0.2707	0.3026	0.3364		0.1503	0.2284	0.2686	0.3114		0.1291	0.2129	0.2601	0.3178	
0.907	0.2129	0.2781	0.3084	0.3408		0.1587	0.2347	0.2738	0.3154		0.1367	0.2191	0.2654	0.3221	
1.007	0.2187	0.2837	0.3131	0.3446		0.1632	0.2395	0.278	0.3189		0.1411	0.2239	0.2697	0.3258	
1.107	0.2199	0.2877	0.3169	0.3477		0.1644	0.243	0.2813	0.3217		0.1426	0.2276	0.2732	0.329	
1.207	0.2175	0.2901	0.3197	0.3503		0.1628	0.2451	0.2838	0.3241		0.142	0.2301	0.276	0.3317	
1.307	0.2126	0.2912	0.3216	0.3524		0.1593	0.2461	0.2855	0.326		0.1396	0.2316	0.2781	0.3339	
1.407	0.206	0.291	0.3228	0.354		0.1545	0.246	0.2865	0.3274		0.1362	0.2322	0.2796	0.3358	
1.507	0.1984	0.2898	0.3232	0.3552		0.149	0.2451	0.2868	0.3284		0.1321	0.232	0.2804	0.3372	
1.607	0.1903	0.2877	0.3229	0.356		0.143	0.2434	0.2866	0.329		0.1277	0.2312	0.2808	0.3382	
1.707	0.1822	0.2848	0.3221	0.3564		0.137	0.2411	0.2858	0.3293		0.1231	0.2298	0.2806	0.3389	
1.807	0.1741	0.2813	0.3208	0.3564		0.131	0.2382	0.2846	0.3292		0.1185	0.2279	0.28	0.3393	

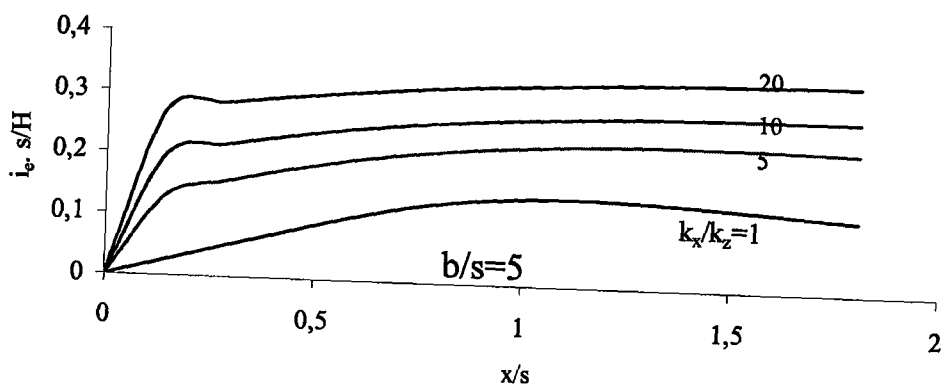
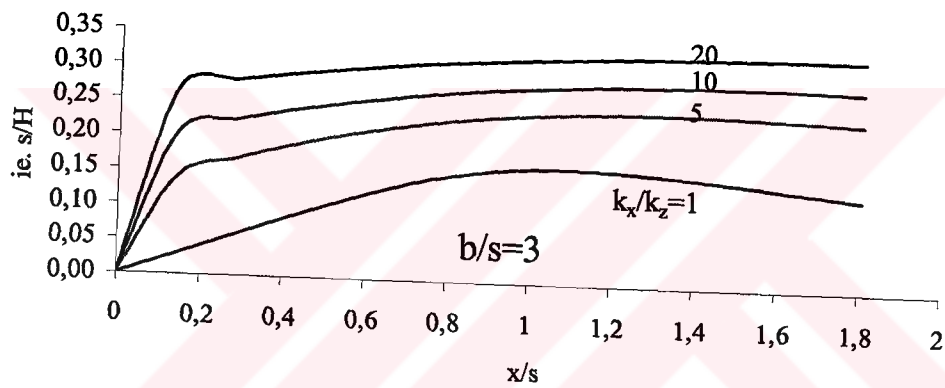
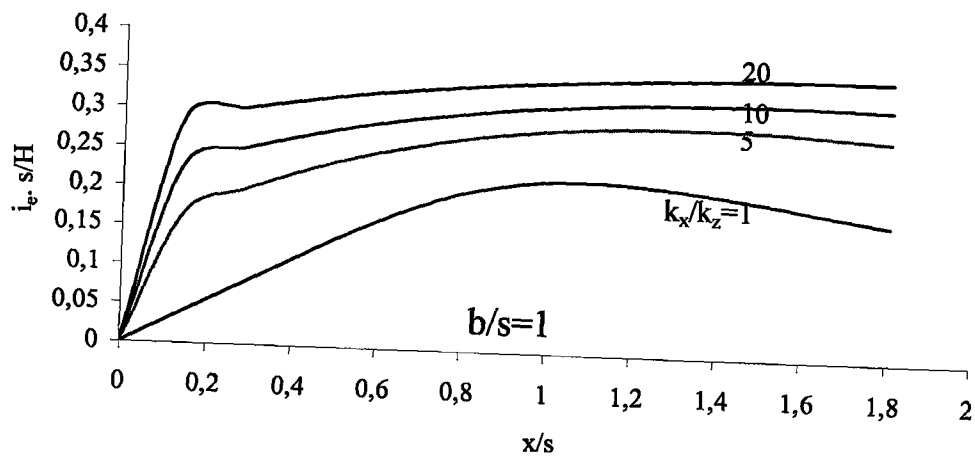


Fig. 6.12c Effect of anisotropy ratio (k_x/k_z) on exit gradient for $\theta = 45^\circ$ and for different values of b/s

It should be noted that the variation of the exit gradient along the downstream side of the structure in the previous figures is shown for a distance of only $x/s = 1.8$ in order to be able to do the comparison with the available analytical solution

6.3.3.2 Effect of k_x / k_z on Uplift Pressure

Concerning the effect of the anisotropy ratio on the uplift pressure, the analysis is conducted for the previous inclination angles with variation of the k_x / k_z ratio as 1, 5, 10, and 20 while keeping the value of b / s constant at 1, 3, and 5, respectively. In each case, the uplift pressure at the inner corner of the boundary node between the dam base and the toe sheet pile is calculated as the ratio of the piezometric head at that point to the total head difference. A summary of the uplift pressures from the computer runs for the studied cases is given in Table 6.9. This table is used to draw Fig. 6.13, which shows the relation between the uplift pressures against the sheet pile inclination angles. From this figure it, can be seen that the uplift pressure increases as the anisotropy ratio increases. For a change of k_x / k_z from 1 to 10, the uplift pressures increase by approximately 15, 25, and 30 % for b / s equal to 1, 3, and 5, respectively.

Table 6.9 Summary of the uplift pressure at the inner corner point between the dam base and the toe sheet pile, expressed as the piezometric head to total head difference, for different angles of inclination and anisotropy ratios

		values of h_c/H			
b/s	θ (deg.)	k_x/k_z			
		1	5	10	20
1	0	0.77	0.888	0.921	0.947
	30	0.71	0.85	0.899	0.937
	45	0.66	0.809	0.866	0.913
3	0	0.52	0.7	0.77	0.82
	30	0.48	0.65	0.73	0.802
	45	0.46	0.608	0.688	0.77
5	0	0.42	0.595	0.675	0.753
	30	0.38	0.547	0.629	0.714
	45	0.36	0.507	0.586	0.671

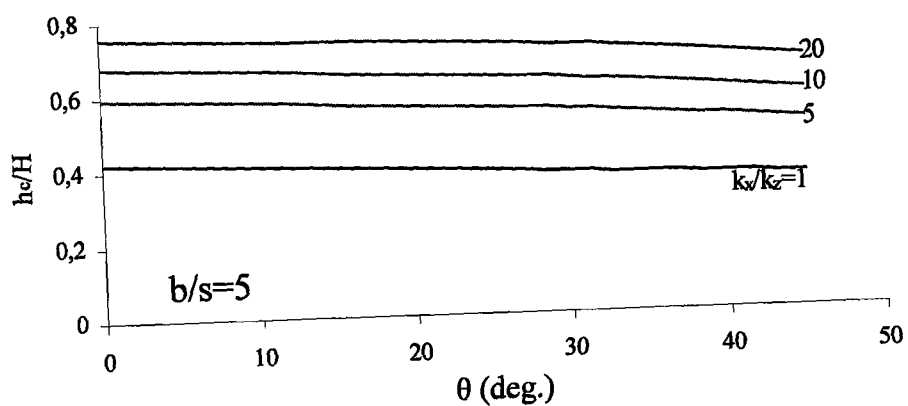
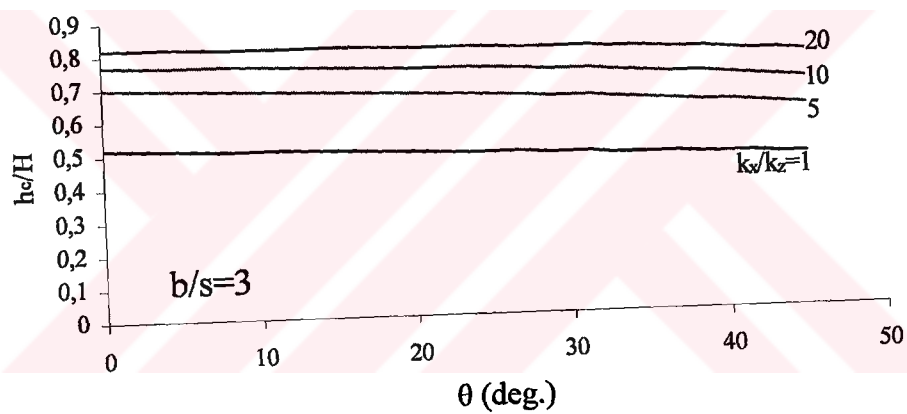
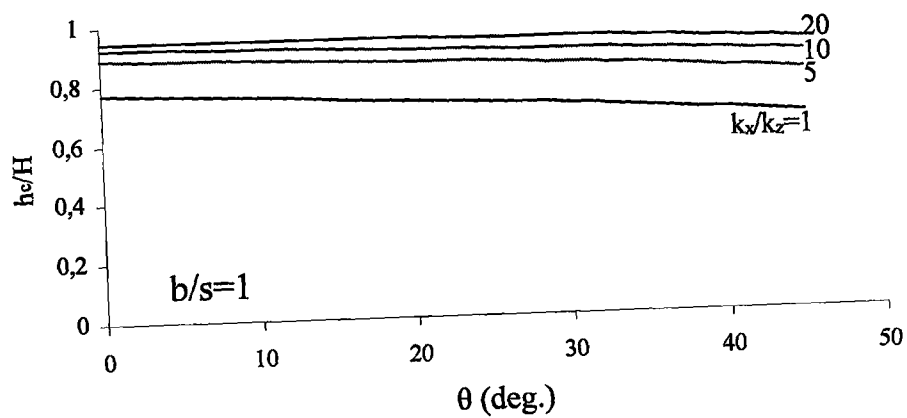


Fig. 6.13 Effect of k_x/k_z on uplift pressure at the inner corner point between the toe sheet pile and the dam base for different values of b/s

6.6.4 Effect of FD on Exit Gradient and Uplift Pressure

6.6.4.1 Effect of FD on Exit Gradient

Another set of analysis is conducted on the exit gradient by varying the foundation soil depth, where the slope angle (θ), the anisotropy ratio (k_x/k_z), and the dam base to the sheet pile depths ratio (b/s) are kept constants. The model computations are presented for three sheet pile inclinations as $\theta = 0^\circ, 30^\circ$, and 45° , and for base width to piling depth as $b/s = 1, 3$, and 5 , respectively. The results of the studied cases are summarized in Tables 6.10, 6.11, and 6.12, respectively. These results are used to draw non-dimensional graphs for the exit gradients along the ordinate against the distance from the toe along the abscissa, (Figs. 6.14a, 6.14b, and 6.14c). To estimate the exit gradient from these charts, one has to fix the dam width, the sheet pile depth, the thickness of the soil layer, the total head and the angle of the sheet pile inclination. Interpolation between the presented curves still provides a good estimate if the exact case is not met by any given chart.

The exit gradient is seen to increase with increasing the foundation soil depth. This outcome agrees totally with the expectation and intuition since the smallest embedded sheet pile would increase the seepage path and consequently increase the exit gradient. An example for such increment can be observed from Fig. 6.14a in which s/FD varies between 0.066 and 0.833. However, in all cases, the computations show that the variation in exit gradients may be neglected for $s/FD < 0.2$. From this observation, it is seen that no material change is gained in the exit gradient by increasing the foundation soil depth beyond a certain limit. This agrees well with Harza (1935) statement, that the water flowing under the dam tends to concentrate as much as possible along the shortest path with the profile of the base. In other words, it follows a narrow strip along the dam base. Therefore, it is possible to assume that a relatively deeper depth in the numerical solution can replace the infinite soil depth in the analytical one. It is assumed that the error will not be great as most of the seepage flow will concentrate close to the foundation profile of the dam.

Table 6.10 Summary of the exit gradients along the downstream side for different s/FD and b/s values where $\theta = 0^\circ$

x/s	i _e s/H for $\theta = 0^\circ$ and b/s=1						i _e s/H for $\theta = 0^\circ$ and b/s=3						i _e s/H for $\theta = 0^\circ$ and b/s=5					
	s/FD						s/FD						s/FD					
	0.833	0.5	0.2	0.1	0.066		0.833	0.5	0.2	0.1	0.066		0.833	0.5	0.2	0.1	0.066	
0	0.2573	0.294	0.3056	0.3085	0.3136		0.1949	0.215	0.2315	0.2362	0.238		0.1568	0.1676	0.1870	0.1947	0.1975	
0.1	0.255	0.2922	0.3041	0.307	0.3121		0.1932	0.2137	0.2304	0.2352	0.2369		0.1554	0.1666	0.1861	0.1939	0.1966	
0.2	0.2482	0.2871	0.2997	0.3027	0.3079		0.1881	0.2101	0.2273	0.2322	0.2339		0.1513	0.1638	0.1837	0.1914	0.1942	
0.3	0.2376	0.2791	0.2929	0.296	0.3013		0.18	0.2042	0.2224	0.2274	0.2292		0.1448	0.1592	0.1797	0.1876	0.1904	
0.4	0.2241	0.2688	0.284	0.2874	0.2927		0.1698	0.1967	0.216	0.2212	0.223		0.1366	0.1534	0.1747	0.1827	0.1855	
0.5	0.2085	0.2567	0.2737	0.2773	0.2827		0.158	0.1879	0.2085	0.214	0.2159		0.1271	0.1465	0.1687	0.1769	0.1798	
0.6	0.1918	0.2435	0.2625	0.2663	0.2719		0.1453	0.1783	0.2004	0.2061	0.2081		0.1169	0.1390	0.1623	0.1706	0.1736	
0.7	0.1748	0.2297	0.2507	0.2549	0.2606		0.1324	0.1682	0.1919	0.1979	0.1999		0.1065	0.1312	0.1555	0.1641	0.1671	
0.8	0.1581	0.2157	0.2389	0.2434	0.2492		0.1197	0.158	0.1833	0.1896	0.1917		0.0963	0.1232	0.1486	0.1574	0.1605	
0.9	0.142	0.2019	0.2272	0.232	0.2381		0.1075	0.1479	0.1748	0.1814	0.1836		0.0865	0.1153	0.1418	0.1509	0.1540	
1	0.127	0.1884	0.2159	0.2211	0.2273		0.0961	0.138	0.1665	0.1734	0.1757		0.0773	0.1076	0.1352	0.1445	0.1476	
1.1	0.1131	0.1754	0.205	0.2105	0.2169		0.0855	0.1285	0.1585	0.1658	0.1681		0.0688	0.1002	0.1288	0.1383	0.1416	
1.2	0.1003	0.1631	0.1947	0.2006	0.2071		0.0759	0.1195	0.1509	0.1585	0.1609		0.0610	0.0932	0.1227	0.1325	0.1358	
1.3	0.0889	0.1515	0.1849	0.1912	0.1979		0.0672	0.111	0.1436	0.1516	0.1541		0.0540	0.0865	0.1170	0.1270	0.1303	
1.4	0.0785	0.1405	0.1757	0.1823	0.1893		0.0593	0.103	0.1368	0.1451	0.1477		0.0477	0.0803	0.1115	0.1217	0.1252	
1.5	0.0693	0.1303	0.1671	0.1741	0.1812		0.0523	0.0954	0.1304	0.139	0.1417		0.0421	0.0744	0.1063	0.1168	0.1203	
1.6	0.0611	0.1207	0.159	0.1663	0.1737		0.0461	0.0884	0.1243	0.1333	0.1361		0.0371	0.0689	0.1015	0.1122	0.1158	
1.7	0.0538	0.1118	0.1515	0.1591	0.1667		0.0406	0.0819	0.1187	0.1279	0.1308		0.0326	0.0638	0.0969	0.1079	0.1115	
1.8	0.0473	0.1036	0.1444	0.1524	0.1602		0.0357	0.0758	0.1133	0.1229	0.1259		0.0287	0.0591	0.0926	0.1038	0.1076	

The anisotropy ratio, k_x/k_z , in the present Sub-Section is assumed to be 1. However, any desired value can easily be incorporated into the model. Application of other anisotropy ratios will lead to curves with similar trends as that in Figs. 6.14 and 6.15, but with different values.

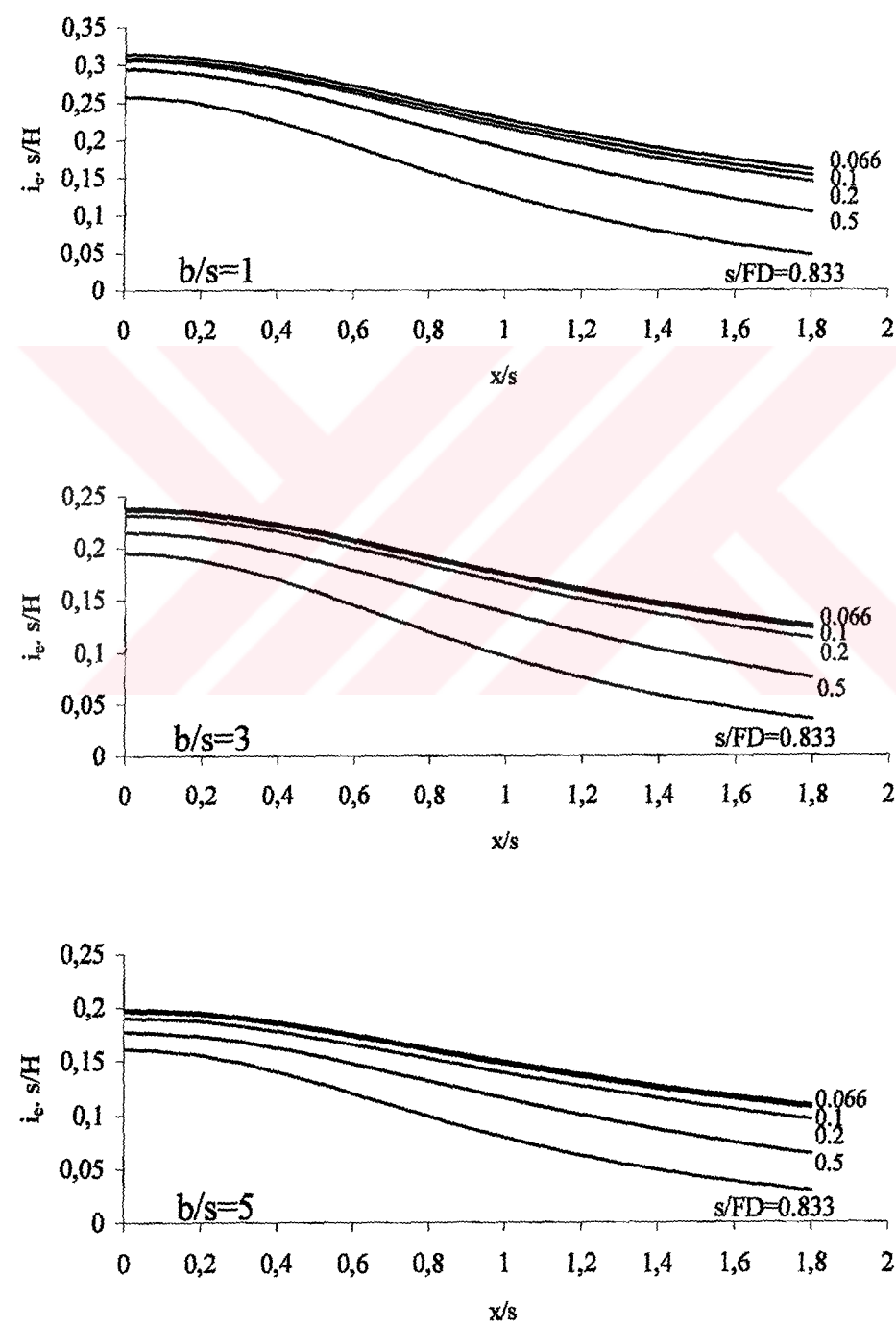


Fig. 6.14a Effect of foundation soil depth on exit gradient for $\theta = 0^\circ$, $k_x/k_z=1$, and for different values of b/s

Table 6.11 Summary of the exit gradients along the downstream side for different s/FD and b/s values where $\theta = 30^\circ$

x/s	$i_e \cdot s/H$ for $\theta = 30^\circ$ and $b/s=1$						$i_e \cdot s/H$ for $\theta = 30^\circ$ and $b/s=3$						$i_e \cdot s/H$ for $\theta = 30^\circ$ and $b/s=5$					
	s/FD						s/FD						s/FD					
	0.833	0.5	0.2	0.1	0.066		0.833	0.5	0.2	0.1	0.066		0.833	0.5	0.2	0.1	0.066	
0	0	0	0	0	0		0	0	0	0	0		0	0	0	0	0	
0.1	0.0789	0.0912	0.0956	0.0997	0.1032		0.0554	0.0624	0.0692	0.0708	0.0713		0.0431	0.0484	0.0568	0.0597	0.0608	
0.2	0.1052	0.1215	0.1275	0.1329	0.1376		0.0739	0.0832	0.0923	0.0944	0.0951		0.0575	0.0645	0.0757	0.0796	0.0811	
0.3	0.1263	0.146	0.1534	0.16	0.1657		0.0887	0.1	0.1111	0.1136	0.1144		0.069	0.0776	0.0911	0.0959	0.0977	
0.4	0.1428	0.1658	0.1745	0.1821	0.1886		0.1002	0.1136	0.1264	0.1294	0.1303		0.078	0.0881	0.1036	0.1092	0.1112	
0.5	0.1546	0.1809	0.1913	0.1998	0.207		0.1086	0.124	0.1386	0.1419	0.143		0.0845	0.0962	0.1136	0.1198	0.1221	
0.6	0.1616	0.1916	0.2037	0.2131	0.221		0.1135	0.1313	0.1476	0.1514	0.1526		0.0883	0.1019	0.1211	0.1279	0.1304	
0.7	0.1638	0.1977	0.2119	0.2221	0.2306		0.115	0.1354	0.1537	0.1578	0.1592		0.0895	0.1051	0.1261	0.1334	0.1361	
0.8	0.1616	0.1996	0.2162	0.2273	0.2362		0.1135	0.1367	0.1568	0.1615	0.163		0.0883	0.1061	0.1288	0.1366	0.1395	
0.9	0.1558	0.198	0.2171	0.229	0.2383		0.1094	0.1355	0.1576	0.1627	0.1643		0.0851	0.1051	0.1294	0.1378	0.1408	
1	0.1474	0.1935	0.2153	0.228	0.2377		0.1035	0.1323	0.1564	0.162	0.1637		0.0805	0.1027	0.1285	0.1373	0.1405	
1.1	0.1372	0.187	0.2114	0.2248	0.2348		0.0964	0.1277	0.1536	0.1597	0.1616		0.075	0.0991	0.1263	0.1356	0.1389	
1.2	0.1262	0.179	0.2061	0.2202	0.2304		0.0886	0.1221	0.1498	0.1563	0.1584		0.069	0.0947	0.1233	0.133	0.1364	
1.3	0.1149	0.1701	0.1998	0.2145	0.225		0.0807	0.1158	0.1452	0.1522	0.1544		0.0628	0.0899	0.1196	0.1297	0.1332	
1.4	0.1038	0.1608	0.1929	0.2082	0.2189		0.0729	0.1093	0.1402	0.1476	0.1499		0.0567	0.0848	0.1156	0.126	0.1296	
1.5	0.0932	0.1514	0.1857	0.2017	0.2124		0.0654	0.1026	0.135	0.1428	0.1452		0.0509	0.0797	0.1114	0.1221	0.1258	
1.6	0.0833	0.1421	0.1785	0.195	0.2059		0.0584	0.0961	0.1296	0.1379	0.1404		0.0455	0.0746	0.1071	0.1181	0.1219	
1.7	0.0741	0.1331	0.1714	0.1883	0.1994		0.052	0.0897	0.1244	0.133	0.1357		0.0405	0.0696	0.1029	0.1141	0.118	
1.8	0.0658	0.122	0.165	0.182	0.1931		0.0461	0.0836	0.1192	0.1282	0.131		0.0359	0.0649	0.0987	0.1102	0.1141	

The developed computer program as well as the design charts for this dam base could be used efficiently to approximate the exit gradient, and the uplift pressure values. While the available analytical solution is limited to a soil layer of infinite thickness, the current model is capable of providing exit gradients and uplift pressures for a soil layer with variable thickness. Varying the thickness of the permeable layers simulates a more realistic situation for various design conditions.

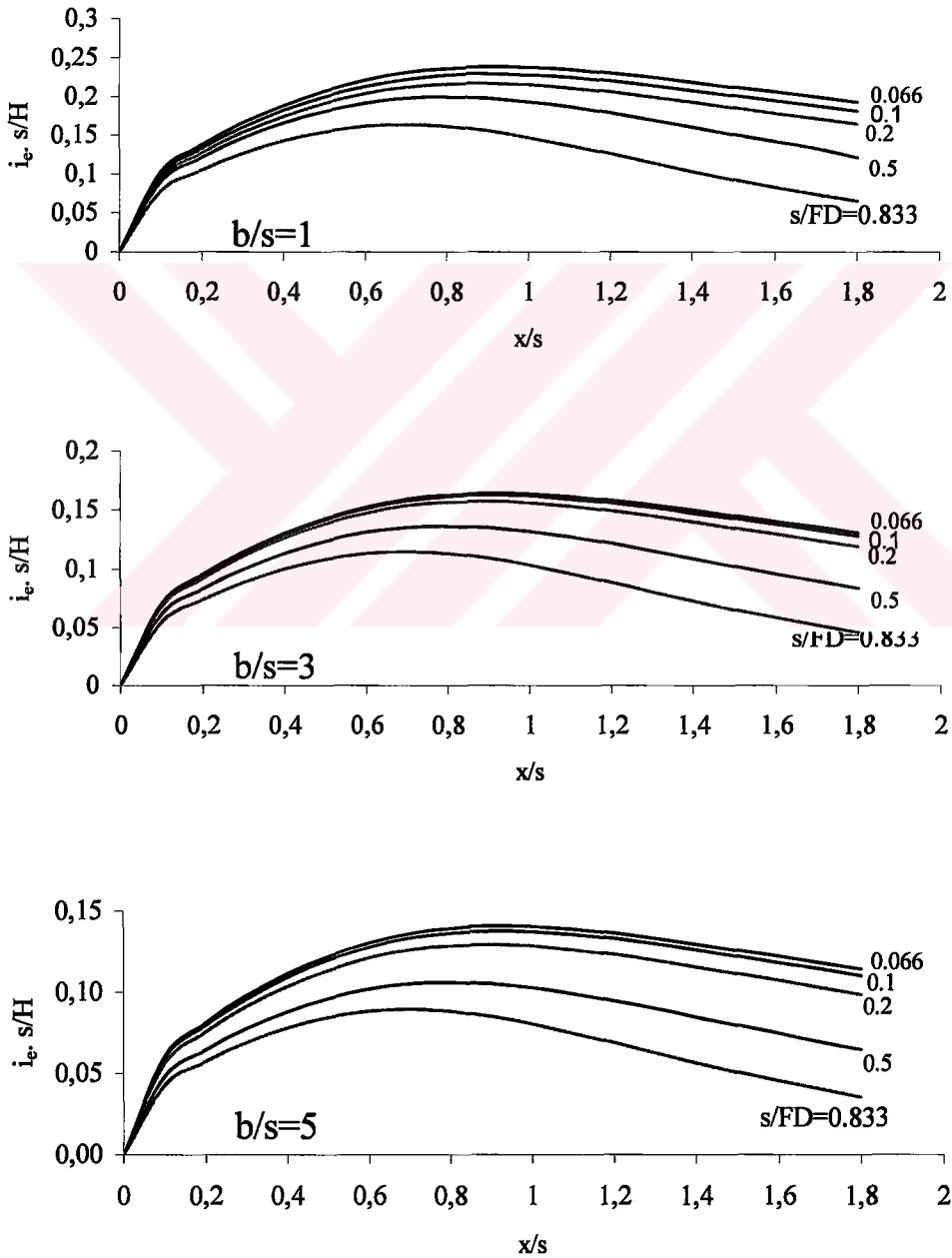


Fig. 6.14b Effect of foundation soil depth on exit gradient for $\theta = 30^\circ$, $k_x/k_z=1$, and for different values of b/s

Table 6.12 Summary of the exit gradients along the downstream side for different s/FD and b/s values where $\theta = 45^\circ$

i _e .s/H for θ = 45° and b/s=1					i _e .s/H for θ = 45° and b/s=3					i _e .s/H for θ = 45° and b/s=5						
x/s	s/FD				0.066	s/FD				0.066	s/FD				0.1	0.066
	0.833	0.5	0.2	0.1		0.833	0.5	0.2	0.1		0.833	0.5	0.2	0.1		
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0.1414	0.034	0.0373	0.0393	0.04	0.0402	0.0231	0.026	0.029	0.0297	0.0299	0.0177	0.0201	0.0238	0.0251	0.0262	
0.2828	0.068	0.0746	0.0787	0.08	0.0804	0.0463	0.052	0.058	0.0595	0.0599	0.0354	0.0402	0.0475	0.0501	0.0524	
0.4242	0.1014	0.1113	0.1175	0.1195	0.12	0.069	0.0776	0.0866	0.0888	0.0894	0.0527	0.0599	0.0709	0.0748	0.0782	
0.5656	0.1323	0.1457	0.154	0.1567	0.1574	0.0899	0.1015	0.1136	0.1164	0.1173	0.0687	0.0784	0.093	0.0982	0.1027	
0.707	0.157	0.1744	0.1852	0.1886	0.1895	0.1068	0.1216	0.1367	0.1402	0.1413	0.0816	0.0939	0.1119	0.1183	0.1238	
0.807	0.1682	0.1889	0.2017	0.2057	0.2067	0.1144	0.1317	0.1489	0.153	0.1543	0.0874	0.1017	0.122	0.1291	0.1353	
0.907	0.1733	0.1977	0.2129	0.2175	0.2187	0.1179	0.1378	0.1573	0.1619	0.1634	0.0901	0.1065	0.1289	0.1367	0.1434	
1.007	0.1726	0.2009	0.2187	0.2241	0.2253	0.1174	0.1401	0.1618	0.1669	0.1685	0.0897	0.1082	0.1326	0.1411	0.1482	
1.107	0.167	0.1993	0.2199	0.226	0.2275	0.1135	0.139	0.1628	0.1686	0.1703	0.0868	0.1074	0.1336	0.1426	0.1502	
1.207	0.1579	0.194	0.2175	0.2244	0.226	0.1074	0.1354	0.1612	0.1675	0.1694	0.0821	0.1046	0.1324	0.142	0.1498	
1.307	0.1467	0.1863	0.2126	0.2202	0.2219	0.0998	0.13	0.1577	0.1645	0.1666	0.0762	0.1004	0.1296	0.1396	0.1477	
1.407	0.1345	0.177	0.206	0.2143	0.2161	0.0914	0.1235	0.153	0.1602	0.1624	0.0699	0.0954	0.1258	0.1362	0.1444	
1.507	0.122	0.1668	0.1984	0.2074	0.2093	0.0829	0.1165	0.1474	0.1551	0.1575	0.0634	0.09	0.1214	0.1321	0.1405	
1.607	0.1098	0.1564	0.1903	0.1999	0.202	0.0747	0.1092	0.1415	0.1496	0.1521	0.0571	0.0844	0.1166	0.1277	0.1362	
1.707	0.0983	0.1461	0.1822	0.1924	0.1945	0.0668	0.102	0.1355	0.144	0.1466	0.0511	0.0788	0.1118	0.1231	0.1318	
1.807	0.0875	0.136	0.1741	0.185	0.1872	0.0595	0.0949	0.1295	0.1384	0.1411	0.0455	0.0734	0.1069	0.1185	0.1273	

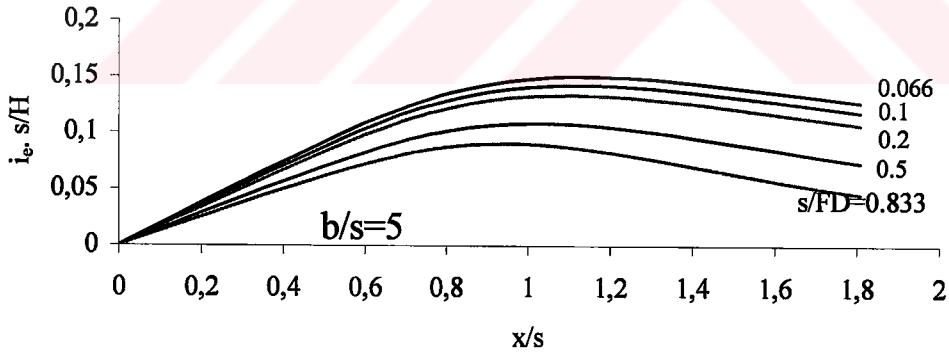
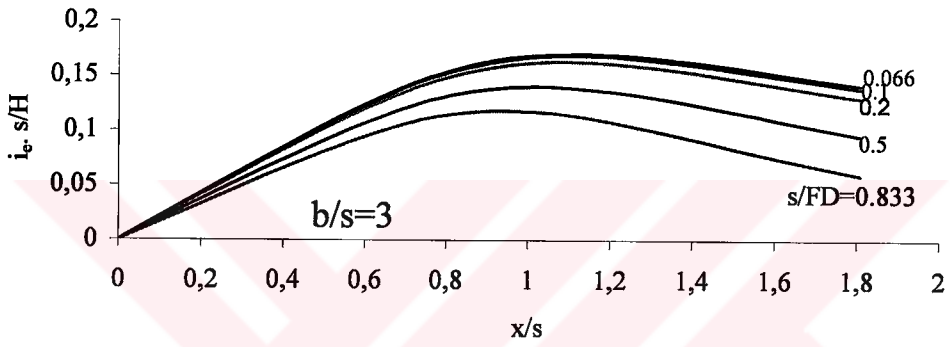
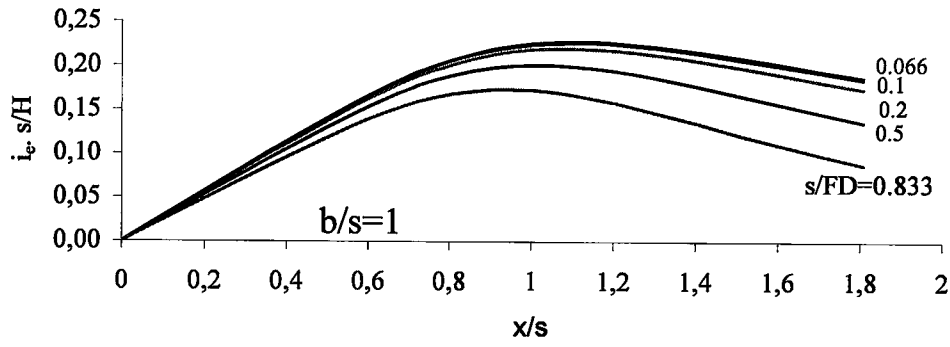


Fig. 6.14c Effect of foundation soil depth on exit gradient for $\theta = 45^\circ$, $k_x/k_z=1$, and for different values of b/s

6.6.4.2 Effect of FD on Uplift Pressure

Similarly, the effect of the foundation soil depth on uplift pressure is studied and the model results are collected in Table 6.13. These data are drawn in Fig. 6.15. From this figure, it is observed that the pressure drop at some key points e.g. at point C in Fig. 6.2 will not be appreciably affected by the finite thickness of the permeable

stratum unless the sheet pile penetrates through more than 50 % of the stratum thickness. The ratio $s/FD \approx 0$ corresponds to the limiting case of an infinitely thick permeable stratum. It is also seen that the deviations from this limiting curve do not become large until $s/FD > 0.5$. This gives the justification for using the curve where $s/FD \approx 0$ in the study of the uplift pressure in dams with inclined sheet piles. In any case, this approximation should be safe from the point of view of the practical design of a dam with an inclined sheet pile at its toe, as it will lead to uplift pressures that are higher than those that will actually occur.

Table 6.13 Summary of uplift pressure results at the inner corner point between the toe sheet pile and the dam base for different b/s and s/FD ratios

values of h_c/H from the numerical model					
θ (deg.)	b/s	s/FD			
		0.833	0.5	0.25	0.1
0	1	0.902	0.824	0.8	0.796
	2	0.816	0.69	0.658	0.65
	3	0.746	0.598	0.564	0.558
	5	0.638	0.474	0.448	0.448
	8	0.524	0.362	0.348	0.348
30	1	0.776	0.728	0.714	0.712
	2	0.652	0.592	0.58	0.58
	3	0.562	0.506	0.498	0.498
	5	0.442	0.392	0.392	0.392
	8	0.338	0.294	0.294	0.294
45	1	0.722	0.69	0.678	0.676
	2	0.59	0.558	0.55	0.55
	3	0.5	0.474	0.47	0.47
	5	0.384	0.364	0.364	0.364
	8	0.286	0.274	0.274	0.274

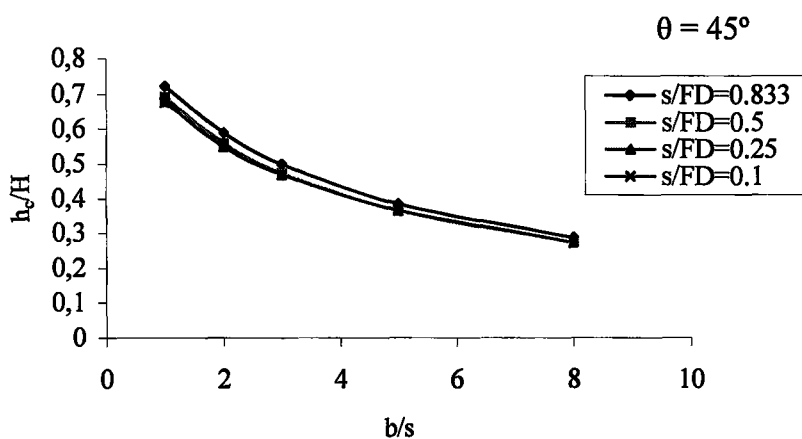
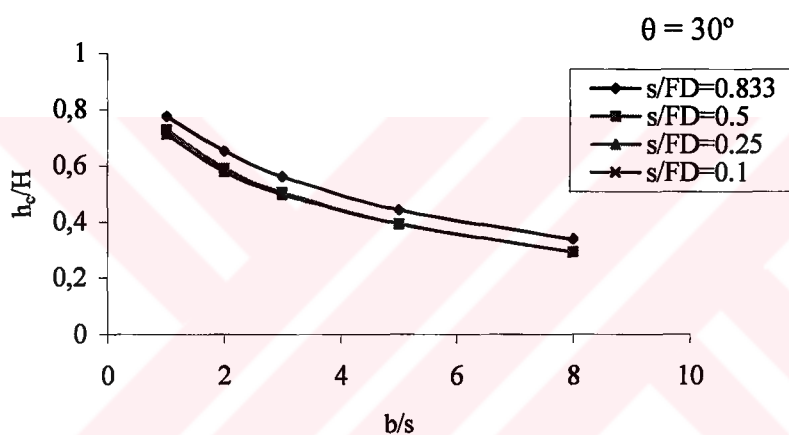
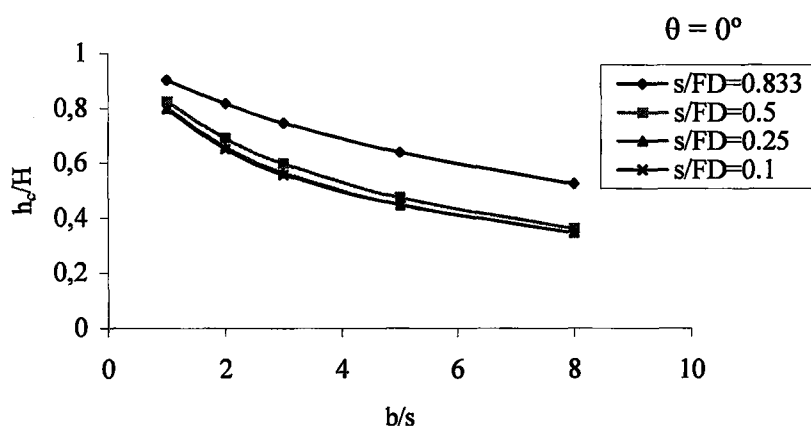


Fig. 6.15 Effect of FD on uplift pressures at the inner corner point between the sheet pile and the dam base for different values of θ where $k_x/k_z=1$

7. INVESTIGATION OF A FLAT DAM BASE WITH SYMMETRICAL INCLINED HEEL AND TOE SHEETINGS

7.1 Introduction

Referring to the dam base configuration analyzed in Chapter 6 for a flat dam base with an inclined sheet pile at its toe, it is found that using inclined piles at the dam toe is more effective in reducing the exit gradient and the uplift pressure than vertical ones. Moreover, the danger from undermining is shifted further downstream from the dam toe. For these reasons, the problem concerning the relative advantages of using another symmetrical inclined sheet pile at the heel in addition to that at the toe is considered in the present Chapter, (Fig. 7.1). A modified version of the numerical model used in the case of a dam with an inclined toe sheet pile alone is applied in the present investigation (as presented in the Appendix).

In order to facilitate the plotting and comparison of the results of the present study with that for a flat dam base with an inclined toe sheet pile alone, an identical set of parameters is selected and analyzed. The present investigation assumes:

- a flat dam base flanked on either side by vertical or inclined symmetrical sheet piling as shown in Fig. 7.1. The heel and toe sheetings inclined toward the upstream and downstream sides, respectively,
- a homogeneous and isotropic soil where Laplace equation (Eq. 3.14) is the governing equation for the seepage flow, and
- the ratio of foundation soil depth to dam base width is assumed to be $FD/b > 2$. Using similar or higher ratios in the numerical solution shows, as presented in Section 6.6.4 about the effect of sub-soil location on seepage flow, that the finite thickness of the permeable stratum will not have a significant effect on exit gradient and uplift pressure values.

The basic numerical model as illustrated in Chapter 6 remains almost unchanged. The only new feature concerns the introduction of the heel sheet pile. In order to take

The rectangular area ABCDEF in Fig. 7.1 is the homogeneous porous medium within which the seepage flow is considered. For a grid with non-uniform spacings superimposed over the flow region, the finite difference forms of the governing equation and boundary conditions used in the present case are given in Fig. 7.2. For every node (i,j) in this area, except those along the boundary, Eq. (1) in Fig. 7.2 is applicable. The horizontal surfaces AB and CD are constant head boundaries along which the heads are assumed to be 5 and 0, respectively. However, BC, EF and AF, DE are impervious horizontal and vertical boundaries, respectively, for which Eq. (5) in Fig. 7.2 is applicable. The heel and toe sheet piles are assumed to be impervious inclined boundaries having zero thickness where Eq. (4) in Fig. 7.2 is applicable for their left and right sides. For more details of the boundary nodes such as those at the corners between the dam base and the heel or the toe sheet piles, between the vertical impervious boundaries and the impervious bedrock, and others see Fig. 7.2. It should be noted that the equations in Fig. 7.2 are applied in the present investigation after using the proper indices representing the exact nodal locations.

(1) For Interior nodes

$$h_0 = (Ah_1 + Bh_3 + Ch_2 + Dh_4) / E$$

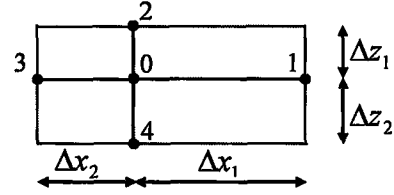
where $A = 1/(\Delta x_1 (\Delta x_1 + \Delta x_2))$,

$$B = 1/(\Delta x_2 (\Delta x_1 + \Delta x_2)),$$

$$C = 1/(\Delta z_1 (\Delta z_1 + \Delta z_2)),$$

$$D = 1/(\Delta z_2 (\Delta z_1 + \Delta z_2)), \text{ and}$$

$$E = A + B + C + D$$



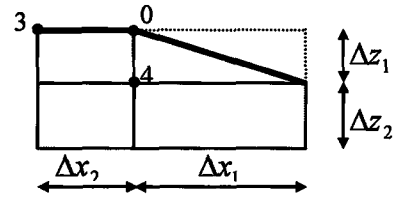
(2) For Corner nodes between :

dam base and inclined heel and toe sheetings,
after using the proper indices for nodal locations.

$$h_0 = (Ah_3 + Bh_4) / (A + B)$$

where $A = \Delta z_1 / (\Delta x_2 (\Delta x_1 + \Delta x_2))$, and

$$B = 1 / \Delta z_1$$



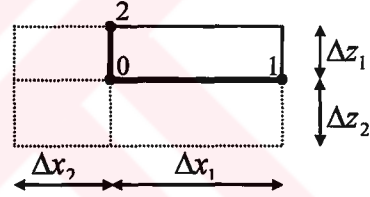
(3) For Corner nodes between :

- (1) dam base and vertical heel and toe sheetings.
 - (2) left and right corner nodes at the bedrock.
- after using the proper indices for nodal locations.

$$h_0 = (Ah_1 + Bh_2) / (A + B)$$

where $A = \Delta z_1 / \Delta x_1$, and

$$B = \Delta x_1 / \Delta z_1$$

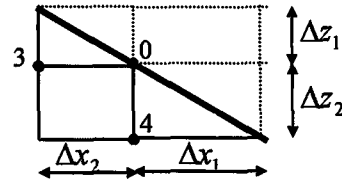


(4) For left and right sides of inclined sheetings,
after using the proper indices for nodal locations.

$$h_0 = (Ah_3 + Bh_4) / (A + B)$$

where $A = 1/(\Delta x_2 (\Delta x_1 + \Delta x_2))$, and

$$B = 1/(\Delta z_2 (\Delta z_1 + \Delta z_2))$$



(5) For (1) left, right, bottom, and dam base boundaries

- (2) left and right sides of vertical sheetings
- after using the proper indices for nodal locations.

$$h_0 = (Ah_1 + Bh_3 + Ch_2) / (A + B + C)$$

where $A = \Delta z_1 / (\Delta x_1 (\Delta x_1 + \Delta x_2))$,

$$B = \Delta z_1 / (\Delta x_2 (\Delta x_1 + \Delta x_2)), \text{ and}$$

$$C = 1 / \Delta z_1$$

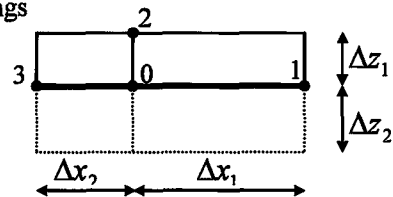


Fig. 7.2 Finite difference equations for interior and boundary nodes
(over a grid with non-uniform spacings where $k_x/k_z = 1$)

7.2 Results and Discussion

7.2.1 Comparison of the Model Results with a Previous Study

The numerical solution for the case of equal and vertical sheet piles placed at either end of the dam base is verified against the available solution of Harza (1935). In his analogy procedure, he investigated four cases. The first one used a 4-in. depth of sheeting on a 20-in. base width, all the others an 8-in. sheeting on base widths of 20, 16, and 12-in.

A brief description of an electric analog model is given here. Experiments may be conducted in a wooden tank with arbitrary selected dimensions (say 40 in. by 30 in. by 2.5 in. deep). The tank is normally coated inside with a waterproof non-conductive paint. The model floor and cutoff walls are constructed of 1/32 in. copper plates when the flow lines are mapped and of 1/32 in. bakelite plates when the potential lines are mapped. The electrolyte, $\frac{3}{4}$ in. deep, which represents a continuous conductive medium, is a solution of copper sulfate in distilled water acidulated with sulfuric acid (50 g of copper sulfate and 25 cc of sulfuric acid per 5 gal of water). During experimentation, the various equipotential lines are detected by means of a probe attached to a pantograph. For example, if the 90% line is to be plotted, the voltmeter is maintained at 10 v, and the probe is moved to various closely spaced points in order to trace the line of points where the electric potential is equal to the one adjusted on the middle terminal of the variable resistor. When the probe is connected to such a point, the ammeter indicates zero current. Fig. 7.5a shows an electric analogy model arrangement for mapping the equipotential lines under a horizontal dam floor. Impermeable boundaries are simulated by the edges of the tank. The sides of the tank upstream and downstream from the dam base are covered with copper plates (electrodes). Therefore, current flows through the electrolyte from one plate to the other in a manner analogous to that of the flow of water from the upstream bed to the downstream bed. In order to map the flow lines of the same example, the position of the copper plates and that of the non-conductive surfaces is reversed. That is, the base of the dam is constructed of a copper plate and the entire perimeter of the tank is covered with copper, with the exception of those parts of the sides that represent the upstream bed and downstream bed. In order to plot the flow lines, the large current required by the relatively large copper area

necessitated a dilution of the solution to about half the foregoing concentration. Fig. 7.5b shows the general wiring diagram of the apparatus. Alternating current is used in order to avoid polarizing effects of the electrodes. The supply of the electric current is regulated by a rheostat and measured by a milliammeter. The control switch is set on “L” (low) when zeroing the voltmeter, set on “H” (high) when the overall voltage between high and low is selected, and set on “P” (probe) while tracing the equipotential lines.

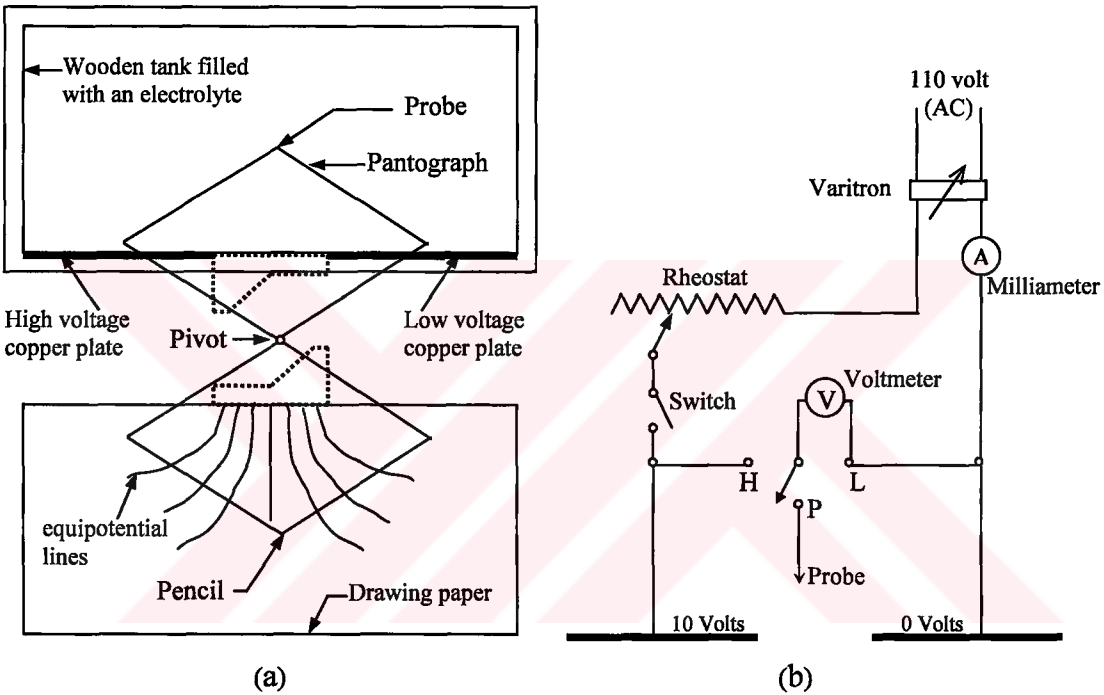


Fig. 7.3 Electric analog model, (a) apparatus for mapping the equipotential lines under a horizontal floor, and (b) general wiring diagram of the apparatus

In this study, the comparison is done for the first two cases, namely, those of 4 and 8-in. depth of sheeting on a 20-in. base width. The ratio of b/s is 5 and 2.5, respectively (b is the horizontal run between sheet piling and s is the length of sheeting). In order to perform the comparison, the previous dimensions are inserted in the numerical model in addition to the foundation soil depth and the permeability ratio which are taken as $FD = 2.5b$ and $k_x/k_z = 1$, respectively. Since no formal rules about the choice of the mesh spacing are available, guidance can only be gained by considering a number of runs for the developed program with different grid dimensions. The first run uses a mesh interval of unity in the vertical and horizontal directions. However, it is observed that using smaller intervals leads to better and

closer solutions in comparison with the analog one. Therefore, In the considered situations of $b/s = 5$ and 2.5 , the grid dimensions as $\Delta x = \Delta z = 0.25$ unit are found to give acceptable results as can be seen from Table 7.1.

Table 7.1 Comparison of uplift pressures along the horizontal run between sheetings using the computer results and Harza (1935) solution

dam with symmetrical and vertical heel and toe sheet piles											
$\theta = 0^\circ$, and $b/s = 5$						$\theta = 0^\circ$, and $b/s = 2.5$					
Computer results				Harza solution		Computer results				Harza solution	
x	h/H	x	h/H	x	h/H	x	h/H	x	h/H	x	h/H
0	0.682	10.25	0.52	0	0.68	0	0.608	10.25	0.530	0	0.59
0.25	0.682	10.5	0.515	5	0.62	0.25	0.608	10.5	0.527	5	0.575
0.5	0.682	10.75	0.509	10	0.5	0.5	0.608	10.75	0.524	10	0.52
0.75	0.681	11	0.504	15	0.42	0.75	0.607	11	0.521	15	0.47
1	0.679	11.25	0.499	20	0.37	1	0.607	11.25	0.519	20	0.45
1.25	0.678	11.5	0.494			1.25	0.606	11.5	0.516		
1.5	0.676	11.75	0.489			1.5	0.606	11.75	0.513		
1.75	0.673	12	0.484			1.75	0.605	12	0.510		
2	0.671	12.25	0.48			2	0.604	12.25	0.508		
2.25	0.668	12.5	0.475			2.25	0.603	12.5	0.505		
2.5	0.665	12.75	0.47			2.5	0.601	12.75	0.503		
2.75	0.661	13	0.465			2.75	0.600	13	0.500		
3	0.658	13.25	0.46			3	0.599	13.25	0.498		
3.25	0.654	13.5	0.455			3.25	0.597	13.5	0.495		
3.5	0.65	13.75	0.45			3.5	0.596	13.75	0.493		
3.75	0.646	14	0.446			3.75	0.594	14	0.490		
4	0.642	14.25	0.441			4	0.592	14.25	0.488		
4.25	0.637	14.5	0.437			4.25	0.590	14.5	0.486		
4.5	0.633	14.75	0.432			4.5	0.588	14.75	0.483		
4.75	0.628	15	0.428			4.75	0.586	15	0.481		
5	0.624	15.25	0.423			5	0.584	15.25	0.479		
5.25	0.619	15.5	0.419			5.25	0.582	15.5	0.477		
5.5	0.614	15.75	0.415			5.5	0.580	15.75	0.475		
5.75	0.61	16	0.411			5.75	0.577	16	0.474		
6	0.605	16.25	0.407			6	0.575	16.25	0.472		
6.25	0.6	16.5	0.403			6.25	0.573	16.5	0.470		
6.5	0.595	16.75	0.399			6.5	0.570	16.75	0.469		
6.75	0.59	17	0.396			6.75	0.568	17	0.467		
7	0.585	17.25	0.393			7	0.565	17.25	0.466		
7.25	0.58	17.5	0.389			7.25	0.562	17.5	0.465		
7.5	0.575	17.75	0.387			7.5	0.560	17.75	0.463		
7.75	0.57	18	0.384			7.75	0.557	18	0.462		
8	0.565	18.25	0.381			8	0.554	18.25	0.462		
8.25	0.56	18.5	0.379			8.25	0.552	18.5	0.461		
8.5	0.555	18.75	0.378			8.5	0.549	18.75	0.460		
8.75	0.55	19	0.376			8.75	0.546	19	0.459		
9	0.545	19.25	0.375			9	0.544	19.25	0.459		
9.25	0.54	19.5	0.374			9.25	0.541	19.5	0.459		
9.5	0.535	19.75	0.373			9.5	0.538	19.75	0.459		
9.75	0.53	20	0.373			9.75	0.535	20	0.458		
10	0.525					10	0.532				

From the results of the numerical and analog solutions, the uplift pressure, expressed as the ratio of the piezometric head to the total head difference, along the horizontal run between the sheeting is drawn in Fig. 7.4. This figure corresponded to two b/s ratios as 5 and 2.5, respectively. Observation of this figure reveals that the agreement between the two solutions is reasonably good. The calculated maximum percentage differences between these solutions is found to be 4.7 % for the case of $b/s = 5$ and occurred at the middle of the dam base, and 2.9 % for the case of $b/s = 2.5$ and occurred at the dam heel.

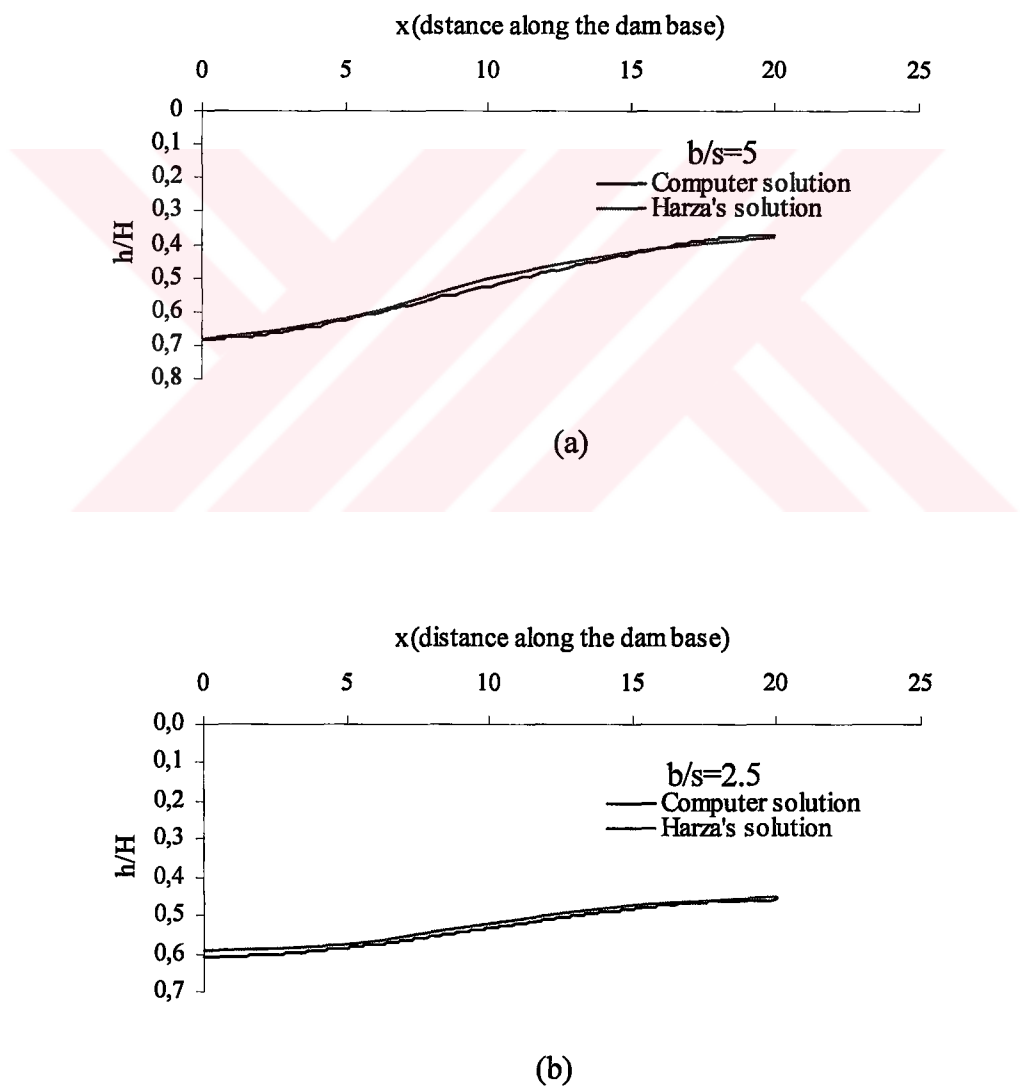


Fig. 7.4 Uplift pressures along the horizontal run between sheetings, a dam base flanked with two equal and vertical sheet piles at either end (a) for $b/s = 5$ and (b) for $b/s = 2.5$

A comparison of the numerical results of the piezometric head distribution along the upstream and downstream sides of the vertical heel and toe sheetings is also performed against Harza's solution for the same configuration. The results are not presented here because the percentage difference between the two solutions does not exceed the previously mentioned limits.

Since analytical or approximate solutions for the cases of symmetrical inclined heel and toe sheetings are not available, several checks on the correctness of the present numerical solution for such cases are made. These include,

- (1) a check on the sensitivity of the solution to the choice of nodal spacings and error tolerance. For this purpose, several computer runs are made with successively smaller nodal spacing and error tolerance until the solution does not change within the desired range of accuracy. The choice of the values of these parameters follows the same procedure explained in Chapter 6,
- (2) a nodal mass balance calculation routine is incorporated into the computer program. This depends on the fact that, at the steady state, the flow-in must equal the flow-out, and
- (3) A total mass balance can also be made.

7.2.2 Exit Gradient

As described in Chapter 6, the exit gradient, i_e , is taken to be the rate of piezometric head change at the exit points closest to the soil surface along the downstream side and computed using a two-point numerical differentiation as shown in Fig. 6.5. In order to investigate the effect of introducing a symmetrical heel sheet pile in addition to that at the toe on exit gradient along the downstream bed, the computer results are collected in Tables 7.2, 7.3, and 7.4 and plotted into non-dimensional graphical forms, (Figs. 7.5a, 7.5b, and 7.5c). These figures show the relation between the exit gradients along the downstream side expressed as $i_e.s/h$ and the distance from the dam toe expressed as x/s . The exit gradients for the case of a flat dam base with an inclined toe sheet pile alone are also inserted in the same figure. The comparison showed a general decrease in $i_e.s/h$ with the application of the heel and toe sheet piles in comparison to the dam base with a toe sheet pile alone.

Table 7.2 Exit gradients along the downstream side, expressed as $i_e \cdot s/H$, for $\theta = 0^\circ$ and different b/s values

x/s	values of $i_e \cdot s/H$			%	values of $i_e \cdot s/H$			%	values of $i_e \cdot s/H$			%
	$\theta = 0^\circ$, and $b/s = 1$				$\theta = 0^\circ$, and $b/s = 3$				$\theta = 0^\circ$, and $b/s = 5$			
	heel & toe sheet piles	toe sheet pile alone			heel & toe sheet piles	toe sheet pile alone			heel & toe sheet piles	toe sheet pile alone		
0	0.2608	0.3056		-17.18	0.2043	0.2339		-14.49	0.17	0.187		-10.00
0.1	0.2596	0.3041		-17.14	0.2033	0.2328		-14.51	0.1692	0.1861		-9.99
0.2	0.2561	0.2997		-17.02	0.2006	0.2296		-14.46	0.1669	0.1836		-10.01
0.3	0.2507	0.2929		-16.83	0.1964	0.2247		-14.41	0.1634	0.1797		-9.98
0.4	0.2436	0.284		-16.58	0.1908	0.2183		-14.41	0.1588	0.1746		-9.95
0.5	0.2353	0.2737		-16.32	0.1844	0.2108		-14.32	0.1534	0.1687		-9.97
0.6	0.2262	0.2625		-16.05	0.1773	0.2026		-14.27	0.1476	0.1622		-9.89
0.7	0.2168	0.2507		-15.64	0.17	0.194		-14.12	0.1414	0.1555		-9.97
0.8	0.2072	0.2389		-15.30	0.1625	0.1854		-14.09	0.1352	0.1486		-9.91
0.9	0.1976	0.2272		-14.98	0.1551	0.1768		-13.99	0.1291	0.1418		-9.84
1	0.1884	0.2159		-14.60	0.1479	0.1685		-13.93	0.1231	0.1352		-9.83
1.1	0.1794	0.205		-14.27	0.1409	0.1604		-13.84	0.1173	0.1288		-9.80
1.2	0.1709	0.1947		-13.93	0.1343	0.1528		-13.78	0.1118	0.1227		-9.75
1.3	0.1628	0.1849		-13.57	0.128	0.1455		-13.67	0.1065	0.1169		-9.77
1.4	0.1551	0.1757		-13.28	0.122	0.1387		-13.69	0.1016	0.1115		-9.74
1.5	0.1479	0.1671		-12.98	0.1164	0.1322		-13.57	0.0969	0.1063		-9.70
1.6	0.1411	0.159		-12.69	0.1111	0.1261		-13.50	0.0925	0.1015		-9.73
1.7	0.1347	0.1515		-12.47	0.1061	0.1204		-13.48	0.0884	0.0969		-9.62
1.8	0.1287	0.1444		-12.20	0.1014	0.1151		-13.51	0.0845	0.0926		-9.59

The outcome agrees totally with the expectation and intuition since the analysis in the present model incorporates another sheet pile at the heel. It is to be noted that, for the same angle of inclination, as the ratio of the dam base width to the sheet pile depth increases the exit gradient decreases (Fig. 7.5a). The smallest embedded sheet pile would increase the seepage path and therefore would increase both the piezometric head and the exit gradient.

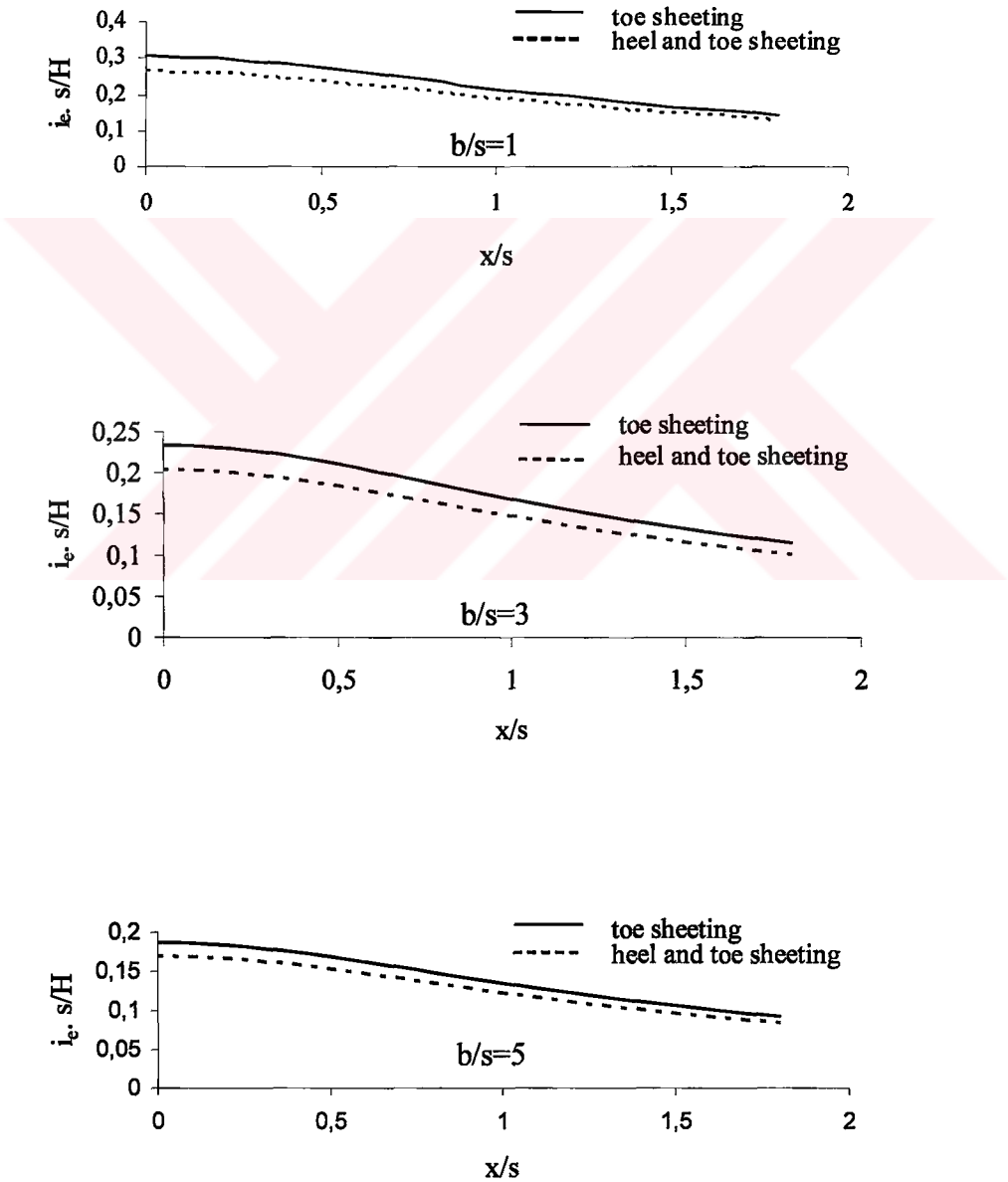


Fig. 7.5a Variation of exit gradient with x/s for $\theta = 0^\circ$ and $k_x/k_z=1$ and for different values of b/s

Table 7.3 Exit gradients along the downstream side, expressed as $i_e s/H$, for $\theta = 30^\circ$ and different b/s values

x/s	values of $i_e s/H$			% reduction	values of $i_e s/H$			% reduction	values of $i_e s/H$			% reduction
	$\theta = 30^\circ$, and $b/s = 1$				$\theta = 30^\circ$, and $b/s = 3$				$\theta = 30^\circ$, and $b/s = 5$			
	heel & toe sheet piles	toe sheet pile alone			heel & toe sheet piles	toe sheet pile alone			heel & toe sheet piles	toe sheet pile alone		
0	0	0	0	0	0	0	0	0	0	0	0	
0.1	0.0858	0.0956	-11.42		0.0659	0.0699	-6.07		0.0539	0.0568	-5.38	
0.2	0.1145	0.1275	-11.35		0.0879	0.0931	-5.92		0.0719	0.0757	-5.29	
0.3	0.1377	0.1534	-11.40		0.1058	0.1121	-5.95		0.0865	0.0911	-5.32	
0.4	0.1567	0.1745	-11.36		0.1204	0.1276	-5.98		0.0984	0.1036	-5.28	
0.5	0.1717	0.1913	-11.42		0.132	0.1398	-5.91		0.1079	0.1136	-5.28	
0.6	0.1829	0.2037	-11.37		0.1406	0.149	-5.97		0.115	0.1211	-5.30	
0.7	0.1902	0.2119	-11.41		0.1463	0.1551	-6.02		0.1197	0.1261	-5.35	
0.8	0.194	0.2162	-11.44		0.1494	0.1583	-5.96		0.1223	0.1288	-5.31	
0.9	0.1947	0.2171	-11.50		0.1501	0.1591	-6.00		0.1229	0.1294	-5.29	
1	0.1929	0.2153	-11.61		0.149	0.1579	-5.97		0.122	0.1285	-5.33	
1.1	0.1893	0.2114	-11.67		0.1464	0.1551	-5.94		0.12	0.1263	-5.25	
1.2	0.1843	0.2061	-11.83		0.1428	0.1513	-5.95		0.117	0.1233	-5.38	
1.3	0.1785	0.1998	-11.93		0.1384	0.1467	-6.00		0.1135	0.1196	-5.37	
1.4	0.1721	0.1929	-12.09		0.1337	0.1417	-5.98		0.1097	0.1156	-5.38	
1.5	0.1654	0.1857	-12.27		0.1287	0.1364	-5.98		0.1057	0.1114	-5.39	
1.6	0.1587	0.1785	-12.48		0.1237	0.1311	-5.98		0.1016	0.1071	-5.41	
1.7	0.1521	0.1714	-12.69		0.1187	0.1258	-5.98		0.0975	0.1029	-5.54	
1.8	0.1456	0.1644	-12.91		0.1138	0.1207	-6.06		0.0936	0.0987	-5.45	

The model can calculate the piezometric head distribution for any desired angle of inclination for the sheet piles where $0^\circ < \theta < 90^\circ$. However, for clear presentation, only three angles of inclination such as $\theta = 0^\circ$, 30° , and 45° are selected and drawn separately for three b/s ratios as 1, 3, and 5, respectively.

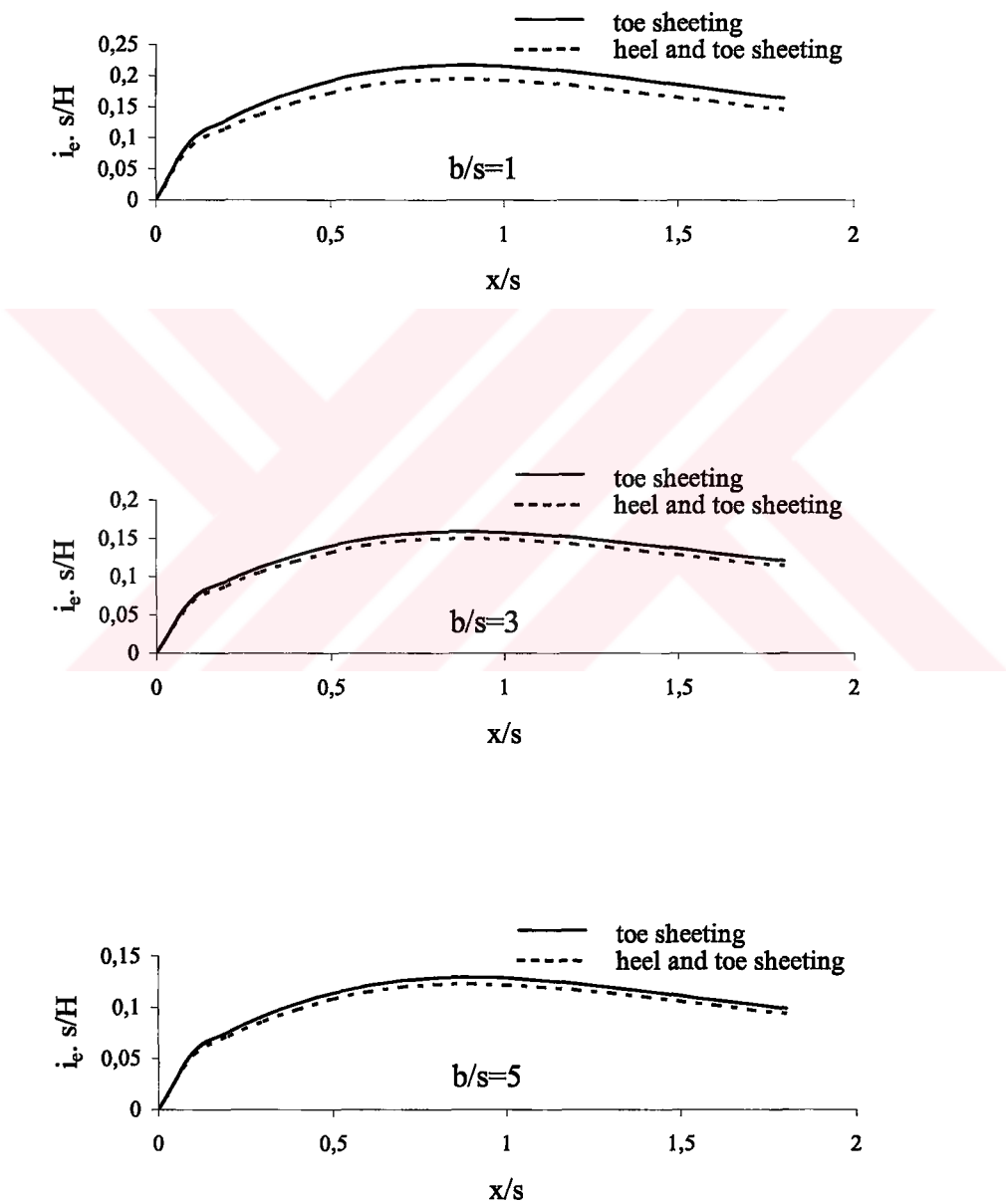


Fig. 7.5b Variation of exit gradient with x/s for $\theta = 30^\circ$ and $k_x/k_z=1$ and for different values of b/s

Table 7.4 Exit gradients along the downstream side, expressed as i_e s/H, for $\theta = 45^\circ$ and different b/s values

x/s	values of i_e s/H $\theta = 45^\circ$, and b/s = 1			% reduction	values of i_e s/H $\theta = 45^\circ$, and b/s = 3			% reduction	values of i_e s/H $\theta = 45^\circ$, and b/s = 5			% reduction
	heel & toe sheet piles	toe sheet pile alone			heel & toe sheet piles	toe sheet pile alone			heel & toe sheet piles	toe sheet pile alone		
	0	0	0	0	0	0	0	0	0	0	0	0
0.1414	0.0354	0.039	-10.16	0.0272	0.0293	-7.72	0.0223	0.0238	-6.72	0.0238	-6.72	
0.2828	0.0707	0.078	-10.32	0.0543	0.0585	-7.73	0.0446	0.0475	-7.73	0.0475	-6.50	
0.4242	0.1056	0.1164	-10.22	0.0811	0.0874	-7.76	0.0666	0.0709	-7.76	0.0709	-6.45	
0.5656	0.1384	0.1526	-10.26	0.1064	0.1146	-7.70	0.0873	0.093	-7.70	0.093	-6.52	
0.707	0.1664	0.1834	-10.21	0.128	0.1379	-7.73	0.105	0.1119	-7.73	0.1119	-6.57	
0.807	0.1813	0.1997	-10.14	0.1395	0.1503	-7.74	0.1145	0.122	-7.74	0.122	-6.55	
0.907	0.1913	0.2106	-10.0	0.1474	0.1587	-7.66	0.121	0.1289	-7.66	0.1289	-6.52	
1.007	0.1966	0.2163	-10.02	0.1516	0.1632	-7.65	0.1245	0.1326	-7.65	0.1326	-6.50	
1.107	0.1976	0.2173	-9.96	0.1526	0.1644	-7.73	0.1254	0.1336	-7.73	0.1336	-6.53	
1.207	0.1955	0.2147	-9.82	0.1512	0.1628	-7.67	0.1242	0.1324	-7.67	0.1324	-6.60	
1.307	0.191	0.2096	-9.73	0.1479	0.1593	-7.70	0.1216	0.1296	-7.70	0.1296	-6.57	
1.407	0.185	0.2028	-9.62	0.1435	0.1545	-7.66	0.118	0.1258	-7.66	0.1258	-6.61	
1.507	0.1781	0.195	-9.48	0.1384	0.149	-7.65	0.1139	0.1214	-7.65	0.1214	-6.58	
1.607	0.1707	0.1868	-9.43	0.1329	0.143	-7.59	0.1094	0.1166	-7.59	0.1166	-6.58	
1.707	0.1632	0.1784	-9.31	0.1272	0.137	-7.70	0.1048	0.1118	-7.70	0.1118	-6.67	
1.807	0.1558	0.1702	-9.24	0.1217	0.131	-7.64	0.1003	0.1069	-7.64	0.1069	-6.58	

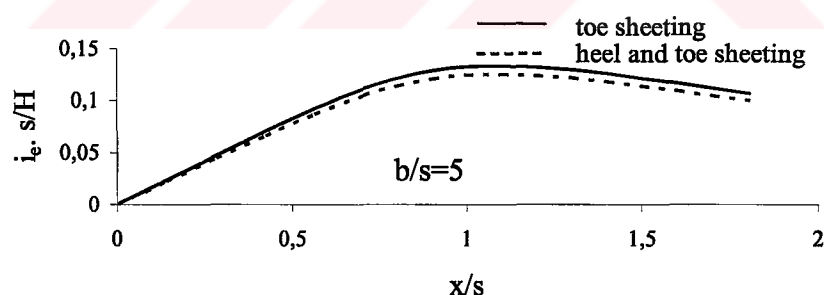
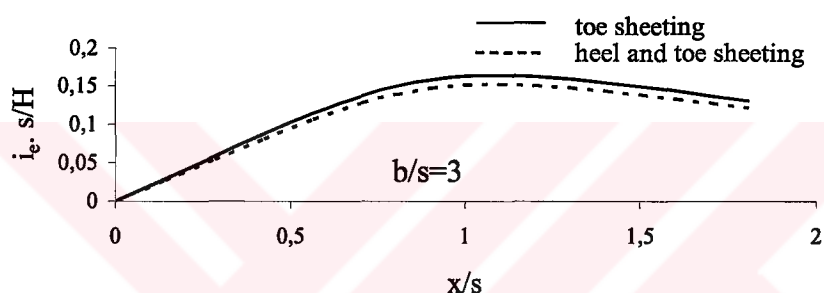
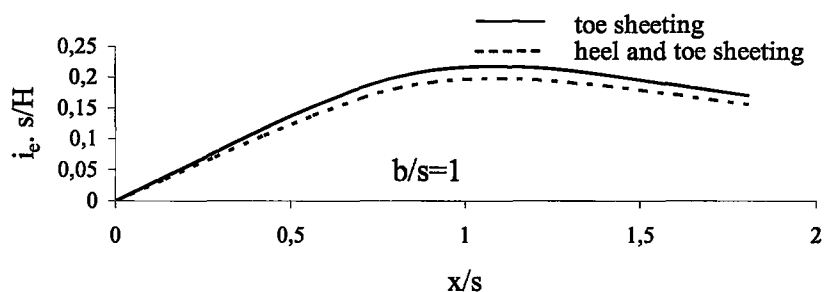


Fig. 7.5c Variation of exit gradient with x/s for $\theta = 45^\circ$ and $k_x/k_z=1$ and for different values of b/s

From Figs. 7.5a, 7.5b, and 7.5c, it appears that the largest, and hence the most critical exit gradient lies at the immediate toe when vertical sheet piles are used. However, for the situations of using inclined sheet piles this is not the case, on an average the location of critical exit gradient occurs at a distance equal to the sheet pile length away from the dam toe.

7.2.3 Uplift Pressure

The uplift pressure at the inner corner of the boundary node between the toe sheet pile and the dam base (point C in Fig. 7.1), expressed as the ratio of the piezometric head to the total head difference, is collected from the computer results and tabulated as shown in Table 7.5. These results are drawn against the angle of inclination of the sheet pile as shown in Fig. 7.6. Every curve in that figure is derived numerically for a certain ratio of base width to sheet pile depth (i.e., for $b/s = 1, 3$, and 5). The uplift pressures from the case of a toe sheet pile alone are also inserted in the same figure. The general effect of the upstream sheet pile on the uplift pressure at point C for a particular θ and b/s can be found by comparing the two curves in each figure.

Table 7.5 Uplift pressures at the inner corner of the boundary node between the dam base and the toe sheet pile, expressed as h_c/H , for different θ and b/s values

θ	values of h_c/H					
	toe sheet pile alone			heel and toe sheet piles		
	b/s			b/s		
	1	3	5	1	3	5
0°	0.77	0.52	0.42	0.66	0.459	0.382
30°	0.71	0.48	0.38	0.631	0.456	0.368
45°	0.66	0.46	0.36	0.582	0.425	0.345

From Figs 7.5 and 7.6, for the case of a flat dam base with symmetrical inclined heel and toe sheetings, the overall trend indicates a small reduction in exit gradient and uplift pressure values as compared with the case of a toe sheet pile alone. The percentage reduction in the exit gradient and uplift pressure values for the cases of using a toe sheet pile alone and heel and toe sheetings are calculated and tabulated as shown in Table 7.6.

Table 7.6 Percentage reduction in exit gradient and uplift pressure between the cases of a toe sheet pile alone and heel and toe sheetings

θ	% reduction of $i_e, s/H$			% reduction of h_c/H		
	$b/s=1$	$b/s=3$	$b/s=5$	$b/s=1$	$b/s=3$	$b/s=5$
0°	17	14	10	14	11	10
30°	11	6	5	11	5	7
45°	10	7	6	9	3	4

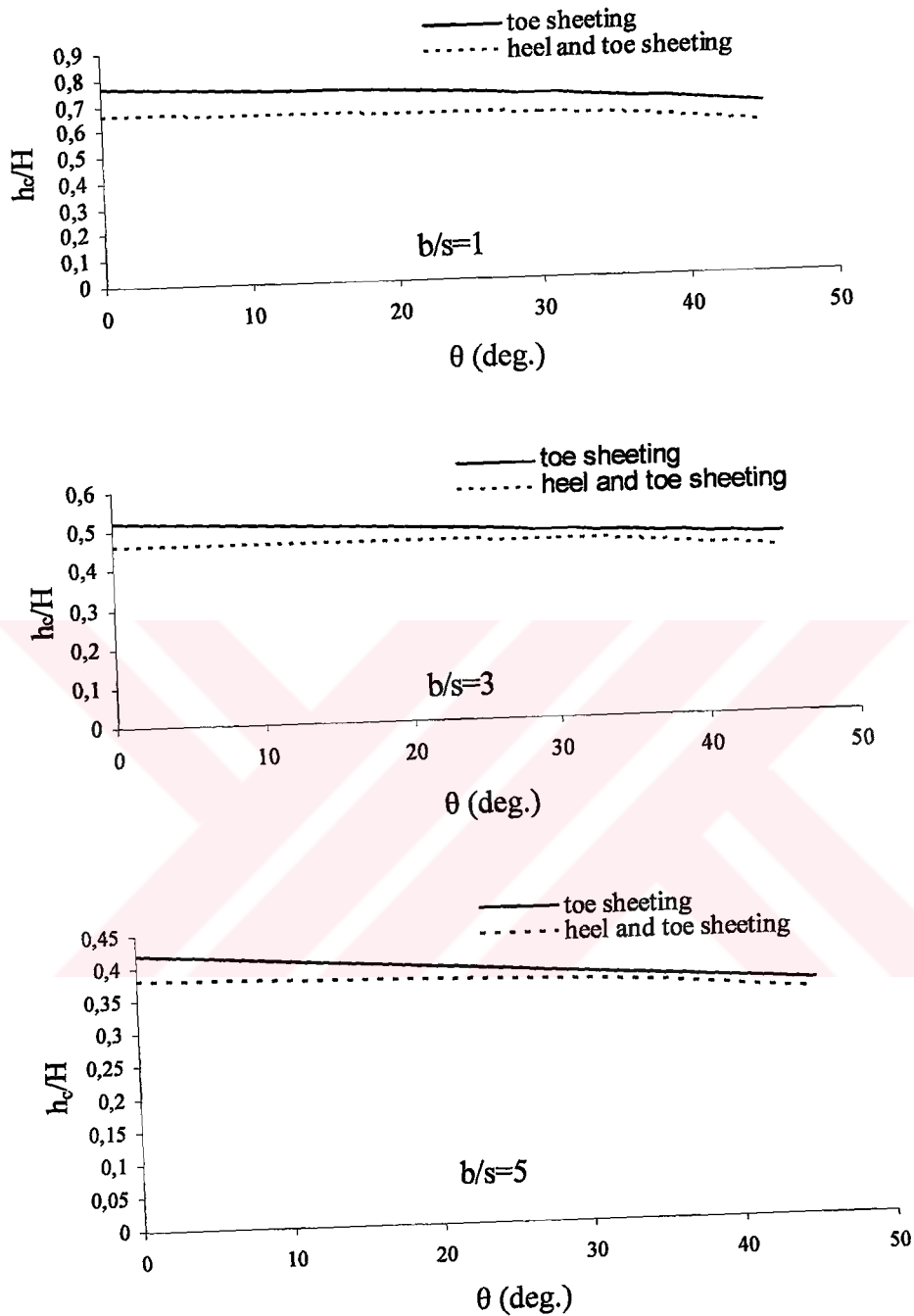


Fig. 7.6 Variation of h_c/H for different θ and b/s values, where $k_x/k_z=1$

From Table 7.6 it is seen that the difference in results is relatively small. Therefore, it could be argued that the analysis of a dam base with only an inclined toe sheet pile is simpler, computationally less, and provides a reasonable design from the practical and economical point of views.

However, it shows more reduction in exit gradient and uplift pressure, the results of the dam flanked on either side by symmetrical inclined sheet pilings yield similar characteristic trends as those for a flat dam base with an inclined toe sheet pile alone. Still, the main conclusions derived from the piling at toe are also applicable to the piling at heel and toe.

7.3 Effectiveness of Different Cut-offs in Reducing Exit Gradients

In the present section, the numerical results of the exit gradients along the downstream side for a dam with: a depressed base, vertical toe sheet pile, vertical heel and toe sheet piles, inclined toe sheet pile ($\theta = 30^\circ$), and inclined heel and toe sheet piles ($\theta = 30^\circ$) are assembled in Table 7.7. In all of the previously selected dam base configurations, the seepage zone is assumed to be homogenous and isotropic where $FD > 2b$. Moreover, the ratios of b/s and b/d are kept constants in case of a dam with sheet pile(s) or with a depressed base, respectively. A collection of the corresponding exit gradient diagrams is prepared from these results as shown in Fig. 7.7. In this figure, the ordinate represents the exit gradient along the downstream side and is computed as $i_e \cdot s / H$, however, the abscissa is considered as the distance along the downstream side measured from the dam toe and computed as x/s . Fig. 7.7 offers a direct comparison between those configurations and shows the effect of the cut-off type in reducing the exit gradients along the downstream side.

Observation of Fig. 7.7 for a dam with heel and toe sheetings, vertical or inclined, reveals that there is slightly larger dissipation of head and consequently a small reduction of exit gradients in comparison with toe sheeting alone. The result is that combined heel and toe sheeting acts much like toe sheeting alone with little gain in effectiveness. Moreover, the resulting loss of head in the case of using a dam with a depressed base is much greater than with vertical heel and toe sheetings having the same ratio of base width to cut-off depth, which served to reduce the exit gradients along the downstream side. The explanation is apparently to be found in the fact that the water flowing under the dam tends to concentrate as much as possible along the shortest path. With the depressed base, this flow is restricted by the structure from diffusing upward and, hence, follows largely a narrow strip along the base. With only

sheeting in this same depth, the flow lines can spread both upward and downward between the lines of sheeting.

Table 7.7 Exit gradients for different dam base configurations

x/s	$i_e s/H$				
	Depressed base	Toe vertical sheeting	Toe and heel vertical sheetings	Toe inclined sheeting	Toe and heel inclined sheetings
0	0.1422	0.1968	0.17	0	0
0.1	0.1416	0.1959	0.1692	0.0597	0.0539
0.2	0.1401	0.1935	0.1669	0.0796	0.0719
0.3	0.1376	0.1896	0.1634	0.0959	0.0865
0.4	0.1345	0.1845	0.1588	0.1092	0.0984
0.5	0.1308	0.1786	0.1534	0.1198	0.1079
0.6	0.1269	0.1722	0.1476	0.1279	0.115
0.7	0.1227	0.1655	0.1414	0.1334	0.1197
0.8	0.1184	0.1587	0.1352	0.1366	0.1223
0.9	0.1142	0.152	0.1291	0.1378	0.1229
1	0.1101	0.1455	0.1231	0.1373	0.122
1.1	0.106	0.1392	0.1173	0.1356	0.12
1.2	0.1022	0.1332	0.1118	0.133	0.117
1.3	0.0985	0.1276	0.1065	0.1297	0.1135
1.4	0.095	0.1222	0.1016	0.126	0.1097
1.5	0.0917	0.1172	0.0969	0.1221	0.1057
1.6	0.0885	0.1125	0.0925	0.1181	0.1016
1.7	0.0855	0.108	0.0884	0.1141	0.0975
1.8	0.0826	0.1038	0.0845	0.1102	0.0936

For structures with vertical toe or heel and toe sheet pile(s) or with a depressed base having vertical sidewalls, it is clear that the exit gradient at the immediate toe has a maximum value and decreases with distance along the downstream side of the riverbed. However, this is not the case with the inclined toe or heel and toe sheet piles, which show a zero exit gradient at the immediate toe and an increase with distance along the downstream side until it reaches a maximum at a distance equal to approximately the sheet pile length. The exit gradient is then decreases with distance along the downstream riverbed. Attention is called to the almost negligible difference in exit gradients calculated from any of the previous dam base configurations after a

distance $x/s = 1$ (i.e. after a distance greater than approximately the sheet pile length). This proves the efficiency of the inclined toe sheet piles in increasing the factor of safety against piping and shifting the danger of such a phenomenon further downstream from the toe of the structure.

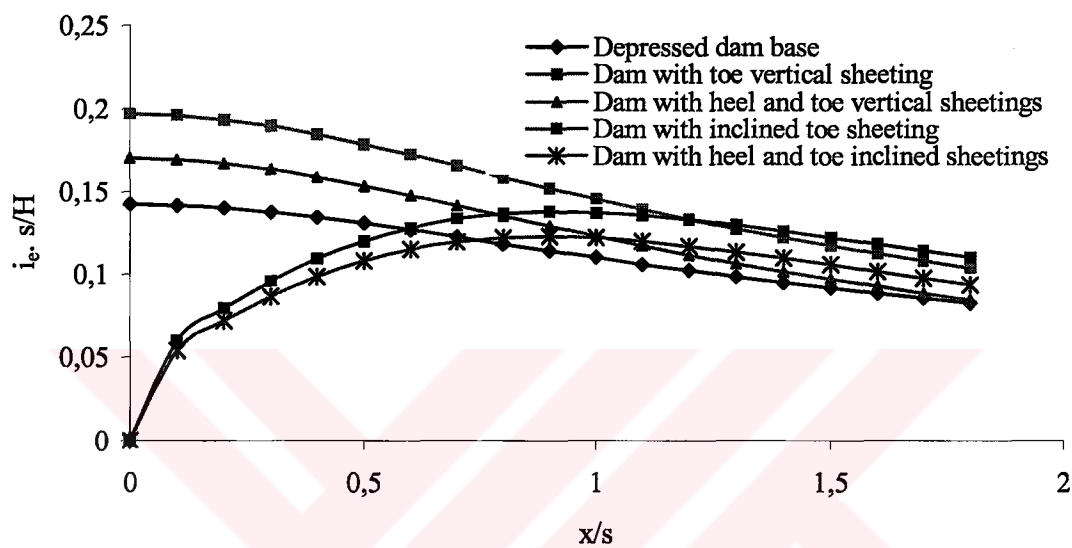


Fig. 7.7 Comparison of exit gradient diagrams for different dam base configurations

8. SUMMARY AND CONCLUSIONS

In the study of confined seepage through pervious soils beneath water retaining structures, three important quantities need to be assessed by the designers, namely, seepage quantity, exit gradient, and uplift pressure. These quantities play a great role in determining the safety of such structures against failure due to piping or uplift. Cut-offs like depressed bases or sheet piles can be provided to reduce uplift pressure and seepage forces, resulting in an appreciable saving in dimensions of the structure. Many researchers have examined this type of flow (confined seepage under hydraulic structures) and gave solutions for various problems including the cases of a depressed dam base with vertical sidewalls, a flat bottom dam with a toe sheet pile (vertical or inclined), and a flat bottom dam with vertical symmetrical heel and toe sheet piles. These solutions are based on the assumption of a homogeneous and isotropic foundation soil having an infinite depth. However, this assumption is usually not met depending on various in situ conditions that possess anisotropy, heterogeneity, finite soil depth, etc. It appears that no solution is available in the literature to determine the effect of using inclined sidewalls or sheet piles with different anisotropy ratios and foundation soil depths on seepage flow under the previously mentioned dam base configurations. Numerical methods such as finite difference can solve these problems easily and accurately on a case-by-case basis.

Therefore, on the basis of finite difference approximations, a two-dimensional numerical model is developed to solve the governing equations of groundwater seepage under each of these structures. The goal was to study the effects of inclined cut-offs, permeability ratio, and foundation soil depth on exit gradient, uplift pressure and flow rate. The model calculates the piezometric head at all nodal points in the problem solution domain. Using these head values, the exit gradient, uplift pressure and flow rate are determined. The model results indicated that the finite difference technique gives comparable values for similar cases solved with other solution methods and offers a more general approach that permits the usage of the desired

anisotropy ratio and foundation soil depth. From the model results, the following remarks can be concluded:

For the Depressed Dam Base:

- 1- The exit gradient at the dam toe decreases as the slope angle of the base sidewalls increases, and becomes steeper in stratified than in homogeneous and isotropic soil,
- 2- In computing dam safety against piping for a depressed base having sloping surfaces, the creep distance along the sloping surfaces within the dam base should be given different weights in accordance with their slope angle, θ , ratio of base to foundation soil depths, d/FD , and permeability ratio, k_x/k_z . The weighted creep distance along the bottom profile of the depressed dam base would be better defined, if it is taken as the sum of the horizontal creep distances divided by $\sqrt{k_x/k_z}$ plus the sum of the sloping surfaces multiplied by their corresponding correction factors, λ ,
- 3- The plot of the correction factors, λ , of the sloping surfaces within a depressed base versus the angles of the base sidewalls, θ , shows that the variation is in the form of a sine curve, for $0^\circ \leq \theta \leq 90^\circ$. For calculating the creep length, a general formulation including a new parameter, λ , is obtained. This parameter is a function of the angle of the sidewalls, the ratio of horizontal to vertical permeability, and the ratio of dam base to foundation soil depths. For a special case of k_x/k_z and FD , the correction factor, λ , is found as, $\lambda = \sin\theta$ (for k_x/k_z equal to 10 and $FD > 2b$, b is the dam base width),
- 4- A comparison of the results for the calculation of the weighted creep length, L_w , using Lane's method and the approach of the present study, to the same two-dimensional problem, shows that the % differences between these solutions become greater (with a maximum value of 35%) as θ is around 45° . However, as θ approaches 0° or 90° , the % differences decrease and reach its minimum (with a maximum value of 4.6%). The creep length calculated using the approach of the present study is longer than that using Lane's method in case of $0^\circ < \theta < 45^\circ$

and shorter in case of $45^\circ < \theta < 90^\circ$. That is because, in the present study, the sloping sidewalls of the dam base are considered to offer more resistance to flow than purely horizontal ones and corresponding to their slope angle,

- 5- Stratification, in general, reduces the seepage discharge as compared with that of a homogeneous and isotropic soil. Such soil conditions reduce the safety factor against piping, if compared with the safety of a similar structure on homogeneous and isotropic soil, which yield the same amount of seepage,
- 6- The seepage discharge is found to be insensitive to changes in the sidewall angle, θ . However, it increases as the soil depth, FD , increases. The increment becomes very small as FD becomes greater than $2b$. The relatively small difference of seepage discharge for $FD > 2b$ is due to the tendency of the flow to concentrate within a narrow strip along the bottom profile of the dam base, and
- 7- The depressed dam base configuration which gives the highest safety factor against piping and lowest seepage discharge is that for θ equal to 90° . The effect of the impermeable sub-soil in the calculation of the uplift pressure under such a dam base could be disregarded for values of $FD/b \geq 2$. Moreover, increasing the permeability ratio causes a decrease in the uplift pressure at the upstream and an increase at the downstream sides of this base. The designer must bear this point in mind, when calculating the required weight of the solid dam or apron.

For Flat Dam Base with an Inclined Toe Sheet Pile:

- 1- The exit gradient, after a distance $x/s \approx 0.85$ downstream from the dam toe, decreases with increasing the sheet pile inclination and the reduction increases as θ increases. The result is an increase in the safety factor against piping phenomenon,
- 2- Using an inclined sheet pile towards the downstream side with θ less than approximately 30° is beneficial in increasing such a safety factor. On the other hand, the danger from undermining will be shifted further downstream from the toe of the structure. The uplift pressure at the inner corner between the toe sheet pile and the dam base, expressed as a percent of the total head, decreases as the sheet pile inclination increases. However, this reduction in pressure is small,

- 3- Placing the sheet pile at the dam heel is not recommended under any angle of inclination. The same conclusion is also valid for the case of toe sheeting inclined towards the upstream side. Doing so produces a singularity at the end of the dam, with a mathematically infinite velocity. This will produce unstable soil condition in close proximity to the toe,
- 4- Considerable differences are observed in exit gradients when the numerical results, using the proposed anisotropy ratios, are compared with that for a homogeneous and isotropic soil. These differences may be approximately doubled for anisotropy ratio of $k_x/k_z = 10$. The exit gradients and uplift pressures are found to increase as the anisotropy ratio increases,
- 5- The exit gradient is seen to increase with increasing the foundation soil depth. However, the variations may be neglected for $s/FD < 0.2$. Moreover, the pressure drop at some key points will not be appreciably affected by the finite thickness of the permeable stratum unless the sheet pile penetrates through more than 50 % of the thickness of the stratum, and
- 6- The developed design charts for this dam base could be used efficiently to approximate the exit gradient, and the uplift pressure values. Unlike the available charts, which are based on a homogeneous and isotropic soil of infinite depth, the design charts presented herein can account for variable anisotropy ratios and depths of strata.

For Flat Dam Base with Symmetrical Inclined Heel and Toe Sheeting:

- 1- Analytical solution or approximation for the exit gradient and uplift pressure in the case of a flat bottom dam flanked with symmetrical inclined sheet piles on either end are non-existent. Such a solution, for the case of homogeneous and isotropic soil, is presented in this thesis using the finite difference technique,
- 2- The largest, and hence the most critical exit gradient occurs at the immediate toe when $\theta = 0^\circ$. However, for $0^\circ \leq \theta \leq 90^\circ$ this is not the case, on average the location of critical exit gradient occurs at a distance equal to the sheet pile length away from the dam toe.

- 3- For the case of symmetrical heel and toe sheetings (vertical or inclined), the overall trend indicates a small reduction in exit gradient and uplift pressure values as compared with the case of a toe sheet pile alone. The maximum percentage reduction in the exit gradient and uplift pressure values for the cases of using a toe sheet pile alone and heel and toe sheet piles are found to be 17 % and 14% respectively. Since the difference in results between these two configurations is not great, it could be argued that the analysis of a dam base with only an inclined toe sheet pile is simpler, computationally less time consuming, and provides a reasonable design from the practical and economical point of views, and finally,
- 4- For the same angle of inclination, the results obtained from the dam flanked on either side by symmetrical inclined sheet piling yield similar characteristic trends as those for a flat dam base with an inclined toe sheet pile alone but with different values. Still, the main conclusions derived from the piling at toe alone are also applicable to the piling at heel and toe.

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