ISTANBUL TECHNICAL UNIVERSITY ★ INSTITUTE OF SCIENCE AND TECHNOLOGY

A KALMAN FILTER APPROACH TO MULTISITE PRECIPITATION MODELING IN METEOROLOGY

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METEOROLOJİDE ÇOK İSTASYONLU YAĞIŞ MODELİNE KALMAN FİLTRESİ YAKLAŞIMI

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LIST OF ABBREVIATIONS

1-L	: One Layer
2-L	: Two Layer
AR	: Autoregressive
ARIMA	: Autoregressive Integrated Moving Average
ARMA	: Autoregressive Moving Average
EKF	: Extended Kalman Filter
EnKF	: Ensemble Kalman Filter
FLKS	: Fixed-Lag Kalman Smoother
KF	: Kalman Filter
MA	: Moving Average
MIT	: Massachusetts Institute of Technology
NWP	: Numerical Weather Prediction
OI	: Optimal Interpolation
QGM	: Quasigeostrophic Model
RIAS	: Research Institute for Advanced Study
SH	: Sum-of-Harmonics
SOSs	: Suboptimal Schemes
SWM	: Shallow-Water Model
WMO	: World Meteorological Organization

LIST OF SYMBOLS

X_k	: random variable at time t_k
X_{k-1}	: random variable at time t_{k-1}
φ	: parameter relating X_k to X_{k-1}
W_{k-1}	: white noise with zero mean and variance Q
$\hat{X}_{k/k-1}$	: initial estimate
$e_{k/k-1}$	: initial estimation error
$P_{k/k-1}$	: variance of estimation error
E	: expected value operator
Z	: noisy measurement of X
V	: white noisy with zero mean and variance R
H	: measurement parameter
Z_k	: noisy measurement of X at time t_k
$\hat{X}_{k/k}$	: updated estimate
K_k	: Kalman gain
$P_{k/k}$	: variance of updated estimation error
$\hat{X}_{k+1/k}$	: projected ahead estimate
$P_{k+1/k}$	: variance of projected ahead estimation error
$\mathbf{X}_k$	: state vectors at time t_k
$\mathbf{X}_{k-1}$	: state vectors at time t_{k-1}
Φ_{kk-1}	: transition matrix relating $\mathbf{X}_{k-1}$ to $\mathbf{X}_k$
$\mathbf{W}_{k-1}$	: white sequence noise vector with known covariance structure
$\mathbf{Q}$	: system noise covariance matrix
$\mathbf{Z}_k$	: measurement vector at time t_k
$\mathbf{H}_k$	: connection matrix, between the measurement and the state vectors at time t_k
$\mathbf{V}_k$	: white noise sequence with known covariance structure
$\mathbf{R}$	: measurement noise covariance matrix
$\hat{\mathbf{X}}_{k/k-1}$	: initial estimation matrix
$\mathbf{e}_{k/k-1}$	: initial estimation error matrix
$\mathbf{e}^T_{k/k-1}$	: transpose of initial estimation error
$\mathbf{P}_{k/k-1}$	: error covariance matrix associated with initial estimate

$\hat{\mathbf{X}}_{k/k}$	: updated estimate
$\mathbf{e}_{k/k}$	: updated estimation error
$\mathbf{e}^{T}_{k/k}$	: transpose of updated estimation error
$\mathbf{P}_{k/k}$	: error covariance matrix associated with the updated estimate
$\mathbf{K}_{k}$	: Kalman gain matrix
$\mathbf{H}^{T}_{k}$	: transpose of the connection matrix
$\mathbf{K}^{T}_{k}$	: transpose of the Kalman gain matrix
$\hat{\mathbf{X}}_{k+1/k}$	: projected ahead estimate
$\mathbf{e}_{k+1/k}$	: projected ahead estimation error
$\mathbf{P}_{k+1/k}$	: error covariance matrix associated with the projected ahead estimate
Φ_{k+1k}	: transition matrix relating $\mathbf{X}_k$ to $\mathbf{X}_{k+1}$
Φ^{T}_{k+1k}	: transpose of transition matrix relating $\mathbf{X}_k$ to $\mathbf{X}_{k+1}$
A, B	: square matrices
C	: symmetric matrix
s	: scalar quantity
f	: vector whose components f_{ix} are non-linear functions of $\mathbf{X}_1$, $\mathbf{X}_2$, …, etc.
$\mathbf{X}_{nom}$	: nominal value of $\mathbf{X}$
X_t	: stochastic variable at time t
X_{t-1}	: stochastic variable at time t-1
ε	: pure-random variable having zero mean and finite variance
$b_0, b_1, ..., b_m$	: weights with $\sum b_j^2$ being convergent usually equal to one
M	: extent of the moving average
$\overline{X}$	: mean of X
A_j and B_j	: Fourier coefficients or amplitudes
f	: the fundamental frequency
$2\pi fjt$	: cyclicity with j=1, 2, …, m
m	: total number of cycles involved in the model
ε_t	: pure-random component
a_1	: Markov coefficient
a_j	: autoregression coefficient with j = 1, 2, …., m
n	: order of the model

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A KALMAN FILTER APPROACH TO MULTISITE PRECIPITATION MODELING IN METEOROLOGY

SUMMARY

Precipitation is characterized by variability in space and time. In addition, there are many factors affecting the magnitude and distribution of precipitation, such as altitude, various air mass movements, distance from the moisture sources, temperature, pressure, and topography. The magnitude and distribution of precipitation vary temporally and spatially even in small areas. However, the precipitation predictor should not be fixed with time and space, but adapt itself to the evolving meteorological conditions. Describing and predicting the precipitation variability in space and/or time are fundamental requirements for a wide variety of human activities and water project designs.

Forecasting models can be classified into two categories, those models that have fixed parameters and variances, and likewise another group of models with varying parameters and variances. Models with fixed parameters require stationarity in both the mean and variance throughout the entire range of observations. That is why so much effort is spent to make the data stationary in the mean and variance. Otherwise, the results are not meaningful from a statistical point of view. For example, when the data pattern changes as with a step or trend, or when there are transient shifts, classical statistical theory will treat those as random effects or temporary shifts. If the changes are continuous, a new forecasting model will have to be specified to deal with the new equilibrium conditions. However, the model will be good for those new equilibrium conditions only when fixed patterns exist. Kalman filter **(KF)**, can deal with step changes and transient situations because they update their parameters in a way that takes account of changes in pattern.

The objective of this thesis is to investigate and develop a **KF** model approach to multisite precipitation modeling. In order to see the effectiveness of the **KF** model developed in this thesis, 30 year records (1956-1985) of annual rainfall for the 52 different meteorology stations are used, and these stations are distributed approximately covering all of Turkey with more concentration in the northwestern part. The necessary contour maps of observed and estimated precipitation amounts are attuned through the results of the software developed during the course of this study. Furthermore, regional error distribution maps are also attuned for any year. The results indicates that **KF** provide an efficient method for modelling annual rainfall in both time and space dimensions.

METEOROLOJİDE ÇOK İSTASYONLU YAĞIŞ MODELİNE KALMAN FİLTRESİ YAKLAŞIMI

ÖZET

Yağış karaktaristik olarak zaman ve konumla değişir. Ayrıca, yağışın şiddet ve dağılımını etkileyen birçok etken vardır. Bunlar arasında yükseklik, değişik hava kütlelerinin hareketi, nem kaynağından olan uzaklık, sıcaklık, basınç ve topografya gelir. Küçük alanlar üzerinde bile yağışın şiddet ve dağılımı alansal ve zamansal olarak değişir. Ancak, yağış tahmin edicileri, zaman veya konum ile sabitleştirilmemeli, fakat değişen meteoroloji şartlarına göre kendisini yenileyebilmelidir. Birçok insan faaliyeti ve su kaynakları tasarımları için yağış değişkenliğinin zaman ve konumla önceden tahmin edilebilmesi temel ihtiyaçlar arasında gelir.

Tahmin yöntemleri biri sabit parametre ve varyanslı diğeri ise değişken parametre ve varyanslı olmak üzere iki grupta sınıflandırılabilir. Sabit parametreli olanlar, yapılan gözlemlerin tümünde ortalama ve varyans olarak durağan (stasyoner) olması gerekir. İşte bu nedenle verilen bir veri dizisinin ortalama ve varyans bakımından durağan hale dönüştürülmesine çalışılır. Aksi taktirde varılan sonuçlar istatistik bakımından anlamlı olmaz. Mesela, veri gidişi bir basamak veya trend olarak değişiklik gösterirse, veya geçici kaymalar bulunuyorsa klasik istatistiksel yöntemler bunları geçici kaymalar veya rastgele etkiler olarak algılayarak işler. Eğer değişimler sürekli olursa, yeni denge durumları için yeni tahmin modellerinin geliştirilmesi gerekir. Eğer sabit paternler söz konosu olursa yeni modeller yeni denge şartları için geçerli olur. Kalman filtreleri **(KF)** adım adım değişiklikleri modellemede ve geçici durumlarda başarılıdır. Çünkü parametrelerini değişen durumlara göre ayarlar.

Bu tezin amacı çok istasyondaki yağışların modellenmesi yaklaşımına **KF** geliştirilerek uygulanmasıdır. Bu tezde geliştirilen **KF** etkinliğini görebilmek için 52 ayrı meteoroloji istasyonunda 1965-1985 yılları arasında yapılan 30 yıllık kayıtlardan yararlanılmıştır. Bu istasyonlar, çoğunlukla Türkiye'nin kuzeybatı kısımlarında olmak üzere oldukça üniform bir şekilde yayılmıştır. Bu çalışma esnasında geliştirilmiş yazılım sayesinde gözlenmiş ve tahmin edilmiş yağışların eş yağış eğrisi haritaları çizilerek kıyaslamalar yapılmıştır. Buna ilave olarak bölgesel hata dağılımı haritaları da her sene için geliştirilmiştir. Elde edilen sonuçlar **KF** zaman ve uzay boyutlarında yağışları modellemede etkin olduğunu gösterir.

1. INTRODUCTION

1.1 Overview

Estimation is the process of extracting information from data, which can be used to infer the desired information and may contain errors. An optimal estimator is a computational algorithm that processes measurements to deduce a minimum error estimate of the state of a system by utilizing knowledge of the system and measurement dynamics, assumed statistics of system noises and measurement error, in addition to conditional initial information. At the beginning of the last century, Gauss developed estimation theory, in order to determine the orbits of comets from the few available astronomical observations. In essence, the atmospheric data assimilation problem is just a much larger version of Gauss's problem. In fact, modern methods of data assimilation such as Kalman filters, can be directly traced to the ideas of Gauss.

As shown in Figure 1.1 the three types of estimation problems are of interest:

1. As shown in Figure 1.1.a when the time at which an estimate is desired coincides with the last measurement point, the problem is referred to as filtering (estimating the state vector at the current time, based on all measurements taken up to the current time),
2. When the time of interest falls within the span of available measurement data, the problem is termed smoothing (estimating the value of the state at some prior time), (see Figure 1.1.b)
3. When the time of interest occurs after the last available measurement, the problem is then called prediction (estimating the state vector at a future time), (see Figure 1.1.c)

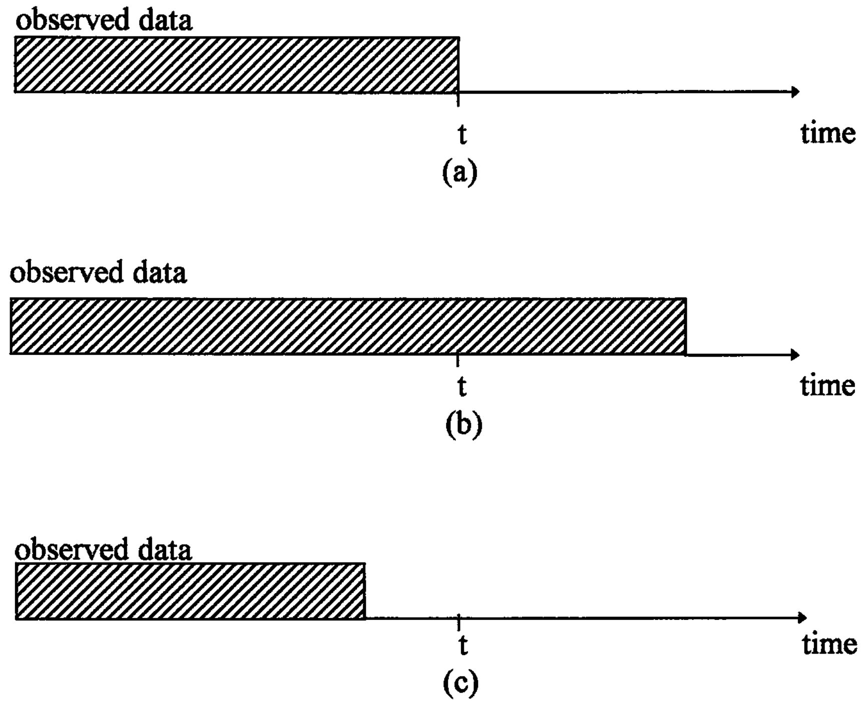

Figure 1.1 Three types of estimation problems (a) filtering, (b) smoothing, (c) prediction.

1.2 Historical Perspective

The development of data estimation methods can be traced back to Gauss (1809), who invented the technique of deterministic least-squares approach and employed it in a relatively simple orbit measurement problem. The next significant contribution to the extensive subject of estimation theory occurred more than 100 years later when Fisher (1912), working with probability density function, introduced the approach of maximum likelihood estimation. However, Wiener (1942, 1949) set forth a procedure for the frequency domain design of statistically optimal filters. The technique addressed the continuous-time problem in terms of correlation functions and the continuous filter impulse response. Moreover, the Wiener solution does not lend itself very well to the corresponding discrete-data problem nor is it easily extended to more complicated time-variable, multiple-input/output problems. It was limited to statistically stationary processes and provided optimal estimates only in the steady-state regime. In the same time period, Kolmogorov (1941) treated the discrete-time problem. He was primarily interested in the mathematics rather than in the concepts inherent in the problem. Kolmogorov's solution was difficult as well as elegant. Therefore, mathematicians automatically assumed that his formulation must be a relevant one. Kalman (1960) re-examined the Wiener-Kolmogorov theory of filtering and prediction by using the Bode-Shannon representation of random processes and the state-transition method of analysis in a dynamic systems. The new results are :

1. The formulation and methods of solution of the problem apply without modification to stationary and nonstationary statistics and to growing-memory and infinite-memory filter,
2. A nonlinear differential equation is derived for the covariance matrix of the optimal estimation error. From the solution of this equation the coefficients of the differential equation of the optimal linear filter are obtained without further calculations,
3. The filtering problem is shown to be the dual of the noise-free regulator problem. This new method applied to well-known problems, confirming and extending earlier results.

About a year after Kalman and Bucy (1961) presented a paper on continuous filtering which proved to be a turning point in the area of optimal filtering. Kalman filtering **(KF)** has been widely used in many areas of industrial and government applications such as video and laser tracking systems, satellite navigation, ballistic missile trajectory estimation, radar, and fire control. With the recent development of high-speed computers, **KF** has become more useful even for complicated real-time applications. Later, **KF** found applications in such varied disciplines as the environmental and earth sciences, hydrology, economics and the social sciences. Unfortunately, its use in the domain of atmospheric sciences and meteorology is rather few but increasing significantly in recent years. For more than 25 years, **KF** has been an established technique in the design and operation of real-time systems. This powerful technique has also been applied to various hydrologic problems, following the pioneering work by Hino (1973). The **KF** has been applied to water-related problems among many others by, Todini and Bouillot (1975), Szöllösi-Nagy (1976), Szöllösi-Nagy et al. (1977), Chiu (1978), Şen (1980a, 1980b, 1984), O'Connell and Clarke (1981) and Bras and Rodriguez-Iturbe (1985).

1.3 Definition of Filtering

Human beings have been filtering things practically for the entire history. Water filtering is a simple example. Foreign matter can be filtered from water as simply as by using our hands to skim dirt and leaves off the top of the water. Another example is filtering out noise from our surroundings. If we paid attention to all the little noises around us we would go crazy. We learn to neglect needless sounds (traffic, appliances, etc.) and get detail on important sounds, like the voice of the person.

There are also many examples in engineering where filtering is desirable. Radio communication signals are often corrupted with noise. A good filtering algorithm can remove the noise from electromagnetic signals while still keeping the useful information. Another example is voltages. Many countries require in-home filtering of line voltages in order to power personal computers and peripherals. Without filtering, the power fluctuation would extremely shorten the life length of the devices.

The word "filter" is a relic from the early history of electrical engineering and it is concerned with the extraction of signals from noise. Kalman (1978) defined the filtering as any mathematical operation which uses past data or measurements on a given dynamical system (that is, systems which vary with time) to make more accurate statements about present, future, or past variables in that system. When and how this can be done is the concern of a new subscience of system theory, usually called filtering (or estimation) theory.

KF is an optimal state estimation process applied to a dynamic system that involves random perturbations. The atmosphere can be regarded as an uncertain dynamical system (Dee, 1991). More precisely, **KF** gives a linear, equitable, and minimum error variance recursive algorithm to optimally estimate the unknown state of a dynamic system from noisy data taken at continuous or discrete real-time intervals.

1.4 Problem Statement

Precipitation is characterized by variability in space and time. In addition, there are many factors affecting the magnitude and distribution of precipitation, such as altitude, various air mass movements, distance from the moisture sources, temperature, pressure, and topography. The magnitude and distribution of precipitation vary from place to place and from time to time even in small areas. Describing and predicting the precipitation variability in space and/or time are fundamental requirements for a wide variety of human activities and water project designs.

Forecasting models can be classified into two categories, those models that have fixed parameters and variances, and likewise another group of models with varying parameters and variances (Makridakis et al. 1983). Models with fixed parameters require stationarity in both the mean and variance throughout the entire range of observations. That is why so much effort is spent to make the data stationary in the mean (through differencing) and variance (through appropriate power logarithmic, square root, etc. transformations).

Otherwise, the results are not meaningful from a statistical point of view. Classical statistical estimation theory has not been able directly with nonstationarity, and this can cause problems of significant practical consequences. For example, when the data pattern changes as with a step or trend, or when there are transient shifts, classical statistical theory will treat those as random effects or temporary shifts. If the changes are continuous, a new forecasting model will have to be specified to deal with the new equilibrium conditions. However, the model will be good for those new equilibrium conditions only when fixed patterns exist.

Classical statistical methods must be used in conjunction with other control processes (Page, 1957, 1961, and Barnard, 1959), if permanent or significant changes in the data are to be identified. Methods based on classical statistics cannot sense a shift by themselves and once a shift has taken place, the model will not do well, because it will still be tuned to the specification of the old data set .

Adaptive-response-rate exponential smoothing, and filtering, on the other hand, can deal with step changes and transient situations because they update their parameters in a way that takes account of changes in pattern. Furthermore, they can deal with changes in trend better than fixed model / fixed parameter methods. However, even these two methods cannot do as well as the **KF**, which can deal with variable models, parameters, and variances all together.

1.5 The Aim of Thesis

KF is the most general approach to statistical estimation and prediction. It has been shown by Harrison and Stevens (1975a) that all forecasting methods are special cases of **KF**. This filter can deal with changes in the model, the parameters, and the variances. The difficulty with **KF** is that many technical questions have not yet been answered satisfactorily. The approach itself has grown out of engineering practices. Consequently, many statisticians and operation researchers know little about it, or find it difficult to understand, because it is most often described in state space notation. Furthermore, many practical difficulties still exist as the initial estimates for parameters, variances, covariances and for the transition matrix.

The objective of this thesis is to investigate and develop a **KF** model approach to multisite precipitation modeling, and prediction in addition to the assessment of associated errors. In order to have an on-line prediction operation, it is desirable to be able to deal with a multitude of rainfall events. The precipitation predictor should not be fixed with time and space, but adapt itself to the evolving meteorological conditions. Any stochastic model is associated with various uncertainties. **KF** consists of combining two independent estimates to form a weighted estimate or prediction. One estimate can be a prior prediction or an estimate based on prior knowledge, and other a prediction based on new information (new data). The purpose of the **KF** is to combine these two pieces of information to obtain an improved estimate.

In order to see the effectiveness of the **KF** model developed in this thesis, annual precipitation data are used that are recorded at 52 different meteorology stations scattered all over Turkey. The application of **KF** to Turkish data will be presented in Chapter 6 after the development and explanation of suitable **KF** model.

2. LITERATURE REVIEW

2.1 Overview

The precipitation data which consist of long time series at various locations in space, have both spatial and temporal variations. However, beginning with cluster model framework introduced by Le Cam (1961), research on modeling rainfall process in space and time has been conducted. Subyani (1997) reviewed most of the publications that have studied precipitation characteristics using common statistical methods (Mejia and Rodriguez-Iturbe, 1974; Bras and Rodriguze-Iturbe, 1976; Eagleson, 1984; Rodriguze-Iturbe et al., 1984; Stein, 1986; Sivapalan and Wood, 1987; Obeysekera et al., 1987; Smith and Krajewski, 1987; Rouhani and Myersa, 1990; Smith et al., 1994 and Zhang et al., 1995).

The theoretical foundation of KF is described in detail in numerous textbooks and articles. The following literature review is a discussion of the KF and its development fundamentals. With the recent development of high-speed computers, KF has become more useful even for complicated real-time applications. Unfortunately, its use in the domain of atmospheric sciences and meteorology is rather few but increasing significantly in recent years. This powerful technique has also been applied to various hydrologic problems, following the pioneering work by Hino (1973).

2.2 Fundamentals of Kalman Filter

Jazwinski (1970) wrote a book which presents a united treatment of linear and nonlinear filtering theory for engineers, with sufficient importance on applications to make possible the reader to use the theory. A review of probability and stochastic processes is included in the first chapters of the book. Of particular interest might be the relatively complete treatment of the mean square calculus and the chapter on stochastic differential equations. Nonlinear filtering results are derived first, and these are then specialized to linear systems. The treatment of linear filtering includes filter stability and model error sensitivity. The last chapter deals with the development of approximate nonlinear filters and presents real applications in nonlinear problems. The performance of these nonlinear filters is critically analyzed.

On the other hand, Gelb (1974) presented the first book on optimal estimation that places its major importance on practical applications. Even so, theoretical and mathematical concepts are introduced and developed sufficiently to make the book a self-contained source of teaching for readers without former knowledge of the basic principles of the field. Numerical examples, based on actual applications, have been interspersed throughout the text to lead the readers to an actual understanding of the theoretical material. After a short historical preface, the book introduces the mathematics underlying random process theory and state-space performance of linear dynamic systems. The theory and practice of optimal estimation are then presented, including filtering, smoothing, and prediction. Both linear and nonlinear systems, and continuous and discrete-time cases are covered in important detail. New results are described about the application of covariance analysis to nonlinear systems and the connection between observers and optimal estimators. The final chapters treat such practical and frequently central issues as suboptimal filtering, sensitivity analysis, algorithm structure, and computer loading considerations.

The mathematical theory of **KF** and its implications are not well understood even among many applied mathematicians and engineers. In fact, most practitioners are just told what the filtering algorithms are without knowing why they work so well. Chui and Chen (1987) presented a book to answer there questions by presenting a fairly thorough discussion of its mathematical theory and applications to various elementary real-time problems. A very elementary derivation of filtering equations is first presented to understand the optimality of the **KF**. This filtering for nonlinear systems with an application to adaptive system identification is also discussed in the same book. Moreover, the limiting or steady-state **KF** theory and efficient computational schemes such as the sequential and square-root algorithms are included for real-time application purposes.

2.3 Kalman Filter in Atmospheric Sciences and Hydrology

Chiu (1978) presented a book that consists of lectures and papers exhibited at the American Geophysical Union, Chapman Conference on Applications of **KF** Theory and Technique to Hydrology, Hydraulics and Water Resources. The objective of the conference was to give the **KF** a significant amount of disclosure to water scientists by giving an introduction to the fundamental **KF** theory and technique; to identify and illustrate cases using **KF** in hydrology; hydraulics and water resources; and to determine directions of future study; and finally, to investigate areas where applications of **KF** are most effective. This book covers **KF** and other estimators, along with applications of **KF** to an extensive spectrum of subjects in hydrology, hydraulics and water resources, such as design of experimental, or monitoring systems, rainfall-runoff system studies, streamflow modeling and forecasting, hydraulics of flow and other transport processes in streams and rivers, water quality studies, groundwater problems, and other areas of water resources and geophysics.

Effective planning, design and operation of any water resources system depend on the available water volume which can be determined by studying the statistical characteristics of many hydrologic series such as precipitation, runoff, ground water levels etc., and hence, the concept of multivariate models became needed. The multivariate models involve large matrices and hence, their inversion creates mathematical as well as computational difficulties. Şen (1980a) presented a multisite recursive disaggregation model which led to the optimum prediction of lower-level (seasonal) events from a given series of higher-level (annual) events at these sites. In his model, a partiotioning technique has been employed together with the Kalman-Bucy linear filtering theory which incurs computational difficulties, especially, in the application to large dimensional systems.

The **KF** differs from optimal interpolation (**OI**) primarily in that it determines the forecast error covariance accurately, by actually evolving it in time according to the forecast model dynamics. To do so, the **KF** requires knowledge of the observation error covariance and also of the model error covariance. The model error is the forecast error committed over one time step, starting from perfect initial data. Dee et al. (1985) showed that the model and observation error covariances can both be determined in a systematic and mathematically sound fashion. They described a two part algorithm which estimates these covariances directly from the observed data, during the coming data assimilation cycle. Authors demonstrated the effectiveness and accuracy of the algorithm by applying it to a simple one-dimensional shallow-water model (**SWM**). The first part of the algorithm is a **KF** which evolves the forecast error covariance. The second part is a filter which estimates the observation and model error covariances required by the primary **KF**. The altogether algorithm is considerably more effective computationally than other adaptive filters which have come into view previously in the estimation literature.

The problem of determining the best initial conditions for numerical weather prediction (**NWP**) is of great practical importance, and has been the subject of many studies by people from different backgrounds. Lorenc (1986) used Bayesian probabilistic arguments to derive idealized equations for finding the best analysis for **NWP**. These equations are compared with those from other published methods in the light of the physical characteristics of the **NWP** analysis problem. Methods discussed include variational techniques, smoothing splines, Kriging, **OI**, successive corrections, constrained initialization, the Kalman-Bucy filter, and adjoint model data assimilation. These are all shown to be related to the idealized analysis, and hence to each other.

The **KF** supplies a measure of the accuracy of an analysis in the form of its error covariance. Therefore, it allows the impact of different observation sets to be compared. Cohn and Parrish (1991) implemented the **KF** algorithm for a linearized **SWM** over the continental United States to assimilate simulated data from the existing radiosonde network, from the demonstration network of 31 Doppler wind profilers in the central United States, and finally, from the hypothetical radiometers located at five of profiler sites. They provided some theoretical justification of Phillips (1986) hypothesis and used it with some modification, to formulate the model error covariance matrix required by the **KF**. The results show that the wind profiler observation can particularly reduce forecast / analysis errors in heights as in winds when compared with the results of assimilating the radiosonde data alone. The forecast error covariance matrices, that the **KF** calculates to obtain this error reduction, differ from these prescribed by the **OI** schemes that are employed for data assimilation at operational centers. They expect the **KF** to yield substantially more accurate analyses and forecasts than **OI** method.

Şen (1991) combined the **KF** with the orthogonal Walsh series which are proposed as an effective model to account for periodicities in observed hydrologic series. This combination led to a real-time prediction procedure of the state variables which are monthly hydrologic variables. General application is performed for monthly flow and rainfall volume sequences. The method has been applied to monthly stream flow data from Turkey and U.S.A., and monthly rainfall data from Saudi Arabia as representatives of extremely arid zones. Comparison with the already available results indicates that the combination of the Walsh functions with **KF** leads to, a better adaptive predictions than the Fourier series. In terms of computer memory and time, Walsh functions are the most economical approach, because their piecewise linearity, orthogonality and symmetry properties result in the basic operations being additions and subtractions only.

Dee (1991) presented a new statistical method of data assimilation that is based on a simplification of the **KF** equations. The forecast error covariance evolution is approximated simply by advecting the mass error covariance field, deriving the remaining covariances geostrophically, and accounting for external model-error forcing only at the end of each forecast cycle. This greatly reduces the cost of computation of the forecast error covariance. In simulations with a linear one-dimensional **SWM** and data generated artificially, the performance of the simplified filter is compared with that of the **KF** and **OI** method. This simplified filter produces analyses that are nearly optimal, and represents a significant improvement over **OI** method.

Daley (1992a) introduced lagged innovation covariances, which is defined as the covariance of observed-minus-forecast differences at different observation stations at selected time lags, through a discussion or **KF** theory and simplified models (a scalar model, a linear univariate one-dimensional model, and a linear quasi-geostrophic model) and then gone on to discuss an observational study with operational data assimilation system. The model results are compared with actual lagged innovation covariances derived from the innovation sequences of an operational data assimilation system and then extensions and limitations of the procedure are discussed.

Over certain special observation networks, such as the North American radiosonde network, forecast-error statistics can be obtained by the zero lag innovation covariance technique. However, over data-sparse regions of the tropics, Southern Hemisphere, and oceans, these techniques cannot be applied and much more ad hoc methods must be employed. Daley (1992b) attempted to examine this problem through the use of **KF** system to actually generate forecast-error statistics for a hierarchy of wind-height observation network. The forecast-error statistics are characterized by their variance and measures of their spatial scale and anisotropy. Several methods of generating forecast-error statistics in data-sparse regions are compared with optimal results. All methods produce results similar to those of the optimal **KF** for very low or very high observation densities.

The spectral characteristics of optimal (statistical) interpolation were examined by Daley (1983, 1985) and by Hollingsworth (1987) using eigenvector decomposition techniqes. Ikawa (1984) introduced the concept of a continuous analog and it was exploited by Daley (1991) to examine the spectral respones for successive correction and statistical interpolation methods. Recently, there has been considerable interest in the application of **KF** methods to data assimilation (Miller, 1986; Cohn and Parrish, 1991; and Daley, 1992a). Daley and Menard (1993) extended the earlier studies of the spectral characteristics of statistical interpolation and successive correction methods to a simple **KF** system. The system employed a one-dimensional advective-diffusive equation on a uniform grid that coincided with the observation network. They described the process to obtain the complete covariance structure at any time from knowledge of the observation and model as well as the initial forecast error covariances. The results are discussed for specified model and observation error spectra and certain asymptotic results. The theory is extended to the multivariate case, and a simple method of determining model error statistics from forecast error statistics are also discussed.

Ahsan and Connor (1994) reviewed some of application of the **KF** technique in river flow forecasting. It is argued that the minimum mean-square error forecasts obtained by using the **KF** technique are identical with those obtained by using the conventional Box and Jenkins-type time series forecasting method when the flow forecasting model assumed to be autoregressive moving average **(ARMA)** model and the corresponding flow data are considered to be free of measurement errors. However, with the assumption of the presence of measurement errors in the river flow time series, the use of the **KF** technique assumes relevance.

Todling and Ghil (1994) implemented a **KF** for a two-dimensional **SWM**, with one layer **(1-L)** or two layer **(2-L)** situations. Both a **1-L** and a **2-L SWM** linearized about a meridionally or vertically dependent basic flow were used to estimate a barotropic and a baroclinic atmosphere, respectively. The model-error covariance matrix for both the **1-L** and the **2-L** systems is constructed by using Phillips (1986) basic hypothesis that these errors are limited to an ensemble of slow modes. The major conclusions of this work are physically reasonable energy partition among the modes in specifying the model's error which is fundamental in producing small forecast and analysis errors bounded in the presence of strong barotropic instability and very few observation. Furthermore, forecast-error correlations can be strongly influenced by the system's instability, causing them to became quite anisotropic and inhomogeneous.

Todling and Cohn (1994) studied the performance of different algorithms based on the simplification of the standard **KF** to approximating the evolution of forecast error covariances for data assimilation. These are several versions of **OI** and suboptimal schemes **(SOSs)** when compared to the **KF**, which is optimal for linear problems with known statistics. They presented the methodology of estimation theory for comparing linear **SOSs**, in which a bets-plane **SWM** linearized about a constant zonal flow is chosen for the test-bed dynamics. The results suggest that even modest enhancements of **OI** go a long way toward achieving the performance of the **KF**, provided that dynamically balanced cross-covariances are constructed and model errors are accounted for properly. The results indicate that such enhancements are necessary if unconventional data are to have a positive impact.

Cohn et al. (1994) introduced the fixed-lag Kalman smoother (**FLKS**) as a means to perform retrospective analysis and to produce model-assimilated datasets for climate studies. They derived the linear **FLKS** in a method that directly generalizes the **KF**. Moreover, in order to incorporate current and past observations, the smoother has the ability to make use of future data to improve current analyses. An application of the **FLKS** to the linear **SWM** of Cohn and Parrish (1991) served as a test-bed for understanding the behavior of the **FLKS** in somewhat more realistic situations. The numerical experiments also demonstrated the ability of **FLKS** to propagate information upstream as well as downstream, thus improving analysis quality substantially in data voids.

Bouttier (1994) used an approximation of the extended Kalman filter (**EKF**) to estimate the forecast and analysis error covariances of an operational assimilation system. The estimation error covariances for the model state are updated during the analysis and prediction cycles. Although no model error term is specified, the estimation error variances grow according to the dynamics on poorly observed areas. The behaviors of the error variances and correlations are shown to be particularly interesting over and around the oceans. A comparison with observation minus analysis and forecast statistics provides an estimate of model error, which is then introduced into the covariance estimation procedure.

Miller et al. (1994) applied advanced data assimilation methods to simple but highly nonlinear problems. The dynamical systems studied here are the stochastically forced double well and Lorenz models. Three generalizations of the **EKF** are described. The first is based on inspection of the successive differences between observation and forecasts. It works very well for the double-well problem. The second is an extension to fourth-order moments and yields excellent results for the Lorenz model but will be unwieldy when applied to models with high-dimensional state spaces. A third but more practical method based on an empirical statistical model is derived from Monte Carlo simulation, which is formulated and shown to work very well.

Analyses of the atmospheric circulation could be substantially improved, if it were possible to estimate the wind field from chemical constituent observations. The modern data assimilation algorithms such as the **EKF** or the four-dimensional variational algorithm have this capability because of the coupling in the transport equation between the wind and the constituent. Daley (1995) examined this possibility by applying an **EKF** to one-dimensional constituent transport equation and to a prognostic linear wind model. The transport and wind models are assumed to be perfect. **EKF** experiments with constituent observations only showed that the wind field can indeed be sufficient structure in the constituent field, the observations are sufficiently frequent and accurate, and data voids are small.

The detection of trends in climatological data has become central to the discussion on climate change due to the enhanced greenhouse effect. To prove detection, flexible mathematical tools are needed. Visser and Molenaar (1995) proposed a structural time series model with which a stochastic trend, a deterministic trend, and regression coefficients can be estimated simultaneously. The stochastic trend component is described using the class of autoregressive integrated moving average (**ARIMA**) models. The regression component is assumed to be linear. However, the regression coefficients corresponding with the explanatory variables may be time dependent to validate this assumption. The **KF** technique is used to estimate the trend-regression model. The authors discussed the main features of the filter and gave some examples of trend estimations.

Todling and Ghil (1994) set up **1-L** and **2-L** versions of a two-dimensional **SWM**. The performance of the **KF** in assimilating sparse and inaccurate data was studied for the **1-L** version both for stable and unstable basic flow profile. Ghil and Todling

(1996) presented the part II of this work. They studied **KF** performance for the model's **2-L** version in a stable and an unstable cases. Baroclinic instability was induced by vertical shear between the two layers with no horizontal shear present. The authors experiments showed that both cases were quite similar to their barotropic counterparts. Once again, the **KF** is shown to keep the estimated flow's error bars bounded, even when a small number of observations taken with realistic frequency was utilized.

Evensen and Leeuwen (1996) studied the ring-shedding process in Agulhas Current using the ensemble Kalman filter **(EnKF)**, which is based on the theory of stochastic dynamic prediction that described the evolution of error statistics and proposed by Evensen (1994) to assimilate the data into a **2-L** quasi-geostrophic model **(QGM)**. They found that the method produced results consistent with the data and that the assimilation of data provided a means to correct for deficiencies or neglected physics in the **QGM**. The method limited too fast meander growth observed in quasi-geostrophic models compared to more advanced models, and the eddy shedding was enhanced by the data. These results suggested that a data assimilation system could indeed be useful for pure physical process studies and could be used to account for ageostrophic effects contained in the data that are missing in the **QGM**. The conclusion of this work is that by allowing the model to contain errors, it is possible to introduce to results physics contained in the data were neglected in model formulation.

The **KF** is the optimal linear assimilation scheme only if the first-and second-order statistics of the observational and system noise are correctly specified. If not, optimality can be reached in principle by using an adaptive filter that estimates both the state vector and the system error statistics. Blanchet et al. (1997) developed and tested a reduced space adaptive **KF** for linear model of the tropical Pacific Ocean. The authors tested three different adaptive algorithms. The first two, the empirical and the maximum-likelihood estimators of Maybeck (1982) were shown to be equivalent if the system noise has zero mean. Both algorithms showed similar performance in forecasting the state of the ocean when using an averaged system noise covariance matrix based on the last part of the adaptive runs. The third algorithm is a maximum-likelihood estimator inspired by Dee (1995), designed to estimate a few parameters of the system error covariance matrix using the entire sequence of the innovation vector. Its performance is comparable even though the estimated system errors covariance matrix differs notably from those estimated with the algorithm Maybeck.

Advanced data assimilation methods become extremely complicated and challenging when used with strongly nonlinear models. Evensen (1997) examined and compared the properties of three advanced data assimilation methods when used with highly nonlinear Lorenz equations. Ensemble smoother method and a gradient decent method are used to minimize two different weak constraint formulation, and the **EnKF** is used for sequential data assimilation. **EnKF** does a good job in tracking the reference solution. The filter estimate is actually better than the ensemble smoother estimate in reproducing the peaks of the reference solution, and with low data density the ensemble smoother gives a rather poor result.

Houtekamer and Mitchell (1998) examined the possibility of performing data assimilation using the flow-dependent statistics calculated from an ensemble of short-range forecasts (a technique referred to an **EnKF**) in an idealized environment. These flow-dependent statistics are calculated at each point directly from the ensemble. They are not parameterized in terms of simple correlation models, as normally done, and, they need not be either homogeneous or isotropic. A series of 30-day data assimilation cycles is performed using ensembles of different sizes. It was found that, the root-mean-square analysis error decreases as the size of ensembles increases, and ensembles having on the order 100 members are sufficient to accurately describe local anisotropic and baroclinic correlation structures. The estimation of small correlations associated with remote observations, is much more difficult and may require very large ensembles. To deal with these small correlations at large distances, authors implemented a cutoff radius beyond which observations were not used. It was found that the optimal value of this cutoff radius increased as the number of available ensemble members increased.

3. THE DISCRETE KALMAN FILTER

3.1. Overview

The discrete-time processes may arise in either of two ways. First, there is the situation where a sequence of event takes place in discrete steps. The length of each step may be either a fixed or random variable. In either case, the random variable of interest is the distance from the origin after taking n steps. In this problem, there is no such thing as fractional steps. The time variable moves in discrete jumps. The discrete-time processes may also arise from sampling a continuous process at discrete times.

The discrete **KF** is a recursive predictive update technique used to determine the correct parameters of a process model. Given some initial estimates, it allows the parameters of a model to be predicted and adjusted with each new measurement, providing an estimate of error at each update. Its ability to incorporate the effects of noise (from both measurement and modelling), and its computational structure, have made it very popular for use.

In 1960, Rudolph Emil Kalman published his famous paper describing a recursive solution to the discrete-data linear filtering problem (Kalman, 1960). Since that time, due in large part advances in digital computing, the **KF** has been the subject of extensive research and application, particularly in the area of autonomous or assisted navigation. Unfortunately, its use in the domain of atmospheric sciences and meteorology is rather few but increasing significantly recent years. Maybeck (1979), introduced the general idea of the discrete **KF**, while a more complete introductory discussion which can be found in Sorenson (1970) with some interesting historical narratives. More extensive references include Gelb (1974), Lewis (1986), Brown and Hwang (1992), and Jacobs (1993). It has been suggested that, in the right situations, the performance of discrete **KF** is better than any other linear filters (Meditch, 1969; Gelb, 1974; Mendel, 1973).

3.2. Description of the Discrete Kalman Filter

As shown in Figure 3.1 the discrete **KF** is an iterative procedure containing several elements which are described in the next sections.

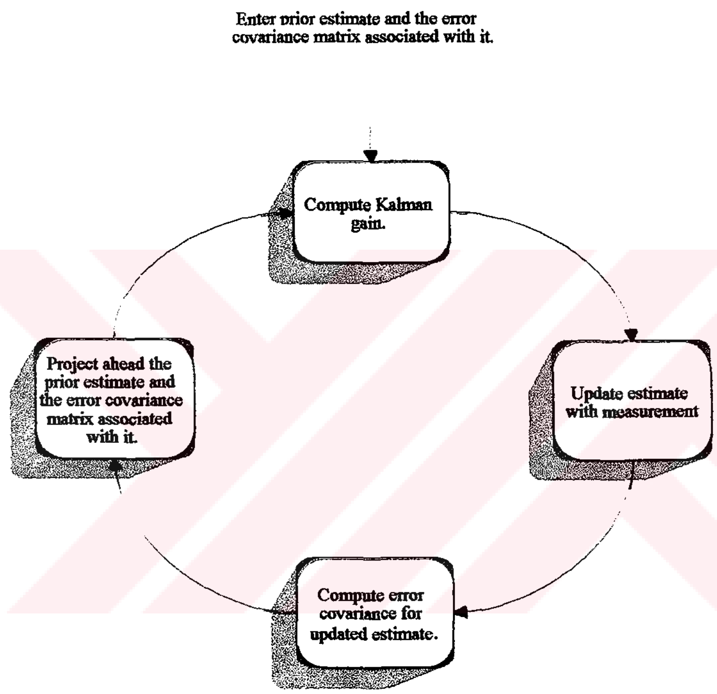

Figure 3.1 Kalman filter iterative procedure.

The filter is supplied with initial information, including the prior estimate of initial parameters, which is based on all the knowledge about the process, and the error covariance associated with it, and these are used to calculate a Kalman gain. The error between the parameter estimation and the measured data is determined and multiplied by Kalman gain to update the parameter estimate and estimation error. The updated error and parameters are used as inputs to a model, in order to predict the projected error and parameters at the next time instance. The first derivation of the **KF** recursive equations is the "filter" equation. The equations used in the discrete **KF** are given in detailed by Brown and Hwang (1992).

3.3. One-Dimensional Kalman Filter

Suppose that there is a random variable, X_k, whose values should be estimated at a set of certain times t_0 , t_1, t_2, ..., t_n. Also, suppose that X_{k-1} satisfies the dynamic equation

$$X_k = \varphi\ X_{k-1} + W_{k-1} \tag{3.1}$$

In this expression φ is a known parameter relating X_k to X_{k-1}, and W_{k-1} is a random number selected by picking a number randomly. Suppose the numbers are with the mean of $W_{k-1} = 0$ and the variance of W_{k-1} is Q. W_{k-1} is called white noise, which means that it is not correlated with any other random variable and especially not correlated with past values of W.

Let an initial estimate $\hat{X}_{k/k-1}$ be based on all the available knowledge about the process prior to t_{k-1}. The estimation error then becomes by definition

$$e_{k/k-1} = X_k - \hat{X}_{k/k-1} \tag{3.2}$$

and the variance of the estimation error is equal to $P_{k/k-1}$.

$$P_{k/k-1} = E\left[(X_k - \hat{X}_{k/k-1})^2\right] \tag{3.3}$$

where E is the expected value operator.

Now, let us assume a noisy measurement of X and call it as Z.

$$Z_k = H_k X_k + V_k \tag{3.4}$$

where V_k is white noisy with variance, R, and H is measurement parameter. In order to improve the prior estimate $\hat{X}_{k/k-1}$, the noisy measurement at time k, Z_k is used as

$$\hat{X}_{k/k} = \hat{X}_{k/k-1} + K_k(Z_k - H_k \hat{X}_{k/k-1}) \tag{3.5}$$

where:

$\hat{X}_{k/k}$: updated estimate,

K_k : the Kalman gain.

Notice, that $(Z_k - H_k \hat{X}_{k/k-1})$ is just the error in estimating Z_k. Part of this is due to the noise, V_k and part due to error in estimating X. If, all the errors were due to error source in estimating X, then it would be convinced that $\hat{X}_{k/k}$ was lessened by the amount $(Z_k - H_k \hat{X}_{k/k-1})$. However, since some of this error is due to V_k, a correction of less than $(Z_k - H_k \hat{X}_{k/k-1})$ is needed to come up with $\hat{X}_{k/k}$. For deciding on the value of K, let the variance of the error be computed as

$$E\left[(X_k - \hat{X}_{k/k})^2\right] = E\left[\left(X_k - \hat{X}_{k/k-1} - K_k(Z_k - H_k \hat{X}_{k/k-1})\right)^2\right]$$

$$= E\left[(1 - K_k H_k)(X_k - \hat{X}_{k/k-1}) + K_k V_k\right]^2$$

$$= P_{k/k-1}(1-K_kH_k)^2 + R\ K_k^2 \tag{3.6}$$

where the cross product terms drop out because V_k is assumed to be uncorrelated with X_k and $\hat{X}_{k/k-1}$. So, the variance of updated estimation error is given by

$$P_{k/k} = P_{k/k-1}(1-K_kH_k)^2 + R\ K_k^2 \tag{3.7}$$

If it is necessary to minimize the estimation error, then minimization of $P_{k/k}$ is required by differentiating $P_{k/k}$ with respect to K_k and setting the derivatives equal to zero. A little algebra shows that the optimal K_k is obtained as

$$K = H\ P_{k/k-1}\left[P_{k/k-1}(K_k)^2 + R\right]^{-1} \tag{3.8}$$

The updated estimate $\hat{X}_{k/k}$ can be projected one-time step ahead as

$$\hat{X}_{k+1/k} = \varphi\ \hat{X}_{k/k} \tag{3.9}$$

The variance of the error of this estimate is

$$E\left[(X_k - \hat{X}_{k/k})^2\right] = E\left[\left(X_k - \hat{X}_{k/k-1} - K_k(Z_k - H_k\hat{X}_{k/k-1})\right)^2\right]$$

$$= E\left[(1-K_kH_k)(X_k - \hat{X}_{k/k-1}) + K_kV_k\right]^2$$

$$= P_{k/k-1}(1-K_kH_k)^2 + R\ K_k^2 \tag{3.10}$$

The last term is equal to zero because W_k is assumed to be uncorrelated with X_k and $\hat{X}_{k/k}$. So, the remaining term becomes

$$P_{k+1/k} = P_{k/k}\varphi^2 + Q \tag{3.11}$$

Consequently, equations (3.5), (3.7), (3.8), (3.9) and (3.11) are the five expressions necessary for the application of the **KF** procedure. At next time step, it is necessary again to use the projected ahead values to be the value of prior estimate. And this goes on computer cycle after cycle. In the case of X_k is a column matrix with many components. Then equations (3.1) through (3.8) become matrix equations and the simplicity as well as the intuitive logic of the **KF** becomes obscured. This multi-dimensional **KF** is explained in the next section

3.4. Multi-Dimensional Kalman Filter Recursive Equations

Let any random process be modeled through the following recursive equation as

$$\mathbf{X}_k = \mathbf{\Phi}_{kk-1}\mathbf{X}_{k-1} + \mathbf{W}_{k-1} \tag{3.12}$$

where:

$\mathbf{X}_{k-1}$, $\mathbf{X}_k$: (n x 1) state vectors at times t_{k-1} and t_k, respectively,

$\mathbf{\Phi}_{kk-1}$: (n x n) transition matrix relating $\mathbf{X}_{k-1}$ to $\mathbf{X}_k$,

$\mathbf{W}_{k-1}$: (n x 1) white noise sequence vector with known covariance structure.

The measurement of the process is assumed to occur at discrete points along time in accordance with linear relationship

$$\mathbf{Z}_k = \mathbf{H}_k\mathbf{X}_k + \mathbf{V}_k \tag{3.13}$$

where:

$\mathbf{Z}_k$: (m x 1) vector measurement at time t_k,

$\mathbf{H}_k$: (m x n) matrix giving the ideal (noiseless) connection between the measurement and the state vectors at time t_k,

$\mathbf{V}_k$: (m x 1) measurement error assumed to be a white noise sequence with known covariance structure and zero cross-correlation with the $\mathbf{W}_{k-1}$ sequence.

The covariance matrices for $\mathbf{W}_{k-1}$ and $\mathbf{V}_k$ vectors are given by

$$E\left[\mathbf{W}_k \mathbf{W}_i^T\right] = \begin{cases} \mathbf{Q}_k, & i = k \\ 0, & i \neq k \end{cases} \tag{3.14}$$

$$E\left[\mathbf{V}_k \mathbf{V}_i^T\right] = \begin{cases} \mathbf{R}_k, & i = k \\ 0, & i \neq k \end{cases}. \tag{3.15}$$

$$E\left[\mathbf{W}_k \mathbf{V}_i^T\right] = 0, \qquad \text{for all k and i.} \tag{3.16}$$

where subscript T denotes transpose of the vector.

Let an initial estimate of the process at same point in time t_{k-1} is available, and that this estimate is based on all the previous knowledge about the process prior to t_{k-1}. This initial estimate will be denoted as $\hat{\mathbf{X}}_{k/k-1}$. It is also assumed that the error covariance matrix associated with $\hat{\mathbf{X}}_{k/k-1}$ is known. Hence, the estimation error becomes by definition as

$$\mathbf{e}_{k/k-1} = \mathbf{X}_k - \hat{\mathbf{X}}_{k/k-1} \tag{3.17}$$

and the error covariance matrix associated with initial estimate is

$$\mathbf{P}_{k/k-1} = \mathrm{E}\left[\mathbf{e}_{k/k-1}\mathbf{e}_{k/k-1}^{T}\right]$$

$$= \mathrm{E}\left[(\mathbf{X}_k - \hat{\mathbf{X}}_{k/k-1})(\mathbf{X}_k - \hat{\mathbf{X}}_{k/k-1})^T\right] \tag{3.18}$$

In the case of no prior measurements, if the process mean is equal to zero, it implies that the initial estimate is zero, and therefore, the associated error covariance matrix is just the covariance matrix of $\mathbf{X}$ itself.

With the assumption of a prior estimate $\hat{\mathbf{X}}_{k/k-1}$, the measurement $\mathbf{Z}_k$ is used to improve this prior estimate by a linear blending of the noisy measurement and the posterior estimate appears in accordance with the following equation

$$\hat{\mathbf{X}}_{k/k} = \hat{\mathbf{X}}_{k/k-1} + \mathbf{K}_k(\mathbf{Z}_k - \mathbf{H}_k\hat{\mathbf{X}}_{k/k-1}) \tag{3.19}$$

where:

$\hat{\mathbf{X}}_{k/k}$: updated estimate,

$\mathbf{K}_k$: blending factor, which is referred to as the Kalman gain.

The problem now is to find the particular blending factor $\mathbf{K}_k$, that yields an updated estimate that is optimal in some sense. For this purpose, it is necessary first to form an expression for the error covariance matrix $\mathbf{P}_{k/k}$ associated with the updated estimate. Since, the updated estimation error is by definition

$$\mathbf{e}_{k/k} = \mathbf{X}_k - \hat{\mathbf{X}}_{k/k} \tag{3.20}$$

Therefore,

$$\mathbf{P}_{k/k} = \mathrm{E}\left[\mathbf{e}_{k/k}\mathbf{e}_{k/k}^{T}\right]$$

$$= \mathrm{E}\left[(\mathbf{X}_k - \hat{\mathbf{X}}_{k/k})(\mathbf{X}_k - \hat{\mathbf{X}}_{k/k})^{T}\right] \tag{3.21}$$

Furthermore, substituting Eq. (3.13) into Eq. (3.19) and then substituting the equation obtained into the expression for $\hat{\mathbf{X}}_{k/k}$ in Eq. (3.21), it is possible to get after some algebra

$$\hat{\mathbf{X}}_{k/k} = \hat{\mathbf{X}}_{k/k-1} + \mathbf{K}_k(\mathbf{H}_k\mathbf{X}_k + \mathbf{V}_k - \mathbf{H}_k\hat{\mathbf{X}}_{k/k-1}) \tag{3.22}$$

with the following covariance matrices

$$\mathbf{P}_{k/k} = \mathrm{E}\left\{\begin{matrix}\left[(\mathbf{X}_k - \hat{\mathbf{X}}_{k/k-1}) - \mathbf{K}_k(\mathbf{H}_k\mathbf{X}_k + \mathbf{V}_k - \mathbf{H}_k\hat{\mathbf{X}}_{k/k-1})\right] \\ \left[(\mathbf{X}_k - \hat{\mathbf{X}}_{k/k-1}) - \mathbf{K}_k(\mathbf{H}_k\mathbf{X}_k + \mathbf{V}_k - \mathbf{H}_k\hat{\mathbf{X}}_{k/k-1})\right]^{T}\end{matrix}\right\}$$

or in terms of expectation multiplication

$$\mathbf{P}_{k/k} = \mathrm{E}\left[(\mathbf{X}_k - \hat{\mathbf{X}}_{k/k-1}) - \mathbf{K}_k\mathbf{H}_k(\mathbf{X}_k - \hat{\mathbf{X}}_{k/k-1}) - \mathbf{K}_k\mathbf{V}_k)\right]$$
$$\bullet\, \mathrm{E}\left[(\mathbf{X}_k - \hat{\mathbf{X}}_{k/k-1}) - \mathbf{K}_k\mathbf{H}_k(\mathbf{X}_k - \hat{\mathbf{X}}_{k/k-1}) - \mathbf{K}_k\mathbf{V}_k)\right]^{T}$$

and finally,

$$\mathbf{P}_{k/k} = \mathrm{E}\left[(\mathbf{I}-\mathbf{K}_k\mathbf{H}_k)(\mathbf{X}_k-\hat{\mathbf{X}}_{k/k-1})-\mathbf{K}_k\mathbf{V}_k)\right]$$

$$\bullet\, \mathrm{E}\left[(\mathbf{I}-\mathbf{K}_k\mathbf{H}_k)(\mathbf{X}_k-\hat{\mathbf{X}}_{k/k-1})-\mathbf{K}_k\mathbf{V}_k)\right]^{\mathrm{T}} \tag{3.23}$$

after performing the necessary expectation operation and noting that $(\mathbf{X}_k-\hat{\mathbf{X}}_{k/k-1})$ is a prior estimation error which is uncorrelated with the measurement error $\mathbf{V}_k$ and further consideration $\mathrm{E}\left[\mathbf{V}_k\mathbf{V}_i^{\mathrm{T}}\right]=\mathbf{R}_k$, it is possible to reach to the following expression.

$$\mathbf{P}_{k/k} = (\mathbf{I}-\mathbf{K}_k\mathbf{H}_k)\mathbf{P}_{k/k-1}(\mathbf{I}-\mathbf{K}_k\mathbf{H}_k)^{\mathrm{T}} + \mathbf{K}_k\mathbf{R}_k\mathbf{K}_k^{\mathrm{T}} \tag{3.24}$$

This is a general expression for the updated error covariance matrix, and it is valid for any gain, whether suboptimal or otherwise.

Returning to the optimization problem, it is desired to find the particular $\mathbf{K}_k$ that minimizes the individual terms along the major diagonal of $\mathbf{P}_{k/k}$, because these terms represent the estimation error covariance for the elements of the state vector being estimated. It is achieved using a straightforward differentiation calculus approach through the following matrix operations

$$\frac{d\left[\mathrm{trace}(\mathbf{AB})\right]}{d\mathbf{A}} = \mathbf{B}^{\mathrm{T}}, \qquad (\mathbf{A}\text{ and }\mathbf{B}\text{ must be square matrices}) \tag{3.25}$$

or, likewise

$$\frac{d\left[\mathrm{trace}(\mathbf{ACA}^{\mathrm{T}})\right]}{d\mathbf{A}} = 2\mathbf{AC}, \qquad (\mathbf{C}\text{ must be symmetric matrix}) \tag{3.26}$$

The derivative of a scalar quantity (s) with respect to a matrix is defined, in general, as

$$\frac{ds}{dA} = \begin{bmatrix} \frac{ds}{da_{11}} & \cdots & \frac{ds}{da_{1n}} \\ \cdots & \cdots & \cdots \\ \frac{ds}{da_{n1}} & \cdots & \frac{ds}{da_{mn}} \end{bmatrix} \tag{3.27}$$

With these points in mind, Eq. (3.24) may be expanded leading to the following form

$$\begin{aligned} \mathbf{P}_{k/k} &= \mathbf{P}_{k/k-1} - \mathbf{K}_k \mathbf{H}_k \mathbf{P}_{k/k-1} - \mathbf{P}_{k/k-1} \mathbf{H}_k^T \mathbf{K}_k^T \\ &\quad + \mathbf{K}_k (\mathbf{H}_k \mathbf{P}_{k/k-1} \mathbf{H}^T + \mathbf{R}_k) \mathbf{K}_k^T \end{aligned} \tag{3.28}$$

The second and third terms on the right hand side are linear, however, the fourth term is quadratic in $\mathbf{K}$. Eqs. (3.25) and (3.26) may now be combined with Eq. (3.28). It is desired to minimize the trace of $\mathbf{P}_{k/k}$ which is the sum of the mean-square errors in the estimates of all the elements in the state vector. The argument that can be used at this stage, is that the individual mean-square errors are also minimized when the total is minimized. Hence, one can proceed to differentiate the trace of $\mathbf{P}_{k/k}$ with respect to $\mathbf{K}_k$, noting that the trace of $\mathbf{P}_{k/k-1}\mathbf{H}_k^T\mathbf{K}_k^T$ is equal to the trace of its transpose, $\mathbf{K}_k\mathbf{H}_k\mathbf{P}_{k/k-1}$. After the derivation the result becomes

$$\frac{d\left[\text{trace } \mathbf{p}_{k/k}\right]}{d\mathbf{K}_k} = -2\ (\mathbf{H}_k \mathbf{P}_{k/k-1})^T + 2\ \mathbf{K}_k (\mathbf{H}_k \mathbf{P}_{k/k-1} \mathbf{H}_k^T + \mathbf{R}_k) \tag{3.29}$$

It is necessary, for minimization to set this expression equal to zero and then its solution yields the optimal gain as

$$\mathbf{K}_k = \mathbf{P}_{k/k-1}\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_{k/k-1}\mathbf{H}_k^T + \mathbf{R}_k)^{-1} \tag{3.30}$$

This particular $\mathbf{K}_k$, namely, the one that minimizes the mean-square estimation error, is called Kalman gain. Now, updated estimate $\hat{\mathbf{X}}_{k/k}$ can be calculated by the use of Eq. (3.19) with $\mathbf{K}_k$ set equal to the Kalman gain as given previously by Eq. (3.30). The covariance matrix associated with the updated estimate may now be computed by substituting Eq. (3.30) into Eq. (3.28) which leads to

$$\begin{aligned}\mathbf{P}_{k/k} = \mathbf{P}_{k/k-1} - \mathbf{P}_{k/k-1}\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_{k/k-1}\mathbf{H}_k^T + \mathbf{R}_k)^{-1}\mathbf{H}_k\mathbf{P}_{k/k-1} - \mathbf{P}_{k/k-1}\mathbf{H}_k^T\mathbf{K}_k^T \\ + \mathbf{P}_{k/k-1}\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_{k/k-1}\mathbf{H}_k^T + \mathbf{R}_k)^{-1}(\mathbf{H}_k\mathbf{P}_{k/k-1}\mathbf{H}^T + \mathbf{R}_k)\mathbf{K}_k^T\end{aligned}$$

$$\begin{aligned}\mathbf{P}_{k/k} = \mathbf{P}_{k/k-1} - \mathbf{P}_{k/k-1}\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_{k/k-1}\mathbf{H}_k^T + \mathbf{R}_k)^{-1}\mathbf{H}_k\mathbf{P}_{k/k-1} \\ - \mathbf{P}_{k/k-1}\mathbf{H}_k^T\mathbf{K}_k^T + \mathbf{P}_{k/k-1}\mathbf{H}_k^T\mathbf{K}_k^T\end{aligned}$$

$$\mathbf{P}_{k/k} = \mathbf{P}_{k/k-1} - \mathbf{P}_{k/k-1}\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_{k/k-1}\mathbf{H}_k^T + \mathbf{R}_k)^{-1}\mathbf{H}_k\mathbf{P}_{k/k-1}$$

$$\mathbf{P}_{k/k} = \mathbf{P}_{k/k-1} - \mathbf{K}_k\mathbf{H}_k\mathbf{P}_{k/k-1} = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_{k/k-1} \tag{3.31}$$

It is to be noticed that this equation is valid only for Kalman gain. The updated estimation $\hat{\mathbf{X}}_{k/k}$ can be projected ahead via the following transition matrix

$$\hat{\mathbf{X}}_{k+1/k} = \mathbf{\Phi}_{k+1k}\hat{\mathbf{X}}_{k/k} \tag{3.32}$$

where the contribution of $\mathbf{W}_k$ in Eq. (3.12) is ignored, because it has zero mean and is not correlated with any of the previous $\mathbf{W}$' s.

The error covariance matrix associated with $\hat{\mathbf{X}}_{k+1/k}$ is obtained by first forming the expression for the updated estimation error defined as

$$\mathbf{e}_{k+1/k} = \mathbf{X}_{k+1} - \hat{\mathbf{X}}_{k+1/k} \tag{3.33}$$

Substitution of Eqs. (3.12) and (3.32) into this final expression leads to

$$\mathbf{e}_{k+1/k} = \mathbf{\Phi}_{k+1k}\mathbf{X}_k + \mathbf{W}_k - \mathbf{\Phi}_{k+1k}\hat{\mathbf{X}}_{k/k}$$

$$\mathbf{e}_{k+1/k} = \mathbf{\Phi}_{k+1k}(\mathbf{X}_k - \hat{\mathbf{X}}_{k/k}) + \mathbf{W}_k$$

$$\mathbf{e}_{k+1/k} = \mathbf{\Phi}_{k+1k}\mathbf{e}_{k/k} + \mathbf{W}_k \tag{3.34}$$

Herein, again $\mathbf{W}_k$ and $\mathbf{e}_k$ have zero cross-correlations, because $\mathbf{W}_k$ is the process noise for the one step ahead of t_k. Thus, one can write the expression for $\mathbf{P}_{k+1/k}$ as

$$\mathbf{P}_{k+1/k} = \mathrm{E}\left[\mathbf{e}_{k+1/k}\mathbf{e}_{k+1/k}^{T}\right]$$

$$\mathbf{P}_{k+1/k} = \mathrm{E}\left[(\mathbf{\Phi}_{k+1k}\mathbf{e}_{k/k} + \mathbf{W}_k)(\mathbf{\Phi}_{k+1k}\mathbf{e}_{k/k} + \mathbf{W}_k)^{T}\right]$$

$$\mathbf{P}_{k+1/k} = \mathbf{\Phi}_{k+1k}\mathbf{P}_{k/k}\mathbf{\Phi}_{k+1k}^{T} + \mathbf{Q}_k \tag{3.35}$$

Hence, finally the necessary expressions for prediction at time t_{k+1} are available as equations (3.19), (3.30), (3.31), (3.32) and (3.35) which comprise the multi-dimensional **KF** recursive equations.

These equations fall into two groups, time and measurement update equations. The time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain the a priori estimates for the next time step. The measurement update equations are responsible for incorporating a new measurement into the a priori estimate to obtain an improved estimate. The time update equations can also be thought of as predictor equations, while the measurement update equations can be thought of as corrector equations. Indeed the final estimation algorithm resembles that of a predictor algorithm for solving numerical problems as shown in the Figure 3.2.

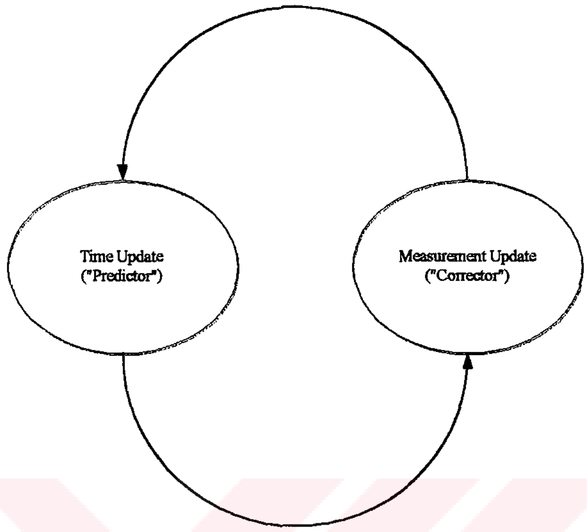

Figure 3.2 The ongoing discrete **KF** cycle.

The specific equations for the time and measurement updates are presented below in Tables 3.1 and Table 3.2 .

Table 3.1 The time update (predictor) equations.

Time update equations
$\hat{\mathbf{X}}_{k+1/k} = \mathbf{\Phi}_{k+1k}\hat{\mathbf{X}}_{k/k}$
$\mathbf{P}_{k+1/k} = \mathbf{\Phi}_{k+1k}\mathbf{P}_{k/k}\mathbf{\Phi}^{T}_{k+1k} + \mathbf{Q}_{k}$

Table 3.2 The measurement update (corrector) equations.

Measurement update equations
$\mathbf{K}_{k} = \mathbf{P}_{k/k-1}\mathbf{H}_{k}^{T}(\mathbf{H}_{k}\mathbf{P}_{k/k-1}\mathbf{H}_{k}^{T} + \mathbf{R}_{k})^{-1}$
$\hat{\mathbf{X}}_{k/k} = \hat{\mathbf{X}}_{k/k-1} + \mathbf{K}_{k}(\mathbf{Z}_{k} - \mathbf{H}_{k}\hat{\mathbf{X}}_{k/k-1})$
$\mathbf{P}_{k/k} = \mathbf{P}_{k/k-1} - \mathbf{K}_{k}\mathbf{H}_{k}\mathbf{P}_{k/k-1}$

The pertinent equations and the sequence of computational steps will be shown in the next section.

3.5. Kalman Filter Loop

Kalman filter estimation requires the execution of the following steps. These are :

1. Enter prior estimate $\hat{\mathbf{X}}_{k/k-1}$ which is based on all our knowledge about the process prior to time t_{k-1}, and also suggest the error covariance matrix associated with it $\mathbf{P}_{k/k-1}$,

2. Compute the Kalman gain as,

$$\mathbf{K}_k = \mathbf{P}_{k/k-1}\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_{k/k-1}\mathbf{H}_k^T + \mathbf{R}_k)^{-1} \tag{3.30}$$

3. Update estimate with measurement $\mathbf{Z}_k$,

$$\hat{\mathbf{X}}_{k/k} = \hat{\mathbf{X}}_{k/k-1} + \mathbf{K}_k(\mathbf{Z}_k - \mathbf{H}_k\hat{\mathbf{X}}_{k/k-1}) \tag{3.19}$$

4. Compute error covariance for updated estimate,

$$\mathbf{P}_{k/k} = \mathbf{P}_{k/k-1} - \mathbf{K}_k\mathbf{H}_k\mathbf{P}_{k/k-1} \tag{3.31}$$

5. Project ahead the updated estimate $\hat{\mathbf{X}}_{k/k}$, and the error covariance matrix associated with it $\mathbf{P}_{k/k}$, to use it as a prior estimation for the next time step,

$$\hat{\mathbf{X}}_{k+1/k} = \Phi_{k+1k}\hat{\mathbf{X}}_{k/k} \tag{3.32}$$

$$\mathbf{P}_{k+1/k} = \Phi_{k+1k}\mathbf{P}_{k/k}\Phi_{k+1k}^T + \mathbf{Q}_k \tag{3.35}$$

Once the loop is entered as shown in Figure 3.1 then it can be continued as much as necessary. Initially, when the model parameters are only rough estimates, the gain matrix ensures that the measurement data is highly influential in estimating the state parameters. Then, as confidence in the accuracy of the parameters grows with each iteration, the gain matrix values decrease, causing the influence of the measurement data in updating the parameters and associated error to reduce.

3.6. The Extended Kalman Filter

As described before, the **KF** addresses the general problem of trying to estimate the state of a first-order, discrete-time process that is covered by a linear difference equation. But what happens, if the process to be estimated and / or the measurement relationship to the process is non-linear ? Some of the most interesting and successful applications of Kalman filtering have appeared in such situations. A **KF** that is linearized about the current mean and covariance is referred to as an extended Kalman filter or **EKF**. In standard Kalman filter , we started with a system whose equations were,

$$\mathbf{X}_k = \Phi_{kk-1}\mathbf{X}_{k-1} + \mathbf{W}_{k-1} \tag{3.12}$$

and

$$\mathbf{Z}_k = \mathbf{H}_k\mathbf{X}_k + \mathbf{V}_k \tag{3.13}$$

These equations are linear in the state vector, $\mathbf{X}$. Now, we will extend the Kalman filter to these non-linear problems by using the linearization tricks,

$$\mathbf{X}_k = f(\mathbf{X}_{k-1}) + \mathbf{W}_{k-1} \tag{3.36}$$

and

$$\mathbf{Z}_k = g(\mathbf{X}_k) + \mathbf{V}_k \tag{337}$$

where f is a vector whose components f_{ix} are non-linear functions of $\mathbf{X}_1$, $\mathbf{X}_2$, etc. Choose some nominal value of $\mathbf{X}$ and call it $\mathbf{X}_{nom}$ and write

$$\Phi_{(i,j)} = \frac{\partial f_i}{\partial x_j} \tag{3.38}$$

where $\mathbf{X}_j$ is evaluated at $\mathbf{X}_{nom}$. Now, expansion of $\mathbf{X}_{k+1}$ in a Taylor series about $\mathbf{X}_{nom}$ gives

$$\mathbf{X}_{k+1} = f(\mathbf{X}_{nom}) + \Phi_{k+1k}(\mathbf{X}_k - \mathbf{X}_{nom}) + \ldots\ldots \tag{3.39}$$

where the higher order terms are ignored Without the higher order terms, this equation is linear in $\mathbf{X}$ and, therefore, the optimum estimate of $\mathbf{X}_{k+1}$ is given by

$$\hat{\mathbf{X}}_{k+1/k} = f(\mathbf{X}_{nom}) + \Phi_{k+1k}(\hat{\mathbf{X}}_{k/k} - \mathbf{X}_{nom}) \tag{3.40}$$

where as usual $\hat{\mathbf{X}}_{k/k}$ is the best estimate of $\mathbf{X}_k$ inherited from the previous computing cycle. Now using trick of setting $\mathbf{X}_{nom} = \hat{\mathbf{X}}_{k/k}$, gives

$$\hat{\mathbf{X}}_{k+1/k} = f(\hat{\mathbf{X}}_{k/k}) \tag{3.41}$$

The equation for the covariance matrix of the error in this estimate is just

$$\mathbf{P}_{k+1/k} = \Phi_{k+1k}\mathbf{P}_{k/k}\Phi^{T}_{k+1k} + \mathbf{Q}_k \tag{3.42}$$

However, by defining that

$$\mathbf{H}_{(i,j)} = \frac{\partial g_i}{\partial x_j} \tag{3.43}$$

it is possible to expand $\mathbf{Z}_k$ into a Taylor series as follows

$$\mathbf{Z}_k = g(\mathbf{X}_{nom}) + \mathbf{H}_k(\mathbf{X}_k - \mathbf{X}_{nom}) + \ldots \tag{3.44}$$

Again, by ignoring the higher order terms, it is possible to get $\mathbf{Z}$ as a linear function of $\mathbf{X}$ and, therefore, the optimal estimate after using linearization trick becomes

$$\hat{\mathbf{X}}_{k/k} = \hat{\mathbf{X}}_{k/k-1} + \mathbf{K}_k[\mathbf{Z}_k - g(\hat{\mathbf{X}}_{k/k-1})] \tag{3.45}$$

The covariance matrix associated with this estimation and the Kalman gain matrix equations unchanged from those of the standard **KF** are obtained as follows

$$\mathbf{P}_{k/k} = \mathbf{P}_{k/k-1} - \mathbf{K}_k\mathbf{H}_k\mathbf{P}_{k/k-1} = (\mathrm{I} - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_{k/k-1} \tag{3.46}$$

$$\mathbf{K}_k = \mathbf{P}_{k/k-1}\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_{k/k-1}\mathbf{H}_k^T + \mathbf{R}_k)^{-1} \tag{3.47}$$

4. KALMAN FILTER DESIGN AND TESTING

4.1. Overview

Prediction models are powerful tools which estimate the most likely state of hydrologic variable at a certain future time lag such that the estimation error is expected to be minimal, i.e. an optimal estimation of a future state variable. The main purpose of prediction is to yield the optimum state variable for a certain lead time (hours, days, weeks, months, years, etc.) on the basis of the available historic record. The most efficient category of prediction models that appear to have adaptive behavior is based on the Kalman (1960) procedure (Şen, 1991).

For the design of Kalman filters certain guidelines can be developed based on practical experience. Mehra (1978) discussed these under the headings of model selection, parameter specification, algorithm selection, sensitivity analysis, validation and testing.

4.2. Model Selection

This is the most important step in **KF** design since, it ultimately determines the usefulness, accuracy, and computational requirements of the filter. It is recommended that the state variables of the model have physical significance and that the model is not overly complex unless great confidence can be placed on it. In some applications, there may be so little a priori modeling information that it would be better to identify the state vector model directly from the historical data. Mehra and Comeron (1977), discussed this problem and described a general technique for identifying state vector models from historical data.

Annual floods and alike hydrologic extremes may be treated as isolated independent events and hence can be considered as pure-random variables (Chow, 1978a). A complete-duration series of hydrologic data consists of daily, weekly, monthly, seasonal or mean annual observations. It has trend components, whether periodical or of straight-line type, which are dependent sequentially.

The dependence among sequential values of a series of observations cannot be satisfactorily modeled by probabilistic distributions of pure-random variables. In attempts to consider this dependence, progressive average values have been used. Hoyt (1936) provided graphs showing the annual and ten-year progressive average precipitation for selected drainage basins in the United States. This graphical approach eventually led to the use of the moving-average (**MA**) model for a stochastic hydrologic process. This model may be expressed as

$$X_t = b_0\varepsilon_t + b_1\varepsilon_{t-1} + \ldots\ldots + b_m\varepsilon_{t-m} = \sum_{j=0}^{m} b_j\varepsilon_{t-j} \quad (4.1)$$

where:

X_t	: the stochastic variable at time t,
ε	: a pure-random variable having zero mean and finite variance,
$b_0, b_1, \ldots, b_m$	: the weights with $\sum b_j^2$ being convergent usually equal to one,
M	: the extent of the moving average.

Harmonic or spectral analysis has been applied to hydrometeorological data to ascertain their periodicity. Chow and Kareliotis (1970) have applied the analysis to determine the periodical dependence in the precipitation, evaportranspiration, and streamflow components in a stochastic watershed system. The general form of the so-called sum-of-harmonics (**SH**) model for stochastic hydrologic process may be expressed by

$$X_t = \overline{X} + \sum_{j=1}^{m}(A_j \cos\ 2\pi fjt + B_j \sin\ 2\pi fjt) + \varepsilon_t \tag{4.2}$$

where:

X_t	: the stochastic variable at time t of the hydrologic process,
$\overline{X}$	: the mean of X_t,
A_j and B_j	: Fourier coefficients or amplitudes,
f	: the fundamental frequency,
$2\pi fjt$	: cyclicity with j=1, 2, ..., m,
m	: the total number of cycles involved in the model,
ε_t	: the pure-random component.

The Russian mathematician A. A. Markov, 1856-1922, introduced a concept that the outcome of any one of a series of trials depends only on the outcome of the directly preceding trial. Following this concept, a simple Markov model for stochastic processes may be written as

$$X_t = a_1 X_{t-1} + \varepsilon_t \tag{4.3}$$

where:

X_{t-1}	: the stochastic variable at time t-1,
a_1	: a Markov coefficient.

This model simply says that the value of a variable at any time depends only upon its preceding value plus a probabilistic component. It has been first applied to the annual flows of the Colorado River at Lees Ferry, Arizona by Brittan (1961).

The simple Markov model is, in fact, a special case of a more general linear autoregressive (**AR**) mode, which may expressed as

$$X_t = \varepsilon_t + a_1 X_{t-1} + \ldots + a_m X_{t-m} \tag{4.4}$$

or

$$X_t = \sum_{j=1}^{n} a_j X_{t-j} + \varepsilon_t \tag{4.5}$$

where:

a_j : an autoregression coefficient with j = 1, 2, …, m,

n : the order of the model.

The simple Markov model is a first-order or lag one **AR** model. Julian (1960) and Yevjevich (1971), respectively, have shown that the **AR** model has its roots in the **SH** model and the **MA** model, and can be so derived.

To extend the Markovian concept, the **MA**, **SH**, and **AR** models can also be classified as Markovian approaches. They have been variously used in the analysis of stochastic hydrologic data in the last three decades (Kisiel, 1969; Fiering and Jackson, 1971; Chow, 1978b). These models contain a random component. Once a model is formulated from the given hydrologic data, its random component can be generated by the Monte Carlo methods. Therefore, given the initial condition, the models can be used to generate pseudo-random or synthetic hydrologic sequences. This procedure was utilized first in the Harvard Water Resources Program (Julian, 1960) and also in the Colorado River study (Brittan, 1961). It was given the name of "Synthetic Hydrology" (Julian, 1960) or "Operational Hydrology" (Fiering, 1966). However, the term "Operational Hydrology" was later adopted by the World Meteorology Organization (**WMO**) with the meaning practically equivalent to applied hydrology.

4.3 Parameter Specification

Once a Markov model has been selected, then **KF** design requires specification of matrices $\mathbf{\Phi}$, $\mathbf{H}$, $\mathbf{Q}$, $\mathbf{R}$, $\hat{\mathbf{X}}_{k/k-1}$ and $\mathbf{P}_{k/k-1}$. For theoretically based models, most of this information comes from the physical understanding of the process. Maximum likelihood, Bayesian and other techniques have also been developed for estimating unknown parameters using past historical data. For black-box models, special canonical forms have to be assumed so that the parameters can be identified uniquely from the historical input-output data.

However, there are several applications where no or very little historical information is available to specify the above matrices. Indeed, **KF** may be started with very little objective information and adapted as data becomes available. For example, if the initial state is known very poorly, then the **KF** is started with large diagonal elements in the covariance matrix. Similarly, if the other matrices are known poorly, then the process noise covariance matrix is assumed to be large. Both of these choices have the relative effects of increasing the filter covariance matrix and the Kalman gain matrix.

The implication of the above interpretation for **KF** design is that the forecaster can use his judgement regarding the relative accuracy of the model versus the observations to select appropriate values for the noise covariance matrices, $\mathbf{Q}$ and $\mathbf{R}$. He can then examine the actual operation of the filter and adjust these values on-line, if the situation changes at a later time.

4.4 Algorithm Selection

Different types of numerical algorithms may be used to implement Kalman filters. The square-root algorithms are numerically most stable, but the initial effort in developing them may be more than that for the covariance algorithms. If it is anticipated that a number of bias terms would be included in the model and the flexibility of neglecting some of these terms is required, the square-root filters with triangular factorizations are very attractive (Bierman, 1976).

Cases in which the measurement noise is correlated or some of the measurements are noise-free has been considered by Bryson and Henrickson (1968). It is possible to reduce the size of the **KF** in these cases. Gelb (1974) discussed other techniques for reducing the dimension of Kalman filtering.

For time-invariant systems, if it is not necessary to compute the state covariance matrix at each step, Chandrashekhar-type algorithms may be used to reduce the computational requirements (Kailath, 1973; and Lindquist, 1974).

A number of practical algorithms for nonlinear state estimation are discussed by Mehra (1970) and Wishner et al. (1971).

4.5 Sensitivity Analysis

The effect of the modeling and parameter errors on the performance of the **KF** can be determined via sensitivity analysis. It is only necessary to solve the covariance equations to study the effect of errors, but the covariance equations for the suboptimal **KF** are different. In general, a 2x2 linear matrix equation has to be solved to study the large scale sensitivity of the **KF**.

4.6 Validation and Testing

An optimal **KF** has the property that the innovation sequence has zero mean white noise with constant covariance. Statistical tests for checking the randomness property are:

1. Correlation tests for testing local linear dependence (Mehra, 1970),
2. Integrated spectrum test for periodic linear dependence, (Jenkins and Watts, 1968),
3. Run tests for linear and nonlinear dependence, (Fama, 1965).

5. STUDY AREA AND DATA CHARACTERISTICS

5.1 Overview

To investigate and develop a KF model approach to multisite precipitation modeling, 30 year records (1956-1985) of annual rainfall for the 52 stations used. As shown in Figure 5.1 these stations are distributed over an area approximately covering all of Turkey with more concentration in the northwestern part.

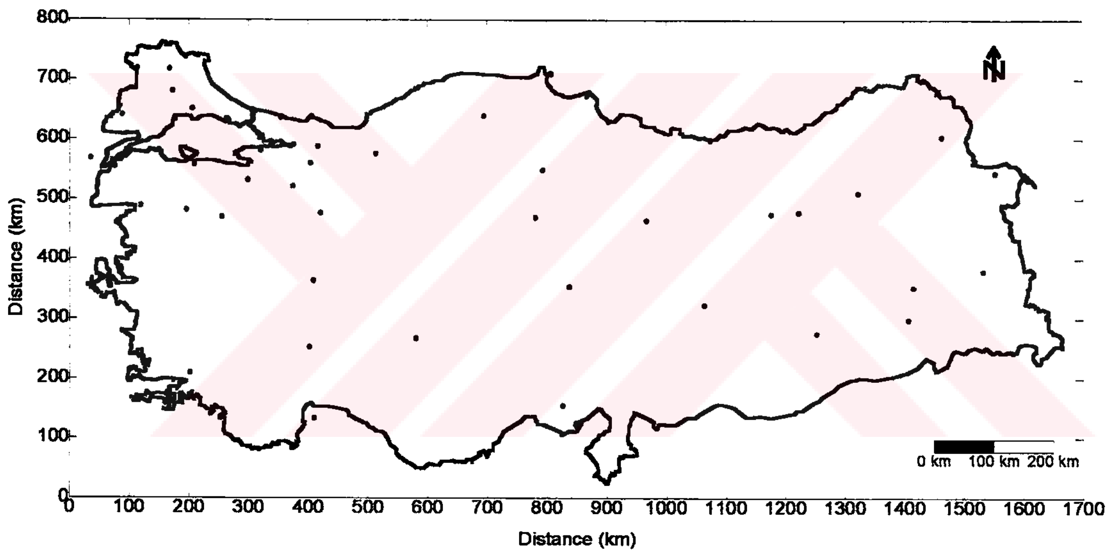

Figure 5.1 Distribution of rainfall stations over Turkey.

5.2 Geographical Setting of Study Area

The study area lies between latitudes $36°:70'$ N and $42°:03'$ N and longitude $25°:90'$ E and $44°:03'$ E, extended over an area of about 780,576 square kilometers. The geographic locations (latitude, longitude and elevation in meter above mean sea level) of precipitation stations are represented in Table 5.1. However, Figure 5.2 shows rough contour line of equal height of stations for the study area.

Table 5.1 Station locations and elevation above mean sea level.

No.	Station No.	Station Name	Latitude (N)	Longitude (E)	Elevation (meter)
01	068	Adapazarı	40° : 78′	30° : 42′	31
02	662	Ali Fuat Paşa – Adapazarı	40° : 52′	30° : 30′	100
03	102	Ağrı	39° : 72′	40° : 05′	1631
04	350	Adana	37° : 00′	35° :33′	20
05	620	Bahçeköy – İstanbul	41° : 17′	29° : 05′	130
06	070	Bolu	40° : 73′	31° : 60′	742
07	150	Balıkesir	39° : 62′	27° : 92′	120
08	116	Bursa	40° : 18′	29° : 07′	100
09	115	Bandırma – Balıksir	40° : 32′	27° : 97′	58
10	122	Bilecık	40° : 15′	29° : 98′	539
11	164	Bitlis	38° : 40′	42° : 12′	1578
12	112	Çanakkale	40° : 13′	26° : 40′	3
13	054	Çorlu – Tekirdağ	41° : 17′	27° : 80′	183
14	084	Çorum	40° : 55′	34° : 97′	776
15	280	Diyarbakır	37° : 88′	40° : 18′	677
16	651	Dursunbey – Balıkesir	39° : 58′	28° : 63′	639
17	653	Edremit – Balıkesir	39° : 60′	27° : 02′	21
18	096	Erzurum	39° : 92′	41° : 27′	1869
19	050	Edirne	41° : 67′	26° : 57′	51
20	124	Eskişehir	39° : 78′	30° : 57′	789
21	058	Florya – İstanbul	40° : 98′	28° : 80′	36
22	673	Gökçeada – Çanakkale	40° : 20′	25° : 90′	72
23	062	Göztepe – İstanbul	40° : 97′	29° : 08′	33
24	674	İpsala – Edirne	40° : 93′	26° : 40′	10
25	010	İzmit	40° : 78′	29° : 93′	76
26	063	Kartal – İstanbul	40° : 90′	29° : 18′	28
27	059	Kumköy – İstanbul	41° : 25′	29° : 02′	30
28	601	Kırklareli	41° : 73′	27° : 23′	232
29	011	Kandilli	41° : 10′	29° : 06′	114
30	098	Kars	40° : 60′	43° : 08′	1775
31	052	Luleburgaz – Kirklareli	41° : 40′	27° : 35′	46
32	210	Siirt	37° : 93′	41° : 95′	896
33	020	Şile – İstanbul	41° : 18′	29° : 62′	31
34	026	Sinop	42° : 03′	35° : 17′	32
35	056	Tekirdağ	40° : 98′	27° : 48′	4
36	118	Yalova – İstanbul	40° : 65′	29° : 27′	4
37	132	Yozgat	39° : 83′	34° : 82′	1298
38	170	Van	38° : 50′	43° : 50′	1671
39	190	Afyon	38° : 75′	30° : 53′	1034
40	195	Kayseri – Erkilet	38° : 78′	35° : 48′	1053
41	240	Isparta	37° : 75′	30° : 55′	997
42	244	Konya	37° : 97′	32° : 55′	1032
43	292	Muğla	37° : 20′	28° : 35′	646
44	030	Samsun	41° : 28′	36° : 33′	4
45	300	Antalya	36° : 70′	30° : 73′	50
46	034	Giresun	40° : 92′	38° : 40′	38
47	074	Kastamonu	41° : 37′	33° : 77′	799
48	090	Sivas	39° : 75′	37° : 02′	1285
49	092	Erzincan	39° : 73′	39° : 50′	1156
50	061	Sarıyer – İstanbul	41° : 17′	29° : 05′	56
51	100	Iğdır	39° : 93′	44° : 03′	858
52	200	Malatya – Erhav	38° : 43′	38° : 08′	0862

5.3 Topographical Structure

Turkey has different topographic structures, and it can be divided into eight land regions as follows (The World Book Encyclopedia, 1988)

1. The Northern Plains: They cover Thrace and extend along the Black Sea coast of Anatolia,
2. The Western Valleys: These are broad, fertile river valleys along the Aegean Sea coast,
3. The Southern Plains: Narrow strips of land along the Mediterranean Sea,
4. The Western Plateau: Region of highlands with scattered river valleys that extend across central Anatolia,
5. The Eastern Plateau: Rugged terrain of towering mountains and barren plains. It extends from the Western Plateau to Turkey's eastern border. The Taurus and Pontic mountains meet in this region. Ararat, the country's highest point, rises 5,185 meters above mean sea level near the Iranian border,
6. The Northern Mountains or Pontic Mountains: They rise between the Northern Plains and the Anatolian Plateau,
7. The Southern Mountains: This region consist of the Taurus Mountains and several smaller ranges on the southern edge of the Anatolian Plateau. These mountains almost completely cut off the plateau from the Mediterranean Sea,
8. The Mesopotamian Lowlands: They are fertile plains and river valleys in southeastern Anatolia.

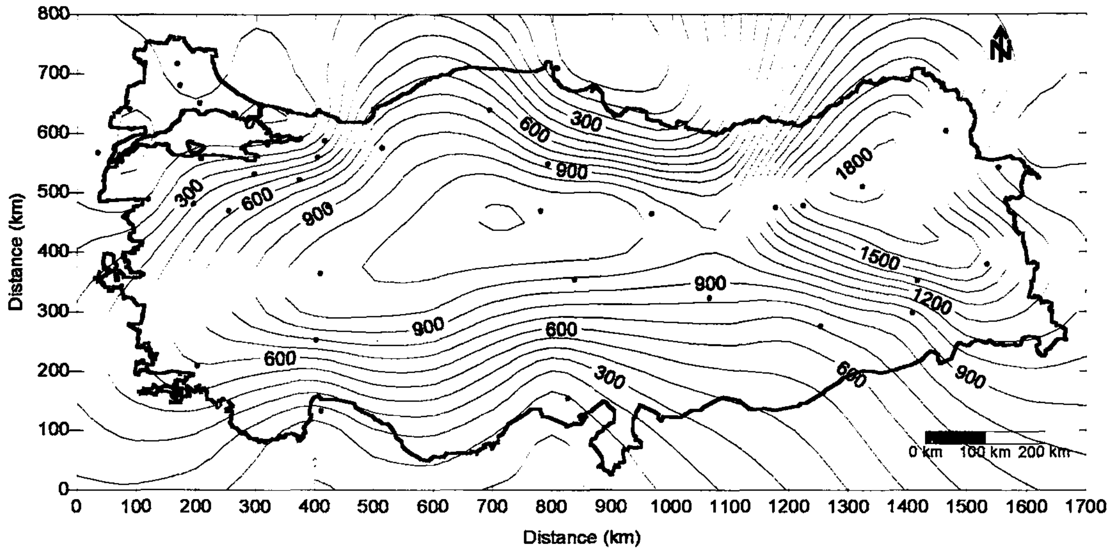

Figure 5.2 Rough contour map of equal height for Turkey (meters above mean sea level).

5.4 Synoptic Situation

The winter atmospheric circulation in this part of Asia is determined by the Asian High to the north-east and ridges of the Azores High to the west. The effect of the Asian High is experienced more in the east. Common too is advection of tropical air from north Africa and Mesopotamia which are drawn there by depressions travelling along more southerly tracks.

In spring, there is a rapid increase in the inflow of solar energy and lower layers of the atmosphere become overheated. Cyclonic activity then comes alive again and depressions from the north Atlantic and central Europe arrive more often. Thunder clouds form rapidly and these bring heavy rain, mainly to mountain areas.

The summer atmospheric circulation is associated with the Azores High and Asian Low which is centered over Baluchistan and the Indus Plain. Polar air masses arriving from the north-west and north are transformed here into continental air. From May to October, winds over the Aegean Sea and Turkey are northerly, and because of their regularity of occurrence, they have been known since ancient times as annual winds (etesian winds-meltem in Turkish).

5.5 Climate Condition

The term "Climate" has a very wide variety of meanings. To many of us "Climate" often first suggests temperature although rainfall and humidity may also come to mind. A useful definition might be "all of the statistics of a climatic state determined over an agreed time interval (seasons, decades or longer), computed for the global or possibly for a selected region.

The climate differs greatly from one region of Turkey to another. Danuta (1992) classified the climate conditions of Turkey, due to its position in the subtropical climate zone, into some of the categories as follows

1. The climate is moist subtropical in north-east and south-west Turkey and on the northern slopes of the Elburz Mountains,

2. In the rest of coastal Turkey the climate is intermediate between moist and continental,
3. Western Anatolia experiences continental climates,
4. Towards eastern Turkey the dryness and continently of climate increases,
5. Each mountain area has a mountain climate of its own.

5.5.1 Temperature

The coldest month of the year is January (February on Mediterranean). Temperatures are below zero at altitudes above 1000 m with temperatures of over 8 °C in south-west and south-east coasts. The maximum temperature varies from 10 °C on the Black Sea but 13-15 °C on the Mediterranean. Night-time temperature drops are particularly severe at high altitudes and in interior depressions, but are much less so on coasts. Negative temperature have been recorded all over the study area. In central Anatolia the absolute minimum temperature has fallen to -28 °C.

In summer, because of the great aridity and the intensity of the solar radiation at this time, absolute maximum temperatures exceed 40 °C in many spots in Anatolia and Mediterranean coast. On north coasts, they are lower, 27 °C on Black Sea and 34 °C on the Mediterranean coast.

5.5.2 Relative humidity

The annual variation in relative humidity corresponds to the seasonal inflow of air masses. Over most of the study area, the humidity is highest in winter, also because of the lower temperatures. In January, the relative humidity is 70-80 % on north and west coasts.

The very high temperatures of summer in central Anatolia make these area very dry. In many places, the July and August relative humidity drops to 20-30 % (Malatya, Urfa) and even 15 % in the afternoon. The frequent inflow of moist air masses with sea breezes or the etesian (meltém) or monsoon winds on the appropriate coasts raises the relative humidity in summer (Rize), in July to over 60 %.

5.5.3 Cloud

The cloudiest area throughout the year is the northern part of study area especially east of Samsun, where at Rize 30-40 % of days per month are cloudy. In winter (January), cloudiness ranges from 75 % in the north to 50 % in the south. July is the finest month of the year, and on the north-eastern part of the Turkish coast cloudiness is over 60 %. Cloudiness falls off rapidly westwards and southwards from that area even as low as 5-6 %, eastwards to 5-20 %. Away from north-east Turkey, there is extremely little cloudiness, from June to September, the monthly percentage is no more than 20 %, and 70-80 % of days are clear.

5.6 Some Statistical Characteristics

The dry and the wet seasons are immediately obvious. All year round precipitation is recorded only on eastern Black Sea coast. Autumn-winter precipitation is rather higher than in remainder of the year. On the west and south coasts, the precipitation increases from October to December-January and decreases from March-April. Onwards the Anatolia receives precipitation associated with frequent lows in May. The duration of the dry season varies from 1-2 months (July-August) in central Turkey and four months (June-September) in southern Turkey.

The some statistical parameters of the annual (1956-1985) rainfall values observed at the selected precipitation stations are summarized in Table 5.2 .

Table 5.2 Some statistical parameters of observed annual rainfall values (1956-1985)

No	Station Name	Mean (mm)	St. Dev. (mm)	Skewness
1	Adapazarı	835.5	183.4	2.18615
2	Ali Fuat Paşa – Adapazarı	660.6	107.7	0.36170
3	Ağrı	506.7	91.9	0.13271
4	Adana	690.4	221.3	0.86862
5	Bahçeköy – İstanbul	1287	478.6	1.73873
6	Bolu	549.2	83.3	0.14283
7	Balıkesir	606.7	163.6	1.59256
8	Bursa	680.6	109.1	0.16262
9	Bandırma – Balıkesir	719	149.3	0.71735
10	Bilecik	445.1	75.4	0.62528
11	Bitlis	1164	287.5	0.27270
12	Çanakkale	636.6	139.9	0.64583
13	Çorlu – Tekirdağ	570.6	104.1	0.58876
14	Çorum	432.1	79.7	-0.05457
15	Diyarbakır	490	134.5	-0.10185
16	Dursun bey – Balıkesir	779.1	171.6	0.36409
17	Edremit – Balikesir	696.2	144.1	1.36238
18	Erzurum	410.1	85.3	0.92707
19	Edirne	593.2	102.2	0.85735
20	Eskişehir	393.1	94.8	-0.18598
21	Florya – Istanbul	656.4	117.3	1.02088
22	Gökçeada – Çanakkale	792.5	212.2	1.28517
23	Göztepe – Istanbul	698.7	127.4	0.87054
24	İpsala – Edirne	612.5	98.7	0.15228
25	İzmit	766.6	150.2	0.37969
26	Kartal – İstanbul	651.1	118.5	0.16123
27	Kumköy – İstanbul	796.1	185.8	1.02989
28	Kırklareli	589.4	133.9	1.17951
29	Kandilli	827.7	148.7	0.54675
30	Kars	470.5	106.1	0.75373
31	Luleburgaz – Kırklareli	652.3	182.2	2.00788
32	Siirt	684.5	183.4	1.02095
33	Şile – İstanbul	800.8	251.5	1.74568
34	Sinop	640.2	134.2	0.83072
35	Tekirdağ	604.2	192.6	3.1807
36	Yalova – İstanbul	770.4	266.9	3.216
37	Yozgat	565.8	109.2	0.72179
38	Van	377.9	62.7	-0.1035
39	Afyon	408.6	89.5	0.14329
40	Kayseri – Erkilet	364.8	60	0.75670
41	Isparta	557.6	160	0.70164
42	Konya	326.1	77.6	0.95732
43	Muğla	1165	298.3	0.53785
44	Samsun	693.7	129.4	0.45975
45	Antalya	1074	293.4	0.29616
46	Giresun	1240	153.2	1.17704
47	Kastamonu	457.6	82.2	0.17054
48	Sivas	411.8	79.2	0.03905
49	Erzincan	368.8	71.7	0.86580
50	Sarıyer – İstanbul	796.6	150.8	0.91685
51	Iğdır	251.9	77.3	1.0145
52	Malatya – Erhav	402.7	100	0.42067

6. APPLICATION OF KALMAN FILTER

6.1 Overview

Hydrologic variables occur in nature as a result of interconnected physical elements, among which unknown climatic and physiographic factors play a dominant role. Hence, hydrologic variables such as rainfall are products of complex time-varying phenomena which can be measured by a finite number of observations. These observations indicate that hydrologic variables are stochastic in nature (Yevjevich, 1971) and that they are nonlinear (Amorcho and Orlob, 1961), as a result of

1. time -variable geologic processes of erosion, deposition, weathering, etc.,
2. climatic changes,
3. state uncertainty in time,
4. energy transfer of the hydrologic cycle, which is nonlinear in its character.

In practice, the inputs and outputs of the hydrologic phenomena are measurable with time at a fixed point. However, the data obtained include a noise component with statistical parameters that can be estimated from the same data.

A detailed study of hydrologic phenomena requires mathematical models that should take into account time variability. Hence, the planner might want either to simulate the underlying generating mechanism of the phenomenon concerned or to make future predictions.

6.2 State Space Formulation of the Problem

Consider the problem of estimating the variable of some system. In dynamic systems (that is, system which vary with time) the system variables are often denoted by the term state variables. An adaptive prediction algorithm for the estimation of the state variables and the unknown parameters of a time-varying hydrologic variable with noisy data requires a system model and measurement of its behavior, i.e. observation, in addition to statistical models which characterize the system and measurement errors as well as the initial conditions.

In the application of Kalman filtering theory, the mathematical formulation of the problem and computational techniques used may depend heavily on the computational simplicities of the system model used (Şen, 1991). Such an inference is particularly important in real-time predictions.

As the problem here is to develop a real-time prediction scheme for annual rainfall, their system and measurement models must first be identified. Assume that the system variables are governed by the equation

$$\underbrace{\begin{bmatrix} X_1 \\ X_2 \\ . \\ . \\ X_{52} \end{bmatrix}_k}_{(1)} = \underbrace{\begin{bmatrix} \varphi_{1,1} & \varphi_{1,2} & . & . & \varphi_{1,52} \\ \varphi_{2,1} & \varphi_{2,2} & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ \varphi_{52,1} & . & . & . & \varphi_{52,52} \end{bmatrix}_{kk-1}}_{(2)} \underbrace{\begin{bmatrix} X_1 \\ X_2 \\ . \\ . \\ X_{52} \end{bmatrix}_{k-1}}_{(3)} + \underbrace{\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ . \\ . \\ \varepsilon_{52} \end{bmatrix}_{k-1}}_{(4)} \quad (6.1)$$

where:

(1) : State variable vector at time t_k,

(2) : Transition matrix relating (1) to (3),

(3) : State variable vector at time t_{k-1},

(4) : System error vector.

The measurement of the process is assumed to occur at discrete points along time in accordance with linear relationship

$$
\underbrace{\begin{bmatrix} Z_1 \\ Z_2 \\ \cdot \\ \cdot \\ Z_{52} \end{bmatrix}_k}_{(1)} = \underbrace{\begin{bmatrix} h_{1,1} & h_{1,2} & \cdot & \cdot & h_{1,52} \\ h_{2,1} & h_{2,2} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ h_{52,1} & \cdot & \cdot & \cdot & h_{52,52} \end{bmatrix}}_{(2)} \underbrace{\begin{bmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ X_{52} \end{bmatrix}_k}_{(3)} + \underbrace{\begin{bmatrix} \eta_1 \\ \eta_2 \\ \cdot \\ \cdot \\ \eta_{52} \end{bmatrix}_k}_{(4)} \qquad (6.2)
$$

where:

(1) :Vector measurement at time t_k,

(2) :Connection matrix between the measurement and the state vectors at

time t_k,

(3) :State variable vector at time t_k,

(4) :Measurement error vector.

Assume that the system randomness is white Gaussian noise with a covariance matrix $\mathbf{Q}$. Further, assume that the measurement noise is also random with a covariance matrix $\mathbf{R}$, and that is not correlated with the system noise. We might want to formulate an estimation algorithm such that the following statistical conditions hold

1. The expected value of our estimate is equal to the expected value of the state. That is, "on average," our estimate of the state will equal the true state,
2. We want an estimation algorithm such that all possible estimation algorithms, our algorithm minimizes the expected value of the estimation error. That is, "on average," our algorithm gives the "smallest" possible estimation error.

6.3 Multisite Kalman Filter Algorithm

The Kalman filtering algorithm is applied to the rainfall measured data, at 52 stations, in a forward direction (Figure 6.1). As discussed in previous chapters, once a model has been selected, **KF** processing requires specification of

1. initial state vector,
2. error covariance matrix associated with this initial state vector,
3. system noise covariance,
4. measurement noise covariance,
5. state transition matrix,
6. connection matrix.

Most of this information should be based on physical understanding and all the previous knowledge about the process prior to t_{k-1}. If little historical information is available to specify the above matrices, then **KF** may be started with very little objective information and adapted as data becomes available. However, the less the initial information, greater diagonal elements should be selected in the covariance matrices. In this manner, the algorithm will have flexibility to adjust itself to sensible values in a relatively short space of time.

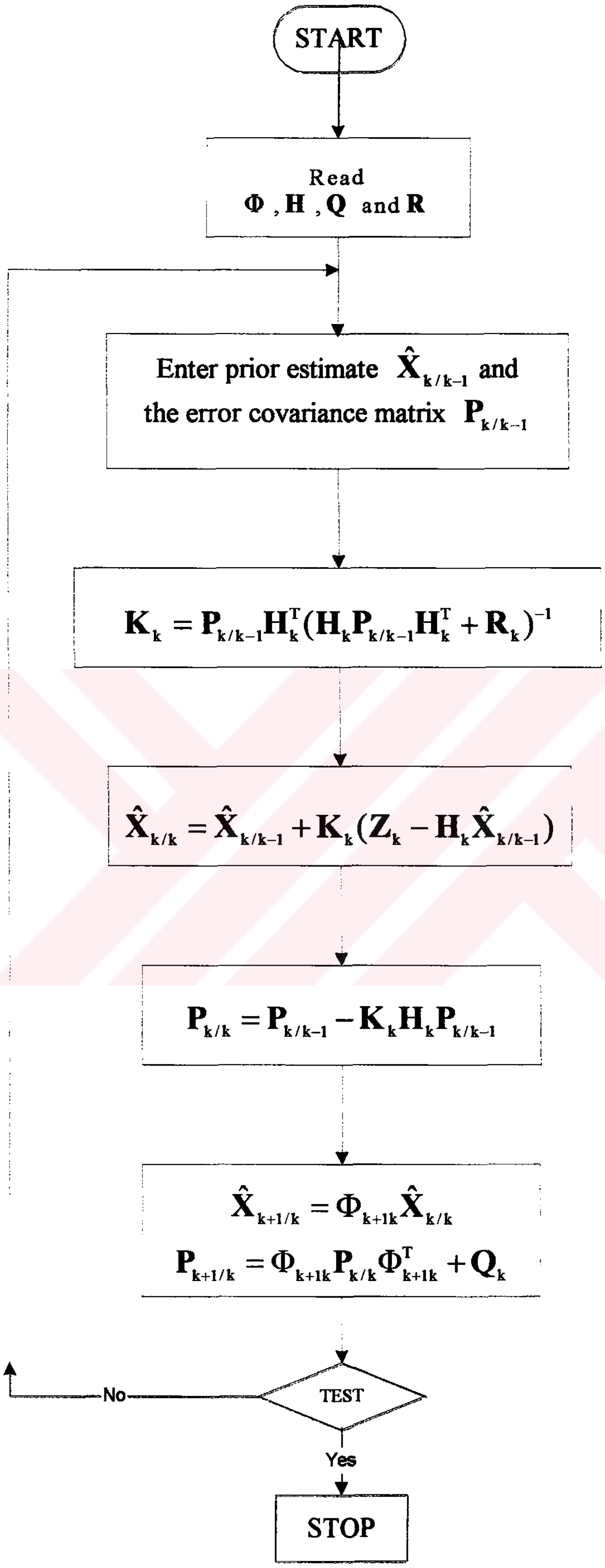

Figure 6.1 Kalman filter processing algorithm

6.3.1 Initial State Description

The average amount of rainfall values at the selected stations ($\overline{X}_1$, $\overline{X}_2$, ..., $\overline{X}_{52}$) are used as the elements of initial state vector $\hat{\mathbf{X}}_{k/k-1}$ as follows

$$\hat{\mathbf{X}}_{1/0} = \begin{bmatrix} \overline{X}_1 \\ \overline{X}_2 \\ .. \\ .. \\ \overline{X}_{52} \end{bmatrix} \tag{6.3}$$

As discussed throughout the thesis, the way to approach the problem is to make sure that whatever initial values are, a sufficiently great diagonal elements of error covariance matrix $\mathbf{P}_{k/k-1}$ are needed with initial state vector provided in the initial moment.

$$\mathbf{P}_{1/0} = \begin{bmatrix} 2000 & . & . & . & 0 \\ . & 2000 & . & . & . \\ . & . & .... & . & . \\ . & . & . & .... & 0 \\ 0 & . & . & 0 & 2000 \end{bmatrix} \tag{6.4}$$

As shown in Figure 6.2, the prediction error covariance steadily decreases with time and arrives at a stable value after some steps, indicating the efficiency of the prediction algorithm .

The **KF** is initially influenced by its initial conditions, but eventually ignores them, paying much greater attention to model parameters and the measurements

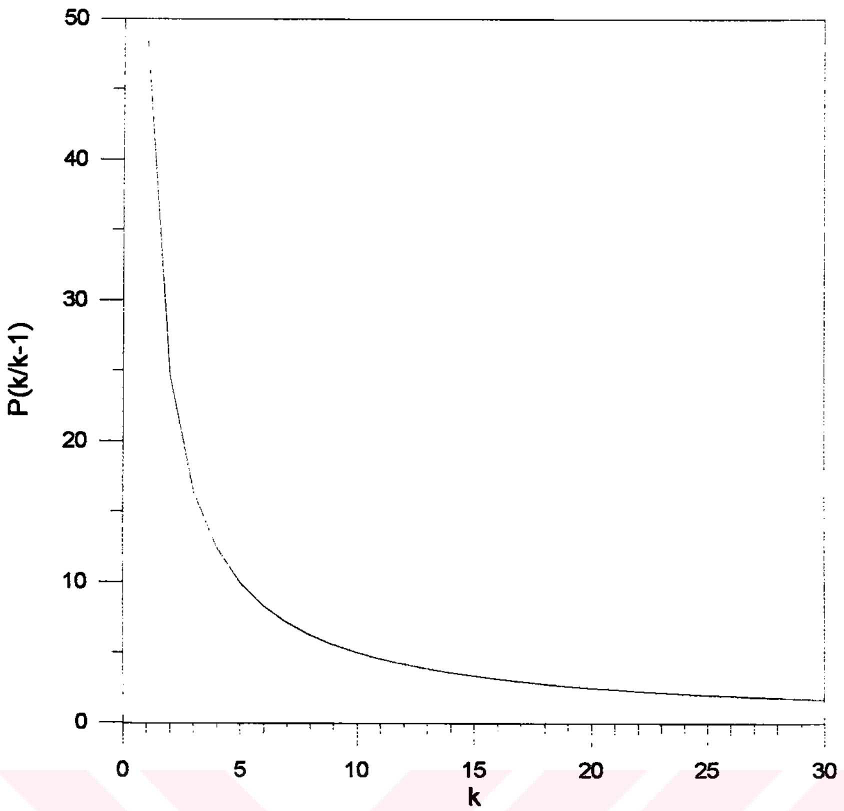

Figure 6.2 Estimation error variance

6.3.2 Kalman Gain Matrix

After the estimates, the initial state description are read into the program as first step. Kalman gain matrix for the one-step prediction can be computed by modified Eq. (3.30) with necessary assumptions as

$$\mathbf{K}_1 = \begin{bmatrix} 2000 & 0 & . & 0 \\ 0 & .. & . & . \\ . & . & .. & 0 \\ 0 & . & 0 & 2000 \end{bmatrix} \left[\begin{bmatrix} 2000 & 0 & . & 0 \\ 0 & .. & . & . \\ . & . & .. & 0 \\ 0 & . & 0 & 2000 \end{bmatrix} + \begin{bmatrix} 50 & 0 & . & 0 \\ 0 & .. & . & . \\ . & . & .. & 0 \\ 0 & . & 0 & 50 \end{bmatrix} \right]^{-1} \tag{6.5}$$

where: connection matrix $\mathbf{H}$ is unity, i.e., all stations are reporting their observations and the diagonal elements of measurement noise covariance matrix $\mathbf{R}$ is taken as 50, which is smaller than those of $\mathbf{Q}$ because the observed value are relatively noise free compared with the errors which result from the system. This implies that the measurements are expected to have small errors.

$$\mathbf{R} = \begin{bmatrix} 50 & 0 & . & . & 0 \\ 0 & 50 & . & . & . \\ . & . & .. & . & . \\ . & . & . & .. & 0 \\ 0 & . & . & 0 & 50 \end{bmatrix} \tag{6.6}$$

Initially, when the model parameters are only rough estimates, with little objective information, the Kalman gain matrix ensures that the measurement data is highly influential in estimating the state parameters. Then, as confidence in the accuracy of the parameters grows with each iteration, the gain matrix values decrease, causing the influence of the measurement data in updating the parameters and associated error (Figure 6.3).

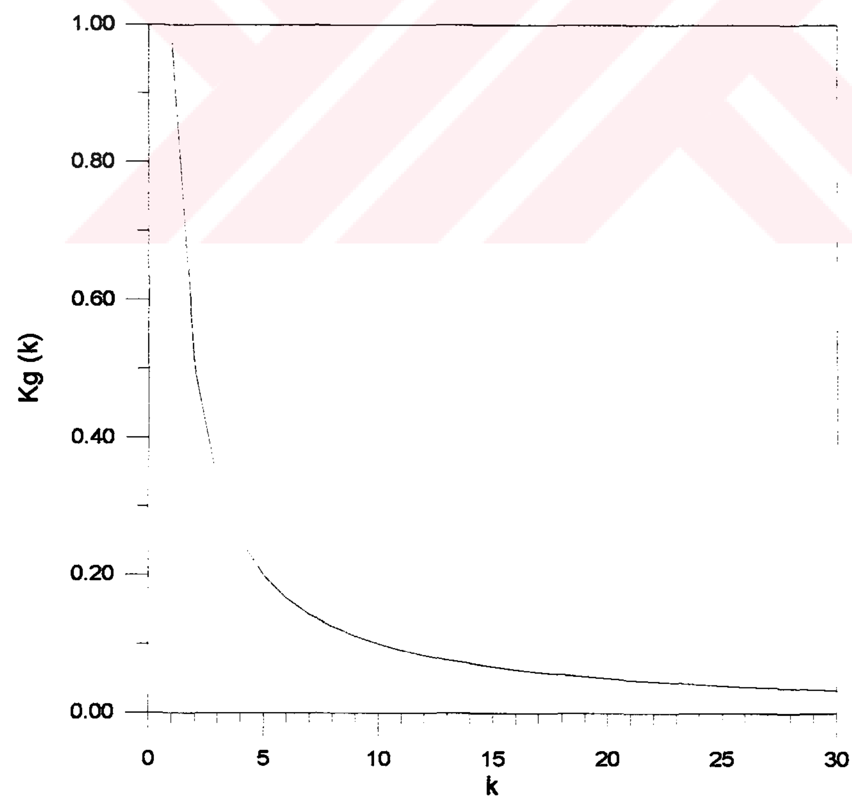

Figure 6.3 Kalman gain

6.3.3 Update Estimate With Measurement

With the assumption of an initial estimate, $\hat{\mathbf{X}}_{1/0}$, the measurement $\mathbf{Z}_k$ is used to improve the initial estimate by a linear blending of the noisy measurement and the prior estimate in accordance with the following equation

$$\hat{\mathbf{X}}_{1/1} = \begin{bmatrix} \overline{X}_1 \\ \overline{X}_2 \\ .. \\ .. \\ \overline{X}_{52} \end{bmatrix} + \begin{bmatrix} k_{1,1} & k_{1,2} & .. & .. & k_{1,52} \\ k_{2,1} & k_{2,2} & .. & .. & .. \\ .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \\ k_{52,1} & .. & .. & .. & k_{52,52} \end{bmatrix}_1 \left(\begin{bmatrix} Z_1 \\ Z_2 \\ .. \\ .. \\ Z_{52} \end{bmatrix}_1 - \begin{bmatrix} \overline{X}_1 \\ \overline{X}_2 \\ .. \\ .. \\ \overline{X}_{52} \end{bmatrix} \right) \tag{6.7}$$

6.3.4 Error Covariance for Updated Estimate

The covariance matrix associated with the updated estimate may now be computed by substituting the error covariance matrix associated with initial state vector, connection and Kalman gain matrices as given previously by Eq.(6.5) which when substitute into Eq. (3.31) leads to

$$\mathbf{P}_{1/1} = \begin{bmatrix} 2000 & 0 & . & 0 \\ 0 & .. & & . \\ . & . & .. & 0 \\ 0 & . & 0 & 2000 \end{bmatrix}_{1/0} - \begin{bmatrix} k_{1,1} & . & k_{1,52} \\ . & . & . \\ k_{52,1} & . & k_{52,52} \end{bmatrix}_1 \begin{bmatrix} 2000 & 0 & . & 0 \\ 0 & .. & & . \\ . & . & .. & 0 \\ 0 & . & 0 & 2000 \end{bmatrix}_{1/0} \tag{6.8}$$

6.3.5 Project Ahead the Updated Estimate

The contribution of $\boldsymbol{\varepsilon}_k$ in Eq. (6.1) can be ignored, because it has zero mean and is not correlated with any of the previous $\boldsymbol{\varepsilon}$'s. However, the updated estimated $\hat{\mathbf{X}}_{1/1}$ can be projected ahead via the following transition matrix

$$\begin{bmatrix} X_1 \\ X_2 \\ . \\ . \\ X_{52} \end{bmatrix}_{2/1} = \begin{bmatrix} \varphi_{1,1} & \varphi_{1,2} & . & . & \varphi_{1,52} \\ \varphi_{2,1} & \varphi_{2,2} & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ \varphi_{52,1} & . & . & . & \varphi_{52,52} \end{bmatrix}_{12} \begin{bmatrix} X_1 \\ X_2 \\ . \\ . \\ X_{52} \end{bmatrix}_{1/1} \tag{6.9}$$

In fact, the estimate of transition matrix may be difficult, but Harrison and Stevens (1971, 1975a,b) claim that the system is quite robust to the transition matrix values, and that they can, therefore, be set to fixed values. They have a minimal effect on the results. However, for simplicity transition matrix assumed to be unity which leads to

$$\begin{bmatrix} X_1 \\ X_2 \\ . \\ . \\ X_{52} \end{bmatrix}_{2/1} = \begin{bmatrix} X \\ X_2 \\ . \\ . \\ X_{52} \end{bmatrix}_{1/1} \tag{6.10}$$

6.3.6 Project Ahead the Error Covariance Matrix

The error covariance matrix associated with projected ahead estimation $\mathbf{P}_{2/1}$ may be obtained by modified Eq. (3.35) to satisfy our assumption as

$$\mathbf{P}_{2/1} = \begin{bmatrix} P_{1,1} & P_{1,2} & . & . & P_{1,52} \\ P_{2,1} & P_{2,2} & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ P_{52,1} & . & . & . & P_{52,52} \end{bmatrix}_{1/1} + \begin{bmatrix} 100 & 0 & . & . & 0 \\ 0 & 100 & . & . & . \\ . & . & .. & . & . \\ . & . & . & .. & 0 \\ 0 & . & . & 0 & 100 \end{bmatrix} \tag{6.11}$$

where: $\mathbf{P}_{1/1}$ given previously by Eq.(6.8), and for the system noise covariance matrix $\mathbf{Q}$, one can use his judgement to select appropriate values. He can then examine the actual operation of the filter and adjust these values on-line if the situation changes at a later time. Similar to the covariance of initial values **KF** is started with large diagonal elements in the system noise covariance matrix.

$$\mathbf{Q} = \begin{bmatrix} 100 & 0 & . & . & 0 \\ 0 & 100 & . & . & . \\ . & . & \cdots & . & . \\ . & . & . & \cdots & 0 \\ 0 & . & . & 0 & 100 \end{bmatrix} \qquad (6.12)$$

The projected ahead estimation $\mathbf{X}_{2/1}$ and the error covariance matrix $\mathbf{P}_{2/1}$ are used as initial estimation for the next time step.

Equations (6.5), (6.7), (6.8), (6.10) and (6.11) comprise the multisite **KF** recursive equations. Once the loop is entered as shown in Figure 6.1, then it can be continued as much as necessary. All the calculations needed in this work are carried out by computer program in Quick Basic written by the author and provided in Appendix A.

6.4 Result and Discussion

Precipitation is characterized by variability in space and time. In addition, there are many factors affecting the magnitude and distribution of precipitation, elevation of station above mean sea level, various air mass movement, moisture, temperature, pressure, and topography. The magnitude and distribution of precipitation vary from place to place and from time to time, even in small areas. The application of multisite **KF** model as developed in this thesis approach to multisite precipitation modeling which illustrates some interesting points in the annual precipitation pattern.

Observed and estimated annual rainfall values time variation at each selected station over Turkey for the 30 year time period (1956-1985) are presented in Figures

D.1-D.52 in Appendix D. Figure 6.4 provides the observed and estimated annual rainfall values at Adapazarı from 1956 to 1984.

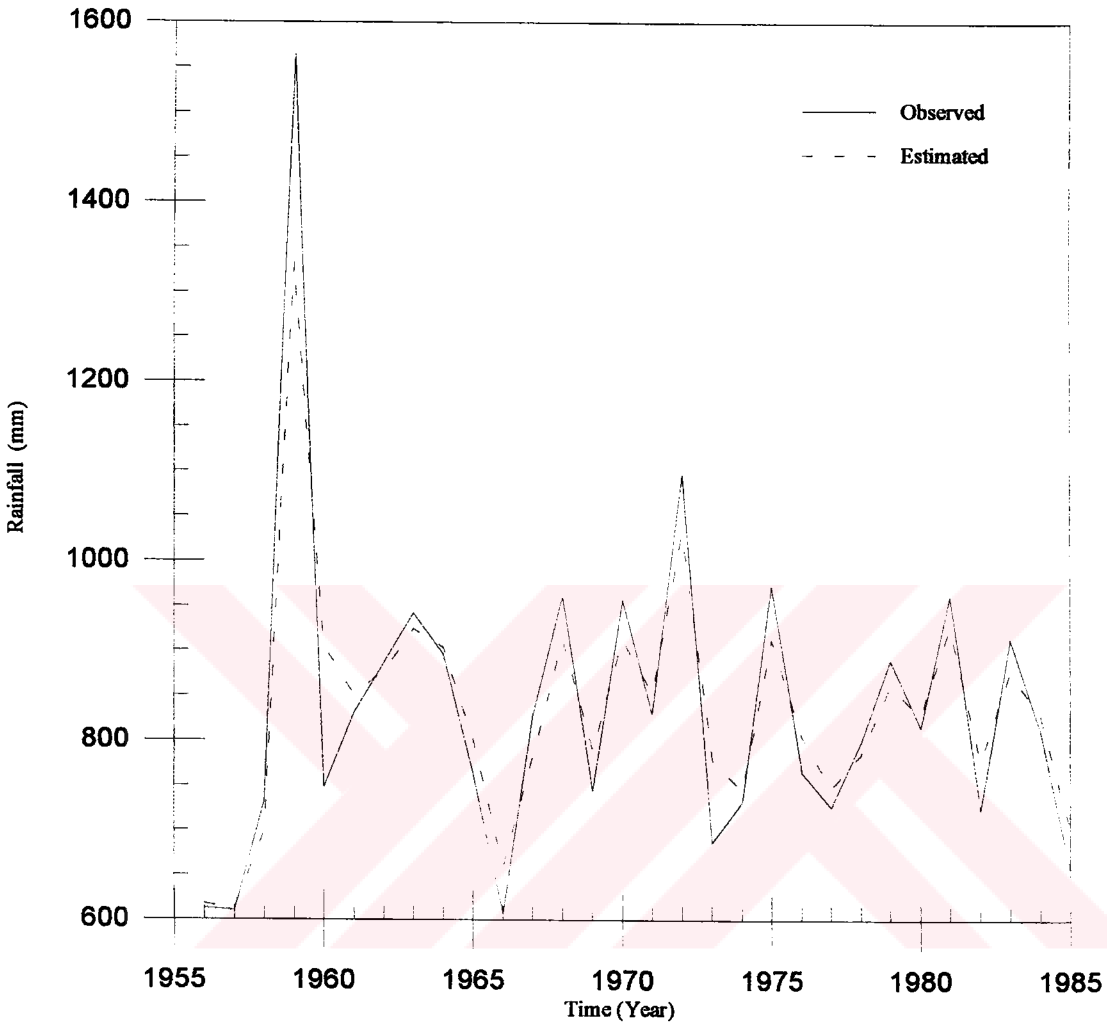

Figure 6.4 Observed and estimated annual rainfall values at Adapazarı from 1956 to 1985.

It is to be noticed from this figure that the observed and estimated values follow each other closely, which indicates that **KF** provides an efficient method for modelling of annual rainfall.

Some statistical parameters of annual observed and estimated rainfall values during the time period (1956-1985) are summarized in Table 6.1.

Table 6.1 Statistics parameters of observed and estimated annual rainfall values (1956-1985)

No	Station Name	Mean (mm)		St. Dev. (mm)		Minimum (mm)		Maximum (mm)		Range (mm)		Skewness	
		Obs.	*Est.*	*Obs.*	*Est.*	*Obs.*	*Est.*	*Obs.*	*Est.*	*Obs.*	*Est.*	*Obs.*	*Est.*
1	Adapazarı	835.5	834.6	183.4	135.3	606.3	611.7	1564	1332	957.7	720.3	2.18615	1.50665
2	Ali Fuat Paşa – Adapazarı	660.6	661	107.7	85.1	484.9	519.1	922.5	851.1	437.6	332	0.36170	0.13996
3	Ağrı	506.7	504.7	91.9	66.6	352	389.1	693.1	612.7	341.1	223.6	0.13271	-0.0978
4	Adana	690.4	688.7	221.3	184.4	319.4	360	1265	1149	945.6	789	0.86862	0.66023
5	Bahçeköy – İstanbul	1287	1284	478.6	354.7	848.1	866.2	2698	2325	1850	1459	1.73873	1.56783
6	Bolu	549.2	549.2	83.3	59.3	377.4	429.1	716.8	659.7	339.4	230.6	0.14283	0.16372
7	Balıkesir	606.7	606.1	163.6	126.3	362.6	385.6	1192	1036	829.4	650.4	1.59256	1.25458
8	Bursa	680.6	680.1	109.1	82.7	447.8	521.1	871	835	423.2	313.9	0.16262	-0.0062
9	Bandırma – Balıkesir	719	718.9	149.3	115	456.2	494.7	1086	988.2	629.8	493.5	0.71735	0.55033
10	Bilecik	445.1	444.5	75.4	57.5	320.6	332.8	624.7	563.1	304.1	230.3	0.62528	0.39746
11	Bitlis	1164	1160	287.5	235.8	564.5	734.2	1866	1738	1301	1004	0.27270	0.51462
12	Çanakkale	636.6	640.4	139.9	108.9	414.8	459.8	977.7	868.5	562.9	408.7	0.64583	0.45769
13	Çorlu – Tekirdağ	570.6	571.1	104.1	81.5	431.2	456.4	801.3	759.9	370.1	303.5	0.58876	0.64617
14	Çorum	432.1	430.7	79.7	59.1	285.5	296.8	606.7	542.9	321.2	246.1	-0.05457	-0.2570
15	Diyarbakır	490	489	134.5	99.3	146.3	276	748.8	671	602.5	395	-0.10185	0.24572
16	Dursun bey – Balıkesir	779.1	782.2	171.6	130.6	479.3	575	1199	1079	719.7	504	0.36409	0.15470
17	Edremit – Balikesir	696.2	697.2	144.1	111.6	512.9	548.4	1175	1061	662.1	512.6	1.36238	1.2798
18	Erzurum	410.1	409.8	85.3	61.3	291.1	308.1	638.5	551.8	347.4	243.7	0.92707	0.49036
19	Edirne	593.2	594.4	102.2	72.1	430.5	496.6	863.6	792.2	433.1	295.6	0.85735	0.87714
20	Eskişehir	393.1	391.8	94.8	51.7	215.8	220.1	524	486.8	326.2	266.7	-0.18598	-0.9474
21	Florya – Istanbul	656.4	655.5	117.3	89.3	500.8	540	1026	942.6	525.2	402.6	1.02088	1.07883
22	Gökçeada – Çanakkale	792.5	796.4	212.2	162.8	483	543.8	1451	1235	968	691.2	1.28517	0.92635
23	Göztepe – Istanbul	698.7	696.7	127.4	98.3	538.8	559	1047	990.9	508.2	431.9	0.87054	0.94386
24	İpsala – Edirne	612.5	616.3	98.7	77.5	386.5	434.7	808.9	763.9	422.4	329.2	0.15228	0.11054

Table 6.1 Continued.

25	İzmit	766.6	764.9	150.2	119.9	554.9	557.2	1088	1012	533.1	454.8	0.37969	0.12170
26	Kartal – İstanbul	651.1	649.5	118.5	90.9	475.6	496.9	871.9	832.4	396.3	335.5	0.16123	0.05385
27	Kumköy – İstanbul	796.1	793.2	185.8	143.7	519.5	581.9	1278	1145	758.5	563.1	1.02989	0.88983
28	Kirklareli	589.4	592.5	133.9	97.5	371.5	417.5	1000	876.9	628.5	459.4	1.17951	0.99290
29	Kandilli	827.7	825.5	148.7	117.5	600.5	631.5	1230	1143	629.5	511.5	0.54675	0.42092
30	Kars	470.5	472.2	106.1	78.2	298.5	361.7	718.8	661.1	420.3	299.4	0.75373	0.81958
31	Luleburgaz – Kırklareli	652.3	655.4	182.2	138.8	399.7	437.4	1360	1159	960.3	721.6	2.00788	1.57797
32	Siirt	684.5	682.4	183.4	141.5	420.8	474.5	1229	1060	808.2	585.5	1.02095	0.91065
33	Şile – İstanbul	800.8	796.2	251.5	211.6	454.6	491.5	1697	1537	1242	1045	1.74568	1.66792
34	Sinop	640.2	639.7	134.2	109.6	414.7	455.2	990.4	917.1	575.7	461.9	0.83072	0.77820
35	Tekirdağ	604.2	606.6	192.6	141.2	405.2	434.8	1464	1192	1059	757.2	3.1807	2.50102
36	Yalova – İstanbul	770.4	769.2	266.9	202.7	473.2	484.7	1959	1617	1486	1132	3.216	2.60607
37	Yozgat	565.8	562.6	109.2	86	391	412.9	858.2	771.3	467.2	358.4	0.72179	0.44662
38	Van	377.9	377.9	62.7	47.6	267.9	292.5	486	461.1	218.1	168.6	-0.1035	0.04482
39	Afyon	408.6	407.7	89.5	69	239	259.3	618	576.6	379	317.3	0.14329	-0.0192
40	Kayseri – Erkilet	364.8	365.2	60	45.3	263	278.6	535	475.3	272	196.7	0.75670	0.2949
41	Isparta	557.6	557.8	160	122.7	332	371.4	968	891.5	636	520.1	0.70164	0.85167
42	Konya	326.1	325	77.6	63.7	193	225	545	483.7	352	258.7	0.95732	0.66484
43	Muğla	1165	1164	298.3	229.3	658	767.4	1805	1672	1147	904.6	0.53785	0.44423
44	Samsun	693.7	694.2	129.4	103.2	442.2	503.5	1011	955	568.8	451.5	0.45975	0.72549
45	Antalya	1074	1072	293.4	228.3	554	608.7	1914	1747	1360	1138	0.29616	0.28473
46	Giresun	1240	1244	153.2	125.5	1057	1090	1679	1668	622	578	1.17704	1.63857
47	Kastamonu	457.6	455	82.2	67.3	308	312.1	617	574.9	309	262.8	0.17054	-0.1840
48	Sivas	411.8	410.1	79.2	56.4	286	298.8	575	519.8	289	221	0.03905	0.09282
49	Erzincan	368.8	367.6	71.7	56.8	257	277.9	563	504.1	306	226.2	0.86580	0.62331
50	Sarıyer – İstanbul	796.6	794.3	150.8	130.6	574.9	597.8	1171	1102	596.1	504.2	0.91685	0.72847
51	Iğdır	251.9	253.2	77.3	55.5	114.5	154.9	501.2	422.9	386.7	268	1.0145	1.00863
52	Malatya – Erhav	402.7	403.1	100	72.1	248	292.1	593	533.9	345	241.8	0.42067	0.33824

From another point of view, Figure 6.5 provides the observed and estimated annual rainfall values at 52 selected station in Turkey for 1985 (Figures for other years are presented in appendix E). From Figure 6.5 and Table 6.2, again as noticed before, in the case of one station (Adapazarı), the observed and estimated values follow each other closely, which indicates that **KF** provides an efficient method for modelling of annual rainfall in both time and space dimension.

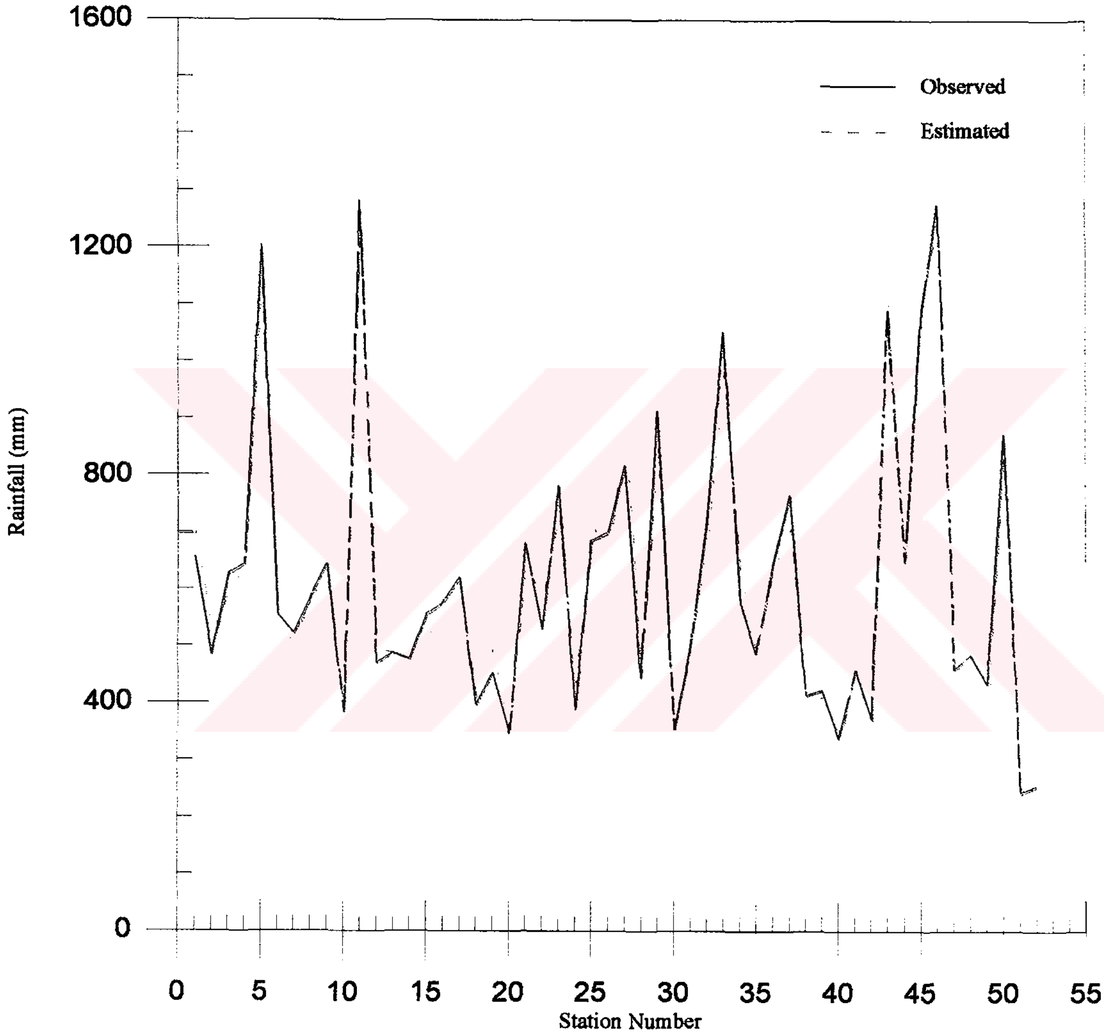

Figure 6.5 Observed and estimated annual rainfall values at selected stations in Turkey for 1985.

Table 6.2 Statistical parameters of observed and estimated areal annual rainfall values at 52 station (1956-1985)

No	Year	Mean (mm)		St. Dev. (mm)		Minimum (mm)		Maximum (mm)		Range (mm)		Skewness	
		Obs.	*Est.*	*Obs.*	*Est.*	*Obs.*	*Est.*	*Obs.*	*Est.*	*Obs.*	*Est.*	*Obs.*	*Est.*
1	1956	581.98	583.56	256.35	254.8	215.8	220.12	1679	1668.3	1463.2	1448.2	1.71258	1.6906
2	1957	528.47	542.35	183.08	188.77	222.07	259.32	1194.2	1257.5	972.13	998.22	1.70045	1.65099
3	1958	629.20	606.05	248.87	228.08	188.8	207.61	1473.6	1416	1284.8	1208.3	1.249	1.37515
4	1959	672.692	654.83	432.87	285.23	225	240.8	1913.8	1651.4	1688.8	1410.6	1.90109	1.55056
5	1960	614.955	625.64	247.43	244.99	249.44	248.76	1576.4	1596.5	1327	1347.7	1.9651	1.7717
6	1961	583.665	594.91	250.09	237.81	120.6	154.94	1269.6	1231.8	1149	1076.8	0.814375	0.900957
7	1962	710.046	679.2	316.86	283.18	228.9	209.08	1736	1560.8	1507.1	1351.7	0.83743	0.723771
8	1963	787.594	758.55	342.92	303.48	364	338.6	2698.1	2325.4	2334.1	1986.8	3.5524	2.76322
9	1964	573.223	622.88	184.49	198.69	188	250.95	1151	1243.9	963	992.98	0.676102	0.856109
10	1965	734.521	704.61	385.07	326.98	218.4	227.12	2450.3	2127.1	2231.9	1899.9	2.22336	2.02445
11	1966	665.531	676.01	272.87	273.38	273.3	260.93	1506	1491.5	1232.7	1230.6	0.763362	0.891475
12	1967	680.248	679.11	29.12	274.3	274.2	270.64	1865.8	1691.5	1591.6	1420.6	1.67997	1.48092
13	1968	787.95	758.79	335.76	306.32	250	255.6	1958.7	1737.9	1708.7	1482.3	1.40347	1.24604
14	1969	672.21	695.41	322.47	303.85	308.3	327.71	1914	1747.3	1605.7	1419.6	2.32248	1.90751
15	1970	641.648	656.05	285.44	267.42	114.5	171.63	1288.5	1246.1	1174	1074.4	0.254917	0.515159
16	1971	661.038	659.7	313.8	289.96	211.8	201.04	2308.7	2024	2096.9	1822.9	3.04718	2.25793
17	1972	586.592	606.18	206	215.25	280	293.17	1217.1	1324.1	937.1	1031	1.14517	1.26685
18	1973	548.151	563.7	218.8	210.48	193	225	1348.7	1282.5	1155.7	1057.5	1.05955	1.02553
19	1974	599.869	590.18	251.66	232.46	239	250.55	1302	1188	1063	937.45	1.08167	0.952766
20	1975	698.027	669.13	254.24	243.92	293.4	287.19	1404.7	1346.6	1111.3	1059.4	0.63316	0.699324
21	1976	642.687	649.77	253.2	238.15	243.8	255.43	1507.6	1407.4	1263.8	1152	1.37822	1.18679
22	1977	531.986	558.28	188.2	197.44	138.56	131.86	1138.9	1210.9	1000.3	1079	0.869633	0.949575
23	1978	686.331	658.14	279.43	256.71	298	306.19	1438	1331.6	1140	1025.4	0.979061	1.03373
24	1979	686.331	678.48	265.29	255.41	222.9	245.22	1682	1546.2	1459.1	1300.9	1.40993	1.29747
25	1980	673.44	674.79	255.82	243.56	142.4	169.95	1317.7	1275.2	1175.3	1105.2	0.342582	0.453326
26	1981	794.467	762.39	354.91	320.5	244.7	224.67	1696.8	1536.9	1452.1	1312.2	0.657066	0.622057
27	1982	570.467	621.8	217.26	236.04	270.94	284.14	1285.7	1302	1014.8	1017.8	1.17979	0.899031
28	1983	637.03	632.95	245.06	234.46	260.9	267.13	1333.2	1276.9	1072.3	1009.8	1.01769	0.983406
29	1984	580.785	594.76	251.22	236.51	211.3	226.26	1278	1183	1066.7	956.71	1.23873	1.07806
30	1985	612.898	608.04	250.86	240.22	244.2	239.39	1282.3	1250.8	1038.1	1011.4	1.17209	1.17431

Contour maps of observed and estimated annual rainfall for 1956-1985, and percentage error of estimated annual rainfall are shown in Figures B.1-B.30 in Appendix B and in Figure C.1-C.30 in Appendix C, respectively. In this section, maps of observed and estimated annual rainfall values and percentage errors of estimated annual rainfall for 1985 are presented in Figure 6.6 and Figure 6.7, respectively.

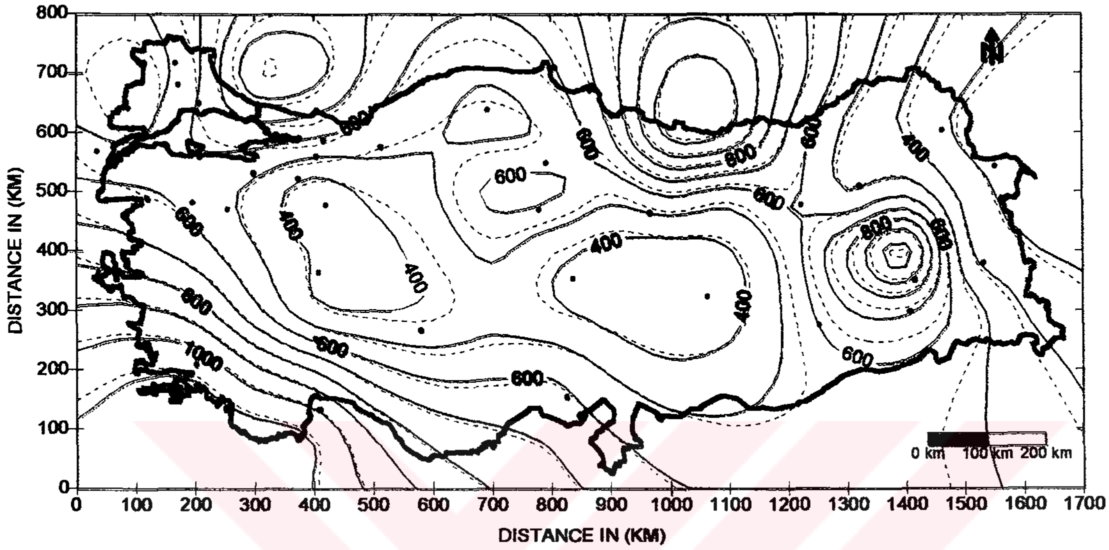

Figure 6.6 Contour map of observed and estimated annual rainfall for 1985

According to the areal values of observed and estimated annual rainfalls, the multisite **KF** method has a slight tendency toward underestimation. Standard deviation of estimated value is smaller than that of observed one (Figure 6.8). It means less variability. Therefore, more smoothed than observed values.

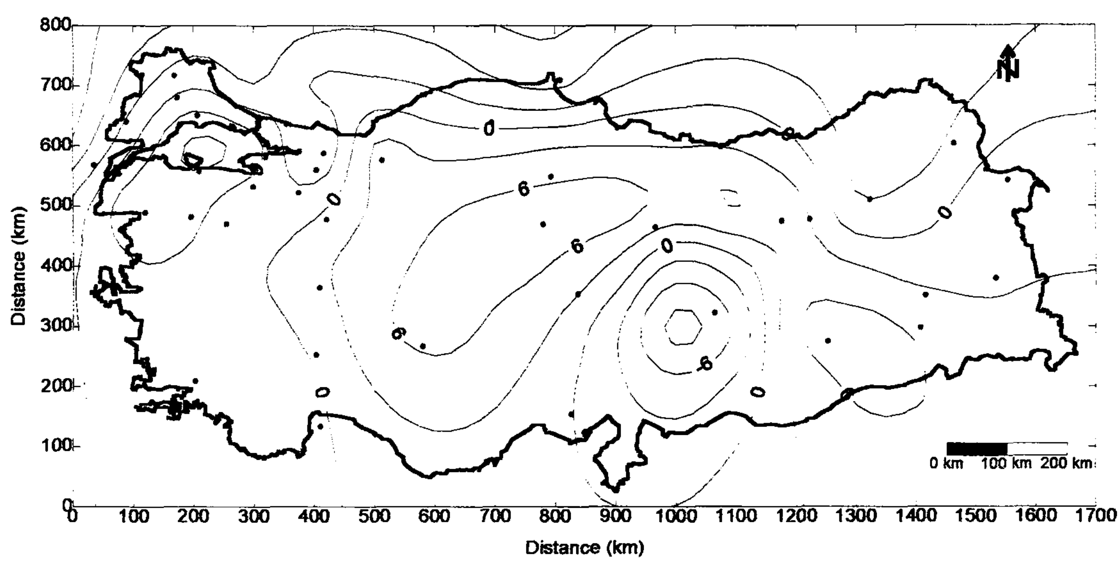

Figure 6.7 Contour map of the percentage error of estimated annual rainfall for 1985.

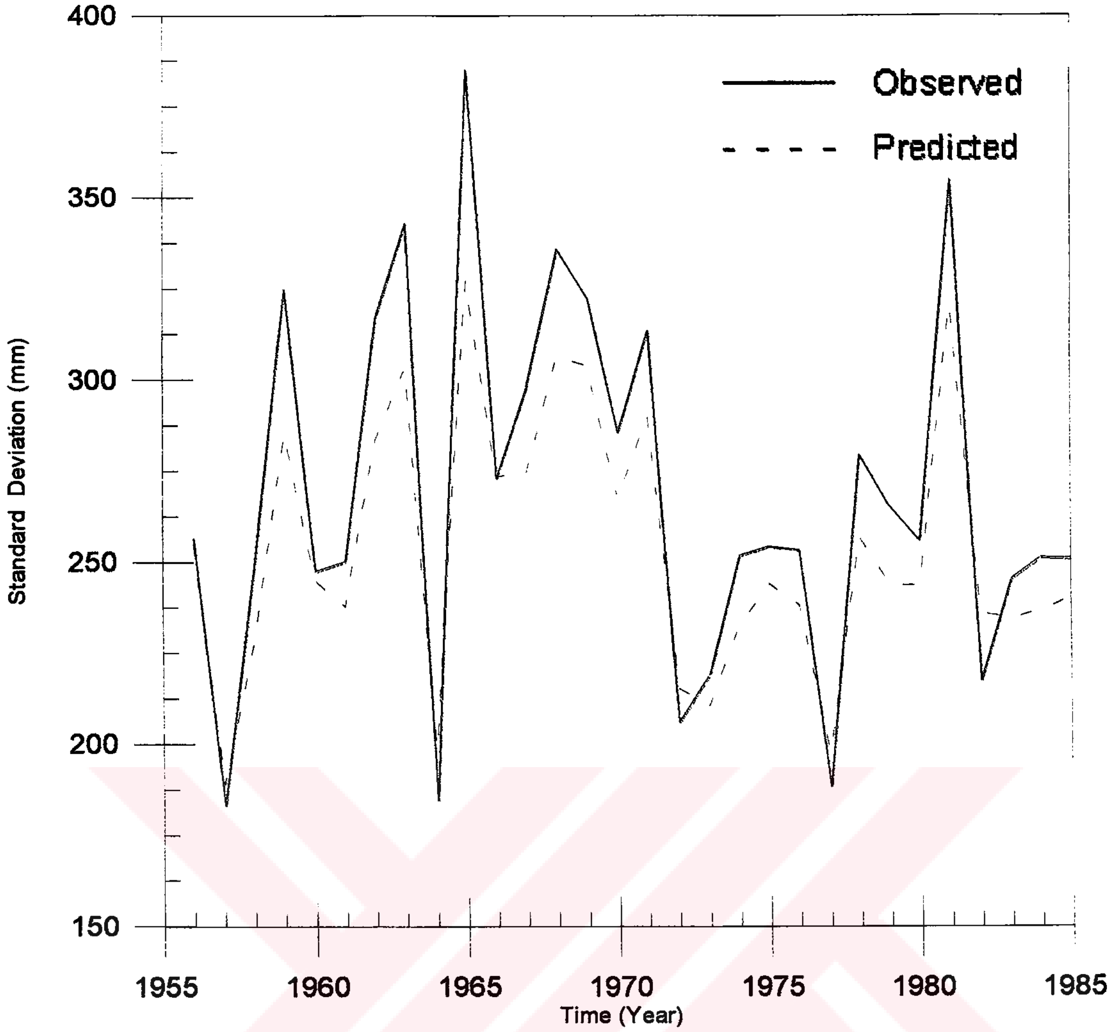

Figure 6.8 Standard deviation of observed and estimated areal rainfall values for 1956-1985

Furthermore, Figure 6.7 proves that the estimated values of annual rainfall at most of the sites in the study area are close to the observed values, specially in the part where more stations are available such as in the northwestern part of Turkey. The percentage error of estimated values vary from -6 in station number 52 (underestimation) to 6 in station number 49 (overestimation) with overall average about 0.12 %.

7. CONCLUSIONS AND RECOMMENDATIONS

The development of data estimation methods can be traced back to Gauss (1809), who invented the technique of deterministic least-squares approach and employed it in a relatively simple orbit measurement problem. Estimation is the process of extracting information from data, which can be used to infer the desired information and may contain errors. when the time at which an estimate is desired coincides with the last measurement point, the problem is referred to as filtering

The word ''filter'' is a relic from the early history of electrical engineering and it is concerned with the extraction of signals from noise. Kalman (1978) defined the filtering as any mathematical operation which uses past data or measurements on a given dynamical system to make more accurate statements about present, future, or past variables in that system.

Precipitation is characterized by variability in space and time. In addition, there are many factors affecting the magnitude and distribution of precipitation, such as altitude, various air mass movements, distance from the moisture sources, temperature, pressure, and topography.

The magnitude and distribution of precipitation vary from place to place and from time to time even in small areas. Describing and predicting the precipitation variability in space and / or time are fundamental requirements for a wide variety of human activities and water project designs.

Classical statistical estimation theory has not been able to predict directly nonstationarity. For reliable predictions it is necessary to have stationarity both in the mean and variance throughout the entire range of observations. That is why so much effort is spent to make the data stationary in the mean and variance. Otherwise, the results are not meaningful from a statistical point of view. For example, when the data pattern changes as with a step or trend, or when there are transient shifts, classical statistical theory will treat those as random effects or temporary shifts. If the changes are continuous, a new forecasting model will have to be specified to deal with the new equilibrium conditions. However, the model will be good for those new equilibrium conditions only when fixed patterns exist.

Classical statistical methods must be used in conjunction with other control processes if permanent or significant changes in the data are to be identified. Methods based on classical statistics cannot sense a shift by themselves and once a shift has taken place, the model will not do well, because it will still be tuned to the specification of the old data set.

Kalman filtering, on the other hand, can deal with step changes and transient situations because they update their parameters in a way that takes in to account the changes in pattern. All forecasting methods are special cases of **KF**. This filter can deal with changes in the model, the parameters, and the variances. The difficulty with **KF** is that many technical questions have not yet been answered satisfactorily.

Kalman filter consists of combining two independent estimates to form a weighted estimate or prediction. One estimate can be a prior prediction or an estimate based on prior knowledge, and other a prediction based on new information (new data). The purpose of the **KF** is to combine these two pieces of information to obtain an improved estimate.

The precipitation data which consist of long time series at various locations in space, have both spatial and temporal variations. In order to see the effectiveness of the **KF** model developed in this thesis, 30 year records (1956-1985) of annual rainfall data for the 52 different meteorology stations scattered all over Turkey with more concentration in the northwestern part are used. In practice, the inputs and outputs of the hydrologic phenomena are measurable with time at a fixed point. However, the data obtained include a noise component with statistical parameters that can be estimated from the same data.

The discrete **KF** is a recursive predictive updating technique used to determine the correct parameters of a process model. Given some initial estimates, it allows the parameters of a model to be predicted and adjusted with each new measurement, providing an estimate of error at each update. Its ability to incorporate the effects of measurement and system noises, and its computational structure, have made it very popular for use.

An adaptive prediction algorithm for the estimation of the state variables and the unknown parameters of a time-varying annual rainfall with noisy data requires a system model and measurement of its behavior in addition to statistical models which characterize the system and measurement errors as well as the initial conditions.

In the application of Kalman filtering theory, the mathematical formulation of the problem and computational techniques used may depend heavily on the computational simplicities of the system model used. Such an inference is particularly important in real-time predictions. The system randomness is assumed white Gaussian noise with a covariance matrix $\mathbf{Q}$. Also the measurement noise is assumed to be random with a covariance matrix $\mathbf{R}$, and that is not correlated with the system noise

Once a model has been selected, then **KF** processing requires specification of initial state vector, error covariance matrix associated with this initial state vector, system noise covariance, measurement noise covariance, state transition matrix and connection matrix. Most of this information should be based on physical understanding and all the previous knowledge about the process prior to t_{k-1}. If little historical information is available to specify the above matrices, then **KF** may be started with very little objective information and adapted as data becomes available. However, the less the initial information, greater diagonal elements should be selected in the covariance matrices. In this manner, the algorithm will have flexibility to adjust itself to sensible values in a relatively short space of time.

The average amount of rainfall values at the selected stations are used as the elements of initial state vector. A sufficiently great diagonal elements of error covariance matrix are needed with initial state vector provided in the initial moment. Then the prediction error covariance steadily decreases with time and arrives at a stable value after some steps, indicating the efficiency of the prediction algorithm.

After the initial state descriptions are read as first step, then the Kalman gain matrix for the one-step prediction can be computed, with necessary assumptions, as the connection matrix $\mathbf{H}$ is unity, i.e., all stations are reporting their observations. The diagonal elements of measurement noise covariance matrix $\mathbf{R}$ is taken smaller than those of $\mathbf{Q}$ because the observed values are relatively noise free compared with the errors which result from the system.

Initially, when the model parameters are only rough estimates, with little objective information, the Kalman gain matrix ensures that the measurement data is highly influential in estimating the state parameters. Then, as confidence in the accuracy of the parameters grows with each iteration, the gain matrix values decrease, causing the influence of the measurement data in updating the parameters and associated error

With the assumption of an initial estimate, the measurement $\mathbf{Z}_k$ is used to improve the initial estimate by a linear blending of the noisy measurement and the prior estimate. The covariance matrix associated with the updated estimate computed by using the error covariance matrix associated with initial state vector, connection and Kalman gain matrices.

The updated estimates can be projected ahead via the transition matrix, where the contribution of system error can be ignored, because it has zero mean. However, the estimate of transition matrix may be difficult, but the system is quite robust to the transition matrix values, and that they can, therefore, be set to fixed values. They have a minimal effect on the results. However, for simplicity transition matrix is assumed to be unity.

Project ahead the error covariance matrix, where the updated error covariance matrix can be computed, and for the system noise covariance matrix $\mathbf{Q}$, one can use his judgement to select appropriate values. He can then examine the actual operation of the filter and adjust these values on-line if the situation changes at a later time. Similar to the covariance of initial values **KF** is started with large diagonal elements in the system noise covariance matrix.

The projected ahead estimation and the error covariance matrix are used as initial estimation for the next time step. Once the multisite **KF** loop is entered, then it can be continued as much as necessary.

It is to be noticed that for one station the observed and estimated values follow each other closely, which indicates that **KF** provides an efficient method for modelling of annual rainfall. The observed and estimated annual rainfall values at 52 selected stations in Turkey for 1985, follow each other closely, which indicates that **KF** provides an efficient method for modelling of annual rainfall in both time and space dimensions.

According to the areal values of observed and estimated annual rainfalls, the multisite **KF** method has a slight tendency toward underestimation. Standard deviation of estimated value is smaller than that of observed one. It means less variability. Therefore, more smoothed than observed values.

The estimated values of annual rainfall at most of the sites in the study area are close to the observed values, specially in the part where more stations are available such as in the northwestern part of Turkey. The percentage error of estimated values vary from -6 (underestimation) to 6 (overestimation) with overall average about 0.12 %.

Although a lot of papers and textbooks have been written on Kalman filtering since its inception in 1960 some issues which complicate the application of the **KF** are as follows. These points are open for future researches

1. It is assumed that the system equation is linear. What if the equation is

nonlinear?

2. What if the measurement noise and process noise are not Gaussian, and not independent of each other?
3. What if the statistics (for example, the covariance matrix) of the noise is not known?
4. What if, rather than estimating the state of system as measurements are made, we already have all the measurements and we want to reconstruct a time history of the state? Can we do better than a **KF**? It would seem that we could, since we have more information available (that is, we have future measurements) to estimate the state at a given time. This is called the smoothing problem.
5. **KF** recursive equations are matrix expressions, and as such can impose a large computational burden for high-dimensional systems. Is there a way to approximate the **KF** for large systems, reducing the computational load while still approaching the theoretical optimum of the **KF**?
6. What if the noise characteristics change with time? Can we somehow formulate a **KF** that adapts over time to changes in the noise characteristics?
7. What if, rather than desired to minimize the "average" estimation error, we desire to minimize the "worst case" estimation error? This is known as the minimax or H-infinity estimation problem.

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APPENDIX A

Computer Program For Multi-Dimensional Kalman Filter

```
   CLS
   NS   = 52                                                    ' NUMBER OF STATIONS
   NM = 52                                                  ' NUMBER OF MEASUREMENT
   ND  = 30                                                         ' NUMBER OF DATA
'------------------------------------------------------------------------------------------
DIM Q(NS,NS), R(NM,NM), PHI(NS,NS), PHIT(NS,NS), IX(NS,1), UX(NS,1),X(NS,1), IP(NS,NS)
DIM UP(NS,NS), P(NS,NS), H(NM,NS), HT(NS,NM), KG(NS,NM), Z(NM,1), RAIN(ND,NM)
'------------------------------------------------------------------------------------------
   FOR I = 1 TO NS
       FOR J = 1 TO NS
           IF J <> I THEN
             IP(I,J) = 0                        ' INITIAL ERROR COVARIANCE MATRIX
             Q(I,J) = 0                     ' COVARIANCE MATRIX OF SYSTEM ERROR
             PHI(I,J) = 0                                    ' TRANSATION MATRIX
           END IF
           IP(I,I) = 2000
           Q(I,I) = 100
           PHI(I,I) = 1
           PHIT(I,J) = PHI(J,I)                ' TRANSPOSE OF TRANSATION MATRIX
       NEXT J
   NEXT I
'------------------------------------------------------------------------------------------
   FOR I = 1 TO NM                  ' COVARIANCE MATRIX OF MEASUREMENT ERROR
       FOR J = 1 TO NM
           IF J <> I THEN
              R(I,J) = 0
           END IF
           R(I,I) = .50
       NEXT J
   NEXT I
'------------------------------------------------------------------------------------------
   FOR I = 1 TO NM                                           ' CONNECTION MATRIX
       FOR J = 1 TO NS
           IF J <> I THEN
             H(I,J) = 0
           END IF
           H(I,I) = 1
           HT(I,J) = H(J,I)                   ' TRANSPOSE OF CONNECTION MATRIX
       NEXT J
   NEXT I
'------------------------------------------------------------------------------------------
   OPEN "DATA.DAT" FOR INPUT AS #1              ' RAINFALL DATA OF NS STATIONS
       FOR I = 1 TO ND
           FOR J = 1 TO NM
               INPUT #1, RAIN(I,J)
           NEXT J
       NEXT I
'------------------------------------------------------------------------------------------
   OPEN "MEAN.DAT" FOR INPUT AS #2         ' INITIAL ESTIMATE OF THE PROCESS X
       FOR I = 1 TO NS
           INPUT #2, IX(I,1)
       NEXT I
'------------------------------------------------------------------------------------------
   OPEN "KG.DAT" FOR OUTPUT AS #3                               ' OPEN OUTPUT FILES
   OPEN "X.DAT" FOR OUTPUT AS #4
   OPEN "P.DAT" FOR OUTPUT AS #5

FOR K = 1 TO ND
```

```
    FOR I = 1 TO NM                                              ' MEASEREMENT MATRIX
        Z(I,1) = RAIN(K,I)
    NEXT I
'-----------------------------------------------------------------------------------------
  CALL KALGAIN(NS,NM,IP(),H(),HT(),R(),KG())           ' COMPUTE KALMAN GAINMATRIX
    FOR I = 1 TO NS
        FOR J = 1 TO NM
            IF I = J THEN
                PRINT #3,USING "#.#######"; KG(I,J);
            END IF
        NEXT J
    NEXT I
     PRINT #3,
'-----------------------------------------------------------------------------------------
  CALL UPDATEX(NS,NM,IX(),KG(),Z(),H(),UX())               ' UPDATE INITIAL ESTIMATE
  CALL UPDATEP(NS,NM,IP(),KG(),H(),UP())                  ' UPDATE ERROR COVARIANCE
  CALL PRAHEAD(NS,NM,PHI(),PHIT(),Q(),UX(),UP(),P(),X())          ' PROJECT AHEAD
    FOR I = 1 TO NS
        FOR J = 1 TO NS
            PRINT #4, USING "##########"; X(I,1);
            IF I = J THEN
                PRINT #5,USING "#####.#####";P(I,J);
            END IF
        NEXT J
    NEXT I
    PRINT #4,
    PRINT #5,
    FOR I = 1 TO NS
        FOR J = 1 TO NS
            IX(I,1) = X(I,1)
            IP(I,J) = P(I,J)
        NEXT J
    NEXT I
NEXT K
'-----------------------------------------------------------------------------------------
  SUB GAUSSJ (A(),N,NP,B(),M,MP)
    DIM IPIV(N),INDXR(N),INDXC(N)
    FOR J = 1 TO N
        IPIV(J) = 0
    NEXT J
    FOR I = 1 TO N
        BIG = 0!
        FOR J = 1 TO N
            IF IPIV(J) <> 1 THEN
                FOR K = 1 TO N
                    IF IPIV(K) = 0 THEN
                        IF ABS(A(J,K)) >= BIG THEN
                            BIG = ABS(A(J,K)):  IROW = J:  ICOL = K
                        END IF
                        ELSEIF IPIV(K) > 1 THEN
                            PRINT "SINGULAR MATRIX"
                            EXIT SUB
                    END IF
                NEXT K
            END IF
        NEXT J
        IPIV(ICOL) = IPIV(ICOL) + 1
        IF IROW <> ICOL THEN
```

```
            FOR L = 1 TO N
                DUM = A(IROW,L): A(IROW,L) = A(ICOL,L): A(ICOL,L) = DUM
            NEXT L
            FOR L = 1 TO M
                DUM = B(IROW,L): B(IROW,L) = B(ICOL,L):  B(ICOL,L) = DUM
            NEXT L
        END IF
        INDXR(I) = IROW:  INDXC(I) = ICOL
        IF A(ICOL,ICOL) = 0! THEN
            PRINT "SINGULAR MATRIX": EXIT SUB
            PIVINV = 1! / A(ICOL,ICOL)
            A(ICOL,ICOL) = 1!
            FOR L = 1 TO N
                A(ICOL,L) = A(ICOL,L) * PIVINV
            NEXT L
            FOR L = 1 TO M
                B(ICOL,L) = B(ICOL,L) * PIVINV
            NEXT L
        FOR LL = 1 TO N
            IF LL <> ICOL THEN
                DUM = A(LL,ICOL):  A(LL, ICOL) = 0!
                FOR L = 1 TO N
                    A(LL,L) = A(LL,L) - A(ICOL,L) * DUM
                NEXT L
                FOR L = 1 TO M
                    B(LL,L) = B(LL,L)-B(ICOL,L) * DUM
                NEXT L
            END IF
        NEXT LL
    NEXT I
    FOR L = N TO 1 STEP -1
        IF INDXR(L) <> INDXC(L) THEN
            FOR K = 1 TO N
                DUM = A(K, INDXR(L)): A(K, INDXR(L)) = A(K, INDXC(L))
                A(K, INDXC(L)) = DUM
            NEXT K
        END IF
    NEXT L
    ERASE INDXC, INDXR, IPIV
END SUB
'-----------------------------------------------------------------------------------------------
SUB KALGAIN (NS,NM,IP( ),HT( ),H( ),R( ),KG( ))
    DIM M1(NS,NM), M2(NM,NS), M3(NM,NM), M4(NM,NM)
    FOR I = 1 TO NS                                                ' M1 = P.HT
        FOR J = 1 TO NM
            SUM = 0
            FOR K = 1 TO NS
                SUM = SUM + IP(I,K) * HT(K,J)
                M1(I,J) = SUM
            NEXT K
        NEXT J
    NEXT I
    FOR I = 1 TO NM                                                ' M2 = H.P
        FOR J = 1 TO NS
            SUM = 0
            FOR K = 1 TO NS
                SUM = SUM + H(I,K) * IP(K,J)
                M2(I,J) = SUM
```

```
            NEXT K
        NEXT J
    NEXT I
    FOR I = 1 TO NM                                         ' M3 = M2.HT
        FOR J = 1 TO NM
            SUM = 0
            FOR K = 1 TO NS
                SUM = SUM + M2(I,K) * HT(K,J)
                M3(I,J) = SUM
            NEXT K
        NEXT J
    NEXT I
    FOR I =1 TO NM                                          ' M4 = M3 + R
        FOR J = 1 TO NM
           M4(I,J) = M3(I,J) + R(I,J)
        NEXT J
    NEXT I
    CALL GAUSSJ(M4(),NM,NP,M1(),NM,MP)
'   M4 IS REPLACED BY ITS MATRIX INVERSE, AND M1 IS REPLACED BY THE
'   CORRESPONDING SET OF SOLUTION  VECTORS (KG).
    FOR I = 1 TO NS
        FOR J = 1 TO NM
            KG(I,J) = M1(I,J)
        NEXT J
    NEXT I
END SUB
'-----------------------------------------------------------------------------------
SUB PRAHEAD (N,M,PHI(),PHIT(),Q(),UX(),UP( ),P(),X())
    DIM M1(N,N), M2(N,N)
    FOR I = 1 TO NS                                         ' X = PHI.X
        SUM = 0
        FOR K = 1 TO NS
            SUM = SUM + PHI(I,K) * UX(K,1)
            X(I,1) = SUM
        NEXT K
    NEXT I
    FOR I = 1 TO NS                                         ' M1 = PHI.P
        FOR J = 1 TO NS
            SUM = 0
            FOR K = 1 TO NS
                SUM = SUM + PHI(I,K) * UP(K,J)
                M1(I,J) = SUM
            NEXT K
        NEXT J
    NEXT I
    FOR I = 1 TO NS                                         ' M2 = M1.PHIT
        FOR J = 1 TO NS
            SUM = 0
            FOR K = 1 TO NS
                SUM = SUM + M1(I,K) * PHIT(K,J)
                M2(I, J) = SUM
            NEXT K
        NEXT J
    NEXT I
    FOR I = 1 TO NS                                         ' P = PHI.P.PHIT + Q
        FOR J = 1 TO NS
            P(I,J) = M2(I,J) + Q(I,J)
        NEXT J
```

```
   NEXT I
END SUB
'-----------------------------------------------------------------------------------------------------------
SUB UPDATEP (NS,NM,IP(),KG(),H(),UP())
   DIM M1(NS,NS), M2(NS,NS)
   FOR I = 1 TO NS                                                              ' M1 = K.H
       FOR J = 1 TO NS
           SUM = 0
           FOR K = 1 TO NM
               SUM = SUM + KG(I,K) * H(K,J)
               M1(I, J) = SUM
           NEXT K
       NEXT J
   NEXT I
   FOR I = 1 TO NS                                                              ' M2 = M1.P
       FOR J = 1 TO NS
           SUM = 0
           FOR K = 1 TO NS
               SUM = SUM + M1(I,K) * IP(K,J)
               M2(I,J) = SUM
           NEXT K
       NEXT J
   NEXT I
   FOR I = 1 TO NS                                                              ' UP = P - M2
       FOR J = 1 TO NS
           UP(I,J) = IP(I,J) - M2(I,J)
       NEXT J
   NEXT I
END SUB
'-----------------------------------------------------------------------------------------------------------
SUB UPDATEX (NS,NM,IX( ),KG( ),Z( ),H( ),UX( ))
   DIM M1(NM,1),  M2(NM,1),  M3(NS,1)
   FOR I = 1 TO NM                                                              ' M1 = H.X
       SUM = 0
       FOR K = 1 TO NS
           SUM = SUM + H(I,K) * IX(K,1)
           M1(I,1) = SUM
       NEXT K
   NEXT I
   FOR I = 1 TO NM                                                              ' M2 = Z - M1
       M2(I,1) = Z(I,1) - M1(I,1)
   NEXT I
   FOR I = 1 TO NS                                                              ' M3 = KG.M2
       SUM = 0
       FOR K = 1 TO NM
           SUM = SUM + KG(I,K) * M2(K,1)
           M3(I,1) = SUM
       NEXT K
   NEXT I
   FOR I = 1 TO NS                                                              ' UX = X + M3
       UX(I,J) = IX(I,1) + M3(I,1)
   NEXT I
END SUB
```

APPENDIX B

Contour Maps Of Observed (——)And Estimated (- - -)Annual Precipitation

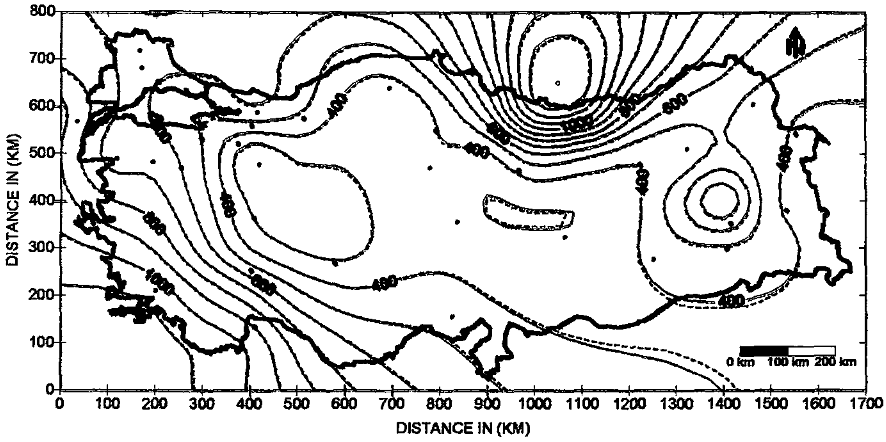

Figure B.1 Contour map of observed and estimated annual precipitation for 1956.

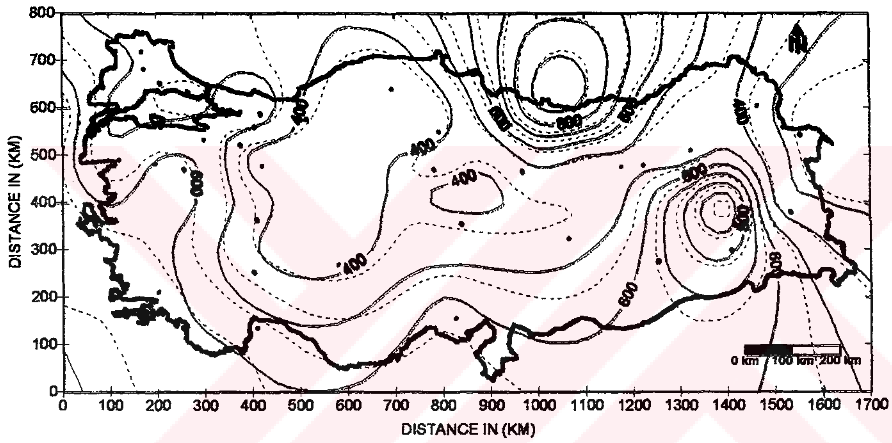

Figure B.2 Contour map of observed and estimated annual precipitation for 1957.

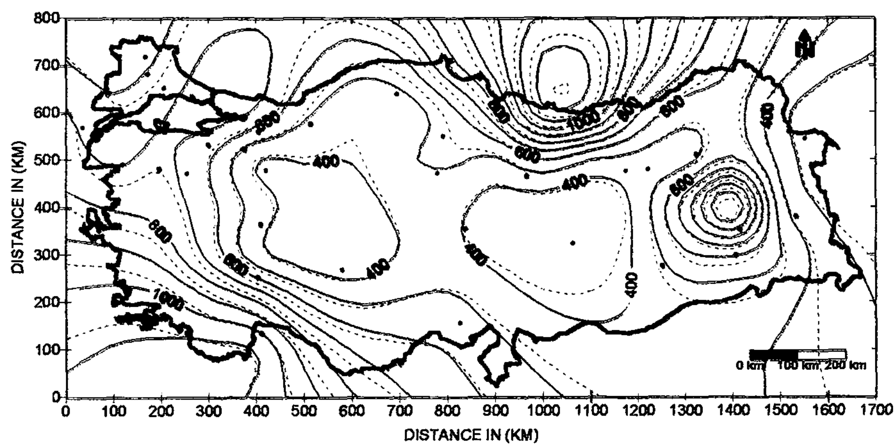

Figure B.3 Contour map of observed and estimated annual precipitation for 1958.

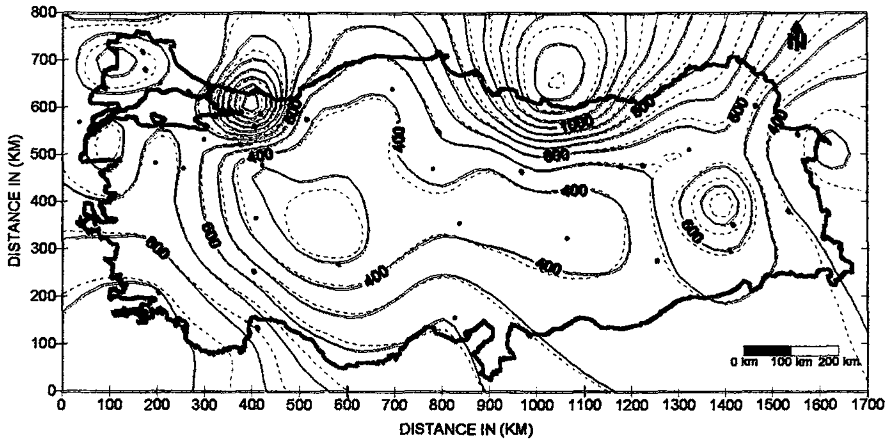

Figure B.4 Contour map of observed and estimated annual precipitation for 1959.

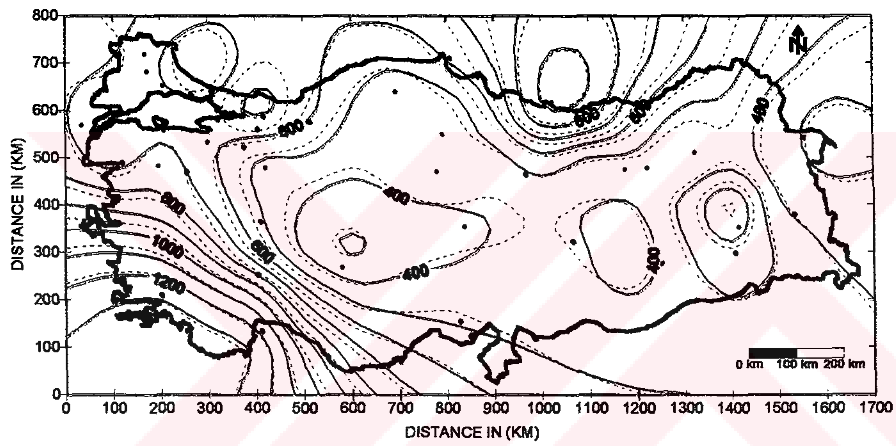

Figure B.5 Contour map of observed and estimated annual precipitation for 1960.

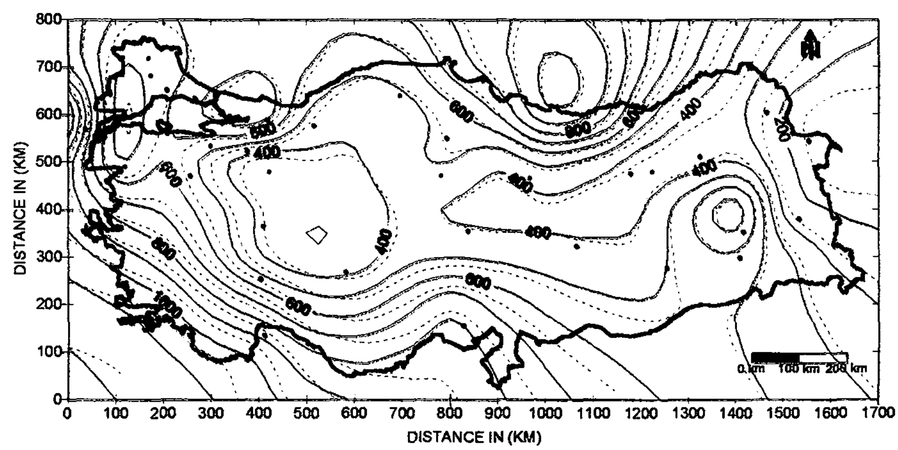

Figure B.6 Contour map of observed and estimated annual precipitation for 1961.

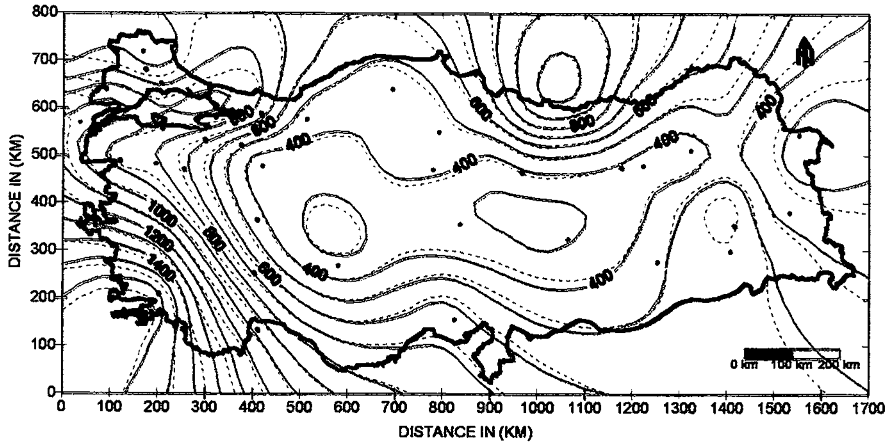

Figure B.7 Contour map of observed and estimated annual precipitation for 1962.

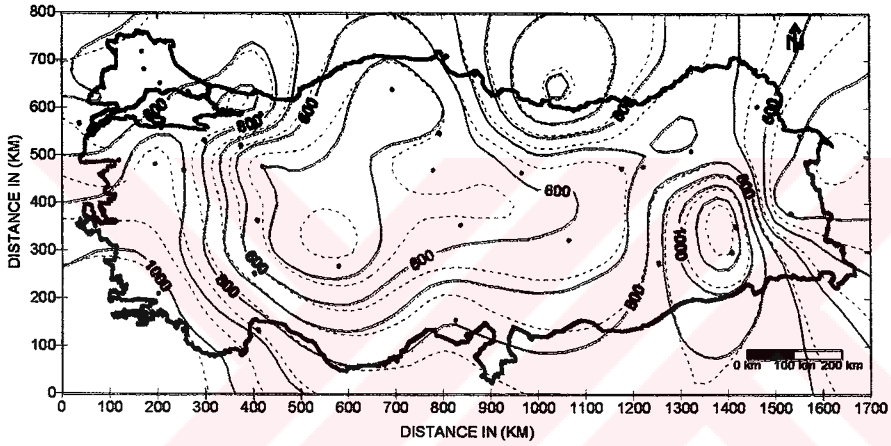

Figure B.8 Contour map of observed and estimated annual precipitation for 1963.

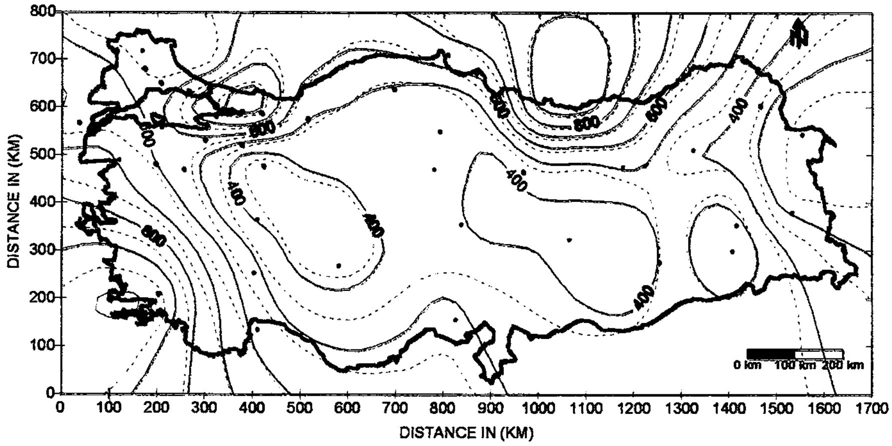

Figure B.9 Contour map of observed and estimated annual precipitation for 1964.

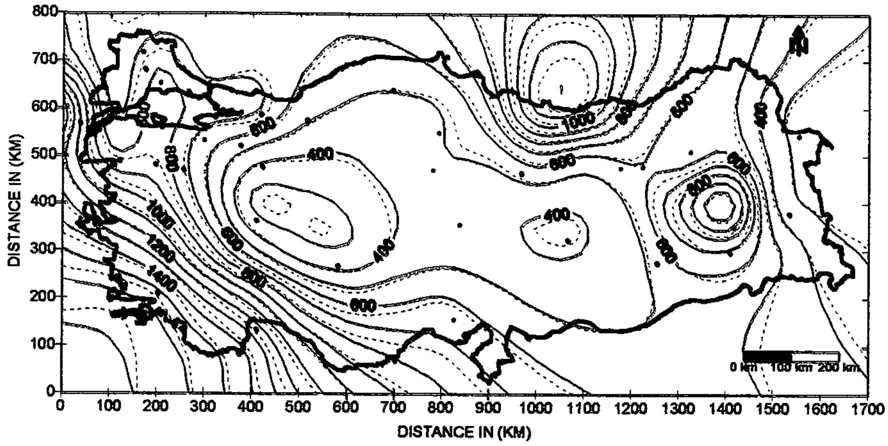

Figure B.10 Contour map of observed and estimated annual precipitation for 1965.

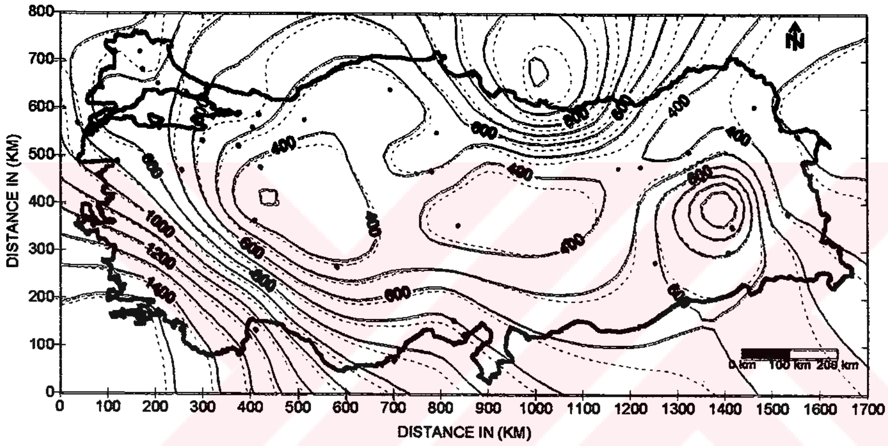

Figure B.11 Contour map of observed and estimated annual precipitation for 1966.

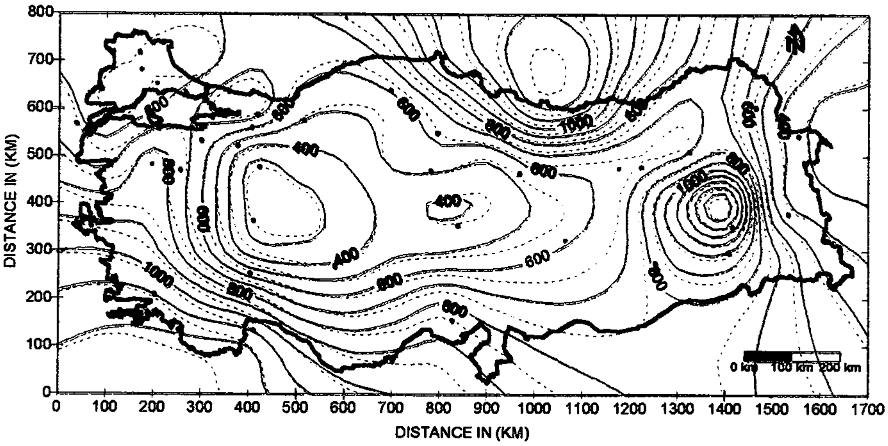

Figure B.12 Contour map of observed and estimated annual precipitation for 1967.

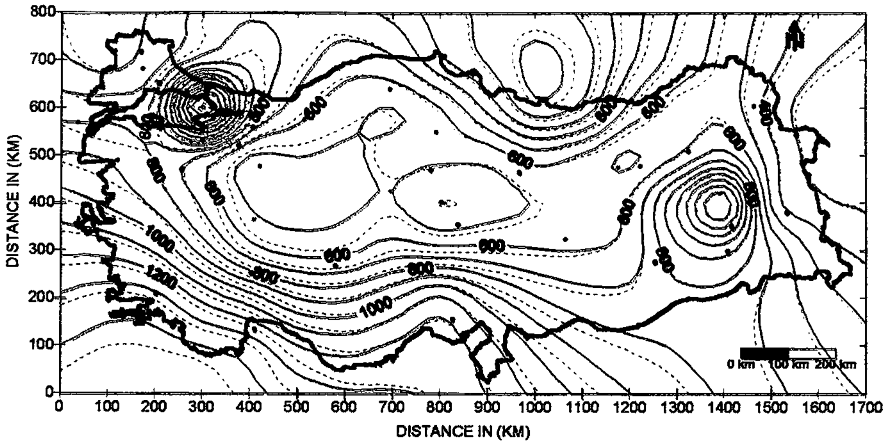

Figure B.13Contour map of observed and estimated annual precipitation for 1968.

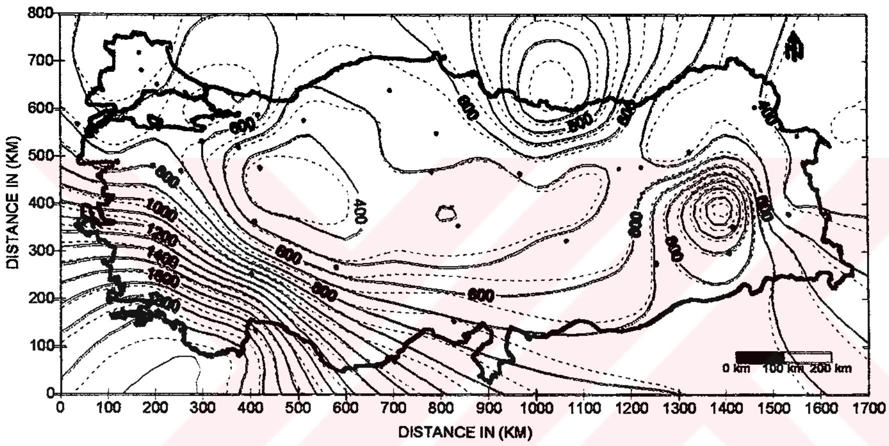

Figure B.14 Contour map of observed and estimated annual precipitation for 1969.

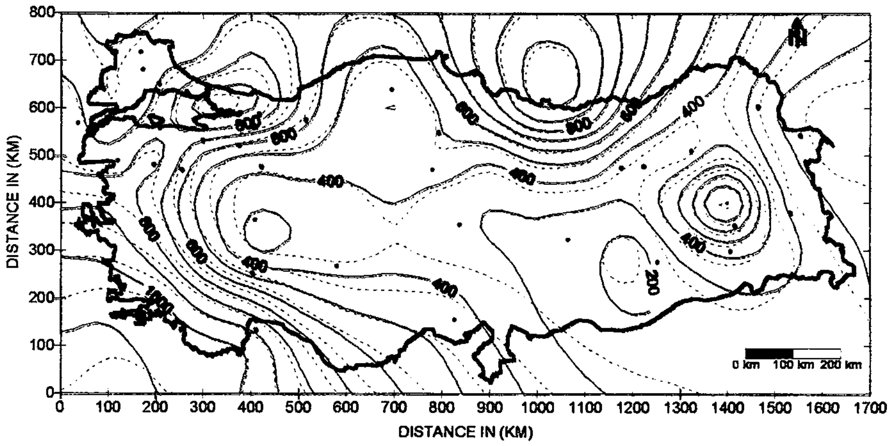

Figure B.15 Contour map of observed and estimated annual precipitation for 1970.

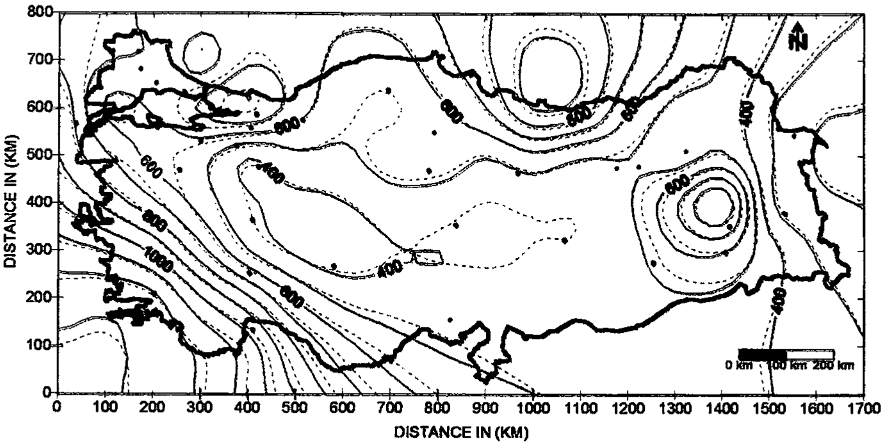

Figure B.16 Contour map of observed and estimated annual precipitation for 1971.

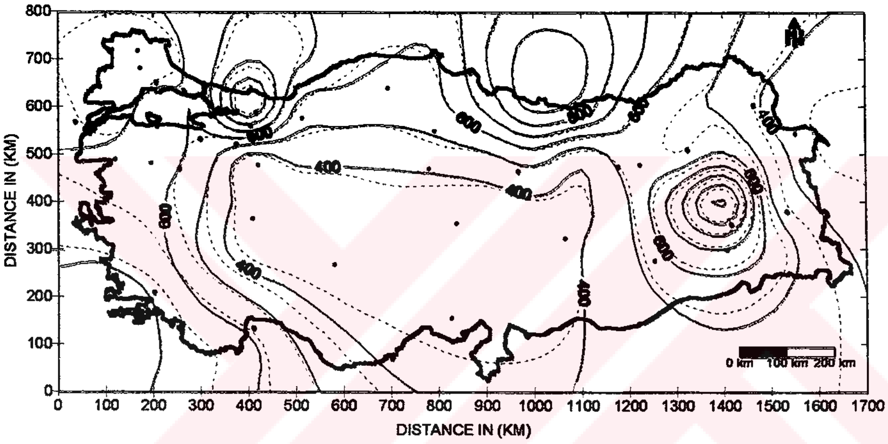

Figure B.17 Contour map of observed and estimated annual precipitation for 1972.

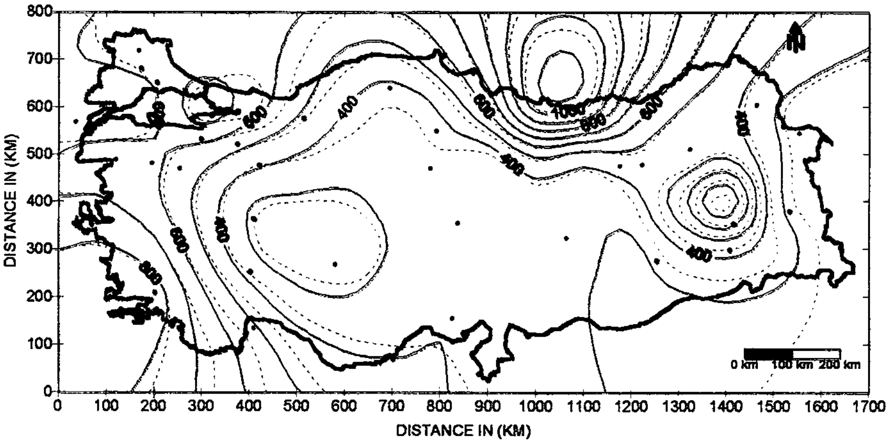

Figure B.18 Contour map of observed and estimated annual precipitation for 1973.

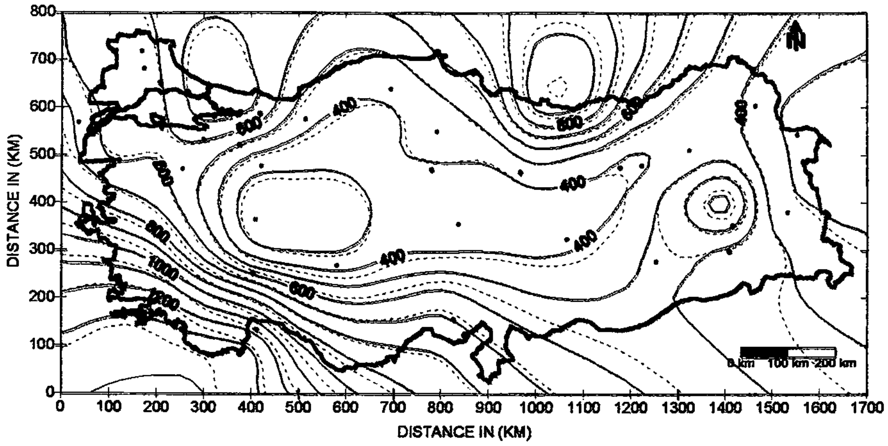

Figure B.19 Contour map of observed and estimated annual precipitation for 1974.

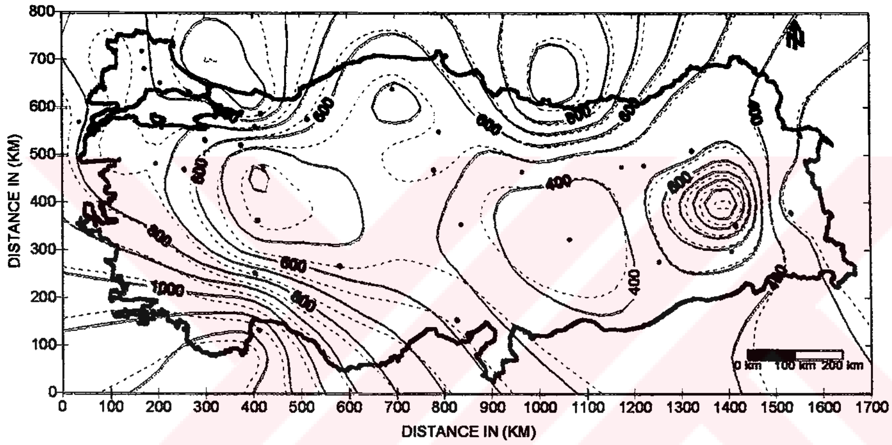

Figure B.20 Contour map of observed and estimated annual precipitation for 1975.

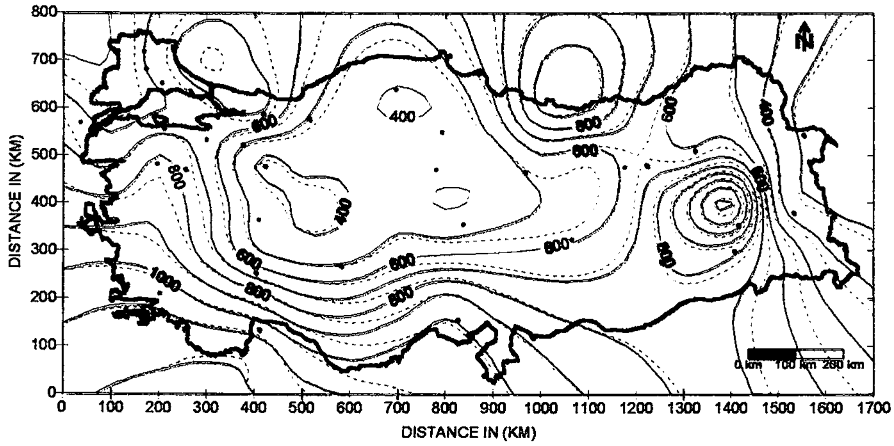

Figure B.21 Contour map of observed and estimated annual precipitation for 1976.

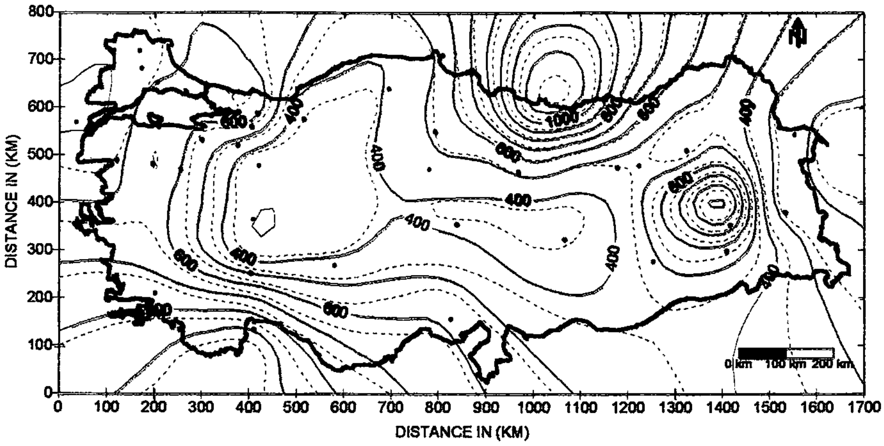

Figure B.22 Contour map of observed and estimated annual precipitation for 1977.

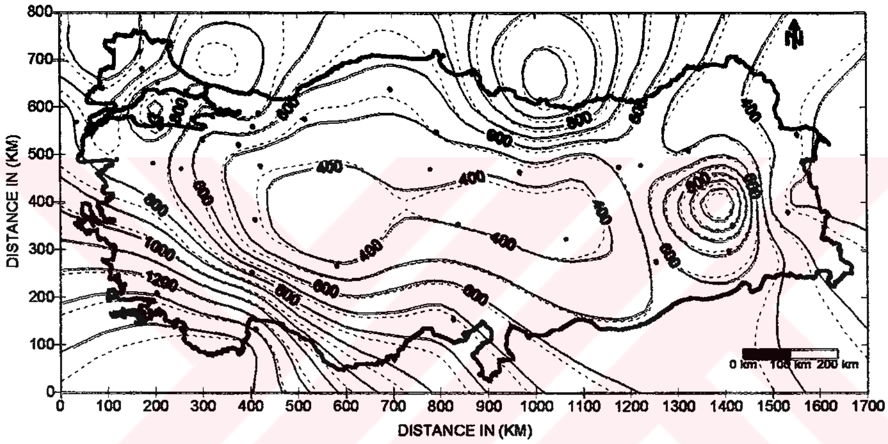

Figure B.23 Contour map of observed and estimated annual precipitation for 1978.

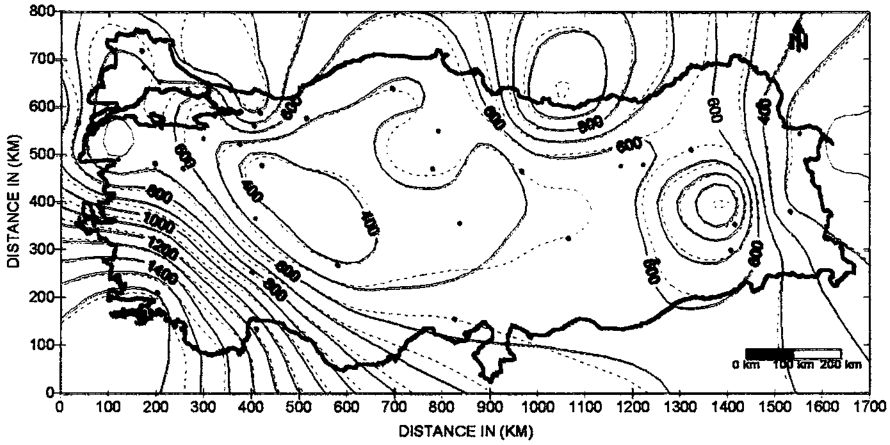

Figure B.24 Contour map of observed and estimated annual precipitation for 1979.

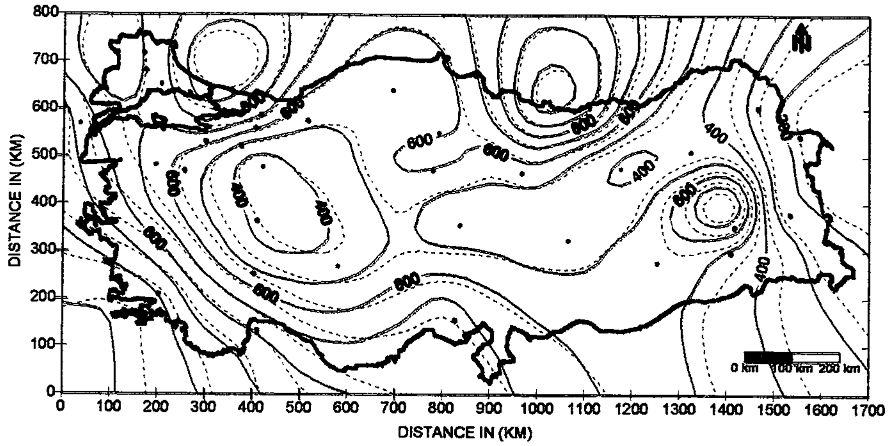

Figure B.25 Contour map of observed and estimated annual precipitation for 1980.

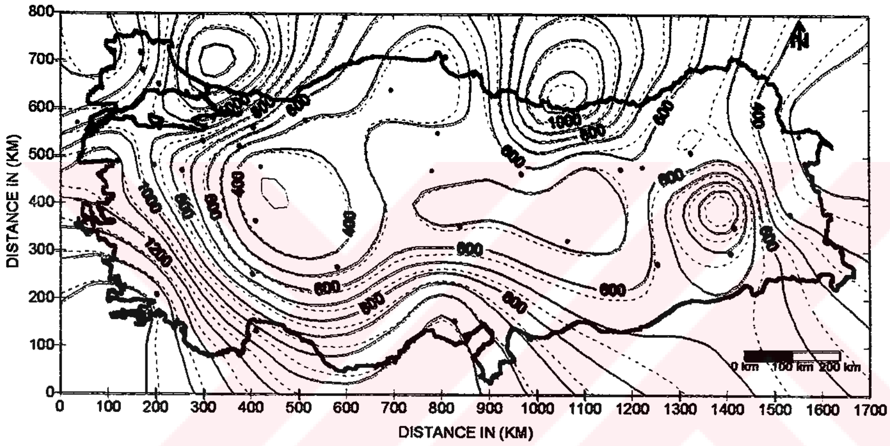

Figure B.26 Contour map of observed and estimated annual precipitation for 1981.

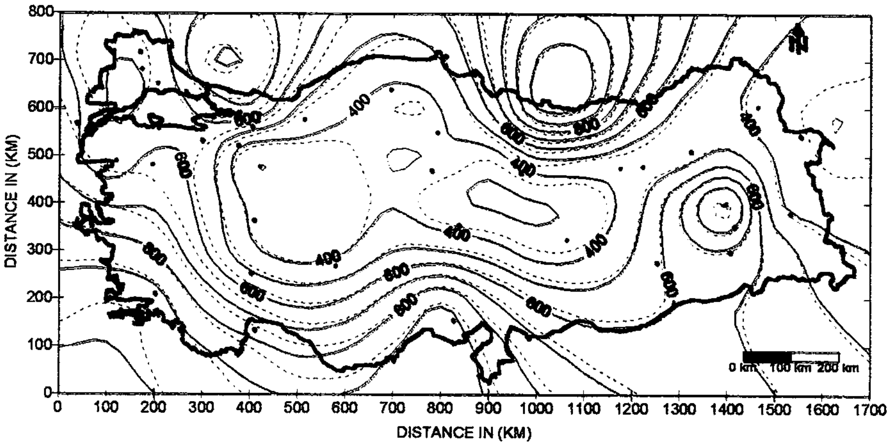

Figure B.27 Contour map of observed and estimated annual precipitation for 1982.

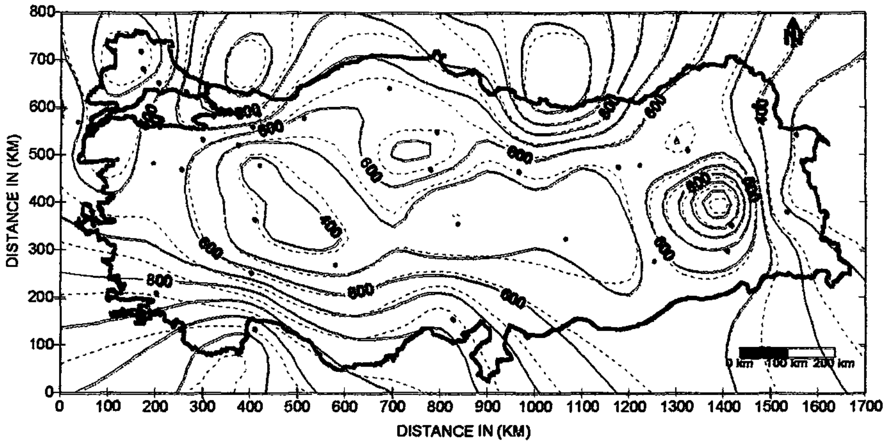

Figure B.28 Contour map of observed and estimated annual precipitation for 1983.

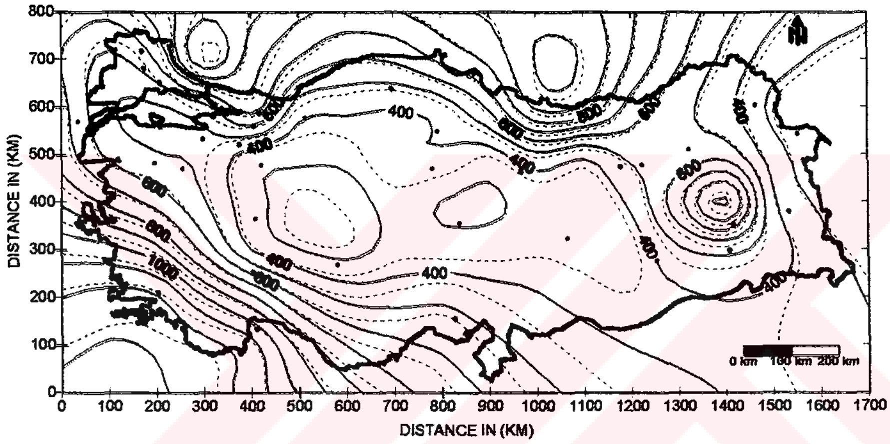

Figure B.29 Contour map of observed and estimated annual precipitation for 1984.

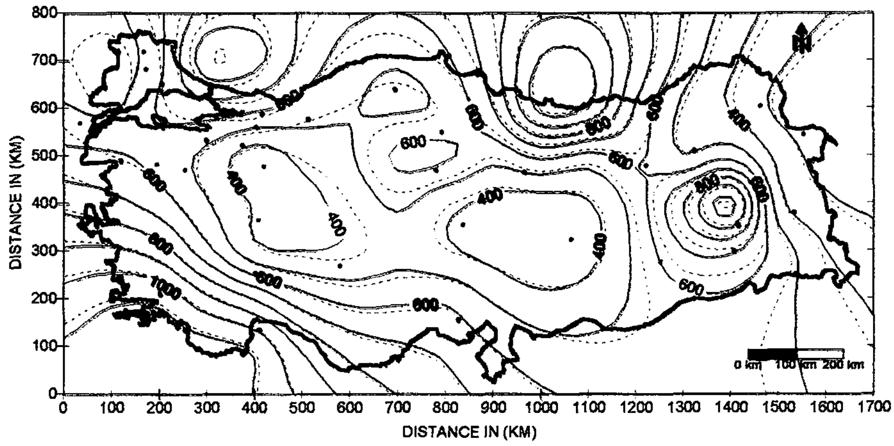

Figure B.30 Contour map of observed and estimated annual precipitation for 1985.

APPENDIX C

Contour Maps Of Percentage Error Of Estimated Annual Rainfall

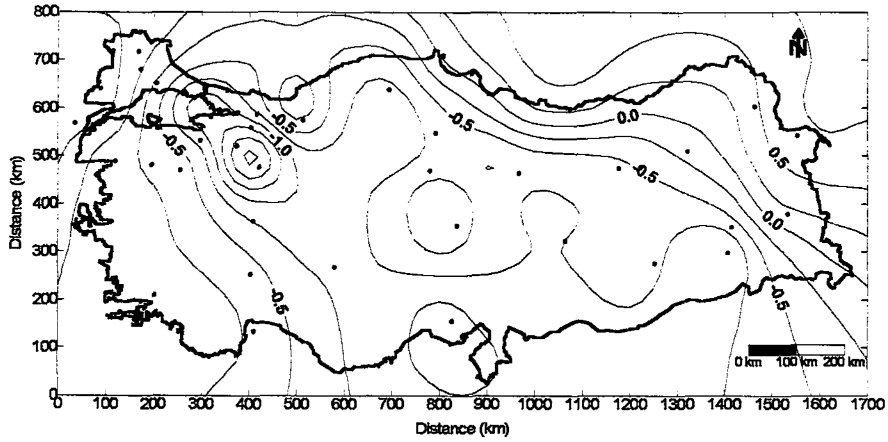

Figure C.1 Contour map of percentage error of estimated annual rainfall for 1956.

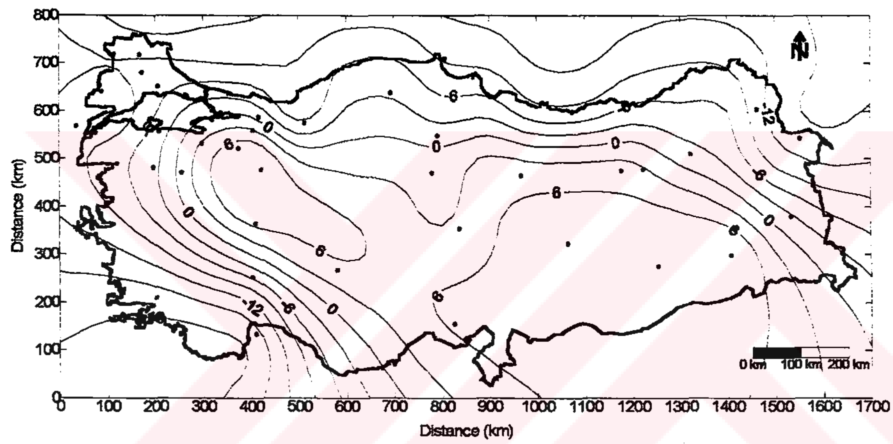

Figure C.2 Contour map of percentage error of estimated annual rainfall for 1957.

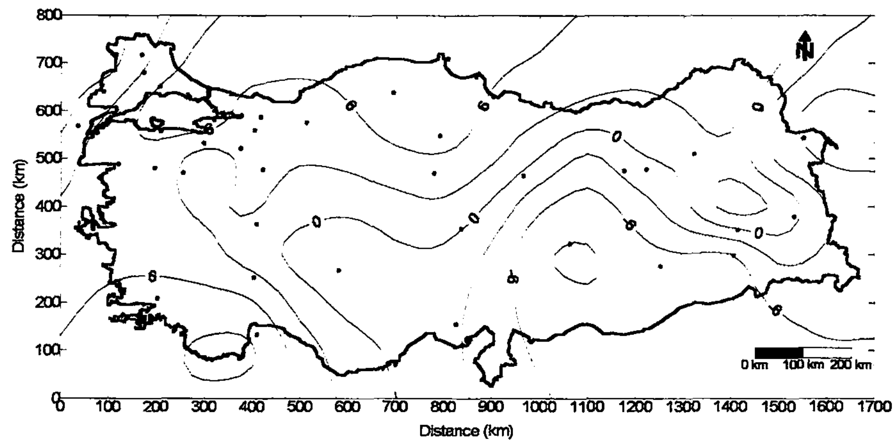

Figure C.3 Contour map of percentage error of estimated annual rainfall for 1958.

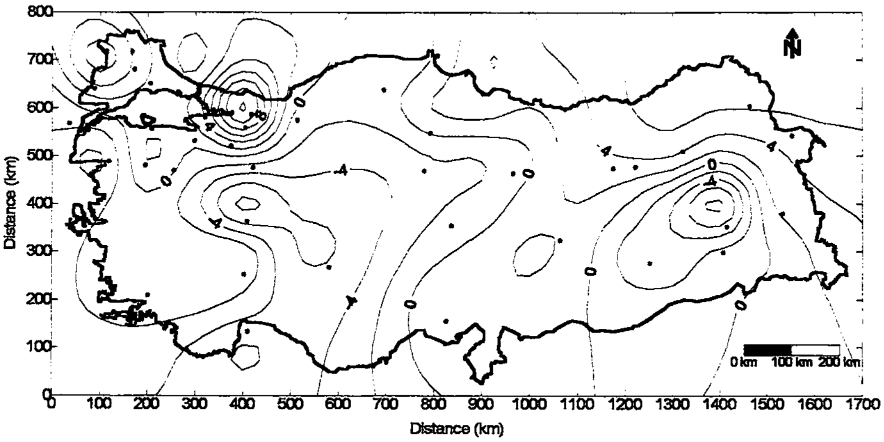

Figure C.4 Contour map of percentage error of estimated annual rainfall for 1959.

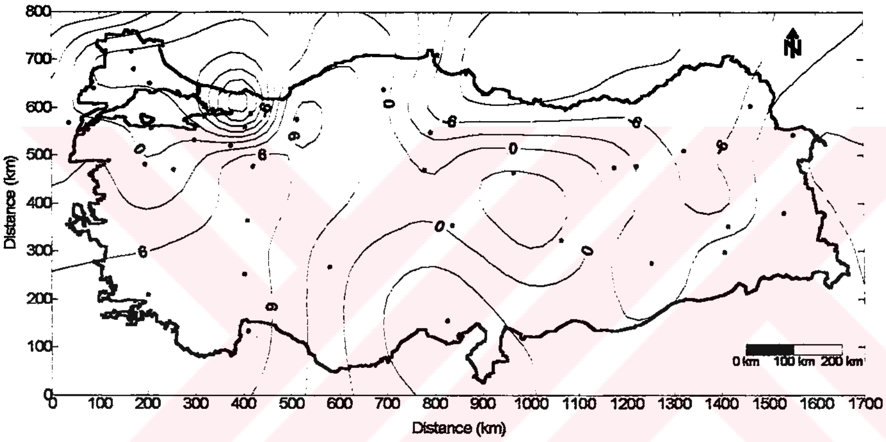

Figure C.5 Contour map of percentage error of estimated annual rainfall for 1960.

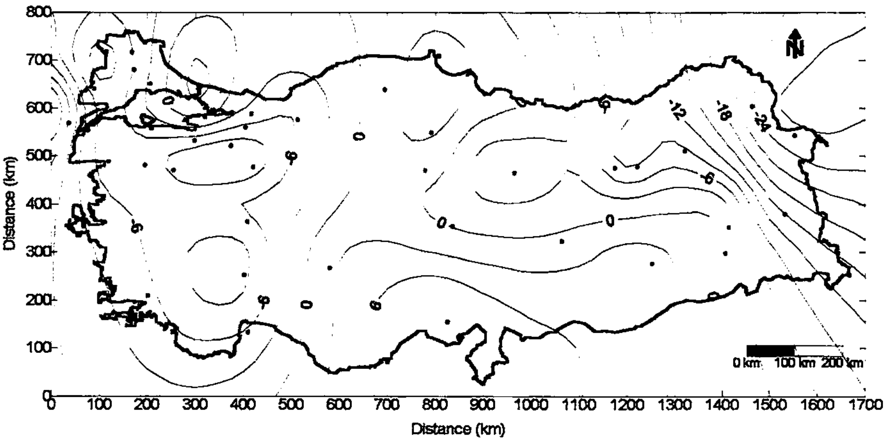

Figure C.6 Contour map of percentage error of estimated annual rainfall for 1961.

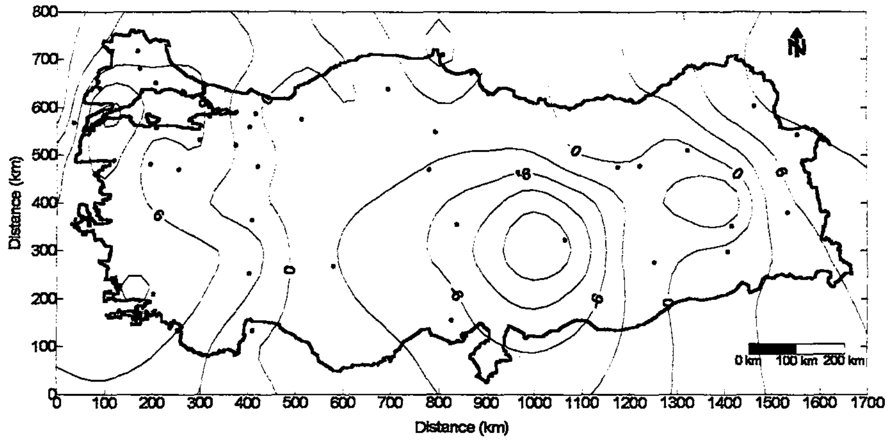

Figure C.7 Contour map of percentage error of estimated annual rainfall for 1962.

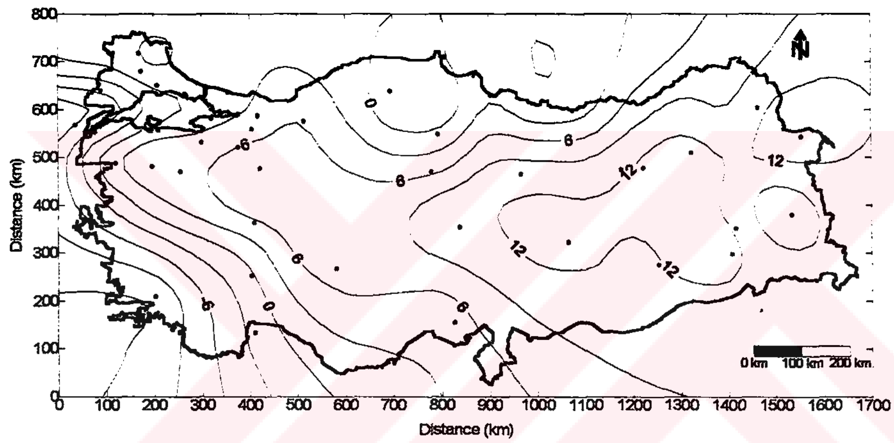

Figure C.8 Contour map of percentage error of estimated annual rainfall for 1963.

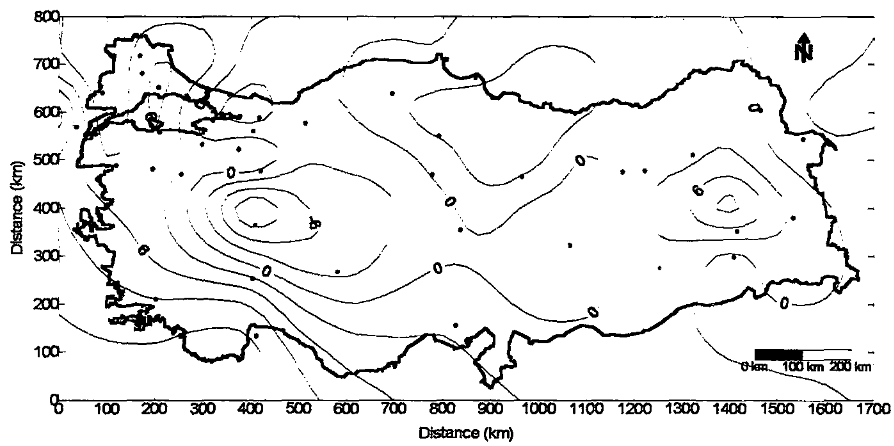

Figure C.9 Contour map of percentage error of estimated annual rainfall for 1964.

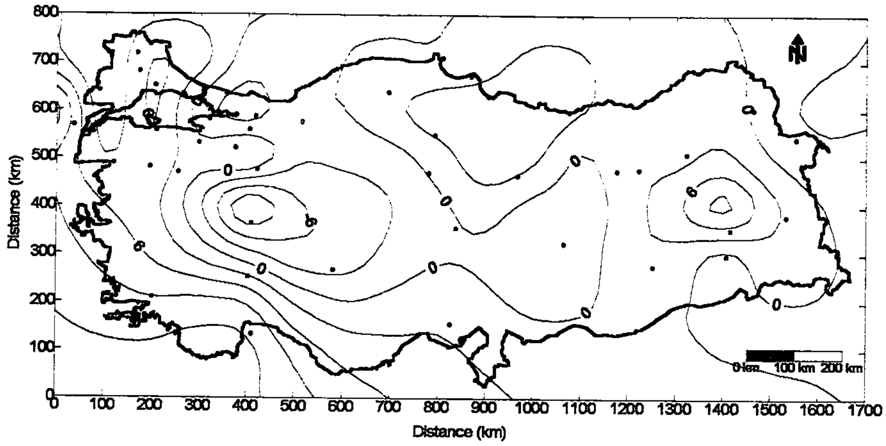

Figure C.10 Contour map of percentage error of estimated annual rainfall for 1965.

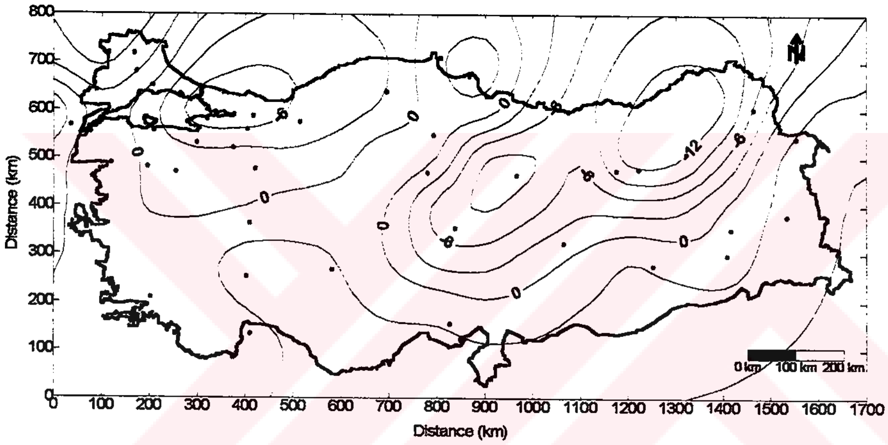

Figure C.11 Contour map of percentage error of estimated annual rainfall for 1966.

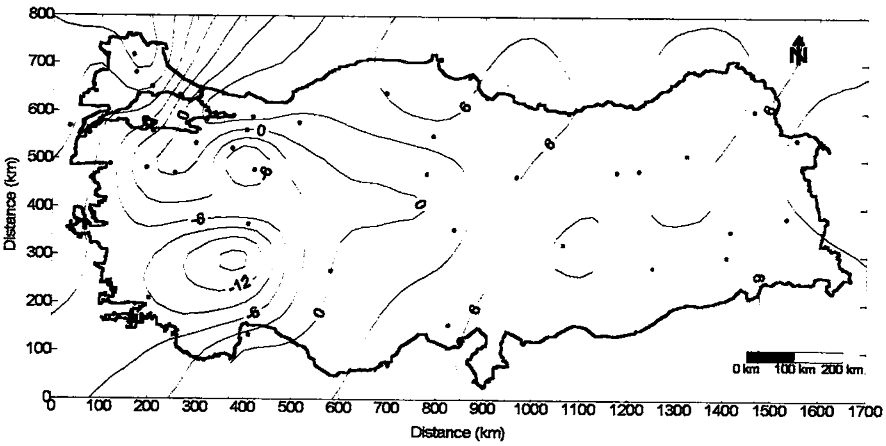

Figure C.12 Contour map of percentage error of estimated annual rainfall for 1967.

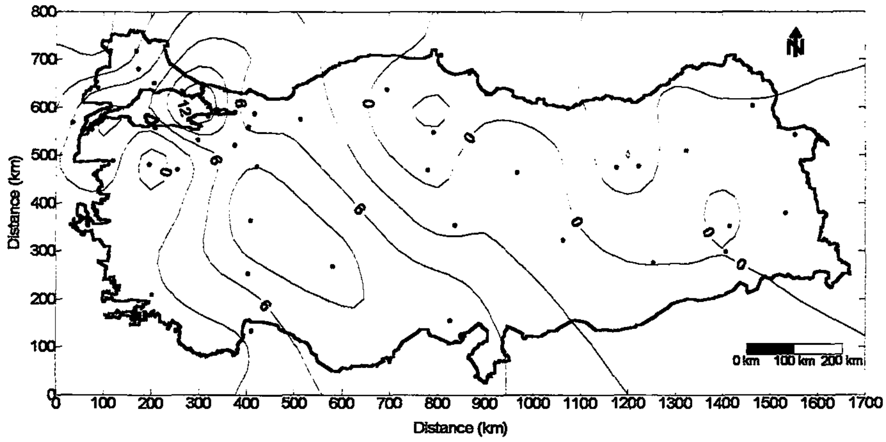

Figure C.13 Contour map of percentage error of estimated annual rainfall for 1968.

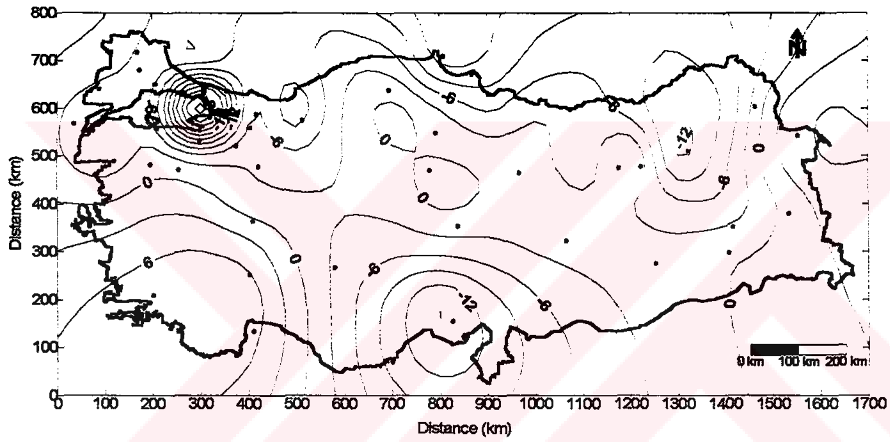

Figure C.14 Contour map of percentage error of estimated annual rainfall for 1969.

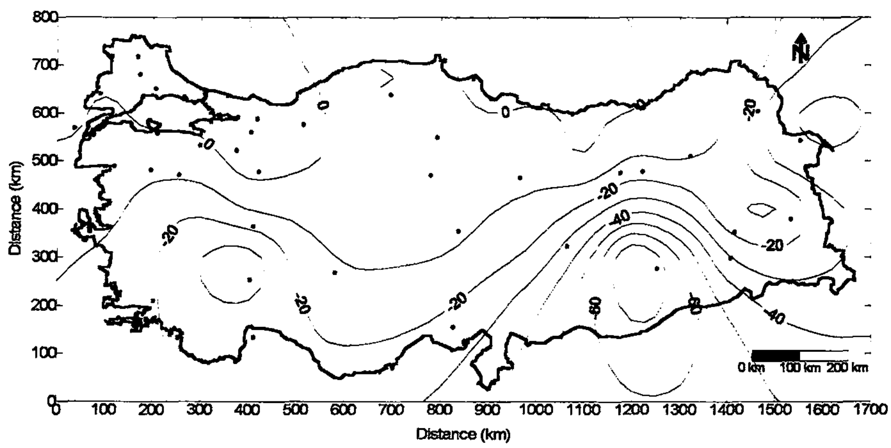

Figure C.15 Contour map of percentage error of estimated annual rainfall for 1970.

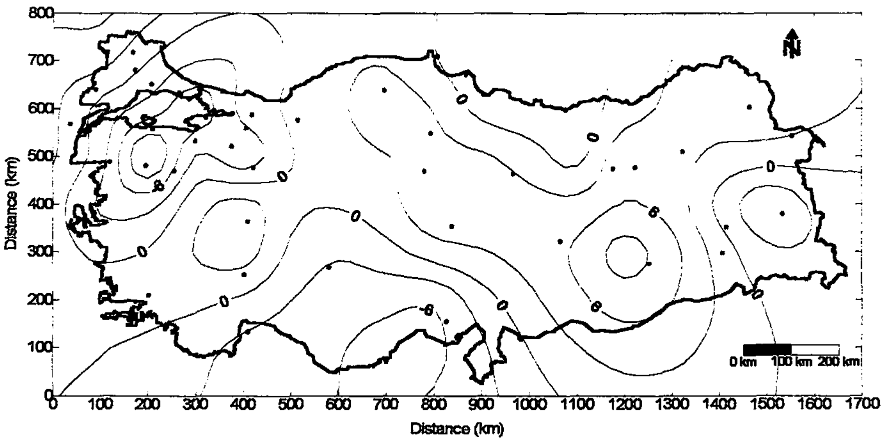

Figure C.16 Contour map of percentage error of estimated annual rainfall for 1971.

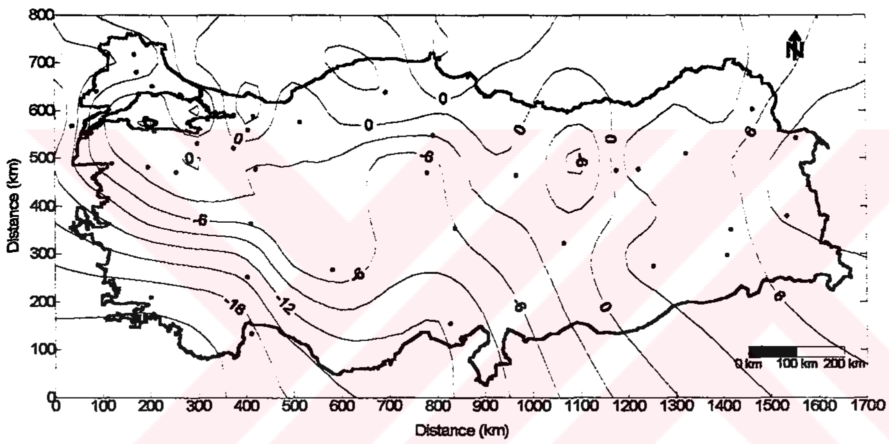

Figure C.17 Contour map of percentage error of estimated annual rainfall for 1972.

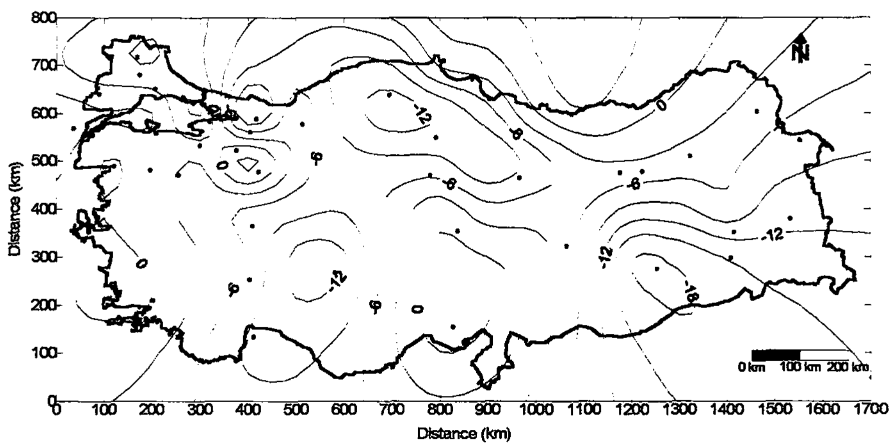

Figure C.18 Contour map of percentage error of estimated annual rainfall for 1973.

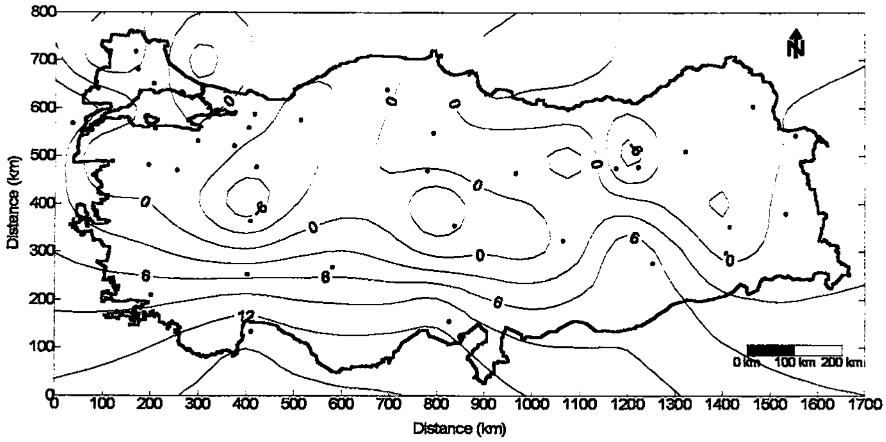

Figure C.19 Contour map of percentage error of estimated annual rainfall for 1974.

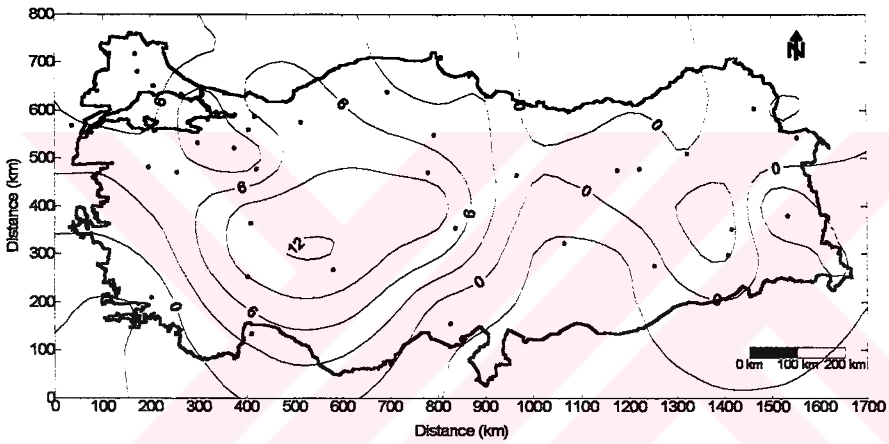

Figure C.20 Contour map of percentage error of estimated annual rainfall for 1975.

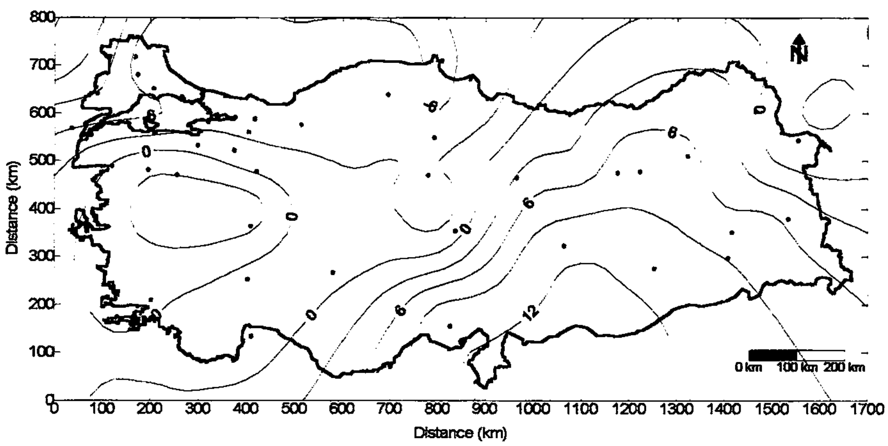

Figure C.21 Contour map of percentage error of estimated annual rainfall for 1976.

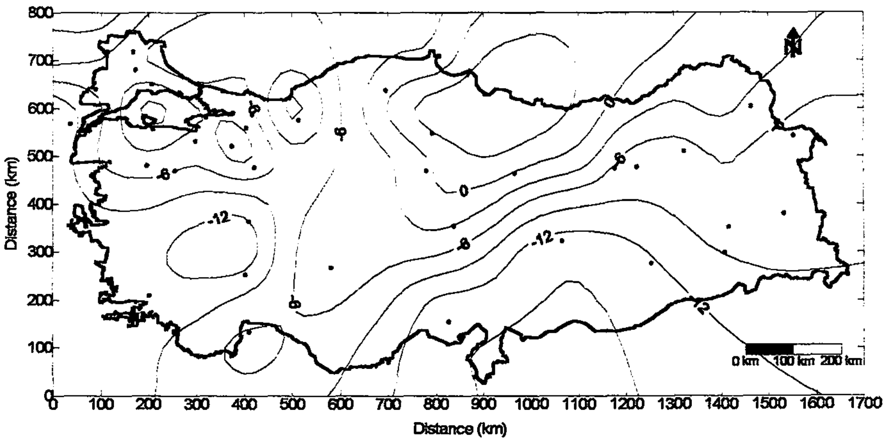

Figure C.22 Contour map of percentage error of estimated annual rainfall for 1977.

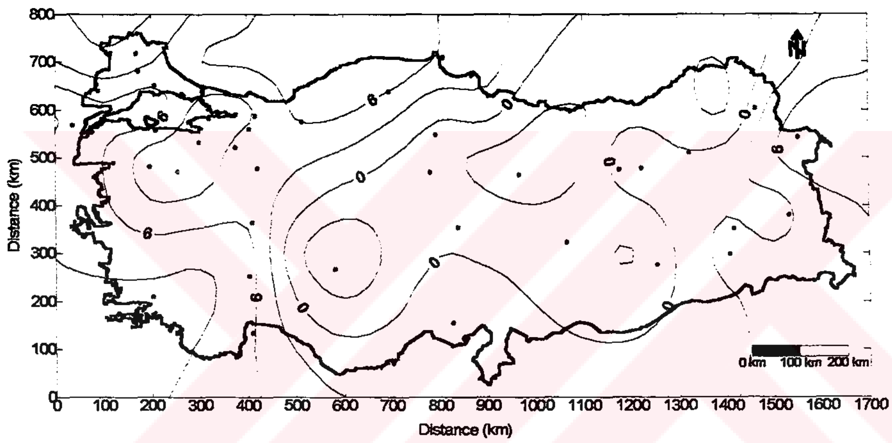

Figure C.23 Contour map of percentage error of estimated annual rainfall for 1978.

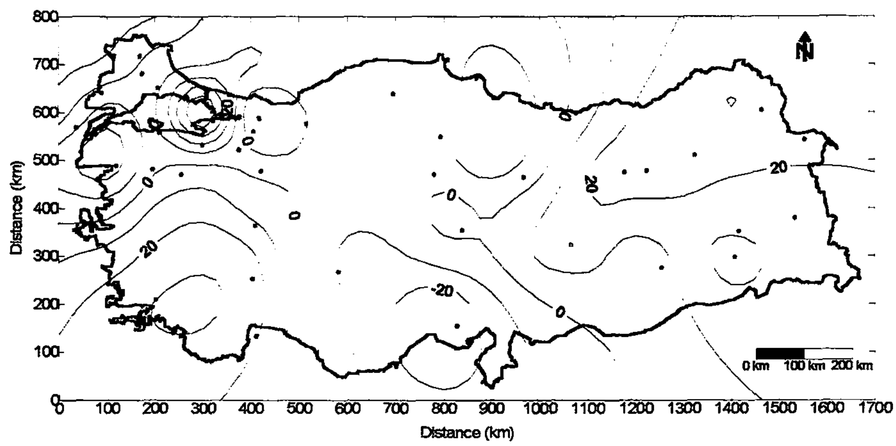

Figure C.24 Contour map of percentage error of estimated annual rainfall for 1979.

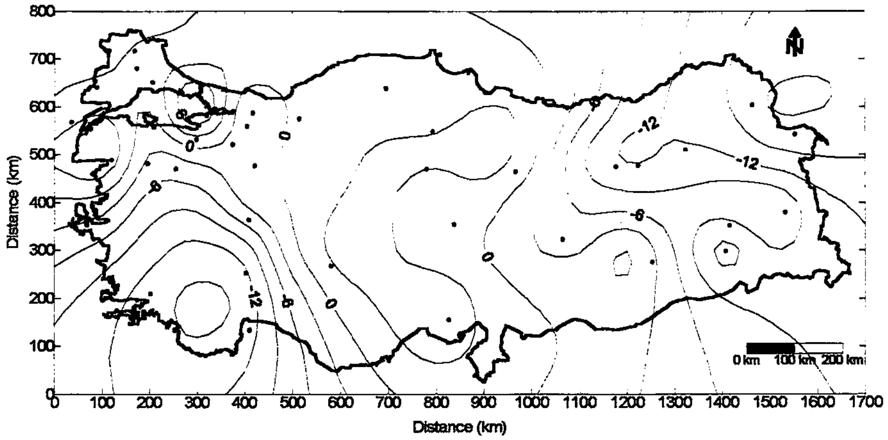

Figure C.25 Contour map of percentage error of estimated annual rainfall for 1980.

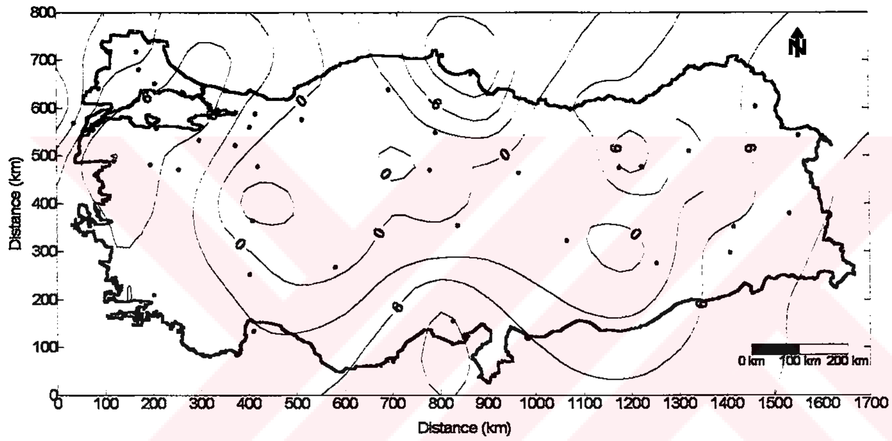

Figure C.26 Contour map of percentage error of estimated annual rainfall for 1981.

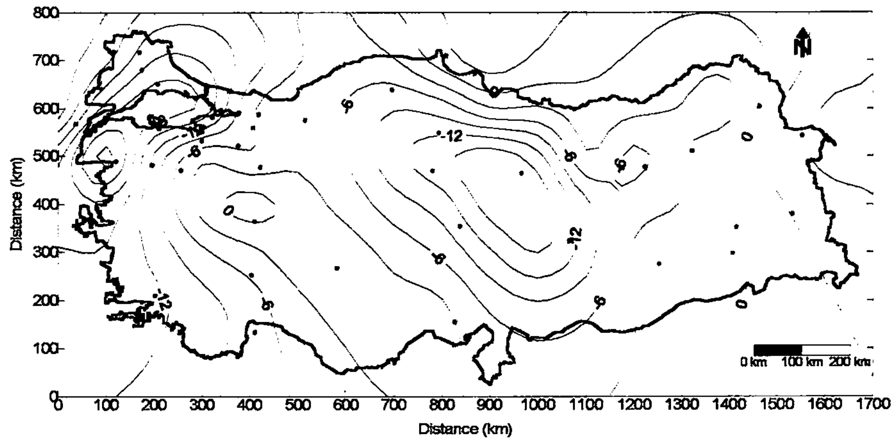

Figure C.27 Contour map of percentage error of estimated annual rainfall for 1982.

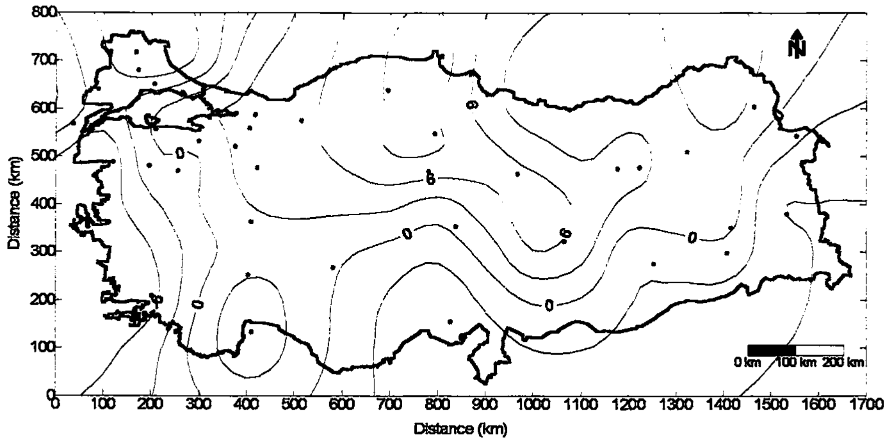

Figure C.28 Contour map of percentage error of estimated annual rainfall for 1983.

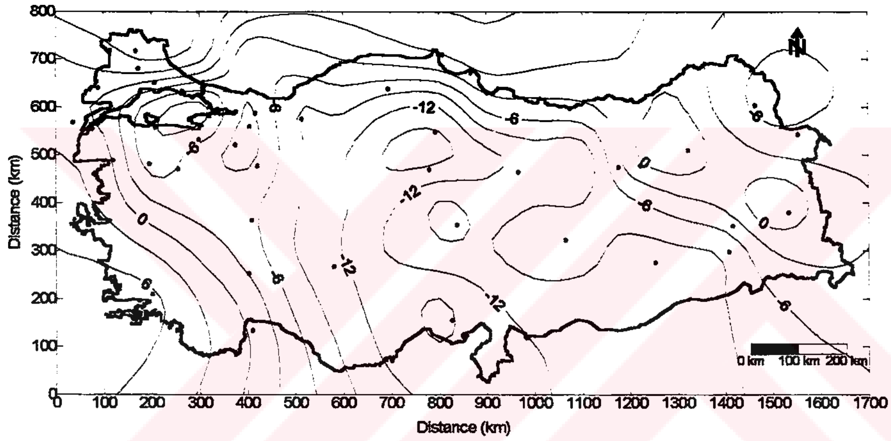

Figure C.29 Contour map of percentage error of estimated annual rainfall for 1984.

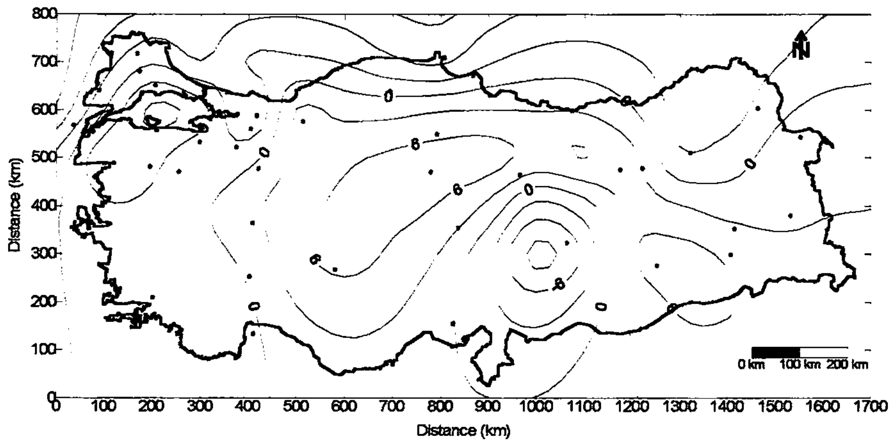

Figure C.30 Contour maps of percentage error of estimated annual rainfall for 1985.

APPENDIX D

Observed And Estimated Annual Rainfall Values

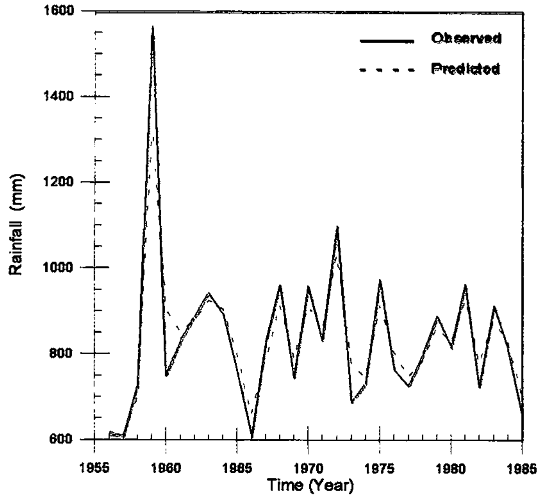

Figure D.1 Observed and estimated annual rainfall values at Adapazarı.

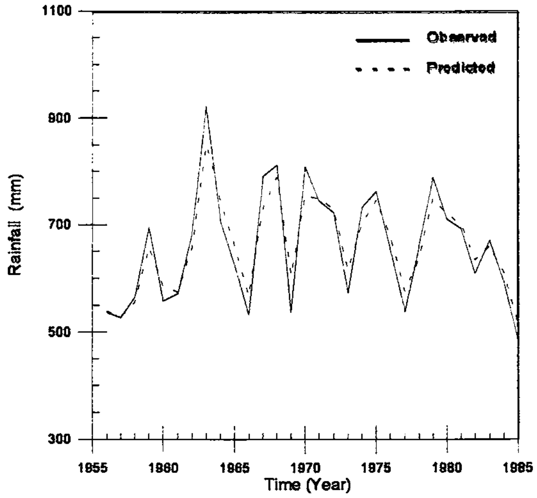

Figure D.2 Observed and estimated annual rainfall values at Ali Fuat Paşa.

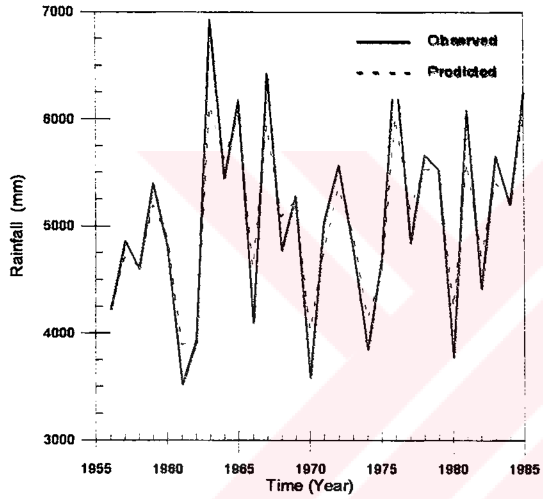

Figure D.3 Observed and estimated annual rainfall values at Ağrı.

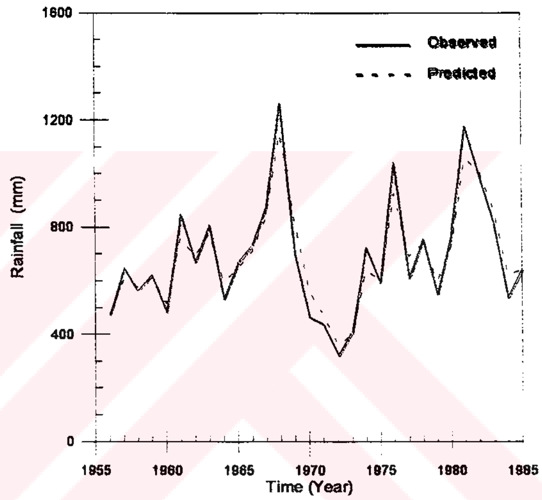

Figure D.4 Observed and estimated annual rainfall values at Adana.

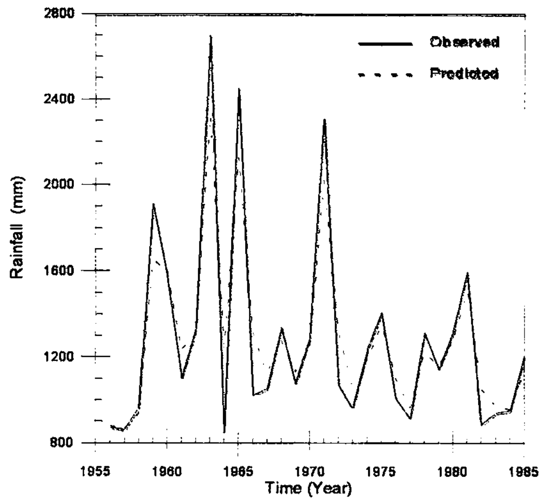

Figure D.5 Observed and estimated annual rainfall values at Bahçeköy.

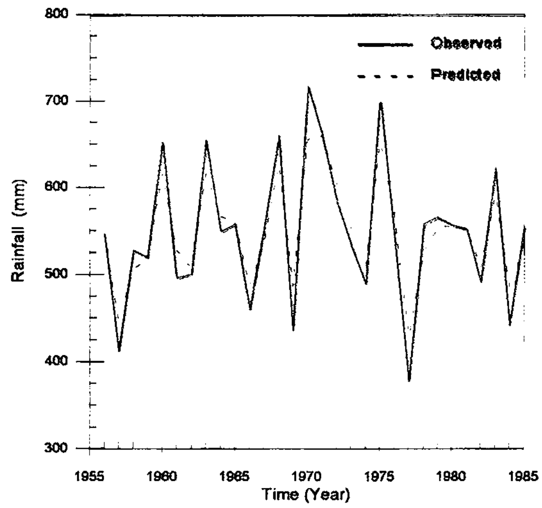

Figure D.6 Observed and estimated annual rainfall values at Bolu.

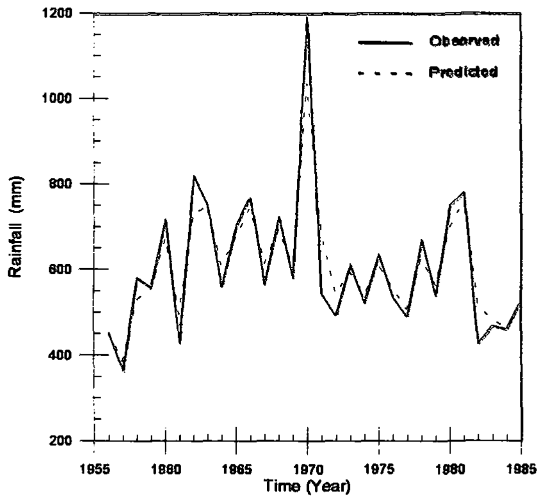

Figure D.7 Observed and estimated annual rainfall values at Balıkesir.

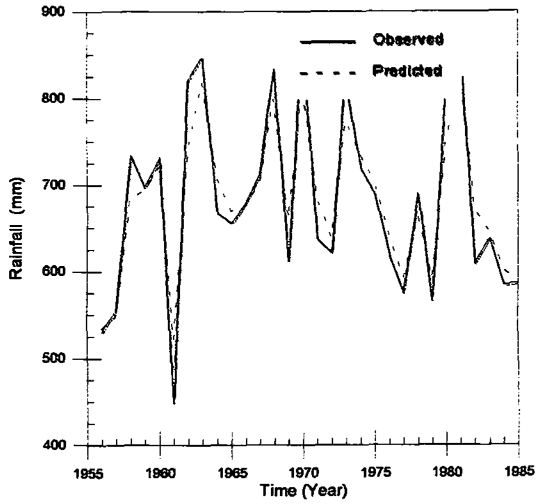

Figure D.8 Observed and estimated annual rainfall values at Bursa.

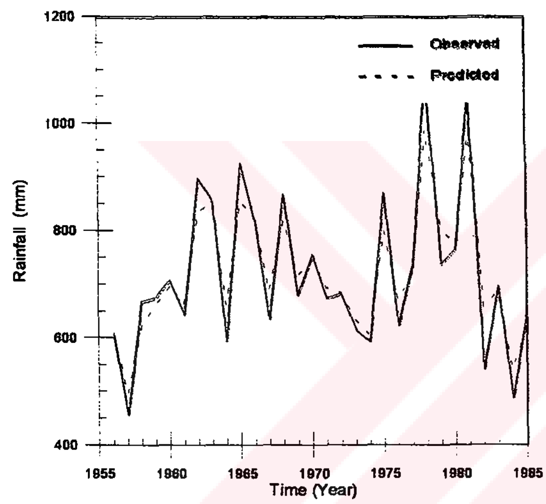

Figure D.9 Observed and estimated annual rainfall values at Bandırma.

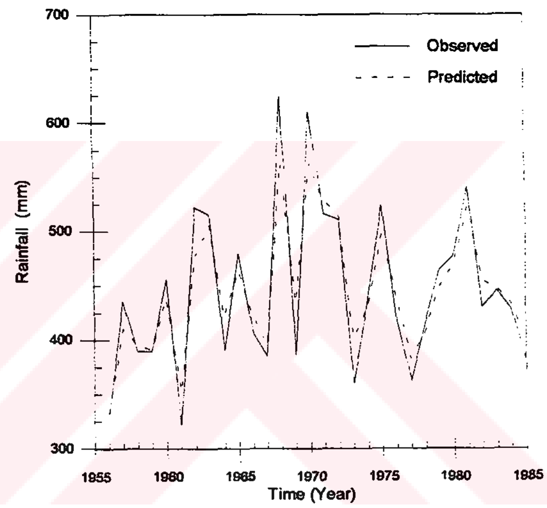

Figure D.10 Observed and estimated annual rainfall values at Bilecik.

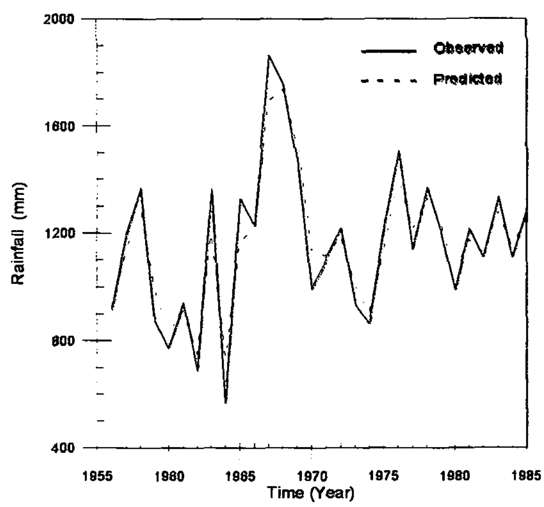

Figure D.11 Observed and estimated annual rainfall values at Bitils.

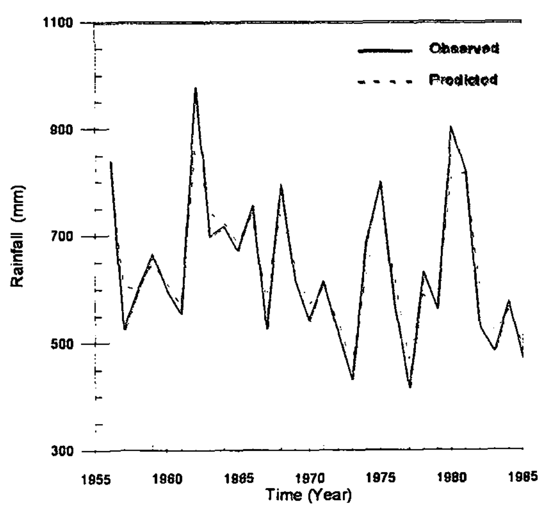

Figure D.12 Observed and estimated annual rainfall values at Çanakkale.

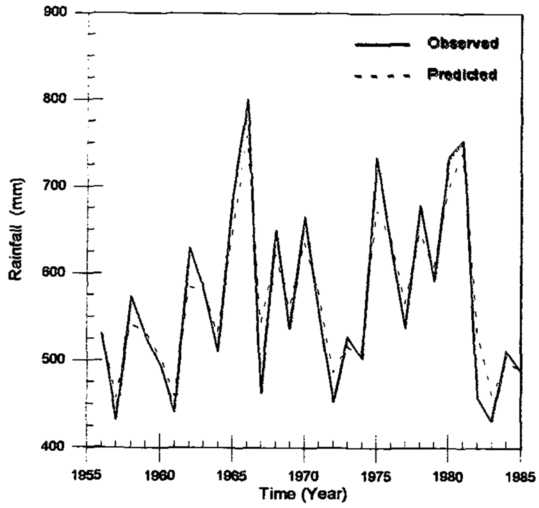

Figure D.13 Observed and estimated annual rainfall values at Çorlu.

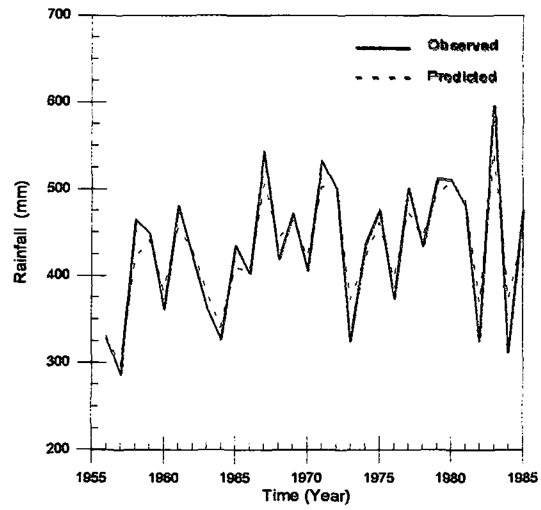

Figure D.14 Observed and estimated annual rainfall values at Çorum.

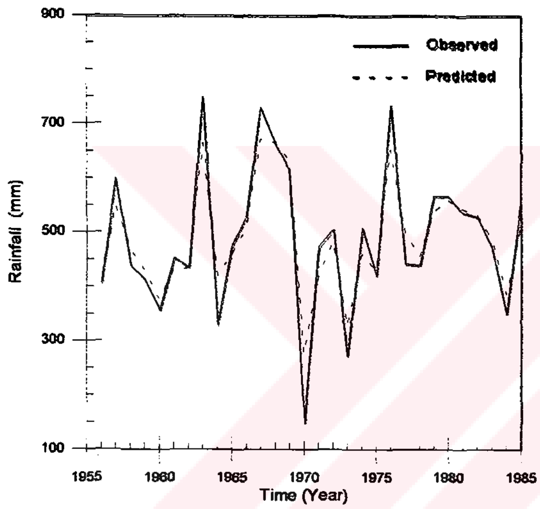

Figure D.15 Observed and estimated annual rainfall values at Diyarbakır.

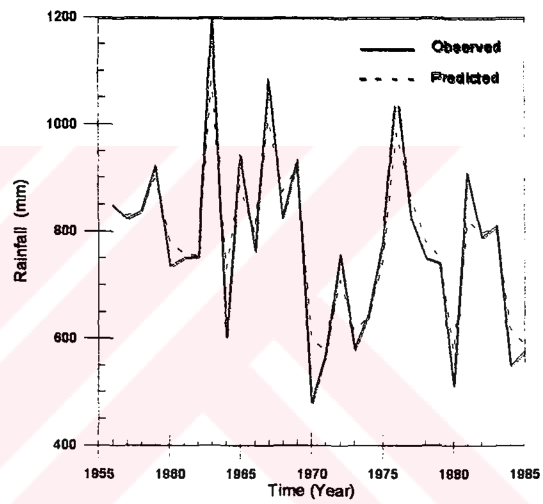

Figure D.16 Observed and estimated annual rainfall values at Dursun bey.

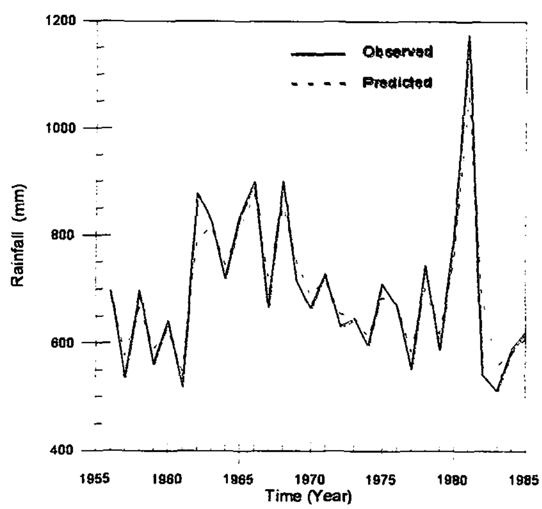

Figure D.17 Observed and estimated annual rainfall values at Edremit.

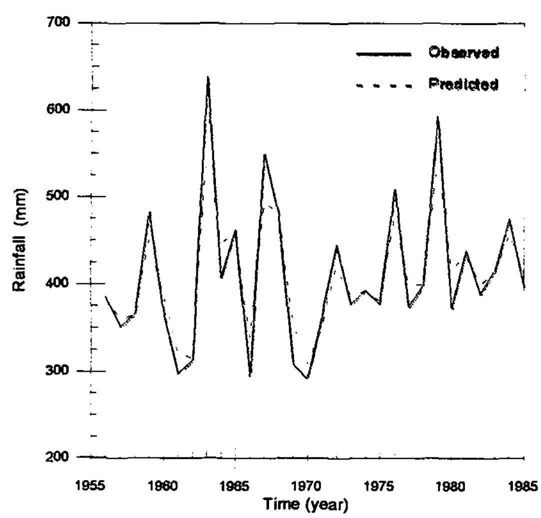

Figure D.18 Observed and estimated annual rainfall values at Erzurum.

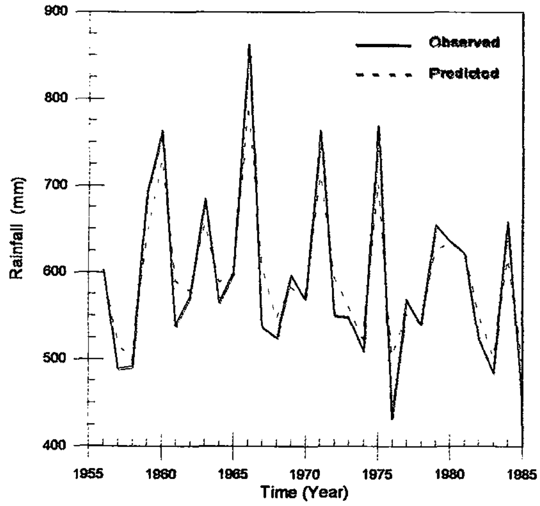

Figure D.19 Observed and estimated annual rainfall values at Edirne.

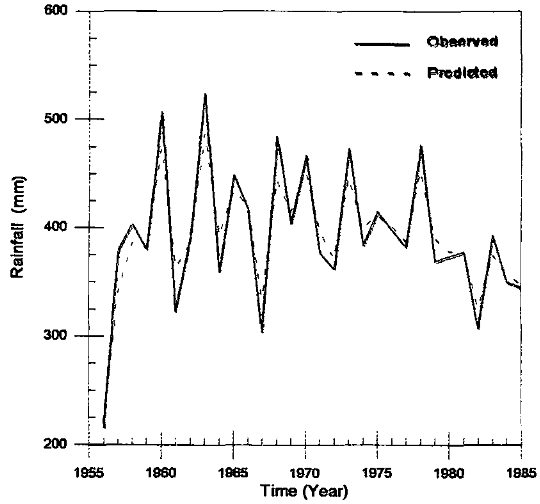

Figure D.20 Observed and estimated annual rainfall values at Eskişehir.

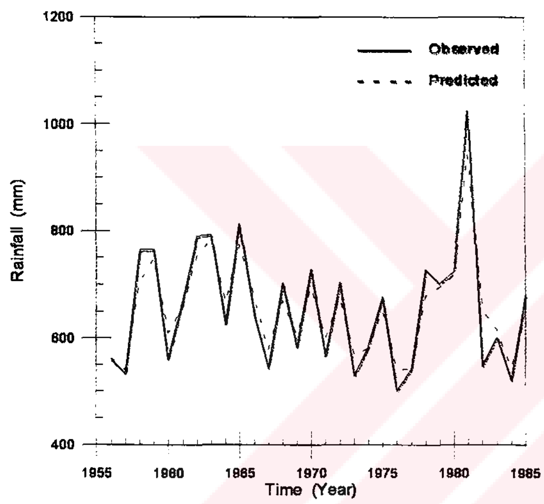

Figure D.21 Observed and estimated annual rainfall values at Florya.

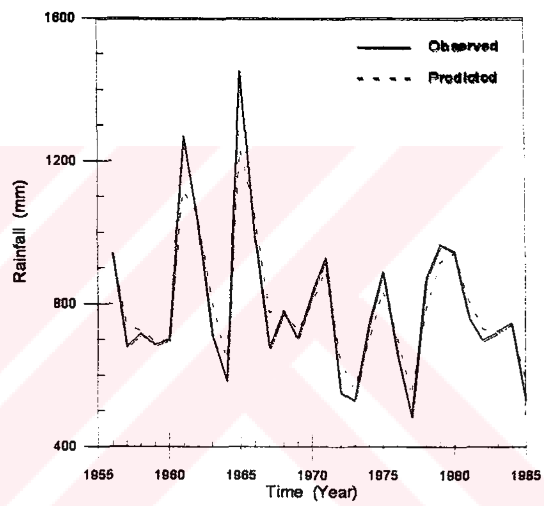

Figure D.22 Observed and estimated annual rainfall values at Gökçeada.

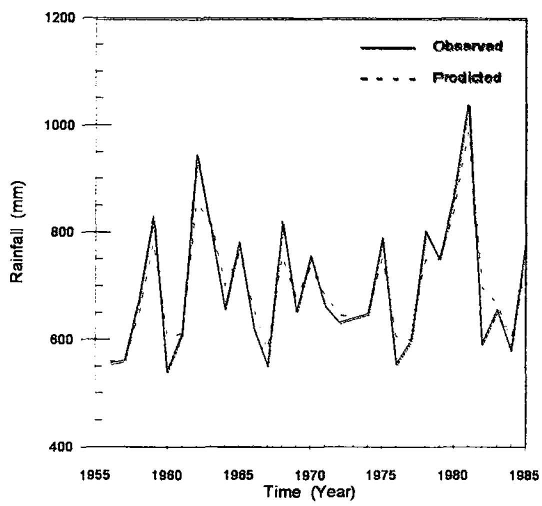

Figure D.23 Observed and estimated annual rainfall values at Göztepe.

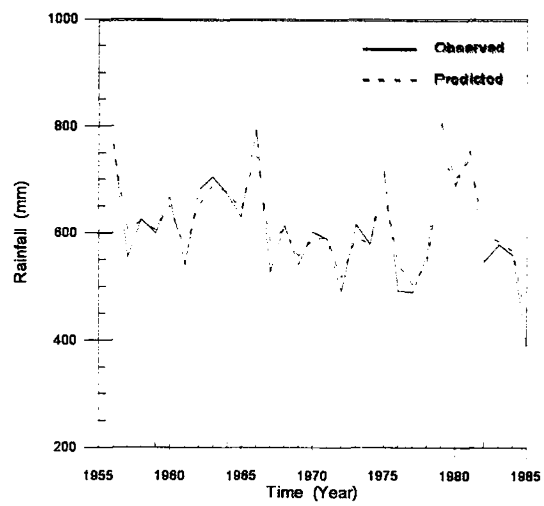

Figure D.24 Observed and estimated annual rainfall values at İpsala.

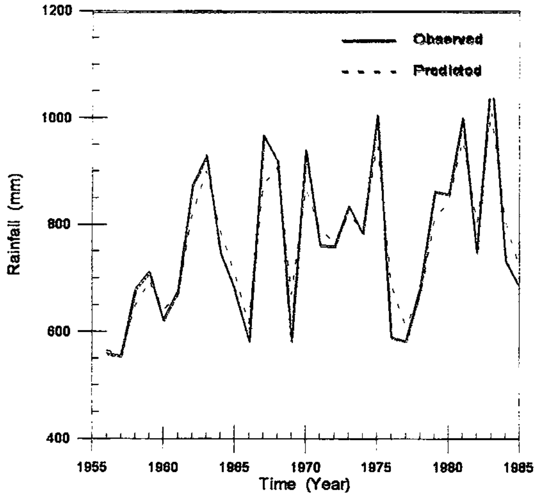

Figure D.25 Observed and estimated annual rainfall values at İzmit.

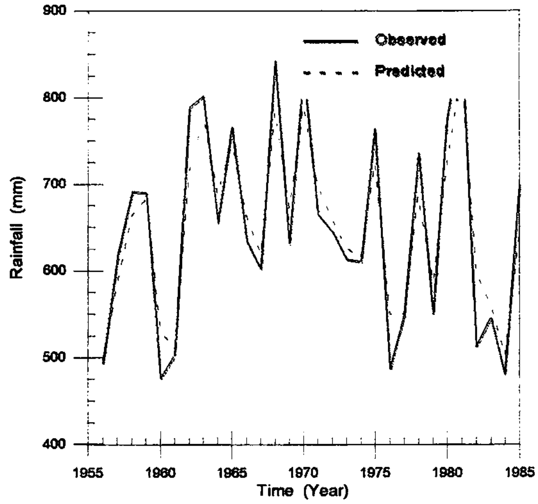

Figure D.26 Observed and estimated annual rainfall values at Kartal.

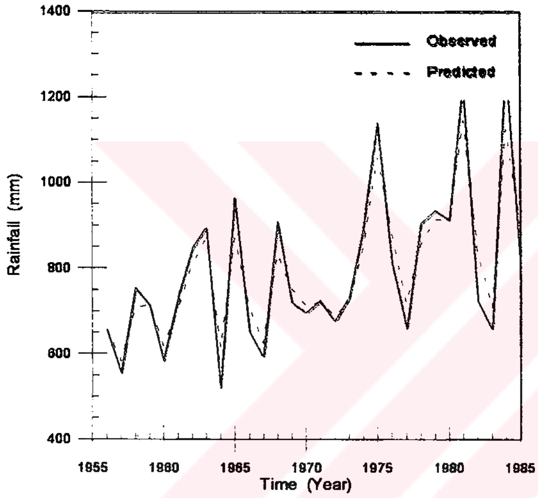

Figure D.27 Observed and estimated annual rainfall values at Kumköy.

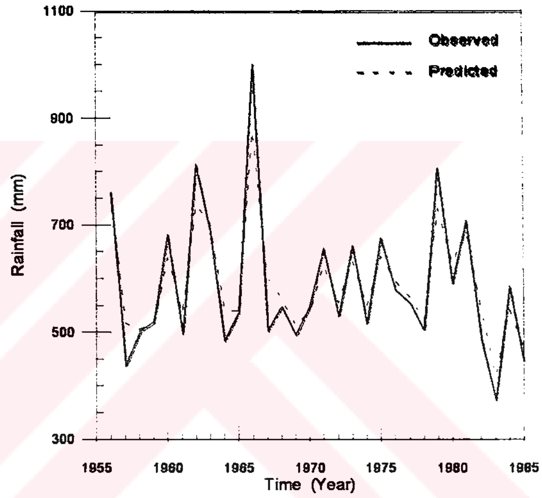

Figure D.28 Observed and estimated annual rainfall values at Kırklareli.

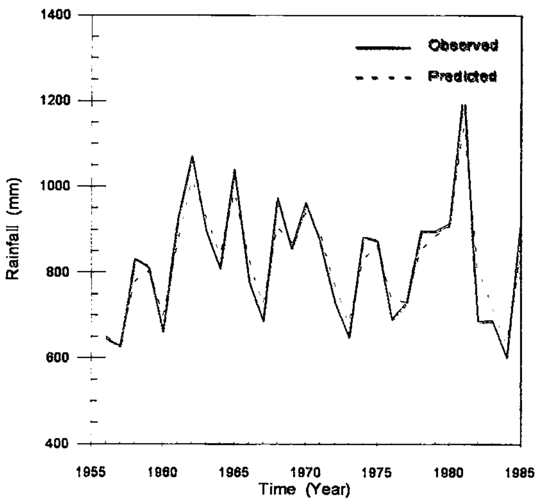

Figure D.29 Observed and estimated annual rainfall values at Kandilli.

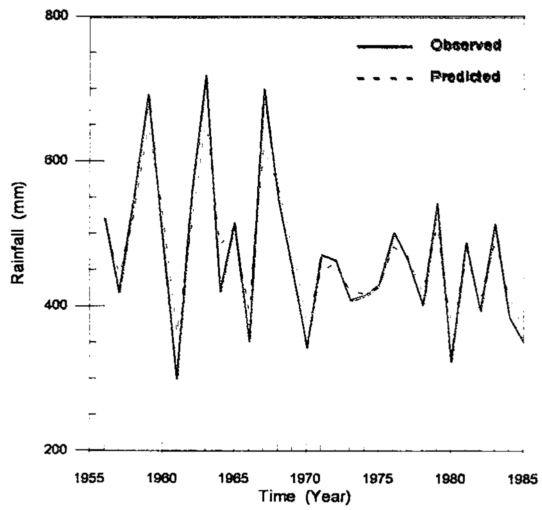

Figure D.30 Observed and estimated annual rainfall values at Kars.

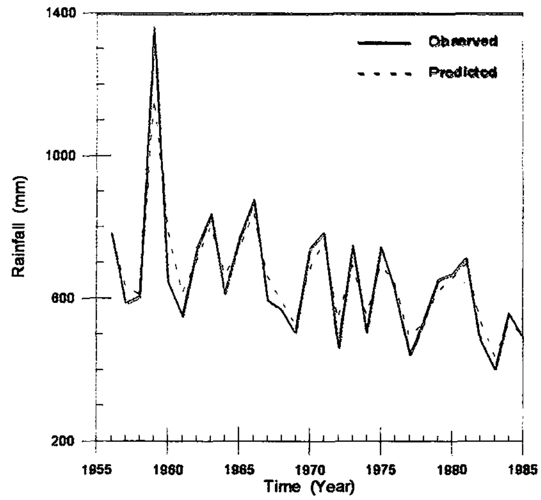

Figure D.31 Observed and estimated annual rainfall values at Luleburgaz.

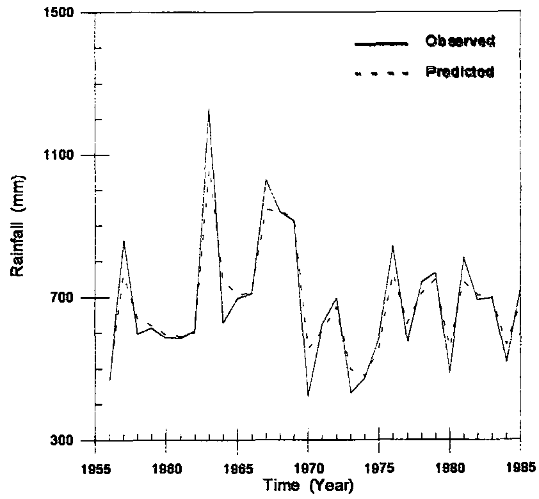

Figure D.32 Observed and estimated annual rainfall values at Siirt.

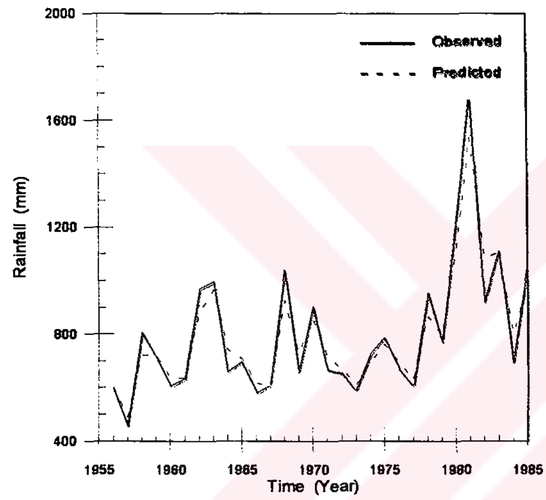

Figure D.33 Observed and estimated annual rainfall values at Şile.

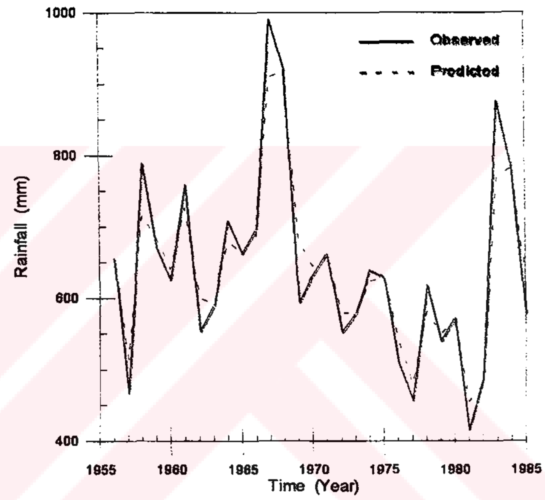

Figure D.34 Observed and estimated annual rainfall values at Sinop.

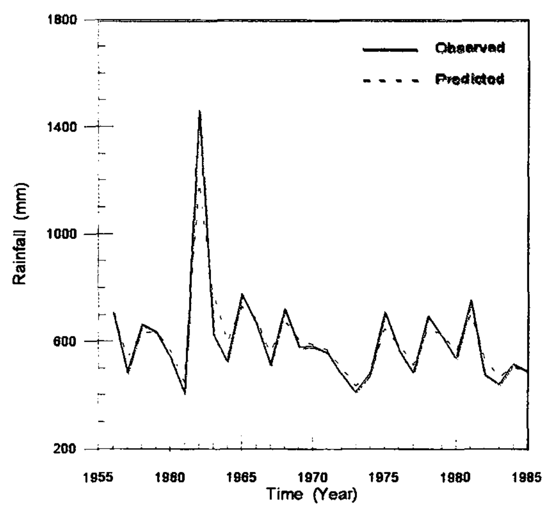

Figure D.35 Observed and estimated annual rainfall values at Tekirdağ.

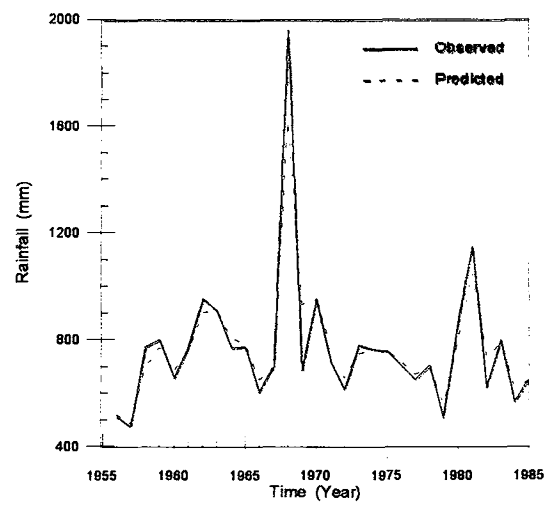

Figure D.36 Observed and estimated annual rainfall values at Yalova.

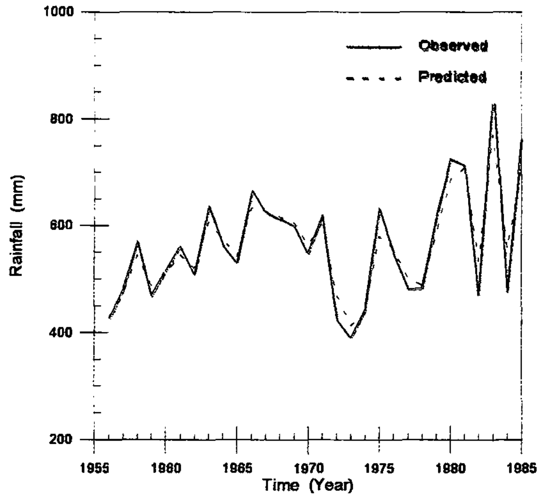

Figure D.37 Observed and estimated annual rainfall values at Yozgat.

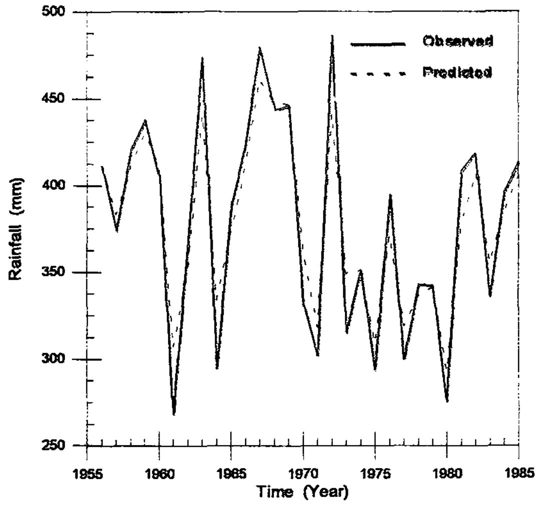

Figure D.38 Observed and estimated annual rainfall values at Van.

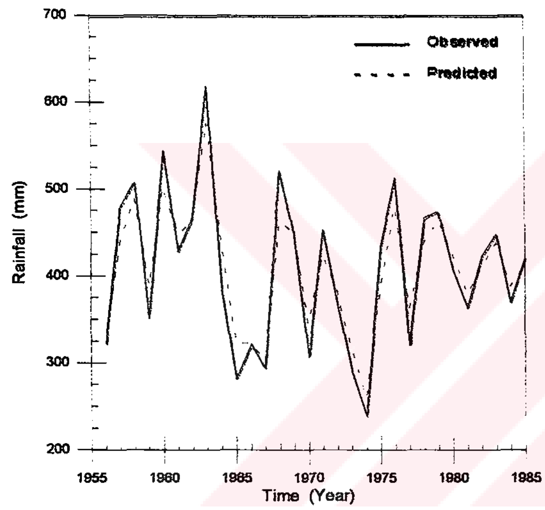

Figure D.39 Observed and estimated annual rainfall values at Afyon.

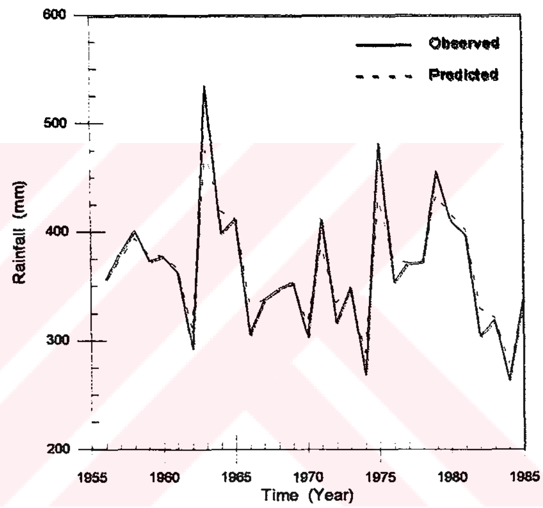

Figure D.40 Observed and estimated annual rainfall values at Kayseri.

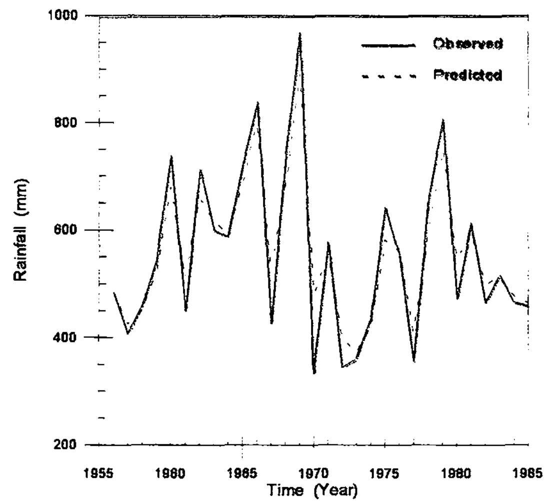

Figure D.41 Observed and estimated annual rainfall values at Isparta.

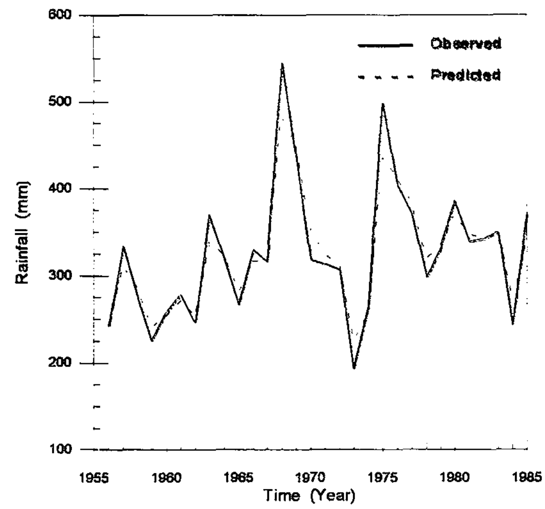

Figure D.42 Observed and estimated annual rainfall values at Konya.

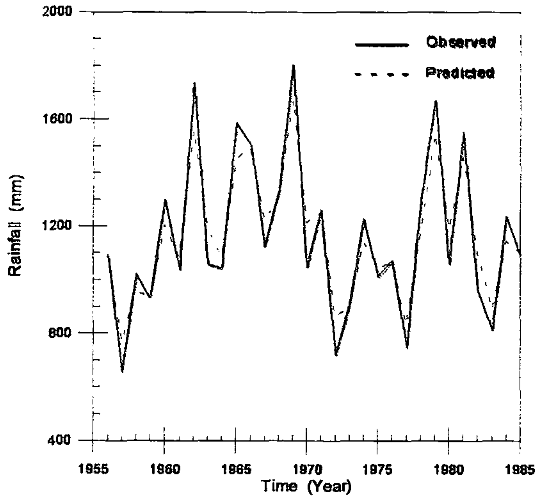

Figure D.43 Observed and estimated annual rainfall values at Muğla.

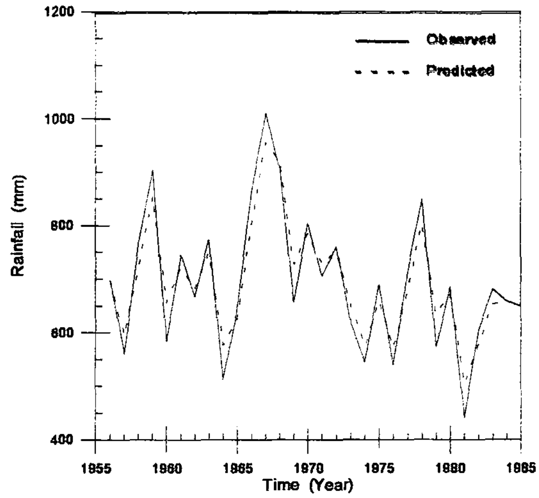

Figure D.44 Observed and estimated annual rainfall values at Samsun.

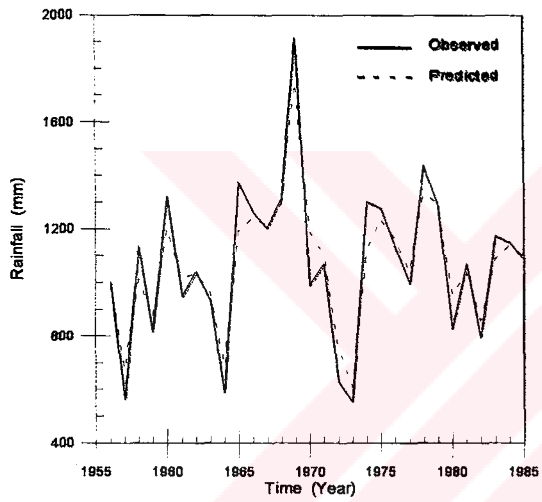

Figure D.45 Observed and estimated annual rainfall values at Antalya.

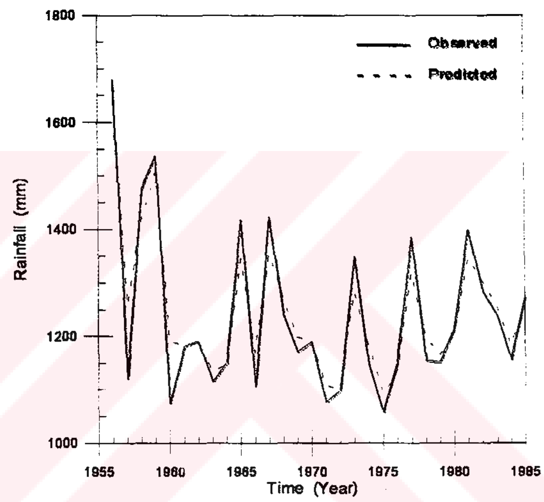

Figure D.46 Observed and estimated annual rainfall values at Giresun.

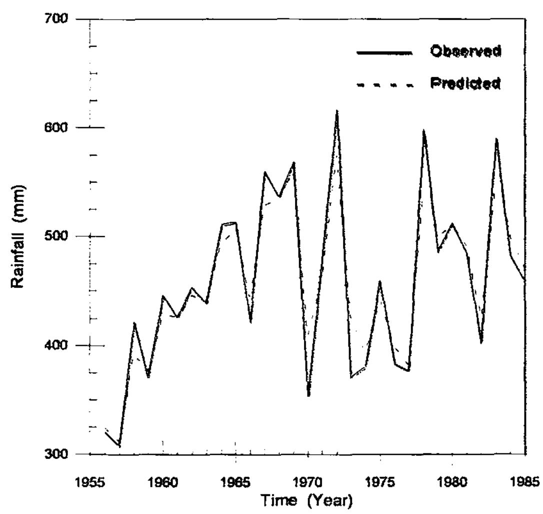

Figure D.47 Observed and estimated annual rainfall values at Kastamonu.

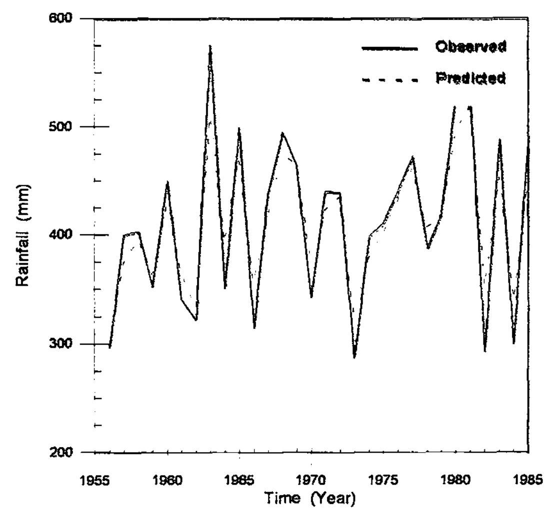

Figure D.48 Observed and estimated annual rainfall values at Sivas.

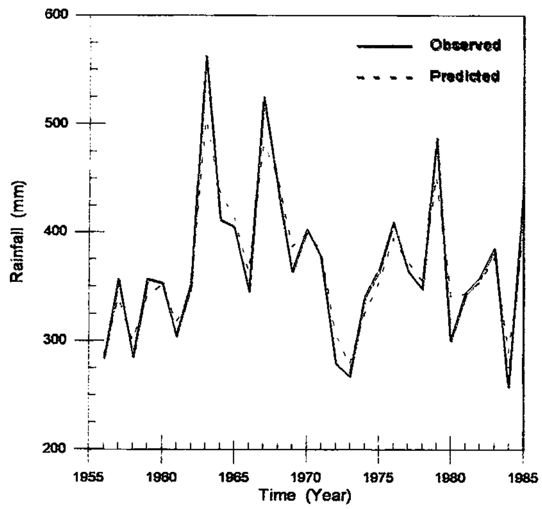

Figure D.49 Observed and estimated annual rainfall values at Erzincan.

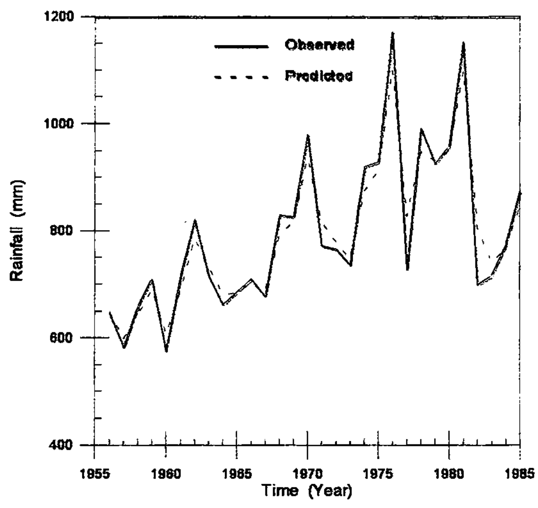

Figure D.50 Observed and estimated annual rainfall values at Sarıyer.

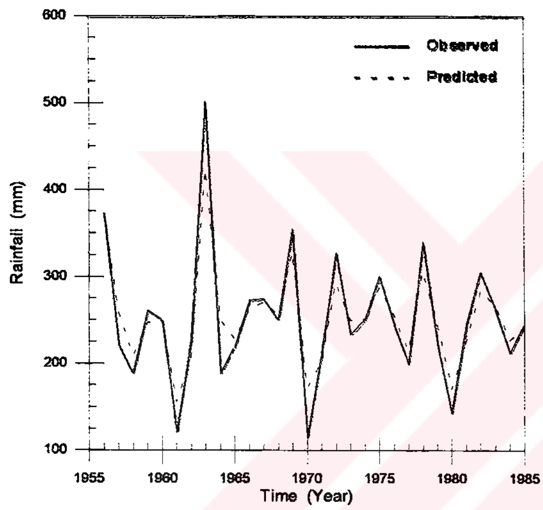

Figure D.51 Observed and estimated annual rainfall values at Iğdır.

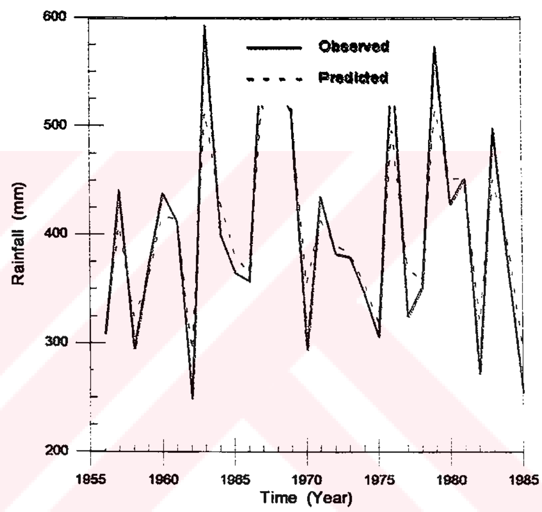

Figure D.52 Observed and estimated annual rainfall values at Malatya.

APPENDIX E

Observed And Estimated Annual Rainfall Values At Selected Stations In Turkey For 1956-1985.

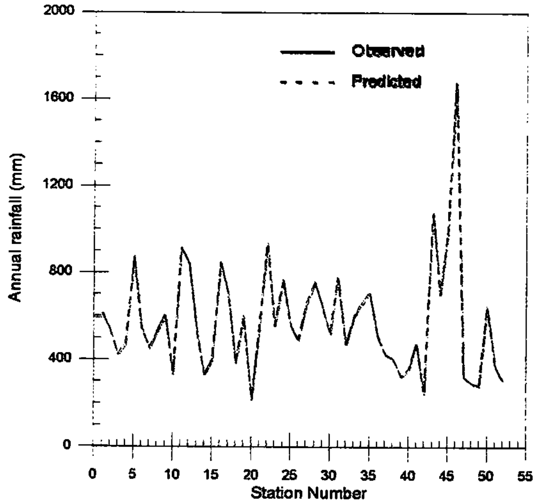

Figure E.1 Observed and estimated annual rainfall values for 1956.

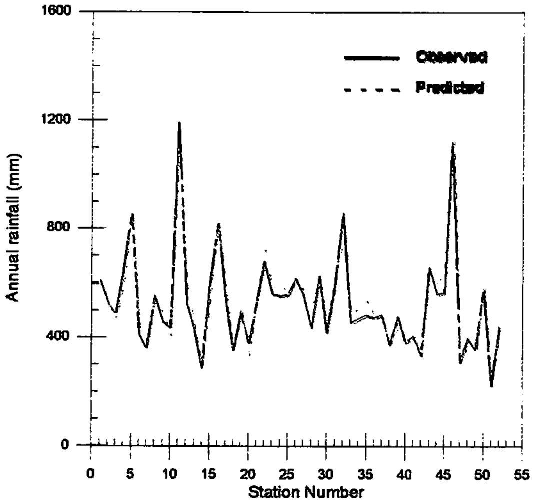

Figure E.2 Observed and estimated annual rainfall values for 1957.

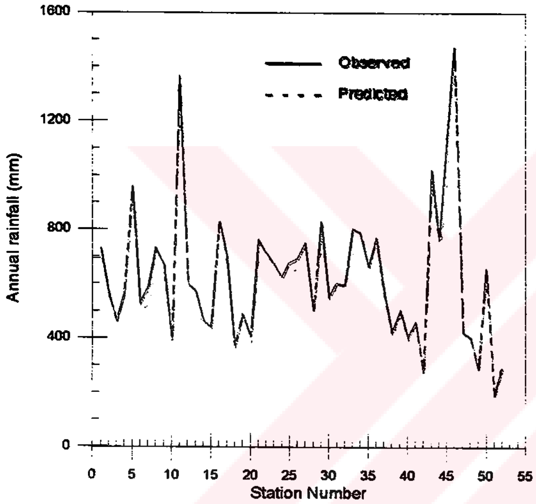

Figure E.3 Observed and estimated annual rainfall values for 1958.

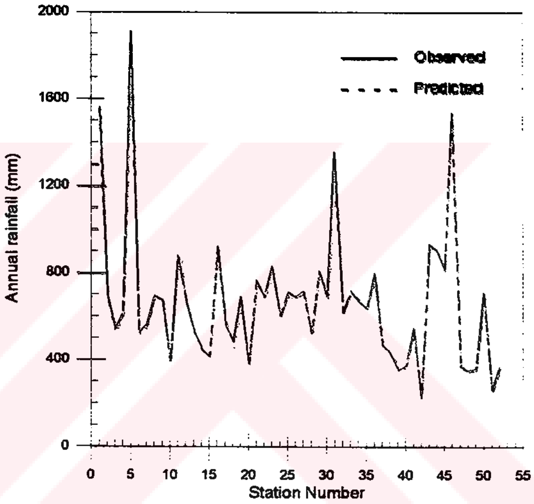

Figure E.4 Observed and estimated annual rainfall values for 1959.

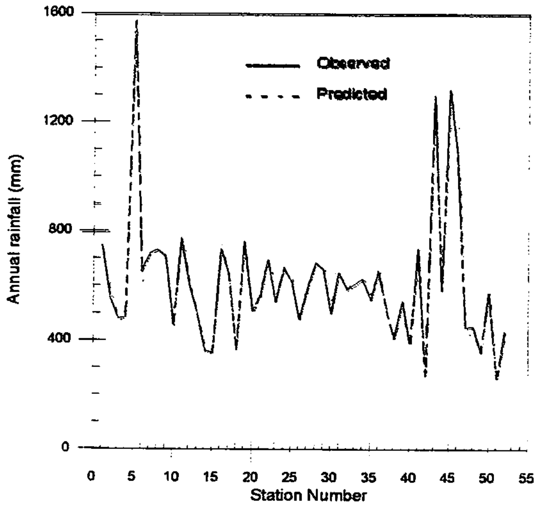

Figure E.5 Observed and estimated annual rainfall values for 1960.

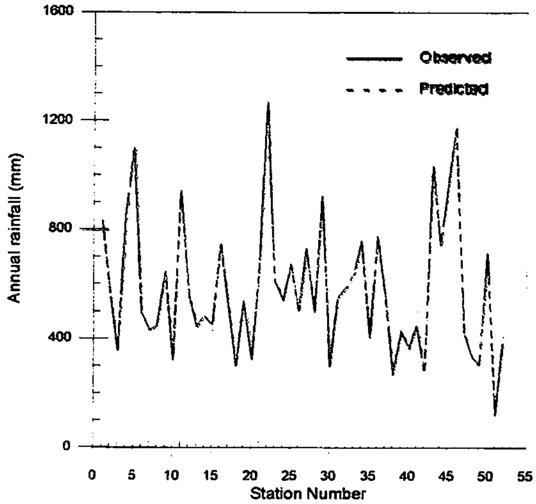

Figure E.6 Observed and estimated annual rainfall values for 1961.

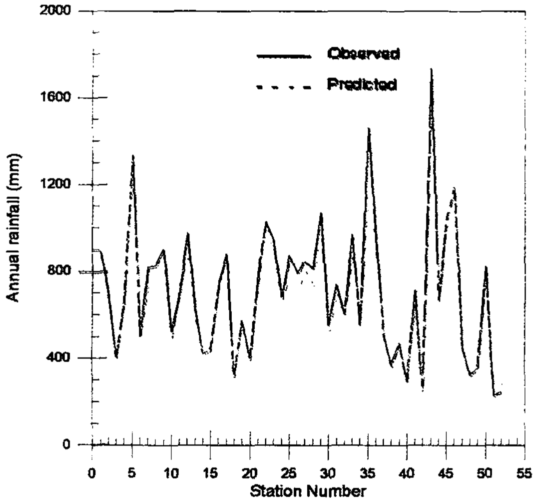

Figure E.7 Observed and estimated annual rainfall values for 1962.

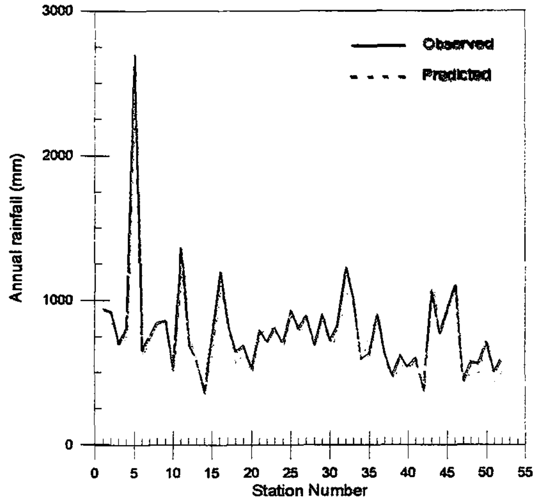

Figure E.8 Observed and estimated annual rainfall values for 1963.

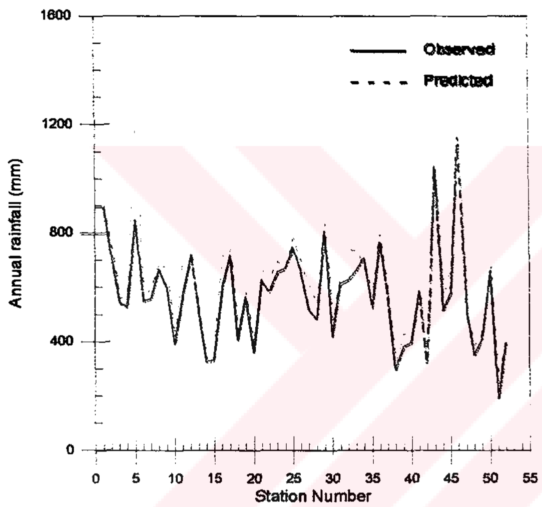

Figure E.9 Observed and estimated annual rainfall values for 1964.

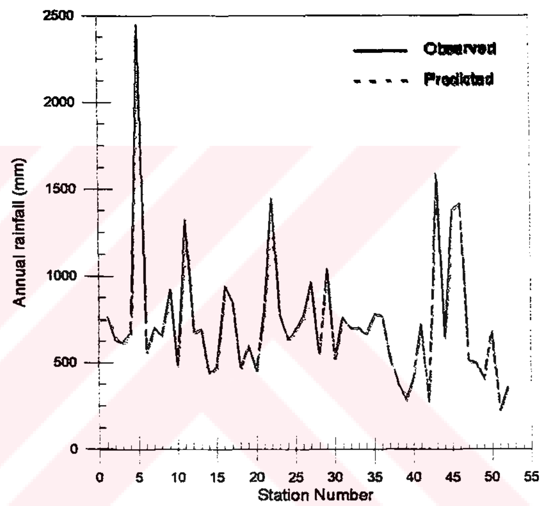

Figure E.10 Observed and estimated annual rainfall values for 1965.

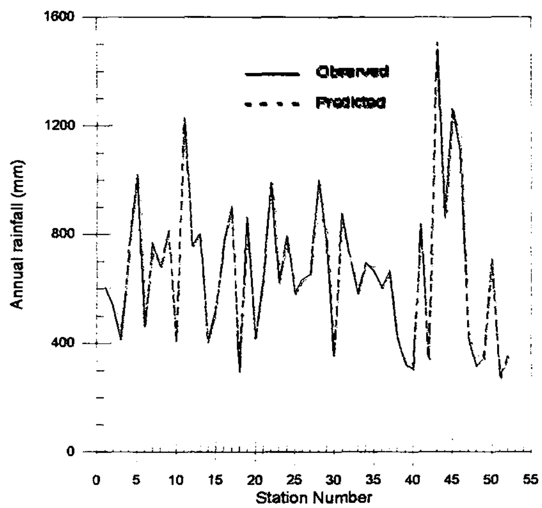

Figure E.11 Observed and estimated annual rainfall values for 1966.

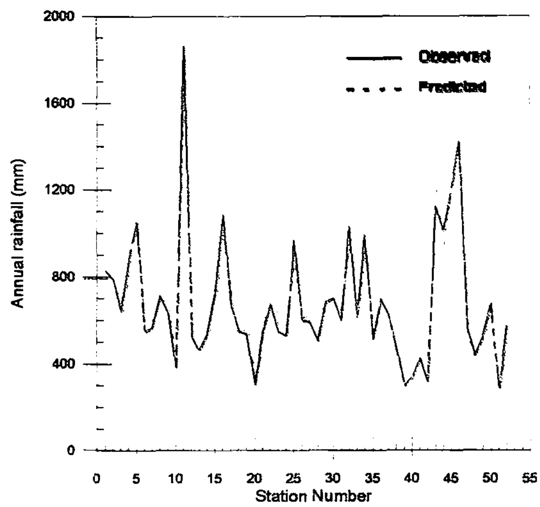

Figure E.12 Observed and estimated annual rainfall values for 1967.

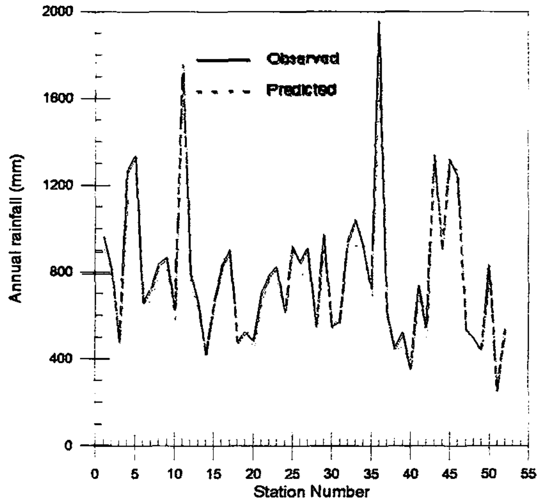

Figure E.13 Observed and estimated annual rainfall values for 1968.

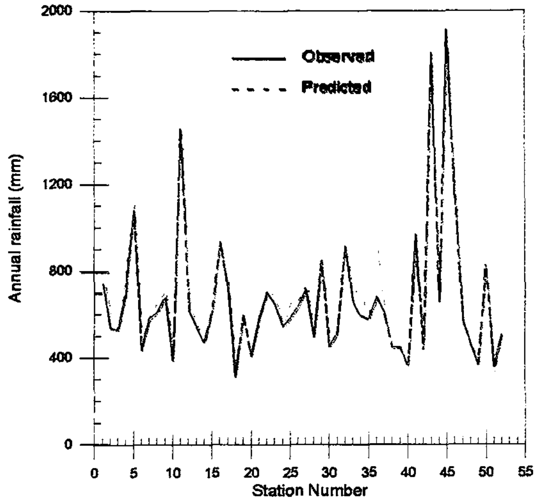

Figure E.14 Observed and estimated annual rainfall values for 1969.

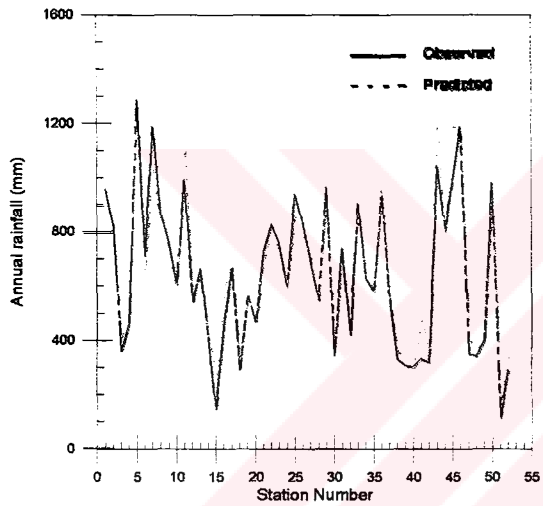

Figure E.15 Observed and estimated annual rainfall values for 1970.

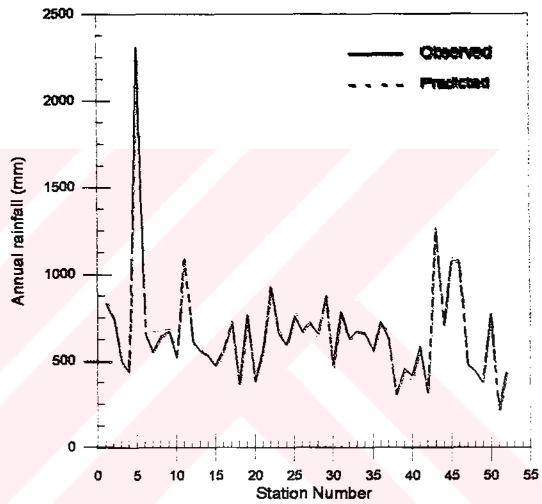

Figure E.16 Observed and estimated annual rainfall values for 1971.

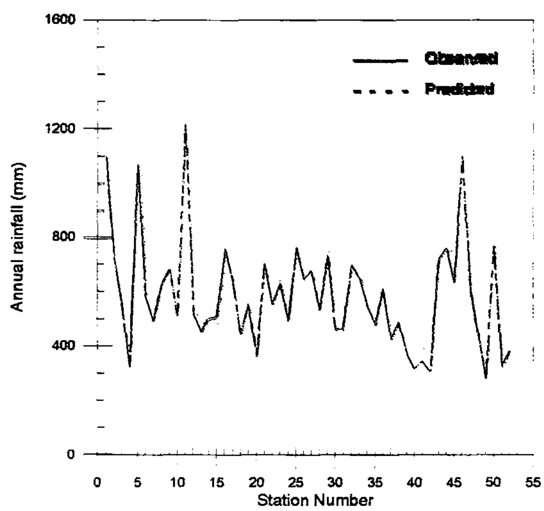

Figure E.17 Observed and estimated annual rainfall values for 1972.

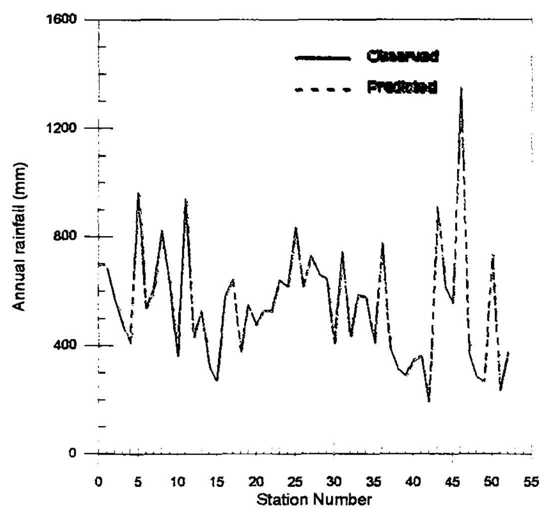

Figure E.18 Observed and estimated annual rainfall values for 1973.

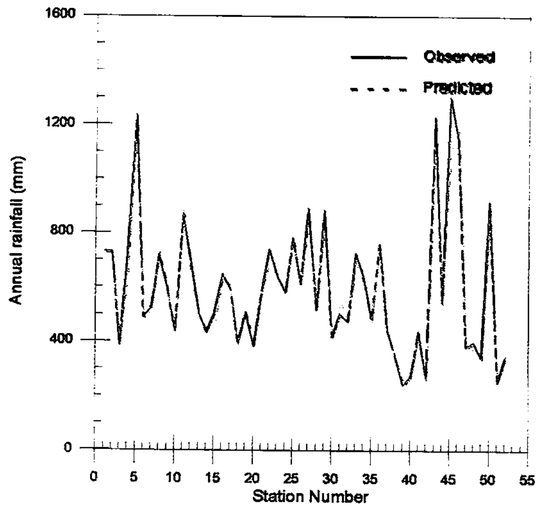

Figure E.19 Observed and estimated annual rainfall values for 1974.

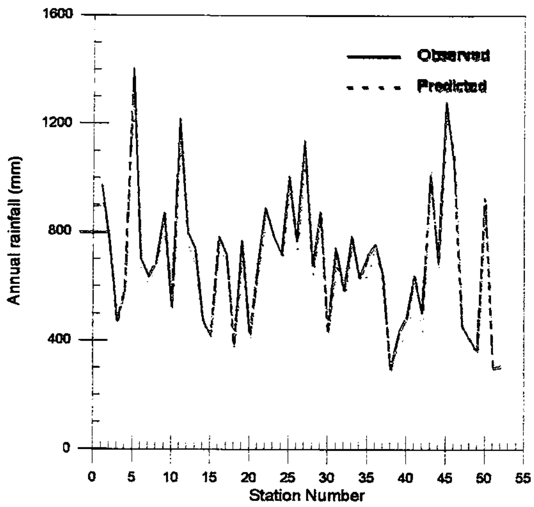

Figure E.20 Observed and estimated annual rainfall values for 1975.

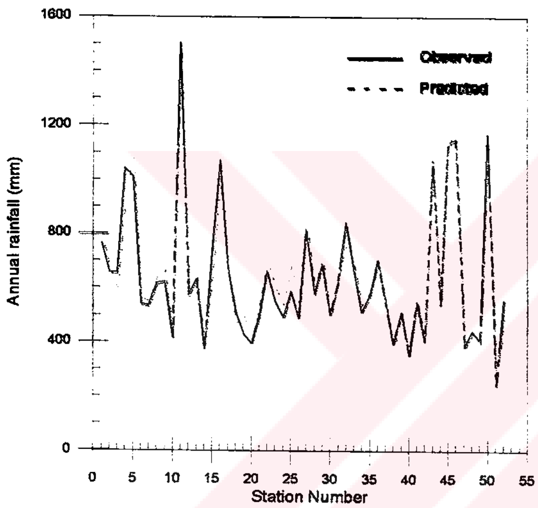

Figure E.21 Observed and estimated annual rainfall values for 1976.

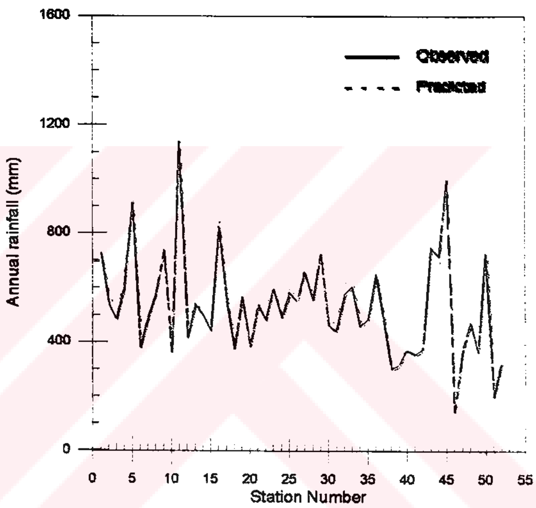

Figure E.22 Observed and estimated annual rainfall values for 1977.

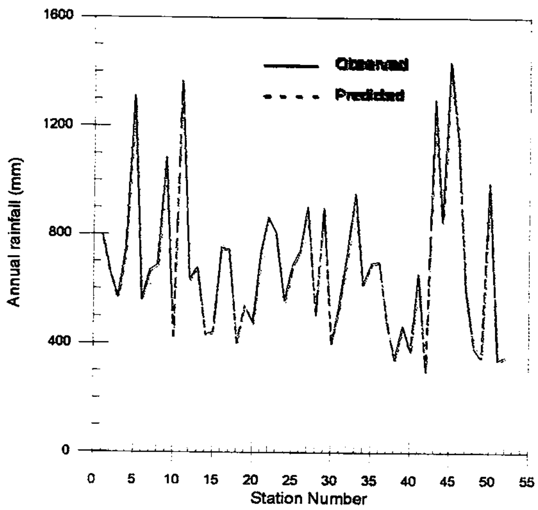

Figure E.23 Observed and estimated annual rainfall values for 1978.

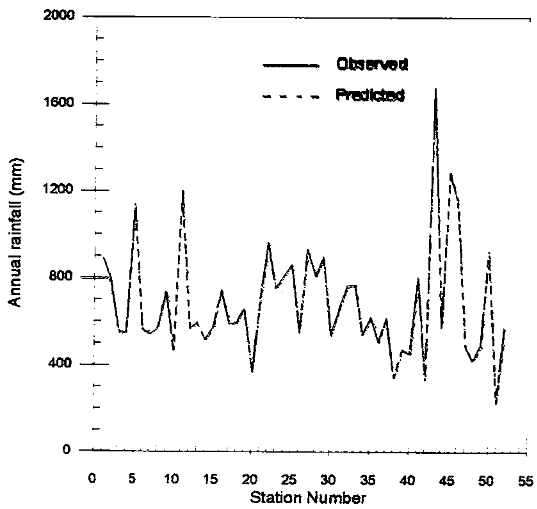

Figure E.24 Observed and estimated annual rainfall values for 1979.

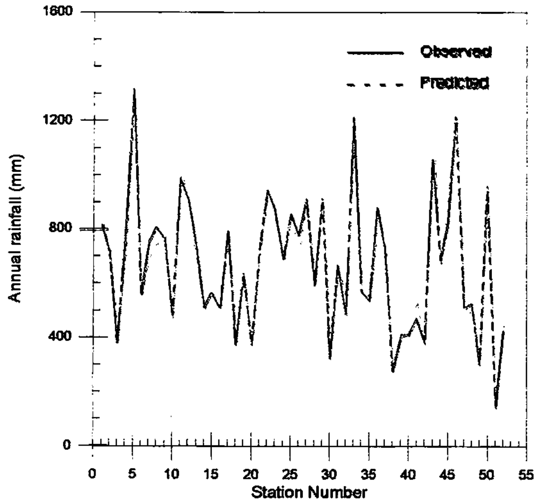

Figure E.25 Observed and estimated annual rainfall values for 1980.

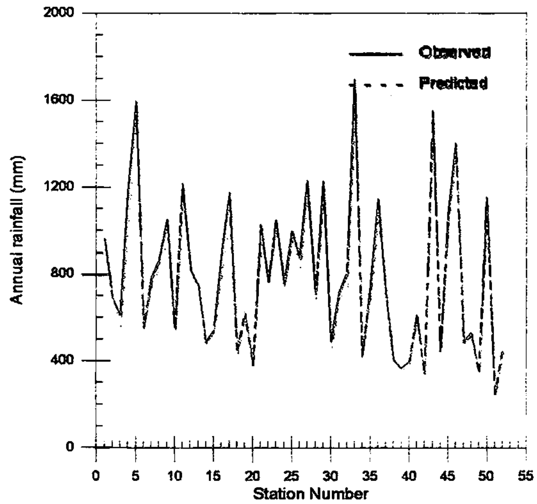

Figure E.26 Observed and estimated annual rainfall values for 1981.

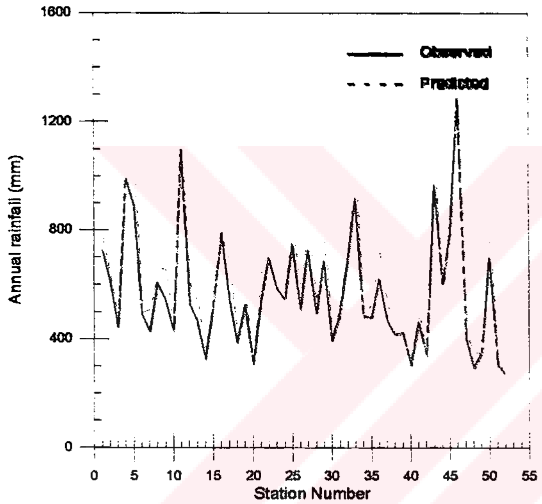

Figure E.27 Observed and estimated annual rainfall values for 1982.

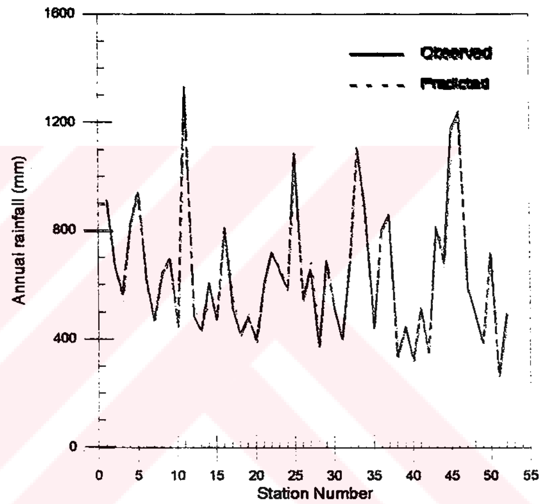

Figure E.28 Observed and estimated annual rainfall values for 1983.

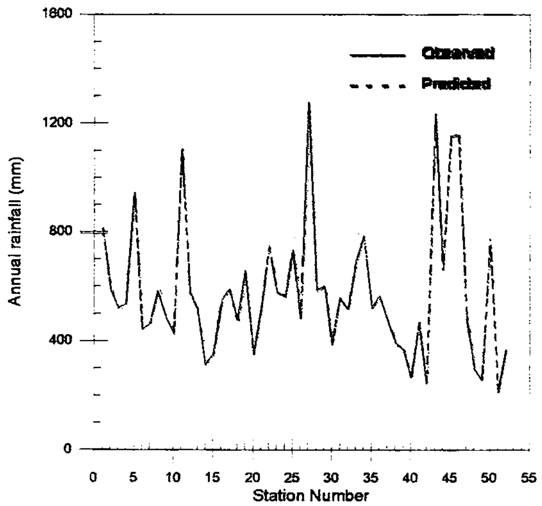

Figure E.29 Observed and estimated annual rainfall values for 1984.

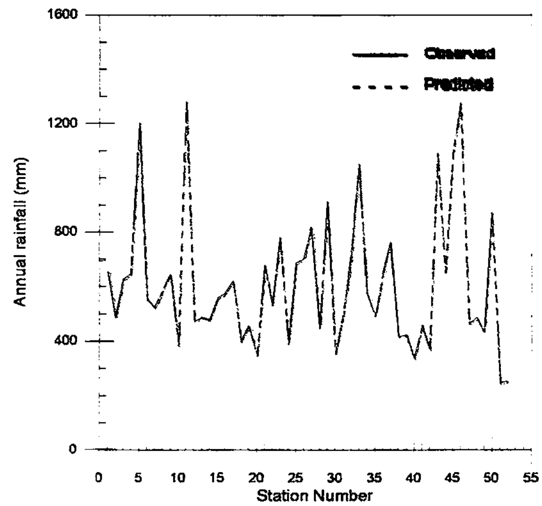

Figure E.30 Observed and estimated annual rainfall values for 1985.

CURRICULUM VITAE

Abdullatif M. LATIF was born in B. Gisher, Tripoli, Libya on December 10, 1962. Completed primary and secondary education in Tripoli, Libya. In 1980 he entered Department of Meteorology, Faculty of Science, University of Al-fateh as an undergraduate student and in 1984 graduated with a degree of Bachelor of Science in Meteorology.

Since 1984, he was working as Demonstrator in the same Department. In September 1991, he started his Post graduate program and received his Master Degree on July 1993 from the Meteorological Engineering Department, Faculty of Aeronautics and Space Science, Istanbul Technical University. In September 1993, he start his Ph. D. program at the same department and university. He is expected to fulfill all the requirements for Ph.D. degree in Meteorology Engineering by the end of August, 1999. He is married.