İSTANBUL TECHNICAL UNIVERSITY ★ INSTITUTE OF SOCIAL SCIENCES

R&D EXPENDITURES AND PUBLIC POLICY IN OLIGOPOLY

M.A. Thesis by

Evren PALABIYIK, B.Sc.

Department : Economics

Programme: Economics

JUNE 2006

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Supervisor (Chairman): Prof. Dr. Benan ZEKİORBAY

Members of the Examining Committee: Assoc. Prof. Dr. M. Özgür KAYALICA (İTÜ)

Assoc. Prof. Dr. Y. İlker TOPÇU (İTÜ)

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YÜKSEK LİSANS TEZİ

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Diğer Jüri Üyeleri : Doç. Dr. M. Özgür KAYALICA (İTÜ)

Doç. Dr. Y. İlker TOPÇU (İTÜ)

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PREFACE

I wish to thank my supervisor Prof. Dr. Benan Zeki Orbay for comments and discussions during the study and for introducing me to Industrial Organization. My interest in Industrial Organization began with her course and matured with this thesis again under her supervising.

I also wish to thank the teachers attended to the presentation of this study and shared their ideas with me.

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June, 2006 Evren PALABIYIK

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SUMMARY

This study analyzes the subsidy effects on cooperative R&D and noncooperative R&D in a duopoly with uncertain spillover rates. Cooperative R&D, where firms invest in R&D to maximize their joint profits, is compared with noncooperative R&D, where firms invest in R&D to maximize individual profits. R&D level, production level, market prices, firm profits and social welfare are compared for cooperative and competitive cases. The effects of subsidy policies on these variables are investigated. Our model differs from previous works in two important ways: The spillover rates are uncertain, that is, the firms do not know how large the incoming and outgoing spillovers will be before investing in R&D. Secondly, the government maximizes the social welfare by subsidizing the R&D investments of firms. When the spillover rate is high, that is, the leakage of R&D knowledge is large, then competing firms get higher subsidies than cooperating firms. Moreover, the profit levels of competing firms are higher than cooperating firms due to these higher subsidy rates. On the other hand cooperative and noncooperative R&D leads to the same level of output, market prices and social welfare, different from the previous studiesô results, since public policy is included in the model.

Keywords: Research and development, spillovers, public policy, subsidy, cooperation, noncooperation, uncertainty.

ÖZET

Bu çalışma, işbirliği altındaki AR-GE yatırımları ve rekabet altındaki AR-GE yatırımlarında sübvansiyonun etkilerini inceliyor. Firmaların toplam kârlarını ençoklayacak olan AR-GE yatırım miktarına beraber karar verdikleri durum olan AR-GE işbirliği ile firmaların her birinin kendi kârını ençoklayacak olan AR-GE yatırım miktarına tek başına karar verdiği durum olan AR-GE rekabeti karşılaştırılıyor. Firmaların AR-GE düzeyi, çıktı miktarı, fiyat düzeyi, firma kârları ve sosyal refah, işbirliği ve rekabet durumları için karşılaştırılıyor. Devletin sübvansiyon politikalarının bu değişkenler üzerindeki etkileri inceleniyor. Çalışmamızın, önceki çalışmalardan iki önemli farkı bulunuyor: Öncelikle, bilgi yayılım oranı belirsizdir, diğer bir deyişle firmalar içeri ve dışarı sızacak bilgi oranından AR-GE yatırımını yapmadan önce haberdar değiller. İkinci olarak, devlet firmaların AR-GE yatırımlarını sübvanse ederek sosyal refahı maksimize etmeyi amaçlıyor. Bilgi yayılım oranının yüksek olduğu durumda, yani R&D bilgisinin önemli bölümü dışarı sızdığında, AR-GE rekabeti yapan firmalar, AR-GE işbirliği yapan firmalardan daha fazla sübvansiyon alır. Ayrıca, AR-GE rekabeti yapan firmaların kâr düzeyi, AR-GE işbirliği yapan firmaların kâr düzeyinden bu sübvansiyonun etkisiyle daha yüksek olur. Diğer yandan AR-GE rekabeti ve AR-GE işbirliği aynı düzeyde çıktı miktarı, fiyat düzeyi ve sosyal refaha yol açar. Modele devletin R&D politikası dâhil edildiğinden dolayı, bu sonuçlar önceki çalışmaların sonuçlarından farklıdır.

Anahtar Kelimeler: Araştırma ve geliştirme, bilgi yayılım oranı, devlet politikası, sübvansiyon, işbirliği, rekabet, belirsizlik.

1. INTRODUCTION

Firms engage in Research and Development to develop new products, reduce the costs or improve the quality of existing products. R&D is generally classified into two types: a) product innovation, searching for producing new products, and b) process innovation, searching for cost-reducing technologies for producing a certain product. In this study, we will consider process innovation, where firms invest in R&D to reduce their production costs.

However, R&D efforts of a firm do not reduce only its own production costs but also its competitorsô costs. Since the discoveries cannot be kept entirely secret, there is always a leakage of R&D information from the innovator firm to the others, this information leakage is called spillover. Spillovers can occur in cases such as reverse engineering, movement of R&D personnel, input suppliers, scientific meetings or publication of scientific papers. The spillover rate indicates the proportion of the leakage to the entire R&D knowledge. Hence it takes a value between zero and unity. A high level of spillover rate would prevent the firm from investing in R&D. Because if the spillover rate is high, then the knowledge produced by a firmôs R&D project will soon become available to all of its competitors. So the R&D effort will reduce all firmsô production costs, this will only make them stronger competitors in the product market. Therefore the firm will not gain an advantage over its competitors in the product market although it is the one who expended on R&D. On the other hand, if the spillover rate is low, only the investor firm will make pretty use of its R&D knowledge. This will be an incentive for the firm to invest in R&D.

In the case of high spillover rate, firms can coordinate their R&D investments or form a Research Joint Venture (RJV) and share their R&D information completely to avoid this negative externality.

Since R&D activities raise the welfare, public policy should always encourage the firms to invest in R&D. Governments have various policy tools to encourage the firms for R&D activities. First policy is giving a patent protection to the innovator firm that ensures earning monopoly profits for several years. However, this is not an effective way since it is very difficult to block spillovers in practice. Mansfield (1985) investigates the data obtained from a random sample of 100 US firms and he states that the information about R&D decisions of a firm generally leaks out to its rivals within about 12 to 18 months, and information about the detailed nature and operation of a new product or process generally leaks out within about a year. Second policy to stimulate R&D efforts is to allow firms to cooperate in R&D activities. In our model, firms are not forced to cooperate in R&D. They are free to act in the way to maximize their profits. The third policy is to subsidize the innovating firms. In our model, the government pays the innovator firm a subsidy, which will maximize the social welfare. This subsidy is some proportion of the cost of R&D faced by the firm. These subsidy rates depend on the firmsô R&D cooperation decisions in the pre-production stage.

Free-ride effect is an important issue conditioning the interrelation between the profitability of R&D cooperation and spillovers. Free-riding is the incentive of a firm to benefit from the R&D investment of its R&D partner instead of investing itself. Free-riding reduces profitability and threaten the stability of a cooperative R&D agreement. Higher spillover rates, although they increase the profits of cooperating firms, also increase the risk of free-riding. An investing firm can avoid being free-ridden by guarding its successful R&D knowledge.

The study is organized as follows. In Section 2, previous studies on R&D environment with spillovers are investigated. In section 3, the model is presented and solved for the states of R&D cooperation and R&D competition. In section 4, the outcomes for both states are compared and the results are discussed. Section 5 concludes the study.

2. LITERATURE REVIEW ON R&D COOPERATION AND SPILLOVERS

The interest in cooperative and noncooperative R&D activities with spillovers has arisen in the last decade, after the paper of dôAspremont and Jacquemin (1988), henceforth AJ. The papers following AJ are usually the extensions or modifications of their model. The examples are De Bondt and Veugelers (1991), Kamien et al. (1992), Motta (1992), Suzumura (1992), Poyago-Theotoky (1995), Leahy and Neary (1997), Salant and Shaffer (1998), Beath et al. (1989), Petit and Tolwinski (1999), Hinloopen (2000), Amir et al. (2003).

AJ was the first to analyze cooperative and noncooperative R&D with spillovers. They focused on the comparison of the level of R&D when firms carry out their R&D efforts competitively versus cooperatively, in the presence of spillovers. Cooperative R&D is the case when firms invest in R&D taking into account their overall profits. Competitive (or noncooperative) R&D is the case when a firm invests in R&D taking only its own profit into account. AJ considered a two-stage duopoly game with homogeneous products and symmetric firms. In the first stage, firms simultaneously decide how much to invest in R&D. In the second stage, they compete in the product market over quantities. AJ model showed that, for large spillovers R&D cooperation leads to higher welfare, R&D and output level than noncooperative R&D leads to. The opposite holds for sufficiently small spillovers. It was surprising to see that R&D investments were greater in R&D cooperation than in noncooperation. Before this paper, it was commonly expected that R&D cooperation agreements would lead to reduction in R&D expenditure, since the duplication of R&D would be prevented. The important factor in this result is the spillovers in R&D from one firm to another.

Suzumura (1992, p.1308) explains the reason of this surprising result:

> The R&D incentive of a single firm hinges squarely on the extent of appropriability of the R&D benefits, so that the presence of large R&D spillovers may drastically reduce the incentives for cost reduction, with the result that the R&D commitment made voluntarily by a firm tends to be socially too small. From this viewpoint, an enforceable agreement on cooperative R&D efforts

seems to facilitate more commitments. The result of the net effect of the R&D cooperation hinges on the relative strength of these competing effects.

Suzumura (1992) also showed that AJ model results are valid not only in the duopoly example with linear demand function and linear marginal cost, but also in a much wider class of oligopolistic industries.

A detailed stability analysis of AJ model is presented by Henriques (1990). Henriques assigned specific values to the parameters and discovered small unstable regions in the spillover parameter space. In particular, it is shown that the equilibrium is unstable for low levels of spillover. De Bondt and Veugelers (1991) showed that R&D investments in cooperation are greater than in noncooperation when spillovers are substantial, supporting the results of AJ.

Kamien et al. (1992), henceforth KMZ, extended the AJ model to more firms than two and the linear cost and demand functions to general function forms. AJ model was extended to this form also by Suzumura (1992). KMZ used a richer set of R&D cooperation scenarios. They considered four cases: R&D competition, R&D cartelization, RJV competition and RJV cartelization. In R&D competition, each firm decides its own R&D level to maximize its individual profit. In R&D cartelization, firms coordinate their R&D investments to maximize the sum of their profits. In RJV competition, firms behave like the case R&D competition, except that the outcomes of their R&D research are fully shared. So the duplication of R&D efforts is avoided and the spillover rate is at its maximum. In RJV cartelization, firms coordinate their R&D investments to maximize the sum of their profits as they do in case of R&D cartelization, but later they share R&D information completely, thus the spillover rate is again at its maximum. KMZ concluded that the RJV cartelization dominates all other scenarios, since it leads to the highest profit per firm, the lowest prices in the product market, the highest level of R&D and the highest social welfare. This implies that it also achieves the highest total consumer plus producer surplus among the four possible scenarios. On the other hand, RJV competition leads to the least reduction per unit cost and the highest product prices. KMZ described two types of externalities explaining the result of AJ and also theirs. First type of externality is called competitive-advantage externality: A firmõs R&D

investment has a negative external effect on its own profit, via reducing the marginal costs of competitor firms and hence making them tougher competitors. This externality inhibits a firmǿs R&D expenditure. Second is the combined-profits externality, which can be negative or positive. A firmǿs R&D investment has an external effect on the profits of all firms. This externality is ignored when each firm chooses its expenditure to maximize only its own profit and internalized when the firms coordinate their R&D expenditures to maximize the sum of their profits. The total effect of the two externalities is positive when the spillover rate is sufficiently high. In this case, R&D cartelization reduces marginal costs more than R&D competition, combined profits are higher and market prices are lower. Thus, both producers and consumers benefit as a result of this type of cooperation in R&D. The same result is obtained when comparing RJV competition to RJV cartelization, the latter being the more socially desirable.

Salant and Shaffer (1998) extended AJ model for asymmetric strategies and showed that the overall joint profits can be larger if the firms make unequal investments in R&D. They showed that, for a particular region in the parameter space, the joint profit maximizing solution in AJ model is not symmetric. Reallocating the same total investment between the two firms can increase joint profits. Joint profit maximizing solution for a research cartel is to choose asymmetric investments at the R&D stage.

Hauenschild (2003) introduced uncertainty of successful completion of R&D projects into AJ and KMZ models. He analyzed how this uncertainty influences technological performance in the sense of expected effective cost reductions. It is assumed that the R&D projects of both firms may fail independent of each other with some probability. Intended cost reduction only becomes effective with some probability between zero and one while the R&D project may also fail with positive probability. When deciding its R&D investment, each firm has to take into account the possibility of failing and the possibility of rivalǿs failure.

Although the literature on R&D activities is enormous, the studies analyzing the effects of subsidies on R&D is limited. Main studies on R&D subsidies are by

Brander and Spencer (1983), Spence (1984), Romano (1989), Leahy and Neary (1997), Cassiman (2000), Hinloopen (2001).

Brander and Spencer (1983) showed that oligopolistic firms that invested strategically in R&D in order to improve their position in the competitive market in the future, would carry out more R&D than the cost-minimizing level.

Leahy and Neary (1997) introduced strategic behavior beside R&D cooperation. They considered two types of subsidy: subsidy to R&D and subsidy to output, where both of them are per unit subsidies. They presented three assumptions about the move orders in the oligopoly. The first assumption, full commitment equilibrium (FCE), has two stages. Firstly the government chooses both types of subsidies and in the second stage firms simultaneously choose their R&D levels and outputs (or prices). Second assumption, government-only commitment equilibrium (GCE), has three stages. First, government chooses both subsidies, then firms choose their R&D levels and at the last stage they choose the output/price levels. The third assumption about the move order is sequence equilibrium (SE), which is a four-stage game. In the first stage government chooses its R&D subsidy, then each firm chooses its R&D level, next the government chooses its output subsidy and in the last stage, firms choose their output/price levels. Leahy and Neary used the term ñstrategic behavioriò referring to the R&D investments in GCE and SE, because of affecting the environment in which the output/price game is played. They showed that R&D cooperation raises output, R&D level and welfare when firms do not behave strategically. With strategic behavior, R&D cooperation raises welfare and requires a lower subsidy only when the spillover rates are high. Strategic behavior tends to reduce output, R&D and welfare and to lead to higher subsidies in all cases except the firms behave noncooperatively while spillovers are low and firmsô actions are strategic substitutes. Moreover, industry profits are always higher when firms choose their R&D level strategically and cooperatively. When the spillover is high, cooperation is more desirable from both private and social perspectives.

Cassiman (2000) considered lump-sum subsidies and set the spillover level of firms as private information of the firms within the industry. In his model, the firms submit their claims about their spillover levels, and then the government commits to its R&D policy.

Hinloopen (2001) compared two R&D stimulating policy tools: sustaining R&D cooperatives and providing R&D subsidies. He showed that the latter enhances more private R&D. In his work the advantages and disadvantages of sustaining R&D cooperatives and providing subsidies are explained. Some of the advantages of sustaining R&D cooperatives are: it internalizes spillovers, it eliminates the free-rider problem, and risk pooling can increase the research efforts. The disadvantages are: An agreement to cooperate in R&D can bring collusion in production stage, R&D cooperatives can act as a barrier to entry, efforts of an innovating firm can lower the profits of other firms more than it increases the profits of the innovating firm and hence widens the gap between actual R&D investments and socially optimal levels. Providing subsidies has some superiority over sustaining R&D cooperatives. Firstly, entry barriers for the research market are lowered. Secondly economies of scale are more easily realized since the cost of R&D on a sufficient scale are lower. However, there are some drawbacks of providing subsidies such as the tax imposed to finance the R&D subsidy carries deadweight loss. Another drawback is that the firms may deceive the authorities to obtain the R&D subsidy and the last one is that it is not clear before the subsidy is given that if government is subsidizing only successful research projects. Hinloopen showed that subsidizing cooperative R&D or noncooperative R&D leads to the same level of R&D activity, which is a conclusion similar to ours. However, Hinloopen sets a three-stage game, where in the first stage the authorities decide whether or not to provide an R&D subsidy, in the second stage firms determine their R&D investments either cooperatively or noncooperatively and in the last stage firms compete. Another difference is that the subsidies are per unit of R&D and firms are taxed in the product market to finance the R&D subsidies. Firms consider this corporate tax rate as given while determining their optimal level of R&D investment and optimal level of production. Hence, firmsô R&D investment and production decisions are influenced only by the R&D subsidy. The tax affects only the final profits. Hinloopen also states that the general effect of the subsidy and tax scheme is a shift from producer surplus to consumer surplus, which always leads to a gain in net total surplus.

3. THE MODEL

Our model is an extension of AJ model, where uncertainty of spillover rates and public policy are included. We consider a four-stage game. The timing of the game is as follows:

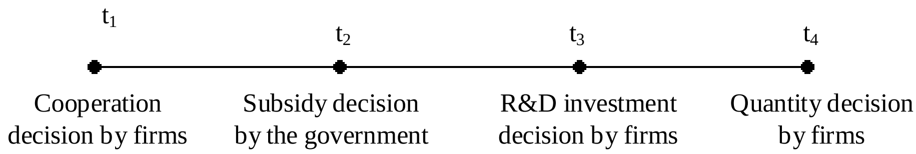

Figure 3.1: Timing of the game

In the first stage, firms simultaneously decide whether to cooperate or compete in R&D investments. If they choose to cooperate, then in the third stage they will invest in R&D the optimal amount for maximizing their joint profits. If they choose to compete, they will invest the optimal amount for maximizing their individual profits. In the second stage, the government chooses the optimal R&D subsidy rate that maximizes the social welfare. In the third stage, firms decide how much to invest in R&D. Their investment depends on their decision of cooperation they made in the first stage. In the fourth stage, firms engage in Cournot competition in the product market so as to maximize their expected profits. We exclude the possibility of cooperation in the production stage, since it violates the antitrust laws.

We analyze how firms determine their research efforts, taking into consideration that they compete in the final good's market after the research is completed. The analysis has the following properties: It is a duopoly, i.e. there are two firms. Firms are symmetric. R&D efforts are directed to reducing unit costs. As in the AJ model, there is spillover from each firm's R&D effort into the other, but we also assume that there is uncertainty in the spillover rates. The firms produce homogenous products.

There are no fixed costs and unit costs are constant. The market demand function and production cost functions are linear. R&D cost function is quadratic, reflecting the existence of diminishing returns to R&D expenditures.

Our model differs from previous works in two important ways: The spillover rate is uncertain, that is, the firms do not know how large the incoming and outgoing spillovers will be. Secondly, the government subsidizes the R&D investments of firms, under this uncertainty.

In the product market, the firms are faced with an inverse demand function of

$$P(q_i, q_j) = a - b(q_i + q_j) \tag{1}$$

with $a, b > 0$, $q_i + q_j \leq a / b$, where q_i and q_j is the output of the firm i and firm j, respectively. Firm i's cost reduction provided by its successful R&D project is $x_i > 0$. On the other hand, it also reduces the production cost of firm j by an amount of $\beta_j x_i$, where β_j is the incoming spillover rate of firm j. Hence, the production cost function of firm i is

$$TC_i(q_i, x_i, x_j) = (c - x_i - \beta_i x_j) q_i \qquad i = 1,2 \ , \ i \neq j \tag{2}$$

with $0 < c < a$, $x_i + \beta_i x_j \leq c$, where the original unit production cost c, which is a constant, is reduced by the successful R&D project that the firm itself carried out and also the competitor firm carried out. The R&D cost is

$$C_i(x_i) = \gamma \frac{(x_i)^2}{2} \tag{3}$$

where $\gamma > 0$ represents a parameter for cost of R&D whose high values mean that R&D cost is high. Therefore the parameter γ is inversely related to the cost effectiveness in R&D. The quadratic form reflects the diminishing returns to R&D expenditure.

In our model, the government subsidizes the innovator firms in order to maximize the social welfare. The government chooses a subsidy rate that is a portion of R&D cost faced by the firm. Firstly, the firms decide whether to cooperate or compete and how much to invest in R&D in either states. Then, the government chooses the subsidy rate in the second stage. Therefore, the subsidy rate depends on the firmsô R&D cooperation decisions on the previous stage.

The subsidy given by the government to support firm i's R&D expenditure is

$$S_i(x_i) = s\gamma \frac{(x_i)^2}{2} \tag{4}$$

where s is the proportion of the subsidy to the total R&D expenditure $C_i(x_i)$ of the firm and hence between zero and unity ($0 \le s \le 1$).

The profit functions of the firms are:

$$\pi_i = (a - b(q_i + q_j))q_i - (c - x_i - \beta_i x_j)q_i - (1-s)\gamma \frac{x_i^2}{2} \tag{5}$$

$$\pi_j = (a - b(q_i + q_j))q_j - (c - x_j - \beta_j x_i)q_j - (1-s)\gamma \frac{x_j^2}{2} \tag{6}$$

These are the general forms of profit functions valid for both R&D cooperation case and R&D competition case.

The firms compete in the product market by choosing their optimal outputs to maximize their expected individual profits. The Nash-Cournot equilibrium quantities are determined by

$$\max_{q_i} \ \pi_i \text{ and } \max_{q_j} \ \pi_j . \tag{7}$$

This maximization process leads to the following first order conditions:

$$\frac{\partial \pi_i}{\partial q_i} = a - c - 2bq_i - bq_j + x_i + \beta_i x_j = 0 \tag{8}$$

$$\frac{\partial \pi_j}{\partial q_j} = a - c - 2bq_j - bq_i + x_j + \beta_j x_i = 0 \tag{9}$$

Solving (8) and (9) simultaneously, we find the optimal quantities as follows:

$$q_i(x_i, x_j) = \frac{(a-c) + (2-\beta_j)x_i + (2\beta_i - 1)x_j}{3b} \tag{10}$$

$$q_j(x_i, x_j) = \frac{(a-c) + (2-\beta_i)x_j + (2\beta_j - 1)x_i}{3b} \tag{11}$$

We obtain the equilibrium profits by inserting (10) and (11) into equations (5) and (6):

$$\pi_i(x_i, x_j) = \frac{1}{9b}\left[(a-c) + (2-\beta_j)x_i + (2\beta_i - 1)x_j\right]^2 - (1-s)\gamma\frac{x_i^2}{2} \tag{12}$$

$$\pi_j(x_i, x_j) = \frac{1}{9b}\left[(a-c) + (2-\beta_i)x_j + (2\beta_j - 1)x_i\right]^2 - (1-s)\gamma\frac{x_j^2}{2} \tag{13}$$

However, the profit functions depend on spillover rates. Since the spillover rates are uncertain in our model, we will refer to expected profits instead of definite profits.

Spillover rate parameter is between zero and unity ($0 \le \beta \le 1$). The spillover rate does not fall below 0 since a successful cost reducing R&D project of a firm does not cause any detrimental change in its competitorôs original production technology. The spillover rate also does not exceed 1 since a firm is unable to benefit from a successful cost reducing R&D project more than its original innovator.

We consider a duopoly where both firmsô R&D spillover rates are uncertain. R&D spillover rates can either be low (β^L) or high (β^H). The firms decide their R&D investments and production quantities taking into account the expected spillover rates of their own and the rivalôs. The government carries out the optimal R&D policy under this spillover rates uncertainty. The probability of high spillover rate is λ and low spillover rate is $1-\lambda$ for both firms.

Hence, the expected spillover rate of firm i is

$$E(\beta_i) = \lambda\beta^H + (1-\lambda)\beta^L \tag{14}$$

where $0 \le \beta^L < \beta^H \le 1$, $i = 1,2$.

There are four possible scenarios regarding the expected spillover rates. In the first one, both firms have a high spillover rate, $\beta_i = \beta_j = \beta^H$, with the probability of λ^2. In the second scenario, firm i has a high spillover rate and firm j has a low spillover rate, $\beta_i = \beta^H, \beta_j = \beta^L$, with the probability of $\lambda(1-\lambda)$. In the third scenario, firm i has a low spillover rate and firm j has a high spillover rate, $\beta_i = \beta^L, \beta_j = \beta^H$, with the probability of $\lambda(1-\lambda)$. In the last scenario, both firms have low spillover rates, $\beta_i = \beta_j = \beta^L$ with the probability of $(1-\lambda)^2$.

Considering four possible scenarios regarding the expected spillover rates, we can compute the following expected profits:

$$\begin{aligned}
E(\pi_i(x_i, x_j, s)) &= \lambda^2\left\{\frac{1}{9b}\left[(a-c)+(2-\beta^H)x_i+(2\beta^H-1)x_j\right]^2\right\} \\
&+\lambda(1-\lambda)\left\{\frac{1}{9b}\left[(a-c)+(2-\beta^L)x_i+(2\beta^H-1)x_j\right]^2\right\} \\
&+\lambda(1-\lambda)\left\{\frac{1}{9b}\left[(a-c)+(2-\beta^H)x_i+(2\beta^L-1)x_j\right]^2\right\} \\
&+(1-\lambda)^2\left\{\frac{1}{9b}\left[(a-c)+(2-\beta^L)x_i+(2\beta^L-1)x_j\right]^2\right\}-(1-s)\gamma\frac{x_i^2}{2}
\end{aligned} \tag{15}$$

$$
\begin{aligned}
E(\pi_j(x_i,x_j,s)) &= \lambda^2\left\{\frac{1}{9b}\left[(a-c)+(2-\beta^H)x_j+(2\beta^H-1)x_i\right]^2\right\} \\
&+\lambda(1-\lambda)\left\{\frac{1}{9b}\left[(a-c)+(2-\beta^L)x_j+(2\beta^H-1)x_i\right]^2\right\} \\
&+\lambda(1-\lambda)\left\{\frac{1}{9b}\left[(a-c)+(2-\beta^H)x_j+(2\beta^L-1)x_i\right]^2\right\} \\
&+(1-\lambda)^2\left\{\frac{1}{9b}\left[(a-c)+(2-\beta^L)x_j+(2\beta^L-1)x_i\right]^2\right\}-(1-s)\gamma\frac{x_j^2}{2}
\end{aligned}
\qquad (16)
$$

The equations (1) to (16) are valid for both cases of R&D cooperation and R&D competition. However, the profit maximization process of firms will alter with respect to their decision in R&D cooperation.

3.1. R&D Competition (Case N)

In case of competition in R&D investments, each firm decides its own R&D expenditure to maximize its expected individual profit. To determine the subgame perfect Nash equilibrium we maximize the expected profits:

$$
\max_{x_i} \; E(\pi_i) \;\text{ and }\; \max_{x_j} \; E(\pi_j) \qquad (17)
$$

and obtain the following best response functions[1] of firms i and j:

$$
x_i^N(x_j,s)=\frac{2(2+\beta^L(\lambda-1)-\beta^H\lambda)(c-a+x_j(1+2\beta^L(\lambda-1)-2\beta^H\lambda))}{8-9\,\mathrm{b}\,(1\text{-}\mathrm{s})\gamma+2\,\beta^L\,(4-\beta^L)\,(\lambda-1)-2\beta^H\,(4-\beta^H)\,\lambda}
$$

$$
i\neq j, i=1,2 \qquad (18)
$$

Simultaneously solving the above conditions for x_i and x_j, we obtain the equilibrium values[2] of cooperative R&D levels of firm i and firm j as follows:

[1] See Appendix B for the second order conditions

[2] See Appendix B for the stability conditions.

$$x_i^N(s) = \frac{2(a-c)(2+\beta^L(\lambda-1)-\beta^H\lambda)}{-4-2\beta^L(1-\beta^L)+9b\gamma(1-s)-2\lambda(1+\beta^H-3\beta^L)(\beta^H-\beta^L)+4(\beta^H-\beta^L)^2\lambda^2}$$

$$i = 1,2 \qquad (19)$$

Inserting (19) into equations (15) and (16), we obtain the following expected profits:

$$\begin{aligned}E(\pi_i^N(s)) = {} & ((a-c)^2(81b^2(s-1)^2\gamma^2 + 18b(s-1)\gamma((\beta^L-2)^2 + 2(\beta^H-2)(\beta^H-\beta^L)\lambda - (\beta^H-\beta^L)^2\lambda^2) \\ & -4(\beta^H-\beta^L)^2(\lambda-1)\lambda(5(\beta^L-2)^2 + (\beta^H-\beta^L)(\beta^H+9\beta^L-20)\lambda + 4(\beta^H-\beta^L)^2\lambda^2)))/ \\ & 9b(4+2\beta^L(1-\beta^L)-9b\gamma(1-s)+2\lambda(1+\beta^H-3\beta^L)(\beta^H-\beta^L)-4(\beta^H-\beta^L)^2\lambda^2)^2\end{aligned}$$

$$i = 1,2 \qquad (20)$$

The government subsidizes the cost of the R&D project so as to maximize the social welfare, which is the sum of producer surplus and consumer surplus from which the government transfers are subtracted.

Producer surplus is the sum of expected profits:

$$PS = E(\pi_i^N(s)) + E(\pi_j^N(s)) \qquad (21)$$

Consumer surplus is the measure for the consumersô gain from trade, which can be computed by the triangular are in figure 3.1.

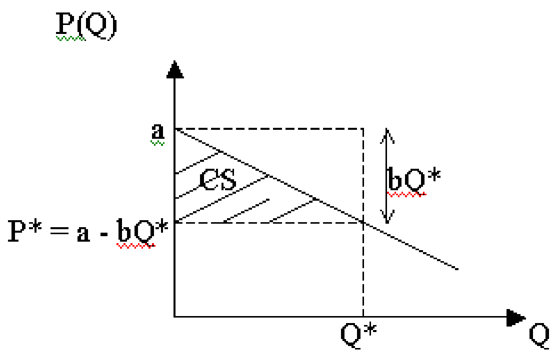

Figure 3.2: Consumer Surplus

P is the market price and Q is the total output where ñ*ò indicates the equilibrium levels. The area beneath the demand curve and above the market price defines consumer surplus as $CS = \frac{bQ*^2}{2}$. However, in our model we can speak only of expected quantities since the quantities depend on spillover rates that are uncertain:

$$CS = \frac{b\left[E(qi) + E(qj)\right]^2}{2} \tag{22}$$

where $E(qi), E(qj)$ are the expected optimal quantities obtained from equations (10) and (11) by substituting expected spillover rates described in equation (14) and also inserting x_i, x_j given in equation (19):

$$E(qi) = \frac{(a-c)(9b(s-1)\gamma - 2(\beta^H - \beta^L)^2(\lambda-1)\lambda)}{3b(4+2\beta^L(1-\beta^L) + 9b(s-1)\gamma + 2(1+\beta^H - 3\beta^L)(\beta^H - \beta^L)\lambda - 4(\beta^H - \beta^L)^2\lambda^2)}$$

$$i = 1,2 \tag{23}$$

$$CS = \frac{2(a-c)^2(9b(s-1)\gamma - 2(\beta^H - \beta^L)^2(\lambda-1)\lambda)^2}{9b(4+2\beta^L(1-\beta^L) + 9b(s-1)\gamma + 2(1+\beta^H - 3\beta^L)(\beta^H - \beta^L)\lambda - 4(\beta^H - \beta^L)^2\lambda^2)^2} \tag{24}$$

Thus, the social welfare can be computed as

$$W^N(x_i, x_j, s) = E(\pi_i^N(s)) + E(\pi_j^N(s)) + \frac{b\left[E(qi) + E(qj)\right]^2}{2} - s\gamma\left\{\frac{\left[x_i^N(s)\right]^2 + \left[x_j^N(s)\right]^2}{2}\right\}$$

$$i \neq j, i = 1,2 \tag{25}$$

$$\begin{aligned}W^N(s) = {} & (4(a-c)^2(81b^2(s-1)^2\gamma^2 - 2(\beta^H - \beta^L)^2(\lambda-1)\lambda)(5(\beta^L-2)^2 \\ & + 2(\beta^H - \beta^L)(\beta^H + 4\beta^L - 10)\lambda + 3(\beta^H - \beta^L)^2\lambda^2 + 9b\gamma(-(\beta^L-2)^2 \\ & + 2(\beta^H - \beta^L)(2 - 2\beta^H + \beta^L + 2s(\beta^H - \beta^L)\lambda - (-3+4s)(\beta^H - \beta^L)^2\lambda^2)))/ \\ & (9b(4+2\beta^L(1-\beta^L) + 9b(s-1)\gamma + 2(1+\beta^H - 3\beta^L)(\beta^H - \beta^L)\lambda - 4(\beta^H - \beta^L)^2\lambda^2)^2)\end{aligned} \tag{26}$$

Government determines the optimal subsidy rate that will be given to firms i and j by solving the maximization problem:

$$\max_{s} W^N(s) \tag{27}$$

as

$$s_i^N = \frac{2(\beta^H - \beta^L)^2(\lambda - 1)\lambda(8 + 7\beta^L(\lambda - 1) - 7\beta^H\lambda) + 27b\gamma(\beta^L(\lambda - 1) - \beta^H\lambda}{18b\gamma(-1 + \beta^L(\lambda - 1) - \beta^H\lambda))} \qquad i = 1,2 \tag{28}$$

Inserting (28) into (19), we obtain the noncooperative equilibrium R&D levels of firma as follows:

$$x_i^N = \frac{4(a - c)(1 + \beta^L + \lambda(\beta^H - \beta^L))}{-4 - 4\beta^L(2 + \beta^L) + 9b\gamma + 2(-4 - 5\beta^H + \beta^L)(\beta^H - \beta^L)\lambda + 6(\beta^H - \beta^L)^2\lambda^2} \qquad i = 1,2 \tag{29}$$

Inserting (28) into (20), we obtain the noncooperative expected equilibrium profit of firms:

$$\begin{aligned}\pi_i^N = {} & ((\mathrm{a} - \mathrm{c})^2(81b^2\gamma^2 + 36b\gamma(-2 - \beta^L + (\beta^L)^2 - (1 + 5\beta^H - 7\beta^L)(\beta^H - \beta^L)\lambda \\ & + 6(\beta^H - \beta^L)^2\lambda^2) + 4(\beta^H - \beta^L)^2(\lambda - 1)\lambda(-6(1 + \beta^L)(6 + \beta^L) \\ & - (42 + 25\beta^H - 13\beta^L)(\beta^H - \beta^L)\lambda + 19(\beta^H - \beta^L)^2\lambda^2)))/ \\ & (9b(4 + 4\beta^L(2 + \beta^L) - 9b\gamma + 2(-4 - 5\beta^H + \beta^L)(-\beta^H + \beta^L)\lambda - 6(\beta^H - \beta^L)^2\lambda^2)^2)\end{aligned} \qquad i = 1,2 \tag{30}$$

Inserting (28) into (26), we obtain the equilibrium level of social welfare for the R&D competition:

$$W^N = \frac{4(\mathrm{a} - \mathrm{c})^2(9b\gamma + 10(\beta^H - \beta^L)^2(\lambda - 1)\lambda)}{9b(-4 - 4\beta^L(2 + \beta^L) + 9b\gamma - 2(-4 - 5\beta^H + \beta^L)(-\beta^H + \beta^L)\lambda + 6(\beta^H - \beta^L)\lambda^2)} \tag{31}$$

3.2. R&D Cooperation (Case C)

In case of cooperation in R&D investments, firms coordinate their R&D expenditures to maximize the expected overall industry profits, not their individual profits. Although firms take R&D decisions jointly, they do not share the results of their R&D efforts; there is no cooperation in R&D knowledge. So the marginal cost of production is decreased only by the firmös own R&D effort and by the spillover from competitorös R&D effort, in the same way it occurs in R&D competition.

To determine the subgame perfect Nash equilibrium, firms maximize their total profits:

$$\max_{x_i} \; E(\pi_i + \pi_j) \text{ and } \max_{x_j} \; E(\pi_i + \pi_j) \tag{32}$$

and obtain the following best response functions[3]:

$$x_i^C(x_j, s) = \frac{2(c + 4x_j + a(-1 + \beta^L(\lambda - 1) - \beta^H\lambda) - (\beta^L(\lambda-1) - \beta^H\lambda)(c + 2x_j(-5 - 2\beta^L(\lambda-1) + 2\beta^H\lambda)))}{10 + 9\,\mathrm{b}\,(\mathrm{s}-1)\gamma - 2\,\beta^L\,(-8 + 5\beta^L)(\lambda - 1) + 2\beta^H\,(-8 + 5\,\beta^H)\,\lambda} = 0$$

$$i \neq j, i = 1,2 \tag{33}$$

Simultaneously solving for x_i, x_j, we obtain the equilibrium cooperative R&D levels[4] as follows:

$$x_i^C(s) = \frac{2(a-c)(-1 + \beta^L(\lambda - 1) - \beta^H\lambda)}{2 + 2\beta^L(2 + \beta^L) - 9b\gamma(1-s) + 2\lambda(2 + 5\beta^H - 3\beta^L)(\beta^H - \beta^L) - 8(\beta^H - \beta^L)^2\lambda^2}$$

$$i = 1,2 \tag{34}$$

Inserting (34) into equations (15) and (16), we obtain the expected profits:

$$E(\pi_i^C(s)) = \frac{(a-c)^2(9b(s-1)\gamma - 10(\beta^H - \beta^L)^2(\lambda - 1)\lambda}{9b(2 + 2\beta^L(2 + \beta^L) - 9b\gamma(1-s) + 2\lambda(2 + 5\beta^H - 3\beta^L)(\beta^H - \beta^L) - 8(\beta^H - \beta^L)^2\lambda^2)}$$

$$i = 1,2 \tag{35}$$

---

[3] See Appendix B for the second order conditions.
[4] See Appendix B for the stability conditions.

As in the noncooperative case, the government subsidizes the cost of the R&D project so as to maximize the social welfare:

$$W^C(x_i, x_j, s) = E(\pi_i^C(s)) + E(\pi_j^C(s)) + \frac{b[E(qi)+E(qj)]^2}{2} - s\gamma\left\{\frac{[x_i^C(s)]^2 + [x_j^C(s)]^2}{2}\right\}$$

$$i \neq j, i = 1,2 \qquad (36)$$

$$\begin{aligned} W^C(s) = {} & (2(a-c)^2(-18s\gamma(1+\beta^L+(\beta^H-\beta^L)\lambda)^2 + \frac{1}{b}(9(s-1)\gamma - 10(\beta^H-\beta^L)^2(\lambda-1)\lambda)^2 \\ & + \frac{1}{b}((9(s-1)\gamma - 10(\beta^H-\beta^L)^2(\lambda-1)\lambda)(2+4\beta^L+2(\beta^L)^2 - 9b\gamma(1-s) \\ & + 2(2+5\beta^H-3\beta^L)(\beta^H-\beta^L)\lambda - 8(\beta^H-\beta^L)^2\lambda^2))))/ \\ & (9(2+2\beta^L(2+\beta^L)+9b(s-1)\gamma + 2(2+5\beta^H-3\beta^L)(\beta^H-\beta^L)\lambda - 8(\beta^H-\beta^L)^2\lambda^2)^2) \end{aligned} \qquad (37)$$

Government decides the optimal subsidy rate for cooperative R&D by maximizing the social welfare function given in (37):

$$\max_s \qquad W^C(s) \qquad (38)$$

$$s_i^C = \frac{9b\gamma + 10(\beta^H-\beta^L)^2(\lambda-1)\lambda}{18b\gamma} \qquad i = 1,2 \qquad (39)$$

Inserting (39) into (34), we obtain the level of R&D that cooperating firms decide to invest after the government decides the optimal subsidy rate:

$$x_i^C = \frac{4(\text{a - c})(-1+\beta^L(\lambda-1)-\beta^H\lambda)}{4+4\beta^L(2+\beta^L)-9b\gamma+2(-4-5\beta^H+\beta^L)(-\beta^H+\beta^L)\lambda-6(\beta^H-\beta^L)\lambda^2}$$

$$i = 1,2 \qquad (40)$$

Inserting (39) into (35), we obtain the expected profit of cooperating firms:

$$\pi_i^C = \frac{(\text{a - c})^2(9b\gamma+10(\beta^H-\beta^L)^2(\lambda-1)\lambda)}{9b(-4-8\beta^L-4\beta^{L^2}+9b\gamma+2(-4-5\beta^H+\beta^L)(\beta^H-\beta^L)\lambda+6(\beta^H-\beta^L)^2\lambda^2)}$$

$$i = 1,2 \qquad (41)$$

Inserting (39) into (37), we obtain the social welfare, which is maximized by the government, in case of R&D cooperation:

$$W^C = \frac{4(\mathrm{a}-\mathrm{c})^2(9b\gamma + 10(\beta^H - \beta^L)^2(\lambda-1)\lambda)}{9b(-4-4\beta^L(2+\beta^L)+9b\gamma+2(-4-5\beta^H+\beta^L)(\beta^H-\beta^L)\lambda+6(\beta^H-\beta^L)\lambda^2)} \quad (42)$$

4. COMPARISON OF CASES

The effect of R&D cooperation on the social welfare, profits, prices, output quantities and subsidies will be analyzed by comparing the two cases.

Proposition 1: Cooperating or competing in R&D investment stage yields the same levels of R&D provided that the government subsidizes firmsô R&D costs.

Comparing the equations (29) and (40), we conclude that

$$x_i^N = x_i^C \qquad i = 1,2 \quad (43)$$

We know from AJôs result that, for large spillovers, the level of R&D increases when firms cooperate in R&D. It is interesting to observe that with subsidized R&D, without depending on the spillover rate, we obtain the same R&D level for both of the cooperative and noncooperative cases.

Proposition 2: Cooperating or competing in R&D investment stage yields the same levels of social welfare provided that the government subsidizes firmsô R&D costs.

The welfare functions defined in the equations (31) and (42) are equal to each other:

$$W^N = W^C \qquad i = 1,2 \quad (44)$$

Government adjusts subsidies so that it will maximize the social welfare, where the optimal levels of social welfare are the same in both cases.

Proposition 3: Cooperating or competing in R&D investment stage does not lead to different levels of output and price provided that the government subsidizes firmsô R&D costs.

The amount of outputs q_i^N, q_i^C are symmetric and they are functions of R&D levels $x_i^N, x_j^N, x_i^C, x_j^C$ as defined in equations (10) and (11). The equality of R&D levels in two cases ($x_i^N = x_i^C$), leads to the equality of outputs:

$$q_i^N = q_i^C \qquad i = 1,2 \tag{45}$$

Since the market prices are functions of output quantities
$P^N = a - b(q_i^N + q_j^N)$, $P^C = a - b(q_i^C + q_j^C)$
and the quantities in two cases are equal to each other, the market prices in both cases will also be equal:

$$P^N = P^C \qquad i = 1,2 \tag{46}$$

An analytical comparison of subsidies and firm profits is mostly inconclusive because of the large number of parameters of the model. So we restrict ourselves to a numerical and graphical analysis. For these comparisons, the following numerical values are assigned to the parameters: $a - c = 1$, $b = 2$, $\gamma = 1$. Now the subsidies and profits are functions of only β^H, β^L and λ. These parameter values are chosen to satisfy the first and second order conditions and the stability conditions of R&D levels and profit levels[5]. Although we assign numerical values to these three parameters, all relevant results are qualitatively robust against variations in these

[5] See Appendix C for second order and stability conditions satisfied by the numerical values.

parameters. Different values of the parameter $a-c$ do not lead to any change in results since it is only a scaling factor.

Assigning the values $b=2$, $\gamma=1$ to the parameters in equations (28) and (39), we obtain the firm-symmetric optimal subsidy rates:

$$s^N = -\frac{(\beta^H-\beta^L)^2(\lambda-1)\lambda(8+7\beta^L(\lambda-1)-7\beta^H\lambda)-27(\beta^L(1-\lambda)+\beta^H\lambda)}{18(1+\beta^L(1-\lambda)+\beta^H\lambda)} \tag{47}$$

$$s^C = \frac{1}{18}(9+5(\beta^H-\beta^L)^2(\lambda-1)\lambda) \tag{48}$$

Assigning the values $a-c=1$, $b=2$, $\gamma=1$ to the parameters in equations (30) and (41), we obtain the firm profits:

$$\begin{aligned}\pi_i^N &= (324+72(-2-\beta^L+(\beta^L)^2-(1+5\beta^H-7\beta^L)(\beta^H-\beta^L)\lambda+6(\beta^H-\beta^L)^2\lambda^2)\\ &+4(\beta^H-\beta^L)^2(\lambda-1)\lambda(-6(1+\beta^L)(6+\beta^L)-(42+25\beta^H-13\beta^L)(\beta^H-\beta^L)\lambda+19(\beta^H-\beta^L)^2\lambda^2))/\\ &(18(-14+8\beta^L+4\beta^{L^2}+2(-4-5\beta^H+\beta^L)(-\beta^H+\beta^L)\lambda-6(\beta^H-\beta^L)^2\lambda^2)^2)\end{aligned} \tag{49}$$

$$\pi_i^C = \frac{9+5(\beta^H-\beta^L)^2(\lambda-1)\lambda}{18(7-2\beta^L(2+\beta^L))+(-4-5\beta^H+\beta^L)(\beta^H-\beta^L)\lambda+3(\beta^H-\beta^L)^2\lambda^2)} \tag{50}$$

Now the subsidies and profits are dependent only on the parameters $\beta^H, \beta^L, \lambda$. Due to the numerical analysis above we reach the following propositions:

Proposition 4: For sufficiently large values of expected spillover rates ($E(\beta)\geq 1/2$), the optimal subsidy rate chosen by the government in noncooperative case is greater than the rate chosen in cooperative case.

The subsidies in both cases of cooperation and competition in R&D, that is, s^N and s^C will be compared via analyzing the difference $s^C - s^N$ graphically. When the difference is positive it indicates that $s^C > s^N$, which means the optimal subsidy rate chosen in case of R&D cooperation is greater than the subsidy rate in R&D noncooperation. The opposite holds for a negative difference. We will repeat this analysis for different values of λ, i.e. probability of high spillover rate. Afterwards, we will attain some general implications regarding the subsidy rates under different spillover rates.

Proposition 4 can be verified by analyzing the figures 3.3-3.5 and A.1-A.3. Figures 3.3-3.5 are the three-dimensional graphics of $s^C - s^N$ across β^H and β^L for three different values of λ. Notice that assigning a value to the probability of high spillover rate (λ), also means assigning a value to the probability of low spillover rate ($1-\lambda$).

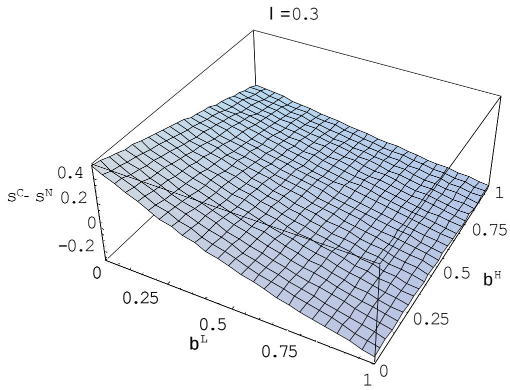

Figure 3.3: s^C ï s^N as a function of ϐ^H and ϐ^L when ϡ = 0.3

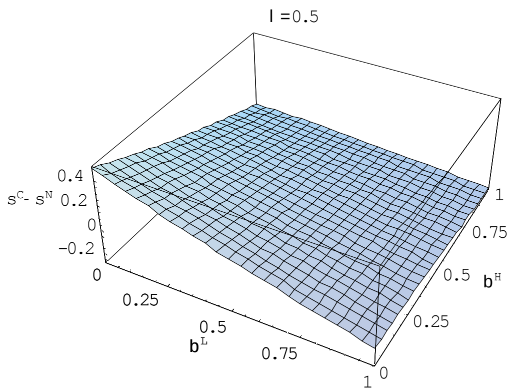

Figure 3.4: s^C ï s^N as a function of ϐ^H and ϐ^L when ϡ = 0.5

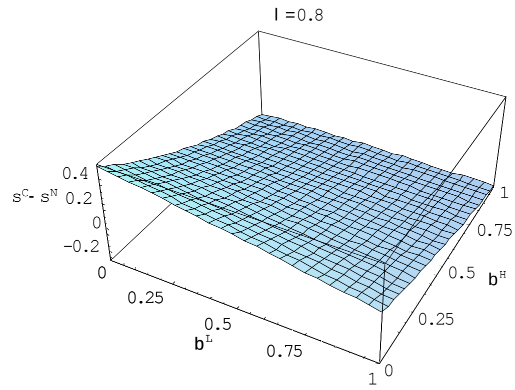

Figure 3.5: s^C ï s^N as a function of ϐ^H and ϐ^L when ঌ = 0.8

It is observed that $s^C - s^N$ is negative for some values of β^H and β^L and positive for some other values. Now we will analyze these values to see how β^H, β^L affect $s^C - s^N$. The analysis is carried out by drawing two-dimensional graphics of $s^C - s^N$ across β^L. Three-dimensional graphics are reduced to two-dimensional graphics by assigning consecutive β^H values between 0 and 1. The procedure is repeated for other values of λ to find a general implication about the relation between spillover rates and subsidy policies. Two-dimensional graphics of $s^C - s^N$ across β^L is shown in figures A.1-A.3 in Appendix A. β^L values which makes $s^C - s^N$ positive/negative are detected for each λ and then expected spillover rates are computed via equation (14). The results are shown in Tables 3.1-3.3.

Table 3.1: The relation between expected spillover rate and subsidy rate when $\lambda = 0.3$

β^H	β^L	$E(\beta) = \lambda\beta^H + (1-\lambda)\beta^L$	Subsidy rate comparison
≤ 0.5	All β^L [6]	≤ 0.493	$s^C > s^N$
$= 0.6$	≤ 0.45	≤ 0.495	$s^C > s^N$
$= 0.6$	> 0.45	≥ 0.495	$s^C < s^N$
$= 0.7$	≤ 0.39	≤ 0.483	$s^C > s^N$
$= 0.7$	> 0.39	> 0.483	$s^C < s^N$
$= 0.8$	≤ 0.32	≤ 0.464	$s^C > s^N$
$= 0.8$	> 0.32	> 0.464	$s^C < s^N$
$= 0.9$	≤ 0.24	≤ 0.438	$s^C > s^N$
$= 0.9$	> 0.24	> 0.438	$s^C < s^N$
$= 1.0$	≤ 0.13	≤ 0.391	$s^C > s^N$
$= 1.0$	> 0.13	> 0.391	$s^C < s^N$

Table 3.2: The relation between expected spillover rate and subsidy rate when $\lambda = 0.5$

β^H	β^L	$E(\beta) = \lambda\beta^H + (1-\lambda)\beta^L$	Subsidy rate comparison
≤ 0.5	All β^L	≤ 0.495	$s^C > s^N$
$= 0.6$	≤ 0.38	≤ 0.490	$s^C > s^N$
$= 0.6$	> 0.38	> 0.490	$s^C < s^N$
$= 0.7$	≤ 0.22	≤ 0.460	$s^C > s^N$
$= 0.7$	> 0.22	> 0.460	$s^C < s^N$
≥ 0.8	All β^L	> 0.400	$s^C < s^N$

Table 3.3: The relation between expected spillover rate and subsidy rate when $\lambda = 0.8$

β^H	β^L	$E(\beta) = \lambda\beta^H + (1-\lambda)\beta^L$	Subsidy rate comparison
≤ 0.5	All β^L	≤ 0.498	$s^C > s^N$
> 0.5	All β^L	≥ 0.480	$s^C < s^N$

For all λ, the optimal subsidy rate chosen in case of cooperative R&D is less than the optimal subsidy rate chosen in case of noncooperative R&D, as long as the expected spillover rate is sufficiently large, that is $E(\beta) \geq 0.5$. The opposite holds when the expected spillover rate is sufficiently small, such that $E(\beta) \leq 0.39$.

[6] Notice that $0 \leq \beta^L < \beta^H \leq 1$. Hence, the statement "all β^L" refers to β^L in the interval $[0, \beta^H]$

Proposition 5: For sufficiently large values of expected spillover rates ($E(\beta) \geq 1/2$), the profit levels of noncooperative firms are greater than the profit levels of cooperative firms.

The expected profit levels in both cases of cooperation and competition in R&D, that is π^N and π^C, will be compared via analyzing the difference $\pi^C - \pi^N$ graphically, in exactly the same way as subsidy rate is compared. When the difference is positive, i.e. $\pi^C > \pi^N$, the profits of firms behaving cooperatively in R&D is greater than the profits of noncooperative firms. The opposite holds for a negative difference. Again the analysis will be repeated for different values of λ to draw general implications regarding the profits under different spillover rates.

Proposition 5 can be verified by analyzing the figures 3.6-3.8 and A.4-A.6. The three-dimensional graphics of $\pi^C - \pi^N$ across β^H and β^L is shown in figures 3.6-3.8, where each graphic is drawn for a different value of λ.

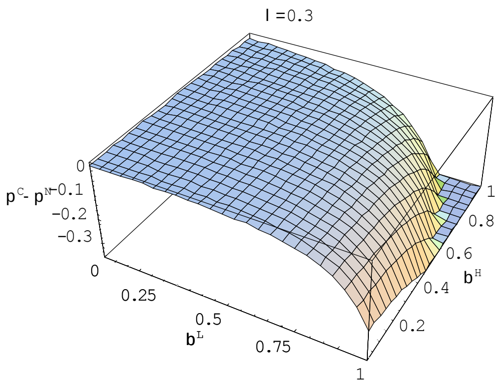

Figure 3.6: $\pi^C - \pi^N$ as a function of β^H and β^L when $\lambda = 0.3$

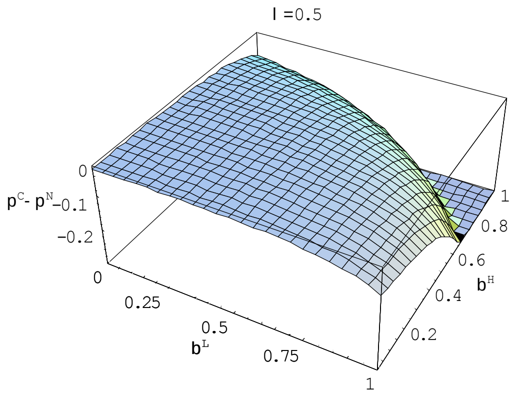

Figure 3.7: $\pi^C - \pi^N$ as a function of β^H and β^L when $\lambda = 0.5$

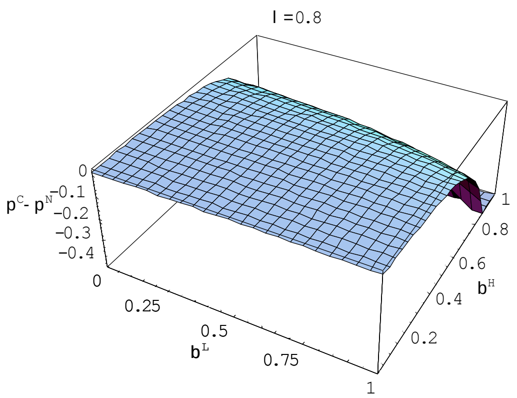

Figure 3.8: $\pi^C - \pi^N$ as a function of β^H and β^L when $\lambda = 0.8$

To explore the regions where $\pi^C - \pi^N$ becomes negative, three-dimensional graphics are reduced to two-dimensional graphics by assigning β^H values between 0 and 1. When the procedure is repeated for other values of λ, figures A.4-A.6 are obtained. β^L values which makes $\pi^C - \pi^N$ positive/negative in these figures are detected for each λ and then the corresponding expected spillover rates are computed via equation (14) to find a general implication about the relation between spillover rates and profits. The results are shown in Tables 3.4-3.6.

Table 3.4: The relation between expected spillover rate and profit level when λ = 0.3

β^H	β^L	**E(β) = $\lambda\beta^H$+(1-λ) β^L**	**Profit comparison**
$\leq$0.5	All β^L	$\leq$0.493	$\pi^C > \pi^N$
= 0.6	$\leq$0.45	$\leq$0.495	$\pi^C > \pi^N$
= 0.6	> 0.45	$\geq$0.495	$\pi^C < \pi^N$
= 0.7	$\leq$0.39	$\leq$0.483	$\pi^C > \pi^N$
= 0.7	> 0.39	> 0.483	$\pi^C < \pi^N$
= 0.8	$\leq$0.32	$\leq$0.464	$\pi^C > \pi^N$
= 0.8	> 0.32	> 0.464	$\pi^C < \pi^N$
= 0.9	$\leq$0.24	$\leq$0.438	$\pi^C > \pi^N$
= 0.9	> 0.24	> 0.438	$\pi^C < \pi^N$
= 1.0	$\leq$0.13	$\leq$0.391	$\pi^C > \pi^N$
= 1.0	> 0.13	> 0.391	$\pi^C < \pi^N$

Table 3.5: The relation between expected spillover rate and profit level when λ = 0.5

β^H	β^L	**E(β) = $\lambda\beta^H$+(1-λ) β^L**	**Profit comparison**
$\leq$0.5	All β^L	$\leq$0.495	$\pi^C > \pi^N$
= 0.6	$\leq$0.38	$\leq$0.490	$\pi^C > \pi^N$
= 0.6	> 0.38	> 0.490	$\pi^C < \pi^N$
= 0.7	$\leq$0.22	$\leq$0.460	$\pi^C > \pi^N$
= 0.7	> 0.22	> 0.460	$\pi^C < \pi^N$
$\geq$0.8	All β^L	> 0.400	$\pi^C < \pi^N$

Table 3.6: The relation between expected spillover rate and profit level when λ = 0.8

β^H	β^L	**E(β) = $\lambda\beta^H$+(1-λ) β^L**	**Profit comparison**
$\leq$0.5	All β^L	$\leq$0.498	$\pi^C > \pi^N$
> 0.5	All β^L	$\geq$0.480	$\pi^C < \pi^N$

For sufficiently large values of expected spillover rates, that is $E(\beta) \geq 0.5$, the profits in noncooperative R&D is higher than the profits in cooperative R&D. The opposite holds when the expected spillover rate is sufficiently small, such that $E(\beta) \leq 0.39$. These results hold for all λ.

This result can be explained by analyzing the components of profit function. Profit level is the total revenue where production cost and R&D cost is subtracted from and subsidy is added on. Since the prices and output quantities are equal for two cases, total revenue is also the same. Since the R&D levels are equal, R&D costs are also equal. Then only subsidy is left as the reason of different profit levels. Hence, for large spillovers, the higher subsidy rates in noncooperative R&D lead to also higher profits. The opposite holds for small spillovers. As a result, the firms will tend to compete in R&D when the spillover rate is sufficiently large, and to cooperate when it is small, which is the result of the first stage of the game.

5. CONCLUSIONS

The cooperative and competitive R&D with uncertain spillover rates are analyzed in a duopoly where R&D costs are subsidized by the government. The effects of subsidy policies on production level, market prices, social welfare and firm profits are investigated. Although the model is based on AJ model, our results are different from theirs, because of the public policy extension of the model.

One of the common results on R&D literature is the fact that when the spillovers are sufficiently large, cooperative R&D leads to higher R&D level than the noncooperative R&D does. But we get rather different results since public policy is introduced to our model. In our model, subsidy policies of government lead to equal levels of welfare and R&D investment in both cases of cooperative and competitive R&D. The spillover rate does not affect welfare or R&D level, as it does in AJ or KMZ models, since in our model there is an optimal level of R&D that maximizes social welfare and government adjusts the subsidy rate such that firms choose this R&D level in both cases for any rate of spillover. As a result of the subsidy policy, R&D levels in both cases are optimal and equal to each other.

Since the government chooses the subsidy rate which will lead to the same maximized welfare in both cases, it chooses a higher subsidy in noncooperative case as long as the spillover rate is sufficiently high. This result is compatible with AJôs result: ñfor large spillovers the level of R&D increases when firms cooperate in R&Dò. This difference of R&D levels in two cases mentioned in AJ model is compensated by subsidies in our model. Therefore the R&D levels in both cases are equal and optimal subsidy rate increases when firms compete in R&D with large spillovers.

Another common result on R&D literature is that when the spillovers are sufficiently large, cooperative R&D leads to higher profit levels than noncooperative R&D does, as proved in KMZ. However, this is the state where there is no subsidy. When we introduce subsidy policy into the model, the profits increase when firms compete in case of large spillovers and decrease when spillovers are low. Analyzing the components of the profit level, we concluded that different subsidy rates is the reason of different profit levels in two cases. Hence when spillover is large, the subsidy rate chosen for competing firms is higher than the subsidy rate chosen for cooperating firms that, this subsidy difference provides the competing firms more profit than cooperating firms. The opposite holds for small spillovers. As a result, the firms will tend to compete when the expected spillover rate is high and to cooperate when it is low.

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APPENDIX A

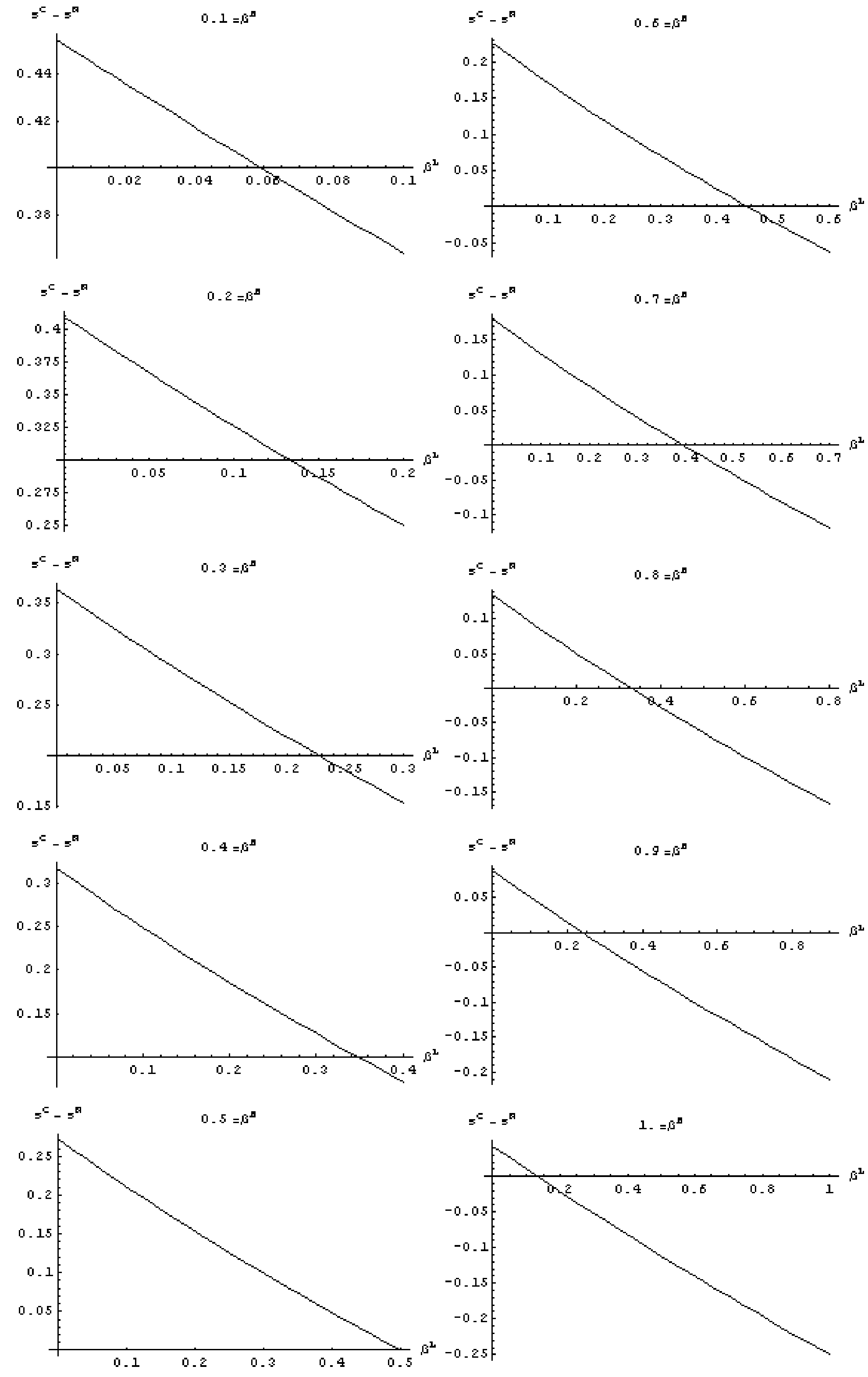

Figure A.1: $s^C - s^N$ as a function of β^L for different values of β^H when $\theta = 0.3$

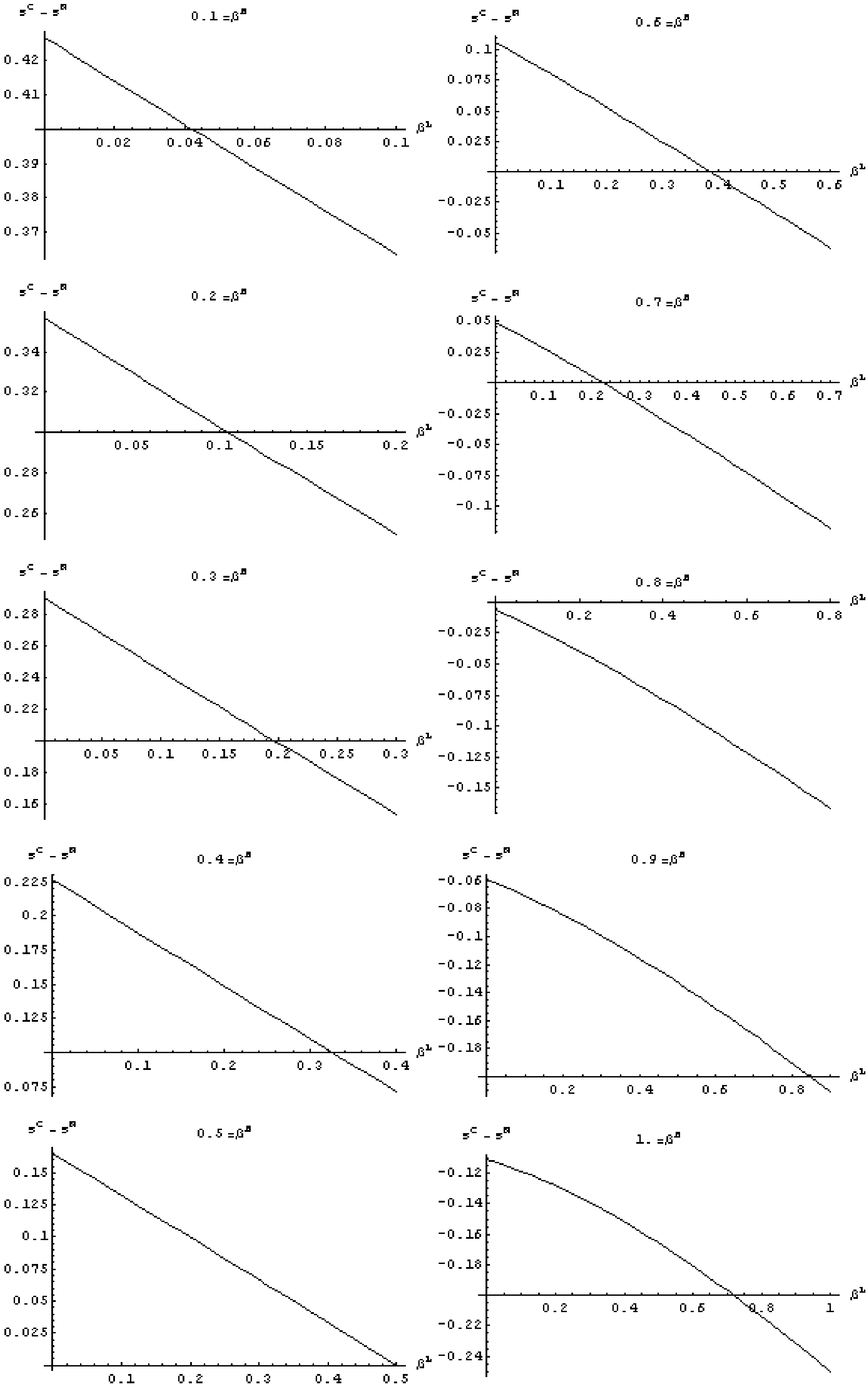

Figure A.2: $s^C - s^N$ as a function of β^L for different values of β^H when $\alpha = 0.5$

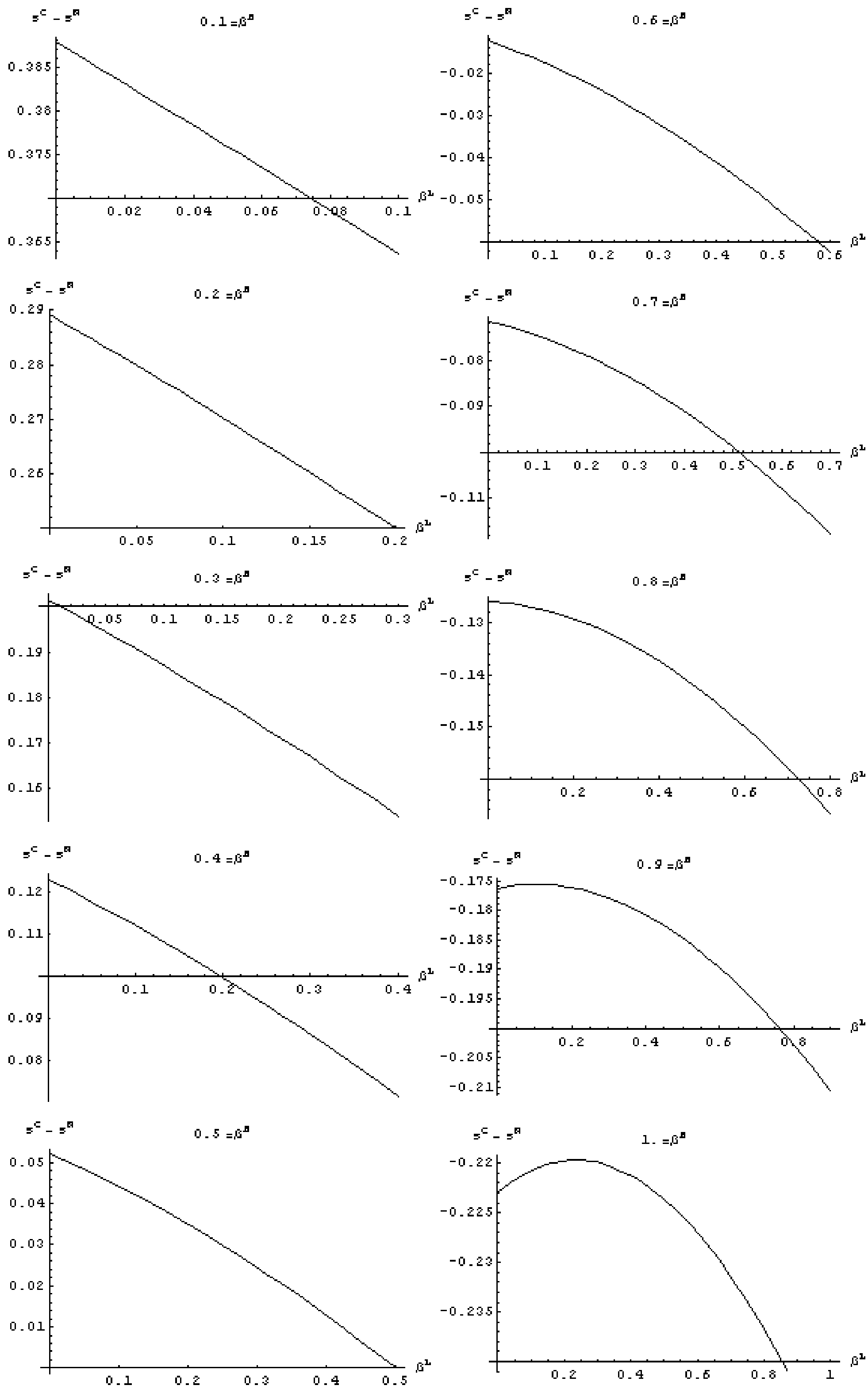

Figure A.3: $s^C - s^N$ as a function of β^L for different values of β^H when ə = 0.8

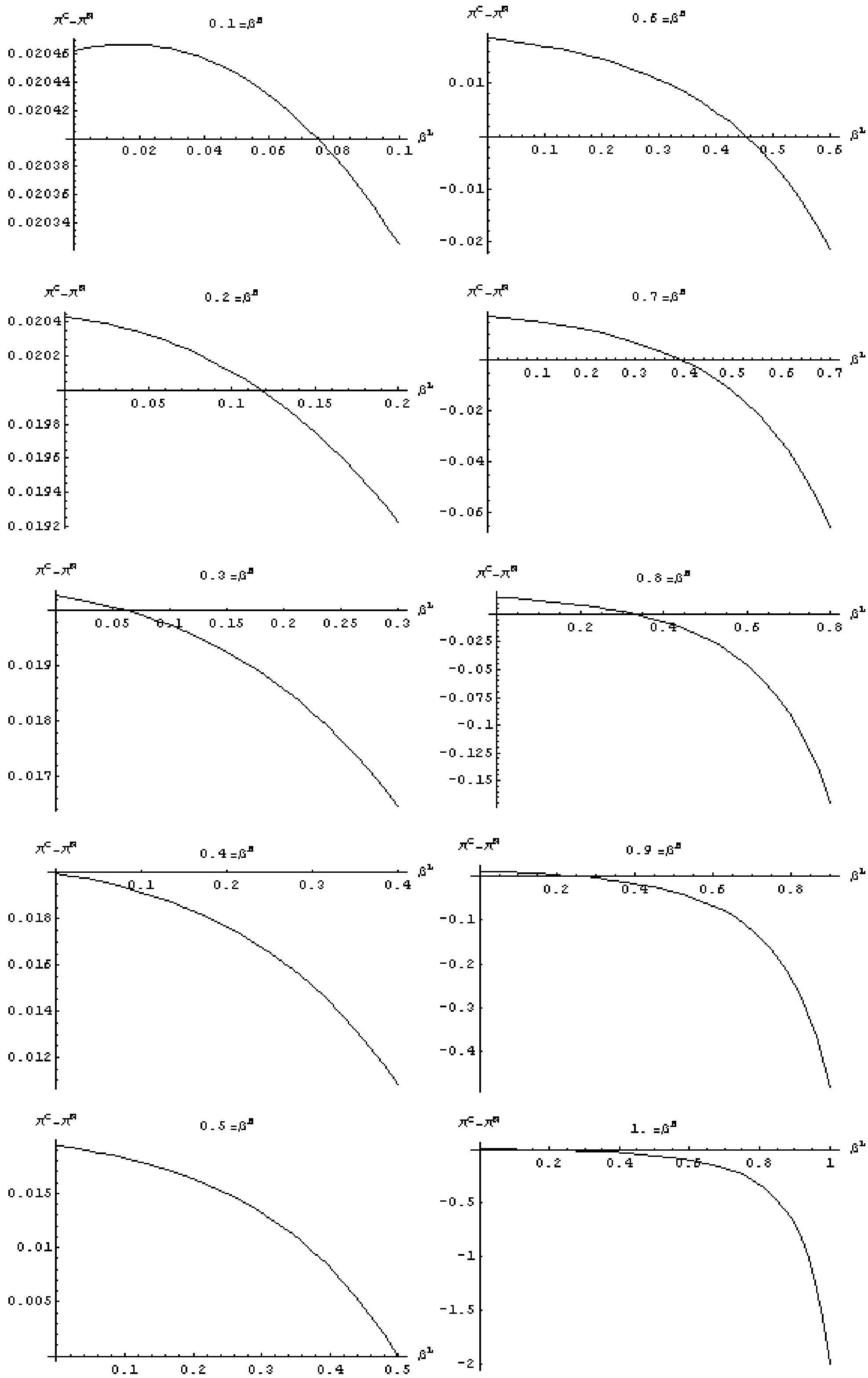

Figure A.4: $\pi^C - \pi^N$ as a function of β^L for different values of β^H when ɚ = 0.3

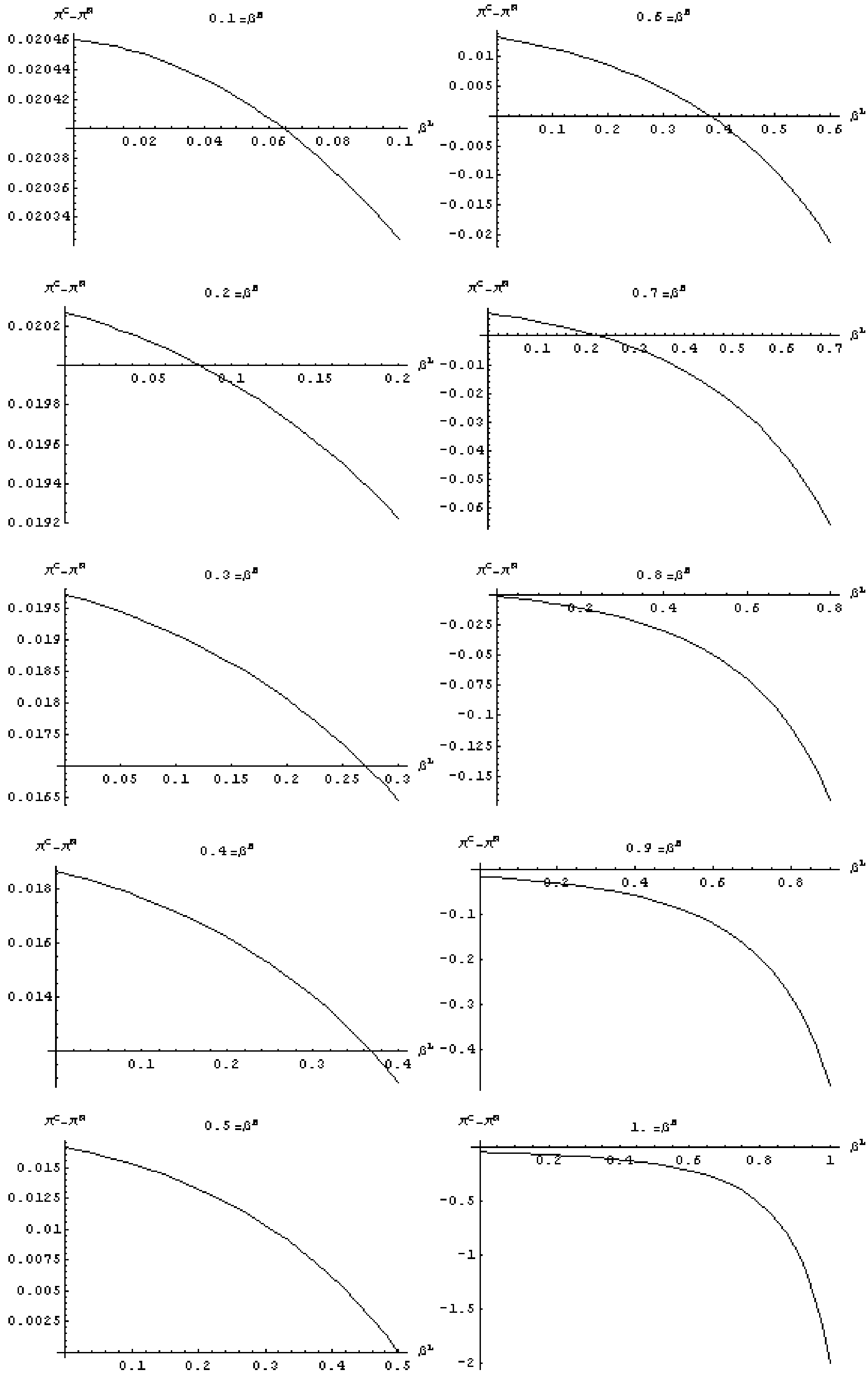

Figure A.5: $\pi^C - \pi^N$ as a function of β^L for different values of β^H when ə = 0.5

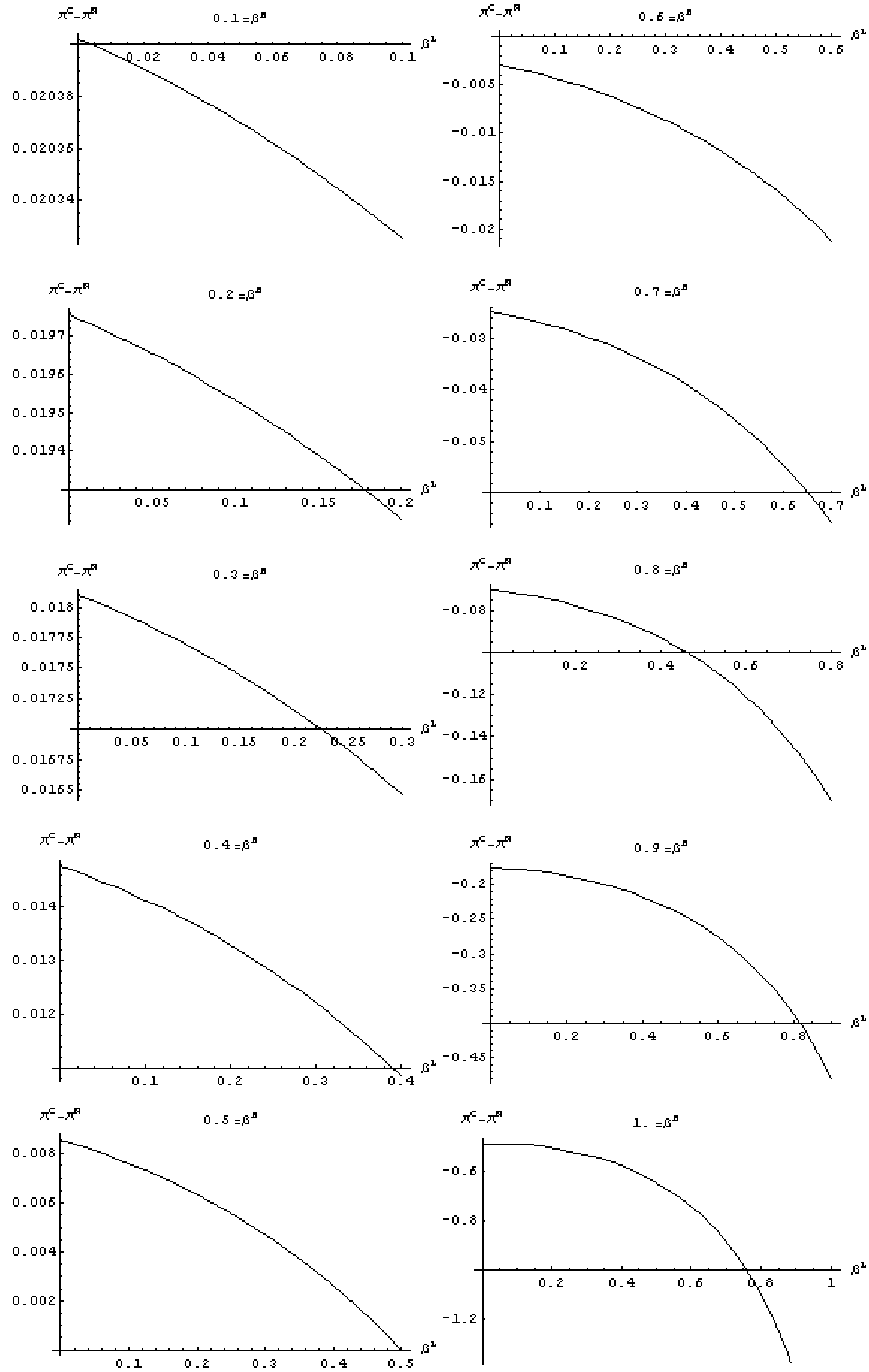

Figure A.6: $\pi^C - \pi^N$ as a function of β^L for different values of β^H when ɚ = 0.8

APPENDIX B

B.1. Second Order Conditions

$$\frac{\partial^2 E(\pi_i(x_i,x_j))}{\partial x_i^2}<0 \qquad \frac{\partial^2 E(\pi_j(x_i,x_j))}{\partial x_j^2}<0 \quad i\neq j, i=1,2 \qquad \text{for cases N and C.}$$

$E(\pi_i(x_i,x_j))$ and $E(\pi_j(x_i,x_j))$ is obtained by inserting (28) into (15) and (16) for case N, and inserting (39) into (15) and (16) for case C.

Case N:

$$\frac{\partial^2 E(\pi_i(x_i,x_j))}{\partial x_i^2}=\frac{1}{36b}(92+60\beta^H-8\beta^L(4-\beta^L)+18b\gamma-4(23+5\beta^H-9\beta^L)(\beta^H-\beta^L)\lambda$$
$$+28(\beta^H-\beta^L)^2\lambda^2+(6(-10(\beta^H+1)(1+\beta^L)-9b\gamma))/(1+\lambda\beta^H+\beta^L(1-\lambda)))<0 \tag{51}$$

$$\frac{\partial^2 E(\pi_j(x_i,x_j))}{\partial x_i^2}=\frac{1}{36b}(92+60\beta^H-8\beta^L(4-\beta^L)+18b\gamma-4(23+5\beta^H-9\beta^L)(\beta^H-\beta^L)\lambda$$
$$+28(\beta^H-\beta^L)^2\lambda^2+(6(-10(\beta^H+1)(1+\beta^L)-9b\gamma))/(1+\lambda\beta^H+\beta^L(1-\lambda)))<0 \tag{52}$$

Case C:

$$\frac{\partial^2 E(\pi_i(x_i,x_j))}{\partial x_i^2}=\frac{1}{36b}(32-8\beta^L(4-\beta^L)-18b\gamma-4(8+3\beta^H-7\beta^L)(\beta^H-\beta^L)\lambda+20(\beta^H-\beta^L)^2\lambda^2<0 \tag{53}$$

$$\frac{\partial^2 E(\pi_j(x_i,x_j))}{\partial x_i^2}=\frac{1}{36b}(32-8\beta^L(4-\beta^L)-18b\gamma-4(8+3\beta^H-7\beta^L)(\beta^H-\beta^L)\lambda+20(\beta^H-\beta^L)^2\lambda^2<0 \tag{54}$$

B.2. Stability Conditions

$$\left|\frac{\partial x_i(x_j)}{\partial x_j}\right| < 1 \qquad \left|\frac{\partial x_j(x_i)}{\partial x_i}\right| < 1 \qquad i \neq j, i = 1,2 \tag{55}$$

$x_i(x_j)$ and $x_j(x_i)$ is obtained by inserting (28) into (18) for case N and (39) into (33) for case C.

$$\left|\frac{\partial x_i(x_j)}{\partial x_j}\right| = \left|\frac{4(1+2\beta^L(\lambda-1)-2\beta^H\lambda)(-1+\beta^L(\lambda-1)-\beta^H\lambda)}{(-8-4\beta^L(1-\beta^L)+9b\gamma-2(2+5\beta^H-9\beta^L)(\beta^H-\beta^L)\lambda+14(\beta^H-\beta^L)^2\lambda^2}\right| < 1 \tag{56}$$

$$\left|\frac{\partial x_j(x_j)}{\partial x_i}\right| = \left|\frac{4(1+2\beta^L(\lambda-1)-2\beta^H\lambda)(-1+\beta^L(\lambda-1)-\beta^H\lambda)}{(-8-4\beta^L(1-\beta^L)+9b\gamma-2(2+5\beta^H-9\beta^L)(\beta^H-\beta^L)\lambda+14(\beta^H-\beta^L)^2\lambda^2}\right| < 1 \tag{57}$$

APPENDIX C

(Mathematica Outputs)

Second Order Conditions are satisfied for numerical values assigned to the parameters :

epro1 =

$$\frac{1}{18\,b}\Big(\big(2\,(-a+c+x2+x1\,(-2+\beta L)-2\,x2\,\beta L)^2+9\,b\,(-1+s)\,x1^2\,\gamma\big)\,(-1+\lambda)^2+\big(2\,(-a+c+x2-2\,x2\,\beta H+x1\,(-2+\beta L))^2+9\,b\,(-1+s)\,x1^2\,\gamma\big)\,(1-\lambda)\,\lambda+\big(2\,(-a+c+x2+x1\,(-2+\beta H)-2\,x2\,\beta L)^2+9\,b\,(-1+s)\,x1^2\,\gamma\big)\,(1-\lambda)\,\lambda+\big(2\,(-a+c+x2+x1\,(-2+\beta H)-2\,x2\,\beta H)^2+9\,b\,(-1+s)\,x1^2\,\gamma\big)\,\lambda^2\Big);$$

epro2 =

$$\frac{1}{18\,b}\Big(\big(2\,(-a+c+x1+x2\,(-2+\beta L)-2\,x1\,\beta L)^2+9\,b\,(-1+s)\,x2^2\,\gamma\big)\,(-1+\lambda)^2+\big(2\,(-a+c+x1-2\,x1\,\beta H+x2\,(-2+\beta L))^2+9\,b\,(-1+s)\,x2^2\,\gamma\big)\,(1-\lambda)\,\lambda+\big(2\,(-a+c+x1+x2\,(-2+\beta H)-2\,x1\,\beta L)^2+9\,b\,(-1+s)\,x2^2\,\gamma\big)\,(1-\lambda)\,\lambda+\big(2\,(-a+c+x1+x2\,(-2+\beta H)-2\,x1\,\beta H)^2+9\,b\,(-1+s)\,x2^2\,\gamma\big)\,\lambda^2\Big);$$

$$\mathbf{sn}=\frac{2\,(\beta H-\beta L)^2\,(-1+\lambda)\,\lambda\,(8+7\,\beta L\,(-1+\lambda)-7\,\beta H\,\lambda)+27\,b\,\gamma\,(\beta L\,(-1+\lambda)-\beta H\,\lambda)}{18\,b\,\gamma\,(-1+\beta L\,(-1+\lambda)-\beta H\,\lambda)};$$

$$\mathbf{sc}=\frac{9\,b\,\gamma+10\,(\beta H-\beta L)^2\,(-1+\lambda)\,\lambda}{18\,b\,\gamma};$$

sepro1n = FullSimplify[epro1 /. s → sn]

$$\frac{1}{36\,b}\Big(4\,a^2+4\,c^2+46\,x1^2-16\,x1\,x2+4\,x2^2+30\,x1^2\,\beta H-16\,x1^2\,\beta L+40\,x1\,x2\,\beta L-16\,x2^2\,\beta L+4\,x1^2\,\beta L^2-16\,x1\,x2\,\beta L^2+16\,x2^2\,\beta L^2-8\,a\,(c+x2+x1\,(-2+\beta L)-2\,x2\,\beta L)+9\,b\,x1^2\,\gamma+8\,a\,(x1-2\,x2)\,(-\beta H+\beta L)\,\lambda-2\,(\beta H-\beta L)\,(x1^2\,(23+5\,\beta H-9\,\beta L)-8\,x2^2\,(-1+\beta H+\beta L)+4\,x1\,x2\,(-5+4\,\beta L))\,\lambda+2\,x1\,(7\,x1-8\,x2)\,(\beta H-\beta L)^2\,\lambda^2+8\,c\,(-2\,x1+x2+x1\,\beta L-2\,x2\,\beta L+(x1-2\,x2)\,(\beta H-\beta L)\,\lambda)+\frac{3\,x1^2\,(-10\,(1+\beta H)\,(1+\beta L)-9\,b\,\gamma)}{1+\beta L+\beta H\,\lambda-\beta L\,\lambda}\Big)$$

sepro2n = FullSimplify[epro2 /. s → sn]

$$\frac{1}{36\,b}\Big(4\,a^2+4\,c^2+8\,c\,x1+4\,x1^2-16\,c\,x2-16\,x1\,x2+46\,x2^2+30\,x2^2\,\beta H-16\,c\,x1\,\beta L-16\,x1^2\,\beta L+8\,c\,x2\,\beta L+40\,x1\,x2\,\beta L-16\,x2^2\,\beta L+16\,x1^2\,\beta L^2-16\,x1\,x2\,\beta L^2+4\,x2^2\,\beta L^2-8\,a\,(c+x1+x2\,(-2+\beta L)-2\,x1\,\beta L)+9\,b\,x2^2\,\gamma+8\,a\,(2\,x1-x2)\,(\beta H-\beta L)\,\lambda+2\,(\beta H-\beta L)\,(c\,(-8\,x1+4\,x2)+8\,x1^2\,(-1+\beta H+\beta L)-4\,x1\,x2\,(-5+4\,\beta L)+x2^2\,(-23-5\,\beta H+9\,\beta L))\,\lambda-2\,(8\,x1-7\,x2)\,x2\,(\beta H-\beta L)^2\,\lambda^2+\frac{3\,x2^2\,(-10\,(1+\beta H)\,(1+\beta L)-9\,b\,\gamma)}{1+\beta L+\beta H\,\lambda-\beta L\,\lambda}\Big)$$

sepro1c = FullSimplify[epro1 /. s → sc]

$$\frac{1}{36\,b}\,(4\,a^2+4\,c^2+16\,x1^2-16\,x1\,x2+4\,x2^2-16\,x1^2\,\beta L+40\,x1\,x2\,\beta L-16\,x2^2\,\beta L+4\,x1^2\,\beta L^2-16\,x1\,x2\,\beta L^2+16\,x2^2\,\beta L^2-8\,a\,(c+x2+x1\,(-2+\beta L)-2\,x2\,\beta L)-9\,b\,x1^2\,\gamma+8\,a\,(x1-2\,x2)\,(-\beta H+\beta L)\,\lambda-2\,(\beta H-\beta L)\,(x1^2\,(8+3\,\beta H-7\,\beta L)-8\,x2^2\,(-1+\beta H+\beta L)+4\,x1\,x2\,(-5+4\,\beta L))\,\lambda+2\,x1\,(5\,x1-8\,x2)\,(\beta H-\beta L)^2\,\lambda^2+8\,c\,(-2\,x1+x2+x1\,\beta L-2\,x2\,\beta L+(x1-2\,x2)\,(\beta H-\beta L)\,\lambda))$$

sepro2c = FullSimplify[epro2 /. s → sc]

$$\frac{1}{36\,b}\,(4\,a^2 + 4\,c^2 + 8\,c\,x1 + 4\,x1^2 - 16\,c\,x2 - 16\,x1\,x2 + 16\,x2^2 - 16\,c\,x1\,\beta L - 16\,x1^2\,\beta L + 8\,c\,x2\,\beta L + 40\,x1\,x2\,\beta L - 16\,x2^2\,\beta L + 16\,x1^2\,\beta L^2 - 16\,x1\,x2\,\beta L^2 + 4\,x2^2\,\beta L^2 - 8\,a\,(c + x1 + x2\,(-2 + \beta L) - 2\,x1\,\beta L) - 9\,b\,x2^2\,\gamma + 8\,a\,(2\,x1 - x2)\,(\beta H - \beta L)\,\lambda + 2\,(\beta H - \beta L)\,(c\,(-8\,x1 + 4\,x2) + 8\,x1^2\,(-1 + \beta H + \beta L) - 4\,x1\,x2\,(-5 + 4\,\beta L) + x2^2\,(-8 - 3\,\beta H + 7\,\beta L))\,\lambda - 2\,(8\,x1 - 5\,x2)\,x2\,(\beta H - \beta L)^2\,\lambda^2)$$

$\partial_{x1,x1}$sepro1n

$$\frac{1}{36\,b}\left(92 + 60\,\beta H - 32\,\beta L + 8\,\beta L^2 + 18\,b\,\gamma - 4\,(23 + 5\,\beta H - 9\,\beta L)\,(\beta H - \beta L)\,\lambda + 28\,(\beta H - \beta L)^2\,\lambda^2 + \frac{6\,(-10\,(1 + \beta H)\,(1 + \beta L) - 9\,b\,\gamma)}{1 + \beta L + \beta H\,\lambda - \beta L\,\lambda}\right)$$

$\partial_{x2,x2}$sepro2n

$$\frac{1}{36\,b}\left(92 + 60\,\beta H - 32\,\beta L + 8\,\beta L^2 + 18\,b\,\gamma + 4\,(\beta H - \beta L)\,(-23 - 5\,\beta H + 9\,\beta L)\,\lambda + 28\,(\beta H - \beta L)^2\,\lambda^2 + \frac{6\,(-10\,(1 + \beta H)\,(1 + \beta L) - 9\,b\,\gamma)}{1 + \beta L + \beta H\,\lambda - \beta L\,\lambda}\right)$$

FullSimplify[$\partial_{x1,x1}$sepro1n < 0, {b == 2, γ == 1, a - c == 1, 0 ≤ λ ≤ 1, 0 ≤ βL < βH ≤ 1}]

True

FullSimplify[$\partial_{x2,x2}$sepro2n < 0, {b == 2, γ == 1, a - c == 1, 0 ≤ λ ≤ 1, 0 ≤ βL < βH ≤ 1}]

True

$\partial_{x1,x1}$sepro1c

$$\frac{32 - 32\,\beta L + 8\,\beta L^2 - 18\,b\,\gamma - 4\,(8 + 3\,\beta H - 7\,\beta L)\,(\beta H - \beta L)\,\lambda + 20\,(\beta H - \beta L)^2\,\lambda^2}{36\,b}$$

$\partial_{x2,x2}$sepro2c

$$\frac{32 - 32\,\beta L + 8\,\beta L^2 - 18\,b\,\gamma + 4\,(\beta H - \beta L)\,(-8 - 3\,\beta H + 7\,\beta L)\,\lambda + 20\,(\beta H - \beta L)^2\,\lambda^2}{36\,b}$$

FullSimplify[$\partial_{x1,x1}$sepro1c < 0, {b == 2, γ == 1, a - c == 1, 0 ≤ λ ≤ 1, 0 ≤ βL < βH ≤ 1}]

True

FullSimplify[$\partial_{x2,x2}$sepro2c < 0, {b == 2, γ == 1, a - c == 1, 0 ≤ λ ≤ 1, 0 ≤ βL < βH ≤ 1}]

True

Stability Conditions are satisfied for numerical values assigned to the parameters :

$$\mathbf{eqx1n} = \frac{2\,(2 + \beta L\,(-1 + \lambda) - \beta H\,\lambda)\,(-a + c + x2\,(1 - 2\,\beta L - 2\,\beta H\,\lambda + 2\,\beta L\,\lambda))}{8 + 9\,b\,(-1 + s)\,\gamma + 8\,\beta L\,(-1 + \lambda) - 2\,\beta L^2\,(-1 + \lambda) - 8\,\beta H\,\lambda + 2\,\beta H^2\,\lambda};$$

$$\mathbf{eqx2n} = \frac{2\,(2 + \beta L\,(-1 + \lambda) - \beta H\,\lambda)\,(-a + c + x1\,(1 - 2\,\beta L - 2\,\beta H\,\lambda + 2\,\beta L\,\lambda))}{8 + 9\,b\,(-1 + s)\,\gamma + 8\,\beta L\,(-1 + \lambda) - 2\,\beta L^2\,(-1 + \lambda) - 8\,\beta H\,\lambda + 2\,\beta H^2\,\lambda};$$

eqx1c = (2 (c + 4 x2 + a (-1 + βL (-1 + λ) - βH λ) - (βL (-1 + λ) - βH λ) (c + 2 x2 (-5 - 2 βL (-1 + λ) + 2 βH λ)))) / (10 + 9 b (-1 + s) γ - 2 βL (-8 + 5 βL) (-1 + λ) + 2 βH (-8 + 5 βH) λ);

eqx2c = (2 (c + 4 x1 + a (-1 + βL (-1 + λ) - βH λ) - (βL (-1 + λ) - βH λ) (c + 2 x1 (-5 - 2 βL (-1 + λ) + 2 βH λ)))) / (10 + 9 b (-1 + s) γ - 2 βL (-8 + 5 βL) (-1 + λ) + 2 βH (-8 + 5 βH) λ);

```
seqx1n = FullSimplify[eqx1n /. s → sn]
```

$$\frac{4\,(-1+\beta L\,(-1+\lambda)-\beta H\,\lambda)\,(-a+c+x2\,(1+2\,\beta L\,(-1+\lambda)-2\,\beta H\,\lambda))}{-8-4\beta L+4\beta L^2+9\,b\,\gamma-2\,(2+5\,\beta H-9\,\beta L)\,(\beta H-\beta L)\,\lambda+14\,(\beta H-\beta L)^2\,\lambda^2}$$

```
seqx2n = FullSimplify[eqx2n /. s → sn]
```

$$\frac{4\,(-1+\beta L\,(-1+\lambda)-\beta H\,\lambda)\,(-a+c+x1\,(1+2\,\beta L\,(-1+\lambda)-2\,\beta H\,\lambda))}{-8-4\beta L+4\beta L^2+9\,b\,\gamma-2\,(2+5\,\beta H-9\,\beta L)\,(\beta H-\beta L)\,\lambda+14\,(\beta H-\beta L)^2\,\lambda^2}$$

```
seqx1c = FullSimplify[eqx1c /. s → sc]
```

$$(4\,(c+4\,x2+a\,(-1+\beta L\,(-1+\lambda)-\beta H\,\lambda)-(\beta L\,(-1+\lambda)-\beta H\,\lambda)\,(c+2\,x2\,(-5-2\,\beta L\,(-1+\lambda)+2\,\beta H\,\lambda))))\,/\,(20-9\,b\,\gamma-4\,\beta L\,(-8+5\,\beta L)\,(-1+\lambda)+4\,\beta H\,(-8+5\,\beta H)\,\lambda+10\,(\beta H-\beta L)^2\,(-1+\lambda)\,\lambda)$$

```
seqx2c = FullSimplify[eqx2c /. s → sc]
```

$$(4\,(c+4\,x1+a\,(-1+\beta L\,(-1+\lambda)-\beta H\,\lambda)-(\beta L\,(-1+\lambda)-\beta H\,\lambda)\,(c+2\,x1\,(-5-2\,\beta L\,(-1+\lambda)+2\,\beta H\,\lambda))))\,/\,(20-9\,b\,\gamma-4\,\beta L\,(-8+5\,\beta L)\,(-1+\lambda)+4\,\beta H\,(-8+5\,\beta H)\,\lambda+10\,(\beta H-\beta L)^2\,(-1+\lambda)\,\lambda)$$

∂_{x2} **seqx1n**

$$\frac{4\,(1+2\,\beta L\,(-1+\lambda)-2\,\beta H\,\lambda)\,(-1+\beta L\,(-1+\lambda)-\beta H\,\lambda)}{-8-4\beta L+4\beta L^2+9\,b\,\gamma-2\,(2+5\,\beta H-9\,\beta L)\,(\beta H-\beta L)\,\lambda+14\,(\beta H-\beta L)^2\,\lambda^2}$$

∂_{x1} **seqx2n**

$$\frac{4\,(1+2\,\beta L\,(-1+\lambda)-2\,\beta H\,\lambda)\,(-1+\beta L\,(-1+\lambda)-\beta H\,\lambda)}{-8-4\beta L+4\beta L^2+9\,b\,\gamma-2\,(2+5\,\beta H-9\,\beta L)\,(\beta H-\beta L)\,\lambda+14\,(\beta H-\beta L)^2\,\lambda^2}$$

FullSimplify[∂_{x1} **seqx2n /. {b → 2, γ → 1}]**

$$\frac{2\,(1+2\,\beta L\,(-1+\lambda)-2\,\beta H\,\lambda)\,(-1+\beta L\,(-1+\lambda)-\beta H\,\lambda)}{5-2\,\beta L+2\,\beta L^2-(2+5\,\beta H-9\,\beta L)\,(\beta H-\beta L)\,\lambda+7\,(\beta H-\beta L)^2\,\lambda^2}$$

FullSimplify[Abs[∂_{x2} **seqx1n] < 1, {b == 2, γ == 1, 0 ≤ λ ≤ 1, 0 ≤ βL < βH ≤ 1}]**

True

FullSimplify[Abs[∂_{x1} **seqx2n] < 1, {b == 2, γ == 1, 0 ≤ λ ≤ 1, 0 ≤ βL < βH ≤ 1}]**

True

$x^C > 0$, $x^N > 0$ **conditions are satisfied for numerical values assigned to the parameters :**

$$x1n = -\frac{4\,(a-c)\,(-1+\beta L\,(-1+\lambda)-\beta H\,\lambda)}{-4-8\,\beta L-4\,\beta L^2+9\,b\,\gamma-2\,(-4-5\,\beta H+\beta L)\,(-\beta H+\beta L)\,\lambda+6\,(\beta H-\beta L)^2\,\lambda^2};$$

FullSimplify[x1n > 0, {b > 0, a > c > 0, γ > 0, s ≥ 0, 0 ≤ λ ≤ 1, 0 ≤ βL < βH ≤ 1}]

$$9\,b\,\gamma+6\,(\beta H-\beta L)^2\,\lambda^2>4\,(1+\beta L)^2+2\,(-4-5\,\beta H+\beta L)\,(-\beta H+\beta L)\,\lambda$$

FullSimplify[x1n > 0, {a - c == 1, b == 2, γ == 1, 0 ≤ λ ≤ 1, 0 ≤ βL < βH ≤ 1}]

True

$$x1c = -\frac{4\,(a-c)\,(-1+\beta L\,(-1+\lambda)-\beta H\,\lambda)}{-4-8\,\beta L-4\,\beta L^2+9\,b\,\gamma-2\,(-4-5\,\beta H+\beta L)\,(-\beta H+\beta L)\,\lambda+6\,(\beta H-\beta L)^2\,\lambda^2};$$

FullSimplify[x1c > 0, {b > 0, a > c > 0, γ > 0, s ≥ 0, 0 ≤ λ ≤ 1, 0 ≤ βL < βH ≤ 1}]

$$9\,b\,\gamma+6\,(\beta H-\beta L)^2\,\lambda^2>4\,(1+\beta L)^2+2\,(-4-5\,\beta H+\beta L)\,(-\beta H+\beta L)\,\lambda$$

FullSimplify[x1c > 0, {a - c == 1, b == 2, γ == 1, 0 ≤ λ ≤ 1, 0 ≤ βL < βH ≤ 1}]

True

$\pi^C > 0$, $\pi^N > 0$ **conditions are satisfied for numerical values assigned to the parameters :**

```
pro1n =
  ((a - c)^2 (81 b^2 γ^2 + 36 b γ (-2 - βL + βL^2 - (1 + 5 βH - 7 βL) (βH - βL) λ + 6 (βH - βL)^2 λ^2) +
       4 (βH - βL)^2 (-1 + λ) λ (-6 (1 + βL) (6 + βL) - (42 + 25 βH - 13 βL) (βH - βL) λ + 19 (βH - βL)^2 λ^2))) /
   (9 b (4 + 8 βL + 4 βL^2 - 9 b γ + 2 (-4 - 5 βH + βL) (-βH + βL) λ - 6 (βH - βL)^2 λ^2)^2);

FullSimplify[pro1n > 0, {b > 0, a > c > 0, γ > 0, s ≥ 0, 0 ≤ λ ≤ 1, 0 ≤ βL < βH ≤ 1}]

(4 + 8 βL + 4 βL^2 - 9 b γ + 2 (-4 - 5 βH + βL) (-βH + βL) λ - 6 (βH - βL)^2 λ^2)^2
  (81 b^2 γ^2 + 36 b γ (-2 - βL + βL^2 - (1 + 5 βH - 7 βL) (βH - βL) λ + 6 (βH - βL)^2 λ^2) +
    4 (βH - βL)^2 (-1 + λ) λ (-6 (1 + βL) (6 + βL) - (42 + 25 βH - 13 βL) (βH - βL) λ + 19 (βH - βL)^2 λ^2)) > 0

FullSimplify[pro1n > 0, {a - c == 1, b == 2, γ == 1, 0 ≤ λ ≤ 1, 0 ≤ βL < βH ≤ 1}]

True

pro1c = ((a - c)^2 (9 b γ + 10 (βH - βL)^2 (-1 + λ) λ)) /
   (9 b (-4 - 8 βL - 4 βL^2 + 9 b γ - 2 (-4 - 5 βH + βL) (-βH + βL) λ + 6 (βH - βL)^2 λ^2));

FullSimplify[pro1c > 0, {b > 0, a > c > 0, γ > 0, s ≥ 0, 0 ≤ λ ≤ 1, 0 ≤ βL < βH ≤ 1}]

(9 b γ + 10 (βH - βL)^2 (-1 + λ) λ) (-4 - 8 βL - 4 βL^2 + 9 b γ - 2 (-4 - 5 βH + βL) (-βH + βL) λ + 6 (βH - βL)^2 λ^2) > 0

FullSimplify[pro1c > 0, {a - c == 1, b == 2, γ == 1, 0 ≤ λ ≤ 1, 0 ≤ βL < βH ≤ 1}]

True
```

RESUME

Evren Palabēyēk was born in 1981 in Ķstanbul. She graduated from Bahçeкehir College in 1999. She got a B.Sc. degree in Mathematics Engineering at Istanbul Technical University in 2004 and attended Master of Economics at Istanbul Technical University the same year.