İSTANBUL TECHNICAL UNIVERSITY ★ INSTITUTE OF SCIENCE AND TECHNOLOGY

NON PERTURBATIVE INVESTIGATION OF A FERMIONIC MODEL

Ph.D. Thesis by
Bekir Can LÜTFÜOĞLU

Department : Physics Engineering

Programme : Physics Engineering

MAY 2009

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Prof. Dr. Cihan Saçlıoğlu (S.U.)

MAY 2009

İSTANBUL TEKNİK ÜNİVERSİTESİ ★ FEN BİLİMLERİ ENSTİTÜSÜ

BİR FERMİYONİK MODELİN NON PERTÜRBATİF İNCELENMESİ

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MAYIS 2009

I would hereby like to dedicate my thesis to my mother,

FOREWORD

There are lots of people I would like to thank for a huge variety of reasons.

Firstly, I would like to express my sincere gratitude to my advisor, Mahmut Hortaçsu, for his patience, guidance, enthusiasm, encouragement, inspiration, and humor.

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May 2009 Bekir Can LÜTFÜOĞLU

TABLE OF CONTENTS

ABBREVIATIONS

NJL	**:**	Nambu Jona-Lasinio
gNJL	**:**	Gauged Nambu Jona-Lasinio
YM	**:**	Yang-Mills
gHY	**:**	Gauged Higgs-Yukawa
RG	**:**	Renormalization Group
QFT	**:**	Quantum Field Theory
QED	**:**	Quantum Electrodynamics
ERG	**:**	Exact Renormalization Group
BCS	**:**	Bardeen-Cooper-Schrieffer
GN	**:**	Gross-Neveu
QCD	**:**	Quantum Chromodynamics

LIST OF FIGURES

Page

LIST OF SYMBOLS

$\psi, \overline{\psi}$	:	Spinor fields.
Φ, ϕ	:	Composite scalar fields.
Λ, λ	:	Composite scalar fields.
A_μ, A_μ^α	:	Vector fields.
$g^{'}, g$	:	Coupling constant of spinor fields to composite scalar fields.
$a^{'}, a$	:	Coupling constant of four composite scalar fields.
e	:	Coupling constant of spinor fields to vector fields.
$\mathscr{L}$	:	Lagrangian density.
$\mathscr{H}$	:	Hamiltonian density.
U	:	Path integral transition amplitude.
$\triangle_F$	:	Faddeev-Popov determinant.

NON PERTURBATIVE INVESTIGATION OF A FERMIONIC MODEL

SUMMARY

To find a nontrivial field theoretical model is one of the outstanding problems in theoretical high energy physics. The perturbatively nontrivial ϕ^4 in four dimensions was shown to go to a free theory as the cut-off is lifted a while ago. During the last twenty years, many papers were written on making sense out of "trivial models", interpreting them as effective theories without taking the cutoff to infinity. One of these models is the Nambu Jona-Lasinio model, hereafter NJL. Although this model is shown to be a trivial in four dimensions, since the coupling constant goes to zero with a negative power of the logarithm of the ultraviolet cut-off, as an effective model in low energies it gives us important insight to several processes.

There were also attempts, by Bardeen et al., to couple the NJL model to a gauge field, the so called gauged NJL model, to be able to get a non-trivial field theory. It was shown that if one has sufficient number of fermion flavors, such a construction is indeed possible.

There are other models, made out of only spinors, which were constructed as alternatives of the original Heisenberg model, the first model given as "a theory of everything", using only spinors. The Gürsey model was proposed, before the NJL model, as a substitute for the Heisenberg model, which could not be renormalized using standard methods. The Gürsey model had the conformal symmetry, when the model is taken in a classical sense. It had classical solutions, which were interpreted as instantons and merons, much like the solutions of the Yang-Mills (YM) theories. It had one important defect, though. Its non-polynomial Lagrangian made the use of standard methods in its quantization not feasible.

M.Hortaçsu, with collaborators, tried to make quantum sense of this model a while ago. He concluded that the result was a "trivial model", which means that the processes involving the constituent spinors resulted in the free result.

Using a new interpretation of the model and taking hints from the work of Bardeen et al., we studied a model, which classically simulates the Gürsey model, by coupling constituent $U(1)$ gauge field to the spinors. We investigated whether this new coupling makes this new model a truly interacting one. We found that we are mimicking a gauge Higgs Yukawa (gHY) system, which had the known problems of the Landau pole, with all of its connotations of triviality.

Then we studied our original model, coupled to a $SU(N)$ gauge field, instead. We derived the renormalization group (RG) equations in one loop, and tried to derive the criteria for obtaining nontrivial fixed points for the coupling constants of the theory. Finally we showed that the renormalization group equations give indications of a nontrivial field theory when it is gauged with a $SU(N)$ field.

BİR FERMİYONİK MODELİN NON PERTÜRBATİF İNCELENMESİ

ÖZET

Teorik yüksek enerji fiziğinin çözülememiş bir problemi triviyal olmayan alan teorisi bulmaktır. Yakın bir zaman önce pertürbatif olarak triviyal olmayan ϕ^4 teorisinin cut-off kaldırıldığında serbest teoriye gittiği dört boyutta gösterilmiştir. Son yirmi yıl içerisinde, triviyal modelleri cut-off u sonsuza götürme gereği duyulmadan etkin teoriler olarak açıklayan modeller birçok makalenin konusu olmuştur. Bu modellerden biri Nambu Jona-Lasanio modelidir. Her ne kadar dört boyutta bu modelin triviyal olduğu, morötesi cut-off da kuplaj sabitinin logaritmanın negatif kuvveti olarak sıfıra gittiği, gösterilse de bu modelin düşük enerjilerde etkin model olarak önemli süreçlere ışık tutabileceği düşünülmektedir.

Triviyal olmayan bir alan teorisi elde etmek için bir takım çalışmalardan bahsedilebilinir. Bunlardan biri Bardeen ve arkadaşları tarafından denenen NJL modeline bir ayar alanı bağlanmasıdır. Ayar NJL (gNJL) modeli de denilen bu modelin, yeteri kadar fermiyon çeşnisi bulunması durumunda triviyal olmadığı gösterilmiştir.

Sadece spinörlerden oluşan başka modeller de vardır. Bu modeller Heisenberg'in "her şeyin teorisi" olarak sunduğu, yalnızca fermiyonlardan oluşan modele alternatif olarak sunulmuştur. Bu modellerden biri de Gürsey modelidir. Bu model NJL modelinden önce ortaya konulmasına rağmen standart metodlarla renormalize edilememektedir. Klasik olarak incelendiğinde Gürsey modelinin konformal simetrisi vardır. Klasik çözümleri Yang-Mills (YM) teorilerinin çözümlerine benzemektedir. Bu çözümler insantonik ve meronik çözümler olarak adlandırılmıştır. Modelin önemli sorunu ise standart yöntemlerle kuantizasyonuna olanak vermeyen polinomik olmayan lagranjiyen ifadesidir.

Bir süre önce M.Hortaçsu ve arkadaşları modelin kuantum anlamlandırması üzerine çalışmalar yapmışlardır. Ulaştıkları sonuca göre salt spinorlerden oluşan süreçler etkileşmemektedir. Dolayısıyla model triviyaldir.

Bu doktora tezinde, Bardeen ve arkadaşlarının çalışmalarından da esinlenerek Gürsey modelinin yeni bir yorumu üzerine çalıştık. Buna göre $U(1)$ ayar alanını spinörlere bağlayarak ayar Gürsey modeli üzerinde durduk. Bu bağlanmanın modeli gerçekten etkileşen bir model yapıp yapmadığını araştırdık. Modelin ayar Higgs-Yukawa (gHY) sistemini taklit ettiğini bulduk. Triviyallik anlamında ise bu modelin Landau kutbu olarak da bilinen sorunlara sahip olduğunu gördük.

Bunun üzerine orijinal modelimize $U(1)$ ayar alanı yerine $SU(N)$ ayar alanı bağladık. Tek halka için renormalizasyon grup (RG) denklemlerini ve teorinin bağlanma sabitleri için triviyal olmayan sabit noktalar verme şartlarını türettik. Modelin $SU(N)$ ayar

alanlarına bağlanması sonucunda, RG denklemlerinden modelin triviyal olmayan alan teorisi olduğunu gösteren kanıtlar elde ettik.

1. INTRODUCTION

Quantum field theory(QFT) is a basic mathematical language. It helps us to describe and analyze the dynamical systems of fields, in other words the physics of the elementary particles. It has a very long history. It started with the quantization of the electromagnetic field by Dirac in 1927 [1]. That work, named as quantum electrodynamics(QED), is the part of QFT that has been developed first. Since that time, millions of studies had been completed and millions will be done in the future. Honestly we have to accept that we have a better understanding comparing the beginning, but surely there is a long way to go. By this study, we want to contribute to the human beings struggle with a small amount.

1.1 Purpose of the Thesis

Mainly there are two objectives of this study. The first objective of the present thesis is building a toy model, which is classically equivalent to Gürsey model that is only constructed by fermions [2]. The second one is constructing a nontrivial field theoretical model out of our toy model by coupling gauge fields for abelian [3] and non-abelian cases [4]. We analyze different cases and summarize the criteria which is required for a nontrivial field theoretical model. To achieve these purposes we use perturbative and nonperturbative techniques.

1.2 Background

Historically, there has always been a continuing interest in building nontrivial field theoretical models. The ϕ^4 theory is a "laboratory" where different methods in quantum field theory are first applied. A while ago it was shown that perturbative expansions are not adequate in deciding whether a model is nontrivial or not. Baker et al. showed that the ϕ^4 theory, although perturbatively nontrivial, went to a free theory as the cutoff was lifted in four dimensions [5, 6]. Continuing research is going on this subject [7]. Alternative methods become popular. RG methods which were first introduced by Wilson et al. [8], are the most commonly used one [9]. Another method is using exact RG (ERG) algorithm which were proposed by Polchinski [10]. Recent studies gave important insights on both methods [11–13].

Another endeavor is building a model of nature using only fermions. Here all the observed bosons are constructed as composites of these ingredient spinors. In solid state physics, electrons come together to form bosonic particles which is known as Bardeen - Cooper - Schrieffer (BCS) theory of superconductivity [14, 15]. This theory is honored with a Nobel Prize in 1972.

Historically, the first work on models with only spinors goes back to the work of Heisenberg. He spent years to formulate a "theory of everything" for particle physics, using only fermions [16]. Two years later Gürsey proposed his model as a substitute for the Heisenberg model [17]. This Gürsey's spinor model is important since it is conformally invariant classically and has classical solutions [18] which may be interpreted as instantons and merons [19], similar to the solutions of pure YM theories in four dimensions [20]. This original model can be generalized to include vector, pseudovector and pseudoscalar interactions.

There are also other four fermion interacting models. The Thirring model is one of them [21]. In particular, if ψ is a Dirac spinor field, the Lagrangian density is given by

$$\mathscr{L} = \overline{\psi}\,(i\partial\!\!\!/ - m)\,\psi - \frac{g}{2}\,(\overline{\psi}\gamma^{\mu}\psi)\,(\overline{\psi}\gamma_{\mu}\psi) \tag{1.1}$$

where g is the coupling constant, γ is the gamma matrix and m is the mass. Another one is the Gross-Neveu (GN) model which is a quantum field theoretical model of Dirac fermions interacting via four fermion interactions in two dimensions [22]. Here we have N Dirac fermions, ψ_1,..., ψ_N. The Lagrangian density is

$$\mathscr{L} = \overline{\psi}_a\,(i\partial\!\!\!/ - m)\,\psi^a + \frac{g}{2N}\,[\overline{\psi}_a\psi^a]^2 \tag{1.2}$$

where g is the coupling constant. If the mass m is nonzero, the model is massive. This model has an $U(N)$ internal symmetry. GN model is similar to NJL model except for the presence of chiral symmetry in the latter. In QFT, the NJL model is a theory of interacting Dirac fermions with chiral symmetry [23]. This model is constructed based on an analogy with the BCS theory of superconductivity. The Lagrangian density is given at the case with N flavors

$$\mathscr{L} = \overline{\psi}_a i\partial\!\!\!/ \psi^a + \frac{\lambda}{4N}\left[(\overline{\psi}_a\psi^b)(\overline{\psi}_b\psi^a) - (\overline{\psi}_a\gamma^5\psi^b)(\overline{\psi}_b\gamma^5\psi^a)\right] \tag{1.3}$$

where the flavor indices represented by the Latin letters a, b, c... Here chiral symmetry forbids a bare mass term.

During the last twenty years, many papers were written on making sense out of "trivial models", interpreting them as effective theories without taking the cutoff to infinity. One of these models is the NJL model. Although this model is shown to be a trivial in four dimensions [24–26], since the coupling constant goes to zero with a negative power of the logarithm of the ultraviolet cut-off, as an effective model in low energies it gives us important insight to several processes [27]. This model is sometimes used as a phenomenological model of quantum chromodynamics (QCD) in the chiral limit. In QCD, the studies of hadron mass generation through spontaneous symmetry breaking, important clues to results of the nuclear pairing interaction and the approximate validity of the interacting boson model can be cited as some examples.

There were also attempts to couple the NJL model to a gauge field, gNJL model [28, 29], to be able to get a non-trivial field theory. It was shown that if one has sufficient number of fermion flavors, such a construction is indeed possible [30]. Recent attempts to gauge this model to obtain a nontrivial theory are also given in references [31–35]. Both functional and diagram summing methods were used in these papers. ERG methods proposed by Wilson and Polchinski, [8, 10], are often employed for this purpose.

1.3 Hypothesis

With this motivation we want to give a new interpretation of the old work of Akdeniz et al. [36]. First we attempt to write the polynomial form of the original Gürsey model. We try to show the equivalency of these two models, at least classically. Then we attempt to quantize the equivalent model and study on some of the fundamental processes. We investigate whether this model is a trivial one or an interacting one.

We try to extend the model by coupling with vector fields for abelian and non-abelian cases. For both cases we seem to get different processes which are varying wildly from the latter one. These new models may give the indications of a nontrivial field theory under certain conditions.

For the analysis, we use perturbative and non-perturbative techniques.

2. THE MODEL

Gürsey proposed a four dimensional conformal invariant spinor model in mid-fifties [17]. This model is defined by the Lagrangian density

$$\mathscr{L}_G = \overline{\psi} i \partial\!\!\!/ \psi + g^{4/3} \left(\overline{\psi}\psi\right)^{4/3}. \qquad \textbf{(2.1)}$$

This is the only possible conformally invariant spinor model which contains no derivatives higher than the first. A class of exact solutions of this model was found by Kortel on the same year [18]. Later they are shown to be instanton and meron solutions by Akdeniz [19].

2.1 Equivalent Model

The Gürsey model, as it stands classically, does not make sense in the context of quantum theory because the composite operator $\left(\overline{\psi}\psi\right)^{4/3}$ does not exist in perturbation theory for the fermion field ψ. Therefore, a transformation is needed to turn it into an equivalent polynomial form. In their study, Akdeniz et al. inspired by the work of Gross-Neveu [22] and introduced auxiliary scalar fields to linearize the nonlinear spinor interaction [36]. This can be shown as follows:

$$\mathscr{L}_G = \overline{\psi} i \partial\!\!\!/ \psi + g\overline{\psi}\psi \left(g\overline{\psi}\psi\right)^{1/3}. \qquad \textbf{(2.2)}$$

By introducing

$$g\overline{\psi}\psi = a\phi^3, \qquad \textbf{(2.3)}$$

we can write an equivalent Lagrangian density as

$$\begin{aligned} \mathscr{L}_{eq} &= \overline{\psi} i \partial\!\!\!/ \psi + g a^{1/3} \overline{\psi}\psi\phi + a^{1/3}\lambda \left(g\overline{\psi}\psi - a\phi^3\right), && \textbf{(2.4)} \\ &= \overline{\psi} i \partial\!\!\!/ \psi + g a^{1/3} \overline{\psi}\psi\phi + \lambda \left(g a^{1/3}\overline{\psi}\psi - a^{4/3}\phi^3\right). && \textbf{(2.5)} \end{aligned}$$

Renaming the coupling constants $g^{'} = ga^{1/3}$ and $a^{'} = a^{4/3}$, we get

$$\mathscr{L}_{eq} = \overline{\psi} i \partial\!\!\!/ \psi + g^{'}\overline{\psi}\psi\phi + \lambda \left(g^{'}\overline{\psi}\psi - a^{'}\phi^3\right). \qquad \textbf{(2.6)}$$

From now on we will call our dimensionless coupling constants as g and a, instead of $g^{'}$ and $a^{'}$. Before ending this section we have to mention that at classic level one auxiliary field ϕ is enough to generate the nonlinear fermion interaction. But in quantum level, an additional auxiliary lagrange multiplier field λ is needed to impose the constraint on the model.

2.2 Equivalence of the Models

The equivalence of the models, namely this equivalent Lagrangian

$$\mathscr{L}_{eq} = \overline{\psi} i\partial\!\!\!/ \psi + g\overline{\psi}\psi\phi + \lambda\left(g\overline{\psi}\psi - a\phi^3\right), \tag{2.7}$$

to the original Gürsey Lagrangian equation given (**2.1**), at the classical level, can be seen with the help of the Euler-Lagrange equations for the ϕ and λ fields, which are constraint equations

$$g\overline{\psi}\psi - 3a\lambda\phi^2 = 0, \tag{2.8}$$

$$g\overline{\psi}\psi - a\phi^3 = 0. \tag{2.9}$$

They impose the following conditions classically,

$$\lambda = \frac{\phi}{3}, \tag{2.10}$$

$$\phi = \left(\frac{g}{a}\overline{\psi}\psi\right)^{1/3}. \tag{2.11}$$

For the proof we start from the equivalent Lagrangian density and try to show that it can be rewritten as the Gürsey Lagrangian density.

$$\begin{aligned}
\mathscr{L}_{eq} &= \overline{\psi} i\partial\!\!\!/ \psi + g\overline{\psi}\psi\phi + \lambda(g\overline{\psi}\psi - a\phi^3), && (2.12)\\
&= \overline{\psi} i\partial\!\!\!/ \psi + g\overline{\psi}\psi\phi + \frac{\phi}{3}(g\overline{\psi}\psi - a\phi^3), && (2.13)\\
&= \overline{\psi} i\partial\!\!\!/ \psi + \frac{4g}{3}\overline{\psi}\psi\phi - \frac{a}{3}\phi^4, && (2.14)\\
&= \overline{\psi} i\partial\!\!\!/ \psi + \frac{4g}{3}\overline{\psi}\psi\left(\frac{g}{a}\overline{\psi}\psi\right)^{1/3} - \frac{a}{3}\left(\frac{g}{a}\overline{\psi}\psi\right)^{4/3}, && (2.15)\\
&= \overline{\psi} i\partial\!\!\!/ \psi + g\left(\frac{g}{a}\right)^{1/3}(\overline{\psi}\psi)^{4/3}. && (2.16)
\end{aligned}$$

Recalling the redefinition of the coupling constants which are given above as $g = ga^{1/3}$ and $a = a^{4/3}$, we get the Gürsey Lagrangian denstity

$$\mathscr{L}_{eq} = \overline{\psi} i \partial\!\!\!/ \psi + g^{4/3} \left(\overline{\psi} \psi \right)^{4/3}. \qquad (\mathbf{2.17})$$

At least classically, this shows us that both Lagrangian densities can be treated as they are equal.

2.3 γ^5 Symmetry

In the previous section we claim the equivalency of the systems. If so, they should obey the same symmetries. In this section we will check a discrete symmetry. We know that under the γ^5 symmetry the fields transform as

$$\begin{aligned} \psi &\rightarrow \gamma^5 \psi, \\ \overline{\psi} &\rightarrow -\overline{\psi} \gamma^5, \\ \overline{\psi} i \partial\!\!\!/ \psi &\rightarrow \overline{\psi} i \partial\!\!\!/ \psi, \\ \overline{\psi} \psi &\rightarrow -\overline{\psi} \psi. \end{aligned}$$

We see that the γ^5 invariance of the Gürsey Lagrangian equation (**2.1**) is retained in the equivalent Lagrangian written in equation (**2.7**). In this polynomial Lagrangian form, when ψ is sent to $\gamma^5 \psi$, the scalar fields ϕ and λ are sent to their negatives

$$\begin{aligned} \phi &\rightarrow -\phi, \\ \lambda &\rightarrow -\lambda. \end{aligned}$$

This discrete symmetry prevents ψ from acquiring a finite mass in higher orders.

2.4 Constraint Analysis

To quantize the system consistently we proceed through the path integral method. Since we introduced two auxiliary fields to turn Lagrangian into an equivalent polynomial form naively, we end up in a constrained system. The only non-trivial part in the quantization of constrained system is the calculation of the Faddeev-Popov determinant [37, 38]. Due to the recipe given by P.Senjanovic, originally it is given by Dirac [39], we start the analysis with the Lagrangian density

$$\mathscr{L} = \overline{\psi} i \partial\!\!\!/ \psi + g \overline{\psi} \psi \phi + \lambda (g \overline{\psi} \psi - a \phi^3). \qquad (\mathbf{2.18})$$

The conjugate momenta are

$$\pi^{\bar{\psi}} = \frac{\partial \mathscr{L}}{\partial(\partial_0 \overline{\psi})} = 0, \tag{2.19}$$

$$\pi^{\psi} = \frac{\partial \mathscr{L}}{\partial(\partial_0 \psi)} = i\,\overline{\psi}\gamma^0, \tag{2.20}$$

$$\pi^{\phi} = \frac{\partial \mathscr{L}}{\partial(\partial_0 \phi)} = 0, \tag{2.21}$$

$$\pi^{\lambda} = \frac{\partial \mathscr{L}}{\partial(\partial_0 \lambda)} = 0. \tag{2.22}$$

The canonical Hamiltonian density is

$$\mathscr{H}_C = i\,\overline{\psi}\gamma^i \partial_i \psi - g\overline{\psi}\psi\phi - \lambda\left(g\overline{\psi}\psi - a\phi^3\right). \tag{2.23}$$

This canonical Hamiltonian generates the wrong Hamiltonian equations of motion. Namely, it generates the time derivative of the ϕ and λ fields as zero due to the absence of the π^{ϕ} and π^{λ}. Therefore we need to define the true Hamiltonian density. Before introducing that Hamiltonian density, we want to remark the constraints in the model. Basically we have four primary constraints as

$$\varphi_1 = \pi^{\bar{\psi}}, \tag{2.24}$$

$$\varphi_2 = \pi^{\psi} - i\,\overline{\psi}\gamma^0, \tag{2.25}$$

$$\varphi_3 = \pi^{\phi}, \tag{2.26}$$

$$\varphi_4 = \pi^{\lambda}. \tag{2.27}$$

We add the primary constraints and define the new Hamiltonian density. We name it as total Hamiltonian density.

$$\mathscr{H}_T = \mathscr{H}_C + u_m \varphi_m(q,p), \tag{2.28}$$

$$= i\,\overline{\psi}\gamma^i \partial_i \psi - g\overline{\psi}\psi\phi - \lambda(g\overline{\psi}\psi - a\phi^3) + u_1 \pi^{\bar{\psi}} + (\pi^{\psi} - i\,\overline{\psi}\gamma^0)u_2$$
$$+ u_3 \pi^{\phi} + u_4 \pi^{\lambda}. \tag{2.29}$$

where u_1 and u_2 are four component spinor coefficients and u_3 and u_4 are scalar coefficients. The constraints that should be consistent with this condition, can be given by

$$\dot{\varphi}_n = \{\varphi_n, \int \mathscr{H}_T\} = 0. \tag{2.30}$$

Here, we use the Poisson brackets convention

$$\{A,B\} = \frac{\partial A}{\partial q_i}\frac{\partial B}{\partial p^i} - \frac{\partial A}{\partial p^i}\frac{\partial B}{\partial q_i}. \tag{2.31}$$

In our model we obtain

$$\begin{aligned} \dot{\varphi}_1 &= \{\pi^{\bar{\psi}}, \int \mathscr{H}_T\}, \\ &= -1\Big(i\,\gamma^i\partial_i\psi - g(\phi+\lambda)\psi - i\,\gamma^0 u_2\Big). \end{aligned} \tag{2.32}$$

The consistency condition can be satisfied if we choose the spinor coefficient such as

$$i\,\gamma^0 u_2 = \big[i\,\gamma^i\partial_i - g(\phi+\lambda)\big]\,\psi. \tag{2.33}$$

Next

$$\begin{aligned} \dot{\varphi}_2 &= \{\pi^{\psi} - i\,\overline{\psi}\gamma^0, \int \mathscr{H}_T\} \\ &= -i\,u_1\gamma^0 - 1\Big(-i\,\partial_i\overline{\psi}\gamma^i - g\overline{\psi}(\phi+\lambda)\Big). \end{aligned} \tag{2.34}$$

The consistency of the second constrainted can be satisfied by choosing

$$-i\,u_1\gamma^0 = \overline{\psi}\big[-i\,\gamma^i\overleftarrow{\partial_i} - g(\phi+\lambda)\big]. \tag{2.35}$$

But the third and fourth primary constraints produce new constraints. Such as

$$\begin{aligned} \dot{\varphi}_3 &= \{\pi^{\phi}, \int \mathscr{H}_T\}, \\ &= g\overline{\psi}\psi - 3a\lambda\phi^2, \end{aligned} \tag{2.36}$$

$$\begin{aligned} \dot{\varphi}_4 &= \{\pi^{\lambda}, \int \mathscr{H}_T\}, \\ &= g\overline{\psi}\psi - a\phi^3. \end{aligned} \tag{2.37}$$

We call them as secondary constraints. Let

$$\varphi_5 = g\overline{\psi}\psi - 3a\lambda\phi^2, \tag{2.38}$$

$$\varphi_6 = g\overline{\psi}\psi - a\phi^3. \tag{2.39}$$

Their consistency condition gives

$$\begin{aligned} \dot{\varphi}_5 &= \{g\overline{\psi}\psi - 3a\lambda\phi^2, \int \mathscr{H}_T\}, \\ &= u_1(g\psi) + (g\overline{\psi})u_2 - 6a\lambda\phi u_3 - 3a\phi^2 u_4, \end{aligned} \tag{2.40}$$

$$\begin{aligned} \dot{\varphi}_6 &= \{g\overline{\psi}\psi - 3a\phi^3, \int \mathscr{H}_T\}, \\ &= u_1(g\psi) + (g\overline{\psi})u_2 - 3a\phi^2 u_3. \end{aligned} \tag{2.41}$$

which can be satisfied if

$$u_3 = \frac{u_1(g\psi)+(g\overline{\psi})u_2}{3a\phi^2}, \quad \textbf{(2.42)}$$

$$u_4 = \frac{u_1(g\psi)+(g\overline{\psi})u_2}{3a\phi^2}\left(1-\frac{6a\lambda\phi}{3a\phi^2}\right). \quad \textbf{(2.43)}$$

We can find all the coefficients. After some algebra we can give the exact solution of them as

$$u_1 = -i\,\overline{\psi}\left[i\,\gamma^i\overleftarrow{\partial_i}+g(\phi+\lambda)\right]\gamma^0, \quad \textbf{(2.44)}$$

$$u_2 = i\,\gamma^0\left[-i\,\gamma^i\overrightarrow{\partial_i}+g(\phi+\lambda)\right]\psi, \quad \textbf{(2.45)}$$

$$u_3 = \frac{-i\,g}{3a\phi^2}\overline{\psi}\left[i\,\gamma^i\gamma^0\overleftarrow{\partial_i}+i\,\gamma^0\gamma^i\overrightarrow{\partial_i}\right]\psi, \quad \textbf{(2.46)}$$

$$u_4 = \frac{-i\,g}{3a\phi^2}\left(1-\frac{6a\lambda\phi}{3a\phi^2}\right)\overline{\psi}\left[i\,\gamma^i\gamma^0\overleftarrow{\partial_i}+i\,\gamma^0\gamma^i\overrightarrow{\partial_i}\right]\psi. \quad \textbf{(2.47)}$$

Then we can write the Hamiltonian density such as

$$\begin{aligned}\mathscr{H}_T = & -i\,\overline{\psi}\left[i\,\gamma^i\overleftarrow{\partial_i}+g(\phi+\lambda)\right]\gamma^0\pi^{\bar{\psi}} \\ & +\pi^{\psi}i\,\gamma^0\left[-i\,\gamma^i\overrightarrow{\partial_i}+g(\phi+\lambda)\right]\psi \\ & +\pi^{\phi}\frac{-i\,g}{3a\phi^2}\overline{\psi}\left[i\,\gamma^i\gamma^0\overleftarrow{\partial_i}+i\,\gamma^0\gamma^i\overrightarrow{\partial_i}\right]\psi \\ & +\pi^{\lambda}\frac{-i\,g}{3a\phi^2}\left(1-\frac{6a\lambda\phi}{3a\phi^2}\right)\overline{\psi}\left[i\,\gamma^i\gamma^0\overleftarrow{\partial_i}+i\,\gamma^0\gamma^i\overrightarrow{\partial_i}\right]\psi \\ & +a\lambda\phi^3.\end{aligned} \quad \textbf{(2.48)}$$

This Hamiltonian density can be derived in a classical sense without using the P.Senjanovic's recipe. That derivation is given in the Appendix A. Although in this thesis Hamiltonian systems are out of our scope, we want to give briefly the right Hamilton equations of motion. They are produced by

$$\dot{q}_i = \{q_i,\int\mathscr{H}_T\} = \frac{\partial H_T}{\partial p^i}, \quad \textbf{(2.49)}$$

$$\dot{p}^i = \{p^i,\int\mathscr{H}_T\} = -\frac{\partial H_T}{\partial q_i}. \quad \textbf{(2.50)}$$

They are

$$\dot{\overline{\psi}} = -i\,\overline{\psi}\left[i\,\gamma^i\overleftarrow{\partial_i} + g(\phi+\lambda)\right]\gamma^0, \tag{2.51}$$

$$\dot{\psi} = i\,\gamma^0\left[-i\,\gamma^i\overrightarrow{\partial_i} + g(\phi+\lambda)\right]\psi, \tag{2.52}$$

$$\dot{\phi} = \frac{-i\,g}{3a\phi^2}\overline{\psi}\left[i\,\gamma^i\gamma^0\overleftarrow{\partial_i} + i\,\gamma^0\gamma^i\overrightarrow{\partial_i}\right]\psi, \tag{2.53}$$

$$\dot{\lambda} = \frac{-i\,g}{3a\phi^2}\left(1-\frac{6a\lambda\phi}{3a\phi^2}\right)\overline{\psi}\left[i\,\gamma^i\gamma^0\overleftarrow{\partial_i} + i\,\gamma^0\gamma^i\overrightarrow{\partial_i}\right]\psi, \tag{2.54}$$

$$\begin{aligned}\dot{\pi}^{\overline{\psi}} &= i\left[-i\,\gamma^i\overrightarrow{\partial_i} + g(\phi+\lambda)\right]\gamma^0\pi^{\overline{\psi}} \\ &\quad + \left(\frac{i\,g}{3a\phi^2}\right)\left[\pi^\phi + \pi^\lambda\left(1-\frac{6a\lambda\phi}{3a\phi^2}\right)\right]\left[2i\,\gamma^0\gamma^i\overrightarrow{\partial_i}\right]\psi,\end{aligned} \tag{2.55}$$

$$\begin{aligned}\dot{\pi}^{\psi} &= -\pi^\psi i\,\gamma^0\left[i\,\gamma^i\overleftarrow{\partial_i} + g(\phi+\lambda)\right] \\ &\quad + \left(\frac{i\,g}{3a\phi^2}\right)\left[\pi^\phi + \pi^\lambda\left(1-\frac{6a\lambda\phi}{3a\phi^2}\right)\right]\overline{\psi}\left[2i\,\gamma^i\gamma^0\overleftarrow{\partial_i}\right],\end{aligned} \tag{2.56}$$

$$\begin{aligned}\dot{\pi}^{\phi} &= ig\left(\overline{\psi}\gamma^0\pi^{\overline{\psi}} - \pi^\psi\gamma^0\psi\right) - 3a\lambda\phi^2 \\ &\quad - \frac{2ig}{3a\phi^4}\left[\phi\pi^\phi + (\phi-3\lambda)\pi^\lambda\right]\overline{\psi}\left[i\,\gamma^i\gamma^0\overleftarrow{\partial_i} + i\,\gamma^0\gamma^i\overrightarrow{\partial_i}\right]\psi,\end{aligned} \tag{2.57}$$

$$\dot{\pi}^{\lambda} = ig\left(\overline{\psi}\gamma^0\pi^{\overline{\psi}} - \pi^\psi\gamma^0\psi\right) - \frac{2ig\phi}{3a\phi^4}\pi^\lambda\overline{\psi}\left[i\,\gamma^i\gamma^0\overleftarrow{\partial_i} + i\,\gamma^0\gamma^i\overrightarrow{\partial_i}\right]\psi. \tag{2.58}$$

We find that there are six constraints in our model. Four of them are primary, two of them are secondary constraints. Years ago Dirac showed that these constraints can be classified into two classes [39,40]. There he defined a function $R(q,p)$ as a *first class quantity* if

$$\{R,\varphi_a\} \approx 0, \quad a = 1,\dots,T. \tag{2.59}$$

$R(q,p)$ as a *second class quantity* if

$$\{R,\varphi_a\} \not\approx 0, \quad a = 1,\dots,T. \tag{2.60}$$

for at least one a.

In our model we find the non zero poisson brackets among all constraints as follow:

$$\{\varphi_1,\varphi_2\} = i\gamma^0, \quad (\mathbf{2.61})$$

$$\{\varphi_1,\varphi_5\} = -g\psi, \quad (\mathbf{2.62})$$

$$\{\varphi_1,\varphi_6\} = -g\psi, \quad (\mathbf{2.63})$$

$$\{\varphi_2,\varphi_5\} = -g\overline{\psi}, \quad (\mathbf{2.64})$$

$$\{\varphi_2,\varphi_6\} = -g\overline{\psi}, \quad (\mathbf{2.65})$$

$$\{\varphi_3,\varphi_5\} = 6a\lambda\phi, \quad (\mathbf{2.66})$$

$$\{\varphi_3,\varphi_6\} = 3a\phi^2, \quad (\mathbf{2.67})$$

$$\{\varphi_4,\varphi_5\} = 3a\phi^2. \quad (\mathbf{2.68})$$

All our constraints are *second class constraints*. Dirac has proven that the second class constraints will give rise to a nonsingular $N \times N$ matrix of Poisson brackets which we write [39]

$$C_{\alpha\beta} = \{\varphi_\alpha, \varphi_\beta\}. \quad (\mathbf{2.69})$$

Next we find the matrix.

$$C_{\alpha\beta} = \begin{bmatrix} 0 & i\gamma^0 & 0 & 0 & -g\psi & -g\psi \\ -i\gamma^0 & 0 & 0 & 0 & -g\overline{\psi} & -g\overline{\psi} \\ 0 & 0 & 0 & 0 & 6a\lambda\phi & 3a\phi^2 \\ 0 & 0 & 0 & 0 & 3a\phi^2 & 0 \\ g\psi & g\overline{\psi} & -6a\lambda\phi & -3a\phi^2 & 0 & 0 \\ g\psi & g\overline{\psi} & -3a\phi^2 & 0 & 0 & 0 \end{bmatrix}.$$

The Faddeev-Popov determinant is defined by the square root of the determinant of the matrix $C_{\alpha\beta}$. In our model we find that the spinor-Dirac constraints, resulting from the canonical momenta of the spinor fields have no field dependent contribution to the Faddeev-Popov determinant. This determinant is given as

$$\triangle_F = |det\{\varphi_\alpha, \varphi_\beta\}|^{1/2} = det\left(9a^2\phi^4\right). \quad (\mathbf{2.70})$$

Here we have omitted the delta functions in space-time. Thus, up to an irrelevant constant factor, we obtained the field dependent contribution coming from the constraints in equations (**2.8**) and (**2.9**).

2.5 Functional Integral Quantization with Second Class Constraints

To quantize the system consistently we proceed via the path integral method. Using the Senjanovic's formula, we can write the path integral transition amplitude as

$$< \text{out} \mid S \mid \text{in} > \equiv U = \int \mathscr{D}\Pi \mathscr{D}\chi \delta(\varphi_i)\Delta_F \exp\left[i\int (\Pi\dot{\chi} - \mathscr{H}_c) d^4x \right]. \qquad \textbf{(2.71)}$$

Here χ is the generic symbol for all the fields, Π is the generic symbol for all momenta and φ is the generic symbol for all the constraints in the model. Explicitly we can write the path integral amplitude as

$$\begin{aligned} U &= \int \mathscr{D}\pi^{\psi}\mathscr{D}\pi^{\bar{\psi}}\mathscr{D}\pi^{\phi}\mathscr{D}\pi^{\lambda}\mathscr{D}\bar{\psi}\mathscr{D}\psi\mathscr{D}\phi\mathscr{D}\lambda \det\left(9a^2\phi^4\right) \\ &\quad \times\delta(\pi^{\bar{\psi}})\delta(\pi^{\psi} - i\overline{\psi}\gamma^0)\delta(\pi^{\phi})\delta(\pi^{\lambda})\delta(g\overline{\psi}\psi - 3a\lambda\phi^2)\delta(g\overline{\psi}\psi - a\phi^3) \\ &\quad \times e^{i\int d^4x\left(\pi^{\psi}\partial_0\psi + \partial_0\overline{\psi}\pi^{\bar{\psi}} + \pi^{\lambda}\partial_0\lambda + \pi^{\phi}\partial_0\phi - i\,\overline{\psi}\gamma^i\partial_i\psi + g\overline{\psi}\psi\phi + \lambda\left(g\overline{\psi}\psi - a\phi^3\right)\right)}. \end{aligned} \qquad \textbf{(2.72)}$$

Performing all the momenta integrals we obtain

$$\begin{aligned} U &= \int \mathscr{D}\bar{\psi}\mathscr{D}\psi\mathscr{D}\phi\mathscr{D}\lambda \det\left(9a^2\phi^4\right)\delta(g\overline{\psi}\psi - 3a\lambda\phi^2)\delta(g\overline{\psi}\psi - a\phi^3) \\ &\quad \times \exp\left[i\int d^4x \left(\overline{\psi} i\partial\!\!\!/\psi + g\overline{\psi}\psi\phi + \lambda(g\overline{\psi}\psi - a\phi^3) \right) \right]. \end{aligned} \qquad \textbf{(2.73)}$$

From the evaluation of λ integral we get a factor,

$$\begin{aligned} U &= \int \mathscr{D}\bar{\psi}\mathscr{D}\psi\mathscr{D}\phi \frac{\det\left(9a^2\phi^4\right)}{\det\left(3a\phi^2\right)}\delta(g\overline{\psi}\psi - a\phi^3) \\ &\quad \times \exp\left[i\int d^4x \left(\overline{\psi} i\partial\!\!\!/\psi + g\overline{\psi}\psi\phi \right) \right], \qquad \textbf{(2.74)} \\ &= \int \mathscr{D}\bar{\psi}\mathscr{D}\psi\mathscr{D}\phi\mathscr{D}\lambda \det\left(3a\phi^2\right) \\ &\quad \times \exp\left[i\int d^4x \left(\overline{\psi} i\partial\!\!\!/\psi + g\overline{\psi}\psi\phi + \lambda(g\overline{\psi}\psi - a\phi^3) \right) \right]. \qquad \textbf{(2.75)} \end{aligned}$$

Here we raised the delta function by introducing λ field. On the other hand there is one more contribution left in the functional integral. This contribution can be inserted into the Lagrangian by using ghost fields c and c^*

$$\det\left(3a\phi^2\right) = \int \mathscr{D}c^*\mathscr{D}c \ \exp\left[-\int c^*\left(3a\phi^2\right) c \ d^4x \right]. \qquad \textbf{(2.76)}$$

Hence, we find the path integral amplitude as

$$U = \int \mathscr{D}\bar{\psi}\mathscr{D}\psi\mathscr{D}\phi\mathscr{D}\lambda\mathscr{D}c^{*}\mathscr{D}c \cdot e^{i\int d^4x\left[\overline{\psi}i\partial\!\!\!/\psi + g\overline{\psi}\psi\phi + \lambda(g\overline{\psi}\psi - a\phi^3) + ic^{*}\left(3a\phi^2\right)c\right]}. \quad \textbf{(2.77)}$$

Since

$$U = \int \mathscr{D}\chi \cdot e^{iS}. \quad \textbf{(2.78)}$$

The resulting action reads

$$S = \int d^4x \left[\overline{\psi}i\partial\!\!\!/\psi + g\overline{\psi}\psi(\phi+\lambda) - a\lambda\phi^3 + 3ia\left(c^{*}\phi^2 c\right)\right] \quad \textbf{(2.79)}$$

Note that the spinor field couples to λ and ϕ fields in the same manner by the same coupling constant g. We can rewrite the action by redefining the fields

$$\Phi = \phi + \lambda, \quad \textbf{(2.80)}$$

$$\Lambda = \phi - \lambda. \quad \textbf{(2.81)}$$

This redefinition changes the transition amplitude with some awkward phase, but it does not mean anything physically. The action becomes

$$S = \int d^4x \left[\overline{\psi}i\partial\!\!\!/\psi + g\overline{\psi}\psi\Phi - \frac{a}{16}(\Phi^2 - \Lambda^2)(\Phi+\Lambda)^2 + \frac{3ia}{4}c^{*}\left(\Phi+\Lambda\right)^2 c\right]. \quad \textbf{(2.82)}$$

Note that by this transformation the Λ field is decoupled from the spinor sector of the Lagrangian.

2.6 Perturbation Expansion of Correlation Functions

Our model consists spinor and composite scalar fields as given in action. To understand the features of the model, we have to derive how these fields propagate or interact with each others. In other words in the following subsections we will find their correlation functions

$$< \Omega \mid T\{\chi_1(x_1)\cdots\chi_N(x_N)\} \mid \Omega > . \quad \textbf{(2.83)}$$

Here $\mid \Omega >$ denotes the ground state where T means "time ordered operator" corresponds to Wick's theorem.

2.6.1 Fermion propagator

The fermion propagator is the usual Dirac propagator in lowest order, as can be seen from the Lagrangian.

$$\frac{i\not{p}}{p^2+i\varepsilon} \tag{2.84}$$

Remark that the propagator is massless.

2.6.2 Composite scalar propagator

The model does not consist a kinetic term for the scalar fields. It can be induced dynamically. We start with performing the gaussian integration over the spinor fields in the functional.

$$\int \mathscr{D}\overline{\psi}\mathscr{D}\psi \;\; \exp\left[i\int d^4x\left(\overline{\psi}\left[i\not{\partial}+g\Phi\right]\psi\right)\right] = \det\left(i\not{\partial}+g\Phi\right). \tag{2.85}$$

Here we can the standard identity from linear algebra. That is, a matrix B which has eigenvalues b_i, can be written as

$$\det B = \prod_i b_i = \exp\left[\sum_i \log b_i\right] = \exp\left[Tr(\log B)\right] \tag{2.86}$$

where the logarithm of a matrix is defined by its power series. We use "Tr" to denote operator trace, while later we will use "tr" to denote Dirac traces. Applying this identity the path integral amplitude gets the final form such as

$$U = \int \mathscr{D}\Phi\mathscr{D}\Lambda\mathscr{D}c^*\mathscr{D}c \;\; e^{Trln(i\not{\partial}+g\Phi)-i\int d^4x\left[\frac{a}{16}\left[\Phi^4+2\Phi\Lambda\left(\Phi^2-\Lambda^2\right)-\Lambda^4\right]+\frac{3ia}{4}c^*(\Phi+\Lambda)^2c\right]} \tag{2.87}$$

This yields the action which is expressed in terms of Φ, Λ and c, c^* fields only. We name it as "effective action".

$$\begin{aligned} S_e &= -i\mathrm{Tr}ln\left(i\not{\partial}+g\Phi\right) \\ &\quad -\int d^4x\left[\frac{a}{16}\left[\Phi^4+2\Phi\Lambda\left(\Phi^2-\Lambda^2\right)-\Lambda^4\right]+\frac{3ia}{4}c^*\left(\Phi+\Lambda\right)^2c\right]. \end{aligned} \tag{2.88}$$

But we can not perform Gaussian functional integrals because of the higher order terms in the effective action. Therefore we need an approximation. Here we use saddle point approximation.

According to this approximation, up to the second order the effective action can be written as

$$
\begin{aligned}
S_e[\chi_i] \;\; &\simeq \;\; S_e[\chi_{i0}] + \left\langle (\chi_i - \chi_{i0}) \frac{\partial S_e}{\partial \chi_i}\bigg|_{\chi_i = \chi_{i0}} \right\rangle \\
&\quad + \frac{1}{2}\left\langle (\chi_i - \chi_{i0}) \frac{\partial^2 S_e}{\partial \chi_i \partial \chi_j}\bigg|_{\chi_{i,j} = \chi_{i,j0}} (\chi_j - \chi_{j0}) \right\rangle. \qquad \textbf{(2.89)}
\end{aligned}
$$

The vacuum expectation values of the fields Φ and Λ will be expressed as $-v$ and s respectively while they are set to zero for the ghost fields. The tadpole contributions are the first derivative of the effective action with respect to the Φ and Λ fields and should be killed by setting them to zero. This gives

$$
\begin{aligned}
Tr\int \frac{d^4p}{(2\pi)^4} \frac{(-ig)}{\not{p} - gv} \;\; &- \;\; \frac{a}{8}(-2v^3 + 3v^2 s - s^3) = 0, \qquad \textbf{(2.90)} \\
&- \;\; \frac{a}{8}(-v^3 + 3vs^2 - 2s^3) = 0. \qquad \textbf{(2.91)}
\end{aligned}
$$

From now on we will use the short notation $\int_p$ for $\int \frac{d^4p}{(2\pi)^4}$ A consistent solution satisfying both equations is

$$
s = v = 0, \qquad \textbf{(2.92)}
$$

which sets the vacuum expectation value of both fields to zero. In this symmetric phase, the γ^5 symmetry is not dynamically broken and no mass is generated for the fermion dynamically. In this respect this model differs from the famous GN model [22], where this dynamical breaking takes place. It also differs from the NJL model [23]. In those models in a broken phase, mass is induced for the fermion due to the existence of a cutoff function. In our model because of the conformal invariance we do not get the same behavior. This can be explained in the Gürsey's original intention in constructing a conformal invariant model, at least classically. As a conclusion we find that upon quantization of our approximate model at least one phase exists which respects the γ^5 symmetry.

The second derivative of the effective action with respect to the Φ field gives us the induced inverse propagator for the Φ field.

$$
\begin{aligned}
{D_\Phi}^{-1}(q) \;\; &= \;\; i\frac{\partial^2 S_e}{\partial \Phi \partial \Phi}\bigg|_{\Phi=0}, \qquad \textbf{(2.93)} \\
&= \;\; (-ig)^2 \mathrm{Tr}\int_p \frac{1}{\not{p}(\not{p} + \not{q})}. \qquad \textbf{(2.94)}
\end{aligned}
$$

Diagrammatically it can be expressed as in figure 2.1.

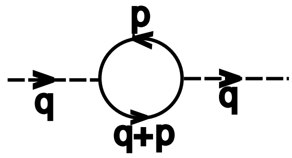


Figure 2.1. The induced composite scalar field propagator.

Before giving the detailed calculation, remark that in this thesis we use the dimensional regularization techniques and study in $D = 4 - \varepsilon$ dimension in Minkowski space.

The induced inverse propagator can be written as

$$= -g^2 \int_p \frac{1}{p^2} \frac{1}{(p+q)^2} \mathrm{Tr}\left[\not{p}\,(\not{p}+\not{q})\right] \quad \textbf{(2.95)}$$

After Feynman parametrization, using the equation (**C.12**), we find

$$= -g^2 \frac{\Gamma(2)}{\Gamma(1)^2} \int_0^1 d\alpha \int_p \frac{1}{[(p+\alpha q)^2 + \alpha(1-\alpha)q^2]^2} \mathrm{Tr}\left[\not{p}\,(\not{p}+\not{q})\right] \quad \textbf{(2.96)}$$

we shift the fields by $P = p + \alpha q$, then

$$= -g^2 \frac{\Gamma(2)}{\Gamma(1)^2} \int_0^1 d\alpha \int_P \frac{1}{[P^2 + \alpha(1-\alpha)q^2]^2} \mathrm{Tr}\left[(\not{P} - \alpha\not{q})\,(\not{P} + (1-\alpha)\not{q})\right], \quad \textbf{(2.97)}$$

$$= -g^2 \frac{\Gamma(2)}{\Gamma(1)^2} \int_0^1 d\alpha \int_P \frac{1}{[P^2 + \alpha(1-\alpha)q^2]^2} \mathrm{Tr}\left[\not{P}\not{P} - \alpha(1-\alpha)\not{q}\not{q}\right]. \quad \textbf{(2.98)}$$

We dropped writing the linear term proportional to $\not{P}$, because it is odd in $\not{P}$ momenta which integrates to zero. Using the definition given in equation (**C.10**), we get

$$= -Dg^2 \frac{\Gamma(2)}{\Gamma(1)^2} \int_0^1 d\alpha \int_P \frac{1}{[P^2 + \alpha(1-\alpha)q^2]^2} \left[P^2 - \alpha(1-\alpha)q^2\right] \quad \textbf{(2.99)}$$

$$= -Dg^2 \frac{\Gamma(2)}{\Gamma(1)^2} \int_0^1 d\alpha \int_P \left[\frac{1}{P^2 + \alpha(1-\alpha)q^2} - \frac{2\alpha(1-\alpha)q^2}{(P^2 + \alpha(1-\alpha)q^2)^2}\right] \textbf{(2.100)}$$

We evaluate the integrals due to the equations given in (**C.18**), we find

$$
\begin{aligned}
&= -Dg^2 \frac{\Gamma(2)}{\Gamma(1)^2} \int_0^1 d\alpha \left[\frac{-i}{(4\pi)^{D/2}} \frac{\Gamma\left(1-\frac{D}{2}\right)}{\Gamma(1)} \frac{1}{[\alpha(\alpha-1)q^2]^{1-D/2}} \right. \\
&\quad \left. - \frac{i}{(4\pi)^{D/2}} \frac{\Gamma\left(2-\frac{D}{2}\right)}{\Gamma(2)} \frac{2\alpha(1-\alpha)q^2}{[\alpha(\alpha-1)q^2]^{2-D/2}} \right], \qquad (2.101) \\
&= -i\frac{Dg^2}{(4\pi)^2} \left[\Gamma\left(1-\frac{D}{2}\right) - 2\Gamma\left(2-\frac{D}{2}\right) \right] \\
&\quad \times \int_0^1 d\alpha\alpha(1-\alpha)q^2 \left(\frac{4\pi}{\alpha(1-\alpha)q^2} \right)^{2-D/2}, \qquad (2.102) \\
&= -i\frac{(4-\varepsilon)g^2}{(4\pi)^2} \left[\Gamma\left(-1+\frac{\varepsilon}{2}\right) - 2\Gamma\left(\frac{\varepsilon}{2}\right) \right] \\
&\quad \times \int_0^1 d\alpha\alpha(1-\alpha)q^2 \left(\frac{4\pi}{\alpha(1-\alpha)q^2} \right)^{\varepsilon/2}, \qquad (2.103) \\
&= -i\frac{(4-\varepsilon)g^2}{(4\pi)^2} \left[-\frac{2}{\varepsilon} + (\gamma-1) - \frac{4}{\varepsilon} + 2\gamma + O(\varepsilon) \right] \\
&\quad \times \int_0^1 d\alpha\alpha(1-\alpha) \left(1 - \frac{\varepsilon}{2} \ln \frac{\alpha(1-\alpha)q^2}{4\pi} \right) + \ldots, \qquad (2.104) \\
&= -i\frac{(4-\varepsilon)g^2q^2}{(4\pi)^2} \left[-\frac{6}{\varepsilon} + (3\gamma-1) \right] \\
&\quad \times \left(\frac{1}{6} - \int_0^1 d\alpha\alpha(1-\alpha) \frac{\varepsilon}{2} \ln \frac{\alpha(1-\alpha)q^2}{4\pi} \right), \qquad (2.105) \\
&= i\frac{4g^2q^2}{(4\pi)^2} \left[\frac{1}{\varepsilon} - \frac{6\gamma+1}{12} - 3\int_0^1 d\alpha\alpha(1-\alpha) \ln \frac{\alpha(1-\alpha)q^2}{4\pi} \right]. \qquad (2.106)
\end{aligned}
$$

The divergent part gives us the induced composite scalar propagator as

$$
-i\frac{4\pi^2}{g^2} \frac{\varepsilon}{q^2} \qquad (2.107)
$$

Here we obtained a massless composite scalar field propagator. This is the crucial point in our work, because it carries a ε factor with itself. Later we will see that, as ε goes to zero, many of diagrams where composite operator takes part as an internal line, becomes convergent.

2.6.3 The other propagators

There is no other propagator in the model, since the linear or quadratic terms in Λ do not exist in the Lagrangian. In other words, because of the decoupling of the Λ field to the fermions, one loop contribution is absent. Similarly the mixed derivatives of the action may lead a mixed composite scalar propagator. But in one loop, they result to zero. This means that there is no mixed composite scalar field propagation in the model. For the ghost fields, we can set the propagators to zero, since they give no contribution in the one loop approximation similarly. The higher loop contributions are absent for all these fields.

2.6.4 Interactions

For the interactions in the model, we can express two different correlation functions. One of them is the three point correlation function which exists between the fermions and the composite scalar field Φ. The other one is four point correlation function of the composite scalar fields which means the self interaction among them. We give the Feynman rules of the model in Appendix (C.1.1). They will be necessary in the following sections.

2.7 $1/N$ Expansion

Before going on with the analysis, we must clarify one point. If our fermion field had a color index i where $i = 1...N$, we could perform an $1/N$ expansion to justify the use of only ladder diagrams for higher orders for the scattering processes. Although in our model the spinor has only one color, we still consider only ladder diagrams anticipating that one can construct a variation of the model with N colors.

2.8 Dressed Fermion Propagator

In this section we want to justify that no mass is generated for the fermion fields in higher orders. Using different techniques, Schwinger [41] and Dyson [42] have derived independently integral equations for Green functions as a consequence of the field equations. When properly renormalized, the Dyson-Schwinger equations may be used as an alternative approach to perturbation theory [43].

2.8.1 Dyson-Schwinger equation of the spinor field propagator

The Dyson-Schwinger equation for the spinor field propagator is shown in figure 2.2. There, a thin solid line corresponds to the free fermion propagator while a bold line corresponds to the full fermion propagator. We use the one loop result for the scalar propagator found in equation (**3.42**).

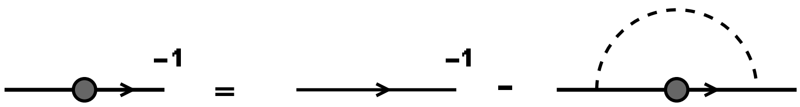


Figure 2.2. Graphical representation of the Schwinger-Dyson equation for fermion propagator.

We can express this representation as

$$-i\left[A(p^2)\not{p}+B(p^2)\right] = -i\not{p} + (-ig)^2 \int d^4q \frac{4\pi^2}{g^2}\frac{-i\varepsilon}{(p-q)^2}\frac{i}{[A(q^2)\not{q}+B(q^2)]}. \quad \textbf{(2.108)}$$

Here $-i\left[A(p^2)\not{p}+B(p^2)\right]$ is the dressed fermion propagator. To find the functions $A(p^2)$ and $B(p^2)$ we first rationalize the denominator,

$$-i\left[A(p^2)\not{p}+B(p^2)\right] = -i\not{p} - 4\pi^2 \int d^4q \frac{\varepsilon}{(p-q)^2}\frac{A(q^2)\not{q}-B(q^2)}{[A(q^2)^2q^2-B(q^2)^2]}. \quad \textbf{(2.109)}$$

Remember the trace of odd numbers of γ matrices are zero (**C.9**). Therefore we can take the trace of this expression in order to leave the $B(p^2)$ function alone. Here we can study in Euclidean space instead of the Minkowski space. This transformation brings an extra i, which cancels the others. We immediately find

$$B(p^2) = -4\pi^2\varepsilon \int d^4q \frac{B(q^2)}{[A(q^2)^2q^2-B(q^2)^2](p-q)^2}. \quad \textbf{(2.110)}$$

We can divide the integral into two parts as

$$B(p^2) = -4\pi^2\varepsilon \left[\int_0^{p^2} \frac{d^4qB(q^2)}{[A(q^2)^2q^2+B(q^2)^2](p-q)^2} + \int_{p^2}^{\infty} \frac{d^4qB(q^2)}{[A(q^2)^2q^2+B(q^2)^2](p-q)^2}\right]. \quad \textbf{(2.111)}$$

Then we explicitly separate the angular integration using by the equation (**C.24**). After performing the angular integral by the help of the equations (**C.27**) and (**C.32**), we get

$$B(p^2) = -4\pi^4\varepsilon \left[\int_0^{p^2} dq^2 \frac{q^2}{p^2}\frac{B(q^2)}{[A(q^2)^2q^2+B(q^2)^2]} + \int_{p^2}^{\infty} dq^2 \frac{B(q^2)}{[A(q^2)^2q^2+B(q^2)^2]}\right]. \quad \textbf{(2.112)}$$

If we differentiate this expression with respect to p^2 by the general integral rule given in equation (**C.23**), we find

$$\frac{dB(p^2)}{dp^2} = -4\pi^4\varepsilon\left[\frac{B(p^2)}{[A(p^2)^2p^2+B(p^2)^2]}\frac{p^2}{p^2}-\int_0^{p^2}dq^2\frac{B(q^2)}{[A(q^2)^2q^2+B(q^2)^2]}\frac{q^2}{(p^2)^2}\right.$$
$$\left.-\frac{B(p^2)}{[A(p^2)^2p^2+B(p^2)^2]}\right], \qquad (\mathbf{2.113})$$
$$= 4\pi^4\varepsilon\int_0^{p^2}dq^2\frac{B(q^2)}{[A^2(q^2)q^2+B^2(q^2)]}\frac{q^2}{(p^2)^2}. \qquad (\mathbf{2.114})$$

This integral is clearly finite. We get zero for the right hand side as ε goes to zero. Since mass is equal to zero in the free case we get this constant equal to zero. This choice satisfies the equation (**2.108**).

The similar argument can be used to show that $A(p^2)$ is the dressed spinor propagator is a constant. We multiply equation (**2.109**) by $i\not{p}$ we get

$$p^2A(p^2)+\not{p}B(p^2) = p^2-4\pi^2 i\int d^4q\frac{\varepsilon}{(p-q)^2}\frac{A(q^2)(\not{p}\not{q})-B(q^2)\not{p}}{[A(q^2)^2q^2-B(q^2)^2]}. \qquad (\mathbf{2.115})$$

We take the trace over the spinor indices. We end up with

$$p^2A(p^2) = p^2-4\pi^2 i\int d^4q\frac{\varepsilon}{(p-q)^2}\frac{A(q^2)}{[A(q^2)^2q^2-B(q^2)^2]}\mathrm{Tr}(\not{p}\not{q}). \qquad (\mathbf{2.116})$$

where the $B(P^2)$ term is absent in the numerator. We can rewrite this term in Euclidean space as

$$-p^2A(p^2) = -p^2+i^2 16\pi^2\varepsilon\int d^4q\frac{p\cdot q}{[A(q^2)^2q^2+B(q^2)^2]}\frac{A(q^2)}{(p-q)^2}. \qquad (\mathbf{2.117})$$

Similarly to what has done above, we can separate the integral into two terms

$$p^2A(p^2)=p^2 \quad + \quad 16\pi^2\varepsilon\left[\int_0^{p^2}d^4q\frac{A(q^2)p\cdot q}{[A(q^2)^2q^2+B(q^2)^2](p-q)^2}\right.$$
$$\left.+\int_{p^2}^{\infty}d^4q\frac{A(q^2)p\cdot q}{[A(q^2)^2q^2+B(q^2)^2](p-q)^2}\right]. \qquad (\mathbf{2.118})$$

We divide the integrand to the angular parts by the help of the equation (**C.24**). Then the angular integration can be performed by equations (**C.29**), (**C.33**). This yields to

$$p^2A(p^2)=p^2 \quad + \quad 8\pi^4\varepsilon\left[\int_0^{p^2}dq^2\frac{(q^2)^2A(q^2)}{p^2[A(q^2)^2q^2+B(q^2)^2]}\right.$$
$$\left.+\int_{p^2}^{\infty}dq^2\frac{p^2A(q^2)}{[A(q^2)^2q^2+B(q^2)^2]}\right]. \qquad (\mathbf{2.119})$$

We can divide both sides by p^2, this leaves the $A(p^2)$ alone.

$$A(p^2) = 1 \quad + \quad 8\pi^4\varepsilon \left[\int_0^{p^2} dq^2 \frac{(q^2)^2 A(q^2)}{(p^2)^2[A(q^2)^2 q^2 + B(q^2)^2]} \right.$$
$$\left. + \int_{p^2}^{\infty} dq^2 \frac{A(q^2)}{[A(q^2)^2 q^2 + B(q^2)^2]} \right]. \tag{2.120}$$

We can differentiate with respect to p^2. The end result is

$$\frac{dA(p^2)}{dp^2} = 8\pi^4\varepsilon \left[\frac{A(p^2)}{[A(p^2)^2 p^2 + B(p^2)^2]} \frac{p^4}{p^4} - 2\int_0^{p^2} dq^2 \frac{A(q^2)}{[A(q^2)^2 q^2 + B(q^2)^2]} \frac{q^4}{p^6} \right.$$
$$\left. - \frac{A(p^2)}{[A(p^2)^2 p^2 + B(p^2)^2]} \right] \tag{2.121}$$
$$= -16\pi^4\varepsilon \int_0^{p^2} dq^2 \frac{A(q^2)}{[A(q^2)^2 q^2 + B(q^2)^2]} \frac{q^4}{p^6}. \tag{2.122}$$

This finite integral shows that $A(p^2)$ is a constant as ε goes to zero. Since the integral is finite, it equals unity for the free case, we take $A(p^2) = 1$.

This result shows that no mass is generated for the fermion in higher orders.

2.9 Interactions

In our model there are two interactions. One of them, namely Yukawa interaction, is between the composite scalar field and two spinors with a dimensionless coupling constant g. The other one is the self interaction of the composite scalar field with a dimensionless coupling constant a. In the next section we will analyze these interaction vertices to understand if they need an infinite coupling constant renormalization. First we will study the Yukawa vertex in one loop correction. Then we will go to higher orders and see whether they need a regularization. After that, we will make a similar analyze the four composite scalar vertex. In the end we will finish the section with the scattering and production processes including the higher order corrections.

2.9.1 Yukawa vertex

In our model to leading order in $1/N$, the contribution to $< \overline{\psi}\psi\phi >$ vertex up to the first order is given by the diagrams in figure (2.3).

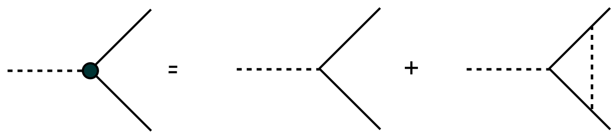


Figure 2.3. The diagrams of the contribution to Yukawa vertex up to the first order.

The one loop correction to the tree vertex involves two fermion and one composite scalar field propagators and one integration.

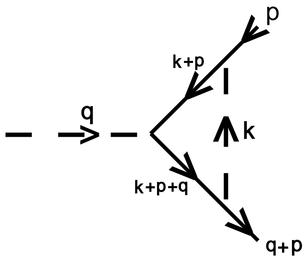


Figure 2.4. The Scalar Correction to the Yukawa vertex in one loop.

Here, we will give some of the basic calculations of these corrections. Then in higher orders we will give the results without showing the explicit calculations which is just a repetition of what we do here with more terms. So we can express this correction by the terms as shown in figure 2.4.

$$
\begin{aligned}
I_1[p,q] &= \int_k (-ig)\frac{i}{\not{k}+\not{p}+\not{q}}(-ig)\frac{i}{\not{k}+\not{p}}(-ig)\frac{4\pi^2\varepsilon}{g^2}\frac{-i}{k^2}, && (\mathbf{2.123})\\
&= (-g^3)\frac{4\pi^2\varepsilon}{g^2}\int_k \frac{(\not{k}+\not{p}+\not{q})(\not{k}+\not{p})}{(k+p+q)^2(k+p)^2k^2}. && (\mathbf{2.124})
\end{aligned}
$$

Here we need a Feynman parametrization. The general form is given in Appendix equation (**C.12**). For two parameters we can use the equation given in equation (**C.14**) and we can express

$$
\begin{aligned}
&\frac{1}{(k+p+q)^2(k+p)^2k^2} = \frac{\Gamma(3)}{\Gamma(1)^3}\times\\
&\int_0^1 d\alpha\int_0^\alpha d\beta\frac{1}{\left[\beta(k+p+q)^2+(\alpha-\beta)(k+p)^2+(1-\alpha)k^2\right]^3}. \qquad (\mathbf{2.125})
\end{aligned}
$$

The term in the denominator can be rewritten as

$$
\begin{aligned}
&\beta(k+p+q)^2+(\alpha-\beta)(k+p)^2+(1-\alpha)k^2\\
&\quad = (k+\beta p+\alpha q)^2+\beta(1-\beta)q^2+\alpha(1-\alpha)p^2+2\beta(1-\alpha)pq, \qquad (\mathbf{2.126})
\end{aligned}
$$

after now we will rename

$$M^2=\beta(1-\beta)q^2+\alpha(1-\alpha)p^2+2\beta(1-\alpha)pq. \tag{2.127}$$

Then we find the vertex correction as

$$=(-g^3)\frac{4\pi^2\varepsilon}{g^2}\frac{\Gamma(3)}{\Gamma(1)^3}\int_0^1 d\alpha\int_0^\alpha d\beta\int_k\frac{(\not{k}+\not{p}+\not{q})(\not{k}+\not{p})}{\left[(k+\beta p+\alpha q)^2+M^2\right]^3}. \tag{2.128}$$

We can shift $K=k+\beta p+\alpha q$,

$$=(-g^3)\frac{4\pi^2\varepsilon}{g^2}\frac{\Gamma(3)}{\Gamma(1)^3}\int_0^1 d\alpha\int_0^\alpha d\beta\int_K\frac{[\not{K}+(1-\alpha)\not{p}+(1-\beta)\not{q}]\,[\not{K}+(1-\alpha)\not{p}-\beta\not{q}]}{(K^2+M^2)^3}.$$

rename

$$\begin{aligned}\not{P} &= [(1-\alpha)\not{p}+(1-\beta)\not{q}], &(2.129)\\ \not{Q} &= [(1-\alpha)\not{p}-\beta\not{q}]. &(2.130)\end{aligned}$$

Then we can drop the odd terms linear to $\not{K}$. We get

$$=(-g^3)\frac{4\pi^2\varepsilon}{g^2}\frac{\Gamma(3)}{\Gamma(1)^3}\int_0^1 d\alpha\int_0^\alpha d\beta\int_K\frac{K^2+\not{P}\not{Q}}{(K^2+M^2)^3}, \tag{2.131}$$

we can write

$$\begin{aligned}&= (-g^3)\frac{4\pi^2\varepsilon}{g^2}\frac{\Gamma(3)}{\Gamma(1)^3}\int_0^1 d\alpha\int_0^\alpha d\beta\int_K\frac{K^2+\not{P}\not{Q}}{(K^2+M^2)^3}, &(2.132)\\ &= (-g^3)\frac{4\pi^2\varepsilon}{g^2}\frac{\Gamma(3)}{\Gamma(1)^3}\int_0^1 d\alpha\int_0^\alpha d\beta\int_K\left[\frac{1}{(K^2+M^2)^2}+\frac{\not{P}\not{Q}-M^2}{(K^2+M^2)^3}\right]. &(2.133)\end{aligned}$$

We perform the integration over K momentum due to the equation **(C.18)**

$$\begin{aligned}&= (-g^3)\frac{4\pi^2\varepsilon}{g^2}\frac{i}{(4\pi)^{D/2}}\int_0^1 d\alpha\int_0^\alpha d\beta\left[2\Gamma\left(2-\frac{D}{2}\right)\frac{1}{(-M^2)^{2-D/2}}\right.\\ &\qquad\left.-\Gamma\left(3-\frac{D}{2}\right)\frac{\not{P}\not{Q}-M^2}{(-M^2)^{3-D/2}}\right], &(2.134)\\ &= -ig\frac{(4\pi^2\varepsilon)}{(4\pi)^2}\int_0^1 d\alpha\int_0^\alpha d\beta\left[2\Gamma\left(2-\frac{D}{2}\right)\right.\\ &\qquad\left.+\Gamma\left(3-\frac{D}{2}\right)\frac{\not{P}\not{Q}-M^2}{M^2}\right]\left(\frac{4\pi}{M^2}\right)^{2-D/2}, &(2.135)\end{aligned}$$

$$= -ig\frac{\varepsilon}{4}\int_0^1 d\alpha \int_0^\alpha d\beta \left[2\Gamma\left(\frac{\varepsilon}{2}\right)+\Gamma\left(1+\frac{\varepsilon}{2}\right)\frac{\not{P}\not{Q}-M^2}{M^2}\right]\left(\frac{4\pi}{M^2}\right)^{\varepsilon/2}, \quad \mathbf{(2.136)}$$

$$= -ig\frac{\varepsilon}{4}\int_0^1 d\alpha \int_0^\alpha d\beta \left[2\left(\frac{2}{\varepsilon}-\gamma\right)+\frac{\not{P}\not{Q}}{M^2}-1\right]\left[1-\frac{\varepsilon}{2}\ln\left(\frac{M^2}{4\pi}\right)+...\right] \mathbf{(2.137)}$$

$$= -ig\frac{\varepsilon}{4}\left[\frac{2}{\varepsilon}-\gamma-\frac{1}{2}+\int_0^1 d\alpha \int_0^\alpha d\beta \left(\frac{\not{P}\not{Q}}{M^2}-2\ln\left(\frac{M^2}{4\pi}\right)\right)\right]. \quad \mathbf{(2.138)}$$

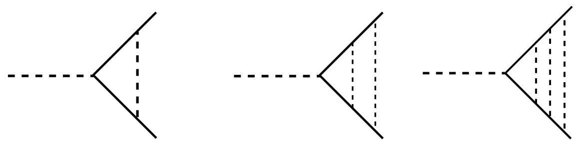
Finally we obtain

$$= \frac{-ig}{2} + O(\varepsilon). \quad \mathbf{(2.139)}$$

This is a finite result as ε goes to zero. This is an important feature of the model. Although we find an infinity from the momentum integral, it is cancelled by the ε in the ϕ propagator. Therefore we do not need an infinite regularization for the Yukawa vertex in one loop.

2.9.2 Higher order corrections of the Yukawa vertex

In this subsection we will check the higher order contributions to the Yukawa vertex. In the previous subsection we gave the detailed calculations of one loop correction. Here we will first check the previous result with counting the dimension of the contribution integrals. When we verify the usage of this method in our calculation, we will analyze the higher orders.



(a) **(b)** **(c)**

Figure 2.5. The ladder diagrams of the Yukawa vertex for higher orders (a) One loop, (b) Two loops, (c) Three loops.

In this power counting method we count every spinor propagator that has one mass dimension in the denominator. The composite scalar propagator counts as two while it has an addition ε factor to the numerator. Every loop brings an integral counts as four mass dimensions to the numerator. If the denominator is higher than the numerator, the integral gives finite results. If they have the same order, we interpret it as an logarithmic divergence which may be renormalized. In the other situation, when the denominator is less than the numerator, we comment that the integral diverges worse than all.

We may start with one loop correction, figure 2.5.a. This diagram is made out of two spinors and one composite scalar lines. There is only loop. So we find that the denominator has four mass dimensions, while the numerator has also four dimensions with an additional ε factor. Therefore the integral gives a logarithmic factor divergence which means $\frac{1}{\varepsilon}$ that cancels the ε in the numerator. While ε goes to zero, we obtain a finite result. This is shown in details in the previous subsection. The finite results was given in equation (**2.139**).

The two loop diagram, figure 2.5.b, contains four spinor and two composite scalar lines. This means that the denominator has eight mass dimensions. The numerator involves two integrals with ε^2 factor. Evaluation of the integrals gives at worst $\frac{1}{\varepsilon^2}$, that cancels the other ones. Finally we get a finite result for the two loop correction.

Similarly the three loop diagram contains, as shown in figure 2.5.c, six spinors and three composite scalar lines. Therefore the denominator has twelve momenta while the numerator has three integrals with ε^3 factor. At worst we end up with a finite result using the dimensional regularization scheme.

We therefore can conclude the following result. The infinity coming from the momentum integration is always canceled by the ε in the ϕ propagator. All the higher order contributions vanish because the powers of ε exceed the number of infinities coming momentum integrations. That is why we do not need an infinite renormalization to the of the spinor scalar coupling constant g.

2.9.3 Four composite scalar vertex

In our model, although the Lagrangian does not possess a kinetic part for the scalar propagator, we have four scalar interaction. This vertex may need an infinite renormalization. In figure 2.6, the correction up to the first order to this vertex is shown.

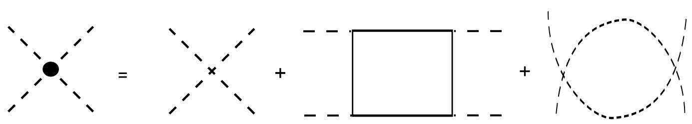


Figure 2.6. The diagrams of the contribution to four Composite Scalar vertex up to the first order.

2.9.4 Fermion box correction

The first correction to the tree diagram is the box diagram shown in figure 2.7.

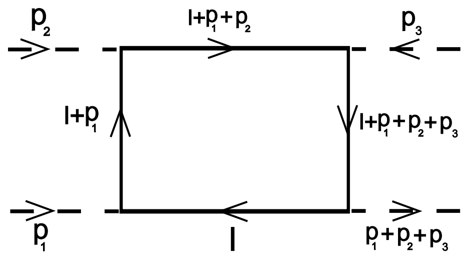


Figure 2.7. One loop correction to the four composite Scalar vertex.

This diagram has four spinor propagators and the contribution can be written as

$$I_2[p_1,p_2,p_3] = -\int_l \mathrm{Tr}\left[\frac{i(-ig)}{\not l+\not p_1+\not p_2+\not p_3}\frac{i(-ig)}{\not l+\not p_1+\not p_2}\frac{i(-ig)}{\not l+\not p_1}\frac{i(-ig)}{\not l}\right] \quad (\mathbf{2.140})$$

$$= -g^4\int_l \frac{\mathrm{Tr}\,[(\not l+\not p_1+\not p_2+\not p_3)(\not l+\not p_1+\not p_2)(\not l+\not p_1)(\not l)]}{(l+p_1+p_2+p_3)^2(l+p_1+p_2)^2(l+p_1)^2l^2} \quad (\mathbf{2.141})$$

Here we need a Feynman parametrization with three parameters. The general form is given in Appendix equation (**C.12**) Using that identity we can express

$$\frac{1}{(l+p_1+p_2+p_3)^2(l+p_1+p_2)^2(l+p_1)^2l^2} = 3!\int_0^1 dx\int_0^x dy\int_0^y dz$$
$$\times\frac{1}{[(l+p_1+p_2+p_3)^2z+(l+p_1+p_2)^2(y-z)+(l+p_1)^2(x-y)+l^2(1-x)]^4} \quad (\mathbf{2.142})$$

The denominator in equation (**2.142**)

$$\left[(l+p_1+p_2+p_3)^2z+(l+p_1+p_2)^2(y-z)+(l+p_1)^2(x-y)+l^2(1-x)\right]$$
$$= \left[(l+xp_1+yp_2+zp_3)^2+M^2\right], \quad (\mathbf{2.143})$$

where

$$M^2 = xp_1^2+y\left[(p_1+p_2)^2-p_1^2\right]+z\left[(p_1+p_2+p_3)^2-(p_1+p_2)^2\right]$$
$$-(xp_1+yp_2+zp_3)^2. \quad (\mathbf{2.144})$$

We replace it to the equation (**2.142**), we find

$$= \frac{1}{\left[(l+xp_1+yp_2+zp_3)^2+M^2\right]^4}. \quad (\mathbf{2.145})$$

We insert equation (**2.142**) into the equation (**2.141**), we get

$$\begin{aligned}I_2[p_1,p_2,p_3] &= -(3!)g^4\int_0^1 dx\int_0^x dy\int_0^y dz\\ &\times\mathrm{Tr}\int_l \frac{[(\not{l}+\not{p}_1+\not{p}_2+\not{p}_3)(\not{l}+\not{p}_1+\not{p}_2)(\not{l}+\not{p}_1)(\not{l})]}{\left[(l+xp_1+yp_2+zp_3)^2+M^2\right]^4}. \end{aligned}\tag{2.146}$$

We need to shift $q=l+xp_1+yp_2+zp_3$, then we find

$$\begin{aligned}I_2[p_1,p_2,p_3] &= -(3!)g^4\int_0^1 dx\int_0^x dy\int_0^y dz\\ &\times\mathrm{Tr}\int_q \frac{\left[(\not{q}-\not{a})(\not{q}-\not{b})(\not{q}-\not{c})(\not{q}-\not{d})\right]}{[q^2+M^2]^4}, \end{aligned}\tag{2.147}$$

where

$$\not{a} = (x-1)\not{p}_1+(y-1)\not{p}_2+(z-1)\not{p}_3 \tag{2.148}$$

$$\not{b} = (x-1)\not{p}_1+(y-1)\not{p}_2+z\not{p}_3 \tag{2.149}$$

$$\not{c} = (x-1)\not{p}_1+y\not{p}_2+z\not{p}_3 \tag{2.150}$$

$$\not{d} = x\not{p}_1+y\not{p}_2+z\not{p}_3 \tag{2.151}$$

The term in trace needs care, we omit to write the odd terms because of the symmetry, we get

$$\begin{aligned}&\mathrm{Tr}\left[(\not{q}-\not{a})(\not{q}-\not{b})(\not{q}-\not{c})(\not{q}-\not{d})\right] = \mathrm{Tr}\Big[\not{q}\not{q}\not{q}\not{q}+\not{q}\not{q}\not{c}\not{d}+\not{q}\not{b}\not{q}\not{d}\\ &+\not{q}\not{b}\not{c}\not{q}+\not{a}\not{q}\not{q}\not{d}+\not{a}\not{q}\not{c}\not{q}+\not{a}\not{b}\not{q}\not{q}+\not{a}\not{b}\not{c}\not{d}\Big]\end{aligned}\tag{2.152}$$

$$\begin{aligned}= D\Big[&q^4+q^2(c\cdot d)+2(q\cdot b)(q\cdot d)-q^2(b\cdot d)+q^2(b\cdot c)+q^2(a\cdot d)+2(q\cdot a)(q\cdot c)\\ &-q^2(a\cdot c)+q^2(a\cdot b)+(a\cdot b)(c\cdot d)-(a\cdot c)(b\cdot b)+(a\cdot d)(b\cdot c)\Big]\end{aligned}\tag{2.153}$$

Here we used the identity in D dimension given in Appendix (**C.11**). We are going to divide these terms by $\frac{1}{[q^2+M^2]^4}$ then integrate them over q. Before evaluation we have to study more, first

$$\int_q \frac{q^4}{[q^2+M^2]^4} = \int_q \frac{q^4-M^4+M^4}{[q^2+M^2]^4}, \tag{2.154}$$

$$= \int_q \frac{(q^2+M^2)(q^2-M^2)}{[q^2+M^2]^4} + \int_q \frac{M^4}{[q^2+M^2]^4}, \tag{2.155}$$

$$= \int_q \frac{q^2-M^2}{[q^2+M^2]^3} + \int_q \frac{M^4}{[q^2+M^2]^4}, \tag{2.156}$$

$$= \int_q \frac{q^2+M^2}{[q^2+M^2]^3} - \int_q \frac{2M^2}{[q^2+M^2]^3} + \int_q \frac{M^4}{[q^2+M^2]^4}, \tag{2.157}$$

$$= \int_q \frac{1}{[q^2+M^2]^2} - \int_q \frac{2M^2}{[q^2+M^2]^3} + \int_q \frac{M^4}{[q^2+M^2]^4}. \tag{2.158}$$

second lets call $S = (c\cdot d)-(b\cdot d)+(b\cdot c)+(a\cdot d)-(a\cdot c)+(a\cdot b)$

$$\int_q \frac{Sq^2}{[q^2+M^2]^4} = S\int_q \frac{q^2+M^2-M^2}{[q^2+M^2]^4}, \tag{2.159}$$

$$= S\int_q \frac{1}{[q^2+M^2]^3} - S\int_q \frac{M^2}{[q^2+M^2]^4}. \tag{2.160}$$

third,

$$\int_q \frac{2\left[(q\cdot b)(q\cdot d)+(q\cdot a)(q\cdot c)\right]}{[q^2+M^2]^4} = 2\int_q \frac{q_\mu q_\nu (b^\mu d^\nu + a^\mu c^\nu)}{[q^2+M^2]^4}, \tag{2.161}$$

$$= \frac{2}{D}\int_q \frac{q^2 g_{\mu\nu}(b^\mu d^\nu + a^\mu c^\nu)}{[q^2+M^2]^4}, \tag{2.162}$$

$$= \frac{2R}{D}\int_q \frac{q^2}{[q^2+M^2]^4}, \tag{2.163}$$

$$= \frac{2R}{D}\int_q \frac{1}{[q^2+M^2]^3} - \frac{2R}{D}\int_q \frac{M^2}{[q^2+M^2]^4}. \tag{2.164}$$

Here we called $R = (b\cdot d + a\cdot c)$ and D is the dimension. Finally last term

$$\int_q \frac{T}{[q^2+M^2]^4} \tag{2.165}$$

where we call $T = (a\cdot b)(c\cdot d)-(a\cdot c)(b\cdot b)+(a\cdot d)(b\cdot c)$. We can combine all of them in a form such as

$$\int_q \frac{1}{[q^2+M^2]^2} + \int_q \frac{-2M^2+S+\frac{2R}{D}}{[q^2+M^2]^3} + \int_q \frac{M^4-\left(S+\frac{2R}{D}\right)M^2+T}{[q^2+M^2]^4} \tag{2.166}$$

Evaluation of the integrals are can be done due to the equation (**C.18**) as follows:

$$\int_q \frac{1}{[q^2+M^2]^2} = \frac{i}{(4\pi)^{D/2}}\frac{\Gamma(2-D/2)}{\Gamma(2)}\frac{1}{(-M^2)^{2-D/2}},$$

$$= \frac{i}{(4\pi)^2}\frac{\Gamma(2-2/D)}{\Gamma(2)}\left(\frac{4\pi}{M^2}\right)^{2-D/2}. \qquad (2.167)$$

$$\int_q \frac{-2M^2+S+\frac{2R}{D}}{[q^2+M^2]^3} = \frac{-i}{(4\pi)^{D/2}}\frac{\Gamma(3-D/2)}{\Gamma(3)}\frac{-2M^2+S+\frac{2R}{D}}{(-M^2)^{3-D/2}},$$

$$= i\frac{-2+\frac{1}{M^2}\left(S+\frac{2R}{D}\right)}{(4\pi)^2}\frac{\Gamma(3-D/2)}{\Gamma(3)}\left(\frac{4\pi}{M^2}\right)^{2-D/2}. \qquad (2.168)$$

$$\int_q \frac{M^4-\left(S+\frac{2R}{D}\right)M^2+T}{[q^2+M^2]^4} = \frac{i}{(4\pi)^{D/2}}\frac{\Gamma(4-D/2)}{\Gamma(4)}\frac{M^4-\left(S+\frac{2R}{D}\right)M^2+T}{(-M^2)^{4-D/2}},$$

$$= i\frac{1-\frac{1}{M^2}\left(S+\frac{2R}{D}\right)+\frac{T}{M^4}}{(4\pi)^2}\frac{\Gamma(4-D/2)}{\Gamma(4)}\left(\frac{4\pi}{M^2}\right)^{2-D/2} \qquad (2.169)$$

We obtain

$$I_2[p_1,p_2,p_3] = -iD\frac{g^4}{(4\pi)^2}\int_0^1 dx\int_0^x dy\int_0^y dz\left(\frac{4\pi}{M^2}\right)^{2-D/2}$$
$$\times\Bigg[6\Gamma(2-D/2)+3\Gamma(3-D/2)\left(-2+\frac{1}{M^2}\left(S+\frac{2R}{D}\right)\right)$$
$$+\Gamma(4-D/2)\left(1-\frac{1}{M^2}\left(S+\frac{2R}{D}\right)+\frac{T}{M^4}\right)\Bigg]. \qquad (2.170)$$

in $D=4-\varepsilon$ dimension we find

$$= - i\frac{g^4}{(4\pi)^2}\int_0^1 dx\int_0^x dy\int_0^y dz\,(4-\varepsilon)\left[1-\frac{\varepsilon}{2}\ln\left(\frac{4\pi}{M^2}\right)+...\right]\left[6\left(\frac{2}{\varepsilon}-\gamma\right)\right.$$
$$\left.+3\left(-2+\frac{1}{M^2}\left(\frac{4S+2R-S\varepsilon}{4-\varepsilon}\right)\right)+\left(1-\frac{1}{M^2}\left(\frac{4S+2R-S\varepsilon}{4-\varepsilon}\right)+\frac{T}{M^4}\right)\right],$$

$$= - i\frac{g^4}{(4\pi)^2}\int_0^1 dx\int_0^x dy\int_0^y dz\,(4-\varepsilon)\left[1-\frac{\varepsilon}{2}\ln\left(\frac{4\pi}{M^2}\right)\right]$$
$$\times \left[\frac{12}{\varepsilon}-6\gamma-5+\frac{2}{M^2}\left(\frac{4S+2R-S\varepsilon}{4-\varepsilon}\right)+\frac{T}{M^4}\right], \qquad (2.171)$$

$$= - i\frac{g^4}{(4\pi)^2}\int_0^1 dx\int_0^x dy\int_0^y dz$$
$$\times (4-\varepsilon)\left[\frac{12}{\varepsilon}-6\gamma-5+\frac{2}{M^2}\frac{4S+2R-S\varepsilon}{(4-\varepsilon)}+\frac{T}{M^4}-6\ln\left(\frac{4\pi}{M^2}\right)\right], \qquad (2.172)$$

$$= - i\frac{g^4}{(4\pi)^2}\int_0^1 dx\int_0^x dy\int_0^y dz\left[\frac{48}{\varepsilon}-24\gamma-32+\frac{2(4S+2R)}{M^2}+\frac{4T}{M^4}-24\ln\left(\frac{4\pi}{M^2}\right)\right]$$

Performing the integrals of the parameter we find

$$= \frac{-ig^4}{(4\pi)^2}\left[\frac{8}{\varepsilon} - \frac{24\gamma+32}{6} + \int_0^1 dx \int_0^x dy \int_0^y dz \left(\frac{8S+4R}{M^2} + \frac{4T}{M^4} - 24\ln\left(\frac{4\pi}{M^2}\right)\right)\right] \quad \textbf{(2.173)}$$

We have 5 other channels, so totally it yields

$$= \frac{-ig^4}{(4\pi)^2}\left[\frac{48}{\varepsilon} - (24\gamma+32) + \int_0^1 dx \int_0^x dy \int_0^y dz \left(\frac{8S+4R}{M^2} + \frac{4T}{M^4} - 24\ln\left(\frac{4\pi}{M^2}\right)\right) + 5 \text{ other channels}\right] \quad \textbf{(2.174)}$$

So we find that this scalar box diagram gives a $1/\varepsilon$ type divergence. We need to renormalize the coupling of this vertex to incorporate this divergence.

2.9.5 Fish diagram correction

The second correction to the tree diagram may be the fish diagram shown in figure 2.8.

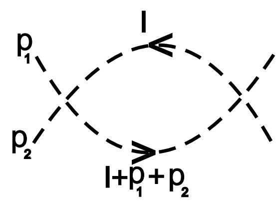


Figure 2.8. The one-loop scalar field correction to the four scalar interaction .

This diagram can be expressed as

$$I_3[p_1,p_2] = \frac{(-ia)^2}{2}\int_l \frac{(-4\pi^2 i)}{g^2}\frac{\varepsilon}{l^2}\frac{(-4\pi^2 i)}{g^2}\frac{\varepsilon}{(l+p_1+p_2)^2}, \quad \textbf{(2.175)}$$

$$= 8\pi^4\frac{a^2}{g^4}\varepsilon^2\int_l \frac{1}{l^2}\frac{1}{(l+p)^2} \quad \textbf{(2.176)}$$

Notice that here we called $p = p_1 + p_2$. There are two more channels: $p = p_1 + p_3$ and $p = p_1 + p_4$. At the end of our calculation we will count them. Now we continue with the Feynman parametrization using the equation (**C.13**)

$$= 8\pi^4\frac{a^2}{g^4}\varepsilon^2\frac{\Gamma(2)}{\Gamma(1)^2}\int_0^1 d\alpha \int_l \frac{1}{[(l+\alpha p)^2 + \alpha(1-\alpha)p^2]^2} \quad \textbf{(2.177)}$$

We shift $l+\alpha p = q$ and call $M^2 = \alpha(1-\alpha)p^2$, then

$$= 8\pi^4\frac{a^2}{g^4}\varepsilon^2\frac{\Gamma(2)}{\Gamma(1)^2}\int_0^1 d\alpha \int_q \frac{1}{[q^2+M^2]^2} \quad \textbf{(2.178)}$$

We evaluated the q integral due to the equation (**C.18**). We find

$$= 8\pi^4\frac{a^2}{g^4}\varepsilon^2\frac{\Gamma(2)}{\Gamma(1)^2}\int_0^1 d\alpha\frac{i}{(4\pi)^{D/2}}\frac{\Gamma(2-D/2)}{\Gamma(2)}\frac{1}{(-M^2)^{2-D/2}}, \qquad (\mathbf{2.179})$$

$$= \frac{\pi^2}{2}\frac{ia^2}{g^4}\varepsilon^2\Gamma\left(2-\frac{D}{2}\right)\int_0^1 d\alpha\left[\frac{4\pi}{M^2}\right]^{2-D/2}, \qquad (\mathbf{2.180})$$

$$= \frac{\pi^2}{2}\frac{ia^2}{g^4}\varepsilon^2\Gamma\left(\frac{\varepsilon}{2}\right)\int_0^1 d\alpha\left[\frac{4\pi}{M^2}\right]^{\varepsilon/2}, \qquad (\mathbf{2.181})$$

$$= \frac{\pi^2}{2}\frac{ia^2}{g^4}\varepsilon^2\left(\frac{2}{\varepsilon}-\gamma\right)\int_0^1 d\alpha\left[1-\frac{\varepsilon}{2}\ln\frac{\alpha(1-\alpha)p^2}{4\pi\mu^2}\right]. \qquad (\mathbf{2.182})$$

This result clearly shows that when we take $\varepsilon \to 0$ limit, fish diagram one loop contribution vanishes.

2.9.6 Higher order corrections of composite scalar vertex

In the previous subsections we find a $1/\varepsilon$ type infinite contribution in the one loop correction of the fermion box. Up to now this is the only diverging interaction. In this section we will go to the higher orders and will try to find the existence of the other types of divergences.

The two loop contribution to the four scalar interaction is shown in figure 2.9. The two loop diagram contains a composite propagator ϕ which makes this diagram finite. Note that the diagram where the internal scalar is connecting adjacent sides, as shown in figure 2.9.b, will be a contribution to the renormalization of the Yukawa interaction.

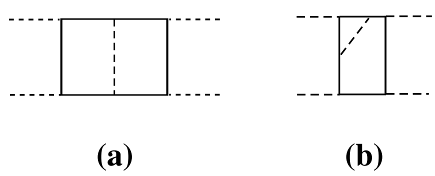
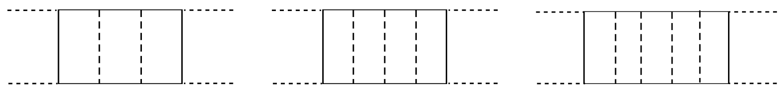


Figure 2.9. (a) The scalar correction to the composite scalar box diagram, (b) The box diagram with one vertex correction.

The other higher order diagrams where the scalar propagators propagates inside the spinor box with nonadjacent spinor fields are shown in figure 2.10.

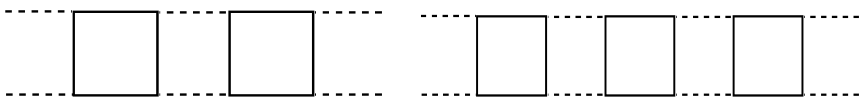


(a) **(b)** **(c)**

Figure 2.10. Composite scalar vertex corrections for (a) Three loops, (b) Four loops, (c) Five loops.

The contributions of the diagrams shown in 2.10.a, 2.10.b, 2.10.c give the result zero with the power of ε, ε^2, ε^3 consequently.

Unlike these non contributing diagrams, there are some other corrections, too. They are shown in figure 2.11.a and 2.11.b.



(a) **(b)**

Figure 2.11. Composite scalar vertex corrections for odd number of loops like (a) Three loops, (b) Five loops.

They are odd loop number diagrams. These corrections also can be interpreted as the connection of one loop diagrams by the composite propagators as intermediates. In the first higher order, at three loop correction figure 2.11.a, we end up with a first order infinity of the form $1/\varepsilon$ at worst. The next order correction is five loop correction given in figure 2.11.b. The dimensional regularization scheme gives this diagram corrections in the $1/\varepsilon$ order of infinity, too. Every higher order of ladder diagrams of this type give at worst the same type of divergence. This divergence for the four composite scalar vertex can be renormalized using the standard means.

2.9.7 Spinor scattering

Here we will study the four spinor diagrams. We do not have four spinor coupling in our Lagrangian. Therefore we need composite scalar particles coupling to spinors to obtain this interaction, which necessitates the use of composite scalar propagators as internal lines.

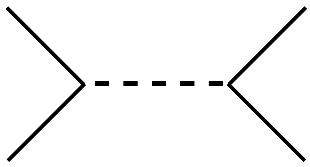
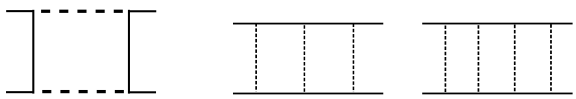


Figure 2.12. Spinor scattering vertex at tree level

Since each composite scalar propagator contains an ε contribution, the four spinor processes go to zero as ε. This tree level diagram indicates that there is no four spinor interaction.

2.9.8 Higher order corrections of spinor scattering

When we go beyond tree diagrams, we need at least two composite scalar propagators to end up with two spinors, which means extra powers of ε. The one loop diagram is given in figure 2.13.a. This process uses two scalar propagators. Since the integral is finite, this process gives a result proportional to ε^2.



(a) **(b)** **(c)**

Figure 2.13. Spinor scattering vertex at the levels of (a) one loop, (b) two loops, (c) three loops.

The two loop and three loop ladder diagrams are shown in figure 2.13.b, and figure 2.13.c. Their contributions are in the order of ε^3 and ε^4, respectively. Therefore, these contributions vanish as ε goes to zero.

2.9.9 Bethe-Salpeter equation for fermion scattering

Although the fermions are the ingredients of our model, above we see that there is no fermion scattering in the model. We can justify this claim by writing the Bethe-Salpeter equations for fermion scattering process.

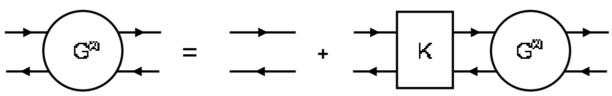


Figure 2.14. Graphical Illustration of Bethe-Salpeter Equation of Four Fermion Scattering

This equation is shown in the figure 2.14 and can be expressed mathematically as

$$\begin{aligned} G^{(2)}(p,q;P) &= G_0^{(2)}(p,q;P) \\ &+ \int \frac{d^4p'}{(2\pi)^4}\frac{d^4q'}{(2\pi)^4} G_0^{(2)}(p,p';P)K(p',q';P)G^{(2)}(q',q;P). \quad \textbf{(2.183)} \end{aligned}$$

Here $G_0^{(2)}(p,q;P)$ is two non-crossing spinor lines, $G^{(2)}(p,q;P)$ is the proper four point function. K is the Bethe-Salpeter kernel.

We note that this expression involves the four spinor kernel which we found to be of order ε. This expression can be written in the quenched ladder approximation [14], where the kernel is separated into a scalar propagator with two spinor legs joining the proper kernel. If the proper kernel is of order ε , the loop involving two spinors and a scalar propagator can be at most finite that makes the whole diagram in first order in ε. This fact also shows that there is no nontrivial spinor-spinor scattering.

2.9.10 Bethe-Salpeter equation for Yukawa interaction

In the previous subsections, we have showed that the Yukawa vertex does not need an infinite renormalization. The finite contributions of the diagrams just renormalize the coupling constant g by a finite amount.

We come to the same result after we write the Bethe-Salpeter equation for this vertex. Diagrammatically it is shown in figure 2.15.

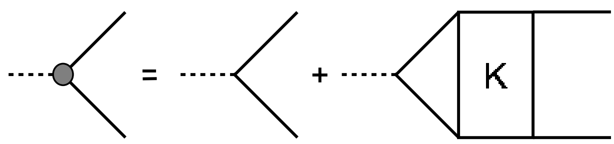
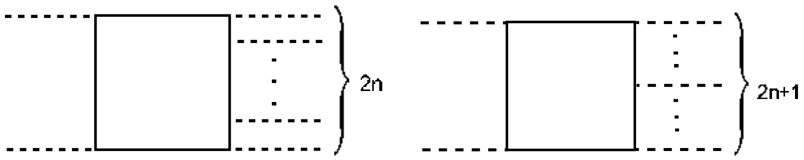


Figure 2.15. Graphical Illustration of Bethe-Salpeter Equation of Yukawa Vertex

We need the result of the four fermion scattering kernel to be able to perform this calculation. Similar to the explanation given above, the kernel will use at least one scalar propagator. Since the scalar particle propagator has a ε factor, all higher orders, including the one loop contribution this process does not give infinite contribution.

2.9.11 Other processes

We can also have scattering processes where two scalar particles go to an even number of scalar particles, shown in figure 2.16.a.

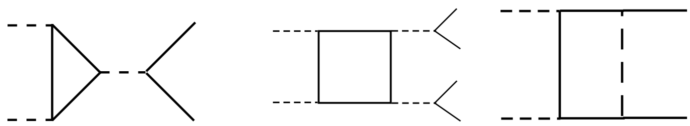


(a) **(b)**

Figure 2.16. Composite scalar fields scatter to (a) Even number of scalar fields, (b) Odd number of scalar fields.

In the one loop approximation all these diagrams give finite results, like the case in the standard Yukawa coupling model. Since going to an odd number of scalars, shown in figure 2.16.a, is forbidden by the γ^5 invariance of the theory, we can also argue that scalar ϕ particles can go to an even number of scalar particles only. This assertion is easily checked by diagrammatic analysis.

Any diagram which describes the process of producing spinor particles out of two scalars contains scalar propagators.



(a) **(b)** **(c)**

Figure 2.17. Two Spinor scatters to spinor fields in one loop (a) With a spinor triangle, (b) With a mixed box, (c) With a spinor box.

The lowest of these diagrams where two scalars, figure 2.17.a, go to two spinors vanish since it either involves a triangle diagram made out of spinors, or a box diagram, figure 2.17.b, made out of three spinors and one scalar. It vanishes due to fall of the scalar propagator in the latter case, although it is not zero unless the cut-off is removed . The diagram which involves the production of four spinors out of two scalars, figure 2.17.c, is zero because of the symmetry of the composite field.

2.10 RG Analysis of the Model

In our model, we have two coupling constants, g and a. We find that the coupling constant a needs renormalization while there is no need for infinite renormalization for coupling constant g. The reason is widely discussed above.

If we take μ_0 as a reference scale at low energies, $t = ln(\mu/\mu_0)$, where μ is the renormalization point by using the language of renormalization group analysis. Since the diagram given in figure 2.4 is finite, due to the presence of ε in the scalar propagator, the first order for the Yukawa vertex can be given by

$$\frac{dg_0}{dt} = 0. \tag{2.184}$$

Higher order calculations using the Bethe-Salpeter equation verify that the right hand side of the equation does not change in higher orders. This has also been discussed.

We see that in the model the only infinite renormalization is needed for the four ϕ vertex; hence the coupling constant for this process *runs*. The first correction to the tree diagram is the box diagram, shown in figure 2.7 . This diagram has four spinor propagators and gives rise to a $\frac{1}{\varepsilon}$ type divergence. The renormalization group equation written for this vertex can be given by

$$16\pi^2 \frac{da(t)}{dt} = -48 g_0^4. \tag{2.185}$$

The solution of the first order renormalization group equations given in equation (**2.184**) is just a constant. This gives the solution of the equation (**2.185**) solution as

$$a(t) = a_0 - \frac{48 g_0^4}{16\pi^2} t. \tag{2.186}$$

Here the coupling constant goes to infinity with the cutoff and should be renormalized.

2.11 Conclusion

In this chapter, we tried to give a new interpretation to the work which was done in [36, 44]. We showed explicitly that the polynomial form of the original model corresponds to the original Gürsey model at least classically. Then we made the constraint analysis and found the propagators of the model via path integral method. We went to higher orders in the calculation, beyond one loop for scattering processes. By using the Dyson-Schwinger and Bethe-Salpeter equations we verified higher order process results. In the end we found that the non-trivial scattering of the fundamental fields was not allowed in this model. Only bound states could scatter from each other.

The essential point in our analysis was the fact that, being proportional to $\frac{\varepsilon}{p^2}$, the composite scalar field propagator cancelled many of the potential infinities that arise while calculating loop integrals. As a result of this cancellation, only composite fields participate in physical processes such as scattering and particle production. The scattering and production of elementary spinor fields were not allowed. This phenomena was an example of treating the bound states, instead of the principal fields, as physical entities.

3. GAUGED SYSTEM MIMICKING THE GÜRSEY MODEL

A further point will be to couple an elementary vector field to the model described in the previous chapter, in line with the process studied for the NJL model [28, 29]. Our final goal is to investigate if we get a non-trivial theory when we couple a YM system with color and flavor degrees of freedom, like it is done in [31–35].

In this chapter we will study the abelian case, as an initial step. First we will summarize the changes in our results when this elementary vector field is coupled to the model. Then we will give our new results in the following sections. Mainly we will conclude that our original model, in which only the composites take part in physical processes like scattering or particle production, is reduced to a gauged-Higgs-Yukawa(gHY) model, where both the composites and the fundamental spinor and vector fields participate in all the processes.

3.1 Gauging with an Elementary Vector Field

We had a Lagrangian density in our model given in equation (**2.18**) with the following form

$$\mathscr{L} = \overline{\psi} i\partial\!\!\!/ \psi + g\overline{\psi}\psi\phi + \lambda(g\overline{\psi}\psi - a\phi^3). \tag{3.1}$$

We can add a constituent $U(1)$ gauge field to the model in a minimal way with a new coupling constant e. The new Lagrangian density can be given as

$$\mathscr{L} = \overline{\psi} iD\!\!\!\!/\, \psi + g\overline{\psi}\psi\phi + \lambda(g\overline{\psi}\psi - a\phi^3) - \frac{1}{4}(F_{\mu\nu}F^{\mu\nu}). \tag{3.2}$$

Here A^μ is the elementary vector field and $F_{\mu\nu}$ is the electromagnetic field strength tensor and D_μ is the covariant derivative defined as in the usual way as follows.

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{3.3}$$

$$D_\mu = \partial_\mu + ieA_\mu. \tag{3.4}$$

3.2 Constraint Analysis

Here we have additional fields, A_0 and A_i, besides our previous four fields ψ, $\overline{\psi}$, ϕ and λ. This leads to new momenta in this modified model in addition to the existing ones which were given in equations (**2.19**), (**2.20**) , (**2.21**), (**2.22**).

$$\pi^0 = \frac{\partial \mathscr{L}}{\partial(\partial_0 A_0)} = 0, \tag{3.5}$$

$$\pi^i = \frac{\partial \mathscr{L}}{\partial(\partial_0 A_i)} = -F^{0i}. \tag{3.6}$$

Here $i = 1,2,3$. By inspecting equations (**3.5**) and (**3.6**), we can conclude that there is only one primary constraint in the modified model. So totally there are five primary constraints,

$$\varphi_1 = \pi, \tag{3.7}$$

$$\varphi_2 = \overline{\pi} - i\,\overline{\psi}\gamma^0, \tag{3.8}$$

$$\varphi_3 = \rho, \tag{3.9}$$

$$\varphi_4 = \eta, \tag{3.10}$$

$$\varphi_5 = \pi^0. \tag{3.11}$$

Here we impose the π^i momentum in the canonical Hamiltonian by

$$\partial^0 A^i = -\pi^i + \partial^i A^0. \tag{3.12}$$

Then the canonical Hamiltonian density of the new model can be expressed as

$$\begin{aligned}\mathscr{H}_c &= -\frac{\pi^i\pi_i}{2} - A_0(\partial_i\pi^i) + \overline{\psi}\left[i\gamma^i\partial_i\psi - g\phi + e(\gamma^0 A_0 - \gamma^i A_i)\right]\psi - \lambda(g\overline{\psi}\psi - a\phi^3) \\ &- \frac{1}{2}[A_j\partial_i(\partial^i A^j) - A_j\partial_i(\partial^j A^i)].\end{aligned} \tag{3.13}$$

Here partial integration is used to be able to write $-A_0(\partial_i\pi^i)$ instead of $\pi^i(\partial_i A_0)$. Similar to the work shown in the previous chapter, this Hamiltonian generates the Hamiltonian equations of motion. These equations are not unique too. Therefore we replace this canonical Hamiltonian density with the total Hamiltonian density as follows.

$$
\begin{aligned}
\mathscr{H}_T &= -\frac{\pi^i\pi_i}{2} - A_0(\partial_i\pi^i) + \overline{\psi}\left[i\gamma^i\partial_i\psi - g\phi + e(\gamma^0 A_0 - \gamma^i A_i)\right]\psi - \lambda(g\overline{\psi}\psi - a\phi^3) \\
&- \frac{1}{2}[A_j\partial_i(\partial^i A^j) - A_j\partial_i(\partial^j A^i)] + u_1\pi^{\overline{\psi}} + (\pi^{\psi} - i\,\overline{\psi}\gamma^0)\overline{u}_2 + u_3\pi^{\phi} \\
&+ u_4\pi^{\lambda} + u_5\pi^0. \qquad \textbf{(3.14)}
\end{aligned}
$$

Here u_1 and $\overline{u}_2$ are four component spinor field coefficients, u_3 and u_4 are scalar field coefficients and u_5 is the four component vector field coefficient. This new $\mathscr{H}_T$ generates new equations of motion which are unique. From the consistency requirement of the constraints we find

$$\dot{\varphi}_1 = -[i\,\gamma^i\overrightarrow{\partial_i} - g(\phi+\lambda) + e\not{A}]\psi + i\,\gamma^0\overline{u}_2, \qquad \textbf{(3.15)}$$

$$\dot{\varphi}_2 = -i\,u_1\gamma^0 - \overline{\psi}[-i\gamma^i\overleftarrow{\partial_i} - g\overline{\psi}(\phi+\lambda) + e\not{A}], \qquad \textbf{(3.16)}$$

$$\dot{\varphi}_3 = g\overline{\psi}\psi - 3a\lambda\phi^2, \qquad \textbf{(3.17)}$$

$$\dot{\varphi}_4 = g\overline{\psi}\psi - a\phi^3, \qquad \textbf{(3.18)}$$

$$\dot{\varphi}_5 = \partial_i\pi^i - e\overline{\psi}\gamma^0\psi, \qquad \textbf{(3.19)}$$

The first two equations give a relation between the spinor coefficients. The last three ones imply a new relation among the canonical momenta and fields. These are the *secondary constraints*.

$$\varphi_6 = g\overline{\psi}\psi - 3a\lambda\phi^2, \qquad \textbf{(3.20)}$$

$$\varphi_7 = g\overline{\psi}\psi - a\phi^3, \qquad \textbf{(3.21)}$$

$$\varphi_8 = \partial_i\pi^i - e\overline{\psi}\gamma^0\psi. \qquad \textbf{(3.22)}$$

Repeating the same processes, taking the Poisson brackets of these constraints with the total Hamiltonian density of the system, we find relations between the coefficients u_1, $\overline{u}_2$, u_3 and u_4. But there is no relation for the u_5 coefficient. Remember that this coefficient is related to the π^0 constraint which arises from the fact that the Lagrangian density is independent of the time derivative of A_0. This is called a *primary constraint*, because it follows directly from the structure of the Lagrangian. We can impose new constraint to our system by hand depends on our choice. Here we will study in the

Landau gauge. Our new two constraints are as follows.

$$\varphi_9 = A_0, \quad \textbf{(3.23)}$$

$$\varphi_{10} = \partial^\mu A_\mu. \quad \textbf{(3.24)}$$

This new constraints let us to determine all the relations between the coefficients. Therefore we find a closed constraint algebra. To classify the constraints we need to take the Poisson brackets of all constraints among themselves.

$$\{\varphi_1, \varphi_2\} = i\gamma^0, \quad \textbf{(3.25)}$$

$$\{\varphi_1, \varphi_6\} = -g\psi, \quad \textbf{(3.26)}$$

$$\{\varphi_1, \varphi_7\} = -g\psi, \quad \textbf{(3.27)}$$

$$\{\varphi_1, \varphi_8\} = e\gamma^0\psi, \quad \textbf{(3.28)}$$

$$\{\varphi_2, \varphi_6\} = -g\overline{\psi}, \quad \textbf{(3.29)}$$

$$\{\varphi_2, \varphi_7\} = -g\overline{\psi}, \quad \textbf{(3.30)}$$

$$\{\varphi_2, \varphi_8\} = e\overline{\psi}\gamma^0, \quad \textbf{(3.31)}$$

$$\{\varphi_3, \varphi_6\} = 6a\lambda\phi, \quad \textbf{(3.32)}$$

$$\{\varphi_3, \varphi_7\} = 3a\phi^2, \quad \textbf{(3.33)}$$

$$\{\varphi_4, \varphi_6\} = 3a\phi^2, \quad \textbf{(3.34)}$$

$$\{\varphi_5, \varphi_9\} = -1, \quad \textbf{(3.35)}$$

$$\{\varphi_8, \varphi_{10}\} = \partial^2. \quad \textbf{(3.36)}$$

This result indicates the class of the constraints as *second class*. We can express the matrix $C_{\alpha\beta}$ as,

$$C_{\alpha\beta} = \begin{bmatrix} 0 & i\gamma^0 & 0 & 0 & 0 & -g\psi & -g\psi & e\gamma^0\psi & 0 & 0 \\ -i\gamma^0 & 0 & 0 & 0 & 0 & -g\overline{\psi} & -g\overline{\psi} & e\overline{\psi}\gamma^0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6a\lambda\phi & 3a\phi^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3a\phi^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ g\psi & g\overline{\psi} & -6a\lambda\phi & -3a\phi^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ g\psi & g\overline{\psi} & -3a\phi^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -e\gamma^0\psi & -e\overline{\psi}\gamma^0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \partial^2 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\partial^2 & 0 & 0 \end{bmatrix}$$

We find the Fadeev-Popov determinant as

$$\triangle_F = \left[\det\{\varphi_\alpha, \varphi_\beta\}\right]^{1/2} = \det\left(9a^2\phi^4\partial^2\right), \tag{3.37}$$

$$= \det\left(9a^2\phi^4\right)\det\left(\partial^2\right). \tag{3.38}$$

This result is not an unexpected one. Since the vector field couples only with the fermions, it has nothing to do with the constraint system. So quantization of the model can be considered in two different parts. First one is the constrained one, which we have shown the quantization in details in the previous chapter. The other one is the electromagnetic field quantization. We may show this quantization, but it is useless since one can find it in basic QFT text books [25, 43, 45].

3.3 Perturbation Expansion of Correlation Functions

We end up with an effective Lagrangian density for the gauged model as follow.

$$\mathscr{L} = -\frac{1}{4}(F_{\mu\nu}F^{\mu\nu}) + \overline{\psi}(i\partial\!\!\!/ - eA\!\!\!/ + g\Phi)\psi - \frac{a}{4}\Phi^4 + \mathscr{L}_{\text{ghost}} + \mathscr{L}_{\text{gau.fix.}}. \tag{3.39}$$

The ghost fields and one of the scalar field Λ is decoupled from the model like before. Note that the spinor field couples to composite scalar field Λ by the coupling constant g where it couples to vector field by the coupling constant e. We also have a self interaction for the composite scalar field with the coupling a.

3.3.1 Photon propagator

We have the same photon two point correlation function as in QED.

$$\frac{-ig^{\mu\nu}}{k^2 + i\varepsilon}. \tag{3.40}$$

Remark that we take the vector field propagator in the Feynman gauge. We will clarify this choice in the following subsection.

3.3.2 Spinor propagator

We have no essential change in the spinor propagator in Feynman gauge. The massless Dirac propagator is

$$\frac{ip\!\!\!/}{p^2 + i\varepsilon}. \tag{3.41}$$

Dynamical mass generations is studied in various ways. In reference [14] Miransky explains how for coupling constant α less than $\pi/3$, there is no mass generation in the quenched approximation. Here $\alpha = \frac{e^2}{4\pi}$. J.C.R. Bloch, in his Durham thesis, [46], explores the range where this result is valid when the calculation is done without this approximation. He states that the quenched and the rainbow approximations, used by Miransky and collaborators, have non physical features, namely they are not gauge invariant, making the calculated value vary wildly depending on the particular gauge used. Bloch, himself, uses the Ball-Chiu vertex, [47], instead of the bare one, where the exact longitudinal part of the full QED vertex, is uniquely determined by the Ward-Takahashi identity relating the vertex with the propagator. The transverse part of the vertex, however, is still arbitrary. Bloch then considers a special form of the Curtis-Pennington vertex [48] in which the transverse part of the vertex is constructed by requiring the multiplicative renormalizability of the fermion propagator with additional assumptions.

Bloch claims that for the different gauges used with this choice, he gets rather close values for the critical coupling [49]. He also performs numerical calculations where the approximations are kept to a minimum. The results are given in the table on pg. 202 of hep-ph/0208074.

Using on the arguments in the Bloch's thesis, also using the results of his numerical calculations, we conclude that at least for $\alpha < 0.5$ we can safely claim that there will be no mass generation or the assumed γ_5 symmetry will not be broken. Since we do not study heavy ion processes, the numerical value we have for α will be much smaller than this limit. Hence, our results will be valid. Note that in QCD mass generation occurs at relatively low energies, where the coupling constant has already increased.

3.3.3 Composite scalar propagator

As we find that, there is no mixing between the vector field and the composite scalar fields. We have the same composite propagator found in the previous chapter in equation (**3.42**).

$$-i\frac{4\pi^2}{g^2}\frac{\varepsilon}{q^2}. \tag{3.42}$$

As before, the ε factor in this correlation function will play an important role.

3.4 Interactions

In the crude model there were two interactions. They were namely Yukawa interaction and four composite scalar interaction. In the gauged model one more interaction appears. This interaction is between the vector field and spinor fields with the coupling constant e.

In Appendix (C.1.2) we give the Feynman rules for the gauged model in Minkowski space.

In the next subsections we will analyze the effects of the new interaction to the previous vertices. We will also check the higher order corrections of these vertices.

3.4.1 Yukawa vertex

In our model to leading order in $1/N$, the contribution to the Yukawa vertex up to the first order was shown by the diagrams in figure (2.3). In gauged version we have one additional correction which is not finite. All the corrections up to the first order to the vertex is shown in figure (3.1).

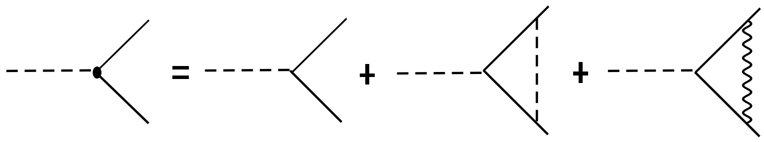


Figure 3.1. The diagram of the contribution to Yukawa Vertex up to the first order in the gauged model.

The scalar correction to the Yukawa vertex is shown to give finite result in the previous chapter. Here we will check the vector field correction to the vertex given in figure (3.2).

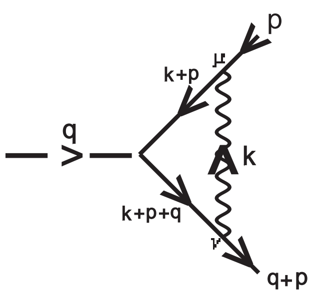


Figure 3.2. One loop vector field correction to the Yukawa vertex

Vector field contribution can be expressed by

$$I_4[p,q] = \int_k (-ie\gamma^\beta) \frac{i}{\not{k}+\not{p}+\not{q}} (-ig) \frac{i}{\not{k}+\not{p}} (-ie\gamma^\alpha) \frac{-ig_{\beta\alpha}}{k^2}, \tag{3.43}$$

$$= -(ge^2) \int_k \frac{\gamma^\beta (\not{k}+\not{p}+\not{q})(\not{k}+\not{p})\gamma_\beta}{(k+p+q)^2 (k+p)^2 k^2}. \tag{3.44}$$

Very similar calculation has done for the Yukawa vertex correction in the previous section. Therefore briefly we give the calculations here. The Feynman parametrization is the same with given equation (**2.125**). We find immediately

$$= -(ge^2) \frac{\Gamma(3)}{\Gamma(1)^3} \int_0^1 d\alpha \int_0^\alpha d\beta \int_k \frac{\gamma^\beta (\not{k}+\not{p}+\not{q})(\not{k}+\not{p})\gamma_\beta}{\left[(k+\beta p+\alpha q)^2 + M^2\right]^3}. \tag{3.45}$$

Here M^2 has the same definition with the equation (**2.127**).

$$M^2 = \beta(1-\beta)q^2 + \alpha(1-\alpha)p^2 + 2\beta(1-\alpha)pq. \tag{3.46}$$

We can make a shift $K = k + \beta p + \alpha q$. The odd terms of momenta will not contribute, so we can ignore them here. Up to now everything is same as what we have done in the previous chapter except some coefficients. The main difference comes from the numerator.

$$\gamma^\beta (\not{K}+\not{P})(\not{K}+\not{Q})\gamma_\beta. \tag{3.47}$$

Here

$$\not{P} = [(1-\alpha)\not{p} + (1-\beta)\not{q}], \tag{3.48}$$

$$\not{Q} = [(1-\alpha)\not{p} - \beta\not{q}]. \tag{3.49}$$

Using the properties of the gamma functions given in appendix equation (**C.8**), we get

$$\gamma^\beta (\not{K}+\not{P})(\not{K}+\not{Q})\gamma_\beta = \frac{D^2}{D} K^2 + \gamma^\beta \not{P}\not{Q}\gamma_\beta. \tag{3.50}$$

Here D is the dimension.

$$= -(ge^2) \frac{\Gamma(3)}{\Gamma(1)^3} \int_0^1 d\alpha \int_0^\alpha d\beta \int_K \frac{DK^2 + \gamma^\beta \not{P}\not{Q}\gamma_\beta}{(K^2+M^2)^3}, \tag{3.51}$$

$$= -(ge^2) \frac{\Gamma(3)}{\Gamma(1)^3} \int_0^1 d\alpha \int_0^\alpha d\beta \int_K \left[\frac{D}{(K^2+M^2)^2} + \frac{\gamma^\beta \not{P}\not{Q}\gamma_\beta - DM^2}{(K^2+M^2)^3}\right]. \tag{3.52}$$

Since we are looking for the divergency we omit the finite part here. The evaluation of the integrals, due to the equation (**C.18**), give us

$$I_4[p,q] = -(ge^2)\frac{2iD}{(4\pi)^2}\left[\frac{2}{\varepsilon}+\text{finite contributions}\right], \tag{3.53}$$

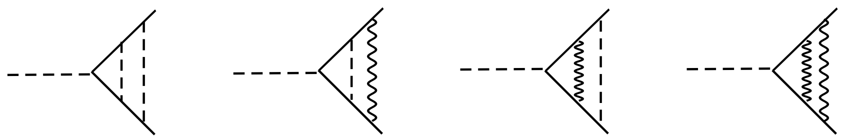
$$= (-ig)\frac{16e^2}{16\pi^2}\frac{1}{\varepsilon}+\text{finite contributions}. \tag{3.54}$$

In the original model we do not need an infinite renormalization for the Yukawa vertex. This result is changed in the gauged model. We will proceed renormalization to this vertex in the following sections.

3.4.2 Higher order corrections of the Yukawa vertex

In this subsection we will check the higher order contributions to the Yukawa vertex in the gauged model. Previously we showed that the finite renormalization of this vertex has been changed to infinite renormalization by the vector field contribution.

The two loop planar corrections to the Yukawa vertex can be given as follows.



(a) **(b)** **(c)** **(d)**

Figure 3.3. Two loop corrections of the Yukawa vertex (a) Two composite scalar field correction, (b) Scalar vector field correction, (c) Vector Scalar field correction, (d) Two vector field correction

We have previously showed that two composite scalar field correction shown in figure 3.3.a is finite. The mixed corrections, figure 3.3.b and figure 3.3.c, give a $1/\varepsilon$ type infinity. The last correction, figure 3.3.d, is the worst term which gives a $(1/\varepsilon)^2$ type infiniteness. Every higher vector field insertion gives another loop and this results with another $1/\varepsilon$ type correction to the vertex. Naively the planar vector field contributions can be summed up as a geometric series [50].

3.4.3 Vector spinor vertex

This new vertex was absent in the ungauged model. In literature this vertex , for the purely electromagnetic case, is vastly studied [47, 48]. Here we give the corrections to the vector spinor field interaction corrections up to the first order as follows.

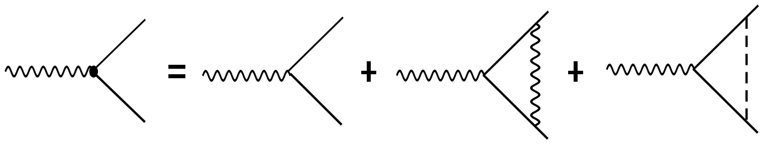


Figure 3.4. Vector spinor field vertex

First we should find the vector field correction to the vector spinor field vertex. We can define the momenta of these vertex as follows.

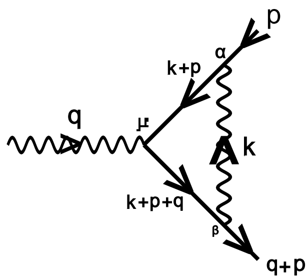


Figure 3.5. Vector field correction to the Vector Spinor field Vertex in one loop.

This one loop correction can be expressed by the terms as,

$$I_5[p,q] \;=\; \int_k (-ie\gamma^{\beta})\frac{i}{\not{k}+\not{p}+\not{q}}(-ie\gamma_{\mu})\frac{i}{\not{k}+\not{p}}(-ie\gamma^{\alpha})\frac{-ig_{\alpha\beta}}{k^2} \tag{3.55}$$

$$= \; -e^3\int_q \frac{\gamma^{\beta}(\not{k}+\not{p}+\not{q})\gamma_{\mu}(\not{k}+\not{p})\gamma_{\beta}}{(k+p+q)^2(k+p)^2k^2} \tag{3.56}$$

We realize that, we have done a very similar calculation before in equation (**3.44**).

$$= -e^3\frac{\Gamma(3)}{\Gamma(1)^3}\int_0^1 d\alpha\int_0^{\alpha} d\beta\int_k \frac{\gamma^{\beta}\,(\not{k}+\not{p}+\not{q})\,\gamma_{\mu}\,(\not{k}+\not{p})\,\gamma_{\beta}}{\left[(k+\beta p+\alpha q)^2+M^2\right]^3}. \tag{3.57}$$

After shifting the momentum as usual,

$$= -e^3\frac{\Gamma(3)}{\Gamma(1)^3}\int_0^1 d\alpha\int_0^{\alpha} d\beta\int_K \frac{\gamma^{\beta}\,(\not{K}+\not{P})\,\gamma_{\mu}\,(\not{K}+\not{Q})\,\gamma_{\beta}}{[K^2+M^2]^3}. \tag{3.58}$$

We see only one difference from the previous one. The numerator is as follows.

$$\gamma^{\beta}\,(\not{K}+\not{P})\,\gamma_{\mu}\,(\not{K}+\not{Q})\,\gamma_{\beta} = \frac{(2-D)^2}{D}K^2\gamma_{\mu}+\gamma^{\beta}\not{P}\gamma_{\mu}\not{Q}\gamma_{\beta} \tag{3.59}$$

Replacing this numerator we obtain,

$$= \; -e^3\frac{\Gamma(3)}{\Gamma(1)^3}\int_0^1 d\alpha\int_0^{\alpha} d\beta\int_K \frac{\frac{(2-D)^2}{D}K^2\gamma_{\mu}+\gamma^{\beta}\not{P}\gamma_{\mu}\not{Q}\gamma_{\beta}}{(K^2+M^2)^3}, \tag{3.60}$$

$$= \; -e^3\frac{\Gamma(3)}{\Gamma(1)^3}\int_0^1 d\alpha\int_0^{\alpha} d\beta\int_K \left[\frac{\frac{(2-D)^2}{D}\gamma_{\mu}}{(K^2+M^2)^2}+\frac{\gamma^{\beta}\not{P}\gamma_{\mu}\not{Q}\gamma_{\beta}-\frac{(2-D)^2}{D}M^2\gamma_{\mu}}{(K^2+M^2)^3}\right] \tag{3.61}$$

Since we are looking for the divergency we omit the finite part here. The evaluation of the integrals, due to the equation (**C.18**), give us

$$I_5[p,q] = -e^2 \frac{i}{(4\pi)^2} \frac{(2-D)^2}{D} \gamma_\mu \left[\frac{2}{\varepsilon} + \text{finite contributions} \right], \tag{3.62}$$

$$= (-ie\gamma_\mu) \frac{2e^2}{16\pi^2} \frac{1}{\varepsilon} + \text{finite contributions}. \tag{3.63}$$

Here we find an infinite contribution of the vector correction to the spinor vector vertex.

The second correction we can deal with is the composite field correction. We give this correction in a diagrammatic way in figure 3.6.

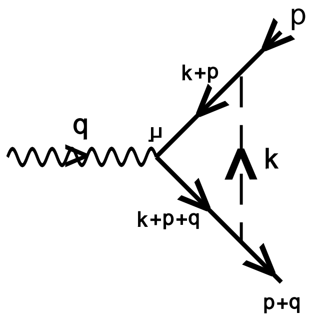


Figure 3.6. Composite scalar field correction to the Vector Spinor field Vertex in one loop.

This correction can be expressed as

$$I_6[p,q] = \int_k (-ig) \frac{i}{\not{k} + \not{p} + \not{q}} (-ie\gamma_\mu) \frac{i}{\not{k} + \not{p}} (-ig) \frac{4\pi^2 \varepsilon}{g^2} \frac{-i}{k^2}, \tag{3.64}$$

$$= (-eg^2) \frac{4\pi^2 \varepsilon}{g^2} \int_k \frac{(\not{k} + \not{p} + \not{q})\, \gamma_\mu\, (\not{k} + \not{p})}{(k+p+q)^2 (k+p)^2 k^2}. \tag{3.65}$$

After the usual Feynman parametrization, given in equation (**C.14**) ,we get another similar expression. We make a shift in the momentum $K = k + \beta p + \alpha q$,

$$= (-eg^2) \frac{4\pi^2 \varepsilon}{g^2} \frac{\Gamma(3)}{\Gamma(1)^3} \int_0^1 d\alpha \int_0^\alpha d\beta \int_K \frac{(\not{K} + \not{P}) \gamma_\mu (\not{K} + \not{Q})}{[K^2 + M^2]^3}. \tag{3.66}$$

Again the only difference comes from the numerator.

$$(\not{K} + \not{P}) \gamma_\mu (\not{K} + \not{Q}) = \frac{(2-D)}{D} K^2 \gamma_\mu + \not{P} \gamma_\mu \not{Q}. \tag{3.67}$$

Then we get,

$$= (-eg^2)\frac{4\pi^2\varepsilon}{g^2}\frac{\Gamma(3)}{\Gamma(1)^3}\int_0^1 d\alpha\int_0^\alpha d\beta\int_K \frac{\frac{(2-D)}{D}K^2\gamma_\mu + \not{P}\gamma_\mu\not{Q}}{[K^2+M^2]^3}, \quad \textbf{(3.68)}$$

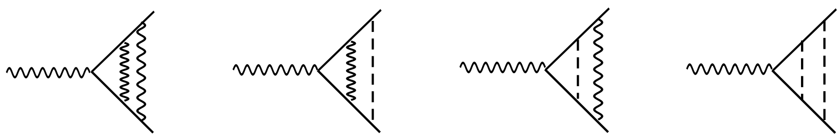
$$= (-eg^2)\frac{4\pi^2\varepsilon}{g^2}\frac{\Gamma(3)}{\Gamma(1)^3}\int_0^1 d\alpha\int_0^\alpha d\beta\int_K \left[\frac{\frac{(2-D)}{D}\gamma_\mu}{[K^2+M^2]^2} + \frac{\not{P}\gamma_\mu\not{Q} - \frac{(2-D)}{D}M^2\gamma_\mu}{[K^2+M^2]^3}\right] \textbf{(3.69)}$$

Then we get a finite result as follows.

$$I_6[p,q] = \frac{-ie\gamma_\mu}{8} + O(\varepsilon). \quad \textbf{(3.70)}$$

3.4.4 Higher order corrections of the vector spinor vertex

In the previous subsection we find that only vector field correction needs an infinite renormalization while the scalar correction needs finite renormalization. We have discussed this result up to the first order. Here we will beyond the one loop and give the two loop planar corrections to the Vector Spinor vertex as follows.



(a) **(b)** **(c)** **(d)**

Figure 3.7. Two loop corrections of the Yukawa vertex (a) Two vector field correction, (b) Vector Scalar field correction, (c) Scalar vector field correction, (d) Two composite scalar field correction

We have previously showed that composite scalar field correction is finite. So the two loop composite scalar field correction shown in figure 3.7.d is also finite. The mixed corrections, figure 3.7.b and figure 3.7.c, give a $1/\varepsilon$ type infinity. Only the correction, figure 3.7.a, is the worst term which gives a $(1/\varepsilon)^2$ type infiniteness. Because every higher vector field insertion gives another loop and which produces another $1/\varepsilon$ type correction to the vertex. Naively the planar vector field contributions can be summed up as a geometric series [50], too.

3.4.5 Four composite scalar vertex

In our crude model, we have showed that the first order correction was the scalar box diagram. This vertex was the only vertex that needed an infinite renormalization. In the gauged model we have an additional correction to the box diagram starts from two loop. In figure 3.8, the correction up to the second order to this vertex is shown.

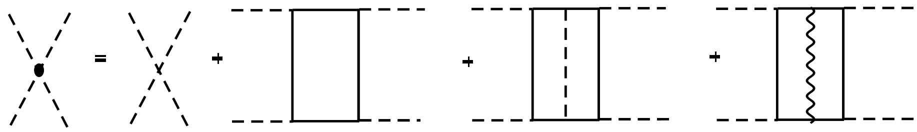


Figure 3.8. Four composite Scalar vertex with planar corrections.

As a conclusion up to the first order, there is no new correction to the four composite scalar vertex. The infinite contribution has been discussed in the previous chapter in detailed.

3.4.6 Higher order corrections of composite scalar vertex

In the crude model we showed that the scalar box diagram needed an infinite renormalization in one loop correction. The two loop contribution to the four scalar interaction, shown in figure 3.9.a, gives a finite result. The higher order corrections does not contribute to the vertex in the absence of the vector field.

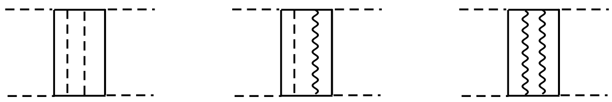
When additional the vector particle contributions are added, this expression is modified. The process where two scalar particles goes to two scalar particles gets further infinite contributions from the box type diagrams with vector field insertions, where one part of the diagram is connected to the non-adjacent part with a vector field as shown in the figure 3.9.b. All these diagrams go as $\frac{1}{\varepsilon}$ where ε is the parameter in dimensional regularization scheme. Figure 3.9.c., contributes to the box diagram renormalization. Also note that the diagram where the internal photon is connecting adjacent sides, as shown figure 3.9.d., will be a contribution to the coupling constant renormalization of one of the vertices. Since this is not a new contribution, we will not consider it separately.

(a) **(b)** **(c)** **(d)**

Figure 3.9. Correction to the fermion box diagram (a) Composite scalar field to non adjacent part, (b) Vector field to non adjacent part, (c) Composite scalar field to adjacent part, (d) Vector field to adjacent part.

For the three loop contribution, we have already showed that figure 3.10.a results to zero with the power of ε. Figure 3.10.c give an infinite contribution to the power of

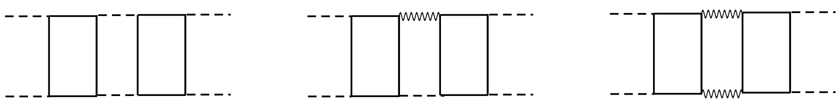
$\frac{1}{\varepsilon}$. Note that mixed scalar and vector insertions, figure 3.10, do not give additional infinities, since the scalar propagator reduces the degree of divergence.



(a) **(b)** **(c)**

Figure 3.10. Three loop correction to the fermion box diagram (a) Two scalar field correction, (b) Scalar and vector field correction, (c) Two vector field correction

Another three loop correction can be diagrammatically expressed as in figure 3.11. We already showed the contribution given in figure 3.11.a. Due to that result this diagram contributes as $1/\varepsilon$. Figure 3.11.b, does not contribute since the odd number of gamma matrices gives a zero result. Finally figure 3.11.c has no contribution due to the current conservation.



(a) **(b)** **(c)**

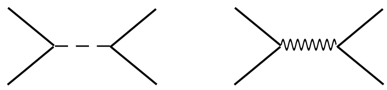
Figure 3.11. Three loop correction to the fermion box diagram (a) Two scalar field channel correction, (b) One scalar and one vector field channel correction, (c) Two vector field channel correction.

Therefore we conlude that there are no higher divergences for this process.

3.4.7 Spinor scattering

As a result of previous analysis in the second chapter, in the ungauged version, we end up with a model where there is no scattering of the fundamental fields, i.e. the spinors, whereas the composite scalar fields can take part in a scattering process. The coupling constant for the scattering of the composite particles runs, whereas the coupling constant for the spinor-scalar interaction does not run.

This result changes drastically when the gauged model is studied instead of the original one. The process in figure 3.12.a , which is prohibited in the previous model, now is possible due to the presence of the vector field channel. In lowest order this process goes through the tree diagram given in figure 3.12.b.

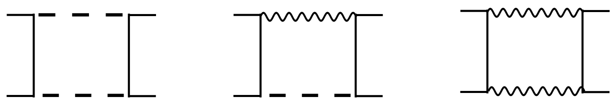


(a) **(b)**

Figure 3.12. Two fermion scattering (a) Through the scalar particle channel, (b) Through the vector channel.

3.4.8 Higher order corrections of spinor scattering

The spinor scattering processes, figure 3.13.a, was prohibited in the previous model. The process is finite though, since at the next higher order the QED box diagram with two spinors and two vector particles, figure 3.13.c, is ultraviolet finite from dimensional analysis, and is calculated in reference [51].



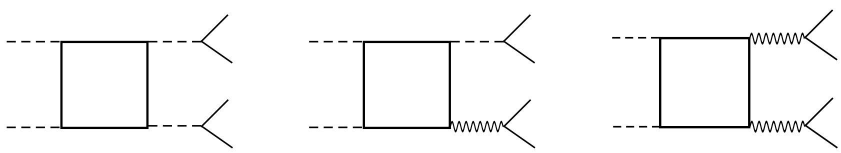
(a) **(b)** **(c)**

Figure 3.13. Higher order diagram for spinor scattering (a) Via two scalar field channel, (b) Via one scalar one vector field channel, (c) Via two vector field channel,

The mixed diagrams, figure3.13.b, do not give a contribution. We conclude that higher orders of spinor scattering do not give new type of ultra violet divergences.

3.4.9 Spinor production

In the crude model, we showed that if the composite scalar particles are used as intermediaries, there is no spinor production, figure 3.14.a.



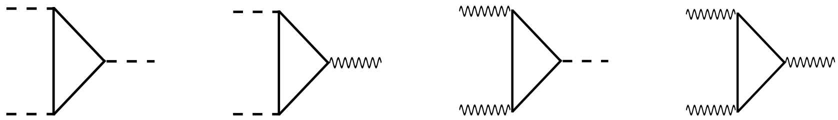
(a) **(b)** **(c)**

Figure 3.14. Spinor production (a) Via scalar particles are used as intermediaries, (b) Via scalar particle are used as intermediaries, (c) Via vector particles are used as intermediaries.

In the gauged model, instead of the composite scalar particles, vector particles can be used as intermediaries, figure 3.14.c. Then spinor production becomes possible. If we use one vector and one scalar particles as intermediaries, we have a zero contribution due to the trace of the odd number of gamma matrices.

3.4.10 Triangle interactions

In the crude model three composite scalar interaction, figure 3.15.a, is not allowed due trace rule. In the gauged model we have additional interactions. One of them, the interaction given in figure 3.15.c, is not allowed with the same reason.



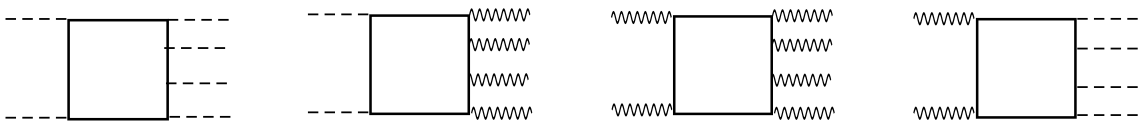
(a) **(b)** **(c)** **(d)**

Figure 3.15. Triangle Interactions (a) Three scalar fields, (b) Two scalar and one vector field, (c) Two vector and one scalar field, (d) Three vector fields.

The other interactions, given in figure 3.15.b and figure 3.15.d, vanish due to the Furry's theorem [52].

3.4.11 Multi scattering processes

In the crude model, we have demonstrated that we can have scattering processes where two scalar particles go to an even number of scalar particles bigger than two. In the one loop approximation all these diagrams give finite results, figure 3.16.a.



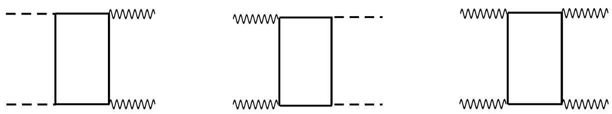
(a) **(b)** **(c)** **(d)**

Figure 3.16. (a) Two composite scalar field scatter to even number of scalar fields, (b) Two composite scalar field scatter to even number of vector fields, (c) Two vector field scatter to even number of vector fields, (d) Two vector field scatter to even number of composite scalar fields.

In the gauged model, we also have scattering processes where two vector particles go to an even number of vector particles, like the case in the standard electrodynamics, figure 3.16.c. Since going to an odd number of vector particles is forbidden by the Furry theorem, we can also argue that vector ϕ particles can go to an even number of vector particles only. This assertion is easily checked by diagrammatic analysis.

3.4.12 Other scattering processes

The $< A_\mu A^\mu \phi^2 >$ can be generated by the spinor box diagram, figure 3.17.a and figure 3.17.b, thus making contact between the scalar and the vector particles in the theory, although such an interaction does not exist in the original Lagrangian. They do not contribute due to the conservation of current.



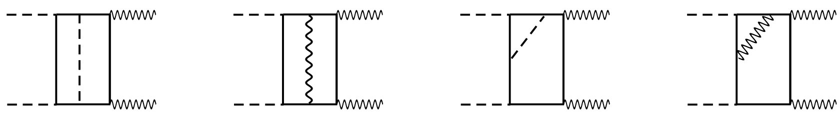
(a) **(b)** **(c)**

Figure 3.17. (a) Two composite scalar fields scatter to two vector fields, (b) Two vector fields scatter to two composite scalar fields, (c) Two vector fields scatter to two vector fields.

Another scattering is between four vector fields. They also use fermion box diagram. This diagram gives a finite contribution which has been wildly discussed in the literature [53–55].

3.4.13 Higher order corrections to the other processes

We can analyze the higher order corrections of the scattering of two vector particles from two composite scalar particles. These corrections can be done by a scalar or vector field to the adjacent and non adjacent part of the fermion box, given in figure 3.18.



(a) **(b)** **(c)** **(d)**

Figure 3.18. Two composite scalar fields scatter to two vector fields, (a) One scalar field correction to the nonadjacent part, (b) One vector field correction to the nonadjacent part, (c) One scalar field correction to the adjacent part, (d) One vector field correction to the adjacent part.

All these two loop corrections do not contribute to the model due to the current conservation. The three loop corrections are given in figure 3.19.

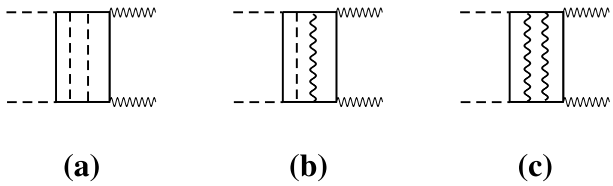


Figure 3.19. Two composite scalar fields scatter to two vector fields, (a) Two scalar fields correction to the nonadjacent part, (b) One vector and one scalar fields correction to the nonadjacent part, (c) Two vector fields correction to the adjacent part.

In the Appendix B we gave some other possible interactions in higher orders. Some of them are finite, some of them are zero. We can conclude that there is no contribution to the infinite renormalization of the interaction in higher orders.

3.5 RG Analysis of the Gauged Model

The original model had two coupling constants, g and a. In the new model, where an elementary vector field is added to the model, we add a new coupling constant e which describes the coupling of the vector field to the spinors. Here all three coupling constants are renormalized.

We can write the three first order renormalization group equations for these three coupling constants similar to the analysis in section 2.10.

$$16\pi^2 \frac{de(t)}{dt} = be^3(t), \qquad \textbf{(3.71)}$$

$$16\pi^2 \frac{dg(t)}{dt} = -cg(t)e^2(t), \qquad \textbf{(3.72)}$$

$$16\pi^2 \frac{da(t)}{dt} = -ug^4(t). \qquad \textbf{(3.73)}$$

where b, c, u are numerical constants. These values are found as $b = 2$, $c = 16$, $u = 48$. These processes are illustrated in diagrams shown in figures (2.7), (3.2), (3.5) above.

Here we take μ_0 as a reference scale at low energies, $t = ln(\mu/\mu_0)$, where μ is the renormalization point.

The first RG equation, equation (**3.71**), can be rewritten as

$$\frac{de}{e^3} = \frac{b}{16\pi^2} dt. \qquad \textbf{(3.74)}$$

The integration of the equation let us to the solution as

$$\frac{1}{e_0^2} - \frac{1}{e^2} = \frac{2b}{16\pi^2}t. \tag{3.75}$$

Finally we can write the solution of the vector coupling constant as

$$e(t)^2 = \frac{e_0^2}{1 - \frac{2be_0^2}{16\pi^2}t}. \tag{3.76}$$

We will use this solution for solving the next equation, equation (**3.72**). If we replace it, we get

$$\frac{dg}{g} = -\frac{ce_0^2}{16\pi^2}\frac{dt}{1 - \frac{2be_0^2}{16\pi^2}t}. \tag{3.77}$$

This integral can be done easily by a variable change. The final result is

$$g(t) = g_0\left(1 - \frac{2be_0^2}{16\pi^2}t\right)^{c/2b}. \tag{3.78}$$

This solution let us to find the final coupling constant solution. If we replace the solution of the g coupling constant to equation (**3.73**), we have

$$da = -\frac{ug_0^4}{16\pi^2}\left(1 - \frac{2be_0^2}{16\pi^2}t\right)^{2c/b} dt. \tag{3.79}$$

Integration of this solution we find the final result as,

$$a = a_0 + \frac{ug_0^4}{2e_0^2(2c+b)}\left(1 - \frac{2be_0^2}{16\pi^2}t\right)^{\frac{2c}{b}+1}. \tag{3.80}$$

We can substitute the values of b, c and u constants, but instead of this substitution we have to point a crucial problem. Here we face with the main problem of models with U(1) coupling, namely the Landau pole. The coupling constant $e(t)$ diverges with a finite value of $t = 16\pi^2/2be_0^2$. This pole makes our new gauged model a trivial one.

3.6 Conclusion

In this chapter we discuss the differences between the ungauged and gauged model. These models are widely analyzed in this and previous chapters. We find out that many of the features of the original model are not true anymore. Like in the crude model only composite scalar particles are taking part in physical processes but now the constituent fields too. As far as renormalizations are concerned, we have essentially QED, with corrections coming from the scalar part mimicking the Yukawa interactions with the Φ^4 term added. We end up with a system mimicking, the gHY system, although our starting point is gauging a constrained model.

We also have scattering processes where two scalar particles go to an even number of scalar particles, or scattering of spinor particles from each other. In the one loop approximation all these diagrams give finite results, like the case in the standard Yukawa coupling model. We also have creation of spinor particles from the interaction of scalars, as well as scattering of spinors with each other, and all the other processes in the gHY system.

In the renormalization of the couplings, we encountered a Landau pole, which means at a certain finite energy, the coupling constant of the vector fields diverges. This divergency makes the model a trivial one.

4. NON-PERTURBATIVE RENORMALIZATION GROUP AND RENORMALIZABILITY OF A GÜRSEY MODEL INSPIRED FIELD THEORY

We encounter with a Landau pole in the gauged model. We expect that coupling to a non-Abelian gauge theory will remedy this defect by new contributions to the RGE's. Thus, obtaining a nontrivial model will be possible. Coupling to a non-Abelian gauge field will also give us more degrees of freedom in studying the behavior of the beta function. This may allow us to find the critical number of gauge and fermion fields to obtain a zero of this function at a nontrivial values of the coupling constants of the model. We will study this in the this chapter.

4.1 Gauging with a non Abelian Gauge Field

Here we couple an $SU(N_C)$ gauge field to the model instead of $U(1)$. We also take spinors with different flavors, up to N_f. Similar to the Lagrangian density given in the previous chapter in equation (**3.2**), we consider the new Lagrangian density as follows

$$\mathscr{L} = \sum_{i=1}^{N_f} i\overline{\Psi}_i \not{D} \psi_i + g\sum_{i=1}^{N_f} \overline{\Psi}_i \psi_i \phi + \lambda \left(g\sum_{i=1}^{N_f} \overline{\Psi}_i \psi_i - a\phi^3 \right) - \frac{1}{4} Tr[F_{\mu\nu}F^{\mu\nu}]. \qquad \textbf{(4.1)}$$

Upon performing constraint analysis similar to the one performed in section 3.2, we see that we have to satisfy

$$g\sum_{i=1}^{N_f} \overline{\Psi}_i \psi_i - a\phi^3 = 0, \qquad \textbf{(4.2)}$$

$$3a\lambda\phi^2 - g\sum_{i=1}^{N_f} \overline{\Psi}_i \psi_i = 0. \qquad \textbf{(4.3)}$$

After calculating the constraint matrix, raising the result to the exponential by using ghost field, and performing the transformations $\Phi = \phi + \lambda$ and $\Lambda = \phi - \lambda$ we get similar equations. We see that both the Λ and the ghost fields coming from the compositeness constraint decouple from our model.

At this point we have to note that there are two kinds of ghost contributions in the new model. The ghosts coming from the gauge condition on the vector field do not decouple, and contribute to the renormalization group equations in the usual way. We impose these constraints on equation (**4.1**).

These steps are done in the previous chapters, so we omit to redo it explicitly. Similar to equation (**3.39**), we end up with the effective Lagrangian density given as

$$\mathscr{L}' = -\frac{1}{4}Tr[F_{\mu\nu}F^{\mu\nu}] + \sum_{i=1}^{N_f} \overline{\psi}_i[i\not{D}\psi_i + g\Phi]\psi_i - \frac{a}{4}\Phi^4 + \mathscr{L}_{\text{ghost}} + \mathscr{L}_{\text{gau.fix.}}. \quad \textbf{(4.4)}$$

Here N_f is the number of flavors. The gauge field belongs to the adjoint representation of the color group $SU(N_C)$ where D_μ is the color covariant derivative. g, a, e are the Yukawa, quartic scalar and gauge coupling constants, respectively. We take N_f in the same order as N_C.

In Appendix (C.1.3) we give the Feynman rules of the model.

4.2 RG Equations

In the one loop approximation, the renormalization group equations read as

$$16\pi^2 \frac{d}{dt}e(t) = -be^3(t), \quad \textbf{(4.5)}$$

$$16\pi^2 \frac{d}{dt}g(t) = -cg(t)e^2(t), \quad \textbf{(4.6)}$$

$$16\pi^2 \frac{d}{dt}a(t) = -ug^4(t), \quad \textbf{(4.7)}$$

where b, c and u are positive constants given as

$$b = \frac{11N_C - 4T(R)N_f}{3}, \qquad c = 6C_2(R), \qquad u = 8N_fN_C. \quad \textbf{(4.8)}$$

Here $C_2(R)$ is the second Casimir, $C_2(R) = \frac{(N_C^2-1)}{2N_C}$ and R is the fundamental representation with $T(R) = \frac{1}{2}$. We take μ_0 as a reference scale at low energies, $t = ln(\mu/\mu_0)$, where μ is the renormalization point.

In the RGE we see that the diagrams, where scalar propagators take part, are down by powers of ε. Hence we do not have contributions proportional to $a^2(t)$, $g^3(t)$ and $a(t)g^2(t)$, as one would have in the gHY system as described in the work of [31].

Since the diagrams, omitted in [31] via a $\frac{1}{N_c}$ analysis, are down by an order of ε in our analysis, we do not need a relation between N_C, N_f and the coupling constants at this point.

In the next section we will solve the RGE's and then analyze the solution in the following sections.

4.3 Solutions of the RGE's

The solution for the first RG equation (**4.5**) can be obtained easily as follows. First we take the integral,

$$16\pi^2 \int \frac{de(t)}{e^3(t)} = -b \int dt. \tag{4.9}$$

We find

$$\frac{1}{e^2} = \frac{1}{e_0^2}\left(1 + \frac{b}{2\pi}\frac{e_0^2}{4\pi}t\right). \tag{4.10}$$

Here we define $\alpha_0 = \frac{e_0^2}{4\pi}$. Then we can rewrite the solution of the RG equation as

$$e^2(t) = e_0^2\left(1 + \frac{b\alpha_0}{2\pi}t\right)^{-1}. \tag{4.11}$$

We define

$$\eta(t) \equiv \frac{\alpha(t)}{\alpha_0} \equiv \frac{e^2(t)}{e_0^2} = \left(1 + \frac{b\alpha_0}{2\pi}t\right)^{-1} \tag{4.12}$$

where $e_0 = e(t=0)$ which is the initial value at the reference scale μ_0. Note that $\eta(0) = 1$ and $\eta(t \to \infty) = +0$.

We can solve the RG equations by defining RG invariants. To observe a RG invariant for the RG equation (**4.6**), we use the equality.

$$\frac{d}{dt}\left(\frac{e^2(t)}{g^2(t)}\right) = 2\left(\frac{e'}{e} - \frac{g'}{g}\right)\frac{e^2}{g^2}. \tag{4.13}$$

If we subtract equation (**4.6**) from equation (**4.5**), we find

$$16\pi^2\left(\frac{e'}{e} - \frac{g'}{g}\right) = (c-b)e^2. \tag{4.14}$$

If we multiply both sides with $\frac{e^2(t)}{g^2(t)}$, we find a similar equation with the equation (**4.13**),

$$\begin{aligned}\frac{d}{dt}\left(\frac{e^2(t)}{g^2(t)}\right) &= \frac{(c-b)}{2\pi}\frac{e_0^2}{4\pi}\frac{e^2(t)}{e_0^2(t)}\frac{e^2(t)}{g^2(t)} \\ &= \alpha_0\frac{(c-b)}{2\pi}\eta(t)\frac{e^2(t)}{g^2(t)}. \end{aligned} \tag{4.15}$$

Here we define a RG invariant $H(t)$ for the solution of the equation (**4.15**) where A is a constant.

$$H(t) = A(c-b)\eta^{\omega}\frac{e^2(t)}{g^2(t)}. \tag{4.16}$$

We need to determine ω for the solution. Since $H(t)$ is a constant, we have $\frac{dH(t)}{dt} = 0$. Therefore we can take the derivative of the constant as

$$\omega\eta^{\omega-1}\frac{d\eta(t)}{dt}\frac{e^2}{g^2} + \eta^{\omega}\frac{d}{dt}\left(\frac{e^2}{g^2}\right) = 0. \tag{4.17}$$

We need the derivative of the function $\eta(t)$ which is defined in equation (**4.12**). We find it as

$$\frac{d\eta(t)}{dt} = -\frac{b\alpha_0}{2\pi}\eta^2(t). \tag{4.18}$$

When we use the equations (**4.18**) and (**4.17**) in equation (**4.15**), we determine a relation for the ω constant as

$$-b\omega + c - b = 0. \tag{4.19}$$

Determination of the ω for the RG invariant lets us to write the RG invariant solution as,

$$H(t) = (c-b)\eta^{-1+\frac{c}{b}}(t)\frac{e^2(t)}{g^2(t)}. \tag{4.20}$$

Since $H(t)$ is a constant, we call it H_0. Then, the solution of the gauge coupling constant can be written as

$$g^2(t) = \frac{c-b}{H_0}\eta^{-1+\frac{c}{b}}(t)e^2(t). \tag{4.21}$$

This solution can be expressed in another form as

$$g^2(t) = \frac{c-b}{H_0} e_0^2 \eta^{\frac{c}{b}}(t). \quad \textbf{(4.22)}$$

For the solution of the last RG equation (**4.7**), we need the derivative of

$$\frac{d}{dt}\left(\frac{a(t)}{g^2(t)}\frac{e^2(t)}{g^2(t)}\right) = \left(\frac{a'}{a} + 2\frac{e'}{e} - 4\frac{g'}{g}\right)\left(\frac{a}{g^2}\frac{e^2}{g^2}\right). \quad \textbf{(4.23)}$$

To obtain this equation we need to multiply the equation (**4.5**) by two and equation (**4.6**) by minus four. Then we need to add them with the equation (**4.7**). Finally we have

$$16\pi^2\left(2\frac{e'}{e} - 4\frac{g'}{g} + \frac{a'}{a}\right) = (4c-2b)e^2 - u\frac{g^4}{a}. \quad \textbf{(4.24)}$$

In order to have a total derivative we need to multiply both sides with $\frac{a}{g^2}\frac{e^2}{g^2}$. Then we can write it as

$$\frac{d}{dt}\left(\frac{a(t)}{g^2(t)}\frac{e^2(t)}{g^2(t)}\right) = \frac{-u\alpha_0}{4\pi}\eta(t)\left[1 - \frac{2(2c-b)}{u}\frac{a(t)}{g^2(t)}\frac{e^2(t)}{g^2(t)}\right] \quad \textbf{(4.25)}$$

Here another RG invariant $K(t)$ can be defined by

$$K(t) = B\eta^{\lambda}\left[1 - \frac{2(2c-b)}{u}\frac{a(t)}{g^2(t)}\frac{e^2(t)}{g^2(t)}\right], \quad \textbf{(4.26)}$$

where B is a constant. Here λ should be determined. If we use the definition of the invariance, the following equation should be equal to zero.

$$\lambda\eta^{\lambda-1}\frac{d\eta}{dt}\left[1 - \frac{2(2c-b)}{u}\frac{a(t)}{g^2(t)}\frac{e^2(t)}{g^2(t)}\right] + \eta^{\lambda}\left(\frac{-2(2c-b)}{u}\right)\frac{d}{dt}\left(\frac{a(t)}{g^2(t)}\frac{e^2(t)}{g^2(t)}\right) \quad \textbf{(4.27)}$$

If the equations (**4.18**) and (**4.25**) are used in equation (**4.27**), we get a relation for the λ coefficient as follows.

$$\lambda = -1 + \frac{2c}{b} \quad \textbf{(4.28)}$$

Now we can define the RG invariant $K(t)$ for the last RG equation (**4.7**) as

$$K(t) = -u\eta^{-1+\frac{2c}{b}}\left[1 - \frac{2(2c-b)}{u}\frac{a(t)}{g^2(t)}\frac{e^2(t)}{g^2(t)}\right]. \quad \textbf{(4.29)}$$

We can then write the solution for the coupling constant $a(t)$ as

$$a(t) = \frac{u}{2(2c-b)}\frac{g^2(t)}{e^2(t)}g^2(t)\left[1+\frac{K_0}{u}\eta^{1-\frac{2c}{b}}(t)\right]. \tag{4.30}$$

Here K_0 is the value of the RG invariant. We can rewrite equation (**4.30**) in terms of all RG invariants as following.

$$a(t) = \frac{u(c-b)^2e_0^2}{2H_0^2(2c-b)}\left[\eta^{-1+\frac{2c}{b}}(t)+\frac{K_0}{u}\right]. \tag{4.31}$$

4.4 Some Limiting Cases

Before entering the detailed analysis in the next section, let us briefly see how the coupling constant solutions look like in some limiting cases. When we check the ultraviolet limit now, we find

$$\eta(t\to\infty)\to \quad +0, \quad b>0; \tag{4.32}$$

$$\eta^{\frac{c}{b}}(t\to\infty)\to \quad +0, \quad c,b>0; \tag{4.33}$$

and

$$\eta^{-1+\frac{2c}{b}}(t\to\infty)\to \begin{cases} +0, & 2c>b; \\ +0, & 2c>b>c; \\ +\infty, & b>2c. \end{cases} \tag{4.34}$$

We see that the constants H_0 and K_0 play important roles on the behavior of solutions of coupling equations (**4.11**), (**4.22**), (**4.31**). For $c>b$, H_0 should be positive; for $c<b$, H_0 should be negative to have the Yukawa coupling take a real value. This is necessary to have a unitary theory. Also for a region $c<b<2c$, with $H_0<0$, the unitarity condition is satisfied for all coupling constants. $K_0 \geq 0$ condition is also needed for stability of the vacuum. If $K_0<0$, we get $a(t\to\infty)<0$, which raises the problem of the vacuum instability.

Next we study the different limits our parameters can take:

4.4.1 $b\to+0$ limit case for finite t

In this limit if we analyze the equation (**4.12**), we see that $\eta(t)$ goes a constant, $\eta(t)=1$. But we have to clarify $\eta(t)^{-\frac{c}{b}}$ expression. If we expand it carefully in the $b\to 0$ limit, we find

$$\eta(t)^{-\frac{c}{b}} \longrightarrow exp\left(\frac{\alpha}{\alpha_c}t\right) = \left(\frac{\mu}{\mu_0}\right)^{\alpha/\alpha_c}, \tag{4.35}$$

where $\frac{c}{2\pi} = \frac{1}{\alpha_c}$ and $\alpha_0 = \alpha$. Then we obtain from the coupling constant equations (**4.11**), (**4.22**), (**4.31**) as

$$\begin{aligned} e^2(t) &= e_0^2, && \textbf{(4.36)} \\ g^2(t) &= \frac{ce_0^2}{H_0} exp\left(-\frac{\alpha}{\alpha_c}t\right), && \textbf{(4.37)} \\ a(t) &= \frac{uce_0^2}{4H_0^2}\left[exp\left(-\frac{2\alpha}{\alpha_c}t\right) + \frac{K_0}{u}\right]. && \textbf{(4.38)} \end{aligned}$$

This means that when we set the b term to zero, the Yukawa running coupling constant decreases exponentially to zero. For this limit the gauge and the quadratic coupling constants go just to a constant. Next we consider the limiting case $b = c$.

4.4.2 $c \to b$ limit case for finite t

Although the Yukawa coupling (**4.22**) appears to vanish in this limit case, a careful consideration of RGE leads to a non-vanishing result. If c approaches b, the limit depends on the value of H_0. If H_0 is non zero, $g^2(t)$ goes to zero. If H_0 goes to zero as a constant times $c - b$, i.e. $H_0 = H_1(c - b)$, we can write the Yukawa coupling as

$$g^2(t) = H_1 e^{ln\eta^{-1+c/b}} e^2(t). \qquad \textbf{(4.39)}$$

We can expand the exponential term, that gives us

$$g^2(t) = H_1\left[1 + \left(-1 + \frac{c}{b}\right) ln\eta + \ldots\right] e^2(t). \qquad \textbf{(4.40)}$$

In the limit, $g^2(t)$ yields to an expression which is proportional to the $e^2(t)$ coupling.

$$g^2(t) = H_1 e^2(t), \quad H_1 > 0. \qquad \textbf{(4.41)}$$

We also find that $a(t)$ is proportional to $e^2(t)$ like the $g^2(t)$ coupling as follows

$$\begin{aligned} e^2(t) &= e_0^2 \eta(t), && \textbf{(4.42)} \\ a(t) &= \frac{ue_0^2 H_1^2}{2b}\left[\eta(t) + \frac{K_0}{u}\right]. && \textbf{(4.43)} \end{aligned}$$

4.4.3 $2c \to b$ limit case for finite t

The case $b = 2c$ can be treated in a similar manner. Here, when $2c$ approaches b, the behavior of $a(t)$ changes. If we set

$$\frac{K_0}{u} = -1 + \frac{2c-b}{b} K_1, \tag{4.44}$$

then we see that a(t) coupling constant goes as $ln\eta(t)$. Therefore it yields

$$a(t) = \frac{ube_0^2}{8H_0^2}\left[K_1 + ln\eta(t)\right]. \tag{4.45}$$

This behavior is not allowed since $a(t)$ diverges as $t \to +\infty$. The other couplings are given as

$$e^2(t) = e_0^2 \eta(t), \tag{4.46}$$

$$g^2(t) = -\frac{b}{2H_0} e_0^2 \eta^{\frac{1}{2}}(t). \tag{4.47}$$

4.5 Nontriviality of the system

We now use the solutions of the RGE's found in the preceding section to investigate under what circumstances our model is nontrivial. We adopt the following criteria for the nontriviality:

All the running coupling constants:

- should not diverge at finite t>0 (no Landau poles);
- should not vanish identically;
- should not violate the consistency of the theory such as unitarity and/or vacuum stability.

Since the composite scalar field is the novel feature of our model, we will not consider the case when the scalar field is completely decoupled from the theory with $a(t) = g(t) = 0$, which is in fact a nontrivial QCD-like theory.

4.6 Fixed Point Solution

We derive the expressions given below from the RGE equations.

$$8\pi^2 \frac{d}{dt}\left[\frac{g^2(t)}{e^2(t)}\right] = (b-c)\left[\frac{g^2(t)}{e^2(t)}\right] e^2(t), \tag{4.48}$$

$$8\pi^2 \frac{d}{dt}\left[\frac{e^2(t)}{g^2(t)}\frac{a(t)}{g^2(t)}\right] = (2c-b)\left[\frac{e^2(t)}{g^2(t)}\frac{a(t)}{g^2(t)} - \frac{u}{2(2c-b)}\right] e^2(t). \tag{4.49}$$

For the fixed point solution, b equals c in equation (**4.48**). For this value, there is a single solution which satisfies both equations (**4.48**) and (**4.49**). This solution is given as,

$$\frac{e^2(t)}{g^2(t)} = \frac{1}{H_1}, \tag{4.50}$$

where H_1 is a constant, and

$$\frac{a(t)}{g^2(t)} = \frac{uH_1}{2c}. \tag{4.51}$$

If we take $H_0 = H_1(c-b)$ approaching zero as c approaches to b, while $K_0 = 0$ in equation (**4.31**) , then we find

$$g^2(t) = H_1 e^2(t), \tag{4.52}$$

$$a(t) = \frac{uH_1}{2c} g^2(t). \tag{4.53}$$

Since $\frac{g^2(t)}{e^2(t)}$ and $\frac{a(t)}{g^2(t)}$ are constants, the behavior of the Yukawa and quartic scalar couplings are completely determined by the gauge coupling. This corresponds to "coupling constant reduction" in the sense of Kubo, Sibold and Zimmermann [56]. In the context of the RGE, it corresponds to the Pendleton-Ross fixed point [57].

4.7 Yukawa Coupling

As seen from the previous sections the behavior of the Yukawa coupling depends on whether $c > b$ or $c < b$. The point where $c = b$ needs a special care. Moreover the sign of the H_0 is important.

4.7.1 c>b case

In this case H_0 should not equal to zero. Then we find in the UV limits

$$g^2(t \to \infty) \to \begin{cases} +0, & H_0 > 0; \\ -0, & H_0 < 0. \end{cases} \tag{4.54}$$

So the Yukawa coupling is asymptotically free. As it is seen, the sign of the RG invariant is important. It should be chosen positive not to cause the violation of stability of the vacuum.

In figure 4.1 we plot $g^2(t)$ vs. $e^2(t)$ for $c = 8, b = 7$. Both coupling constants approach the origin as t goes to infinity. Thus, our model fulfills the condition required by the asymptotic freedom criterion.

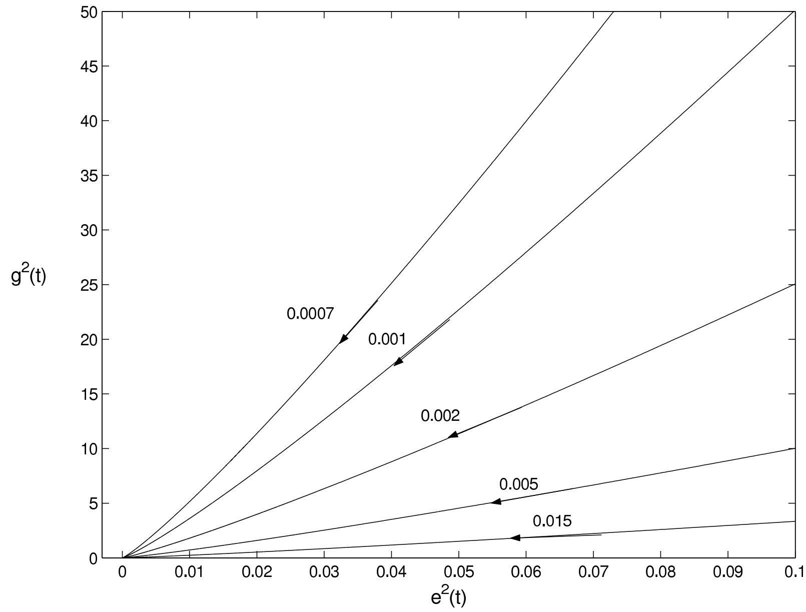


Figure 4.1. Plot of $g^2(t)$ vs. $e^2(t)$ for different values of H_0. The arrows denote the flow directions toward the UV region.

4.7.2 c<b case

In this case with a non zero value of H_0

$$g^2(t \to \infty) \to \begin{cases} -0, & H_0 > 0; \\ +0, & H_0 < 0. \end{cases} \tag{4.55}$$

For $H_0 < 0$, our system satisfies the asymptotic freedom condition. Our system does not have a Landau pole. In this respect it differs from the gHY system [31]. As shown below, there is a restriction on the value of b in this case.

4.7.3 c=b case

This is the fixed point solution analyzed above.

$$g^2(t) = H_1 e^2(t). \tag{4.56}$$

4.7.4 Quartic scalar coupling

$a(t)$ can be analyzed with four non trivial limits of the Yukawa coupling.

- $c > b$ with $H_0 > 0$,
- $c < b < 2c$ with $H_0 < 0$,
- $b > 2c$ with $H_0 < 0$,
- $c = b$ with $H_0 = 0$.

For $c > b$ case, we should have $H_0 > 0$, whereas in the $c < b < 2c$ case we have $H_0 < 0$. In both cases K_0 should be greater or equal to zero for the stability of the vacuum. In the third case, $b > 2c$ with $H_0 < 0$, for all the real values of K_0, $a(t)$ diverges in the UV limit. This means that there is no chance for a nontrivial theory in that region. Finally the $c = b$ case with $H_0 = 0$ has already be shown in equation (**4.45**). It is clear that in the UV limits K_0 should not take negative values.

As seen above these constraints give different relations between numbers of color and flavor. Note that in all the cases studied, if we take $K_0 < 0$, one can deduce from equation (**4.31**) that $a(t)$ can be made equal to zero for a finite value of t, a situation which should not be allowed. Therefore, we can use only the option with $K_0 \geq 0$. The standard model with three colors and six flavors satisfies the $c > b$ case.

For $K_0 = 0$ at the UV limit, the equation (**4.31**)

$$a(t) = \frac{u(c-b)^2 e_0^2}{2H_0^2(2c-b)} \eta^{-1+\frac{2c}{b}}(t) \to +0, \tag{4.57}$$

shows that the coupling constant is asymptotically free. Also for a non zero K_0, we find in the UV limit

$$a(t) \to \frac{(c-b)^2 e_0^2 K_0}{2H_0^2(2c-b)}. \tag{4.58}$$

Then the sign of the K_0 is crucial for the stability of the vacuum.

Although for $K_0 > 0$ we do not violate unitarity, we see that the asymptotic freedom criterion is not satisfied. The requirement of this criterion fixes K_0 at the value zero. In figure 4.2, we plot the RG flows in $(a(t), g^2(t))$ plane for different values of H_0 higher than zero while the gauge coupling $\alpha(t = 0)$ is fixed to 1. The origin is the limit where t goes to infinity, there both coupling constants approach zero when $K_0 = 0$.

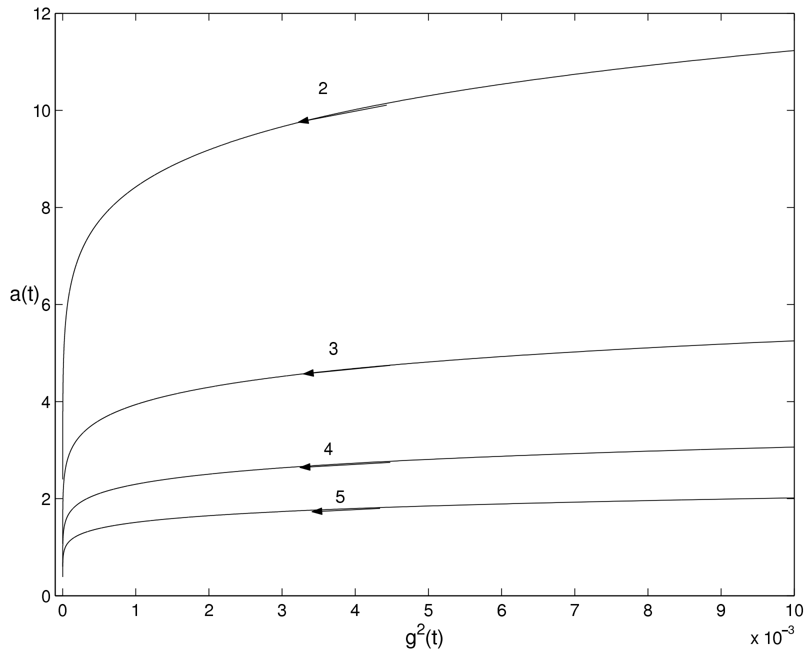


Figure 4.2. Plot of $a(t)$ vs. $g^2(t)$ for different values of H_0 while $K_0 = 0$.

4.8 Conclusion

Here we write the $SU(N)$ gauge version of the polynomial Lagrangian inspired by the Gürsey model. In the second chapter we find an interacting model, where only the composites take part in scattering processes, if only perturbative calculations are done. Gauging it with a constituent $U(1)$ field, in the third chapter, resulted in a model which looked like the gHY system, with all the problems associated with the Landau pole. In this chapter, when a $SU(N)$ gauge field is coupled, instead, we find that the renormalization group equations for the three coupling constants indicate that this model is nontrivial. All the coupling constants go to zero asymptotically as the cutoff parameter goes to infinity, exhibiting the behavior dictated by asymptotic freedom.

In equations (**4.48**) and (**4.49**) we give the equations for the ratios of the coupling constants and find the fixed points. We see that we can have nontrivial fixed points.

5. RESULTS AND DISCUSSION

With this thesis, we give a new interpretation of the old work of Akdeniz et al. [36]. First we see that the non polynomial original Gürsey model can be expressed in a polynomial form. We show that both models are equivalent, at least classically. We use the path integral method to quantize the system.

In the equivalent model, the interaction between the scalar and the fermions is a Yukawa interaction. Naturally it looks like a massless Yukawa model. But we find that this is not quite true. In the equivalent model, the scalar field is not elementary.

In the new version, we go to higher orders in our calculation beyond the one loop for the scattering processes. It is shown that by using the Dyson-Schwinger and Bethe-Salpeter equations some of the fundamental processes can be better understood. We see that while the non-trivial scattering of the fundamental fields is not allowed, bound states can scatter from each other with non-trivial amplitudes. This phenomena can be interpreted as another example of treating the bound states, instead of the principal fields, as physical entities, that go through physical processes such as scattering.

In our model we find that we need an infinite renormalization in one of the diagrams. Further renormalization is necessary at each higher loop, like any other renormalizable model. The difference between our model and other renormalizable models lies in the fact that, although our model is a renormalizable one using naive dimensional counting arguments, we have only one set of diagrams which are divergent. We need to renormalize only one of the coupling constants by an infinite amount. This set of diagrams, corresponding to the scattering of two bound states to two bound states, have the same type of divergence, i.e. $\frac{1}{\varepsilon}$ in the dimensional regularization scheme for all odd number of loops. The contributions from even number of diagrams are finite, hence require no infinite renormalization. The scattering of two scalars to four, or to any higher even number of scalars is finite, as expected to have a renormalizable model, whereas production of spinors from the scattering of scalars goes to zero as the cut-off is lifted [2].

After renormalization a further point would be to couple an elementary vector field to the model described in reference [2], in line with the process studied for the NJL model [28,29]. Our final goal was to investigate if we get a non-trivial theory when we couple a Yang-Mills system with color and flavor degrees of freedom. We study the abelian case, as an initial step.

In the third chapter, we summarize the changes in our results when this elementary vector field is coupled to the model described in reference [2]. The main conclusion is that our original model, in which only the composites take part in physical processes like scattering or particle production, is reduced to a gauged gHY model, where both the composites and the fundamental spinor and vector fields participate in all the processes. We have the known problems of the Landau pole of a mimicked gHY system, with all of its connotations of triviality, [3].

Finally, in the fourth chapter, we study our original model [2], coupled to a $SU(N)$ gauge field and use solely RG techniques. We derive the RG equations in one loop, and try to derive the criteria for obtaining non-trivial fixed points for the coupling constants of the theory. There we closely follow the line of discussion followed in our reference [31]. In our model, however, there is a composite scalar field with a propagator completely different from a constituent scalar field used in this reference. This gives rise to RG equations in our case which are different from those given by Harada et al. Since our starting models are different the motivation of our work is different from that of this reference. We show that the renormalization group equations point to the non-triviality of the model when it is coupled to an $SU(N)$ gauge field with a few remarks, [4].

5.1 Further studies

The most essential point of this thesis is the fact that the propagator of the composite scalar field is proportional to ε. This parameter comes from the dimensional regularization method. But physics should not depend on the regularization method. So checking these results in alternative methods, like inserting a cut off function, can be one of the important issue for the further studies.

Also one can study the beta functions and the anomalous dimensions of the models. Although the equivalent model looks like Yukawa model, we explain that it does not. Therefore one can find beta functions and the anomalous dimensions differently from the Yukawa model.

Alternatively one can study the models in ERG methods. Since the absence of the kinetic term of the composite scalar field in the equivalent Lagrangian density, we can not study in perturbative ERG. Also the parameter ε does not correspond anything. But we believe, it is still possible to study our model in unperturbative ERG method. Sonoda gave a lecture series for two weeks in 2007, in Istanbul. There he gave important clues for the non-perturbative aspects of the Wilson ERG [58], in the chapter 5.

Finally one can study the models in non-commutative space. At high energies the composite scalar propagator may give interesting results. All the different versions of the model, may give different processes.

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APPENDICES

A. CONSTRAINT ANALYSIS OF THE GÜRSEY MODEL

We start the analysis with the Lagrangian density

$$\mathscr{L}(q_i, \dot{q}_i) \;=\; i\,\overline{\psi}\partial\!\!\!/\psi + g\overline{\psi}\psi\phi + \lambda(g\overline{\psi}\psi - a\phi^3). \qquad \textbf{(A.1)}$$

The Euler Lagrange equations of the model are

$$\overline{\psi}\left[i\gamma^\mu \overleftarrow{\partial_\mu} - g(\phi+\lambda)\right] \;=\; 0, \qquad \textbf{(A.2)}$$

$$g\overline{\psi}\psi - a\phi^3 \;=\; 0, \qquad \textbf{(A.3)}$$

$$g\overline{\psi}\psi - 3a\lambda\phi^2 \;=\; 0. \qquad \textbf{(A.4)}$$

We can find solution to these equations of motions. We start with

$$i\partial_\mu\overline{\psi}\gamma^\mu - g\overline{\psi}(\phi+\lambda) \;=\; 0, \qquad \textbf{(A.5)}$$

$$i\partial_0\overline{\psi}\gamma^0 - i\partial_i\overline{\psi}\gamma^i - g\overline{\psi}(\phi+\lambda) \;=\; 0. \qquad \textbf{(A.6)}$$

We find

$$\partial_0\overline{\psi} \;=\; -i\left[i\partial_i\overline{\psi}\gamma^i + g\overline{\psi}(\phi+\lambda)\right]\gamma^0, \qquad \textbf{(A.7)}$$

We keep doing the simple algebra, the complex conjugation gives

$$\left[\partial_0\psi^\dagger\gamma^0\right]^\dagger \;=\; i\gamma^0\left[i\partial_i\psi\dagger\gamma^0\gamma^i + g\psi^\dagger\gamma^0(\phi+\lambda)\right]^\dagger, \qquad \textbf{(A.8)}$$

$$\gamma^0\partial_0\psi \;=\; i\gamma^0\left[-i\partial_i\gamma^0\gamma^i\psi + g\gamma^0(\phi+\lambda)\psi\right], \qquad \textbf{(A.9)}$$

$$\partial_0\psi \;=\; i\gamma^0\left[-i\partial_i\gamma^i\psi + g(\phi+\lambda)\psi\right]. \qquad \textbf{(A.10)}$$

Here we used $(\gamma^i)^\dagger = -\gamma^i$, $(\gamma^0)^\dagger = \gamma^0$ and $\{\gamma^0, \gamma^i\} = 0$.

Briefly we can rewrite the solutions as follows:

$$\partial_0\overline{\psi} \;=\; -i\,\overline{\psi}\left[i\,\gamma^i\overleftarrow{\partial_i} + g(\phi+\lambda)\right]\gamma^0, \qquad \textbf{(A.11)}$$

$$\partial_0\psi \;=\; i\,\gamma^0\left[-i\,\gamma^i\overrightarrow{\partial_i} + g(\phi+\lambda)\right]\psi. \qquad \textbf{(A.12)}$$

Next we take the time derivative of the second equation of motion given above. We get

$$\partial_0\left[g\overline{\psi}\psi - a\phi^3 \;=\; 0\right], \qquad \textbf{(A.13)}$$

$$g\left[(\partial_0\overline{\psi}) + \overline{\psi}(\partial_0\psi)\right] - 3a(\partial_0\phi)\phi^2 \;=\; 0. \qquad \textbf{(A.14)}$$

Using the above result we get

$$\partial_0\phi = \frac{g\Big[-i\,\overline{\psi}\{i\,\gamma^i\overleftarrow{\partial_i}+g(\phi+\lambda)\}\gamma^0\psi+i\overline{\psi}\gamma^0\{-i\,\gamma^i\overrightarrow{\partial_i}+g(\phi+\lambda)\}\psi\Big]}{3a\phi^2}, \qquad \textbf{(A.15)}$$

$$\partial_0\phi = \frac{-i\,g}{3a\phi^2}\overline{\psi}\Big[i\,\gamma^i\gamma^0\overleftarrow{\partial_i}+i\,\gamma^0\gamma^i\overrightarrow{\partial_i}\Big]\psi. \qquad \textbf{(A.16)}$$

Similarly the last equation of motion

$$\partial_0\Big[g\overline{\psi}\psi-3a\lambda\phi^2 = 0\Big], \qquad \textbf{(A.17)}$$

$$g\Big[(\partial_0\overline{\psi})+\overline{\psi}(\partial_0\psi)\Big]-6a(\partial_0\phi)\lambda\phi-3a(\partial_0\lambda)\phi^2 = 0. \qquad \textbf{(A.18)}$$

Using the above results we immediately find,

$$3a\phi^2(\partial_0\lambda) = g\Big[(\partial_0\overline{\psi})+\overline{\psi}(\partial_0\psi)\Big]\Big[1-\frac{6a\lambda\phi}{3a\phi^2}\Big], \qquad \textbf{(A.19)}$$

which can be given as

$$\partial_0\lambda = \frac{-i\,g}{3a\phi^2}\overline{\psi}\Big[i\,\gamma^i\gamma^0\overleftarrow{\partial_i}+i\,\gamma^0\gamma^i\overrightarrow{\partial_i}\Big]\psi\Big[1-\frac{6a\lambda\phi}{3a\phi^2}\Big]. \qquad \textbf{(A.20)}$$

The Hamiltonian density is

$$\begin{aligned}\mathscr{H} &= (\partial_0\overline{\psi})\pi_{\bar{\psi}}+\pi_\psi(\partial_0\psi)+\pi_\phi(\partial_0\phi)+\pi_\lambda(\partial_0\lambda)-i\,\overline{\psi}\gamma^0\partial_0\psi \\ &\quad +i\,\overline{\psi}\gamma^i\partial_i\psi-g\overline{\psi}\psi\phi-\lambda(g\overline{\psi}\psi-a\phi^3).\end{aligned} \qquad \textbf{(A.21)}$$

We use the time derivatives of the fields as found, we obtain

$$\mathscr{H} = (\partial_0\overline{\psi})\pi_{\bar{\psi}}+\pi_\psi(\partial_0\psi)+\pi_\phi(\partial_0\phi)+\pi_\lambda(\partial_0\lambda)+a\lambda\phi^3. \qquad \textbf{(A.22)}$$

Terms are canceled due to

$$\begin{aligned}-i\,\overline{\psi}\gamma^0\partial_0\psi &= \Big(-i\,\overline{\psi}\gamma^0\Big)\Big(i\,\gamma^0\Big[-i\,\gamma^i\overrightarrow{\partial_i}+g(\phi+\lambda)\Big]\psi\Big) \\ &= -i\,\overline{\psi}\gamma^i\overrightarrow{\partial_i}\,\psi+g\overline{\psi}(\phi+\lambda)\psi.\end{aligned} \qquad \textbf{(A.23)}$$

Explicitly we can give the final Hamiltonian as

$$\begin{aligned}\mathscr{H} &= -i\,\overline{\psi}\Big[i\,\gamma^i\overleftarrow{\partial_i}+g(\phi+\lambda)\Big]\gamma^0\pi_{\bar{\psi}} \\ &\quad +\pi_\psi i\,\gamma^0\Big[-i\,\gamma^i\overrightarrow{\partial_i}+g(\phi+\lambda)\Big]\psi \\ &\quad +\pi_\phi\frac{-i\,g}{3a\phi^2}\overline{\psi}\Big[i\,\gamma^i\gamma^0\overleftarrow{\partial_i}+i\,\gamma^0\gamma^i\overrightarrow{\partial_i}\Big]\psi \\ &\quad +\pi_\lambda\frac{-i\,g}{3a\phi^2}\Big(1-\frac{6a\lambda\phi}{3a\phi^2}\Big)\overline{\psi}\Big[i\,\gamma^i\gamma^0\overleftarrow{\partial_i}+i\,\gamma^0\gamma^i\overrightarrow{\partial_i}\Big]\psi \\ &\quad +a\lambda\phi^3.\end{aligned} \qquad \textbf{(A.24)}$$

B. SOME OF THE HIGHER ORDER PROCESSES

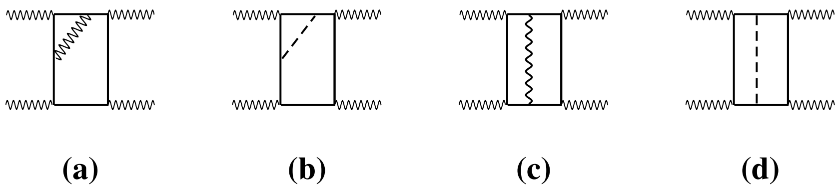


Figure B.1. Two vector fields scatter to two vector fields, (a) One vector field correction to the nonadjacent part, (b) One scalar field correction to the adjacent part, (c) One vector field correction to the nonadjacent part, (d) One scalar field correction to the nonadjacent part.

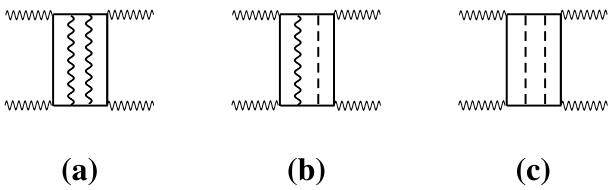


Figure B.2. Two vector fields scatter to two vector fields, (a) Two vector fields correction to the adjacent part, (b) One vector and one scalar fields correction to the nonadjacent part, (c) Two scalar fields correction to the nonadjacent part.

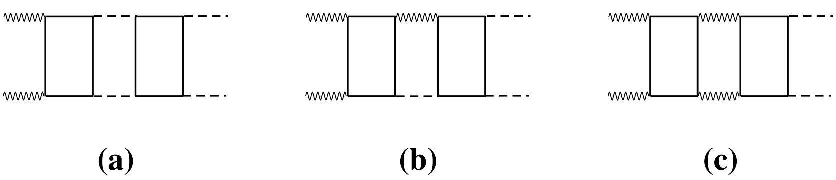
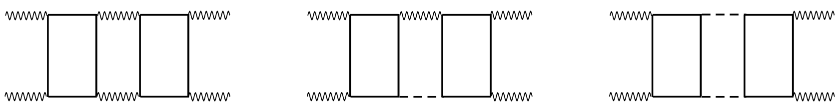
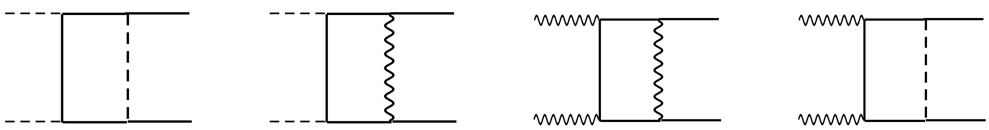


Figure B.3. Two scalar two vector interaction at three loop (a) Via two scalar field channel, (b) Via one scalar and one vector field channel, (c) Via two vector field channel.



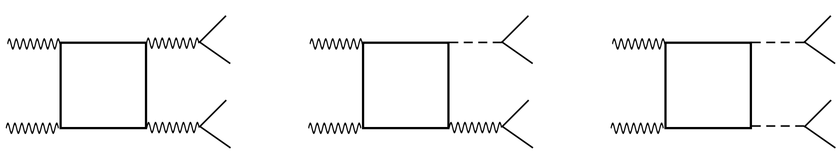
(a) **(b)** **(c)**

Figure B.4. Two vector field scatters to two vector field at three loop, (a) Via two vector field channel, (b) Via one scalar and one vector field channel, (c) Via two vector field channel.



(a) **(b)** **(c)** **(d)**

Figure B.5. Some other spinor production processes at one loop (a) Two scalars scatters using another scalar field as intermediaries, (b) Two scalars scatter using vector field as intermediaries, (c) Two scalars scatter using vector field as intermediaries, (d) Two scalars scatter using another scalar field as intermediaries.



(a) **(b)** **(c)**

Figure B.6. Four spinor field production from two vector fields (a) Via vector particle are used as intermediaries, (b) Via scalar and vector particle are used as intermediaries, (c) Via scalar particles are used as intermediaries.

C. REFERENCE FORMULAE

This Appendix collects together some of the formulae that are most commonly used in Feynman diagram calculations.

C.1 Feynman Rules

In all the models,

- the momentum is conserved at each vertex,
- the loop momenta are integrated over

$$\int \frac{d^D p}{(2\pi)^D} \tag{C.1}$$

- Fermion loops (including the ghost loops) receive an additional factor (-1),
- Each diagram may have a symmetry factor.

We find the following effective Lagrangian density and the Feynman rules:

C.1.1 Equivalent model

$$\mathscr{L} \;=\; \overline{\psi}(i\partial\!\!\!/ + g\Phi)\psi - \frac{a}{4!}\Phi^4 + \mathscr{L}_{\text{ghost}} \tag{C.2}$$

- Fermion propagator: $\frac{i p\!\!\!/}{p^2+i\varepsilon}$
- Composite scalar propagator: $-i\frac{4\pi^2}{g^2}\frac{\varepsilon}{q^2+i\varepsilon}$
- Yukawa vertex: $-ig$
- Φ^4 vertex: $-ia$

C.1.2 $U(1)$ gauged model

$$\mathscr{L} = -\frac{1}{4}(F_{\mu\nu}F^{\mu\nu}) + \overline{\psi}(i\partial\!\!\!/ - eA\!\!\!/ + g\Phi)\psi - \frac{a}{4!}\Phi^4 + \mathscr{L}_{\text{ghost}} + \mathscr{L}_{\text{gau.fix.}}. \quad \text{(C.3)}$$

In Feynman gauge,

- Fermion propagator: $\frac{ip\!\!\!/}{p^2+i\varepsilon}$
- Photon propagator: $\frac{-ig^{\mu\nu}}{k^2+i\varepsilon}$
- Composite scalar propagator: $-i\frac{4\pi^2}{g^2}\frac{\varepsilon}{q^2+i\varepsilon}$
- QED vertex:$-ie\gamma^{\mu}$
- Yukawa vertex: $-ig$
- Φ^4 vertex: $-ia$

C.1.3 $SU(N)$ gauged model

$$\mathscr{L} = -\frac{1}{4}\text{Tr}[F_{\mu\nu}F^{\mu\nu}] + \sum_{i=1}^{N_f}\overline{\psi}_i[iD\!\!\!/\psi_i + g\Phi]\psi_i - \frac{a}{4!}\Phi^4 + \mathscr{L}_{\text{ghost}} + \mathscr{L}_{\text{gau.fix.}}. \quad \text{(C.4)}$$

- Fermion propagator: $\frac{ip\!\!\!/}{p^2+i\varepsilon}\delta_{ij}$
- Gauge boson propagator: $\frac{-ig^{\mu\nu}}{k^2+i\varepsilon}\delta^{ab}$
- Composite scalar propagator: $-i\frac{4\pi^2}{g^2}\frac{\varepsilon}{q^2+i\varepsilon}$
- Fermion vertex: $ig\gamma^{\mu}t^{\alpha}$
- 3-boson vertex: $gf^{abc}[g^{\mu\nu}(k-p)^{\rho} + g^{\nu\rho}(p-q)^{\mu} + g^{\rho\mu}(q-k)^{\nu}]$
- 4-boson vertex: $-ig^2[f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{ace}f^{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\mu\sigma})]$
- Yukawa vertex: $-ig$
- Φ^4 vertex: $-ia$
- Ghost propagator: $-gf^{abc}p^{\mu}$
- Ghost vertex: $\frac{i\delta^{ab}}{p^2+i\varepsilon}$

C.2 Numerator Algebra

In d-dimension we will give a brief summary of the γ matrices.

C.2.1 Miscellaneous identities of gamma matrices

$$\{\gamma^{\mu},\gamma^{\nu}\} = 2g^{\mu\nu}, \tag{C.5}$$
$$\gamma^{\mu}\gamma^{\nu}\gamma_{\mu} = -(D-2)\gamma^{\nu}, \tag{C.6}$$
$$\gamma^{\mu}\gamma^{\nu}\gamma^{\rho}\gamma_{\mu} = 4g^{\nu\rho}-(4-D)\gamma^{\nu}\gamma^{\rho}, \tag{C.7}$$
$$\gamma^{\mu}\gamma^{\nu}\gamma^{\rho}\gamma^{\sigma}\gamma_{\mu} = -2\gamma^{\sigma}\gamma^{\rho}\gamma^{\nu}+(4-D)\gamma^{\nu}\gamma^{\rho}\gamma^{\sigma}. \tag{C.8}$$

C.2.2 Traces of gamma matrices

Traces of γ matrices can be evaluated as follows:

$$\mathrm{tr}(\text{odd number of }\gamma) = 0, \tag{C.9}$$
$$\mathrm{tr}\left[\gamma^{\mu}\gamma^{\nu}\right] = \mathrm{D}g_{\mu\nu}, \tag{C.10}$$
$$\mathrm{tr}\left[\gamma^{\mu}\gamma^{\alpha}\gamma^{\nu}\gamma^{\beta}\right] = \mathrm{D}\Big(g_{\mu\alpha}g_{\nu\beta}-g_{\mu\nu}g_{\alpha\beta}-g_{\mu\beta}g_{\alpha\nu}\Big). \tag{C.11}$$

C.3 Loops Integrals and Dimensional Regularization

To combine propagator denominators, introduce integrals over Feynman parameters:

$$\frac{1}{A_1A_2\cdots A_n} = \int_0^1 dx_1\int_0^{x_1}dx_2\cdots\int_0^{x_{n-2}}dx_{n-1} \cdot\frac{\Gamma(n)}{\Gamma(1)^n}\frac{1}{[x_{n-1}A_1+(x_{n-2}-x_{n-1})A_2+\cdots+(1-x_1)A_n]^n}. \tag{C.12}$$

In this thesis we use $n=2$, 3, 4 cases. This formula reduces to for $n=2$,

$$\frac{1}{AB} == \frac{\Gamma(2)}{\Gamma(1)^2}\int_0^1 dx\frac{1}{[Ax+B(1-x)]^2}. \tag{C.13}$$

For $n=3$,

$$\frac{1}{ABC} = \frac{\Gamma(3)}{\Gamma(1)^3}\int_0^1 dx\int_0^x dy\frac{1}{[Ay+B(y-z)+C(1-x)]^3}. \tag{C.14}$$

For $n=4$,

$$\frac{1}{ABCD} = \frac{\Gamma(4)}{\Gamma(1)^4}\int_0^1 dx\int_0^x dy\int_0^y dz\frac{1}{[Az+B(y-z)+C(x-y)+D(1-x)]^4}. \tag{C.15}$$

C.3.1 Symmetry

We can always use the symmetry properties as follows:

$$\int \frac{d^Dk}{(2\pi)^D} k_\mu k_\nu f\left(k^2\right) = \int \frac{d^Dk}{(2\pi)^D} \frac{k^2 g_{\mu\nu} f\left(k^2\right)}{D}, \tag{C.16}$$

$$\int \frac{d^Dk}{(2\pi)^D} k_\mu k_\nu k_\theta k_\sigma f\left(k^2\right) = \int \frac{d^Dk}{(2\pi)^D} \frac{k^4 (g_{\mu\nu} g_{\theta\sigma} + g_{\mu\theta} g_{\nu\sigma} + g_{\mu\sigma} g_{\nu\theta}) f\left(k^2\right)}{D(D+2)}. \tag{C.17}$$

C.3.2 D-dimensional integrals

In Minkowski space d-dimensional integrals can be solved by

$$\int \frac{d^Dk}{(2\pi)^D} \frac{1}{(k^2+M^2)^n} = \frac{(-1)^n i}{(4\pi)^{D/2}} \frac{\Gamma(n-D/2)}{\Gamma(n)} \frac{1}{(-M^2)^{n-D/2}}. \tag{C.18}$$

C.3.3 Gamma functions

After the d-dimensional integration we get Γ functions. We need the expansion near its pole. The general expression is

$$\Gamma(\varepsilon - n) = \frac{(-1)^n}{n!} \left[\frac{1}{\varepsilon} + \left(\sum_{k=1}^{n} \frac{1}{k} - \gamma \right) \right] + O(\varepsilon^2) \tag{C.19}$$

where γ, Euler's constant, and γ_n are given by

$$\gamma \simeq 0.577 \qquad \text{and} \qquad \gamma_n = 1 + \frac{1}{2} + \cdots - \gamma \tag{C.20}$$

In the thesis we often use

$$\Gamma\left(\frac{\varepsilon}{2}\right) = \frac{2}{\varepsilon} - \gamma + \frac{1}{4}\left(\gamma^2 + \frac{\pi^2}{6}\right)\varepsilon + O(\varepsilon^2) \tag{C.21}$$

$$\Gamma\left(-1+\frac{\varepsilon}{2}\right) = -\frac{2}{\varepsilon} + (\gamma - 1) - \frac{1}{4}\left(\gamma^2 - 2\gamma + \frac{\pi^2}{6}\right)\varepsilon + O(\varepsilon^2) \tag{C.22}$$

C.4 Integrals

C.4.1 Basic rule

Before starting to the Angular integration rules, here we want to remind a very basic integration rule that we use.

$$\frac{d}{dx}\int_{u(x)}^{v(x)} f(x,t)dt = \frac{dv(x)}{dx} f\left(x, v(x)\right) - \frac{du(x)}{dx} f\left(x, u(x)\right) + \int_{u(x)}^{v(x)} \frac{\partial}{\partial x} f(x,t)dt \quad \textbf{(C.23)}$$

C.5 Angular Integration in 4 Dimensional Euclidean Space

Here are some useful formulas for the angle integrations in 4-dimensional Euclidean space [14].

Let k_μ and p_μ be two momenta and $pk = |p||k|cos\theta; |p| = (p_0^2 + \mathbf{p}^2)^{1/2}$. Then

$$\begin{aligned}
\int d^4kF\left[(p-k)^2\right] &= \pi^2 \int dk^2 d\Omega_k k^2 F\left(k^2 - 2|k||p|cos\theta + p^2\right) && \textbf{(C.24)} \\
d\Omega_k &= \frac{2}{\pi} sin^2\theta d\theta; && \textbf{(C.25)} \\
\int d\Omega_k &= 1 && \textbf{(C.26)}
\end{aligned}$$

The general structure of the integrals we use takes the following form:

$$\begin{aligned}
I_n[(pk)^m] &\equiv \int d\Omega_k \frac{(pk)^m}{(k-p)^{2n}}, && \textbf{(C.27)} \\
I_n[k_\mu (pk)^m] &\equiv \int d\Omega_k \frac{k_\mu (pk)^m}{(k-p)^{2n}}, && \textbf{(C.28)} \\
I_n[k_\mu k_\nu (pk)^m] &\equiv \int d\Omega_k \frac{k_\mu k_\nu (pk)^m}{(k-p)^{2n}}. && \textbf{(C.29)}
\end{aligned}$$

Some useful explicit expressions are $\left(x \equiv p^2\right)$, $\left(y \equiv k^2\right)$

$$\begin{aligned}
I_{-1} &= k^2 + p^2, && \textbf{(C.30)} \\
I_0 &= 1, && \textbf{(C.31)} \\
I_1[(pk)^0] &= \frac{1}{max(x,y)}, && \textbf{(C.32)} \\
I_1[(pk)^1] &= \frac{1}{2}\frac{min(x,y)}{max(x,y)}, && \textbf{(C.33)} \\
I_1[(pk)^2] &= \frac{1}{4}\frac{(x+y)min(x,y)}{max(x,y)}, && \textbf{(C.34)} \\
I_2[(pk)^1] &= \frac{min(x,y)}{|x-y|max(x,y)}, && \textbf{(C.35)} \\
I_2[(pk)^2] &= \frac{xy + 3min(x^2,y^2)}{4|x-y|max(x,y)}, && \textbf{(C.36)} \\
I_2[(pk)^3] &= \frac{(x+y)min(x^2,y^2)}{2|x-y|max(x,y)}. && \textbf{(C.37)}
\end{aligned}$$

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